

# Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.4-Inverse-hyperbolic-cotangent/7.4.1-  
Inverse-hyperbolic-cotangent-functions

Nasser M. Abbasi

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3.239	$\int \frac{\coth^{-1}(c+d \tan(a+bx))}{x} dx$	.1152
3.240	$\int x^2 \coth^{-1}(1 - id + d \tan(a + bx)) dx$	.1155
3.241	$\int x \coth^{-1}(1 - id + d \tan(a + bx)) dx$	.1161
3.242	$\int \coth^{-1}(1 - id + d \tan(a + bx)) dx$	.1166
3.243	$\int \frac{\coth^{-1}(1-id+d \tan(a+bx))}{x} dx$	.1171
3.244	$\int x^2 \coth^{-1}(1 + id - d \tan(a + bx)) dx$	.1174
3.245	$\int x \coth^{-1}(1 + id - d \tan(a + bx)) dx$	.1180
3.246	$\int \coth^{-1}(1 + id - d \tan(a + bx)) dx$	.1185
3.247	$\int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$	.1189
3.248	$\int (e + fx)^3 \coth^{-1}(\cot(a + bx)) dx$	.1192
3.249	$\int (e + fx)^2 \coth^{-1}(\cot(a + bx)) dx$	.1198
3.250	$\int (e + fx) \coth^{-1}(\cot(a + bx)) dx$	.1203
3.251	$\int \coth^{-1}(\cot(a + bx)) dx$	.1209
3.252	$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$	.1213
3.253	$\int x^2 \coth^{-1}(c + d \cot(a + bx)) dx$	.1216
3.254	$\int x \coth^{-1}(c + d \cot(a + bx)) dx$	.1222
3.255	$\int \coth^{-1}(c + d \cot(a + bx)) dx$	.1228

3.256	$\int \frac{\coth^{-1}(c+d \cot(ax))}{x} dx$	1235
3.257	$\int x^2 \coth^{-1}(1+id+d \cot(a+bx)) dx$	1238
3.258	$\int x \coth^{-1}(1+id+d \cot(a+bx)) dx$	1244
3.259	$\int \coth^{-1}(1+id+d \cot(a+bx)) dx$	1249
3.260	$\int \frac{\coth^{-1}(1+id+d \cot(ax))}{x} dx$	1253
3.261	$\int x^2 \coth^{-1}(1-id-d \cot(a+bx)) dx$	1256
3.262	$\int x \coth^{-1}(1-id-d \cot(a+bx)) dx$	1262
3.263	$\int \coth^{-1}(1-id-d \cot(a+bx)) dx$	1267
3.264	$\int \frac{\coth^{-1}(1-id-d \cot(ax))}{x} dx$	1271
3.265	$\int \frac{(a+b \coth^{-1}(cx^n))(d+e \log(fx^m))}{x} dx$	1274
3.266	$\int x^5 (a+b \coth^{-1}(cx)) (d+e \log(1-c^2x^2)) dx$	1279
3.267	$\int x^3 (a+b \coth^{-1}(cx)) (d+e \log(1-c^2x^2)) dx$	1288
3.268	$\int x (a+b \coth^{-1}(cx)) (d+e \log(1-c^2x^2)) dx$	1296
3.269	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x} dx$	1302
3.270	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^3} dx$	1308
3.271	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^5} dx$	1314
3.272	$\int x^4 (a+b \coth^{-1}(cx)) (d+e \log(1-c^2x^2)) dx$	1320
3.273	$\int x^2 (a+b \coth^{-1}(cx)) (d+e \log(1-c^2x^2)) dx$	1329
3.274	$\int (a+b \coth^{-1}(cx)) (d+e \log(1-c^2x^2)) dx$	1337
3.275	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^2} dx$	1343
3.276	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^4} dx$	1348
3.277	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^6} dx$	1354
3.278	$\int x (a+b \coth^{-1}(cx)) (d+e \log(f+gx^2)) dx$	1361
3.279	$\int (a+b \coth^{-1}(cx)) (d+e \log(f+gx^2)) dx$	1369
3.280	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x} dx$	1377
3.281	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x^2} dx$	1380
3.282	$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x^3} dx$	1389
3.283	$\int \coth^{-1}(e^x) dx$	1397
3.284	$\int x \coth^{-1}(e^x) dx$	1400
3.285	$\int x^2 \coth^{-1}(e^x) dx$	1404
3.286	$\int \coth^{-1}(e^{a+bx}) dx$	1408

3.287	$\int x \coth^{-1}(e^{a+bx}) dx$	.1411
3.288	$\int x^2 \coth^{-1}(e^{a+bx}) dx$	.1415
3.289	$\int \coth^{-1}(a + bf^{c+dx}) dx$	.1419
3.290	$\int x \coth^{-1}(a + bf^{c+dx}) dx$	.1424
3.291	$\int x^2 \coth^{-1}(a + bf^{c+dx}) dx$	.1429
3.292	$\int \frac{1}{(a-ax^2)(b-2b \coth^{-1}(x))} dx$	.1435
3.293	$\int x^3 \coth^{-1}(a + bx^4) dx$	.1438
3.294	$\int x^{-1+n} \coth^{-1}(a + bx^n) dx$	.1442
3.295	$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac + bcx)) dx$	.1445
3.296	$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac + bcx)) dx$	.1450
3.297	$\int e^{c(a+bx)} \coth^{-1}(\tanh(ac + bcx)) dx$	.1454
3.298	$\int e^{c(a+bx)} \coth^{-1}(\coth(ac + bcx)) dx$	.1457
3.299	$\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac + bcx)) dx$	.1460
3.300	$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac + bcx)) dx$	.1464
<b>4</b>	<b>Listing of Grading functions</b>	<b>1469</b>
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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 300 ]. This is test number [ 198 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 300 )	% 0.00 ( 0 )
Mathematica	% 98.00 ( 294 )	% 2.00 ( 6 )
Maple	% 91.00 ( 273 )	% 9.00 ( 27 )
Maxima	% 82.00 ( 246 )	% 18.00 ( 54 )
Fricas	% 74.33 ( 223 )	% 25.67 ( 77 )
Sympy	% 33.33 ( 100 )	% 66.67 ( 200 )
Giac	% 10.00 ( 30 )	% 90.00 ( 270 )
Mupad	% 51.00 ( 153 )	% 49.00 ( 147 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

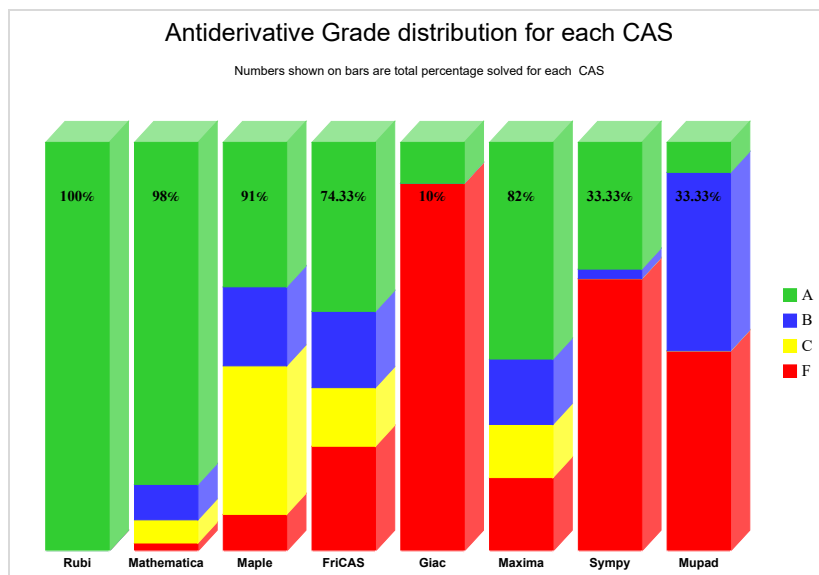
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

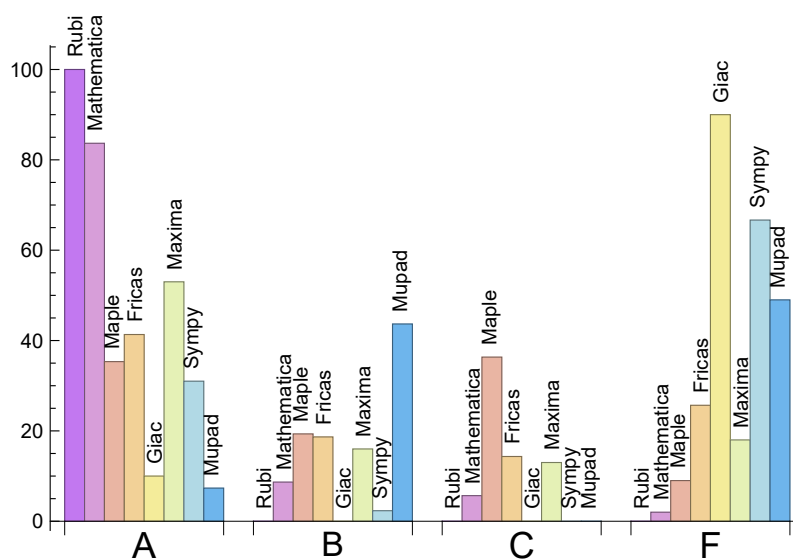
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	83.67	8.67	5.67	2.00
Maple	35.33	19.33	36.33	9.00
Maxima	53.00	16.00	13.00	18.00
Fricas	41.33	18.67	14.33	25.67
Sympy	31.00	2.33	0.00	66.67
Giac	10.00	0.00	0.00	90.00
Mupad	7.33	43.67	0.00	49.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.



The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	100.00 %	0.00 %	0.00 %
Maple	27	59.26 %	29.63 %	11.11 %
Maxima	54	98.15 %	0.00 %	1.85 %
Fricas	77	100.00 %	0.00 %	0.00 %
Sympy	200	85.50 %	11.00 %	3.50 %
Giac	270	99.26 %	0.00 %	0.74 %
Mupad	147	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

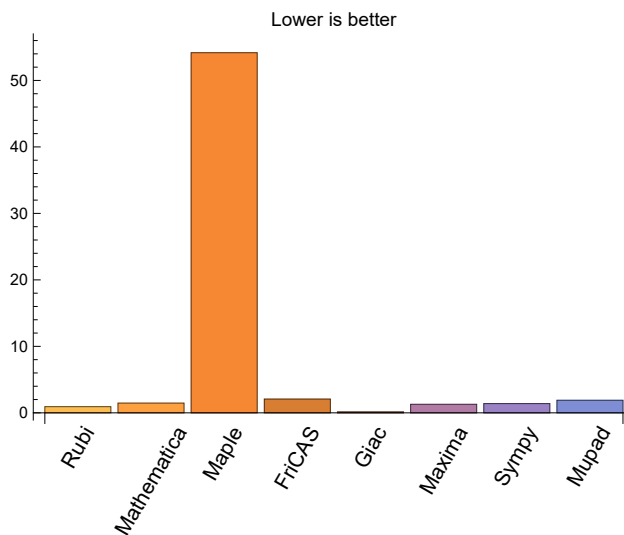
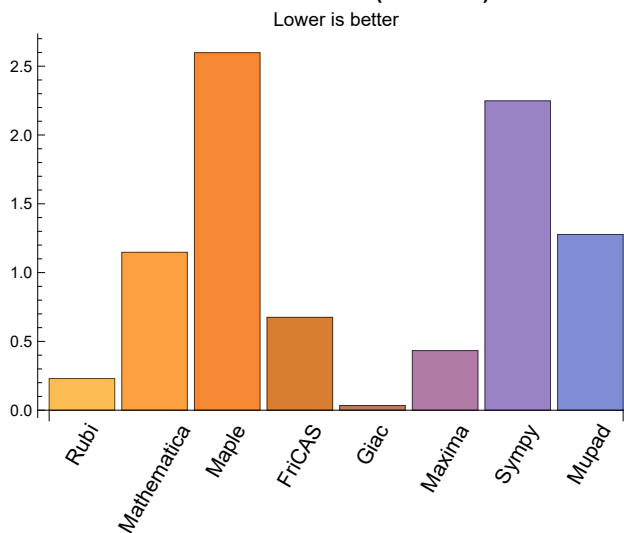
## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	125.80	0.93	79.00	1.00
Mathematica	1.15	234.41	1.46	78.00	0.94
Maple	2.60	5663.65	54.17	306.00	2.61
Maxima	0.43	128.09	1.28	78.50	1.09
Fricas	0.67	244.84	2.07	92.00	1.37
Sympy	2.25	95.50	1.39	41.00	1.14
Giac	0.03	3.47	0.14	0.00	0.00
Mupad	1.28	145.89	1.89	48.00	1.05

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.

**Normalized mean size of antiderivative****Mean time used (seconds)**

## 1.4 list of integrals that has no closed form antiderivative

{42, 43, 120, 121, 122, 126, 127, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {76,78}

**Mathematica** {13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 48, 49, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 118, 125, 210, 215, 217, 218, 224, 229, 233, 238, 242, 246, 250, 255, 259, 263, 270, 271, 278, 281, 282, 295, 296, 297, 298, 299, 300}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at <https://>

[ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](http://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

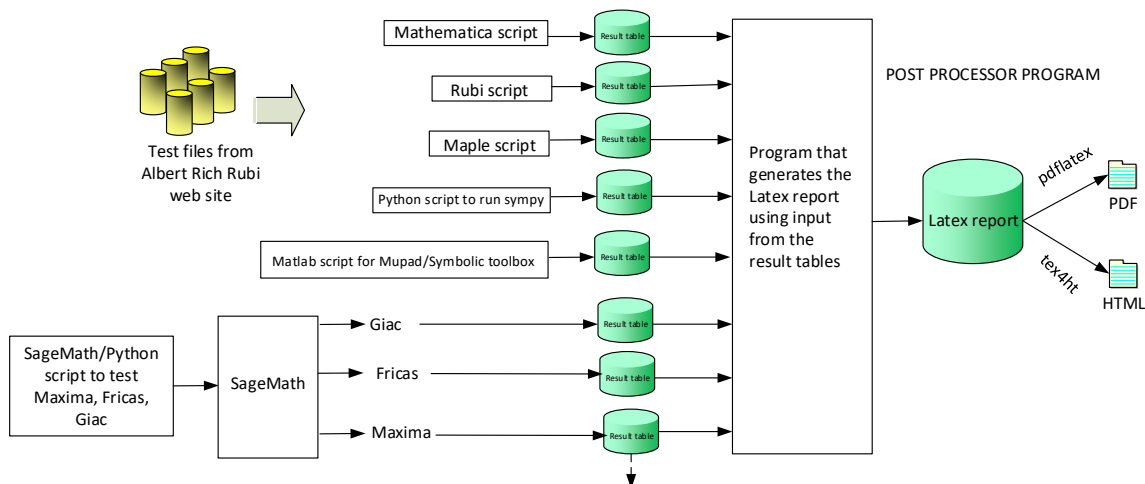
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer, the problem number.
  2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
  3. integer. Leaf size of result.
  4. integer. Leaf size of the optimal antiderivative.
  5. number. CPU time used to solve this integral. 0 if failed.
  6. string. The integral in Latex format
  7. string. The input used in CAS own syntax.
  8. string. The result (antiderivative) produced by CAS in Latex format
  9. string. The optimal antiderivative in Latex format.
  10. integer. 0 or 1. Indicates if problem has known antiderivative or not
  11. String. The result (antiderivative) in CAS own syntax.
  12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
  15. integer. Integrand leaf size.
  16. real number. Ratio of field 14 over field 15
  17. integer. 1 if result was verified or 0 if not verified.
  18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**

Nassier M. Abbasi  
May 11, 2021



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 71, 72, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94,

96, 97, 99, 102, 103, 104, 105, 106, 107, 108, 110, 111, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 243, 244, 245, 247, 249, 250, 251, 252, 253, 254, 256, 257, 258, 260, 261, 262, 264, 266, 267, 268, 270, 271, 272, 273, 274, 278, 280, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

B grade: { 41, 70, 95, 98, 100, 101, 109, 139, 150, 210, 215, 224, 229, 231, 238, 242, 246, 248, 255, 259, 263, 275, 276, 279, 281, 283 }

C grade: { 24, 26, 34, 66, 73, 74, 75, 78, 79, 112, 113, 114, 115, 116, 118, 265, 282 }

F grade: { 117, 119, 123, 124, 269, 277 }

## 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 35, 36, 37, 38, 42, 43, 48, 49, 50, 51, 52, 53, 54, 57, 58, 60, 64, 65, 66, 67, 68, 72, 74, 75, 77, 78, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 95, 97, 98, 99, 100, 101, 105, 106, 107, 108, 113, 120, 121, 122, 125, 126, 127, 133, 134, 135, 140, 151, 162, 171, 180, 189, 197, 198, 199, 200, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 282, 283, 284, 285, 286, 288, 289, 292, 293, 298 }

B grade: { 12, 14, 16, 17, 19, 20, 21, 22, 28, 39, 40, 41, 56, 59, 61, 62, 63, 69, 70, 71, 76, 82, 86, 96, 102, 103, 104, 109, 110, 111, 116, 123, 124, 129, 130, 131, 185, 194, 195, 196, 205, 210, 215, 219, 224, 229, 234, 238, 242, 246, 251, 255, 259, 263, 287, 290, 291, 294 }

C grade: { 18, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 55, 73, 79, 93, 112, 114, 115, 117, 118, 128, 132, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 161, 163, 167, 168, 169, 170, 176, 177, 178, 179, 186, 187, 188, 193, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 266, 267, 268, 269, 272, 273, 274, 278, 295, 296, 297, 299, 300 }

F grade: { 44, 45, 46, 47, 119, 158, 164, 165, 166, 172, 173, 174, 175, 181, 182, 183, 184, 190, 191, 192, 270, 271, 275, 276, 277, 279, 281 }

## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 20, 21, 23, 35, 36, 37, 38, 39, 40, 42, 43, 50, 51, 52, 53, 54, 57, 58, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 77, 78, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 103, 104, 105, 107, 108, 110, 120, 121, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215,

216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 239, 243, 247, 252, 256, 260, 264, 268, 280, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299 }

B grade: { 7, 16, 17, 19, 22, 25, 27, 31, 33, 41, 44, 45, 46, 47, 55, 56, 59, 61, 86, 93, 95, 98, 100, 101, 102, 109, 140, 151, 234, 238, 240, 241, 242, 244, 245, 246, 251, 255, 257, 258, 259, 261, 262, 263, 283, 286, 295, 300 }

C grade: { 76, 79, 141, 152, 153, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 266, 267, 269, 272, 273, 274 }

F grade: { 18, 24, 26, 28, 29, 30, 32, 34, 48, 49, 73, 80, 81, 82, 106, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 231, 232, 233, 236, 237, 248, 249, 250, 253, 254, 265, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 42, 43, 50, 51, 52, 55, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 104, 105, 120, 121, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 159, 160, 163, 193, 194, 195, 196, 197, 198, 199, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 259, 260, 263, 264, 266, 267, 268, 272, 273, 274, 280, 289, 292, 293, 296, 297, 298, 299 }

B grade: { 44, 45, 46, 47, 53, 54, 57, 95, 102, 103, 107, 108, 151, 161, 162, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 200, 205, 210, 215, 219, 224, 229, 234, 238, 242, 246, 251, 255, 283, 286, 294, 295, 300 }

C grade: { 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 217, 218, 221, 222, 223, 226, 227, 228, 231, 232, 233, 236, 237, 240, 241, 244, 245, 248, 249, 250, 253, 254, 257, 258, 261, 262, 265, 284, 285, 287, 288, 290, 291 }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 48, 49, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 93, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282 }

## 2.1.6 SymPy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 42, 43, 53, 54, 57, 58, 62, 63, 64, 65, 67, 68, 90, 94, 96, 97, 99, 102, 103, 104, 105, 107, 122, 126, 127, 129, 130, 131, 133, 134, 135, 137, 138, 139, 140, 143, 144, 145, 147, 148, 149, 150, 151, 155, 157, 189, 194, 195, 196, 198, 199, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 266, 267, 268, 272, 273, 274, 292, 293, 297 }

B grade: { 60, 87, 88, 89, 91, 92, 156 }

C grade: { }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 59, 61, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 95, 98, 100, 101, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 128, 132, 136, 141, 142, 146, 152, 153, 154, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 197, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 298, 299, 300 }

## 2.1.7 Giac

A grade: { 42, 43, 54, 120, 121, 122, 126, 127, 194, 195, 196, 197, 198, 199, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280, 298 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300 }

## 2.1.8 Mupad

A grade: { 42, 43, 120, 121, 122, 126, 127, 206, 211, 216, 220, 225, 230, 235, 239, 243, 247, 252, 256, 260, 264, 280 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 16, 20, 22, 35, 36, 37, 38, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 83, 84, 85, 87, 88, 89, 91, 92, 94, 96, 97, 99, 102, 103, 104, 105, 107, 108, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 198, 199, 266, 267, 268, 272, 273, 274, 292, 293, 294, 295, 296, 297, 298, 299, 300 }

C grade: { }

F grade: { 7, 13, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 90, 93, 95, 98, 100, 101, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 158, 166, 175, 184, 190, 191, 192, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217, 218, 219, 221, 222, 223, 224, 226, 227, 228, 229, 231, 232, 233, 234, 236, 237, 238, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 253, 254, 255, 257, 258, 259, 261, 262, 263, 265, 269, 270, 271, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	67	55	61	51	49	0	41
normalized size	1	1.00	1.31	1.08	1.20	1.00	0.96	0.00	0.80
time (sec)	N/A	0.028	0.009	0.030	0.302	0.703	1.525	0.000	1.313
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	46	55	54	0	43
normalized size	1	1.00	1.00	0.98	0.92	1.10	1.08	0.00	0.86
time (sec)	N/A	0.037	0.009	0.034	0.300	0.584	1.180	0.000	1.267
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	57	47	52	43	41	0	33
normalized size	1	1.00	1.39	1.15	1.27	1.05	1.00	0.00	0.80
time (sec)	N/A	0.024	0.009	0.033	0.297	0.422	0.868	0.000	1.219

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	41	35	44	46	0	35
normalized size	1	1.00	1.00	1.02	0.88	1.10	1.15	0.00	0.88
time (sec)	N/A	0.030	0.008	0.030	0.305	0.519	0.665	0.000	1.225

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	47	39	41	34	32	0	26
normalized size	1	1.00	1.52	1.26	1.32	1.10	1.03	0.00	0.84
time (sec)	N/A	0.013	0.007	0.032	0.302	0.581	0.454	0.000	1.180

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	25	33	27	0	22
normalized size	1	1.00	1.00	0.92	1.00	1.32	1.08	0.00	0.88
time (sec)	N/A	0.007	0.003	0.044	0.301	0.611	0.332	0.000	1.149

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	37	86	0	0	0	-1
normalized size	1	1.00	0.93	1.32	3.07	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.010	0.008	0.049	0.305	0.444	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	35	30	39	26	0	27
normalized size	1	1.00	1.00	1.17	1.00	1.30	0.87	0.00	0.90
time (sec)	N/A	0.021	0.008	0.037	0.305	0.449	0.375	0.000	1.163

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	47	39	36	35	24	0	40
normalized size	1	1.00	1.52	1.26	1.16	1.13	0.77	0.00	1.29
time (sec)	N/A	0.016	0.008	0.037	0.307	0.525	0.545	0.000	1.190

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	48	40	50	46	0	39
normalized size	1	1.00	1.00	1.02	0.85	1.06	0.98	0.00	0.83
time (sec)	N/A	0.030	0.009	0.039	0.310	1.044	0.736	0.000	1.183

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	57	47	51	43	32	0	60
normalized size	1	1.00	1.39	1.15	1.24	1.05	0.78	0.00	1.46
time (sec)	N/A	0.022	0.009	0.038	0.306	0.515	1.159	0.000	1.606

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	80	196	135	98	114	0	85
normalized size	1	1.00	0.76	1.87	1.29	0.93	1.09	0.00	0.81
time (sec)	N/A	0.246	0.022	0.059	0.305	0.531	3.307	0.000	1.365

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	87	196	155	0	0	0	-1
normalized size	1	1.00	0.69	1.54	1.22	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.451	0.059	0.319	0.741	0.000	0.000	0.000



Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	62	176	118	81	90	0	65
normalized size	1	1.00	0.77	2.17	1.46	1.00	1.11	0.00	0.80
time (sec)	N/A	0.163	0.019	0.058	0.320	0.532	1.443	0.000	1.265

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	66	176	134	0	0	0	-1
normalized size	1	1.00	0.64	1.71	1.30	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.244	0.059	0.311	0.429	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	155	97	62	60	0	44
normalized size	1	1.00	0.80	2.87	1.80	1.15	1.11	0.00	0.81
time (sec)	N/A	0.078	0.012	0.060	0.316	0.531	0.783	0.000	1.209

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	122	135	0	0	0	-1
normalized size	1	1.00	0.79	2.10	2.33	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.084	0.303	0.323	0.426	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	114	487	0	0	0	0	-1
normalized size	1	1.00	1.18	5.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.058	0.683	0.000	0.380	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	159	146	0	0	0	-1
normalized size	1	1.00	0.89	2.89	2.65	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.101	0.062	0.319	0.518	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	164	96	79	56	0	145
normalized size	1	1.00	0.93	2.69	1.57	1.30	0.92	0.00	2.38
time (sec)	N/A	0.099	0.016	0.063	0.311	0.548	1.577	0.000	1.447

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	87	224	176	0	0	0	-1
normalized size	1	1.00	0.84	2.17	1.71	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.212	0.068	0.322	0.457	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	82	185	154	97	90	0	196
normalized size	1	1.00	0.91	2.06	1.71	1.08	1.00	0.00	2.18
time (sec)	N/A	0.172	0.021	0.065	0.322	0.657	2.636	0.000	1.550

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	117	1141	289	0	0	0	-1
normalized size	1	1.00	0.63	6.13	1.55	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.717	0.535	3.610	0.327	1.266	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	175	806	0	0	0	0	-1
normalized size	1	1.00	0.89	4.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.580	0.588	2.917	0.000	1.954	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	88	684	262	0	0	0	-1
normalized size	1	1.00	0.63	4.92	1.88	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.417	0.308	1.586	0.330	0.481	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	140	765	0	0	0	0	-1
normalized size	1	1.00	0.94	5.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.333	0.382	1.490	0.000	0.677	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	68	3070	215	0	0	0	-1
normalized size	1	1.00	0.72	32.32	2.26	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.139	0.757	0.326	0.548	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	79	180	0	0	0	0	-1
normalized size	1	1.00	0.93	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.100	0.381	0.000	0.386	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	156	564	0	0	0	0	-1
normalized size	1	1.00	1.04	3.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.350	0.076	0.549	0.000	0.474	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	796	0	0	0	0	-1
normalized size	1	1.00	0.91	10.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.131	0.700	0.000	0.810	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	3673	252	0	0	0	-1
normalized size	1	1.00	0.83	38.66	2.65	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.168	0.862	0.335	0.557	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	142	895	0	0	0	0	-1
normalized size	1	1.00	0.92	5.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.369	0.217	2.412	0.000	0.636	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	118	661	342	0	0	0	-1
normalized size	1	1.00	0.84	4.69	2.43	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.464	0.238	1.495	0.340	0.510	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	565	926	0	0	0	0	-1
normalized size	1	1.00	3.45	5.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	7.804	1.286	0.000	0.546	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	213	334	276	247	427	0	296
normalized size	1	1.00	0.87	1.36	1.13	1.01	1.74	0.00	1.21
time (sec)	N/A	0.181	0.116	0.041	0.307	0.563	6.987	0.000	1.509

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	233	198	177	282	0	190
normalized size	1	1.00	0.89	1.38	1.17	1.05	1.67	0.00	1.12
time (sec)	N/A	0.127	0.083	0.034	0.306	0.599	4.748	0.000	1.519

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	148	131	118	182	0	115
normalized size	1	1.00	0.89	1.35	1.19	1.07	1.65	0.00	1.05
time (sec)	N/A	0.133	0.052	0.036	0.304	0.455	2.127	0.000	1.370

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	76	65	64	87	0	60
normalized size	1	1.00	1.21	1.33	1.14	1.12	1.53	0.00	1.05
time (sec)	N/A	0.066	0.011	0.037	0.303	0.494	1.064	0.000	1.274

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	671	785	406	0	0	0	-1
normalized size	1	1.00	1.72	2.01	1.04	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.947	1.391	0.678	0.537	0.521	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	755	2218	550	0	0	0	-1
normalized size	1	1.00	1.28	3.76	0.93	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.870	7.783	0.639	0.514	0.598	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	657	657	1846	4128	1084	0	0	0	-1
normalized size	1	1.00	2.81	6.28	1.65	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.951	12.947	0.837	0.584	0.800	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	26.742	1.267	0.000	0.503	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	4.384	1.008	0.000	0.425	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	153	354	0	0	-1
normalized size	1	1.00	1.92	0.00	2.47	5.71	0.00	0.00	-0.02
time (sec)	N/A	0.113	0.112	0.937	0.340	0.504	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	226	0	223	728	0	0	-1
normalized size	1	1.00	1.77	0.00	1.74	5.69	0.00	0.00	-0.01
time (sec)	N/A	0.340	0.297	0.944	0.415	0.692	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	329	0	401	1278	0	0	-1
normalized size	1	1.00	1.64	0.00	2.00	6.39	0.00	0.00	-0.00
time (sec)	N/A	1.047	0.567	0.948	0.423	0.749	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	431	0	639	2004	0	0	-1
normalized size	1	1.00	1.52	0.00	2.26	7.08	0.00	0.00	-0.00
time (sec)	N/A	1.343	0.937	0.943	0.431	0.932	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	125	199	0	0	0	0	-1
normalized size	1	1.00	0.67	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.970	0.658	0.000	0.807	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	77	190	0	0	0	0	-1
normalized size	1	1.00	0.53	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.119	0.557	0.000	0.511	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	52	63	41	0	0	-1
normalized size	1	1.00	0.81	1.41	1.70	1.11	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.047	0.485	0.408	0.469	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	45	112	67	61	0	0	-1
normalized size	1	1.00	0.54	1.35	0.81	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.052	0.513	0.319	0.471	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	55	176	99	81	0	0	-1
normalized size	1	1.00	0.44	1.42	0.80	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.063	0.522	0.323	0.554	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	0	3
normalized size	1	1.00	1.00	1.33	1.00	3.67	1.00	0.00	1.00
time (sec)	N/A	0.023	0.025	0.057	0.304	0.466	0.284	0.000	0.304



Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	62	15	1	22
normalized size	1	1.00	1.00	1.08	1.00	5.17	1.25	0.08	1.83
time (sec)	N/A	0.026	0.009	0.059	0.303	0.600	1.722	0.123	1.363

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	707	171	63	0	0	201
normalized size	1	1.00	0.98	11.40	2.76	1.02	0.00	0.00	3.24
time (sec)	N/A	0.051	0.066	2.003	0.309	0.530	0.000	0.000	2.680

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	75	76	0	0	0	-1
normalized size	1	1.00	0.92	2.03	2.05	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	0.051	0.053	0.304	0.435	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	13	6	14	5	0	21
normalized size	1	1.00	1.00	1.62	0.75	1.75	0.62	0.00	2.62
time (sec)	N/A	0.014	0.004	0.040	0.298	0.413	0.701	0.000	1.203

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	44	39	34	29	31	0	21
normalized size	1	1.00	1.22	1.08	0.94	0.81	0.86	0.00	0.58
time (sec)	N/A	0.031	0.028	0.038	0.298	0.388	0.558	0.000	1.155

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	99	76	42	0	0	81
normalized size	1	1.00	0.74	2.61	2.00	1.11	0.00	0.00	2.13
time (sec)	N/A	0.017	0.033	0.065	0.303	0.522	0.000	0.000	1.216

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	53	47	47	88	0	34
normalized size	1	1.00	1.00	1.06	0.94	0.94	1.76	0.00	0.68
time (sec)	N/A	0.034	0.053	0.038	0.302	0.610	0.908	0.000	1.210

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	131	118	66	0	0	112
normalized size	1	1.00	0.64	1.96	1.76	0.99	0.00	0.00	1.67
time (sec)	N/A	0.035	0.055	0.070	0.307	0.399	0.000	0.000	1.318

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	81	199	106	112	153	0	134
normalized size	1	1.00	0.80	1.97	1.05	1.11	1.51	0.00	1.33
time (sec)	N/A	0.126	0.042	0.036	0.306	0.501	1.841	0.000	1.473

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	92	146	79	84	117	0	98
normalized size	1	1.00	1.18	1.87	1.01	1.08	1.50	0.00	1.26
time (sec)	N/A	0.102	0.024	0.035	0.307	0.556	1.270	0.000	1.364

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	89	61	66	76	0	62
normalized size	1	1.00	0.86	1.37	0.94	1.02	1.17	0.00	0.95
time (sec)	N/A	0.072	0.023	0.033	0.303	0.842	0.814	0.000	2.004

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	36	31	48	41	0	42
normalized size	1	1.00	1.23	1.03	0.89	1.37	1.17	0.00	1.20
time (sec)	N/A	0.016	0.015	0.029	0.310	0.669	0.513	0.000	1.707

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	259	81	128	0	0	0	-1
normalized size	1	1.00	2.82	0.88	1.39	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.169	0.055	0.309	0.699	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	63	54	68	144	0	62
normalized size	1	1.00	0.86	0.98	0.84	1.06	2.25	0.00	0.97
time (sec)	N/A	0.051	0.056	0.038	0.314	0.679	1.482	0.000	1.732

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	82	85	111	410	0	247
normalized size	1	1.00	0.84	0.91	0.94	1.23	4.56	0.00	2.74
time (sec)	N/A	0.100	0.117	0.041	0.305	0.722	2.430	0.000	1.951

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	203	967	320	0	0	0	-1
normalized size	1	1.00	0.77	3.68	1.22	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	1.703	0.066	0.341	0.772	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	607	729	259	0	0	0	-1
normalized size	1	1.00	2.98	3.57	1.27	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	4.539	0.063	0.333	0.481	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	106	365	202	0	0	0	-1
normalized size	1	1.00	0.78	2.68	1.49	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.260	0.062	0.336	0.682	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	151	139	0	0	0	-1
normalized size	1	1.00	0.68	1.86	1.72	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.075	0.305	0.330	0.436	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	547	985	0	0	0	0	-1
normalized size	1	1.00	3.70	6.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	2.871	1.151	0.000	0.467	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	206	342	244	0	0	0	-1
normalized size	1	1.00	0.82	1.36	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.719	1.124	0.073	0.322	0.472	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	291	467	360	0	0	0	-1
normalized size	1	1.00	0.79	1.26	0.97	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.830	2.350	0.076	0.337	0.407	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	673	597	529	1230	589	0	0	0	-1
normalized size	1	0.89	0.79	1.83	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.099	0.620	0.753	0.595	0.412	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	185	176	192	0	0	0	-1
normalized size	1	1.00	1.54	1.47	1.60	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.073	0.084	0.332	0.714	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	360	502	297	192	0	0	0	-1
normalized size	1	1.23	1.72	1.02	0.66	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	4.494	0.088	0.340	0.583	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	738	738	5552	19686	647	0	0	0	-1
normalized size	1	1.00	7.52	26.67	0.88	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.530	36.346	1.877	0.550	0.495	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	619	575	738	0	0	0	0	-1
normalized size	1	1.00	0.93	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.199	0.707	0.128	0.000	0.470	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	738	738	719	970	0	0	0	0	-1
normalized size	1	1.00	0.97	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.371	0.745	0.118	0.000	0.653	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	596	2098	0	0	0	0	-1
normalized size	1	1.00	1.78	6.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.749	0.962	1.113	0.000	0.650	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	59	42	41	38	0	0	31
normalized size	1	1.00	1.16	0.82	0.80	0.75	0.00	0.00	0.61
time (sec)	N/A	0.015	0.019	0.045	0.301	0.466	0.000	0.000	1.305

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	52	37	36	31	0	0	26
normalized size	1	1.00	1.24	0.88	0.86	0.74	0.00	0.00	0.62
time (sec)	N/A	0.011	0.014	0.046	0.308	0.569	0.000	0.000	1.263

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	27	26	24	0	0	16
normalized size	1	1.00	1.00	1.23	1.18	1.09	0.00	0.00	0.73
time (sec)	N/A	0.006	0.007	0.046	0.301	0.622	0.000	0.000	1.237

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	33	66	0	0	0	-1
normalized size	1	1.00	1.00	1.74	3.47	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	0.010	0.063	0.310	0.656	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	45	32	31	30	92	0	18
normalized size	1	1.00	1.80	1.28	1.24	1.20	3.68	0.00	0.72
time (sec)	N/A	0.014	0.021	0.051	0.305	0.493	2.118	0.000	1.272

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	58	37	36	38	160	0	45
normalized size	1	1.00	1.38	0.88	0.86	0.90	3.81	0.00	1.07
time (sec)	N/A	0.015	0.022	0.053	0.311	0.507	5.450	0.000	1.487

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	35	24	35	121	0	24
normalized size	1	1.00	0.82	0.92	0.63	0.92	3.18	0.00	0.63
time (sec)	N/A	0.018	0.016	0.046	0.309	1.268	5.352	0.000	1.259

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	19	30	39	0	-1
normalized size	1	1.00	0.81	0.97	0.61	0.97	1.26	0.00	-0.03
time (sec)	N/A	0.014	0.013	0.045	0.307	0.645	1.069	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	16	24	87	0	14
normalized size	1	1.00	1.00	0.75	0.80	1.20	4.35	0.00	0.70
time (sec)	N/A	0.009	0.010	0.047	0.304	0.649	0.535	0.000	1.289

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	18	36	126	0	22
normalized size	1	1.00	1.00	1.21	0.75	1.50	5.25	0.00	0.92
time (sec)	N/A	0.010	0.022	0.050	0.313	0.489	1.376	0.000	1.252

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	85	104	0	0	0	-1
normalized size	1	1.00	0.93	3.04	3.71	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	0.015	0.166	0.306	0.508	0.000	0.000	0.000



Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	30	15	23	15	0	26
normalized size	1	1.00	1.00	1.58	0.79	1.21	0.79	0.00	1.37
time (sec)	N/A	0.006	0.002	0.069	0.303	0.992	0.198	0.000	1.135

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	97	61	147	128	0	0	-1
normalized size	1	1.00	2.55	1.61	3.87	3.37	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.054	0.092	0.417	0.581	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	66	70	62	44	56	0	50
normalized size	1	1.00	1.69	1.79	1.59	1.13	1.44	0.00	1.28
time (sec)	N/A	0.022	0.033	0.032	0.306	0.568	0.785	0.000	2.024

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	95	81	86	97	0	114
normalized size	1	1.00	0.78	1.76	1.50	1.59	1.80	0.00	2.11
time (sec)	N/A	0.048	0.041	0.033	0.306	0.575	1.224	0.000	1.542

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	286	59	112	0	0	0	-1
normalized size	1	1.00	8.17	1.69	3.20	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.026	0.048	0.320	0.831	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	43	54	53	67	136	0	93
normalized size	1	1.00	0.90	1.12	1.10	1.40	2.83	0.00	1.94
time (sec)	N/A	0.045	0.029	0.038	0.311	0.543	1.400	0.000	1.406

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	117	34	58	0	0	0	-1
normalized size	1	1.00	4.68	1.36	2.32	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.023	0.015	0.046	0.306	0.551	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	312	59	132	0	0	0	-1
normalized size	1	1.00	8.91	1.69	3.77	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.030	0.028	0.048	0.326	1.090	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	270	786	333	385	644	0	742
normalized size	1	1.00	1.61	4.68	1.98	2.29	3.83	0.00	4.42
time (sec)	N/A	0.342	0.302	0.043	0.311	0.551	7.456	0.000	2.173

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	174	477	207	241	369	0	386
normalized size	1	1.00	1.45	3.98	1.72	2.01	3.08	0.00	3.22
time (sec)	N/A	0.204	0.171	0.040	0.299	0.430	4.449	0.000	1.926

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	138	184	109	133	173	0	136
normalized size	1	1.00	1.42	1.90	1.12	1.37	1.78	0.00	1.40
time (sec)	N/A	0.171	0.048	0.034	0.301	0.454	2.265	0.000	2.404

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	42	36	60	46	0	48
normalized size	1	1.00	1.20	1.05	0.90	1.50	1.15	0.00	1.20
time (sec)	N/A	0.025	0.017	0.030	0.297	0.876	0.585	0.000	1.754

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	206	202	0	0	0	0	-1
normalized size	1	1.00	1.58	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.117	0.085	0.000	0.644	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	125	141	121	262	1658	0	175
normalized size	1	1.00	1.09	1.23	1.05	2.28	14.42	0.00	1.52
time (sec)	N/A	0.169	0.193	0.042	0.311	0.797	9.478	0.000	2.080

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	174	236	291	833	0	0	422
normalized size	1	1.00	1.04	1.41	1.74	4.99	0.00	0.00	2.53
time (sec)	N/A	0.234	0.331	0.046	0.333	1.700	0.000	0.000	3.296

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	1054	2694	791	0	0	0	-1
normalized size	1	1.00	2.82	7.20	2.11	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.637	7.362	0.078	0.618	0.583	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	220	295	857	400	0	0	0	-1
normalized size	1	1.00	1.33	3.88	1.81	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	0.609	0.069	0.612	0.396	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	226	0	0	0	0	-1
normalized size	1	1.00	1.14	2.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.166	0.237	0.000	0.453	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	3759	1845	0	0	0	0	-1
normalized size	1	1.00	17.57	8.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.153	31.059	1.495	0.000	0.486	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	485	470	783	0	0	0	0	-1
normalized size	1	1.01	0.98	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.743	8.899	0.077	0.000	0.527	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	2594	10477	0	0	0	0	-1
normalized size	1	1.00	4.75	19.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.043	10.598	16.593	0.000	0.440	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	325	600	12285	0	0	0	0	-1
normalized size	1	1.00	1.84	37.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.721	1.457	1.423	0.000	0.649	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	208	485	0	0	0	0	-1
normalized size	1	1.00	1.58	3.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.330	0.303	0.000	0.595	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	0	3796	0	0	0	0	-1
normalized size	1	1.00	0.00	12.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	28.571	1.546	0.000	0.672	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1089	1094	3937	4619	0	0	0	0	-1
normalized size	1	1.00	3.62	4.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.782	31.312	1.316	0.000	0.556	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	2.528	1.926	0.000	0.394	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	2.652	1.793	0.000	0.481	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.065	0.351	2.065	0.000	0.621	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.097	1.025	0.000	0.842	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	0	1492	0	0	0	0	-1
normalized size	1	1.00	0.00	3.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	0.289	1.674	0.000	0.436	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	0	696	0	0	0	0	-1
normalized size	1	1.00	0.00	2.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.541	0.727	0.000	0.617	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	98	119	0	0	0	0	-1
normalized size	1	1.00	1.10	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.401	0.721	0.000	0.712	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.095	0.988	0.000	0.558	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.792	1.073	0.000	0.481	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	38	33	0	0	96
normalized size	1	1.00	0.92	18.27	1.03	0.89	0.00	0.00	2.59
time (sec)	N/A	0.027	0.065	0.165	0.339	0.538	0.000	0.000	1.610

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	59	19	13	19	0	19
normalized size	1	1.00	0.87	2.57	0.83	0.57	0.83	0.00	0.83
time (sec)	N/A	0.009	0.017	0.383	0.378	0.637	0.400	0.000	0.090

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	48	19	13	19	0	19
normalized size	1	1.00	0.87	2.09	0.83	0.57	0.83	0.00	0.83
time (sec)	N/A	0.007	0.015	0.382	0.378	0.446	0.228	0.000	1.127

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	32	16	10	19	0	16
normalized size	1	1.00	1.12	2.00	1.00	0.62	1.19	0.00	1.00
time (sec)	N/A	0.003	0.008	0.057	0.381	0.428	0.164	0.000	1.121

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	354	34	8	0	0	59
normalized size	1	1.00	0.90	16.86	1.62	0.38	0.00	0.00	2.81
time (sec)	N/A	0.044	0.014	0.425	0.326	0.475	0.000	0.000	0.180

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	17	13	14	0	17
normalized size	1	1.00	1.06	1.18	1.00	0.76	0.82	0.00	1.00
time (sec)	N/A	0.009	0.016	0.378	0.386	0.939	0.253	0.000	0.086



Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	19	11	19	0	16
normalized size	1	1.00	0.78	0.87	0.83	0.48	0.83	0.00	0.70
time (sec)	N/A	0.010	0.014	0.387	0.378	1.207	0.510	0.000	1.135

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	20	0	19
normalized size	1	1.00	0.87	0.87	0.83	0.57	0.87	0.00	0.83
time (sec)	N/A	0.009	0.016	0.397	0.377	0.571	0.793	0.000	1.123

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	9175	73	101	0	0	203
normalized size	1	1.00	0.87	129.23	1.03	1.42	0.00	0.00	2.86
time (sec)	N/A	0.035	0.168	1.002	0.389	0.501	0.000	0.000	1.325

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	3418	36	30	78	0	36
normalized size	1	1.00	0.88	81.38	0.86	0.71	1.86	0.00	0.86
time (sec)	N/A	0.026	0.034	0.389	0.445	0.575	2.620	0.000	1.175

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	3418	36	30	60	0	36
normalized size	1	1.00	0.88	81.38	0.86	0.71	1.43	0.00	0.86
time (sec)	N/A	0.025	0.055	0.388	0.445	0.527	1.263	0.000	1.182

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	74	3418	36	30	41	0	36
normalized size	1	1.00	2.18	100.53	1.06	0.88	1.21	0.00	1.06
time (sec)	N/A	0.025	0.076	0.395	0.448	0.667	0.671	0.000	1.154

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	33	27	20	0	33
normalized size	1	1.00	1.00	0.94	2.06	1.69	1.25	0.00	2.06
time (sec)	N/A	0.005	0.006	0.092	0.443	0.631	0.300	0.000	1.121

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	3774	38	27	0	0	183
normalized size	1	1.00	1.08	77.02	0.78	0.55	0.00	0.00	3.73
time (sec)	N/A	0.026	0.064	0.331	0.727	0.617	0.000	0.000	0.292

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	1095	54	29	0	0	207
normalized size	1	1.00	0.95	28.08	1.38	0.74	0.00	0.00	5.31
time (sec)	N/A	0.026	0.049	0.269	0.385	0.581	0.000	0.000	1.247

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	42	3213	34	29	32	0	34
normalized size	1	1.00	1.17	89.25	0.94	0.81	0.89	0.00	0.94
time (sec)	N/A	0.023	0.037	0.431	0.458	0.421	0.566	0.000	1.158

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	34	3217	36	29	37	0	32
normalized size	1	1.00	1.10	103.77	1.16	0.94	1.19	0.00	1.03
time (sec)	N/A	0.014	0.046	0.388	0.451	1.439	0.869	0.000	1.119

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	37	3217	36	29	39	0	36
normalized size	1	1.00	0.58	50.27	0.56	0.45	0.61	0.00	0.56
time (sec)	N/A	0.035	0.031	0.387	0.454	0.615	1.295	0.000	1.169

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	63382	109	209	0	0	332
normalized size	1	1.00	0.88	576.20	0.99	1.90	0.00	0.00	3.02
time (sec)	N/A	0.059	0.276	9.364	0.463	0.616	0.000	0.000	1.452

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	18111	54	52	97	0	53
normalized size	1	1.00	0.89	296.90	0.89	0.85	1.59	0.00	0.87
time (sec)	N/A	0.041	0.035	1.142	0.519	0.624	6.881	0.000	1.251

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	18111	54	52	80	0	53
normalized size	1	1.00	0.89	296.90	0.89	0.85	1.31	0.00	0.87
time (sec)	N/A	0.041	0.026	1.281	0.516	0.461	4.087	0.000	1.232

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	18111	54	52	60	0	53
normalized size	1	1.00	1.02	341.72	1.02	0.98	1.13	0.00	1.00
time (sec)	N/A	0.031	0.025	1.133	0.516	0.714	2.383	0.000	1.207

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	99	18111	54	52	41	0	53
normalized size	1	1.00	2.91	532.68	1.59	1.53	1.21	0.00	1.56
time (sec)	N/A	0.015	0.074	1.128	0.524	0.550	1.214	0.000	0.117

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	51	49	20	0	47
normalized size	1	1.00	1.00	0.94	3.19	3.06	1.25	0.00	2.94
time (sec)	N/A	0.005	0.008	0.092	0.524	0.928	0.606	0.000	1.181

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	104	21848	75	49	0	0	306
normalized size	1	1.00	1.35	283.74	0.97	0.64	0.00	0.00	3.97
time (sec)	N/A	0.096	0.097	0.900	0.756	0.673	0.000	0.000	0.144

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	7683	124	51	0	0	372
normalized size	1	1.00	0.91	112.99	1.82	0.75	0.00	0.00	5.47
time (sec)	N/A	0.044	0.042	0.462	0.613	0.461	0.000	0.000	1.203

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	66	7366	72	51	0	0	383
normalized size	1	1.00	1.10	122.77	1.20	0.85	0.00	0.00	6.38
time (sec)	N/A	0.041	0.038	0.549	0.462	0.514	0.000	0.000	1.325

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	17237	52	51	51	0	51
normalized size	1	1.00	1.09	313.40	0.95	0.93	0.93	0.00	0.93
time (sec)	N/A	0.038	0.026	1.587	0.587	0.925	0.848	0.000	1.180

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	50	17235	53	49	56	0	48
normalized size	1	1.00	1.61	555.97	1.71	1.58	1.81	0.00	1.55
time (sec)	N/A	0.014	0.024	1.471	0.524	0.468	1.318	0.000	1.193

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	17234	54	49	60	0	53
normalized size	1	1.00	0.84	269.28	0.84	0.77	0.94	0.00	0.83
time (sec)	N/A	0.034	0.037	1.417	0.537	0.507	2.121	0.000	0.118

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.093	0.981	0.000	0.549	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	79	130774	86	127	0	0	354
normalized size	1	1.00	0.98	1614.49	1.06	1.57	0.00	0.00	4.37
time (sec)	N/A	0.059	0.047	4.805	0.531	0.592	0.000	0.000	0.129

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	28786	51	97	0	0	234
normalized size	1	1.00	0.98	514.04	0.91	1.73	0.00	0.00	4.18
time (sec)	N/A	0.035	0.039	1.184	0.529	0.476	0.000	0.000	1.328

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	4303	30	79	0	0	108
normalized size	1	1.00	1.00	138.81	0.97	2.55	0.00	0.00	3.48
time (sec)	N/A	0.015	0.026	0.408	0.525	0.596	0.000	0.000	0.144

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	28	0	0	12
normalized size	1	1.00	1.00	1.08	1.33	2.33	0.00	0.00	1.00
time (sec)	N/A	0.004	0.049	0.071	0.411	0.684	0.000	0.000	1.178

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	972	37	87	0	0	113
normalized size	1	1.00	0.66	22.09	0.84	1.98	0.00	0.00	2.57
time (sec)	N/A	0.028	0.028	10.105	0.523	0.521	0.000	0.000	2.906

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	0	65	137	0	0	220
normalized size	1	1.00	0.69	0.00	1.00	2.11	0.00	0.00	3.38
time (sec)	N/A	0.038	0.025	180.000	0.523	0.588	0.000	0.000	3.119

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	0	108	199	0	0	300
normalized size	1	1.00	0.72	0.00	1.17	2.16	0.00	0.00	3.26
time (sec)	N/A	0.063	0.027	180.000	0.546	0.676	0.000	0.000	3.595

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.538	2.949	0.000	0.846	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	106	131085	178	326	0	0	669
normalized size	1	1.00	1.08	1337.60	1.82	3.33	0.00	0.00	6.83
time (sec)	N/A	0.081	0.091	5.951	0.769	0.518	0.000	0.000	1.286

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	83	29109	124	244	0	0	490
normalized size	1	1.00	1.11	388.12	1.65	3.25	0.00	0.00	6.53
time (sec)	N/A	0.054	0.054	1.481	0.763	0.503	0.000	0.000	0.171

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	4626	80	189	0	0	302
normalized size	1	1.00	1.12	92.52	1.60	3.78	0.00	0.00	6.04
time (sec)	N/A	0.031	0.070	0.480	0.748	0.456	0.000	0.000	1.293

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	625	47	97	0	0	28
normalized size	1	1.00	0.96	22.32	1.68	3.46	0.00	0.00	1.00
time (sec)	N/A	0.013	0.055	0.273	0.755	0.746	0.000	0.000	0.088

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	18	36	0	0	14
normalized size	1	1.00	1.00	1.07	1.29	2.57	0.00	0.00	1.00
time (sec)	N/A	0.005	0.006	0.076	0.412	0.646	0.000	0.000	1.136

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	0	77	306	0	0	421
normalized size	1	1.00	0.76	0.00	1.10	4.37	0.00	0.00	6.01
time (sec)	N/A	0.048	0.074	180.000	0.768	0.458	0.000	0.000	4.025

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	70	0	135	480	0	0	453
normalized size	1	1.00	0.69	0.00	1.32	4.71	0.00	0.00	4.44
time (sec)	N/A	0.063	0.061	180.000	0.776	0.685	0.000	0.000	3.799



Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	92	0	191	644	0	0	689
normalized size	1	1.00	0.64	0.00	1.34	4.50	0.00	0.00	4.82
time (sec)	N/A	0.090	0.045	180.000	0.757	0.548	0.000	0.000	4.905

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.543	3.505	0.000	0.508	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	114	29456	197	493	0	0	867
normalized size	1	1.00	1.24	320.17	2.14	5.36	0.00	0.00	9.42
time (sec)	N/A	0.071	0.043	1.510	1.150	1.144	0.000	0.000	1.378

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	86	4977	144	418	0	0	620
normalized size	1	1.00	1.21	70.10	2.03	5.89	0.00	0.00	8.73
time (sec)	N/A	0.049	0.046	0.782	1.146	0.646	0.000	0.000	1.432

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	952	94	250	0	0	46
normalized size	1	1.00	1.04	20.26	2.00	5.32	0.00	0.00	0.98
time (sec)	N/A	0.029	0.035	0.294	1.123	0.509	0.000	0.000	1.219

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	634	62	124	0	0	25
normalized size	1	1.00	0.79	18.65	1.82	3.65	0.00	0.00	0.74
time (sec)	N/A	0.014	0.050	0.268	1.112	0.844	0.000	0.000	0.088

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	30	107	0	0	14
normalized size	1	1.00	1.00	0.94	1.88	6.69	0.00	0.00	0.88
time (sec)	N/A	0.005	0.006	0.076	0.430	0.567	0.000	0.000	0.066

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	0	173	811	0	0	902
normalized size	1	1.00	0.76	0.00	1.78	8.36	0.00	0.00	9.30
time (sec)	N/A	0.066	0.110	180.000	1.132	0.615	0.000	0.000	6.493

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	93	0	243	1078	0	0	1074
normalized size	1	1.00	0.71	0.00	1.85	8.23	0.00	0.00	8.20
time (sec)	N/A	0.087	0.045	180.000	1.152	0.536	0.000	0.000	5.262

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	107	0	332	1316	0	0	1251
normalized size	1	1.00	0.63	0.00	1.95	7.74	0.00	0.00	7.36
time (sec)	N/A	0.127	0.046	180.000	1.154	0.623	0.000	0.000	8.121

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.140	14.578	0.000	0.423	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	146	504228	380	583	0	0	546
normalized size	1	1.00	0.88	3055.93	2.30	3.53	0.00	0.00	3.31
time (sec)	N/A	0.133	0.108	33.060	0.536	0.721	0.000	0.000	2.187

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	129477	255	411	0	0	418
normalized size	1	1.00	0.88	1070.06	2.11	3.40	0.00	0.00	3.45
time (sec)	N/A	0.080	0.078	18.412	0.534	1.476	0.000	0.000	1.500

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	25561	166	287	0	0	304
normalized size	1	1.00	0.87	311.72	2.02	3.50	0.00	0.00	3.71
time (sec)	N/A	0.046	0.065	14.728	0.543	0.464	0.000	0.000	1.361

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	480	101	210	0	0	205
normalized size	1	1.00	0.85	10.00	2.10	4.38	0.00	0.00	4.27
time (sec)	N/A	0.020	0.045	13.674	0.537	0.574	0.000	0.000	1.307

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	65	164	51	0	121
normalized size	1	1.00	1.00	1.05	3.25	8.20	2.55	0.00	6.05
time (sec)	N/A	0.007	0.016	0.084	0.525	0.573	0.737	0.000	1.268

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.089	0.681	0.000	0.918	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.041	13.987	0.000	0.540	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	67	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.044	13.911	0.000	0.590	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	38	33	0	0	96
normalized size	1	1.00	0.92	18.27	1.03	0.89	0.00	0.00	2.59
time (sec)	N/A	0.010	0.035	0.164	0.322	0.474	0.000	0.000	0.003

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	59	13	13	39	13	19
normalized size	1	1.00	0.87	2.57	0.57	0.57	1.70	0.57	0.83
time (sec)	N/A	0.014	0.038	0.442	0.309	0.392	11.343	0.118	0.079

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	48	13	13	39	13	19
normalized size	1	1.00	0.87	2.09	0.57	0.57	1.70	0.57	0.83
time (sec)	N/A	0.008	0.017	0.391	0.314	0.358	5.601	0.131	0.056

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	32	10	10	37	10	16
normalized size	1	1.00	1.12	2.00	0.62	0.62	2.31	0.62	1.00
time (sec)	N/A	0.003	0.007	0.064	0.306	0.507	2.928	0.118	1.207

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	8	8	0	9	58
normalized size	1	1.00	0.90	1.29	0.38	0.38	0.00	0.43	2.76
time (sec)	N/A	0.031	0.018	0.309	0.307	0.499	0.000	0.132	0.545

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	20	11	13	34	12	17
normalized size	1	1.00	1.06	1.18	0.65	0.76	2.00	0.71	1.00
time (sec)	N/A	0.009	0.017	0.393	0.311	0.506	5.736	0.140	0.074

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	11	11	39	11	16
normalized size	1	1.00	0.78	0.87	0.48	0.48	1.70	0.48	0.70
time (sec)	N/A	0.009	0.015	0.380	0.309	0.536	10.803	0.119	1.143

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	47	21	33	57	0	0	-1
normalized size	1	1.00	1.74	0.78	1.22	2.11	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.022	0.367	0.366	1.388	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	449	56	87	0	0	-1
normalized size	1	1.00	1.59	8.80	1.10	1.71	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.015	0.519	0.375	2.014	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	109	471	78	117	0	0	-1
normalized size	1	1.00	1.42	6.12	1.01	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.034	0.503	0.373	1.561	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	345	5294	281	899	0	0	-1
normalized size	1	1.00	1.12	17.24	0.92	2.93	0.00	0.00	-0.00
time (sec)	N/A	0.465	0.504	10.107	0.709	0.796	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	259	4990	215	745	0	0	-1
normalized size	1	1.00	1.12	21.60	0.93	3.23	0.00	0.00	-0.00
time (sec)	N/A	0.379	0.181	3.578	0.714	0.749	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	142	551	0	0	-1
normalized size	1	1.00	0.87	2.04	0.95	3.67	0.00	0.00	-0.01
time (sec)	N/A	0.232	1.423	0.351	0.688	0.854	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.066	9.537	0.559	0.000	0.774	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	144	1726	149	450	0	0	-1
normalized size	1	1.00	0.93	11.14	0.96	2.90	0.00	0.00	-0.01
time (sec)	N/A	0.307	0.205	5.739	1.124	1.751	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	1667	125	381	0	0	-1
normalized size	1	1.00	0.92	13.02	0.98	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.119	5.319	1.094	3.990	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	1584	101	322	0	0	-1
normalized size	1	1.00	0.90	15.68	1.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.102	5.091	1.099	0.753	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	201	247	72	238	0	0	-1
normalized size	1	1.00	2.91	3.58	1.04	3.45	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.913	0.460	1.099	0.969	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.077	3.351	0.961	0.000	0.686	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	144	1802	146	423	0	0	-1
normalized size	1	1.00	0.86	10.73	0.87	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.303	0.218	5.434	1.100	0.693	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	119	1745	123	359	0	0	-1
normalized size	1	1.00	0.86	12.55	0.88	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.115	5.313	1.107	1.861	0.000	0.000	0.000



Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	93	1664	100	305	0	0	-1
normalized size	1	1.00	0.85	15.13	0.91	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.101	4.902	1.098	1.013	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	200	271	73	227	0	0	-1
normalized size	1	1.00	2.63	3.57	0.96	2.99	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.985	0.396	1.102	0.800	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.065	3.391	0.950	0.000	0.524	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	353	5222	277	879	0	0	-1
normalized size	1	1.00	1.17	17.23	0.91	2.90	0.00	0.00	-0.00
time (sec)	N/A	0.465	0.449	10.362	0.701	0.565	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	267	4918	213	729	0	0	-1
normalized size	1	1.00	1.17	21.48	0.93	3.18	0.00	0.00	-0.00
time (sec)	N/A	0.382	0.165	3.775	0.738	0.622	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	142	539	0	0	-1
normalized size	1	1.00	0.87	2.04	0.95	3.59	0.00	0.00	-0.01
time (sec)	N/A	0.235	1.263	0.528	0.713	0.483	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.147	6.228	0.844	0.000	1.066	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	141	1698	146	423	0	0	-1
normalized size	1	1.00	0.93	11.17	0.96	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.308	0.201	5.863	1.092	0.528	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	116	1641	123	359	0	0	-1
normalized size	1	1.00	0.92	13.02	0.98	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.115	5.589	1.107	0.872	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	1560	100	305	0	0	-1
normalized size	1	1.00	0.90	15.60	1.00	3.05	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.103	5.092	1.100	0.639	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	197	247	72	226	0	0	-1
normalized size	1	1.00	2.86	3.58	1.04	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.869	0.560	1.098	0.485	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.061	3.505	1.270	0.000	0.696	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	147	1830	149	450	0	0	-1
normalized size	1	1.00	0.89	11.09	0.90	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.310	0.215	6.464	1.099	0.570	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	1771	125	381	0	0	-1
normalized size	1	1.00	0.88	12.93	0.91	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.122	5.458	1.104	0.532	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	1688	101	322	0	0	-1
normalized size	1	1.00	0.86	15.49	0.93	2.95	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.119	4.881	1.115	0.527	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	208	271	73	239	0	0	-1
normalized size	1	1.00	2.74	3.57	0.96	3.14	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.791	0.522	1.111	0.836	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.083	3.633	1.201	0.000	0.421	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	1808	0	0	-1
normalized size	1	1.00	2.17	24.60	0.00	5.99	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.351	50.403	0.000	0.772	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	1278	0	0	-1
normalized size	1	1.00	1.75	23.69	0.00	5.46	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.196	36.632	0.000	0.914	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2543	0	830	0	0	-1
normalized size	1	1.00	1.62	15.70	0.00	5.12	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.139	4.738	0.000	0.656	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	180	182	498	0	0	-1
normalized size	1	1.00	0.99	2.28	2.30	6.30	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.022	0.547	0.463	1.010	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.041	0.990	4.151	0.000	0.559	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	346	6892	0	2160	0	0	-1
normalized size	1	1.00	0.88	17.45	0.00	5.47	0.00	0.00	-0.00
time (sec)	N/A	0.500	0.340	34.684	0.000	0.962	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	257	6518	0	1688	0	0	-1
normalized size	1	1.00	0.87	22.09	0.00	5.72	0.00	0.00	-0.00
time (sec)	N/A	0.397	0.156	4.678	0.000	0.828	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	4654	612	372	1188	0	0	-1
normalized size	1	1.00	23.99	3.15	1.92	6.12	0.00	0.00	-0.01
time (sec)	N/A	0.251	13.415	0.330	0.515	0.827	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.146	0.393	1.270	0.000	0.593	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	155	2339	341	348	0	0	-1
normalized size	1	1.00	0.91	13.76	2.01	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.224	5.882	0.351	0.506	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2249	247	296	0	0	-1
normalized size	1	1.00	0.89	16.91	1.86	2.23	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.117	5.056	0.343	0.578	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	766	292	265	221	0	0	-1
normalized size	1	1.00	8.24	3.14	2.85	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.155	3.851	0.521	0.430	0.633	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.089	0.811	1.823	0.000	0.712	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	156	2449	340	348	0	0	-1
normalized size	1	1.00	0.91	14.32	1.99	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.214	5.932	0.350	0.673	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	120	2351	246	296	0	0	-1
normalized size	1	1.00	0.90	17.54	1.84	2.21	0.00	0.00	-0.01
time (sec)	N/A	0.245	0.119	4.913	0.342	0.741	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	723	297	263	222	0	0	-1
normalized size	1	1.00	7.69	3.16	2.80	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.156	3.239	0.506	0.424	0.481	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.085	0.803	1.819	0.000	0.633	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	1566	0	0	-1
normalized size	1	1.00	2.17	24.60	0.00	5.19	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.311	49.151	0.000	0.800	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	1080	0	0	-1
normalized size	1	1.00	1.75	23.69	0.00	4.62	0.00	0.00	-0.00
time (sec)	N/A	0.171	0.198	40.199	0.000	0.722	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2543	0	676	0	0	-1
normalized size	1	1.00	1.62	15.70	0.00	4.17	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.123	4.354	0.000	0.560	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	267	184	388	0	0	-1
normalized size	1	1.00	0.99	3.38	2.33	4.91	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.033	0.552	0.469	0.694	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.042	0.116	4.189	0.000	0.593	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	339	6698	0	1798	0	0	-1
normalized size	1	1.00	0.87	17.13	0.00	4.60	0.00	0.00	-0.00
time (sec)	N/A	0.503	0.333	39.339	0.000	0.833	0.000	0.000	0.000



Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	253	6348	0	1462	0	0	-1
normalized size	1	1.00	0.86	21.67	0.00	4.99	0.00	0.00	-0.00
time (sec)	N/A	0.403	0.128	4.915	0.000	0.893	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	4463	629	392	1098	0	0	-1
normalized size	1	1.00	23.01	3.24	2.02	5.66	0.00	0.00	-0.01
time (sec)	N/A	0.249	13.100	0.585	0.542	0.750	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.093	0.441	1.455	0.000	0.578	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	155	2449	342	179	0	0	-1
normalized size	1	1.00	0.92	14.58	2.04	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.306	0.205	6.234	0.378	0.665	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	119	2351	248	156	0	0	-1
normalized size	1	1.00	0.90	17.81	1.88	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.117	5.269	0.366	0.725	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	709	299	286	121	0	0	-1
normalized size	1	1.00	7.62	3.22	3.08	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.159	3.785	0.757	0.451	0.521	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.088	0.783	2.065	0.000	0.640	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	155	2339	343	179	0	0	-1
normalized size	1	1.00	0.92	13.84	2.03	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.295	0.213	5.973	0.406	0.681	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2249	249	156	0	0	-1
normalized size	1	1.00	0.89	16.91	1.87	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.110	5.512	0.377	0.627	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	605	304	288	121	0	0	-1
normalized size	1	1.00	6.44	3.23	3.06	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.150	2.942	0.703	0.454	0.712	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.080	0.829	2.073	0.000	0.596	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	131	920	0	326	0	0	-1
normalized size	1	1.00	0.82	5.75	0.00	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.574	0.302	0.536	0.000	0.598	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	236	4034	328	246	362	0	510
normalized size	1	1.00	0.79	13.58	1.10	0.83	1.22	0.00	1.72
time (sec)	N/A	0.386	0.175	16.949	0.351	0.666	23.397	0.000	2.288

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	192	3320	269	195	286	0	414
normalized size	1	1.00	0.85	14.76	1.20	0.87	1.27	0.00	1.84
time (sec)	N/A	0.256	0.151	12.627	0.349	0.522	9.881	0.000	2.280

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	2616	171	138	209	0	329
normalized size	1	1.00	0.92	18.69	1.22	0.99	1.49	0.00	2.35
time (sec)	N/A	0.119	0.110	3.357	0.344	0.579	3.929	0.000	2.048

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	C	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	0	864	167	0	0	0	-1
normalized size	1	1.00	0.00	2.27	0.44	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	0.223	4.559	0.536	0.502	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	161	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	0.159	11.444	0.000	0.865	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	307	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.717	0.153	180.000	0.000	0.509	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	236	4194	314	249	345	0	497
normalized size	1	1.00	0.75	13.31	1.00	0.79	1.10	0.00	1.58
time (sec)	N/A	0.749	0.159	4.748	0.339	0.608	15.301	0.000	2.339

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	3514	252	198	265	0	414
normalized size	1	1.00	0.74	14.23	1.02	0.80	1.07	0.00	1.68
time (sec)	N/A	0.621	0.128	3.720	0.331	0.477	6.778	0.000	2.328

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	144	2210	178	130	155	0	315
normalized size	1	1.00	1.38	21.25	1.71	1.25	1.49	0.00	3.03
time (sec)	N/A	0.198	0.019	1.798	0.332	0.572	2.303	0.000	2.089

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	94	332	0	0	0	0	0	-1
normalized size	1	0.90	3.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.269	0.206	5.649	0.000	0.572	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	191	457	0	0	0	0	0	-1
normalized size	1	0.97	2.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.463	0.367	180.000	0.000	0.618	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F(-2)	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	250	0	0	0	0	0	0	-1
normalized size	1	0.98	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.670	0.297	180.000	0.000	0.442	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	677	8491	0	0	0	0	-1
normalized size	1	1.00	1.32	16.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.767	4.434	3.145	0.000	0.597	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	1287	0	0	0	0	0	-1
normalized size	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.387	3.223	2.963	0.000	0.645	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.240	1.488	0.000	0.498	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	1236	0	0	0	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.261	3.586	2.197	0.000	0.635	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	1318	936	0	0	0	0	-1
normalized size	1	1.00	1.85	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.113	5.904	4.682	0.000	0.612	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	51	31	58	64	0	0	-1
normalized size	1	1.00	2.04	1.24	2.32	2.56	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.035	0.100	0.318	0.577	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	71	62	59	94	0	0	-1
normalized size	1	1.00	1.39	1.22	1.16	1.84	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.025	0.065	0.322	0.582	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	93	79	76	119	0	0	-1
normalized size	1	1.00	1.33	1.13	1.09	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.026	0.064	0.319	0.687	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	68	67	107	137	0	0	-1
normalized size	1	1.00	1.66	1.63	2.61	3.34	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.083	0.069	0.330	0.630	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	113	153	108	198	0	0	-1
normalized size	1	1.00	1.36	1.84	1.30	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.045	0.075	0.343	0.643	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	149	185	142	247	0	0	-1
normalized size	1	1.00	1.25	1.55	1.19	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.042	0.073	0.346	0.573	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	108	164	202	283	0	0	-1
normalized size	1	1.00	0.64	0.98	1.20	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.086	0.150	0.333	0.579	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	177	590	194	395	0	0	-1
normalized size	1	1.00	0.82	2.73	0.90	1.83	0.00	0.00	-0.00
time (sec)	N/A	2.714	0.112	0.185	0.376	0.659	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	235	666	254	479	0	0	-1
normalized size	1	1.00	0.87	2.48	0.94	1.78	0.00	0.00	-0.00
time (sec)	N/A	2.586	0.076	0.184	0.375	0.705	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	21	21	14	0	15
normalized size	1	1.00	1.00	1.12	1.24	1.24	0.82	0.00	0.88
time (sec)	N/A	0.044	0.058	0.066	0.331	0.730	0.567	0.000	1.395

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	46	37	58	60	0	107
normalized size	1	1.00	0.89	1.05	0.84	1.32	1.36	0.00	2.43
time (sec)	N/A	0.052	0.018	0.068	0.325	0.607	4.217	0.000	1.561



Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	118	40	108	0	0	58
normalized size	1	1.00	0.89	2.51	0.85	2.30	0.00	0.00	1.23
time (sec)	N/A	0.055	0.038	0.129	0.304	0.637	0.000	0.000	2.510

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	153	794	184	233	0	0	187
normalized size	1	1.00	1.43	7.42	1.72	2.18	0.00	0.00	1.75
time (sec)	N/A	0.155	0.181	0.513	0.415	0.590	0.000	0.000	1.672

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	824	64	92	0	0	119
normalized size	1	1.00	1.22	16.82	1.31	1.88	0.00	0.00	2.43
time (sec)	N/A	0.079	0.093	0.430	0.326	0.496	0.000	0.000	1.830

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	349	43	25	66	0	28
normalized size	1	1.00	1.02	7.76	0.96	0.56	1.47	0.00	0.62
time (sec)	N/A	0.060	0.095	0.384	0.335	0.584	3.118	0.000	0.103

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	68	42	25	0	35	28
normalized size	1	1.00	1.02	1.51	0.93	0.56	0.00	0.78	0.62
time (sec)	N/A	0.059	0.092	0.294	0.315	0.526	0.000	0.137	1.205

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	59	939	64	93	0	0	111
normalized size	1	1.00	1.20	19.16	1.31	1.90	0.00	0.00	2.27
time (sec)	N/A	0.071	0.089	0.482	0.334	0.667	0.000	0.000	1.437

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	150	920	184	234	0	0	179
normalized size	1	1.00	1.40	8.60	1.72	2.19	0.00	0.00	1.67
time (sec)	N/A	0.148	0.167	0.646	0.421	0.460	0.000	0.000	1.604

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [75] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	8	0.375
2	A	4	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
4	A	4	3	1.00	8	0.375
5	A	3	3	1.00	6	0.500
6	A	2	2	1.00	4	0.500
7	A	1	1	1.00	8	0.125
8	A	5	5	1.00	8	0.625
9	A	3	3	1.00	8	0.375
10	A	4	3	1.00	8	0.375
11	A	4	3	1.00	8	0.375
12	A	15	7	1.00	10	0.700
13	A	14	9	1.00	10	0.900
14	A	10	7	1.00	10	0.700
15	A	9	8	1.00	10	0.800
16	A	5	5	1.00	8	0.625
17	A	5	5	1.00	6	0.833
18	A	6	5	1.00	10	0.500
19	A	4	4	1.00	10	0.400
20	A	8	7	1.00	10	0.700
21	A	8	7	1.00	10	0.700
22	A	13	8	1.00	10	0.800
23	A	33	11	1.00	10	1.100
24	A	22	11	1.00	10	1.100
25	A	18	10	1.00	10	1.000
26	A	11	9	1.00	10	0.900
27	A	8	8	1.00	8	1.000
28	A	5	6	1.00	6	1.000
29	A	8	6	1.00	10	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
30	A	5	6	1.00	10	0.600
31	A	7	6	1.00	10	0.600
32	A	14	11	1.00	10	1.100
33	A	16	8	1.00	10	0.800
34	A	1	1	1.00	14	0.071
35	A	4	4	1.00	14	0.286
36	A	4	4	1.00	14	0.286
37	A	5	5	1.00	14	0.357
38	A	5	4	1.00	12	0.333
39	A	27	13	1.00	14	0.929
40	A	25	13	1.00	14	0.929
41	A	23	11	1.00	14	0.786
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	5	6	1.00	16	0.375
45	A	7	9	1.00	16	0.562
46	A	8	9	1.00	16	0.562
47	A	8	9	1.00	16	0.562
48	A	3	3	1.00	15	0.200
49	A	2	2	1.00	15	0.133
50	A	1	1	1.00	15	0.067
51	A	2	2	1.00	15	0.133
52	A	3	2	1.00	15	0.133
53	A	1	1	1.00	14	0.071
54	A	1	1	1.00	14	0.071
55	A	4	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	4	4	1.00	13	0.308
57	A	1	1	1.00	12	0.083
58	A	3	3	1.00	13	0.231
59	A	2	2	1.00	12	0.167
60	A	4	3	1.00	13	0.231
61	A	3	3	1.00	12	0.250
62	A	7	5	1.00	10	0.500
63	A	7	5	1.00	10	0.500
64	A	7	5	1.00	8	0.625
65	A	3	3	1.00	6	0.500
66	A	5	5	1.00	10	0.500
67	A	7	5	1.00	10	0.500
68	A	5	4	1.00	10	0.400
69	A	19	15	1.00	12	1.250
70	A	15	13	1.00	12	1.083
71	A	12	10	1.00	10	1.000
72	A	6	6	1.00	8	0.750
73	A	2	2	1.00	12	0.167
74	A	17	15	1.00	12	1.250
75	A	21	16	1.00	12	1.333
76	A	37	7	0.89	16	0.438
77	A	5	5	1.00	14	0.357
78	A	37	10	1.23	16	0.625
79	A	57	11	1.00	16	0.688
80	A	55	16	1.00	18	0.889
81	A	65	19	1.00	18	1.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	12	8	1.00	19	0.421
83	A	6	4	1.00	10	0.400
84	A	5	4	1.00	8	0.500
85	A	4	4	1.00	6	0.667
86	A	2	2	1.00	10	0.200
87	A	4	4	1.00	10	0.400
88	A	5	4	1.00	10	0.400
89	A	3	2	1.00	12	0.167
90	A	3	2	1.00	12	0.167
91	A	2	2	1.00	12	0.167
92	A	4	4	1.00	12	0.333
93	A	2	2	1.00	10	0.200
94	A	3	3	1.00	4	0.750
95	A	2	2	1.00	10	0.200
96	A	4	4	1.00	12	0.333
97	A	5	4	1.00	14	0.286
98	A	2	2	1.00	14	0.143
99	A	6	6	1.00	14	0.429
100	A	3	3	1.00	12	0.250
101	A	3	3	1.00	19	0.158
102	A	7	5	1.00	18	0.278
103	A	7	5	1.00	18	0.278
104	A	7	5	1.00	16	0.312
105	A	4	3	1.00	10	0.300
106	A	5	5	1.00	18	0.278
107	A	7	5	1.00	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	5	4	1.00	18	0.222
109	A	16	13	1.00	20	0.650
110	A	13	10	1.00	18	0.556
111	A	6	6	1.00	12	0.500
112	A	2	2	1.00	20	0.100
113	A	21	19	1.01	20	0.950
114	A	21	14	1.00	20	0.700
115	A	15	11	1.00	18	0.611
116	A	6	7	1.00	12	0.583
117	A	2	2	1.00	20	0.100
118	A	30	18	1.00	20	0.900
119	A	6	4	1.00	18	0.222
120	A	0	0	0.00	0	0.000
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	9	7	1.00	40	0.175
124	A	7	6	1.00	40	0.150
125	A	2	3	1.00	38	0.079
126	A	0	0	0.00	0	0.000
127	A	0	0	0.00	0	0.000
128	A	2	2	1.00	11	0.182
129	A	2	2	1.00	11	0.182
130	A	2	2	1.00	9	0.222
131	A	2	2	1.00	7	0.286
132	A	2	2	1.00	11	0.182
133	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
134	A	2	2	1.00	11	0.182
135	A	2	2	1.00	11	0.182
136	A	3	2	1.00	13	0.154
137	A	3	2	1.00	13	0.154
138	A	3	2	1.00	13	0.154
139	A	3	3	1.00	11	0.273
140	A	2	2	1.00	9	0.222
141	A	3	3	1.00	13	0.231
142	A	3	3	1.00	13	0.231
143	A	3	2	1.00	13	0.154
144	A	1	1	1.00	13	0.077
145	A	2	2	1.00	13	0.154
146	A	4	2	1.00	13	0.154
147	A	4	2	1.00	13	0.154
148	A	4	2	1.00	13	0.154
149	A	4	3	1.00	13	0.231
150	A	3	3	1.00	11	0.273
151	A	2	2	1.00	9	0.222
152	A	4	3	1.00	13	0.231
153	A	4	4	1.00	13	0.308
154	A	4	3	1.00	13	0.231
155	A	4	2	1.00	13	0.154
156	A	1	1	1.00	13	0.077
157	A	2	2	1.00	13	0.154
158	A	1	1	1.00	13	0.077
159	A	5	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	4	4	1.00	13	0.308
161	A	3	3	1.00	11	0.273
162	A	2	2	1.00	9	0.222
163	A	4	3	1.00	13	0.231
164	A	5	4	1.00	13	0.308
165	A	6	4	1.00	13	0.308
166	A	2	2	1.00	13	0.154
167	A	6	5	1.00	13	0.385
168	A	5	5	1.00	13	0.385
169	A	4	4	1.00	13	0.308
170	A	3	3	1.00	11	0.273
171	A	2	2	1.00	9	0.222
172	A	5	4	1.00	13	0.308
173	A	6	5	1.00	13	0.385
174	A	7	5	1.00	13	0.385
175	A	3	2	1.00	13	0.154
176	A	6	5	1.00	13	0.385
177	A	5	4	1.00	13	0.308
178	A	4	3	1.00	13	0.231
179	A	3	3	1.00	11	0.273
180	A	2	2	1.00	9	0.222
181	A	6	4	1.00	13	0.308
182	A	7	5	1.00	13	0.385
183	A	8	5	1.00	13	0.385
184	A	1	1	1.00	13	0.077
185	A	6	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
186	A	5	3	1.00	13	0.231
187	A	4	3	1.00	13	0.231
188	A	3	3	1.00	11	0.273
189	A	2	2	1.00	9	0.222
190	A	1	1	1.00	13	0.077
191	A	2	2	1.00	13	0.154
192	A	3	2	1.00	13	0.154
193	A	2	2	1.00	11	0.182
194	A	2	2	1.00	11	0.182
195	A	2	2	1.00	9	0.222
196	A	2	2	1.00	7	0.286
197	A	2	2	1.00	11	0.182
198	A	2	2	1.00	11	0.182
199	A	2	2	1.00	11	0.182
200	A	6	4	1.00	3	1.333
201	A	8	5	1.00	5	1.000
202	A	10	6	1.00	7	0.857
203	A	11	6	1.00	15	0.400
204	A	9	5	1.00	13	0.385
205	A	7	4	1.00	11	0.364
206	A	0	0	0.00	0	0.000
207	A	8	7	1.00	16	0.438
208	A	7	7	1.00	16	0.438
209	A	6	6	1.00	14	0.429
210	A	5	5	1.00	12	0.417
211	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	8	7	1.00	19	0.368
213	A	7	7	1.00	19	0.368
214	A	6	6	1.00	17	0.353
215	A	5	5	1.00	15	0.333
216	A	0	0	0.00	0	0.000
217	A	11	6	1.00	15	0.400
218	A	9	5	1.00	13	0.385
219	A	7	4	1.00	11	0.364
220	A	0	0	0.00	0	0.000
221	A	8	7	1.00	16	0.438
222	A	7	7	1.00	16	0.438
223	A	6	6	1.00	14	0.429
224	A	5	5	1.00	12	0.417
225	A	0	0	0.00	0	0.000
226	A	8	7	1.00	19	0.368
227	A	7	7	1.00	19	0.368
228	A	6	6	1.00	17	0.353
229	A	5	5	1.00	15	0.333
230	A	0	0	0.00	0	0.000
231	A	12	6	1.00	15	0.400
232	A	10	6	1.00	15	0.400
233	A	8	5	1.00	13	0.385
234	A	6	4	1.00	7	0.571
235	A	0	0	0.00	0	0.000
236	A	11	6	1.00	15	0.400
237	A	9	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
238	A	7	4	1.00	11	0.364
239	A	0	0	0.00	0	0.000
240	A	7	7	1.00	20	0.350
241	A	6	6	1.00	18	0.333
242	A	5	5	1.00	16	0.312
243	A	0	0	0.00	0	0.000
244	A	7	7	1.00	21	0.333
245	A	6	6	1.00	19	0.316
246	A	5	5	1.00	17	0.294
247	A	0	0	0.00	0	0.000
248	A	12	6	1.00	15	0.400
249	A	10	6	1.00	15	0.400
250	A	8	5	1.00	13	0.385
251	A	6	4	1.00	7	0.571
252	A	0	0	0.00	0	0.000
253	A	11	6	1.00	15	0.400
254	A	9	5	1.00	13	0.385
255	A	7	4	1.00	11	0.364
256	A	0	0	0.00	0	0.000
257	A	7	7	1.00	20	0.350
258	A	6	6	1.00	18	0.333
259	A	5	5	1.00	16	0.312
260	A	0	0	0.00	0	0.000
261	A	7	7	1.00	21	0.333
262	A	6	6	1.00	19	0.316
263	A	5	5	1.00	17	0.294
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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	0	0	0.00	0	0.000
265	A	11	8	1.00	24	0.333
266	A	23	11	1.00	27	0.407
267	A	14	11	1.00	27	0.407
268	A	7	8	1.00	25	0.320
269	A	21	11	1.00	27	0.407
270	A	13	13	1.00	27	0.482
271	A	17	14	1.00	27	0.518
272	A	26	15	1.00	27	0.556
273	A	21	15	1.00	27	0.556
274	A	9	8	1.00	24	0.333
275	A	8	8	0.90	27	0.296
276	A	17	16	0.97	27	0.593
277	A	26	18	0.98	27	0.667
278	A	22	17	1.00	22	0.773
279	A	38	20	1.00	21	0.952
280	A	0	0	0.00	0	0.000
281	A	38	22	1.00	24	0.917
282	A	32	17	1.00	24	0.708
283	A	2	2	1.00	4	0.500
284	A	7	4	1.00	6	0.667
285	A	9	5	1.00	8	0.625
286	A	2	2	1.00	8	0.250
287	A	7	4	1.00	10	0.400
288	A	9	5	1.00	12	0.417
289	A	6	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
290	A	25	8	1.00	14	0.571
291	A	29	9	1.00	16	0.562
292	A	1	1	1.00	20	0.050
293	A	4	4	1.00	12	0.333
294	A	4	4	1.00	14	0.286
295	A	8	7	1.00	20	0.350
296	A	5	5	1.00	20	0.250
297	A	3	2	1.00	20	0.100
298	A	3	2	1.00	20	0.100
299	A	5	5	1.00	20	0.250
300	A	8	7	1.00	20	0.350

# Chapter 3

## Listing of integrals

### 3.1 $\int x^5 \coth^{-1}(ax) dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}(ax)}{6a^6} + \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{1}{6}x^6 \coth^{-1}(ax) + \frac{x^5}{30a}$$

[Out]  $1/6*x/a^5+1/18*x^3/a^3+1/30*x^5/a+1/6*x^6*\operatorname{arccoth}(a*x)-1/6*\operatorname{arctanh}(a*x)/a^6$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 302, 206}

$$\frac{x^3}{18a^3} + \frac{x}{6a^5} - \frac{\tanh^{-1}(ax)}{6a^6} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^5\*ArcCoth[a\*x],x]

[Out]  $x/(6*a^5) + x^3/(18*a^3) + x^5/(30*a) + (x^6*ArcCoth[a*x])/6 - ArcTanh[a*x]/(6*a^6)$

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int x^5 \coth^{-1}(ax) dx &= \frac{1}{6}x^6 \coth^{-1}(ax) - \frac{1}{6}a \int \frac{x^6}{1 - a^2x^2} dx \\ &= \frac{1}{6}x^6 \coth^{-1}(ax) - \frac{1}{6}a \int \left( -\frac{1}{a^6} - \frac{x^2}{a^4} - \frac{x^4}{a^2} + \frac{1}{a^6(1 - a^2x^2)} \right) dx \\ &= \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax) - \frac{\int \frac{1}{1 - a^2x^2} dx}{6a^5} \\ &= \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{1}{6}x^6 \coth^{-1}(ax) - \frac{\tanh^{-1}(ax)}{6a^6} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 67, normalized size = 1.31

$$\frac{\log(1 - ax)}{12a^6} - \frac{\log(ax + 1)}{12a^6} + \frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{1}{6}x^6 \coth^{-1}(ax) + \frac{x^5}{30a}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*ArcCoth[a\*x], x]

[Out] x/(6\*a^5) + x^3/(18\*a^3) + x^5/(30\*a) + (x^6\*ArcCoth[a\*x])/6 + Log[1 - a\*x]/(12\*a^6) - Log[1 + a\*x]/(12\*a^6)

**fricas** [A] time = 0.70, size = 51, normalized size = 1.00

$$\frac{6 a^5 x^5 + 10 a^3 x^3 + 30 a x + 15 (a^6 x^6 - 1) \log\left(\frac{ax+1}{ax-1}\right)}{180 a^6}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x^5\*arccoth(a\*x),x, algorithm="fricas")

[Out] 1/180\*(6\*a^5\*x^5 + 10\*a^3\*x^3 + 30\*a\*x + 15\*(a^6\*x^6 - 1)\*log((a\*x + 1)/(a\*x - 1)))/a^6

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate(x^5\*arccoth(a\*x), x)

**maple** [A] time = 0.03, size = 55, normalized size = 1.08

$$\frac{x^6 \operatorname{arccoth}(ax)}{6} + \frac{x^5}{30a} + \frac{x^3}{18a^3} + \frac{x}{6a^5} + \frac{\ln(ax-1)}{12a^6} - \frac{\ln(ax+1)}{12a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*arccoth(a\*x),x)

[Out] 1/6\*x^6\*arccoth(a\*x)+1/30\*x^5/a+1/18\*x^3/a^3+1/6\*x/a^5+1/12/a^6\*ln(a\*x-1)-1/12\*ln(a\*x+1)/a^6

**maxima** [A] time = 0.30, size = 61, normalized size = 1.20

$$\frac{1}{6} x^6 \operatorname{arccoth}(ax) + \frac{1}{180} a \left( \frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*arccoth(a\*x),x, algorithm="maxima")

[Out] 1/6\*x^6\*arccoth(a\*x) + 1/180\*a\*(2\*(3\*a^4\*x^5 + 5\*a^2\*x^3 + 15\*x)/a^6 - 15\*log(a\*x + 1)/a^7 + 15\*log(a\*x - 1)/a^7)

**mupad** [B] time = 1.31, size = 41, normalized size = 0.80

$$\frac{\frac{ax}{6} - \frac{\operatorname{acoth}(ax)}{6} + \frac{a^3x^3}{18} + \frac{a^5x^5}{30}}{a^6} + \frac{x^6 \operatorname{acoth}(ax)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*acoth(a\*x),x)

[Out]  $((a*x)/6 - \operatorname{acoth}(a*x)/6 + (a^3*x^3)/18 + (a^5*x^5)/30)/a^6 + (x^6*\operatorname{acoth}(a*x))/6$

sympy [A] time = 1.53, size = 49, normalized size = 0.96

$$\begin{cases} \frac{x^6 \operatorname{acoth}(ax)}{6} + \frac{x^5}{30a} + \frac{x^3}{18a^3} + \frac{x}{6a^5} - \frac{\operatorname{acoth}(ax)}{6a^6} & \text{for } a \neq 0 \\ \frac{i\pi x^6}{12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*acoth(a*x),x)`

[Out] `Piecewise((x**6*acoth(a*x)/6 + x**5/(30*a) + x**3/(18*a**3) + x/(6*a**5) - acoth(a*x)/(6*a**6), Ne(a, 0)), (I*pi*x**6/12, True))`

## 3.2 $\int x^4 \coth^{-1}(ax) dx$

Optimal. Leaf size=50

$$\frac{x^2}{10a^3} + \frac{\log(1 - a^2x^2)}{10a^5} + \frac{1}{5}x^5 \coth^{-1}(ax) + \frac{x^4}{20a}$$

[Out]  $1/10*x^2/a^3+1/20*x^4/a+1/5*x^5*\operatorname{arccoth}(a*x)+1/10*\ln(-a^2*x^2+1)/a^5$

**Rubi** [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 266, 43}

$$\frac{x^2}{10a^3} + \frac{\log(1 - a^2x^2)}{10a^5} + \frac{x^4}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^4\*ArcCoth[a\*x], x]

[Out]  $x^2/(10*a^3) + x^4/(20*a) + (x^5*ArcCoth[a*x])/5 + \text{Log}[1 - a^2*x^2]/(10*a^5)$

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int x^4 \coth^{-1}(ax) dx &= \frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{5}a \int \frac{x^5}{1-a^2x^2} dx \\
&= \frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a \operatorname{Subst}\left(\int \frac{x^2}{1-a^2x} dx, x, x^2\right) \\
&= \frac{1}{5}x^5 \coth^{-1}(ax) - \frac{1}{10}a \operatorname{Subst}\left(\int \left(-\frac{1}{a^4} - \frac{x}{a^2} - \frac{1}{a^4(-1+a^2x)}\right) dx, x, x^2\right) \\
&= \frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{10a^5}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 50, normalized size = 1.00

$$\frac{x^2}{10a^3} + \frac{\log(1-a^2x^2)}{10a^5} + \frac{1}{5}x^5 \coth^{-1}(ax) + \frac{x^4}{20a}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcCoth[a\*x], x]

[Out] x^2/(10\*a^3) + x^4/(20\*a) + (x^5\*ArcCoth[a\*x])/5 + Log[1 - a^2\*x^2]/(10\*a^5)

**fricas** [A] time = 0.58, size = 55, normalized size = 1.10

$$\frac{2a^5x^5 \log\left(\frac{ax+1}{ax-1}\right) + a^4x^4 + 2a^2x^2 + 2 \log(a^2x^2 - 1)}{20a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x), x, algorithm="fricas")

[Out] 1/20\*(2\*a^5\*x^5\*log((a\*x + 1)/(a\*x - 1)) + a^4\*x^4 + 2\*a^2\*x^2 + 2\*log(a^2\*x^2 - 1))/a^5

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x), x, algorithm="giac")

[Out] integrate(x^4\*arccoth(a\*x), x)

**maple [A]** time = 0.03, size = 49, normalized size = 0.98

$$\frac{x^5 \operatorname{arccoth}(ax)}{5} + \frac{x^4}{20a} + \frac{x^2}{10a^3} + \frac{\ln(ax-1)}{10a^5} + \frac{\ln(ax+1)}{10a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arccoth(a\*x), x)

[Out] 1/5\*x^5\*arccoth(a\*x)+1/20\*x^4/a+1/10\*x^2/a^3+1/10/a^5\*ln(a\*x-1)+1/10\*ln(a\*x+1)/a^5

**maxima [A]** time = 0.30, size = 46, normalized size = 0.92

$$\frac{1}{5} x^5 \operatorname{arccoth}(ax) + \frac{1}{20} a \left( \frac{a^2 x^4 + 2 x^2}{a^4} + \frac{2 \log(a^2 x^2 - 1)}{a^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x), x, algorithm="maxima")

[Out] 1/5\*x^5\*arccoth(a\*x) + 1/20\*a\*((a^2\*x^4 + 2\*x^2)/a^4 + 2\*log(a^2\*x^2 - 1)/a^6)

**mupad [B]** time = 1.27, size = 43, normalized size = 0.86

$$\frac{\frac{\ln(a^2 x^2 - 1)}{10} + \frac{a^2 x^2}{10} + \frac{a^4 x^4}{20}}{a^5} + \frac{x^5 \operatorname{acoth}(ax)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*acoth(a\*x), x)

[Out] (log(a^2\*x^2 - 1)/10 + (a^2\*x^2)/10 + (a^4\*x^4)/20)/a^5 + (x^5\*acoth(a\*x))/5

**sympy [A]** time = 1.18, size = 54, normalized size = 1.08

$$\begin{cases} \frac{x^5 \operatorname{acoth}(ax)}{5} + \frac{x^4}{20a} + \frac{x^2}{10a^3} + \frac{\log(ax+1)}{5a^5} - \frac{\operatorname{acoth}(ax)}{5a^5} & \text{for } a \neq 0 \\ \frac{i\pi x^5}{10} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x),x)
```

```
[Out] Piecewise((x**5*acoth(a*x)/5 + x**4/(20*a) + x**2/(10*a**3) + log(a*x + 1)/  
(5*a**5) - acoth(a*x)/(5*a**5), Ne(a, 0)), (I*pi*x**5/10, True))
```

### 3.3 $\int x^3 \coth^{-1}(ax) dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}(ax)}{4a^4} + \frac{x}{4a^3} + \frac{1}{4}x^4 \coth^{-1}(ax) + \frac{x^3}{12a}$$

[Out]  $1/4*x/a^3+1/12*x^3/a+1/4*x^4*\operatorname{arccoth}(a*x)-1/4*\operatorname{arctanh}(a*x)/a^4$

**Rubi [A]** time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 302, 206}

$$\frac{x}{4a^3} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[a\*x], x]

[Out]  $x/(4*a^3) + x^3/(12*a) + (x^4*ArcCoth[a*x])/4 - ArcTanh[a*x]/(4*a^4)$

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^n), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

#### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m+1)), x] - Dist[(b\*c\*p)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcCoth[c\*x])^(p-1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(ax) dx &= \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{1}{4}a \int \frac{x^4}{1-a^2x^2} dx \\
&= \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{1}{4}a \int \left( -\frac{1}{a^4} - \frac{x^2}{a^2} + \frac{1}{a^4(1-a^2x^2)} \right) dx \\
&= \frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{\int \frac{1}{1-a^2x^2} dx}{4a^3} \\
&= \frac{x}{4a^3} + \frac{x^3}{12a} + \frac{1}{4}x^4 \coth^{-1}(ax) - \frac{\tanh^{-1}(ax)}{4a^4}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 57, normalized size = 1.39

$$\frac{\log(1-ax)}{8a^4} - \frac{\log(ax+1)}{8a^4} + \frac{x}{4a^3} + \frac{1}{4}x^4 \coth^{-1}(ax) + \frac{x^3}{12a}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[a\*x],x]

[Out] x/(4\*a^3) + x^3/(12\*a) + (x^4\*ArcCoth[a\*x])/4 + Log[1 - a\*x]/(8\*a^4) - Log[1 + a\*x]/(8\*a^4)

**fricas** [A] time = 0.42, size = 43, normalized size = 1.05

$$\frac{2a^3x^3 + 6ax + 3(a^4x^4 - 1)\log\left(\frac{ax+1}{ax-1}\right)}{24a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x),x, algorithm="fricas")

[Out] 1/24\*(2\*a^3\*x^3 + 6\*a\*x + 3\*(a^4\*x^4 - 1)\*log((a\*x + 1)/(a\*x - 1)))/a^4

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate(x^3\*arccoth(a\*x), x)



**maple** [A] time = 0.03, size = 47, normalized size = 1.15

$$\frac{x^4 \operatorname{arccoth}(ax)}{4} + \frac{x^3}{12a} + \frac{x}{4a^3} + \frac{\ln(ax-1)}{8a^4} - \frac{\ln(ax+1)}{8a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arccoth(a*x),x)`

[Out] `1/4*x^4*arccoth(a*x)+1/12*x^3/a+1/4*x/a^3+1/8/a^4*ln(a*x-1)-1/8*ln(a*x+1)/a^4`

**maxima** [A] time = 0.30, size = 52, normalized size = 1.27

$$\frac{1}{4} x^4 \operatorname{arccoth}(ax) + \frac{1}{24} a \left( \frac{2(a^2 x^3 + 3x)}{a^4} - \frac{3 \log(ax+1)}{a^5} + \frac{3 \log(ax-1)}{a^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(a*x),x, algorithm="maxima")`

[Out] `1/4*x^4*arccoth(a*x) + 1/24*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*log(a*x + 1)/a^5 + 3*log(a*x - 1)/a^5)`

**mupad** [B] time = 1.22, size = 33, normalized size = 0.80

$$\frac{\frac{ax}{4} - \frac{\operatorname{acoth}(ax)}{4} + \frac{a^3 x^3}{12}}{a^4} + \frac{x^4 \operatorname{acoth}(ax)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*acoth(a*x),x)`

[Out] `((a*x)/4 - acoth(a*x)/4 + (a^3*x^3)/12)/a^4 + (x^4*acoth(a*x))/4`

**sympy** [A] time = 0.87, size = 41, normalized size = 1.00

$$\begin{cases} \frac{x^4 \operatorname{acoth}(ax)}{4} + \frac{x^3}{12a} + \frac{x}{4a^3} - \frac{\operatorname{acoth}(ax)}{4a^4} & \text{for } a \neq 0 \\ \frac{i\pi x^4}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(a*x),x)`

[Out] `Piecewise((x**4*acoth(a*x)/4 + x**3/(12*a) + x/(4*a**3) - acoth(a*x)/(4*a**4), Ne(a, 0)), (I*pi*x**4/8, True))`

### 3.4 $\int x^2 \coth^{-1}(ax) dx$

Optimal. Leaf size=40

$$\frac{\log(1 - a^2x^2)}{6a^3} + \frac{1}{3}x^3 \coth^{-1}(ax) + \frac{x^2}{6a}$$

[Out] 1/6\*x^2/a+1/3\*x^3\*arccoth(a\*x)+1/6\*ln(-a^2\*x^2+1)/a^3

**Rubi [A]** time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 266, 43}

$$\frac{\log(1 - a^2x^2)}{6a^3} + \frac{x^2}{6a} + \frac{1}{3}x^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[a\*x], x]

[Out] x^2/(6\*a) + (x^3\*ArcCoth[a\*x])/3 + Log[1 - a^2\*x^2]/(6\*a^3)

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(ax) dx &= \frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{3}a \int \frac{x^3}{1-a^2x^2} dx \\
&= \frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \operatorname{Subst}\left(\int \frac{x}{1-a^2x} dx, x, x^2\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(ax) - \frac{1}{6}a \operatorname{Subst}\left(\int \left(-\frac{1}{a^2} - \frac{1}{a^2(-1+a^2x)}\right) dx, x, x^2\right) \\
&= \frac{x^2}{6a} + \frac{1}{3}x^3 \coth^{-1}(ax) + \frac{\log(1-a^2x^2)}{6a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 1.00

$$\frac{\log(1-a^2x^2)}{6a^3} + \frac{1}{3}x^3 \coth^{-1}(ax) + \frac{x^2}{6a}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[a\*x], x]

[Out] x^2/(6\*a) + (x^3\*ArcCoth[a\*x])/3 + Log[1 - a^2\*x^2]/(6\*a^3)

**fricas [A]** time = 0.52, size = 44, normalized size = 1.10

$$\frac{a^3 x^3 \log\left(\frac{ax+1}{ax-1}\right) + a^2 x^2 + \log(a^2 x^2 - 1)}{6 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x), x, algorithm="fricas")

[Out] 1/6\*(a^3\*x^3\*log((a\*x + 1)/(a\*x - 1)) + a^2\*x^2 + log(a^2\*x^2 - 1))/a^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x), x, algorithm="giac")

[Out] integrate(x^2\*arccoth(a\*x), x)

**maple** [A] time = 0.03, size = 41, normalized size = 1.02

$$\frac{x^3 \operatorname{arccoth}(ax)}{3} + \frac{x^2}{6a} + \frac{\ln(ax-1)}{6a^3} + \frac{\ln(ax+1)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(a*x),x)`

[Out] `1/3*x^3*arccoth(a*x)+1/6*x^2/a+1/6/a^3*ln(a*x-1)+1/6/a^3*ln(a*x+1)`

**maxima** [A] time = 0.31, size = 35, normalized size = 0.88

$$\frac{1}{3} x^3 \operatorname{arccoth}(ax) + \frac{1}{6} a \left( \frac{x^2}{a^2} + \frac{\log(a^2 x^2 - 1)}{a^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(a*x),x, algorithm="maxima")`

[Out] `1/3*x^3*arccoth(a*x) + 1/6*a*(x^2/a^2 + log(a^2*x^2 - 1)/a^4)`

**mupad** [B] time = 1.22, size = 35, normalized size = 0.88

$$\frac{\frac{\ln(a^2 x^2 - 1)}{6} + \frac{a^2 x^2}{6}}{a^3} + \frac{x^3 \operatorname{acoth}(ax)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(a*x),x)`

[Out] `(log(a^2*x^2 - 1)/6 + (a^2*x^2)/6)/a^3 + (x^3*acoth(a*x))/3`

**sympy** [A] time = 0.67, size = 46, normalized size = 1.15

$$\begin{cases} \frac{x^3 \operatorname{acoth}(ax)}{3} + \frac{x^2}{6a} + \frac{\log(ax+1)}{3a^3} - \frac{\operatorname{acoth}(ax)}{3a^3} & \text{for } a \neq 0 \\ \frac{i\pi x^3}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(a*x),x)`

[Out] `Piecewise((x**3*acoth(a*x)/3 + x**2/(6*a) + log(a*x + 1)/(3*a**3) - acoth(a*x)/(3*a**3), Ne(a, 0)), (I*pi*x**3/6, True))`

### 3.5 $\int x \coth^{-1}(ax) dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}(ax)}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax) + \frac{x}{2a}$$

[Out]  $1/2*x/a+1/2*x^2*\operatorname{arccoth}(a*x)-1/2*\operatorname{arctanh}(a*x)/a^2$

**Rubi** [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5917, 321, 206}

$$-\frac{\tanh^{-1}(ax)}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax) + \frac{x}{2a}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[a*x],x]`

[Out]  $x/(2*a) + (x^2*ArcCoth[a*x])/2 - ArcTanh[a*x]/(2*a^2)$

#### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

#### Rule 321

`Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]`

#### Rule 5917

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]`

#### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(ax) dx &= \frac{1}{2}x^2 \coth^{-1}(ax) - \frac{1}{2}a \int \frac{x^2}{1-a^2x^2} dx \\
 &= \frac{x}{2a} + \frac{1}{2}x^2 \coth^{-1}(ax) - \frac{\int \frac{1}{1-a^2x^2} dx}{2a} \\
 &= \frac{x}{2a} + \frac{1}{2}x^2 \coth^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2a^2}
 \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 47, normalized size = 1.52

$$\frac{\log(1-ax)}{4a^2} - \frac{\log(ax+1)}{4a^2} + \frac{1}{2}x^2 \coth^{-1}(ax) + \frac{x}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[a\*x], x]

[Out] x/(2\*a) + (x^2\*ArcCoth[a\*x])/2 + Log[1 - a\*x]/(4\*a^2) - Log[1 + a\*x]/(4\*a^2)

**fricas** [A] time = 0.58, size = 34, normalized size = 1.10

$$\frac{2ax + (a^2x^2 - 1) \log\left(\frac{ax+1}{ax-1}\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x), x, algorithm="fricas")

[Out] 1/4\*(2\*a\*x + (a^2\*x^2 - 1)\*log((a\*x + 1)/(a\*x - 1)))/a^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x), x, algorithm="giac")

[Out] integrate(x\*arccoth(a\*x), x)

**maple** [A] time = 0.03, size = 39, normalized size = 1.26

$$\frac{x^2 \operatorname{arccoth}(ax)}{2} + \frac{x}{2a} + \frac{\ln(ax-1)}{4a^2} - \frac{\ln(ax+1)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(a*x),x)`

[Out]  $1/2*x^2*arccoth(a*x)+1/2*x/a+1/4/a^2*\ln(a*x-1)-1/4/a^2*\ln(a*x+1)$

**maxima** [A] time = 0.30, size = 41, normalized size = 1.32

$$\frac{1}{2}x^2 \operatorname{arccoth}(ax) + \frac{1}{4}a \left( \frac{2x}{a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(a*x),x, algorithm="maxima")`

[Out]  $1/2*x^2*arccoth(a*x) + 1/4*a*(2*x/a^2 - \log(a*x + 1)/a^3 + \log(a*x - 1)/a^3)$

**mupad** [B] time = 1.18, size = 26, normalized size = 0.84

$$\frac{x^2 \operatorname{acoth}(ax)}{2} - \frac{\operatorname{acoth}(ax) - \frac{ax}{2}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(a*x),x)`

[Out]  $(x^2*acoth(a*x))/2 - (acoth(a*x)/2 - (a*x)/2)/a^2$

**sympy** [A] time = 0.45, size = 32, normalized size = 1.03

$$\begin{cases} \frac{x^2 \operatorname{acoth}(ax)}{2} + \frac{x}{2a} - \frac{\operatorname{acoth}(ax)}{2a^2} & \text{for } a \neq 0 \\ \frac{i\pi x^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(a*x),x)`

[Out] `Piecewise((x**2*acoth(a*x)/2 + x/(2*a) - acoth(a*x)/(2*a**2), Ne(a, 0)), (I*pi*x**2/4, True))`

### 3.6 $\int \coth^{-1}(ax) dx$

Optimal. Leaf size=25

$$\frac{\log(1 - a^2x^2)}{2a} + x \coth^{-1}(ax)$$

[Out] x\*arccoth(a\*x)+1/2\*ln(-a^2\*x^2+1)/a

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5911, 260}

$$\frac{\log(1 - a^2x^2)}{2a} + x \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x], x]

[Out] x\*ArcCoth[a\*x] + Log[1 - a^2\*x^2]/(2\*a)

Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x]))^(p - 1)]/(1 - c^2\*x^2), x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \coth^{-1}(ax) dx &= x \coth^{-1}(ax) - a \int \frac{x}{1 - a^2x^2} dx \\ &= x \coth^{-1}(ax) + \frac{\log(1 - a^2x^2)}{2a} \end{aligned}$$

**Mathematica [A]** time = 0.00, size = 25, normalized size = 1.00

$$\frac{\log(1 - a^2x^2)}{2a} + x \coth^{-1}(ax)$$



Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x], x]

[Out] x\*ArcCoth[a\*x] + Log[1 - a^2\*x^2]/(2\*a)

**fricas** [A] time = 0.61, size = 33, normalized size = 1.32

$$\frac{ax \log\left(\frac{ax+1}{ax-1}\right) + \log(a^2x^2 - 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x), x, algorithm="fricas")

[Out] 1/2\*(a\*x\*log((a\*x + 1)/(a\*x - 1)) + log(a^2\*x^2 - 1))/a

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x), x, algorithm="giac")

[Out] integrate(arccoth(a\*x), x)

**maple** [A] time = 0.04, size = 23, normalized size = 0.92

$$x \operatorname{arccoth}(ax) + \frac{\ln(a^2x^2 - 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x), x)

[Out] x\*arccoth(a\*x)+1/2/a\*ln(a^2\*x^2-1)

**maxima** [A] time = 0.30, size = 25, normalized size = 1.00

$$\frac{2ax \operatorname{arccoth}(ax) + \log(-a^2x^2 + 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x), x, algorithm="maxima")

[Out]  $1/2*(2*a*x*\operatorname{arccoth}(a*x) + \log(-a^2*x^2 + 1))/a$

**mupad** [B] time = 1.15, size = 22, normalized size = 0.88

$$x \operatorname{acoth}(ax) + \frac{\ln(a^2 x^2 - 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x),x)`

[Out]  $x*\operatorname{acoth}(a*x) + \log(a^2*x^2 - 1)/(2*a)$

**sympy** [A] time = 0.33, size = 27, normalized size = 1.08

$$\begin{cases} x \operatorname{acoth}(ax) + \frac{\log(ax+1)}{a} - \frac{\operatorname{acoth}(ax)}{a} & \text{for } a \neq 0 \\ \frac{i\pi x}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x),x)`

[Out] `Piecewise((x*acoth(a*x) + log(a*x + 1)/a - acoth(a*x)/a, Ne(a, 0)), (I*pi*x/2, True))`

$$3.7 \quad \int \frac{\coth^{-1}(ax)}{x} dx$$

Optimal. Leaf size=28

$$\frac{1}{2}\text{Li}_2\left(-\frac{1}{ax}\right) - \frac{1}{2}\text{Li}_2\left(\frac{1}{ax}\right)$$

[Out] 1/2\*polylog(2,-1/a/x)-1/2\*polylog(2,1/a/x)

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {5913}

$$\frac{1}{2}\text{PolyLog}\left(2, -\frac{1}{ax}\right) - \frac{1}{2}\text{PolyLog}\left(2, \frac{1}{ax}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/x,x]

[Out] PolyLog[2, -(1/(a\*x))]/2 - PolyLog[2, 1/(a\*x)]/2

Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])]/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])]/2, x) /; FreeQ[{a, b, c}, x]

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{x} dx = \frac{1}{2}\text{Li}_2\left(-\frac{1}{ax}\right) - \frac{1}{2}\text{Li}_2\left(\frac{1}{ax}\right)$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$\frac{1}{2}\left(\text{Li}_2\left(-\frac{1}{ax}\right) - \text{Li}_2\left(\frac{1}{ax}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x,x]

[Out] (PolyLog[2, -(1/(a\*x))] - PolyLog[2, 1/(a\*x)])/2

**fricas** [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arccoth}(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/x, x)

**maple** [A] time = 0.05, size = 37, normalized size = 1.32

$$\ln(ax) \operatorname{arccoth}(ax) - \frac{\operatorname{dilog}(ax)}{2} - \frac{\operatorname{dilog}(ax+1)}{2} - \frac{\ln(ax) \ln(ax+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/x,x)

[Out] ln(a\*x)\*arccoth(a\*x)-1/2\*dilog(a\*x)-1/2\*dilog(a\*x+1)-1/2\*ln(a\*x)\*ln(a\*x+1)

**maxima** [B] time = 0.30, size = 86, normalized size = 3.07

$$-\frac{1}{2}a\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\log(x) - \frac{1}{2}a\left(\frac{\log(ax-1)\log(ax) + \operatorname{Li}_2(-ax+1)}{a} - \frac{\log(ax+1)\log(-ax) + \operatorname{Li}_2(ax+1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x,x, algorithm="maxima")

[Out] -1/2\*a\*(log(a\*x + 1)/a - log(a\*x - 1)/a)\*log(x) - 1/2\*a\*((log(a\*x - 1)\*log(a\*x) + dilog(-a\*x + 1))/a - (log(a\*x + 1)\*log(-a\*x) + dilog(a\*x + 1))/a) + arccoth(a\*x)\*log(x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/x,x)
```

```
[Out] int(acoth(a*x)/x, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\operatorname{acoth}(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/x,x)
```

```
[Out] Integral(acoth(a*x)/x, x)
```

### 3.8 $\int \frac{\coth^{-1}(ax)}{x^2} dx$

Optimal. Leaf size=30

$$-\frac{1}{2}a \log(1 - a^2x^2) + a \log(x) - \frac{\coth^{-1}(ax)}{x}$$

[Out]  $-\operatorname{arccoth}(a*x)/x+a*\ln(x)-1/2*a*\ln(-a^2*x^2+1)$

**Rubi [A]** time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {5917, 266, 36, 29, 31}

$$-\frac{1}{2}a \log(1 - a^2x^2) + a \log(x) - \frac{\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[a*x]/x^2, x]$

[Out]  $-(\operatorname{ArcCoth}[a*x]/x) + a*\operatorname{Log}[x] - (a*\operatorname{Log}[1 - a^2*x^2])/2$

Rule 29

$\operatorname{Int}[(x_)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[x], x]$

Rule 31

$\operatorname{Int}[((a_) + (b_)*(x_))^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

Rule 36

$\operatorname{Int}[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x\_Symbol] \rightarrow \operatorname{Dist}[b/(b*c - a*d), \operatorname{Int}[1/(a + b*x), x], x] - \operatorname{Dist}[d/(b*c - a*d), \operatorname{Int}[1/(c + d*x), x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0]$

Rule 266

$\operatorname{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, m, n, p\}, x] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m + 1)/n]]$

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax)}{x^2} dx &= -\frac{\coth^{-1}(ax)}{x} + a \int \frac{1}{x(1-a^2x^2)} dx \\ &= -\frac{\coth^{-1}(ax)}{x} + \frac{1}{2}a \operatorname{Subst}\left(\int \frac{1}{x(1-a^2x)} dx, x, x^2\right) \\ &= -\frac{\coth^{-1}(ax)}{x} + \frac{1}{2}a \operatorname{Subst}\left(\int \frac{1}{x} dx, x, x^2\right) + \frac{1}{2}a^3 \operatorname{Subst}\left(\int \frac{1}{1-a^2x} dx, x, x^2\right) \\ &= -\frac{\coth^{-1}(ax)}{x} + a \log(x) - \frac{1}{2}a \log(1-a^2x^2) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 30, normalized size = 1.00

$$-\frac{1}{2}a \log(1-a^2x^2) + a \log(x) - \frac{\coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x^2,x]

[Out] -(ArcCoth[a\*x]/x) + a\*Log[x] - (a\*Log[1 - a^2\*x^2])/2

**fricas [A]** time = 0.45, size = 39, normalized size = 1.30

$$\frac{ax \log(a^2x^2 - 1) - 2ax \log(x) + \log\left(\frac{ax+1}{ax-1}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^2,x, algorithm="fricas")

[Out] -1/2\*(a\*x\*log(a^2\*x^2 - 1) - 2\*a\*x\*log(x) + log((a\*x + 1)/(a\*x - 1)))/x

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^2,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/x^2, x)

maple [A] time = 0.04, size = 35, normalized size = 1.17

$$-\frac{\operatorname{arccoth}(ax)}{x} + a \ln(ax) - \frac{a \ln(ax-1)}{2} - \frac{a \ln(ax+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/x^2,x)

[Out] -arccoth(a\*x)/x+a\*ln(a\*x)-1/2\*a\*ln(a\*x-1)-1/2\*a\*ln(a\*x+1)

maxima [A] time = 0.31, size = 30, normalized size = 1.00

$$-\frac{1}{2} a (\log(a^2 x^2 - 1) - \log(x^2)) - \frac{\operatorname{arccoth}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^2,x, algorithm="maxima")

[Out] -1/2\*a\*(log(a^2\*x^2 - 1) - log(x^2)) - arccoth(a\*x)/x

mupad [B] time = 1.16, size = 27, normalized size = 0.90

$$a \ln(x) - \frac{a \ln(a^2 x^2 - 1)}{2} - \frac{\operatorname{acoth}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)/x^2,x)

[Out] a\*log(x) - (a\*log(a^2\*x^2 - 1))/2 - acoth(a\*x)/x

sympy [A] time = 0.37, size = 26, normalized size = 0.87

$$a \log(x) - a \log(ax+1) + a \operatorname{acoth}(ax) - \frac{\operatorname{acoth}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/x\*\*2,x)

[Out] a\*log(x) - a\*log(a\*x + 1) + a\*acoth(a\*x) - acoth(a\*x)/x



### 3.9 $\int \frac{\coth^{-1}(ax)}{x^3} dx$

Optimal. Leaf size=31

$$\frac{1}{2}a^2 \tanh^{-1}(ax) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

[Out]  $-1/2*a/x - 1/2*\operatorname{arccoth}(a*x)/x^2 + 1/2*a^2*\operatorname{arctanh}(a*x)$

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 325, 206}

$$\frac{1}{2}a^2 \tanh^{-1}(ax) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[a*x]/x^3, x]`

[Out]  $-a/(2*x) - \operatorname{ArcCoth}[a*x]/(2*x^2) + (a^2*\operatorname{ArcTanh}[a*x])/2$

#### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

#### Rule 325

`Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]`

#### Rule 5917

`Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]`

#### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax)}{x^3} dx &= -\frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a \int \frac{1}{x^2(1-a^2x^2)} dx \\ &= -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^3 \int \frac{1}{1-a^2x^2} dx \\ &= -\frac{a}{2x} - \frac{\coth^{-1}(ax)}{2x^2} + \frac{1}{2}a^2 \tanh^{-1}(ax) \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 47, normalized size = 1.52

$$-\frac{1}{4}a^2 \log(1-ax) + \frac{1}{4}a^2 \log(ax+1) - \frac{\coth^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x^3,x]

[Out] -1/2\*a/x - ArcCoth[a\*x]/(2\*x^2) - (a^2\*Log[1 - a\*x])/4 + (a^2\*Log[1 + a\*x])/4

**fricas** [A] time = 0.52, size = 35, normalized size = 1.13

$$-\frac{2ax - (a^2x^2 - 1) \log\left(\frac{ax+1}{ax-1}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*a\*x - (a^2\*x^2 - 1)\*log((a\*x + 1)/(a\*x - 1)))/x^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^3,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/x^3, x)

**maple** [A] time = 0.04, size = 39, normalized size = 1.26

$$-\frac{\operatorname{arccoth}(ax)}{2x^2} - \frac{a}{2x} - \frac{a^2 \ln(ax-1)}{4} + \frac{a^2 \ln(ax+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(a*x)/x^3,x)`

[Out]  $-1/2*\operatorname{arccoth}(a*x)/x^2-1/2*a/x-1/4*a^2*\ln(a*x-1)+1/4*a^2*\ln(a*x+1)$

**maxima** [A] time = 0.31, size = 36, normalized size = 1.16

$$\frac{1}{4} \left( a \log(ax + 1) - a \log(ax - 1) - \frac{2}{x} \right) a - \frac{\operatorname{arccoth}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)/x^3,x, algorithm="maxima")`

[Out]  $1/4*(a*\log(a*x + 1) - a*\log(a*x - 1) - 2/x)*a - 1/2*\operatorname{arccoth}(a*x)/x^2$

**mupad** [B] time = 1.19, size = 40, normalized size = 1.29

$$\frac{a \operatorname{atan}\left(\frac{a^2 x}{\sqrt{-a^2}}\right) \sqrt{-a^2}}{2} - \frac{\operatorname{acoth}(ax)}{x^2} + \frac{ax}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)/x^3,x)`

[Out]  $(a*\operatorname{atan}((a^2*x)/(-a^2)^{(1/2)})*(-a^2)^{(1/2)})/2 - (\operatorname{acoth}(a*x)/2 + (a*x)/2)/x^2$

**sympy** [A] time = 0.54, size = 24, normalized size = 0.77

$$\frac{a^2 \operatorname{acoth}(ax)}{2} - \frac{a}{2x} - \frac{\operatorname{acoth}(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)/x**3,x)`

[Out]  $a**2*\operatorname{acoth}(a*x)/2 - a/(2*x) - \operatorname{acoth}(a*x)/(2*x**2)$

### 3.10 $\int \frac{\coth^{-1}(ax)}{x^4} dx$

Optimal. Leaf size=47

$$\frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log(1 - a^2x^2) - \frac{\coth^{-1}(ax)}{3x^3} - \frac{a}{6x^2}$$

[Out]  $-1/6*a/x^2-1/3*\operatorname{arccoth}(a*x)/x^3+1/3*a^3*\ln(x)-1/6*a^3*\ln(-a^2*x^2+1)$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 266, 44}

$$-\frac{1}{6}a^3 \log(1 - a^2x^2) + \frac{1}{3}a^3 \log(x) - \frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/x^4,x]

[Out]  $-a/(6*x^2) - \operatorname{ArcCoth}[a*x]/(3*x^3) + (a^3*\operatorname{Log}[x])/3 - (a^3*\operatorname{Log}[1 - a^2*x^2])/6$

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & & NeQ[b\*c - a\*d, 0] & & ILtQ[m, 0] & & IntegerQ[n] & & !(IGtQ[n, 0] & & LtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] & & IntegerQ[Simplify[(m + 1)/n]]

#### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] & & IGtQ[p, 0] & & (EqQ[p, 1] || IntegerQ[m]) & & NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{x^4} dx &= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a \int \frac{1}{x^3(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{6}a \operatorname{Subst} \left( \int \frac{1}{x^2(1-a^2x)} dx, x, x^2 \right) \\
&= -\frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{6}a \operatorname{Subst} \left( \int \left( \frac{1}{x^2} + \frac{a^2}{x} - \frac{a^4}{-1+a^2x} \right) dx, x, x^2 \right) \\
&= -\frac{a}{6x^2} - \frac{\coth^{-1}(ax)}{3x^3} + \frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log(1-a^2x^2)
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 47, normalized size = 1.00

$$\frac{1}{3}a^3 \log(x) - \frac{1}{6}a^3 \log(1-a^2x^2) - \frac{\coth^{-1}(ax)}{3x^3} - \frac{a}{6x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x^4,x]

[Out] -1/6\*a/x^2 - ArcCoth[a\*x]/(3\*x^3) + (a^3\*Log[x])/3 - (a^3\*Log[1 - a^2\*x^2])/6

**fricas** [A] time = 1.04, size = 50, normalized size = 1.06

$$-\frac{a^3x^3 \log(a^2x^2 - 1) - 2a^3x^3 \log(x) + ax + \log\left(\frac{ax+1}{ax-1}\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^4,x, algorithm="fricas")

[Out] -1/6\*(a^3\*x^3\*log(a^2\*x^2 - 1) - 2\*a^3\*x^3\*log(x) + a\*x + log((a\*x + 1)/(a\*x - 1)))/x^3

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^4,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/x^4, x)

**maple** [A] time = 0.04, size = 48, normalized size = 1.02

$$-\frac{\operatorname{arccoth}(ax)}{3x^3} - \frac{a}{6x^2} + \frac{a^3 \ln(ax)}{3} - \frac{a^3 \ln(ax-1)}{6} - \frac{a^3 \ln(ax+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/x^4,x)

[Out] -1/3\*arccoth(a\*x)/x^3-1/6\*a/x^2+1/3\*a^3\*ln(a\*x)-1/6\*a^3\*ln(a\*x-1)-1/6\*a^3\*ln(a\*x+1)

**maxima** [A] time = 0.31, size = 40, normalized size = 0.85

$$-\frac{1}{6} \left( a^2 \log(a^2 x^2 - 1) - a^2 \log(x^2) + \frac{1}{x^2} \right) a - \frac{\operatorname{arccoth}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^4,x, algorithm="maxima")

[Out] -1/6\*(a^2\*log(a^2\*x^2 - 1) - a^2\*log(x^2) + 1/x^2)\*a - 1/3\*arccoth(a\*x)/x^3

**mupad** [B] time = 1.18, size = 39, normalized size = 0.83

$$\frac{a^3 \ln(x)}{3} - \frac{\operatorname{acoth}(ax)}{3} + \frac{ax}{6} - \frac{a^3 \ln(a^2 x^2 - 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)/x^4,x)

[Out] (a^3\*log(x))/3 - (acoth(a\*x)/3 + (a\*x)/6)/x^3 - (a^3\*log(a^2\*x^2 - 1))/6

**sympy** [A] time = 0.74, size = 46, normalized size = 0.98

$$\frac{a^3 \log(x)}{3} - \frac{a^3 \log(ax+1)}{3} + \frac{a^3 \operatorname{acoth}(ax)}{3} - \frac{a}{6x^2} - \frac{\operatorname{acoth}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/x\*\*4,x)

[Out] a\*\*3\*log(x)/3 - a\*\*3\*log(a\*x + 1)/3 + a\*\*3\*acoth(a\*x)/3 - a/(6\*x\*\*2) - acot  
h(a\*x)/(3\*x\*\*3)

### 3.11 $\int \frac{\coth^{-1}(ax)}{x^5} dx$

Optimal. Leaf size=41

$$\frac{1}{4}a^4 \tanh^{-1}(ax) - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} - \frac{a}{12x^3}$$

[Out]  $-1/12*a/x^3-1/4*a^3/x-1/4*\operatorname{arccoth}(a*x)/x^4+1/4*a^4*\operatorname{arctanh}(a*x)$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5917, 325, 206}

$$-\frac{a^3}{4x} + \frac{1}{4}a^4 \tanh^{-1}(ax) - \frac{a}{12x^3} - \frac{\coth^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/x^5, x]

[Out]  $-a/(12*x^3) - a^3/(4*x) - \operatorname{ArcCoth}[a*x]/(4*x^4) + (a^4*\operatorname{ArcTanh}[a*x])/4$

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 325

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 5917

Int[((a\_) + ArcCoth[(c\_)\*(x\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a+b\*ArcCoth[c\*x])^p)/(d\*(m+1)), x] - Dist[(b\*c\*p)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a+b\*ArcCoth[c\*x])^(p-1))/(1-c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{x^5} dx &= -\frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a \int \frac{1}{x^4(1-a^2x^2)} dx \\
&= -\frac{a}{12x^3} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^3 \int \frac{1}{x^2(1-a^2x^2)} dx \\
&= -\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^5 \int \frac{1}{1-a^2x^2} dx \\
&= -\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} + \frac{1}{4}a^4 \tanh^{-1}(ax)
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 57, normalized size = 1.39

$$-\frac{1}{8}a^4 \log(1-ax) + \frac{1}{8}a^4 \log(ax+1) - \frac{a^3}{4x} - \frac{\coth^{-1}(ax)}{4x^4} - \frac{a}{12x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/x^5,x]

[Out]  $-\frac{1}{12}a/x^3 - a^3/(4*x) - \text{ArcCoth}[a*x]/(4*x^4) - (a^4*\text{Log}[1 - a*x])/8 + (a^4*\text{Log}[1 + a*x])/8$

**fricas** [A] time = 0.52, size = 43, normalized size = 1.05

$$\frac{6a^3x^3 + 2ax - 3(a^4x^4 - 1)\log\left(\frac{ax+1}{ax-1}\right)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^5,x, algorithm="fricas")

[Out]  $-1/24*(6*a^3*x^3 + 2*a*x - 3*(a^4*x^4 - 1)*\log((a*x + 1)/(a*x - 1)))/x^4$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^5,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/x^5, x)



**maple [A]** time = 0.04, size = 47, normalized size = 1.15

$$-\frac{\operatorname{arccoth}(ax)}{4x^4} - \frac{a}{12x^3} - \frac{a^3}{4x} - \frac{a^4 \ln(ax-1)}{8} + \frac{a^4 \ln(ax+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/x^5,x)

[Out] -1/4\*arccoth(a\*x)/x^4-1/12\*a/x^3-1/4\*a^3/x-1/8\*a^4\*ln(a\*x-1)+1/8\*a^4\*ln(a\*x+1)

**maxima [A]** time = 0.31, size = 51, normalized size = 1.24

$$\frac{1}{24} \left( 3a^3 \log(ax+1) - 3a^3 \log(ax-1) - \frac{2(3a^2x^2+1)}{x^3} \right) a - \frac{\operatorname{arccoth}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/x^5,x, algorithm="maxima")

[Out] 1/24\*(3\*a^3\*log(a\*x + 1) - 3\*a^3\*log(a\*x - 1) - 2\*(3\*a^2\*x^2 + 1)/x^3)\*a - 1/4\*arccoth(a\*x)/x^4

**mupad [B]** time = 1.61, size = 60, normalized size = 1.46

$$\frac{\ln\left(1 - \frac{1}{ax}\right)}{8x^4} - \frac{\ln\left(\frac{1}{ax} + 1\right)}{8x^4} - \frac{a^3x^2 + \frac{a}{3}}{4x^3} - \frac{a^4 \operatorname{atan}(ax \operatorname{li} 1) \operatorname{li} 1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoath(a\*x)/x^5,x)

[Out] log(1 - 1/(a\*x))/(8\*x^4) - (a^4\*atan(a\*x\*1i)\*1i)/4 - log(1/(a\*x) + 1)/(8\*x^4) - (a/3 + a^3\*x^2)/(4\*x^3)

**sympy [A]** time = 1.16, size = 32, normalized size = 0.78

$$\frac{a^4 \operatorname{acoath}(ax)}{4} - \frac{a^3}{4x} - \frac{a}{12x^3} - \frac{\operatorname{acoath}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoath(a\*x)/x\*\*5,x)

[Out] a\*\*4\*acoath(a\*x)/4 - a\*\*3/(4\*x) - a/(12\*x\*\*3) - acoath(a\*x)/(4\*x\*\*4)

### 3.12 $\int x^5 \coth^{-1}(ax)^2 dx$

Optimal. Leaf size=105

$$-\frac{\coth^{-1}(ax)^2}{6a^6} + \frac{x \coth^{-1}(ax)}{3a^5} + \frac{4x^2}{45a^4} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^4}{60a^2} + \frac{23 \log(1 - a^2x^2)}{90a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 + \frac{x^5 \coth^{-1}(ax)}{15a}$$

[Out]  $4/45*x^2/a^4 + 1/60*x^4/a^2 + 1/3*x*\operatorname{arccoth}(a*x)/a^5 + 1/9*x^3*\operatorname{arccoth}(a*x)/a^3 + 1/15*x^5*\operatorname{arccoth}(a*x)/a - 1/6*\operatorname{arccoth}(a*x)^2/a^6 + 1/6*x^6*\operatorname{arccoth}(a*x)^2 + 23/90*\ln(-a^2*x^2+1)/a^6$

**Rubi [A]** time = 0.25, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {5917, 5981, 266, 43, 5911, 260, 5949}

$$\frac{x^4}{60a^2} + \frac{4x^2}{45a^4} + \frac{23 \log(1 - a^2x^2)}{90a^6} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x \coth^{-1}(ax)}{3a^5} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 + \frac{x^5 \coth^{-1}(ax)}{15a}$$

Antiderivative was successfully verified.

[In] Int[x^5\*ArcCoth[a\*x]^2, x]

[Out]  $(4*x^2)/(45*a^4) + x^4/(60*a^2) + (x*\operatorname{ArcCoth}[a*x])/(3*a^5) + (x^3*\operatorname{ArcCoth}[a*x])/(9*a^3) + (x^5*\operatorname{ArcCoth}[a*x])/(15*a) - \operatorname{ArcCoth}[a*x]^2/(6*a^6) + (x^6*\operatorname{ArcCoth}[a*x]^2)/6 + (23*\operatorname{Log}[1 - a^2*x^2])/(90*a^6)$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

### Rubi steps

$$\begin{aligned}
\int x^5 \coth^{-1}(ax)^2 dx &= \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{3}a \int \frac{x^6 \coth^{-1}(ax)}{1 - a^2x^2} dx \\
&= \frac{1}{6}x^6 \coth^{-1}(ax)^2 + \frac{\int x^4 \coth^{-1}(ax) dx}{3a} - \frac{\int \frac{x^4 \coth^{-1}(ax)}{1 - a^2x^2} dx}{3a} \\
&= \frac{x^5 \coth^{-1}(ax)}{15a} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{15} \int \frac{x^5}{1 - a^2x^2} dx + \frac{\int x^2 \coth^{-1}(ax) dx}{3a^3} - \frac{\int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx}{3a^3} \\
&= \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{30} \text{Subst} \left( \int \frac{x^2}{1 - a^2x} dx, x, x^2 \right) + \int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx \\
&= \frac{x \coth^{-1}(ax)}{3a^5} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{30} \text{Subst} \left( \int \frac{x^2}{1 - a^2x} dx, x, x^2 \right) + \int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx \\
&= \frac{x^2}{30a^4} + \frac{x^4}{60a^2} + \frac{x \coth^{-1}(ax)}{3a^5} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{30} \text{Subst} \left( \int \frac{x^2}{1 - a^2x} dx, x, x^2 \right) + \int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx \\
&= \frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x \coth^{-1}(ax)}{3a^5} + \frac{x^3 \coth^{-1}(ax)}{9a^3} + \frac{x^5 \coth^{-1}(ax)}{15a} - \frac{\coth^{-1}(ax)^2}{6a^6} + \frac{1}{6}x^6 \coth^{-1}(ax)^2 - \frac{1}{30} \text{Subst} \left( \int \frac{x^2}{1 - a^2x} dx, x, x^2 \right) + \int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 80, normalized size = 0.76

$$\frac{30(a^6x^6 - 1) \coth^{-1}(ax)^2 + 3a^4x^4 + 16a^2x^2 + 46 \log(1 - a^2x^2) + 4ax(3a^4x^4 + 5a^2x^2 + 15) \coth^{-1}(ax)}{180a^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*ArcCoth[a\*x]^2,x]

[Out] (16\*a^2\*x^2 + 3\*a^4\*x^4 + 4\*a\*x\*(15 + 5\*a^2\*x^2 + 3\*a^4\*x^4)\*ArcCoth[a\*x] + 30\*(-1 + a^6\*x^6)\*ArcCoth[a\*x]^2 + 46\*Log[1 - a^2\*x^2])/(180\*a^6)

**fricas [A]** time = 0.53, size = 98, normalized size = 0.93

$$\frac{6a^4x^4 + 32a^2x^2 + 15(a^6x^6 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2 + 4(3a^5x^5 + 5a^3x^3 + 15ax) \log\left(\frac{ax+1}{ax-1}\right) + 92 \log(a^2x^2 - 1)}{360a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*arccoth(a\*x)^2,x, algorithm="fricas")

[Out]  $\frac{1}{360}(6a^4x^4 + 32a^2x^2 + 15(a^6x^6 - 1))\log\left(\frac{ax+1}{ax-1}\right)^2 + 4(3a^5x^5 + 5a^3x^3 + 15ax)\log\left(\frac{ax+1}{ax-1}\right) + 92\log(a^2x^2 - 1)/a^6$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arccoth(a*x)^2,x, algorithm="giac")`

[Out] `integrate(x^5*arccoth(a*x)^2, x)`

**maple** [B] time = 0.06, size = 196, normalized size = 1.87

$$\frac{x^6 \operatorname{arccoth}(ax)^2}{6} + \frac{x^5 \operatorname{arccoth}(ax)}{15a} + \frac{x^3 \operatorname{arccoth}(ax)}{9a^3} + \frac{x \operatorname{arccoth}(ax)}{3a^5} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{6a^6} - \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{6a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*arccoth(a*x)^2,x)`

[Out]  $\frac{1}{6}x^6 \operatorname{arccoth}(ax)^2 + \frac{1}{15}x^5 \operatorname{arccoth}(ax)/a + \frac{1}{9}x^3 \operatorname{arccoth}(ax)/a^3 + \frac{1}{3}x \operatorname{arccoth}(ax)/a^5 + \frac{1}{6} \operatorname{arccoth}(ax) \ln(ax-1) - \frac{1}{6} \operatorname{arccoth}(ax) \ln(ax+1) + \frac{1}{24} \ln^2(ax-1) - \frac{1}{12} \ln(ax-1) \ln\left(\frac{1}{2} + \frac{1}{2}ax\right) + \frac{1}{24} \ln^2(ax+1) - \frac{1}{12} \ln(ax+1) \ln\left(-\frac{1}{2}ax + \frac{1}{2}\right) + \frac{1}{12} \ln\left(-\frac{1}{2}ax + \frac{1}{2}\right) \ln\left(\frac{1}{2} + \frac{1}{2}ax\right) + \frac{1}{60}x^4/a^2 + \frac{4}{45}x^2/a^4 + \frac{23}{90} \operatorname{arccoth}(ax) \ln(ax-1) + \frac{23}{90} \operatorname{arccoth}(ax) \ln(ax+1)/a^6$

**maxima** [A] time = 0.31, size = 135, normalized size = 1.29

$$\frac{1}{6}x^6 \operatorname{arccoth}(ax)^2 + \frac{1}{90}a \left( \frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right) \operatorname{arccoth}(ax) + \frac{6a^4x^4 + 3a^2x^2 - 2(15 \log(ax-1) - 46) \log(ax+1) + 15 \log(ax+1)^2 + 15 \log(ax-1)^2 + 92 \log(ax-1)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*arccoth(a*x)^2,x, algorithm="maxima")`

[Out]  $\frac{1}{6}x^6 \operatorname{arccoth}(ax)^2 + \frac{1}{90}a(2(3a^4x^5 + 5a^2x^3 + 15x)/a^6 - 15 \log(ax+1)/a^7 + 15 \log(ax-1)/a^7) \operatorname{arccoth}(ax) + \frac{1}{360}(6a^4x^4 + 32a^2x^2 - 2(15 \log(ax-1) - 46) \log(ax+1) + 15 \log(ax+1)^2 + 15 \log(ax-1)^2 + 92 \log(ax-1))/a^6$

**mupad [B]** time = 1.36, size = 85, normalized size = 0.81

$$\frac{x^6 \operatorname{acoth}(ax)^2}{6} + \frac{23 \ln(a^2 x^2 - 1)}{90} + \frac{4a^2 x^2}{45} + \frac{a^4 x^4}{60} - \frac{\operatorname{acoth}(ax)^2}{6} + \frac{a^3 x^3 \operatorname{acoth}(ax)}{9} + \frac{a^5 x^5 \operatorname{acoth}(ax)}{15} + \frac{ax \operatorname{acoth}(ax)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*acoth(a*x)^2,x)`

[Out]  $(x^6 \operatorname{acoth}(ax)^2)/6 + ((23 \log(a^2 x^2 - 1))/90 + (4a^2 x^2)/45 + (a^4 x^4)/60 - \operatorname{acoth}(ax)^2/6 + (a^3 x^3 \operatorname{acoth}(ax))/9 + (a^5 x^5 \operatorname{acoth}(ax))/15 + (ax \operatorname{acoth}(ax))/3)/a^6$

**sympy [A]** time = 3.31, size = 114, normalized size = 1.09

$$\begin{cases} \frac{x^6 \operatorname{acoth}^2(ax)}{6} + \frac{x^5 \operatorname{acoth}(ax)}{15a} + \frac{x^4}{60a^2} + \frac{x^3 \operatorname{acoth}(ax)}{9a^3} + \frac{4x^2}{45a^4} + \frac{x \operatorname{acoth}(ax)}{3a^5} + \frac{23 \log(ax+1)}{45a^6} - \frac{\operatorname{acoth}^2(ax)}{6a^6} - \frac{23 \operatorname{acoth}(ax)}{45a^6} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^6}{24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*acoth(a*x)**2,x)`

[Out] `Piecewise((x**6*acoth(a*x)**2/6 + x**5*acoth(a*x)/(15*a) + x**4/(60*a**2) + x**3*acoth(a*x)/(9*a**3) + 4*x**2/(45*a**4) + x*acoth(a*x)/(3*a**5) + 23*log(a*x + 1)/(45*a**6) - acoth(a*x)**2/(6*a**6) - 23*acoth(a*x)/(45*a**6), Ne(a, 0)), (-pi**2*x**6/24, True))`

### 3.13 $\int x^4 \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=127

$$-\frac{\operatorname{Li}_2\left(1 - \frac{2}{1-ax}\right)}{5a^5} - \frac{3 \tanh^{-1}(ax)}{10a^5} + \frac{\coth^{-1}(ax)^2}{5a^5} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{5a^5} + \frac{3x}{10a^4} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^3}{30a^2} + \frac{1}{5} x^5 \coth^{-1}(ax)$$

[Out] 3/10\*x/a^4+1/30\*x^3/a^2+1/5\*x^2\*arccoth(a\*x)/a^3+1/10\*x^4\*arccoth(a\*x)/a+1/5\*arccoth(a\*x)^2/a^5+1/5\*x^5\*arccoth(a\*x)^2-3/10\*arctanh(a\*x)/a^5-2/5\*arccoth(a\*x)\*ln(2/(-a\*x+1))/a^5-1/5\*polylog(2,1-2/(-a\*x+1))/a^5

**Rubi [A]** time = 0.23, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$ , Rules used = {5917, 5981, 302, 206, 321, 5985, 5919, 2402, 2315}

$$-\frac{\operatorname{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{5a^5} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{3x}{10a^4} - \frac{3 \tanh^{-1}(ax)}{10a^5} + \frac{\coth^{-1}(ax)^2}{5a^5} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{5a^5} + \frac{1}{5} x^5 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[x^4\*ArcCoth[a\*x]^2,x]

[Out] (3\*x)/(10\*a^4) + x^3/(30\*a^2) + (x^2\*ArcCoth[a\*x])/(5\*a^3) + (x^4\*ArcCoth[a\*x])/(10\*a) + ArcCoth[a\*x]^2/(5\*a^5) + (x^5\*ArcCoth[a\*x]^2)/5 - (3\*ArcTanh[a\*x])/(10\*a^5) - (2\*ArcCoth[a\*x]\*Log[2/(1 - a\*x)])/(5\*a^5) - PolyLog[2, 1 - 2/(1 - a\*x)]/(5\*a^5)

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p]

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\_.\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\_/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5981

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\_.\*((f\_.)\*(x\_))^(m\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\_.\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]



Rubi steps

$$\begin{aligned}
\int x^4 \coth^{-1}(ax)^2 dx &= \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{5}(2a) \int \frac{x^5 \coth^{-1}(ax)}{1-a^2x^2} dx \\
&= \frac{1}{5}x^5 \coth^{-1}(ax)^2 + \frac{2 \int x^3 \coth^{-1}(ax) dx}{5a} - \frac{2 \int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2} dx}{5a} \\
&= \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{10} \int \frac{x^4}{1-a^2x^2} dx + \frac{2 \int x \coth^{-1}(ax) dx}{5a^3} - \frac{2 \int \frac{x \coth^{-1}(ax)}{1-a^2x^2} dx}{5a} \\
&= \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{1}{10} \int \left( -\frac{1}{a^4} - \frac{x^2}{a^2} + \right. \\
&= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax)}{5a} \\
&= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{3 \coth^{-1}(ax)}{5a} \\
&= \frac{3x}{10a^4} + \frac{x^3}{30a^2} + \frac{x^2 \coth^{-1}(ax)}{5a^3} + \frac{x^4 \coth^{-1}(ax)}{10a} + \frac{\coth^{-1}(ax)^2}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^2 - \frac{3 \coth^{-1}(ax)}{5a}
\end{aligned}$$

**Mathematica [A]** time = 0.45, size = 87, normalized size = 0.69

$$\frac{6(a^5x^5 - 1) \coth^{-1}(ax)^2 + ax(a^2x^2 + 9) + 3 \coth^{-1}(ax) \left( a^4x^4 + 2a^2x^2 - 4 \log(1 - e^{-2 \coth^{-1}(ax)}) - 3 \right) + 6 \text{Li}_2 \left( e^{-2 \coth^{-1}(ax)} \right)}{30a^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*ArcCoth[a\*x]^2,x]

[Out] (a\*x\*(9 + a^2\*x^2) + 6\*(-1 + a^5\*x^5)\*ArcCoth[a\*x]^2 + 3\*ArcCoth[a\*x]\*(-3 + 2\*a^2\*x^2 + a^4\*x^4 - 4\*Log[1 - E^(-2\*ArcCoth[a\*x])]) + 6\*PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(30\*a^5)

**fricas [F]** time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}(x^4 \operatorname{arccoth}(ax)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x)^2,x, algorithm="fricas")

[Out] integral(x^4\*arccoth(a\*x)^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x)^2,x, algorithm="giac")

[Out] integrate(x^4\*arccoth(a\*x)^2, x)

**maple** [A] time = 0.06, size = 196, normalized size = 1.54

$$\frac{x^5 \operatorname{arccoth}(ax)^2}{5} + \frac{x^4 \operatorname{arccoth}(ax)}{10a} + \frac{x^2 \operatorname{arccoth}(ax)}{5a^3} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{5a^5} + \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{5a^5} + \frac{x^3}{30a^2} + \frac{3}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arccoth(a\*x)^2,x)

[Out] 1/5\*x^5\*arccoth(a\*x)^2+1/10\*x^4\*arccoth(a\*x)/a+1/5\*x^2\*arccoth(a\*x)/a^3+1/5/a^5\*arccoth(a\*x)\*ln(a\*x-1)+1/5/a^5\*arccoth(a\*x)\*ln(a\*x+1)+1/30\*x^3/a^2+3/10\*x/a^4+3/20/a^5\*ln(a\*x-1)-3/20\*ln(a\*x+1)/a^5+1/20/a^5\*ln(a\*x-1)^2-1/5/a^5\*dilog(1/2+1/2\*a\*x)-1/10/a^5\*ln(a\*x-1)\*ln(1/2+1/2\*a\*x)-1/20/a^5\*ln(a\*x+1)^2-1/10/a^5\*ln(-1/2\*a\*x+1/2)\*ln(1/2+1/2\*a\*x)+1/10/a^5\*ln(-1/2\*a\*x+1/2)\*ln(a\*x+1)

**maxima** [A] time = 0.32, size = 155, normalized size = 1.22

$$\frac{1}{5} x^5 \operatorname{arccoth}(ax)^2 + \frac{1}{60} a^2 \left( \frac{2a^3 x^3 + 18ax - 3 \log(ax+1)^2 + 6 \log(ax+1) \log(ax-1) + 3 \log(ax-1)^2 + 9 \log(ax-1) \log(1/2ax+1/2) + \operatorname{dilog}(-1/2ax+1/2)}{a^7} - 12 \log(ax-1) \log(1/2ax+1/2) + \operatorname{dilog}(-1/2ax+1/2) \right) / a^7 - 9 \log(ax+1) / a^7 + 1/10 * a * ((a^2 * x^4 + 2 * x^2) / a^4 + 2 * \log(a^2 * x^2 - 1) / a^6) * \operatorname{arccoth}(a * x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x)^2,x, algorithm="maxima")

[Out] 1/5\*x^5\*arccoth(a\*x)^2 + 1/60\*a^2\*((2\*a^3\*x^3 + 18\*a\*x - 3\*log(a\*x + 1)^2 + 6\*log(a\*x + 1)\*log(a\*x - 1) + 3\*log(a\*x - 1)^2 + 9\*log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a^7 - 12\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a^7 - 9\*log(a\*x + 1)/a^7) + 1/10\*a\*((a^2\*x^4 + 2\*x^2)/a^4 + 2\*log(a^2\*x^2 - 1)/a^6)\*arccoth(a\*x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{acoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acoth(a*x)^2, x)
```

```
[Out] int(x^4*acoth(a*x)^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x^4 \operatorname{acoth}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x)**2, x)
```

```
[Out] Integral(x**4*acoth(a*x)**2, x)
```

### 3.14 $\int x^3 \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=81

$$-\frac{\coth^{-1}(ax)^2}{4a^4} + \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^2}{12a^2} + \frac{\log(1-a^2x^2)}{3a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{x^3 \coth^{-1}(ax)}{6a}$$

[Out]  $1/12*x^2/a^2+1/2*x*\operatorname{arccoth}(a*x)/a^3+1/6*x^3*\operatorname{arccoth}(a*x)/a-1/4*\operatorname{arccoth}(a*x)^2/a^4+1/4*x^4*\operatorname{arccoth}(a*x)^2+1/3*\ln(-a^2*x^2+1)/a^4$

**Rubi [A]** time = 0.16, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {5917, 5981, 266, 43, 5911, 260, 5949}

$$\frac{x^2}{12a^2} + \frac{\log(1-a^2x^2)}{3a^4} + \frac{x \coth^{-1}(ax)}{2a^3} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{x^3 \coth^{-1}(ax)}{6a}$$

Antiderivative was successfully verified.

[In] `Int[x^3*ArcCoth[a*x]^2,x]`

[Out]  $x^2/(12*a^2) + (x*ArcCoth[a*x])/(2*a^3) + (x^3*ArcCoth[a*x])/(6*a) - ArcCoth[a*x]^2/(4*a^4) + (x^4*ArcCoth[a*x]^2)/4 + Log[1 - a^2*x^2]/(3*a^4)$

#### Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

#### Rule 260

`Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]`

#### Rule 266

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

#### Rule 5911

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -`

$c^2x^2), x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

### Rule 5917

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)(x_.)](b_.)]^{(p_.)}((d_.)(x_.))^{(m_.)}, x\_Symbol]$   
 $:\> \text{Simp}[(d*x)^{(m+1)}(a + b*\text{ArcCoth}[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c$   
 $*p)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}(a + b*\text{ArcCoth}[c*x])^{(p-1)}]/(1 - c^2*$   
 $x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{In}$   
 $\text{tegerQ}[m]) \ \&\& \ \text{NeQ}[m, -1]$

### Rule 5949

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)(x_.)](b_.)]^{(p_.)}/((d_.) + (e_.)(x_.)^2), x\_Symbol]$   
 $:\> \text{Simp}[(a + b*\text{ArcCoth}[c*x])^{(p+1)}/(b*c*d*(p+1)), x] /; \text{FreeQ}[\{a, b$   
 $, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[c^2*d + e, 0] \ \&\& \ \text{NeQ}[p, -1]$

### Rule 5981

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)(x_.)](b_.)]^{(p_.)}((f_.)(x_.))^{(m_.)}/((d_.) + ($   
 $e_.)(x_.)^2), x\_Symbol] :\> \text{Dist}[f^2/e, \text{Int}[(f*x)^{(m-2)}(a + b*\text{ArcCoth}[c*x$   
 $)]^p, x], x] - \text{Dist}[(d*f^2)/e, \text{Int}[(f*x)^{(m-2)}(a + b*\text{ArcCoth}[c*x])^p]/($   
 $d + e*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{GtQ}[m, 1$   
 $]$

### Rubi steps

$$\begin{aligned} \int x^3 \coth^{-1}(ax)^2 dx &= \frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{2}a \int \frac{x^4 \coth^{-1}(ax)}{1 - a^2x^2} dx \\ &= \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{\int x^2 \coth^{-1}(ax) dx}{2a} - \frac{\int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx}{2a} \\ &= \frac{x^3 \coth^{-1}(ax)}{6a} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{6} \int \frac{x^3}{1 - a^2x^2} dx + \frac{\int \coth^{-1}(ax) dx}{2a^3} - \frac{\int \frac{\coth^{-1}(ax)}{1 - a^2x^2} dx}{2a^3} \\ &= \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^3 \coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 - \frac{1}{12} \text{Subst} \left( \int \frac{x}{1 - a^2x^2} dx \right) \\ &= \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^3 \coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{\log(1 - a^2x^2)}{4a^4} - \frac{1}{12} \log(1 - a^2x^2) \\ &= \frac{x^2}{12a^2} + \frac{x \coth^{-1}(ax)}{2a^3} + \frac{x^3 \coth^{-1}(ax)}{6a} - \frac{\coth^{-1}(ax)^2}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^2 + \frac{\log(1 - a^2x^2)}{3a^4} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 62, normalized size = 0.77

$$\frac{3(a^4x^4 - 1) \coth^{-1}(ax)^2 + a^2x^2 + 4 \log(1 - a^2x^2) + 2ax(a^2x^2 + 3) \coth^{-1}(ax)}{12a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[a\*x]^2,x]

[Out] (a^2\*x^2 + 2\*a\*x\*(3 + a^2\*x^2)\*ArcCoth[a\*x] + 3\*(-1 + a^4\*x^4)\*ArcCoth[a\*x]^2 + 4\*Log[1 - a^2\*x^2])/(12\*a^4)

**fricas [A]** time = 0.53, size = 81, normalized size = 1.00

$$\frac{4a^2x^2 + 3(a^4x^4 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2 + 4(a^3x^3 + 3ax) \log\left(\frac{ax+1}{ax-1}\right) + 16 \log(a^2x^2 - 1)}{48a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x)^2,x, algorithm="fricas")

[Out] 1/48\*(4\*a^2\*x^2 + 3\*(a^4\*x^4 - 1)\*log((a\*x + 1)/(a\*x - 1))^2 + 4\*(a^3\*x^3 + 3\*a\*x)\*log((a\*x + 1)/(a\*x - 1)) + 16\*log(a^2\*x^2 - 1))/a^4

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x)^2,x, algorithm="giac")

[Out] integrate(x^3\*arccoth(a\*x)^2, x)

**maple [B]** time = 0.06, size = 176, normalized size = 2.17

$$\frac{x^4 \operatorname{arccoth}(ax)^2}{4} + \frac{x^3 \operatorname{arccoth}(ax)}{6a} + \frac{x \operatorname{arccoth}(ax)}{2a^3} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{4a^4} - \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{4a^4} + \frac{\ln(ax-1)}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(a\*x)^2,x)

[Out] 1/4\*x^4\*arccoth(a\*x)^2+1/6\*x^3\*arccoth(a\*x)/a+1/2\*x\*arccoth(a\*x)/a^3+1/4/a^4\*arccoth(a\*x)\*ln(a\*x-1)-1/4/a^4\*arccoth(a\*x)\*ln(a\*x+1)+1/16/a^4\*ln(a\*x-1)^

$2-1/8/a^4*\ln(a*x-1)*\ln(1/2+1/2*a*x)+1/16/a^4*\ln(a*x+1)^2+1/8/a^4*\ln(-1/2*a*x+1/2)*\ln(1/2+1/2*a*x)-1/8/a^4*\ln(-1/2*a*x+1/2)*\ln(a*x+1)+1/12*x^2/a^2+1/3/a^4*\ln(a*x-1)+1/3*\ln(a*x+1)/a^4$

**maxima [A]** time = 0.32, size = 118, normalized size = 1.46

$$\frac{1}{4}x^4 \operatorname{arccoth}(ax)^2 + \frac{1}{12}a \left( \frac{2(a^2x^3 + 3x)}{a^4} - \frac{3 \log(ax + 1)}{a^5} + \frac{3 \log(ax - 1)}{a^5} \right) \operatorname{arccoth}(ax) + \frac{4a^2x^2 - 2(3 \log(ax - 1))}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x)^2,x, algorithm="maxima")

[Out]  $1/4*x^4*\operatorname{arccoth}(a*x)^2 + 1/12*a*(2*(a^2*x^3 + 3*x)/a^4 - 3*\log(a*x + 1)/a^5 + 3*\log(a*x - 1)/a^5)*\operatorname{arccoth}(a*x) + 1/48*(4*a^2*x^2 - 2*(3*\log(a*x - 1) - 8)*\log(a*x + 1) + 3*\log(a*x + 1)^2 + 3*\log(a*x - 1)^2 + 16*\log(a*x - 1))/a^4$

**mupad [B]** time = 1.26, size = 65, normalized size = 0.80

$$\frac{x^4 \operatorname{acoth}(ax)^2}{4} + \frac{\frac{\ln(a^2x^2-1)}{3} + \frac{a^2x^2}{12} - \frac{\operatorname{acoth}(ax)^2}{4} + \frac{a^3x^3 \operatorname{acoth}(ax)}{6} + \frac{ax \operatorname{acoth}(ax)}{2}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*acoth(a\*x)^2,x)

[Out]  $(x^4*\operatorname{acoth}(a*x)^2)/4 + (\log(a^2*x^2 - 1)/3 + (a^2*x^2)/12 - \operatorname{acoth}(a*x)^2/4 + (a^3*x^3*\operatorname{acoth}(a*x))/6 + (a*x*\operatorname{acoth}(a*x))/2)/a^4$

**sympy [A]** time = 1.44, size = 90, normalized size = 1.11

$$\begin{cases} \frac{x^4 \operatorname{acoth}^2(ax)}{4} + \frac{x^3 \operatorname{acoth}(ax)}{6a} + \frac{x^2}{12a^2} + \frac{x \operatorname{acoth}(ax)}{2a^3} + \frac{2 \log(ax+1)}{3a^4} - \frac{\operatorname{acoth}^2(ax)}{4a^4} - \frac{2 \operatorname{acoth}(ax)}{3a^4} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(a\*x)\*\*2,x)

[Out] Piecewise((x\*\*4\*acoth(a\*x)\*\*2/4 + x\*\*3\*acoth(a\*x)/(6\*a) + x\*\*2/(12\*a\*\*2) + x\*acoth(a\*x)/(2\*a\*\*3) + 2\*log(a\*x + 1)/(3\*a\*\*4) - acoth(a\*x)\*\*2/(4\*a\*\*4) - 2\*acoth(a\*x)/(3\*a\*\*4), Ne(a, 0)), (-pi\*\*2\*x\*\*4/16, True))

### 3.15 $\int x^2 \coth^{-1}(ax)^2 dx$

**Optimal.** Leaf size=103

$$-\frac{\operatorname{Li}_2\left(1 - \frac{2}{1-ax}\right)}{3a^3} - \frac{\tanh^{-1}(ax)}{3a^3} + \frac{\coth^{-1}(ax)^2}{3a^3} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{3a^3} + \frac{x}{3a^2} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 + \frac{x^2 \coth^{-1}(ax)}{3a}$$

[Out]  $1/3*x/a^2+1/3*x^2*\operatorname{arccoth}(a*x)/a+1/3*\operatorname{arccoth}(a*x)^2/a^3+1/3*x^3*\operatorname{arccoth}(a*x)^2-1/3*\operatorname{arctanh}(a*x)/a^3-2/3*\operatorname{arccoth}(a*x)*\ln(2/(-a*x+1))/a^3-1/3*\operatorname{polylog}(2, 1-2/(-a*x+1))/a^3$

**Rubi [A]** time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$ , Rules used = {5917, 5981, 321, 206, 5985, 5919, 2402, 2315}

$$-\frac{\operatorname{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{3a^3} + \frac{x}{3a^2} - \frac{\tanh^{-1}(ax)}{3a^3} + \frac{\coth^{-1}(ax)^2}{3a^3} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 + \frac{x^2 \coth^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{ArcCoth}[a*x]^2, x]$

[Out]  $x/(3*a^2) + (x^2*\operatorname{ArcCoth}[a*x])/(3*a) + \operatorname{ArcCoth}[a*x]^2/(3*a^3) + (x^3*\operatorname{ArcCoth}[a*x]^2)/3 - \operatorname{ArcTanh}[a*x]/(3*a^3) - (2*\operatorname{ArcCoth}[a*x]*\operatorname{Log}[2/(1 - a*x)])/(3*a^3) - \operatorname{PolyLog}[2, 1 - 2/(1 - a*x)]/(3*a^3)$

#### Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ \|\ \operatorname{LtQ}[b, 0])$

#### Rule 321

$\operatorname{Int}[(c_+)*(x_+)^m*((a_+ + (b_+)*(x_+)^n)^{p_+}), x\_Symbol] \rightarrow \operatorname{Simp}[(c^{n-1}*(c*x)^{m-n+1}*(a + b*x^n)^{p+1})/(b*(m + n*p + 1)), x] - \operatorname{Dist}[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \operatorname{Int}[(c*x)^{m-n}*(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{GtQ}[m, n - 1] \ \&\& \operatorname{NeQ}[m + n*p + 1, 0] \ \&\& \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 2315

$\operatorname{Int}[\operatorname{Log}[(c_+)*(x_+)]/((d_+ + (e_+)*(x_+))), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /; \operatorname{FreeQ}\{c, d, e\}, x \ \&\& \operatorname{EqQ}[e + c*d, 0]$



Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 5981

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_)^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p]/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5985

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(ax)^2 dx &= \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{1}{3}(2a) \int \frac{x^3 \coth^{-1}(ax)}{1-a^2x^2} dx \\
&= \frac{1}{3}x^3 \coth^{-1}(ax)^2 + \frac{2 \int x \coth^{-1}(ax) dx}{3a} - \frac{2 \int \frac{x \coth^{-1}(ax)}{1-a^2x^2} dx}{3a} \\
&= \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{1}{3} \int \frac{x^2}{1-a^2x^2} dx - \frac{2 \int \frac{\coth^{-1}(ax)}{1-ax} dx}{3a^2} \\
&= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{3a^3} - \frac{\int \frac{1}{1-ax} dx}{3} \\
&= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)}{3a^3} - \frac{2 \coth^{-1}(ax) \log}{3a^3} \\
&= \frac{x}{3a^2} + \frac{x^2 \coth^{-1}(ax)}{3a} + \frac{\coth^{-1}(ax)^2}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)}{3a^3} - \frac{2 \coth^{-1}(ax) \log}{3a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 66, normalized size = 0.64

$$\frac{(a^3x^3 - 1) \coth^{-1}(ax)^2 + \coth^{-1}(ax) \left( a^2x^2 - 2 \log\left(1 - e^{-2 \coth^{-1}(ax)}\right) - 1 \right) + \text{Li}_2\left(e^{-2 \coth^{-1}(ax)}\right) + ax}{3a^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcCoth[a\*x]^2,x]

[Out] (a\*x + (-1 + a^3\*x^3)\*ArcCoth[a\*x]^2 + ArcCoth[a\*x]\*(-1 + a^2\*x^2 - 2\*Log[1 - E^(-2\*ArcCoth[a\*x])]) + PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(3\*a^3)

**fricas [F]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}(x^2 \operatorname{arccoth}(ax)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x)^2,x, algorithm="fricas")

[Out] integral(x^2\*arccoth(a\*x)^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x)^2,x, algorithm="giac")

[Out] integrate(x^2\*arccoth(a\*x)^2, x)

**maple** [A] time = 0.06, size = 176, normalized size = 1.71

$$\frac{x^3 \operatorname{arccoth}(ax)^2}{3} + \frac{x^2 \operatorname{arccoth}(ax)}{3a} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{3a^3} + \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{3a^3} + \frac{x}{3a^2} + \frac{\ln(ax-1)}{6a^3} - \frac{\ln(ax+1)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(a\*x)^2,x)

[Out] 1/3\*x^3\*arccoth(a\*x)^2+1/3\*x^2\*arccoth(a\*x)/a+1/3/a^3\*arccoth(a\*x)\*ln(a\*x-1)+1/3/a^3\*arccoth(a\*x)\*ln(a\*x+1)+1/3\*x/a^2+1/6/a^3\*ln(a\*x-1)-1/6/a^3\*ln(a\*x+1)+1/12/a^3\*ln(a\*x-1)^2-1/3/a^3\*dilog(1/2+1/2\*a\*x)-1/6/a^3\*ln(a\*x-1)\*ln(1/2+1/2\*a\*x)-1/12/a^3\*ln(a\*x+1)^2-1/6/a^3\*ln(-1/2\*a\*x+1/2)\*ln(1/2+1/2\*a\*x)+1/6/a^3\*ln(-1/2\*a\*x+1/2)\*ln(a\*x+1)

**maxima** [A] time = 0.31, size = 134, normalized size = 1.30

$$\frac{1}{3} x^3 \operatorname{arccoth}(ax)^2 + \frac{1}{12} a^2 \left( \frac{4ax - \log(ax+1)^2 + 2 \log(ax+1) \log(ax-1) + \log(ax-1)^2 + 2 \log(ax-1)}{a^5} - \frac{4}{a^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x)^2,x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(a\*x)^2 + 1/12\*a^2\*((4\*a\*x - log(a\*x + 1)^2 + 2\*log(a\*x + 1)\*log(a\*x - 1) + log(a\*x - 1)^2 + 2\*log(a\*x - 1))/a^5 - 4\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a^5 - 2\*log(a\*x + 1)/a^5) + 1/3\*a\*(x^2/a^2 + log(a^2\*x^2 - 1)/a^4)\*arccoth(a\*x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*acoth(a\*x)^2,x)

[Out] int(x^2\*acoth(a\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(a*x)**2,x)`

[Out] `Integral(x**2*acoth(a*x)**2, x)`

### 3.16 $\int x \coth^{-1}(ax)^2 dx$

Optimal. Leaf size=54

$$\frac{\log(1 - a^2x^2)}{2a^2} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{x \coth^{-1}(ax)}{a}$$

[Out]  $x \operatorname{arccoth}(a x) / a - 1/2 \operatorname{arccoth}(a x)^2 / a^2 + 1/2 x^2 \operatorname{arccoth}(a x)^2 + 1/2 \ln(-a^2 x^2 + 1) / a^2$

**Rubi** [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {5917, 5981, 5911, 260, 5949}

$$\frac{\log(1 - a^2x^2)}{2a^2} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{x \coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[a\*x]^2,x]

[Out]  $(x \operatorname{ArcCoth}[a x]) / a - \operatorname{ArcCoth}[a x]^2 / (2 a^2) + (x^2 \operatorname{ArcCoth}[a x]^2) / 2 + \operatorname{Log}[1 - a^2 x^2] / (2 a^2)$

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 5911

Int[((a\_) + ArcCoth[(c\_)\*(x\_)])\*(b\_)^(p\_), x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5917

Int[((a\_) + ArcCoth[(c\_)\*(x\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol]
:= Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_)^(m_.)))/((d_.) + (e_.)*(x_)^2), x_Symbol]
:= Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& GtQ[p, 0] && GtQ[m, 1]
```

### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(ax)^2 dx &= \frac{1}{2}x^2 \coth^{-1}(ax)^2 - a \int \frac{x^2 \coth^{-1}(ax)}{1 - a^2x^2} dx \\ &= \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{\int \coth^{-1}(ax) dx}{a} - \frac{\int \frac{\coth^{-1}(ax)}{1 - a^2x^2} dx}{a} \\ &= \frac{x \coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 - \int \frac{x}{1 - a^2x^2} dx \\ &= \frac{x \coth^{-1}(ax)}{a} - \frac{\coth^{-1}(ax)^2}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^2 + \frac{\log(1 - a^2x^2)}{2a^2} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{\log(1 - a^2x^2) + (a^2x^2 - 1) \coth^{-1}(ax)^2 + 2ax \coth^{-1}(ax)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[a\*x]^2, x]

[Out] (2\*a\*x\*ArcCoth[a\*x] + (-1 + a^2\*x^2)\*ArcCoth[a\*x]^2 + Log[1 - a^2\*x^2])/(2\*a^2)

**fricas** [A] time = 0.53, size = 62, normalized size = 1.15

$$\frac{4ax \log\left(\frac{ax+1}{ax-1}\right) + (a^2x^2 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2 + 4 \log(a^2x^2 - 1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^2,x, algorithm="fricas")

[Out]  $\frac{1}{8}*(4*a*x*\log((a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*\log((a*x + 1)/(a*x - 1))^2 + 4*\log(a^2*x^2 - 1))/a^2$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^2,x, algorithm="giac")

[Out] integrate(x\*arccoth(a\*x)^2, x)

**maple** [B] time = 0.06, size = 155, normalized size = 2.87

$$\frac{x^2 \operatorname{arccoth}(ax)^2}{2} + \frac{x \operatorname{arccoth}(ax)}{a} + \frac{\operatorname{arccoth}(ax) \ln(ax-1)}{2a^2} - \frac{\operatorname{arccoth}(ax) \ln(ax+1)}{2a^2} + \frac{\ln(ax-1)^2}{8a^2} - \frac{\ln(ax-1) \ln(ax+1)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(a\*x)^2,x)

[Out]  $\frac{1}{2}*x^2*\operatorname{arccoth}(a*x)^2 + x*\operatorname{arccoth}(a*x)/a + 1/2/a^2*\operatorname{arccoth}(a*x)*\ln(a*x-1) - 1/2/a^2*\operatorname{arccoth}(a*x)*\ln(a*x+1) + 1/8/a^2*\ln(a*x-1)^2 - 1/4/a^2*\ln(a*x-1)*\ln(1/2+1/2*a*x) + 1/2/a^2*\ln(a*x-1) + 1/2/a^2*\ln(a*x+1) + 1/8/a^2*\ln(a*x+1)^2 + 1/4/a^2*\ln(-1/2*a*x+1/2)*\ln(1/2+1/2*a*x) - 1/4/a^2*\ln(-1/2*a*x+1/2)*\ln(a*x+1)$

**maxima** [B] time = 0.32, size = 97, normalized size = 1.80

$$\frac{1}{2} x^2 \operatorname{arccoth}(ax)^2 + \frac{1}{2} a \left( \frac{2x}{a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3} \right) \operatorname{arccoth}(ax) - \frac{2(\log(ax-1) - 2)\log(ax+1) - \log(ax-1)\log(ax+1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}*x^2*\operatorname{arccoth}(a*x)^2 + 1/2*a*(2*x/a^2 - \log(a*x + 1)/a^3 + \log(a*x - 1)/a^3)*\operatorname{arccoth}(a*x) - 1/8*(2*(\log(a*x - 1) - 2)*\log(a*x + 1) - \log(a*x + 1)^2 - \log(a*x - 1)^2 - 4*\log(a*x - 1))/a^2$

**mupad** [B] time = 1.21, size = 44, normalized size = 0.81

$$\frac{x^2 \operatorname{arccoth}(ax)^2}{2} + \frac{-\operatorname{arccoth}(ax)^2}{2} + \frac{ax \operatorname{arccoth}(ax)}{a^2} + \frac{\ln(a^2 x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(a*x)^2,x)
```

```
[Out] (x^2*acoth(a*x)^2)/2 + (log(a^2*x^2 - 1)/2 - acoth(a*x)^2/2 + a*x*acoth(a*x))/a^2
```

**sympy** [A] time = 0.78, size = 60, normalized size = 1.11

$$\begin{cases} \frac{x^2 \operatorname{acoth}^2(ax)}{2} + \frac{x \operatorname{acoth}(ax)}{a} + \frac{\log(ax+1)}{a^2} - \frac{\operatorname{acoth}^2(ax)}{2a^2} - \frac{\operatorname{acoth}(ax)}{a^2} & \text{for } a \neq 0 \\ -\frac{\pi^2 x^2}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(a*x)**2,x)
```

```
[Out] Piecewise((x**2*acoth(a*x)**2/2 + x*acoth(a*x)/a + log(a*x + 1)/a**2 - acot h(a*x)**2/(2*a**2) - acoth(a*x)/a**2, Ne(a, 0)), (-pi**2*x**2/8, True))
```



### 3.17 $\int \coth^{-1}(ax)^2 dx$

Optimal. Leaf size=58

$$-\frac{\operatorname{Li}_2\left(1 - \frac{2}{1-ax}\right)}{a} + x \coth^{-1}(ax)^2 + \frac{\coth^{-1}(ax)^2}{a} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{a}$$

[Out]  $\operatorname{arccoth}(a*x)^2/a + x*\operatorname{arccoth}(a*x)^2 - 2*\operatorname{arccoth}(a*x)*\ln(2/(-a*x+1))/a - \operatorname{polylog}(2, 1-2/(-a*x+1))/a$

**Rubi [A]** time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$ , Rules used = {5911, 5985, 5919, 2402, 2315}

$$-\frac{\operatorname{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a} + x \coth^{-1}(ax)^2 + \frac{\coth^{-1}(ax)^2}{a} - \frac{2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2, x]

[Out]  $\operatorname{ArcCoth}[a*x]^2/a + x*\operatorname{ArcCoth}[a*x]^2 - (2*\operatorname{ArcCoth}[a*x]*\operatorname{Log}[2/(1 - a*x)])/a - \operatorname{PolyLog}[2, 1 - 2/(1 - a*x)]/a$

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*

p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(ax)^2 dx &= x \coth^{-1}(ax)^2 - (2a) \int \frac{x \coth^{-1}(ax)}{1 - a^2x^2} dx \\
 &= \frac{\coth^{-1}(ax)^2}{a} + x \coth^{-1}(ax)^2 - 2 \int \frac{\coth^{-1}(ax)}{1 - ax} dx \\
 &= \frac{\coth^{-1}(ax)^2}{a} + x \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a} + 2 \int \frac{\log\left(\frac{2}{1-ax}\right)}{1 - a^2x^2} dx \\
 &= \frac{\coth^{-1}(ax)^2}{a} + x \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a} - \frac{2 \operatorname{Subst}\left(\int \frac{\log(2x)}{1-2x} dx, x, \frac{1}{1-ax}\right)}{a} \\
 &= \frac{\coth^{-1}(ax)^2}{a} + x \coth^{-1}(ax)^2 - \frac{2 \coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{a} - \frac{\operatorname{Li}_2\left(1 - \frac{2}{1-ax}\right)}{a}
 \end{aligned}$$

**Mathematica** [A] time = 0.08, size = 46, normalized size = 0.79

$$\frac{\operatorname{Li}_2\left(e^{-2 \coth^{-1}(ax)}\right) + \coth^{-1}(ax) \left( (ax - 1) \coth^{-1}(ax) - 2 \log\left(1 - e^{-2 \coth^{-1}(ax)}\right) \right)}{a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^2, x]

[Out] (ArcCoth[a\*x]\*((-1 + a\*x)\*ArcCoth[a\*x] - 2\*Log[1 - E^(-2\*ArcCoth[a\*x])]) + PolyLog[2, E^(-2\*ArcCoth[a\*x])])/a

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\operatorname{arcoth}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2, x)

**maple** [B] time = 0.30, size = 122, normalized size = 2.10

$$x \operatorname{arccoth}(ax)^2 + \frac{\operatorname{arccoth}(ax)^2}{a} - \frac{2 \operatorname{arccoth}(ax) \ln\left(1 - \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{a} - \frac{2 \operatorname{arccoth}(ax) \ln\left(1 + \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{a} - \frac{2 \operatorname{polylog}\left(2, \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2,x)

[Out] x\*arccoth(a\*x)^2+arccoth(a\*x)^2/a-2/a\*arccoth(a\*x)\*ln(1-1/((a\*x-1)/(a\*x+1))^(1/2))-2/a\*arccoth(a\*x)\*ln(1+1/((a\*x-1)/(a\*x+1))^(1/2))-2/a\*polylog(2,1/((a\*x-1)/(a\*x+1))^(1/2))-2/a\*polylog(2,-1/((a\*x-1)/(a\*x+1))^(1/2))

**maxima** [B] time = 0.32, size = 135, normalized size = 2.33

$$x \operatorname{arccoth}(ax)^2 + \frac{1}{4} \left( a \left( \frac{\log(ax+1)^2 + 2 \log(ax+1) \log(ax-1) - \log(ax-1)^2}{a^3} - \frac{4 \left( \log(ax-1) \log\left(\frac{1}{2}ax + \frac{1}{2}\right)\right)}{a^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2,x, algorithm="maxima")

[Out] x\*arccoth(a\*x)^2 + 1/4\*(a\*((log(a\*x + 1))^2 + 2\*log(a\*x + 1)\*log(a\*x - 1) - log(a\*x - 1)^2)/a^3 - 4\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a^3) - 2\*(log(a\*x + 1)/a - log(a\*x - 1)/a)\*log(a^2\*x^2 - 1)/a\*a + a\*arccoth(a\*x)\*log(a^2\*x^2 - 1)/a

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{acoth}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)^2,x)`

[Out] `int(acoth(a*x)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)**2,x)`

[Out] `Integral(acoth(a*x)**2, x)`

$$3.18 \quad \int \frac{\coth^{-1}(ax)^2}{x} dx$$

**Optimal.** Leaf size=97

$$\frac{1}{2}\text{Li}_3\left(1 - \frac{2}{ax+1}\right) - \frac{1}{2}\text{Li}_3\left(1 - \frac{2ax}{ax+1}\right) + \text{Li}_2\left(1 - \frac{2}{ax+1}\right)\coth^{-1}(ax) - \text{Li}_2\left(1 - \frac{2ax}{ax+1}\right)\coth^{-1}(ax) + 2\coth^{-1}\left(1 - \frac{2}{ax+1}\right)$$

[Out] 2\*arccoth(a\*x)^2\*arccoth(1-2/(-a\*x+1))+arccoth(a\*x)\*polylog(2,1-2/(a\*x+1))-arccoth(a\*x)\*polylog(2,1-2\*a\*x/(a\*x+1))+1/2\*polylog(3,1-2/(a\*x+1))-1/2\*polylog(3,1-2\*a\*x/(a\*x+1))

**Rubi [A]** time = 0.23, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5915, 6053, 5949, 6057, 6610}

$$\frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2}{ax+1}\right) - \frac{1}{2}\text{PolyLog}\left(3, 1 - \frac{2ax}{ax+1}\right) + \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{ax+1}\right) - \coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2ax}{ax+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2/x, x]

[Out] 2\*ArcCoth[a\*x]^2\*ArcCoth[1 - 2/(1 - a\*x)] + ArcCoth[a\*x]\*PolyLog[2, 1 - 2/(1 + a\*x)] - ArcCoth[a\*x]\*PolyLog[2, 1 - (2\*a\*x)/(1 + a\*x)] + PolyLog[3, 1 - 2/(1 + a\*x)]/2 - PolyLog[3, 1 - (2\*a\*x)/(1 + a\*x)]/2

**Rule 5915**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_)/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcCoth[c\*x])^p\*ArcCoth[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*ArcCoth[1 - 2/(1 - c\*x)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]

**Rule 5949**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

**Rule 6053**

Int[(ArcCoth[u\_] \* ((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]])\*(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegrand[1 - 1/u, x]])\*(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6057

Int[(Log[u\_]\*((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^p\_]/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] - Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax)^2}{x} dx &= 2 \coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1 - ax}\right) - (4a) \int \frac{\coth^{-1}(ax) \coth^{-1}\left(1 - \frac{2}{1 - ax}\right)}{1 - a^2x^2} dx \\ &= 2 \coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1 - ax}\right) + (2a) \int \frac{\coth^{-1}(ax) \log\left(\frac{2}{1 + ax}\right)}{1 - a^2x^2} dx - (2a) \int \frac{\coth^{-1}(ax)}{1 - a^2x^2} dx \\ &= 2 \coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1 - ax}\right) + \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2}{1 + ax}\right) - \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2ax}{1 + ax}\right) \\ &= 2 \coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1 - ax}\right) + \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2}{1 + ax}\right) - \coth^{-1}(ax) \text{Li}_2\left(1 - \frac{2ax}{1 + ax}\right) \end{aligned}$$

**Mathematica** [A] time = 0.06, size = 114, normalized size = 1.18

$$-\coth^{-1}(ax) \text{Li}_2\left(-e^{-2\coth^{-1}(ax)}\right) - \coth^{-1}(ax) \text{Li}_2\left(e^{2\coth^{-1}(ax)}\right) - \frac{1}{2} \text{Li}_3\left(-e^{-2\coth^{-1}(ax)}\right) + \frac{1}{2} \text{Li}_3\left(e^{2\coth^{-1}(ax)}\right) + \frac{2}{3} \coth^{-1}(ax)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^2/x, x]

[Out] (2\*ArcCoth[a\*x]^3)/3 + ArcCoth[a\*x]^2\*Log[1 + E^(-2\*ArcCoth[a\*x])] - ArcCoth[a\*x]^2\*Log[1 - E^(2\*ArcCoth[a\*x])] - ArcCoth[a\*x]\*PolyLog[2, -E^(-2\*ArcCoth[a\*x])]

th[a\*x])) - ArcCoth[a\*x]\*PolyLog[2, E^(2\*ArcCoth[a\*x])] - PolyLog[3, -E^(-2\*ArcCoth[a\*x])]/2 + PolyLog[3, E^(2\*ArcCoth[a\*x])]/2

**fricas** [F] time = 0.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arccoth}(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^2/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2/x, x)

**maple** [C] time = 0.68, size = 487, normalized size = 5.02

$$\ln(ax) \operatorname{arccoth}(ax)^2 + \frac{i\pi \operatorname{csgn}\left(i\left(1 + \frac{ax+1}{ax-1}\right)\right) \operatorname{csgn}\left(\frac{i}{\frac{ax+1}{ax-1}-1}\right) \operatorname{csgn}\left(\frac{i\left(1 + \frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1}-1}\right) \operatorname{arccoth}(ax)^2}{2} - \frac{i\pi \operatorname{csgn}\left(i\left(1 + \frac{ax+1}{ax-1}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x,x)

[Out] ln(a\*x)\*arccoth(a\*x)^2+1/2\*I\*Pi\*csgn(I\*(1+(a\*x+1)/(a\*x-1)))\*csgn(I/((a\*x+1)/(a\*x-1)-1))\*csgn(I/((a\*x+1)/(a\*x-1)-1)\*(1+(a\*x+1)/(a\*x-1)))\*arccoth(a\*x)^2-1/2\*I\*Pi\*csgn(I\*(1+(a\*x+1)/(a\*x-1)))\*csgn(I/((a\*x+1)/(a\*x-1)-1)\*(1+(a\*x+1)/(a\*x-1)))^2\*arccoth(a\*x)^2-1/2\*I\*Pi\*csgn(I/((a\*x+1)/(a\*x-1)-1))\*csgn(I/((a\*x+1)/(a\*x-1)-1)\*(1+(a\*x+1)/(a\*x-1)))^2\*arccoth(a\*x)^2+1/2\*I\*Pi\*csgn(I/((a\*x+1)/(a\*x-1)-1)\*(1+(a\*x+1)/(a\*x-1)))^3\*arccoth(a\*x)^2+arccoth(a\*x)^2\*ln((a\*x+1)/(a\*x-1)-1)-arccoth(a\*x)^2\*ln(1-1/((a\*x-1)/(a\*x+1))^(1/2))-2\*arccoth(a\*x)\*polylog(2,1/((a\*x-1)/(a\*x+1))^(1/2))+2\*polylog(3,1/((a\*x-1)/(a\*x+1))^(1/2))-arccoth(a\*x)^2\*ln(1+1/((a\*x-1)/(a\*x+1))^(1/2))-2\*arccoth(a\*x)\*polylog(2,-1/((a\*x-1)/(a\*x+1))^(1/2))+2\*polylog(3,-1/((a\*x-1)/(a\*x+1))^(1/2))+arccoth(a\*x)\*polylog(2,-(a\*x+1)/(a\*x-1))-1/2\*polylog(3,-(a\*x+1)/(a\*x-1))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x,x, algorithm="maxima")

[Out] integrate(arccoth(a\*x)^2/x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)^2/x,x)

[Out] int(acoth(a\*x)^2/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*2/x,x)

[Out] Integral(acoth(a\*x)\*\*2/x, x)



$$3.19 \quad \int \frac{\coth^{-1}(ax)^2}{x^2} dx$$

Optimal. Leaf size=55

$$-a\text{Li}_2\left(\frac{2}{ax+1}-1\right) + a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a\log\left(2 - \frac{2}{ax+1}\right)\coth^{-1}(ax)$$

[Out] a\*arccoth(a\*x)^2-arccoth(a\*x)^2/x+2\*a\*arccoth(a\*x)\*ln(2-2/(a\*x+1))-a\*polylog(2,-1+2/(a\*x+1))

**Rubi [A]** time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {5917, 5989, 5933, 2447}

$$-a\text{PolyLog}\left(2, \frac{2}{ax+1}-1\right) + a\coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a\log\left(2 - \frac{2}{ax+1}\right)\coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2/x^2,x]

[Out] a\*ArcCoth[a\*x]^2 - ArcCoth[a\*x]^2/x + 2\*a\*ArcCoth[a\*x]\*Log[2 - 2/(1 + a\*x)] - a\*PolyLog[2, -1 + 2/(1 + a\*x)]

#### Rule 2447

Int[Log[u]\*(Pq\_)^(m\_), x\_Symbol] :> With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

#### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 5933

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] :> Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^

$2*d^2 - e^2, 0]$

### Rule 5989

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)),  
 x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/  
 d, Int[(a + b\*ArcCoth[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e  
 }, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax)^2}{x^2} dx &= -\frac{\coth^{-1}(ax)^2}{x} + (2a) \int \frac{\coth^{-1}(ax)}{x(1 - a^2x^2)} dx \\ &= a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + (2a) \int \frac{\coth^{-1}(ax)}{x(1 + ax)} dx \\ &= a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a \coth^{-1}(ax) \log\left(2 - \frac{2}{1 + ax}\right) - (2a^2) \int \frac{\log\left(2 - \frac{2}{1 + ax}\right)}{1 - a^2x^2} dx \\ &= a \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{x} + 2a \coth^{-1}(ax) \log\left(2 - \frac{2}{1 + ax}\right) - a \operatorname{Li}_2\left(-1 + \frac{2}{1 + ax}\right) \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 49, normalized size = 0.89

$$-a \operatorname{Li}_2\left(-e^{-2 \coth^{-1}(ax)}\right) + \frac{(ax - 1) \coth^{-1}(ax)^2}{x} + 2a \coth^{-1}(ax) \log\left(e^{-2 \coth^{-1}(ax)} + 1\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^2/x^2,x]

[Out] ((-1 + a\*x)\*ArcCoth[a\*x]^2)/x + 2\*a\*ArcCoth[a\*x]\*Log[1 + E^(-2\*ArcCoth[a\*x])]  
 )] - a\*PolyLog[2, -E^(-2\*ArcCoth[a\*x])]

**fricas [F]** time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^2,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^2/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^2,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2/x^2, x)

**maple** [B] time = 0.06, size = 159, normalized size = 2.89

$$-\frac{\operatorname{arccoth}(ax)^2}{x} + 2a \operatorname{arccoth}(ax) \ln(ax) - a \operatorname{arccoth}(ax) \ln(ax-1) - a \operatorname{arccoth}(ax) \ln(ax+1) - \frac{a \ln(ax-1)^2}{4} + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x^2,x)

[Out] -arccoth(a\*x)^2/x+2\*a\*arccoth(a\*x)\*ln(a\*x)-a\*arccoth(a\*x)\*ln(a\*x-1)-a\*arccoth(a\*x)\*ln(a\*x+1)-1/4\*a\*ln(a\*x-1)^2+a\*dilog(1/2+1/2\*a\*x)+1/2\*a\*ln(a\*x-1)\*ln(1/2+1/2\*a\*x)+1/4\*a\*ln(a\*x+1)^2+1/2\*a\*ln(-1/2\*a\*x+1/2)\*ln(1/2+1/2\*a\*x)-1/2\*a\*ln(-1/2\*a\*x+1/2)\*ln(a\*x+1)-a\*dilog(a\*x)-a\*dilog(a\*x+1)-a\*ln(a\*x)\*ln(a\*x+1)

**maxima** [B] time = 0.32, size = 146, normalized size = 2.65

$$\frac{1}{4} a^2 \left( \frac{\log(ax+1)^2 - 2 \log(ax+1) \log(ax-1) - \log(ax-1)^2}{a} + \frac{4 \left( \log(ax-1) \log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}ax + \frac{1}{2}\right) \right)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^2,x, algorithm="maxima")

[Out] 1/4\*a^2\*((log(a\*x + 1)^2 - 2\*log(a\*x + 1)\*log(a\*x - 1) - log(a\*x - 1)^2)/a + 4\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a - 4\*(log(a\*x + 1)\*log(x) + dilog(-a\*x))/a + 4\*(log(-a\*x + 1)\*log(x) + dilog(a\*x))/a - a\*(log(a^2\*x^2 - 1) - log(x^2))\*arccoth(a\*x) - arccoth(a\*x)^2/x

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acoth}(ax)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoath(a*x)^2/x^2, x)
```

```
[Out] int(acoath(a*x)^2/x^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\operatorname{acoth}^2(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(a*x)**2/x**2, x)
```

```
[Out] Integral(acoath(a*x)**2/x**2, x)
```

### 3.20 $\int \frac{\coth^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=61

$$-\frac{1}{2}a^2 \log(1 - a^2x^2) + a^2 \log(x) + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} - \frac{a \coth^{-1}(ax)}{x}$$

[Out]  $-a \operatorname{arccoth}(ax)/x + 1/2 a^2 \operatorname{arccoth}(ax)^2 - 1/2 \operatorname{arccoth}(ax)^2/x^2 + a^2 \ln(x) - 1/2 a^2 \ln(-a^2 x^2 + 1)$

**Rubi [A]** time = 0.10, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {5917, 5983, 266, 36, 29, 31, 5949}

$$-\frac{1}{2}a^2 \log(1 - a^2x^2) + a^2 \log(x) + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} - \frac{a \coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2/x^3,x]

[Out]  $-((a \operatorname{ArcCoth}[a*x])/x) + (a^2 \operatorname{ArcCoth}[a*x]^2)/2 - \operatorname{ArcCoth}[a*x]^2/(2*x^2) + a^2 \operatorname{Log}[x] - (a^2 \operatorname{Log}[1 - a^2*x^2])/2$

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_) +
(e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^3} dx &= -\frac{\coth^{-1}(ax)^2}{2x^2} + a \int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^2}{2x^2} + a \int \frac{\coth^{-1}(ax)}{x^2} dx + a^3 \int \frac{\coth^{-1}(ax)}{1-a^2x^2} dx \\
&= -\frac{a \coth^{-1}(ax)}{x} + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + a^2 \int \frac{1}{x(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)}{x} + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2 \text{Subst} \left( \int \frac{1}{x(1-a^2x)} dx, x, x^2 \right) \\
&= -\frac{a \coth^{-1}(ax)}{x} + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2 \text{Subst} \left( \int \frac{1}{x} dx, x, x^2 \right) + \frac{1}{2}a^4 \text{Subst} \\
&= -\frac{a \coth^{-1}(ax)}{x} + \frac{1}{2}a^2 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{2x^2} + a^2 \log(x) - \frac{1}{2}a^2 \log(1-a^2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 57, normalized size = 0.93

$$-\frac{1}{2}a^2 \log(1-a^2x^2) + \frac{(a^2x^2-1)\coth^{-1}(ax)^2}{2x^2} + a^2 \log(x) - \frac{a \coth^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]^2/x^3,x]

[Out]  $-\left(\frac{a \operatorname{ArcCoth}[a x]}{x}\right) + \left(\frac{(-1 + a^2 x^2) \operatorname{ArcCoth}[a x]^2}{2 x^2}\right) + a^2 \operatorname{Log}[x] - \left(\frac{a^2 \operatorname{Log}[1 - a^2 x^2]}{2}\right)$

**fricas** [A] time = 0.55, size = 79, normalized size = 1.30

$$\frac{4 a^2 x^2 \log \left(a^2 x^2 - 1\right) - 8 a^2 x^2 \log (x) + 4 a x \log \left(\frac{a x+1}{a x-1}\right) - \left(a^2 x^2 - 1\right) \log \left(\frac{a x+1}{a x-1}\right)^2}{8 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^3,x, algorithm="fricas")

[Out]  $-1/8*(4*a^2*x^2*\log(a^2*x^2 - 1) - 8*a^2*x^2*\log(x) + 4*a*x*\log((a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*\log((a*x + 1)/(a*x - 1))^2)/x^2$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(a x)^2}{x^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^3,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2/x^3, x)

**maple** [B] time = 0.06, size = 164, normalized size = 2.69

$$\frac{\operatorname{arccoth}(a x)^2}{2 x^2} - \frac{a \operatorname{arccoth}(a x)}{x} - \frac{a^2 \operatorname{arccoth}(a x) \ln (a x - 1)}{2} + \frac{a^2 \operatorname{arccoth}(a x) \ln (a x + 1)}{2} - \frac{a^2 \ln (a x - 1)^2}{8} + \frac{a^2 \ln (a x + 1)^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x^3,x)

[Out]  $-1/2*\operatorname{arccoth}(a*x)^2/x^2 - a*\operatorname{arccoth}(a*x)/x - 1/2*a^2*\operatorname{arccoth}(a*x)*\ln(a*x-1) + 1/2*a^2*\operatorname{arccoth}(a*x)*\ln(a*x+1) - 1/8*a^2*\ln(a*x-1)^2 + 1/4*a^2*\ln(a*x-1)*\ln(1/2+1/2*a*x) + a^2*\ln(a*x) - 1/2*a^2*\ln(a*x-1) - 1/2*a^2*\ln(a*x+1) - 1/8*a^2*\ln(a*x+1)^2 + 1/4*a^2*\ln(-1/2*a*x+1/2)*\ln(a*x+1) - 1/4*a^2*\ln(-1/2*a*x+1/2)*\ln(1/2+1/2*a*x)$

**maxima** [A] time = 0.31, size = 96, normalized size = 1.57

$$\frac{1}{8} \left( 2 \left( \log (a x - 1) - 2 \right) \log (a x + 1) - \log (a x + 1)^2 - \log (a x - 1)^2 - 4 \log (a x - 1) + 8 \log (x) \right) a^2 + \frac{1}{2} \left( a \log (a x - 1) - a \log (a x + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^3,x, algorithm="maxima")

[Out] 1/8\*(2\*(log(a\*x - 1) - 2)\*log(a\*x + 1) - log(a\*x + 1)^2 - log(a\*x - 1)^2 - 4\*log(a\*x - 1) + 8\*log(x))\*a^2 + 1/2\*(a\*log(a\*x + 1) - a\*log(a\*x - 1) - 2/x)\*a\*arccoth(a\*x) - 1/2\*arccoth(a\*x)^2/x^2

**mupad [B]** time = 1.45, size = 145, normalized size = 2.38

$$a^2 \ln(x) + \ln\left(\frac{1}{ax} + 1\right)^2 \left(\frac{a^2}{8} - \frac{1}{8x^2}\right) + \ln\left(1 - \frac{1}{ax}\right)^2 \left(\frac{a^2}{8} - \frac{1}{8x^2}\right) - \frac{a^2 \ln(a^2 x^2 - 1)}{2} + \ln\left(1 - \frac{1}{ax}\right) \left(\frac{4ax - 2}{16x^2} + \frac{4ax}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoath(a\*x)^2/x^3,x)

[Out] a^2\*log(x) + log(1/(a\*x) + 1)^2\*(a^2/8 - 1/(8\*x^2)) + log(1 - 1/(a\*x))^2\*(a^2/8 - 1/(8\*x^2)) - (a^2\*log(a^2\*x^2 - 1))/2 + log(1 - 1/(a\*x))\*((4\*a\*x - 2)/(16\*x^2) + (4\*a\*x + 2)/(16\*x^2) - log(1/(a\*x) + 1)\*(a^2/4 - 1/(4\*x^2))) - (a\*log(1/(a\*x) + 1))/(2\*x)

**sympy [A]** time = 1.58, size = 56, normalized size = 0.92

$$a^2 \log(x) - a^2 \log(ax + 1) + \frac{a^2 \operatorname{acoth}^2(ax)}{2} + a^2 \operatorname{acoth}(ax) - \frac{a \operatorname{acoth}(ax)}{x} - \frac{\operatorname{acoth}^2(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoath(a\*x)\*\*2/x\*\*3,x)

[Out] a\*\*2\*log(x) - a\*\*2\*log(a\*x + 1) + a\*\*2\*acoath(a\*x)\*\*2/2 + a\*\*2\*acoath(a\*x) - a\*acoath(a\*x)/x - acoath(a\*x)\*\*2/(2\*x\*\*2)



### 3.21 $\int \frac{\coth^{-1}(ax)^2}{x^4} dx$

**Optimal.** Leaf size=103

$$-\frac{1}{3}a^3 \text{Li}_2\left(\frac{2}{ax+1} - 1\right) + \frac{1}{3}a^3 \tanh^{-1}(ax) + \frac{1}{3}a^3 \coth^{-1}(ax)^2 + \frac{2}{3}a^3 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{a^2}{3x} - \frac{\coth^{-1}(ax)^2}{3x^3}$$

[Out]  $-1/3*a^2/x - 1/3*a*\text{arccoth}(a*x)/x^2 + 1/3*a^3*\text{arccoth}(a*x)^2 - 1/3*\text{arccoth}(a*x)^2/x^3 + 1/3*a^3*\text{arctanh}(a*x) + 2/3*a^3*\text{arccoth}(a*x)*\ln(2 - 2/(a*x+1)) - 1/3*a^3*\text{polylog}(2, -1 + 2/(a*x+1))$

**Rubi [A]** time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {5917, 5983, 325, 206, 5989, 5933, 2447}

$$-\frac{1}{3}a^3 \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{a^2}{3x} + \frac{1}{3}a^3 \tanh^{-1}(ax) + \frac{1}{3}a^3 \coth^{-1}(ax)^2 + \frac{2}{3}a^3 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{a \coth^{-1}(ax)^2}{3x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a*x]^2/x^4, x]$

[Out]  $-a^2/(3*x) - (a*\text{ArcCoth}[a*x])/(3*x^2) + (a^3*\text{ArcCoth}[a*x]^2)/3 - \text{ArcCoth}[a*x]^2/(3*x^3) + (a^3*\text{ArcTanh}[a*x])/3 + (2*a^3*\text{ArcCoth}[a*x]*\text{Log}[2 - 2/(1 + a*x)]) / 3 - (a^3*\text{PolyLog}[2, -1 + 2/(1 + a*x)]) / 3$

#### Rule 206

$\text{Int}[(a + (b*x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2]) / (\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 325

$\text{Int}[(c*x)^m * (a + (b*x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(c*x)^{m+1} * (a + b*x^n)^{p+1} / (a*c*(m+1)), x] - \text{Dist}[(b*(m+n*(p+1)+1)) / (a*c^n*(m+1)), \text{Int}[(c*x)^{m+n} * (a + b*x^n)^p, x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 2447

$\text{Int}[\text{Log}[u] * (\text{Pq})^m, x\_Symbol] \rightarrow \text{With}[\{C = \text{FullSimplify}[(\text{Pq}^m * (1 - u)) / \text{D}[u, x]]\}, \text{Simp}[C*\text{PolyLog}[2, 1 - u], x] /;$  FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,

x][[2]], Expon[Pq, x]]

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.))), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]
)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

### Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_.) + (
e_.)*(x_.)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

### Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.)^2)),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^4} dx &= -\frac{\coth^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a) \int \frac{\coth^{-1}(ax)}{x^3(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^2}{3x^3} + \frac{1}{3}(2a) \int \frac{\coth^{-1}(ax)}{x^3} dx + \frac{1}{3}(2a^3) \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)}{3x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^2 \int \frac{1}{x^2(1-a^2x^2)} dx + \frac{1}{3}(2a^3) \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= -\frac{a^2}{3x} - \frac{a \coth^{-1}(ax)}{3x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{3x^3} + \frac{2}{3}a^3 \coth^{-1}(ax) \log\left(2 - \frac{2}{1+ax}\right) \\
&= -\frac{a^2}{3x} - \frac{a \coth^{-1}(ax)}{3x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{3x^3} + \frac{1}{3}a^3 \tanh^{-1}(ax) + \frac{2}{3}a^3 \coth^{-1}(ax)
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 87, normalized size = 0.84

$$\frac{-a^3 x^3 \operatorname{Li}_2\left(-e^{-2 \coth^{-1}(ax)}\right) + \left(a^3 x^3 - 1\right) \coth^{-1}(ax)^2 - a^2 x^2 + ax \coth^{-1}(ax) \left(a^2 x^2 + 2a^2 x^2 \log\left(e^{-2 \coth^{-1}(ax)} + 1\right)\right)}{3x^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^2/x^4, x]

[Out]  $(-(a^2 x^2) + (-1 + a^3 x^3) \operatorname{ArcCoth}[a x]^2 + a x \operatorname{ArcCoth}[a x] * (-1 + a^2 x^2 + 2 a^2 x^2 \operatorname{Log}[1 + E^{(-2 \operatorname{ArcCoth}[a x])}])) - a^3 x^3 \operatorname{PolyLog}[2, -E^{(-2 \operatorname{ArcCoth}[a x])}]) / (3 x^3)$

**fricas [F]** time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(ax)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^4, x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^2/x^4, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^4,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2/x^4, x)

**maple [B]** time = 0.07, size = 224, normalized size = 2.17

$$-\frac{\operatorname{arccoth}(ax)^2}{3x^3} - \frac{a \operatorname{arccoth}(ax)}{3x^2} + \frac{2a^3 \operatorname{arccoth}(ax) \ln(ax)}{3} - \frac{a^3 \operatorname{arccoth}(ax) \ln(ax-1)}{3} - \frac{a^3 \operatorname{arccoth}(ax) \ln(ax+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x^4,x)

[Out]  $-1/3 \operatorname{arccoth}(a*x)^2/x^3 - 1/3*a*\operatorname{arccoth}(a*x)/x^2 + 2/3*a^3*\operatorname{arccoth}(a*x)*\ln(a*x) - 1/3*a^3*\operatorname{arccoth}(a*x)*\ln(a*x-1) - 1/3*a^3*\operatorname{arccoth}(a*x)*\ln(a*x+1) - 1/3*a^2/x - 1/6*a^3*\ln(a*x-1) + 1/6*a^3*\ln(a*x+1) - 1/12*a^3*\ln(a*x-1)^2 + 1/3*a^3*\operatorname{dilog}(1/2+1/2*a*x) + 1/6*a^3*\ln(a*x-1)*\ln(1/2+1/2*a*x) + 1/12*a^3*\ln(a*x+1)^2 + 1/6*a^3*\ln(-1/2*a*x+1/2)*\ln(1/2+1/2*a*x) - 1/6*a^3*\ln(-1/2*a*x+1/2)*\ln(a*x+1) - 1/3*a^3*\operatorname{dilog}(a*x) - 1/3*a^3*\operatorname{dilog}(a*x+1) - 1/3*a^3*\ln(a*x)*\ln(a*x+1)$

**maxima [A]** time = 0.32, size = 176, normalized size = 1.71

$$\frac{1}{12} \left( 4 \left( \log(ax-1) \log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}ax + \frac{1}{2}\right) \right) a - 4 \left( \log(ax+1) \log(x) + \operatorname{Li}_2(-ax) \right) a + 4 \left( \log(-ax+1) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^4,x, algorithm="maxima")

[Out]  $1/12*(4*(\log(a*x-1)*\log(1/2*a*x+1/2)+\operatorname{dilog}(-1/2*a*x+1/2))*a-4*(\log(a*x+1)*\log(x)+\operatorname{dilog}(-a*x))*a+4*(\log(-a*x+1)*\log(x)+\operatorname{dilog}(a*x))*a+2*a*\log(a*x+1)-2*a*\log(a*x-1)+(a*x*\log(a*x+1)^2-2*a*x*\log(a*x+1)*\log(a*x-1)-a*x*\log(a*x-1)^2-4)/x)*a^2-1/3*(a^2*\log(a^2*x^2-1)-a^2*\log(x^2)+1/x^2)*a*\operatorname{arccoth}(a*x)-1/3*\operatorname{arccoth}(a*x)^2/x^3$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(ax)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)^2/x^4,x)

[Out] int(acoth(a\*x)^2/x^4, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*2/x\*\*4, x)

[Out] Integral(acoth(a\*x)\*\*2/x\*\*4, x)

$$3.22 \quad \int \frac{\coth^{-1}(ax)^2}{x^5} dx$$

**Optimal.** Leaf size=90

$$\frac{2}{3}a^4 \log(x) + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{a^3 \coth^{-1}(ax)}{2x} - \frac{a^2}{12x^2} - \frac{1}{3}a^4 \log(1 - a^2x^2) - \frac{\coth^{-1}(ax)^2}{4x^4} - \frac{a \coth^{-1}(ax)}{6x^3}$$

[Out]  $-1/12*a^2/x^2 - 1/6*a*arccoth(a*x)/x^3 - 1/2*a^3*arccoth(a*x)/x + 1/4*a^4*arccoth(a*x)^2 - 1/4*arccoth(a*x)^2/x^4 + 2/3*a^4*\ln(x) - 1/3*a^4*\ln(-a^2*x^2 + 1)$

**Rubi [A]** time = 0.17, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$ , Rules used = {5917, 5983, 266, 44, 36, 29, 31, 5949}

$$-\frac{a^2}{12x^2} - \frac{1}{3}a^4 \log(1 - a^2x^2) + \frac{2}{3}a^4 \log(x) + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{a^3 \coth^{-1}(ax)}{2x} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{\coth^{-1}(ax)^2}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^2/x^5, x]

[Out]  $-a^2/(12*x^2) - (a*ArcCoth[a*x])/(6*x^3) - (a^3*ArcCoth[a*x])/(2*x) + (a^4*ArcCoth[a*x]^2)/4 - ArcCoth[a*x]^2/(4*x^4) + (2*a^4*Log[x])/3 - (a^4*Log[1 - a^2*x^2])/3$

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m

+ n + 2, 0])

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5983

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^2}{x^5} dx &= -\frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{2}a \int \frac{\coth^{-1}(ax)}{x^4(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{2}a \int \frac{\coth^{-1}(ax)}{x^4} dx + \frac{1}{2}a^3 \int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)}{6x^3} - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^2 \int \frac{1}{x^3(1-a^2x^2)} dx + \frac{1}{2}a^3 \int \frac{\coth^{-1}(ax)}{x^2} dx + \frac{1}{2}a^5 \int \frac{\coth^{-1}(ax)}{1-a^2x^2} dx \\
&= -\frac{a \coth^{-1}(ax)}{6x^3} - \frac{a^3 \coth^{-1}(ax)}{2x} + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{12}a^2 \text{Subst} \left( \int \frac{1}{x^2(1-a^2x^2)} dx \right) \\
&= -\frac{a \coth^{-1}(ax)}{6x^3} - \frac{a^3 \coth^{-1}(ax)}{2x} + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{12}a^2 \text{Subst} \left( \int \left( \frac{1}{x^2} + \frac{a^2}{1-a^2x^2} \right) dx \right) \\
&= -\frac{a^2}{12x^2} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{a^3 \coth^{-1}(ax)}{2x} + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{1}{6}a^4 \log(x) - \frac{1}{12}a^4 \log(1-a^2x^2) \\
&= -\frac{a^2}{12x^2} - \frac{a \coth^{-1}(ax)}{6x^3} - \frac{a^3 \coth^{-1}(ax)}{2x} + \frac{1}{4}a^4 \coth^{-1}(ax)^2 - \frac{\coth^{-1}(ax)^2}{4x^4} + \frac{2}{3}a^4 \log(x) - \frac{1}{3}a^4 \log(1-a^2x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 82, normalized size = 0.91

$$\frac{(a^4x^4 - 1) \coth^{-1}(ax)^2}{4x^4} + \frac{2}{3}a^4 \log(x) - \frac{a^2}{12x^2} - \frac{a(3a^2x^2 + 1) \coth^{-1}(ax)}{6x^3} - \frac{1}{3}a^4 \log(1 - a^2x^2)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]^2/x^5,x]

[Out] -1/12\*a^2/x^2 - (a\*(1 + 3\*a^2\*x^2)\*ArcCoth[a\*x])/(6\*x^3) + ((-1 + a^4\*x^4)\*ArcCoth[a\*x]^2)/(4\*x^4) + (2\*a^4\*Log[x])/3 - (a^4\*Log[1 - a^2\*x^2])/3

**fricas [A]** time = 0.66, size = 97, normalized size = 1.08

$$\frac{16a^4x^4 \log(a^2x^2 - 1) - 32a^4x^4 \log(x) + 4a^2x^2 - 3(a^4x^4 - 1) \log\left(\frac{ax+1}{ax-1}\right)^2 + 4(3a^3x^3 + ax) \log\left(\frac{ax+1}{ax-1}\right)}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^5,x, algorithm="fricas")

[Out] -1/48\*(16\*a^4\*x^4\*log(a^2\*x^2 - 1) - 32\*a^4\*x^4\*log(x) + 4\*a^2\*x^2 - 3\*(a^4\*x^4 - 1)\*log((a\*x + 1)/(a\*x - 1))^2 + 4\*(3\*a^3\*x^3 + a\*x)\*log((a\*x + 1)/(a\*x - 1)))/x^4



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^5,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^2/x^5, x)

**maple** [B] time = 0.06, size = 185, normalized size = 2.06

$$\frac{\operatorname{arccoth}(ax)^2}{4x^4} - \frac{a \operatorname{arccoth}(ax)}{6x^3} - \frac{a^3 \operatorname{arccoth}(ax)}{2x} - \frac{a^4 \operatorname{arccoth}(ax) \ln(ax-1)}{4} + \frac{a^4 \operatorname{arccoth}(ax) \ln(ax+1)}{4} - \frac{a^4 \ln^2(ax-1)}{4} + \frac{a^4 \ln^2(ax+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^2/x^5,x)

[Out]  $-1/4 \operatorname{arccoth}(a*x)^2/x^4 - 1/6 a \operatorname{arccoth}(a*x)/x^3 - 1/2 a^3 \operatorname{arccoth}(a*x)/x - 1/4 a^4 \operatorname{arccoth}(a*x) \ln(a*x-1) + 1/4 a^4 \operatorname{arccoth}(a*x) \ln(a*x+1) - 1/16 a^4 \ln^2(a*x-1) + 1/8 a^4 \ln^2(a*x-1) \ln(1/2 + 1/2 a*x) - 1/16 a^4 \ln^2(a*x+1) - 1/8 a^4 \ln^2(-1/2 a*x + 1/2) \ln(1/2 + 1/2 a*x) + 1/8 a^4 \ln(-1/2 a*x + 1/2) \ln(a*x+1) - 1/12 a^2/x^2 + 2/3 a^4 \ln(a*x) - 1/3 a^4 \ln(a*x-1) - 1/3 a^4 \ln(a*x+1)$

**maxima** [B] time = 0.32, size = 154, normalized size = 1.71

$$\frac{1}{48} \left( 32 a^2 \log(x) - \frac{3 a^2 x^2 \log(ax+1)^2 + 3 a^2 x^2 \log(ax-1)^2 + 16 a^2 x^2 \log(ax-1) - 2(3 a^2 x^2 \log(ax-1) - 8 a^2 x^2 \log(ax+1))}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^2/x^5,x, algorithm="maxima")

[Out]  $1/48 (32 a^2 \log(x) - (3 a^2 x^2 \log(ax+1)^2 + 3 a^2 x^2 \log(ax-1)^2 + 16 a^2 x^2 \log(ax-1) - 2(3 a^2 x^2 \log(ax-1) - 8 a^2 x^2 \log(ax+1)))/x^2 + 1/12 (3 a^3 \log(ax+1) - 3 a^3 \log(ax-1) - 2(3 a^2 x^2 + 1)/x^3) a \operatorname{arccoth}(a*x) - 1/4 \operatorname{arccoth}(a*x)^2/x^4$

**mupad** [B] time = 1.55, size = 196, normalized size = 2.18

$$\frac{2 a^4 \ln(x)}{3} + \ln\left(\frac{1}{ax} + 1\right)^2 \left(\frac{a^4}{16} - \frac{1}{16 x^4}\right) + \ln\left(1 - \frac{1}{ax}\right)^2 \left(\frac{a^4}{16} - \frac{1}{16 x^4}\right) + \ln\left(1 - \frac{1}{ax}\right) \left(\frac{24 a^3 x^3 - 12 a^2 x^2 + 8 a x - 8}{192 x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)^2/x^5,x)`

[Out]  $(2a^4 \log(x))/3 + \log(1/(ax) + 1)^2(a^4/16 - 1/(16x^4)) + \log(1 - 1/(ax))^2(a^4/16 - 1/(16x^4)) + \log(1 - 1/(ax)) * ((8ax - 12a^2x^2 + 24a^3x^3 - 6)/(192x^4) + (8ax + 12a^2x^2 + 24a^3x^3 + 6)/(192x^4)) - \log(1/(ax) + 1) * (a^4/8 - 1/(8x^4)) - (a^4 \log(a^2x^2 - 1))/3 - a^2/(12x^2) - (a \log(1/(ax) + 1) * ((a^2x^2)/4 + 1/12))/x^3$

**sympy** [A] time = 2.64, size = 90, normalized size = 1.00

$$\frac{2a^4 \log(x)}{3} - \frac{2a^4 \log(ax+1)}{3} + \frac{a^4 \operatorname{acoth}^2(ax)}{4} + \frac{2a^4 \operatorname{acoth}(ax)}{3} - \frac{a^3 \operatorname{acoth}(ax)}{2x} - \frac{a^2}{12x^2} - \frac{a \operatorname{acoth}(ax)}{6x^3} - \frac{\operatorname{acoth}^2(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)**2/x**5,x)`

[Out]  $2a^4 \log(x)/3 - 2a^4 \log(ax + 1)/3 + a^4 \operatorname{acoth}(ax)^2/4 + 2a^4 \operatorname{acoth}(ax)/3 - a^3 \operatorname{acoth}(ax)/(2x) - a^2/(12x^2) - a \operatorname{acoth}(ax)/(6x^3) - \operatorname{acoth}(ax)^2/(4x^4)$

### 3.23 $\int x^5 \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=186

$$-\frac{23\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{30a^6} - \frac{19 \tanh^{-1}(ax)}{60a^6} - \frac{\coth^{-1}(ax)^3}{6a^6} + \frac{23 \coth^{-1}(ax)^2}{30a^6} - \frac{23 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)}{15a^6} + \frac{19x}{60a^5} + \frac{x \coth^{-1}(ax)}{2a^5}$$

[Out]  $19/60*x/a^5+1/60*x^3/a^3+4/15*x^2*\text{arccoth}(a*x)/a^4+1/20*x^4*\text{arccoth}(a*x)/a^2+23/30*\text{arccoth}(a*x)^2/a^6+1/2*x*\text{arccoth}(a*x)^2/a^5+1/6*x^3*\text{arccoth}(a*x)^2/a^3+1/10*x^5*\text{arccoth}(a*x)^2/a-1/6*\text{arccoth}(a*x)^3/a^6+1/6*x^6*\text{arccoth}(a*x)^3-19/60*\text{arctanh}(a*x)/a^6-23/15*\text{arccoth}(a*x)*\ln(2/(-a*x+1))/a^6-23/30*\text{polylog}(2,1-2/(-a*x+1))/a^6$

**Rubi [A]** time = 0.72, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$ , Rules used = {5917, 5981, 302, 206, 321, 5985, 5919, 2402, 2315, 5911, 5949}

$$-\frac{23\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{30a^6} + \frac{x^3}{60a^3} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{19x}{60a^5} - \frac{19 \tanh^{-1}(ax)}{60a^6} + \frac{x \coth^{-1}(ax)}{2a^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5*\text{ArcCoth}[a*x]^3,x]$

[Out]  $(19*x)/(60*a^5) + x^3/(60*a^3) + (4*x^2*\text{ArcCoth}[a*x])/(15*a^4) + (x^4*\text{ArcCoth}[a*x])/(20*a^2) + (23*\text{ArcCoth}[a*x]^2)/(30*a^6) + (x*\text{ArcCoth}[a*x]^2)/(2*a^5) + (x^3*\text{ArcCoth}[a*x]^2)/(6*a^3) + (x^5*\text{ArcCoth}[a*x]^2)/(10*a) - \text{ArcCoth}[a*x]^3/(6*a^6) + (x^6*\text{ArcCoth}[a*x]^3)/6 - (19*\text{ArcTanh}[a*x])/(60*a^6) - (23*\text{ArcCoth}[a*x]*\text{Log}[2/(1-a*x)])/(15*a^6) - (23*\text{PolyLog}[2,1-2/(1-a*x)])/(30*a^6)$

#### Rule 206

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 302

$\text{Int}[(x_+)^{m_+}/((a_+ + (b_+)*(x_+)^{n_+})^2), x\_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /;$  FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

Rule 321

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2
*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^ (p_.)*((f_.)*(x_.))^(m_.))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

### Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^ (p_.)*(x_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^5 \coth^{-1}(ax)^3 dx &= \frac{1}{6}x^6 \coth^{-1}(ax)^3 - \frac{1}{2}a \int \frac{x^6 \coth^{-1}(ax)^2}{1-a^2x^2} dx \\
&= \frac{1}{6}x^6 \coth^{-1}(ax)^3 + \frac{\int x^4 \coth^{-1}(ax)^2 dx}{2a} - \frac{\int \frac{x^4 \coth^{-1}(ax)^2}{1-a^2x^2} dx}{2a} \\
&= \frac{x^5 \coth^{-1}(ax)^2}{10a} + \frac{1}{6}x^6 \coth^{-1}(ax)^3 - \frac{1}{5} \int \frac{x^5 \coth^{-1}(ax)}{1-a^2x^2} dx + \frac{\int x^2 \coth^{-1}(ax)^2 dx}{2a^3} - \frac{\int x^2}{2a^3} \\
&= \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a} + \frac{1}{6}x^6 \coth^{-1}(ax)^3 + \frac{\int \coth^{-1}(ax)^2 dx}{2a^5} - \frac{\int \frac{\coth^{-1}(ax)^2}{1-a^2x^2} dx}{2a^5} \\
&= \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5 \coth^{-1}(ax)^2}{10a} - \frac{\coth^{-1}(ax)^3}{6a^6} + \frac{1}{6}x^6 \\
&= \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3 \coth^{-1}(ax)^2}{6a^3} + \frac{x^5}{60a^5} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3}{60a^3} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3}{60a^3} \\
&= \frac{19x}{60a^5} + \frac{x^3}{60a^3} + \frac{4x^2 \coth^{-1}(ax)}{15a^4} + \frac{x^4 \coth^{-1}(ax)}{20a^2} + \frac{23 \coth^{-1}(ax)^2}{30a^6} + \frac{x \coth^{-1}(ax)^2}{2a^5} + \frac{x^3}{60a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.54, size = 117, normalized size = 0.63

$$\frac{10(a^6x^6 - 1) \coth^{-1}(ax)^3 + ax(a^2x^2 + 19) + 2(3a^5x^5 + 5a^3x^3 + 15ax - 23) \coth^{-1}(ax)^2 + \coth^{-1}(ax)(3a^4x^4 + 19a^2x^2 + 19)}{60a^6}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5\*ArcCoth[a\*x]^3,x]

[Out] (a\*x\*(19 + a^2\*x^2) + 2\*(-23 + 15\*a\*x + 5\*a^3\*x^3 + 3\*a^5\*x^5)\*ArcCoth[a\*x]^2 + 10\*(-1 + a^6\*x^6)\*ArcCoth[a\*x]^3 + ArcCoth[a\*x]\*(-19 + 16\*a^2\*x^2 + 3\*a^4\*x^4 - 92\*Log[1 - E^(-2\*ArcCoth[a\*x])])) + 46\*PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(60\*a^6)

**fricas [F]** time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}(x^5 \operatorname{arccoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^5*arccoth(a*x)^3, x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arccoth(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*arccoth(a*x)^3, x)
```

**maple** [C] time = 3.61, size = 1141, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arccoth(a*x)^3,x)
```

```
[Out] 23/30*arccoth(a*x)^2/a^6-1/6*arccoth(a*x)^3/a^6+1/6*x^6*arccoth(a*x)^3+1/10
*x^5*arccoth(a*x)^2/a+1/6*x^3*arccoth(a*x)^2/a^3+1/20*x^4*arccoth(a*x)/a^2+
4/15*x^2*arccoth(a*x)/a^4+1/2*x*arccoth(a*x)^2/a^5-19/60/a^6*arccoth(a*x)+2
3/15/a^6*dilog(1/((a*x-1)/(a*x+1))^(1/2))+1/80/a^6/(((a*x-1)/(a*x+1))^(1/2)
*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x)-1/80/a^6/(((a*x-1)/(a*x+1))^(1/2)*a*x+((a
*x-1)/(a*x+1))^(1/2)+a*x)-23/15/a^6*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))-1/8*
I/a^6*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2+1/120/a^6*((a*x-1)/(a*x+1)
)^(1/2)/(2*((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+2*a*x-1)-1/
80/a^5/(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-a*x)*x+1/80/a^5
/(((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+a*x)*x-1/8*I/a^6*Pi*c
sgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/(
(a*x+1)/(a*x-1)-1))*arccoth(a*x)^2-1/120/a^5*((a*x-1)/(a*x+1))^(1/2)/(2*((a
*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)-2*a*x+1)*x-1/120/a^5*((a*x
-1)/(a*x+1))^(1/2)/(2*((a*x-1)/(a*x+1))^(1/2)*a*x+((a*x-1)/(a*x+1))^(1/2)+2
*a*x-1)*x+1/8*I/a^6*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/(
(a*x+1)/(a*x-1)-1))^2*arccoth(a*x)^2+1/8*I/a^6*Pi*csgn(I*(a*x+1)/(a*x-1))*c
sgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)^2-1/8*I/a^6*Pi*cs
gn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2+1/4*
I/a^6*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(
a*x)^2+1/120/a^6*((a*x-1)/(a*x+1))^(1/2)/(2*((a*x-1)/(a*x+1))^(1/2)*a*x+((a
*x-1)/(a*x+1))^(1/2)-2*a*x+1)+1/4/a^6*arccoth(a*x)^2*ln(a*x-1)-1/4/a^6*arcc
oth(a*x)^2*ln(a*x+1)-23/15/a^6*arccoth(a*x)*ln(1+1/((a*x-1)/(a*x+1))^(1/2))
```

$$-41/120/a^6/(((a*x-1)/(a*x+1))^{(1/2)+1})*((a*x-1)/(a*x+1))^{(1/2)}-1/4/a^6*\text{arccoth}(a*x)^2*\ln((a*x-1)/(a*x+1))-41/120/a^6/(-1+((a*x-1)/(a*x+1))^{(1/2)})*((a*x-1)/(a*x+1))^{(1/2)}-1/8*I/a^6*\text{Pi}*c\text{sgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^{3*}\text{arccoth}(a*x)^2$$

**maxima** [A] time = 0.33, size = 289, normalized size = 1.55

$$\frac{1}{6}x^6 \text{arccoth}(ax)^3 + \frac{1}{60}a \left( \frac{2(3a^4x^5 + 5a^2x^3 + 15x)}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right) \text{arccoth}(ax)^2 + \frac{1}{240}a \left( \frac{4a^3}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right) \text{arccoth}(ax) + \frac{1}{240}a \left( \frac{4a^3}{a^6} - \frac{15 \log(ax+1)}{a^7} + \frac{15 \log(ax-1)}{a^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*arccoth(a\*x)^3,x, algorithm="maxima")

[Out]  $\frac{1}{6}x^6*\text{arccoth}(a*x)^3 + \frac{1}{60}*a*(2*(3*a^4*x^5 + 5*a^2*x^3 + 15*x)/a^6 - 15*\log(a*x + 1)/a^7 + 15*\log(a*x - 1)/a^7)*\text{arccoth}(a*x)^2 + \frac{1}{240}*a*((4*a^3*x^3 + (15*\log(a*x - 1) - 46)*\log(a*x + 1)^2 - 5*\log(a*x + 1)^3 + 5*\log(a*x - 1)^3 + 76*a*x - (15*\log(a*x - 1)^2 - 92*\log(a*x - 1))*\log(a*x + 1) + 46*\log(a*x - 1)^2 + 38*\log(a*x - 1))/a - 184*(\log(a*x - 1)*\log(1/2*a*x + 1/2) + \text{dilog}(-1/2*a*x + 1/2))/a - 38*\log(a*x + 1)/a)/a^6 + 2*(6*a^4*x^4 + 32*a^2*x^2 - 2*(15*\log(a*x - 1) - 46)*\log(a*x + 1) + 15*\log(a*x + 1)^2 + 15*\log(a*x - 1)^2 + 92*\log(a*x - 1))*\text{arccoth}(a*x)/a^7$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \text{acoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*acoth(a\*x)^3,x)

[Out] int(x^5\*acoth(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \text{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*acoth(a\*x)\*\*3,x)

[Out] Integral(x\*\*5\*acoth(a\*x)\*\*3, x)



### 3.24 $\int x^4 \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=196

$$\frac{3\text{Li}_3\left(1 - \frac{2}{1-ax}\right)}{10a^5} - \frac{3\text{Li}_2\left(1 - \frac{2}{1-ax}\right) \coth^{-1}(ax)}{5a^5} + \frac{\coth^{-1}(ax)^3}{5a^5} - \frac{9 \coth^{-1}(ax)^2}{20a^5} - \frac{3 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)^2}{5a^5} + \frac{9x \coth^{-1}(ax)}{10a^4}$$

[Out]  $1/20*x^2/a^3+9/10*x*\text{arccoth}(a*x)/a^4+1/10*x^3*\text{arccoth}(a*x)/a^2-9/20*\text{arccoth}(a*x)^2/a^5+3/10*x^2*\text{arccoth}(a*x)^2/a^3+3/20*x^4*\text{arccoth}(a*x)^2/a+1/5*\text{arccoth}(a*x)^3/a^5+1/5*x^5*\text{arccoth}(a*x)^3-3/5*\text{arccoth}(a*x)^2*\ln(2/(-a*x+1))/a^5+1/2*\ln(-a^2*x^2+1)/a^5-3/5*\text{arccoth}(a*x)*\text{polylog}(2,1-2/(-a*x+1))/a^5+3/10*\text{polylog}(3,1-2/(-a*x+1))/a^5$

**Rubi [A]** time = 0.58, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$ , Rules used = {5917, 5981, 266, 43, 5911, 260, 5949, 5985, 5919, 6059, 6610}

$$\frac{3\text{PolyLog}\left(3,1 - \frac{2}{1-ax}\right)}{10a^5} - \frac{3 \coth^{-1}(ax)\text{PolyLog}\left(2,1 - \frac{2}{1-ax}\right)}{5a^5} + \frac{x^2}{20a^3} + \frac{\log(1 - a^2x^2)}{2a^5} + \frac{x^3 \coth^{-1}(ax)}{10a^2} + \frac{3x^2 \coth^{-1}(ax)}{10a^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*\text{ArcCoth}[a*x]^3,x]$

[Out]  $x^2/(20*a^3) + (9*x*\text{ArcCoth}[a*x])/(10*a^4) + (x^3*\text{ArcCoth}[a*x])/(10*a^2) - (9*\text{ArcCoth}[a*x]^2)/(20*a^5) + (3*x^2*\text{ArcCoth}[a*x]^2)/(10*a^3) + (3*x^4*\text{ArcCoth}[a*x]^2)/(20*a) + \text{ArcCoth}[a*x]^3/(5*a^5) + (x^5*\text{ArcCoth}[a*x]^3)/5 - (3*\text{ArcCoth}[a*x]^2*\text{Log}[2/(1 - a*x)])/(5*a^5) + \text{Log}[1 - a^2*x^2]/(2*a^5) - (3*\text{ArcCoth}[a*x]*\text{PolyLog}[2, 1 - 2/(1 - a*x)])/(5*a^5) + (3*\text{PolyLog}[3, 1 - 2/(1 - a*x)])/(10*a^5)$

#### Rule 43

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0])) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0]$

#### Rule 260

$\text{Int}[(x_.)^(m_.)/((a_.) + (b_.)*(x_.)^(n_.)), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n\}, x \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5911

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5917

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)*((d_)*(x_))^(m_), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 5919

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol]
:= -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/e, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 5949

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symbol]
:= Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5981

```
Int((((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)*((f_)*(x_))^(m_))/((d_) + (e_)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5985

```
Int((((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)*(x_))/((d_) + (e_)*(x_)^2),
```

```
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int x^4 \coth^{-1}(ax)^3 dx &= \frac{1}{5}x^5 \coth^{-1}(ax)^3 - \frac{1}{5}(3a) \int \frac{x^5 \coth^{-1}(ax)^2}{1 - a^2x^2} dx \\
&= \frac{1}{5}x^5 \coth^{-1}(ax)^3 + \frac{3 \int x^3 \coth^{-1}(ax)^2 dx}{5a} - \frac{3 \int \frac{x^3 \coth^{-1}(ax)^2}{1 - a^2x^2} dx}{5a} \\
&= \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{1}{5}x^5 \coth^{-1}(ax)^3 - \frac{3}{10} \int \frac{x^4 \coth^{-1}(ax)}{1 - a^2x^2} dx + \frac{3 \int x \coth^{-1}(ax)^2 dx}{5a^3} \\
&= \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^3 - \frac{3 \int \frac{\coth^{-1}(ax)^2}{1 - ax} dx}{5a^4} \\
&= \frac{x^3 \coth^{-1}(ax)}{10a^2} + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} + \frac{\coth^{-1}(ax)^3}{5a^5} + \frac{1}{5}x^5 \coth^{-1}(ax)^3 \\
&= \frac{9x \coth^{-1}(ax)}{10a^4} + \frac{x^3 \coth^{-1}(ax)}{10a^2} - \frac{9 \coth^{-1}(ax)^2}{20a^5} + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} \\
&= \frac{9x \coth^{-1}(ax)}{10a^4} + \frac{x^3 \coth^{-1}(ax)}{10a^2} - \frac{9 \coth^{-1}(ax)^2}{20a^5} + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a} \\
&= \frac{x^2}{20a^3} + \frac{9x \coth^{-1}(ax)}{10a^4} + \frac{x^3 \coth^{-1}(ax)}{10a^2} - \frac{9 \coth^{-1}(ax)^2}{20a^5} + \frac{3x^2 \coth^{-1}(ax)^2}{10a^3} + \frac{3x^4 \coth^{-1}(ax)^2}{20a}
\end{aligned}$$

**Mathematica** [C] time = 0.59, size = 175, normalized size = 0.89

$$8a^5x^5 \coth^{-1}(ax)^3 + 6a^4x^4 \coth^{-1}(ax)^2 + 4a^3x^3 \coth^{-1}(ax) + 2a^2x^2 - 40 \log \left( \frac{1}{ax \sqrt{1 - \frac{1}{a^2x^2}}} \right) + 12a^2x^2 \coth^{-1}(ax)^2 -$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*ArcCoth[a\*x]^3,x]

[Out] (-2 - I\*Pi^3 + 2\*a^2\*x^2 + 36\*a\*x\*ArcCoth[a\*x] + 4\*a^3\*x^3\*ArcCoth[a\*x] - 1  
8\*ArcCoth[a\*x]^2 + 12\*a^2\*x^2\*ArcCoth[a\*x]^2 + 6\*a^4\*x^4\*ArcCoth[a\*x]^2 + 8  
\*ArcCoth[a\*x]^3 + 8\*a^5\*x^5\*ArcCoth[a\*x]^3 - 24\*ArcCoth[a\*x]^2\*Log[1 - E^(2  
\*ArcCoth[a\*x])] - 40\*Log[1/(a\*Sqrt[1 - 1/(a^2\*x^2)]]\*x] - 24\*ArcCoth[a\*x]\*P  
olyLog[2, E^(2\*ArcCoth[a\*x])] + 12\*PolyLog[3, E^(2\*ArcCoth[a\*x])])/(40\*a^5)

**fricas** [F] time = 1.95, size = 0, normalized size = 0.00

$$\text{integral}(x^4 \operatorname{arccoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x)^3,x, algorithm="fricas")

[Out] integral(x^4\*arccoth(a\*x)^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x)^3,x, algorithm="giac")

[Out] integrate(x^4\*arccoth(a\*x)^3, x)

**maple** [C] time = 2.92, size = 806, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arccoth(a\*x)^3,x)

[Out] -9/20\*arccoth(a\*x)^2/a^5+1/5\*arccoth(a\*x)^3/a^5+1/10\*x^3\*arccoth(a\*x)/a^2+3  
/10\*x^2\*arccoth(a\*x)^2/a^3+3/20\*x^4\*arccoth(a\*x)^2/a+9/10\*x\*arccoth(a\*x)/a^

$4-1/20/a^5+1/5*x^5*\operatorname{arccoth}(a*x)^3+1/20*x^2/a^3+3/10/a^5*\operatorname{arccoth}(a*x)^2*\ln(a*x+1)-3/5/a^5*\operatorname{arccoth}(a*x)^2*\ln(1-1/((a*x-1)/(a*x+1))^{(1/2)})-6/5/a^5*\operatorname{arccoth}(a*x)*\operatorname{polylog}(2,1/((a*x-1)/(a*x+1))^{(1/2)})-3/5/a^5*\operatorname{arccoth}(a*x)^2*\ln(1+1/((a*x-1)/(a*x+1))^{(1/2)})-6/5/a^5*\operatorname{arccoth}(a*x)*\operatorname{polylog}(2,-1/((a*x-1)/(a*x+1))^{(1/2)})+3/5/a^5*\operatorname{arccoth}(a*x)^2*\ln((a*x+1)/(a*x-1)-1)+3/10/a^5*\operatorname{arccoth}(a*x)^2*\ln((a*x-1)/(a*x+1))+3/20*I/a^5*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*\operatorname{arccoth}(a*x)^2+1/a^5*\operatorname{arccoth}(a*x)-1/a^5*\ln(1+1/((a*x-1)/(a*x+1))^{(1/2)})-1/a^5*\ln(1/((a*x-1)/(a*x+1))^{(1/2)})+6/5/a^5*\operatorname{polylog}(3,-1/((a*x-1)/(a*x+1))^{(1/2)})+6/5/a^5*\operatorname{polylog}(3,1/((a*x-1)/(a*x+1))^{(1/2)})+3/20*I/a^5*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*\operatorname{arccoth}(a*x)^2+3/20*I/a^5*Pi*csgn(I*(a*x+1)/(a*x-1))^3*\operatorname{arccoth}(a*x)^2-3/20*I/a^5*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*\operatorname{arccoth}(a*x)^2-3/20*I/a^5*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*\operatorname{arccoth}(a*x)^2+3/20*I/a^5*Pi*csgn(I/((a*x-1)/(a*x+1))^{(1/2)})^2*csgn(I*(a*x+1)/(a*x-1))*\operatorname{arccoth}(a*x)^2-3/10*I/a^5*Pi*csgn(I/((a*x-1)/(a*x+1))^{(1/2)})*csgn(I*(a*x+1)/(a*x-1))^2*\operatorname{arccoth}(a*x)^2-3/5/a^5*\operatorname{arccoth}(a*x)^2*\ln(2)+3/10/a^5*\operatorname{arccoth}(a*x)^2*\ln(a*x-1)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2(a^5x^5 + 1)\log(ax + 1)^3 + 3(a^4x^4 + 2a^2x^2 - 2(a^5x^5 - 1)\log(ax - 1))\log(ax + 1)^2}{80a^5} + \frac{1}{8} \int -\frac{5(a^5x^5 + a^4x^4)}{ax^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(a\*x)^3,x, algorithm="maxima")

[Out] 1/80\*(2\*(a^5\*x^5 + 1)\*log(a\*x + 1)^3 + 3\*(a^4\*x^4 + 2\*a^2\*x^2 - 2\*(a^5\*x^5 - 1)\*log(a\*x - 1))\*log(a\*x + 1)^2)/a^5 + 1/8\*integrate(-1/5\*(5\*(a^5\*x^5 + a^4\*x^4)\*log(a\*x - 1)^3 + 3\*(a^4\*x^4 + 2\*a^2\*x^2 - 5\*(a^5\*x^5 + a^4\*x^4))\*log(a\*x - 1)^2 - 2\*(a^5\*x^5 - 1)\*log(a\*x - 1))\*log(a\*x + 1))/(a^5\*x + a^4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \operatorname{acoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*acoth(a\*x)^3,x)

[Out] int(x^4\*acoth(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(a*x)**3,x)
```

```
[Out] Integral(x**4*acoth(a*x)**3, x)
```

### 3.25 $\int x^3 \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=139

$$-\frac{\operatorname{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^4} - \frac{\tanh^{-1}(ax)}{4a^4} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{\coth^{-1}(ax)^2}{a^4} - \frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^4} + \frac{x}{4a^3} + \frac{3x\coth^{-1}(ax)^2}{4a^3} + \frac{x^2\coth^{-1}(ax)}{4a^3}$$

[Out]  $1/4*x/a^3+1/4*x^2*\operatorname{arccoth}(a*x)/a^2+\operatorname{arccoth}(a*x)^2/a^4+3/4*x*\operatorname{arccoth}(a*x)^2/a^3+1/4*x^3*\operatorname{arccoth}(a*x)^2/a-1/4*\operatorname{arccoth}(a*x)^3/a^4+1/4*x^4*\operatorname{arccoth}(a*x)^3-1/4*\operatorname{arctanh}(a*x)/a^4-2*\operatorname{arccoth}(a*x)*\ln(2/(-a*x+1))/a^4-\operatorname{polylog}(2,1-2/(-a*x+1))/a^4$

**Rubi [A]** time = 0.42, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {5917, 5981, 321, 206, 5985, 5919, 2402, 2315, 5911, 5949}

$$-\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4} + \frac{x^2\coth^{-1}(ax)}{4a^2} + \frac{x}{4a^3} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{3x\coth^{-1}(ax)^2}{4a^3} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{\coth^{-1}(ax)^2}{a^4} - \frac{2\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[a\*x]^3,x]

[Out]  $x/(4*a^3) + (x^2*ArcCoth[a*x])/(4*a^2) + ArcCoth[a*x]^2/a^4 + (3*x*ArcCoth[a*x]^2)/(4*a^3) + (x^3*ArcCoth[a*x]^2)/(4*a) - ArcCoth[a*x]^3/(4*a^4) + (x^4*ArcCoth[a*x]^3)/4 - ArcTanh[a*x]/(4*a^4) - (2*ArcCoth[a*x]*Log[2/(1 - a*x)])/a^4 - PolyLog[2, 1 - 2/(1 - a*x)]/a^4$

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 2315

`Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]`

### Rule 2402

`Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]`

### Rule 5911

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]`

### Rule 5917

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]`

### Rule 5919

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]`

### Rule 5949

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]`

### Rule 5981

`Int[(((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_)^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]`



Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_.)^2),  
 x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/  
 (c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e  
 }, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int x^3 \coth^{-1}(ax)^3 dx &= \frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{1}{4}(3a) \int \frac{x^4 \coth^{-1}(ax)^2}{1 - a^2x^2} dx \\
 &= \frac{1}{4}x^4 \coth^{-1}(ax)^3 + \frac{3 \int x^2 \coth^{-1}(ax)^2 dx}{4a} - \frac{3 \int \frac{x^2 \coth^{-1}(ax)^2}{1 - a^2x^2} dx}{4a} \\
 &= \frac{x^3 \coth^{-1}(ax)^2}{4a} + \frac{1}{4}x^4 \coth^{-1}(ax)^3 - \frac{1}{2} \int \frac{x^3 \coth^{-1}(ax)}{1 - a^2x^2} dx + \frac{3 \int \coth^{-1}(ax)^2 dx}{4a^3} - \frac{3 \int}{4a^3} \\
 &= \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \coth^{-1}(ax)^3 + \frac{\int x \coth^{-1}(ax) dx}{2a^2} \\
 &= \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} + \frac{1}{4}x^4 \\
 &= \frac{x}{4a^3} + \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} \\
 &= \frac{x}{4a^3} + \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4} \\
 &= \frac{x}{4a^3} + \frac{x^2 \coth^{-1}(ax)}{4a^2} + \frac{\coth^{-1}(ax)^2}{a^4} + \frac{3x \coth^{-1}(ax)^2}{4a^3} + \frac{x^3 \coth^{-1}(ax)^2}{4a} - \frac{\coth^{-1}(ax)^3}{4a^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 88, normalized size = 0.63

$$\frac{(a^4x^4 - 1) \coth^{-1}(ax)^3 + (a^3x^3 + 3ax - 4) \coth^{-1}(ax)^2 + \coth^{-1}(ax) (a^2x^2 - 8 \log(1 - e^{-2 \coth^{-1}(ax)}) - 1) + 4 \operatorname{Li}_2(-e^{-2 \coth^{-1}(ax)})}{4a^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*ArcCoth[a\*x]^3,x]

[Out]  $(a*x + (-4 + 3*a*x + a^3*x^3)*\text{ArcCoth}[a*x]^2 + (-1 + a^4*x^4)*\text{ArcCoth}[a*x]^3 + \text{ArcCoth}[a*x]*(-1 + a^2*x^2 - 8*\text{Log}[1 - E^{(-2*\text{ArcCoth}[a*x])}])) + 4*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[a*x])}])/(4*a^4)$

**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}(x^3 \text{arccoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(a*x)^3,x, algorithm="fricas")`

[Out] `integral(x^3*arccoth(a*x)^3, x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \text{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(a*x)^3,x, algorithm="giac")`

[Out] `integrate(x^3*arccoth(a*x)^3, x)`

**maple** [C] time = 1.59, size = 684, normalized size = 4.92

$$\frac{\text{arccoth}(ax)}{4a^4} + \frac{\text{arccoth}(ax)^2}{a^4} + \frac{3\text{arccoth}(ax)^2 \ln(ax-1)}{8a^4} - \frac{3\text{arccoth}(ax)^2 \ln(ax+1)}{8a^4} - \frac{2 \text{arccoth}(ax) \ln\left(1 + \frac{\sqrt{a}}{\sqrt{a}}\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arccoth(a*x)^3,x)`

[Out]  $-1/4/a^4*\text{arccoth}(a*x)+\text{arccoth}(a*x)^2/a^4+3/8/a^4*\text{arccoth}(a*x)^2*\ln(a*x-1)-3/8/a^4*\text{arccoth}(a*x)^2*\ln(a*x+1)-2/a^4*\text{arccoth}(a*x)*\ln(1+1/((a*x-1)/(a*x+1))^{(1/2)})-1/4/a^4/(((a*x-1)/(a*x+1))^{(1/2)+1}*((a*x-1)/(a*x+1))^{(1/2)}-3/8/a^4*\text{arccoth}(a*x)^2*\ln((a*x-1)/(a*x+1))-1/4/a^4/(-1+((a*x-1)/(a*x+1))^{(1/2)})*(((a*x-1)/(a*x+1))^{(1/2)}+2/a^4*\text{dilog}(1/((a*x-1)/(a*x+1))^{(1/2)})-1/4*\text{arccoth}(a*x)^3/a^4+1/4*x^4*\text{arccoth}(a*x)^3+3/4*x*\text{arccoth}(a*x)^2/a^3+1/4*x^3*\text{arccoth}(a*x)^2/a+1/4*x^2*\text{arccoth}(a*x)/a^2+3/16*I/a^4*\text{Pi}*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*\text{arccoth}(a*x)^2-3/16*I/a^4*\text{Pi}*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*\text{arccoth}(a*x)^2+3/8*I/a^4*\text{Pi}*csgn(I/((a*x-1)/(a*x+1))^{(1/2)})*csgn(I*(a*x+1)/(a*x-1))^2*\text{arccoth}(a*x)^2-2/a^4*\text{dilog}(1+1/((a*x-1)/(a*x+1))^{(1/2)})+3/16*I/a^4*\text{Pi}*csgn(I*(a*x+1)/(a*x-1))*$

```

csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)^2-3/16*I/a^4*Pi*
csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2-3/
16*I/a^4*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)^2-3/16*I/a^4*Pi*csgn(I*(
a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)
/(a*x-1)-1))*arccoth(a*x)^2

```

**maxima** [B] time = 0.33, size = 262, normalized size = 1.88

$$\frac{1}{4} x^4 \operatorname{arccoth}(ax)^3 + \frac{1}{8} a \left( \frac{2(a^2 x^3 + 3x)}{a^4} - \frac{3 \log(ax+1)}{a^5} + \frac{3 \log(ax-1)}{a^5} \right) \operatorname{arccoth}(ax)^2 + \frac{1}{32} a \left( \frac{(3 \log(ax-1)-8) \log(ax)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(a\*x)^3,x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(a\*x)^3 + 1/8\*a\*(2\*(a^2\*x^3 + 3\*x)/a^4 - 3\*log(a\*x + 1)/a^5 + 3\*log(a\*x - 1)/a^5)\*arccoth(a\*x)^2 + 1/32\*a\*(((3\*log(a\*x - 1) - 8)\*log(a\*x + 1)^2 - log(a\*x + 1)^3 + log(a\*x - 1)^3 + 8\*a\*x - (3\*log(a\*x - 1)^2 - 16\*log(a\*x - 1))\*log(a\*x + 1) + 8\*log(a\*x - 1)^2 + 4\*log(a\*x - 1))/a - 32\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a - 4\*log(a\*x + 1)/a)/a^4 + 2\*(4\*a^2\*x^2 - 2\*(3\*log(a\*x - 1) - 8)\*log(a\*x + 1) + 3\*log(a\*x + 1)^2 + 3\*log(a\*x - 1)^2 + 16\*log(a\*x - 1))\*arccoth(a\*x)/a^5)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*acoth(a\*x)^3,x)

[Out] int(x^3\*acoth(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(a\*x)\*\*3,x)

[Out] Integral(x\*\*3\*acoth(a\*x)\*\*3, x)

### 3.26 $\int x^2 \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=149

$$\frac{\operatorname{Li}_3\left(1 - \frac{2}{1-ax}\right)}{2a^3} - \frac{\operatorname{Li}_2\left(1 - \frac{2}{1-ax}\right) \coth^{-1}(ax)}{a^3} + \frac{\coth^{-1}(ax)^3}{3a^3} - \frac{\coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right) \coth^{-1}(ax)^2}{2a^3} + \frac{x \coth^{-1}(ax)}{a^2} + \dots$$

[Out]  $x \operatorname{arccoth}(ax)/a^2 - 1/2 \operatorname{arccoth}(ax)^2/a^3 + 1/2 x^2 \operatorname{arccoth}(ax)^2/a + 1/3 \operatorname{arccoth}(ax)^3/a^3 + 1/3 x^3 \operatorname{arccoth}(ax)^3 - \operatorname{arccoth}(ax)^2 \ln(2/(-ax+1))/a^3 + 1/2 \ln(-a^2 x^2 + 1)/a^3 - \operatorname{arccoth}(ax) \operatorname{polylog}(2, 1 - 2/(-ax+1))/a^3 + 1/2 \operatorname{polylog}(3, 1 - 2/(-ax+1))/a^3$

**Rubi [A]** time = 0.33, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$ , Rules used = {5917, 5981, 5911, 260, 5949, 5985, 5919, 6059, 6610}

$$\frac{\operatorname{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{2a^3} - \frac{\coth^{-1}(ax) \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a^3} + \frac{\log(1 - a^2 x^2)}{2a^3} + \frac{\coth^{-1}(ax)^3}{3a^3} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x \coth^{-1}(ax)}{a^2} + \dots$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 \operatorname{ArcCoth}[a*x]^3, x]$

[Out]  $(x \operatorname{ArcCoth}[a*x])/a^2 - \operatorname{ArcCoth}[a*x]^2/(2*a^3) + (x^2 \operatorname{ArcCoth}[a*x]^2)/(2*a) + \operatorname{ArcCoth}[a*x]^3/(3*a^3) + (x^3 \operatorname{ArcCoth}[a*x]^3)/3 - (\operatorname{ArcCoth}[a*x]^2 \operatorname{Log}[2/(1 - a*x)])/a^3 + \operatorname{Log}[1 - a^2 x^2]/(2*a^3) - (\operatorname{ArcCoth}[a*x] \operatorname{PolyLog}[2, 1 - 2/(1 - a*x)])/a^3 + \operatorname{PolyLog}[3, 1 - 2/(1 - a*x)]/(2*a^3)$

#### Rule 260

$\operatorname{Int}[(x_)^m / ((a_) + (b_)*(x_)^n), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \operatorname{FreeQ}\{a, b, m, n, x\} \ \&\& \ \operatorname{EqQ}[m, n - 1]$

#### Rule 5911

$\operatorname{Int}[(a_) + \operatorname{ArcCoth}[(c_)*(x_)]*(b_)^p, x\_Symbol] \rightarrow \operatorname{Simp}[x*(a + b \operatorname{ArcCoth}[c*x])^p, x] - \operatorname{Dist}[b*c^p, \operatorname{Int}[(x*(a + b \operatorname{ArcCoth}[c*x])^p - 1)/(1 - c^2 x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{IGtQ}[p, 0]$

#### Rule 5917

$\operatorname{Int}[(a_) + \operatorname{ArcCoth}[(c_)*(x_)]*(b_)^p * ((d_)*(x_))^m, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{m+1}*(a + b \operatorname{ArcCoth}[c*x])^p / (d*(m+1)), x] - \operatorname{Dist}[(b*c^p)/(d*(m+1)), \operatorname{Int}[(d*x)^{m+1}*(a + b \operatorname{ArcCoth}[c*x])^p - 1 / (1 - c^2 x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, m, x\} \ \&\& \ \operatorname{IGtQ}[p, 0] \ \&\& \ (\operatorname{EqQ}[p, 1] \ || \ \operatorname{In})$

tegerQ[m]) && NeQ[m, -1]

### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_)^(m_)))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

### Rule 5985

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol]
:> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(ax)^3 dx &= \frac{1}{3}x^3 \coth^{-1}(ax)^3 - a \int \frac{x^3 \coth^{-1}(ax)^2}{1-a^2x^2} dx \\
&= \frac{1}{3}x^3 \coth^{-1}(ax)^3 + \frac{\int x \coth^{-1}(ax)^2 dx}{a} - \frac{\int \frac{x \coth^{-1}(ax)^2}{1-a^2x^2} dx}{a} \\
&= \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\int \frac{\coth^{-1}(ax)^2}{1-ax} dx}{a^2} - \int \frac{x^2 \coth^{-1}(ax)}{1-a^2x^2} dx \\
&= \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a^3} + \frac{\int \coth^{-1}(ax)}{a^2} \\
&= \frac{x \coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)}{a^2} \\
&= \frac{x \coth^{-1}(ax)}{a^2} - \frac{\coth^{-1}(ax)^2}{2a^3} + \frac{x^2 \coth^{-1}(ax)^2}{2a} + \frac{\coth^{-1}(ax)^3}{3a^3} + \frac{1}{3}x^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)}{a^2}
\end{aligned}$$

**Mathematica [C]** time = 0.38, size = 140, normalized size = 0.94

$$\frac{8a^3x^3 \coth^{-1}(ax)^3 - 24 \log\left(\frac{1}{ax\sqrt{1-\frac{1}{a^2x^2}}}\right) + 12a^2x^2 \coth^{-1}(ax)^2 - 24 \coth^{-1}(ax) \text{Li}_2\left(e^{2\coth^{-1}(ax)}\right) + 12 \text{Li}_3\left(e^{2\coth^{-1}(ax)}\right)}{24a^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcCoth[a\*x]^3,x]

[Out]  $((-1)*\text{Pi}^3 + 24*a*x*\text{ArcCoth}[a*x] - 12*\text{ArcCoth}[a*x]^2 + 12*a^2*x^2*\text{ArcCoth}[a*x]^2 + 8*\text{ArcCoth}[a*x]^3 + 8*a^3*x^3*\text{ArcCoth}[a*x]^3 - 24*\text{ArcCoth}[a*x]^2*\text{Log}[1 - E^{(2*\text{ArcCoth}[a*x])}] - 24*\text{Log}[1/(a*\text{Sqrt}[1 - 1/(a^2*x^2)]*x)] - 24*\text{ArcCoth}[a*x]*\text{PolyLog}[2, E^{(2*\text{ArcCoth}[a*x])}] + 12*\text{PolyLog}[3, E^{(2*\text{ArcCoth}[a*x])}]) / (24*a^3)$

**fricas [F]** time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}(x^2 \text{arcoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x)^3,x, algorithm="fricas")

[Out] integral(x^2\*arccoth(a\*x)^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x)^3,x, algorithm="giac")

[Out] integrate(x^2\*arccoth(a\*x)^3, x)

**maple** [C] time = 1.49, size = 765, normalized size = 5.13

$$\frac{\operatorname{arccoth}(ax)}{a^3} - \frac{\ln\left(\frac{1}{\sqrt{\frac{ax-1}{ax+1}}} - 1\right)}{a^3} - \frac{\ln\left(1 + \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{a^3} - \frac{\operatorname{arccoth}(ax)^2}{2a^3} + \frac{\operatorname{arccoth}(ax)^3}{3a^3} + \frac{x^2 \operatorname{arccoth}(ax)^2}{2a} + \frac{\operatorname{arccoth}(ax)^2}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(a\*x)^3,x)

[Out]  $\frac{1}{a^3} \operatorname{arccoth}(ax) - \frac{1}{a^3} \ln\left(\frac{1}{\left(\frac{ax-1}{ax+1}\right)^{1/2}} - 1\right) - \frac{1}{a^3} \ln\left(1 + \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{1/2}}\right) + \frac{2}{a^3} \operatorname{polylog}\left(3, \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{1/2}}\right) + \frac{2}{a^3} \operatorname{polylog}\left(3, -\frac{1}{\left(\frac{ax-1}{ax+1}\right)^{1/2}}\right) - \frac{1}{2} \operatorname{arccoth}(ax)^2 / a^3 + \frac{1}{3} \operatorname{arccoth}(ax)^3 / a^3 + \frac{1}{2} x^2 \operatorname{arccoth}(ax)^2 / a + \frac{1}{2} \operatorname{arccoth}(ax)^2 \ln(ax-1) + \frac{1}{2} \operatorname{arccoth}(ax)^2 \ln(ax+1) + \frac{1}{2} \operatorname{arccoth}(ax)^2 \ln\left(\frac{ax-1}{ax+1}\right) + \frac{1}{a^3} \operatorname{arccoth}(ax)^2 \ln\left(\frac{ax+1}{ax-1} - 1\right) - \frac{1}{a^3} \operatorname{arccoth}(ax)^2 \ln\left(1 - \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{1/2}}\right) - \frac{2}{a^3} \operatorname{arccoth}(ax) \operatorname{polylog}\left(2, \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{1/2}}\right) - \frac{1}{a^3} \operatorname{arccoth}(ax)^2 \ln\left(1 + \frac{1}{\left(\frac{ax-1}{ax+1}\right)^{1/2}}\right) - \frac{2}{a^3} \operatorname{arccoth}(ax) \operatorname{polylog}\left(2, -\frac{1}{\left(\frac{ax-1}{ax+1}\right)^{1/2}}\right) - \frac{1}{a^3} \operatorname{arccoth}(ax)^2 \ln(2) + x \operatorname{arccoth}(ax) / a^2 + \frac{1}{3} x^3 \operatorname{arccoth}(ax)^3 + \frac{1}{4} I / a^3 \operatorname{arccoth}(ax)^2 \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right)^3 + \frac{1}{4} I / a^3 \operatorname{arccoth}(ax)^2 \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right) \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right) \operatorname{Picsgn}\left(I / \left(\frac{ax+1}{ax-1} - 1\right)\right) + \frac{1}{4} I / a^3 \operatorname{arccoth}(ax)^2 \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right)^3 - \frac{1}{4} I / a^3 \operatorname{arccoth}(ax)^2 \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right)^2 \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right) - \frac{1}{4} I / a^3 \operatorname{arccoth}(ax)^2 \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right)^2 \operatorname{Picsgn}\left(I / \left(\frac{ax+1}{ax-1} - 1\right)\right) - \frac{1}{2} I / a^3 \operatorname{arccoth}(ax)^2 \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right)^2 \operatorname{Picsgn}\left(I / \left(\frac{ax-1}{ax+1}\right)^{1/2}\right) + \frac{1}{4} I / a^3 \operatorname{arccoth}(ax)^2 \operatorname{Picsgn}\left(I \frac{ax+1}{ax-1} / \left(\frac{ax+1}{ax-1} - 1\right)\right) \operatorname{Picsgn}\left(I / \left(\frac{ax-1}{ax+1}\right)^{1/2}\right)^2$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a^3 x^3 + 1) \log(ax + 1)^3 + 3(a^2 x^2 - (a^3 x^3 - 1) \log(ax - 1)) \log(ax + 1)^2}{24 a^3} + \frac{1}{8} \int -\frac{(a^3 x^3 + a^2 x^2) \log(ax - 1)^3 + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a\*x)^3,x, algorithm="maxima")

[Out] 1/24\*((a^3\*x^3 + 1)\*log(a\*x + 1)^3 + 3\*(a^2\*x^2 - (a^3\*x^3 - 1)\*log(a\*x - 1))\*log(a\*x + 1)^2)/a^3 + 1/8\*integrate(-((a^3\*x^3 + a^2\*x^2)\*log(a\*x - 1)^3 + (2\*a^2\*x^2 - 3\*(a^3\*x^3 + a^2\*x^2))\*log(a\*x - 1)^2 - 2\*(a^3\*x^3 - 1)\*log(a\*x - 1))\*log(a\*x + 1))/(a^3\*x + a^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*acoth(a\*x)^3,x)

[Out] int(x^2\*acoth(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(a\*x)\*\*3,x)

[Out] Integral(x\*\*2\*acoth(a\*x)\*\*3, x)



### 3.27 $\int x \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=95

$$-\frac{3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{3\coth^{-1}(ax)^2}{2a^2} - \frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 + \frac{3x\coth^{-1}(ax)^2}{2a}$$

[Out]  $3/2*\text{arccoth}(a*x)^2/a^2+3/2*x*\text{arccoth}(a*x)^2/a-1/2*\text{arccoth}(a*x)^3/a^2+1/2*x^2*\text{arccoth}(a*x)^3-3*\text{arccoth}(a*x)*\ln(2/(-a*x+1))/a^2-3/2*\text{polylog}(2,1-2/(-a*x+1))/a^2$

**Rubi [A]** time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {5917, 5981, 5911, 5985, 5919, 2402, 2315, 5949}

$$-\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{3\coth^{-1}(ax)^2}{2a^2} - \frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)}{a^2} + \frac{1}{2}x^2\coth^{-1}(ax)^3 + \frac{3x\coth^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[a*x]^3,x]`

[Out]  $(3*\text{ArcCoth}[a*x]^2)/(2*a^2) + (3*x*\text{ArcCoth}[a*x]^2)/(2*a) - \text{ArcCoth}[a*x]^3/(2*a^2) + (x^2*\text{ArcCoth}[a*x]^3)/2 - (3*\text{ArcCoth}[a*x]*\text{Log}[2/(1-a*x)])/a^2 - (3*\text{PolyLog}[2,1-2/(1-a*x)])/(2*a^2)$

#### Rule 2315

`Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]`

#### Rule 2402

`Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1-2*d*x), x], x, 1/(d+e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]`

#### Rule 5911

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)^p, x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p-1))/(1-c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]`

#### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
  :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
  :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x]
)]^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
]
```

### Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
  x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(ax)^3 dx &= \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{1}{2}(3a) \int \frac{x^2 \coth^{-1}(ax)^2}{1-a^2x^2} dx \\
&= \frac{1}{2}x^2 \coth^{-1}(ax)^3 + \frac{3 \int \coth^{-1}(ax)^2 dx}{2a} - \frac{3 \int \frac{\coth^{-1}(ax)^2}{1-a^2x^2} dx}{2a} \\
&= \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - 3 \int \frac{x \coth^{-1}(ax)}{1-a^2x^2} dx \\
&= \frac{3 \coth^{-1}(ax)^2}{2a^2} + \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{3 \int \frac{\coth^{-1}(ax)}{1-ax} dx}{a} \\
&= \frac{3 \coth^{-1}(ax)^2}{2a^2} + \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax) \log\left(\frac{1-\coth^{-1}(ax)}{1+\coth^{-1}(ax)}\right)}{a^2} \\
&= \frac{3 \coth^{-1}(ax)^2}{2a^2} + \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax) \log\left(\frac{1-\coth^{-1}(ax)}{1+\coth^{-1}(ax)}\right)}{a^2} \\
&= \frac{3 \coth^{-1}(ax)^2}{2a^2} + \frac{3x \coth^{-1}(ax)^2}{2a} - \frac{\coth^{-1}(ax)^3}{2a^2} + \frac{1}{2}x^2 \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax) \log\left(\frac{1-\coth^{-1}(ax)}{1+\coth^{-1}(ax)}\right)}{a^2}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 68, normalized size = 0.72

$$\frac{\coth^{-1}(ax) \left( (a^2x^2 - 1) \coth^{-1}(ax)^2 + 3(ax - 1) \coth^{-1}(ax) - 6 \log\left(1 - e^{-2 \coth^{-1}(ax)}\right) \right) + 3 \text{Li}_2\left(e^{-2 \coth^{-1}(ax)}\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*ArcCoth[a\*x]^3,x]

[Out] (ArcCoth[a\*x]\*(3\*(-1 + a\*x)\*ArcCoth[a\*x] + (-1 + a^2\*x^2)\*ArcCoth[a\*x]^2 - 6\*Log[1 - E^(-2\*ArcCoth[a\*x])]) + 3\*PolyLog[2, E^(-2\*ArcCoth[a\*x])])/(2\*a^2)

**fricas [F]** time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(x \operatorname{arccoth}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^3,x, algorithm="fricas")

[Out] integral(x\*arccoth(a\*x)^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^3,x, algorithm="giac")

[Out] integrate(x\*arccoth(a\*x)^3, x)

maple [C] time = 0.76, size = 3070, normalized size = 32.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(a\*x)^3,x)

[Out]  $\frac{3}{8}I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3\operatorname{arccoth}(a*x)*\ln(1-1/((a*x-1)/(a*x+1))^{(1/2)})+3/8I/a^2\pi\operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2\operatorname{dilog}(1+1/((a*x-1)/(a*x+1))^{(1/2)})+3/8I/a^2\pi\operatorname{csgn}(I/((a*x-1)/(a*x+1))^{(1/2)})^2*\operatorname{csgn}(I*(a*x+1)/(a*x-1))*\operatorname{polylog}(2,-1/((a*x-1)/(a*x+1))^{(1/2)})+3/8I/a^2\pi\operatorname{csgn}(I/((a*x-1)/(a*x+1))^{(1/2)})^2*\operatorname{csgn}(I*(a*x+1)/(a*x-1))*\operatorname{polylog}(2,1/((a*x-1)/(a*x+1))^{(1/2)})-3/8I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1))*\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*\operatorname{polylog}(2,-1/((a*x-1)/(a*x+1))^{(1/2)})-3/4I/a^2\pi\operatorname{csgn}(I/((a*x-1)/(a*x+1))^{(1/2)})*\operatorname{csgn}(I*(a*x+1)/(a*x-1))^2*\operatorname{dilog}(1/((a*x-1)/(a*x+1))^{(1/2)})-3/8I/a^2\pi\operatorname{csgn}(I/((a*x-1)/(a*x+1))^{(1/2)})^2*\operatorname{csgn}(I*(a*x+1)/(a*x-1))*\operatorname{arccoth}(a*x)^2+3/4I/a^2\pi\operatorname{csgn}(I/((a*x-1)/(a*x+1))^{(1/2)})*\operatorname{csgn}(I*(a*x+1)/(a*x-1))^2*\operatorname{arccoth}(a*x)^2-1/2*\operatorname{arccoth}(a*x)^3/a^2+3/8I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*\operatorname{polylog}(2,-1/((a*x-1)/(a*x+1))^{(1/2)})+3/8I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*\operatorname{dilog}(1/((a*x-1)/(a*x+1))^{(1/2)})-3/8I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*\operatorname{dilog}(1+1/((a*x-1)/(a*x+1))^{(1/2)})+3/8I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*\operatorname{polylog}(2,1/((a*x-1)/(a*x+1))^{(1/2)})-3/2/a^2*\operatorname{arccoth}(a*x)*\ln(1-1/((a*x-1)/(a*x+1))^{(1/2)})+3/4/a^2*\operatorname{arccoth}(a*x)^2*\ln(a*x-1)-3/4/a^2*\operatorname{arccoth}(a*x)^2*\ln(a*x+1)-3/a^2*\operatorname{arccoth}(a*x)*\ln(1+1/((a*x-1)/(a*x+1))^{(1/2)})-3/4/a^2*\operatorname{arccoth}(a*x)^2*\ln((a*x-1)/(a*x+1))+3/2*\operatorname{arccoth}(a*x)^2/a^2+3/2*x*\operatorname{arccoth}(a*x)^2/a^2+3/8I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1))*\operatorname{csgn}(I/((a*x+1)/(a*x-1)-1))*\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*\operatorname{arccoth}(a*x)*\ln(1-1/((a*x-1)/(a*x+1))^{(1/2)})-3/8I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1))^3*\operatorname{arccoth}(a*x)^2+1/2*x^2*\operatorname{arccoth}(a*x)^3-3/2/a^2*\operatorname{polylog}(2,1/((a*x-1)/(a*x+1))^{(1/2)})-3/2/a^2*\operatorname{dilog}(1+1/((a*x-1)/(a*x+1))^{(1/2)})+3/2/a^2*\operatorname{dilog}(1/((a*x-1)/(a*x+1))^{(1/2)})-3/2/a^2*\operatorname{polylog}(2,-1/((a*x-1)/(a*x+1))^{(1/2)})-3/8I/a^2\pi\operatorname{csgn}(I*(a*x+1)/(a*x-1))*\operatorname{csgn}(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*\operatorname{dilog}(1/((a*x-1)/(a*x+1))^{(1/2)})+3/$

$$\begin{aligned}
& 4*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*dilog( \\
& 1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a \\
& *x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/8 \\
& *I/a^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x- \\
& 1)-1))^2*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I/((a*x+1)/(a*x \\
& -1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*polylog(2,1/((a*x-1)/ \\
& (a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a* \\
& x-1)/((a*x+1)/(a*x-1)-1))^2*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/4*I/a^2 \\
& *Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*polylog(2,-1/ \\
& ((a*x-1)/(a*x+1))^(1/2))-3/4*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn( \\
& I*(a*x+1)/(a*x-1))^2*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn \\
& (I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*arcco \\
& th(a*x)^2+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x \\
& +1)/(a*x-1)-1))^2*arccoth(a*x)^2+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arc \\
& coth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x- \\
& 1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*polylog(2,1/((a*x-1)/(a*x \\
& +1))^(1/2))+3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/( \\
& a*x-1))*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+ \\
& 1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))-3/8 \\
& *I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)^2+3/8* \\
& I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*dilog(1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a \\
& ^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*dilog(1+1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^ \\
& 2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/ \\
& a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*polylog(2,1/((a*x-1)/(a*x+1))^(1/2))+3/8*I \\
& /a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/ \\
& (a*x-1)/((a*x+1)/(a*x-1)-1))*polylog(2,-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^ \\
& 2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1) \\
& )^2*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1 \\
& )/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)*ln(1- \\
& 1/((a*x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a* \\
& x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*dilog(1+1/((a* \\
& x-1)/(a*x+1))^(1/2))-3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/( \\
& a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*arccoth(a*x)^2+3/8*I \\
& /a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/ \\
& (a*x-1)/((a*x+1)/(a*x-1)-1))*dilog(1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi* \\
& csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)*ln(1 \\
& -1/((a*x-1)/(a*x+1))^(1/2))+3/8*I/a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a \\
& *x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))*polylog(2,1/( \\
& (a*x-1)/(a*x+1))^(1/2))-3/4*I/a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I \\
& *(a*x+1)/(a*x-1))^2*arccoth(a*x)*ln(1-1/((a*x-1)/(a*x+1))^(1/2))
\end{aligned}$$

**maxima** [B] time = 0.33, size = 215, normalized size = 2.26

$$\frac{1}{2} x^2 \operatorname{arccoth}(ax)^3 + \frac{3}{4} a \left( \frac{2x}{a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3} \right) \operatorname{arccoth}(ax)^2 + \frac{1}{16} a \left( \frac{3(\log(ax-1)-2)\log(ax+1)^2 - \log(ax+1)^3 + \log(ax+1)^3 + \log(ax-1)^3 - 3(\log(ax-1)^2 - 4\log(ax-1))\log(ax+1) + 6\log(ax-1)^2}{a} - 24(\log(ax-1)\log(1/2*ax+1/2) + \operatorname{dilog}(-1/2*ax+1/2))/a \right) / a^2 - 6(2(\log(ax-1)-2)\log(ax+1) - \log(ax+1)^2 - \log(ax-1)^2 - 4\log(ax-1))\operatorname{arccoth}(ax) / a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a\*x)^3,x, algorithm="maxima")

[Out] 1/2\*x^2\*arccoth(a\*x)^3 + 3/4\*a\*(2\*x/a^2 - log(a\*x + 1)/a^3 + log(a\*x - 1)/a^3)\*arccoth(a\*x)^2 + 1/16\*a\*((3\*(log(a\*x - 1) - 2)\*log(a\*x + 1)^2 - log(a\*x + 1)^3 + log(a\*x - 1)^3 - 3\*(log(a\*x - 1)^2 - 4\*log(a\*x - 1))\*log(a\*x + 1) + 6\*log(a\*x - 1)^2)/a - 24\*(log(a\*x - 1)\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a)/a^2 - 6\*(2\*(log(a\*x - 1) - 2)\*log(a\*x + 1) - log(a\*x + 1)^2 - log(a\*x - 1)^2 - 4\*log(a\*x - 1))\*arccoth(a\*x)/a^3

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(a\*x)^3,x)

[Out] int(x\*acoth(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(a\*x)\*\*3,x)

[Out] Integral(x\*acoth(a\*x)\*\*3, x)

### 3.28 $\int \coth^{-1}(ax)^3 dx$

**Optimal.** Leaf size=85

$$\frac{3\text{Li}_3\left(1 - \frac{2}{1-ax}\right)}{2a} - \frac{3\text{Li}_2\left(1 - \frac{2}{1-ax}\right)\coth^{-1}(ax)}{a} + x\coth^{-1}(ax)^3 + \frac{\coth^{-1}(ax)^3}{a} - \frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}$$

[Out]  $\text{arccoth}(a*x)^3/a + x*\text{arccoth}(a*x)^3 - 3*\text{arccoth}(a*x)^2*\ln(2/(-a*x+1))/a - 3*\text{arccoth}(a*x)*\text{polylog}(2, 1-2/(-a*x+1))/a + 3/2*\text{polylog}(3, 1-2/(-a*x+1))/a$

**Rubi [A]** time = 0.17, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {5911, 5985, 5919, 5949, 6059, 6610}

$$\frac{3\text{PolyLog}\left(3, 1 - \frac{2}{1-ax}\right)}{2a} - \frac{3\coth^{-1}(ax)\text{PolyLog}\left(2, 1 - \frac{2}{1-ax}\right)}{a} + x\coth^{-1}(ax)^3 + \frac{\coth^{-1}(ax)^3}{a} - \frac{3\log\left(\frac{2}{1-ax}\right)\coth^{-1}(ax)^2}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3, x]

[Out]  $\text{ArcCoth}[a*x]^3/a + x*\text{ArcCoth}[a*x]^3 - (3*\text{ArcCoth}[a*x]^2*\text{Log}[2/(1 - a*x)])/a - (3*\text{ArcCoth}[a*x]*\text{PolyLog}[2, 1 - 2/(1 - a*x)])/a + (3*\text{PolyLog}[3, 1 - 2/(1 - a*x)])/(2*a)$

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

#### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rule 5985

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
  x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \coth^{-1}(ax)^3 dx &= x \coth^{-1}(ax)^3 - (3a) \int \frac{x \coth^{-1}(ax)^2}{1 - a^2x^2} dx \\
&= \frac{\coth^{-1}(ax)^3}{a} + x \coth^{-1}(ax)^3 - 3 \int \frac{\coth^{-1}(ax)^2}{1 - ax} dx \\
&= \frac{\coth^{-1}(ax)^3}{a} + x \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a} + 6 \int \frac{\coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{1 - a^2x^2} dx \\
&= \frac{\coth^{-1}(ax)^3}{a} + x \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a} - \frac{3 \coth^{-1}(ax) \operatorname{Li}_2\left(1 - \frac{2}{1-ax}\right)}{a} + 3 \int \frac{\coth^{-1}(ax) \log\left(\frac{2}{1-ax}\right)}{1 - a^2x^2} dx \\
&= \frac{\coth^{-1}(ax)^3}{a} + x \coth^{-1}(ax)^3 - \frac{3 \coth^{-1}(ax)^2 \log\left(\frac{2}{1-ax}\right)}{a} - \frac{3 \coth^{-1}(ax) \operatorname{Li}_2\left(1 - \frac{2}{1-ax}\right)}{a} + \frac{3 \operatorname{Li}_3\left(e^{2 \coth^{-1}(ax)}\right)}{2a}
\end{aligned}$$

**Mathematica** [A] time = 0.10, size = 79, normalized size = 0.93

$$-\frac{3 \coth^{-1}(ax) \operatorname{Li}_2\left(e^{2 \coth^{-1}(ax)}\right)}{a} + \frac{3 \operatorname{Li}_3\left(e^{2 \coth^{-1}(ax)}\right)}{2a} + x \coth^{-1}(ax)^3 + \frac{\coth^{-1}(ax)^3}{a} - \frac{3 \coth^{-1}(ax)^2 \log\left(1 - e^{2 \coth^{-1}(ax)}\right)}{a}$$

Warning: Unable to verify antiderivative.



[In] Integrate[ArcCoth[a\*x]^3,x]

[Out] ArcCoth[a\*x]^3/a + x\*ArcCoth[a\*x]^3 - (3\*ArcCoth[a\*x]^2\*Log[1 - E^(2\*ArcCoth[a\*x])])/a - (3\*ArcCoth[a\*x]\*PolyLog[2, E^(2\*ArcCoth[a\*x])])/a + (3\*PolyLog[3, E^(2\*ArcCoth[a\*x])])/(2\*a)

**fricas** [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}(\text{arccoth}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{arccoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3, x)

**maple** [B] time = 0.38, size = 180, normalized size = 2.12

$$x \text{arccoth}(ax)^3 + \frac{\text{arccoth}(ax)^3}{a} - \frac{3 \text{arccoth}(ax)^2 \ln\left(1 - \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{a} - \frac{3 \text{arccoth}(ax)^2 \ln\left(1 + \frac{1}{\sqrt{\frac{ax-1}{ax+1}}}\right)}{a} + 6 \text{arccoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3,x)

[Out] x\*arccoth(a\*x)^3+arccoth(a\*x)^3/a-3/a\*arccoth(a\*x)^2\*ln(1-1/((a\*x-1)/(a\*x+1))^(1/2))-3/a\*arccoth(a\*x)^2\*ln(1+1/((a\*x-1)/(a\*x+1))^(1/2))-6/a\*arccoth(a\*x)\*polylog(2,1/((a\*x-1)/(a\*x+1))^(1/2))-6/a\*arccoth(a\*x)\*polylog(2,-1/((a\*x-1)/(a\*x+1))^(1/2))+6/a\*polylog(3,1/((a\*x-1)/(a\*x+1))^(1/2))+6/a\*polylog(3,-1/((a\*x-1)/(a\*x+1))^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ax+1)\log(ax+1)^3 - 3(ax-1)\log(ax+1)^2\log(ax-1)}{8a} + \frac{1}{8} \int -\frac{(ax+1)\log(ax-1)^3 - 3((ax+1)\log(ax-1))}{ax-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3,x, algorithm="maxima")

[Out] 1/8\*((a\*x + 1)\*log(a\*x + 1)^3 - 3\*(a\*x - 1)\*log(a\*x + 1)^2\*log(a\*x - 1))/a + 1/8\*integrate(-((a\*x + 1)\*log(a\*x - 1)^3 - 3\*((a\*x + 1)\*log(a\*x - 1)^2 + 2\*(a\*x - 1)\*log(a\*x - 1))\*log(a\*x + 1))/(a\*x + 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)^3,x)

[Out] int(acoth(a\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*3,x)

[Out] Integral(acoth(a\*x)\*\*3, x)

$$3.29 \quad \int \frac{\coth^{-1}(ax)^3}{x} dx$$

**Optimal.** Leaf size=150

$$\frac{3}{4}\text{Li}_4\left(1 - \frac{2}{ax+1}\right) - \frac{3}{4}\text{Li}_4\left(1 - \frac{2ax}{ax+1}\right) + \frac{3}{2}\text{Li}_2\left(1 - \frac{2}{ax+1}\right)\coth^{-1}(ax)^2 - \frac{3}{2}\text{Li}_2\left(1 - \frac{2ax}{ax+1}\right)\coth^{-1}(ax)^2 + \frac{3}{2}\text{Li}_3\left(1 - \frac{2}{ax+1}\right)\coth^{-1}(ax) - \frac{3}{2}\text{Li}_3\left(1 - \frac{2ax}{ax+1}\right)\coth^{-1}(ax)$$

[Out] 2\*arccoth(a\*x)^3\*arccoth(1-2/(-a\*x+1))+3/2\*arccoth(a\*x)^2\*polylog(2,1-2/(a\*x+1))-3/2\*arccoth(a\*x)^2\*polylog(2,1-2\*a\*x/(a\*x+1))+3/2\*arccoth(a\*x)\*polylog(3,1-2/(a\*x+1))-3/2\*arccoth(a\*x)\*polylog(3,1-2\*a\*x/(a\*x+1))+3/4\*polylog(4,1-2/(a\*x+1))-3/4\*polylog(4,1-2\*a\*x/(a\*x+1))

**Rubi [A]** time = 0.35, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {5915, 6053, 5949, 6057, 6061, 6610}

$$\frac{3}{4}\text{PolyLog}\left(4,1 - \frac{2}{ax+1}\right) - \frac{3}{4}\text{PolyLog}\left(4,1 - \frac{2ax}{ax+1}\right) + \frac{3}{2}\coth^{-1}(ax)^2\text{PolyLog}\left(2,1 - \frac{2}{ax+1}\right) - \frac{3}{2}\coth^{-1}(ax)^2\text{PolyLog}\left(2,1 - \frac{2ax}{ax+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3/x,x]

[Out] 2\*ArcCoth[a\*x]^3\*ArcCoth[1 - 2/(1 - a\*x)] + (3\*ArcCoth[a\*x]^2\*PolyLog[2, 1 - 2/(1 + a\*x)])/2 - (3\*ArcCoth[a\*x]^2\*PolyLog[2, 1 - (2\*a\*x)/(1 + a\*x)])/2 + (3\*ArcCoth[a\*x]\*PolyLog[3, 1 - 2/(1 + a\*x)])/2 - (3\*ArcCoth[a\*x]\*PolyLog[3, 1 - (2\*a\*x)/(1 + a\*x)])/2 + (3\*PolyLog[4, 1 - 2/(1 + a\*x)])/4 - (3\*PolyLog[4, 1 - (2\*a\*x)/(1 + a\*x)])/4

**Rule 5915**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_)/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcCoth[c\*x])^p\*ArcCoth[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*ArcCoth[1 - 2/(1 - c\*x)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]

**Rule 5949**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

**Rule 6053**

Int[(ArcCoth[u\_] \* ((a\_.) + ArcCoth[(c\_.)\*(x\_)]) \* (b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]])\*(a +

```

b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegr
and[1 - 1/u, x]]*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b
, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 -
c*x))^2, 0]

```

### Rule 6057

```

Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^p_)/((d_) + (e_.)*(x_)^
2), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]

```

### Rule 6061

```

Int[(((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^p_*PolyLog[k_, u_)/((d_) + (e_
.)*(x_)^2), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]

```

### Rule 6610

```

Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x} dx &= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) - (6a) \int \frac{\coth^{-1}(ax)^2 \coth^{-1}\left(1 - \frac{2}{1-ax}\right)}{1-a^2x^2} dx \\
&= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) + (3a) \int \frac{\coth^{-1}(ax)^2 \log\left(\frac{2}{1+ax}\right)}{1-a^2x^2} dx - (3a) \int \frac{\coth^{-1}(ax)}{1-a^2x^2} dx \\
&= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) + \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1+ax}\right) - \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1-ax}\right) \\
&= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) + \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1+ax}\right) - \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1-ax}\right) \\
&= 2 \coth^{-1}(ax)^3 \coth^{-1}\left(1 - \frac{2}{1-ax}\right) + \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1+ax}\right) - \frac{3}{2} \coth^{-1}(ax)^2 \text{Li}_2\left(1 - \frac{2}{1-ax}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 156, normalized size = 1.04

$$\frac{1}{64} \left( -96 \coth^{-1}(ax)^2 \text{Li}_2\left(-e^{-2 \coth^{-1}(ax)}\right) - 96 \coth^{-1}(ax)^2 \text{Li}_2\left(e^{2 \coth^{-1}(ax)}\right) - 96 \coth^{-1}(ax) \text{Li}_3\left(-e^{-2 \coth^{-1}(ax)}\right) + \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x,x]

[Out]  $(-\text{Pi}^4 + 32 \text{ArcCoth}[a*x]^4 + 64 \text{ArcCoth}[a*x]^3 \text{Log}[1 + \text{E}^{-2 \text{ArcCoth}[a*x]}]) - 64 \text{ArcCoth}[a*x]^3 \text{Log}[1 - \text{E}^{2 \text{ArcCoth}[a*x]}] - 96 \text{ArcCoth}[a*x]^2 \text{PolyLog}[2, -\text{E}^{-2 \text{ArcCoth}[a*x]}] - 96 \text{ArcCoth}[a*x]^2 \text{PolyLog}[2, \text{E}^{2 \text{ArcCoth}[a*x]}] - 96 \text{ArcCoth}[a*x] \text{PolyLog}[3, -\text{E}^{-2 \text{ArcCoth}[a*x]}] + 96 \text{ArcCoth}[a*x] \text{PolyLog}[3, \text{E}^{2 \text{ArcCoth}[a*x]}] - 48 \text{PolyLog}[4, -\text{E}^{-2 \text{ArcCoth}[a*x]}] - 48 \text{PolyLog}[4, \text{E}^{2 \text{ArcCoth}[a*x]}]) / 64$

**fricas [F]** time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(ax)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^3/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3/x, x)

**maple** [C] time = 0.55, size = 564, normalized size = 3.76

$$\ln(ax) \operatorname{arccoth}(ax)^3 + \operatorname{arccoth}(ax)^3 \ln\left(\frac{ax+1}{ax-1} - 1\right) + \frac{3 \operatorname{arccoth}(ax)^2 \operatorname{polylog}\left(2, -\frac{ax+1}{ax-1}\right)}{2} - \frac{3 \operatorname{arccoth}(ax) \operatorname{polylog}\left(3, -\frac{ax+1}{ax-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3/x,x)

[Out] ln(a\*x)\*arccoth(a\*x)^3+arccoth(a\*x)^3\*ln((a\*x+1)/(a\*x-1)-1)+3/2\*arccoth(a\*x)^2\*polylog(2,-(a\*x+1)/(a\*x-1))-3/2\*arccoth(a\*x)\*polylog(3,-(a\*x+1)/(a\*x-1))+3/4\*polylog(4,-(a\*x+1)/(a\*x-1))+1/2\*I\*Pi\*csgn(I\*(1+(a\*x+1)/(a\*x-1)))\*csgn(I/((a\*x+1)/(a\*x-1)-1))\*csgn(I/((a\*x+1)/(a\*x-1)-1)\*(1+(a\*x+1)/(a\*x-1)))\*arccoth(a\*x)^3-1/2\*I\*Pi\*csgn(I/((a\*x+1)/(a\*x-1)-1))\*csgn(I/((a\*x+1)/(a\*x-1)-1)\*(1+(a\*x+1)/(a\*x-1)))^2\*arccoth(a\*x)^3-1/2\*I\*Pi\*csgn(I\*(1+(a\*x+1)/(a\*x-1)))\*csgn(I/((a\*x+1)/(a\*x-1)-1)\*(1+(a\*x+1)/(a\*x-1)))^2\*arccoth(a\*x)^3+1/2\*I\*Pi\*csgn(I/((a\*x+1)/(a\*x-1)-1)\*(1+(a\*x+1)/(a\*x-1)))^3\*arccoth(a\*x)^3-arccoth(a\*x)^3\*ln(1-1/((a\*x-1)/(a\*x+1))^(1/2))-3\*arccoth(a\*x)^2\*polylog(2,1/((a\*x-1)/(a\*x+1))^(1/2))+6\*arccoth(a\*x)\*polylog(3,1/((a\*x-1)/(a\*x+1))^(1/2))-6\*polylog(4,1/((a\*x-1)/(a\*x+1))^(1/2))-arccoth(a\*x)^3\*ln(1+1/((a\*x-1)/(a\*x+1))^(1/2))-3\*arccoth(a\*x)^2\*polylog(2,-1/((a\*x-1)/(a\*x+1))^(1/2))+6\*arccoth(a\*x)\*polylog(3,-1/((a\*x-1)/(a\*x+1))^(1/2))-6\*polylog(4,-1/((a\*x-1)/(a\*x+1))^(1/2))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x,x, algorithm="maxima")

[Out] integrate(arccoth(a\*x)^3/x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{arccoth}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)^3/x, x)

[Out] int(acoth(a\*x)^3/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}^3(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*3/x, x)

[Out] Integral(acoth(a\*x)\*\*3/x, x)

$$3.30 \quad \int \frac{\coth^{-1}(ax)^3}{x^2} dx$$

**Optimal.** Leaf size=79

$$-\frac{3}{2}a\text{Li}_3\left(\frac{2}{ax+1}-1\right)-3a\text{Li}_2\left(\frac{2}{ax+1}-1\right)\coth^{-1}(ax)+a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)$$

[Out] a\*arccoth(a\*x)^3-arccoth(a\*x)^3/x+3\*a\*arccoth(a\*x)^2\*ln(2-2/(a\*x+1))-3\*a\*arccoth(a\*x)\*polylog(2,-1+2/(a\*x+1))-3/2\*a\*polylog(3,-1+2/(a\*x+1))

**Rubi [A]** time = 0.20, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {5917, 5989, 5933, 5949, 6057, 6610}

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\coth^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+a\coth^{-1}(ax)^3-\frac{\coth^{-1}(ax)^3}{x}+3a\log\left(2-\frac{2}{ax+1}\right)\coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3/x^2,x]

[Out] a\*ArcCoth[a\*x]^3 - ArcCoth[a\*x]^3/x + 3\*a\*ArcCoth[a\*x]^2\*Log[2 - 2/(1 + a\*x)] - 3\*a\*ArcCoth[a\*x]\*PolyLog[2, -1 + 2/(1 + a\*x)] - (3\*a\*PolyLog[3, -1 + 2/(1 + a\*x)])/2

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)/((d_.) + (e_.)*(x_)), x
_Symbol] :> Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]]
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)/((d_.) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```



, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5989

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)), x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcCoth[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 6057

Int[(Log[u\_]\*((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] - Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax)^3}{x^2} dx &= -\frac{\coth^{-1}(ax)^3}{x} + (3a) \int \frac{\coth^{-1}(ax)^2}{x(1 - a^2x^2)} dx \\ &= a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + (3a) \int \frac{\coth^{-1}(ax)^2}{x(1 + ax)} dx \\ &= a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a \coth^{-1}(ax)^2 \log\left(2 - \frac{2}{1 + ax}\right) - (6a^2) \int \frac{\coth^{-1}(ax) \log}{1 - a^2} \\ &= a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a \coth^{-1}(ax)^2 \log\left(2 - \frac{2}{1 + ax}\right) - 3a \coth^{-1}(ax) \text{Li}_2\left(-1 + \right. \\ &= a \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{x} + 3a \coth^{-1}(ax)^2 \log\left(2 - \frac{2}{1 + ax}\right) - 3a \coth^{-1}(ax) \text{Li}_2\left(-1 + \right. \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 72, normalized size = 0.91

$$-3a \coth^{-1}(ax) \text{Li}_2\left(-e^{-2 \coth^{-1}(ax)}\right) - \frac{3}{2} a \text{Li}_3\left(-e^{-2 \coth^{-1}(ax)}\right) + \frac{(ax - 1) \coth^{-1}(ax)^3}{x} + 3a \coth^{-1}(ax)^2 \log\left(e^{-2 \coth^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x^2,x]

[Out]  $((-1 + a*x)*\text{ArcCoth}[a*x]^3)/x + 3*a*\text{ArcCoth}[a*x]^2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[a*x])}] - 3*a*\text{ArcCoth}[a*x]*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[a*x])}] - (3*a*\text{PolyLog}[3, -E^{(-2*\text{ArcCoth}[a*x])}])/2$

**fricas** [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(ax)^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^2,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^3/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(ax)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^2,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3/x^2, x)

**maple** [C] time = 0.70, size = 796, normalized size = 10.08

$$-\frac{\text{arccoth}(ax)^3}{x} + 3a\text{arccoth}(ax)^2 \ln(ax) - \frac{3a\text{arccoth}(ax)^2 \ln(ax-1)}{2} - \frac{3a\text{arccoth}(ax)^2 \ln(ax+1)}{2} - \frac{3a\text{arccoth}(ax)^2 \ln\left(\frac{ax-1}{ax+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3/x^2,x)

[Out]  $-\text{arccoth}(a*x)^3/x + 3*a*\text{arccoth}(a*x)^2*\ln(a*x) - 3/2*a*\text{arccoth}(a*x)^2*\ln(a*x-1) - 3/2*a*\text{arccoth}(a*x)^2*\ln(a*x+1) - 3/2*a*\text{arccoth}(a*x)^2*\ln\left(\frac{a*x-1}{a*x+1}\right) - a*\text{arccoth}(a*x)^3 + 3*a*\text{arccoth}(a*x)*\text{polylog}(2, -(a*x+1)/(a*x-1)) - 3/2*a*\text{polylog}(3, -(a*x+1)/(a*x-1)) - 3/4*I*a*\text{arccoth}(a*x)^2*\text{Pi}*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1)) - 3/2*I*a*\text{arccoth}(a*x)^2*\text{Pi}*csgn(I/((a*x+1)/(a*x-1)-1))*(1+(a*x+1)/(a*x-1))^2*csgn(I*$

$$1+(a*x+1)/(a*x-1))-3/2*I*a*arccoth(a*x)^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))^2*csgn(I/((a*x+1)/(a*x-1)-1))+3/4*I*a*arccoth(a*x)^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2-3/4*I*a*arccoth(a*x)^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3+3/2*I*a*arccoth(a*x)^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))^3+3/4*I*a*arccoth(a*x)^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2-3/4*I*a*arccoth(a*x)^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3+3/2*I*a*arccoth(a*x)^2*Pi*csgn(I*(a*x+1)/(a*x-1))^2*csgn(I/((a*x-1)/(a*x+1)))^(1/2))-3/4*I*a*arccoth(a*x)^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x-1)/(a*x+1)))^(1/2))^2+3/2*I*a*arccoth(a*x)^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1)*(1+(a*x+1)/(a*x-1)))^2*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(1+(a*x+1)/(a*x-1)))+3*a*arccoth(a*x)^2*ln(2)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^2,x, algorithm="maxima")

[Out]  $1/8*a*(\log(a*x + 1) - \log(x))*\log(a)^3 + 3/8*a*\int(x*\log(a*x - 1)/(a*x^3 + x^2), x)*\log(a)^2 - 3/8*a*\int(x*\log(x)/(a*x^3 + x^2), x)*\log(a)^2 - 1/8*(a*\log(a*x + 1) - a*\log(x) - 1/x)*\log(a)^3 + 3/4*a^2*\int(x^2*\log(a*x + 1)*\log(a*x - 1)/(a*x^3 + x^2), x) - 3/2*a^2*\int(x^2*\log(a*x + 1)*\log(x)/(a*x^3 + x^2), x) + 3/4*a*\int(x*\log(a*x - 1)*\log(x)/(a*x^3 + x^2), x)*\log(a) - 3/8*a*\int(x*\log(x)^2/(a*x^3 + x^2), x)*\log(a) + 3/8*\int(\log(a*x - 1)/(a*x^3 + x^2), x)*\log(a)^2 - 3/8*\int(\log(x)/(a*x^3 + x^2), x)*\log(a)^2 + 3/8*a*\int(x*\log(a*x + 1)*\log(a*x - 1)^2/(a*x^3 + x^2), x) - 3/8*a*\int(x*\log(a*x - 1)^2*\log(x)/(a*x^3 + x^2), x) + 3/8*a*\int(x*\log(a*x - 1)*\log(x)^2/(a*x^3 + x^2), x) - 1/8*a*\int(x*\log(x)^3/(a*x^3 + x^2), x) - 3/4*a*\int(x*\log(a*x + 1)*\log(a*x - 1)/(a*x^3 + x^2), x) - 3/8*\int(a*x*\log(a*x - 1)^2/(a*x^3 + x^2), x)*\log(a) - 3/8*\int(\log(a*x - 1)^2/(a*x^3 + x^2), x)*\log(a) + 3/4*\int(\log(a*x - 1)*\log(x)/(a*x^3 + x^2), x)*\log(a) - 3/8*\int(\log(x)^2/(a*x^3 + x^2), x)*\log(a) - 3/8*(a^2*\log(a*x - 1) - a^2*\log(x) + a/x)*\log(-1/(a*x) + 1)^2/a + 1/8*\log(-1/(a*x) + 1)^3/x - 1/8*((a*x + 1)*\log(a*x + 1)^3 - 3*(2*a*x*\log(x) - (a*x - 1)*\log(a*x - 1))*\log(a*x + 1)^2)/x + 1/8*(3*(a^3*x*\log(a*x - 1)^2 + a^3*x*\log(x)^2 - 2*a^3*x*\log(x) + 2*a^2 - 2*(a^3*x*\log(x) - a^3*x)*\log(a*x - 1))*\log(-1/(a*x) + 1)/(a*x) - (a^4*x*\log(a*x - 1)^3 - a^4*x*\log(x)^3 + 3*a^4*x*\log(x)^2 - 6*a^4*x*\log(x) + 6*a^3 - 3*(a^4*x*\log(x) - a^4*x)*\log(a*x - 1)^2 + 3*(a^4*x*\log(x)^2 - 2*a^4*x*\log(x) + 2*a^4*x)*\log(a*x - 1))/(a^2*x))/a + 3/8*\int(\log(a*x + 1)*\log(a*x - 1)^2/(a*x^3 + x^2), x) - 3/8*\int(\log(a*x - 1)^2*\log(x)/(a*x^3 + x^2), x) + 3/8*\int(\log(a*x - 1)*\log(x)^2/(a*x^3 + x^2), x) - 1/8*\int(\log(x)^3/(a*x^3 + x^2), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(ax)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)^3/x^2, x)`

[Out] `int(acoth(a*x)^3/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)**3/x**2, x)`

[Out] `Integral(acoth(a*x)**3/x**2, x)`

$$3.31 \quad \int \frac{\coth^{-1}(ax)^3}{x^3} dx$$

**Optimal.** Leaf size=95

$$-\frac{3}{2}a^2 \operatorname{Li}_2\left(\frac{2}{ax+1} - 1\right) + \frac{1}{2}a^2 \coth^{-1}(ax)^3 + \frac{3}{2}a^2 \coth^{-1}(ax)^2 + 3a^2 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{\coth^{-1}(ax)^3}{2x^2} - \frac{3a^2}{2x^2}$$

[Out]  $3/2*a^2*\operatorname{arccoth}(a*x)^2 - 3/2*a*\operatorname{arccoth}(a*x)^2/x + 1/2*a^2*\operatorname{arccoth}(a*x)^3 - 1/2*a*\operatorname{arccoth}(a*x)^3/x^2 + 3*a^2*\operatorname{arccoth}(a*x)*\ln(2-2/(a*x+1)) - 3/2*a^2*\operatorname{polylog}(2, -1+2/(a*x+1))$

**Rubi [A]** time = 0.22, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {5917, 5983, 5989, 5933, 2447, 5949}

$$-\frac{3}{2}a^2 \operatorname{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) + \frac{1}{2}a^2 \coth^{-1}(ax)^3 + \frac{3}{2}a^2 \coth^{-1}(ax)^2 + 3a^2 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{\coth^{-1}(ax)^3}{2x^2}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[a*x]^3/x^3, x]`

[Out]  $(3*a^2*\operatorname{ArcCoth}[a*x]^2)/2 - (3*a*\operatorname{ArcCoth}[a*x]^2)/(2*x) + (a^2*\operatorname{ArcCoth}[a*x]^3)/2 - \operatorname{ArcCoth}[a*x]^3/(2*x^2) + 3*a^2*\operatorname{ArcCoth}[a*x]*\operatorname{Log}[2 - 2/(1 + a*x)] - (3*a^2*\operatorname{PolyLog}[2, -1 + 2/(1 + a*x)])/2$

**Rule 2447**

`Int[Log[u]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]`

**Rule 5917**

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]`

**Rule 5933**

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((x_)*((d_.) + (e_.)*(x_))), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -`

Dist[(b\*c\*p)/d, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5983

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((f\_.)\*(x\_.))^(m\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 5989

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcCoth[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)^3}{x^3} dx &= -\frac{\coth^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a) \int \frac{\coth^{-1}(ax)^2}{x^2(1 - a^2x^2)} dx \\
 &= -\frac{\coth^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a) \int \frac{\coth^{-1}(ax)^2}{x^2} dx + \frac{1}{2}(3a^3) \int \frac{\coth^{-1}(ax)^2}{1 - a^2x^2} dx \\
 &= -\frac{3a \coth^{-1}(ax)^2}{2x} + \frac{1}{2}a^2 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{2x^2} + (3a^2) \int \frac{\coth^{-1}(ax)}{x(1 - a^2x^2)} dx \\
 &= \frac{3}{2}a^2 \coth^{-1}(ax)^2 - \frac{3a \coth^{-1}(ax)^2}{2x} + \frac{1}{2}a^2 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{2x^2} + (3a^2) \int \frac{\coth^{-1}(ax)}{x(1 + ax)} dx \\
 &= \frac{3}{2}a^2 \coth^{-1}(ax)^2 - \frac{3a \coth^{-1}(ax)^2}{2x} + \frac{1}{2}a^2 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{2x^2} + 3a^2 \coth^{-1}(ax) \log \left( \frac{1 + ax}{1 - ax} \right) \\
 &= \frac{3}{2}a^2 \coth^{-1}(ax)^2 - \frac{3a \coth^{-1}(ax)^2}{2x} + \frac{1}{2}a^2 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{2x^2} + 3a^2 \coth^{-1}(ax) \log \left( \frac{1 + ax}{1 - ax} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 79, normalized size = 0.83

$$\frac{1}{2} \left( \frac{\coth^{-1}(ax) \left( (a^2x^2 - 1) \coth^{-1}(ax)^2 + 6a^2x^2 \log \left( e^{-2\coth^{-1}(ax)} + 1 \right) + 3ax(ax - 1) \coth^{-1}(ax) \right)}{x^2} - 3a^2 \text{Li}_2 \left( -e^{-2\coth^{-1}(ax)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x^3,x]

[Out] ((ArcCoth[a\*x]\*(3\*a\*x\*(-1 + a\*x)\*ArcCoth[a\*x] + (-1 + a^2\*x^2)\*ArcCoth[a\*x]^2 + 6\*a^2\*x^2\*Log[1 + E^(-2\*ArcCoth[a\*x])]))/x^2 - 3\*a^2\*PolyLog[2, -E^(-2\*ArcCoth[a\*x])])/2

**fricas [F]** time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\text{arccoth}(ax)^3}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^3,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^3/x^3, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(ax)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^3,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3/x^3, x)

**maple [C]** time = 0.86, size = 3673, normalized size = 38.66

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3/x^3,x)

[Out] 1/2\*a^2\*arccoth(a\*x)^3-3/2\*a^2\*arccoth(a\*x)^2-3/2\*a\*arccoth(a\*x)^2/x+3/2\*a^2\*arccoth(a\*x)\*ln(1+I/((a\*x-1)/(a\*x+1))^(1/2))+3/2\*a^2\*arccoth(a\*x)\*ln(1-I/((a\*x-1)/(a\*x+1))^(1/2))+3/2\*a^2\*arccoth(a\*x)\*ln(1+(a\*x+1)/(a\*x-1))-3/4\*a^2

$$\begin{aligned}
& * \operatorname{arccoth}(ax)^2 \ln(ax-1) + 3/4 a^2 \operatorname{arccoth}(ax)^2 \ln(ax+1) + 3/4 a^2 \operatorname{arccoth}(ax) \\
& * \ln((ax-1)/(ax+1)) + 3/8 I a^2 \pi \operatorname{csgn}(I((ax+1)/(ax-1)-1)) * \operatorname{csgn}(I * \\
& (ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^{1/2} \operatorname{arccoth}(ax) * \ln(1+(ax+1)/(ax-1)) + 3 \\
& /4 I a^2 \pi \operatorname{csgn}(I((ax-1)/(ax+1))^{1/2}) * \operatorname{csgn}(I * (ax+1)/(ax-1))^{1/2} \operatorname{arcco} \\
& \operatorname{th}(ax) * \ln(1+(ax+1)/(ax-1)) - 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I * ( \\
& ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^{1/2} \operatorname{arccoth}(ax) * \ln(1-I/((ax-1)/(ax+1)) \\
& ^{1/2}) - 3/4 I a^2 \pi \operatorname{csgn}(I((ax-1)/(ax+1))^{1/2}) * \operatorname{csgn}(I * (ax+1)/(ax-1) \\
& )^{1/2} \operatorname{arccoth}(ax) * \ln(1-I/((ax-1)/(ax+1))^{1/2}) - 3/8 I a^2 \pi \operatorname{csgn}(I((ax+ \\
& 1)/(ax-1)-1)) * \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^{1/2} \operatorname{arccoth}(ax) * \ln \\
& (1+I/((ax-1)/(ax+1))^{1/2}) - 3/8 I a^2 \pi \operatorname{csgn}(I((ax+1)/(ax-1)-1)) * \operatorname{csgn} \\
& (I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^{1/2} \operatorname{arccoth}(ax) * \ln(1-I/((ax-1)/(ax \\
& +1))^{1/2}) - 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I((ax+1)/(ax-1)-1 \\
& )) * \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1)) \operatorname{arccoth}(ax) * \ln(1+(ax+1)/(a \\
& x-1)) + 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I((ax+1)/(ax-1)-1)) * \operatorname{csgn} \\
& (I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1)) \operatorname{arccoth}(ax) * \ln(1-I/((ax-1)/(ax+ \\
& 1))^{1/2}) + 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I((ax+1)/(ax-1)-1)) \\
& * \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1)) \operatorname{arccoth}(ax) * \ln(1+I/((ax-1)/( \\
& ax+1))^{1/2}) - 3/16 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^{3/2} \\
& \operatorname{polylog}(2, -(ax+1)/(ax-1)) + 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1))^{3/2} \operatorname{arccoth}( \\
& ax)^2 + 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^{3/2} \operatorname{arccoth}(a \\
& x)^2 - 3/16 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1))^{3/2} \operatorname{polylog}(2, -(ax+1)/(ax-1)) + 3 \\
& /8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1))^{3/2} \operatorname{dilog}(1-I/((ax-1)/(ax+1))^{1/2}) + 3/ \\
& 8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1))^{3/2} \operatorname{dilog}(1+I/((ax-1)/(ax+1))^{1/2}) + 3/8 \\
& * I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I((ax+1)/(ax-1)-1)) * \operatorname{csgn}(I * (ax+1) \\
& )/(ax-1)/((ax+1)/(ax-1)-1)) \operatorname{dilog}(1+I/((ax-1)/(ax+1))^{1/2}) + 3/2 a^2 \operatorname{d} \\
& \operatorname{ilog}(1-I/((ax-1)/(ax+1))^{1/2}) + 3/2 a^2 \operatorname{dilog}(1+I/((ax-1)/(ax+1))^{1/2}) \\
& ) + 3/4 a^2 \operatorname{polylog}(2, -(ax+1)/(ax-1)) + 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)/ \\
& (ax+1)/(ax-1)-1))^{3/2} \operatorname{dilog}(1+I/((ax-1)/(ax+1))^{1/2}) + 3/8 I a^2 \pi \operatorname{csgn}( \\
& I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1))^{3/2} \operatorname{dilog}(1-I/((ax-1)/(ax+1))^{1/2}) \\
& - 1/2 \operatorname{arccoth}(ax)^3/x^2 - 3/8 I a^2 \pi \operatorname{csgn}(I((ax-1)/(ax+1))^{1/2})^{1/2} \operatorname{csgn} \\
& (I * (ax+1)/(ax-1)) \operatorname{arccoth}(ax) * \ln(1+(ax+1)/(ax-1)) + 3/8 I a^2 \pi \operatorname{csgn}(I/ \\
& ((ax-1)/(ax+1))^{1/2})^{1/2} \operatorname{csgn}(I * (ax+1)/(ax-1)) \operatorname{arccoth}(ax) * \ln(1-I/((ax \\
& -1)/(ax+1))^{1/2}) - 3/4 I a^2 \pi \operatorname{csgn}(I((ax-1)/(ax+1))^{1/2}) * \operatorname{csgn}(I * (a \\
& x+1)/(ax-1))^{1/2} \operatorname{arccoth}(ax) * \ln(1+I/((ax-1)/(ax+1))^{1/2}) + 3/8 I a^2 \pi \operatorname{csgn} \\
& (I((ax-1)/(ax+1))^{1/2})^{1/2} \operatorname{csgn}(I * (ax+1)/(ax-1)) \operatorname{arccoth}(ax) * \ln(1 \\
& +I/((ax-1)/(ax+1))^{1/2}) - 3/16 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I(( \\
& ax+1)/(ax-1)-1)) * \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1)) \operatorname{polylog}(2, -( \\
& ax+1)/(ax-1)) + 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I((ax+1)/(ax-1) \\
& -1)) * \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax+1)/(ax-1)-1)) \operatorname{dilog}(1-I/((ax-1)/(ax+1) \\
& ))^{1/2}) + 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax \\
& +1)/(ax-1)-1))^{1/2} \operatorname{arccoth}(ax) * \ln(1+(ax+1)/(ax-1)) + 3/8 I a^2 \pi \operatorname{csgn}(I * (a \\
& x+1)/(ax-1)) * \operatorname{csgn}(I((ax+1)/(ax-1)-1)) * \operatorname{csgn}(I * (ax+1)/(ax-1)/((ax+1) \\
& / \\
& (ax-1)-1)) \operatorname{arccoth}(ax)^2 - 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1)) * \operatorname{csgn}(I * (ax \\
& +1)/(ax-1)/((ax+1)/(ax-1)-1))^{1/2} \operatorname{arccoth}(ax) * \ln(1+I/((ax-1)/(ax+1))^{1 \\
& /2}) + 3/8 I a^2 \pi \operatorname{csgn}(I * (ax+1)/(ax-1))^{3/2} \operatorname{arccoth}(ax) * \ln(1-I/((ax-1)/(a
\end{aligned}$$



```

*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a*x-1)/
(a*x+1)/(a*x-1)-1))^2*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*csgn(
I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*arccoth(a*x)^2-3/8*I*a
^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-
1))^2*arccoth(a*x)^2-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(a*x+1)/(a
*x-1)/((a*x+1)/(a*x-1)-1))^2*arccoth(a*x)^2+3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a
*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))-3
/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth(a*x)*ln(1+(a*x+1)/(a*x-1))+3/
8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x+1)/(a*x-1))*dilog(
1+I/((a*x-1)/(a*x+1))^(1/2))-3/4*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*c
sgn(I*(a*x+1)/(a*x-1))^2*dilog(1+I/((a*x-1)/(a*x+1))^(1/2))-3/4*I*a^2*Pi*cs
gn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*dilog(1-I/((a*x-1)/
(a*x+1))^(1/2))-3/16*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))^2*csgn(I*(a*x
+1)/(a*x-1))*polylog(2,-(a*x+1)/(a*x-1))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1
))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*dilog(1-I/((a*x-1)/(a*x+1
))^(1/2))+3/8*I*a^2*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1
))^2*polylog(2,-(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/((a*x+
1)/(a*x-1)-1))^3*arccoth(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))+3/8*I*a^2*Pi*
csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^3*arccoth(a*x)*ln(1-I/((a*x-1)/
(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a
*x-1)/((a*x+1)/(a*x-1)-1))^2*dilog(1-I/((a*x-1)/(a*x+1))^(1/2))+3/16*I*a^2*P
i*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2
*polylog(2,-(a*x+1)/(a*x-1))+3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))^3*arccoth
(a*x)*ln(1+I/((a*x-1)/(a*x+1))^(1/2))-3/8*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1)/
(a*x+1)/(a*x-1)-1))^3*arccoth(a*x)*ln(1+(a*x+1)/(a*x-1))-3/8*I*a^2*Pi*csgn(
I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*dilog(
1+I/((a*x-1)/(a*x+1))^(1/2))+3/16*I*a^2*Pi*csgn(I*(a*x+1)/(a*x-1))*csgn(I*(
a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1))^2*polylog(2,-(a*x+1)/(a*x-1))-3/4*I*a^2
*Pi*csgn(I/((a*x-1)/(a*x+1))^(1/2))*csgn(I*(a*x+1)/(a*x-1))^2*arccoth(a*x)^
2

```

**maxima [B]** time = 0.33, size = 252, normalized size = 2.65

$$\frac{3}{4} \left( a \log(ax+1) - a \log(ax-1) - \frac{2}{x} \right) a \operatorname{arccoth}(ax)^2 - \frac{1}{16} \left( a^2 \left( \frac{3(\log(ax-1)-2)\log(ax+1)^2 - \log(ax+1)^3 + \log(ax-1)^3 - 3(\log(ax-1)^2 - 4\log(ax-1))\log(ax+1) + 6\log(ax-1)^2}{a} - 24(1 - \log(ax-1))\log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{dilog}\left(-\frac{1}{2}ax + \frac{1}{2}\right)\right) \right) / a + 24(\log(ax+1) - \log(ax-1))\log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \operatorname{dilog}\left(-\frac{1}{2}ax + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^3,x, algorithm="maxima")

[Out] 3/4\*(a\*log(a\*x + 1) - a\*log(a\*x - 1) - 2/x)\*a\*arccoth(a\*x)^2 - 1/16\*(a^2\*((3\*(log(a\*x - 1) - 2)\*log(a\*x + 1)^2 - log(a\*x + 1)^3 + log(a\*x - 1)^3 - 3\*(log(a\*x - 1)^2 - 4\*log(a\*x - 1))\*log(a\*x + 1) + 6\*log(a\*x - 1)^2)/a - 24\*(1 - log(a\*x - 1))\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))/a + 24\*(log(a\*x + 1) - log(a\*x - 1))\*log(1/2\*a\*x + 1/2) + dilog(-1/2\*a\*x + 1/2))

) $\log(x) + \operatorname{dilog}(-ax)/a - 24(\log(-ax + 1)\log(x) + \operatorname{dilog}(ax))/a) - 6(2(\log(ax - 1) - 2)\log(ax + 1) - \log(ax + 1)^2 - \log(ax - 1)^2 - 4\log(ax - 1) + 8\log(x))a\operatorname{arccoth}(ax))a - 1/2\operatorname{arccoth}(ax)^3/x^2$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(ax)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(ax)^3/x^3,x)`

[Out] `int(acoth(ax)^3/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(ax)**3/x**3,x)`

[Out] `Integral(acoth(ax)**3/x**3, x)`

$$3.32 \quad \int \frac{\coth^{-1}(ax)^3}{x^4} dx$$

**Optimal.** Leaf size=154

$$-\frac{1}{2}a^3\text{Li}_3\left(\frac{2}{ax+1}-1\right)-a^3\text{Li}_2\left(\frac{2}{ax+1}-1\right)\coth^{-1}(ax)+a^3\log(x)+\frac{1}{3}a^3\coth^{-1}(ax)^3+\frac{1}{2}a^3\coth^{-1}(ax)^2+a^3\log\left(2\right)$$

[Out]  $-a^2\text{arccoth}(a*x)/x+1/2*a^3\text{arccoth}(a*x)^2-1/2*a*\text{arccoth}(a*x)^2/x^2+1/3*a^3*\text{arccoth}(a*x)^3-1/3*\text{arccoth}(a*x)^3/x^3+a^3*\ln(x)-1/2*a^3*\ln(-a^2*x^2+1)+a^3*\text{arccoth}(a*x)^2*\ln(2-2/(a*x+1))-a^3*\text{arccoth}(a*x)*\text{polylog}(2,-1+2/(a*x+1))-1/2*a^3*\text{polylog}(3,-1+2/(a*x+1))$

**Rubi [A]** time = 0.37, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$ , Rules used = {5917, 5983, 266, 36, 29, 31, 5949, 5989, 5933, 6057, 6610}

$$-\frac{1}{2}a^3\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-a^3\coth^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{1}{2}a^3\log(1-a^2x^2)+a^3\log(x)+\frac{1}{3}a^3\coth^{-1}(ax)^3$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3/x^4,x]

[Out]  $-((a^2*\text{ArcCoth}[a*x])/x) + (a^3*\text{ArcCoth}[a*x]^2)/2 - (a*\text{ArcCoth}[a*x]^2)/(2*x^2) + (a^3*\text{ArcCoth}[a*x]^3)/3 - \text{ArcCoth}[a*x]^3/(3*x^3) + a^3*\text{Log}[x] - (a^3*\text{Log}[1 - a^2*x^2])/2 + a^3*\text{ArcCoth}[a*x]^2*\text{Log}[2 - 2/(1 + a*x)] - a^3*\text{ArcCoth}[a*x]*\text{PolyLog}[2, -1 + 2/(1 + a*x)] - (a^3*\text{PolyLog}[3, -1 + 2/(1 + a*x)])/2$

**Rule 29**

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

**Rule 31**

Int[((a\_) + (b\_)\*(x\_))^(-1), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 36**

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5917

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)*((d_)*(x_))^(m_), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5933

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)/((x_)*((d_) + (e_)*(x_))), x
_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]
)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5949

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)/((d_) + (e_)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5983

```
Int[(((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)*((f_)*(x_))^(m_))/((d_) + (
e_)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5989

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_)/((x_)*((d_) + (e_)*(x_)^2)),
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6057

```
Int[(Log[u_]*((a_) + ArcCoth[(c_)*(x_)]*(b_))^(p_))/((d_) + (e_)*(x_)^
```

```

2), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]

```

### Rule 6610

```

Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x^4} dx &= -\frac{\coth^{-1}(ax)^3}{3x^3} + a \int \frac{\coth^{-1}(ax)^2}{x^3(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^3}{3x^3} + a \int \frac{\coth^{-1}(ax)^2}{x^3} dx + a^3 \int \frac{\coth^{-1}(ax)^2}{x(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^2 \int \frac{\coth^{-1}(ax)}{x^2(1-a^2x^2)} dx + a^3 \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^3 \coth^{-1}(ax)^2 \log\left(2 - \frac{2}{1+ax}\right) + a^2 \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= -\frac{a^2 \coth^{-1}(ax)}{x} + \frac{1}{2}a^3 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^2 \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= -\frac{a^2 \coth^{-1}(ax)}{x} + \frac{1}{2}a^3 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^2 \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= -\frac{a^2 \coth^{-1}(ax)}{x} + \frac{1}{2}a^3 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^2 \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx \\
&= -\frac{a^2 \coth^{-1}(ax)}{x} + \frac{1}{2}a^3 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^3 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{3x^3} + a^2 \int \frac{\coth^{-1}(ax)}{x(1-a^2x^2)} dx
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 142, normalized size = 0.92

$$\frac{1}{6} \left( -6a^3 \coth^{-1}(ax) \text{Li}_2\left(-e^{-2 \coth^{-1}(ax)}\right) - 3a^3 \text{Li}_3\left(-e^{-2 \coth^{-1}(ax)}\right) + 2a^3 \coth^{-1}(ax)^3 + 3a^3 \coth^{-1}(ax)^2 + 6a^3 \coth^{-1}(ax) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x^4,x]

[Out]  $((-6*a^2*ArcCoth[a*x])/x + 3*a^3*ArcCoth[a*x]^2 - (3*a*ArcCoth[a*x]^2)/x^2 + 2*a^3*ArcCoth[a*x]^3 - (2*ArcCoth[a*x]^3)/x^3 + 6*a^3*ArcCoth[a*x]^2*Log[1 + E^{(-2*ArcCoth[a*x])}] + 6*a^3*Log[1/Sqrt[1 - 1/(a^2*x^2)]] - 6*a^3*ArcCoth[a*x]*PolyLog[2, -E^{(-2*ArcCoth[a*x])}] - 3*a^3*PolyLog[3, -E^{(-2*ArcCoth[a*x])}])/6$

**fricas** [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arccoth}(ax)^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^4,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^3/x^4, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^4,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3/x^4, x)

**maple** [C] time = 2.41, size = 895, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3/x^4,x)

[Out]  $-1/3*a^3*\operatorname{arccoth}(a*x)^3 + 1/2*a^3*\operatorname{arccoth}(a*x)^2 - 1/2*a*\operatorname{arccoth}(a*x)^2/x^2 - a^3*\operatorname{arccoth}(a*x) - 1/2*a^3*\operatorname{polylog}(3, -(a*x+1)/(a*x-1)) + a^3*\ln(1+(a*x+1)/(a*x-1)) - a^2*\operatorname{arccoth}(a*x)/x - 1/3*\operatorname{arccoth}(a*x)^3/x^3 - 1/4*I*a^3*\operatorname{arccoth}(a*x)^2*\operatorname{Pi}*csgn(I*(a*x+1)/(a*x-1))^3 - 1/4*I*a^3*\operatorname{arccoth}(a*x)^2*\operatorname{Pi}*csgn(I*(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1))*csgn(I*(a*x+1)/(a*x-1)/((a*x+1)/(a*x-1)-1)) + 1/2*I*a^3*\operatorname{arccoth}(a*x)^2*\operatorname{Pi}*csgn(I/((a*x+1)/(a*x-1)-1))*(1+(a*x+1)/(a*x-1))*csgn(I*(1+(a*x+1)/(a*x-1))*csgn(I/((a*x+1)/(a*x-1)-1)) - 1/4*I*a^3*\operatorname{arccoth}(a*x)$

$$\begin{aligned}
& 2\pi \operatorname{csgn}\left(\frac{I(a^3x+1)}{a^3x-1}\right) \left(\frac{a^3x+1}{a^3x-1}-1\right)^{3+1/2} I a^3 \operatorname{arccoth}(a^3x)^{2\pi} \\
& 2\pi \operatorname{csgn}\left(\frac{I}{\left(\frac{a^3x+1}{a^3x-1}-1\right)\left(1+\frac{a^3x+1}{a^3x-1}\right)}\right)^{3+1/2} I a^3 \operatorname{arccoth}(a^3x)^{2\pi} \\
& \operatorname{csgn}\left(\frac{I}{\left(\frac{a^3x-1}{a^3x+1}\right)^{1/2}}\right) \operatorname{csgn}\left(\frac{I(a^3x+1)}{a^3x-1}\right)^{2+1/4} I a^3 \\
& \operatorname{arccoth}(a^3x)^{2\pi} \operatorname{csgn}\left(\frac{I(a^3x+1)}{a^3x-1}\right) \operatorname{csgn}\left(\frac{I(a^3x+1)}{a^3x-1}\right) \left(\frac{a^3x+1}{a^3x-1}-1\right)^{2-1/4} \\
& I a^3 \operatorname{arccoth}(a^3x)^{2\pi} \operatorname{csgn}\left(\frac{I}{\left(\frac{a^3x-1}{a^3x+1}\right)^{1/2}}\right)^{2\pi} \operatorname{csgn}\left(\frac{I(a^3x+1)}{a^3x-1}\right) + \frac{1}{4} I a^3 \operatorname{arccoth}(a^3x)^{2\pi} \\
& \operatorname{csgn}\left(\frac{I(a^3x+1)}{a^3x-1}\right) \left(\frac{a^3x+1}{a^3x-1}-1\right)^{2-1/2} I a^3 \operatorname{arccoth}(a^3x)^{2\pi} \operatorname{csgn}\left(\frac{I}{\left(\frac{a^3x+1}{a^3x-1}-1\right)\left(1+\frac{a^3x+1}{a^3x-1}\right)}\right)^{2\pi} \\
& \operatorname{csgn}\left(\frac{I(a^3x+1)}{a^3x-1}\right) \left(\frac{a^3x+1}{a^3x-1}-1\right)^{-1/2} I a^3 \operatorname{arccoth}(a^3x)^{2\pi} \operatorname{csgn}\left(\frac{I}{\left(\frac{a^3x+1}{a^3x-1}-1\right)\left(1+\frac{a^3x+1}{a^3x-1}\right)}\right)^{2\pi} \\
& \operatorname{csgn}\left(\frac{I(1+\frac{a^3x+1}{a^3x-1})}{a^3x-1}\right) + a^3 \operatorname{arccoth}(a^3x)^{2\pi} \ln(a^3x) - \frac{1}{2} a^3 a \\
& \operatorname{arccoth}(a^3x)^{2\pi} \ln(a^3x-1) + a^3 \operatorname{arccoth}(a^3x) \operatorname{polylog}\left(2, -\frac{a^3x+1}{a^3x-1}\right) - \frac{1}{2} a^3 a \\
& \operatorname{arccoth}(a^3x)^{2\pi} \ln(a^3x+1) - \frac{1}{2} a^3 a \operatorname{arccoth}(a^3x)^{2\pi} \ln\left(\frac{a^3x-1}{a^3x+1}\right) + a^3 a \\
& \operatorname{arccoth}(a^3x)^{2\pi} \ln(2)
\end{aligned}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^4,x, algorithm="maxima")

[Out]  $\frac{1}{4} a^4 \operatorname{integrate}(x^4 \log(a^3x+1) \log(a^3x-1) / (a^3x^5+x^4), x) - \frac{1}{2} a^4 \operatorname{integrate}(x^4 \log(a^3x+1) \log(x) / (a^3x^5+x^4), x) + \frac{1}{16} (2a^2 \log(a^3x+1) - 2a^2 \log(x) - (2a^3x-1)/x^2) a^3 \log(a)^3 + \frac{3}{8} a \operatorname{integrate}(x \log(a^3x-1) / (a^3x^5+x^4), x) \log(a)^2 - \frac{3}{8} a \operatorname{integrate}(x \log(x) / (a^3x^5+x^4), x) \log(a)^2 - \frac{1}{48} (6a^3 \log(a^3x+1) - 6a^3 \log(x) - (6a^2x^2-3a^3x+2)/x^3) \log(a)^3 + \frac{1}{4} a^2 \operatorname{integrate}(x^2 \log(a^3x+1) / (a^3x^5+x^4), x) + \frac{3}{4} a \operatorname{integrate}(x \log(a^3x-1) \log(x) / (a^3x^5+x^4), x) \log(a) - \frac{3}{8} a \operatorname{integrate}(x \log(x)^2 / (a^3x^5+x^4), x) \log(a) + \frac{3}{8} \operatorname{integrate}(\log(a^3x-1) / (a^3x^5+x^4), x) \log(a)^2 - \frac{3}{8} \operatorname{integrate}(\log(x) / (a^3x^5+x^4), x) \log(a)^2 + \frac{3}{8} a \operatorname{integrate}(x \log(a^3x+1) \log(a^3x-1)^2 / (a^3x^5+x^4), x) - \frac{3}{8} a \operatorname{integrate}(x \log(a^3x-1)^2 \log(x) / (a^3x^5+x^4), x) + \frac{3}{8} a \operatorname{integrate}(x \log(a^3x-1) \log(x)^2 / (a^3x^5+x^4), x) - \frac{1}{8} a \operatorname{integrate}(x \log(x)^3 / (a^3x^5+x^4), x) - \frac{1}{4} a \operatorname{integrate}(x \log(a^3x+1) \log(a^3x-1) / (a^3x^5+x^4), x) - \frac{3}{8} \operatorname{integrate}(a^3x \log(a^3x-1)^2 / (a^3x^5+x^4), x) \log(a) - \frac{3}{8} \operatorname{integrate}(\log(a^3x-1)^2 / (a^3x^5+x^4), x) \log(a) + \frac{3}{4} \operatorname{integrate}(\log(a^3x-1) \log(x) / (a^3x^5+x^4), x) \log(a) - \frac{3}{8} \operatorname{integrate}(\log(x)^2 / (a^3x^5+x^4), x) \log(a) - \frac{1}{48} (6a^4 \log(a^3x-1) - 6a^4 \log(x) + (6a^3x^2+3a^2x+2a)/x^3) \log(-1/(a^3x)+1)^2/a + \frac{1}{864} (6(18a^5x^3 \log(a^3x-1)^2 + 18a^5x^3 \log(x)^2 - 66a^5x^3 \log(x) + 66a^4x^2 + 15a^3x + 4a^2 - 6(6a^5x^3 \log(x) - 11a^5x^3) \log(a^3x-1)) \log(-1/(a^3x)+1) / (a^3x^3) - (36a^6x^3 \log(a^3x-1)^3 - 36a^6x^3 \log(x)^3 + 198a^6x^3 \log(x)^2 - 510a^6x^3 \log(x) + 510a^5x^2 + 57a^4x + 8a^3 - 18(6a^6x^3 \log(x) - 11a^6x^3) \log(a^3x-1)^2 + 6(18a^6x^3 \log(x)^2 - 66a^6x^3 \log(x) + 85a^6x^3) \log(a^3x-1)) / (a^2x^3)) / a + \frac{1}{24} \log(-1/(a^3x)+1)^3/x^3 - \frac{1}{24} ((a^3x^3$

```
+ 1)*log(a*x + 1)^3 - 3*(2*a^3*x^3*log(x) - a*x - (a^3*x^3 - 1)*log(a*x -
1))*log(a*x + 1)^2)/x^3 + 3/8*integrate(log(a*x + 1)*log(a*x - 1)^2/(a*x^5
+ x^4), x) - 3/8*integrate(log(a*x - 1)^2*log(x)/(a*x^5 + x^4), x) + 3/8*in
tegrate(log(a*x - 1)*log(x)^2/(a*x^5 + x^4), x) - 1/8*integrate(log(x)^3/(a
*x^5 + x^4), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)^3/x^4,x)

[Out] int(acoth(a\*x)^3/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*3/x\*\*4,x)

[Out] Integral(acoth(a\*x)\*\*3/x\*\*4, x)



$$3.33 \quad \int \frac{\coth^{-1}(ax)^3}{x^5} dx$$

Optimal. Leaf size=141

$$-a^4 \text{Li}_2\left(\frac{2}{ax+1} - 1\right) + \frac{1}{4}a^4 \tanh^{-1}(ax) + \frac{1}{4}a^4 \coth^{-1}(ax)^3 + a^4 \coth^{-1}(ax)^2 + 2a^4 \log\left(2 - \frac{2}{ax+1}\right) \coth^{-1}(ax) - \frac{a^3}{4x}$$

[Out]  $-1/4*a^3/x - 1/4*a^2*\text{arccoth}(a*x)/x^2 + a^4*\text{arccoth}(a*x)^2 - 1/4*a*\text{arccoth}(a*x)^2/x^3 - 3/4*a^3*\text{arccoth}(a*x)^2/x + 1/4*a^4*\text{arccoth}(a*x)^3 - 1/4*\text{arccoth}(a*x)^3/x^4 + 1/4*a^4*\text{arctanh}(a*x) + 2*a^4*\text{arccoth}(a*x)*\ln(2 - 2/(a*x + 1)) - a^4*\text{polylog}(2, -1 + 2/(a*x + 1))$

Rubi [A] time = 0.46, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$ , Rules used = {5917, 5983, 325, 206, 5989, 5933, 2447, 5949}

$$-a^4 \text{PolyLog}\left(2, \frac{2}{ax+1} - 1\right) - \frac{a^2 \coth^{-1}(ax)}{4x^2} - \frac{a^3}{4x} + \frac{1}{4}a^4 \tanh^{-1}(ax) + \frac{1}{4}a^4 \coth^{-1}(ax)^3 + a^4 \coth^{-1}(ax)^2 - \frac{3a^3 \coth^{-1}(ax)}{4x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]^3/x^5, x]

[Out]  $-a^3/(4*x) - (a^2*\text{ArcCoth}[a*x])/(4*x^2) + a^4*\text{ArcCoth}[a*x]^2 - (a*\text{ArcCoth}[a*x]^2)/(4*x^3) - (3*a^3*\text{ArcCoth}[a*x]^2)/(4*x) + (a^4*\text{ArcCoth}[a*x]^3)/4 - \text{ArcCoth}[a*x]^3/(4*x^4) + (a^4*\text{ArcTanh}[a*x])/4 + 2*a^4*\text{ArcCoth}[a*x]*\text{Log}[2 - 2/(1 + a*x)] - a^4*\text{PolyLog}[2, -1 + 2/(1 + a*x)]$

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]]
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5983

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

### Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^ (p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)^3}{x^5} dx &= -\frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a) \int \frac{\coth^{-1}(ax)^2}{x^4(1-a^2x^2)} dx \\
&= -\frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{4}(3a) \int \frac{\coth^{-1}(ax)^2}{x^4} dx + \frac{1}{4}(3a^3) \int \frac{\coth^{-1}(ax)^2}{x^2(1-a^2x^2)} dx \\
&= -\frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2 \int \frac{\coth^{-1}(ax)}{x^3(1-a^2x^2)} dx + \frac{1}{4}(3a^3) \int \frac{\coth^{-1}(ax)^2}{x^2} dx + \frac{1}{4} \\
&= -\frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{3a^3 \coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4 \coth^{-1}(ax)^3 - \frac{\coth^{-1}(ax)^3}{4x^4} + \frac{1}{2}a^2 \int \frac{\coth^{-1}(ax)}{x^3} dx \\
&= -\frac{a^2 \coth^{-1}(ax)}{4x^2} + a^4 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{3a^3 \coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4 \coth^{-1}(ax)^3 - \\
&= -\frac{a^3}{4x} - \frac{a^2 \coth^{-1}(ax)}{4x^2} + a^4 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{3a^3 \coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4 \coth^{-1}(ax)^3 \\
&= -\frac{a^3}{4x} - \frac{a^2 \coth^{-1}(ax)}{4x^2} + a^4 \coth^{-1}(ax)^2 - \frac{a \coth^{-1}(ax)^2}{4x^3} - \frac{3a^3 \coth^{-1}(ax)^2}{4x} + \frac{1}{4}a^4 \coth^{-1}(ax)^3
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 118, normalized size = 0.84

$$\frac{-4a^4x^4\text{Li}_2\left(-e^{-2\coth^{-1}(ax)}\right) + \left(a^4x^4 - 1\right)\coth^{-1}(ax)^3 - a^3x^3 + a^2x^2\coth^{-1}(ax)\left(a^2x^2 + 8a^2x^2\log\left(e^{-2\coth^{-1}(ax)} + 1\right)\right)}{4x^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]^3/x^5,x]

[Out]  $(-(a^3x^3) + a*x*(-1 - 3a^2x^2 + 4a^3x^3)*\text{ArcCoth}[a*x]^2 + (-1 + a^4x^4)*\text{ArcCoth}[a*x]^3 + a^2x^2*\text{ArcCoth}[a*x]*(-1 + a^2x^2 + 8a^2x^2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[a*x])}])) - 4a^4x^4*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[a*x])}]))/(4x^4)$

**fricas [F]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcoth}(ax)^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoath(a\*x)^3/x^5,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)^3/x^5, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^5,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)^3/x^5, x)

**maple** [C] time = 1.50, size = 661, normalized size = 4.69

$$\frac{a^4 \operatorname{arccoth}(ax)}{4} + \frac{a^4}{4} - a^4 \operatorname{arccoth}(ax)^2 + \frac{a^4 \operatorname{arccoth}(ax)^3}{4} - \frac{3a^4 \operatorname{arccoth}(ax)^2 \ln(ax-1)}{8} + \frac{3a^4 \operatorname{arccoth}(ax)^2 \ln(ax+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)^3/x^5,x)

[Out]  $\frac{1}{4}a^4 \operatorname{arccoth}(a*x) + \frac{1}{4}a^4 - a^4 \operatorname{arccoth}(a*x)^2 + \frac{1}{4}a^4 \operatorname{arccoth}(a*x)^3 - \frac{3}{8}a^4 \operatorname{arccoth}(a*x)^2 \ln(a*x-1) + \frac{3}{8}a^4 \operatorname{arccoth}(a*x)^2 \ln(a*x+1) + 2a^4 \operatorname{arccoth}(a*x) \ln(1 + I/\sqrt{(a*x-1)/(a*x+1)}) + 2a^4 \operatorname{arccoth}(a*x) \ln(1 - I/\sqrt{(a*x-1)/(a*x+1)}) + \frac{3}{8}a^4 \operatorname{arccoth}(a*x)^2 \ln((a*x-1)/(a*x+1)) - \frac{1}{4}a^4 \operatorname{arccoth}(a*x)^3/x^4 - \frac{1}{4}a^4 \operatorname{arccoth}(a*x)^2/x^3 - \frac{3}{4}a^4 \operatorname{arccoth}(a*x)^2/x - \frac{1}{4}a^4/x^3 - \frac{3}{16}Ia^4 \operatorname{arccoth}(a*x)^2 \operatorname{csgn}(I/\sqrt{(a*x+1)/(a*x-1)}) \operatorname{csgn}(I*(a*x+1)/(a*x-1)/\sqrt{(a*x+1)/(a*x-1)})^2 \operatorname{arccoth}(a*x)^2 - \frac{1}{4}a^4 \operatorname{arccoth}(a*x)/x^2 + 2a^4 \operatorname{dilog}(1 + I/\sqrt{(a*x-1)/(a*x+1)}) + 2a^4 \operatorname{dilog}(1 - I/\sqrt{(a*x-1)/(a*x+1)}) + \frac{3}{16}Ia^4 \operatorname{arccoth}(a*x)^2 \operatorname{csgn}(I/\sqrt{(a*x-1)/(a*x+1)})^2 \operatorname{csgn}(I*(a*x+1)/(a*x-1)) \operatorname{arccoth}(a*x)^2 - \frac{3}{8}Ia^4 \operatorname{arccoth}(a*x)^2 \operatorname{csgn}(I/\sqrt{(a*x-1)/(a*x+1)}) \operatorname{csgn}(I*(a*x+1)/(a*x-1))^2 \operatorname{arccoth}(a*x)^2 + \frac{3}{16}Ia^4 \operatorname{arccoth}(a*x)^2 \operatorname{csgn}(I*(a*x+1)/(a*x-1))^3 \operatorname{arccoth}(a*x)^2 + \frac{3}{16}Ia^4 \operatorname{arccoth}(a*x)^2 \operatorname{csgn}(I*(a*x+1)/(a*x-1)/\sqrt{(a*x+1)/(a*x-1)})^3 \operatorname{arccoth}(a*x)^2 + \frac{3}{16}Ia^4 \operatorname{arccoth}(a*x)^2 \operatorname{csgn}(I*(a*x+1)/(a*x-1)) \operatorname{csgn}(I/\sqrt{(a*x+1)/(a*x-1)}) \operatorname{csgn}(I*(a*x+1)/(a*x-1)/\sqrt{(a*x+1)/(a*x-1)}) \operatorname{arccoth}(a*x)^2 - \frac{3}{16}Ia^4 \operatorname{arccoth}(a*x)^2 \operatorname{csgn}(I*(a*x+1)/(a*x-1)) \operatorname{csgn}(I*(a*x+1)/(a*x-1)/\sqrt{(a*x+1)/(a*x-1)})^2 \operatorname{arccoth}(a*x)^2$

**maxima** [B] time = 0.34, size = 342, normalized size = 2.43

$$\frac{1}{8} \left( 3a^3 \log(ax+1) - 3a^3 \log(ax-1) - \frac{2(3a^2x^2+1)}{x^3} \right) a \operatorname{arccoth}(ax)^2 + \frac{1}{32} \left( 32 \left( \log(ax-1) \log\left(\frac{1}{2}ax + \frac{1}{2}\right) + \log(ax+1) \log\left(\frac{1}{2}ax - \frac{1}{2}\right) \right) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)^3/x^5,x, algorithm="maxima")

[Out]  $\frac{1}{8}(3a^3\log(ax+1) - 3a^3\log(ax-1) - 2(3a^2x^2 + 1)/x^3) \operatorname{arccoth}(ax)^2 + \frac{1}{32}((32(\log(ax-1)\log(1/2ax + 1/2) + \operatorname{dilog}(-1/2ax + 1/2))a - 32(\log(ax+1)\log(x) + \operatorname{dilog}(-ax))a + 32(\log(-ax+1)\log(x) + \operatorname{dilog}(ax))a + 4a\log(ax+1) - 4a\log(ax-1) + (ax\log(ax+1))^3 - ax\log(ax-1)^3 - 8ax\log(ax-1)^2 - (3ax\log(ax-1) - 8ax)\log(ax+1)^2 + (3ax\log(ax-1)^2 - 16ax\log(ax-1))\log(ax+1) - 8/x)a^2 + 2(32a^2\log(x) - (3a^2x^2\log(ax+1)^2 + 3a^2x^2\log(ax-1)^2 + 16a^2x^2\log(ax-1) - 2(3a^2x^2\log(ax-1) - 8a^2x^2)\log(ax+1) + 4)/x^2) \operatorname{arccoth}(ax))a - \frac{1}{4}\operatorname{arccoth}(ax)^3/x^4$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(ax)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)^3/x^5,x)

[Out] int(acoth(a\*x)^3/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)\*\*3/x\*\*5,x)

[Out] Integral(acoth(a\*x)\*\*3/x\*\*5, x)

$$3.34 \quad \int \frac{\coth^{-1}(cx)^2}{d+ex} dx$$

**Optimal.** Leaf size=164

$$\frac{\operatorname{Li}_3\left(1 - \frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{2e} - \frac{\coth^{-1}(cx)\operatorname{Li}_2\left(1 - \frac{2c(d+ex)}{(cd+e)(cx+1)}\right)}{e} + \frac{\coth^{-1}(cx)^2 \log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e} + \frac{\operatorname{Li}_3\left(1 - \frac{2}{cx+1}\right)}{2e} + \frac{\operatorname{Li}_2\left(1 - \frac{2}{cx+1}\right)}{e}$$

[Out]  $-\operatorname{arccoth}(c*x)^2*\ln(2/(c*x+1))/e+\operatorname{arccoth}(c*x)^2*\ln(2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,1-2/(c*x+1))/e-\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e+1/2*\operatorname{polylog}(3,1-2/(c*x+1))/e-1/2*\operatorname{polylog}(3,1-2*c*(e*x+d)/(c*d+e)/(c*x+1))/e$

**Rubi [A]** time = 0.03, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {5923}

$$\frac{\operatorname{PolyLog}\left(3,1 - \frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e} - \frac{\coth^{-1}(cx)\operatorname{PolyLog}\left(2,1 - \frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e} + \frac{\operatorname{PolyLog}\left(3,1 - \frac{2}{cx+1}\right)}{2e} + \frac{\coth^{-1}(cx)\operatorname{PolyLog}\left(2,1 - \frac{2}{cx+1}\right)}{e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[c*x]^2/(d + e*x), x]$

[Out]  $-\left(\operatorname{ArcCoth}[c*x]^2*\operatorname{Log}[2/(1 + c*x)]\right)/e + \left(\operatorname{ArcCoth}[c*x]^2*\operatorname{Log}[(2*c*(d + e*x))/(c*d + e)*(1 + c*x)]\right)/e + \left(\operatorname{ArcCoth}[c*x]*\operatorname{PolyLog}[2, 1 - 2/(1 + c*x)]\right)/e - \left(\operatorname{ArcCoth}[c*x]*\operatorname{PolyLog}[2, 1 - (2*c*(d + e*x))/(c*d + e)*(1 + c*x)]\right)/e + \operatorname{PolyLog}[3, 1 - 2/(1 + c*x)]/(2*e) - \operatorname{PolyLog}[3, 1 - (2*c*(d + e*x))/(c*d + e)*(1 + c*x)]/(2*e)$

**Rule 5923**

$\operatorname{Int}[\left((a_.) + \operatorname{ArcCoth}[(c_.)*(x_.)]*(b_.)\right)^2/\left((d_.) + (e_.)*(x_.)\right), x\_Symbol] :>$   
 $-\operatorname{Simp}[\left((a + b*\operatorname{ArcCoth}[c*x])^2*\operatorname{Log}[2/(1 + c*x)]\right)/e, x] + \left(\operatorname{Simp}[\left((a + b*\operatorname{ArcCoth}[c*x])^2*\operatorname{Log}[(2*c*(d + e*x))/(c*d + e)*(1 + c*x)]\right)/e, x] + \operatorname{Simp}[(b*(a + b*\operatorname{ArcCoth}[c*x])^2*\operatorname{Log}[2, 1 - 2/(1 + c*x)])]/e, x] - \operatorname{Simp}[(b*(a + b*\operatorname{ArcCoth}[c*x])^2*\operatorname{Log}[2, 1 - (2*c*(d + e*x))/(c*d + e)*(1 + c*x)])]/e, x] + \operatorname{Simp}[(b^2*\operatorname{PolyLog}[3, 1 - 2/(1 + c*x)])/(2*e), x] - \operatorname{Simp}[(b^2*\operatorname{PolyLog}[3, 1 - (2*c*(d + e*x))/(c*d + e)*(1 + c*x)])/(2*e), x]) /; \operatorname{FreeQ}\{a, b, c, d, e, x\} \&\& \operatorname{NeQ}[c^2*d^2 - e^2, 0]$

**Rubi steps**

$$\int \frac{\coth^{-1}(cx)^2}{d+ex} dx = -\frac{\coth^{-1}(cx)^2 \log\left(\frac{2}{1+cx}\right)}{e} + \frac{\coth^{-1}(cx)^2 \log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{\coth^{-1}(cx) \operatorname{Li}_2\left(1 - \frac{2}{1+cx}\right)}{e} - \dots$$

**Mathematica [C]** time = 7.80, size = 565, normalized size = 3.45

$$24(e-cd)(cd+e) \left( 2cd \sqrt{1-\frac{e^2}{c^2d^2}} \coth^{-1}(cx)^3 e^{-\tanh^{-1}\left(\frac{e}{cd}\right)} + 3e \coth^{-1}(cx)^2 \log\left(\frac{d+ex}{x \sqrt{1-\frac{1}{c^2x^2}}}\right) - 3i\pi e \log\left(\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\right) \coth^{-1}(cx) - 6e \coth^{-1}(cx) \operatorname{Li}_2\left(-e^{\coth^{-1}(cx)}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[c\*x]^2/(d + e\*x), x]

[Out]  $((-I)*e*\pi^3 + 8*c*d*\operatorname{ArcCoth}[c*x]^3 + 8*e*\operatorname{ArcCoth}[c*x]^3 - 24*e*\operatorname{ArcCoth}[c*x]^2*\log[1 - E^{(2*\operatorname{ArcCoth}[c*x])}] - 24*e*\operatorname{ArcCoth}[c*x]*\operatorname{PolyLog}[2, E^{(2*\operatorname{ArcCoth}[c*x])}] + 12*e*\operatorname{PolyLog}[3, E^{(2*\operatorname{ArcCoth}[c*x])}] + (24*(-(c*d) + e)*(c*d + e)*(-(c*d*\operatorname{ArcCoth}[c*x]^3) + 3*e*\operatorname{ArcCoth}[c*x]^3 + (2*c*d*\sqrt{1 - e^2/(c^2*d^2)})*\operatorname{ArcCoth}[c*x]^3)/E^{\operatorname{ArcTanh}[e/(c*d)]} + (3*I)*e*\pi*\operatorname{ArcCoth}[c*x]*\log[(E^{(-\operatorname{ArcCoth}[c*x])} + E^{\operatorname{ArcCoth}[c*x]})/2] - 3*e*\operatorname{ArcCoth}[c*x]^2*\log[1 - E^{(\operatorname{ArcCoth}[c*x] + \operatorname{ArcTanh}[e/(c*d)])}] - 3*e*\operatorname{ArcCoth}[c*x]^2*\log[1 + E^{(\operatorname{ArcCoth}[c*x] + \operatorname{ArcTanh}[e/(c*d)])}] - 6*e*\operatorname{ArcCoth}[c*x]*\operatorname{ArcTanh}[e/(c*d)]*\log[(I/2)*E^{(-\operatorname{ArcCoth}[c*x] - \operatorname{ArcTanh}[e/(c*d)])}*(-1 + E^{(2*(\operatorname{ArcCoth}[c*x] + \operatorname{ArcTanh}[e/(c*d)])})]) - 3*e*\operatorname{ArcCoth}[c*x]^2*\log[(c*d*(-1 + E^{(2*\operatorname{ArcCoth}[c*x])}) + e*(1 + E^{(2*\operatorname{ArcCoth}[c*x])})]/(2*E^{\operatorname{ArcCoth}[c*x]})] - (3*I)*e*\pi*\operatorname{ArcCoth}[c*x]*\log[1/\sqrt{1 - 1/(c^2*x^2)}] + 3*e*\operatorname{ArcCoth}[c*x]^2*\log[(d + e*x)/(sqrt{1 - 1/(c^2*x^2)})*x] + 6*e*\operatorname{ArcCoth}[c*x]*\operatorname{ArcTanh}[e/(c*d)]*\log[I*\sinh[\operatorname{ArcCoth}[c*x] + \operatorname{ArcTanh}[e/(c*d)]]] - 6*e*\operatorname{ArcCoth}[c*x]*\operatorname{PolyLog}[2, -E^{(\operatorname{ArcCoth}[c*x] + \operatorname{ArcTanh}[e/(c*d)])}] - 6*e*\operatorname{ArcCoth}[c*x]*\operatorname{PolyLog}[2, E^{(\operatorname{ArcCoth}[c*x] + \operatorname{ArcTanh}[e/(c*d)])}] + 6*e*\operatorname{PolyLog}[3, -E^{(\operatorname{ArcCoth}[c*x] + \operatorname{ArcTanh}[e/(c*d)])}] + 6*e*\operatorname{PolyLog}[3, E^{(\operatorname{ArcCoth}[c*x] + \operatorname{ArcTanh}[e/(c*d)])}])/(3*c^2*d^2 - 3*e^2))/(24*e^2)$

**fricas [F]** time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(cx)^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c\*x)^2/(e\*x+d), x, algorithm="fricas")

[Out] integral(arccoth(c\*x)^2/(e\*x + d), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(cx)^2}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c\*x)^2/(e\*x+d),x, algorithm="giac")

[Out] integrate(arccoth(c\*x)^2/(e\*x + d), x)

**maple** [C] time = 1.29, size = 926, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c\*x)^2/(e\*x+d),x)

[Out]  $\ln(c*e*x+c*d)/e*\operatorname{arccoth}(c*x)^2-1/e*\operatorname{arccoth}(c*x)^2*\ln(((c*x+1)/(c*x-1)-1)*d*c+e*(1+(c*x+1)/(c*x-1)))-1/2*I/e*Pi*\operatorname{arccoth}(c*x)^2*csgn(I*(((c*x+1)/(c*x-1)-1)*d*c+e*(1+(c*x+1)/(c*x-1))))*csgn(I*(((c*x+1)/(c*x-1)-1)*d*c+e*(1+(c*x+1)/(c*x-1)))/(c*x-1)))/(c*x+1)/(c*x-1)-1)^2-1/2*I/e*Pi*\operatorname{arccoth}(c*x)^2*csgn(I/(c*x+1)/(c*x-1)-1))*csgn(I*(((c*x+1)/(c*x-1)-1)*d*c+e*(1+(c*x+1)/(c*x-1)))/(c*x+1)/(c*x-1)-1))^2+1/2*I/e*Pi*\operatorname{arccoth}(c*x)^2*csgn(I/(c*x+1)/(c*x-1)-1))*csgn(I*(((c*x+1)/(c*x-1)-1)*d*c+e*(1+(c*x+1)/(c*x-1)))/(c*x+1)/(c*x-1)-1))*d*c+e*(1+(c*x+1)/(c*x-1)))/(c*x+1)/(c*x-1)-1)+1/2*I/e*Pi*\operatorname{arccoth}(c*x)^2*csgn(I*(((c*x+1)/(c*x-1)-1)*d*c+e*(1+(c*x+1)/(c*x-1)))/(c*x+1)/(c*x-1)-1))^3+1/e*\operatorname{arccoth}(c*x)^2*\ln((c*x+1)/(c*x-1)-1)-1/e*\operatorname{arccoth}(c*x)^2*\ln(1-1/(c*x-1)/(c*x+1))^(1/2))-2/e*\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,1/((c*x-1)/(c*x+1))^(1/2))+2/e*\operatorname{polylog}(3,1/((c*x-1)/(c*x+1))^(1/2))-1/e*\operatorname{arccoth}(c*x)^2*\ln(1+1/((c*x-1)/(c*x+1))^(1/2))-2/e*\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,-1/((c*x-1)/(c*x+1))^(1/2))+2/e*\operatorname{polylog}(3,-1/((c*x-1)/(c*x+1))^(1/2))+1/(c*d+e)*\operatorname{arccoth}(c*x)^2*\ln(1-(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))+1/(c*d+e)*\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))-1/2/(c*d+e)*\operatorname{polylog}(3,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))+c/e*d/(c*d+e)*\operatorname{arccoth}(c*x)^2*\ln(1-(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))+c/e*d/(c*d+e)*\operatorname{arccoth}(c*x)*\operatorname{polylog}(2,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1))-1/2*c/e*d/(c*d+e)*\operatorname{polylog}(3,(c*d+e)/(c*d-e)*(c*x+1)/(c*x-1)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(cx)^2}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c\*x)^2/(e\*x+d),x, algorithm="maxima")



[Out] integrate(arccoth(c\*x)^2/(e\*x + d), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(cx)^2}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(c\*x)^2/(d + e\*x), x)

[Out] int(acoth(c\*x)^2/(d + e\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(cx)}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c\*x)\*\*2/(e\*x+d), x)

[Out] Integral(acoth(c\*x)\*\*2/(d + e\*x), x)

### 3.35 $\int (c + dx^2)^4 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=245

$$\frac{d^3x^6(36a^2c + 7d)}{378a^3} + \frac{d^2x^4(378a^4c^2 + 180a^2cd + 35d^2)}{1260a^5} + \frac{dx^2(420a^6c^3 + 378a^4c^2d + 180a^2cd^2 + 35d^3)}{630a^7} + \frac{(315a^8c^4 + 420a^6c^3d + 378a^4c^2d^2 + 180a^2cd^3 + 35d^4)}{630a^9} \ln(-a^2x^2 + 1)$$

[Out] 1/630\*d\*(420\*a^6\*c^3+378\*a^4\*c^2\*d+180\*a^2\*c\*d^2+35\*d^3)\*x^2/a^7+1/1260\*d^2\*(378\*a^4\*c^2+180\*a^2\*c\*d+35\*d^2)\*x^4/a^5+1/378\*d^3\*(36\*a^2\*c+7\*d)\*x^6/a^3+1/72\*d^4\*x^8/a+c^4\*x\*arccoth(a\*x)+4/3\*c^3\*d\*x^3\*arccoth(a\*x)+6/5\*c^2\*d^2\*x^5\*arccoth(a\*x)+4/7\*c\*d^3\*x^7\*arccoth(a\*x)+1/9\*d^4\*x^9\*arccoth(a\*x)+1/630\*(315\*a^8\*c^4+420\*a^6\*c^3\*d+378\*a^4\*c^2\*d^2+180\*a^2\*c\*d^3+35\*d^4)\*ln(-a^2\*x^2+1)/a^9

**Rubi [A]** time = 0.18, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {194, 5977, 1810, 260}

$$\frac{d^2x^4(378a^4c^2 + 180a^2cd + 35d^2)}{1260a^5} + \frac{dx^2(378a^4c^2d + 420a^6c^3 + 180a^2cd^2 + 35d^3)}{630a^7} + \frac{(378a^4c^2d^2 + 420a^6c^3d + 315a^8c^4 + 420a^6c^3d + 378a^4c^2d^2 + 180a^2cd^3 + 35d^4)}{630a^9} \ln(-a^2x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)^4\*ArcCoth[a\*x], x]

[Out] (d\*(420\*a^6\*c^3 + 378\*a^4\*c^2\*d + 180\*a^2\*c\*d^2 + 35\*d^3)\*x^2)/(630\*a^7) + (d^2\*(378\*a^4\*c^2 + 180\*a^2\*c\*d + 35\*d^2)\*x^4)/(1260\*a^5) + (d^3\*(36\*a^2\*c + 7\*d)\*x^6)/(378\*a^3) + (d^4\*x^8)/(72\*a) + c^4\*x\*ArcCoth[a\*x] + (4\*c^3\*d\*x^3\*ArcCoth[a\*x])/3 + (6\*c^2\*d^2\*x^5\*ArcCoth[a\*x])/5 + (4\*c\*d^3\*x^7\*ArcCoth[a\*x])/7 + (d^4\*x^9\*ArcCoth[a\*x])/9 + ((315\*a^8\*c^4 + 420\*a^6\*c^3\*d + 378\*a^4\*c^2\*d^2 + 180\*a^2\*c\*d^3 + 35\*d^4)\*Log[1 - a^2\*x^2])/(630\*a^9)

#### Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 1810

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 5977

```
Int[((a_) + ArcCoth[(c_)*(x_)])*(b_))*((d_) + (e_)*(x_)^2)^(q_), x_Symbol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

### Rubi steps

$$\begin{aligned} \int (c + dx^2)^4 \coth^{-1}(ax) dx &= c^4 x \coth^{-1}(ax) + \frac{4}{3} c^3 dx^3 \coth^{-1}(ax) + \frac{6}{5} c^2 d^2 x^5 \coth^{-1}(ax) + \frac{4}{7} cd^3 x^7 \coth^{-1}(ax) \\ &= c^4 x \coth^{-1}(ax) + \frac{4}{3} c^3 dx^3 \coth^{-1}(ax) + \frac{6}{5} c^2 d^2 x^5 \coth^{-1}(ax) + \frac{4}{7} cd^3 x^7 \coth^{-1}(ax) \\ &= \frac{d(420a^6c^3 + 378a^4c^2d + 180a^2cd^2 + 35d^3)x^2}{630a^7} + \frac{d^2(378a^4c^2 + 180a^2cd + 35d^2)}{1260a^5} \\ &= \frac{d(420a^6c^3 + 378a^4c^2d + 180a^2cd^2 + 35d^3)x^2}{630a^7} + \frac{d^2(378a^4c^2 + 180a^2cd + 35d^2)}{1260a^5} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 213, normalized size = 0.87

$$\frac{24a^9x \coth^{-1}(ax) (315c^4 + 420c^3dx^2 + 378c^2d^2x^4 + 180cd^3x^6 + 35d^4x^8) + a^2dx^2 (3a^6 (1680c^3 + 756c^2dx^2 + 240cd^3x^4 + 35d^4x^6)) + 24a^9x^9 (315c^4 + 420c^3d^2x^2 + 378c^2d^2x^4 + 180cd^3x^6 + 35d^4x^8) \operatorname{ArcCoth}[ax] + 12(315a^8c^4 + 420a^6c^3d + 378a^4c^2d^2 + 180a^2cd^3 + 35d^4) \operatorname{Log}[1 - a^2x^2]}{(7560a^9)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcCoth[a*x], x]
```

```
[Out] (a^2*d*x^2*(420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) + 4*a^4*d*(1134*c^2 + 270*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4 + 35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*d^3*x^6 + 35*d^4*x^8)*ArcCoth[a*x] + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(7560*a^9)
```

**fricas [A]** time = 0.56, size = 247, normalized size = 1.01

$$105 a^8 d^4 x^8 + 20 (36 a^8 c d^3 + 7 a^6 d^4) x^6 + 6 (378 a^8 c^2 d^2 + 180 a^6 c d^3 + 35 a^4 d^4) x^4 + 12 (420 a^8 c^3 d + 378 a^6 c^2 d^2 + 240 a^4 c d^3 + 35 d^4) x^2 + 12 (315 a^8 c^4 + 420 a^6 c^3 d + 378 a^4 c^2 d^2 + 180 a^2 c d^3 + 35 d^4) \operatorname{Log}[1 - a^2 x^2]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^4\*arccoth(a\*x),x, algorithm="fricas")

[Out]  $\frac{1}{7560} \cdot (105a^8d^4x^8 + 20(36a^8cd^3 + 7a^6d^4)x^6 + 6(378a^8c^2d^2 + 180a^6cd^3 + 35a^4d^4)x^4 + 12(420a^8c^3d + 378a^6c^2d^2 + 180a^4cd^3 + 35a^2d^4)x^2 + 12(315a^8c^4 + 420a^6c^3d + 378a^4c^2d^2 + 180a^2cd^3 + 35d^4) \cdot \log(a^2x^2 - 1) + 12(35a^9d^4x^9 + 180a^9cd^3x^7 + 378a^9c^2d^2x^5 + 420a^9c^3dx^3 + 315a^9c^4x) \cdot \log((ax + 1)/(ax - 1))) / a^9$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^2 + c)^4 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^4\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate((d\*x^2 + c)^4\*arccoth(a\*x), x)

**maple** [A] time = 0.04, size = 334, normalized size = 1.36

$$\frac{3c^2d^2x^2}{5a^3} + \frac{2cd^3x^6}{21a} + \frac{3c^2d^2x^4}{10a} + \frac{2c^3dx^2}{3a} + \frac{2\ln(ax-1)c^3d}{3a^3} + \frac{x^4cd^3}{7a^3} + \frac{2x^2cd^3}{7a^5} + \frac{3\ln(ax-1)c^2d^2}{5a^5} + \frac{2\ln(ax-1)cd^3}{7a^7} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^2+c)^4\*arccoth(a\*x),x)

[Out]  $\frac{3}{5} \frac{1}{a^3} c^2 d^2 x^2 + \frac{2}{21} \frac{1}{a} c d^3 x^6 + \frac{3}{10} \frac{1}{a} c^2 d^2 x^4 + \frac{2}{3} \frac{1}{a} c^3 d x^2 + \frac{2}{3} \frac{1}{a^3} \ln(ax-1) c^3 d + \frac{1}{7} \frac{1}{a^3} x^4 c d^3 + \frac{2}{7} \frac{1}{a^5} x^2 c d^3 + \frac{3}{5} \frac{1}{a^5} \ln(ax-1) c^2 d^2 + \frac{2}{7} \frac{1}{a^7} \ln(ax-1) c d^3 + \frac{2}{3} \frac{1}{a^3} \ln(ax+1) c^3 d + \frac{3}{5} \frac{1}{a^5} \ln(ax+1) c^2 d^2 + \frac{2}{7} \frac{1}{a^7} \ln(ax+1) c d^3 + \frac{1}{18} \frac{1}{a^9} \ln(ax-1) d^4 + \frac{1}{18} \frac{1}{a^9} \ln(ax+1) d^4 + \frac{1}{2} \frac{1}{a} \ln(ax-1) c^4 + \frac{1}{54} \frac{1}{a^3} x^6 d^4 + \frac{1}{18} \frac{1}{a^7} x^2 d^4 + \frac{1}{36} \frac{1}{a^5} x^4 d^4 + \frac{1}{9} d^4 x^9 \operatorname{arccoth}(ax) + \frac{1}{2} c^4 \ln(ax+1) / a + c^4 x \operatorname{arccoth}(ax) + \frac{1}{72} d^4 x^8 / a + \frac{4}{3} c^3 d x^3 \operatorname{arccoth}(ax) + \frac{6}{5} c^2 d^2 x^5 \operatorname{arccoth}(ax) + \frac{4}{7} c d^3 x^7 \operatorname{arccoth}(ax)$

**maxima** [A] time = 0.31, size = 276, normalized size = 1.13

$$\frac{1}{7560} a \left( \frac{105 a^6 d^4 x^8 + 20 (36 a^6 c d^3 + 7 a^4 d^4) x^6 + 6 (378 a^6 c^2 d^2 + 180 a^4 c d^3 + 35 a^2 d^4) x^4 + 12 (420 a^6 c^3 d + 378 a^4 c^2 d^2 + 180 a^2 c d^3 + 35 d^4) x^2 + 12 (315 a^8 c^4 + 420 a^6 c^3 d + 378 a^4 c^2 d^2 + 180 a^2 c d^3 + 35 d^4) \log(a^2 x^2 - 1) + 12 (35 a^9 d^4 x^9 + 180 a^9 c d^3 x^7 + 378 a^9 c^2 d^2 x^5 + 420 a^9 c^3 d x^3 + 315 a^9 c^4 x) \log((ax + 1)/(ax - 1))}{a^9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^4\*arccoth(a\*x),x, algorithm="maxima")

```
[Out] 1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 + 7*a^4*d^4)*x^6 + 6*(378*a^6*c^2*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x + 1)/a^10 + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x - 1)/a^10) + 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^4*x)*arccoth(a*x)
```

**mupad [B]** time = 1.51, size = 296, normalized size = 1.21

$$\ln\left(\frac{1}{ax} + 1\right) \left(\frac{c^4 x}{2} + \frac{2c^3 dx^3}{3} + \frac{3c^2 d^2 x^5}{5} + \frac{2cd^3 x^7}{7} + \frac{d^4 x^9}{18}\right) - \ln\left(1 - \frac{1}{ax}\right) \left(\frac{c^4 x}{2} + \frac{2c^3 dx^3}{3} + \frac{3c^2 d^2 x^5}{5} + \frac{2cd^3 x^7}{7} + \frac{d^4 x^9}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoath(a*x)*(c + d*x^2)^4, x)
```

```
[Out] log(1/(a*x) + 1)*((c^4*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^7)/7 + (3*c^2*d^2*x^5)/5) - log(1 - 1/(a*x))*((c^4*x)/2 + (d^4*x^9)/18 + (2*c^3*d*x^3)/3 + (2*c*d^3*x^7)/7 + (3*c^2*d^2*x^5)/5) + x^2*(((d^4/(9*a^3) + (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5*a))/(2*a^2) + (2*c^3*d)/(3*a)) + x^6*(d^4/(54*a^3) + (2*c*d^3)/(21*a)) + x^4*((d^4/(9*a^3) + (4*c*d^3)/(7*a))/(4*a^2) + (3*c^2*d^2)/(10*a)) + (log(a^2*x^2 - 1)*(35*d^4 + 315*a^8*c^4 + 180*a^2*c*d^3 + 420*a^6*c^3*d + 378*a^4*c^2*d^2))/(630*a^9) + (d^4*x^8)/(72*a)
```

**sympy [A]** time = 6.99, size = 427, normalized size = 1.74

$$\left\{ \begin{array}{l} c^4 x \operatorname{acoath}(ax) + \frac{4c^3 dx^3 \operatorname{acoath}(ax)}{3} + \frac{6c^2 d^2 x^5 \operatorname{acoath}(ax)}{5} + \frac{4cd^3 x^7 \operatorname{acoath}(ax)}{7} + \frac{d^4 x^9 \operatorname{acoath}(ax)}{9} + \frac{c^4 \log\left(x - \frac{1}{a}\right)}{a} + \frac{c^4 \operatorname{acoath}(ax)}{a} + \frac{2c^3}{3} \\ \frac{i\pi\left(c^4 x + \frac{4c^3 dx^3}{3} + \frac{6c^2 d^2 x^5}{5} + \frac{4cd^3 x^7}{7} + \frac{d^4 x^9}{9}\right)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*acoath(a*x), x)
```

```
[Out] Piecewise((c**4*x*acoath(a*x) + 4*c**3*d*x**3*acoath(a*x)/3 + 6*c**2*d**2*x**5*acoath(a*x)/5 + 4*c*d**3*x**7*acoath(a*x)/7 + d**4*x**9*acoath(a*x)/9 + c**4*log(x - 1/a)/a + c**4*acoath(a*x)/a + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**4/(10*a) + 2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) + 4*c**3*d*log(x - 1/a)/(3*a**3) + 4*c**3*d*acoath(a*x)/(3*a**3) + 3*c**2*d**2*x**2/(5*a**3) + c*d**3*x**4/(7*a**3) + d**4*x**6/(54*a**3) + 6*c**2*d**2*log(x - 1/a)/(5*a**5) +
```

```

6*c**2*d**2*acoth(a*x)/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*x**4/(36*a
**5) + 4*c*d**3*log(x - 1/a)/(7*a**7) + 4*c*d**3*acoth(a*x)/(7*a**7) + d**4
*x**2/(18*a**7) + d**4*log(x - 1/a)/(9*a**9) + d**4*acoth(a*x)/(9*a**9), Ne
(a, 0)), (I*pi*(c**4*x + 4*c**3*d*x**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x*
*7/7 + d**4*x**9/9)/2, True))

```

### 3.36 $\int (c + dx^2)^3 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=169

$$\frac{d^2x^4(21a^2c + 5d)}{140a^3} + \frac{dx^2(35a^4c^2 + 21a^2cd + 5d^2)}{70a^5} + \frac{(35a^6c^3 + 35a^4c^2d + 21a^2cd^2 + 5d^3) \log(1 - a^2x^2)}{70a^7} + c^3x \coth^{-1}(ax)$$

[Out] 1/70\*d\*(35\*a^4\*c^2+21\*a^2\*c\*d+5\*d^2)\*x^2/a^5+1/140\*d^2\*(21\*a^2\*c+5\*d)\*x^4/a^3+1/42\*d^3\*x^6/a+c^3\*x\*arccoth(a\*x)+c^2\*d\*x^3\*arccoth(a\*x)+3/5\*c\*d^2\*x^5\*a\*arccoth(a\*x)+1/7\*d^3\*x^7\*arccoth(a\*x)+1/70\*(35\*a^6\*c^3+35\*a^4\*c^2\*d+21\*a^2\*c\*d^2+5\*d^3)\*ln(-a^2\*x^2+1)/a^7

**Rubi [A]** time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {194, 5977, 1810, 260}

$$\frac{dx^2(35a^4c^2 + 21a^2cd + 5d^2)}{70a^5} + \frac{(35a^4c^2d + 35a^6c^3 + 21a^2cd^2 + 5d^3) \log(1 - a^2x^2)}{70a^7} + \frac{d^2x^4(21a^2c + 5d)}{140a^3} + c^2dx^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)^3\*ArcCoth[a\*x], x]

[Out] (d\*(35\*a^4\*c^2 + 21\*a^2\*c\*d + 5\*d^2)\*x^2)/(70\*a^5) + (d^2\*(21\*a^2\*c + 5\*d)\*x^4)/(140\*a^3) + (d^3\*x^6)/(42\*a) + c^3\*x\*ArcCoth[a\*x] + c^2\*d\*x^3\*ArcCoth[a\*x] + (3\*c\*d^2\*x^5\*ArcCoth[a\*x])/5 + (d^3\*x^7\*ArcCoth[a\*x])/7 + ((35\*a^6\*c^3 + 35\*a^4\*c^2\*d + 21\*a^2\*c\*d^2 + 5\*d^3)\*Log[1 - a^2\*x^2])/(70\*a^7)

#### Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 1810

Int[(Pq\_)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[Pq\*(a + b\*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

#### Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symbol]
:> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x]
- Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

### Rubi steps

$$\begin{aligned} \int (c + dx^2)^3 \coth^{-1}(ax) dx &= c^3x \coth^{-1}(ax) + c^2dx^3 \coth^{-1}(ax) + \frac{3}{5}cd^2x^5 \coth^{-1}(ax) + \frac{1}{7}d^3x^7 \coth^{-1}(ax) - a \int \\ &= c^3x \coth^{-1}(ax) + c^2dx^3 \coth^{-1}(ax) + \frac{3}{5}cd^2x^5 \coth^{-1}(ax) + \frac{1}{7}d^3x^7 \coth^{-1}(ax) - a \int \\ &= \frac{d(35a^4c^2 + 21a^2cd + 5d^2)x^2}{70a^5} + \frac{d^2(21a^2c + 5d)x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x \coth^{-1}(ax) + c^2d \\ &= \frac{d(35a^4c^2 + 21a^2cd + 5d^2)x^2}{70a^5} + \frac{d^2(21a^2c + 5d)x^4}{140a^3} + \frac{d^3x^6}{42a} + c^3x \coth^{-1}(ax) + c^2d \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 150, normalized size = 0.89

$$\frac{12a^7x \coth^{-1}(ax) (35c^3 + 35c^2dx^2 + 21cd^2x^4 + 5d^3x^6) + a^2dx^2 (a^4(210c^2 + 63cdx^2 + 10d^2x^4) + 3a^2d(42c + 5dx^2))}{420a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcCoth[a*x], x]
```

```
[Out] (a^2*d*x^2*(30*d^2 + 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2 +
10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6)
*ArcCoth[a*x] + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1
- a^2*x^2])/(420*a^7)
```

**fricas [A]** time = 0.60, size = 177, normalized size = 1.05

$$\frac{10a^6d^3x^6 + 3(21a^6cd^2 + 5a^4d^3)x^4 + 6(35a^6c^2d + 21a^4cd^2 + 5a^2d^3)x^2 + 6(35a^6c^3 + 35a^4c^2d + 21a^2cd^2 + 5d^3)}{420a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccoth(a*x), x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 + 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
+ 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d
```



$$^2 + 5*d^3)*\log(a^2*x^2 - 1) + 6*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7*c^2*d*x^3 + 35*a^7*c^3*x)*\log((a*x + 1)/(a*x - 1))/a^7$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^2 + c)^3 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^3\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate((d\*x^2 + c)^3\*arccoth(a\*x), x)

**maple** [A] time = 0.03, size = 233, normalized size = 1.38

$$\frac{d^3 x^7 \operatorname{arccoth}(ax)}{7} + \frac{3c d^2 x^5 \operatorname{arccoth}(ax)}{5} + c^2 d x^3 \operatorname{arccoth}(ax) + c^3 x \operatorname{arccoth}(ax) + \frac{3c d^2 x^2}{10a^3} + \frac{x^2 c^2 d}{2a} + \frac{3c d^2 x^4}{20a} + \frac{d^3 x^6}{42a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^2+c)^3\*arccoth(a\*x),x)

[Out] 1/7\*d^3\*x^7\*arccoth(a\*x)+3/5\*c\*d^2\*x^5\*arccoth(a\*x)+c^2\*d\*x^3\*arccoth(a\*x)+c^3\*x\*arccoth(a\*x)+3/10/a^3\*c\*d^2\*x^2+1/2/a\*x^2\*c^2\*d+3/20/a\*c\*d^2\*x^4+1/42\*d^3\*x^6/a+1/28/a^3\*x^4\*d^3+1/2/a\*ln(a\*x-1)\*c^3+1/2/a^3\*ln(a\*x-1)\*c^2\*d+3/10/a^5\*ln(a\*x-1)\*c\*d^2+1/14/a^7\*ln(a\*x-1)\*d^3+1/14/a^5\*d^3\*x^2+1/2\*c^3\*ln(a\*x+1)/a+1/2/a^3\*ln(a\*x+1)\*c^2\*d+3/10/a^5\*ln(a\*x+1)\*c\*d^2+1/14/a^7\*ln(a\*x+1)\*d^3

**maxima** [A] time = 0.31, size = 198, normalized size = 1.17

$$\frac{1}{420} a \left( \frac{10 a^4 d^3 x^6 + 3 (21 a^4 c d^2 + 5 a^2 d^3) x^4 + 6 (35 a^4 c^2 d + 21 a^2 c d^2 + 5 d^3) x^2}{a^6} + \frac{6 (35 a^6 c^3 + 35 a^4 c^2 d + 21 a^2 c d^2 + 5 d^3)}{a^8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^3\*arccoth(a\*x),x, algorithm="maxima")

[Out] 1/420\*a\*((10\*a^4\*d^3\*x^6 + 3\*(21\*a^4\*c\*d^2 + 5\*a^2\*d^3)\*x^4 + 6\*(35\*a^4\*c^2\*d + 21\*a^2\*c\*d^2 + 5\*d^3)\*x^2)/a^6 + 6\*(35\*a^6\*c^3 + 35\*a^4\*c^2\*d + 21\*a^2\*c\*d^2 + 5\*d^3)\*log(a\*x + 1)/a^8 + 6\*(35\*a^6\*c^3 + 35\*a^4\*c^2\*d + 21\*a^2\*c\*d^2 + 5\*d^3)\*log(a\*x - 1)/a^8 + 1/35\*(5\*d^3\*x^7 + 21\*c\*d^2\*x^5 + 35\*c^2\*d\*x^3 + 35\*c^3\*x)\*arccoth(a\*x)

**mupad** [B] time = 1.52, size = 190, normalized size = 1.12

$$c^3 x \operatorname{acoth}(ax) + \frac{d^3 x^7 \operatorname{acoth}(ax)}{7} + \frac{c^3 \ln(a^2 x^2 - 1)}{2a} + \frac{d^3 \ln(a^2 x^2 - 1)}{14a^7} + \frac{d^3 x^6}{42a} + \frac{d^3 x^4}{28a^3} + \frac{d^3 x^2}{14a^5} + \frac{c^2 d \ln(a^2 x^2 - 1)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)*(c + d*x^2)^3,x)`

[Out]  $c^3 x \operatorname{acoth}(a x) + (d^3 x^7 \operatorname{acoth}(a x))/7 + (c^3 \log(a^2 x^2 - 1))/(2 a) + (d^3 \log(a^2 x^2 - 1))/(14 a^7) + (d^3 x^6)/(42 a) + (d^3 x^4)/(28 a^3) + (d^3 x^2)/(14 a^5) + (c^2 d \log(a^2 x^2 - 1))/(2 a^3) + (3 c d^2 \log(a^2 x^2 - 1))/(10 a^5) + (c^2 d x^2)/(2 a) + (3 c d^2 x^4)/(20 a) + (3 c d^2 x^2)/(10 a^3) + c^2 d x^3 \operatorname{acoth}(a x) + (3 c d^2 x^5 \operatorname{acoth}(a x))/5$

**sympy** [A] time = 4.75, size = 282, normalized size = 1.67

$$\left\{ \begin{array}{l} c^3 x \operatorname{acoth}(a x) + c^2 d x^3 \operatorname{acoth}(a x) + \frac{3 c d^2 x^5 \operatorname{acoth}(a x)}{5} + \frac{d^3 x^7 \operatorname{acoth}(a x)}{7} + \frac{c^3 \log\left(x - \frac{1}{a}\right)}{a} + \frac{c^3 \operatorname{acoth}(a x)}{a} + \frac{c^2 d x^2}{2 a} + \frac{3 c d^2 x^4}{20 a} + \frac{d^3 x^2}{14 a^5} \\ \frac{i \pi \left( c^3 x + c^2 d x^3 + \frac{3 c d^2 x^5}{5} + \frac{d^3 x^7}{7} \right)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**2+c)**3*acoth(a*x),x)`

[Out] `Piecewise((c**3*x*acoth(a*x) + c**2*d*x**3*acoth(a*x) + 3*c*d**2*x**5*acoth(a*x)/5 + d**3*x**7*acoth(a*x)/7 + c**3*log(x - 1/a)/a + c**3*acoth(a*x)/a + c**2*d*x**2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) + c**2*d*log(x - 1/a)/a**3 + c**2*d*acoth(a*x)/a**3 + 3*c*d**2*x**2/(10*a**3) + d**3*x**4/(28*a**3) + 3*c*d**2*log(x - 1/a)/(5*a**5) + 3*c*d**2*acoth(a*x)/(5*a**5) + d**3*x**2/(14*a**5) + d**3*log(x - 1/a)/(7*a**7) + d**3*acoth(a*x)/(7*a**7), Ne(a, 0)), (I*pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7)/2, True))`

### 3.37 $\int (c + dx^2)^2 \coth^{-1}(ax) dx$

**Optimal.** Leaf size=110

$$\frac{dx^2(10a^2c + 3d)}{30a^3} + \frac{(15a^4c^2 + 10a^2cd + 3d^2) \log(1 - a^2x^2)}{30a^5} + c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax)$$

[Out] 1/30\*d\*(10\*a^2\*c+3\*d)\*x^2/a^3+1/20\*d^2\*x^4/a+c^2\*x\*arccoth(a\*x)+2/3\*c\*d\*x^3\*arccoth(a\*x)+1/5\*d^2\*x^5\*arccoth(a\*x)+1/30\*(15\*a^4\*c^2+10\*a^2\*c\*d+3\*d^2)\*Log[1-a^2\*x^2]/a^5

**Rubi [A]** time = 0.13, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {194, 5977, 1594, 1247, 698}

$$\frac{(15a^4c^2 + 10a^2cd + 3d^2) \log(1 - a^2x^2)}{30a^5} + \frac{dx^2(10a^2c + 3d)}{30a^3} + c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{d^2x^4}{20a} + \frac{1}{5}d^2x^5 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)^2\*ArcCoth[a\*x], x]

[Out] (d\*(10\*a^2\*c + 3\*d)\*x^2)/(30\*a^3) + (d^2\*x^4)/(20\*a) + c^2\*x\*ArcCoth[a\*x] + (2\*c\*d\*x^3\*ArcCoth[a\*x])/3 + (d^2\*x^5\*ArcCoth[a\*x])/5 + ((15\*a^4\*c^2 + 10\*a^2\*c\*d + 3\*d^2)\*Log[1 - a^2\*x^2])/(30\*a^5)

#### Rule 194

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 698

Int[((d\_) + (e\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

#### Rule 1247

Int[(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (b\_)\*(x\_)^2 + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] := Dist[1/2, Subst[Int[(d + e\*x)^q\*(a + b\*x + c\*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 1594

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.) + (c\_.)\*(x\_)^(r\_.))^(n\_.), x\_Symbol] :> Int[u\*x^(n\*p)\*(a + b\*x^(q - p) + c\*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 5977

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(q\_.), x\_Symbol] :> With[{u = IntHide[(d + e\*x^2)^q, x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[u/(1 - c^2\*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \int (c + dx^2)^2 \coth^{-1}(ax) dx &= c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) - a \int \frac{c^2x + \frac{2}{3}cdx^3 + \frac{d^2x^5}{5}}{1 - a^2x^2} \\
 &= c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) - a \int \frac{x \left( c^2 + \frac{2}{3}cdx^2 + \frac{d^2x^4}{5} \right)}{1 - a^2x^2} \\
 &= c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) - \frac{1}{2}a \operatorname{Subst} \left( \int \frac{c^2 + \frac{2cdx}{3} + \frac{d^2x^3}{5}}{1 - a^2x^2} \right) \\
 &= c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax) - \frac{1}{2}a \operatorname{Subst} \left( \int \left( -\frac{d(10a^2c + 3d)}{15} \right) \right) \\
 &= \frac{d(10a^2c + 3d)x^2}{30a^3} + \frac{d^2x^4}{20a} + c^2x \coth^{-1}(ax) + \frac{2}{3}cdx^3 \coth^{-1}(ax) + \frac{1}{5}d^2x^5 \coth^{-1}(ax)
 \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 98, normalized size = 0.89

$$\frac{4a^5x \coth^{-1}(ax) (15c^2 + 10cdx^2 + 3d^2x^4) + a^2dx^2 (a^2(20c + 3dx^2) + 6d) + (30a^4c^2 + 20a^2cd + 6d^2) \log(1 - a^2x^2)}{60a^5}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^2)^2\*ArcCoth[a\*x], x]

[Out] (a^2\*d\*x^2\*(6\*d + a^2\*(20\*c + 3\*d\*x^2)) + 4\*a^5\*x\*(15\*c^2 + 10\*c\*d\*x^2 + 3\*d^2\*x^4)\*ArcCoth[a\*x] + (30\*a^4\*c^2 + 20\*a^2\*c\*d + 6\*d^2)\*Log[1 - a^2\*x^2]) / (60\*a^5)

**fricas** [A] time = 0.46, size = 118, normalized size = 1.07

$$\frac{3a^4d^2x^4 + 2(10a^4cd + 3a^2d^2)x^2 + 2(15a^4c^2 + 10a^2cd + 3d^2)\log(a^2x^2 - 1) + 2(3a^5d^2x^5 + 10a^5cdx^3 + 15a^5c^2x)}{60a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^2\*arccoth(a\*x),x, algorithm="fricas")

[Out] 1/60\*(3\*a^4\*d^2\*x^4 + 2\*(10\*a^4\*c\*d + 3\*a^2\*d^2)\*x^2 + 2\*(15\*a^4\*c^2 + 10\*a^2\*c\*d + 3\*d^2)\*log(a^2\*x^2 - 1) + 2\*(3\*a^5\*d^2\*x^5 + 10\*a^5\*c\*d\*x^3 + 15\*a^5\*c^2\*x)\*log((a\*x + 1)/(a\*x - 1)))/a^5

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^2 + c)^2 \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^2\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate((d\*x^2 + c)^2\*arccoth(a\*x), x)

**maple** [A] time = 0.04, size = 148, normalized size = 1.35

$$\frac{d^2x^5 \operatorname{arccoth}(ax)}{5} + \frac{2cdx^3 \operatorname{arccoth}(ax)}{3} + c^2x \operatorname{arccoth}(ax) + \frac{d^2x^4}{20a} + \frac{cdx^2}{3a} + \frac{x^2d^2}{10a^3} + \frac{\ln(ax-1)c^2}{2a} + \frac{\ln(ax-1)cd}{3a^3} + \frac{\ln(ax-1)d^2}{10a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^2+c)^2\*arccoth(a\*x),x)

[Out] 1/5\*d^2\*x^5\*arccoth(a\*x)+2/3\*c\*d\*x^3\*arccoth(a\*x)+c^2\*x\*arccoth(a\*x)+1/20\*d^2\*x^4/a+1/3/a\*c\*d\*x^2+1/10/a^3\*x^2\*d^2+1/2/a\*ln(a\*x-1)\*c^2+1/3/a^3\*ln(a\*x-1)\*c\*d+1/10/a^5\*ln(a\*x-1)\*d^2+1/2\*c^2\*ln(a\*x+1)/a+1/3/a^3\*ln(a\*x+1)\*c\*d+1/10/a^5\*ln(a\*x+1)\*d^2

**maxima** [A] time = 0.30, size = 131, normalized size = 1.19

$$\frac{1}{60}a \left( \frac{3a^2d^2x^4 + 2(10a^2cd + 3d^2)x^2}{a^4} + \frac{2(15a^4c^2 + 10a^2cd + 3d^2)\log(ax+1)}{a^6} + \frac{2(15a^4c^2 + 10a^2cd + 3d^2)}{a^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^2\*arccoth(a\*x),x, algorithm="maxima")

[Out]  $\frac{1}{60}a^4 \left( \frac{3a^2 d^2 x^4 + 2(10a^2 c d + 3d^2)x^2}{a^4} + 2 \frac{(15a^4 c^2 + 10a^2 c d + 3d^2) \log(ax + 1)}{a^6} + 2 \frac{(15a^4 c^2 + 10a^2 c d + 3d^2) \log(ax - 1)}{a^6} + \frac{1}{15} (3d^2 x^5 + 10c d x^3 + 15c^2 x) \operatorname{arccoth}(ax) \right)$

**mupad [B]** time = 1.37, size = 115, normalized size = 1.05

$$\frac{a^4 \left( \frac{c^2 \ln(a^2 x^2 - 1)}{2} + \frac{d^2 x^4}{20} + \frac{c d x^2}{3} \right) + a^2 \left( \frac{d^2 x^2}{10} + \frac{c d \ln(a^2 x^2 - 1)}{3} \right) + \frac{d^2 \ln(a^2 x^2 - 1)}{10}}{a^5} + c^2 x \operatorname{acoth}(ax) + \frac{d^2 x^5 \operatorname{acoth}(ax)}{5} + \frac{2c}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)*(c + d*x^2)^2,x)`

[Out]  $(a^4((c^2 \log(a^2 x^2 - 1))/2 + (d^2 x^4)/20 + (c d x^2)/3) + a^2((d^2 x^2)/10 + (c d \log(a^2 x^2 - 1))/3) + (d^2 \log(a^2 x^2 - 1))/10)/a^5 + c^2 x \operatorname{acoth}(ax) + (d^2 x^5 \operatorname{acoth}(ax))/5 + (2c d x^3 \operatorname{acoth}(ax))/3$

**sympy [A]** time = 2.13, size = 182, normalized size = 1.65

$$\left\{ \begin{array}{l} c^2 x \operatorname{acoth}(ax) + \frac{2c d x^3 \operatorname{acoth}(ax)}{3} + \frac{d^2 x^5 \operatorname{acoth}(ax)}{5} + \frac{c^2 \log\left(x - \frac{1}{a}\right)}{a} + \frac{c^2 \operatorname{acoth}(ax)}{a} + \frac{c d x^2}{3a} + \frac{d^2 x^4}{20a} + \frac{2c d \log\left(x - \frac{1}{a}\right)}{3a^3} + \frac{2c d \operatorname{acoth}(ax)}{3a^3} \\ \frac{i\pi \left( c^2 x + \frac{2c d x^3}{3} + \frac{d^2 x^5}{5} \right)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**2+c)**2*acoth(a*x),x)`

[Out] `Piecewise((c**2*x*acoth(a*x) + 2*c*d*x**3*acoth(a*x)/3 + d**2*x**5*acoth(a*x)/5 + c**2*log(x - 1/a)/a + c**2*acoth(a*x)/a + c*d*x**2/(3*a) + d**2*x**4/(20*a) + 2*c*d*log(x - 1/a)/(3*a**3) + 2*c*d*acoth(a*x)/(3*a**3) + d**2*x**2/(10*a**3) + d**2*log(x - 1/a)/(5*a**5) + d**2*acoth(a*x)/(5*a**5), Ne(a, 0)), (I*pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, True))`

### 3.38 $\int (c + dx^2) \coth^{-1}(ax) dx$

**Optimal.** Leaf size=57

$$\frac{(3a^2c + d) \log(1 - a^2x^2)}{6a^3} + cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) + \frac{dx^2}{6a}$$

[Out] 1/6\*d\*x^2/a+c\*x\*arccoth(a\*x)+1/3\*d\*x^3\*arccoth(a\*x)+1/6\*(3\*a^2\*c+d)\*ln(-a^2\*x^2+1)/a^3

**Rubi [A]** time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5977, 1593, 444, 43}

$$\frac{(3a^2c + d) \log(1 - a^2x^2)}{6a^3} + cx \coth^{-1}(ax) + \frac{dx^2}{6a} + \frac{1}{3} dx^3 \coth^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)\*ArcCoth[a\*x], x]

[Out] (d\*x^2)/(6\*a) + c\*x\*ArcCoth[a\*x] + (d\*x^3\*ArcCoth[a\*x])/3 + ((3\*a^2\*c + d)\*Log[1 - a^2\*x^2])/(6\*a^3)

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 444

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

#### Rule 1593

Int[(u\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symbol]
:> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

### Rubi steps

$$\begin{aligned}
\int (c + dx^2) \coth^{-1}(ax) dx &= cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) - a \int \frac{cx + \frac{dx^3}{3}}{1 - a^2x^2} dx \\
&= cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) - a \int \frac{x \left( c + \frac{dx^2}{3} \right)}{1 - a^2x^2} dx \\
&= cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) - \frac{1}{2} a \operatorname{Subst} \left( \int \frac{c + \frac{dx}{3}}{1 - a^2x} dx, x, x^2 \right) \\
&= cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) - \frac{1}{2} a \operatorname{Subst} \left( \int \left( -\frac{d}{3a^2} + \frac{-3a^2c - d}{3a^2(-1 + a^2x)} \right) dx, x, x^2 \right) \\
&= \frac{dx^2}{6a} + cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) + \frac{(3a^2c + d) \log(1 - a^2x^2)}{6a^3}
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 69, normalized size = 1.21

$$\frac{c \log(1 - a^2x^2)}{2a} + \frac{d \log(1 - a^2x^2)}{6a^3} + cx \coth^{-1}(ax) + \frac{1}{3} dx^3 \coth^{-1}(ax) + \frac{dx^2}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcCoth[a*x], x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcCoth[a*x] + (d*x^3*ArcCoth[a*x])/3 + (c*Log[1 - a^2*x^2])/(2*a) + (d*Log[1 - a^2*x^2])/(6*a^3)
```

**fricas** [A] time = 0.49, size = 64, normalized size = 1.12

$$\frac{a^2 dx^2 + (3a^2c + d) \log(a^2x^2 - 1) + (a^3 dx^3 + 3a^3 cx) \log\left(\frac{ax+1}{ax-1}\right)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccoth(a*x), x, algorithm="fricas")
```



[Out]  $\frac{1}{6}*(a^2*d*x^2 + (3*a^2*c + d)*\log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*\log((a*x + 1)/(a*x - 1)))/a^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^2 + c) \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)*arccoth(a*x),x, algorithm="giac")`

[Out] `integrate((d*x^2 + c)*arccoth(a*x), x)`

**maple** [A] time = 0.04, size = 76, normalized size = 1.33

$$\frac{dx^3 \operatorname{arccoth}(ax)}{3} + cx \operatorname{arccoth}(ax) + \frac{dx^2}{6a} + \frac{\ln(ax-1)c}{2a} + \frac{\ln(ax-1)d}{6a^3} + \frac{c \ln(ax+1)}{2a} + \frac{\ln(ax+1)d}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^2+c)*arccoth(a*x),x)`

[Out]  $\frac{1}{3}d*x^3*\operatorname{arccoth}(a*x)+c*x*\operatorname{arccoth}(a*x)+\frac{1}{6}d*x^2/a+\frac{1}{2}/a*\ln(a*x-1)*c+\frac{1}{6}/a^3*\ln(a*x-1)*d+\frac{1}{2}c*\ln(a*x+1)/a+\frac{1}{6}/a^3*\ln(a*x+1)*d$

**maxima** [A] time = 0.30, size = 65, normalized size = 1.14

$$\frac{1}{6}a\left(\frac{dx^2}{a^2} + \frac{(3a^2c + d)\log(ax + 1)}{a^4} + \frac{(3a^2c + d)\log(ax - 1)}{a^4}\right) + \frac{1}{3}(dx^3 + 3cx)\operatorname{arccoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)*arccoth(a*x),x, algorithm="maxima")`

[Out]  $\frac{1}{6}*a*(d*x^2/a^2 + (3*a^2*c + d)*\log(a*x + 1)/a^4 + (3*a^2*c + d)*\log(a*x - 1)/a^4) + \frac{1}{3}*(d*x^3 + 3*c*x)*\operatorname{arccoth}(a*x)$

**mupad** [B] time = 1.27, size = 60, normalized size = 1.05

$$\frac{\frac{d \ln(a^2 x^2 - 1)}{6} + a^2 \left( \frac{c \ln(a^2 x^2 - 1)}{2} + \frac{dx^2}{6} \right)}{a^3} + \frac{dx^3 \operatorname{acoth}(ax)}{3} + cx \operatorname{acoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)*(c + d*x^2),x)`

[Out]  $((d \cdot \log(a^2 x^2 - 1))/6 + a^2((c \cdot \log(a^2 x^2 - 1))/2 + (d x^2)/6))/a^3 + (d x^3 \operatorname{acoth}(a x))/3 + c x \operatorname{acoth}(a x)$

sympy [A] time = 1.06, size = 87, normalized size = 1.53

$$\begin{cases} cx \operatorname{acoth}(ax) + \frac{dx^3 \operatorname{acoth}(ax)}{3} + \frac{c \log\left(x - \frac{1}{a}\right)}{a} + \frac{c \operatorname{acoth}(ax)}{a} + \frac{dx^2}{6a} + \frac{d \log\left(x - \frac{1}{a}\right)}{3a^3} + \frac{d \operatorname{acoth}(ax)}{3a^3} & \text{for } a \neq 0 \\ \frac{i\pi\left(cx + \frac{dx^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**2+c)*acoth(a*x),x)`

[Out] `Piecewise((c*x*acoth(a*x) + d*x**3*acoth(a*x)/3 + c*log(x - 1/a)/a + c*acoth(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*acoth(a*x)/(3*a**3), Ne(a, 0)), (I*pi*(c*x + d*x**3/3)/2, True))`

$$3.39 \quad \int \frac{\coth^{-1}(ax)}{c+dx^2} dx$$

**Optimal.** Leaf size=390

$$\frac{i\text{Li}_2\left(\frac{2\sqrt{c}\sqrt{d}(1-ax)}{(i\sqrt{c}-\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}+1\right)}{4\sqrt{c}\sqrt{d}} + \frac{i\text{Li}_2\left(1-\frac{2\sqrt{c}\sqrt{d}(ax+1)}{(i\sqrt{c}+\sqrt{d})(\sqrt{c}-i\sqrt{d}x)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\log\left(1-\frac{1}{ax}\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\log\left(\frac{1}{ax}+1\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}}$$

[Out]  $-1/2*\arctan(x*d^{(1/2)}/c^{(1/2)})*\ln(1-1/a/x)/c^{(1/2)}/d^{(1/2)}+1/2*\arctan(x*d^{(1/2)}/c^{(1/2)})*\ln(1+1/a/x)/c^{(1/2)}/d^{(1/2)}+1/2*\arctan(x*d^{(1/2)}/c^{(1/2)})*\ln(-2*(-a*x+1)*c^{(1/2)*d^{(1/2)}}/(I*a*c^{(1/2)}-d^{(1/2)})/(c^{(1/2)}-I*x*d^{(1/2)}))/c^{(1/2)}/d^{(1/2)}-1/2*\arctan(x*d^{(1/2)}/c^{(1/2)})*\ln(2*(a*x+1)*c^{(1/2)*d^{(1/2)}}/(I*a*c^{(1/2)}+d^{(1/2)})/(c^{(1/2)}-I*x*d^{(1/2)}))/c^{(1/2)}/d^{(1/2)}-1/4*I*\text{polylog}(2, 1+2*(-a*x+1)*c^{(1/2)*d^{(1/2)}}/(I*a*c^{(1/2)}-d^{(1/2)})/(c^{(1/2)}-I*x*d^{(1/2)}))/c^{(1/2)}/d^{(1/2)}+1/4*I*\text{polylog}(2, 1-2*(a*x+1)*c^{(1/2)*d^{(1/2)}}/(I*a*c^{(1/2)}+d^{(1/2)})/(c^{(1/2)}-I*x*d^{(1/2)}))/c^{(1/2)}/d^{(1/2)}$

**Rubi [A]** time = 0.95, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$ , Rules used = {5973, 205, 2470, 12, 260, 6688, 4876, 4848, 2391, 4856, 2402, 2315, 2447}

$$\frac{i\text{PolyLog}\left(2, 1 + \frac{2\sqrt{c}\sqrt{d}(1-ax)}{(-\sqrt{d}+ia\sqrt{c})(\sqrt{c}-i\sqrt{d}x)}\right)}{4\sqrt{c}\sqrt{d}} + \frac{i\text{PolyLog}\left(2, 1 - \frac{2\sqrt{c}\sqrt{d}(ax+1)}{(\sqrt{d}+ia\sqrt{c})(\sqrt{c}-i\sqrt{d}x)}\right)}{4\sqrt{c}\sqrt{d}} - \frac{\log\left(1-\frac{1}{ax}\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}} + \frac{\log\left(\frac{1}{ax}+1\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2), x]

[Out]  $-(\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[1-1/(a*x)])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) + (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[1+1/(a*x)])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) + (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[(-2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1-a*x))/((I*a*\text{Sqrt}[c]-\text{Sqrt}[d])*(\text{Sqrt}[c]-I*\text{Sqrt}[d]*x)))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) - (\text{ArcTan}[(\text{Sqrt}[d]*x)/\text{Sqrt}[c]]*\text{Log}[(2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1+a*x))/((I*a*\text{Sqrt}[c]+\text{Sqrt}[d])*(\text{Sqrt}[c]-I*\text{Sqrt}[d]*x)))])/(2*\text{Sqrt}[c]*\text{Sqrt}[d]) - ((I/4)*\text{PolyLog}[2, 1+(2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1-a*x))/((I*a*\text{Sqrt}[c]-\text{Sqrt}[d])*(\text{Sqrt}[c]-I*\text{Sqrt}[d]*x)))]/(\text{Sqrt}[c]*\text{Sqrt}[d])) + ((I/4)*\text{PolyLog}[2, 1-(2*\text{Sqrt}[c]*\text{Sqrt}[d]*(1+a*x))/((I*a*\text{Sqrt}[c]+\text{Sqrt}[d])*(\text{Sqrt}[c]-I*\text{Sqrt}[d]*x)))]/(\text{Sqrt}[c]*\text{Sqrt}[d]))$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 205

$\text{Int}[(a_.) + (b_.) \cdot (x_.)^2]^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]]/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 260

$\text{Int}[(x_.)^{(m_.)} / ((a_.) + (b_.) \cdot (x_.)^{(n_.)})], x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x^n, x]] / (b \cdot n), x] /; \text{FreeQ}\{a, b, m, n\}, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 2315

$\text{Int}[\text{Log}[(c_.) \cdot (x_.)] / ((d_.) + (e_.) \cdot (x_.)^2)], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c \cdot x] / e, x] /; \text{FreeQ}\{c, d, e\}, x\} \ \&\& \ \text{EqQ}[e + c \cdot d, 0]$

Rule 2391

$\text{Int}[\text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_.)^{(n_.)}))] / (x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)] / n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c \cdot d, 1]$

Rule 2402

$\text{Int}[\text{Log}[(c_.) / ((d_.) + (e_.) \cdot (x_.)^2)] / ((f_.) + (g_.) \cdot (x_.)^2)], x\_Symbol] \rightarrow -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2 \cdot d \cdot x] / (1 - 2 \cdot d \cdot x)], x], x, 1 / (d + e \cdot x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x\} \ \&\& \ \text{EqQ}[c, 2 \cdot d] \ \&\& \ \text{EqQ}[e^2 \cdot f + d^2 \cdot g, 0]$

Rule 2447

$\text{Int}[\text{Log}[u_.] \cdot (Pq_.)^{(m_.)}], x\_Symbol] \rightarrow \text{With}\{C = \text{FullSimplify}[(Pq^m \cdot (1 - u)) / D[u, x]]\}, \text{Simp}[C \cdot \text{PolyLog}[2, 1 - u], x] /; \text{FreeQ}[C, x] /; \text{IntegerQ}[m] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{RationalFunctionQ}[u, x] \ \&\& \ \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]$

Rule 2470

$\text{Int}[(a_.) + \text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_.)^{(n_.)}))^{(p_.)}] \cdot (b_.)] / ((f_.) + (g_.) \cdot (x_.)^2), x\_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[1 / (f + g \cdot x^2), x]\}, \text{Simp}[u \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p]), x] - \text{Dist}[b \cdot e \cdot n \cdot p, \text{Int}[(u \cdot x^{(n-1)}) / (d + e \cdot x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p\}, x\} \ \&\& \ \text{IntegerQ}[n]$

Rule 4848

$\text{Int}[(a_.) + \text{ArcTan}[(c_.) \cdot (x_.)] \cdot (b_.)] / (x_.), x\_Symbol] \rightarrow \text{Simp}[a \cdot \text{Log}[x], x] + (\text{Dist}[(I \cdot b) / 2, \text{Int}[\text{Log}[1 - I \cdot c \cdot x] / x, x], x] - \text{Dist}[(I \cdot b) / 2, \text{Int}[\text{Log}[1 +$

$I*c*x]/x, x], x]) /; \text{FreeQ}\{a, b, c\}, x]$

### Rule 4856

$\text{Int}[(a_.) + \text{ArcTan}[c_.*(x_.)]*(b_.)]/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow -\text{Simp}[(a + b*\text{ArcTan}[c*x])*\text{Log}[2/(1 - I*c*x)]]/e, x] + (\text{Dist}[(b*c)/e, \text{Int}[\text{Log}[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - \text{Dist}[(b*c)/e, \text{Int}[\text{Log}[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)]]/(1 + c^2*x^2), x], x] + \text{Simp}[(a + b*\text{ArcTan}[c*x])*\text{Log}[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)]]/e, x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[c^2*d^2 + e^2, 0]$

### Rule 4876

$\text{Int}[(a_.) + \text{ArcTan}[c_.*(x_.)]*(b_.)]^{(p_.)}*((f_.)*(x_.))^{(m_.)}*((d_.) + (e_.)*(x_.))^{(q_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcTan}[c*x])^p, (f*x)^m*(d + e*x)^q], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{IntegerQ}[q] \&\& (\text{GtQ}[q, 0] \parallel \text{NeQ}[a, 0] \parallel \text{IntegerQ}[m])$

### Rule 5973

$\text{Int}[\text{ArcCoth}[c_.*(x_.)]/((d_.) + (e_.)*(x_.)^2), x\_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[\text{Log}[1 + 1/(c*x)]/(d + e*x^2), x], x] - \text{Dist}[1/2, \text{Int}[\text{Log}[1 - 1/(c*x)]/(d + e*x^2), x], x] /; \text{FreeQ}\{c, d, e\}, x]$

### Rule 6688

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{SimplifyIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SimplerIntegrandQ}[v, u, x]$

### Rubi steps



**Mathematica [A]** time = 1.39, size = 671, normalized size = 1.72

$$a \left( i \left( \operatorname{Li}_2 \left( \frac{(ca^2-d+2i\sqrt{a^2cd})(iadx+\sqrt{a^2cd})}{(ca^2+d)(\sqrt{a^2cd}-iadx)} \right) \right) - \operatorname{Li}_2 \left( \frac{(ca^2-d-2i\sqrt{a^2cd})(iadx+\sqrt{a^2cd})}{(ca^2+d)(\sqrt{a^2cd}-iadx)} \right) \right) - 2i \cos^{-1} \left( \frac{a^2c-d}{a^2c+d} \right) \tan^{-1} \left( \frac{ac}{x\sqrt{a^2cd}} \right) + 4 \operatorname{coth}^{-1} \left( \frac{ax}{\sqrt{a^2cd}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2), x]

[Out] (a\*((-2\*I)\*ArcCos[(a^2\*c - d)/(a^2\*c + d)]\*ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x]) + 4\*ArcCoth[a\*x]\*ArcTan[(a\*d\*x)/Sqrt[a^2\*c\*d]] - (ArcCos[(a^2\*c - d)/(a^2\*c + d)] + 2\*ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x]))\*Log[(2\*d\*(a^2\*c - I\*Sqrt[a^2\*c\*d])\*(-1 + a\*x))/((a^2\*c + d)\*(I\*Sqrt[a^2\*c\*d] + a\*d\*x))] - (ArcCos[(a^2\*c - d)/(a^2\*c + d)] - 2\*ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x]))\*Log[(2\*d\*(a^2\*c + I\*Sqrt[a^2\*c\*d])\*(1 + a\*x))/((a^2\*c + d)\*(I\*Sqrt[a^2\*c\*d] + a\*d\*x))] + (ArcCos[(a^2\*c - d)/(a^2\*c + d)] + 2\*(ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x)] + ArcTan[(a\*d\*x)/Sqrt[a^2\*c\*d]]))\*Log[(Sqrt[2]\*Sqrt[a^2\*c\*d])/(Sqrt[a^2\*c + d]\*E^ArcCoth[a\*x]\*Sqrt[-(a^2\*c) + d + (a^2\*c + d)\*Cosh[2\*ArcCoth[a\*x]])]) + (ArcCos[(a^2\*c - d)/(a^2\*c + d)] - 2\*(ArcTan[(a\*c)/(Sqrt[a^2\*c\*d]\*x)] + ArcTan[(a\*d\*x)/Sqrt[a^2\*c\*d]]))\*Log[(Sqrt[2]\*Sqrt[a^2\*c\*d]\*E^ArcCoth[a\*x])/(Sqrt[a^2\*c + d]\*Sqrt[-(a^2\*c) + d + (a^2\*c + d)\*Cosh[2\*ArcCoth[a\*x]])]) + I\*(-PolyLog[2, ((a^2\*c - d - (2\*I)\*Sqrt[a^2\*c\*d])\*(Sqrt[a^2\*c\*d] + I\*a\*d\*x))/((a^2\*c + d)\*(Sqrt[a^2\*c\*d] - I\*a\*d\*x))] + PolyLog[2, ((a^2\*c - d + (2\*I)\*Sqrt[a^2\*c\*d])\*(Sqrt[a^2\*c\*d] + I\*a\*d\*x))/((a^2\*c + d)\*(Sqrt[a^2\*c\*d] - I\*a\*d\*x))]))/(4\*Sqrt[a^2\*c\*d])

**fricas [F]** time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\operatorname{arccoth}(ax)}{dx^2 + c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c), x, algorithm="fricas")

[Out] integral(arccoth(a\*x)/(d\*x^2 + c), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c), x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/(d\*x^2 + c), x)

**maple [B]** time = 0.68, size = 785, normalized size = 2.01

$$\frac{a^3 \ln\left(1 - \frac{(a^2c+d)(ax+1)}{(ax-1)(a^2c-2\sqrt{-a^2cd}-d)}\right) \operatorname{arccoth}(ax) \sqrt{-a^2cd} c}{2d(a^4c^2 + 2a^2cd + d^2)} - \frac{a \ln\left(1 - \frac{(a^2c+d)(ax+1)}{(ax-1)(a^2c-2\sqrt{-a^2cd}-d)}\right) \operatorname{arccoth}(ax) \sqrt{-a^2cd}}{a^4c^2 + 2a^2cd + d^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c), x)

[Out] 
$$-1/2*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*\operatorname{arccoth}(a*x)*(-a^2*c*d)^{(1/2)}*c-a/(a^4*c^2+2*a^2*c*d+d^2)*\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*\operatorname{arccoth}(a*x)*(-a^2*c*d)^{(1/2)}+1/2*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)^2*(-a^2*c*d)^{(1/2)}*c+a/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)^2*(-a^2*c*d)^{(1/2)}-1/4*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}*c-1/2*a/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}-1/2/a/c/(a^4*c^2+2*a^2*c*d+d^2)*\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*\operatorname{arccoth}(a*x)*(-a^2*c*d)^{(1/2)}*d+1/2/a/c/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{arccoth}(a*x)^2*(-a^2*c*d)^{(1/2)}*d-1/4/a/c/(a^4*c^2+2*a^2*c*d+d^2)*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}*d+1/2/a*(-a^2*c*d)^{(1/2)}/c/d*\operatorname{arccoth}(a*x)*\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-1/2/a*(-a^2*c*d)^{(1/2)}/c/d*\operatorname{arccoth}(a*x)^2+1/4/a*(-a^2*c*d)^{(1/2)}/c/d*\operatorname{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))$$

**maxima [A]** time = 0.54, size = 406, normalized size = 1.04

$$\frac{\operatorname{arccoth}(ax) \operatorname{arctan}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}} + \frac{\left(\operatorname{arctan}\left(\frac{(a^2x+a)\sqrt{c}\sqrt{d}}{a^2c+d}, \frac{adx+d}{a^2c+d}\right) - \operatorname{arctan}\left(\frac{(a^2x-a)\sqrt{c}\sqrt{d}}{a^2c+d}, -\frac{adx-d}{a^2c+d}\right)\right) \log(dx^2 + c) - \operatorname{arctan}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c), x, algorithm="maxima")

[Out] 
$$\operatorname{arccoth}(a*x)*\operatorname{arctan}(d*x/\sqrt{c*d})/\sqrt{c*d} + 1/4*((\operatorname{arctan}2((a^2*x + a)*\sqrt{c})*\sqrt{d}/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - \operatorname{arctan}2((a^2*x - a)*\sqrt{c})*\sqrt{d}/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*\log(d*x^2 + c) - \operatorname{arctan}(\sqrt{d}*x/\sqrt{c})*\log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) + \operatorname{arctan}(\sqrt{d}*x/\sqrt{c})*\log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) - I*\operatorname{dilog}((a^2*c + a*d*x - (I*a^2*x - I*a)*\sqrt{c})*\sqrt{d})/(a^2*c + 2*I*a*\sqrt{c})*\sqrt{d})$$



d) - d)) - I\*dilog((a^2\*c - a\*d\*x + (I\*a^2\*x + I\*a)\*sqrt(c)\*sqrt(d))/(a^2\*c + 2\*I\*a\*sqrt(c)\*sqrt(d) - d)) + I\*dilog((a^2\*c + a\*d\*x + (I\*a^2\*x - I\*a)\*sqrt(c)\*sqrt(d))/(a^2\*c - 2\*I\*a\*sqrt(c)\*sqrt(d) - d)) + I\*dilog((a^2\*c - a\*d\*x - (I\*a^2\*x + I\*a)\*sqrt(c)\*sqrt(d))/(a^2\*c - 2\*I\*a\*sqrt(c)\*sqrt(d) - d))) /sqrt(c\*d)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(ax)}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)/(c + d\*x^2), x)

[Out] int(acoth(a\*x)/(c + d\*x^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{c + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/(d\*x\*\*2+c), x)

[Out] Integral(acoth(a\*x)/(c + d\*x\*\*2), x)

$$3.40 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^2} dx$$

**Optimal.** Leaf size=590

$$\frac{a \log(1 - a^2 x^2)}{4c(a^2 c + d)} - \frac{a \log(c + dx^2)}{4c(a^2 c + d)} + \frac{i \operatorname{Li}_2\left(\frac{a(\sqrt{c} - i\sqrt{d}x)}{a\sqrt{c} - i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \operatorname{Li}_2\left(\frac{a(\sqrt{c} - i\sqrt{d}x)}{\sqrt{c}a + i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} + \frac{i \operatorname{Li}_2\left(\frac{a(i\sqrt{d}x + \sqrt{c})}{a\sqrt{c} - i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \operatorname{Li}_2\left(\frac{a(i\sqrt{d}x + \sqrt{c})}{\sqrt{c}a + i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} + \dots$$

[Out]  $\frac{1}{2}x \operatorname{arccoth}(ax)/c/(dx^2+c) + \frac{1}{4}a \ln(-a^2x^2+1)/c/(a^2c+d) - \frac{1}{4}a \ln(dx^2+c)/c/(a^2c+d) + \frac{1}{2} \operatorname{arccoth}(ax) \operatorname{arctan}(x\sqrt{d}/c^{1/2})/c^{3/2}/d^{1/2} - \frac{1}{8}I \ln(-(ax+1)d^{1/2}/(Iac^{1/2}-d^{1/2}))/c^{3/2}/d^{1/2} + \frac{1}{8}I \ln(1-Ixd^{1/2}/c^{1/2})/c^{3/2}/d^{1/2} + \frac{1}{8}I \ln((-ax+1)d^{1/2}/(Iac^{1/2}+d^{1/2}))/c^{3/2}/d^{1/2} - \frac{1}{8}I \ln(1-Ixd^{1/2}/c^{1/2})/c^{3/2}/d^{1/2} - \frac{1}{8}I \ln(-(-ax+1)d^{1/2}/(Iac^{1/2}-d^{1/2}))/c^{3/2}/d^{1/2} + \frac{1}{8}I \ln(1+Ixd^{1/2}/c^{1/2})/c^{3/2}/d^{1/2} + \frac{1}{8}I \ln((ax+1)d^{1/2}/(Iac^{1/2}+d^{1/2}))/c^{3/2}/d^{1/2} + \frac{1}{8}I \operatorname{polylog}(2, a(c^{1/2}-Ixd^{1/2})/(ac^{1/2}-I d^{1/2}))/c^{3/2}/d^{1/2} - \frac{1}{8}I \operatorname{polylog}(2, a(c^{1/2}-Ixd^{1/2})/(ac^{1/2}+I d^{1/2}))/c^{3/2}/d^{1/2} + \frac{1}{8}I \operatorname{polylog}(2, a(c^{1/2}+Ixd^{1/2})/(ac^{1/2}-I d^{1/2}))/c^{3/2}/d^{1/2} - \frac{1}{8}I \operatorname{polylog}(2, a(c^{1/2}+Ixd^{1/2})/(ac^{1/2}+I d^{1/2}))/c^{3/2}/d^{1/2}$

**Rubi [A]** time = 0.87, antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$ , Rules used = {199, 205, 5977, 6725, 517, 444, 36, 31, 4908, 2409, 2394, 2393, 2391}

$$\frac{i \operatorname{PolyLog}\left(2, \frac{a(\sqrt{c} - i\sqrt{d}x)}{a\sqrt{c} - i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \operatorname{PolyLog}\left(2, \frac{a(\sqrt{c} - i\sqrt{d}x)}{a\sqrt{c} + i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} + \frac{i \operatorname{PolyLog}\left(2, \frac{a(\sqrt{c} + i\sqrt{d}x)}{a\sqrt{c} - i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \operatorname{PolyLog}\left(2, \frac{a(\sqrt{c} + i\sqrt{d}x)}{a\sqrt{c} + i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}} + \dots$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[ax]/(c + dx^2)^2, x]$

[Out]  $\frac{(x \operatorname{ArcCoth}[ax])/(2c(c + dx^2)) + (\operatorname{ArcCoth}[ax] \operatorname{ArcTan}[(\sqrt{d}x)/\sqrt{c}])/(2c^{3/2}\sqrt{d}) + ((I/8) \operatorname{Log}[(\sqrt{d}(1 - ax))/(\sqrt{c} + \sqrt{d})]) \operatorname{Log}[1 - (I\sqrt{d}x)/\sqrt{c}]/(c^{3/2}\sqrt{d}) - ((I/8) \operatorname{Log}[-(\sqrt{d}(1 + ax))/(\sqrt{c} - \sqrt{d})]) \operatorname{Log}[1 - (I\sqrt{d}x)/\sqrt{c}]/(c^{3/2}\sqrt{d}) - ((I/8) \operatorname{Log}[-(\sqrt{d}(1 - ax))/(\sqrt{c} - \sqrt{d})]) \operatorname{Log}[1 + (I\sqrt{d}x)/\sqrt{c}]/(c^{3/2}\sqrt{d}) + ((I/8) \operatorname{Log}[(\sqrt{d}(1 + ax))/(\sqrt{c} + \sqrt{d})]) \operatorname{Log}[1 + (I\sqrt{d}x)/\sqrt{c}]/(c^{3/2}\sqrt{d}) + (a \operatorname{Log}[1 - a^2x^2])/(4c(a^2c + d)) - (a \operatorname{Log}[c + dx^2])/(4c(a^2c + d))$

$$(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(3/2)*Sqrt[d])$$

### Rule 31

$$\text{Int}[(a + b*x)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ ; FreeQ}\{a, b, x\}$$

### Rule 36

$$\text{Int}[1/((a + b*x)*(c + d*x)), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] \text{ ; FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0]$$

### Rule 199

$$\text{Int}[(a + b*x^n)^p, x\_Symbol] \rightarrow -\text{Simp}[(x*(a + b*x^n)^{p+1})/(a*n*(p+1)), x] + \text{Dist}[(n*(p+1) + 1)/(a*n*(p+1)), \text{Int}[(a + b*x^n)^{p+1}, x], x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ (\text{IntegerQ}[2*p] \ || \ (n == 2 \ \&\& \ \text{IntegerQ}[4*p]) \ || \ (n == 2 \ \&\& \ \text{IntegerQ}[3*p]) \ || \ \text{Denominator}[p + 1/n] < \text{Denominator}[p])$$

### Rule 205

$$\text{Int}[(a + b*x^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$$

### Rule 444

$$\text{Int}[(x^m)*(a + b*x^n)^p*(c + d*x^n)^q, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] \text{ ; FreeQ}\{a, b, c, d, m, n, p, q, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m - n + 1, 0]$$

### Rule 517

$$\text{Int}[(u*(c + d*x^n)^q)*((a1 + b1*x^{non2})^p)*((a2 + b2*x^{non2})^p), x\_Symbol] \rightarrow \text{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] \text{ ; FreeQ}\{a1, b1, a2, b2, c, d, n, p, q, x\} \ \&\& \ \text{EqQ}[non2, n/2] \ \&\& \ \text{EqQ}[a2*b1 + a1*b2, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[a1, 0] \ \&\& \ \text{GtQ}[a2, 0]))$$

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[I/2, Int[Log[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^2} dx &= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - a \int \frac{\frac{x}{2c(c+dx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1-a^2x^2} dx \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - a \int \left( -\frac{x}{2c(-1+ax)(1+ax)(c+dx^2)} - \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} \right) dx \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a \int \frac{x}{(-1+ax)(1+ax)(c+dx^2)} dx}{2c} + \frac{a \int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{-1+a^2x^2} dx}{2c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a \int \frac{x}{(-1+a^2x^2)(c+dx^2)} dx}{2c} + \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{-1+a^2x^2} dx}{4c^{3/2}\sqrt{d}} - \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1-ax} dx}{8c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{a \text{Subst}\left(\int \frac{1}{(-1+a^2x)(c+dx)} dx, x, x^2\right)}{4c} + \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{-1+a^2x^2} dx}{4c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} - \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1-ax} dx}{8c^{3/2}\sqrt{d}} - \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1+ax} dx}{8c^{3/2}\sqrt{d}} + \frac{(ia) \int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1+ax} dx}{8c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i \log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i \log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} \\
&= \frac{x \coth^{-1}(ax)}{2c(c+dx^2)} + \frac{\coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}} + \frac{i \log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i \log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right) \log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}}
\end{aligned}$$

**Mathematica [A]** time = 7.78, size = 755, normalized size = 1.28

$$a \left( \frac{i \operatorname{Li}_2 \left( \frac{(ca^2 - d - 2i\sqrt{a^2cd})(iadx + \sqrt{a^2cd})}{(ca^2 + d)(\sqrt{a^2cd} - iadx)} \right) - \operatorname{Li}_2 \left( \frac{(ca^2 - d + 2i\sqrt{a^2cd})(iadx + \sqrt{a^2cd})}{(ca^2 + d)(\sqrt{a^2cd} - iadx)} \right)}{+2i \cos^{-1} \left( \frac{a^2c - d}{a^2c + d} \right) \tan^{-1} \left( \frac{ac}{x\sqrt{a^2cd}} \right) - 4 \coth^{-1}(ax) \tan^{-1} \left( \frac{adx}{\sqrt{a^2cd}} \right) + \log \left( \frac{2}{\dots} \right)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^2, x]

[Out] 
$$\begin{aligned} & -1/8*(a*((2*\operatorname{Log}[1 - ((a^2*c + d)*\operatorname{Cosh}[2*\operatorname{ArcCoth}[a*x]])/(a^2*c - d)])/(a^2*c \\ & + d) + ((2*I)*\operatorname{ArcCos}[(a^2*c - d)/(a^2*c + d)]*\operatorname{ArcTan}[(a*c)/(\operatorname{Sqrt}[a^2*c*d]* \\ & x)] - 4*\operatorname{ArcCoth}[a*x]*\operatorname{ArcTan}[(a*d*x)/\operatorname{Sqrt}[a^2*c*d]] + (\operatorname{ArcCos}[(a^2*c - d)/(a \\ & ^2*c + d)] + 2*\operatorname{ArcTan}[(a*c)/(\operatorname{Sqrt}[a^2*c*d]*x)])*\operatorname{Log}[(2*d*(a^2*c - I*\operatorname{Sqrt}[a^ \\ & 2*c*d))*(-1 + a*x)]/((a^2*c + d)*(I*\operatorname{Sqrt}[a^2*c*d] + a*d*x))] + (\operatorname{ArcCos}[(a^2 \\ & *c - d)/(a^2*c + d)] - 2*\operatorname{ArcTan}[(a*c)/(\operatorname{Sqrt}[a^2*c*d]*x)])*\operatorname{Log}[(2*d*(a^2*c + \\ & I*\operatorname{Sqrt}[a^2*c*d])*(1 + a*x)]/((a^2*c + d)*(I*\operatorname{Sqrt}[a^2*c*d] + a*d*x))] - (\operatorname{Ar \\ & cCos}[(a^2*c - d)/(a^2*c + d)] + 2*(\operatorname{ArcTan}[(a*c)/(\operatorname{Sqrt}[a^2*c*d]*x)] + \operatorname{ArcTan} \\ & [(a*d*x)/\operatorname{Sqrt}[a^2*c*d]]))*\operatorname{Log}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a^2*c*d])/(\operatorname{Sqrt}[a^2*c + d]*E^{\operatorname{Ar \\ & cCoth}[a*x]*\operatorname{Sqrt}[-(a^2*c) + d + (a^2*c + d)*\operatorname{Cosh}[2*\operatorname{ArcCoth}[a*x]])}] - (\operatorname{ArcCo \\ & s}[(a^2*c - d)/(a^2*c + d)] - 2*(\operatorname{ArcTan}[(a*c)/(\operatorname{Sqrt}[a^2*c*d]*x)] + \operatorname{ArcTan}[(a \\ & *d*x)/\operatorname{Sqrt}[a^2*c*d]]))*\operatorname{Log}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a^2*c*d]*E^{\operatorname{ArcCoth}[a*x]})/(\operatorname{Sqrt}[a^2 \\ & *c + d]*\operatorname{Sqrt}[-(a^2*c) + d + (a^2*c + d)*\operatorname{Cosh}[2*\operatorname{ArcCoth}[a*x]])]) + I*(\operatorname{PolyLo \\ & g}[2, ((a^2*c - d - (2*I)*\operatorname{Sqrt}[a^2*c*d])*(\operatorname{Sqrt}[a^2*c*d] + I*a*d*x))/((a^2*c \\ & + d)*(\operatorname{Sqrt}[a^2*c*d] - I*a*d*x))] - \operatorname{PolyLog}[2, ((a^2*c - d + (2*I)*\operatorname{Sqrt}[a^2* \\ & c*d])*(\operatorname{Sqrt}[a^2*c*d] + I*a*d*x))/((a^2*c + d)*(\operatorname{Sqrt}[a^2*c*d] - I*a*d*x))]) \\ & / \operatorname{Sqrt}[a^2*c*d] - (4*\operatorname{ArcCoth}[a*x]*\operatorname{Sinh}[2*\operatorname{ArcCoth}[a*x]])/(-(a^2*c) + d + (a^2 \\ & *c + d)*\operatorname{Cosh}[2*\operatorname{ArcCoth}[a*x]])))/c \end{aligned}$$

**fricas [F]** time = 0.60, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\operatorname{arccoth}(ax)}{d^2x^4 + 2cdx^2 + c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^2,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)/(d^2\*x^4 + 2\*c\*d\*x^2 + c^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^2,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/(d\*x^2 + c)^2, x)

**maple [B]** time = 0.64, size = 2218, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^2,x)

[Out] 
$$\frac{3}{4}a^3\operatorname{arccoth}(ax)^2/(a^4c^2+2a^2cd+d^2)/(a^2c+d)(-a^2cd)^{1/2}-1/2a^3\operatorname{arccoth}(ax)/c/(a^2c+d)/(a^2dx^2+a^2c)dx^2+1/2a^2\operatorname{arccoth}(ax)/c/(a^2c+d)/(a^2dx^2+a^2c)xd-1/8a/(a^2c+d)/c^2d^2/(a^4c^2+2a^2cd+d^2)\operatorname{polylog}(2,(a^2c+d)(ax+1)/(ax-1)/(a^2c-2(-a^2cd)^{1/2}-d))(-a^2cd)^{1/2}+1/4a/(a^2c+d)/c^2d^2/(a^4c^2+2a^2cd+d^2)\operatorname{arccoth}(ax)^2(-a^2cd)^{1/2}-1/8a^5d/(a^2c+d)/(a^4c^2+2a^2cd+d^2)\operatorname{polylog}(2,(a^2c+d)(ax+1)/(ax-1)/(a^2c-2(-a^2cd)^{1/2}-d))(-a^2cd)^{1/2}c+1/4a(-a^2cd)^{1/2}/cd/(a^2c+d)\operatorname{arccoth}(ax)\ln(1-(a^2c+d)(ax+1)/(ax-1)/(a^2c+2(-a^2cd)^{1/2}-d))-3/8a/(a^2c+d)/cd/(a^4c^2+2a^2cd+d^2)\operatorname{polylog}(2,(a^2c+d)(ax+1)/(ax-1)/(a^2c-2(-a^2cd)^{1/2}-d))(-a^2cd)^{1/2}+1/4a^5d/(a^2c+d)/(a^4c^2+2a^2cd+d^2)\operatorname{arccoth}(ax)^2(-a^2cd)^{1/2}c+3/4a/(a^2c+d)/cd/(a^4c^2+2a^2cd+d^2)\operatorname{arccoth}(ax)^2(-a^2cd)^{1/2}-1/4a^5d/(a^2c+d)/(a^4c^2+2a^2cd+d^2)\ln(1-(a^2c+d)(ax+1)/(ax-1)/(a^2c-2(-a^2cd)^{1/2}-d))\operatorname{arccoth}(ax)(-a^2cd)^{1/2}c-1/4a/(a^2c+d)/c^2d^2/(a^4c^2+2a^2cd+d^2)\ln(1-(a^2c+d)(ax+1)/(ax-1)/(a^2c-2(-a^2cd)^{1/2}-d))\operatorname{arccoth}(ax)(-a^2cd)^{1/2}-3/4ad\ln(1-(a^2c+d)(ax+1)/(ax-1)/(a^2c-2(-a^2cd)^{1/2}-d))\operatorname{arccoth}(ax)/(a^2c+d)/c/(a^4c^2+2a^2cd+d^2)(-a^2cd)^{1/2}-1/4a^4(cd)^{1/2}/d/(a^2c+d)^2\arctan(a/d(cd)^{1/2})+1/4(cd)^{1/2}/c^2d/(a^2c+d)^2\arctan(a/d(cd)^{1/2})-1/4a/(a^2c+d)^2/cd\ln(a^2c/(ax-1)^2(ax+1)^2-2a^2c(ax+1)/(ax-1)+d/(ax-1)^2(ax+1)^2+a^2c+2(ax+1)/(ax-1)d+d)+1/2a^4\operatorname{arccoth}(ax)/(a^2c+d)/(a^2dx^2+a^2c)xd-1/4a(-a^2cd)^{1/2}/(a^2c+d)/c^2\operatorname{arccoth}(ax)^2+1/8a(-a^2cd)^{1/2}/(a^2c+d)/c^2\operatorname{polylog}(2,(a^2c+d)(ax+1)/(ax-1)/(a^2c+2(-a^2cd)^{1/2}-d))-1/2a/(a^2c+d)^2/cd\ln((ax-1)/(ax+1))-1/4a^4(cd)^{1/2}/d/(a^2c+d)^2\arctan(1/(a^2c+d)d^2/(cd)^{1/2})xd+1/(a^2c+d)d/(cd)^{1/2}ac+a^2/(a^2c+d)(cd)^{1/2}xd-a/(a^2c+d)(cd)^{1/2}-3/8a^3\operatorname{polylog}(2,(a^2c+d)(ax+1)/(ax-1)/(a^2c-2(-a^2cd)^{1/2}-d))/(a^4c^2+2a^2cd+d^2)/(a^2c+d)(-a^2cd)^{1/2}+1/4(cd)^{1/2}/c^2d/(a^2c+d)^2\arctan(1/(a^2c+d)d^2/(cd)^{1/2})xd+1/(a^2c+d)d/(cd)^{1/2}ac+a^2/(a^2c+d)(cd)^{1/2}xd-a/(a^2c+d)(cd)^{1/2})-1/4(cd)^{1/2}/c^2/(a^2c+d)\arctan(a/d(cd)^{1/2})-1/4(cd)^{1/2}/c^2/(a^2c+d)\arctan(1/(a^2c+d)d^2/(cd)^{1/2})xd+1/(a^2c+d)d/(cd)^{1/2}a$$



```
*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-1/2*a^3*arccoth(a*x
)/(a^2*c+d)/(a^2*d*x^2+a^2*c)-1/2*a^3/(a^2*c+d)^2*ln((a*x-1)/(a*x+1))-1/4*a
^3/(a^2*c+d)^2*ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+1)/(a*x-1)+d/(a*x-
1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)-3/4*a^3/(a^2*c+d)/(a^4*c^2+2*a^
2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^(1/2)-d))*arc
coth(a*x)*(-a^2*c*d)^(1/2)+1/4*a^2*(c*d)^(1/2)/c/d/(a^2*c+d)*arctan(1/(a^2*
c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1
/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-1/4*a*(-a^2*c*d)^(1/2)/c/d/(a^2*c+d)*arccoth
(a*x)^2+1/4/a*(-a^2*c*d)^(1/2)/(a^2*c+d)/c^2*arccoth(a*x)*ln(1-(a^2*c+d)*(a
*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))+1/8*a*(-a^2*c*d)^(1/2)/c/d/(a^2
*c+d)*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^(1/2)-d))+1/4
*a^2*(c*d)^(1/2)/c/d/(a^2*c+d)*arctan(a/d*(c*d)^(1/2))
```

**maxima** [A] time = 0.51, size = 550, normalized size = 0.93

$$\frac{1}{2} \left( \frac{x}{cdx^2 + c^2} + \frac{\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c} \right) \operatorname{arccoth}(ax) - \frac{\left( 2acd \log(dx^2 + c) - 2acd \log(ax + 1) - 2acd \log(ax - 1) + \left( a^2 \right. \right.}{\left. \left. \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^2,x, algorithm="maxima")

```
[Out] 1/2*(x/(c*d*x^2 + c^2) + arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c))*arccoth(a*x)
- 1/8*(2*a*c*d*log(d*x^2 + c) - 2*a*c*d*log(a*x + 1) - 2*a*c*d*log(a*x - 1)
+ ((a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^
2*c + d)) - (a^2*c + d)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x
+ d)/(a^2*c + d)) + (I*a^2*c + I*d)*dilog((a^2*c + a*d*x - (I*a^2*x - I*a)*
sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (I*a^2*c + I*d)*dil
og((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)
*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilog((a^2*c + a*d*x + (I*a^2*x - I*a)*sq
rt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) + (-I*a^2*c - I*d)*dilo
g((a^2*c - a*d*x - (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*
sqrt(d) - d)) - ((a^2*c + d)*arctan2((a^2*x + a)*sqrt(c)*sqrt(d)/(a^2*c + d
), (a*d*x + d)/(a^2*c + d)) - (a^2*c + d)*arctan2((a^2*x - a)*sqrt(c)*sqrt(
d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d)))*log(d*x^2 + c))*sqrt(c)*sqrt(d))
*a/(a^3*c^3*d + a*c^2*d^2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(ax)}{(dx^2 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a*x)/(c + d*x^2)^2,x)
```

```
[Out] int(acoth(a*x)/(c + d*x^2)^2, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\operatorname{acoth}(ax)}{(c + dx^2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Integral(acoth(a*x)/(c + d*x**2)**2, x)
```

$$3.41 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^3} dx$$

**Optimal.** Leaf size=657

$$\frac{a(5a^2c+3d)\log(1-a^2x^2)}{16c^2(a^2c+d)^2} - \frac{a(5a^2c+3d)\log(c+dx^2)}{16c^2(a^2c+d)^2} + \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{3i\text{Li}_2\left(\frac{a(\sqrt{c-i\sqrt{d}x})}{a\sqrt{c-i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\text{Li}_2\left(\frac{a(\sqrt{c+i\sqrt{d}x})}{a\sqrt{c+i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}}$$

[Out] 1/8\*a/c/(a^2\*c+d)/(d\*x^2+c)+1/4\*x\*arccoth(a\*x)/c/(d\*x^2+c)^2+3/8\*x\*arccoth(a\*x)/c^2/(d\*x^2+c)+1/16\*a\*(5\*a^2\*c+3\*d)\*ln(-a^2\*x^2+1)/c^2/(a^2\*c+d)^2-1/16\*a\*(5\*a^2\*c+3\*d)\*ln(d\*x^2+c)/c^2/(a^2\*c+d)^2+3/8\*arccoth(a\*x)\*arctan(x\*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)-3/32\*I\*ln(-(a\*x+1)\*d^(1/2)/(I\*a\*c^(1/2)-d^(1/2)))\*ln(1-I\*x\*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)+3/32\*I\*ln((-a\*x+1)\*d^(1/2)/(I\*a\*c^(1/2)+d^(1/2)))\*ln(1-I\*x\*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)-3/32\*I\*ln(-(a\*x+1)\*d^(1/2)/(I\*a\*c^(1/2)-d^(1/2)))\*ln(1+I\*x\*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)+3/32\*I\*ln((a\*x+1)\*d^(1/2)/(I\*a\*c^(1/2)+d^(1/2)))\*ln(1+I\*x\*d^(1/2)/c^(1/2))/c^(5/2)/d^(1/2)+3/32\*I\*polylog(2,a\*(c^(1/2)-I\*x\*d^(1/2))/(a\*c^(1/2)-I\*d^(1/2)))/c^(5/2)/d^(1/2)-3/32\*I\*polylog(2,a\*(c^(1/2)-I\*x\*d^(1/2))/(a\*c^(1/2)+I\*d^(1/2)))/c^(5/2)/d^(1/2)+3/32\*I\*polylog(2,a\*(c^(1/2)+I\*x\*d^(1/2))/(a\*c^(1/2)-I\*d^(1/2)))/c^(5/2)/d^(1/2)-3/32\*I\*polylog(2,a\*(c^(1/2)+I\*x\*d^(1/2))/(a\*c^(1/2)+I\*d^(1/2)))/c^(5/2)/d^(1/2)

**Rubi [A]** time = 0.95, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$ , Rules used = {199, 205, 5977, 6725, 571, 77, 4908, 2409, 2394, 2393, 2391}

$$\frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c-i\sqrt{d}x})}{a\sqrt{c-i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c+i\sqrt{d}x})}{a\sqrt{c+i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} + \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c+i\sqrt{d}x})}{a\sqrt{c-i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}} - \frac{3i\text{PolyLog}\left(2, \frac{a(\sqrt{c-i\sqrt{d}x})}{a\sqrt{c+i\sqrt{d}}}\right)}{32c^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^3, x]

[Out] a/(8\*c\*(a^2\*c + d)\*(c + d\*x^2)) + (x\*ArcCoth[a\*x])/(4\*c\*(c + d\*x^2)^2) + (3\*x\*ArcCoth[a\*x])/(8\*c^2\*(c + d\*x^2)) + (3\*ArcCoth[a\*x]\*ArcTan[(Sqrt[d]\*x)/Sqrt[c]])/(8\*c^(5/2)\*Sqrt[d]) + (((3\*I)/32)\*Log[(Sqrt[d]\*(1 - a\*x)/(I\*a\*Sqrt[c] + Sqrt[d])]\*Log[1 - (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(5/2)\*Sqrt[d]) - (((3\*I)/32)\*Log[-((Sqrt[d]\*(1 + a\*x))/(I\*a\*Sqrt[c] - Sqrt[d]))]\*Log[1 - (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(5/2)\*Sqrt[d]) - (((3\*I)/32)\*Log[-((Sqrt[d]\*(1 - a\*x))/(I\*a\*Sqrt[c] - Sqrt[d]))]\*Log[1 + (I\*Sqrt[d]\*x)/Sqrt[c]])/(c^(5/2)\*Sqrt[d]) +

$$\begin{aligned} & \left( \frac{((3I)/32) \cdot \text{Log}[\text{Sqrt}[d] \cdot (1 + a \cdot x)] / (I \cdot a \cdot \text{Sqrt}[c] + \text{Sqrt}[d]) \cdot \text{Log}[1 + (I \cdot \text{Sqrt}[d] \cdot x) / \text{Sqrt}[c]]}{(c^{5/2} \cdot \text{Sqrt}[d])} + (a \cdot (5 \cdot a^2 \cdot c + 3 \cdot d) \cdot \text{Log}[1 - a^2 \cdot x^2]) \right. \\ & / (16 \cdot c^2 \cdot (a^2 \cdot c + d)^2) - (a \cdot (5 \cdot a^2 \cdot c + 3 \cdot d) \cdot \text{Log}[c + d \cdot x^2]) / (16 \cdot c^2 \cdot (a^2 \cdot c \\ & + d)^2) + \left. \left( \frac{((3I)/32) \cdot \text{PolyLog}[2, (a \cdot (\text{Sqrt}[c] - I \cdot \text{Sqrt}[d] \cdot x)) / (a \cdot \text{Sqrt}[c] - I \cdot \text{Sqrt}[d])]}{c^{5/2} \cdot \text{Sqrt}[d]} \right. \right. \\ & - \left. \left( \frac{((3I)/32) \cdot \text{PolyLog}[2, (a \cdot (\text{Sqrt}[c] - I \cdot \text{Sqrt}[d] \cdot x)) / (a \cdot \text{Sqrt}[c] + I \cdot \text{Sqrt}[d])]}{c^{5/2} \cdot \text{Sqrt}[d]} \right) \right. \\ & + \left. \left( \frac{((3I)/32) \cdot \text{PolyLog}[2, (a \cdot (\text{Sqrt}[c] + I \cdot \text{Sqrt}[d] \cdot x)) / (a \cdot \text{Sqrt}[c] - I \cdot \text{Sqrt}[d])]}{c^{5/2} \cdot \text{Sqrt}[d]} \right) \right. \\ & - \left. \left( \frac{((3I)/32) \cdot \text{PolyLog}[2, (a \cdot (\text{Sqrt}[c] + I \cdot \text{Sqrt}[d] \cdot x)) / (a \cdot \text{Sqrt}[c] + I \cdot \text{Sqrt}[d])]}{c^{5/2} \cdot \text{Sqrt}[d]} \right) \right) \end{aligned}$$
Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

### Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

### Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

### Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^3} dx &= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} - a \int \frac{\frac{x}{4c(c+dx^2)^2} + \frac{3x}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}}}{1-a^2x^2} dx \\
&= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} - a \int \left( \frac{x(5c+3dx^2)}{8c^2(-1+a^2x^2)(c+dx^2)} \right) dx \\
&= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a \int \frac{x(5c+3dx^2)}{(-1+a^2x^2)(c+dx^2)^2} dx}{8c^2} + \frac{(3a)}{8c^2} \\
&= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a \operatorname{Subst}\left(\int \frac{5c+3dx}{(-1+a^2x)(c+dx)^2} dx, x\right)}{16c^2} \\
&= \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a \operatorname{Subst}\left(\int \left( \frac{a^2(5a^2c+3d)}{(a^2c+d)^2(-1+a^2x)} - \frac{3d}{(a^2c+d)(-1+a^2x)} \right) dx, x\right)}{16c^2} \\
&= \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{a(5a^2c+3d)}{16c^2} \\
&= \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{3i \log\left(\frac{a^2(5a^2c+3d)}{(a^2c+d)^2(-1+a^2x)} - \frac{3d}{(a^2c+d)(-1+a^2x)}\right)}{16c^2} \\
&= \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{3i \log\left(\frac{a^2(5a^2c+3d)}{(a^2c+d)^2(-1+a^2x)} - \frac{3d}{(a^2c+d)(-1+a^2x)}\right)}{16c^2} \\
&= \frac{a}{8c(a^2c+d)(c+dx^2)} + \frac{x \coth^{-1}(ax)}{4c(c+dx^2)^2} + \frac{3x \coth^{-1}(ax)}{8c^2(c+dx^2)} + \frac{3 \coth^{-1}(ax) \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}\sqrt{d}} + \frac{3i \log\left(\frac{a^2(5a^2c+3d)}{(a^2c+d)^2(-1+a^2x)} - \frac{3d}{(a^2c+d)(-1+a^2x)}\right)}{16c^2}
\end{aligned}$$

**Mathematica [B]** time = 12.95, size = 1846, normalized size = 2.81

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^3,x]

[Out]  $a^5 \left( (-5 \operatorname{Log}[1 - ((-a^2c) - d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a*x]]] / (-a^2c) + d) \right) / (16 a^2 c (a^2 c + d)^2) - (3 d \operatorname{Log}[1 - ((-a^2c) - d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a*x]]] / (-a^2c) + d) / (16 a^4 c^2 (a^2 c + d)^2) + (3 ((-2I) \operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)] + 4 \operatorname{ArcCoth}[a*x] \operatorname{ArcTan}[(a*d*x) / \operatorname{Sqrt}[a^2c*d]] - (\operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] - 2 \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)]) \operatorname{Log}[1 - ((-a^2c) + d - (2I) \operatorname{Sqrt}[a^2c*d]) * (2d - ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x))] / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))] + (-\operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] - 2 \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)]) \operatorname{Log}[1 - ((-a^2c) + d + (2I) \operatorname{Sqrt}[a^2c*d]) * (2d - ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x))] / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))] + (\operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] + (2I) * ((-I) \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)] - I \operatorname{ArcTan}[(a*d*x) / \operatorname{Sqrt}[a^2c*d]]) \operatorname{Log}[(\operatorname{Sqrt}[2] \operatorname{Sqrt}[a^2c*d]) / (\operatorname{Sqrt}[a^2c + d] \operatorname{E}^{\operatorname{ArcCoth}[a*x]} \operatorname{Sqrt}[-(a^2c) + d + (a^2c + d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a*x]]])]) + (\operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] - (2I) * ((-I) \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)] - I \operatorname{ArcTan}[(a*d*x) / \operatorname{Sqrt}[a^2c*d]]) \operatorname{Log}[(\operatorname{Sqrt}[2] \operatorname{Sqrt}[a^2c*d] \operatorname{E}^{\operatorname{ArcCoth}[a*x]}) / (\operatorname{Sqrt}[a^2c + d] \operatorname{Sqrt}[-(a^2c) + d + (a^2c + d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a*x]]])]) + I * (\operatorname{PolyLog}[2, ((-a^2c) + d - (2I) \operatorname{Sqrt}[a^2c*d]) * (2d - ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x))] / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))] - \operatorname{PolyLog}[2, ((-a^2c) + d + (2I) \operatorname{Sqrt}[a^2c*d]) * (2d - ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x))] / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))])) / (32 a^2 c \operatorname{Sqrt}[a^2c*d] * (a^2c + d)) + (3 d * ((-2I) \operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)] + 4 \operatorname{ArcCoth}[a*x] \operatorname{ArcTan}[(a*d*x) / \operatorname{Sqrt}[a^2c*d]] - (\operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] - 2 \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)]) \operatorname{Log}[1 - ((-a^2c) + d - (2I) \operatorname{Sqrt}[a^2c*d]) * (2d - ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x))] / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))] + (-\operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] - 2 \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)]) \operatorname{Log}[1 - ((-a^2c) + d + (2I) \operatorname{Sqrt}[a^2c*d]) * (2d - ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x))] / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))] + (\operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] + (2I) * ((-I) \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)] - I \operatorname{ArcTan}[(a*d*x) / \operatorname{Sqrt}[a^2c*d]]) \operatorname{Log}[(\operatorname{Sqrt}[2] \operatorname{Sqrt}[a^2c*d]) / (\operatorname{Sqrt}[a^2c + d] \operatorname{E}^{\operatorname{ArcCoth}[a*x]} \operatorname{Sqrt}[-(a^2c) + d + (a^2c + d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a*x]]])]) + (\operatorname{ArcCos}[(a^2c - d) / (a^2c + d)] - (2I) * ((-I) \operatorname{ArcTan}[(a*c) / (\operatorname{Sqrt}[a^2c*d]*x)] - I \operatorname{ArcTan}[(a*d*x) / \operatorname{Sqrt}[a^2c*d]]) \operatorname{Log}[(\operatorname{Sqrt}[2] \operatorname{Sqrt}[a^2c*d] \operatorname{E}^{\operatorname{ArcCoth}[a*x]}) / (\operatorname{Sqrt}[a^2c + d] \operatorname{Sqrt}[-(a^2c) + d + (a^2c + d) \operatorname{Cosh}[2 \operatorname{ArcCoth}[a*x]]])]) + I * (\operatorname{PolyLog}[2, ((-a^2c) + d - (2I) \operatorname{Sqrt}[a^2c*d]) * (2d - ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x))] / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))] - \operatorname{PolyLog}[2, ((-a^2c) + d + (2I) \operatorname{Sqrt}[a^2c*d]) * (2d - ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x))] / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))])) / ((a^2c + d) * (2d + ((2I) \operatorname{Sqrt}[a^2c*d]) / (a*x)))]$

$$2*I)*\text{Sqrt}[a^2*c*d]/(a*x)))))))/(32*a^4*c^2*\text{Sqrt}[a^2*c*d]*(a^2*c + d)) - (d$$

$$*\text{ArcCoth}[a*x]*\text{Sinh}[2*\text{ArcCoth}[a*x]]/(2*a^2*c*(a^2*c + d)*(-(a^2*c) + d + a^$$

$$2*c*\text{Cosh}[2*\text{ArcCoth}[a*x]] + d*\text{Cosh}[2*\text{ArcCoth}[a*x]])^2) - (2*a^2*c*d - 5*a^4*$$

$$c^2*\text{ArcCoth}[a*x]*\text{Sinh}[2*\text{ArcCoth}[a*x]] - 8*a^2*c*d*\text{ArcCoth}[a*x]*\text{Sinh}[2*\text{ArcCo}$$

$$\text{th}[a*x]] - 3*d^2*\text{ArcCoth}[a*x]*\text{Sinh}[2*\text{ArcCoth}[a*x]]/(8*a^4*c^2*(a^2*c + d)^$$

$$2*(-(a^2*c) + d + a^2*c*\text{Cosh}[2*\text{ArcCoth}[a*x]] + d*\text{Cosh}[2*\text{ArcCoth}[a*x]]))$$

**fricas** [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(ax)}{d^3x^6 + 3cd^2x^4 + 3c^2dx^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^3,x, algorithm="fricas")

[Out] integral(arccoth(a\*x)/(d^3\*x^6 + 3\*c\*d^2\*x^4 + 3\*c^2\*d\*x^2 + c^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(ax)}{(dx^2 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^3,x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/(d\*x^2 + c)^3, x)

**maple** [B] time = 0.84, size = 4128, normalized size = 6.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^3,x)

$$[\text{Out}] -3/16*a^4*(c*d)^{(1/2)}/c/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*\text{arctan}(a/d*(c*d)^{(1/2)})+1/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d-5/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*\ln((a*x-1)/(a*x+1))-5/16*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*\ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)+3/4*a^5*\text{arccoth}(a*x)^2/(a^4*c^2+2*a^2*c*d+d^2)^2*(-a^2*c*d)^{(1/2)}-3/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)^2*\text{polylog}(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}-5/16*a^6*(c*d)^{(1/2)}/d/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*\text{arctan}(a/d*(c*d)^{(1/2)})+5/16*a^4*(c*d)^{(1/2)}/c/d/(a^4*c^2+2*a^2*c*d+d^2)*\text{arctan}(a/d*(c*d)^{(1/2)})+3/16*(c*d)^{(1/2)}/c^3*d^2/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*\text{arctan}(a/$$



$$\begin{aligned}
& d*(c*d)^{(1/2)}+5/8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*c*arccoth \\
& h(a*x)*x+3/8*a*(-a^2*c*d)^{(1/2)}/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)*\ln \\
& (1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-3/32*a^7/d/(a^4* \\
& c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d) \\
& )^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}*c+3/16*a^7/d/(a^4*c^2+2*a^2*c*d+d^2)^2*arccoth \\
& (a*x)^2*(-a^2*c*d)^{(1/2)}*c-3/16/a*(-a^2*c*d)^{(1/2)}/c^3*d/(a^4*c^2+2*a^2*c*d \\
& +d^2)*arccoth(a*x)^2+3/32/a*(-a^2*c*d)^{(1/2)}/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)* \\
& polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-3/32/a/c^ \\
& 3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c- \\
& 2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}+3/16/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d \\
& ^2)^2*arccoth(a*x)^2*(-a^2*c*d)^{(1/2)}+3/8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2* \\
& d*x^2+a^2*c)^2*arccoth(a*x)*x^3*d-5/4*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^ \\
& 2+a^2*c)^2*arccoth(a*x)*x^2*d+5/4*a^6/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^ \\
& 2*c)^2*arccoth(a*x)*x*d-3/16*a^4*(c*d)^{(1/2)}/c/(a^4*c^2+2*a^2*c*d+d^2)/(a^2 \\
& *c+d)*arctan(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^ \\
& 2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1/2)})+3/32*a^3*(-a^2*c*d)^{(1/2) \\
& )/c/d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2* \\
& (-a^2*c*d)^{(1/2)}-d))-3/16*a^3*(-a^2*c*d)^{(1/2)}/c/d/(a^4*c^2+2*a^2*c*d+d^2)* \\
& arccoth(a*x)^2-9/16*a^3/c/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a* \\
& x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*(-a^2*c*d)^{(1/2)}*d+3/4*a*d^2*arc \\
& coth(a*x)^2/c^2/(a^4*c^2+2*a^2*c*d+d^2)^2*(-a^2*c*d)^{(1/2)}-3/8*a*d^2*polylo \\
& g(2,(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))/c^2/(a^4*c^2+2* \\
& a^2*c*d+d^2)^2*(-a^2*c*d)^{(1/2)}-5/16*a^6*(c*d)^{(1/2)}/d/(a^4*c^2+2*a^2*c*d+d \\
& ^2)/(a^2*c+d)*arctan(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/2) \\
& )*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1/2)})-3/16*a/c^2/(a^4* \\
& c^2+2*a^2*c*d+d^2)*d^2/(a^2*c+d)*\ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+ \\
& 1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)-3/8*a/c^2/(a^ \\
& 4*c^2+2*a^2*c*d+d^2)*d^2/(a^2*c+d)*\ln((a*x-1)/(a*x+1))-a^3/c/(a^4*c^2+2*a^2 \\
& *c*d+d^2)*d/(a^2*c+d)*\ln((a*x-1)/(a*x+1))+5/16*a^4*(c*d)^{(1/2)}/c/d/(a^4*c^2 \\
& +2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1 \\
& /2)*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1/2)})-1/2*a^3/c/(a^4 \\
& *c^2+2*a^2*c*d+d^2)*d/(a^2*c+d)*\ln(a^2*c/(a*x-1)^2*(a*x+1)^2-2*a^2*c*(a*x+ \\
& 1)/(a*x-1)+d/(a*x-1)^2*(a*x+1)^2+a^2*c+2*(a*x+1)/(a*x-1)*d+d)+9/8*a^3*d*arcc \\
& oth(a*x)^2/c/(a^4*c^2+2*a^2*c*d+d^2)^2*(-a^2*c*d)^{(1/2)}+3/16*(c*d)^{(1/2)}/c^ \\
& 3*d^2/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*arctan(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}* \\
& x+1/(a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c \\
& d)^{(1/2)})-1/8*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*d^2*x^4+1/8 \\
& *a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*d^2*x^2-1/8*a^2*(c*d)^{(1 \\
& /2)}/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^{(1/2)})-3/16*(c*d)^{(1/2)}/c^ \\
& 3*d/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^{(1/2)})-1/8*a^7/(a^4*c^2+2*a^2* \\
& c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d*x^2-3/4*a^5*\ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/( \\
& a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*arccoth(a*x)/(a^4*c^2+2*a^2*c*d+d^2)^2*(-a^2*c \\
& *d)^{(1/2)}-5/8*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*c*arccoth(a*x \\
& )-1/8*a^2*(c*d)^{(1/2)}/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c \\
& *d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^
\end{aligned}$$

$$\begin{aligned}
& 2*c*d)*(c*d)^{(1/2)}-3/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d*a \\
& rccoth(a*x)-3/8*a*(-a^2*c*d)^{(1/2)}/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x) \\
& ^2+3/16*a*(-a^2*c*d)^{(1/2)}/c^2/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)* \\
& (a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-3/16*(c*d)^{(1/2)}/c^3*d/(a^4*c \\
& ^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^{(1/2)}*x+1/(a^2*c+d)*d/(c*d)^ \\
& (1/2)*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d)*(c*d)^{(1/2)})+3/16/a*(-a^2 \\
& *c*d)^{(1/2)}/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth(a*x)*ln(1-(a^2*c+d)*(a*x+ \\
& 1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-9/8*a^3/c*d/(a^4*c^2+2*a^2*c*d+d^2 \\
& )^2*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*arccoth(a* \\
& x)*(-a^2*c*d)^{(1/2)}-3/4*a/c^2*d^2/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)* \\
& (a*x+1)/(a*x-1)/(a^2*c-2*(-a^2*c*d)^{(1/2)}-d))*arccoth(a*x)*(-a^2*c*d)^{(1/2)} \\
& -3/16*a^7/d/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c \\
& -2*(-a^2*c*d)^{(1/2)}-d))*arccoth(a*x)*(-a^2*c*d)^{(1/2)}*c-5/8*a^7/(a^4*c^2+2* \\
& a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arccoth(a*x)*x^4*d^2-3/8*a^5/(a^4*c^2+2* \\
& a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c^2*arccoth(a*x)*x^4*d^3+3/4*a^6/(a^4*c^2+ \\
& 2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arccoth(a*x)*x^3*d^2+3/8*a^4/(a^4*c^2+ \\
& 2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c^2*arccoth(a*x)*x^3*d^3-3/4*a^5/(a^4*c^ \\
& 2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arccoth(a*x)*x^2*d^2+5/8*a^4/(a^4*c^ \\
& 2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*arccoth(a*x)*x*d^2+5/16*a^2*(c*d)^{(1 \\
& /2)}/c^2*d/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*arctan(1/(a^2*c+d)*d^2/(c*d)^{(1 \\
& /2)}*x+1/(a^2*c+d)*d/(c*d)^{(1/2)}*a*c+a^2/(a^2*c+d)*(c*d)^{(1/2)}*x-a/(a^2*c+d) \\
& *(c*d)^{(1/2)})+3/16*a^3*(-a^2*c*d)^{(1/2)}/c/d/(a^4*c^2+2*a^2*c*d+d^2)*arccoth \\
& (a*x)*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c+2*(-a^2*c*d)^{(1/2)}-d))-3/16/a/c \\
& ^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)/(a*x-1)/(a^2*c-2*(- \\
& a^2*c*d)^{(1/2)}-d))*arccoth(a*x)*(-a^2*c*d)^{(1/2)}+5/16*a^2*(c*d)^{(1/2)}/c^2*d \\
& /((a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*arctan(a/d*(c*d)^{(1/2)})
\end{aligned}$$

**maxima** [B] time = 0.58, size = 1084, normalized size = 1.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^3,x, algorithm="maxima")

[Out]  $1/8*((3*d*x^3 + 5*c*x)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4) + 3*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2))*arccoth(a*x) + 1/32*(4*a^3*c^3*d + 4*a*c^2*d^2 - (3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 + 2*a*d*x + d)/(a^2*c + d)) - 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan(sqrt(d)*x/sqrt(c))*log((a^2*d*x^2 - 2*a*d*x + d)/(a^2*c + d)) + (3*I*a^4*c^3 + 6*I*a^2*c^2*d + 3*I*c*d^2 + (3*I*a^4*c^2*d + 6*I*a^2*c*d^2 + 3*I*d^3)*x^2)*dilog((a^2*c + a*d*x - (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (3*I*a^4*c^3 + 6*I*a^2*c^2*d + 3*I*c*d^2 + (3*I*a^4*c^2*d + 6*I*a^2*c*d^2 + 3*I*d^3)*x^2)*dilog((a^2*c - a*d*x + (I*a^2*x + I*a)*sqrt(c)*sqrt(d))/(a^2*c + 2*I*a*sqrt(c)*sqrt(d) - d)) + (-3*I*a^4*c^3 - 6*I$

```

*a^2*c^2*d - 3*I*c*d^2 + (-3*I*a^4*c^2*d - 6*I*a^2*c*d^2 - 3*I*d^3)*x^2)*di
log((a^2*c + a*d*x + (I*a^2*x - I*a)*sqrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)
)*sqrt(d) - d)) + (-3*I*a^4*c^3 - 6*I*a^2*c^2*d - 3*I*c*d^2 + (-3*I*a^4*c^2
*d - 6*I*a^2*c*d^2 - 3*I*d^3)*x^2)*dilog((a^2*c - a*d*x - (I*a^2*x + I*a)*s
qrt(c)*sqrt(d))/(a^2*c - 2*I*a*sqrt(c)*sqrt(d) - d)) - 3*((a^4*c^3 + 2*a^2*
c^2*d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan2((a^2*x + a)*sq
rt(c)*sqrt(d)/(a^2*c + d), (a*d*x + d)/(a^2*c + d)) - (a^4*c^3 + 2*a^2*c^2*
d + c*d^2 + (a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^2)*arctan2((a^2*x - a)*sqrt(c)
)*sqrt(d)/(a^2*c + d), -(a*d*x - d)/(a^2*c + d))*log(d*x^2 + c))*sqrt(c)*s
qrt(d) - 2*(5*a^3*c^3*d + 3*a*c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x^2)*lo
g(d*x^2 + c) + 2*(5*a^3*c^3*d + 3*a*c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^3)*x
^2)*log(a*x + 1) + 2*(5*a^3*c^3*d + 3*a*c^2*d^2 + (5*a^3*c^2*d^2 + 3*a*c*d^
3)*x^2)*log(a*x - 1))*a/(a^5*c^6*d + 2*a^3*c^5*d^2 + a*c^4*d^3 + (a^5*c^5*d
^2 + 2*a^3*c^4*d^3 + a*c^3*d^4)*x^2)

```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(ax)}{(dx^2 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)/(c + d\*x^2)^3,x)

[Out] int(acoth(a\*x)/(c + d\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/(d\*x\*\*2+c)\*\*3,x)

[Out] Timed out

### 3.42 $\int \sqrt{c + dx^2} \coth^{-1}(ax) dx$

**Optimal.** Leaf size=19

$$\text{Int}\left(\coth^{-1}(ax)\sqrt{c + dx^2}, x\right)$$

[Out] Unintegrable((d\*x^2+c)^(1/2)\*arccoth(a\*x), x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \sqrt{c + dx^2} \coth^{-1}(ax) dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

[Out] Defer[Int][Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

Rubi steps

$$\int \sqrt{c + dx^2} \coth^{-1}(ax) dx = \int \sqrt{c + dx^2} \coth^{-1}(ax) dx$$

**Mathematica [A]** time = 26.74, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2} \coth^{-1}(ax) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

[Out] Integrate[Sqrt[c + d\*x^2]\*ArcCoth[a\*x], x]

**fricas [A]** time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{dx^2 + c} \operatorname{arccoth}(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^(1/2)\*arccoth(a\*x), x, algorithm="fricas")

[Out] integral(sqrt(d\*x^2 + c)\*arccoth(a\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c} \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^(1/2)\*arccoth(a\*x),x, algorithm="giac")

[Out] integrate(sqrt(d\*x^2 + c)\*arccoth(a\*x), x)

**maple** [A] time = 1.27, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c} \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^2+c)^(1/2)\*arccoth(a\*x),x)

[Out] int((d\*x^2+c)^(1/2)\*arccoth(a\*x),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c} \operatorname{arccoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)^(1/2)\*arccoth(a\*x),x, algorithm="maxima")

[Out] integrate(sqrt(d\*x^2 + c)\*arccoth(a\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{acoth}(ax) \sqrt{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)\*(c + d\*x^2)^(1/2),x)

[Out] int(acoth(a\*x)\*(c + d\*x^2)^(1/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2} \operatorname{acoth}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*2+c)\*\*(1/2)\*acoth(a\*x),x)

[Out] Integral(sqrt(c + d\*x\*\*2)\*acoth(a\*x), x)

$$3.43 \quad \int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

**Optimal.** Leaf size=19

$$\text{Int}\left(\frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}}, x\right)$$

[Out] Unintegrable(arccoth(a\*x)/(d\*x^2+c)^(1/2), x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

[Out] Defer[Int][ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

Rubi steps

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx = \int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

**Mathematica [A]** time = 4.38, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(ax)}{\sqrt{c+dx^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

[Out] Integrate[ArcCoth[a\*x]/Sqrt[c + d\*x^2], x]

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(ax)}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(1/2),x, algorithm="fricas")

[Out] integral(arccoth(a\*x)/sqrt(d\*x^2 + c), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{\sqrt{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(1/2),x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/sqrt(d\*x^2 + c), x)

**maple** [A] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{\sqrt{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^(1/2),x)

[Out] int(arccoth(a\*x)/(d\*x^2+c)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{\sqrt{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(arccoth(a\*x)/sqrt(d\*x^2 + c), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acoth}(ax)}{\sqrt{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x)/(c + d\*x^2)^(1/2),x)

[Out] int(acoth(a\*x)/(c + d\*x^2)^(1/2), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{\sqrt{c+dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x)/(d\*x\*\*2+c)\*\*(1/2),x)

[Out] Integral(acoth(a\*x)/sqrt(c + d\*x\*\*2), x)



$$3.44 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}} dx$$

Optimal. Leaf size=62

$$\frac{x \coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

[Out]  $-\operatorname{arctanh}(a*(d*x^2+c)^{(1/2)/(a^2*c+d)^{(1/2)})/c/(a^2*c+d)^{(1/2)}+x*\operatorname{arccoth}(a*x)/c/(d*x^2+c)^{(1/2)}$

**Rubi [A]** time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {191, 5977, 12, 444, 63, 208}

$$\frac{x \coth^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[a*x]/(c + d*x^2)^(3/2), x]`

[Out]  $(x*\operatorname{ArcCoth}[a*x])/(c*\operatorname{Sqrt}[c + d*x^2]) - \operatorname{ArcTanh}[(a*\operatorname{Sqrt}[c + d*x^2])/ \operatorname{Sqrt}[a^2*c + d]]/(c*\operatorname{Sqrt}[a^2*c + d])$

### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 191

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]`

Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m - n + 1, 0]

Rule 5977

Int[((a\_) + ArcCoth[(c\_)\*(x\_)])\*(b\_)\*((d\_) + (e\_)\*(x\_)^2)^(q\_.), x\_Symbol] := With[{u = IntHide[(d + e\*x^2)^q, x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[u/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)}{(c + dx^2)^{3/2}} dx &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - a \int \frac{x}{c(1 - a^2x^2)\sqrt{c + dx^2}} dx \\
 &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - \frac{a \int \frac{x}{(1 - a^2x^2)\sqrt{c + dx^2}} dx}{c} \\
 &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{1}{(1 - a^2x)\sqrt{c + dx}} dx, x, x^2\right)}{2c} \\
 &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{1}{1 + \frac{a^2c}{d} - \frac{a^2x^2}{d}} dx, x, \sqrt{c + dx^2}\right)}{cd} \\
 &= \frac{x \coth^{-1}(ax)}{c\sqrt{c + dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c + dx^2}}{\sqrt{a^2c + d}}\right)}{c\sqrt{a^2c + d}}
 \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 119, normalized size = 1.92

$$\frac{-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2+ac-dx}\right)-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2+ac+dx}\right)+\log(1-ax)+\log(ax+1)}{\sqrt{a^2c+d}} + \frac{2x \operatorname{coth}^{-1}(ax)}{\sqrt{c+dx^2}}$$

$$2c$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^(3/2), x]

[Out] ((2\*x\*ArcCoth[a\*x])/Sqrt[c + d\*x^2] + (Log[1 - a\*x] + Log[1 + a\*x] - Log[a\*c - d\*x + Sqrt[a^2\*c + d]\*Sqrt[c + d\*x^2]] - Log[a\*c + d\*x + Sqrt[a^2\*c + d]\*Sqrt[c + d\*x^2]])/Sqrt[a^2\*c + d])/(2\*c)

**fricas [B]** time = 0.50, size = 354, normalized size = 5.71

$$\left[ \frac{2(a^2c + d)\sqrt{dx^2 + c}x \log\left(\frac{ax+1}{ax-1}\right) + \sqrt{a^2c + d}(dx^2 + c) \log\left(\frac{a^4d^2x^4 + 8a^4c^2 + 8a^2cd + 2(4a^4cd + 3a^2d^2)x^2 - 4(a^3dx^2 + 2a^3c + ad)\sqrt{c+dx^2}}{a^4x^4 - 2a^2x^2 + 1}\right)}{4(a^2c^3 + c^2d + (a^2c^2d + cd^2)x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(3/2), x, algorithm="fricas")

[Out] [1/4\*(2\*(a^2\*c + d)\*sqrt(d\*x^2 + c)\*x\*log((a\*x + 1)/(a\*x - 1)) + sqrt(a^2\*c + d)\*(d\*x^2 + c)\*log((a^4\*d^2\*x^4 + 8\*a^4\*c^2 + 8\*a^2\*c\*d + 2\*(4\*a^4\*c\*d + 3\*a^2\*d^2)\*x^2 - 4\*(a^3\*d\*x^2 + 2\*a^3\*c + a\*d)\*sqrt(a^2\*c + d)\*sqrt(d\*x^2 + c) + d^2)/(a^4\*x^4 - 2\*a^2\*x^2 + 1)))/(a^2\*c^3 + c^2\*d + (a^2\*c^2\*d + c\*d^2)\*x^2), 1/2\*((a^2\*c + d)\*sqrt(d\*x^2 + c)\*x\*log((a\*x + 1)/(a\*x - 1)) + sqrt(-a^2\*c - d)\*(d\*x^2 + c)\*arctan(1/2\*(a^2\*d\*x^2 + 2\*a^2\*c + d)\*sqrt(-a^2\*c - d)\*sqrt(d\*x^2 + c)/(a^3\*c^2 + a\*c\*d + (a^3\*c\*d + a\*d^2)\*x^2)))/(a^2\*c^3 + c^2\*d + (a^2\*c^2\*d + c\*d^2)\*x^2)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(3/2), x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/(d\*x^2 + c)^(3/2), x)

**maple** [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(a*x)/(d*x^2+c)^(3/2),x)`

[Out] `int(arccoth(a*x)/(d*x^2+c)^(3/2),x)`

**maxima** [B] time = 0.34, size = 153, normalized size = 2.47

$$\frac{a^2 \left( \frac{\operatorname{arsinh}\left(-\frac{2a^2c}{\sqrt{cd}|2a^2x+2a|} + \frac{2adx}{\sqrt{cd}|2a^2x+2a|}\right)}{a^3 \sqrt{c + \frac{d}{a^2}}} - \frac{\operatorname{arsinh}\left(\frac{2a^2c}{\sqrt{cd}|2a^2x-2a|} + \frac{2adx}{\sqrt{cd}|2a^2x-2a|}\right)}{a^3 \sqrt{c + \frac{d}{a^2}}} \right)}{2c} + \frac{x \operatorname{arccoth}(ax)}{\sqrt{dx^2 + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")`

[Out] `1/2*a^2*(arcsinh(-2*a^2*c/(sqrt(c*d)*abs(2*a^2*x + 2*a)) + 2*a*d*x/(sqrt(c*d)*abs(2*a^2*x + 2*a)))/(a^3*sqrt(c + d/a^2)) - arcsinh(2*a^2*c/(sqrt(c*d)*abs(2*a^2*x - 2*a)) + 2*a*d*x/(sqrt(c*d)*abs(2*a^2*x - 2*a)))/(a^3*sqrt(c + d/a^2)))/c + x*arccoth(a*x)/(sqrt(d*x^2 + c)*c)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acoth}(ax)}{(dx^2 + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)/(c + d*x^2)^(3/2),x)`

[Out] `int(acoth(a*x)/(c + d*x^2)^(3/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{(c + dx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)/(d*x**2+c)**(3/2),x)`

[Out] `Integral(acoth(a*x)/(c + d*x**2)**(3/2),x)`

$$3.45 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}} dx$$

**Optimal.** Leaf size=128

$$-\frac{(3a^2c + 2d) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c + d)^{3/2}} + \frac{a}{3c(a^2c + d)\sqrt{c + dx^2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c + dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c + dx^2)^{3/2}}$$

[Out] 1/3\*x\*arccoth(a\*x)/c/(d\*x^2+c)^(3/2)-1/3\*(3\*a^2\*c+2\*d)\*arctanh(a\*(d\*x^2+c)^(1/2)/(a^2\*c+d)^(1/2))/c^2/(a^2\*c+d)^(3/2)+1/3\*a/c/(a^2\*c+d)/(d\*x^2+c)^(1/2)+2/3\*x\*arccoth(a\*x)/c^2/(d\*x^2+c)^(1/2)

**Rubi [A]** time = 0.34, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {192, 191, 5977, 6688, 12, 571, 78, 63, 208}

$$-\frac{(3a^2c + 2d) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c + d)^{3/2}} + \frac{a}{3c(a^2c + d)\sqrt{c + dx^2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c + dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c + dx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^(5/2), x]

[Out] a/(3\*c\*(a^2\*c + d)\*Sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(3\*c\*(c + d\*x^2)^(3/2)) + (2\*x\*ArcCoth[a\*x])/(3\*c^2\*Sqrt[c + d\*x^2]) - ((3\*a^2\*c + 2\*d)\*ArcTanh[(a\*Sqrt[c + d\*x^2])/Sqrt[a^2\*c + d]])/(3\*c^2\*(a^2\*c + d)^(3/2))

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 78**

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

### Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

### Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

### Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}} dx &= \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a \int \frac{\frac{x}{3c(c+dx^2)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1-a^2x^2} dx \\
&= \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a \int \frac{x(3c+2dx^2)}{3c^2(1-a^2x^2)(c+dx^2)^{3/2}} dx \\
&= \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a \int \frac{x(3c+2dx^2)}{(1-a^2x^2)(c+dx^2)^{3/2}} dx}{3c^2} \\
&= \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{3c+2dx}{(1-a^2x)(c+dx)^{3/2}} dx, x, x^2\right)}{6c^2} \\
&= \frac{a}{3c(a^2c+d)\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(a(3a^2c+2d)) \operatorname{Subst}\left(\int \frac{1}{(1-a^2x)} dx, x, x^2\right)}{6c^2(a^2c+d)} \\
&= \frac{a}{3c(a^2c+d)\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(a(3a^2c+2d)) \operatorname{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}} dx, x, x^2\right)}{3c^2d(a^2c+d)} \\
&= \frac{a}{3c(a^2c+d)\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{3c(c+dx^2)^{3/2}} + \frac{2x \coth^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{(3a^2c+2d) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 226, normalized size = 1.77

$$\frac{2ac}{(a^2c+d)\sqrt{c+dx^2}} - \frac{(3a^2c+2d) \log(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac-dx)}{(a^2c+d)^{3/2}} - \frac{(3a^2c+2d) \log(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac+dx)}{(a^2c+d)^{3/2}} + \frac{(3a^2c+2d) \log(1-ax)}{(a^2c+d)^{3/2}} + \frac{(3a^2c+2d)}{(a^2c+d)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^(5/2), x]

[Out] ((2\*a\*c)/((a^2\*c + d)\*Sqrt[c + d\*x^2]) + (2\*x\*(3\*c + 2\*d\*x^2)\*ArcCoth[a\*x])/(c + d\*x^2)^(3/2) + ((3\*a^2\*c + 2\*d)\*Log[1 - a\*x])/(a^2\*c + d)^(3/2) + ((3\*a^2\*c + 2\*d)\*Log[1 + a\*x])/(a^2\*c + d)^(3/2) - ((3\*a^2\*c + 2\*d)\*Log[a\*c -

$d*x + \text{Sqrt}[a^2*c + d]*\text{Sqrt}[c + d*x^2]]/(a^2*c + d)^{(3/2)} - ((3*a^2*c + 2*d) * \text{Log}[a*c + d*x + \text{Sqrt}[a^2*c + d]*\text{Sqrt}[c + d*x^2]]/(a^2*c + d)^{(3/2)})/(6*c^2)$

**fricas** [B] time = 0.69, size = 728, normalized size = 5.69

$$\left[ \frac{(3a^2c^3 + (3a^2cd^2 + 2d^3)x^4 + 2c^2d + 2(3a^2c^2d + 2cd^2)x^2)\sqrt{a^2c + d} \log\left(\frac{a^4d^2x^4 + 8a^4c^2 + 8a^2cd + 2(4a^4cd + 3a^2d^2)x^2 - 4(a^4x^4 - 2a^2x^2 + d^2)}{a^4x^4 - 2a^2x^2 + d^2}\right)}{12(a^4c^6 + 2a^2c^5d + c^4d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(5/2),x, algorithm="fricas")

[Out] [1/12\*((3\*a^2\*c^3 + (3\*a^2\*c\*d^2 + 2\*d^3)\*x^4 + 2\*c^2\*d + 2\*(3\*a^2\*c^2\*d + 2\*c\*d^2)\*x^2)\*sqrt(a^2\*c + d)\*log((a^4\*d^2\*x^4 + 8\*a^4\*c^2 + 8\*a^2\*c\*d + 2\*(4\*a^4\*c\*d + 3\*a^2\*d^2)\*x^2 - 4\*(a^3\*d\*x^2 + 2\*a^3\*c + a\*d)\*sqrt(a^2\*c + d)\*sqrt(d\*x^2 + c) + d^2)/(a^4\*x^4 - 2\*a^2\*x^2 + 1)) + 2\*(2\*a^3\*c^3 + 2\*a\*c^2\*d + 2\*(a^3\*c^2\*d + a\*c\*d^2)\*x^2 + (2\*(a^4\*c^2\*d + 2\*a^2\*c\*d^2 + d^3)\*x^3 + 3\*(a^4\*c^3 + 2\*a^2\*c^2\*d + c\*d^2)\*x)\*log((a\*x + 1)/(a\*x - 1))\*sqrt(d\*x^2 + c))/(a^4\*c^6 + 2\*a^2\*c^5\*d + c^4\*d^2 + (a^4\*c^4\*d^2 + 2\*a^2\*c^3\*d^3 + c^2\*d^4)\*x^4 + 2\*(a^4\*c^5\*d + 2\*a^2\*c^4\*d^2 + c^3\*d^3)\*x^2), 1/6\*((3\*a^2\*c^3 + (3\*a^2\*c\*d^2 + 2\*d^3)\*x^4 + 2\*c^2\*d + 2\*(3\*a^2\*c^2\*d + 2\*c\*d^2)\*x^2)\*sqrt(-a^2\*c - d)\*arctan(1/2\*(a^2\*d\*x^2 + 2\*a^2\*c + d)\*sqrt(-a^2\*c - d)\*sqrt(d\*x^2 + c)/(a^3\*c^2 + a\*c\*d + (a^3\*c\*d + a\*d^2)\*x^2)) + (2\*a^3\*c^3 + 2\*a\*c^2\*d + 2\*(a^3\*c^2\*d + a\*c\*d^2)\*x^2 + (2\*(a^4\*c^2\*d + 2\*a^2\*c\*d^2 + d^3)\*x^3 + 3\*(a^4\*c^3 + 2\*a^2\*c^2\*d + c\*d^2)\*x)\*log((a\*x + 1)/(a\*x - 1))\*sqrt(d\*x^2 + c))/(a^4\*c^6 + 2\*a^2\*c^5\*d + c^4\*d^2 + (a^4\*c^4\*d^2 + 2\*a^2\*c^3\*d^3 + c^2\*d^4)\*x^4 + 2\*(a^4\*c^5\*d + 2\*a^2\*c^4\*d^2 + c^3\*d^3)\*x^2)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(ax)}{(dx^2 + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(5/2),x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/(d\*x^2 + c)^(5/2), x)

**maple** [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(ax)}{(dx^2 + c)^{\frac{5}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(a*x)/(d*x^2+c)^(5/2),x)`

[Out] `int(arccoth(a*x)/(d*x^2+c)^(5/2),x)`

**maxima** [B] time = 0.41, size = 223, normalized size = 1.74

$$\frac{1}{6} a \left( \frac{ad \log\left(\frac{\sqrt{dx^2+c}a^2 - \sqrt{a^2c+da}}{\sqrt{dx^2+c}a^2 + \sqrt{a^2c+da}}\right) + \frac{2d}{(a^2c^2+cd)\sqrt{dx^2+c}}}{\frac{(a^2c^2+cd)\sqrt{a^2c+d}}{d}} + \frac{2 \log\left(\frac{\sqrt{dx^2+c}a^2 - \sqrt{a^2c+da}}{\sqrt{dx^2+c}a^2 + \sqrt{a^2c+da}}\right)}{\sqrt{a^2c+d}ac^2} \right) + \frac{1}{3} \left( \frac{2x}{\sqrt{dx^2+c}c^2} + \frac{x}{(dx^2+c)^{\frac{3}{2}}c} \right) \operatorname{arccoth}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")`

[Out] `1/6*a*((a*d*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/((a^2*c^2 + c*d)*sqrt(a^2*c + d)) + 2*d/((a^2*c^2 + c*d)*sqrt(d*x^2 + c)))/d + 2*log((sqrt(d*x^2 + c)*a^2 - sqrt(a^2*c + d)*a)/(sqrt(d*x^2 + c)*a^2 + sqrt(a^2*c + d)*a))/(sqrt(a^2*c + d)*a*c^2) + 1/3*(2*x/(sqrt(d*x^2 + c)*c^2) + x/((d*x^2 + c)^(3/2)*c))*arccoth(a*x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(ax)}{(dx^2+c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)/(c + d*x^2)^(5/2),x)`

[Out] `int(acoth(a*x)/(c + d*x^2)^(5/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{(c + dx^2)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)/(d*x**2+c)**(5/2),x)`

[Out] `Integral(acoth(a*x)/(c + d*x**2)**(5/2),x)`

$$3.46 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}} dx$$

Optimal. Leaf size=200

$$\frac{a(7a^2c + 4d)}{15c^2(a^2c + d)^2 \sqrt{c + dx^2}} + \frac{a}{15c(a^2c + d)(c + dx^2)^{3/2}} - \frac{(15a^4c^2 + 20a^2cd + 8d^2) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3(a^2c + d)^{5/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c + dx^2}}$$

[Out] 1/15\*a/c/(a^2\*c+d)/(d\*x^2+c)^(3/2)+1/5\*x\*arccoth(a\*x)/c/(d\*x^2+c)^(5/2)+4/15\*x\*arccoth(a\*x)/c^2/(d\*x^2+c)^(3/2)-1/15\*(15\*a^4\*c^2+20\*a^2\*c\*d+8\*d^2)\*arc tanh(a\*(d\*x^2+c)^(1/2)/(a^2\*c+d)^(1/2))/c^3/(a^2\*c+d)^(5/2)+1/15\*a\*(7\*a^2\*c+4\*d)/c^2/(a^2\*c+d)^2/(d\*x^2+c)^(1/2)+8/15\*x\*arccoth(a\*x)/c^3/(d\*x^2+c)^(1/2)

**Rubi [A]** time = 1.05, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {192, 191, 5977, 6688, 12, 6715, 897, 1261, 208}

$$-\frac{(15a^4c^2 + 20a^2cd + 8d^2) \tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3(a^2c + d)^{5/2}} + \frac{a(7a^2c + 4d)}{15c^2(a^2c + d)^2 \sqrt{c + dx^2}} + \frac{a}{15c(a^2c + d)(c + dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c + dx^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^(7/2), x]

[Out] a/(15\*c\*(a^2\*c + d)\*(c + d\*x^2)^(3/2)) + (a\*(7\*a^2\*c + 4\*d))/(15\*c^2\*(a^2\*c + d)^2\*sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(5\*c\*(c + d\*x^2)^(5/2)) + (4\*x\*ArcCoth[a\*x])/(15\*c^2\*(c + d\*x^2)^(3/2)) + (8\*x\*ArcCoth[a\*x])/(15\*c^3\*sqrt[c + d\*x^2]) - ((15\*a^4\*c^2 + 20\*a^2\*c\*d + 8\*d^2)\*ArcTanh[(a\*sqrt[c + d\*x^2])/sqrt[a^2\*c + d]])/(15\*c^3\*(a^2\*c + d)^(5/2))

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 192

```
Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)
)/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
&& NeQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 897

```
Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_)^(n_))*((a_) + (b_)*(x_)
+ (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)
^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5977

```
Int[((a_) + ArcCoth[(c_)*(x_)])*(b_))*((d_) + (e_)*(x_)^2)^(q_), x_Sym
bol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function0
```

fQ[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{7/2}} dx &= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a \int \frac{\frac{x}{5c(c+dx^2)^{5/2}} + \frac{4x}{15c^2(c+dx^2)^{3/2}} + \frac{8x}{15c^3\sqrt{c+dx^2}}}{1-a^2x^2} dx \\
 &= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a \int \frac{x(15c^2+20cdx^2+8d^2x^4)}{15c^3(1-a^2x^2)(c+dx^2)^{5/2}} dx \\
 &= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a \int \frac{x(15c^2+20cdx^2+8d^2x^4)}{(1-a^2x^2)(c+dx^2)^{5/2}} dx}{15c^3} \\
 &= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{(1-a^2x)(c+dx)^{5/2}} dx, x, x^2\right)}{30c^3} \\
 &= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{a^2c+d}{d}-\frac{a^2x^2}{d}\right)} dx, x, \sqrt{c+dx^2}\right)}{15c^3d} \\
 &= \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x \coth^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \left(\frac{3c^2d}{(a^2c+d)x^4} + \frac{cd(7a^2c+4d)}{(a^2c+d)^2x^2} + \frac{cd^2}{(a^2c+d)^2}\right) dx, x, \sqrt{c+dx^2}\right)}{15c^3} \\
 &= \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}} \\
 &= \frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}} + \frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}} + \frac{x \coth^{-1}(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x \coth^{-1}(ax)}{15c^2(c+dx^2)^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.57, size = 329, normalized size = 1.64

$$\frac{2x(a^2c+d)^{5/2} \coth^{-1}(ax)(15c^2+20cdx^2+8d^2x^4) + 2ac\sqrt{a^2c+d}(c+dx^2)(a^2c(8c+7dx^2)+d(5c+4dx^2))}{15c^3(a^2c+d)^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^(7/2), x]

[Out]  $(2*a*c*\sqrt{a^2*c + d}*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(8*c + 7*d*x^2)) + 2*(a^2*c + d)^{(5/2)}*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*\text{ArcCoth}[a*x] + (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^{(5/2)}*\text{Log}[1 - a*x] + (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^{(5/2)}*\text{Log}[1 + a*x] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^{(5/2)}*\text{Log}[a*c - d*x + \sqrt{a^2*c + d}]*\sqrt{c + d*x^2}] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^{(5/2)}*\text{Log}[a*c + d*x + \sqrt{a^2*c + d}]*\sqrt{c + d*x^2}]/(30*c^3*(a^2*c + d)^{(5/2)}*(c + d*x^2)^{(5/2)})$

**fricas** [B] time = 0.75, size = 1278, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(7/2), x, algorithm="fricas")

[Out]  $[1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*\sqrt{a^2*c + d}*\log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*\sqrt{a^2*c + d}*\sqrt{d*x^2 + c} + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*\log((a*x + 1)/(a*x - 1))*\sqrt{d*x^2 + c}]/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*\sqrt{-a^2*c - d}*\arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*\sqrt{-a^2*c - d}*\sqrt{d*x^2 + c}]/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*\log((a*x + 1)/(a*x - 1))*\sqrt{d*x^2 + c}]/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2)]$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(7/2),x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/(d\*x^2 + c)^(7/2), x)

**maple** [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^(7/2),x)

[Out] int(arccoth(a\*x)/(d\*x^2+c)^(7/2),x)

**maxima** [B] time = 0.42, size = 401, normalized size = 2.00

$$\frac{1}{30} a \left( \frac{3 a^3 d \log\left(\frac{\sqrt{dx^2+c} a^2 - \sqrt{a^2c+da}}{\sqrt{dx^2+c} a^2 + \sqrt{a^2c+da}}\right) + \frac{2(3(dx^2+c)a^2d+a^2cd+d^2)}{(a^4c^3+2a^2c^2d+cd^2)(dx^2+c)^{\frac{3}{2}}}}{d} + \frac{4 \left( \frac{ad \log\left(\frac{\sqrt{dx^2+c} a^2 - \sqrt{a^2c+da}}{\sqrt{dx^2+c} a^2 + \sqrt{a^2c+da}}\right) + \frac{2d}{(a^2c^3+c^2d)\sqrt{dx^2+c}} \right)}{d} + \frac{8 \log\left(\frac{\sqrt{dx^2+c} a^2 - \sqrt{a^2c+da}}{\sqrt{dx^2+c} a^2 + \sqrt{a^2c+da}}\right)}{\sqrt{dx^2+c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(7/2),x, algorithm="maxima")

[Out] 1/30\*a\*((3\*a^3\*d\*log((sqrt(d\*x^2 + c)\*a^2 - sqrt(a^2\*c + d)\*a)/(sqrt(d\*x^2 + c)\*a^2 + sqrt(a^2\*c + d)\*a)))/((a^4\*c^3 + 2\*a^2\*c^2\*d + c\*d^2)\*sqrt(a^2\*c + d)) + 2\*(3\*(d\*x^2 + c)\*a^2\*d + a^2\*c\*d + d^2)/((a^4\*c^3 + 2\*a^2\*c^2\*d + c\*d^2)\*(d\*x^2 + c)^(3/2))/d + 4\*(a\*d\*log((sqrt(d\*x^2 + c)\*a^2 - sqrt(a^2\*c + d)\*a)/(sqrt(d\*x^2 + c)\*a^2 + sqrt(a^2\*c + d)\*a)))/((a^2\*c^3 + c^2\*d)\*sqrt(a^2\*c + d)) + 2\*d/((a^2\*c^3 + c^2\*d)\*sqrt(d\*x^2 + c))/d + 8\*log((sqrt(d\*x^2 + c)\*a^2 - sqrt(a^2\*c + d)\*a)/(sqrt(d\*x^2 + c)\*a^2 + sqrt(a^2\*c + d)\*a))/(sqrt(a^2\*c + d)\*a\*c^3) + 1/15\*(8\*x/(sqrt(d\*x^2 + c)\*c^3) + 4\*x/((d\*x^2 + c)^(3/2)\*c^2) + 3\*x/((d\*x^2 + c)^(5/2)\*c))\*arccoth(a\*x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(ax)}{(dx^2 + c)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x)/(c + d*x^2)^(7/2), x)`

[Out] `int(acoth(a*x)/(c + d*x^2)^(7/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{(c + dx^2)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x)/(d*x**2+c)**(7/2), x)`

[Out] `Integral(acoth(a*x)/(c + d*x**2)**(7/2), x)`

$$3.47 \quad \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}} dx$$

Optimal. Leaf size=283

$$\frac{a(11a^2c + 6d)}{105c^2(a^2c + d)^2(c + dx^2)^{3/2}} + \frac{a}{35c(a^2c + d)(c + dx^2)^{5/2}} + \frac{a(19a^4c^2 + 22a^2cd + 8d^2)}{35c^3(a^2c + d)^3\sqrt{c + dx^2}} - \frac{(35a^6c^3 + 70a^4c^2d + 56a^2cd^2 + 16d^3)}{35c^4(a^2c + d)^{7/2}}$$

[Out] 1/35\*a/c/(a^2\*c+d)/(d\*x^2+c)^(5/2)+1/105\*a\*(11\*a^2\*c+6\*d)/c^2/(a^2\*c+d)^2/(d\*x^2+c)^(3/2)+1/7\*x\*arccoth(a\*x)/c/(d\*x^2+c)^(7/2)+6/35\*x\*arccoth(a\*x)/c^2/(d\*x^2+c)^(5/2)+8/35\*x\*arccoth(a\*x)/c^3/(d\*x^2+c)^(3/2)-1/35\*(35\*a^6\*c^3+70\*a^4\*c^2\*d+56\*a^2\*c\*d^2+16\*d^3)\*arctanh(a\*(d\*x^2+c)^(1/2)/(a^2\*c+d)^(1/2))/c^4/(a^2\*c+d)^(7/2)+1/35\*a\*(19\*a^4\*c^2+22\*a^2\*c\*d+8\*d^2)/c^3/(a^2\*c+d)^3/(d\*x^2+c)^(1/2)+16/35\*x\*arccoth(a\*x)/c^4/(d\*x^2+c)^(1/2)

**Rubi [A]** time = 1.34, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {192, 191, 5977, 6688, 12, 6715, 1619, 63, 208}

$$\frac{a(19a^4c^2 + 22a^2cd + 8d^2)}{35c^3(a^2c + d)^3\sqrt{c + dx^2}} - \frac{(70a^4c^2d + 35a^6c^3 + 56a^2cd^2 + 16d^3)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4(a^2c + d)^{7/2}} + \frac{a(11a^2c + 6d)}{105c^2(a^2c + d)^2(c + dx^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x]/(c + d\*x^2)^(9/2), x]

[Out] a/(35\*c\*(a^2\*c + d)\*(c + d\*x^2)^(5/2)) + (a\*(11\*a^2\*c + 6\*d))/(105\*c^2\*(a^2\*c + d)^2\*(c + d\*x^2)^(3/2)) + (a\*(19\*a^4\*c^2 + 22\*a^2\*c\*d + 8\*d^2))/(35\*c^3\*(a^2\*c + d)^3\*sqrt[c + d\*x^2]) + (x\*ArcCoth[a\*x])/(7\*c\*(c + d\*x^2)^(7/2)) + (6\*x\*ArcCoth[a\*x])/(35\*c^2\*(c + d\*x^2)^(5/2)) + (8\*x\*ArcCoth[a\*x])/(35\*c^3\*(c + d\*x^2)^(3/2)) + (16\*x\*ArcCoth[a\*x])/(35\*c^4\*sqrt[c + d\*x^2]) - ((35\*a^6\*c^3 + 70\*a^4\*c^2\*d + 56\*a^2\*c\*d^2 + 16\*d^3)\*ArcTanh[(a\*sqrt[c + d\*x^2])/sqrt[a^2\*c + d]])/(35\*c^4\*(a^2\*c + d)^(7/2))

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 63



```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 191

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

### Rule 192

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)
)/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
&& NeQ[p, -1]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 1619

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] := I
nt[ExpandIntegrand[1/Sqrt[c + d*x], (Px*(c + d*x)^(n + 1/2))/(a + b*x), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

### Rule 5977

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] := With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcCoth[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

### Rule 6715

Int[(u\_)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}} dx &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a \int \frac{x}{7c(c+dx^2)^{7/2}} \\
 &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a \int \frac{x(35c^3+7c^2)}{35c^4} \\
 &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a \int \frac{x(35c^3+70c^2dx^2)}{(1-a^2x^2)^{7/2}}}{3} \\
 &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \frac{35c^3+70c^2x^2}{(a^2c-x^2)^{7/2}}\right)}{3} \\
 &= \frac{x \coth^{-1}(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x \coth^{-1}(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x \coth^{-1}(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x \coth^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a \operatorname{Subst}\left(\int \left(\frac{35c^3+70c^2x^2}{(a^2c-x^2)^{7/2}}\right)\right)}{3} \\
 &= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{a}{7c} \\
 &= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{a}{7c} \\
 &= \frac{a}{35c(a^2c+d)(c+dx^2)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(c+dx^2)^{3/2}} + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{c+dx^2}} + \frac{a}{7c}
 \end{aligned}$$

**Mathematica [A]** time = 0.94, size = 431, normalized size = 1.52

$$6x(a^2c + d)^{7/2} \coth^{-1}(ax) (35c^3 + 70c^2dx^2 + 56cd^2x^4 + 16d^3x^6) + 2ac\sqrt{a^2c + d} (c + dx^2) (3c^2(a^2c + d)^2 + c(1$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x]/(c + d\*x^2)^(9/2), x]

[Out] (2\*a\*c\*Sqrt[a^2\*c + d]\*(c + d\*x^2)\*(3\*c^2\*(a^2\*c + d)^2 + c\*(a^2\*c + d)\*(11\*a^2\*c + 6\*d)\*(c + d\*x^2) + 3\*(19\*a^4\*c^2 + 22\*a^2\*c\*d + 8\*d^2)\*(c + d\*x^2)^2) + 6\*(a^2\*c + d)^(7/2)\*x\*(35\*c^3 + 70\*c^2\*d\*x^2 + 56\*c\*d^2\*x^4 + 16\*d^3\*x^6)\*ArcCoth[a\*x] + 3\*(35\*a^6\*c^3 + 70\*a^4\*c^2\*d + 56\*a^2\*c\*d^2 + 16\*d^3)\*(c + d\*x^2)^(7/2)\*Log[1 - a\*x] + 3\*(35\*a^6\*c^3 + 70\*a^4\*c^2\*d + 56\*a^2\*c\*d^2 + 16\*d^3)\*(c + d\*x^2)^(7/2)\*Log[1 + a\*x] - 3\*(35\*a^6\*c^3 + 70\*a^4\*c^2\*d + 56\*a^2\*c\*d^2 + 16\*d^3)\*(c + d\*x^2)^(7/2)\*Log[a\*c - d\*x + Sqrt[a^2\*c + d]\*Sqrt[c + d\*x^2]] - 3\*(35\*a^6\*c^3 + 70\*a^4\*c^2\*d + 56\*a^2\*c\*d^2 + 16\*d^3)\*(c + d\*x^2)^(7/2)\*Log[a\*c + d\*x + Sqrt[a^2\*c + d]\*Sqrt[c + d\*x^2]]/(210\*c^4\*(a^2\*c + d)^(7/2)\*(c + d\*x^2)^(7/2))

**fricas [B]** time = 0.93, size = 2004, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(9/2), x, algorithm="fricas")

[Out] [1/420\*(3\*(35\*a^6\*c^7 + 70\*a^4\*c^6\*d + 56\*a^2\*c^5\*d^2 + (35\*a^6\*c^3\*d^4 + 70\*a^4\*c^2\*d^5 + 56\*a^2\*c\*d^6 + 16\*d^7)\*x^8 + 16\*c^4\*d^3 + 4\*(35\*a^6\*c^4\*d^3 + 70\*a^4\*c^3\*d^4 + 56\*a^2\*c^2\*d^5 + 16\*c\*d^6)\*x^6 + 6\*(35\*a^6\*c^5\*d^2 + 70\*a^4\*c^4\*d^3 + 56\*a^2\*c^3\*d^4 + 16\*c^2\*d^5)\*x^4 + 4\*(35\*a^6\*c^6\*d + 70\*a^4\*c^5\*d^2 + 56\*a^2\*c^4\*d^3 + 16\*c^3\*d^4)\*x^2)\*sqrt(a^2\*c + d)\*log((a^4\*d^2\*x^4 + 8\*a^4\*c^2 + 8\*a^2\*c\*d + 2\*(4\*a^4\*c\*d + 3\*a^2\*d^2)\*x^2 - 4\*(a^3\*d\*x^2 + 2\*a^3\*c + a\*d)\*sqrt(a^2\*c + d)\*sqrt(d\*x^2 + c) + d^2)/(a^4\*x^4 - 2\*a^2\*x^2 + 1)) + 2\*(142\*a^7\*c^7 + 320\*a^5\*c^6\*d + 244\*a^3\*c^5\*d^2 + 66\*a\*c^4\*d^3 + 6\*(19\*a^7\*c^4\*d^3 + 41\*a^5\*c^3\*d^4 + 30\*a^3\*c^2\*d^5 + 8\*a\*c\*d^6)\*x^6 + 2\*(182\*a^7\*c^5\*d^2 + 397\*a^5\*c^4\*d^3 + 293\*a^3\*c^3\*d^4 + 78\*a\*c^2\*d^5)\*x^4 + 2\*(196\*a^7\*c^6\*d + 434\*a^5\*c^5\*d^2 + 325\*a^3\*c^4\*d^3 + 87\*a\*c^3\*d^4)\*x^2 + 3\*(16\*(a^8\*c^4\*d^3 + 4\*a^6\*c^3\*d^4 + 6\*a^4\*c^2\*d^5 + 4\*a^2\*c\*d^6 + d^7)\*x^7 + 56\*(a^8\*c^5\*d^2 + 4\*a^6\*c^4\*d^3 + 6\*a^4\*c^3\*d^4 + 4\*a^2\*c^2\*d^5 + c\*d^6)\*x^5 + 70\*(a^8\*c^6\*d + 4\*a^6\*c^5\*d^2 + 6\*a^4\*c^4\*d^3 + 4\*a^2\*c^3\*d^4 + c^2\*d^5)\*x^3 + 35\*(a^8\*c^7 + 4\*a^6\*c^6\*d + 6\*a^4\*c^5\*d^2 + 4\*a^2\*c^4\*d^3 + c^3\*d^4)\*x)\*log((a\*x + 1)/(a\*x - 1))\*sqrt(d\*x^2 + c)]/(a^8\*c^12 + 4\*a^6\*c^11\*d + 6\*a^4\*c^10\*d^2 + 4\*a^2\*c^9\*d^3 + c^8\*d^4 + (a^8\*c^8\*d^4 + 4\*a^6\*c^7\*d^5 + 6

```

*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*d^
4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^6*
c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d + 4*
a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/210*(3*(35*
a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 70*a^4*c^2*d^5
+ 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 + 70*a^4*c^3*
d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70*a^4*c^4*d^3 +
56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*c^5*d^2 + 56*a
^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^
2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d
^2)*x^2)) + (142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 +
6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*(
182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 + 2
*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 + 3
*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7
+ 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*
x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d
^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d
^4)*x)*log((a*x + 1)/(a*x - 1))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d
+ 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 +
6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*
d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^
6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d +
4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x)/(d\*x^2+c)^(9/2),x, algorithm="giac")

[Out] integrate(arccoth(a\*x)/(d\*x^2 + c)^(9/2), x)

**maple** [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x)/(d\*x^2+c)^(9/2),x)

[Out]  $\int \operatorname{arccoth}(ax)/(dx^2+c)^{9/2}, x$

**maxima** [B] time = 0.43, size = 639, normalized size = 2.26

$$\frac{1}{210} a \left[ \frac{15 a^5 d \log\left(\frac{\sqrt{dx^2+c}a^2 - \sqrt{a^2c+da}}{\sqrt{dx^2+c}a^2 + \sqrt{a^2c+da}}\right)}{(a^6c^4+3a^4c^3d+3a^2c^2d^2+cd^3)\sqrt{a^2c+d}} + \frac{2\left(15(dx^2+c)^2a^4d+3a^4c^2d+6a^2cd^2+3d^3+5(a^4cd+a^2d^2)(dx^2+c)\right)}{(a^6c^4+3a^4c^3d+3a^2c^2d^2+cd^3)(dx^2+c)^{\frac{5}{2}}}\right] + \frac{6\left(3a^3d \log\left(\frac{\sqrt{dx^2+c}a^2 - \sqrt{dx^2+c}a^2 + \sqrt{a^2c+da}}{\sqrt{dx^2+c}a^2 + \sqrt{a^2c+da}}\right)\right)}{(a^4c^4+2a^2c^3d+c^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(\operatorname{arccoth}(ax)/(dx^2+c)^{9/2}, x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{210} a \left( \frac{15 a^5 d \log\left(\frac{\sqrt{dx^2+c}a^2 - \sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2 + \sqrt{a^2c+d}a}\right)}{(a^6c^4+3a^4c^3d+3a^2c^2d^2+cd^3)\sqrt{a^2c+d}} + \frac{2\left(15(dx^2+c)^2a^4d+3a^4c^2d+6a^2cd^2+3d^3+5(a^4cd+a^2d^2)(dx^2+c)\right)}{(a^6c^4+3a^4c^3d+3a^2c^2d^2+cd^3)(dx^2+c)^{5/2}} \right) + \frac{6\left(3a^3d \log\left(\frac{\sqrt{dx^2+c}a^2 - \sqrt{a^2c+d}a}{\sqrt{dx^2+c}a^2 + \sqrt{a^2c+d}a}\right)\right)}{(a^4c^4+2a^2c^3d+c^2d^2)}$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(ax)}{(dx^2+c)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int \operatorname{acoth}(ax)/(c+dx^2)^{9/2}, x$

[Out]  $\int \operatorname{acoth}(ax)/(c+dx^2)^{9/2}, x$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax)}{(c+dx^2)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoath(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral(acoath(a*x)/(c + d*x**2)**(9/2), x)
```

### 3.48 $\int \sqrt{a - ax^2} \coth^{-1}(x) dx$

**Optimal.** Leaf size=186

$$-\frac{ia\sqrt{1-x^2} \operatorname{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}} + \frac{ia\sqrt{1-x^2} \operatorname{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}} + \frac{1}{2}\sqrt{a-ax^2} + \frac{1}{2}x\sqrt{a-ax^2} \coth^{-1}(x) - \frac{a\sqrt{1-x^2} \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}$$

[Out]  $-a*\operatorname{arccoth}(x)*\arctan((1-x)^{(1/2)}/(1+x)^{(1/2)})*(-x^2+1)^{(1/2)}/(-a*x^2+a)^{(1/2)} - 1/2*I*a*\operatorname{polylog}(2, -I*(1-x)^{(1/2)}/(1+x)^{(1/2)})*(-x^2+1)^{(1/2)}/(-a*x^2+a)^{(1/2)} + 1/2*I*a*\operatorname{polylog}(2, I*(1-x)^{(1/2)}/(1+x)^{(1/2)})*(-x^2+1)^{(1/2)}/(-a*x^2+a)^{(1/2)} + 1/2*(-a*x^2+a)^{(1/2)} + 1/2*x*\operatorname{arccoth}(x)*(-a*x^2+a)^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {5943, 5955, 5951}

$$-\frac{ia\sqrt{1-x^2} \operatorname{PolyLog}\left(2, -\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}} + \frac{ia\sqrt{1-x^2} \operatorname{PolyLog}\left(2, \frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}} + \frac{1}{2}\sqrt{a-ax^2} + \frac{1}{2}x\sqrt{a-ax^2} \coth^{-1}(x) - \frac{a\sqrt{1-x^2} \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - a\*x^2]\*ArcCoth[x], x]

[Out]  $\operatorname{Sqrt}[a - a*x^2]/2 + (x*\operatorname{Sqrt}[a - a*x^2]*\operatorname{ArcCoth}[x])/2 - (a*\operatorname{Sqrt}[1 - x^2]*\operatorname{ArcCoth}[x]*\operatorname{ArcTan}[\operatorname{Sqrt}[1 - x]/\operatorname{Sqrt}[1 + x]])/\operatorname{Sqrt}[a - a*x^2] - ((I/2)*a*\operatorname{Sqrt}[1 - x^2]*\operatorname{PolyLog}[2, ((-I)*\operatorname{Sqrt}[1 - x])/\operatorname{Sqrt}[1 + x]])/\operatorname{Sqrt}[a - a*x^2] + ((I/2)*a*\operatorname{Sqrt}[1 - x^2]*\operatorname{PolyLog}[2, (I*\operatorname{Sqrt}[1 - x])/\operatorname{Sqrt}[1 + x]])/\operatorname{Sqrt}[a - a*x^2]$

Rule 5943

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/((d\_) + (e\_.)\*(x\_)^2)^(q\_.), x\_Symbol] :> Simp[(b\*(d + e\*x^2)^q)/(2\*c\*q\*(2\*q + 1)), x] + (Dist[(2\*d\*q)/(2\*q + 1), Int[(d + e\*x^2)^(q - 1)\*(a + b\*ArcCoth[c\*x]), x], x] + Simp[(x\*(d + e\*x^2)^q\*(a + b\*ArcCoth[c\*x])]/(2\*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[q, 0]

Rule 5951

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] :> Simp[(-2\*(a + b\*ArcCoth[c\*x])\*ArcTan[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/(c\*Sqrt[d]), x] + (-Simp[(I\*b\*PolyLog[2, -((I\*Sqrt[1 - c\*x])/Sqrt[1 + c\*x])])]/(c\*Sqrt[d]), x] + Simp[(I\*b\*PolyLog[2, (I\*Sqrt[1 - c\*x])/Sqrt[1 + c\*x]])]/(c\*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[d,

0]

Rule 5955

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/Sqrt[(d\_.) + (e\_.)\*(x\_.)^2], x\_  
 Symbol] :> Dist[Sqrt[1 - c^2\*x^2]/Sqrt[d + e\*x^2], Int[(a + b\*ArcCoth[c\*x])  
 ^p/Sqrt[1 - c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e,  
 0] && IGtQ[p, 0] && !GtQ[d, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{a - ax^2} \coth^{-1}(x) dx &= \frac{1}{2} \sqrt{a - ax^2} + \frac{1}{2} x \sqrt{a - ax^2} \coth^{-1}(x) + \frac{1}{2} a \int \frac{\coth^{-1}(x)}{\sqrt{a - ax^2}} dx \\ &= \frac{1}{2} \sqrt{a - ax^2} + \frac{1}{2} x \sqrt{a - ax^2} \coth^{-1}(x) + \frac{\left(a \sqrt{1 - x^2}\right) \int \frac{\coth^{-1}(x)}{\sqrt{1 - x^2}} dx}{2 \sqrt{a - ax^2}} \\ &= \frac{1}{2} \sqrt{a - ax^2} + \frac{1}{2} x \sqrt{a - ax^2} \coth^{-1}(x) - \frac{a \sqrt{1 - x^2} \coth^{-1}(x) \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a - ax^2}} - \frac{ia \sqrt{1 - x^2}}{2} \end{aligned}$$

**Mathematica** [A] time = 0.97, size = 125, normalized size = 0.67

$$\frac{\sqrt{a - ax^2} \left( -4 \operatorname{Li}_2 \left( -e^{-\coth^{-1}(x)} \right) + 4 \operatorname{Li}_2 \left( e^{-\coth^{-1}(x)} \right) - 2 \coth \left( \frac{1}{2} \coth^{-1}(x) \right) - 4 \coth^{-1}(x) \log \left( 1 - e^{-\coth^{-1}(x)} \right) + 4 \right)}{8}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a - a\*x^2]\*ArcCoth[x],x]

[Out] -1/8\*(Sqrt[a - a\*x^2]\*(-2\*Coth[ArcCoth[x]/2] - ArcCoth[x]\*Csch[ArcCoth[x]/2]  
 ]^2 - 4\*ArcCoth[x]\*Log[1 - E^(-ArcCoth[x])] + 4\*ArcCoth[x]\*Log[1 + E^(-ArcC  
 oth[x])] - 4\*PolyLog[2, -E^(-ArcCoth[x])] + 4\*PolyLog[2, E^(-ArcCoth[x])] -  
 ArcCoth[x]\*Sech[ArcCoth[x]/2]^2 + 2\*Tanh[ArcCoth[x]/2]))/(Sqrt[1 - x^(-2)]  
 \*x)

**fricas** [F] time = 0.81, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{-ax^2 + a} \operatorname{arccoth}(x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((-a\*x^2+a)^(1/2)\*arccoth(x),x, algorithm="fricas")

[Out] integral(sqrt(-a\*x^2 + a)\*arccoth(x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-ax^2 + a} \operatorname{arccoth}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a\*x^2+a)^(1/2)\*arccoth(x),x, algorithm="giac")

[Out] integrate(sqrt(-a\*x^2 + a)\*arccoth(x), x)

**maple** [A] time = 0.66, size = 199, normalized size = 1.07

$$\frac{(\operatorname{arccoth}(x)x + 1) \sqrt{-(-1+x)(1+x)a}}{2} + \frac{\sqrt{-(-1+x)(1+x)a} \sqrt{\frac{-1+x}{1+x}} \operatorname{arccoth}(x) \ln\left(1 - \frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)}{-2 + 2x} + \frac{\sqrt{-(-1+x)(1+x)a}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a\*x^2+a)^(1/2)\*arccoth(x),x)

[Out] 1/2\*(arccoth(x)\*x+1)\*(-(-1+x)\*(1+x)\*a)^(1/2)+1/2\*(-(-1+x)\*(1+x)\*a)^(1/2)\*((  
-1+x)/(1+x))^(1/2)/(-1+x)\*arccoth(x)\*ln(1-1/(((-1+x)/(1+x))^(1/2)))+1/2\*(-(-1  
+x)\*(1+x)\*a)^(1/2)\*(((-1+x)/(1+x))^(1/2)/(-1+x)\*polylog(2,1/(((-1+x)/(1+x))^(  
1/2)))-1/2\*(-(-1+x)\*(1+x)\*a)^(1/2)\*(((-1+x)/(1+x))^(1/2)/(-1+x)\*arccoth(x)\*ln  
(1+1/(((-1+x)/(1+x))^(1/2)))-1/2\*(-(-1+x)\*(1+x)\*a)^(1/2)\*(((-1+x)/(1+x))^(1/2)  
/(-1+x)\*polylog(2,-1/(((-1+x)/(1+x))^(1/2))))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-ax^2 + a} \operatorname{arccoth}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a\*x^2+a)^(1/2)\*arccoth(x),x, algorithm="maxima")

[Out] integrate(sqrt(-a\*x^2 + a)\*arccoth(x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{arccoth}(x) \sqrt{a - ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(x)*(a - a*x^2)^(1/2),x)`

[Out] `int(acoth(x)*(a - a*x^2)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a(x-1)(x+1)} \operatorname{acoth}(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a*x**2+a)**(1/2)*acoth(x),x)`

[Out] `Integral(sqrt(-a*(x - 1)*(x + 1))*acoth(x), x)`

$$3.49 \quad \int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}} dx$$

**Optimal.** Leaf size=144

$$-\frac{i\sqrt{1-x^2} \operatorname{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2} \operatorname{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}} - \frac{2\sqrt{1-x^2} \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right) \coth^{-1}(x)}{\sqrt{a-ax^2}}$$

[Out]  $-2*\operatorname{arccoth}(x)*\arctan((1-x)^{(1/2)}/(1+x)^{(1/2)})*(-x^2+1)^{(1/2)}/(-a*x^2+a)^{(1/2)}-I*\operatorname{polylog}(2,-I*(1-x)^{(1/2)}/(1+x)^{(1/2)})*(-x^2+1)^{(1/2)}/(-a*x^2+a)^{(1/2)}+I*\operatorname{polylog}(2,I*(1-x)^{(1/2)}/(1+x)^{(1/2)})*(-x^2+1)^{(1/2)}/(-a*x^2+a)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {5955, 5951}

$$-\frac{i\sqrt{1-x^2} \operatorname{PolyLog}\left(2, -\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2} \operatorname{PolyLog}\left(2, \frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}} - \frac{2\sqrt{1-x^2} \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right) \coth^{-1}(x)}{\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[x]/Sqrt[a - a*x^2], x]`

[Out]  $(-2*\operatorname{Sqrt}[1-x^2]*\operatorname{ArcCoth}[x]*\operatorname{ArcTan}[\operatorname{Sqrt}[1-x]/\operatorname{Sqrt}[1+x]])/\operatorname{Sqrt}[a-a*x^2] - (I*\operatorname{Sqrt}[1-x^2]*\operatorname{PolyLog}[2, ((-I)*\operatorname{Sqrt}[1-x])/\operatorname{Sqrt}[1+x]])/\operatorname{Sqrt}[a-a*x^2] + (I*\operatorname{Sqrt}[1-x^2]*\operatorname{PolyLog}[2, (I*\operatorname{Sqrt}[1-x])/\operatorname{Sqrt}[1+x]])/\operatorname{Sqrt}[a-a*x^2]$

#### Rule 5951

`Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-2*(a + b*ArcCoth[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])]/(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])]/(c*Sqrt[d]), x) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]`

#### Rule 5955

`Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcCoth[c*x])^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && !GtQ[d, 0]`

Rubi steps

$$\int \frac{\coth^{-1}(x)}{\sqrt{a-ax^2}} dx = \frac{\sqrt{1-x^2} \int \frac{\coth^{-1}(x)}{\sqrt{1-x^2}} dx}{\sqrt{a-ax^2}}$$

$$= -\frac{2\sqrt{1-x^2} \coth^{-1}(x) \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} - \frac{i\sqrt{1-x^2} \operatorname{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}} + \frac{i\sqrt{1-x^2} \operatorname{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}$$

**Mathematica** [A] time = 0.12, size = 77, normalized size = 0.53

$$\frac{\sqrt{a-ax^2} \left( \operatorname{Li}_2\left(-e^{-\coth^{-1}(x)}\right) - \operatorname{Li}_2\left(e^{-\coth^{-1}(x)}\right) + \coth^{-1}(x) \left( \log\left(1 - e^{-\coth^{-1}(x)}\right) - \log\left(e^{-\coth^{-1}(x)} + 1\right) \right) \right)}{a\sqrt{1 - \frac{1}{x^2}} x}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[x]/Sqrt[a - a\*x^2], x]

[Out] (Sqrt[a - a\*x^2]\*(ArcCoth[x]\*(Log[1 - E^(-ArcCoth[x])]) - Log[1 + E^(-ArcCoth[x])]) + PolyLog[2, -E^(-ArcCoth[x])] - PolyLog[2, E^(-ArcCoth[x])]))/(a\*Sqrt[1 - x^(-2)]\*x)

**fricas** [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-ax^2+a} \operatorname{arccoth}(x)}{ax^2-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(1/2), x, algorithm="fricas")

[Out] integral(-sqrt(-a\*x^2 + a)\*arccoth(x)/(a\*x^2 - a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(x)}{\sqrt{-ax^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(1/2), x, algorithm="giac")

[Out] integrate(arccoth(x)/sqrt(-a\*x^2 + a), x)

**maple** [A] time = 0.56, size = 190, normalized size = 1.32

$$\frac{\ln\left(1 - \frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right) \operatorname{arccoth}(x) \sqrt{\frac{-1+x}{1+x}} \sqrt{-(-1+x)(1+x)a}}{(-1+x)a} + \frac{\operatorname{polylog}\left(2, \frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right) \sqrt{\frac{-1+x}{1+x}} \sqrt{-(-1+x)(1+x)a}}{(-1+x)a} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-a\*x^2+a)^(1/2), x)

[Out]  $\ln(1-1/((-1+x)/(1+x))^{(1/2)}) \operatorname{arccoth}(x) * ((-1+x)/(1+x))^{(1/2)} * (-(-1+x) * (1+x) * a)^{(1/2)} / (-1+x) / a + \operatorname{polylog}(2, 1/((-1+x)/(1+x))^{(1/2)}) * ((-1+x)/(1+x))^{(1/2)} * (-(-1+x) * (1+x) * a)^{(1/2)} / (-1+x) / a - \ln(1+1/((-1+x)/(1+x))^{(1/2)}) \operatorname{arccoth}(x) * ((-1+x)/(1+x))^{(1/2)} * (-(-1+x) * (1+x) * a)^{(1/2)} / (-1+x) / a - \operatorname{polylog}(2, -1/((-1+x)/(1+x))^{(1/2)}) * ((-1+x)/(1+x))^{(1/2)} * (-(-1+x) * (1+x) * a)^{(1/2)} / (-1+x) / a$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(x)}{\sqrt{-ax^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(1/2), x, algorithm="maxima")

[Out] integrate(arccoth(x)/sqrt(-a\*x^2 + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(x)}{\sqrt{a - ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(x)/(a - a\*x^2)^(1/2), x)

[Out] int(acoth(x)/(a - a\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(x)}{\sqrt{-a(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)/(-a\*x\*\*2+a)\*\*(1/2), x)

[Out] Integral(acoth(x)/sqrt(-a\*(x - 1)\*(x + 1)), x)

$$3.50 \quad \int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx$$

Optimal. Leaf size=37

$$\frac{x \coth^{-1}(x)}{a\sqrt{a-ax^2}} - \frac{1}{a\sqrt{a-ax^2}}$$

[Out]  $-1/a/(-a*x^2+a)^{(1/2)}+x*\operatorname{arccoth}(x)/a/(-a*x^2+a)^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {5959}

$$\frac{x \coth^{-1}(x)}{a\sqrt{a-ax^2}} - \frac{1}{a\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[x]/(a - a*x^2)^{(3/2)}, x]$

[Out]  $-(1/(a*\operatorname{Sqrt}[a - a*x^2])) + (x*\operatorname{ArcCoth}[x])/(a*\operatorname{Sqrt}[a - a*x^2])$

Rule 5959

$\operatorname{Int}[(a_.) + \operatorname{ArcCoth}[(c_.)*(x_.)]*(b_.)]/((d_.) + (e_.)*(x_.)^2)^{(3/2)}, x\_Symbol] \rightarrow -\operatorname{Simp}[b/(c*d*\operatorname{Sqrt}[d + e*x^2]), x] + \operatorname{Simp}[(x*(a + b*\operatorname{ArcCoth}[c*x]))/(d*\operatorname{Sqrt}[d + e*x^2]), x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x] \&\& \operatorname{EqQ}[c^2*d + e, 0]$

Rubi steps

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx = -\frac{1}{a\sqrt{a-ax^2}} + \frac{x \coth^{-1}(x)}{a\sqrt{a-ax^2}}$$

**Mathematica [A]** time = 0.05, size = 30, normalized size = 0.81

$$\frac{\sqrt{a-ax^2} (1 - x \coth^{-1}(x))}{a^2 (x^2 - 1)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[\operatorname{ArcCoth}[x]/(a - a*x^2)^{(3/2)}, x]$

[Out]  $(\sqrt{a - ax^2} * (1 - x * \text{ArcCoth}[x])) / (a^2 * (-1 + x^2))$

**fricas** [A] time = 0.47, size = 41, normalized size = 1.11

$$-\frac{\sqrt{-ax^2 + a} \left( x \log\left(\frac{x+1}{x-1}\right) - 2 \right)}{2(a^2x^2 - a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="fricas")`

[Out]  $-1/2 * \sqrt{-ax^2 + a} * (x * \log((x + 1)/(x - 1)) - 2) / (a^2 * x^2 - a^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(x)}{(-ax^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")`

[Out] `integrate(arccoth(x)/(-a*x^2 + a)^(3/2), x)`

**maple** [A] time = 0.48, size = 52, normalized size = 1.41

$$-\frac{(\text{arccoth}(x) - 1) \sqrt{-(-1 + x)(1 + x)a}}{2(-1 + x)a^2} - \frac{(\text{arccoth}(x) + 1) \sqrt{-(-1 + x)(1 + x)a}}{2(1 + x)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(x)/(-a*x^2+a)^(3/2),x)`

[Out]  $-1/2 * (\text{arccoth}(x) - 1) * (-(-1 + x) * (1 + x) * a)^{(1/2)} / (-1 + x) / a^2 - 1/2 * (\text{arccoth}(x) + 1) * (-(-1 + x) * (1 + x) * a)^{(1/2)} / (1 + x) / a^2$

**maxima** [A] time = 0.41, size = 63, normalized size = 1.70

$$\frac{x \text{ arccoth}(x)}{\sqrt{-ax^2 + a} a} - \frac{\sqrt{-ax^2 + a}}{ax + a} - \frac{\sqrt{-ax^2 + a}}{ax - a} - \frac{1}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="maxima")`

[Out]  $x \cdot \operatorname{arccoth}(x) / (\sqrt{-a x^2 + a}) \cdot a - 1/2 \cdot (\sqrt{-a x^2 + a}) / (a x + a) - \sqrt{-a x^2 + a} / (a x - a) / a$

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acoth}(x)}{(a - a x^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(x)/(a - a*x^2)^(3/2), x)`

[Out] `int(acoth(x)/(a - a*x^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(x)}{(-a(x-1)(x+1))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x)/(-a*x**2+a)**(3/2), x)`

[Out] `Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(3/2), x)`



$$3.51 \quad \int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}} dx$$

Optimal. Leaf size=83

$$-\frac{2}{3a^2\sqrt{a-ax^2}} + \frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}} - \frac{1}{9a(a-ax^2)^{3/2}} + \frac{x\coth^{-1}(x)}{3a(a-ax^2)^{3/2}}$$

[Out]  $-1/9/a/(-a*x^2+a)^{(3/2)}+1/3*x*\operatorname{arccoth}(x)/a/(-a*x^2+a)^{(3/2)}-2/3/a^2/(-a*x^2+a)^{(1/2)}+2/3*x*\operatorname{arccoth}(x)/a^2/(-a*x^2+a)^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {5961, 5959}

$$-\frac{2}{3a^2\sqrt{a-ax^2}} + \frac{2x\coth^{-1}(x)}{3a^2\sqrt{a-ax^2}} - \frac{1}{9a(a-ax^2)^{3/2}} + \frac{x\coth^{-1}(x)}{3a(a-ax^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]/(a - a\*x^2)^(5/2), x]

[Out]  $-1/(9*a*(a - a*x^2)^{(3/2)}) - 2/(3*a^2*\operatorname{Sqrt}[a - a*x^2]) + (x*\operatorname{ArcCoth}[x])/(3*a*(a - a*x^2)^{(3/2)}) + (2*x*\operatorname{ArcCoth}[x])/(3*a^2*\operatorname{Sqrt}[a - a*x^2])$

Rule 5959

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/((d\_) + (e\_.)\*(x\_)^2)^(3/2), x\_Symbol] := -Simp[b/(c\*d\*Sqrt[d + e\*x^2]), x] + Simp[(x\*(a + b\*ArcCoth[c\*x]))/(d\*Sqrt[d + e\*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0]

Rule 5961

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(q\_), x\_Symbol] := -Simp[(b\*(d + e\*x^2)^(q + 1))/(4\*c\*d\*(q + 1)^2), x] + (Dist[(2\*q + 3)/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x]), x], x] - Simp[(x\*(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x]))/(2\*d\*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]

Rubi steps

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}} dx = -\frac{1}{9a(a-ax^2)^{3/2}} + \frac{x \coth^{-1}(x)}{3a(a-ax^2)^{3/2}} + \frac{2 \int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx}{3a}$$

$$= -\frac{1}{9a(a-ax^2)^{3/2}} - \frac{2}{3a^2\sqrt{a-ax^2}} + \frac{x \coth^{-1}(x)}{3a(a-ax^2)^{3/2}} + \frac{2x \coth^{-1}(x)}{3a^2\sqrt{a-ax^2}}$$

**Mathematica [A]** time = 0.05, size = 45, normalized size = 0.54

$$\frac{\sqrt{a-ax^2} \left( (6x^3 - 9x) \coth^{-1}(x) - 6x^2 + 7 \right)}{9a^3(x^2 - 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(a - a\*x^2)^(5/2), x]

[Out] -1/9\*(Sqrt[a - a\*x^2]\*(7 - 6\*x^2 + (-9\*x + 6\*x^3)\*ArcCoth[x]))/(a^3\*(-1 + x^2)^2)

**fricas [A]** time = 0.47, size = 61, normalized size = 0.73

$$\frac{\sqrt{-ax^2 + a} \left( 12x^2 - 3(2x^3 - 3x) \log\left(\frac{x+1}{x-1}\right) - 14 \right)}{18(a^3x^4 - 2a^3x^2 + a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(5/2), x, algorithm="fricas")

[Out] 1/18\*sqrt(-a\*x^2 + a)\*(12\*x^2 - 3\*(2\*x^3 - 3\*x)\*log((x + 1)/(x - 1)) - 14)/(a^3\*x^4 - 2\*a^3\*x^2 + a^3)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(x)}{(-ax^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(5/2), x, algorithm="giac")

[Out] integrate(arccoth(x)/(-a\*x^2 + a)^(5/2), x)

**maple** [A] time = 0.51, size = 112, normalized size = 1.35

$$\frac{(1+x)(-1+3\operatorname{arccoth}(x))\sqrt{-(-1+x)(1+x)a}}{72(-1+x)^2a^3} - \frac{3(\operatorname{arccoth}(x)-1)\sqrt{-(-1+x)(1+x)a}}{8(-1+x)a^3} - \frac{3(\operatorname{arccoth}(x)+1)}{8(1+x)^2/a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-a\*x^2+a)^(5/2),x)

[Out] 1/72\*(1+x)\*(-1+3\*arccoth(x))\*(-(-1+x)\*(1+x)\*a)^(1/2)/(-1+x)^2/a^3-3/8\*(arccoth(x)-1)\*(-(-1+x)\*(1+x)\*a)^(1/2)/(-1+x)/a^3-3/8\*(arccoth(x)+1)\*(-(-1+x)\*(1+x)\*a)^(1/2)/(1+x)/a^3+1/72\*(1+3\*arccoth(x))\*(-1+x)\*(-(-1+x)\*(1+x)\*a)^(1/2)/(1+x)^2/a^3

**maxima** [A] time = 0.32, size = 67, normalized size = 0.81

$$\frac{1}{3} \left( \frac{2x}{\sqrt{-ax^2 + a}a^2} + \frac{x}{(-ax^2 + a)^{\frac{3}{2}}a} \right) \operatorname{arccoth}(x) - \frac{2}{3\sqrt{-ax^2 + a}a^2} - \frac{1}{9(-ax^2 + a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(5/2),x, algorithm="maxima")

[Out] 1/3\*(2\*x/(sqrt(-a\*x^2 + a)\*a^2) + x/((-a\*x^2 + a)^(3/2)\*a))\*arccoth(x) - 2/3/(sqrt(-a\*x^2 + a)\*a^2) - 1/9/((-a\*x^2 + a)^(3/2)\*a)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(x)}{(a - ax^2)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(x)/(a - a\*x^2)^(5/2),x)

[Out] int(acoth(x)/(a - a\*x^2)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(x)}{(-a(x-1)(x+1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-a*x**2+a)**(5/2),x)
```

```
[Out] Integral(acoth(x)/(-a*(x - 1)*(x + 1))**(5/2), x)
```

$$3.52 \quad \int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}} dx$$

**Optimal.** Leaf size=124

$$-\frac{8}{15a^3\sqrt{a-ax^2}} + \frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} + \frac{4x\coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}} - \frac{1}{25a(a-ax^2)^{5/2}} + \frac{x\coth^{-1}(x)}{5a(a-ax^2)^{5/2}}$$

[Out]  $-1/25/a/(-a*x^2+a)^{(5/2)}-4/45/a^2/(-a*x^2+a)^{(3/2)}+1/5*x*\operatorname{arccoth}(x)/a/(-a*x^2+a)^{(5/2)}+4/15*x*\operatorname{arccoth}(x)/a^2/(-a*x^2+a)^{(3/2)}-8/15/a^3/(-a*x^2+a)^{(1/2)}+8/15*x*\operatorname{arccoth}(x)/a^3/(-a*x^2+a)^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {5961, 5959}

$$-\frac{8}{15a^3\sqrt{a-ax^2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} + \frac{8x\coth^{-1}(x)}{15a^3\sqrt{a-ax^2}} + \frac{4x\coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}} - \frac{1}{25a(a-ax^2)^{5/2}} + \frac{x\coth^{-1}(x)}{5a(a-ax^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]/(a - a\*x^2)^(7/2), x]

[Out]  $-1/(25*a*(a - a*x^2)^{(5/2)}) - 4/(45*a^2*(a - a*x^2)^{(3/2)}) - 8/(15*a^3*\operatorname{Sqrt}[a - a*x^2]) + (x*\operatorname{ArcCoth}[x])/(5*a*(a - a*x^2)^{(5/2)}) + (4*x*\operatorname{ArcCoth}[x])/(15*a^2*(a - a*x^2)^{(3/2)}) + (8*x*\operatorname{ArcCoth}[x])/(15*a^3*\operatorname{Sqrt}[a - a*x^2])$

**Rule 5959**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/((d\_) + (e\_.)\*(x\_)^2)^(3/2), x\_Symbol] :> -Simp[b/(c\*d\*Sqrt[d + e\*x^2]), x] + Simp[(x\*(a + b\*ArcCoth[c\*x]))/(d\*Sqrt[d + e\*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0]

**Rule 5961**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^2)^(q\_), x\_Symbol] :> -Simp[(b\*(d + e\*x^2)^(q + 1))/(4\*c\*d\*(q + 1)^2), x] + (Dist[(2\*q + 3)/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x]), x], x] - Simp[(x\*(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x]))/(2\*d\*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(x)}{(a-ax^2)^{7/2}} dx &= -\frac{1}{25a(a-ax^2)^{5/2}} + \frac{x \coth^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4 \int \frac{\coth^{-1}(x)}{(a-ax^2)^{5/2}} dx}{5a} \\
&= -\frac{1}{25a(a-ax^2)^{5/2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} + \frac{x \coth^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4x \coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}} + \frac{8 \int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx}{15a^2} \\
&= -\frac{1}{25a(a-ax^2)^{5/2}} - \frac{4}{45a^2(a-ax^2)^{3/2}} - \frac{8}{15a^3\sqrt{a-ax^2}} + \frac{x \coth^{-1}(x)}{5a(a-ax^2)^{5/2}} + \frac{4x \coth^{-1}(x)}{15a^2(a-ax^2)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 55, normalized size = 0.44

$$\frac{\sqrt{a-ax^2} (120x^4 - 260x^2 - 15(8x^4 - 20x^2 + 15)x \coth^{-1}(x) + 149)}{225a^4(x^2 - 1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(a - a\*x^2)^(7/2), x]

[Out] (Sqrt[a - a\*x^2]\*(149 - 260\*x^2 + 120\*x^4 - 15\*x\*(15 - 20\*x^2 + 8\*x^4)\*ArcCoth[x]))/(225\*a^4\*(-1 + x^2)^3)

**fricas [A]** time = 0.55, size = 81, normalized size = 0.65

$$\frac{\left(240x^4 - 520x^2 - 15(8x^5 - 20x^3 + 15x) \log\left(\frac{x+1}{x-1}\right) + 298\right) \sqrt{-ax^2 + a}}{450(a^4x^6 - 3a^4x^4 + 3a^4x^2 - a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(7/2), x, algorithm="fricas")

[Out] 1/450\*(240\*x^4 - 520\*x^2 - 15\*(8\*x^5 - 20\*x^3 + 15\*x)\*log((x + 1)/(x - 1)) + 298)\*sqrt(-a\*x^2 + a)/(a^4\*x^6 - 3\*a^4\*x^4 + 3\*a^4\*x^2 - a^4)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(x)}{(-ax^2 + a)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(7/2),x, algorithm="giac")

[Out] integrate(arccoth(x)/(-a\*x^2 + a)^(7/2), x)

**maple [A]** time = 0.52, size = 176, normalized size = 1.42

$$\frac{(1+x)^2(-1+5\operatorname{arccoth}(x))\sqrt{-(-1+x)(1+x)a}}{800(-1+x)^3a^4} + \frac{5(1+x)(-1+3\operatorname{arccoth}(x))\sqrt{-(-1+x)(1+x)a}}{288(-1+x)^2a^4} - \frac{5(\operatorname{arccoth}(x)-1)\sqrt{-(-1+x)(1+x)a}}{288(-1+x)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-a\*x^2+a)^(7/2),x)

[Out] 
$$-1/800*(1+x)^2*(-1+5*\operatorname{arccoth}(x))*(-(-1+x)*(1+x)*a)^{(1/2)}/(-1+x)^3/a^4+5/288*(1+x)*(-1+3*\operatorname{arccoth}(x))*(-(-1+x)*(1+x)*a)^{(1/2)}/(-1+x)^2/a^4-5/16*(\operatorname{arccoth}(x)-1)*(-(-1+x)*(1+x)*a)^{(1/2)}/(-1+x)/a^4-5/16*(\operatorname{arccoth}(x)+1)*(-(-1+x)*(1+x)*a)^{(1/2)}/(1+x)/a^4+5/288*(1+3*\operatorname{arccoth}(x))*(-1+x)*(-(-1+x)*(1+x)*a)^{(1/2)}/(1+x)^2/a^4-1/800*(1+5*\operatorname{arccoth}(x))*(-1+x)^2*(-(-1+x)*(1+x)*a)^{(1/2)}/(1+x)^3/a^4$$

**maxima [A]** time = 0.32, size = 99, normalized size = 0.80

$$\frac{1}{15} \left( \frac{8x}{\sqrt{-ax^2+a}a^3} + \frac{4x}{(-ax^2+a)^{\frac{3}{2}}a^2} + \frac{3x}{(-ax^2+a)^{\frac{5}{2}}a} \right) \operatorname{arccoth}(x) - \frac{8}{15\sqrt{-ax^2+a}a^3} - \frac{4}{45(-ax^2+a)^{\frac{3}{2}}a^2} - \frac{1}{25(-ax^2+a)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-a\*x^2+a)^(7/2),x, algorithm="maxima")

[Out] 
$$1/15*(8*x/(\operatorname{sqrt}(-a*x^2+a)*a^3) + 4*x/((-a*x^2+a)^{(3/2)}*a^2) + 3*x/((-a*x^2+a)^{(5/2)}*a))*\operatorname{arccoth}(x) - 8/15/(\operatorname{sqrt}(-a*x^2+a)*a^3) - 4/45/((-a*x^2+a)^{(3/2)}*a^2) - 1/25/((-a*x^2+a)^{(5/2)}*a)$$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(x)}{(a-ax^2)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(x)/(a - a\*x^2)^(7/2),x)

[Out] int(acoth(x)/(a - a\*x^2)^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(x)}{(-a(x-1)(x+1))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)/(-a\*x\*\*2+a)\*\*(7/2),x)

[Out] Integral(acoth(x)/(-a\*(x - 1)\*(x + 1))\*\*(7/2), x)



$$3.53 \quad \int \frac{1}{(1-x^2) \coth^{-1}(x)} dx$$

Optimal. Leaf size=3

$$\log(\coth^{-1}(x))$$

[Out] ln(arccoth(x))

**Rubi** [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {5947}

$$\log(\coth^{-1}(x))$$

Antiderivative was successfully verified.

[In] Int[1/((1 - x^2)\*ArcCoth[x]), x]

[Out] Log[ArcCoth[x]]

Rule 5947

Int[1/(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + (e\_.)\*(x\_)^2)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*ArcCoth[c\*x], x]]/(b\*c\*d), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0]

Rubi steps

$$\int \frac{1}{(1-x^2) \coth^{-1}(x)} dx = \log(\coth^{-1}(x))$$

**Mathematica** [A] time = 0.03, size = 3, normalized size = 1.00

$$\log(\coth^{-1}(x))$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 - x^2)\*ArcCoth[x]), x]

[Out] Log[ArcCoth[x]]

**fricas** [B] time = 0.47, size = 11, normalized size = 3.67

$$\log\left(\log\left(\frac{x+1}{x-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="fricas")

[Out] log(log((x + 1)/(x - 1)))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(x^2 - 1) \operatorname{arccoth}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="giac")

[Out] integrate(-1/((x^2 - 1)\*arccoth(x)), x)

**maple** [A] time = 0.06, size = 4, normalized size = 1.33

$$\ln(\operatorname{arccoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+1)/arccoth(x),x)

[Out] ln(arccoth(x))

**maxima** [A] time = 0.30, size = 3, normalized size = 1.00

$$\log(\operatorname{arccoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/arccoth(x),x, algorithm="maxima")

[Out] log(arccoth(x))

**mupad** [B] time = 0.30, size = 3, normalized size = 1.00

$$\ln(\operatorname{acoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(acoth(x)\*(x^2 - 1)),x)

[Out] log(acoth(x))

**sympy** [A] time = 0.28, size = 3, normalized size = 1.00

$$\log(\operatorname{acoth}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)/acoth(x),x)
```

```
[Out] log(acoth(x))
```

$$3.54 \quad \int \frac{\coth^{-1}(x)^n}{1-x^2} dx$$

Optimal. Leaf size=12

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

[Out] arccoth(x)^(1+n)/(1+n)

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {5949}

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]^n/(1 - x^2), x]

[Out] ArcCoth[x]^(1 + n)/(1 + n)

Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{\coth^{-1}(x)^n}{1-x^2} dx = \frac{\coth^{-1}(x)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{\coth^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]^n/(1 - x^2), x]

[Out] ArcCoth[x]^(1 + n)/(1 + n)

**fricas** [B] time = 0.60, size = 62, normalized size = 5.17

$$\frac{\cosh\left(n \log\left(\frac{1}{2} \log\left(\frac{x+1}{x-1}\right)\right)\right) \log\left(\frac{x+1}{x-1}\right) + \log\left(\frac{x+1}{x-1}\right) \sinh\left(n \log\left(\frac{1}{2} \log\left(\frac{x+1}{x-1}\right)\right)\right)}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="fricas")

[Out] 1/2\*(cosh(n\*log(1/2\*log((x + 1)/(x - 1))))\*log((x + 1)/(x - 1)) + log((x + 1)/(x - 1))\*sinh(n\*log(1/2\*log((x + 1)/(x - 1)))))/(n + 1)

**giac** [A] time = 0.12, size = 1, normalized size = 0.08

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="giac")

[Out] +Infinity

**maple** [A] time = 0.06, size = 13, normalized size = 1.08

$$\frac{\operatorname{arccoth}(x)^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)^n/(-x^2+1),x)

[Out] arccoth(x)^(1+n)/(1+n)

**maxima** [A] time = 0.30, size = 12, normalized size = 1.00

$$\frac{\operatorname{arccoth}(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^n/(-x^2+1),x, algorithm="maxima")

[Out] arccoth(x)^(n + 1)/(n + 1)

**mupad** [B] time = 1.36, size = 22, normalized size = 1.83

$$\begin{cases} \ln(\operatorname{acoth}(x)) & \text{if } n = -1 \\ \frac{\operatorname{acoth}(x)^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-acoth(x)^n/(x^2 - 1), x)`

[Out] `piecewise(n == -1, log(acoth(x)), n != -1, acoth(x)^(n + 1)/(n + 1))`

sympy [A] time = 1.72, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{acoth}(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x)**n/(-x**2+1), x)`

[Out] `Piecewise((acoth(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(acoth(x)), True))`

$$3.55 \quad \int \frac{\coth^{-1}(x)^2}{(1-x^2)^2} dx$$

Optimal. Leaf size=62

$$\frac{x}{4(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \tanh^{-1}(x) + \frac{1}{6} \coth^{-1}(x)^3$$

[Out] 1/4\*x/(-x^2+1)-1/2\*arccoth(x)/(-x^2+1)+1/2\*x\*arccoth(x)^2/(-x^2+1)+1/6\*arccoth(x)^3+1/4\*arctanh(x)

**Rubi [A]** time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {5957, 5995, 199, 206}

$$\frac{x}{4(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \tanh^{-1}(x) + \frac{1}{6} \coth^{-1}(x)^3$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]^2/(1 - x^2)^2,x]

[Out] x/(4\*(1 - x^2)) - ArcCoth[x]/(2\*(1 - x^2)) + (x\*ArcCoth[x]^2)/(2\*(1 - x^2)) + ArcCoth[x]^3/6 + ArcTanh[x]/4

### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 5957

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2)^2, x\_Symbol] := Simp[(x\*(a + b\*ArcCoth[c\*x])^p)/(2\*d\*(d + e\*x^2)), x] + (-Dist[(b\*c\*p)/2, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(d + e\*x^2)^2, x], x] + Simp[(

$a + b \cdot \text{ArcCoth}[c \cdot x])^{(p + 1)/(2 \cdot b \cdot c \cdot d^2 \cdot (p + 1))}$ ,  $x$ ) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2 \cdot d + e, 0] && GtQ[p, 0]

### Rule 5995

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(q\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(2\*e\*(q + 1)), x] + Dist[(b\*p)/(2\*c\*(q + 1)), Int[(d + e\*x^2)^q\*(a + b\*ArcCoth[c\*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0] && NeQ[q, -1]

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(x)^2}{(1-x^2)^2} dx &= \frac{x \coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6} \coth^{-1}(x)^3 - \int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx \\ &= -\frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6} \coth^{-1}(x)^3 + \frac{1}{2} \int \frac{1}{(1-x^2)^2} dx \\ &= \frac{x}{4(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6} \coth^{-1}(x)^3 + \frac{1}{4} \int \frac{1}{1-x^2} dx \\ &= \frac{x}{4(1-x^2)} - \frac{\coth^{-1}(x)}{2(1-x^2)} + \frac{x \coth^{-1}(x)^2}{2(1-x^2)} + \frac{1}{6} \coth^{-1}(x)^3 + \frac{1}{4} \tanh^{-1}(x) \end{aligned}$$

**Mathematica** [A] time = 0.07, size = 61, normalized size = 0.98

$$\frac{-3(x^2 - 1) \log(1 - x) + 3(x^2 - 1) \log(x + 1) + 4(x^2 - 1) \coth^{-1}(x)^3 - 6x - 12x \coth^{-1}(x)^2 + 12 \coth^{-1}(x)}{24(x^2 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]^2/(1 - x^2)^2, x]

[Out] (-6\*x + 12\*ArcCoth[x] - 12\*x\*ArcCoth[x]^2 + 4\*(-1 + x^2)\*ArcCoth[x]^3 - 3\*(-1 + x^2)\*Log[1 - x] + 3\*(-1 + x^2)\*Log[1 + x])/(24\*(-1 + x^2))

**fricas** [A] time = 0.53, size = 63, normalized size = 1.02

$$\frac{(x^2 - 1) \log\left(\frac{x+1}{x-1}\right)^3 - 6x \log\left(\frac{x+1}{x-1}\right)^2 + 6(x^2 + 1) \log\left(\frac{x+1}{x-1}\right) - 12x}{48(x^2 - 1)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="fricas")

[Out] 1/48\*((x^2 - 1)\*log((x + 1)/(x - 1))^3 - 6\*x\*log((x + 1)/(x - 1))^2 + 6\*(x^2 + 1)\*log((x + 1)/(x - 1)) - 12\*x)/(x^2 - 1)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(x)^2}{(x^2 - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="giac")

[Out] integrate(arccoth(x)^2/(x^2 - 1)^2, x)

**maple** [C] time = 2.00, size = 707, normalized size = 11.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)^2/(-x^2+1)^2,x)

[Out] -1/4\*arccoth(x)^2/(-1+x)-1/4\*arccoth(x)^2\*ln(-1+x)-1/4\*arccoth(x)^2/(1+x)+1/4\*arccoth(x)^2\*ln(1+x)+1/4\*arccoth(x)^2\*ln((-1+x)/(1+x))+1/24\*(-3\*I\*arccoth(x)^2\*Pi\*csgn(I/((1+x)/(-1+x)-1))\*csgn(I\*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2\*x^2+3\*I\*arccoth(x)^2\*Pi\*csgn(I\*(1+x)/(-1+x))\*csgn(I/((-1+x)/(1+x))^(1/2))^2\*x^2+3\*I\*csgn(I\*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2\*csgn(I/((1+x)/(-1+x)-1))\*arccoth(x)^2\*Pi+3\*I\*arccoth(x)^2\*Pi\*csgn(I\*(1+x)/(-1+x))\*csgn(I/((1+x)/(-1+x)-1))\*csgn(I\*(1+x)/(-1+x)/((1+x)/(-1+x)-1))\*x^2+3\*I\*arccoth(x)^2\*Pi\*csgn(I\*(1+x)/(-1+x))^3\*x^2-3\*I\*csgn(I\*(1+x)/(-1+x)/((1+x)/(-1+x)-1))\*csgn(I/((1+x)/(-1+x)-1))\*csgn(I\*(1+x)/(-1+x))\*arccoth(x)^2\*Pi+6\*I\*csgn(I\*(1+x)/(-1+x))^2\*csgn(I/((-1+x)/(1+x))^(1/2))\*arccoth(x)^2\*Pi+3\*I\*csgn(I\*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2\*csgn(I\*(1+x)/(-1+x))\*arccoth(x)^2\*Pi+3\*I\*arccoth(x)^2\*Pi\*csgn(I\*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^3\*x^2-3\*I\*csgn(I\*(1+x)/(-1+x))\*csgn(I/((-1+x)/(1+x))^(1/2))^2\*arccoth(x)^2\*Pi-6\*I\*arccoth(x)^2\*Pi\*csgn(I\*(1+x)/(-1+x))^2\*csgn(I/((-1+x)/(1+x))^(1/2))\*x^2-3\*I\*csgn(I\*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^3\*arccoth(x)^2\*Pi-3\*I\*arccoth(x)^2\*Pi\*csgn(I\*(1+x)/(-1+x))\*csgn(I\*(1+x)/(-1+x)/((1+x)/(-1+x)-1))^2\*x^2-3\*I\*csgn(I\*(1+x)/(-1+x))^3\*arccoth(x)^2\*Pi+4\*arccoth(x)^3\*x^2-4\*arccoth(x)^3+6\*arccoth(x)\*x^2+6\*arccoth(x)-6\*x)/(-1+x)/(1+x)

**maxima** [B] time = 0.31, size = 171, normalized size = 2.76

$$-\frac{1}{4} \left( \frac{2x}{x^2-1} - \log(x+1) + \log(x-1) \right) \operatorname{arccoth}(x)^2 - \frac{((x^2-1)\log(x+1)^2 - 2(x^2-1)\log(x+1)\log(x-1) + (x^2-1)\log(x-1)^2)}{8(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="maxima")

[Out] -1/4\*(2\*x/(x^2 - 1) - log(x + 1) + log(x - 1))\*arccoth(x)^2 - 1/8\*((x^2 - 1)\*log(x + 1)^2 - 2\*(x^2 - 1)\*log(x + 1)\*log(x - 1) + (x^2 - 1)\*log(x - 1)^2 - 4)\*arccoth(x)/(x^2 - 1) + 1/48\*((x^2 - 1)\*log(x + 1)^3 - 3\*(x^2 - 1)\*log(x + 1)^2\*log(x - 1) - (x^2 - 1)\*log(x - 1)^3 + 3\*((x^2 - 1)\*log(x - 1)^2 + 2\*x^2 - 2)\*log(x + 1) - 6\*(x^2 - 1)\*log(x - 1) - 12\*x)/(x^2 - 1)

**mupad** [B] time = 2.68, size = 201, normalized size = 3.24

$$\frac{\ln\left(\frac{1}{x}+1\right)^3}{48} - \frac{\ln\left(1-\frac{1}{x}\right)^3}{48} - \frac{x}{4(x^2-1)} + \ln\left(1-\frac{1}{x}\right) \left( \frac{\frac{3x}{32}-\frac{1}{8}}{x^2-1} - \frac{\frac{x}{8}+\frac{1}{8}}{x^2-1} - \frac{\ln\left(\frac{1}{x}+1\right)^2}{16} + \frac{x}{32(x^2-1)} + \ln\left(\frac{1}{x}+1\right) \left( \frac{x}{4x^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoath(x)^2/(x^2 - 1)^2,x)

[Out] log(1/x + 1)^3/48 - (atan(x\*1i)\*1i)/4 - log(1 - 1/x)^3/48 - x/(4\*(x^2 - 1)) + log(1 - 1/x)\*(((3\*x)/32 - 1/8)/(x^2 - 1) - (x/8 + 1/8)/(x^2 - 1) - log(1/x + 1)^2/16 + x/(32\*(x^2 - 1))) + log(1/x + 1)\*((x/4 + 1/16)/(x^2 - 1) - 1/(16\*(x^2 - 1))) + log(1 - 1/x)^2\*(log(1/x + 1)/16 - x/(8\*(x^2 - 1))) + log(1/x + 1)/(4\*(x^2 - 1)) - (x\*log(1/x + 1)^2)/(8\*(x^2 - 1))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoath}^2(x)}{(x-1)^2(x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoath(x)\*\*2/(-x\*\*2+1)\*\*2,x)

[Out] Integral(acoath(x)\*\*2/((x - 1)\*\*2\*(x + 1)\*\*2), x)

$$3.56 \quad \int \frac{x \coth^{-1}(x)}{1-x^2} dx$$

Optimal. Leaf size=37

$$\frac{1}{2} \operatorname{Li}_2\left(\frac{x+1}{x-1}\right) - \frac{1}{2} \coth^{-1}(x)^2 + \log\left(\frac{2}{1-x}\right) \coth^{-1}(x)$$

[Out]  $-1/2*\operatorname{arccoth}(x)^2+\operatorname{arccoth}(x)*\ln(2/(1-x))+1/2*\operatorname{polylog}(2,(1+x)/(-1+x))$

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {5985, 5919, 2402, 2315}

$$\frac{1}{2} \operatorname{PolyLog}\left(2, \frac{x+1}{x-1}\right) - \frac{1}{2} \coth^{-1}(x)^2 + \log\left(\frac{2}{1-x}\right) \coth^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x*\operatorname{ArcCoth}[x])/(1-x^2), x]$

[Out]  $-\operatorname{ArcCoth}[x]^2/2 + \operatorname{ArcCoth}[x]*\operatorname{Log}[2/(1-x)] + \operatorname{PolyLog}[2, (1+x)/(-1+x)]/2$

#### Rule 2315

$\operatorname{Int}[\operatorname{Log}[(c_*)(x_)]/((d_*) + (e_*)(x_)), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /; \operatorname{FreeQ}\{c, d, e\}, x \ \&\& \operatorname{EqQ}[e + c*d, 0]$

#### Rule 2402

$\operatorname{Int}[\operatorname{Log}[(c_*)/((d_*) + (e_*)(x_))]/((f_*) + (g_*)(x_)^2), x\_Symbol] \rightarrow -\operatorname{Dist}[e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1-2*d*x), x], x, 1/(d + e*x)], x] /; \operatorname{FreeQ}\{c, d, e, f, g\}, x \ \&\& \operatorname{EqQ}[c, 2*d] \ \&\& \operatorname{EqQ}[e^2*f + d^2*g, 0]$

#### Rule 5919

$\operatorname{Int}[(a_*) + \operatorname{ArcCoth}[(c_*)(x_)]*(b_*)^{(p_*)}/((d_*) + (e_*)(x_)), x\_Symbol] \rightarrow -\operatorname{Simp}[(a + b*\operatorname{ArcCoth}[c*x])^p*\operatorname{Log}[2/(1 + (e*x)/d)]/e, x] + \operatorname{Dist}[(b*c^p)/e, \operatorname{Int}[(a + b*\operatorname{ArcCoth}[c*x])^{(p-1)}*\operatorname{Log}[2/(1 + (e*x)/d)]/(1 - c^2*x^2), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{EqQ}[c^2*d^2 - e^2, 0]$

#### Rule 5985

$\operatorname{Int}[(a_*) + \operatorname{ArcCoth}[(c_*)(x_)]*(b_*)^{(p_*)}(x_)/((d_*) + (e_*)(x_)^2), x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*\operatorname{ArcCoth}[c*x])^{(p+1)}/(b*e*(p+1)), x] + \operatorname{Dist}[1/$

$(c*d), \text{Int}[(a + b*\text{ArcCoth}[c*x])^p/(1 - c*x), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{IGtQ}[p, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{x \coth^{-1}(x)}{1-x^2} dx &= -\frac{1}{2} \coth^{-1}(x)^2 + \int \frac{\coth^{-1}(x)}{1-x} dx \\ &= -\frac{1}{2} \coth^{-1}(x)^2 + \coth^{-1}(x) \log\left(\frac{2}{1-x}\right) - \int \frac{\log\left(\frac{2}{1-x}\right)}{1-x^2} dx \\ &= -\frac{1}{2} \coth^{-1}(x)^2 + \coth^{-1}(x) \log\left(\frac{2}{1-x}\right) + \text{Subst}\left(\int \frac{\log(2x)}{1-2x} dx, x, \frac{1}{1-x}\right) \\ &= -\frac{1}{2} \coth^{-1}(x)^2 + \coth^{-1}(x) \log\left(\frac{2}{1-x}\right) + \frac{1}{2} \text{Li}_2\left(\frac{1+x}{-1+x}\right) \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 34, normalized size = 0.92

$$\frac{1}{2} \left( \coth^{-1}(x) \left( \coth^{-1}(x) + 2 \log\left(1 - e^{-2 \coth^{-1}(x)}\right) \right) - \text{Li}_2\left(e^{-2 \coth^{-1}(x)}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*ArcCoth[x])/(1 - x^2), x]

[Out] (ArcCoth[x]\*(ArcCoth[x] + 2\*Log[1 - E^(-2\*ArcCoth[x])]) - PolyLog[2, E^(-2\*ArcCoth[x])])/2

**fricas** [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{x \operatorname{arccoth}(x)}{x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1), x, algorithm="fricas")

[Out] integral(-x\*arccoth(x)/(x^2 - 1), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x \operatorname{arccoth}(x)}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1),x, algorithm="giac")

[Out] integrate(-x\*arccoth(x)/(x^2 - 1), x)

**maple** [B] time = 0.05, size = 75, normalized size = 2.03

$$-\frac{\operatorname{arccoth}(x) \ln(-1+x)}{2} - \frac{\operatorname{arccoth}(x) \ln(1+x)}{2} - \frac{\ln(-1+x)^2}{8} + \frac{\operatorname{dilog}\left(\frac{1}{2} + \frac{x}{2}\right)}{2} + \frac{\ln(-1+x) \ln\left(\frac{1}{2} + \frac{x}{2}\right)}{4} + \frac{\ln(1+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(x)/(-x^2+1),x)

[Out] -1/2\*arccoth(x)\*ln(-1+x)-1/2\*arccoth(x)\*ln(1+x)-1/8\*ln(-1+x)^2+1/2\*dilog(1/2+1/2\*x)+1/4\*ln(-1+x)\*ln(1/2+1/2\*x)+1/8\*ln(1+x)^2-1/4\*(ln(1+x)-ln(1/2+1/2\*x))\*ln(-1/2\*x+1/2)

**maxima** [B] time = 0.30, size = 76, normalized size = 2.05

$$\frac{1}{4} (\log(x+1) - \log(x-1)) \log(x^2-1) - \frac{1}{2} \operatorname{arccoth}(x) \log(x^2-1) - \frac{1}{8} \log(x+1)^2 - \frac{1}{4} \log(x+1) \log(x-1) + \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1),x, algorithm="maxima")

[Out] 1/4\*(log(x + 1) - log(x - 1))\*log(x^2 - 1) - 1/2\*arccoth(x)\*log(x^2 - 1) - 1/8\*log(x + 1)^2 - 1/4\*log(x + 1)\*log(x - 1) + 1/8\*log(x - 1)^2 + 1/2\*log(x - 1)\*log(1/2\*x + 1/2) + 1/2\*dilog(-1/2\*x + 1/2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{x \operatorname{acoth}(x)}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x\*acoth(x))/(x^2 - 1),x)

[Out] -int((x\*acoth(x))/(x^2 - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x \operatorname{acoth}(x)}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x)/(-x**2+1),x)
```

```
[Out] -Integral(x*acoth(x)/(x**2 - 1), x)
```

$$3.57 \quad \int \frac{\coth^{-1}(x)}{1-x^2} dx$$

Optimal. Leaf size=8

$$\frac{1}{2} \coth^{-1}(x)^2$$

[Out] 1/2\*arccoth(x)^2

**Rubi** [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {5949}

$$\frac{1}{2} \coth^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]/(1 - x^2), x]

[Out] ArcCoth[x]^2/2

Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{\coth^{-1}(x)}{1-x^2} dx = \frac{1}{2} \coth^{-1}(x)^2$$

**Mathematica** [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2} \coth^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(1 - x^2), x]

[Out] ArcCoth[x]^2/2

**fricas** [B] time = 0.41, size = 14, normalized size = 1.75

$$\frac{1}{8} \log\left(\frac{x+1}{x-1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="fricas")

[Out] 1/8\*log((x + 1)/(x - 1))^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arccoth}(x)}{x^2-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="giac")

[Out] integrate(-arccoth(x)/(x^2 - 1), x)

**maple** [A] time = 0.04, size = 13, normalized size = 1.62

$$\operatorname{arctanh}(x)\operatorname{arccoth}(x) - \frac{\operatorname{arctanh}(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-x^2+1),x)

[Out] arctanh(x)\*arccoth(x)-1/2\*arctanh(x)^2

**maxima** [A] time = 0.30, size = 6, normalized size = 0.75

$$\frac{1}{2} \operatorname{arccoth}(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1),x, algorithm="maxima")

[Out] 1/2\*arccoth(x)^2

**mupad** [B] time = 1.20, size = 21, normalized size = 2.62

$$\frac{\left(\ln\left(1 - \frac{1}{x}\right) - \ln\left(\frac{1}{x} + 1\right)\right)^2}{8}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-acoth(x)/(x^2 - 1),x)`

[Out]  $(\log(1 - 1/x) - \log(1/x + 1))^2/8$

sympy [A] time = 0.70, size = 5, normalized size = 0.62

$$\frac{\operatorname{acoth}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x)/(-x**2+1),x)`

[Out]  $\operatorname{acoth}(x)**2/2$

$$3.58 \quad \int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx$$

Optimal. Leaf size=36

$$-\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4} \tanh^{-1}(x)$$

[Out]  $-1/4*x/(-x^2+1)+1/2*\operatorname{arccoth}(x)/(-x^2+1)-1/4*\operatorname{arctanh}(x)$

**Rubi [A]** time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {5995, 199, 206}

$$-\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] `Int[(x*ArcCoth[x])/(1 - x^2)^2,x]`

[Out]  $-x/(4*(1 - x^2)) + \operatorname{ArcCoth}[x]/(2*(1 - x^2)) - \operatorname{ArcTanh}[x]/4$

#### Rule 199

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])`

#### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

#### Rule 5995

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbol] := Simp[((d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(2*e*(q + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcCoth[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && NeQ[q, -1]`

Rubi steps

$$\begin{aligned} \int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx &= \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{2} \int \frac{1}{(1-x^2)^2} dx \\ &= -\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4} \int \frac{1}{1-x^2} dx \\ &= -\frac{x}{4(1-x^2)} + \frac{\coth^{-1}(x)}{2(1-x^2)} - \frac{1}{4} \tanh^{-1}(x) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 44, normalized size = 1.22

$$\frac{x}{4(x^2-1)} - \frac{\coth^{-1}(x)}{2(x^2-1)} + \frac{1}{8} \log(1-x) - \frac{1}{8} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*ArcCoth[x])/(1 - x^2)^2,x]

[Out] x/(4\*(-1 + x^2)) - ArcCoth[x]/(2\*(-1 + x^2)) + Log[1 - x]/8 - Log[1 + x]/8

**fricas [A]** time = 0.39, size = 29, normalized size = 0.81

$$\frac{(x^2 + 1) \log\left(\frac{x+1}{x-1}\right) - 2x}{8(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")

[Out] -1/8\*((x^2 + 1)\*log((x + 1)/(x - 1)) - 2\*x)/(x^2 - 1)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{arccoth}(x)}{(x^2-1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^2,x, algorithm="giac")

[Out] integrate(x\*arccoth(x)/(x^2 - 1)^2, x)

**maple** [A] time = 0.04, size = 39, normalized size = 1.08

$$-\frac{\operatorname{arccoth}(x)}{2(x^2-1)} + \frac{1}{-8+8x} + \frac{\ln(-1+x)}{8} + \frac{1}{8+8x} - \frac{\ln(1+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(x)/(-x^2+1)^2,x)

[Out] -1/2/(x^2-1)\*arccoth(x)+1/8/(-1+x)+1/8\*ln(-1+x)+1/8/(1+x)-1/8\*ln(1+x)

**maxima** [A] time = 0.30, size = 34, normalized size = 0.94

$$\frac{x}{4(x^2-1)} - \frac{\operatorname{arccoth}(x)}{2(x^2-1)} - \frac{1}{8} \log(x+1) + \frac{1}{8} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")

[Out] 1/4\*x/(x^2 - 1) - 1/2\*arccoth(x)/(x^2 - 1) - 1/8\*log(x + 1) + 1/8\*log(x - 1)

**mupad** [B] time = 1.16, size = 21, normalized size = 0.58

$$\frac{\frac{x}{4} - \frac{\operatorname{acoth}(x)}{2}}{x^2-1} - \frac{\operatorname{acoth}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*acoth(x))/(x^2 - 1)^2,x)

[Out] (x/4 - acoth(x)/2)/(x^2 - 1) - acoth(x)/4

**sympy** [A] time = 0.56, size = 31, normalized size = 0.86

$$-\frac{x^2 \operatorname{acoth}(x)}{4x^2-4} + \frac{x}{4x^2-4} - \frac{\operatorname{acoth}(x)}{4x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(x)/(-x\*\*2+1)\*\*2,x)

[Out] -x\*\*2\*acoth(x)/(4\*x\*\*2 - 4) + x/(4\*x\*\*2 - 4) - acoth(x)/(4\*x\*\*2 - 4)

$$3.59 \quad \int \frac{\coth^{-1}(x)}{(1-x^2)^2} dx$$

Optimal. Leaf size=38

$$-\frac{1}{4(1-x^2)} + \frac{x \coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \coth^{-1}(x)^2$$

[Out]  $-1/4/(-x^2+1)+1/2*x*\operatorname{arccoth}(x)/(-x^2+1)+1/4*\operatorname{arccoth}(x)^2$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {5957, 261}

$$-\frac{1}{4(1-x^2)} + \frac{x \coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \coth^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[x]/(1 - x^2)^2,x]`

[Out]  $-1/(4*(1 - x^2)) + (x*\operatorname{ArcCoth}[x])/(2*(1 - x^2)) + \operatorname{ArcCoth}[x]^2/4$

Rule 261

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]`

Rule 5957

`Int[((a_) + ArcCoth[(c_)*(x_)])*(b_)^(p_)/((d_) + (e_)*(x_)^2)^2, x_Symbol] :> Simp[(x*(a + b*ArcCoth[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*c*p)/2, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(a + b*ArcCoth[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]`

Rubi steps

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2} dx = \frac{x \coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \coth^{-1}(x)^2 - \frac{1}{2} \int \frac{x}{(1-x^2)^2} dx$$

$$= -\frac{1}{4(1-x^2)} + \frac{x \coth^{-1}(x)}{2(1-x^2)} + \frac{1}{4} \coth^{-1}(x)^2$$

**Mathematica [A]** time = 0.03, size = 28, normalized size = 0.74

$$\frac{(x^2 - 1) \coth^{-1}(x)^2 - 2x \coth^{-1}(x) + 1}{4(x^2 - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(1 - x^2)^2,x]

[Out] (1 - 2\*x\*ArcCoth[x] + (-1 + x^2)\*ArcCoth[x]^2)/(4\*(-1 + x^2))

**fricas [A]** time = 0.52, size = 42, normalized size = 1.11

$$\frac{(x^2 - 1) \log\left(\frac{x+1}{x-1}\right)^2 - 4x \log\left(\frac{x+1}{x-1}\right) + 4}{16(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="fricas")

[Out] 1/16\*((x^2 - 1)\*log((x + 1)/(x - 1))^2 - 4\*x\*log((x + 1)/(x - 1)) + 4)/(x^2 - 1)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(x)}{(x^2 - 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="giac")

[Out] integrate(arccoth(x)/(x^2 - 1)^2, x)

**maple [B]** time = 0.06, size = 99, normalized size = 2.61

$$\frac{\operatorname{arccoth}(x)}{4(-1+x)} - \frac{\operatorname{arccoth}(x) \ln(-1+x)}{4} - \frac{\operatorname{arccoth}(x)}{4(1+x)} + \frac{\operatorname{arccoth}(x) \ln(1+x)}{4} - \frac{\ln(1+x)^2}{16} + \frac{\left(\ln(1+x) - \ln\left(\frac{1}{2} + \frac{x}{2}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-x^2+1)^2,x)

[Out] -1/4\*arccoth(x)/(-1+x)-1/4\*arccoth(x)\*ln(-1+x)-1/4\*arccoth(x)/(1+x)+1/4\*arccoth(x)\*ln(1+x)-1/16\*ln(1+x)^2+1/8\*(ln(1+x)-ln(1/2+1/2\*x))\*ln(-1/2\*x+1/2)-1/16\*ln(-1+x)^2+1/8\*ln(-1+x)\*ln(1/2+1/2\*x)+1/8/(-1+x)-1/8/(1+x)

**maxima [B]** time = 0.30, size = 76, normalized size = 2.00

$$-\frac{1}{4} \left( \frac{2x}{x^2-1} - \log(x+1) + \log(x-1) \right) \operatorname{arccoth}(x) - \frac{(x^2-1) \log(x+1)^2 - 2(x^2-1) \log(x+1) \log(x-1) + (x^2-1) \log(x-1)^2}{16(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="maxima")

[Out] -1/4\*(2\*x/(x^2 - 1) - log(x + 1) + log(x - 1))\*arccoth(x) - 1/16\*((x^2 - 1)\*log(x + 1)^2 - 2\*(x^2 - 1)\*log(x + 1)\*log(x - 1) + (x^2 - 1)\*log(x - 1)^2 - 4)/(x^2 - 1)

**mupad [B]** time = 1.22, size = 81, normalized size = 2.13

$$\frac{\ln\left(\frac{1}{x}+1\right)^2}{16} - \ln\left(1-\frac{1}{x}\right) \left( \frac{\ln\left(\frac{1}{x}+1\right)}{8} - \frac{x}{4(x^2-1)} \right) + \frac{\ln\left(1-\frac{1}{x}\right)^2}{16} + \frac{1}{4(x^2-1)} - \frac{x \ln\left(\frac{1}{x}+1\right)}{4(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoath(x)/(x^2 - 1)^2,x)

[Out] log(1/x + 1)^2/16 - log(1 - 1/x)\*(log(1/x + 1)/8 - x/(4\*(x^2 - 1))) + log(1 - 1/x)^2/16 + 1/(4\*(x^2 - 1)) - (x\*log(1/x + 1))/(4\*(x^2 - 1))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoath}(x)}{(x-1)^2(x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(x)/(-x**2+1)**2,x)
```

```
[Out] Integral(acoth(x)/((x - 1)**2*(x + 1)**2), x)
```



$$3.60 \quad \int \frac{x \coth^{-1}(x)}{(1-x^2)^3} dx$$

Optimal. Leaf size=50

$$-\frac{3x}{32(1-x^2)} - \frac{x}{16(1-x^2)^2} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{32} \tanh^{-1}(x)$$

[Out]  $-1/16*x/(-x^2+1)^2-3/32*x/(-x^2+1)+1/4*\operatorname{arccoth}(x)/(-x^2+1)^2-3/32*\operatorname{arctanh}(x)$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {5995, 199, 206}

$$-\frac{3x}{32(1-x^2)} - \frac{x}{16(1-x^2)^2} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{32} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x*\operatorname{ArcCoth}[x])/(1-x^2)^3, x]$

[Out]  $-x/(16*(1-x^2)^2) - (3*x)/(32*(1-x^2)) + \operatorname{ArcCoth}[x]/(4*(1-x^2)^2) - (3*\operatorname{ArcTanh}[x])/32$

Rule 199

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^{(n_+)})^{(p_+)}, x\_Symbol] \rightarrow -\operatorname{Simp}[(x_+*(a_+ + b_+*x_+^{n_+})^{(p_+ + 1)})/(a_+*n_+(p_+ + 1)), x] + \operatorname{Dist}[(n_+(p_+ + 1) + 1)/(a_+*n_+(p_+ + 1)), \operatorname{Int}[(a_+ + b_+*x_+^{n_+})^{(p_+ + 1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b\}, x$  &&  $\operatorname{IGtQ}[n, 0]$  &&  $\operatorname{LtQ}[p, -1]$  &&  $(\operatorname{IntegerQ}[2*p] \parallel (n == 2 \ \&\& \operatorname{IntegerQ}[4*p]) \parallel (n == 2 \ \&\& \operatorname{IntegerQ}[3*p]) \parallel \operatorname{Denominator}[p + 1/n] < \operatorname{Denominator}[p])$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$   $\operatorname{FreeQ}\{a, b\}, x$  &&  $\operatorname{NegQ}[a/b]$  &&  $(\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 5995

$\operatorname{Int}[(a_+ + \operatorname{ArcCoth}[(c_+)*(x_+)])*(b_+)^{(p_+)}*(x_+)*((d_+ + (e_+)*(x_+)^2)^{(q_+)})^{(p_+)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d_+ + e_+*x_+^2)^{(q_+ + 1)}*(a_+ + b_+*\operatorname{ArcCoth}[c_+*x_+])^{(p_+)}/(2*e_+(q_+ + 1)), x] + \operatorname{Dist}[(b_+*p_+)/(2*c_+(q_+ + 1)), \operatorname{Int}[(d_+ + e_+*x_+^2)^{q_+}*(a_+ + b_+*\operatorname{ArcCoth}[c_+*x_+])^{(p_+)}, x]$

)]^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0] && NeQ[q, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{x \coth^{-1}(x)}{(1-x^2)^3} dx &= \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{1}{4} \int \frac{1}{(1-x^2)^3} dx \\
 &= -\frac{x}{16(1-x^2)^2} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{16} \int \frac{1}{(1-x^2)^2} dx \\
 &= -\frac{x}{16(1-x^2)^2} - \frac{3x}{32(1-x^2)} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{32} \int \frac{1}{1-x^2} dx \\
 &= -\frac{x}{16(1-x^2)^2} - \frac{3x}{32(1-x^2)} + \frac{\coth^{-1}(x)}{4(1-x^2)^2} - \frac{3}{32} \tanh^{-1}(x)
 \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 50, normalized size = 1.00

$$\frac{1}{64} \left( \frac{6x}{x^2-1} - \frac{4x}{(x^2-1)^2} + \frac{16 \coth^{-1}(x)}{(x^2-1)^2} + 3 \log(1-x) - 3 \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*ArcCoth[x])/(1 - x^2)^3,x]

[Out] ((-4\*x)/(-1 + x^2)^2 + (6\*x)/(-1 + x^2) + (16\*ArcCoth[x])/(-1 + x^2)^2 + 3\*Log[1 - x] - 3\*Log[1 + x])/64

**fricas** [A] time = 0.61, size = 47, normalized size = 0.94

$$\frac{6x^3 - (3x^4 - 6x^2 - 5) \log\left(\frac{x+1}{x-1}\right) - 10x}{64(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")

[Out] 1/64\*(6\*x^3 - (3\*x^4 - 6\*x^2 - 5)\*log((x + 1)/(x - 1)) - 10\*x)/(x^4 - 2\*x^2 + 1)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x \operatorname{arccoth}(x)}{(x^2 - 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^3,x, algorithm="giac")

[Out] integrate(-x\*arccoth(x)/(x^2 - 1)^3, x)

**maple** [A] time = 0.04, size = 53, normalized size = 1.06

$$\frac{\operatorname{arccoth}(x)}{4(x^2 - 1)^2} - \frac{1}{64(-1 + x)^2} + \frac{3}{64(-1 + x)} + \frac{3 \ln(-1 + x)}{64} + \frac{1}{64(1 + x)^2} + \frac{3}{64(1 + x)} - \frac{3 \ln(1 + x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(x)/(-x^2+1)^3,x)

[Out] 1/4/(x^2-1)^2\*arccoth(x)-1/64/(-1+x)^2+3/64/(-1+x)+3/64\*ln(-1+x)+1/64/(1+x)^2+3/64/(1+x)-3/64\*ln(1+x)

**maxima** [A] time = 0.30, size = 47, normalized size = 0.94

$$\frac{3x^3 - 5x}{32(x^4 - 2x^2 + 1)} + \frac{\operatorname{arccoth}(x)}{4(x^2 - 1)^2} - \frac{3}{64} \log(x + 1) + \frac{3}{64} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")

[Out] 1/32\*(3\*x^3 - 5\*x)/(x^4 - 2\*x^2 + 1) + 1/4\*arccoth(x)/(x^2 - 1)^2 - 3/64\*log(x + 1) + 3/64\*log(x - 1)

**mupad** [B] time = 1.21, size = 34, normalized size = 0.68

$$\frac{3 \ln(x - 1)}{64} - \frac{3 \ln(x + 1)}{64} + \frac{\frac{\operatorname{arccoth}(x)}{4} - \frac{5x}{32} + \frac{3x^3}{32}}{(x^2 - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x\*acoth(x))/(x^2 - 1)^3,x)

[Out]  $(3 \cdot \log(x - 1))/64 - (3 \cdot \log(x + 1))/64 + (\operatorname{acoth}(x))/4 - (5 \cdot x)/32 + (3 \cdot x^3)/32$   
 $)/(x^2 - 1)^2$

**sympy [B]** time = 0.91, size = 88, normalized size = 1.76

$$-\frac{3x^4 \operatorname{acoth}(x)}{32x^4 - 64x^2 + 32} + \frac{3x^3}{32x^4 - 64x^2 + 32} + \frac{6x^2 \operatorname{acoth}(x)}{32x^4 - 64x^2 + 32} - \frac{5x}{32x^4 - 64x^2 + 32} + \frac{5 \operatorname{acoth}(x)}{32x^4 - 64x^2 + 32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(x)/(-x**2+1)**3,x)`

[Out]  $-3x^4 \operatorname{acoth}(x)/(32x^4 - 64x^2 + 32) + 3x^3/(32x^4 - 64x^2 + 32)$   
 $+ 6x^2 \operatorname{acoth}(x)/(32x^4 - 64x^2 + 32) - 5x/(32x^4 - 64x^2 + 32)$   
 $+ 5 \operatorname{acoth}(x)/(32x^4 - 64x^2 + 32)$

$$3.61 \quad \int \frac{\coth^{-1}(x)}{(1-x^2)^3} dx$$

Optimal. Leaf size=67

$$-\frac{3}{16(1-x^2)} - \frac{1}{16(1-x^2)^2} + \frac{3x \coth^{-1}(x)}{8(1-x^2)} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3}{16} \coth^{-1}(x)^2$$

[Out]  $-1/16/(-x^2+1)^2-3/16/(-x^2+1)+1/4*x*\operatorname{arccoth}(x)/(-x^2+1)^2+3/8*x*\operatorname{arccoth}(x)/(-x^2+1)+3/16*\operatorname{arccoth}(x)^2$

**Rubi [A]** time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {5961, 5957, 261}

$$-\frac{3}{16(1-x^2)} - \frac{1}{16(1-x^2)^2} + \frac{3x \coth^{-1}(x)}{8(1-x^2)} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3}{16} \coth^{-1}(x)^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[x]/(1 - x^2)^3,x]

[Out]  $-1/(16*(1-x^2)^2) - 3/(16*(1-x^2)) + (x*\operatorname{ArcCoth}[x])/(4*(1-x^2)^2) + (3*x*\operatorname{ArcCoth}[x])/(8*(1-x^2)) + (3*\operatorname{ArcCoth}[x]^2)/16$

#### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^p\_., x\_Symbol] :> Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

#### Rule 5957

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^p\_/((d\_) + (e\_.)\*(x\_)^2)^2, x\_Symbol] :> Simp[(x\*(a + b\*ArcCoth[c\*x])^p)/(2\*d\*(d + e\*x^2)), x] + (-Dist[(b\*c\*p)/2, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(d + e\*x^2)^2, x], x] + Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(2\*b\*c\*d^2\*(p + 1)), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

#### Rule 5961

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^2)^(q\_.), x\_Symbol] :> -Simp[(b\*(d + e\*x^2)^(q + 1))/(4\*c\*d\*(q + 1)^2), x] + (Dist[(2\*q + 3)/(2\*d\*(q + 1)), Int[(d + e\*x^2)^(q + 1)\*(a + b\*ArcCoth[c\*x]), x], x] - Simp

$[(x*(d + e*x^2)^(q + 1)*(a + b*ArcCoth[c*x]))/(2*d*(q + 1)), x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& EqQ[c^2*d + e, 0] \&\& LtQ[q, -1] \&\& NeQ[q, -3/2]$

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(x)}{(1-x^2)^3} dx &= -\frac{1}{16(1-x^2)^2} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3}{4} \int \frac{\coth^{-1}(x)}{(1-x^2)^2} dx \\ &= -\frac{1}{16(1-x^2)^2} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3x \coth^{-1}(x)}{8(1-x^2)} + \frac{3}{16} \coth^{-1}(x)^2 - \frac{3}{8} \int \frac{x}{(1-x^2)^2} dx \\ &= -\frac{1}{16(1-x^2)^2} - \frac{3}{16(1-x^2)} + \frac{x \coth^{-1}(x)}{4(1-x^2)^2} + \frac{3x \coth^{-1}(x)}{8(1-x^2)} + \frac{3}{16} \coth^{-1}(x)^2 \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 43, normalized size = 0.64

$$-\frac{-3x^2 + 2(3x^2 - 5)x \coth^{-1}(x) - 3(x^2 - 1)^2 \coth^{-1}(x)^2 + 4}{16(x^2 - 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x]/(1 - x^2)^3,x]

[Out] -1/16\*(4 - 3\*x^2 + 2\*x\*(-5 + 3\*x^2)\*ArcCoth[x] - 3\*(-1 + x^2)^2\*ArcCoth[x]^2)/(-1 + x^2)^2

**fricas [A]** time = 0.40, size = 66, normalized size = 0.99

$$\frac{3(x^4 - 2x^2 + 1) \log\left(\frac{x+1}{x-1}\right)^2 + 12x^2 - 4(3x^3 - 5x) \log\left(\frac{x+1}{x-1}\right) - 16}{64(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="fricas")

[Out] 1/64\*(3\*(x^4 - 2\*x^2 + 1)\*log((x + 1)/(x - 1))^2 + 12\*x^2 - 4\*(3\*x^3 - 5\*x)\*log((x + 1)/(x - 1)) - 16)/(x^4 - 2\*x^2 + 1)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arccoth}(x)}{(x^2 - 1)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="giac")

[Out] integrate(-arccoth(x)/(x^2 - 1)^3, x)

**maple [B]** time = 0.07, size = 131, normalized size = 1.96

$$\frac{\operatorname{arccoth}(x)}{16(-1+x)^2} - \frac{3 \operatorname{arccoth}(x)}{16(-1+x)} - \frac{3 \operatorname{arccoth}(x) \ln(-1+x)}{16} - \frac{\operatorname{arccoth}(x)}{16(1+x)^2} - \frac{3 \operatorname{arccoth}(x)}{16(1+x)} + \frac{3 \operatorname{arccoth}(x) \ln(1+x)}{16} - \frac{3 \ln}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x)/(-x^2+1)^3,x)

[Out] 1/16\*arccoth(x)/(-1+x)^2-3/16\*arccoth(x)/(-1+x)-3/16\*arccoth(x)\*ln(-1+x)-1/16\*arccoth(x)/(1+x)^2-3/16\*arccoth(x)/(1+x)+3/16\*arccoth(x)\*ln(1+x)-3/64\*ln(-1+x)^2+3/32\*ln(-1+x)\*ln(1/2+1/2\*x)-3/64\*ln(1+x)^2+3/32\*(ln(1+x)-ln(1/2+1/2\*x))\*ln(-1/2\*x+1/2)-1/64/(-1+x)^2+7/64/(-1+x)-1/64/(1+x)^2-7/64/(1+x)

**maxima [B]** time = 0.31, size = 118, normalized size = 1.76

$$-\frac{1}{16} \left( \frac{2(3x^3 - 5x)}{x^4 - 2x^2 + 1} - 3 \log(x + 1) + 3 \log(x - 1) \right) \operatorname{arccoth}(x) - \frac{3(x^4 - 2x^2 + 1) \log(x + 1)^2 - 6(x^4 - 2x^2 + 1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x)/(-x^2+1)^3,x, algorithm="maxima")

[Out] -1/16\*(2\*(3\*x^3 - 5\*x)/(x^4 - 2\*x^2 + 1) - 3\*log(x + 1) + 3\*log(x - 1))\*arccoth(x) - 1/64\*(3\*(x^4 - 2\*x^2 + 1)\*log(x + 1)^2 - 6\*(x^4 - 2\*x^2 + 1)\*log(x + 1)\*log(x - 1) + 3\*(x^4 - 2\*x^2 + 1)\*log(x - 1)^2 - 12\*x^2 + 16)/(x^4 - 2\*x^2 + 1)

**mupad [B]** time = 1.32, size = 112, normalized size = 1.67

$$\frac{3 \ln\left(\frac{1}{x} + 1\right)^2}{64} - \ln\left(1 - \frac{1}{x}\right) \left( \frac{3 \ln\left(\frac{1}{x} + 1\right)}{32} + \frac{\frac{5x}{16} - \frac{3x^3}{16}}{x^4 - 2x^2 + 1} \right) + \frac{3 \ln\left(1 - \frac{1}{x}\right)^2}{64} + \frac{\frac{3x^2}{16} - \frac{1}{4}}{x^4 - 2x^2 + 1} + \frac{\ln\left(\frac{1}{x} + 1\right) \left(\frac{5x}{16} - \frac{3x^3}{16}\right)}{x^4 - 2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-acoth(x)/(x^2 - 1)^3,x)

[Out]  $(3 \cdot \log(1/x + 1)^2)/64 - \log(1 - 1/x) \cdot ((3 \cdot \log(1/x + 1))/32 + ((5x)/16 - (3x^3)/16)/(x^4 - 2x^2 + 1)) + (3 \cdot \log(1 - 1/x)^2)/64 + ((3x^2)/16 - 1/4)/(x^4 - 2x^2 + 1) + (\log(1/x + 1) \cdot ((5x)/16 - (3x^3)/16))/(x^4 - 2x^2 + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{acoth}(x)}{x^6 - 3x^4 + 3x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x)/(-x\*\*2+1)\*\*3,x)

[Out] -Integral(acoth(x)/(x\*\*6 - 3\*x\*\*4 + 3\*x\*\*2 - 1), x)



### 3.62 $\int x^3 \coth^{-1}(a + bx) dx$

**Optimal.** Leaf size=101

$$\frac{(6a^2 + 1)x}{4b^3} + \frac{(a + bx)^3}{12b^4} - \frac{a(a + bx)^2}{2b^4} + \frac{(1 - a)^4 \log(-a - bx + 1)}{8b^4} - \frac{(a + 1)^4 \log(a + bx + 1)}{8b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx)$$

[Out]  $1/4*(6*a^2+1)*x/b^3-1/2*a*(b*x+a)^2/b^4+1/12*(b*x+a)^3/b^4+1/4*x^4*\operatorname{arccoth}(b*x+a)+1/8*(1-a)^4*\ln(-b*x-a+1)/b^4-1/8*(1+a)^4*\ln(b*x+a+1)/b^4$

**Rubi [A]** time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(6a^2 + 1)x}{4b^3} + \frac{(a + bx)^3}{12b^4} - \frac{a(a + bx)^2}{2b^4} + \frac{(1 - a)^4 \log(-a - bx + 1)}{8b^4} - \frac{(a + 1)^4 \log(a + bx + 1)}{8b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*\operatorname{ArcCoth}[a + b*x], x]$

[Out]  $((1 + 6*a^2)*x)/(4*b^3) - (a*(a + b*x)^2)/(2*b^4) + (a + b*x)^3/(12*b^4) + (x^4*\operatorname{ArcCoth}[a + b*x])/4 + ((1 - a)^4*\operatorname{Log}[1 - a - b*x])/(8*b^4) - ((1 + a)^4*\operatorname{Log}[1 + a + b*x])/(8*b^4)$

#### Rule 31

$\operatorname{Int}[(a + (b \cdot x)^{-1}), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

#### Rule 633

$\operatorname{Int}[(d + (e \cdot x))/(a + (c \cdot x)^2), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[-(a \cdot c), 2]\}, \operatorname{Dist}[e/2 + (c \cdot d)/(2 \cdot q), \operatorname{Int}[1/(-q + c \cdot x), x], x] + \operatorname{Dist}[e/2 - (c \cdot d)/(2 \cdot q), \operatorname{Int}[1/(q + c \cdot x), x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \operatorname{NiceSqrtQ}[-(a \cdot c)]$

#### Rule 702

$\operatorname{Int}[(d + (e \cdot x))^m/(a + (c \cdot x)^2), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(d + e \cdot x)^m, a + c \cdot x^2, x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \operatorname{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \operatorname{IGtQ}[m, 1] \ \&\& (\operatorname{NeQ}[d, 0] \ || \ \operatorname{GtQ}[m, 2])$

#### Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned} \int x^3 \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^3 \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{1}{4}x^4 \coth^{-1}(a + bx) - \frac{1}{4} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^4}{1 - x^2} dx, x, a + bx\right) \\ &= \frac{1}{4}x^4 \coth^{-1}(a + bx) - \frac{1}{4} \text{Subst}\left(\int \left(-\frac{1 + 6a^2}{b^4} + \frac{4ax}{b^4} - \frac{x^2}{b^4} + \frac{1 + 6a^2 + a^4 - 4a(1 + a^2)}{b^4(1 - x^2)}\right) dx, x, a + bx\right) \\ &= \frac{(1 + 6a^2)x}{4b^3} - \frac{a(a + bx)^2}{2b^4} + \frac{(a + bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx) - \frac{\text{Subst}\left(\int \frac{1 + 6a^2 + a^4 - 4a(1 + a^2)}{1 - x^2} dx, x, a + bx\right)}{4b^4} \\ &= \frac{(1 + 6a^2)x}{4b^3} - \frac{a(a + bx)^2}{2b^4} + \frac{(a + bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx) - \frac{(1 - a)^4 \text{Subst}\left(\int \frac{1}{1 - x} dx, x, a + bx\right)}{8b^4} \\ &= \frac{(1 + 6a^2)x}{4b^3} - \frac{a(a + bx)^2}{2b^4} + \frac{(a + bx)^3}{12b^4} + \frac{1}{4}x^4 \coth^{-1}(a + bx) + \frac{(1 - a)^4 \log(1 - a - bx)}{8b^4} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 81, normalized size = 0.80

$$\frac{6(3a^2 + 1)bx + 6b^4x^4 \coth^{-1}(a + bx) - 6ab^2x^2 + 3(a - 1)^4 \log(-a - bx + 1) - 3(a + 1)^4 \log(a + bx + 1) + 2b^3x^3}{24b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCoth[a + b*x], x]
```

[Out]  $(6*(1 + 3*a^2)*b*x - 6*a*b^2*x^2 + 2*b^3*x^3 + 6*b^4*x^4*ArcCoth[a + b*x] + 3*(-1 + a)^4*Log[1 - a - b*x] - 3*(1 + a)^4*Log[1 + a + b*x])/(24*b^4)$

**fricas** [A] time = 0.50, size = 112, normalized size = 1.11

$$\frac{3b^4x^4 \log\left(\frac{bx+a+1}{bx+a-1}\right) + 2b^3x^3 - 6ab^2x^2 + 6(3a^2+1)bx - 3(a^4+4a^3+6a^2+4a+1)\log(bx+a+1) + 3(a^4 - 3a^3 + 3a^2 - 3a + 1)\log(bx+a-1)}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(b*x+a),x, algorithm="fricas")`

[Out]  $1/24*(3*b^4*x^4*\log((b*x + a + 1)/(b*x + a - 1)) + 2*b^3*x^3 - 6*a*b^2*x^2 + 6*(3*a^2 + 1)*b*x - 3*(a^4 + 4*a^3 + 6*a^2 + 4*a + 1)*\log(b*x + a + 1) + 3*(a^4 - 4*a^3 + 6*a^2 - 4*a + 1)*\log(b*x + a - 1))/b^4$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(b*x+a),x, algorithm="giac")`

[Out] `integrate(x^3*arccoth(b*x + a), x)`

**maple** [B] time = 0.04, size = 199, normalized size = 1.97

$$\frac{a}{4b^4} + \frac{x}{4b^3} + \frac{x^4 \operatorname{arccoth}(bx + a)}{4} + \frac{\ln(bx + a - 1)}{8b^4} - \frac{\ln(bx + a + 1)}{8b^4} - \frac{\ln(bx + a + 1)a^4}{8b^4} - \frac{\ln(bx + a + 1)a^3}{2b^4} - \frac{3 \ln(bx + a + 1)a^2}{2b^4} - \frac{3 \ln(bx + a + 1)a}{2b^4} - \frac{3 \ln(bx + a + 1)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arccoth(b*x+a),x)`

[Out]  $1/4/b^4*a+1/4*x/b^3+1/4*x^4*arccoth(b*x+a)+1/8/b^4*\ln(b*x+a-1)-1/8/b^4*\ln(b*x+a+1)-1/8/b^4*\ln(b*x+a+1)*a^4-1/2/b^4*\ln(b*x+a+1)*a^3-3/4/b^4*\ln(b*x+a+1)*a^2-1/2/b^4*\ln(b*x+a+1)*a+1/12*x^3/b-1/4/b^2*x^2*a+3/4/b^3*x*a^2+13/12/b^4*a^3+1/8/b^4*\ln(b*x+a-1)*a^4-1/2/b^4*\ln(b*x+a-1)*a^3+3/4/b^4*\ln(b*x+a-1)*a^2-1/2/b^4*\ln(b*x+a-1)*a$

**maxima** [A] time = 0.31, size = 106, normalized size = 1.05

$$\frac{1}{4}x^4 \operatorname{arccoth}(bx + a) + \frac{1}{24}b \left( \frac{2(b^2x^3 - 3abx^2 + 3(3a^2 + 1)x)}{b^4} - \frac{3(a^4 + 4a^3 + 6a^2 + 4a + 1)\log(bx + a + 1)}{b^5} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{4}x^4 \operatorname{arccoth}(bx+a) + \frac{1}{24}b(2(b^2x^3 - 3abx^2 + 3(3a^2 + 1)x)/b^4 - 3(a^4 + 4a^3 + 6a^2 + 4a + 1)\log(bx+a+1)/b^5 + 3(a^4 - 4a^3 + 6a^2 - 4a + 1)\log(bx+a-1)/b^5)$

mupad [B] time = 1.47, size = 134, normalized size = 1.33

$$\frac{x^4 \ln\left(\frac{1}{a+bx} + 1\right)}{8} - x \left( \frac{4a^2 - 4}{16b^3} - \frac{a^2}{b^3} \right) - \frac{x^4 \ln\left(1 - \frac{1}{a+bx}\right)}{8} + \frac{x^3}{12b} - \frac{ax^2}{4b^2} + \frac{\ln(a+bx-1)(a^4 - 4a^3 + 6a^2 - 4a + 1)}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*acoth(a + b\*x),x)

[Out]  $(x^4 \log(1/(a+bx) + 1))/8 - x((4a^2 - 4)/(16b^3) - a^2/b^3) - (x^4 \log(1 - 1/(a+bx)))/8 + x^3/(12b) - (ax^2)/(4b^2) + (\log(a+bx-1)(6a^2 - 4a - 4a^3 + a^4 + 1))/(8b^4) - (\log(a+bx+1)(4a + 6a^2 + 4a^3 + a^4 + 1))/(8b^4)$

sympy [A] time = 1.84, size = 153, normalized size = 1.51

$$\left\{ \begin{array}{l} -\frac{a^4 \operatorname{acoth}(a+bx)}{4b^4} - \frac{a^3 \log(a+bx+1)}{b^4} + \frac{a^3 \operatorname{acoth}(a+bx)}{b^4} + \frac{3a^2x}{4b^3} - \frac{3a^2 \operatorname{acoth}(a+bx)}{2b^4} - \frac{ax^2}{4b^2} - \frac{a \log(a+bx+1)}{b^4} + \frac{a \operatorname{acoth}(a+bx)}{b^4} + \frac{x^4 \operatorname{acoth}(a)}{4} \\ \frac{x^4 \operatorname{acoth}(a)}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*acoth(b\*x+a),x)

[Out] Piecewise((-a\*\*4\*acoth(a + b\*x)/(4\*b\*\*4) - a\*\*3\*log(a + b\*x + 1)/b\*\*4 + a\*\*3\*acoth(a + b\*x)/b\*\*4 + 3\*a\*\*2\*x/(4\*b\*\*3) - 3\*a\*\*2\*acoth(a + b\*x)/(2\*b\*\*4) - a\*x\*\*2/(4\*b\*\*2) - a\*log(a + b\*x + 1)/b\*\*4 + a\*acoth(a + b\*x)/b\*\*4 + x\*\*4\*acoth(a + b\*x)/4 + x\*\*3/(12\*b) + x/(4\*b\*\*3) - acoth(a + b\*x)/(4\*b\*\*4), Ne(b, 0)), (x\*\*4\*acoth(a)/4, True))

### 3.63 $\int x^2 \coth^{-1}(a + bx) dx$

**Optimal.** Leaf size=78

$$\frac{(a + bx)^2}{6b^3} + \frac{(1 - a)^3 \log(-a - bx + 1)}{6b^3} + \frac{(a + 1)^3 \log(a + bx + 1)}{6b^3} - \frac{ax}{b^2} + \frac{1}{3}x^3 \coth^{-1}(a + bx)$$

[Out]  $-a*x/b^2 + 1/6*(b*x+a)^2/b^3 + 1/3*x^3*\operatorname{arccoth}(b*x+a) + 1/6*(1-a)^3*\ln(-b*x-a+1)/b^3 + 1/6*(1+a)^3*\ln(b*x+a+1)/b^3$

**Rubi [A]** time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(a + bx)^2}{6b^3} - \frac{ax}{b^2} + \frac{(1 - a)^3 \log(-a - bx + 1)}{6b^3} + \frac{(a + 1)^3 \log(a + bx + 1)}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{ArcCoth}[a + b*x], x]$

[Out]  $-((a*x)/b^2) + (a + b*x)^2/(6*b^3) + (x^3*\operatorname{ArcCoth}[a + b*x])/3 + ((1 - a)^3*\operatorname{Log}[1 - a - b*x])/(6*b^3) + ((1 + a)^3*\operatorname{Log}[1 + a + b*x])/(6*b^3)$

#### Rule 31

$\operatorname{Int}(((a_) + (b_)*(x_))^{-1}, x\_Symbol) \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

#### Rule 633

$\operatorname{Int}(((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x\_Symbol) \rightarrow \operatorname{With}\{q = \operatorname{Rt}[-(a*c), 2]\}, \operatorname{Dist}[e/2 + (c*d)/(2*q), \operatorname{Int}[1/(-q + c*x), x], x] + \operatorname{Dist}[e/2 - (c*d)/(2*q), \operatorname{Int}[1/(q + c*x), x], x]] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \operatorname{NiceSqrtQ}[-(a*c)]$

#### Rule 702

$\operatorname{Int}(((d_) + (e_)*(x_))^{(m_)}/((a_) + (c_)*(x_)^2), x\_Symbol) \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(d + e*x)^m, a + c*x^2, x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \operatorname{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \operatorname{IGtQ}[m, 1] \ \&\& (\operatorname{NeQ}[d, 0] \ || \ \operatorname{GtQ}[m, 2])$

#### Rule 5927

$\operatorname{Int}(((a_) + \operatorname{ArcCoth}[(c_)*(x_)])*(b_))*((d_) + (e_)*(x_))^{(q_)}, x\_Symbol) \rightarrow \operatorname{Simp}(((d + e*x)^{(q + 1)}*(a + b*\operatorname{ArcCoth}[c*x]))/(e*(q + 1)), x) - \operatorname{Dist}[($

$b*c)/(e*(q + 1)), \text{Int}[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, q\}, x] \&\& \text{NeQ}[q, -1]$

### Rule 6112

$\text{Int}[(a_.) + \text{ArcCoth}[c_.) + (d_.)*(x_.)]*(b_.)^(p_.)*((e_.) + (f_.)*(x_.)^(m_.), x\_Symbol] :> \text{Dist}[1/d, \text{Subst}[\text{Int}[(d*e - c*f)/d + (f*x)/d]^m*(a + b*\text{ArcCoth}[x])^p, x], x, c + d*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{IGTQ}[p, 0]$

### Rubi steps

$$\begin{aligned} \int x^2 \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^2 \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{1}{3}x^3 \coth^{-1}(a + bx) - \frac{1}{3} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^3}{1 - x^2} dx, x, a + bx\right) \\ &= \frac{1}{3}x^3 \coth^{-1}(a + bx) - \frac{1}{3} \text{Subst}\left(\int \left(\frac{3a}{b^3} - \frac{x}{b^3} - \frac{a(3 + a^2) - (1 + 3a^2)x}{b^3(1 - x^2)}\right) dx, x, a + bx\right) \\ &= -\frac{ax}{b^2} + \frac{(a + bx)^2}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx) + \frac{\text{Subst}\left(\int \frac{a(3+a^2) - (1+3a^2)x}{1-x^2} dx, x, a + bx\right)}{3b^3} \\ &= -\frac{ax}{b^2} + \frac{(a + bx)^2}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx) - \frac{(1 - a)^3 \text{Subst}\left(\int \frac{1}{1-x} dx, x, a + bx\right)}{6b^3} - \frac{(1 + a)}{6b^3} \\ &= -\frac{ax}{b^2} + \frac{(a + bx)^2}{6b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx) + \frac{(1 - a)^3 \log(1 - a - bx)}{6b^3} + \frac{(1 + a)^3 \log(1 + a + bx)}{6b^3} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 92, normalized size = 1.18

$$\frac{(-a^3 + 3a^2 - 3a + 1) \log(-a - bx + 1)}{6b^3} + \frac{(a^3 + 3a^2 + 3a + 1) \log(a + bx + 1)}{6b^3} - \frac{2ax}{3b^2} + \frac{1}{3}x^3 \coth^{-1}(a + bx) + \frac{x^2}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[a + b\*x], x]

[Out]  $(-2*a*x)/(3*b^2) + x^2/(6*b) + (x^3*ArcCoth[a + b*x])/3 + ((1 - 3*a + 3*a^2 - a^3)*Log[1 - a - b*x])/(6*b^3) + ((1 + 3*a + 3*a^2 + a^3)*Log[1 + a + b*x])/(6*b^3)$

**fricas** [A] time = 0.56, size = 84, normalized size = 1.08

$$\frac{b^3 x^3 \log\left(\frac{bx+a+1}{bx+a-1}\right) + b^2 x^2 - 4abx + (a^3 + 3a^2 + 3a + 1) \log(bx + a + 1) - (a^3 - 3a^2 + 3a - 1) \log(bx + a - 1)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(b\*x+a),x, algorithm="fricas")

[Out] 1/6\*(b^3\*x^3\*log((b\*x + a + 1)/(b\*x + a - 1)) + b^2\*x^2 - 4\*a\*b\*x + (a^3 + 3\*a^2 + 3\*a + 1)\*log(b\*x + a + 1) - (a^3 - 3\*a^2 + 3\*a - 1)\*log(b\*x + a - 1))/b^3

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(b\*x+a),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(b\*x + a), x)

**maple** [B] time = 0.04, size = 146, normalized size = 1.87

$$\frac{x^3 \operatorname{arccoth}(bx + a)}{3} + \frac{x^2}{6b} - \frac{2ax}{3b^2} - \frac{5a^2}{6b^3} - \frac{\ln(bx + a - 1) a^3}{6b^3} + \frac{\ln(bx + a - 1) a^2}{2b^3} - \frac{\ln(bx + a - 1) a}{2b^3} + \frac{\ln(bx + a - 1)}{6b^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(b\*x+a),x)

[Out] 1/3\*x^3\*arccoth(b\*x+a)+1/6\*x^2/b-2/3\*a\*x/b^2-5/6/b^3\*a^2-1/6/b^3\*ln(b\*x+a-1)\*a^3+1/2/b^3\*ln(b\*x+a-1)\*a^2-1/2/b^3\*ln(b\*x+a-1)\*a+1/6/b^3\*ln(b\*x+a-1)+1/6/b^3\*ln(b\*x+a+1)\*a^3+1/2/b^3\*ln(b\*x+a+1)\*a^2+1/2/b^3\*ln(b\*x+a+1)\*a+1/6/b^3\*ln(b\*x+a+1)

**maxima** [A] time = 0.31, size = 79, normalized size = 1.01

$$\frac{1}{3} x^3 \operatorname{arccoth}(bx + a) + \frac{1}{6} b \left( \frac{bx^2 - 4ax}{b^3} + \frac{(a^3 + 3a^2 + 3a + 1) \log(bx + a + 1)}{b^4} - \frac{(a^3 - 3a^2 + 3a - 1) \log(bx + a - 1)}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{3}x^3 \operatorname{arccoth}(bx + a) + \frac{1}{6}b \left( \frac{bx^2 - 4ax}{b^3} + \frac{a^3 + 3a^2 + 3a + 1}{b^4} \log(bx + a + 1) - \frac{a^3 - 3a^2 + 3a - 1}{b^4} \log(bx + a - 1) \right)$

**mupad [B]** time = 1.36, size = 98, normalized size = 1.26

$$\frac{x^3 \ln\left(\frac{1}{a+bx} + 1\right)}{6} - \frac{x^3 \ln\left(1 - \frac{1}{a+bx}\right)}{6} + \frac{x^2}{6b} - \frac{\ln(a+bx-1)(a^3 - 3a^2 + 3a - 1)}{6b^3} + \frac{\ln(a+bx+1)(a^3 + 3a^2 + 3a + 1)}{6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(a + b*x),x)`

[Out]  $(x^3 \log(1/(a + bx) + 1))/6 - (x^3 \log(1 - 1/(a + bx)))/6 + x^2/(6b) - (\log(a + bx - 1)(3a - 3a^2 + a^3 - 1))/(6b^3) + (\log(a + bx + 1)(3a + 3a^2 + a^3 + 1))/(6b^3) - (2ax)/(3b^2)$

**sympy [A]** time = 1.27, size = 117, normalized size = 1.50

$$\left\{ \begin{array}{l} \frac{a^3 \operatorname{acoth}(a+bx)}{3b^3} + \frac{a^2 \log(a+bx+1)}{b^3} - \frac{a^2 \operatorname{acoth}(a+bx)}{b^3} - \frac{2ax}{3b^2} + \frac{a \operatorname{acoth}(a+bx)}{b^3} + \frac{x^3 \operatorname{acoth}(a+bx)}{3} + \frac{x^2}{6b} + \frac{\log(a+bx+1)}{3b^3} - \frac{\operatorname{acoth}(a+bx)}{3b^3} \\ \frac{x^3 \operatorname{acoth}(a)}{3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(b*x+a),x)`

[Out] `Piecewise((a**3*acoth(a + b*x)/(3*b**3) + a**2*log(a + b*x + 1)/b**3 - a**2*acoth(a + b*x)/b**3 - 2*a*x/(3*b**2) + a*acoth(a + b*x)/b**3 + x**3*acoth(a + b*x)/3 + x**2/(6*b) + log(a + b*x + 1)/(3*b**3) - acoth(a + b*x)/(3*b**3), Ne(b, 0)), (x**3*acoth(a)/3, True))`



### 3.64 $\int x \coth^{-1}(a + bx) dx$

Optimal. Leaf size=65

$$\frac{(1-a)^2 \log(-a-bx+1)}{4b^2} - \frac{(a+1)^2 \log(a+bx+1)}{4b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx) + \frac{x}{2b}$$

[Out]  $1/2*x/b+1/2*x^2*\operatorname{arccoth}(b*x+a)+1/4*(1-a)^2*\ln(-b*x-a+1)/b^2-1/4*(1+a)^2*\ln(b*x+a+1)/b^2$

**Rubi** [A] time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(1-a)^2 \log(-a-bx+1)}{4b^2} - \frac{(a+1)^2 \log(a+bx+1)}{4b^2} + \frac{1}{2}x^2 \coth^{-1}(a+bx) + \frac{x}{2b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{ArcCoth}[a + b*x], x]$

[Out]  $x/(2*b) + (x^2*\operatorname{ArcCoth}[a + b*x])/2 + ((1 - a)^2*\operatorname{Log}[1 - a - b*x])/(4*b^2) - ((1 + a)^2*\operatorname{Log}[1 + a + b*x])/(4*b^2)$

#### Rule 31

$\operatorname{Int}[(a_ + (b_)*(x_))^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

#### Rule 633

$\operatorname{Int}[(d_ + (e_)*(x_))/((a_ + (c_)*(x_)^2), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[-(a*c), 2]\}, \operatorname{Dist}[e/2 + (c*d)/(2*q), \operatorname{Int}[1/(-q + c*x), x], x] + \operatorname{Dist}[e/2 - (c*d)/(2*q), \operatorname{Int}[1/(q + c*x), x], x]] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \operatorname{NiceSqrtQ}[-(a*c)]$

#### Rule 702

$\operatorname{Int}[(d_ + (e_)*(x_))^{(m_)}((a_ + (c_)*(x_)^2), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(d + e*x)^m, a + c*x^2, x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \operatorname{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \operatorname{IGtQ}[m, 1] \ \&\& (\operatorname{NeQ}[d, 0] \ || \ \operatorname{GtQ}[m, 2])$

#### Rule 5927

$\operatorname{Int}[(a_ + \operatorname{ArcCoth}[c_*(x_)]*(b_))*((d_ + (e_)*(x_))^{(q_)}), x\_Symbol] \rightarrow \operatorname{Simp}[(d + e*x)^{(q+1)}*(a + b*\operatorname{ArcCoth}[c*x])]/(e*(q+1)), x] - \operatorname{Dist}[($

$b*c)/(e*(q + 1)), \text{Int}[(d + e*x)^{(q + 1)}/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, q\}, x\} \&\& \text{NeQ}[q, -1]$

### Rule 6112

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.) + (d_.)*(x_.)]*(b_.)]^{(p_.)}*((e_.) + (f_.)*(x_.)^{(m_.)}, x\_Symbol] :> \text{Dist}[1/d, \text{Subst}[\text{Int}[(d*e - c*f)/d + (f*x)/d]^{m*(a + b*\text{ArcCoth}[x])^p, x], x, c + d*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \text{IGtQ}[p, 0]$

### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right) \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{1}{2}x^2 \coth^{-1}(a + bx) - \frac{1}{2} \text{Subst}\left(\int \frac{\left(-\frac{a}{b} + \frac{x}{b}\right)^2}{1 - x^2} dx, x, a + bx\right) \\ &= \frac{1}{2}x^2 \coth^{-1}(a + bx) - \frac{1}{2} \text{Subst}\left(\int \left(-\frac{1}{b^2} + \frac{1 + a^2 - 2ax}{b^2(1 - x^2)}\right) dx, x, a + bx\right) \\ &= \frac{x}{2b} + \frac{1}{2}x^2 \coth^{-1}(a + bx) - \frac{\text{Subst}\left(\int \frac{1 + a^2 - 2ax}{1 - x^2} dx, x, a + bx\right)}{2b^2} \\ &= \frac{x}{2b} + \frac{1}{2}x^2 \coth^{-1}(a + bx) - \frac{(1 - a)^2 \text{Subst}\left(\int \frac{1}{1 - x} dx, x, a + bx\right)}{4b^2} + \frac{(1 + a)^2 \text{Subst}\left(\int \frac{1}{-1 - x} dx, x, a + bx\right)}{4b^2} \\ &= \frac{x}{2b} + \frac{1}{2}x^2 \coth^{-1}(a + bx) + \frac{(1 - a)^2 \log(1 - a - bx)}{4b^2} - \frac{(1 + a)^2 \log(1 + a + bx)}{4b^2} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 56, normalized size = 0.86

$$\frac{2b^2x^2 \coth^{-1}(a + bx) + (a - 1)^2 \log(-a - bx + 1) - (a + 1)^2 \log(a + bx + 1) + 2bx}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[a + b\*x], x]

[Out] (2\*b\*x + 2\*b^2\*x^2\*ArcCoth[a + b\*x] + (-1 + a)^2\*Log[1 - a - b\*x] - (1 + a)^2\*Log[1 + a + b\*x])/(4\*b^2)

**fricas** [A] time = 0.84, size = 66, normalized size = 1.02

$$\frac{b^2 x^2 \log\left(\frac{bx+a+1}{bx+a-1}\right) + 2bx - (a^2 + 2a + 1) \log(bx + a + 1) + (a^2 - 2a + 1) \log(bx + a - 1)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(b\*x+a),x, algorithm="fricas")

[Out] 1/4\*(b^2\*x^2\*log((b\*x + a + 1)/(b\*x + a - 1)) + 2\*b\*x - (a^2 + 2\*a + 1)\*log(b\*x + a + 1) + (a^2 - 2\*a + 1)\*log(b\*x + a - 1))/b^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(b\*x+a),x, algorithm="giac")

[Out] integrate(x\*arccoth(b\*x + a), x)

**maple** [A] time = 0.03, size = 89, normalized size = 1.37

$$\frac{x^2 \operatorname{arccoth}(bx + a)}{2} - \frac{\operatorname{arccoth}(bx + a) a^2}{2b^2} + \frac{x}{2b} + \frac{a}{2b^2} + \frac{\ln(bx + a - 1)}{4b^2} - \frac{\ln(bx + a - 1) a}{2b^2} - \frac{\ln(bx + a + 1)}{4b^2} - \frac{\ln(bx + a + 1) a}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(b\*x+a),x)

[Out] 1/2\*x^2\*arccoth(b\*x+a)-1/2/b^2\*arccoth(b\*x+a)\*a^2+1/2\*x/b+1/2/b^2\*a+1/4/b^2\*ln(b\*x+a-1)-1/2/b^2\*ln(b\*x+a-1)\*a-1/4/b^2\*ln(b\*x+a+1)-1/2/b^2\*ln(b\*x+a+1)\*a

**maxima** [A] time = 0.30, size = 61, normalized size = 0.94

$$\frac{1}{2} x^2 \operatorname{arccoth}(bx + a) + \frac{1}{4} b \left( \frac{2x}{b^2} - \frac{(a^2 + 2a + 1) \log(bx + a + 1)}{b^3} + \frac{(a^2 - 2a + 1) \log(bx + a - 1)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(b\*x+a),x, algorithm="maxima")

[Out] 1/2\*x^2\*arccoth(b\*x + a) + 1/4\*b\*(2\*x/b^2 - (a^2 + 2\*a + 1)\*log(b\*x + a + 1)/b^3 + (a^2 - 2\*a + 1)\*log(b\*x + a - 1)/b^3)

**mupad [B]** time = 2.00, size = 62, normalized size = 0.95

$$\frac{x^2 \operatorname{acoth}(a + bx)}{2} - \frac{\frac{\operatorname{acoth}(a+bx)}{2} - \frac{bx}{2} + \frac{a^2 \operatorname{acoth}(a+bx)}{2} + \frac{a \ln(a^2 + 2abx + b^2x^2 - 1)}{2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(a + b*x),x)`

[Out]  $(x^2 \operatorname{acoth}(a + bx))/2 - (\operatorname{acoth}(a + bx)/2 - (bx)/2 + (a^2 \operatorname{acoth}(a + bx))/2 + (a \log(a^2 + b^2x^2 + 2abx - 1))/2)/b^2$

**sympy [A]** time = 0.81, size = 76, normalized size = 1.17

$$\begin{cases} -\frac{a^2 \operatorname{acoth}(a+bx)}{2b^2} - \frac{a \log(a+bx+1)}{b^2} + \frac{a \operatorname{acoth}(a+bx)}{b^2} + \frac{x^2 \operatorname{acoth}(a+bx)}{2} + \frac{x}{2b} - \frac{\operatorname{acoth}(a+bx)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acoth}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(b*x+a),x)`

[Out] `Piecewise((-a**2*acoth(a + b*x)/(2*b**2) - a*log(a + b*x + 1)/b**2 + a*acoth(a + b*x)/b**2 + x**2*acoth(a + b*x)/2 + x/(2*b) - acoth(a + b*x)/(2*b**2), Ne(b, 0)), (x**2*acoth(a)/2, True))`

### 3.65 $\int \coth^{-1}(a + bx) dx$

Optimal. Leaf size=35

$$\frac{\log(1 - (a + bx)^2)}{2b} + \frac{(a + bx) \coth^{-1}(a + bx)}{b}$$

[Out] (b\*x+a)\*arccoth(b\*x+a)/b+1/2\*ln(1-(b\*x+a)^2)/b

**Rubi** [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.500, Rules used = {6104, 5911, 260}

$$\frac{\log(1 - (a + bx)^2)}{2b} + \frac{(a + bx) \coth^{-1}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x], x]

[Out] ((a + b\*x)\*ArcCoth[a + b\*x])/b + Log[1 - (a + b\*x)^2]/(2\*b)

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 5911

Int[((a\_) + ArcCoth[(c\_)\*(x\_)])\*(b\_)^(p\_), x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 6104

Int[((a\_) + ArcCoth[(c\_) + (d\_)\*(x\_)])\*(b\_)^(p\_), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \coth^{-1}(a + bx)}{b} - \frac{\text{Subst}\left(\int \frac{x}{1-x^2} dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx) \coth^{-1}(a + bx)}{b} + \frac{\log(1 - (a + bx)^2)}{2b} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 43, normalized size = 1.23

$$\frac{(a + 1) \log(a + bx + 1) - (a - 1) \log(-a - bx + 1)}{2b} + x \coth^{-1}(a + bx)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*x], x]

[Out] x\*ArcCoth[a + b\*x] + (-((-1 + a)\*Log[1 - a - b\*x]) + (1 + a)\*Log[1 + a + b\*x])/(2\*b)

**fricas** [A] time = 0.67, size = 48, normalized size = 1.37

$$\frac{bx \log\left(\frac{bx+a+1}{bx+a-1}\right) + (a + 1) \log(bx + a + 1) - (a - 1) \log(bx + a - 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a), x, algorithm="fricas")

[Out] 1/2\*(b\*x\*log((b\*x + a + 1)/(b\*x + a - 1)) + (a + 1)\*log(b\*x + a + 1) - (a - 1)\*log(b\*x + a - 1))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a), x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a), x)

**maple** [A] time = 0.03, size = 36, normalized size = 1.03

$$x \operatorname{arccoth}(bx + a) + \frac{\operatorname{arccoth}(bx + a)a}{b} + \frac{\ln((bx + a)^2 - 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a),x)

[Out] x\*arccoth(b\*x+a)+1/b\*arccoth(b\*x+a)\*a+1/2/b\*ln((b\*x+a)^2-1)

**maxima** [A] time = 0.31, size = 31, normalized size = 0.89

$$\frac{2(bx + a) \operatorname{arccoth}(bx + a) + \log(-(bx + a)^2 + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a),x, algorithm="maxima")

[Out] 1/2\*(2\*(b\*x + a)\*arccoth(b\*x + a) + log(-(b\*x + a)^2 + 1))/b

**mupad** [B] time = 1.71, size = 42, normalized size = 1.20

$$\frac{\frac{\ln(a^2 + 2abx + b^2x^2 - 1)}{2} + a \operatorname{acoth}(a + bx)}{b} + x \operatorname{acoth}(a + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x),x)

[Out] (log(a^2 + b^2\*x^2 + 2\*a\*b\*x - 1)/2 + a\*acoth(a + b\*x))/b + x\*acoth(a + b\*x)

**sympy** [A] time = 0.51, size = 41, normalized size = 1.17

$$\begin{cases} \frac{a \operatorname{acoth}(a+bx)}{b} + x \operatorname{acoth}(a + bx) + \frac{\log(a+bx+1)}{b} - \frac{\operatorname{acoth}(a+bx)}{b} & \text{for } b \neq 0 \\ x \operatorname{acoth}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a),x)

[Out] Piecewise((a\*acoth(a + b\*x)/b + x\*acoth(a + b\*x) + log(a + b\*x + 1)/b - acoth(a + b\*x)/b, Ne(b, 0)), (x\*acoth(a), True))

$$3.66 \quad \int \frac{\coth^{-1}(a+bx)}{x} dx$$

**Optimal.** Leaf size=92

$$\frac{1}{2}\text{Li}_2\left(1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{Li}_2\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \log\left(\frac{2}{a+bx+1}\right)(-\coth^{-1}(a+bx)) + \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)$$

[Out]  $-\text{arccoth}(b*x+a)*\ln(2/(b*x+a+1))+\text{arccoth}(b*x+a)*\ln(2*b*x/(1-a)/(b*x+a+1))+1/2*\text{polylog}(2,1-2/(b*x+a+1))-1/2*\text{polylog}(2,1-2*b*x/(1-a)/(b*x+a+1))$

**Rubi [A]** time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6112, 5921, 2402, 2315, 2447}

$$\frac{1}{2}\text{PolyLog}\left(2,1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{PolyLog}\left(2,1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \log\left(\frac{2}{a+bx+1}\right)(-\coth^{-1}(a+bx)) + \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/x,x]

[Out]  $-(\text{ArcCoth}[a + b*x]*\text{Log}[2/(1 + a + b*x)]) + \text{ArcCoth}[a + b*x]*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))] + \text{PolyLog}[2, 1 - 2/(1 + a + b*x)]/2 - \text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))]/2$

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 2447

Int[Log[u\_]\*(Pq\_)^(m\_.), x\_Symbol] :> With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

#### Rule 5921



```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^p_.*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

### Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{x} dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-\frac{a}{b} + \frac{x}{b}} dx, x, a+bx\right)}{b}$$

$$= -\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx) \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) + \text{Subst}\left(\int \frac{1}{x} dx, x, a+bx\right)$$

$$= -\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx) \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) - \frac{1}{2} \text{Li}_2\left(\frac{2bx}{(1-a)(1+a+bx)}\right)$$

$$= -\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx) \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) + \frac{1}{2} \text{Li}_2\left(\frac{2bx}{(1-a)(1+a+bx)}\right)$$

**Mathematica [C]** time = 0.17, size = 259, normalized size = 2.82

$$\frac{1}{8} \left( -4\text{Li}_2\left(e^{2 \tanh^{-1}(a)} - 2 \tanh^{-1}(a+bx)\right) - 4\text{Li}_2\left(-e^{2 \tanh^{-1}(a+bx)}\right) + 4\left(\tanh^{-1}(a) - \tanh^{-1}(a+bx)\right)^2 - \left(\pi - 2i \tanh^{-1}\left(\frac{2bx}{(1-a)(1+a+bx)}\right)\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/x, x]

[Out] (ArcCoth[a + b\*x] - ArcTanh[a + b\*x])\*Log[x] + ArcTanh[a + b\*x]\*(-Log[1/Sqrt[1 - (a + b\*x)^2]] + Log[(-I)\*Sinh[ArcTanh[a] - ArcTanh[a + b\*x]]]) + 4\*(

$\text{ArcTanh}[a] - \text{ArcTanh}[a + b*x]^2 - (\text{Pi} - (2*I)*\text{ArcTanh}[a + b*x])^2 - 8*(\text{ArcTanh}[a] - \text{ArcTanh}[a + b*x])* \text{Log}[1 - E^{(2*\text{ArcTanh}[a] - 2*\text{ArcTanh}[a + b*x])}] - (4*I)*(\text{Pi} - (2*I)*\text{ArcTanh}[a + b*x])* \text{Log}[1 + E^{(2*\text{ArcTanh}[a + b*x])}] + 4*(I*\text{Pi} + 2*\text{ArcTanh}[a + b*x])* \text{Log}[2/\text{Sqrt}[1 - (a + b*x)^2]] + 8*(\text{ArcTanh}[a] - \text{ArcTanh}[a + b*x])* \text{Log}[(-2*I)*\text{Sinh}[\text{ArcTanh}[a] - \text{ArcTanh}[a + b*x]]] - 4*\text{PolyLog}[2, E^{(2*\text{ArcTanh}[a] - 2*\text{ArcTanh}[a + b*x])}] - 4*\text{PolyLog}[2, -E^{(2*\text{ArcTanh}[a + b*x])}]]/8$

**fricas** [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(bx + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x,x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(bx + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x,x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/x, x)

**maple** [A] time = 0.06, size = 81, normalized size = 0.88

$$\ln(bx) \text{arccoth}(bx + a) - \frac{\text{dilog}\left(\frac{bx+a+1}{1+a}\right) \ln(bx) \ln\left(\frac{bx+a+1}{1+a}\right)}{2} - \frac{\text{dilog}\left(\frac{bx+a-1}{a-1}\right) \ln(bx) \ln\left(\frac{bx+a-1}{a-1}\right)}{2} + \frac{\text{dilog}\left(\frac{bx+a+1}{1+a}\right)}{2} + \frac{\text{dilog}\left(\frac{bx+a-1}{a-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/x,x)

[Out]  $\ln(b*x)*\text{arccoth}(b*x+a) - 1/2*\text{dilog}((b*x+a+1)/(1+a)) - 1/2*\ln(b*x)*\ln((b*x+a+1)/(1+a)) + 1/2*\text{dilog}((b*x+a-1)/(a-1)) + 1/2*\ln(b*x)*\ln((b*x+a-1)/(a-1))$

**maxima** [A] time = 0.31, size = 128, normalized size = 1.39

$$-\frac{1}{2}b\left(\frac{\log(bx + a + 1)}{b} - \frac{\log(bx + a - 1)}{b}\right)\log(x) + \frac{1}{2}b\left(\frac{\log(bx + a + 1)\log\left(-\frac{bx+a+1}{a+1} + 1\right) + \text{Li}_2\left(\frac{bx+a+1}{a+1}\right)}{b} - \frac{\log(bx + a - 1)\log\left(\frac{bx+a-1}{a-1}\right) + \text{Li}_2\left(\frac{bx+a-1}{a-1}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x,x, algorithm="maxima")

[Out]  $-1/2*b*(\log(b*x + a + 1)/b - \log(b*x + a - 1)/b)*\log(x) + 1/2*b*((\log(b*x + a + 1)*\log(-(b*x + a + 1)/(a + 1) + 1) + \operatorname{dilog}((b*x + a + 1)/(a + 1)))/b - (\log(b*x + a - 1)*\log(-(b*x + a - 1)/(a - 1) + 1) + \operatorname{dilog}((b*x + a - 1)/(a - 1)))/b) + \operatorname{arccoth}(b*x + a)*\log(x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)/x,x)

[Out] int(acoth(a + b\*x)/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(a + bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/x,x)

[Out] Integral(acoth(a + b\*x)/x, x)

$$3.67 \quad \int \frac{\coth^{-1}(a+bx)}{x^2} dx$$

**Optimal.** Leaf size=64

$$\frac{b \log(x)}{1-a^2} - \frac{b \log(-a-bx+1)}{2(1-a)} - \frac{b \log(a+bx+1)}{2(a+1)} - \frac{\coth^{-1}(a+bx)}{x}$$

[Out]  $-\operatorname{arccoth}(b*x+a)/x+b*\ln(x)/(-a^2+1)-1/2*b*\ln(-b*x-a+1)/(1-a)-1/2*b*\ln(b*x+a+1)/(1+a)$

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6110, 371, 706, 31, 633}

$$\frac{b \log(x)}{1-a^2} - \frac{b \log(-a-bx+1)}{2(1-a)} - \frac{b \log(a+bx+1)}{2(a+1)} - \frac{\coth^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/x^2, x]

[Out]  $-(\operatorname{ArcCoth}[a + b*x]/x) + (b*\operatorname{Log}[x])/(1 - a^2) - (b*\operatorname{Log}[1 - a - b*x])/(2*(1 - a)) - (b*\operatorname{Log}[1 + a + b*x])/(2*(1 + a))$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 371

Int[((a\_) + (b\_.)\*(v\_)<sup>(n\_)</sup>)<sup>(p\_)</sup>\*(x\_)<sup>(m\_)</sup>, x\_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d<sup>(m + 1)</sup>, Subst[Int[SimplifyIntegrand[(x - c)<sup>m</sup>\*(a + b\*x<sup>n</sup>)<sup>p</sup>, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

### Rule 633

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)<sup>2</sup>), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a\*c)]

### Rule 706

```
Int[1/(((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

### Rule 6110

```
Int[((a_) + ArcCoth[(c_) + (d_)*(x_)])*(b_)^(p_)*((e_) + (f_)*(x_))^(
m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{x^2} dx &= -\frac{\coth^{-1}(a+bx)}{x} + b \int \frac{1}{x(1-(a+bx)^2)} dx \\
&= -\frac{\coth^{-1}(a+bx)}{x} + b \operatorname{Subst}\left(\int \frac{1}{(-a+x)(1-x^2)} dx, x, a+bx\right) \\
&= -\frac{\coth^{-1}(a+bx)}{x} + \frac{b \operatorname{Subst}\left(\int \frac{1}{-a+x} dx, x, a+bx\right)}{1-a^2} + \frac{b \operatorname{Subst}\left(\int \frac{a+x}{1-x^2} dx, x, a+bx\right)}{1-a^2} \\
&= -\frac{\coth^{-1}(a+bx)}{x} + \frac{b \log(x)}{1-a^2} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-x} dx, x, a+bx\right)}{2(1-a)} + \frac{b \operatorname{Subst}\left(\int \frac{1}{-1-x} dx, x, a+bx\right)}{2(1+a)} \\
&= -\frac{\coth^{-1}(a+bx)}{x} + \frac{b \log(x)}{1-a^2} - \frac{b \log(1-a-bx)}{2(1-a)} - \frac{b \log(1+a+bx)}{2(1+a)}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 55, normalized size = 0.86

$$\frac{b((a+1)\log(-a-bx+1) - (a-1)\log(a+bx+1) - 2\log(x))}{2(a^2-1)} - \frac{\coth^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/x^2, x]
```

```
[Out] -(ArcCoth[a + b*x]/x) + (b*(-2*Log[x] + (1 + a)*Log[1 - a - b*x] - (-1 + a)
*Log[1 + a + b*x]))/(2*(-1 + a^2))
```

**fricas** [A] time = 0.68, size = 68, normalized size = 1.06

$$\frac{(a-1)bx \log(bx+a+1) - (a+1)bx \log(bx+a-1) + 2bx \log(x) + (a^2-1) \log\left(\frac{bx+a+1}{bx+a-1}\right)}{2(a^2-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x^2,x, algorithm="fricas")

[Out] -1/2\*((a-1)\*b\*x\*log(b\*x+a+1) - (a+1)\*b\*x\*log(b\*x+a-1) + 2\*b\*x\*log(x) + (a^2-1)\*log((b\*x+a+1)/(b\*x+a-1)))/((a^2-1)\*x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x^2,x, algorithm="giac")

[Out] integrate(arccoth(b\*x+a)/x^2, x)

**maple** [A] time = 0.04, size = 63, normalized size = 0.98

$$-\frac{\operatorname{arccoth}(bx+a)}{x} + \frac{b \ln(bx+a-1)}{2a-2} - \frac{b \ln(bx+a+1)}{2+2a} - \frac{b \ln(bx)}{(a-1)(1+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/x^2,x)

[Out] -arccoth(b\*x+a)/x+b/(2\*a-2)\*ln(b\*x+a-1)-b/(2+2\*a)\*ln(b\*x+a+1)-b/(a-1)/(1+a)\*ln(b\*x)

**maxima** [A] time = 0.31, size = 54, normalized size = 0.84

$$-\frac{1}{2}b\left(\frac{\log(bx+a+1)}{a+1} - \frac{\log(bx+a-1)}{a-1} + \frac{2 \log(x)}{a^2-1}\right) - \frac{\operatorname{arccoth}(bx+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x^2,x, algorithm="maxima")

[Out] -1/2\*b\*(log(b\*x+a+1)/(a+1) - log(b\*x+a-1)/(a-1) + 2\*log(x)/(a^2-1)) - arccoth(b\*x+a)/x

**mupad [B]** time = 1.73, size = 62, normalized size = 0.97

$$-\frac{\operatorname{acoth}(a+bx)}{x} - \frac{bx \ln(x) - \frac{bx \ln(a^2+2abx+b^2x^2-1)}{2} + abx \operatorname{acoth}(a+bx)}{x(a^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)/x^2,x)

[Out] -acoth(a + b\*x)/x - (b\*x\*log(x) - (b\*x\*log(a^2 + b^2\*x^2 + 2\*a\*b\*x - 1))/2 + a\*b\*x\*acoth(a + b\*x))/(x\*(a^2 - 1))

**sympy [A]** time = 1.48, size = 144, normalized size = 2.25

$$\left\{ \begin{array}{ll} \frac{b \operatorname{acoth}(bx-1)}{2} - \frac{\operatorname{acoth}(bx-1)}{x} - \frac{1}{2x} & \text{for } a = -1 \\ -\frac{b \operatorname{acoth}(bx+1)}{2} - \frac{\operatorname{acoth}(bx+1)}{x} + \frac{1}{2x} & \text{for } a = 1 \\ -\frac{a^2 \operatorname{acoth}(a+bx)}{a^2x-x} - \frac{abx \operatorname{acoth}(a+bx)}{a^2x-x} - \frac{bx \log(x)}{a^2x-x} + \frac{bx \log(a+bx+1)}{a^2x-x} - \frac{bx \operatorname{acoth}(a+bx)}{a^2x-x} + \frac{\operatorname{acoth}(a+bx)}{a^2x-x} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/x\*\*2,x)

[Out] Piecewise((b\*acoth(b\*x - 1)/2 - acoth(b\*x - 1)/x - 1/(2\*x), Eq(a, -1)), (-b\*acoth(b\*x + 1)/2 - acoth(b\*x + 1)/x + 1/(2\*x), Eq(a, 1)), (-a\*\*2\*acoth(a + b\*x)/(a\*\*2\*x - x) - a\*b\*x\*acoth(a + b\*x)/(a\*\*2\*x - x) - b\*x\*log(x)/(a\*\*2\*x - x) + b\*x\*log(a + b\*x + 1)/(a\*\*2\*x - x) - b\*x\*acoth(a + b\*x)/(a\*\*2\*x - x) + acoth(a + b\*x)/(a\*\*2\*x - x), True))

$$3.68 \quad \int \frac{\coth^{-1}(a+bx)}{x^3} dx$$

Optimal. Leaf size=90

$$\frac{ab^2 \log(x)}{(1-a^2)^2} - \frac{b}{2(1-a^2)x} - \frac{b^2 \log(-a-bx+1)}{4(1-a)^2} + \frac{b^2 \log(a+bx+1)}{4(a+1)^2} - \frac{\coth^{-1}(a+bx)}{2x^2}$$

[Out]  $-1/2*b/(-a^2+1)/x-1/2*\operatorname{arccoth}(b*x+a)/x^2+a*b^2*\ln(x)/(-a^2+1)^2-1/4*b^2*\ln(-b*x-a+1)/(1-a)^2+1/4*b^2*\ln(b*x+a+1)/(1+a)^2$

**Rubi [A]** time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6110, 371, 710, 801}

$$\frac{ab^2 \log(x)}{(1-a^2)^2} - \frac{b}{2(1-a^2)x} - \frac{b^2 \log(-a-bx+1)}{4(1-a)^2} + \frac{b^2 \log(a+bx+1)}{4(a+1)^2} - \frac{\coth^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/x^3, x]

[Out]  $-b/(2*(1-a^2)*x) - \operatorname{ArcCoth}[a + b*x]/(2*x^2) + (a*b^2*\operatorname{Log}[x])/(1-a^2)^2 - (b^2*\operatorname{Log}[1-a-b*x])/(4*(1-a)^2) + (b^2*\operatorname{Log}[1+a+b*x])/(4*(1+a)^2)$

### Rule 371

Int[((a\_) + (b\_.)\*(v\_)^(n\_))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m+1), Subst[Int[SimplifyIntegrand[(x-c)^m\*(a+b\*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

### Rule 710

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(e\*(d+e\*x)^(m+1))/((m+1)\*(c\*d^2+a\*e^2)), x] + Dist[c/(c\*d^2+a\*e^2), Int[((d+e\*x)^(m+1)\*(d-e\*x))/(a+c\*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2+a\*e^2, 0] && LtQ[m, -1]

### Rule 801

Int((((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_)))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d+e\*x)^m\*(f+g\*x))/(a+c\*x^2), x],



$x] /; \text{FreeQ}[\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IntegerQ}[m]$

### Rule 6110

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.) + (d_.)*(x_.)]*(b_.)]^{(p_.)*((e_.) + (f_.)*(x_.))^{(m_.)}, x\_Symbol] :> \text{Simp}[(e + f*x)^{(m + 1)}*(a + b*\text{ArcCoth}[c + d*x])^p]/(f*(m + 1)), x] - \text{Dist}[(b*d*p)/(f*(m + 1)), \text{Int}[(e + f*x)^{(m + 1)}*(a + b*\text{ArcCoth}[c + d*x])^{(p - 1)}]/(1 - (c + d*x)^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{ILtQ}[m, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(a + bx)}{x^3} dx &= -\frac{\coth^{-1}(a + bx)}{2x^2} + \frac{1}{2}b \int \frac{1}{x^2(1 - (a + bx)^2)} dx \\ &= -\frac{\coth^{-1}(a + bx)}{2x^2} + \frac{1}{2}b^2 \text{Subst} \left( \int \frac{1}{(-a + x)^2(1 - x^2)} dx, x, a + bx \right) \\ &= -\frac{b}{2(1 - a^2)x} - \frac{\coth^{-1}(a + bx)}{2x^2} - \frac{b^2 \text{Subst} \left( \int \frac{-a-x}{(-a+x)(1-x^2)} dx, x, a + bx \right)}{2(1 - a^2)} \\ &= -\frac{b}{2(1 - a^2)x} - \frac{\coth^{-1}(a + bx)}{2x^2} - \frac{b^2 \text{Subst} \left( \int \left( -\frac{2a}{(-1+a^2)(a-x)} + \frac{-1-a}{2(-1+a)(-1+x)} + \frac{-1+a}{2(1+a)(1+x)} \right) dx, x, a + bx \right)}{2(1 - a^2)} \\ &= -\frac{b}{2(1 - a^2)x} - \frac{\coth^{-1}(a + bx)}{2x^2} + \frac{ab^2 \log(x)}{(1 - a^2)^2} - \frac{b^2 \log(1 - a - bx)}{4(1 - a)^2} + \frac{b^2 \log(1 + a + bx)}{4(1 + a)^2} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 76, normalized size = 0.84

$$\frac{1}{4} \left( b \left( \frac{4ab \log(x)}{(a^2 - 1)^2} + \frac{2}{(a^2 - 1)x} - \frac{b \log(-a - bx + 1)}{(a - 1)^2} + \frac{b \log(a + bx + 1)}{(a + 1)^2} \right) - \frac{2 \coth^{-1}(a + bx)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*x]/x^3, x]

[Out] ((-2\*ArcCoth[a + b\*x])/x^2 + b\*(2/((-1 + a^2)\*x) + (4\*a\*b\*Log[x])/(-1 + a^2)^2 - (b\*Log[1 - a - b\*x])/(-1 + a)^2 + (b\*Log[1 + a + b\*x]/(1 + a)^2))/4

**fricas** [A] time = 0.72, size = 111, normalized size = 1.23

$$\frac{(a^2 - 2a + 1)b^2x^2 \log(bx + a + 1) - (a^2 + 2a + 1)b^2x^2 \log(bx + a - 1) + 4ab^2x^2 \log(x) + 2(a^2 - 1)bx - (a^4 - 2a^2 + 1)}{4(a^4 - 2a^2 + 1)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x^3,x, algorithm="fricas")

[Out] 1/4\*((a^2 - 2\*a + 1)\*b^2\*x^2\*log(b\*x + a + 1) - (a^2 + 2\*a + 1)\*b^2\*x^2\*log(b\*x + a - 1) + 4\*a\*b^2\*x^2\*log(x) + 2\*(a^2 - 1)\*b\*x - (a^4 - 2\*a^2 + 1)\*log((b\*x + a + 1)/(b\*x + a - 1)))/((a^4 - 2\*a^2 + 1)\*x^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x^3,x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/x^3, x)

**maple** [A] time = 0.04, size = 82, normalized size = 0.91

$$-\frac{\operatorname{arccoth}(bx + a)}{2x^2} - \frac{b^2 \ln(bx + a - 1)}{4(a - 1)^2} + \frac{b^2 \ln(bx + a + 1)}{4(1 + a)^2} + \frac{b}{2(a - 1)(1 + a)x} + \frac{b^2 a \ln(bx)}{(a - 1)^2(1 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/x^3,x)

[Out] -1/2\*arccoth(b\*x+a)/x^2-1/4\*b^2/(a-1)^2\*ln(b\*x+a-1)+1/4\*b^2\*ln(b\*x+a+1)/(1+a)^2+1/2\*b/(a-1)/(1+a)/x+b^2\*a/(a-1)^2/(1+a)^2\*ln(b\*x)

**maxima** [A] time = 0.30, size = 85, normalized size = 0.94

$$\frac{1}{4} \left( \frac{4ab \log(x)}{a^4 - 2a^2 + 1} + \frac{b \log(bx + a + 1)}{a^2 + 2a + 1} - \frac{b \log(bx + a - 1)}{a^2 - 2a + 1} + \frac{2}{(a^2 - 1)x} \right) b - \frac{\operatorname{arccoth}(bx + a)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/x^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}*(4*a*b*\log(x)/(a^4 - 2*a^2 + 1) + b*\log(b*x + a + 1)/(a^2 + 2*a + 1) - b*\log(b*x + a - 1)/(a^2 - 2*a + 1) + 2/((a^2 - 1)*x))*b - 1/2*\operatorname{arccoth}(b*x + a)/x^2$

**mupad [B]** time = 1.95, size = 247, normalized size = 2.74

$$\ln(x) \left( \frac{b^2}{4(a-1)^2} - \frac{b^2}{4(a+1)^2} \right) - \ln(a^2 + 2abx + b^2x^2 - 1) \left( \frac{b^2}{8(a-1)^2} - \frac{b^2}{8(a+1)^2} \right) - \frac{\operatorname{acoth}(a + bx) \left( \frac{a^2}{2} - \frac{1}{2} \right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a + b*x)/x^3, x)`

[Out]  $\log(x)*(b^2/(4*(a-1)^2) - b^2/(4*(a+1)^2)) - \log(a^2 + b^2*x^2 + 2*a*b*x - 1)*(b^2/(8*(a-1)^2) - b^2/(8*(a+1)^2)) - (\operatorname{acoth}(a + b*x)*(a^2/2 - 1/2) - (b*x)/2 + (b^2*x^2*\operatorname{acoth}(a + b*x))/2 + (x^3*(b^3 + 3*a^2*b^3))/(2*(a^2 - 1)^2) + (a*b^4*x^4)/(a^2 - 1)^2 + a*b*x*\operatorname{acoth}(a + b*x))/(a^2*x^2 - x^2 + b^2*x^4 + 2*a*b*x^3) - (\operatorname{atan}((2*a*b + 2*b^2*x)/(2*(b^2*(a^2 - 1) - a^2*b^2)^{(1/2)})))*(b^3 + a^2*b^3)/((-b^2)^{(1/2)}*(2*a^4 - 4*a^2 + 2))$

**sympy [A]** time = 2.43, size = 410, normalized size = 4.56

$$\left\{ \begin{array}{l} \frac{b^2 \operatorname{acoth}(bx-1)}{8} - \frac{b}{8x} - \frac{\operatorname{acoth}(bx-1)}{2x^2} - \frac{1}{8x^2} \\ \frac{b^2 \operatorname{acoth}(bx+1)}{8} - \frac{b}{8x} - \frac{\operatorname{acoth}(bx+1)}{2x^2} + \frac{1}{8x^2} \\ -\frac{a^4 \operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2} + \frac{a^2b^2x^2 \operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2} + \frac{a^2bx}{2a^4x^2-4a^2x^2+2x^2} + \frac{2a^2 \operatorname{acoth}(a+bx)}{2a^4x^2-4a^2x^2+2x^2} + \frac{2ab^2x^2 \log(x)}{2a^4x^2-4a^2x^2+2x^2} - \frac{2ab^2x^2 \log(a+bx+1)}{2a^4x^2-4a^2x^2+2x^2} + \frac{2ab^2}{2a^4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(b*x+a)/x**3, x)`

[Out]  $\operatorname{Piecewise}((b**2*\operatorname{acoth}(b*x - 1)/8 - b/(8*x) - \operatorname{acoth}(b*x - 1)/(2*x**2) - 1/(8*x**2), \operatorname{Eq}(a, -1)), (b**2*\operatorname{acoth}(b*x + 1)/8 - b/(8*x) - \operatorname{acoth}(b*x + 1)/(2*x**2) + 1/(8*x**2), \operatorname{Eq}(a, 1)), (-a**4*\operatorname{acoth}(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + a**2*b**2*x**2*\operatorname{acoth}(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + a**2*b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a**2*\operatorname{acoth}(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*\log(x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) - 2*a*b**2*x**2*\log(a + b*x + 1)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + 2*a*b**2*x**2*\operatorname{acoth}(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) + b**2*x**2*\operatorname{acoth}(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) - b*x/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2) - \operatorname{acoth}(a + b*x)/(2*a**4*x**2 - 4*a**2*x**2 + 2*x**2), \operatorname{True}))$

### 3.69 $\int x^3 \coth^{-1}(a + bx)^2 dx$

**Optimal.** Leaf size=263

$$\frac{a(a^2 + 1) \operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b^4} + \frac{(6a^2 + 1) \log(1 - (a + bx)^2)}{4b^4} - \frac{a(a^2 + 1) \coth^{-1}(a + bx)^2}{b^4} + \frac{(6a^2 + 1)(a + bx) \coth^{-1}(a + bx)}{2b^4}$$

[Out]  $-a*x/b^3+1/12*(b*x+a)^2/b^4+1/2*(6*a^2+1)*(b*x+a)*\operatorname{arccoth}(b*x+a)/b^4-a*(b*x+a)^2*\operatorname{arccoth}(b*x+a)/b^4+1/6*(b*x+a)^3*\operatorname{arccoth}(b*x+a)/b^4-a*(a^2+1)*\operatorname{arccoth}(b*x+a)^2/b^4-1/4*(a^4+6*a^2+1)*\operatorname{arccoth}(b*x+a)^2/b^4+1/4*x^4*\operatorname{arccoth}(b*x+a)^2+a*\operatorname{arctanh}(b*x+a)/b^4+2*a*(a^2+1)*\operatorname{arccoth}(b*x+a)*\ln(2/(-b*x-a+1))/b^4+1/2*\ln(1-(b*x+a)^2)/b^4+1/4*(6*a^2+1)*\ln(1-(b*x+a)^2)/b^4+a*(a^2+1)*\operatorname{polylog}(2,(-b*x-a-1)/(-b*x-a+1))/b^4$

**Rubi [A]** time = 0.35, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$ , Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 266, 43, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{a(a^2 + 1) \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b^4} + \frac{(6a^2 + 1) \log(1 - (a + bx)^2)}{4b^4} - \frac{a(a^2 + 1) \coth^{-1}(a + bx)^2}{b^4} - \frac{(a^4 + 6a^2 + 1) \coth^{-1}(a + bx)}{4b^4}$$

Antiderivative was successfully verified.

[In] `Int[x^3*ArcCoth[a + b*x]^2,x]`

[Out]  $-\frac{(a*x)}{b^3} + \frac{(a + b*x)^2}{(12*b^4)} + \frac{((1 + 6*a^2)*(a + b*x)*\operatorname{ArcCoth}[a + b*x])}{(2*b^4)} - \frac{(a*(a + b*x)^2*\operatorname{ArcCoth}[a + b*x])}{b^4} + \frac{((a + b*x)^3*\operatorname{ArcCoth}[a + b*x])}{(6*b^4)} - \frac{(a*(1 + a^2)*\operatorname{ArcCoth}[a + b*x]^2)}{b^4} - \frac{((1 + 6*a^2 + a^4)*\operatorname{ArcCoth}[a + b*x]^2)}{(4*b^4)} + \frac{(x^4*\operatorname{ArcCoth}[a + b*x]^2)}{4} + \frac{(a*\operatorname{ArcTanh}[a + b*x])}{b^4} + \frac{(2*a*(1 + a^2)*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 - a - b*x)])}{b^4} + \operatorname{Log}[1 - (a + b*x)^2]/(12*b^4) + \frac{((1 + 6*a^2)*\operatorname{Log}[1 - (a + b*x)^2])}{(4*b^4)} + \frac{(a*(1 + a^2)*\operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x)])}{b^4}$

#### Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

#### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt`

$Q[a, 0] \parallel LtQ[b, 0]$

### Rule 260

$Int[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_.)}), x\_Symbol] \rightarrow Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[\{a, b, m, n\}, x] \&\& EqQ[m, n - 1]$

### Rule 266

$Int[(x_)^{(m_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}], x\_Symbol] \rightarrow Dist[1/n, Subst[Int[x^{(Simplify[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; FreeQ[\{a, b, m, n, p\}, x] \&\& IntegerQ[Simplify[(m + 1)/n]]$

### Rule 321

$Int[((c_.)*(x_))^{(m_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}], x\_Symbol] \rightarrow Simp[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /; FreeQ[\{a, b, c, p\}, x] \&\& IGtQ[n, 0] \&\& GtQ[m, n - 1] \&\& NeQ[m + n*p + 1, 0] \&\& IntBinomialQ[a, b, c, n, m, p, x]$

### Rule 2315

$Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x\_Symbol] \rightarrow -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[\{c, d, e\}, x] \&\& EqQ[e + c*d, 0]$

### Rule 2402

$Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x\_Symbol] \rightarrow -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[\{c, d, e, f, g\}, x] \&\& EqQ[c, 2*d] \&\& EqQ[e^2*f + d^2*g, 0]$

### Rule 5911

$Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^{(p_.)}, x\_Symbol] \rightarrow Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^{(p - 1)})/(1 - c^2*x^2), x], x] /; FreeQ[\{a, b, c\}, x] \&\& IGtQ[p, 0]$

### Rule 5917

$Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^{(p_.)*((d_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow Simp[((d*x)^{(m + 1)}*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^{(m + 1)}*(a + b*ArcCoth[c*x])^{(p - 1)})/(1 - c^2*x^2), x], x] /; FreeQ[\{a, b, c, d, m\}, x] \&\& IGtQ[p, 0] \&\& (EqQ[p, 1] \parallel In$

tegerQ[m]) && NeQ[m, -1]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_.))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5985

Int((((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 6049

Int((((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_.))^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(a+bx)^2 dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^3 \coth^{-1}(x)^2 dx, x, a+bx\right)}{b} \\
&= \frac{1}{4}x^4 \coth^{-1}(a+bx)^2 - \frac{1}{2} \text{Subst}\left(\int \left(-\frac{(1+6a^2)\coth^{-1}(x)}{b^4} + \frac{4ax\coth^{-1}(x)}{b^4} - \frac{x^2\coth^{-1}(x)}{b^4}\right) dx, x, a+bx\right) \\
&= \frac{1}{4}x^4 \coth^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int x^2 \coth^{-1}(x) dx, x, a+bx\right)}{2b^4} - \frac{\text{Subst}\left(\int \frac{(1+6a^2+a^4-4a^2x)}{b^4} dx, x, a+bx\right)}{b^4} \\
&= \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4} \\
&= -\frac{ax}{b^3} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4} \\
&= -\frac{ax}{b^3} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4} \\
&= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4} \\
&= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4} \\
&= -\frac{ax}{b^3} + \frac{(a+bx)^2}{12b^4} + \frac{(1+6a^2)(a+bx)\coth^{-1}(a+bx)}{2b^4} - \frac{a(a+bx)^2\coth^{-1}(a+bx)}{b^4} + \frac{(a+bx)^3\coth^{-1}(a+bx)}{6b^4}
\end{aligned}$$

**Mathematica [A]** time = 1.70, size = 203, normalized size = 0.77

$$12(a^3 + a) \text{Li}_2\left(e^{-2\coth^{-1}(a+bx)}\right) + 36a^2 \log\left(\frac{1}{(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}\right) + 11a^2 - 2\coth^{-1}(a+bx)\left(12(a^3 + a) \log\left(1 - e^{-2\coth^{-1}(a+bx)}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*ArcCoth[a + b\*x]^2,x]

```
[Out] -1/12*(1 + 11*a^2 + 10*a*b*x - b^2*x^2 + 3*(1 - 4*a + 6*a^2 - 4*a^3 + a^4 -
b^4*x^4)*ArcCoth[a + b*x]^2 - 2*ArcCoth[a + b*x]*(9*a + 13*a^3 + 3*b*x + 9
*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 12*(a + a^3)*Log[1 - E^(-2*ArcCoth[a + b
*x]))] + 8*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])] + 36*a^2*Log[1/((a +
b*x)*Sqrt[1 - (a + b*x)^(-2)])] + 12*(a + a^3)*PolyLog[2, E^(-2*ArcCoth[a
+ b*x])))/b^4
```

**fricas** [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}(x^3 \operatorname{arccoth}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arccoth(b*x + a)^2, x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(b*x + a)^2, x)
```

**maple** [B] time = 0.07, size = 967, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(b*x+a)^2,x)
```

```
[Out] 3/2/b^4*ln(b*x+a+1)*a^2+1/2/b^4*ln(b*x+a+1)*a+3/2/b^4*ln(b*x+a-1)*a^2-1/2/b
^4*ln(b*x+a-1)*a+1/4*x^4*arccoth(b*x+a)^2-3/4/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*
x+1/2*a)*a^2+1/2/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a-1/b^4*arccoth(b*x+
a)*ln(b*x+a-1)*a^3-1/8/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a^4-1/2/b^4*ln
(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^3+3/2/b^4*arccoth(b*x+a)*ln(b*x+a-1)*a^2
-1/4/b^4*arccoth(b*x+a)*ln(b*x+a+1)*a^4-1/b^4*arccoth(b*x+a)*ln(b*x+a+1)*a+
1/3/b^4*ln(b*x+a-1)+1/3/b^4*ln(b*x+a+1)+1/4/b^4*arccoth(b*x+a)*ln(b*x+a-1)*
a^4+1/8/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a^4-1/2/b^4*ln(b*x
+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a+1/2/b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*
x+1/2*a)*a^3+1/2/b^4*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a^3-5/6*a*x/b^3+1/8/
b^4*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)+1/4/b^4*arccoth(b*x+a)*ln(
```



$b*x+a-1)-1/8/b^4*\ln(b*x+a-1)*\ln(1/2+1/2*b*x+1/2*a)-1/4/b^4*\ln(b*x+a-1)^2*a+1/b^4*dilog(1/2+1/2*b*x+1/2*a)*a^3+1/16/b^4*\ln(b*x+a+1)^2*a^4+1/4/b^4*\ln(b*x+a+1)^2*a^3+3/8/b^4*\ln(b*x+a+1)^2*a^2+1/4/b^4*\ln(b*x+a+1)^2*a+1/b^4*dilog(1/2+1/2*b*x+1/2*a)*a+1/16/b^4*\ln(b*x+a-1)^2*a^4+1/2/b^3*arccoth(b*x+a)*x-1/8/b^4*\ln(-1/2*b*x-1/2*a+1/2)*\ln(b*x+a+1)-1/4/b^4*arccoth(b*x+a)*\ln(b*x+a+1)+1/6/b*arccoth(b*x+a)*x^3+1/2/b^4*arccoth(b*x+a)*a-1/4/b^4*\ln(b*x+a-1)^2*a^3+3/8/b^4*\ln(b*x+a-1)^2*a^2+13/6/b^4*arccoth(b*x+a)*a^3-1/b^4*arccoth(b*x+a)*\ln(b*x+a-1)*a+1/12*x^2/b^2-3/2/b^4*arccoth(b*x+a)*\ln(b*x+a+1)*a^2-1/b^4*arccoth(b*x+a)*\ln(b*x+a+1)*a^3-1/2/b^2*arccoth(b*x+a)*x^2*a+3/2/b^3*arccoth(b*x+a)*x*a^2+3/4/b^4*\ln(-1/2*b*x-1/2*a+1/2)*\ln(1/2+1/2*b*x+1/2*a)*a^2-3/4/b^4*\ln(b*x+a+1)*\ln(-1/2*b*x-1/2*a+1/2)*a^2-1/8/b^4*\ln(b*x+a+1)*\ln(-1/2*b*x-1/2*a+1/2)*a^4+1/2/b^4*\ln(-1/2*b*x-1/2*a+1/2)*\ln(1/2+1/2*b*x+1/2*a)*a-11/12/b^4*a^2+1/16/b^4*\ln(b*x+a-1)^2+1/16/b^4*\ln(b*x+a+1)^2$

**maxima** [A] time = 0.34, size = 320, normalized size = 1.22

$$\frac{1}{4} x^4 \operatorname{arccoth}(bx+a)^2 + \frac{1}{48} b^2 \left( \frac{48(a^3+a) \left( \log(bx+a-1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right) \right)}{b^6} + \frac{4(13a^3 + 18a^2 + 9a + 4) \log(bx+a+1)}{b^6} + \frac{(4b^2x^2 - 40abx + 3(a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1)^2 - 6(a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1) \log(bx+a-1) + 3(a^4 - 4a^3 + 6a^2 - 4a + 1) \log(bx+a-1)^2 - 4(13a^3 - 18a^2 + 9a - 4) \log(bx+a-1))}{b^6} + \frac{1}{12} b^2 (2(b^2x^3 - 3abx^2 + 3(3a^2 + 1)x) / b^4 - 3(a^4 + 4a^3 + 6a^2 + 4a + 1) \log(bx+a+1) / b^5 + 3(a^4 - 4a^3 + 6a^2 - 4a + 1) \log(bx+a-1) / b^5) \operatorname{arccoth}(bx+a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x+a)^2,x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(b\*x + a)^2 + 1/48\*b^2\*(48\*(a^3 + a)\*(log(b\*x + a - 1)\*log(1/2\*b\*x + 1/2\*a + 1/2) + dilog(-1/2\*b\*x - 1/2\*a + 1/2))/b^6 + 4\*(13\*a^3 + 18\*a^2 + 9\*a + 4)\*log(b\*x + a + 1)/b^6 + (4\*b^2\*x^2 - 40\*a\*b\*x + 3\*(a^4 + 4\*a^3 + 6\*a^2 + 4\*a + 1)\*log(b\*x + a + 1)^2 - 6\*(a^4 + 4\*a^3 + 6\*a^2 + 4\*a + 1)\*log(b\*x + a + 1)\*log(b\*x + a - 1) + 3\*(a^4 - 4\*a^3 + 6\*a^2 - 4\*a + 1)\*log(b\*x + a - 1)^2 - 4\*(13\*a^3 - 18\*a^2 + 9\*a - 4)\*log(b\*x + a - 1))/b^6) + 1/12\*b\*(2\*(b^2\*x^3 - 3\*a\*b\*x^2 + 3\*(3\*a^2 + 1)\*x)/b^4 - 3\*(a^4 + 4\*a^3 + 6\*a^2 + 4\*a + 1)\*log(b\*x + a + 1)/b^5 + 3\*(a^4 - 4\*a^3 + 6\*a^2 - 4\*a + 1)\*log(b\*x + a - 1)/b^5)\*arccoth(b\*x + a)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \operatorname{acoth}(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*acoth(a + b\*x)^2,x)

[Out] int(x^3\*acoth(a + b\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acoth(b*x+a)**2,x)
```

```
[Out] Integral(x**3*acoth(a + b*x)**2, x)
```

### 3.70 $\int x^2 \coth^{-1}(a + bx)^2 dx$

**Optimal.** Leaf size=204

$$\frac{(3a^2 + 1) \operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{3b^3} + \frac{a(a^2 + 3) \coth^{-1}(a + bx)^2}{3b^3} + \frac{(3a^2 + 1) \coth^{-1}(a + bx)^2}{3b^3} - \frac{2(3a^2 + 1) \log\left(\frac{2}{-a-bx+1}\right) \coth^{-1}(a + bx)}{3b^3}$$

[Out]  $1/3*x/b^2-2*a*(b*x+a)*\operatorname{arccoth}(b*x+a)/b^3+1/3*(b*x+a)^2*\operatorname{arccoth}(b*x+a)/b^3+1/3*a*(a^2+3)*\operatorname{arccoth}(b*x+a)^2/b^3+1/3*(3*a^2+1)*\operatorname{arccoth}(b*x+a)^2/b^3+1/3*x^3*\operatorname{arccoth}(b*x+a)^2-1/3*\operatorname{arctanh}(b*x+a)/b^3-2/3*(3*a^2+1)*\operatorname{arccoth}(b*x+a)*\ln(2/(-b*x-a+1))/b^3-a*\ln(1-(b*x+a)^2)/b^3-1/3*(3*a^2+1)*\operatorname{polylog}(2,(-b*x-a-1)/(-b*x-a+1))/b^3$

**Rubi [A]** time = 0.28, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$ , Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{(3a^2 + 1) \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{3b^3} + \frac{a(a^2 + 3) \coth^{-1}(a + bx)^2}{3b^3} + \frac{(3a^2 + 1) \coth^{-1}(a + bx)^2}{3b^3} - \frac{2(3a^2 + 1) \log\left(\frac{2}{-a-bx+1}\right) \coth^{-1}(a + bx)}{3b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{ArcCoth}[a + b*x]^2, x]$

[Out]  $x/(3*b^2) - (2*a*(a + b*x)*\operatorname{ArcCoth}[a + b*x])/b^3 + ((a + b*x)^2*\operatorname{ArcCoth}[a + b*x])/(3*b^3) + (a*(3 + a^2)*\operatorname{ArcCoth}[a + b*x]^2)/(3*b^3) + ((1 + 3*a^2)*\operatorname{ArcCoth}[a + b*x]^2)/(3*b^3) + (x^3*\operatorname{ArcCoth}[a + b*x]^2)/3 - \operatorname{ArcTanh}[a + b*x]/(3*b^3) - (2*(1 + 3*a^2)*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 - a - b*x)])/(3*b^3) - (a*\operatorname{Log}[1 - (a + b*x)^2])/b^3 - ((1 + 3*a^2)*\operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))])/(3*b^3)$

#### Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$   $\operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

#### Rule 260

$\operatorname{Int}[(x_)^m/((a_ + (b_)*(x_)^n)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]/(b*n), x] /;$   $\operatorname{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \operatorname{EqQ}[m, n - 1]$

#### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2
*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] := Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
```

&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 6049

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right)^2 \coth^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
&= \frac{1}{3}x^3 \coth^{-1}(a + bx)^2 - \frac{2}{3} \text{Subst}\left(\int \left(\frac{3a \coth^{-1}(x)}{b^3} - \frac{x \coth^{-1}(x)}{b^3} - \frac{(a(3 + a^2) - (1 + 3a^2)x)}{b^3(1 - x^2)}\right) dx, x, a + bx\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(a + bx)^2 + \frac{2 \text{Subst}\left(\int x \coth^{-1}(x) dx, x, a + bx\right)}{3b^3} + \frac{2 \text{Subst}\left(\int \frac{(a(3 + a^2) - (1 + 3a^2)x)}{1 - x^2} dx, x, a + bx\right)}{3b^3} \\
&= -\frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx)^2 - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{1}{3}x^3 \coth^{-1}(a + bx)^2 - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{a(3 + a^2) \coth^{-1}(a + bx)}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{a(3 + a^2) \coth^{-1}(a + bx)}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{a(3 + a^2) \coth^{-1}(a + bx)}{3b^3} \\
&= \frac{x}{3b^2} - \frac{2a(a + bx) \coth^{-1}(a + bx)}{b^3} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{3b^3} + \frac{a(3 + a^2) \coth^{-1}(a + bx)}{3b^3}
\end{aligned}$$

**Mathematica [B]** time = 4.54, size = 607, normalized size = 2.98

$$(a + bx) \sqrt{1 - \frac{1}{(a + bx)^2}} (1 - (a + bx)^2) \left( \frac{4(3a^2 + 1) \text{Li}_2\left(e^{-2 \coth^{-1}(a + bx)}\right)}{(a + bx)^3 \left(1 - \frac{1}{(a + bx)^2}\right)^{3/2}} + \frac{9a^2 \coth^{-1}(a + bx)^2}{(a + bx) \sqrt{1 - \frac{1}{(a + bx)^2}}} + \frac{-3(a^2 - 1) \coth^{-1}(a + bx)^2 + 6a \coth^{-1}(a + bx)}{\sqrt{1 - \frac{1}{(a + bx)^2}}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcCoth[a + b\*x]^2,x]

```
[Out] -1/12*((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]*(1 - (a + b*x)^2)*((4*ArcCoth[a +
b*x])/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]) + (3*ArcCoth[a + b*x]^2)/((a +
b*x)*Sqrt[1 - (a + b*x)^(-2)])) - (12*a*ArcCoth[a + b*x]^2)/((a + b*x)*Sqrt[
1 - (a + b*x)^(-2)]) + (9*a^2*ArcCoth[a + b*x]^2)/((a + b*x)*Sqrt[1 - (a +
b*x)^(-2)]) + (-1 + 6*a*ArcCoth[a + b*x] - 3*(-1 + a^2)*ArcCoth[a + b*x]^2)
/Sqrt[1 - (a + b*x)^(-2)] + Cosh[3*ArcCoth[a + b*x]] - 6*a*ArcCoth[a + b*x]
*Cosh[3*ArcCoth[a + b*x]] + ArcCoth[a + b*x]^2*Cosh[3*ArcCoth[a + b*x]] + 3
*a^2*ArcCoth[a + b*x]^2*Cosh[3*ArcCoth[a + b*x]] + (6*ArcCoth[a + b*x]*Log[
1 - E^(-2*ArcCoth[a + b*x])])/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)]) + (18*a^
2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])])/((a + b*x)*Sqrt[1 - (a
+ b*x)^(-2)]) - (18*a*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])])/((a + b
*x)*Sqrt[1 - (a + b*x)^(-2)]) + (4*(1 + 3*a^2)*PolyLog[2, E^(-2*ArcCoth[a +
b*x])])/((a + b*x)^3*(1 - (a + b*x)^(-2))^(3/2)) - ArcCoth[a + b*x]^2*Sinh
[3*ArcCoth[a + b*x]] - 3*a^2*ArcCoth[a + b*x]^2*Sinh[3*ArcCoth[a + b*x]] -
2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]*Sinh[3*ArcCoth[a + b*x]
] - 6*a^2*ArcCoth[a + b*x]*Log[1 - E^(-2*ArcCoth[a + b*x])]*Sinh[3*ArcCoth[
a + b*x]] + 6*a*Log[1/((a + b*x)*Sqrt[1 - (a + b*x)^(-2)])]*Sinh[3*ArcCoth[
a + b*x]])/b^3
```

**fricas** [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}(x^2 \operatorname{arccoth}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccoth(b*x + a)^2, x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(b*x + a)^2, x)
```

**maple** [B] time = 0.06, size = 729, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(b*x+a)^2,x)
```

```
[Out] 1/b^3*arccoth(b*x+a)*ln(b*x+a-1)*a^2-1/b^3*arccoth(b*x+a)*ln(b*x+a-1)*a+1/3
/b^3*arccoth(b*x+a)*ln(b*x+a+1)*a^3-4/3/b^2*arccoth(b*x+a)*x*a-1/2/b^3*ln(b
*x+a-1)*ln(1/2+1/2*b*x+1/2*a)*a^2+1/2/b^3*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)
*a-1/2/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a^2-1/2/b^3*ln(-1/2
*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a+1/6/b^3*ln(b*x+a-1)*ln(1/2+1/2*b*x+
1/2*a)*a^3-1/3/b^3*arccoth(b*x+a)*ln(b*x+a-1)*a^3+1/6/b^3*ln(b*x+a+1)*ln(-1
/2*b*x-1/2*a+1/2)*a^3+1/2/b^3*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a^2+1/2/b^
3*ln(b*x+a+1)*ln(-1/2*b*x-1/2*a+1/2)*a-1/b^3*ln(b*x+a+1)*a-1/b^3*ln(b*x+a-1
)*a+1/b^3*arccoth(b*x+a)*ln(b*x+a+1)*a+1/b^3*arccoth(b*x+a)*ln(b*x+a+1)*a^2
-1/6/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)*a^3+1/3*x^3*arccoth(b
*x+a)^2+1/3*x/b^2-1/6/b^3*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)-1/b^3*dilog(1/2
+1/2*b*x+1/2*a)*a^2+1/4/b^3*ln(b*x+a-1)^2*a^2+1/3/b*arccoth(b*x+a)*x^2-5/3/
b^3*arccoth(b*x+a)*a^2+1/3/b^3*arccoth(b*x+a)*ln(b*x+a+1)+1/6/b^3*ln(-1/2*b
*x-1/2*a+1/2)*ln(b*x+a+1)-1/12/b^3*ln(b*x+a+1)^2*a^3-1/4/b^3*ln(b*x+a+1)^2*
a^2-1/4/b^3*ln(b*x+a+1)^2*a-1/12/b^3*ln(b*x+a-1)^2*a^3-1/4/b^3*ln(b*x+a-1)^
2*a-1/6/b^3*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*a)+1/3/b^3*arccoth(b*
x+a)*ln(b*x+a-1)+1/3/b^3*a-1/3/b^3*dilog(1/2+1/2*b*x+1/2*a)+1/12/b^3*ln(b*x
+a-1)^2-1/12/b^3*ln(b*x+a+1)^2+1/6/b^3*ln(b*x+a-1)-1/6/b^3*ln(b*x+a+1)
```

**maxima** [A] time = 0.33, size = 259, normalized size = 1.27

$$\frac{1}{3} x^3 \operatorname{arccoth}(bx+a)^2 - \frac{1}{12} b^2 \left( \frac{4(3a^2+1) \left( \log(bx+a-1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right) \right)}{b^5} + \frac{2(5a^2+6a+1) \log(bx+a+1)}{b^5} + ((a^3+3a^2+3a+1) \log(bx+a+1)^2 - 2(a^3+3a^2+3a+1) \log(bx+a+1) \log(bx+a-1) + (a^3-3a^2+3a-1) \log(bx+a-1)^2 - 4bx - 2(5a^2-6a+1) \log(bx+a-1)) / b^5 + 1/3 * b * ((bx^2 - 4ax) / b^3 + (a^3 + 3a^2 + 3a + 1) \log(bx+a+1) / b^4 - (a^3 - 3a^2 + 3a - 1) \log(bx+a-1) / b^4) * \operatorname{arccoth}(bx+a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(b*x + a)^2 - 1/12*b^2*(4*(3*a^2 + 1)*(log(b*x + a - 1)*log(
1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^5 + 2*(5*a^2 + 6*
a + 1)*log(b*x + a + 1)/b^5 + ((a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)^2 -
2*(a^3 + 3*a^2 + 3*a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a^3 - 3*a^2
+ 3*a - 1)*log(b*x + a - 1)^2 - 4*b*x - 2*(5*a^2 - 6*a + 1)*log(b*x + a -
1))/b^5) + 1/3*b*((b*x^2 - 4*a*x)/b^3 + (a^3 + 3*a^2 + 3*a + 1)*log(b*x + a
+ 1)/b^4 - (a^3 - 3*a^2 + 3*a - 1)*log(b*x + a - 1)/b^4)*arccoth(b*x + a)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(a + b*x)^2,x)
```

```
[Out] int(x^2*acoth(a + b*x)^2, x)
```



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(b*x+a)**2,x)
```

```
[Out] Integral(x**2*acoth(a + b*x)**2, x)
```

### 3.71 $\int x \coth^{-1}(a + bx)^2 dx$

Optimal. Leaf size=136

$$-\frac{(a^2 + 1) \coth^{-1}(a + bx)^2}{2b^2} + \frac{a \operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b^2} + \frac{\log(1 - (a + bx)^2)}{2b^2} - \frac{a \coth^{-1}(a + bx)^2}{b^2} + \frac{(a + bx) \coth^{-1}(a + bx)}{b^2} + \dots$$

[Out] (b\*x+a)\*arccoth(b\*x+a)/b^2-a\*arccoth(b\*x+a)^2/b^2-1/2\*(a^2+1)\*arccoth(b\*x+a)^2/b^2+1/2\*x^2\*arccoth(b\*x+a)^2+2\*a\*arccoth(b\*x+a)\*ln(2/(-b\*x-a+1))/b^2+1/2\*ln(1-(b\*x+a)^2)/b^2+a\*polylog(2,(-b\*x-a-1)/(-b\*x-a+1))/b^2

**Rubi [A]** time = 0.21, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {6112, 5929, 5911, 260, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{a \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b^2} - \frac{(a^2 + 1) \coth^{-1}(a + bx)^2}{2b^2} + \frac{\log(1 - (a + bx)^2)}{2b^2} - \frac{a \coth^{-1}(a + bx)^2}{b^2} + \frac{(a + bx) \coth^{-1}(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[a + b\*x]^2,x]

[Out] ((a + b\*x)\*ArcCoth[a + b\*x])/b^2 - (a\*ArcCoth[a + b\*x]^2)/b^2 - ((1 + a^2)\*ArcCoth[a + b\*x]^2)/(2\*b^2) + (x^2\*ArcCoth[a + b\*x]^2)/2 + (2\*a\*ArcCoth[a + b\*x]\*Log[2/(1 - a - b\*x)])/b^2 + Log[1 - (a + b\*x)^2]/(2\*b^2) + (a\*PolyLog[2, -((1 + a + b\*x)/(1 - a - b\*x))])/b^2

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 2315

Int[Log[(c\_)\*(x\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2402

Int[Log[(c\_)/((d\_) + (e\_)\*(x\_))]/((f\_) + (g\_)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rule 5985

Int((((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

Rule 6049

Int((((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(

```
m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b} + \frac{x}{b}\right) \coth^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
&= \frac{1}{2}x^2 \coth^{-1}(a + bx)^2 - \text{Subst}\left(\int \left(-\frac{\coth^{-1}(x)}{b^2} + \frac{(1 + a^2 - 2ax) \coth^{-1}(x)}{b^2(1 - x^2)}\right) dx, x, a + bx\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(a + bx)^2 + \frac{\text{Subst}\left(\int \coth^{-1}(x) dx, x, a + bx\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{(1 + a^2 - 2ax) \coth^{-1}(x)}{1 - x^2} dx, x, a + bx\right)}{b^2} \\
&= \frac{(a + bx) \coth^{-1}(a + bx)}{b^2} + \frac{1}{2}x^2 \coth^{-1}(a + bx)^2 - \frac{\text{Subst}\left(\int \frac{x}{1 - x^2} dx, x, a + bx\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{1 - x^2} dx, x, a + bx\right)}{b^2} \\
&= \frac{(a + bx) \coth^{-1}(a + bx)}{b^2} + \frac{1}{2}x^2 \coth^{-1}(a + bx)^2 + \frac{\log(1 - (a + bx)^2)}{2b^2} + \frac{(2a) \text{Subst}\left(\int \frac{\coth^{-1}(x)}{1 - x^2} dx, x, a + bx\right)}{b^2} \\
&= \frac{(a + bx) \coth^{-1}(a + bx)}{b^2} - \frac{a \coth^{-1}(a + bx)^2}{b^2} - \frac{(1 + a^2) \coth^{-1}(a + bx)^2}{2b^2} + \frac{1}{2}x^2 \coth^{-1}(a + bx)^2 \\
&= \frac{(a + bx) \coth^{-1}(a + bx)}{b^2} - \frac{a \coth^{-1}(a + bx)^2}{b^2} - \frac{(1 + a^2) \coth^{-1}(a + bx)^2}{2b^2} + \frac{1}{2}x^2 \coth^{-1}(a + bx)^2 \\
&= \frac{(a + bx) \coth^{-1}(a + bx)}{b^2} - \frac{a \coth^{-1}(a + bx)^2}{b^2} - \frac{(1 + a^2) \coth^{-1}(a + bx)^2}{2b^2} + \frac{1}{2}x^2 \coth^{-1}(a + bx)^2 \\
&= \frac{(a + bx) \coth^{-1}(a + bx)}{b^2} - \frac{a \coth^{-1}(a + bx)^2}{b^2} - \frac{(1 + a^2) \coth^{-1}(a + bx)^2}{2b^2} + \frac{1}{2}x^2 \coth^{-1}(a + bx)^2
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 106, normalized size = 0.78

$$\frac{(-a^2 + 2a + b^2x^2 - 1) \coth^{-1}(a + bx)^2 - 2a \text{Li}_2\left(e^{-2 \coth^{-1}(a + bx)}\right) - 2 \log\left(\frac{1}{(a + bx) \sqrt{1 - \frac{1}{(a + bx)^2}}}\right) + 2 \coth^{-1}(a + bx) (2a + b^2x^2 - 1)}{2b^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*ArcCoth[a + b\*x]^2,x]

[Out]  $((-1 + 2*a - a^2 + b^2*x^2)*\text{ArcCoth}[a + b*x]^2 + 2*\text{ArcCoth}[a + b*x]*(a + b*x + 2*a*\text{Log}[1 - E^{(-2*\text{ArcCoth}[a + b*x])}])) - 2*\text{Log}[1/((a + b*x)*\text{Sqrt}[1 - (a + b*x)^{(-2)}])] - 2*a*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[a + b*x])}])/(2*b^2)$

**fricas** [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}(x \operatorname{arccoth}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(b\*x+a)^2,x, algorithm="fricas")

[Out] integral(x\*arccoth(b\*x + a)^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(x\*arccoth(b\*x + a)^2, x)

**maple** [B] time = 0.06, size = 365, normalized size = 2.68

$$\frac{x^2 \operatorname{arccoth}(bx + a)^2}{2} - \frac{\operatorname{arccoth}(bx + a)^2 a^2}{2b^2} + \frac{\operatorname{arccoth}(bx + a) x}{b} + \frac{\operatorname{arccoth}(bx + a) a}{b^2} - \frac{\operatorname{arccoth}(bx + a) \ln(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(b\*x+a)^2,x)

[Out]  $1/2*x^2*\text{arccoth}(b*x+a)^2 - 1/2/b^2*\text{arccoth}(b*x+a)^2*a^2 + 1/b*\text{arccoth}(b*x+a)*x + 1/b^2*\text{arccoth}(b*x+a)*a - 1/b^2*\text{arccoth}(b*x+a)*\ln(b*x+a-1)*a + 1/2/b^2*\text{arccoth}(b*x+a)*\ln(b*x+a-1) - 1/b^2*\text{arccoth}(b*x+a)*\ln(b*x+a+1)*a - 1/2/b^2*\text{arccoth}(b*x+a)*\ln(b*x+a+1) + 1/2/b^2*\ln(b*x+a-1) + 1/2/b^2*\ln(b*x+a+1) + 1/8/b^2*\ln(b*x+a-1)^2 - 1/4/b^2*\ln(b*x+a-1)*\ln(1/2+1/2*b*x+1/2*a) - 1/4/b^2*\ln(b*x+a-1)^2*a + 1/b^2*\text{dilog}(1/2+1/2*b*x+1/2*a)*a + 1/2/b^2*\ln(b*x+a-1)*\ln(1/2+1/2*b*x+1/2*a)*a + 1/4/b^2*\ln(b*x+a+1)^2*a - 1/2/b^2*\ln(b*x+a+1)*\ln(-1/2*b*x-1/2*a+1/2)*a + 1/2/b^2*\ln(-1/2*b*x-1/2*a+1/2)*\ln(1/2+1/2*b*x+1/2*a)*a + 1/8/b^2*\ln(b*x+a+1)^2 - 1/4/b^2*\ln(-1/2*b*x-1/2*a+1/2)*\ln(b*x+a+1) + 1/4/b^2*\ln(-1/2*b*x-1/2*a+1/2)*\ln(1/2+1/2*b*x+1/2*a)$

**maxima** [A] time = 0.34, size = 202, normalized size = 1.49

$$\frac{1}{2} x^2 \operatorname{arccoth}(bx + a)^2 + \frac{1}{8} b^2 \left( \frac{8 \left( \log(bx + a - 1) \log\left(\frac{1}{2} bx + \frac{1}{2} a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2} bx - \frac{1}{2} a + \frac{1}{2}\right)\right) a}{b^4} + \frac{4(a + 1) \log(bx + a)}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(b\*x+a)^2,x, algorithm="maxima")

[Out] 1/2\*x^2\*arccoth(b\*x + a)^2 + 1/8\*b^2\*(8\*(log(b\*x + a - 1)\*log(1/2\*b\*x + 1/2\*a + 1/2) + dilog(-1/2\*b\*x - 1/2\*a + 1/2))\*a/b^4 + 4\*(a + 1)\*log(b\*x + a + 1)/b^4 + ((a^2 + 2\*a + 1)\*log(b\*x + a + 1)^2 - 2\*(a^2 + 2\*a + 1)\*log(b\*x + a + 1)\*log(b\*x + a - 1) + (a^2 - 2\*a + 1)\*log(b\*x + a - 1)^2 - 4\*(a - 1)\*log(b\*x + a - 1))/b^4 + 1/2\*b\*(2\*x/b^2 - (a^2 + 2\*a + 1)\*log(b\*x + a + 1)/b^3 + (a^2 - 2\*a + 1)\*log(b\*x + a - 1)/b^3)\*arccoth(b\*x + a)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(a + b\*x)^2,x)

[Out] int(x\*acoth(a + b\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(b\*x+a)\*\*2,x)

[Out] Integral(x\*acoth(a + b\*x)\*\*2, x)

### 3.72 $\int \coth^{-1}(a + bx)^2 dx$

**Optimal.** Leaf size=81

$$-\frac{\operatorname{Li}_2\left(-\frac{a+bx+1}{-a-bx+1}\right)}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} + \frac{\coth^{-1}(a+bx)^2}{b} - \frac{2\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b}$$

[Out]  $\operatorname{arccoth}(b*x+a)^2/b + (b*x+a)*\operatorname{arccoth}(b*x+a)^2/b - 2*\operatorname{arccoth}(b*x+a)*\ln(2/(-b*x-a+1))/b - \operatorname{polylog}(2, (-b*x-a-1)/(-b*x-a+1))/b$

**Rubi [A]** time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {6104, 5911, 5985, 5919, 2402, 2315}

$$-\frac{\operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{b} + \frac{(a+bx)\coth^{-1}(a+bx)^2}{b} + \frac{\coth^{-1}(a+bx)^2}{b} - \frac{2\log\left(\frac{2}{-a-bx+1}\right)\coth^{-1}(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]^2, x]

[Out]  $\operatorname{ArcCoth}[a + b*x]^2/b + ((a + b*x)*\operatorname{ArcCoth}[a + b*x]^2)/b - (2*\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 - a - b*x)])/b - \operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))]/b$

#### Rule 2315

$\operatorname{Int}[\operatorname{Log}[(c\_)*(x\_)]/((d\_)+(e\_)*(x\_)), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /; \operatorname{FreeQ}\{c, d, e\}, x \ \&\& \operatorname{EqQ}[e + c*d, 0]$

#### Rule 2402

$\operatorname{Int}[\operatorname{Log}[(c\_)/((d\_)+(e\_)*(x_))]/((f\_)+(g\_)*(x_)^2), x\_Symbol] \rightarrow -\operatorname{Dist}[e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \operatorname{FreeQ}\{c, d, e, f, g\}, x \ \&\& \operatorname{EqQ}[c, 2*d] \ \&\& \operatorname{EqQ}[e^2*f + d^2*g, 0]$

#### Rule 5911

$\operatorname{Int}(((a\_)+\operatorname{ArcCoth}[(c\_)*(x_)]*(b\_))^p), x\_Symbol] \rightarrow \operatorname{Simp}[x*(a + b*\operatorname{ArcCoth}[c*x])^p, x] - \operatorname{Dist}[b*c^p, \operatorname{Int}[(x*(a + b*\operatorname{ArcCoth}[c*x])^{p-1})/(1 - c^2*x^2), x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{IGtQ}[p, 0]$

#### Rule 5919

$\operatorname{Int}(((a\_)+\operatorname{ArcCoth}[(c\_)*(x_)]*(b\_))^p)/((d\_)+(e\_)*(x_)), x\_Symbol] \rightarrow -\operatorname{Simp}(((a + b*\operatorname{ArcCoth}[c*x])^p*\operatorname{Log}[2/(1 + (e*x)/d)])/e, x] + \operatorname{Dist}[(b*c^p$

p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 6104

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(a + bx)^2 dx &= \frac{\text{Subst}\left(\int \coth^{-1}(x)^2 dx, x, a + bx\right)}{b} \\
 &= \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \text{Subst}\left(\int \frac{x \coth^{-1}(x)}{1-x^2} dx, x, a + bx\right)}{b} \\
 &= \frac{\coth^{-1}(a + bx)^2}{b} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \text{Subst}\left(\int \frac{\coth^{-1}(x)}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\coth^{-1}(a + bx)^2}{b} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \coth^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} + \frac{2 \text{Subst}\left(\int \frac{1}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\coth^{-1}(a + bx)^2}{b} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \coth^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{2 \text{Subst}\left(\int \frac{1}{1-x} dx, x, a + bx\right)}{b} \\
 &= \frac{\coth^{-1}(a + bx)^2}{b} + \frac{(a + bx) \coth^{-1}(a + bx)^2}{b} - \frac{2 \coth^{-1}(a + bx) \log\left(\frac{2}{1-a-bx}\right)}{b} - \frac{\text{Li}_2\left(1 - \frac{2}{1-a-bx}\right)}{b}
 \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 55, normalized size = 0.68

$$\frac{\text{Li}_2\left(e^{-2 \coth^{-1}(a+bx)}\right) + \coth^{-1}(a + bx) \left( (a + bx - 1) \coth^{-1}(a + bx) - 2 \log\left(1 - e^{-2 \coth^{-1}(a+bx)}\right) \right)}{b}$$



Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]^2,x]

[Out] (ArcCoth[a + b\*x]\*((-1 + a + b\*x)\*ArcCoth[a + b\*x] - 2\*Log[1 - E^(-2\*ArcCoth[a + b\*x])]) + PolyLog[2, E^(-2\*ArcCoth[a + b\*x])])/b

**fricas** [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}(\text{arccoth}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2,x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{arccoth}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2,x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)^2, x)

**maple** [A] time = 0.30, size = 151, normalized size = 1.86

$$x \text{arccoth}(bx + a)^2 + \frac{\text{arccoth}(bx + a)^2 a}{b} + \frac{\text{arccoth}(bx + a)^2}{b} - \frac{2 \text{arccoth}(bx + a) \ln\left(1 - \frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)}{b} - \frac{2 \text{arccoth}(bx + a) \ln\left(1 + \frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)^2,x)

[Out] x\*arccoth(b\*x+a)^2+1/b\*arccoth(b\*x+a)^2\*a+arccoth(b\*x+a)^2/b-2/b\*arccoth(b\*x+a)\*ln(1-1/((b\*x+a-1)/(b\*x+a+1))^(1/2))-2/b\*arccoth(b\*x+a)\*ln(1+1/((b\*x+a-1)/(b\*x+a+1))^(1/2))-2/b\*polylog(2,-1/((b\*x+a-1)/(b\*x+a+1))^(1/2))-2/b\*polylog(2,1/((b\*x+a-1)/(b\*x+a+1))^(1/2))

**maxima** [A] time = 0.33, size = 139, normalized size = 1.72

$$-\frac{1}{4} b^2 \left( \frac{(a+1) \log(bx+a+1)^2 - 2(a+1) \log(bx+a+1) \log(bx+a-1) + (a-1) \log(bx+a-1)^2}{b^3} + \frac{4 \left( \log\left(1 - \frac{1}{\sqrt{\frac{bx+a-1}{bx+a+1}}}\right) \right)^2}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(((a + 1)*log(b*x + a + 1)^2 - 2*(a + 1)*log(b*x + a + 1)*log(b*x + a - 1) + (a - 1)*log(b*x + a - 1)^2)/b^3 + 4*(log(b*x + a - 1)*log(1/2*b*x + 1/2*a + 1/2) + dilog(-1/2*b*x - 1/2*a + 1/2))/b^3) + b*((a + 1)*log(b*x + a + 1)/b^2 - (a - 1)*log(b*x + a - 1)/b^2)*arccoth(b*x + a) + x*arccoth(b*x + a)^2
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(a + bx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)^2,x)
```

```
[Out] int(acoth(a + b*x)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2,x)
```

```
[Out] Integral(acoth(a + b*x)**2, x)
```

$$3.73 \quad \int \frac{\coth^{-1}(a+bx)^2}{x} dx$$

**Optimal.** Leaf size=148

$$\frac{1}{2}\text{Li}_3\left(1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{Li}_3\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \text{Li}_2\left(1 - \frac{2}{a+bx+1}\right)\coth^{-1}(a+bx) - \text{Li}_2\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right)$$

[Out]  $-\text{arccoth}(b*x+a)^2*\ln(2/(b*x+a+1))+\text{arccoth}(b*x+a)^2*\ln(2*b*x/(1-a)/(b*x+a+1))$   
 $+\text{arccoth}(b*x+a)*\text{polylog}(2,1-2/(b*x+a+1))-\text{arccoth}(b*x+a)*\text{polylog}(2,1-2*b*x/(1-a)/(b*x+a+1))+1/2*\text{polylog}(3,1-2/(b*x+a+1))-1/2*\text{polylog}(3,1-2*b*x/(1-a)/(b*x+a+1))$

**Rubi [A]** time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6112, 5923}

$$\frac{1}{2}\text{PolyLog}\left(3,1 - \frac{2}{a+bx+1}\right) - \frac{1}{2}\text{PolyLog}\left(3,1 - \frac{2bx}{(1-a)(a+bx+1)}\right) + \coth^{-1}(a+bx)\text{PolyLog}\left(2,1 - \frac{2}{a+bx+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]^2/x, x]

[Out]  $-(\text{ArcCoth}[a + b*x]^2*\text{Log}[2/(1 + a + b*x)]) + \text{ArcCoth}[a + b*x]^2*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))] + \text{ArcCoth}[a + b*x]*\text{PolyLog}[2, 1 - 2/(1 + a + b*x)] - \text{ArcCoth}[a + b*x]*\text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))] + \text{PolyLog}[3, 1 - 2/(1 + a + b*x)]/2 - \text{PolyLog}[3, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))]/2$

#### Rule 5923

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)^2/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :=  
 $-\text{Simp}[(a + b*\text{ArcCoth}[c*x])^2*\text{Log}[2/(1 + c*x)]/e, x] + (\text{Simp}[(a + b*\text{ArcCoth}[c*x])^2*\text{Log}[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x) + \text{Simp}[(b*(a + b*\text{ArcCoth}[c*x])*\text{PolyLog}[2, 1 - 2/(1 + c*x)])/e, x] - \text{Simp}[(b*(a + b*\text{ArcCoth}[c*x])*\text{PolyLog}[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + \text{Simp}[(b^2*\text{PolyLog}[3, 1 - 2/(1 + c*x)])/(2*e), x] - \text{Simp}[(b^2*\text{PolyLog}[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x)))/(2*e), x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]$

#### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_)])\*(b\_.)^p\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt

Q[p, 0]

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)^2}{x} dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)^2}{-\frac{a}{b} + \frac{x}{b}} dx, x, a+bx\right)}{b}$$

$$= -\coth^{-1}(a+bx)^2 \log\left(\frac{2}{1+a+bx}\right) + \coth^{-1}(a+bx)^2 \log\left(\frac{2bx}{(1-a)(1+a+bx)}\right) + \coth^{-1}(a+bx)$$

**Mathematica** [C] time = 2.87, size = 547, normalized size = 3.70

$$\frac{2}{3} \sqrt{1 - \frac{1}{a^2}} a e^{\tanh^{-1}\left(\frac{1}{a}\right)} \coth^{-1}(a+bx)^3 + 2 \coth^{-1}(a+bx) \text{Li}_2\left(-\sqrt{\frac{a-1}{a+1}} e^{\coth^{-1}(a+bx)}\right) + 2 \coth^{-1}(a+bx) \text{Li}_2\left(\sqrt{\frac{a-1}{a+1}} e^{\coth^{-1}(a+bx)}\right)$$

Warning: Unable to verify antiderivative.

`[In] Integrate[ArcCoth[a + b*x]^2/x, x]`

```
[Out] (-1/24*I)*Pi^3 - (2*ArcCoth[a + b*x]^3)/3 - (2*a*ArcCoth[a + b*x]^3)/3 + (2
*Sqrt[1 - a^(-2)]*a*E^ArcTanh[a^(-1)]*ArcCoth[a + b*x]^3)/3 - I*Pi*ArcCoth[
a + b*x]*Log[(E^(-ArcCoth[a + b*x]) + E^ArcCoth[a + b*x])/2] + ArcCoth[a +
b*x]^2*Log[1 - Sqrt[(-1 + a)/(1 + a)]]*E^ArcCoth[a + b*x]] + ArcCoth[a + b*x
]^2*Log[1 + Sqrt[(-1 + a)/(1 + a)]]*E^ArcCoth[a + b*x]] - ArcCoth[a + b*x]^2
*Log[1 - E^(2*ArcCoth[a + b*x])] - 2*ArcCoth[a + b*x]*ArcTanh[a^(-1)]*Log[(
I/2)*(E^(ArcCoth[a + b*x]) - ArcTanh[a^(-1)]) - E^(-ArcCoth[a + b*x]) + ArcTa
nh[a^(-1)])] + ArcCoth[a + b*x]^2*Log[(-1 - E^(2*ArcCoth[a + b*x]) + a*(-1
+ E^(2*ArcCoth[a + b*x])))/(2*E^ArcCoth[a + b*x])] + I*Pi*ArcCoth[a + b*x]
*Log[1/Sqrt[1 - (a + b*x)^(-2)]] - ArcCoth[a + b*x]^2*Log[-((b*x)/((a + b*x
)*Sqrt[1 - (a + b*x)^(-2)])]] + 2*ArcCoth[a + b*x]*ArcTanh[a^(-1)]*Log[I*Si
nh[ArcCoth[a + b*x] - ArcTanh[a^(-1)]]] + 2*ArcCoth[a + b*x]*PolyLog[2, -(S
qrt[(-1 + a)/(1 + a)]]*E^ArcCoth[a + b*x])] + 2*ArcCoth[a + b*x]*PolyLog[2,
Sqrt[(-1 + a)/(1 + a)]]*E^ArcCoth[a + b*x]] - ArcCoth[a + b*x]*PolyLog[2, E^
(2*ArcCoth[a + b*x])] - 2*PolyLog[3, -(Sqrt[(-1 + a)/(1 + a)]]*E^ArcCoth[a +
b*x])] - 2*PolyLog[3, Sqrt[(-1 + a)/(1 + a)]]*E^ArcCoth[a + b*x]] + PolyLog
[3, E^(2*ArcCoth[a + b*x])]/2
```

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcoth}(bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x, x)
```

```
giac [F]   time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\operatorname{arccoth}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x, x)
```

```
maple [C]   time = 1.15, size = 985, normalized size = 6.66
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x,x)
```

```
[Out] ln(b*x)*arccoth(b*x+a)^2-arccoth(b*x+a)^2*ln(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)+arccoth(b*x+a)^2*ln((b*x+a+1)/(b*x+a-1)-1)-arccoth(b*x+a)^2*ln(1-1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*arccoth(b*x+a)*polylog(2,1/((b*x+a-1)/(b*x+a+1))^(1/2))+2*polylog(3,1/((b*x+a-1)/(b*x+a+1))^(1/2))-arccoth(b*x+a)^2*ln(1+1/((b*x+a-1)/(b*x+a+1))^(1/2))-2*arccoth(b*x+a)*polylog(2,-1/((b*x+a-1)/(b*x+a+1))^(1/2))+2*polylog(3,-1/((b*x+a-1)/(b*x+a+1))^(1/2))-I*Pi*arccoth(b*x+a)^2+I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^2-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^3-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I/((b*x+a+1)/(b*x+a-1)-1))*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^2+1/2*I*Pi*arccoth(b*x+a)^2*csgn(I/((b*x+a+1)/(b*x+a-1)-1))*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1))*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))-1/2*I*Pi*arccoth(b*x+a)^2*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1))*csgn(I*(a*((b*x+a+1)/(b*x+a-1)-1)-(b*x+a+1)/(b*x+a-1)-1)/((b*x+a+1)/(b*x+a-1)-1))^2+a/(a-1)*arccoth(b*x+a)^2*ln(1-(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))+a/(a-1)*arccoth(b*x+a)*polylog(2,(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))-1/2*a/(a-1)*polylog(3,(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))-1/(a-1)*arccoth(b*x+a)^2*ln(1-(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))-1/(a-1)*arccoth(b*x+a)*polylog(2,(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))+1/2/(a-1)*polylog(3,(a-1)*(b*x+a+1)/(b*x+a-1)/(1+a))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x,x, algorithm="maxima")

[Out] integrate(arccoth(b\*x + a)^2/x, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a+bx)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)^2/x,x)

[Out] int(acoth(a + b\*x)^2/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(a+bx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)\*\*2/x,x)

[Out] Integral(acoth(a + b\*x)\*\*2/x, x)

$$3.74 \quad \int \frac{\coth^{-1}(a+bx)^2}{x^2} dx$$

**Optimal.** Leaf size=251

$$\frac{b\text{Li}_2\left(1 - \frac{2}{a+bx+1}\right)}{1-a^2} - \frac{b\text{Li}_2\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2} - \frac{2b \log\left(\frac{2}{a+bx+1}\right) \coth^{-1}(a+bx)}{1-a^2} + \frac{2b \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right) \coth^{-1}(a+bx)}{1-a^2}$$

[Out]  $-\text{arccoth}(b*x+a)^2/x + b*\text{arccoth}(b*x+a)*\ln(2/(-b*x-a+1))/(1-a) + b*\text{arccoth}(b*x+a)*\ln(2/(b*x+a+1))/(1+a) - 2*b*\text{arccoth}(b*x+a)*\ln(2/(b*x+a+1))/(-a^2+1) + 2*b*\text{arccoth}(b*x+a)*\ln(2*b*x/(1-a)/(b*x+a+1))/(-a^2+1) + 1/2*b*\text{polylog}(2, (-b*x-a-1)/(-b*x-a+1))/(1-a) - 1/2*b*\text{polylog}(2, 1-2/(b*x+a+1))/(1+a) + b*\text{polylog}(2, 1-2/(b*x+a+1))/(-a^2+1) - b*\text{polylog}(2, 1-2*b*x/(1-a)/(b*x+a+1))/(-a^2+1)$

**Rubi [A]** time = 0.72, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$ , Rules used = {6110, 371, 706, 31, 633, 6741, 6122, 6688, 12, 6725, 5921, 2402, 2315, 2447, 5919}

$$\frac{b\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{1-a^2} - \frac{b\text{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{1-a^2} + \frac{b\text{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{2(1-a)} - \frac{b\text{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{2(a+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[a + b*x]^2/x^2, x]$

[Out]  $-(\text{ArcCoth}[a + b*x]^2/x) + (b*\text{ArcCoth}[a + b*x]*\text{Log}[2/(1 - a - b*x)])/(1 - a) + (b*\text{ArcCoth}[a + b*x]*\text{Log}[2/(1 + a + b*x)])/(1 + a) - (2*b*\text{ArcCoth}[a + b*x]*\text{Log}[2/(1 + a + b*x)])/(1 - a^2) + (2*b*\text{ArcCoth}[a + b*x]*\text{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2) + (b*\text{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))])/(2*(1 - a)) - (b*\text{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(2*(1 + a)) + (b*\text{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(1 - a^2) - (b*\text{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)$

**Rule 12**

$\text{Int}[(a\_)*(u\_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b\_)*(v\_)] /; \text{FreeQ}[b, x]$

**Rule 31**

$\text{Int}[(a\_ + (b\_)*(x\_))^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```



]

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[
2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6122

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] := Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcCoth
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)^2}{x^2} dx &= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b) \int \frac{\coth^{-1}(a+bx)}{x(1-(a+bx)^2)} dx \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b) \int \frac{\coth^{-1}(a+bx)}{x(1-a^2-2abx-b^2x^2)} dx \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} + 2 \text{Subst} \left( \int \frac{\coth^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} + 2 \text{Subst} \left( \int \frac{b \coth^{-1}(x)}{(-a+x)(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b) \text{Subst} \left( \int \frac{\coth^{-1}(x)}{(-a+x)(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} + (2b) \text{Subst} \left( \int \left( \frac{\coth^{-1}(x)}{(-1+a^2)(a-x)} + \frac{\coth^{-1}(x)}{2(-1+a)(-1+x)} - \frac{\coth^{-1}(x)}{2(1+a)(1+x)} \right) dx, x, a+bx \right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} - \frac{b \text{Subst} \left( \int \frac{\coth^{-1}(x)}{-1+x} dx, x, a+bx \right)}{1-a} - \frac{b \text{Subst} \left( \int \frac{\coth^{-1}(x)}{1+x} dx, x, a+bx \right)}{1+a} \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a} - \frac{2b \coth^{-1}(a+bx)}{1-a^2} \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a} - \frac{2b \coth^{-1}(a+bx)}{1-a^2} \\
&= -\frac{\coth^{-1}(a+bx)^2}{x} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{1-a} + \frac{b \coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{1+a} - \frac{2b \coth^{-1}(a+bx)}{1-a^2}
\end{aligned}$$

**Mathematica [C]** time = 1.12, size = 206, normalized size = 0.82

$$-\left( \left( \sqrt{1 - \frac{1}{a^2}} abxe^{\tanh^{-1}\left(\frac{1}{a}\right)} + a^2 - 1 \right) \coth^{-1}(a+bx)^2 \right) + bx \text{Li}_2 \left( e^{2 \tanh^{-1}\left(\frac{1}{a}\right) - 2 \coth^{-1}(a+bx)} \right) + bx \coth^{-1}(a+bx) \left( -2 \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]^2/x^2, x]

```
[Out] (-((-1 + a^2 + Sqrt[1 - a^(-2)])*a*b*E^ArcTanh[a^(-1)]*x)*ArcCoth[a + b*x]^2
) + b*x*ArcCoth[a + b*x]*((-I)*Pi + 2*ArcTanh[a^(-1)] - 2*Log[1 - E^(-2*Arc
Coth[a + b*x] + 2*ArcTanh[a^(-1)])]) + b*x*(I*Pi*(Log[1 + E^(2*ArcCoth[a +
b*x]]) - Log[1/Sqrt[1 - (a + b*x)^(-2)]]) + 2*ArcTanh[a^(-1)]*(Log[1 - E^(-
2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1)])]) - Log[I*Sinh[ArcCoth[a + b*x] - Ar
cTanh[a^(-1)]]) + b*x*PolyLog[2, E^(-2*ArcCoth[a + b*x] + 2*ArcTanh[a^(-1
)]]))/((-1 + a^2)*x)
```

**fricas** [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arccoth}(bx+a)^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arccoth(b*x + a)^2/x^2, x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccoth(b*x + a)^2/x^2, x)
```

**maple** [A] time = 0.07, size = 342, normalized size = 1.36

$$-\frac{\operatorname{arccoth}(bx+a)^2}{x} + \frac{2b \operatorname{arccoth}(bx+a) \ln(bx+a-1)}{2a-2} - \frac{2b \operatorname{arccoth}(bx+a) \ln(bx+a+1)}{2+2a} - \frac{2b \operatorname{arccoth}(bx+a)}{(a-1)(1+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(b*x+a)^2/x^2,x)
```

```
[Out] -arccoth(b*x+a)^2/x+2*b*arccoth(b*x+a)/(2*a-2)*ln(b*x+a-1)-2*b*arccoth(b*x+
a)/(2+2*a)*ln(b*x+a+1)-2*b*arccoth(b*x+a)/(a-1)/(1+a)*ln(b*x)+b/(a-1)/(1+a)
*dilog((b*x+a+1)/(1+a))+b/(a-1)/(1+a)*ln(b*x)*ln((b*x+a+1)/(1+a))-b/(a-1)/(
1+a)*dilog((b*x+a-1)/(a-1))-b/(a-1)/(1+a)*ln(b*x)*ln((b*x+a-1)/(a-1))+1/4*b
/(1+a)*ln(b*x+a+1)^2+1/2*b/(1+a)*ln(-1/2*b*x-1/2*a+1/2)*ln(1/2+1/2*b*x+1/2*
a)-1/2*b/(1+a)*ln(-1/2*b*x-1/2*a+1/2)*ln(b*x+a+1)+1/2*b/(1+a)*dilog(1/2+1/2
*b*x+1/2*a)+1/4*b/(a-1)*ln(b*x+a-1)^2-1/2*b/(a-1)*dilog(1/2+1/2*b*x+1/2*a)-
1/2*b/(a-1)*ln(b*x+a-1)*ln(1/2+1/2*b*x+1/2*a)
```

**maxima** [A] time = 0.32, size = 244, normalized size = 0.97

$$\frac{1}{4} b^2 \left( \frac{(a-1) \log(bx+a+1)^2 - 2(a-1) \log(bx+a+1) \log(bx+a-1) + (a+1) \log(bx+a-1)^2}{a^2 b - b} - 4 \left( \log(bx \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x^2,x, algorithm="maxima")

[Out] 1/4\*b^2\*((a - 1)\*log(b\*x + a + 1)^2 - 2\*(a - 1)\*log(b\*x + a + 1)\*log(b\*x + a - 1) + (a + 1)\*log(b\*x + a - 1)^2)/(a^2\*b - b) - 4\*(log(b\*x + a - 1)\*log(1/2\*b\*x + 1/2\*a + 1/2) + dilog(-1/2\*b\*x - 1/2\*a + 1/2))/(a^2\*b - b) + 4\*(log(b\*x/(a + 1) + 1)\*log(x) + dilog(-b\*x/(a + 1)))/(a^2\*b - b) - 4\*(log(b\*x/(a - 1) + 1)\*log(x) + dilog(-b\*x/(a - 1)))/(a^2\*b - b) - b\*(log(b\*x + a + 1)/(a + 1) - log(b\*x + a - 1)/(a - 1) + 2\*log(x)/(a^2 - 1))\*arccoth(b\*x + a) - arccoth(b\*x + a)^2/x

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a + bx)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)^2/x^2,x)

[Out] int(acoth(a + b\*x)^2/x^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(a + bx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)\*\*2/x\*\*2,x)

[Out] Integral(acoth(a + b\*x)\*\*2/x\*\*2, x)

$$3.75 \quad \int \frac{\coth^{-1}(a+bx)^2}{x^3} dx$$

**Optimal.** Leaf size=370

$$\frac{ab^2 \operatorname{Li}_2\left(1 - \frac{2}{a+bx+1}\right)}{(1-a^2)^2} - \frac{ab^2 \operatorname{Li}_2\left(1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2} + \frac{b^2 \log(x)}{(1-a^2)^2} - \frac{2ab^2 \log\left(\frac{2}{a+bx+1}\right) \coth^{-1}(a+bx)}{(1-a^2)^2} + \frac{2ab^2 \log\left(\frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2} \quad (1)$$

[Out]  $-b \operatorname{arccoth}(b*x+a)/(-a^2+1)/x - 1/2 \operatorname{arccoth}(b*x+a)^2/x^2 + b^2 \ln(x)/(-a^2+1)^2 + 1/2 * b^2 \operatorname{arccoth}(b*x+a) * \ln(2/(-b*x-a+1))/(1-a)^2 - 1/2 * b^2 \ln(-b*x-a+1)/(1-a)^2 / (1+a) - 1/2 * b^2 \operatorname{arccoth}(b*x+a) * \ln(2/(b*x+a+1))/(1+a)^2 - 2 * a * b^2 \operatorname{arccoth}(b*x+a) * \ln(2/(b*x+a+1))/(-a^2+1)^2 + 2 * a * b^2 \operatorname{arccoth}(b*x+a) * \ln(2 * b*x/(1-a)/(b*x+a+1))/(-a^2+1)^2 - 1/2 * b^2 \ln(b*x+a+1)/(1-a)/(1+a)^2 + 1/4 * b^2 \operatorname{polylog}(2, (-b*x-a-1)/(-b*x-a+1))/(1-a)^2 + 1/4 * b^2 \operatorname{polylog}(2, 1-2/(b*x+a+1))/(1+a)^2 + a * b^2 \operatorname{polylog}(2, 1-2/(b*x+a+1))/(-a^2+1)^2 - a * b^2 \operatorname{polylog}(2, 1-2 * b*x/(1-a)/(b*x+a+1))/(-a^2+1)^2$

**Rubi [A]** time = 0.83, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$ , Rules used = {6110, 371, 710, 801, 6741, 6122, 6725, 5927, 706, 31, 633, 5921, 2402, 2315, 2447, 5919}

$$\frac{ab^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{(1-a^2)^2} - \frac{ab^2 \operatorname{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{(1-a^2)^2} + \frac{b^2 \operatorname{PolyLog}\left(2, -\frac{a+bx+1}{-a-bx+1}\right)}{4(1-a)^2} + \frac{b^2 \operatorname{PolyLog}\left(2, 1 - \frac{2bx}{(1-a)(a+bx+1)}\right)}{4(a+1)^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]^2/x^3, x]

[Out]  $-((b \operatorname{ArcCoth}[a + b*x])/((1 - a^2)*x)) - \operatorname{ArcCoth}[a + b*x]^2/(2*x^2) + (b^2 \operatorname{Log}[x])/((1 - a^2)^2) + (b^2 \operatorname{ArcCoth}[a + b*x] * \operatorname{Log}[2/(1 - a - b*x)])/(2*(1 - a)^2) - (b^2 \operatorname{Log}[1 - a - b*x])/((2*(1 - a)^2*(1 + a))) - (b^2 \operatorname{ArcCoth}[a + b*x] * \operatorname{Log}[2/(1 + a + b*x)])/(2*(1 + a)^2) - (2*a*b^2 \operatorname{ArcCoth}[a + b*x] * \operatorname{Log}[2/(1 + a + b*x)])/(1 - a^2)^2 + (2*a*b^2 \operatorname{ArcCoth}[a + b*x] * \operatorname{Log}[(2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)^2 - (b^2 \operatorname{Log}[1 + a + b*x])/((2*(1 - a)*(1 + a)^2)) + (b^2 \operatorname{PolyLog}[2, -((1 + a + b*x)/(1 - a - b*x))])/(4*(1 - a)^2) + (b^2 \operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(4*(1 + a)^2) + (a*b^2 \operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)])/(1 - a^2)^2 - (a*b^2 \operatorname{PolyLog}[2, 1 - (2*b*x)/((1 - a)*(1 + a + b*x))])/(1 - a^2)^2$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(−1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(d - e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int((((d_.) + (e_.)*(x_))^(m_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^((p_.)/((d_) + (e_.)*(x_))), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] := Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^((p_.)*((e_.) + (f_.)*(x_))^(
m_)), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

Rule 6122

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^((p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] := Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcCoth
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
```

x] && EqQ[B\*(1 - c^2) + 2\*A\*c\*d, 0] && EqQ[2\*c\*C - B\*d, 0]

### Rule 6725

Int[(u\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{v = RationalFunctionE  
xpan[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ  
[n, 0]

### Rule 6741

Int[u\_, x\_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=  
= u]

### Rubi steps



$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)^2}{x^3} dx &= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + b \int \frac{\coth^{-1}(a+bx)}{x^2(1-(a+bx)^2)} dx \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + b \int \frac{\coth^{-1}(a+bx)}{x^2(1-a^2-2abx-b^2x^2)} dx \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + \text{Subst} \left( \int \frac{\coth^{-1}(x)}{\left(-\frac{a}{b} + \frac{x}{b}\right)^2(1-x^2)} dx, x, a+bx \right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2} + \text{Subst} \left( \int \left( -\frac{b^2 \coth^{-1}(x)}{(-1+a^2)(a-x)^2} - \frac{2ab^2 \coth^{-1}(x)}{(-1+a^2)^2(a-x)} - \frac{b^2 \coth^{-1}(x)}{2(-1+a)^2} \right) dx, x, a+bx \right) \\
&= -\frac{\coth^{-1}(a+bx)^2}{2x^2} - \frac{b^2 \text{Subst} \left( \int \frac{\coth^{-1}(x)}{-1+x} dx, x, a+bx \right)}{2(1-a)^2} + \frac{b^2 \text{Subst} \left( \int \frac{\coth^{-1}(x)}{1+x} dx, x, a+bx \right)}{2(1+a)^2} \\
&= -\frac{b \coth^{-1}(a+bx)}{(1-a^2)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \coth^{-1}(a+bx)}{(1-a^2)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \coth^{-1}(a+bx)}{(1-a^2)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \log(x)}{(1-a^2)^2} + \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2} \\
&= -\frac{b \coth^{-1}(a+bx)}{(1-a^2)x} - \frac{\coth^{-1}(a+bx)^2}{2x^2} + \frac{b^2 \log(x)}{(1-a^2)^2} + \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1-a-bx}\right)}{2(1-a)^2} - \frac{b^2 \coth^{-1}(a+bx) \log\left(\frac{2}{1+a-bx}\right)}{2(1+a)^2}
\end{aligned}$$

**Mathematica [C]** time = 2.35, size = 291, normalized size = 0.79

$$2bx \coth^{-1}(a+bx) \left( a^2 + abx + i\pi abx - 2abx \tanh^{-1}\left(\frac{1}{a}\right) + 2abx \log\left(1 - e^{2 \tanh^{-1}\left(\frac{1}{a}\right) - 2 \coth^{-1}(a+bx)}\right) - 1 \right) + \left( -a^4 - \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]^2/x^3,x]

[Out]  $((-1 - a^4 + b^2*x^2 + a^2*(2 + b^2*(-1 + 2*\sqrt{1 - a^{(-2)}})*E^{\text{ArcTanh}[a^{(-1)}]})^2)*\text{ArcCoth}[a + b*x]^2 + 2*b*x*\text{ArcCoth}[a + b*x]*(-1 + a^2 + a*b*x + I*a*b*\text{Pi}*x - 2*a*b*x*\text{ArcTanh}[a^{(-1)}] + 2*a*b*x*\text{Log}[1 - E^{(-2*\text{ArcCoth}[a + b*x] + 2*\text{ArcTanh}[a^{(-1)}])}] + 2*b^2*x^2*((-I)*a*\text{Pi}*\text{Log}[1 + E^{(2*\text{ArcCoth}[a + b*x])}] + I*a*\text{Pi}*\text{Log}[1/\sqrt{1 - (a + b*x)^{(-2)}]]) + \text{Log}[-(b*x)/((a + b*x)*\sqrt{1 - (a + b*x)^{(-2)})})] - 2*a*\text{ArcTanh}[a^{(-1)}]*(\text{Log}[1 - E^{(-2*\text{ArcCoth}[a + b*x] + 2*\text{ArcTanh}[a^{(-1)}])}] - \text{Log}[I*\text{Sinh}[\text{ArcCoth}[a + b*x] - \text{ArcTanh}[a^{(-1)}]]) - 2*a*b^2*x^2*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[a + b*x] + 2*\text{ArcTanh}[a^{(-1)}])})]/(2*(-1 + a^2)^2*x^2)$

**fricas** [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(bx+a)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x^3,x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)^2/x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(bx+a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x^3,x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)^2/x^3, x)

**maple** [A] time = 0.08, size = 467, normalized size = 1.26

$$-\frac{\text{arccoth}(bx+a)^2}{2x^2} - \frac{b^2\text{arccoth}(bx+a)\ln(bx+a-1)}{2(a-1)^2} + \frac{b^2\text{arccoth}(bx+a)\ln(bx+a+1)}{2(1+a)^2} + \frac{b\text{arccoth}(bx+a)}{(a-1)(1+a)x} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)^2/x^3,x)

[Out]  $-1/2*\text{arccoth}(b*x+a)^2/x^2 - 1/2*b^2*\text{arccoth}(b*x+a)/(a-1)^2*\ln(b*x+a-1) + 1/2*b^2*\text{arccoth}(b*x+a)/(1+a)^2*\ln(b*x+a+1) + b*\text{arccoth}(b*x+a)/(a-1)/(1+a)/x + 2*b^2*\text{arccoth}(b*x+a)*a/(a-1)^2/(1+a)^2*\ln(b*x) - 1/8*b^2/(a-1)^2*\ln(b*x+a-1)^2 + 1/4*b^2*$

$$\begin{aligned} & \frac{1}{8} \left( \frac{8 \left( \log(bx + a - 1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right)\right) a}{a^4 - 2a^2 + 1} - \frac{8 \left( \log\left(\frac{bx}{a+1} + 1\right) \log(x) + \operatorname{Li}_2\left(-\frac{bx}{a+1}\right)\right) a}{a^4 - 2a^2 + 1} \right) \\ & - \frac{1}{4} b^2 (a-1)^2 \operatorname{dilog}\left(\frac{1}{2} + \frac{1}{2} \frac{bx+a-1}{a}\right) + \frac{1}{4} b^2 (a-1)^2 \ln(bx+a-1) \ln\left(\frac{1}{2} + \frac{1}{2} \frac{bx+a-1}{a}\right) \\ & - \frac{1}{8} b^2 (1+a)^2 \ln(bx+a+1)^2 - \frac{1}{4} b^2 (1+a)^2 \ln\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right) \ln\left(\frac{1}{2} + \frac{1}{2} \frac{bx+a-1}{a}\right) \\ & + \frac{1}{4} b^2 (1+a)^2 \operatorname{dilog}\left(\frac{1}{2} + \frac{1}{2} \frac{bx+a-1}{a}\right) - \frac{b^2 (a-1)}{(1+a)(2a-2)} \ln(bx+a-1) \\ & + \frac{b^2 (a-1)}{(1+a)(2+2a)} \ln(bx+a+1) + \frac{b^2 (a-1)^2}{(1+a)^2} \ln(bx) - \frac{b^2 a}{(a-1)^2 (1+a)^2} \operatorname{dilog}\left(\frac{bx+a+1}{1+a}\right) \\ & - \frac{b^2 a}{(a-1)^2 (1+a)^2} \ln(bx) \ln\left(\frac{bx+a+1}{1+a}\right) + \frac{b^2 a}{(a-1)^2 (1+a)^2} \operatorname{dilog}\left(\frac{bx+a-1}{a-1}\right) \\ & + \frac{b^2 a}{(a-1)^2 (1+a)^2} \ln(bx) \ln\left(\frac{bx+a-1}{a-1}\right) \end{aligned}$$

**maxima** [A] time = 0.34, size = 360, normalized size = 0.97

$$\frac{1}{8} \left( \frac{8 \left( \log(bx + a - 1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{Li}_2\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right)\right) a}{a^4 - 2a^2 + 1} - \frac{8 \left( \log\left(\frac{bx}{a+1} + 1\right) \log(x) + \operatorname{Li}_2\left(-\frac{bx}{a+1}\right)\right) a}{a^4 - 2a^2 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)^2/x^3,x, algorithm="maxima")

[Out]  $\frac{1}{8} \left( 8 \left( \log(bx + a - 1) \log\left(\frac{1}{2}bx + \frac{1}{2}a + \frac{1}{2}\right) + \operatorname{dilog}\left(-\frac{1}{2}bx - \frac{1}{2}a + \frac{1}{2}\right)\right) a / (a^4 - 2a^2 + 1) - 8 \left( \log(bx / (a + 1) + 1) \log(x) + \operatorname{dilog}\left(-bx / (a + 1)\right)\right) a / (a^4 - 2a^2 + 1) + 8 \left( \log(bx / (a - 1) + 1) \log(x) + \operatorname{dilog}\left(-bx / (a - 1)\right)\right) a / (a^4 - 2a^2 + 1) - ((a^2 - 2a + 1) \log(bx + a + 1)^2 - 2(a^2 - 2a + 1) \log(bx + a + 1) \log(bx + a - 1) + (a^2 + 2a + 1) \log(bx + a - 1)^2) / (a^4 - 2a^2 + 1) + 4 \log(bx + a + 1) / (a^3 + a^2 - a - 1) - 4 \log(bx + a - 1) / (a^3 - a^2 - a + 1) + 8 \log(x) / (a^4 - 2a^2 + 1) \right) b^2 + \frac{1}{2} \left( 4a \log(x) / (a^4 - 2a^2 + 1) + b \log(bx + a + 1) / (a^2 + 2a + 1) - b \log(bx + a - 1) / (a^2 - 2a + 1) + 2 / ((a^2 - 1)x) \right) b \operatorname{arccoth}(bx + a) - \frac{1}{2} \operatorname{arccoth}(bx + a)^2 / x^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a + bx)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)^2/x^3,x)

[Out] int(acoth(a + b\*x)^2/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(a + bx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)**2/x**3,x)
```

```
[Out] Integral(acoth(a + b*x)**2/x**3, x)
```

$$3.76 \quad \int \frac{\coth^{-1}(a+bx)}{c+dx^2} dx$$

**Optimal.** Leaf size=673

$$\frac{\operatorname{Li}_2\left(-\frac{(da^2+b^2c)(-a-bx+1)}{(cb^2-\sqrt{-c}\sqrt{d}b-(1-a)ad)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{Li}_2\left(-\frac{(da^2+b^2c)(-a-bx+1)}{(cb^2+\sqrt{-c}\sqrt{d}b-(1-a)ad)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{Li}_2\left(\frac{(da^2+b^2c)(a+bx+1)}{(cb^2-\sqrt{-c}\sqrt{d}b+a(a+1)d)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{Li}_2\left(\frac{(da^2+b^2c)(a+bx+1)}{(cb^2+\sqrt{-c}\sqrt{d}b+a(a+1)d)(a+bx)}\right)}{4\sqrt{-c}\sqrt{d}}$$

[Out]  $\frac{1}{4} \ln\left(\frac{b^2x+a-1}{b^2x+a}\right) \ln\left(1+\frac{a^2d+b^2c}{-c}\frac{-b^2x-a+1}{b^2x+a}\right) - \frac{1}{4} \ln\left(\frac{b^2x+a+1}{b^2x+a}\right) \ln\left(1+\frac{a^2d+b^2c}{-c}\frac{b^2x+a+1}{b^2x+a}\right) + \frac{1}{4} \operatorname{polylog}\left(2, -\frac{a^2d+b^2c}{-c}\frac{-b^2x-a+1}{b^2x+a}\right) - \frac{1}{4} \operatorname{polylog}\left(2, -\frac{a^2d+b^2c}{-c}\frac{b^2x+a+1}{b^2x+a}\right) + \frac{1}{4} \operatorname{polylog}\left(2, \frac{a^2d+b^2c}{-c}\frac{-b^2x-a+1}{b^2x+a}\right) - \frac{1}{4} \operatorname{polylog}\left(2, \frac{a^2d+b^2c}{-c}\frac{b^2x+a+1}{b^2x+a}\right)$

**Rubi [A]** time = 1.10, antiderivative size = 597, normalized size of antiderivative = 0.89, number of steps used = 37, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6116, 2513, 2409, 2394, 2393, 2391, 205}

$$\frac{\operatorname{PolyLog}\left(2, -\frac{\sqrt{d}(-a-bx+1)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{PolyLog}\left(2, \frac{\sqrt{d}(-a-bx+1)}{(1-a)\sqrt{d}+b\sqrt{-c}}\right)}{4\sqrt{-c}\sqrt{d}} - \frac{\operatorname{PolyLog}\left(2, -\frac{\sqrt{d}(a+bx+1)}{b\sqrt{-c}-(a+1)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}} + \frac{\operatorname{PolyLog}\left(2, \frac{\sqrt{d}(a+bx+1)}{(a+1)\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}$$

Warning: Unable to verify antiderivative.

[In] Int[ArcCoth[a + b\*x]/(c + d\*x^2), x]

[Out]  $\frac{\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \left(\log[-1+a+b^2x] - \log\left[-\frac{(1-a-b^2x)}{a+b^2x}\right]\right) - \log[a+b^2x]}{2\sqrt{c}\sqrt{d}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{d}x}{\sqrt{c}}\right] \left(\log[a+b^2x] - \log[1+a+b^2x] + \log\left[\frac{1+a+b^2x}{a+b^2x}\right]\right)}{2\sqrt{c}\sqrt{d}} - \frac{\log[-1+a+b^2x] \log\left[\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\log[1+a+b^2x] \log\left[\frac{b(\sqrt{-c}+\sqrt{d}x)}{b\sqrt{-c}+(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} + \frac{\log[-1+a+b^2x] \log\left[\frac{b(\sqrt{-c}+\sqrt{d}x)}{b\sqrt{-c}+(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} - \frac{\log[1+a+b^2x] \log\left[\frac{b(\sqrt{-c}-\sqrt{d}x)}{b\sqrt{-c}-(1-a)\sqrt{d}}\right]}{4\sqrt{-c}\sqrt{d}} - \operatorname{PolyLog}\left[2, -\left(\frac{a^2d+b^2c}{-c}\frac{-b^2x-a+1}{b^2x+a}\right)\right] + \operatorname{PolyLog}\left[2, -\left(\frac{a^2d+b^2c}{-c}\frac{b^2x+a+1}{b^2x+a}\right)\right] + \operatorname{PolyLog}\left[2, \frac{a^2d+b^2c}{-c}\frac{-b^2x-a+1}{b^2x+a}\right] - \operatorname{PolyLog}\left[2, \frac{a^2d+b^2c}{-c}\frac{b^2x+a+1}{b^2x+a}\right]$

```

qrt[d]*(1 - a - b*x))/(b*Sqrt[-c] - (1 - a)*Sqrt[d]))/(4*Sqrt[-c]*Sqrt[d])
+ PolyLog[2, (Sqrt[d]*(1 - a - b*x))/(b*Sqrt[-c] + (1 - a)*Sqrt[d])]/(4*Sq
rt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*(1 + a + b*x))/(b*Sqrt[-c] - (1 + a
)*Sqrt[d]))]/(4*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a + b*x))/(b*S
qrt[-c] + (1 + a)*Sqrt[d])]/(4*Sqrt[-c]*Sqrt[d])

```

### Rule 205

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

### Rule 2391

```

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rule 2393

```

Int[((a_.) + Log[(c_)*((d_) + (e_)*(x_))])*(b_.)/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]

```

### Rule 2394

```

Int[((a_.) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_.)/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

```

### Rule 2409

```

Int[((a_.) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_.))^(p_.)*((f_.) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x
)^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))

```

### Rule 2513

```

Int[Log[(e_)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFx, x], x) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]

```

]

Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+dx^2} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+dx^2} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+dx^2} dx \\
&= -\left(\frac{1}{2} \int \frac{\log(-1+a+bx)}{c+dx^2} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+dx^2} dx - \frac{1}{2} \left(-\log(-1+a+bx) + \log(a+bx)\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(a+bx) - \log(-1+a+bx)\right)}{2\sqrt{c}\sqrt{d}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(a+bx) - \log(-1+a+bx)\right)}{2\sqrt{c}\sqrt{d}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(a+bx) - \log(-1+a+bx)\right)}{2\sqrt{c}\sqrt{d}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(a+bx) - \log(-1+a+bx)\right)}{2\sqrt{c}\sqrt{d}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2\sqrt{c}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \left(\log(a+bx) - \log(-1+a+bx)\right)}{2\sqrt{c}\sqrt{d}}
\end{aligned}$$

**Mathematica [A]** time = 0.62, size = 529, normalized size = 0.79

$$\text{Li}_2\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{\sqrt{d}(a-1)+b\sqrt{-c}}\right) - \text{Li}_2\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{\sqrt{d}(a+1)+b\sqrt{-c}}\right) - \text{Li}_2\left(\frac{b(\sqrt{d}x+\sqrt{-c})}{b\sqrt{-c}-(a-1)\sqrt{d}}\right) + \text{Li}_2\left(\frac{b(\sqrt{d}x+\sqrt{-c})}{b\sqrt{-c}-(a+1)\sqrt{d}}\right) + \log(\sqrt{-c}-\sqrt{d}x) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{\sqrt{d}(a-1)+b\sqrt{-c}}\right) - \log(\sqrt{-c}-\sqrt{d}x) \log\left(\frac{b(\sqrt{-c}-\sqrt{d}x)}{\sqrt{d}(a+1)+b\sqrt{-c}}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(c + d\*x^2), x]

[Out] (Log[(Sqrt[d]\*(-1 + a + b\*x))/(b\*Sqrt[-c] + (-1 + a)\*Sqrt[d])] \* Log[Sqrt[-c] - Sqrt[d]\*x] - Log[(-1 + a + b\*x)/(a + b\*x)] \* Log[Sqrt[-c] - Sqrt[d]\*x] - Log[(Sqrt[d]\*(1 + a + b\*x))/(b\*Sqrt[-c] + (1 + a)\*Sqrt[d])] \* Log[Sqrt[-c] - Sqrt[d]\*x] + Log[(1 + a + b\*x)/(a + b\*x)] \* Log[Sqrt[-c] - Sqrt[d]\*x] - Log[-(Sqrt[d]\*(-1 + a + b\*x))/(b\*Sqrt[-c] - (-1 + a)\*Sqrt[d])] \* Log[Sqrt[-c] + Sqrt[d]\*x] + Log[(-1 + a + b\*x)/(a + b\*x)] \* Log[Sqrt[-c] + Sqrt[d]\*x] + Log[-((Sqrt[d]\*(1 + a + b\*x))/(b\*Sqrt[-c] - (1 + a)\*Sqrt[d]))] \* Log[Sqrt[-c] + Sqrt[d]\*x] - Log[(1 + a + b\*x)/(a + b\*x)] \* Log[Sqrt[-c] + Sqrt[d]\*x] + PolyLog[2, (b\*(Sqrt[-c] - Sqrt[d]\*x))/(b\*Sqrt[-c] + (-1 + a)\*Sqrt[d])] - PolyLog[2, (b\*(Sqrt[-c] - Sqrt[d]\*x))/(b\*Sqrt[-c] + (1 + a)\*Sqrt[d])] - PolyLog[2, (b\*(Sqrt[-c] + Sqrt[d]\*x))/(b\*Sqrt[-c] - (-1 + a)\*Sqrt[d])] + PolyLog[2, (b\*(Sqrt[-c] + Sqrt[d]\*x))/(b\*Sqrt[-c] - (1 + a)\*Sqrt[d])])/(4\*Sqrt[-c]\*Sqrt[d])

**fricas** [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arccoth}(bx+a)}{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x^2+c), x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)/(d\*x^2 + c), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)}{dx^2+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x^2+c), x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(d\*x^2 + c), x)

**maple** [B] time = 0.75, size = 1230, normalized size = 1.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(d\*x^2+c), x)



```
[Out] -1/2/b*(-b^2*c*d)^(1/2)/c*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*arccoth(b*x+a)*a^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2*b*(-b^2*c*d)^(1/2)/d*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*arccoth(b*x+a)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)+1/2/b*(-b^2*c*d)^(1/2)/c*arccoth(b*x+a)^2*a^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)+1/2*b*(-b^2*c*d)^(1/2)/d*arccoth(b*x+a)^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/4/b*(-b^2*c*d)^(1/2)/c*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*a^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/4*b*(-b^2*c*d)^(1/2)/d*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-b/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*arccoth(b*x+a)+b/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)*arccoth(b*x+a)^2+1/2/b*(-b^2*c*d)^(1/2)/c*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))*arccoth(b*x+a)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2/b*(-b^2*c*d)^(1/2)/c*arccoth(b*x+a)^2/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)-1/2*b/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))+1/4/b*(-b^2*c*d)^(1/2)/c*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d))/(a^2*d+b^2*c-2*(-b^2*c*d)^(1/2)-d)+1/2/b*(-b^2*c*d)^(1/2)/c/d*arccoth(b*x+a)*ln(1-(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c+2*(-b^2*c*d)^(1/2)-d))-1/2/b*(-b^2*c*d)^(1/2)/c/d*arccoth(b*x+a)^2+1/4/b*(-b^2*c*d)^(1/2)/c/d*polylog(2,(a^2*d+b^2*c-2*a*d+d)*(b*x+a+1)/(b*x+a-1)/(a^2*d+b^2*c+2*(-b^2*c*d)^(1/2)-d))
```

**maxima** [C] time = 0.59, size = 589, normalized size = 0.88

$$\frac{\operatorname{arccoth}(bx+a) \operatorname{arctan}\left(\frac{dx}{\sqrt{cd}}\right) + \left( \operatorname{arctan}\left(\frac{(b^2x+(a+1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2+2a+1)d}\right) - \operatorname{arctan}\left(\frac{(b^2x+(a-1)b)\sqrt{c}\sqrt{d}}{b^2c+(a^2-2a+1)d}\right) \right) \frac{(a-1)bd}{b^2c}}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] arccoth(b*x + a)*arctan(d*x/sqrt(c*d))/sqrt(c*d) + 1/4*((arctan2((b^2*x + (a + 1)*b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 + 2*a + 1)*d), ((a + 1)*b*d*x + (a^2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2*a + 1)*d)) - arctan2((b^2*x + (a - 1)*b)*sqrt(c)*sqrt(d)/(b^2*c + (a^2 - 2*a + 1)*d), ((a - 1)*b*d*x + (a^2 - 2*a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)))*log(d*x^2 + c) - arctan(sqrt(d)*x/sqrt(c))*log((b^2*d*x^2 + 2*(a + 1)*b*d*x + (a^2 + 2*a + 1)*d)/(b^2*c + (a^2 + 2*a + 1)*d)) + arctan(sqrt(d)*x/sqrt(c))*log((b^2*d*x^2 + 2*(a - 1)*b*d*x + (a^2 - 2*a + 1)*d)/(b^2*c + (a^2 - 2*a + 1)*d)) - I*dilog(((a + 1)*b*d*x + b^2*c - (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c + (2*I*a + 2*I)*b*sqrt(c)*sqrt(d) - (a^2 + 2*a + 1)*d)) + I*dilog(((a + 1)*b*d*x + b^2*c + (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/(b^2*c - (2*I*a + 2*I)*b*sqrt(c)*sq
```

```
rt(d) - (a^2 + 2*a + 1)*d)) + I*dilog(((a - 1)*b*d*x + b^2*c - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c + (2*I*a - 2*I)*b*sqrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)) - I*dilog(((a - 1)*b*d*x + b^2*c + (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/(b^2*c - (2*I*a - 2*I)*b*sqrt(c)*sqrt(d) - (a^2 - 2*a + 1)*d)))/sqrt(c*d)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a + bx)}{dx^2 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)/(c + d\*x^2), x)

[Out] int(acoth(a + b\*x)/(c + d\*x^2), x)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/(d\*x\*\*2+c), x)

[Out] Timed out

$$3.77 \quad \int \frac{\coth^{-1}(a+bx)}{c+dx} dx$$

**Optimal.** Leaf size=120

$$\frac{\operatorname{Li}_2\left(1 - \frac{2b(c+dx)}{(bc-ad+d)(a+bx+1)}\right)}{2d} + \frac{\coth^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d} + \frac{\operatorname{Li}_2\left(1 - \frac{2}{a+bx+1}\right)}{2d} - \frac{\log\left(\frac{2}{a+bx+1}\right) \coth^{-1}(a+bx)}{d}$$

[Out]  $-\operatorname{arccoth}(b*x+a)*\ln(2/(b*x+a+1))/d + \operatorname{arccoth}(b*x+a)*\ln(2*b*(d*x+c)/(-a*d+b*c+d)/(b*x+a+1))/d + 1/2*\operatorname{polylog}(2, 1-2/(b*x+a+1))/d - 1/2*\operatorname{polylog}(2, 1-2*b*(d*x+c)/(-a*d+b*c+d)/(b*x+a+1))/d$

**Rubi [A]** time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6112, 5921, 2402, 2315, 2447}

$$-\frac{\operatorname{PolyLog}\left(2, 1 - \frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{2d} + \frac{\operatorname{PolyLog}\left(2, 1 - \frac{2}{a+bx+1}\right)}{2d} + \frac{\coth^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(a+bx+1)(-ad+bc+d)}\right)}{d} - \frac{\log\left(\frac{2}{a+bx+1}\right) \coth^{-1}(a+bx)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[a + b*x]/(c + d*x), x]$

[Out]  $-\left(\left(\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[2/(1 + a + b*x)]\right)/d\right) + \left(\operatorname{ArcCoth}[a + b*x]*\operatorname{Log}[(2*b*(c + d*x))/((b*c + d - a*d)*(1 + a + b*x))]\right)/d + \operatorname{PolyLog}[2, 1 - 2/(1 + a + b*x)]/(2*d) - \operatorname{PolyLog}[2, 1 - (2*b*(c + d*x))/((b*c + d - a*d)*(1 + a + b*x))]/(2*d)$

**Rule 2315**

$\operatorname{Int}[\operatorname{Log}[(c_.)*(x_)]/((d_.) + (e_.)*(x_)), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /;$   $\operatorname{FreeQ}\{c, d, e\}, x \ \&\& \ \operatorname{EqQ}[e + c*d, 0]$

**Rule 2402**

$\operatorname{Int}[\operatorname{Log}[(c_.)/((d_.) + (e_.)*(x_))]/((f_.) + (g_.)*(x_)^2), x\_Symbol] \rightarrow -\operatorname{Dist}[e/g, \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, g\}, x \ \&\& \ \operatorname{EqQ}[c, 2*d] \ \&\& \ \operatorname{EqQ}[e^2*f + d^2*g, 0]$

**Rule 2447**

$\operatorname{Int}[\operatorname{Log}[u_]*(Pq_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{With}\{C = \operatorname{FullSimplify}[(Pq^m*(1 - u))/D[u, x]]\}, \operatorname{Simp}[C*\operatorname{PolyLog}[2, 1 - u], x] /;$   $\operatorname{FreeQ}[C, x] /;$   $\operatorname{IntegerQ}[m] \ \&\& \ \operatorname{PolyQ}[Pq, x] \ \&\& \ \operatorname{RationalFunctionQ}[u, x] \ \&\& \ \operatorname{LeQ}[\operatorname{RationalFunctionExponents}[u,$

x][[2]], Expon[Pq, x]]

### Rule 5921

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> -  
Simp[((a + b\*ArcCoth[c\*x])\*Log[2/(1 + c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log  
[2/(1 + c\*x)]/(1 - c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))  
/((c\*d + e)\*(1 + c\*x))]/(1 - c^2\*x^2), x], x] + Simp[((a + b\*ArcCoth[c\*x])\*  
Log[(2\*c\*(d + e\*x))/((c\*d + e)\*(1 + c\*x))])/e, x]) /; FreeQ[{a, b, c, d, e},  
x] && NeQ[c^2\*d^2 - e^2, 0]

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(  
m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*A  
rcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt  
Q[p, 0]

### Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{c+dx} dx = \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{\frac{bc-ad}{b} + \frac{dx}{b}} dx, x, a+bx\right)}{b}$$

$$= -\frac{\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+x}\right)}{1-x^2} dx, x, a+bx\right)}{2d}$$

$$= -\frac{\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} - \frac{\text{Li}_2\left(1 - \frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{2d}$$

$$= -\frac{\coth^{-1}(a+bx) \log\left(\frac{2}{1+a+bx}\right)}{d} + \frac{\coth^{-1}(a+bx) \log\left(\frac{2b(c+dx)}{(bc+d-ad)(1+a+bx)}\right)}{d} + \frac{\text{Li}_2\left(1 - \frac{2}{1+a+bx}\right)}{2d}$$

**Mathematica** [A] time = 0.07, size = 185, normalized size = 1.54

$$-\frac{\text{Li}_2\left(\frac{b(c+dx)}{bc-ad-d}\right)}{2d} + \frac{\text{Li}_2\left(\frac{b(c+dx)}{bc-ad+d}\right)}{2d} + \frac{\log(c+dx) \log\left(\frac{d(-a-bx+1)}{-ad+bc+d}\right)}{2d} - \frac{\log\left(\frac{a+bx-1}{a+bx}\right) \log(c+dx)}{2d} - \frac{\log(c+dx) \log\left(-\frac{d(a+bx+1)}{-ad+bc-d}\right)}{2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(c + d\*x),x]

[Out] (Log[(d\*(1 - a - b\*x))/(b\*c + d - a\*d)]\*Log[c + d\*x])/(2\*d) - (Log[(-1 + a + b\*x)/(a + b\*x)]\*Log[c + d\*x])/(2\*d) - (Log[-((d\*(1 + a + b\*x))/(b\*c - d - a\*d))]\*Log[c + d\*x])/(2\*d) + (Log[(1 + a + b\*x)/(a + b\*x)]\*Log[c + d\*x])/(2\*d) - PolyLog[2, (b\*(c + d\*x))/(b\*c - d - a\*d)]/(2\*d) + PolyLog[2, (b\*(c + d\*x))/(b\*c + d - a\*d)]/(2\*d)

**fricas** [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arccoth}(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x+c),x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)/(d\*x + c), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x+c),x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(d\*x + c), x)

**maple** [A] time = 0.08, size = 176, normalized size = 1.47

$$\frac{\ln(d(bx+a) - ad + bc) \operatorname{arccoth}(bx+a)}{d} + \frac{\ln(d(bx+a) - ad + bc) \ln\left(\frac{d(bx+a)-d}{ad-bc-d}\right)}{2d} + \frac{\operatorname{dilog}\left(\frac{d(bx+a)-d}{ad-bc-d}\right) \ln\left(\frac{d(bx+a)}{ad-bc-d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(d\*x+c),x)

[Out] ln(d\*(b\*x+a)-a\*d+b\*c)/d\*arccoth(b\*x+a)+1/2/d\*ln(d\*(b\*x+a)-a\*d+b\*c)\*ln((d\*(b\*x+a)-d)/(a\*d-b\*c-d))+1/2/d\*dilog((d\*(b\*x+a)-d)/(a\*d-b\*c-d))-1/2/d\*ln((d\*(b\*x+a)+d)/(a\*d-b\*c+d))\*ln(d\*(b\*x+a)-a\*d+b\*c)-1/2/d\*dilog((d\*(b\*x+a)+d)/(a\*d-b\*c+d))

**maxima** [A] time = 0.33, size = 192, normalized size = 1.60

$$-\frac{1}{2}b \left( \frac{\log(bx+a-1) \log\left(\frac{bdx+ad-d}{bc-ad+d} + 1\right) + \operatorname{Li}_2\left(-\frac{bdx+ad-d}{bc-ad+d}\right)}{bd} - \frac{\log(bx+a+1) \log\left(\frac{bdx+ad+d}{bc-ad-d} + 1\right) + \operatorname{Li}_2\left(-\frac{bdx+ad+d}{bc-ad-d}\right)}{bd} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(d\*x+c),x, algorithm="maxima")

[Out]  $-1/2*b*((\log(b*x + a - 1)*\log((b*d*x + a*d - d)/(b*c - a*d + d) + 1) + \operatorname{dilog}(- (b*d*x + a*d - d)/(b*c - a*d + d)))/(b*d) - (\log(b*x + a + 1)*\log((b*d*x + a*d + d)/(b*c - a*d - d) + 1) + \operatorname{dilog}(- (b*d*x + a*d + d)/(b*c - a*d - d)))/(b*d)) - 1/2*b*(\log(b*x + a + 1)/b - \log(b*x + a - 1)/b)*\log(d*x + c)/d + \operatorname{arccoth}(b*x + a)*\log(d*x + c)/d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)/(c + d\*x),x)

[Out] int(acoth(a + b\*x)/(c + d\*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/(d\*x+c),x)

[Out] Integral(acoth(a + b\*x)/(c + d\*x), x)

$$3.78 \quad \int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}} dx$$

**Optimal.** Leaf size=292

$$\frac{d\text{Li}_2\left(-\frac{b(d+cx)}{ac+c-bd}\right)}{2c^2} - \frac{d\text{Li}_2\left(\frac{b(d+cx)}{-ac+c+bd}\right)}{2c^2} + \frac{d\log\left(-\frac{-a-bx+1}{a+bx}\right)\log(cx+d)}{2c^2} - \frac{d\log(cx+d)\log\left(\frac{c(-a-bx+1)}{-ac+bd+c}\right)}{2c^2} + \frac{d\log(cx+d)\log\left(\frac{c(-a-bx+1)}{-ac+bd+c}\right)}{2c^2}$$

[Out]  $1/2*(-b*x-a+1)*\ln((b*x+a-1)/(b*x+a))/b/c+1/2*\ln(b*x+a)/b/c+1/2*\ln(b*x+a+1)/b/c+1/2*(b*x+a)*\ln((b*x+a+1)/(b*x+a))/b/c-1/2*d*\ln(c*(-b*x-a+1)/(-a*c+b*d+c))*\ln(c*x+d)/c^2+1/2*d*\ln((b*x+a-1)/(b*x+a))*\ln(c*x+d)/c^2+1/2*d*\ln(c*(b*x+a+1)/(a*c-b*d+c))*\ln(c*x+d)/c^2-1/2*d*\ln((b*x+a+1)/(b*x+a))*\ln(c*x+d)/c^2+1/2*d*\text{polylog}(2,-b*(c*x+d)/(a*c-b*d+c))/c^2-1/2*d*\text{polylog}(2,b*(c*x+d)/(-a*c+b*d+c))/c^2$

**Rubi [A]** time = 0.50, antiderivative size = 360, normalized size of antiderivative = 1.23, number of steps used = 37, number of rules used = 10, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {6116, 2513, 2409, 2389, 2295, 2394, 2393, 2391, 193, 43}

$$\frac{d\text{PolyLog}\left(2, \frac{c(-a-bx+1)}{-ac+bd+c}\right)}{2c^2} - \frac{d\text{PolyLog}\left(2, \frac{c(a+bx+1)}{ac-bd+c}\right)}{2c^2} + \frac{d\log(a+bx-1)\log\left(\frac{b(cx+d)}{-ac+bd+c}\right)}{2c^2} - \frac{d\left(\log(a+bx-1) - \log\left(\frac{b(cx+d)}{-ac+bd+c}\right)\right)}{2c^2}$$

Warning: Unable to verify antiderivative.

[In] Int[ArcCoth[a + b\*x]/(c + d/x), x]

[Out]  $((1-a-b*x)*\text{Log}[-1+a+b*x])/(2*b*c) + (x*(\text{Log}[-1+a+b*x] - \text{Log}[-((1-a-b*x)/(a+b*x))] - \text{Log}[a+b*x]))/(2*c) + ((1+a+b*x)*\text{Log}[1+a+b*x])/(2*b*c) + (x*(\text{Log}[a+b*x] - \text{Log}[1+a+b*x] + \text{Log}[(1+a+b*x)/(a+b*x]))) / (2*c) - (d*(\text{Log}[-1+a+b*x] - \text{Log}[-((1-a-b*x)/(a+b*x))] - \text{Log}[a+b*x])* \text{Log}[d+c*x]) / (2*c^2) - (d*(\text{Log}[a+b*x] - \text{Log}[1+a+b*x] + \text{Log}[(1+a+b*x)/(a+b*x]))* \text{Log}[d+c*x]) / (2*c^2) - (d*\text{Log}[1+a+b*x]* \text{Log}[-((b*(d+c*x))/(c+a*c-b*d))]) / (2*c^2) + (d*\text{Log}[-1+a+b*x]* \text{Log}[(b*(d+c*x))/(c-a*c+b*d)]) / (2*c^2) + (d*\text{PolyLog}[2, (c*(1-a-b*x))/(c-a*c+b*d)]) / (2*c^2) - (d*\text{PolyLog}[2, (c*(1+a+b*x))/(c+a*c-b*d)]) / (2*c^2)$

**Rule 43**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 193

$\text{Int}[(a_.) + (b_.) \cdot (x_.)^{(n_.)}]^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[x^{(n \cdot p)} \cdot (b + a/x^n)^p, x] /;$   $\text{FreeQ}\{a, b\}, x \} \ \&\& \ \text{LtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p]$

Rule 2295

$\text{Int}[\text{Log}[(c_.) \cdot (x_.)^{(n_.)}], x\_Symbol] \rightarrow \text{Simp}[x \cdot \text{Log}[c \cdot x^n], x] - \text{Simp}[n \cdot x, x] /;$   $\text{FreeQ}\{c, n\}, x \}$

Rule 2389

$\text{Int}[(a_.) + \text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_.)^{(n_.)})] \cdot (b_.)]^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b \cdot \text{Log}[c \cdot x^n])^p, x], x, d + e \cdot x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, n, p\}, x \}$

Rule 2391

$\text{Int}[\text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_.)^{(n_.)})] / (x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)] / n, x] /;$   $\text{FreeQ}\{c, d, e, n\}, x \} \ \&\& \ \text{EqQ}[c \cdot d, 1]$

Rule 2393

$\text{Int}[(a_.) + \text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_.)^{(n_.)})] \cdot (b_.)] / ((f_.) + (g_.) \cdot (x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b \cdot \text{Log}[1 + (c \cdot e \cdot x)/g]] / x, x], x, f + g \cdot x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g\}, x \} \ \&\& \ \text{NeQ}[e \cdot f - d \cdot g, 0] \ \&\& \ \text{EqQ}[g + c \cdot (e \cdot f - d \cdot g), 0]$

Rule 2394

$\text{Int}[(a_.) + \text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_.)^{(n_.)})] \cdot (b_.)] / ((f_.) + (g_.) \cdot (x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e \cdot (f + g \cdot x)) / (e \cdot f - d \cdot g)] \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n]) / g, x] - \text{Dist}[(b \cdot e \cdot n) / g, \text{Int}[\text{Log}[(e \cdot (f + g \cdot x)) / (e \cdot f - d \cdot g)] / (d + e \cdot x), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n\}, x \} \ \&\& \ \text{NeQ}[e \cdot f - d \cdot g, 0]$

Rule 2409

$\text{Int}[(a_.) + \text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_.)^{(n_.)})] \cdot (b_.)]^{(p_.)} \cdot ((f_.) + (g_.) \cdot (x_.)^{(r_.)})^{(q_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n])^p, (f + g \cdot x^r)^q, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, r\}, x \} \ \&\& \ \text{IntegerQ}[p, 0] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ (\text{GtQ}[q, 0] \ || \ (\text{IntegerQ}[r] \ \&\& \ \text{NeQ}[r, 1]))$

Rule 2513



```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFX_), x_Symbol] := Dist[p*r, Int[RFX*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFX*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r], Int[RFX, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFX, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFX, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n
]

```

### Rule 6116

```

Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x}} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{x}} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{x}} dx \\
&= -\left(\frac{1}{2} \int \frac{\log(-1+a+bx)}{c+\frac{d}{x}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+\frac{d}{x}} dx - \frac{1}{2} \left(-\log(-1+a+bx) + \log\right) \\
&= -\left(\frac{1}{2} \int \left(\frac{\log(-1+a+bx)}{c} - \frac{d \log(-1+a+bx)}{c(d+cx)}\right) dx\right) + \frac{1}{2} \int \left(\frac{\log(1+a+bx)}{c} - \frac{d \log(1+a+bx)}{c(d+cx)}\right) dx \\
&= -\frac{\int \log(-1+a+bx) dx}{2c} + \frac{\int \log(1+a+bx) dx}{2c} + \frac{d \int \frac{\log(-1+a+bx)}{d+cx} dx}{2c} - \frac{d \int \frac{\log(1+a+bx)}{d+cx} dx}{2c} \\
&= \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} + \frac{x \left(\log(a+bx) - \log(1+a+bx)\right)}{2c} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} + \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} +
\end{aligned}$$

**Mathematica** [C] time = 4.49, size = 502, normalized size = 1.72

$$b^2 d^2 \sqrt{1 - \frac{c^2}{(ac-bd)^2}} \coth^{-1}(a+bx)^2 e^{\tanh^{-1}\left(\frac{c}{ac-bd}\right)} - b^2 d^2 \coth^{-1}(a+bx)^2 - abcd \sqrt{1 - \frac{c^2}{(ac-bd)^2}} \coth^{-1}(a+bx)^2 e^{\tanh^{-1}\left(\frac{c}{ac-bd}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(c + d/x), x]

[Out] (2\*a\*c^2\*ArcCoth[a + b\*x] - I\*b\*c\*d\*Pi\*ArcCoth[a + b\*x] + 2\*b\*c^2\*x\*ArcCoth[a + b\*x] + b\*c\*d\*ArcCoth[a + b\*x]^2 + a\*b\*c\*d\*ArcCoth[a + b\*x]^2 - b^2\*d^2\*ArcCoth[a + b\*x]^2 - a\*b\*c\*d\*Sqrt[1 - c^2/(a\*c - b\*d)^2]\*E^ArcTanh[c/(a\*c - b\*d)]\*ArcCoth[a + b\*x]^2 + b^2\*d^2\*Sqrt[1 - c^2/(a\*c - b\*d)^2]\*E^ArcTanh[c/(a\*c - b\*d)]\*ArcCoth[a + b\*x]^2 + 2\*b\*c\*d\*ArcCoth[a + b\*x]\*ArcTanh[c/(a\*c - b\*d)] + 2\*b\*c\*d\*ArcCoth[a + b\*x]\*Log[1 - E^(-2\*ArcCoth[a + b\*x])] + I\*b\*c\*d\*Pi\*Log[1 + E^(2\*ArcCoth[a + b\*x])] - 2\*b\*c\*d\*ArcCoth[a + b\*x]\*Log[1 - E^(-2\*ArcCoth[a + b\*x] + 2\*ArcTanh[c/(a\*c - b\*d)])] + 2\*b\*c\*d\*ArcTanh[c/(a\*c - b\*d)]\*Log[1 - E^(-2\*ArcCoth[a + b\*x] + 2\*ArcTanh[c/(a\*c - b\*d)])] - I\*b\*c\*d\*Pi\*Log[1/Sqrt[1 - (a + b\*x)^(-2)]] - 2\*c^2\*Log[1/((a + b\*x)\*Sqrt[1 - (a + b\*x)^(-2)])] - 2\*b\*c\*d\*ArcTanh[c/(a\*c - b\*d)]\*Log[I\*Sinh[ArcCoth[a + b\*x] - ArcTanh[c/(a\*c - b\*d)]]] - b\*c\*d\*PolyLog[2, E^(-2\*ArcCoth[a + b\*x])] + b\*c\*d\*PolyLog[2, E^(-2\*ArcCoth[a + b\*x] + 2\*ArcTanh[c/(a\*c - b\*d)])])/(2\*b\*c^3)

**fricas** [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x \operatorname{arccoth}(bx+a)}{cx+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x), x, algorithm="fricas")

[Out] integral(x\*arccoth(b\*x + a)/(c\*x + d), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x), x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(c + d/x), x)

**maple** [A] time = 0.09, size = 297, normalized size = 1.02

$$\frac{\operatorname{arccoth}(bx+a)x}{c} + \frac{\operatorname{arccoth}(bx+a)a}{bc} - \frac{\operatorname{arccoth}(bx+a)d \ln(c(bx+a) - ac + bd)}{c^2} + \frac{\ln(a^2c^2 - 2abcd + b^2d^2 + \dots)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(c+d/x), x)

[Out] arccoth(b\*x+a)/c\*x+1/b\*arccoth(b\*x+a)/c\*a-arccoth(b\*x+a)\*d/c^2\*ln(c\*(b\*x+a)-a\*c+b\*d)+1/2/b/c\*ln(a^2\*c^2-2\*a\*b\*c\*d+b^2\*d^2+2\*(c\*(b\*x+a)-a\*c+b\*d)\*a\*c-2\*(c\*(b\*x+a)-a\*c+b\*d)\*b\*d+(c\*(b\*x+a)-a\*c+b\*d)^2-c^2)-1/2/c^2\*d\*ln(c\*(b\*x+a)-a\*c+b\*d)\*ln((c\*(b\*x+a)-c)/(a\*c-b\*d-c))-1/2/c^2\*d\*dilog((c\*(b\*x+a)-c)/(a\*c-b\*d-c))+1/2/c^2\*d\*ln(c\*(b\*x+a)-a\*c+b\*d)\*ln((c\*(b\*x+a)+c)/(a\*c-b\*d+c))+1/2/c^2\*d\*dilog((c\*(b\*x+a)+c)/(a\*c-b\*d+c))

**maxima** [A] time = 0.34, size = 192, normalized size = 0.66

$$\frac{1}{2} b \left( \frac{\left( \log(cx+d) \log\left(\frac{bcx+bd}{ac-bd+c} + 1\right) + \operatorname{Li}_2\left(-\frac{bcx+bd}{ac-bd+c}\right) \right) d}{bc^2} - \frac{\left( \log(cx+d) \log\left(\frac{bcx+bd}{ac-bd-c} + 1\right) + \operatorname{Li}_2\left(-\frac{bcx+bd}{ac-bd-c}\right) \right) d}{bc^2} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x), x, algorithm="maxima")

[Out] 1/2\*b\*((log(c\*x + d)\*log((b\*c\*x + b\*d)/(a\*c - b\*d + c) + 1) + dilog(-(b\*c\*x + b\*d)/(a\*c - b\*d + c)))\*d/(b\*c^2) - (log(c\*x + d)\*log((b\*c\*x + b\*d)/(a\*c - b\*d - c) + 1) + dilog(-(b\*c\*x + b\*d)/(a\*c - b\*d - c)))\*d/(b\*c^2) + (a + 1)\*log(b\*x + a + 1)/(b^2\*c) - (a - 1)\*log(b\*x + a - 1)/(b^2\*c)) + (x/c - d\*log(c\*x + d)/c^2)\*arccoth(b\*x + a)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a + bx)}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)/(c + d/x), x)

[Out] int(acoth(a + b\*x)/(c + d/x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{acoth}(a + bx)}{cx + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/(c+d/x), x)

[Out] Integral(x\*acoth(a + b\*x)/(c\*x + d), x)

$$3.79 \quad \int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}} dx$$

Optimal. Leaf size=738

$$\frac{\sqrt{d} \left( \log(a+bx-1) - \log\left(-\frac{-a-bx+1}{a+bx}\right) - \log(a+bx) \right) \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{3/2}} - \frac{\sqrt{d} \left( \log(a+bx) - \log(a+bx+1) + \log\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \right)}{2c^{3/2}}$$

[Out]  $\frac{1}{2}*(-b*x-a+1)*\ln(b*x+a-1)/b/c+1/2*x*(\ln(b*x+a-1)-\ln((b*x+a-1)/(b*x+a)))-\ln(b*x+a)/c+1/2*(b*x+a+1)*\ln(b*x+a+1)/b/c+1/2*x*(\ln(b*x+a)-\ln(b*x+a+1)+\ln((b*x+a+1)/(b*x+a)))/c-1/2*\arctan(x*c^{(1/2)}/d^{(1/2)})*(\ln(b*x+a-1)-\ln((b*x+a-1)/(b*x+a)))-\ln(b*x+a)*d^{(1/2)}/c^{(3/2)}-1/2*\arctan(x*c^{(1/2)}/d^{(1/2)})*(\ln(b*x+a)-\ln(b*x+a+1)+\ln((b*x+a+1)/(b*x+a)))*d^{(1/2)}/c^{(3/2)}+1/4*\ln(b*x+a-1)*\ln(-b*(-x*(-c)^{(1/2)}+d^{(1/2)})/((1-a)*(-c)^{(1/2)}-b*d^{(1/2)}))*d^{(1/2)}/(-c)^{(3/2)}+1/4*\ln(b*x+a+1)*\ln(-b*(x*(-c)^{(1/2)}+d^{(1/2)})/((1+a)*(-c)^{(1/2)}-b*d^{(1/2)}))*d^{(1/2)}/(-c)^{(3/2)}-1/4*\ln(b*x+a-1)*\ln(b*(x*(-c)^{(1/2)}+d^{(1/2)})/((1-a)*(-c)^{(1/2)}+b*d^{(1/2)}))*d^{(1/2)}/(-c)^{(3/2)}-1/4*\ln(b*x+a+1)*\ln(b*(-x*(-c)^{(1/2)}+d^{(1/2)})/((1+a)*(-c)^{(1/2)}+b*d^{(1/2)}))*d^{(1/2)}/(-c)^{(3/2)}+1/4*\text{polylog}(2,(-b*x-a+1)*(-c)^{(1/2)}/((1-a)*(-c)^{(1/2)}-b*d^{(1/2)}))*d^{(1/2)}/(-c)^{(3/2)}+1/4*\text{polylog}(2,(b*x+a+1)*(-c)^{(1/2)}/((1+a)*(-c)^{(1/2)}-b*d^{(1/2)}))*d^{(1/2)}/(-c)^{(3/2)}-1/4*\text{polylog}(2,(-b*x-a+1)*(-c)^{(1/2)}/((1-a)*(-c)^{(1/2)}+b*d^{(1/2)}))*d^{(1/2)}/(-c)^{(3/2)}-1/4*\text{polylog}(2,(b*x+a+1)*(-c)^{(1/2)}/((1+a)*(-c)^{(1/2)}+b*d^{(1/2)}))*d^{(1/2)}/(-c)^{(3/2)}$

**Rubi [A]** time = 1.53, antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 57, number of rules used = 11, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$ , Rules used = {6116, 2513, 2409, 2389, 2295, 2394, 2393, 2391, 193, 321, 205}

$$\frac{\sqrt{d} \text{PolyLog}\left(2, \frac{\sqrt{-c}(-a-bx+1)}{(1-a)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d} \text{PolyLog}\left(2, \frac{\sqrt{-c}(-a-bx+1)}{(1-a)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}} + \frac{\sqrt{d} \text{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx+1)}{(a+1)\sqrt{-c}-b\sqrt{d}}\right)}{4(-c)^{3/2}} - \frac{\sqrt{d} \text{PolyLog}\left(2, \frac{\sqrt{-c}(a+bx+1)}{(a+1)\sqrt{-c}+b\sqrt{d}}\right)}{4(-c)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/(c + d/x^2), x]

[Out]  $((1-a-b*x)*\text{Log}[-1+a+b*x])/(2*b*c) + (x*(\text{Log}[-1+a+b*x] - \text{Log}[-((1-a-b*x)/(a+b*x))] - \text{Log}[a+b*x]))/(2*c) - (\text{Sqrt}[d]*\text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]])*(\text{Log}[-1+a+b*x] - \text{Log}[-((1-a-b*x)/(a+b*x))] - \text{Log}[a+b*x])/(2*c^{(3/2)}) + ((1+a+b*x)*\text{Log}[1+a+b*x])/(2*b*c) + (x*(\text{Log}[a+b*x] - \text{Log}[1+a+b*x] + \text{Log}[(1+a+b*x)/(a+b*x]])))/(2*c) - (\text{Sqrt}[d]*\text{ArcTan}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]])*(\text{Log}[a+b*x] - \text{Log}[1+a+b*x] + \text{Log}[(1+a+b*x)/(a+b*x]]))/(2*c^{(3/2)})$

$$\begin{aligned} & b*x)/(a + b*x)])))/(2*c^(3/2)) + (\text{Sqrt}[d]*\text{Log}[-1 + a + b*x]*\text{Log}[-((b*(\text{Sqrt}[d] - \text{Sqrt}[-c]*x))/((1 - a)*\text{Sqrt}[-c] - b*\text{Sqrt}[d])))]/(4*(-c)^(3/2)) - (\text{Sqrt}[d]*\text{Log}[1 + a + b*x]*\text{Log}[(b*(\text{Sqrt}[d] - \text{Sqrt}[-c]*x))/((1 + a)*\text{Sqrt}[-c] + b*\text{Sqrt}[d])))]/(4*(-c)^(3/2)) + (\text{Sqrt}[d]*\text{Log}[1 + a + b*x]*\text{Log}[-((b*(\text{Sqrt}[d] + \text{Sqrt}[-c]*x))/((1 + a)*\text{Sqrt}[-c] - b*\text{Sqrt}[d])))]/(4*(-c)^(3/2)) - (\text{Sqrt}[d]*\text{Log}[-1 + a + b*x]*\text{Log}[(b*(\text{Sqrt}[d] + \text{Sqrt}[-c]*x))/((1 - a)*\text{Sqrt}[-c] + b*\text{Sqrt}[d])))]/(4*(-c)^(3/2)) + (\text{Sqrt}[d]*\text{PolyLog}[2, (\text{Sqrt}[-c]*(1 - a - b*x))/((1 - a)*\text{Sqrt}[-c] - b*\text{Sqrt}[d])])/(4*(-c)^(3/2)) - (\text{Sqrt}[d]*\text{PolyLog}[2, (\text{Sqrt}[-c]*(1 - a - b*x))/((1 - a)*\text{Sqrt}[-c] + b*\text{Sqrt}[d])])/(4*(-c)^(3/2)) + (\text{Sqrt}[d]*\text{PolyLog}[2, (\text{Sqrt}[-c]*(1 + a + b*x))/((1 + a)*\text{Sqrt}[-c] - b*\text{Sqrt}[d])])/(4*(-c)^(3/2)) - (\text{Sqrt}[d]*\text{PolyLog}[2, (\text{Sqrt}[-c]*(1 + a + b*x))/((1 + a)*\text{Sqrt}[-c] + b*\text{Sqrt}[d])])/(4*(-c)^(3/2)) \end{aligned}$$
Rule 193

$$\text{Int}[((a_) + (b_)*(x_)^(n_))^(p_), x\_Symbol] \text{ :> Int}[x^(n*p)*(b + a/x^n)^p, x] \text{ /; FreeQ}\{a, b\}, x\} \ \&\& \ \text{LtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p]$$
Rule 205

$$\text{Int}(((a_) + (b_)*(x_)^2)^(-1), x\_Symbol] \text{ :> Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \text{ /; FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$$
Rule 321

$$\text{Int}(((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x\_Symbol] \text{ :> Simp}[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - \text{Dist}[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^(m - n)*(a + b*x^n)^p, x], x] \text{ /; FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$$
Rule 2295

$$\text{Int}[\text{Log}[(c_)*(x_)^(n_)], x\_Symbol] \text{ :> Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] \text{ /; FreeQ}\{c, n\}, x\}$$
Rule 2389

$$\text{Int}(((a_.) + \text{Log}[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_.))^(p_.), x\_Symbol] \text{ :> Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] \text{ /; FreeQ}\{a, b, c, d, e, n, p\}, x\}$$
Rule 2391

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x\_Symbol] \text{ :> -Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] \text{ /; FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$$

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2513

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*(RFx_), x_Symbol] := Dist[p*r, Int[RFx*Log[a + b*x], x], x] + (Dis
t[q*r, Int[RFx*Log[c + d*x], x], x] - Dist[p*r*Log[a + b*x] + q*r*Log[c + d
*x] - Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r, Int[RFx, x], x]) /; FreeQ[{a, b
, c, d, e, f, p, q, r}, x] && RationalFunctionQ[RFx, x] && NeQ[b*c - a*d, 0
] && !MatchQ[RFx, (u_.)*(a + b*x)^(m_.)*(c + d*x)^(n_.) /; IntegersQ[m, n]
]
```

Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist
[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[
Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{x^2}} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{x^2}} dx \\
&= -\left(\frac{1}{2} \int \frac{\log(-1+a+bx)}{c+\frac{d}{x^2}} dx\right) + \frac{1}{2} \int \frac{\log(1+a+bx)}{c+\frac{d}{x^2}} dx - \frac{1}{2} \left(-\log(-1+a+bx) + \log\left(\frac{1+a+bx}{a+bx}\right)\right) \\
&= -\left(\frac{1}{2} \int \left(\frac{\log(-1+a+bx)}{c} - \frac{d \log(-1+a+bx)}{c(d+cx^2)}\right) dx\right) + \frac{1}{2} \int \left(\frac{\log(1+a+bx)}{c} - \frac{d \log(1+a+bx)}{c(d+cx^2)}\right) dx \\
&= \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} + \frac{x \left(\log(a+bx) - \log(1+a+bx) + \log\left(\frac{1+a+bx}{a+bx}\right)\right)}{2c} \\
&= \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^3} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^3} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^3} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^3} \\
&= \frac{(1-a-bx) \log(-1+a+bx)}{2bc} + \frac{x \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c} - \frac{\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \left(\log(-1+a+bx) - \log\left(-\frac{1-a-bx}{a+bx}\right) - \log(a+bx)\right)}{2c^3}
\end{aligned}$$

**Mathematica [C]** time = 36.35, size = 5552, normalized size = 7.52

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(c + d/x^2), x]

[Out] Result too large to show



**fricas** [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2 \operatorname{arccoth}(bx + a)}{cx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^2),x, algorithm="fricas")

[Out] integral(x^2\*arccoth(b\*x + a)/(c\*x^2 + d), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx + a)}{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^2),x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(c + d/x^2), x)

**maple** [C] time = 1.88, size = 19686, normalized size = 26.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(c+d/x^2),x)

[Out] result too large to display

**maxima** [C] time = 0.55, size = 647, normalized size = 0.88

$$-\left(\frac{d \arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c} - \frac{x}{c}\right) \operatorname{arccoth}(bx + a) + \frac{2(a+1)c \log(bx + a + 1) - 2(a-1)c \log(bx + a - 1) + \left(b \arctan\left(\frac{\sqrt{c}}{\sqrt{a}}\right)\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^2),x, algorithm="maxima")

[Out]  $-(d*\arctan(cx/\sqrt{cd}))/(\sqrt{cd}*c) - x/c)*\operatorname{arccoth}(bx + a) + 1/4*(2*(a + 1)*c*\log(b*x + a + 1) - 2*(a - 1)*c*\log(b*x + a - 1) + (b*\arctan(\sqrt{c})*x/\sqrt{d})*\log((b^2*c*x^2 + 2*(a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d + (a^2 + 2*a + 1)*c)) - b*\arctan(\sqrt{c})*x/\sqrt{d})*\log((b^2*c*x^2 + 2*(a - 1$

```

)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 - 2*a + 1)*c)) + I*b*dilog(((a +
1)*b*c*x + b^2*d - (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/((2*I*a + 2*I
)*b*sqrt(c)*sqrt(d) + b^2*d - (a^2 + 2*a + 1)*c)) - I*b*dilog(-((a + 1)*b*c
*x + b^2*d + (I*b^2*x + (-I*a - I)*b)*sqrt(c)*sqrt(d))/((2*I*a + 2*I)*b*sqrt
(c)*sqrt(d) - b^2*d + (a^2 + 2*a + 1)*c)) - I*b*dilog(((a - 1)*b*c*x + b^2
*d - (I*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/((2*I*a - 2*I)*b*sqrt(c)*sqrt
(d) + b^2*d - (a^2 - 2*a + 1)*c)) + I*b*dilog(-((a - 1)*b*c*x + b^2*d + (I
*b^2*x + (-I*a + I)*b)*sqrt(c)*sqrt(d))/((2*I*a - 2*I)*b*sqrt(c)*sqrt(d) -
b^2*d + (a^2 - 2*a + 1)*c)) - (b*arctan2((b^2*x + (a + 1)*b)*sqrt(c)*sqrt(d)
)/(b^2*d + (a^2 + 2*a + 1)*c), ((a + 1)*b*c*x + (a^2 + 2*a + 1)*c)/(b^2*d +
(a^2 + 2*a + 1)*c)) - b*arctan2((b^2*x + (a - 1)*b)*sqrt(c)*sqrt(d)/(b^2*d
+ (a^2 - 2*a + 1)*c), ((a - 1)*b*c*x + (a^2 - 2*a + 1)*c)/(b^2*d + (a^2 -
2*a + 1)*c))) * log(c*x^2 + d) * sqrt(c) * sqrt(d) / (b*c^2)

```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a + bx)}{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)/(c + d/x^2), x)

[Out] int(acoth(a + b\*x)/(c + d/x^2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/(c+d/x\*\*2), x)

[Out] Timed out

$$3.80 \quad \int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}} dx$$

**Optimal.** Leaf size=619

$$\frac{c \operatorname{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2} + \frac{c \operatorname{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{-a-1}d}\right)}{d^2} - \frac{c \operatorname{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-ad}}\right)}{d^2} - \frac{c \operatorname{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c+\sqrt{1-ad}}\right)}{d^2} + \frac{c \log(c+d\sqrt{x}) \log\left(\frac{d(\sqrt{-a-1}-\sqrt{-a-1}d)}{\sqrt{-a-1}d}\right)}{d^2}$$

[Out]  $c \ln((b*x+a-1)/(b*x+a)) * \ln(c+d*x^{(1/2)})/d^2 - c \ln((b*x+a+1)/(b*x+a)) * \ln(c+d*x^{(1/2)})/d^2 + c \ln(c+d*x^{(1/2)}) * \ln(d*((-1-a)^{(1/2)}-b^{(1/2)}*x^{(1/2)})/(d*(-a)^{(1/2)}+c*b^{(1/2)}))/d^2 - c \ln(c+d*x^{(1/2)}) * \ln(d*((1-a)^{(1/2)}-b^{(1/2)}*x^{(1/2)})/(d*(1-a)^{(1/2)}+c*b^{(1/2)}))/d^2 + c \ln(c+d*x^{(1/2)}) * \ln(-d*((-1-a)^{(1/2)}+b^{(1/2)}*x^{(1/2)})/(-d*(-1-a)^{(1/2)}+c*b^{(1/2)}))/d^2 - c \ln(c+d*x^{(1/2)}) * \ln(-d*((1-a)^{(1/2)}+b^{(1/2)}*x^{(1/2)})/(-d*(1-a)^{(1/2)}+c*b^{(1/2)}))/d^2 + c \operatorname{polylog}(2, b^{(1/2)}*(c+d*x^{(1/2)})/(-d*(-1-a)^{(1/2)}+c*b^{(1/2)}))/d^2 + c \operatorname{polylog}(2, b^{(1/2)}*(c+d*x^{(1/2)})/(d*(-1-a)^{(1/2)}+c*b^{(1/2)}))/d^2 - c \operatorname{polylog}(2, b^{(1/2)}*(c+d*x^{(1/2)})/(-d*(1-a)^{(1/2)}+c*b^{(1/2)}))/d^2 - c \operatorname{polylog}(2, b^{(1/2)}*(c+d*x^{(1/2)})/(-d*(1-a)^{(1/2)}+c*b^{(1/2)}))/d^2 - 2 * \operatorname{arctanh}(b^{(1/2)}*x^{(1/2)/(1-a)^{(1/2)})} * (1-a)^{(1/2)}/d/b^{(1/2)} + 2 * \operatorname{arctan}(b^{(1/2)}*x^{(1/2)/(1+a)^{(1/2)})} * (1+a)^{(1/2)}/d/b^{(1/2)} - \ln((b*x+a-1)/(b*x+a)) * x^{(1/2)}/d + \ln((b*x+a+1)/(b*x+a)) * x^{(1/2)}/d$

**Rubi [A]** time = 2.20, antiderivative size = 619, normalized size of antiderivative = 1.00, number of steps used = 55, number of rules used = 16, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$ , Rules used = {6116, 190, 43, 2528, 2523, 12, 481, 205, 2524, 2418, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{-a-1}d}\right)}{d^2} + \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{-a-1}d+\sqrt{b}c}\right)}{d^2} - \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{b}c-\sqrt{1-ad}}\right)}{d^2} - \frac{c \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{1-ad}+\sqrt{b}c}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/(c + d\*Sqrt[x]), x]

[Out]  $(2*\operatorname{Sqrt}[1+a]*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])/(\operatorname{Sqrt}[1+a])]) / (\operatorname{Sqrt}[b]*d) - (2*\operatorname{Sqrt}[1-a]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])/(\operatorname{Sqrt}[1-a])]) / (\operatorname{Sqrt}[b]*d) + (c*\operatorname{Log}[(d*(\operatorname{Sqrt}[-1-a] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])) / (\operatorname{Sqrt}[b]*c + \operatorname{Sqrt}[-1-a]*d)] * \operatorname{Log}[c + d*\operatorname{Sqrt}[x]]) / d^2 - (c*\operatorname{Log}[(d*(\operatorname{Sqrt}[1-a] - \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])) / (\operatorname{Sqrt}[b]*c + \operatorname{Sqrt}[1-a]*d)] * \operatorname{Log}[c + d*\operatorname{Sqrt}[x]]) / d^2 + (c*\operatorname{Log}[-((d*(\operatorname{Sqrt}[-1-a] + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])) / (\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[-1-a]*d))] * \operatorname{Log}[c + d*\operatorname{Sqrt}[x]]) / d^2 - (c*\operatorname{Log}[-((d*(\operatorname{Sqrt}[1-a] + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[x])) / (\operatorname{Sqrt}[b]*c - \operatorname{Sqrt}[1-a]*d))] * \operatorname{Log}[c + d*\operatorname{Sqrt}[x]]) / d^2 - (\operatorname{Sqrt}[x]*\operatorname{Log}[-((1-a-b*x)/(a+b*x))]) / d + (c*\operatorname{Log}[c + d*\operatorname{Sqrt}[x]] * \operatorname{Log}[-((1-a-b*x)/(a+b*x))]) / d^2 + (\operatorname{Sqrt}[x]*\operatorname{Log}[(1+a+b*x)/(a$

+ b\*x))/d - (c\*Log[c + d\*Sqrt[x]]\*Log[(1 + a + b\*x)/(a + b\*x)]/d^2 + (c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[-1 - a]\*d)]/d^2 + (c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-1 - a]\*d)]/d^2 - (c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[1 - a]\*d)]/d^2 - (c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[1 - a]\*d)]/d^2

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 190

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 481

Int[((e\_.)\*(x\_))^(m\_.)/(((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_))), x\_Symbol] := -Dist[(a\*e^n)/(b\*c - a\*d), Int[(e\*x)^(m - n)/(a + b\*x^n), x], x] + Dist[(c\*e^n)/(b\*c - a\*d), Int[(e\*x)^(m - n)/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && LeQ[n, m,

2\*n - 1]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_)^(m\_.))\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2523

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*RFx^p])^n, x] - Dist[b^n\*p, Int[SimplifyIntegrand[(x\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e,
Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol]
:> With[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]]
/; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x]
&& IGtQ[n, 0]
```

### Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol]
:> Dist[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
&& RationalQ[n]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+d\sqrt{x}} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+d\sqrt{x}} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+d\sqrt{x}} dx \\
&= -\text{Subst}\left(\int \frac{x \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c+dx} dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{x \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c+dx} dx, x, \sqrt{x}\right) \\
&= -\text{Subst}\left(\int \left(\frac{\log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d} - \frac{c \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right) dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \left(\frac{\log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d} - \frac{c \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d(c+dx)}\right) dx, x, \sqrt{x}\right) \\
&= -\frac{\text{Subst}\left(\int \log\left(\frac{-1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x}\right)}{d} + \frac{\text{Subst}\left(\int \log\left(\frac{1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x}\right)}{d} + \frac{c \text{Subst}\left(\int \frac{1}{c+dx} dx, x, \sqrt{x}\right)}{d} \\
&= -\frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c \log(c+d\sqrt{x}) \log\left(\frac{1+a+bx}{a+bx}\right)}{d^2} \\
&= -\frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c \log(c+d\sqrt{x}) \log\left(\frac{1+a+bx}{a+bx}\right)}{d^2} \\
&= -\frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} + \frac{\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{d} - \frac{c \log(c+d\sqrt{x}) \log\left(\frac{1+a+bx}{a+bx}\right)}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} - \frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} - \frac{\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{d} + \frac{c \log(c+d\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c \log\left(\frac{d(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right) \log(c+d\sqrt{x})}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c \log\left(\frac{d(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right) \log(c+d\sqrt{x})}{d^2} \\
&= \frac{2\sqrt{1+a} \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}d} - \frac{2\sqrt{1-a} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}d} + \frac{c \log\left(\frac{d(\sqrt{-1-a}-\sqrt{b}\sqrt{x})}{\sqrt{b}c+\sqrt{-1-a}d}\right) \log(c+d\sqrt{x})}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.71, size = 575, normalized size = 0.93

$$c\text{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{bc}-\sqrt{-a-1}d}\right) + c\text{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{bc}+\sqrt{-a-1}d}\right) - c\text{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{bc}-\sqrt{1-ad}}\right) - c\text{Li}_2\left(\frac{\sqrt{b}(c+d\sqrt{x})}{\sqrt{bc}+\sqrt{1-ad}}\right) + c\log(c+d\sqrt{x})\log\left(\frac{d(\sqrt{-a-1}}{\sqrt{-a-1}}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*x]/(c + d\*Sqrt[x]),x]

[Out] ((2\*Sqrt[1 + a]\*d\*ArcTan[(Sqrt[b]\*Sqrt[x])/Sqrt[1 + a]]/Sqrt[b] - (2\*Sqrt[1 - a]\*d\*ArcTanh[(Sqrt[b]\*Sqrt[x])/Sqrt[1 - a]]/Sqrt[b] + c\*Log[(d\*(Sqrt[-1 - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-1 - a]\*d)]\*Log[c + d\*Sqrt[x]] - c\*Log[(d\*(Sqrt[1 - a] - Sqrt[b]\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[1 - a]\*d)]\*Log[c + d\*Sqrt[x]] + c\*Log[(d\*(Sqrt[-1 - a] + Sqrt[b]\*Sqrt[x]))/(-(Sqrt[b]\*c) + Sqrt[-1 - a]\*d)]\*Log[c + d\*Sqrt[x]] - c\*Log[(d\*(Sqrt[1 - a] + Sqrt[b]\*Sqrt[x]))/(-(Sqrt[b]\*c) + Sqrt[1 - a]\*d)]\*Log[c + d\*Sqrt[x]] - d\*Sqrt[x]\*Log[(-1 + a + b\*x)/(a + b\*x)] + c\*Log[c + d\*Sqrt[x]]\*Log[(-1 + a + b\*x)/(a + b\*x)] + d\*Sqrt[x]\*Log[(1 + a + b\*x)/(a + b\*x)] - c\*Log[c + d\*Sqrt[x]]\*Log[(1 + a + b\*x)/(a + b\*x)] + c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[-1 - a]\*d)] + c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[-1 - a]\*d)] - c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c - Sqrt[1 - a]\*d)] - c\*PolyLog[2, (Sqrt[b]\*(c + d\*Sqrt[x]))/(Sqrt[b]\*c + Sqrt[1 - a]\*d)]))/d^2

**fricas [F]** time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d\sqrt{x}\operatorname{arccoth}(bx+a)-c\operatorname{arccoth}(bx+a)}{d^2x-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d\*x^(1/2)),x, algorithm="fricas")

[Out] integral((d\*sqrt(x)\*arccoth(b\*x + a) - c\*arccoth(b\*x + a))/(d^2\*x - c^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)}{d\sqrt{x}+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d\*x^(1/2)),x, algorithm="giac")



[Out] integrate(arccoth(b\*x + a)/(d\*sqrt(x) + c), x)

**maple** [A] time = 0.13, size = 738, normalized size = 1.19

$$\frac{2 \operatorname{arccoth}(bx + a) \sqrt{x}}{d} - \frac{2 \operatorname{arccoth}(bx + a) c \ln(c + d\sqrt{x})}{d^2} + \frac{2 \arctan\left(\frac{2b(c+d\sqrt{x})-2bc}{2\sqrt{ad^2b+bd^2}}\right)}{\sqrt{ad^2b+bd^2}} + \frac{2 \arctan\left(\frac{2b(c+d\sqrt{x})-2bc}{2\sqrt{ad^2b+bd^2}}\right) a}{\sqrt{ad^2b+bd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(c+d\*x^(1/2)), x)

[Out]  $2 \operatorname{arccoth}(bx+a) \cdot x^{1/2} / d - 2 \operatorname{arccoth}(bx+a) \cdot c / d^2 \cdot \ln(c+d \cdot x^{1/2}) + 2 / (a \cdot b \cdot d^2 + b \cdot d^2)^{1/2} \cdot \arctan(1/2 \cdot (2 \cdot b \cdot (c+d \cdot x^{1/2}) - 2 \cdot b \cdot c) / (a \cdot b \cdot d^2 + b \cdot d^2)^{1/2}) + 2 / (a \cdot b \cdot d^2 + b \cdot d^2)^{1/2} \cdot \arctan(1/2 \cdot (2 \cdot b \cdot (c+d \cdot x^{1/2}) - 2 \cdot b \cdot c) / (a \cdot b \cdot d^2 + b \cdot d^2)^{1/2}) \cdot a + 2 / (a \cdot b \cdot d^2 - b \cdot d^2)^{1/2} \cdot \arctan(1/2 \cdot (2 \cdot b \cdot (c+d \cdot x^{1/2}) - 2 \cdot b \cdot c) / (a \cdot b \cdot d^2 - b \cdot d^2)^{1/2}) - 2 / (a \cdot b \cdot d^2 - b \cdot d^2)^{1/2} \cdot \arctan(1/2 \cdot (2 \cdot b \cdot (c+d \cdot x^{1/2}) - 2 \cdot b \cdot c) / (a \cdot b \cdot d^2 - b \cdot d^2)^{1/2}) \cdot a + 1/d^2 \cdot c \cdot \ln(c+d \cdot x^{1/2}) \cdot \ln((-b \cdot (c+d \cdot x^{1/2}) + b \cdot c + (-a \cdot b \cdot d^2 - b \cdot d^2)^{1/2}) / (b \cdot c + (-a \cdot b \cdot d^2 - b \cdot d^2)^{1/2})) + 1/d^2 \cdot c \cdot \ln(c+d \cdot x^{1/2}) \cdot \ln((b \cdot (c+d \cdot x^{1/2}) - b \cdot c + (-a \cdot b \cdot d^2 - b \cdot d^2)^{1/2}) / (-b \cdot c + (-a \cdot b \cdot d^2 - b \cdot d^2)^{1/2})) + 1/d^2 \cdot c \cdot \operatorname{dilog}((-b \cdot (c+d \cdot x^{1/2}) + b \cdot c + (-a \cdot b \cdot d^2 - b \cdot d^2)^{1/2}) / (b \cdot c + (-a \cdot b \cdot d^2 - b \cdot d^2)^{1/2})) + 1/d^2 \cdot c \cdot \operatorname{dilog}((b \cdot (c+d \cdot x^{1/2}) - b \cdot c + (-a \cdot b \cdot d^2 - b \cdot d^2)^{1/2}) / (-b \cdot c + (-a \cdot b \cdot d^2 - b \cdot d^2)^{1/2})) - 1/d^2 \cdot c \cdot \ln(c+d \cdot x^{1/2}) \cdot \ln((-b \cdot (c+d \cdot x^{1/2}) + b \cdot c + (-a \cdot b \cdot d^2 + b \cdot d^2)^{1/2}) / (b \cdot c + (-a \cdot b \cdot d^2 + b \cdot d^2)^{1/2})) - 1/d^2 \cdot c \cdot \ln(c+d \cdot x^{1/2}) \cdot \ln((b \cdot (c+d \cdot x^{1/2}) - b \cdot c + (-a \cdot b \cdot d^2 + b \cdot d^2)^{1/2}) / (-b \cdot c + (-a \cdot b \cdot d^2 + b \cdot d^2)^{1/2})) - 1/d^2 \cdot c \cdot \operatorname{dilog}((-b \cdot (c+d \cdot x^{1/2}) + b \cdot c + (-a \cdot b \cdot d^2 + b \cdot d^2)^{1/2}) / (b \cdot c + (-a \cdot b \cdot d^2 + b \cdot d^2)^{1/2})) - 1/d^2 \cdot c \cdot \operatorname{dilog}((b \cdot (c+d \cdot x^{1/2}) - b \cdot c + (-a \cdot b \cdot d^2 + b \cdot d^2)^{1/2}) / (-b \cdot c + (-a \cdot b \cdot d^2 + b \cdot d^2)^{1/2}))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx + a)}{d\sqrt{x} + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d\*x^(1/2)), x, algorithm="maxima")

[Out] integrate(arccoth(b\*x + a)/(d\*sqrt(x) + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a + bx)}{c + d\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*x)/(c + d*x^(1/2)),x)
```

```
[Out] int(acoth(a + b*x)/(c + d*x^(1/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(c+d*x**(1/2)),x)
```

```
[Out] Timed out
```

$$3.81 \quad \int \frac{\coth^{-1}(a+bx)}{c + \frac{d}{\sqrt{x}}} dx$$

**Optimal.** Leaf size=738

$$\frac{d^2 \operatorname{Li}_2\left(-\frac{\sqrt{b}(\sqrt{xc+d})}{\sqrt{-a-1}c-\sqrt{bd}}\right)}{c^3} + \frac{d^2 \operatorname{Li}_2\left(-\frac{\sqrt{b}(\sqrt{xc+d})}{\sqrt{1-a}c-\sqrt{bd}}\right)}{c^3} - \frac{d^2 \operatorname{Li}_2\left(\frac{\sqrt{b}(\sqrt{xc+d})}{\sqrt{-a-1}c+\sqrt{bd}}\right)}{c^3} + \frac{d^2 \operatorname{Li}_2\left(\frac{\sqrt{b}(\sqrt{xc+d})}{\sqrt{1-a}c+\sqrt{bd}}\right)}{c^3} - \frac{d^2 \log(c\sqrt{x}+d) \log(\dots)}{c^3}$$

[Out]  $\frac{1}{2}(1-a) \ln(-bx-a+1)/b/c - 1/2 * x * \ln((bx+a-1)/(bx+a))/c + 1/2 * (1+a) \ln(bx+a+1)/b/c + 1/2 * x * \ln((bx+a+1)/(bx+a))/c - d^2 * \ln((bx+a-1)/(bx+a)) * \ln(d+cx^{1/2})/c^3 + d^2 * \ln((bx+a+1)/(bx+a)) * \ln(d+cx^{1/2})/c^3 - d^2 * \ln(d+cx^{1/2}) * \ln(c * ((-1-a)^{1/2} - b^{1/2}) * x^{1/2}) / (c * (-1-a)^{1/2} + d * b^{1/2}) / c^3 + d^2 * \ln(d+cx^{1/2}) * \ln(c * ((1-a)^{1/2} - b^{1/2}) * x^{1/2}) / (c * (1-a)^{1/2} + d * b^{1/2}) / c^3 - d^2 * \ln(d+cx^{1/2}) * \ln(c * ((-1-a)^{1/2} + b^{1/2}) * x^{1/2}) / (c * (-1-a)^{1/2} - d * b^{1/2}) / c^3 + d^2 * \ln(d+cx^{1/2}) * \ln(c * ((1-a)^{1/2} + b^{1/2}) * x^{1/2}) / (c * (1-a)^{1/2} - d * b^{1/2}) / c^3 - d^2 * \operatorname{polylog}(2, -b^{1/2} * (d+cx^{1/2}) / (c * (-1-a)^{1/2} - d * b^{1/2})) / c^3 + d^2 * \operatorname{polylog}(2, -b^{1/2} * (d+cx^{1/2}) / (c * (1-a)^{1/2} - d * b^{1/2})) / c^3 - d^2 * \operatorname{polylog}(2, b^{1/2} * (d+cx^{1/2}) / (c * (-1-a)^{1/2} + d * b^{1/2})) / c^3 + d^2 * \operatorname{polylog}(2, b^{1/2} * (d+cx^{1/2}) / (c * (1-a)^{1/2} + d * b^{1/2})) / c^3 + 2 * d * \operatorname{arctanh}(b^{1/2} * x^{1/2} / (1-a)^{1/2}) * (1-a)^{1/2} / c^2 + b^{1/2} - 2 * d * \operatorname{arctan}(b^{1/2} * x^{1/2} / (1+a)^{1/2}) * (1+a)^{1/2} / c^2 + b^{1/2} + d * \ln((bx+a-1)/(bx+a)) * x^{1/2} / c^2 - d * \ln((bx+a+1)/(bx+a)) * x^{1/2} / c^2$

**Rubi [A]** time = 2.37, antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 65, number of rules used = 19, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.056$ , Rules used = {6116, 190, 44, 2528, 2523, 12, 481, 205, 2525, 446, 72, 2524, 2418, 260, 2416, 2394, 2393, 2391, 208}

$$\frac{d^2 \operatorname{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1}c-\sqrt{bd}}\right)}{c^3} + \frac{d^2 \operatorname{PolyLog}\left(2, -\frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-a}c-\sqrt{bd}}\right)}{c^3} - \frac{d^2 \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{-a-1}c+\sqrt{bd}}\right)}{c^3} + \frac{d^2 \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(c\sqrt{x}+d)}{\sqrt{1-a}c+\sqrt{bd}}\right)}{c^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[a + b*x]/(c + d/\operatorname{Sqrt}[x]), x]$

[Out]  $(-2 * \operatorname{Sqrt}[1 + a] * d * \operatorname{ArcTan}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x]) / \operatorname{Sqrt}[1 + a]]) / (\operatorname{Sqrt}[b] * c^2) + (2 * \operatorname{Sqrt}[1 - a] * d * \operatorname{ArcTanh}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[x]) / \operatorname{Sqrt}[1 - a]]) / (\operatorname{Sqrt}[b] * c^2) - (d^2 * \operatorname{Log}[(c * (\operatorname{Sqrt}[-1 - a] - \operatorname{Sqrt}[b] * \operatorname{Sqrt}[x])) / (\operatorname{Sqrt}[-1 - a] * c + \operatorname{Sqrt}[b] * d)] * \operatorname{Log}[d + c * \operatorname{Sqrt}[x]]) / c^3 + (d^2 * \operatorname{Log}[(c * (\operatorname{Sqrt}[1 - a] - \operatorname{Sqrt}[b] * \operatorname{Sqrt}[x])) / (\operatorname{Sqrt}[1 - a] * c + \operatorname{Sqrt}[b] * d)] * \operatorname{Log}[d + c * \operatorname{Sqrt}[x]]) / c^3 - (d^2 * \operatorname{Log}[(c * (\operatorname{Sqrt}[-1 - a] + \operatorname{Sqrt}[b] * \operatorname{Sqrt}[x])) / (\operatorname{Sqrt}[-1 - a] * c - \operatorname{Sqrt}[b] * d)] * \operatorname{Log}[d + c * \operatorname{Sqrt}[x]]) / c^3 +$

$$\begin{aligned} & (d^2 \text{Log}[(c(\text{Sqrt}[1 - a] + \text{Sqrt}[b] \text{Sqrt}[x])) / (\text{Sqrt}[1 - a]c - \text{Sqrt}[b]d)] * \text{Log}[d + c \text{Sqrt}[x]]) / c^3 + ((1 - a) \text{Log}[1 - a - b*x]) / (2*b*c) + (d \text{Sqrt}[x] * \text{Log}[-((1 - a - b*x) / (a + b*x))]) / c^2 - (x \text{Log}[-((1 - a - b*x) / (a + b*x))]) / (2*c) - (d^2 \text{Log}[d + c \text{Sqrt}[x]] * \text{Log}[-((1 - a - b*x) / (a + b*x))]) / c^3 + ((1 + a) \text{Log}[1 + a + b*x]) / (2*b*c) - (d \text{Sqrt}[x] * \text{Log}[(1 + a + b*x) / (a + b*x)]) / c^2 + (x \text{Log}[(1 + a + b*x) / (a + b*x)]) / (2*c) + (d^2 \text{Log}[d + c \text{Sqrt}[x]] * \text{Log}[(1 + a + b*x) / (a + b*x)]) / c^3 - (d^2 \text{PolyLog}[2, -((\text{Sqrt}[b](d + c \text{Sqrt}[x])) / (\text{Sqrt}[-1 - a]c - \text{Sqrt}[b]d))]) / c^3 + (d^2 \text{PolyLog}[2, -((\text{Sqrt}[b](d + c \text{Sqrt}[x])) / (\text{Sqrt}[1 - a]c - \text{Sqrt}[b]d))]) / c^3 - (d^2 \text{PolyLog}[2, (\text{Sqrt}[b](d + c \text{Sqrt}[x])) / (\text{Sqrt}[-1 - a]c + \text{Sqrt}[b]d)]) / c^3 + (d^2 \text{PolyLog}[2, (\text{Sqrt}[b](d + c \text{Sqrt}[x])) / (\text{Sqrt}[1 - a]c + \text{Sqrt}[b]d)]) / c^3 \end{aligned}$$
Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 481

Int[((e\_.)\*(x\_))^(m\_.)/(((a\_) + (b\_.)\*(x\_)^(n\_.))\*((c\_) + (d\_.)\*(x\_)^(n\_.))), x\_Symbol] := -Dist[(a\*e^n)/(b\*c - a\*d), Int[(e\*x)^(m - n)/(a + b\*x^n), x], x] + Dist[(c\*e^n)/(b\*c - a\*d), Int[(e\*x)^(m - n)/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2\*n - 1]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c

, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

### Rule 2523

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*Log[c*Rfx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(Rgx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, Rgx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[Rgx, x] && IGtQ[n, 0]
```

### Rule 6116

```
Int[ArcCoth[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] := Dist[1/2, Int[Log[(1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[(-1 + c + d*x)/(c + d*x)]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x]
```

&& RationalQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(a+bx)}{c+\frac{d}{\sqrt{x}}} dx &= -\left(\frac{1}{2} \int \frac{\log\left(\frac{-1+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}} dx\right) + \frac{1}{2} \int \frac{\log\left(\frac{1+a+bx}{a+bx}\right)}{c+\frac{d}{\sqrt{x}}} dx \\
&= -\text{Subst}\left(\int \frac{x^2 \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{d+cx} dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{x^2 \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{d+cx} dx, x, \sqrt{x}\right) \\
&= -\text{Subst}\left(\int \left(-\frac{d \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c^2} + \frac{x \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c} + \frac{d^2 \log\left(\frac{-1+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)}\right) dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \left(\frac{d \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c^2} + \frac{x \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c} + \frac{d^2 \log\left(\frac{1+a+bx^2}{a+bx^2}\right)}{c^2(d+cx)}\right) dx, x, \sqrt{x}\right) \\
&= -\frac{\text{Subst}\left(\int x \log\left(\frac{-1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x}\right)}{c} + \frac{\text{Subst}\left(\int x \log\left(\frac{1+a+bx^2}{a+bx^2}\right) dx, x, \sqrt{x}\right)}{c} + \frac{d \text{Subst}\left(\int \frac{1}{d+cx} dx, x, \sqrt{x}\right)}{c} \\
&= \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2 \log(d+c\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{c^2} \\
&= \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2 \log(d+c\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{c^2} \\
&= \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} - \frac{d^2 \log(d+c\sqrt{x}) \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^3} - \frac{d\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{c^2} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} + \frac{d\sqrt{x} \log\left(-\frac{1-a-bx}{a+bx}\right)}{c^2} - \frac{x \log\left(-\frac{1-a-bx}{a+bx}\right)}{2c} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} + \frac{(1-a) \log(1-a-bx)}{2bc} + \frac{d\sqrt{x} \log\left(\frac{1+a+bx}{a+bx}\right)}{c^2} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} - \frac{d^2 \log\left(\frac{c(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right) \log(d)}{c^3} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} - \frac{d^2 \log\left(\frac{c(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right) \log(d)}{c^3} \\
&= -\frac{2\sqrt{1+a} d \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1+a}}\right)}{\sqrt{b}c^2} + \frac{2\sqrt{1-a} d \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{1-a}}\right)}{\sqrt{b}c^2} - \frac{d^2 \log\left(\frac{c(\sqrt{1-a}-\sqrt{b}\sqrt{x})}{\sqrt{1-a}c+\sqrt{b}d}\right) \log(d)}{c^3}
\end{aligned}$$



**Mathematica [A]** time = 0.74, size = 719, normalized size = 0.97

$$-ac^2 \log(-a - bx + 1) + c^2 \log(-a - bx + 1) - bc^2 x \log\left(\frac{a+bx-1}{a+bx}\right) + ac^2 \log(a + bx + 1) + c^2 \log(a + bx + 1) + bc$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*x]/(c + d/Sqrt[x]), x]

[Out]  $(-4\sqrt{1+a}\sqrt{b}c*d*\text{ArcTan}[(\sqrt{b}*\sqrt{x})/\sqrt{1+a}] + 4\sqrt{1-a}\sqrt{b}c*d*\text{ArcTanh}[(\sqrt{b}*\sqrt{x})/\sqrt{1-a}] - 2*b*d^2*\text{Log}[(c*(\sqrt{-1-a} - \sqrt{b}*\sqrt{x}))]/(\sqrt{-1-a}*c + \sqrt{b}*d)]*\text{Log}[d + c*\sqrt{x}] + 2*b*d^2*\text{Log}[(c*(\sqrt{1-a} - \sqrt{b}*\sqrt{x}))]/(\sqrt{1-a}*c + \sqrt{b}*d)]*\text{Log}[d + c*\sqrt{x}] - 2*b*d^2*\text{Log}[(c*(\sqrt{-1-a} + \sqrt{b}*\sqrt{x}))]/(\sqrt{-1-a}*c - \sqrt{b}*d)]*\text{Log}[d + c*\sqrt{x}] + 2*b*d^2*\text{Log}[(c*(\sqrt{1-a} + \sqrt{b}*\sqrt{x}))]/(\sqrt{1-a}*c - \sqrt{b}*d)]*\text{Log}[d + c*\sqrt{x}] + c^2*\text{Log}[1 - a - b*x] - a*c^2*\text{Log}[1 - a - b*x] + 2*b*c*d*\sqrt{x}*\text{Log}[(1 - a + b*x)/(a + b*x)] - b*c^2*x*\text{Log}[(1 - a + b*x)/(a + b*x)] - 2*b*d^2*\text{Log}[d + c*\sqrt{x}]*\text{Log}[(1 - a + b*x)/(a + b*x)] + c^2*\text{Log}[1 + a + b*x] + a*c^2*\text{Log}[1 + a + b*x] - 2*b*c*d*\sqrt{x}*\text{Log}[(1 + a + b*x)/(a + b*x)] + b*c^2*x*\text{Log}[(1 + a + b*x)/(a + b*x)] + 2*b*d^2*\text{Log}[d + c*\sqrt{x}]*\text{Log}[(1 + a + b*x)/(a + b*x)] - 2*b*d^2*\text{PolyLog}[2, (\sqrt{b}*(d + c*\sqrt{x}))]/(-(\sqrt{-1-a}*c) + \sqrt{b}*d)] - 2*b*d^2*\text{PolyLog}[2, (\sqrt{b}*(d + c*\sqrt{x}))]/(\sqrt{-1-a}*c + \sqrt{b}*d)] + 2*b*d^2*\text{PolyLog}[2, (\sqrt{b}*(d + c*\sqrt{x}))]/(-(\sqrt{1-a}*c) + \sqrt{b}*d)] + 2*b*d^2*\text{PolyLog}[2, (\sqrt{b}*(d + c*\sqrt{x}))]/(\sqrt{1-a}*c + \sqrt{b}*d)]/(2*b*c^3)$

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{cx \operatorname{arccoth}(bx + a) - d\sqrt{x} \operatorname{arccoth}(bx + a)}{c^2x - d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^(1/2)), x, algorithm="fricas")

[Out] integral((c\*x\*arccoth(b\*x + a) - d\*sqrt(x)\*arccoth(b\*x + a))/(c^2\*x - d^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx + a)}{c + \frac{d}{\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^(1/2)),x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(c + d/sqrt(x)), x)

**maple [A]** time = 0.12, size = 970, normalized size = 1.31

$$\frac{\operatorname{arccoth}(bx+a)x}{c} - \frac{2 \operatorname{arccoth}(bx+a)d\sqrt{x}}{c^2} + \frac{2 \operatorname{arccoth}(bx+a)d^2 \ln(d+c\sqrt{x})}{c^3} - \frac{d^2 \ln(d+c\sqrt{x}) \ln\left(\frac{-b(d+c\sqrt{x})+bd+\sqrt{c^2x+d^2}}{bd+\sqrt{c^2x+d^2}}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(c+d/x^(1/2)),x)

[Out] arccoth(b\*x+a)/c\*x-2\*arccoth(b\*x+a)/c^2\*d\*x^(1/2)+2\*arccoth(b\*x+a)\*d^2/c^3\*ln(d+c\*x^(1/2))-1/c^3\*d^2\*ln(d+c\*x^(1/2))\*ln((-b\*(d+c\*x^(1/2))+b\*d+(-a\*b\*c^2-b\*c^2)^(1/2))/(b\*d+(-a\*b\*c^2-b\*c^2)^(1/2)))-1/c^3\*d^2\*ln(d+c\*x^(1/2))\*ln((b\*(d+c\*x^(1/2))-b\*d+(-a\*b\*c^2-b\*c^2)^(1/2))/(-b\*d+(-a\*b\*c^2-b\*c^2)^(1/2)))-1/c^3\*d^2\*dilog((-b\*(d+c\*x^(1/2))+b\*d+(-a\*b\*c^2-b\*c^2)^(1/2))/(b\*d+(-a\*b\*c^2-b\*c^2)^(1/2)))-1/c^3\*d^2\*dilog((b\*(d+c\*x^(1/2))-b\*d+(-a\*b\*c^2-b\*c^2)^(1/2))/(-b\*d+(-a\*b\*c^2-b\*c^2)^(1/2)))+1/c^3\*d^2\*ln(d+c\*x^(1/2))\*ln((-b\*(d+c\*x^(1/2))+b\*d+(-a\*b\*c^2+b\*c^2)^(1/2))/(b\*d+(-a\*b\*c^2+b\*c^2)^(1/2)))+1/c^3\*d^2\*ln(d+c\*x^(1/2))\*ln((b\*(d+c\*x^(1/2))-b\*d+(-a\*b\*c^2+b\*c^2)^(1/2))/(-b\*d+(-a\*b\*c^2+b\*c^2)^(1/2)))+1/c^3\*d^2\*dilog((-b\*(d+c\*x^(1/2))+b\*d+(-a\*b\*c^2+b\*c^2)^(1/2))/(b\*d+(-a\*b\*c^2+b\*c^2)^(1/2)))+1/c^3\*d^2\*dilog((b\*(d+c\*x^(1/2))-b\*d+(-a\*b\*c^2+b\*c^2)^(1/2))/(-b\*d+(-a\*b\*c^2+b\*c^2)^(1/2)))+1/2/b/c\*ln(b\*(d+c\*x^(1/2))^2-2\*(d+c\*x^(1/2))\*b\*d+a\*c^2+b\*d^2+c^2)-2/c\*d/(a\*b\*c^2+b\*c^2)^(1/2)\*arctan(1/2\*(2\*b\*(d+c\*x^(1/2))-2\*b\*d)/(a\*b\*c^2+b\*c^2)^(1/2))+1/2/b/c\*a\*ln(b\*(d+c\*x^(1/2))^2-2\*(d+c\*x^(1/2))\*b\*d+a\*c^2+b\*d^2+c^2)+2/c\*a\*d/(a\*b\*c^2-b\*c^2)^(1/2)\*arctan(1/2\*(2\*b\*(d+c\*x^(1/2))-2\*b\*d)/(a\*b\*c^2-b\*c^2)^(1/2))+1/2/b/c\*ln(b\*(d+c\*x^(1/2))^2-2\*(d+c\*x^(1/2))\*b\*d+a\*c^2+b\*d^2-c^2)-2/c\*d/(a\*b\*c^2-b\*c^2)^(1/2)\*arctan(1/2\*(2\*b\*(d+c\*x^(1/2))-2\*b\*d)/(a\*b\*c^2-b\*c^2)^(1/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{(bx+a+1)\log(bx+a+1)-(bx+a-1)\log(bx+a-1)}{2bc} - \frac{1}{2} \int \frac{d \log(bx+a+1) - d \log(bx+a-1)}{c^2 \sqrt{x} + cd} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(c+d/x^(1/2)),x, algorithm="maxima")

[Out] 1/2\*((b\*x + a + 1)\*log(b\*x + a + 1) - (b\*x + a - 1)\*log(b\*x + a - 1))/(b\*c) - 1/2\*integrate((d\*log(b\*x + a + 1) - d\*log(b\*x + a - 1))/(c^2\*sqrt(x) + c\*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(a + b x)}{c + \frac{d}{\sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a + b*x)/(c + d/x^(1/2)), x)`

[Out] `int(acoth(a + b*x)/(c + d/x^(1/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(b*x+a)/(c+d/x**(1/2)), x)`

[Out] Timed out

$$3.82 \quad \int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=335

$$\frac{\operatorname{Li}_2\left(\frac{2(2cd-(b-\sqrt{b^2-4ac})e^{-2c(d+ex)})}{(-2dc+2c+be-\sqrt{b^2-4ac})e^{(d+ex+1)}}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\operatorname{Li}_2\left(\frac{2(2cd-(b+\sqrt{b^2-4ac})e^{-2c(d+ex)})}{(2c(1-d)+(b+\sqrt{b^2-4ac})e^{(d+ex+1)}}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\coth^{-1}(d+ex)\log\left(\frac{2e^{(-\sqrt{b^2-4ac})}}{(d+ex+1)(e^{(b-\sqrt{b^2-4ac})})}\right)}{\sqrt{b^2-4ac}}$$

[Out] arccoth(e\*x+d)\*ln(2\*e\*(b+2\*c\*x-(-4\*a\*c+b^2)^(1/2))/(e\*x+d+1)/(2\*c\*(1-d)+e\*(b-(-4\*a\*c+b^2)^(1/2))))/(-4\*a\*c+b^2)^(1/2)-arccoth(e\*x+d)\*ln(2\*e\*(b+2\*c\*x+(-4\*a\*c+b^2)^(1/2))/(e\*x+d+1)/(2\*c\*(1-d)+e\*(b+(-4\*a\*c+b^2)^(1/2))))/(-4\*a\*c+b^2)^(1/2)-1/2\*polylog(2,1+2\*(2\*c\*d-2\*c\*(e\*x+d)-e\*(b-(-4\*a\*c+b^2)^(1/2))))/(e\*x+d+1)/(2\*c-2\*c\*d+b\*e-e\*(-4\*a\*c+b^2)^(1/2))/(-4\*a\*c+b^2)^(1/2)+1/2\*polylog(2,1+2\*(2\*c\*d-2\*c\*(e\*x+d)-e\*(b+(-4\*a\*c+b^2)^(1/2))))/(e\*x+d+1)/(2\*c\*(1-d)+e\*(b+(-4\*a\*c+b^2)^(1/2)))/(-4\*a\*c+b^2)^(1/2)

**Rubi [A]** time = 0.75, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {618, 206, 6728, 6112, 5921, 2402, 2315, 2447}

$$\frac{\operatorname{PolyLog}\left(2, \frac{2(-e^{(b-\sqrt{b^2-4ac})}-2c(d+ex)+2cd)}{(d+ex+1)(-e\sqrt{b^2-4ac}+be-2cd+2c)}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\operatorname{PolyLog}\left(2, \frac{2(-e^{(\sqrt{b^2-4ac}+b)-2c(d+ex)+2cd)})}{(d+ex+1)(e(\sqrt{b^2-4ac}+b)+2c(1-d))}+1\right)}{2\sqrt{b^2-4ac}} + \frac{\coth^{-1}(d+ex)\log\left(\frac{2e^{(-\sqrt{b^2-4ac})}}{(d+ex+1)(e^{(b-\sqrt{b^2-4ac})})}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[d + e\*x]/(a + b\*x + c\*x^2), x]

[Out] (ArcCoth[d + e\*x]\*Log[(2\*e\*(b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/((2\*c\*(1 - d) + (b - Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))])/Sqrt[b^2 - 4\*a\*c] - (ArcCoth[d + e\*x]\*Log[(2\*e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/((2\*c\*(1 - d) + (b + Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))])/Sqrt[b^2 - 4\*a\*c] - PolyLog[2, 1 + (2\*(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e - 2\*c\*(d + e\*x)))/((2\*c - 2\*c\*d + b\*e - Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))]/(2\*Sqrt[b^2 - 4\*a\*c]) + PolyLog[2, 1 + (2\*(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e - 2\*c\*(d + e\*x)))/((2\*c\*(1 - d) + (b + Sqrt[b^2 - 4\*a\*c])\*e)\*(1 + d + e\*x))]/(2\*Sqrt[b^2 - 4\*a\*c])

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + e*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*Log[(2*c*(d + e*x))/(c*d + e*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2\*n] && IGtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(d+ex)}{a+bx+cx^2} dx &= \int \left( \frac{2c \coth^{-1}(d+ex)}{\sqrt{b^2-4ac} (b-\sqrt{b^2-4ac}+2cx)} - \frac{2c \coth^{-1}(d+ex)}{\sqrt{b^2-4ac} (b+\sqrt{b^2-4ac}+2cx)} \right) dx \\
 &= \frac{(2c) \int \frac{\coth^{-1}(d+ex)}{b-\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{\coth^{-1}(d+ex)}{b+\sqrt{b^2-4ac}+2cx} dx}{\sqrt{b^2-4ac}} \\
 &= \frac{(2c) \text{Subst} \left( \int \frac{\coth^{-1}(x)}{\frac{-2cd+(b-\sqrt{b^2-4ac})e}{e} + \frac{2cx}{e}} dx, x, d+ex \right)}{\sqrt{b^2-4ac} e} - \frac{(2c) \text{Subst} \left( \int \frac{\coth^{-1}(x)}{\frac{-2cd+(b+\sqrt{b^2-4ac})e}{e} + \frac{2cx}{e}} dx, x, d+ex \right)}{\sqrt{b^2-4ac} e} \\
 &= \frac{\coth^{-1}(d+ex) \log \left( \frac{2e(b-\sqrt{b^2-4ac}+2cx)}{(2c-2cd+be-\sqrt{b^2-4ac}e)(1+d+ex)} \right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex) \log \left( \frac{2e(b+\sqrt{b^2-4ac}+2cx)}{(2c(1-d)+(b+\sqrt{b^2-4ac}e)(1+d+ex)} \right)}{\sqrt{b^2-4ac}} \\
 &= \frac{\coth^{-1}(d+ex) \log \left( \frac{2e(b-\sqrt{b^2-4ac}+2cx)}{(2c-2cd+be-\sqrt{b^2-4ac}e)(1+d+ex)} \right)}{\sqrt{b^2-4ac}} - \frac{\coth^{-1}(d+ex) \log \left( \frac{2e(b+\sqrt{b^2-4ac}+2cx)}{(2c(1-d)+(b+\sqrt{b^2-4ac}e)(1+d+ex)} \right)}{\sqrt{b^2-4ac}}
 \end{aligned}$$

**Mathematica [A]** time = 0.96, size = 596, normalized size = 1.78

$$\frac{-\text{Li}_2 \left( \frac{e(-b-2cx+\sqrt{b^2-4ac})}{2c(d+1)+(\sqrt{b^2-4ac}-b)e} \right) + \text{Li}_2 \left( \frac{e(b+2cx-\sqrt{b^2-4ac})}{-2dc+2c+be-\sqrt{b^2-4ac}e} \right) - \text{Li}_2 \left( \frac{e(b+2cx+\sqrt{b^2-4ac})}{(b+\sqrt{b^2-4ac})e-2c(d-1)} \right) + \text{Li}_2 \left( \frac{e(b+2cx+\sqrt{b^2-4ac})}{(b+\sqrt{b^2-4ac})e-2c(d+1)} \right) + \text{Li}_2 \left( \frac{e(b+2cx+\sqrt{b^2-4ac})}{(b+\sqrt{b^2-4ac})e-2c(d+1)} \right)}{\sqrt{b^2-4ac}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[d + e\*x]/(a + b\*x + c\*x^2), x]

```
[Out] (Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(-1 + d + e*x))/(2*c*(-1 + d)
+ (-b + Sqrt[b^2 - 4*a*c])*e)] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*
c*(-1 + d + e*x))/(2*c*(-1 + d) - (b + Sqrt[b^2 - 4*a*c])*e)] - Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x]*Log[(-1 + d + e*x)/(d + e*x)] + Log[b + Sqrt[b^2 -
4*a*c] + 2*c*x]*Log[(-1 + d + e*x)/(d + e*x)] - Log[b - Sqrt[b^2 - 4*a*c] +
2*c*x]*Log[(2*c*(1 + d + e*x))/(2*c*(1 + d) + (-b + Sqrt[b^2 - 4*a*c])*e)]
+ Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(2*c*(1 + d + e*x))/(2*c*(1 + d)
- (b + Sqrt[b^2 - 4*a*c])*e)] + Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(1 +
d + e*x)/(d + e*x)] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(1 + d + e*x)
/(d + e*x)] - PolyLog[2, (e*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*(1 + d)
+ (-b + Sqrt[b^2 - 4*a*c])*e)] + PolyLog[2, (e*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x))/(2*c - 2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e)] - PolyLog[2, (e*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x))/(-2*c*(-1 + d) + (b + Sqrt[b^2 - 4*a*c])*e)] + PolyLo
g[2, (e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*(1 + d) + (b + Sqrt[b^2 - 4*
a*c])*e)]/(2*Sqrt[b^2 - 4*a*c])
```

**fricas** [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arccoth}(ex+d)}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ex+d)}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arccoth(e*x + d)/(c*x^2 + b*x + a), x)
```

**maple** [B] time = 1.11, size = 2098, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] -e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c
)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(-e^2*(4*a*c-b^2))^(1/2)-c))*arcco
```

```

th(e*x+d)*a/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)+(e^2*(-4*a*c+b^2
))^1/2/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-
1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d)*b*d/(a*e^
2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a
*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e
*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e*x+d)*c*d^2/(a*e^2-b*e*d+c*d
^2-(e^2*(-4*a*c+b^2))^(1/2)-c)+e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arcco
th(e*x+d)^2*a/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-(e^2*(-4*a*c+b
^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2*b*d/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c
+b^2))^(1/2)-c)+1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2*c
*d^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-1/2*e*(e^2*(-4*a*c+b^2)
)^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*
x+d-1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*a/(a*e^2-b*e*d+c*d^2
-(e^2*(-4*a*c+b^2))^(1/2)-c)+1/2*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polyl
og(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2
-(e^2*(4*a*c-b^2))^(1/2)-c))*b*d/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/
2)-c)-1/2/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d
^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(
1/2)-c))*c*d^2/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)-e/(a*e^2-b*e*
d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e
*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))*arccoth(e
*x+d)+1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*ln(1-(a*e^2-b*d*e+c*d^2+b*e-
2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c)
)*arccoth(e*x+d)*c/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)+e/(a*e^2-
b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*arccoth(e*x+d)^2-1/e*(e^2*(-4*a*c+b
^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2*c/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b
^2))^(1/2)-c)-1/2*e/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)-c)*polylog(
2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(e
^2*(4*a*c-b^2))^(1/2)-c))+1/2/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polyl
og(2,(a*e^2-b*d*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2
-(e^2*(4*a*c-b^2))^(1/2)-c))*c/(a*e^2-b*e*d+c*d^2-(e^2*(-4*a*c+b^2))^(1/2)
-c)+1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)*ln(1-(a*e^2-b*d
*e+c*d^2+b*e-2*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b
^2))^(1/2)-c))-1/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*arccoth(e*x+d)^2+1/
2/e*(e^2*(-4*a*c+b^2))^(1/2)/(4*a*c-b^2)*polylog(2,(a*e^2-b*d*e+c*d^2+b*e-2
*c*d+c)*(e*x+d+1)/(e*x+d-1)/(a*e^2-b*e*d+c*d^2-(e^2*(4*a*c-b^2))^(1/2)-c))

```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(e\*x+d)/(c\*x^2+b\*x+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a



additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4\*a\*c-b^2>0)', see 'assume?' for more details) Is 4\*a\*c-b^2 positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acoth}(d + ex)}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(d + e\*x)/(a + b\*x + c\*x^2), x)

[Out] int(acoth(d + e\*x)/(a + b\*x + c\*x^2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(e\*x+d)/(c\*x\*\*2+b\*x+a), x)

[Out] Timed out

### 3.83 $\int x^2 \coth^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=51

$$\frac{x^{5/2}}{15} + \frac{x^{3/2}}{9} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{3} - \frac{1}{3} \tanh^{-1}(\sqrt{x})$$

[Out]  $1/9*x^{(3/2)}+1/15*x^{(5/2)}+1/3*x^3*\operatorname{arccoth}(x^{(1/2)})-1/3*\operatorname{arctanh}(x^{(1/2)})+1/3*x^{(1/2)}$

**Rubi [A]** time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6098, 50, 63, 206}

$$\frac{x^{5/2}}{15} + \frac{x^{3/2}}{9} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{3} - \frac{1}{3} \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] `Int[x^2*ArcCoth[Sqrt[x]], x]`

[Out] `Sqrt[x]/3 + x^(3/2)/9 + x^(5/2)/15 + (x^3*ArcCoth[Sqrt[x]])/3 - ArcTanh[Sqrt[x]]/3`

#### Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt
```

Q[a, 0] || LtQ[b, 0])

### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
 \int x^2 \coth^{-1}(\sqrt{x}) dx &= \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{6} \int \frac{x^{5/2}}{1-x} dx \\
 &= \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{6} \int \frac{x^{3/2}}{1-x} dx \\
 &= \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{6} \int \frac{\sqrt{x}}{1-x} dx \\
 &= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{6} \int \frac{1}{(1-x)\sqrt{x}} dx \\
 &= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{3} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\
 &= \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15} + \frac{1}{3}x^3 \coth^{-1}(\sqrt{x}) - \frac{1}{3} \tanh^{-1}(\sqrt{x})
 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 59, normalized size = 1.16

$$\frac{1}{90} (6x^{5/2} + 10x^{3/2} + 30x^3 \coth^{-1}(\sqrt{x}) + 30\sqrt{x} + 15 \log(1 - \sqrt{x}) - 15 \log(\sqrt{x} + 1))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Sqrt[x]], x]

[Out] (30\*Sqrt[x] + 10\*x^(3/2) + 6\*x^(5/2) + 30\*x^3\*ArcCoth[Sqrt[x]] + 15\*Log[1 - Sqrt[x]] - 15\*Log[1 + Sqrt[x]])/90

**fricas [A]** time = 0.47, size = 38, normalized size = 0.75

$$\frac{1}{6} (x^3 - 1) \log\left(\frac{x + 2\sqrt{x} + 1}{x - 1}\right) + \frac{1}{45} (3x^2 + 5x + 15)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(x^(1/2)),x, algorithm="fricas")

[Out] 1/6\*(x^3 - 1)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) + 1/45\*(3\*x^2 + 5\*x + 15)\*sqrt(x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(x^(1/2)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(sqrt(x)), x)

**maple** [A] time = 0.04, size = 42, normalized size = 0.82

$$\frac{x^3 \operatorname{arccoth}(\sqrt{x})}{3} + \frac{x^{\frac{5}{2}}}{15} + \frac{x^{\frac{3}{2}}}{9} + \frac{\sqrt{x}}{3} + \frac{\ln(-1 + \sqrt{x})}{6} - \frac{\ln(1 + \sqrt{x})}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(x^(1/2)),x)

[Out] 1/3\*x^3\*arccoth(x^(1/2))+1/15\*x^(5/2)+1/9\*x^(3/2)+1/3\*x^(1/2)+1/6\*ln(-1+x^(1/2))-1/6\*ln(1+x^(1/2))

**maxima** [A] time = 0.30, size = 41, normalized size = 0.80

$$\frac{1}{3} x^3 \operatorname{arccoth}(\sqrt{x}) + \frac{1}{15} x^{\frac{5}{2}} + \frac{1}{9} x^{\frac{3}{2}} + \frac{1}{3} \sqrt{x} - \frac{1}{6} \log(\sqrt{x} + 1) + \frac{1}{6} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(x^(1/2)),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(sqrt(x)) + 1/15\*x^(5/2) + 1/9\*x^(3/2) + 1/3\*sqrt(x) - 1/6\*log(sqrt(x) + 1) + 1/6\*log(sqrt(x) - 1)

**mupad** [B] time = 1.30, size = 31, normalized size = 0.61

$$\frac{x^3 \operatorname{acoth}(\sqrt{x})}{3} - \frac{\operatorname{acoth}(\sqrt{x})}{3} + \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} + \frac{x^{5/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*acoth(x^(1/2)),x)

```
[Out] (x^3*acoth(x^(1/2)))/3 - acoth(x^(1/2))/3 + x^(1/2)/3 + x^(3/2)/9 + x^(5/2)/15
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x^2 \operatorname{acoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(x**(1/2)),x)
```

```
[Out] Integral(x**2*acoth(sqrt(x)), x)
```

### 3.84 $\int x \coth^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=42

$$\frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{2} - \frac{1}{2} \tanh^{-1}(\sqrt{x})$$

[Out] 1/6\*x^(3/2)+1/2\*x^2\*arccoth(x^(1/2))-1/2\*arctanh(x^(1/2))+1/2\*x^(1/2)

**Rubi [A]** time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6098, 50, 63, 206}

$$\frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) + \frac{\sqrt{x}}{2} - \frac{1}{2} \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[Sqrt[x]],x]

[Out] Sqrt[x]/2 + x^(3/2)/6 + (x^2\*ArcCoth[Sqrt[x]])/2 - ArcTanh[Sqrt[x]]/2

#### Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(\sqrt{x}) dx &= \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{4} \int \frac{x^{3/2}}{1-x} dx \\
&= \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{4} \int \frac{\sqrt{x}}{1-x} dx \\
&= \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{4} \int \frac{1}{(1-x)\sqrt{x}} dx \\
&= \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{2} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\
&= \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6} + \frac{1}{2}x^2 \coth^{-1}(\sqrt{x}) - \frac{1}{2} \tanh^{-1}(\sqrt{x})
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 52, normalized size = 1.24

$$\frac{1}{12} (2x^{3/2} + 6x^2 \coth^{-1}(\sqrt{x}) + 6\sqrt{x} + 3 \log(1 - \sqrt{x}) - 3 \log(\sqrt{x} + 1))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCoth[Sqrt[x]], x]
```

```
[Out] (6*Sqrt[x] + 2*x^(3/2) + 6*x^2*ArcCoth[Sqrt[x]] + 3*Log[1 - Sqrt[x]] - 3*Log[1 + Sqrt[x]])/12
```

**fricas [A]** time = 0.57, size = 31, normalized size = 0.74

$$\frac{1}{4}(x^2 - 1) \log\left(\frac{x + 2\sqrt{x} + 1}{x - 1}\right) + \frac{1}{6}(x + 3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(x^(1/2)), x, algorithm="fricas")
```

```
[Out] 1/4*(x^2 - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + 1/6*(x + 3)*sqrt(x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x^(1/2)),x, algorithm="giac")

[Out] integrate(x\*arccoth(sqrt(x)), x)

**maple** [A] time = 0.05, size = 37, normalized size = 0.88

$$\frac{x^2 \operatorname{arccoth}(\sqrt{x})}{2} + \frac{x^{\frac{3}{2}}}{6} + \frac{\sqrt{x}}{2} + \frac{\ln(-1 + \sqrt{x})}{4} - \frac{\ln(1 + \sqrt{x})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(x^(1/2)),x)

[Out] 1/2\*x^2\*arccoth(x^(1/2))+1/6\*x^(3/2)+1/2\*x^(1/2)+1/4\*ln(-1+x^(1/2))-1/4\*ln(1+x^(1/2))

**maxima** [A] time = 0.31, size = 36, normalized size = 0.86

$$\frac{1}{2} x^2 \operatorname{arccoth}(\sqrt{x}) + \frac{1}{6} x^{\frac{3}{2}} + \frac{1}{2} \sqrt{x} - \frac{1}{4} \log(\sqrt{x} + 1) + \frac{1}{4} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(x^(1/2)),x, algorithm="maxima")

[Out] 1/2\*x^2\*arccoth(sqrt(x)) + 1/6\*x^(3/2) + 1/2\*sqrt(x) - 1/4\*log(sqrt(x) + 1) + 1/4\*log(sqrt(x) - 1)

**mupad** [B] time = 1.26, size = 26, normalized size = 0.62

$$\frac{x^2 \operatorname{acoth}(\sqrt{x})}{2} - \frac{\operatorname{acoth}(\sqrt{x})}{2} + \frac{\sqrt{x}}{2} + \frac{x^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(x^(1/2)),x)

[Out] (x^2\*acoth(x^(1/2)))/2 - acoth(x^(1/2))/2 + x^(1/2)/2 + x^(3/2)/6

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(\sqrt{x}) dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(x**(1/2)),x)
```

```
[Out] Integral(x*acoth(sqrt(x)), x)
```

### 3.85 $\int \coth^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=22

$$\sqrt{x} - \tanh^{-1}(\sqrt{x}) + x \coth^{-1}(\sqrt{x})$$

[Out] x\*arccoth(x^(1/2))-arctanh(x^(1/2))+x^(1/2)

**Rubi [A]** time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6092, 50, 63, 206}

$$\sqrt{x} - \tanh^{-1}(\sqrt{x}) + x \coth^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Sqrt[x]],x]

[Out] Sqrt[x] + x\*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]

#### Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

#### Rule 6092

```
Int[ArcCoth[(c_.)*(x_)^(n_)], x_Symbol] := Simp[x*ArcCoth[c*x^n], x] - Dist
[c*n, Int[x^n/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\sqrt{x}) dx &= x \coth^{-1}(\sqrt{x}) - \frac{1}{2} \int \frac{\sqrt{x}}{1-x} dx \\ &= \sqrt{x} + x \coth^{-1}(\sqrt{x}) - \frac{1}{2} \int \frac{1}{(1-x)\sqrt{x}} dx \\ &= \sqrt{x} + x \coth^{-1}(\sqrt{x}) - \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\ &= \sqrt{x} + x \coth^{-1}(\sqrt{x}) - \tanh^{-1}(\sqrt{x}) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 22, normalized size = 1.00

$$\sqrt{x} - \tanh^{-1}(\sqrt{x}) + x \coth^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Sqrt[x]], x]
```

```
[Out] Sqrt[x] + x*ArcCoth[Sqrt[x]] - ArcTanh[Sqrt[x]]
```

**fricas [A]** time = 0.62, size = 24, normalized size = 1.09

$$\frac{1}{2}(x-1) \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)), x, algorithm="fricas")
```

```
[Out] 1/2*(x - 1)*log((x + 2*sqrt(x) + 1)/(x - 1)) + sqrt(x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(x^(1/2)), x, algorithm="giac")
```

[Out] integrate(arccoth(sqrt(x)), x)

**maple** [A] time = 0.05, size = 27, normalized size = 1.23

$$x \operatorname{arccoth}(\sqrt{x}) + \sqrt{x} + \frac{\ln(-1 + \sqrt{x})}{2} - \frac{\ln(1 + \sqrt{x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2)), x)

[Out] x\*arccoth(x^(1/2))+x^(1/2)+1/2\*ln(-1+x^(1/2))-1/2\*ln(1+x^(1/2))

**maxima** [A] time = 0.30, size = 26, normalized size = 1.18

$$x \operatorname{arccoth}(\sqrt{x}) + \sqrt{x} - \frac{1}{2} \log(\sqrt{x} + 1) + \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2)), x, algorithm="maxima")

[Out] x\*arccoth(sqrt(x)) + sqrt(x) - 1/2\*log(sqrt(x) + 1) + 1/2\*log(sqrt(x) - 1)

**mupad** [B] time = 1.24, size = 16, normalized size = 0.73

$$x \operatorname{acoth}(\sqrt{x}) - \operatorname{acoth}(\sqrt{x}) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(x^(1/2)), x)

[Out] x\*acoth(x^(1/2)) - acoth(x^(1/2)) + x^(1/2)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x\*\*(1/2)), x)

[Out] Integral(acoth(sqrt(x)), x)

$$3.86 \quad \int \frac{\coth^{-1}(\sqrt{x})}{x} dx$$

Optimal. Leaf size=19

$$\operatorname{Li}_2\left(-\frac{1}{\sqrt{x}}\right) - \operatorname{Li}_2\left(\frac{1}{\sqrt{x}}\right)$$

[Out] polylog(2,-1/x^(1/2))-polylog(2,1/x^(1/2))

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {6096, 5913}

$$\operatorname{PolyLog}\left(2, -\frac{1}{\sqrt{x}}\right) - \operatorname{PolyLog}\left(2, \frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Sqrt[x]]/x,x]

[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]

Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

Rule 6096

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*ArcCoth[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\sqrt{x})}{x} dx &= 2 \operatorname{Subst}\left(\int \frac{\coth^{-1}(x)}{x} dx, x, \sqrt{x}\right) \\ &= \operatorname{Li}_2\left(-\frac{1}{\sqrt{x}}\right) - \operatorname{Li}_2\left(\frac{1}{\sqrt{x}}\right) \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 19, normalized size = 1.00

$$\operatorname{Li}_2\left(-\frac{1}{\sqrt{x}}\right) - \operatorname{Li}_2\left(\frac{1}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/x,x]

[Out] PolyLog[2, -(1/Sqrt[x])] - PolyLog[2, 1/Sqrt[x]]

**fricas** [F] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(\sqrt{x})}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x,x, algorithm="fricas")

[Out] integral(arccoth(sqrt(x))/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x,x, algorithm="giac")

[Out] integrate(arccoth(sqrt(x))/x, x)

**maple** [B] time = 0.06, size = 33, normalized size = 1.74

$$\ln(x)\operatorname{arccoth}(\sqrt{x}) - \operatorname{dilog}(\sqrt{x}) - \operatorname{dilog}(1 + \sqrt{x}) - \frac{\ln(x)\ln(1 + \sqrt{x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))/x,x)

[Out] ln(x)\*arccoth(x^(1/2))-dilog(x^(1/2))-dilog(1+x^(1/2))-1/2\*ln(x)\*ln(1+x^(1/2))

**maxima** [B] time = 0.31, size = 66, normalized size = 3.47

$$-\frac{1}{2}(\log(\sqrt{x} + 1) - \log(\sqrt{x} - 1))\log(x) + \operatorname{arccoth}(\sqrt{x})\log(x) + \log(-\sqrt{x})\log(\sqrt{x} + 1) - \frac{1}{2}\log(x)\log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))/x,x, algorithm="maxima")`

[Out]  $-1/2*(\log(\sqrt{x} + 1) - \log(\sqrt{x} - 1))*\log(x) + \operatorname{arccoth}(\sqrt{x})*\log(x) + \log(-\sqrt{x})*\log(\sqrt{x} + 1) - 1/2*\log(x)*\log(\sqrt{x} - 1) + \operatorname{dilog}(\sqrt{x} + 1) - \operatorname{dilog}(-\sqrt{x} + 1)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acoth}(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(x^(1/2))/x,x)`

[Out] `int(acoth(x^(1/2))/x, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x**(1/2))/x,x)`

[Out] `Integral(acoth(sqrt(x))/x, x)`

$$3.87 \quad \int \frac{\coth^{-1}(\sqrt{x})}{x^2} dx$$

Optimal. Leaf size=25

$$-\frac{1}{\sqrt{x}} + \tanh^{-1}(\sqrt{x}) - \frac{\coth^{-1}(\sqrt{x})}{x}$$

[Out]  $-\operatorname{arccoth}(x^{(1/2)})/x + \operatorname{arctanh}(x^{(1/2)}) - 1/x^{(1/2)}$

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6098, 51, 63, 206}

$$-\frac{1}{\sqrt{x}} + \tanh^{-1}(\sqrt{x}) - \frac{\coth^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Sqrt[x]]/x^2,x]`

[Out] `-(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x + ArcTanh[Sqrt[x]]`

#### Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```



Q[a, 0] || LtQ[b, 0])

### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\sqrt{x})}{x^2} dx &= -\frac{\coth^{-1}(\sqrt{x})}{x} + \frac{1}{2} \int \frac{1}{(1-x)x^{3/2}} dx \\ &= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{x} + \frac{1}{2} \int \frac{1}{(1-x)\sqrt{x}} dx \\ &= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{x} + \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\ &= -\frac{1}{\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{x} + \tanh^{-1}(\sqrt{x}) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 45, normalized size = 1.80

$$-\frac{1}{\sqrt{x}} - \frac{1}{2} \log(1 - \sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{\coth^{-1}(\sqrt{x})}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/x^2,x]

[Out] -(1/Sqrt[x]) - ArcCoth[Sqrt[x]]/x - Log[1 - Sqrt[x]]/2 + Log[1 + Sqrt[x]]/2

**fricas [A]** time = 0.49, size = 30, normalized size = 1.20

$$\frac{(x-1) \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) - 2\sqrt{x}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^2,x, algorithm="fricas")

[Out]  $1/2*((x - 1)*\log((x + 2*\sqrt{x} + 1)/(x - 1)) - 2*\sqrt{x})/x$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))/x^2,x, algorithm="giac")`

[Out] `integrate(arccoth(sqrt(x))/x^2, x)`

**maple** [A] time = 0.05, size = 32, normalized size = 1.28

$$-\frac{\operatorname{arccoth}(\sqrt{x})}{x} - \frac{1}{\sqrt{x}} - \frac{\ln(-1 + \sqrt{x})}{2} + \frac{\ln(1 + \sqrt{x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(x^(1/2))/x^2,x)`

[Out] `-arccoth(x^(1/2))/x-1/x^(1/2)-1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))`

**maxima** [A] time = 0.30, size = 31, normalized size = 1.24

$$-\frac{\operatorname{arccoth}(\sqrt{x})}{x} - \frac{1}{\sqrt{x}} + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))/x^2,x, algorithm="maxima")`

[Out] `-arccoth(sqrt(x))/x - 1/sqrt(x) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)`

**mupad** [B] time = 1.27, size = 18, normalized size = 0.72

$$\operatorname{atanh}(\sqrt{x}) - \frac{\operatorname{acoth}(\sqrt{x}) + \sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(x^(1/2))/x^2,x)`

[Out] `atanh(x^(1/2)) - (acoth(x^(1/2)) + x^(1/2))/x`

sympy [B] time = 2.12, size = 92, normalized size = 3.68

$$\frac{x^{\frac{5}{2}} \operatorname{acoth}(\sqrt{x})}{x^{\frac{5}{2}} - x^{\frac{3}{2}}} - \frac{2x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{x^{\frac{5}{2}} - x^{\frac{3}{2}}} + \frac{\sqrt{x} \operatorname{acoth}(\sqrt{x})}{x^{\frac{5}{2}} - x^{\frac{3}{2}}} - \frac{x^2}{x^{\frac{5}{2}} - x^{\frac{3}{2}}} + \frac{x}{x^{\frac{5}{2}} - x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(x\*\*(1/2))/x\*\*2,x)

[Out] x\*\*(5/2)\*acoth(sqrt(x))/(x\*\*(5/2) - x\*\*(3/2)) - 2\*x\*\*(3/2)\*acoth(sqrt(x))/(x\*\*(5/2) - x\*\*(3/2)) + sqrt(x)\*acoth(sqrt(x))/(x\*\*(5/2) - x\*\*(3/2)) - x\*\*2/(x\*\*(5/2) - x\*\*(3/2)) + x/(x\*\*(5/2) - x\*\*(3/2))

$$3.88 \quad \int \frac{\coth^{-1}(\sqrt{x})}{x^3} dx$$

Optimal. Leaf size=42

$$-\frac{1}{6x^{3/2}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} - \frac{1}{2\sqrt{x}} + \frac{1}{2} \tanh^{-1}(\sqrt{x})$$

[Out]  $-1/6/x^{(3/2)}-1/2*\operatorname{arccoth}(x^{(1/2)})/x^2+1/2*\operatorname{arctanh}(x^{(1/2)})-1/2/x^{(1/2)}$

**Rubi [A]** time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6098, 51, 63, 206}

$$-\frac{1}{6x^{3/2}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} - \frac{1}{2\sqrt{x}} + \frac{1}{2} \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Sqrt[x]]/x^3,x]

[Out]  $-1/(6*x^{(3/2)}) - 1/(2*\operatorname{Sqrt}[x]) - \operatorname{ArcCoth}[\operatorname{Sqrt}[x]]/(2*x^2) + \operatorname{ArcTanh}[\operatorname{Sqrt}[x]]/2$

### Rule 51

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

### Rule 6098

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(\sqrt{x})}{x^3} dx &= -\frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{4} \int \frac{1}{(1-x)x^{5/2}} dx \\
 &= -\frac{1}{6x^{3/2}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{4} \int \frac{1}{(1-x)x^{3/2}} dx \\
 &= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{4} \int \frac{1}{(1-x)\sqrt{x}} dx \\
 &= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{2} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right) \\
 &= -\frac{1}{6x^{3/2}} - \frac{1}{2\sqrt{x}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} + \frac{1}{2} \tanh^{-1}(\sqrt{x})
 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 58, normalized size = 1.38

$$-\frac{1}{6x^{3/2}} - \frac{\coth^{-1}(\sqrt{x})}{2x^2} - \frac{1}{2\sqrt{x}} - \frac{1}{4} \log(1 - \sqrt{x}) + \frac{1}{4} \log(\sqrt{x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/x^3, x]

[Out] -1/6\*1/x^(3/2) - 1/(2\*Sqrt[x]) - ArcCoth[Sqrt[x]]/(2\*x^2) - Log[1 - Sqrt[x]]/4 + Log[1 + Sqrt[x]]/4

**fricas [A]** time = 0.51, size = 38, normalized size = 0.90

$$\frac{3(x^2 - 1) \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) - 2(3x+1)\sqrt{x}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="fricas")

[Out] 1/12\*(3\*(x^2 - 1)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) - 2\*(3\*x + 1)\*sqrt(x))/x^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="giac")

[Out] integrate(arccoth(sqrt(x))/x^3, x)

**maple** [A] time = 0.05, size = 37, normalized size = 0.88

$$-\frac{\operatorname{arccoth}(\sqrt{x})}{2x^2} - \frac{1}{6x^{\frac{3}{2}}} - \frac{1}{2\sqrt{x}} - \frac{\ln(-1 + \sqrt{x})}{4} + \frac{\ln(1 + \sqrt{x})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))/x^3,x)

[Out] -1/2\*arccoth(x^(1/2))/x^2-1/6/x^(3/2)-1/2/x^(1/2)-1/4\*ln(-1+x^(1/2))+1/4\*ln(1+x^(1/2))

**maxima** [A] time = 0.31, size = 36, normalized size = 0.86

$$-\frac{3x+1}{6x^{\frac{3}{2}}} - \frac{\operatorname{arccoth}(\sqrt{x})}{2x^2} + \frac{1}{4} \log(\sqrt{x} + 1) - \frac{1}{4} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^3,x, algorithm="maxima")

[Out] -1/6\*(3\*x + 1)/x^(3/2) - 1/2\*arccoth(sqrt(x))/x^2 + 1/4\*log(sqrt(x) + 1) - 1/4\*log(sqrt(x) - 1)

**mupad** [B] time = 1.49, size = 45, normalized size = 1.07

$$\frac{\ln\left(1 - \frac{1}{\sqrt{x}}\right)}{4x^2} - \frac{\frac{x}{2} + \frac{1}{6}}{x^{3/2}} - \frac{\ln\left(\frac{1}{\sqrt{x}} + 1\right)}{4x^2} - \frac{\operatorname{atan}(\sqrt{x} \operatorname{li} 1) \operatorname{li} 1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(x^(1/2))/x^3,x)`

[Out]  $\log(1 - 1/x^{(1/2)})/(4*x^2) - (\operatorname{atan}(x^{(1/2)}*1i)*1i)/2 - (x/2 + 1/6)/x^{(3/2)}$   
 $- \log(1/x^{(1/2)} + 1)/(4*x^2)$

**sympy** [B] time = 5.45, size = 160, normalized size = 3.81

$$\frac{3x^{\frac{7}{2}} \operatorname{acoth}(\sqrt{x})}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} - \frac{3x^{\frac{5}{2}} \operatorname{acoth}(\sqrt{x})}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} - \frac{3x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} + \frac{3\sqrt{x} \operatorname{acoth}(\sqrt{x})}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} - \frac{3x^3}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} + \frac{2x^2}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}} + \frac{x}{6x^{\frac{7}{2}} - 6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x**(1/2))/x**3,x)`

[Out]  $3*x^{(7/2)}*\operatorname{acoth}(\operatorname{sqrt}(x))/(6*x^{(7/2)} - 6*x^{(5/2)}) - 3*x^{(5/2)}*\operatorname{acoth}(\operatorname{sqrt}(x))/(6*x^{(7/2)} - 6*x^{(5/2)}) - 3*x^{(3/2)}*\operatorname{acoth}(\operatorname{sqrt}(x))/(6*x^{(7/2)} - 6*x^{(5/2)}) + 3*\operatorname{sqrt}(x)*\operatorname{acoth}(\operatorname{sqrt}(x))/(6*x^{(7/2)} - 6*x^{(5/2)}) - 3*x^{(3/2)}/(6*x^{(7/2)} - 6*x^{(5/2)}) + 2*x^{(2)}/(6*x^{(7/2)} - 6*x^{(5/2)}) + x/(6*x^{(7/2)} - 6*x^{(5/2)})$

### 3.89 $\int x^{3/2} \coth^{-1}(\sqrt{x}) dx$

Optimal. Leaf size=38

$$\frac{2}{5}x^{5/2} \coth^{-1}(\sqrt{x}) + \frac{x^2}{10} + \frac{x}{5} + \frac{1}{5} \log(1-x)$$

[Out] 1/5\*x+1/10\*x^2+2/5\*x^(5/2)\*arccoth(x^(1/2))+1/5\*ln(1-x)

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6098, 43}

$$\frac{x^2}{10} + \frac{2}{5}x^{5/2} \coth^{-1}(\sqrt{x}) + \frac{x}{5} + \frac{1}{5} \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)\*ArcCoth[Sqrt[x]], x]

[Out] x/5 + x^2/10 + (2\*x^(5/2)\*ArcCoth[Sqrt[x]])/5 + Log[1 - x]/5

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 6098

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^{3/2} \coth^{-1}(\sqrt{x}) dx &= \frac{2}{5}x^{5/2} \coth^{-1}(\sqrt{x}) - \frac{1}{5} \int \frac{x^2}{1-x} dx \\ &= \frac{2}{5}x^{5/2} \coth^{-1}(\sqrt{x}) - \frac{1}{5} \int \left(-1 + \frac{1}{1-x} - x\right) dx \\ &= \frac{x}{5} + \frac{x^2}{10} + \frac{2}{5}x^{5/2} \coth^{-1}(\sqrt{x}) + \frac{1}{5} \log(1-x) \end{aligned}$$



**Mathematica** [A] time = 0.02, size = 31, normalized size = 0.82

$$\frac{1}{10} \left( 4x^{5/2} \coth^{-1}(\sqrt{x}) + (x+2)x + 2 \log(1-x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*ArcCoth[Sqrt[x]], x]

[Out] (x\*(2 + x) + 4\*x^(5/2)\*ArcCoth[Sqrt[x]] + 2\*Log[1 - x])/10

**fricas** [A] time = 1.27, size = 35, normalized size = 0.92

$$\frac{1}{5} x^{\frac{5}{2}} \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right) + \frac{1}{10} x^2 + \frac{1}{5} x + \frac{1}{5} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*arccoth(x^(1/2)), x, algorithm="fricas")

[Out] 1/5\*x^(5/2)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) + 1/10\*x^2 + 1/5\*x + 1/5\*log(x - 1)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*arccoth(x^(1/2)), x, algorithm="giac")

[Out] integrate(x^(3/2)\*arccoth(sqrt(x)), x)

**maple** [A] time = 0.05, size = 35, normalized size = 0.92

$$\frac{2x^{\frac{5}{2}} \operatorname{arccoth}(\sqrt{x})}{5} + \frac{x^2}{10} + \frac{x}{5} + \frac{\ln(-1 + \sqrt{x})}{5} + \frac{\ln(1 + \sqrt{x})}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*arccoth(x^(1/2)), x)

[Out] 2/5\*x^(5/2)\*arccoth(x^(1/2))+1/10\*x^2+1/5\*x+1/5\*ln(-1+x^(1/2))+1/5\*ln(1+x^(1/2))

**maxima** [A] time = 0.31, size = 24, normalized size = 0.63

$$\frac{2}{5} x^{\frac{5}{2}} \operatorname{arccoth}(\sqrt{x}) + \frac{1}{10} x^2 + \frac{1}{5} x + \frac{1}{5} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*arccoth(x^(1/2)),x, algorithm="maxima")

[Out] 2/5\*x^(5/2)\*arccoth(sqrt(x)) + 1/10\*x^2 + 1/5\*x + 1/5\*log(x - 1)

mupad [B] time = 1.26, size = 24, normalized size = 0.63

$$\frac{x}{5} + \frac{\ln(x-1)}{5} + \frac{2x^{5/2} \operatorname{arccoth}(\sqrt{x})}{5} + \frac{x^2}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*acoth(x^(1/2)),x)

[Out] x/5 + log(x - 1)/5 + (2\*x^(5/2)\*acoth(x^(1/2)))/5 + x^2/10

sympy [B] time = 5.35, size = 121, normalized size = 3.18

$$\frac{4x^{\frac{7}{2}} \operatorname{arccoth}(\sqrt{x})}{10x-10} - \frac{4x^{\frac{5}{2}} \operatorname{arccoth}(\sqrt{x})}{10x-10} + \frac{x^3}{10x-10} + \frac{x^2}{10x-10} + \frac{4x \log(\sqrt{x}+1)}{10x-10} - \frac{4x \operatorname{arccoth}(\sqrt{x})}{10x-10} - \frac{4 \log(\sqrt{x}+1)}{10x-10} + \frac{4}{10x-10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*acoth(x\*\*(1/2)),x)

[Out] 4\*x\*\*(7/2)\*acoth(sqrt(x))/(10\*x - 10) - 4\*x\*\*(5/2)\*acoth(sqrt(x))/(10\*x - 10) + x\*\*3/(10\*x - 10) + x\*\*2/(10\*x - 10) + 4\*x\*log(sqrt(x) + 1)/(10\*x - 10) - 4\*x\*acoth(sqrt(x))/(10\*x - 10) - 4\*log(sqrt(x) + 1)/(10\*x - 10) + 4\*acoth(sqrt(x))/(10\*x - 10) - 2/(10\*x - 10)

### 3.90 $\int \sqrt{x} \coth^{-1}(\sqrt{x}) dx$

Optimal. Leaf size=31

$$\frac{2}{3}x^{3/2} \coth^{-1}(\sqrt{x}) + \frac{x}{3} + \frac{1}{3} \log(1-x)$$

[Out] 1/3\*x+2/3\*x^(3/2)\*arccoth(x^(1/2))+1/3\*ln(1-x)

**Rubi [A]** time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6098, 43}

$$\frac{2}{3}x^{3/2} \coth^{-1}(\sqrt{x}) + \frac{x}{3} + \frac{1}{3} \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]\*ArcCoth[Sqrt[x]],x]

[Out] x/3 + (2\*x^(3/2)\*ArcCoth[Sqrt[x]])/3 + Log[1 - x]/3

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 6098

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \sqrt{x} \coth^{-1}(\sqrt{x}) dx &= \frac{2}{3}x^{3/2} \coth^{-1}(\sqrt{x}) - \frac{1}{3} \int \frac{x}{1-x} dx \\ &= \frac{2}{3}x^{3/2} \coth^{-1}(\sqrt{x}) - \frac{1}{3} \int \left(-1 + \frac{1}{1-x}\right) dx \\ &= \frac{x}{3} + \frac{2}{3}x^{3/2} \coth^{-1}(\sqrt{x}) + \frac{1}{3} \log(1-x) \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 25, normalized size = 0.81

$$\frac{1}{3} \left( 2x^{3/2} \coth^{-1}(\sqrt{x}) + x + \log(1-x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*ArcCoth[Sqrt[x]],x]

[Out] (x + 2\*x^(3/2)\*ArcCoth[Sqrt[x]] + Log[1 - x])/3

**fricas** [A] time = 0.65, size = 30, normalized size = 0.97

$$\frac{1}{3} x^{\frac{3}{2}} \log\left(\frac{x + 2\sqrt{x} + 1}{x - 1}\right) + \frac{1}{3} x + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))\*x^(1/2),x, algorithm="fricas")

[Out] 1/3\*x^(3/2)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) + 1/3\*x + 1/3\*log(x - 1)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \operatorname{arccoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))\*x^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x)\*arccoth(sqrt(x)), x)

**maple** [A] time = 0.04, size = 30, normalized size = 0.97

$$\frac{2x^{\frac{3}{2}} \operatorname{arccoth}(\sqrt{x})}{3} + \frac{x}{3} + \frac{\ln(-1 + \sqrt{x})}{3} + \frac{\ln(1 + \sqrt{x})}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))\*x^(1/2),x)

[Out] 2/3\*x^(3/2)\*arccoth(x^(1/2))+1/3\*x+1/3\*ln(-1+x^(1/2))+1/3\*ln(1+x^(1/2))

**maxima** [A] time = 0.31, size = 19, normalized size = 0.61

$$\frac{2}{3} x^{\frac{3}{2}} \operatorname{arccoth}(\sqrt{x}) + \frac{1}{3} x + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="maxima")`

[Out] `2/3*x^(3/2)*arccoth(sqrt(x)) + 1/3*x + 1/3*log(x - 1)`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{x} \operatorname{acoth}(\sqrt{x}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*acoth(x^(1/2)),x)`

[Out] `int(x^(1/2)*acoth(x^(1/2)), x)`

sympy [A] time = 1.07, size = 39, normalized size = 1.26

$$\frac{2x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{3} + \frac{x}{3} + \frac{2 \log(\sqrt{x} + 1)}{3} - \frac{2 \operatorname{acoth}(\sqrt{x})}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x**(1/2))*x**(1/2),x)`

[Out] `2*x**(3/2)*acoth(sqrt(x))/3 + x/3 + 2*log(sqrt(x) + 1)/3 - 2*acoth(sqrt(x))/3`

$$3.91 \quad \int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=20

$$\log(1-x) + 2\sqrt{x} \coth^{-1}(\sqrt{x})$$

[Out]  $\ln(1-x)+2*\operatorname{arccoth}(x^{(1/2)})*x^{(1/2)}$

**Rubi [A]** time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6098, 31}

$$\log(1-x) + 2\sqrt{x} \coth^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[\operatorname{Sqrt}[x]]/\operatorname{Sqrt}[x], x]$

[Out]  $2*\operatorname{Sqrt}[x]*\operatorname{ArcCoth}[\operatorname{Sqrt}[x]] + \operatorname{Log}[1-x]$

Rule 31

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(-1)}, x\_Symbol] := \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /;$   $\operatorname{FreeQ}\{a, b\}, x]$

Rule 6098

$\operatorname{Int}[(a_.) + \operatorname{ArcCoth}[(c_.)*(x_.)^{(n_)}]*(b_.)]*((d_.)*(x_.)^{(m_)}), x\_Symbol] := \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcCoth}[c*x^n])/(d*(m+1)), x] - \operatorname{Dist}[(b*c*n)/(d*(m+1)), \operatorname{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \operatorname{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\sqrt{x})}{\sqrt{x}} dx &= 2\sqrt{x} \coth^{-1}(\sqrt{x}) - \int \frac{1}{1-x} dx \\ &= 2\sqrt{x} \coth^{-1}(\sqrt{x}) + \log(1-x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 20, normalized size = 1.00

$$\log(1-x) + 2\sqrt{x} \coth^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/Sqrt[x],x]

[Out] 2\*Sqrt[x]\*ArcCoth[Sqrt[x]] + Log[1 - x]

**fricas** [A] time = 0.65, size = 24, normalized size = 1.20

$$\sqrt{x} \log\left(\frac{x + 2\sqrt{x} + 1}{x - 1}\right) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="fricas")

[Out] sqrt(x)\*log((x + 2\*sqrt(x) + 1)/(x - 1)) + log(x - 1)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="giac")

[Out] integrate(arccoth(sqrt(x))/sqrt(x), x)

**maple** [A] time = 0.05, size = 15, normalized size = 0.75

$$2 \operatorname{arccoth}(\sqrt{x}) \sqrt{x} + \ln(-1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(x^(1/2))/x^(1/2),x)

[Out] 2\*arccoth(x^(1/2))\*x^(1/2)+ln(-1+x)

**maxima** [A] time = 0.30, size = 16, normalized size = 0.80

$$2\sqrt{x} \operatorname{arccoth}(\sqrt{x}) + \log(-x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(x)\*arccoth(sqrt(x)) + log(-x + 1)

**mupad [B]** time = 1.29, size = 14, normalized size = 0.70

$$\ln(x - 1) + 2\sqrt{x} \operatorname{acoth}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(x^(1/2))/x^(1/2), x)`

[Out] `log(x - 1) + 2*x^(1/2)*acoth(x^(1/2))`

**sympy [B]** time = 0.54, size = 87, normalized size = 4.35

$$\frac{2x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{x-1} - \frac{2\sqrt{x} \operatorname{acoth}(\sqrt{x})}{x-1} + \frac{2x \log(\sqrt{x} + 1)}{x-1} - \frac{2x \operatorname{acoth}(\sqrt{x})}{x-1} - \frac{2 \log(\sqrt{x} + 1)}{x-1} + \frac{2 \operatorname{acoth}(\sqrt{x})}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x**(1/2))/x**(1/2), x)`

[Out] `2*x**(3/2)*acoth(sqrt(x))/(x - 1) - 2*sqrt(x)*acoth(sqrt(x))/(x - 1) + 2*x*log(sqrt(x) + 1)/(x - 1) - 2*x*acoth(sqrt(x))/(x - 1) - 2*log(sqrt(x) + 1)/(x - 1) + 2*acoth(sqrt(x))/(x - 1)`



$$3.92 \quad \int \frac{\coth^{-1}(\sqrt{x})}{x^{3/2}} dx$$

Optimal. Leaf size=24

$$-\log(1-x) + \log(x) - \frac{2 \coth^{-1}(\sqrt{x})}{\sqrt{x}}$$

[Out]  $-\ln(1-x) + \ln(x) - 2 \operatorname{arccoth}(x^{(1/2)})/x^{(1/2)}$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6098, 36, 31, 29}

$$-\log(1-x) + \log(x) - \frac{2 \coth^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Sqrt[x]]/x^(3/2), x]`

[Out] `(-2*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]`

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 31

`Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 36

`Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

Rule 6098

`Int[((a_) + ArcCoth[(c_)*(x_)^(n_)])*(b_)*((d_)*(x_))^(m_), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*ArcCoth[c*x^n]))/(d*(m+1)), x] - Dist[(b*c*n)/(d*(m+1)), Int[(x^(n-1)*(d*x)^(m+1))/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]`

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{coth}^{-1}(\sqrt{x})}{x^{3/2}} dx &= -\frac{2 \operatorname{coth}^{-1}(\sqrt{x})}{\sqrt{x}} + \int \frac{1}{(1-x)x} dx \\
&= -\frac{2 \operatorname{coth}^{-1}(\sqrt{x})}{\sqrt{x}} + \int \frac{1}{1-x} dx + \int \frac{1}{x} dx \\
&= -\frac{2 \operatorname{coth}^{-1}(\sqrt{x})}{\sqrt{x}} - \log(1-x) + \log(x)
\end{aligned}$$

**Mathematica** [A] time = 0.02, size = 24, normalized size = 1.00

$$-\log(1-x) + \log(x) - \frac{2 \operatorname{coth}^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Sqrt[x]]/x^(3/2), x]

[Out] (-2\*ArcCoth[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]

**fricas** [A] time = 0.49, size = 36, normalized size = 1.50

$$\frac{x \log(x-1) - x \log(x) + \sqrt{x} \log\left(\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(x^(1/2))/x^(3/2), x, algorithm="fricas")

[Out] -(x\*log(x - 1) - x\*log(x) + sqrt(x)\*log((x + 2\*sqrt(x) + 1)/(x - 1)))/x

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(sqrt(x))/x^(3/2), x, algorithm="giac")

[Out] integrate(arccoth(sqrt(x))/x^(3/2), x)

**maple [A]** time = 0.05, size = 29, normalized size = 1.21

$$-\frac{2 \operatorname{arccoth}(\sqrt{x})}{\sqrt{x}} + \ln(x) - \ln(-1 + \sqrt{x}) - \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(x^(1/2))/x^(3/2), x)`

[Out] `-2*arccoth(x^(1/2))/x^(1/2)+ln(x)-ln(-1+x^(1/2))-ln(1+x^(1/2))`

**maxima [A]** time = 0.31, size = 18, normalized size = 0.75

$$-\frac{2 \operatorname{arccoth}(\sqrt{x})}{\sqrt{x}} - \log(x - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(x^(1/2))/x^(3/2), x, algorithm="maxima")`

[Out] `-2*arccoth(sqrt(x))/sqrt(x) - log(x - 1) + log(x)`

**mupad [B]** time = 1.25, size = 22, normalized size = 0.92

$$2 \ln(\sqrt{x}) - \ln(x - 1) - \frac{2 \operatorname{acoth}(\sqrt{x})}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(x^(1/2))/x^(3/2), x)`

[Out] `2*log(x^(1/2)) - log(x - 1) - (2*acoth(x^(1/2)))/x^(1/2)`

**sympy [B]** time = 1.38, size = 126, normalized size = 5.25

$$-\frac{2x^{\frac{3}{2}} \operatorname{acoth}(\sqrt{x})}{x^2 - x} + \frac{2\sqrt{x} \operatorname{acoth}(\sqrt{x})}{x^2 - x} + \frac{x^2 \log(x)}{x^2 - x} - \frac{2x^2 \log(\sqrt{x} + 1)}{x^2 - x} + \frac{2x^2 \operatorname{acoth}(\sqrt{x})}{x^2 - x} - \frac{x \log(x)}{x^2 - x} + \frac{2x \log(\sqrt{x})}{x^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(x**(1/2))/x**(3/2), x)`

[Out] `-2*x**(3/2)*acoth(sqrt(x))/(x**2 - x) + 2*sqrt(x)*acoth(sqrt(x))/(x**2 - x) + x**2*log(x)/(x**2 - x) - 2*x**2*log(sqrt(x) + 1)/(x**2 - x) + 2*x**2*acoth(sqrt(x))/(x**2 - x) - x*log(x)/(x**2 - x) + 2*x*log(sqrt(x) + 1)/(x**2 - x) - 2*x*acoth(sqrt(x))/(x**2 - x)`

$$3.93 \quad \int \frac{\coth^{-1}(ax^5)}{x} dx$$

Optimal. Leaf size=28

$$\frac{1}{10}\text{Li}_2\left(-\frac{1}{ax^5}\right) - \frac{1}{10}\text{Li}_2\left(\frac{1}{ax^5}\right)$$

[Out] 1/10\*polylog(2,-1/a/x^5)-1/10\*polylog(2,1/a/x^5)

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {6096, 5913}

$$\frac{1}{10}\text{PolyLog}\left(2, -\frac{1}{ax^5}\right) - \frac{1}{10}\text{PolyLog}\left(2, \frac{1}{ax^5}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x^5]/x, x]

[Out] PolyLog[2, -(1/(a\*x^5))]/10 - PolyLog[2, 1/(a\*x^5)]/10

Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)])\*(b\_.))/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])]/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])]/2, x) /; FreeQ[{a, b, c}, x]

Rule 6096

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*ArcCoth[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax^5)}{x} dx &= \frac{1}{5} \text{Subst}\left(\int \frac{\coth^{-1}(ax)}{x} dx, x, x^5\right) \\ &= \frac{1}{10}\text{Li}_2\left(-\frac{1}{ax^5}\right) - \frac{1}{10}\text{Li}_2\left(\frac{1}{ax^5}\right) \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 26, normalized size = 0.93

$$\frac{1}{10} \left( \operatorname{Li}_2 \left( -\frac{1}{ax^5} \right) - \operatorname{Li}_2 \left( \frac{1}{ax^5} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x^5]/x,x]

[Out] (PolyLog[2, -(1/(a\*x^5))] - PolyLog[2, 1/(a\*x^5)])/10

**fricas** [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\operatorname{arccoth}(ax^5)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^5)/x,x, algorithm="fricas")

[Out] integral(arccoth(a\*x^5)/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^5)/x,x, algorithm="giac")

[Out] integrate(arccoth(a\*x^5)/x, x)

**maple** [C] time = 0.17, size = 85, normalized size = 3.04

$$\ln(x) \operatorname{arccoth}(ax^5) - \frac{\left( \sum_{_R1=\operatorname{RootOf}(a\_Z^5+1)} \left( \ln(x) \ln \left( \frac{-R1-x}{-R1} \right) + \operatorname{dilog} \left( \frac{-R1-x}{-R1} \right) \right) \right)}{2} + \frac{\left( \sum_{_R1=\operatorname{RootOf}(a\_Z^5-1)} \left( \ln(x) \ln \left( \frac{-R1-x}{-R1} \right) + \operatorname{dilog} \left( \frac{-R1-x}{-R1} \right) \right) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x^5)/x,x)

[Out] ln(x)\*arccoth(a\*x^5)-1/2\*sum(ln(x)\*ln((R1-x)/R1)+dilog((R1-x)/R1),\_R1=RootOf(\_Z^5\*a+1))+1/2\*sum(ln(x)\*ln((R1-x)/R1)+dilog((R1-x)/R1),\_R1=RootOf(\_Z^5\*a-1))

**maxima** [B] time = 0.31, size = 104, normalized size = 3.71

$$-\frac{1}{2}a\left(\frac{\log(ax^5+1)}{a}-\frac{\log(ax^5-1)}{a}\right)\log(x)-\frac{1}{10}a\left(\frac{\log(ax^5-1)\log(ax^5)+\operatorname{Li}_2(-ax^5+1)}{a}-\frac{\log(ax^5+1)\log(ax^5-1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^5)/x,x, algorithm="maxima")

[Out] -1/2\*a\*(log(a\*x^5 + 1)/a - log(a\*x^5 - 1)/a)\*log(x) - 1/10\*a\*((log(a\*x^5 - 1)\*log(a\*x^5) + dilog(-a\*x^5 + 1))/a - (log(a\*x^5 + 1)\*log(-a\*x^5) + dilog(a\*x^5 + 1))/a) + arccoth(a\*x^5)\*log(x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a\*x^5)/x,x)

[Out] int(acoth(a\*x^5)/x, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(a\*x\*\*5)/x,x)

[Out] Timed out

### 3.94 $\int \coth^{-1}\left(\frac{1}{x}\right) dx$

Optimal. Leaf size=19

$$\frac{1}{2} \log(1 - x^2) + x \coth^{-1}\left(\frac{1}{x}\right)$$

[Out]  $x \operatorname{arccoth}(1/x) + 1/2 \ln(-x^2 + 1)$

**Rubi [A]** time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {6092, 263, 260}

$$\frac{1}{2} \log(1 - x^2) + x \coth^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[x^{-1}], x]$

[Out]  $x \operatorname{ArcCoth}[x^{-1}] + \operatorname{Log}[1 - x^2]/2$

#### Rule 260

$\operatorname{Int}[(x_)^{(m_.)} / ((a_) + (b_.)(x_)^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]] / (b*n), x] /;$   $\operatorname{FreeQ}\{a, b, m, n\}, x \ \&\& \ \operatorname{EqQ}[m, n - 1]$

#### Rule 263

$\operatorname{Int}[(x_)^{(m_.)} * ((a_) + (b_.)(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Int}[x^{(m + n*p)} * (b + a/x^n)^p, x] /;$   $\operatorname{FreeQ}\{a, b, m, n\}, x \ \&\& \ \operatorname{IntegerQ}[p] \ \&\& \ \operatorname{NegQ}[n]$

#### Rule 6092

$\operatorname{Int}[\operatorname{ArcCoth}[(c_.)(x_)^{(n_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[x \operatorname{ArcCoth}[c*x^n], x] - \operatorname{Dist}[c*n, \operatorname{Int}[x^n / (1 - c^2*x^{(2*n)})], x, x] /;$   $\operatorname{FreeQ}\{c, n\}, x]$

#### Rubi steps

$$\begin{aligned}
\int \coth^{-1}\left(\frac{1}{x}\right) dx &= x \coth^{-1}\left(\frac{1}{x}\right) + \int \frac{1}{\left(1 - \frac{1}{x^2}\right)x} dx \\
&= x \coth^{-1}\left(\frac{1}{x}\right) + \int \frac{x}{-1 + x^2} dx \\
&= x \coth^{-1}\left(\frac{1}{x}\right) + \frac{1}{2} \log(1 - x^2)
\end{aligned}$$

**Mathematica** [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{2} \log(1 - x^2) + x \coth^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[x^(-1)], x]

[Out] x\*ArcCoth[x^(-1)] + Log[1 - x^2]/2

**fricas** [A] time = 0.99, size = 23, normalized size = 1.21

$$\frac{1}{2} x \log\left(-\frac{x+1}{x-1}\right) + \frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1/x), x, algorithm="fricas")

[Out] 1/2\*x\*log(-(x + 1)/(x - 1)) + 1/2\*log(x^2 - 1)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}\left(\frac{1}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1/x), x, algorithm="giac")

[Out] integrate(arccoth(1/x), x)

**maple** [A] time = 0.07, size = 30, normalized size = 1.58

$$x \operatorname{arccoth}\left(\frac{1}{x}\right) - \ln\left(\frac{1}{x}\right) + \frac{\ln\left(\frac{1}{x} - 1\right)}{2} + \frac{\ln\left(\frac{1}{x} + 1\right)}{2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(1/x),x)`

[Out] `x*arccoth(1/x)-ln(1/x)+1/2*ln(1/x-1)+1/2*ln(1/x+1)`

**maxima** [A] time = 0.30, size = 15, normalized size = 0.79

$$x \operatorname{arccoth}\left(\frac{1}{x}\right) + \frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1/x),x, algorithm="maxima")`

[Out] `x*arccoth(1/x) + 1/2*log(x^2 - 1)`

**mupad** [B] time = 1.13, size = 26, normalized size = 1.37

$$\frac{\ln(x^2 - 1)}{2} + x \left( \frac{\ln(x + 1)}{2} - \frac{\ln(1 - x)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(1/x),x)`

[Out] `log(x^2 - 1)/2 + x*(log(x + 1)/2 - log(1 - x)/2)`

**sympy** [A] time = 0.20, size = 15, normalized size = 0.79

$$x \operatorname{acoth}\left(\frac{1}{x}\right) + \log(x + 1) - \operatorname{acoth}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(1/x),x)`

[Out] `x*acoth(1/x) + log(x + 1) - acoth(1/x)`

$$3.95 \quad \int \frac{\coth^{-1}(ax^n)}{x} dx$$

Optimal. Leaf size=38

$$\frac{\operatorname{Li}_2\left(-\frac{x^{-n}}{a}\right)}{2n} - \frac{\operatorname{Li}_2\left(\frac{x^{-n}}{a}\right)}{2n}$$

[Out] 1/2\*polylog(2,-1/a/(x^n))/n-1/2\*polylog(2,1/a/(x^n))/n

**Rubi [A]** time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {6096, 5913}

$$\frac{\operatorname{PolyLog}\left(2, -\frac{x^{-n}}{a}\right)}{2n} - \frac{\operatorname{PolyLog}\left(2, \frac{x^{-n}}{a}\right)}{2n}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a\*x^n]/x, x]

[Out] PolyLog[2, -(1/(a\*x^n))]/(2\*n) - PolyLog[2, 1/(a\*x^n)]/(2\*n)

Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

Rule 6096

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*ArcCoth[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(ax^n)}{x} dx &= \frac{\operatorname{Subst}\left(\int \frac{\coth^{-1}(ax)}{x} dx, x, x^n\right)}{n} \\ &= \frac{\operatorname{Li}_2\left(-\frac{x^{-n}}{a}\right)}{2n} - \frac{\operatorname{Li}_2\left(\frac{x^{-n}}{a}\right)}{2n} \end{aligned}$$

**Mathematica [B]** time = 0.05, size = 97, normalized size = 2.55

$$\frac{-\operatorname{Li}_2(1 - ax^n) + \operatorname{Li}_2(ax^n + 1) + n \log(x) \log(ax^n - 1) - n \log(x) \log(ax^n + 1) - \log(ax^n) \log(ax^n - 1) + \log(ax^n) \log(ax^n + 1)}{2n}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a\*x^n]/x,x]

[Out] (2\*n\*ArcCoth[a\*x^n]\*Log[x] + n\*Log[x]\*Log[-1 + a\*x^n] - Log[a\*x^n]\*Log[-1 + a\*x^n] - n\*Log[x]\*Log[1 + a\*x^n] + Log[-(a\*x^n)]\*Log[1 + a\*x^n] - PolyLog[2, 1 - a\*x^n] + PolyLog[2, 1 + a\*x^n])/(2\*n)

**fricas [B]** time = 0.58, size = 128, normalized size = 3.37

$$n \log(a \cosh(n \log(x)) + a \sinh(n \log(x)) + 1) \log(x) - n \log(-a \cosh(n \log(x)) - a \sinh(n \log(x)) + 1) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^n)/x,x, algorithm="fricas")

[Out] -1/2\*(n\*log(a\*cosh(n\*log(x)) + a\*sinh(n\*log(x)) + 1)\*log(x) - n\*log(-a\*cosh(n\*log(x)) - a\*sinh(n\*log(x)) + 1)\*log(x) - n\*log(x)\*log((a\*cosh(n\*log(x)) + a\*sinh(n\*log(x)) + 1)/(a\*cosh(n\*log(x)) + a\*sinh(n\*log(x)) - 1)) - dilog(a\*cosh(n\*log(x)) + a\*sinh(n\*log(x))) + dilog(-a\*cosh(n\*log(x)) - a\*sinh(n\*log(x)))))/n

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a\*x^n)/x,x, algorithm="giac")

[Out] integrate(arccoth(a\*x^n)/x, x)

**maple [A]** time = 0.09, size = 61, normalized size = 1.61

$$\frac{\ln(ax^n) \operatorname{arccoth}(ax^n)}{n} - \frac{\operatorname{dilog}(ax^n)}{2n} - \frac{\operatorname{dilog}(ax^n + 1)}{2n} - \frac{\ln(ax^n) \ln(ax^n + 1)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a\*x^n)/x,x)

[Out]  $1/n*\ln(a*x^n)*\operatorname{arccoth}(a*x^n)-1/2/n*\operatorname{dilog}(a*x^n)-1/2/n*\operatorname{dilog}(a*x^{n+1})-1/2/n*\ln(a*x^n)*\ln(a*x^{n+1})$

**maxima** [B] time = 0.42, size = 147, normalized size = 3.87

$$-\frac{1}{2}an\left(\frac{\log\left(\frac{ax^n+1}{a}\right)}{an}-\frac{\log\left(\frac{ax^n-1}{a}\right)}{an}\right)\log(x)+\frac{1}{2}an\left(\frac{\log(ax^n+1)\log(x)-\log(ax^n-1)\log(x)}{an}-\frac{n\log(ax^n+1)\log(x)}{an^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(a*x^n)/x,x, algorithm="maxima")`

[Out]  $-1/2*a*n*(\log((a*x^n+1)/a)/(a*n)-\log((a*x^n-1)/a)/(a*n))*\log(x)+1/2*a*n*((\log(a*x^n+1)*\log(x)-\log(a*x^n-1)*\log(x))/(a*n)-(n*\log(a*x^n+1)*\log(x)+\operatorname{dilog}(-a*x^n))/(a*n^2)+(n*\log(-a*x^n+1)*\log(x)+\operatorname{dilog}(a*x^n))/(a*n^2))+\operatorname{arccoth}(a*x^n)*\log(x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acoth}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a*x^n)/x,x)`

[Out] `int(acoth(a*x^n)/x, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(a*x**n)/x,x)`

[Out] `Integral(acoth(a*x**n)/x, x)`

### 3.96 $\int (a + bx) \coth^{-1}(a + bx) dx$

Optimal. Leaf size=39

$$-\frac{\tanh^{-1}(a + bx)}{2b} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} + \frac{x}{2}$$

[Out]  $1/2*x+1/2*(b*x+a)^2*\operatorname{arccoth}(b*x+a)/b-1/2*\operatorname{arctanh}(b*x+a)/b$

**Rubi** [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6108, 5917, 321, 206}

$$-\frac{\tanh^{-1}(a + bx)}{2b} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*x)*ArcCoth[a + b*x],x]`

[Out]  $x/2 + ((a + b*x)^2*\operatorname{ArcCoth}[a + b*x])/(2*b) - \operatorname{ArcTanh}[a + b*x]/(2*b)$

#### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

#### Rule 321

`Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]`

#### Rule 5917

`Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]`

#### Rule 6108

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned} \int (a + bx) \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int x \coth^{-1}(x) dx, x, a + bx\right)}{b} \\ &= \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} - \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2} dx, x, a + bx\right)}{2b} \\ &= \frac{x}{2} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, a + bx\right)}{2b} \\ &= \frac{x}{2} + \frac{(a + bx)^2 \coth^{-1}(a + bx)}{2b} - \frac{\tanh^{-1}(a + bx)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 1.69

$$\frac{a^2 \log(a + bx + 1) - (a^2 - 1) \log(-a - bx + 1) - \log(a + bx + 1) + 2bx(2a + bx) \coth^{-1}(a + bx) + 2bx}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*ArcCoth[a + b*x], x]
```

```
[Out] (2*b*x + 2*b*x*(2*a + b*x)*ArcCoth[a + b*x] - (-1 + a^2)*Log[1 - a - b*x] -
Log[1 + a + b*x] + a^2*Log[1 + a + b*x])/(4*b)
```

**fricas [A]** time = 0.57, size = 44, normalized size = 1.13

$$\frac{2bx + (b^2x^2 + 2abx + a^2 - 1) \log\left(\frac{bx+a+1}{bx+a-1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*arccoth(b*x+a), x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x + (b^2*x^2 + 2*a*b*x + a^2 - 1)*log((b*x + a + 1)/(b*x + a - 1))
)/b
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + a) \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*arccoth(b\*x+a),x, algorithm="giac")

[Out] integrate((b\*x + a)\*arccoth(b\*x + a), x)

**maple** [B] time = 0.03, size = 70, normalized size = 1.79

$$\frac{b \operatorname{arccoth}(bx + a) x^2}{2} + \operatorname{arccoth}(bx + a) xa + \frac{\operatorname{arccoth}(bx + a) a^2}{2b} + \frac{x}{2} + \frac{a}{2b} + \frac{\ln(bx + a - 1)}{4b} - \frac{\ln(bx + a + 1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*arccoth(b\*x+a),x)

[Out] 1/2\*b\*arccoth(b\*x+a)\*x^2+arccoth(b\*x+a)\*x\*a+1/2/b\*arccoth(b\*x+a)\*a^2+1/2\*x+1/2\*a/b+1/4/b\*ln(b\*x+a-1)-1/4\*ln(b\*x+a+1)/b

**maxima** [A] time = 0.31, size = 62, normalized size = 1.59

$$\frac{1}{4} b \left( \frac{2x}{b} + \frac{(a^2 - 1) \log(bx + a + 1)}{b^2} - \frac{(a^2 - 1) \log(bx + a - 1)}{b^2} \right) + \frac{1}{2} (bx^2 + 2ax) \operatorname{arccoth}(bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*arccoth(b\*x+a),x, algorithm="maxima")

[Out] 1/4\*b\*(2\*x/b + (a^2 - 1)\*log(b\*x + a + 1)/b^2 - (a^2 - 1)\*log(b\*x + a - 1)/b^2) + 1/2\*(b\*x^2 + 2\*a\*x)\*arccoth(b\*x + a)

**mupad** [B] time = 2.02, size = 50, normalized size = 1.28

$$\frac{x}{2} - \frac{\frac{\operatorname{acoth}(a+bx)}{2} - \frac{a^2 \operatorname{acoth}(a+bx)}{2}}{b} + ax \operatorname{acoth}(a + bx) + \frac{bx^2 \operatorname{acoth}(a + bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)\*(a + b\*x),x)

[Out] x/2 - (acoth(a + b\*x)/2 - (a^2\*acoth(a + b\*x))/2)/b + a\*x\*acoth(a + b\*x) + (b\*x^2\*acoth(a + b\*x))/2

sympy [A] time = 0.78, size = 56, normalized size = 1.44

$$\begin{cases} \frac{a^2 \operatorname{acoth}(a+bx)}{2b} + ax \operatorname{acoth}(a+bx) + \frac{bx^2 \operatorname{acoth}(a+bx)}{2} + \frac{x}{2} - \frac{\operatorname{acoth}(a+bx)}{2b} & \text{for } b \neq 0 \\ ax \operatorname{acoth}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*acoth(b\*x+a),x)

[Out] Piecewise((a\*\*2\*acoth(a + b\*x)/(2\*b) + a\*x\*acoth(a + b\*x) + b\*x\*\*2\*acoth(a + b\*x)/2 + x/2 - acoth(a + b\*x)/(2\*b), Ne(b, 0)), (a\*x\*acoth(a), True))



### 3.97 $\int (a + bx)^2 \coth^{-1}(a + bx) dx$

Optimal. Leaf size=54

$$\frac{(a + bx)^2}{6b} + \frac{\log(1 - (a + bx)^2)}{6b} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b}$$

[Out]  $1/6*(b*x+a)^2/b+1/3*(b*x+a)^3*\operatorname{arccoth}(b*x+a)/b+1/6*\ln(1-(b*x+a)^2)/b$

**Rubi [A]** time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6108, 5917, 266, 43}

$$\frac{(a + bx)^2}{6b} + \frac{\log(1 - (a + bx)^2)}{6b} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^2\*ArcCoth[a + b\*x], x]

[Out]  $(a + b*x)^2/(6*b) + ((a + b*x)^3*\operatorname{ArcCoth}[a + b*x])/(3*b) + \operatorname{Log}[1 - (a + b*x)^2]/(6*b)$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 6108

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
, x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
& IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + bx)^2 \coth^{-1}(a + bx) dx &= \frac{\text{Subst}\left(\int x^2 \coth^{-1}(x) dx, x, a + bx\right)}{b} \\
&= \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b} - \frac{\text{Subst}\left(\int \frac{x^3}{1-x^2} dx, x, a + bx\right)}{3b} \\
&= \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b} - \frac{\text{Subst}\left(\int \frac{x}{1-x} dx, x, (a + bx)^2\right)}{6b} \\
&= \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b} - \frac{\text{Subst}\left(\int \left(-1 + \frac{1}{1-x}\right) dx, x, (a + bx)^2\right)}{6b} \\
&= \frac{(a + bx)^2}{6b} + \frac{(a + bx)^3 \coth^{-1}(a + bx)}{3b} + \frac{\log(1 - (a + bx)^2)}{6b}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 42, normalized size = 0.78

$$\frac{(a + bx)^2 + \log(1 - (a + bx)^2) + 2(a + bx)^3 \coth^{-1}(a + bx)}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^2\*ArcCoth[a + b\*x], x]

[Out] ((a + b\*x)^2 + 2\*(a + b\*x)^3\*ArcCoth[a + b\*x] + Log[1 - (a + b\*x)^2])/(6\*b)

**fricas [A]** time = 0.58, size = 86, normalized size = 1.59

$$\frac{b^2x^2 + 2abx + (a^3 + 1)\log(bx + a + 1) - (a^3 - 1)\log(bx + a - 1) + (b^3x^3 + 3ab^2x^2 + 3a^2bx)\log\left(\frac{bx+a+1}{bx+a-1}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arccoth(b\*x+a), x, algorithm="fricas")

[Out] 1/6\*(b^2\*x^2 + 2\*a\*b\*x + (a^3 + 1)\*log(b\*x + a + 1) - (a^3 - 1)\*log(b\*x + a - 1) + (b^3\*x^3 + 3\*a\*b^2\*x^2 + 3\*a^2\*b\*x)\*log((b\*x + a + 1)/(b\*x + a - 1)))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + a)^2 \operatorname{arccoth}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arccoth(b\*x+a),x, algorithm="giac")

[Out] integrate((b\*x + a)^2\*arccoth(b\*x + a), x)

**maple** [A] time = 0.03, size = 95, normalized size = 1.76

$$\frac{b^2 \operatorname{arccoth}(bx + a) x^3}{3} + b \operatorname{arccoth}(bx + a) x^2 a + \operatorname{arccoth}(bx + a) x a^2 + \frac{\operatorname{arccoth}(bx + a) a^3}{3b} + \frac{bx^2}{6} + \frac{ax}{3} + \frac{a^2}{6b} + \frac{\ln(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*arccoth(b\*x+a),x)

[Out] 1/3\*b^2\*arccoth(b\*x+a)\*x^3+b\*arccoth(b\*x+a)\*x^2\*a+arccoth(b\*x+a)\*x\*a^2+1/3/b\*arccoth(b\*x+a)\*a^3+1/6\*b\*x^2+1/3\*a\*x+1/6/b\*a^2+1/6/b\*ln(b\*x+a-1)+1/6\*ln(b\*x+a+1)/b

**maxima** [A] time = 0.31, size = 81, normalized size = 1.50

$$\frac{1}{6} b \left( \frac{bx^2 + 2ax}{b} + \frac{(a^3 + 1) \log(bx + a + 1)}{b^2} - \frac{(a^3 - 1) \log(bx + a - 1)}{b^2} \right) + \frac{1}{3} (b^2 x^3 + 3abx^2 + 3a^2 x) \operatorname{arccoth}(bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*arccoth(b\*x+a),x, algorithm="maxima")

[Out] 1/6\*b\*((b\*x^2 + 2\*a\*x)/b + (a^3 + 1)\*log(b\*x + a + 1)/b^2 - (a^3 - 1)\*log(b\*x + a - 1)/b^2) + 1/3\*(b^2\*x^3 + 3\*a\*b\*x^2 + 3\*a^2\*x)\*arccoth(b\*x + a)

**mupad** [B] time = 1.54, size = 114, normalized size = 2.11

$$\frac{ax}{3} + \ln\left(\frac{1}{a+bx} + 1\right) \left(\frac{a^2x}{2} + \frac{abx^2}{2} + \frac{b^2x^3}{6}\right) + \frac{bx^2}{6} - \ln\left(1 - \frac{1}{a+bx}\right) \left(\frac{a^2x}{2} + \frac{abx^2}{2} + \frac{b^2x^3}{6}\right) - \frac{\ln(a+bx-1)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)\*(a + b\*x)^2,x)

[Out] (a\*x)/3 + log(1/(a + b\*x) + 1)\*((a^2\*x)/2 + (b^2\*x^3)/6 + (a\*b\*x^2)/2) + (b\*x^2)/6 - log(1 - 1/(a + b\*x))\*((a^2\*x)/2 + (b^2\*x^3)/6 + (a\*b\*x^2)/2) - (log(a + b\*x - 1)\*(a^3 - 1))/(6\*b) + (log(a + b\*x + 1)\*(a^3 + 1))/(6\*b)

sympy [A] time = 1.22, size = 97, normalized size = 1.80

$$\left\{ \begin{array}{l} \frac{a^3 \operatorname{acoth}(a+bx)}{3b} + a^2x \operatorname{acoth}(a+bx) + abx^2 \operatorname{acoth}(a+bx) + \frac{ax}{3} + \frac{b^2x^3 \operatorname{acoth}(a+bx)}{3} + \frac{bx^2}{6} + \frac{\log\left(\frac{a}{b}+x+\frac{1}{b}\right)}{3b} - \frac{\operatorname{acoth}(a+bx)}{3b} \\ a^2x \operatorname{acoth}(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*acoth(b\*x+a),x)

[Out] Piecewise((a\*\*3\*acoth(a + b\*x)/(3\*b) + a\*\*2\*x\*acoth(a + b\*x) + a\*b\*x\*\*2\*acoth(a + b\*x) + a\*x/3 + b\*\*2\*x\*\*3\*acoth(a + b\*x)/3 + b\*x\*\*2/6 + log(a/b + x + 1/b)/(3\*b) - acoth(a + b\*x)/(3\*b), Ne(b, 0)), (a\*\*2\*x\*acoth(a), True))

$$3.98 \quad \int \frac{\coth^{-1}(a+bx)}{a+bx} dx$$

Optimal. Leaf size=35

$$\frac{\operatorname{Li}_2\left(-\frac{1}{a+bx}\right)}{2b} - \frac{\operatorname{Li}_2\left(\frac{1}{a+bx}\right)}{2b}$$

[Out]  $1/2*\operatorname{polylog}(2, -1/(b*x+a))/b - 1/2*\operatorname{polylog}(2, 1/(b*x+a))/b$

**Rubi** [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6108, 5913}

$$\frac{\operatorname{PolyLog}\left(2, -\frac{1}{a+bx}\right)}{2b} - \frac{\operatorname{PolyLog}\left(2, \frac{1}{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[a + b*x]/(a + b*x), x]`

[Out] `PolyLog[2, -(a + b*x)^(-1)]/(2*b) - PolyLog[2, (a + b*x)^(-1)]/(2*b)`

Rule 5913

`Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/2, x]) /;` `FreeQ[{a, b, c}, x]`

Rule 6108

`Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /;` `FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] & IGtQ[p, 0]`

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(a+bx)}{a+bx} dx &= \frac{\operatorname{Subst}\left(\int \frac{\coth^{-1}(x)}{x} dx, x, a+bx\right)}{b} \\ &= \frac{\operatorname{Li}_2\left(-\frac{1}{a+bx}\right)}{2b} - \frac{\operatorname{Li}_2\left(\frac{1}{a+bx}\right)}{2b} \end{aligned}$$

**Mathematica** [B] time = 0.03, size = 286, normalized size = 8.17

$$-\frac{\operatorname{Li}_2(-a-bx)}{2b} + \frac{\operatorname{Li}_2(a+bx)}{2b} - \frac{\log^2\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{4b} + \frac{\log^2\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{4b} - \frac{\log\left(\frac{b(a+bx-1)}{(a-1)b-ab}\right)\log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{2b} + \frac{\log\left(\frac{a+bx-1}{a+bx}\right)\log}{2b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/(a + b\*x), x]

[Out] 
$$-1/2*(\operatorname{Log}[(b*(-1+a+b*x))/((-1+a)*b-a*b)]*\operatorname{Log}[(-((-1+a)*b)+a*b)/(b*(a+b*x))])/b - \operatorname{Log}[(-((-1+a)*b)+a*b)/(b*(a+b*x))]^2/(4*b) + (\operatorname{Log}[(b*(-1-a-b*x))/((-1-a)*b+a*b)]*\operatorname{Log}[(a*b-(1+a)*b)/(b*(a+b*x))])/(2*b) + \operatorname{Log}[(a*b-(1+a)*b)/(b*(a+b*x))]^2/(4*b) + (\operatorname{Log}[(-((-1+a)*b)+a*b)/(b*(a+b*x))]*\operatorname{Log}[(-1+a+b*x)/(a+b*x)])/(2*b) - (\operatorname{Log}[(a*b-(1+a)*b)/(b*(a+b*x))]*\operatorname{Log}[(1+a+b*x)/(a+b*x)])/(2*b) - \operatorname{PolyLog}[2, -a-b*x]/(2*b) + \operatorname{PolyLog}[2, a+b*x]/(2*b)$$

**fricas** [F] time = 0.83, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{arccoth}(bx+a)}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(b\*x+a), x, algorithm="fricas")

[Out] integral(arccoth(b\*x + a)/(b\*x + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(b\*x+a), x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(b\*x + a), x)

**maple** [A] time = 0.05, size = 59, normalized size = 1.69

$$\frac{\ln(bx+a)\operatorname{arccoth}(bx+a)}{b} - \frac{\operatorname{dilog}(bx+a)}{2b} - \frac{\operatorname{dilog}(bx+a+1)}{2b} - \frac{\ln(bx+a)\ln(bx+a+1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(b\*x+a), x)

[Out]  $1/b \cdot \ln(b \cdot x + a) \cdot \operatorname{arccoth}(b \cdot x + a) - 1/2/b \cdot \operatorname{dilog}(b \cdot x + a) - 1/2/b \cdot \operatorname{dilog}(b \cdot x + a + 1) - 1/2/b \cdot \ln(b \cdot x + a) \cdot \ln(b \cdot x + a + 1)$

**maxima** [B] time = 0.32, size = 112, normalized size = 3.20

$$-\frac{1}{2} b \left( \frac{\log(bx + a) \log(bx + a - 1) + \operatorname{Li}_2(-bx - a + 1)}{b^2} - \frac{\log(bx + a + 1) \log(-bx - a) + \operatorname{Li}_2(bx + a + 1)}{b^2} \right) - \frac{1}{2} \left( \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(b*x+a),x, algorithm="maxima")`

[Out]  $-1/2 * b * ((\log(b * x + a) * \log(b * x + a - 1) + \operatorname{dilog}(-b * x - a + 1)) / b^2 - (\log(b * x + a + 1) * \log(-b * x - a) + \operatorname{dilog}(b * x + a + 1)) / b^2) - 1/2 * (\log(b * x + a + 1) / b - \log(b * x + a - 1) / b) * \log(b * x + a) + \operatorname{arccoth}(b * x + a) * \log(b * x + a) / b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acoth}(a + bx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(a + b*x)/(a + b*x),x)`

[Out] `int(acoth(a + b*x)/(a + b*x), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(a + bx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(b*x+a)/(b*x+a),x)`

[Out] `Integral(acoth(a + b*x)/(a + b*x), x)`

$$3.99 \quad \int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx$$

Optimal. Leaf size=48

$$\frac{\log(a+bx)}{b} - \frac{\log(1-(a+bx)^2)}{2b} - \frac{\coth^{-1}(a+bx)}{b(a+bx)}$$

[Out]  $-\operatorname{arccoth}(b*x+a)/b/(b*x+a)+\ln(b*x+a)/b-1/2*\ln(1-(b*x+a)^2)/b$

**Rubi [A]** time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6108, 5917, 266, 36, 31, 29}

$$\frac{\log(a+bx)}{b} - \frac{\log(1-(a+bx)^2)}{2b} - \frac{\coth^{-1}(a+bx)}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/(a + b\*x)^2, x]

[Out]  $-(\operatorname{ArcCoth}[a + b*x]/(b*(a + b*x))) + \operatorname{Log}[a + b*x]/b - \operatorname{Log}[1 - (a + b*x)^2]/(2*b)$

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_))^(n\_)\*((c\_) + (d\_)\*(x\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]



Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
 *p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
 x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
 tegerQ[m]) && NeQ[m, -1]
```

Rule 6108

```
Int[((a_.) + ArcCoth[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
 m_.), x_Symbol] :> Dist[1/d, Subst[Int[((f*x)/d)^m*(a + b*ArcCoth[x])^p, x]
 , x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d*e - c*f, 0] &
 & IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^{-1}(a+bx)}{(a+bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x^2} dx, x, a+bx\right)}{b} \\
 &= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)} dx, x, a+bx\right)}{b} \\
 &= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)x} dx, x, (a+bx)^2\right)}{2b} \\
 &= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-x} dx, x, (a+bx)^2\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, (a+bx)^2\right)}{2b} \\
 &= -\frac{\coth^{-1}(a+bx)}{b(a+bx)} + \frac{\log(a+bx)}{b} - \frac{\log(1-(a+bx)^2)}{2b}
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 43, normalized size = 0.90

$$\frac{-2\log(a+bx) + \log(1-(a+bx)^2) + \frac{2\coth^{-1}(a+bx)}{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[a + b*x]/(a + b*x)^2, x]
```

[Out]  $-1/2*((2*\text{ArcCoth}[a + b*x])/(a + b*x) - 2*\text{Log}[a + b*x] + \text{Log}[1 - (a + b*x)^2])/b$

**fricas** [A] time = 0.54, size = 67, normalized size = 1.40

$$\frac{(bx + a) \log(b^2x^2 + 2abx + a^2 - 1) - 2(bx + a) \log(bx + a) + \log\left(\frac{bx+a+1}{bx+a-1}\right)}{2(b^2x + ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="fricas")`

[Out]  $-1/2*((b*x + a)*\log(b^2*x^2 + 2*a*b*x + a^2 - 1) - 2*(b*x + a)*\log(b*x + a) + \log((b*x + a + 1)/(b*x + a - 1)))/(b^2*x + a*b)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(bx + a)}{(bx + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="giac")`

[Out] `integrate(arccoth(b*x + a)/(b*x + a)^2, x)`

**maple** [A] time = 0.04, size = 54, normalized size = 1.12

$$-\frac{\text{arccoth}(bx + a)}{b(bx + a)} + \frac{\ln(bx + a)}{b} - \frac{\ln(bx + a - 1)}{2b} - \frac{\ln(bx + a + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(b*x+a)/(b*x+a)^2,x)`

[Out]  $-\text{arccoth}(b*x+a)/b/(b*x+a)+\ln(b*x+a)/b-1/2/b*\ln(b*x+a-1)-1/2*\ln(b*x+a+1)/b$

**maxima** [A] time = 0.31, size = 53, normalized size = 1.10

$$-\frac{\log(bx + a + 1)}{2b} + \frac{\log(bx + a)}{b} - \frac{\log(bx + a - 1)}{2b} - \frac{\text{arccoth}(bx + a)}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="maxima")`

[Out]  $-1/2*\log(b*x + a + 1)/b + \log(b*x + a)/b - 1/2*\log(b*x + a - 1)/b - \text{arccoth}(b*x + a)/((b*x + a)*b)$

mupad [B] time = 1.41, size = 93, normalized size = 1.94

$$\frac{\ln(a + bx)}{b} - \frac{\ln(a^2 + 2abx + b^2x^2 - 1)}{2b} - \frac{\ln\left(\frac{a+bx+1}{a+bx}\right)}{2(xb^2 + ab)} + \frac{\ln\left(\frac{a+bx-1}{a+bx}\right)}{2xb^2 + 2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)/(a + b\*x)^2, x)

[Out] log(a + b\*x)/b - log(a^2 + b^2\*x^2 + 2\*a\*b\*x - 1)/(2\*b) - log((a + b\*x + 1)/(a + b\*x))/(2\*(a\*b + b^2\*x)) + log((a + b\*x - 1)/(a + b\*x))/(2\*a\*b + 2\*b^2\*x)

sympy [A] time = 1.40, size = 136, normalized size = 2.83

$$\begin{cases} \frac{a \log\left(\frac{a}{b} + x\right)}{ab + b^2x} - \frac{a \log\left(\frac{a}{b} + x + \frac{1}{b}\right)}{ab + b^2x} + \frac{a \operatorname{acoth}(a + bx)}{ab + b^2x} + \frac{bx \log\left(\frac{a}{b} + x\right)}{ab + b^2x} - \frac{bx \log\left(\frac{a}{b} + x + \frac{1}{b}\right)}{ab + b^2x} + \frac{bx \operatorname{acoth}(a + bx)}{ab + b^2x} - \frac{\operatorname{acoth}(a + bx)}{ab + b^2x} & \text{for } b \neq 0 \\ \frac{x \operatorname{acoth}(a)}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(b\*x+a)/(b\*x+a)\*\*2, x)

[Out] Piecewise((a\*log(a/b + x)/(a\*b + b\*\*2\*x) - a\*log(a/b + x + 1/b)/(a\*b + b\*\*2\*x) + a\*acoth(a + b\*x)/(a\*b + b\*\*2\*x) + b\*x\*log(a/b + x)/(a\*b + b\*\*2\*x) - b\*x\*log(a/b + x + 1/b)/(a\*b + b\*\*2\*x) + b\*x\*acoth(a + b\*x)/(a\*b + b\*\*2\*x) - acoth(a + b\*x)/(a\*b + b\*\*2\*x), Ne(b, 0)), (x\*acoth(a)/a\*\*2, True))

$$3.100 \quad \int \frac{\coth^{-1}(1+x)}{2+2x} dx$$

Optimal. Leaf size=25

$$\frac{1}{4}\text{Li}_2\left(-\frac{1}{x+1}\right) - \frac{1}{4}\text{Li}_2\left(\frac{1}{x+1}\right)$$

[Out] 1/4\*polylog(2,-1/(1+x))-1/4\*polylog(2,1/(1+x))

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {6108, 12, 5913}

$$\frac{1}{4}\text{PolyLog}\left(2, -\frac{1}{x+1}\right) - \frac{1}{4}\text{PolyLog}\left(2, \frac{1}{x+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 + x]/(2 + 2\*x), x]

[Out] PolyLog[2, -(1 + x)^(-1)]/4 - PolyLog[2, (1 + x)^(-1)]/4

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rule 6108

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] && IGtQ[p, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(1+x)}{2+2x} dx &= \text{Subst} \left( \int \frac{\coth^{-1}(x)}{2x} dx, x, 1+x \right) \\ &= \frac{1}{2} \text{Subst} \left( \int \frac{\coth^{-1}(x)}{x} dx, x, 1+x \right) \\ &= \frac{1}{4} \text{Li}_2 \left( -\frac{1}{1+x} \right) - \frac{1}{4} \text{Li}_2 \left( \frac{1}{1+x} \right) \end{aligned}$$

**Mathematica [B]** time = 0.02, size = 117, normalized size = 4.68

$$-\frac{\text{Li}_2(-x-1)}{4} + \frac{\text{Li}_2(x+1)}{4} + \frac{1}{8} \log^2 \left( -\frac{1}{x+1} \right) - \frac{1}{8} \log^2 \left( \frac{1}{x+1} \right) + \frac{1}{4} \log(x+2) \log \left( -\frac{1}{x+1} \right) - \frac{1}{4} \log \left( \frac{x+2}{x+1} \right) \log \left( -\frac{1}{x+1} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[1 + x]/(2 + 2\*x), x]

[Out] Log[-(1 + x)^(-1)]^2/8 - (Log[-x]\*Log[(1 + x)^(-1)])/4 - Log[(1 + x)^(-1)]^2/8 + (Log[(1 + x)^(-1)]\*Log[x/(1 + x)])/4 + (Log[-(1 + x)^(-1)]\*Log[2 + x])/4 - (Log[-(1 + x)^(-1)]\*Log[(2 + x)/(1 + x)])/4 - PolyLog[2, -1 - x]/4 + PolyLog[2, 1 + x]/4

**fricas [F]** time = 0.55, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\text{arccoth}(x+1)}{2(x+1)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+x)/(2+2\*x), x, algorithm="fricas")

[Out] integral(1/2\*arccoth(x + 1)/(x + 1), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(x+1)}{2(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+x)/(2+2\*x), x, algorithm="giac")

[Out] integrate(1/2\*arccoth(x + 1)/(x + 1), x)

**maple** [A] time = 0.05, size = 34, normalized size = 1.36

$$\frac{\ln(1+x) \operatorname{arccoth}(1+x)}{2} - \frac{\operatorname{dilog}(1+x)}{4} - \frac{\operatorname{dilog}(x+2)}{4} - \frac{\ln(1+x) \ln(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+x)/(2+2\*x), x)

[Out] 1/2\*ln(1+x)\*arccoth(1+x)-1/4\*dilog(1+x)-1/4\*dilog(x+2)-1/4\*ln(1+x)\*ln(x+2)

**maxima** [B] time = 0.31, size = 58, normalized size = 2.32

$$-\frac{1}{4} (\log(x+2) - \log(x)) \log(x+1) + \frac{1}{2} \operatorname{arccoth}(x+1) \log(x+1) - \frac{1}{4} \log(x+1) \log(x) + \frac{1}{4} \log(x+2) \log(-x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+x)/(2+2\*x), x, algorithm="maxima")

[Out] -1/4\*(log(x + 2) - log(x))\*log(x + 1) + 1/2\*arccoth(x + 1)\*log(x + 1) - 1/4\*log(x + 1)\*log(x) + 1/4\*log(x + 2)\*log(-x - 1) - 1/4\*dilog(-x) + 1/4\*dilog(x + 2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}(x+1)}{2x+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(x + 1)/(2\*x + 2), x)

[Out] int(acoth(x + 1)/(2\*x + 2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acoth}(x+1)}{x+1} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+x)/(2+2\*x), x)

[Out] Integral(acoth(x + 1)/(x + 1), x)/2

$$3.101 \quad \int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$$

Optimal. Leaf size=35

$$\frac{\operatorname{Li}_2\left(-\frac{1}{a+bx}\right)}{2d} - \frac{\operatorname{Li}_2\left(\frac{1}{a+bx}\right)}{2d}$$

[Out] 1/2\*polylog(2,-1/(b\*x+a))/d-1/2\*polylog(2,1/(b\*x+a))/d

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {6108, 12, 5913}

$$\frac{\operatorname{PolyLog}\left(2, -\frac{1}{a+bx}\right)}{2d} - \frac{\operatorname{PolyLog}\left(2, \frac{1}{a+bx}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[a + b\*x]/((a\*d)/b + d\*x), x]

[Out] PolyLog[2, -(a + b\*x)^(-1)]/(2\*d) - PolyLog[2, (a + b\*x)^(-1)]/(2\*d)

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

Rule 6108

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[d\*e - c\*f, 0] & IGtQ[p, 0]

Rubi steps

$$\int \frac{\coth^{-1}(a+bx)}{\frac{ad}{b} + dx} dx = \frac{\text{Subst}\left(\int \frac{b \coth^{-1}(x)}{dx} dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{x} dx, x, a+bx\right)}{d}$$

$$= \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{2d} - \frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{2d}$$

**Mathematica [B]** time = 0.03, size = 312, normalized size = 8.91

$$b \left( -\frac{\text{Li}_2(-a-bx)}{2bd} + \frac{\text{Li}_2(a+bx)}{2bd} - \frac{\log^2\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{4bd} + \frac{\log^2\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{4bd} - \frac{\log\left(\frac{b(a+bx-1)}{(a-1)b-ab}\right) \log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{2bd} + \frac{\log\left(\frac{a+bx}{a+bx}\right)}{2bd} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[a + b\*x]/((a\*d)/b + d\*x), x]

[Out]  $b \cdot (-1/2 \cdot (\text{Log}[(b \cdot (-1 + a + b \cdot x)) / ((-1 + a) \cdot b - a \cdot b)]) \cdot \text{Log}[(-((-1 + a) \cdot b) + a \cdot b) / (b \cdot (a + b \cdot x))]) / (b \cdot d) - \text{Log}[(-((-1 + a) \cdot b) + a \cdot b) / (b \cdot (a + b \cdot x))]^2 / (4 \cdot b \cdot d) + (\text{Log}[(b \cdot (-1 - a - b \cdot x)) / ((-1 - a) \cdot b + a \cdot b)]) \cdot \text{Log}[(a \cdot b - (1 + a) \cdot b) / (b \cdot (a + b \cdot x))]) / (2 \cdot b \cdot d) + \text{Log}[(a \cdot b - (1 + a) \cdot b) / (b \cdot (a + b \cdot x))]^2 / (4 \cdot b \cdot d) + (\text{Log}[(-((-1 + a) \cdot b) + a \cdot b) / (b \cdot (a + b \cdot x))] \cdot \text{Log}[(1 + a + b \cdot x) / (a + b \cdot x)]) / (2 \cdot b \cdot d) - (\text{Log}[(a \cdot b - (1 + a) \cdot b) / (b \cdot (a + b \cdot x))] \cdot \text{Log}[(1 + a + b \cdot x) / (a + b \cdot x)]) / (2 \cdot b \cdot d) - \text{PolyLog}[2, -a - b \cdot x] / (2 \cdot b \cdot d) + \text{PolyLog}[2, a + b \cdot x] / (2 \cdot b \cdot d)$

**fricas [F]** time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arccoth}(bx+a)}{bdx+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(a\*d/b+d\*x), x, algorithm="fricas")

[Out] integral(b\*arccoth(b\*x + a)/(b\*d\*x + a\*d), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(bx+a)}{dx + \frac{ad}{b}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(a\*d/b+d\*x),x, algorithm="giac")

[Out] integrate(arccoth(b\*x + a)/(d\*x + a\*d/b), x)

**maple** [A] time = 0.05, size = 59, normalized size = 1.69

$$\frac{\ln(bx + a) \operatorname{arccoth}(bx + a)}{d} - \frac{\operatorname{dilog}(bx + a)}{2d} - \frac{\operatorname{dilog}(bx + a + 1)}{2d} - \frac{\ln(bx + a) \ln(bx + a + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(b\*x+a)/(a\*d/b+d\*x),x)

[Out] 1/d\*ln(b\*x+a)\*arccoth(b\*x+a)-1/2/d\*dilog(b\*x+a)-1/2/d\*dilog(b\*x+a+1)-1/2/d\*ln(b\*x+a)\*ln(b\*x+a+1)

**maxima** [B] time = 0.33, size = 132, normalized size = 3.77

$$-\frac{1}{2}b\left(\frac{\log(bx + a)\log(bx + a - 1) + \operatorname{Li}_2(-bx - a + 1)}{bd} - \frac{\log(bx + a + 1)\log(-bx - a) + \operatorname{Li}_2(bx + a + 1)}{bd}\right) - \frac{1}{2}b\left(\frac{\log(bx + a)\log(bx + a - 1) + \operatorname{Li}_2(-bx - a + 1)}{bd} - \frac{\log(bx + a + 1)\log(-bx - a) + \operatorname{Li}_2(bx + a + 1)}{bd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(b\*x+a)/(a\*d/b+d\*x),x, algorithm="maxima")

[Out] -1/2\*b\*((log(b\*x + a)\*log(b\*x + a - 1) + dilog(-b\*x - a + 1))/(b\*d) - (log(b\*x + a + 1)\*log(-b\*x - a) + dilog(b\*x + a + 1))/(b\*d)) - 1/2\*b\*(log(b\*x + a + 1)/b - log(b\*x + a - 1)/b)\*log(d\*x + a\*d/b)/d + arccoth(b\*x + a)\*log(d\*x + a\*d/b)/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acoth}\left(\frac{a + bx}{d}\right)}{dx + \frac{ad}{b}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(a + b\*x)/(d\*x + (a\*d)/b),x)

[Out] int(acoth(a + b\*x)/(d\*x + (a\*d)/b), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b \int \frac{\operatorname{acoth}\left(\frac{a+bx}{d}\right)}{a+bx} dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acoth(a + b*x)/(a + b*x), x)/d
```

### 3.102 $\int (e + fx)^3 (a + b \coth^{-1}(c + dx)) dx$

**Optimal.** Leaf size=168

$$\frac{(e + fx)^4 (a + b \coth^{-1}(c + dx))}{4f} + \frac{bfx((6c^2 + 1)f^2 - 12cdef + 6d^2e^2)}{4d^3} + \frac{bf^2(c + dx)^2(de - cf)}{2d^4} - \frac{b(-cf + de - f)}{8f}$$

[Out]  $\frac{1}{4}bf(6d^2e^2 - 12cde + (6c^2 + 1)f^2)x/d^3 + \frac{1}{2}bf^2(-cf + de)(d*x + c)^2/d^4 + \frac{1}{12}bf^3(d*x + c)^3/d^4 + \frac{1}{4}(f*x + e)^4(a + b*\operatorname{arccoth}(d*x + c))/f + \frac{1}{8}b(-cf + de + f)^4*\ln(-d*x - c + 1)/d^4 - \frac{1}{8}b(-cf + de - f)^4*\ln(d*x + c + 1)/d^4$

**Rubi [A]** time = 0.34, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e + fx)^4 (a + b \coth^{-1}(c + dx))}{4f} + \frac{bfx((6c^2 + 1)f^2 - 12cdef + 6d^2e^2)}{4d^3} + \frac{bf^2(c + dx)^2(de - cf)}{2d^4} - \frac{b(-cf + de - f)}{8f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^3*(a + b*\operatorname{ArcCoth}[c + d*x]), x]$

[Out]  $(b*f*(6*d^2*e^2 - 12*c*d*e*f + (1 + 6*c^2)*f^2)*x)/(4*d^3) + (b*f^2*(d*e - c*f)*(c + d*x)^2)/(2*d^4) + (b*f^3*(c + d*x)^3)/(12*d^4) + ((e + f*x)^4*(a + b*\operatorname{ArcCoth}[c + d*x]))/(4*f) + (b*(d*e + f - c*f)^4*\operatorname{Log}[1 - c - d*x])/(8*d^4*f) - (b*(d*e - f - c*f)^4*\operatorname{Log}[1 + c + d*x])/(8*d^4*f)$

#### Rule 31

$\operatorname{Int}[(a + (b*x)^m)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b, x\}$

#### Rule 633

$\operatorname{Int}[(d + (e*x))/(a + (c*x)^2), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[-(a*c), 2]\}, \operatorname{Dist}[e/2 + (c*d)/(2*q), \operatorname{Int}[1/(-q + c*x), x], x] + \operatorname{Dist}[e/2 - (c*d)/(2*q), \operatorname{Int}[1/(q + c*x), x], x] /; \operatorname{FreeQ}\{a, c, d, e, x\} \&\& \operatorname{NiceSqrtQ}[-(a*c)]$

#### Rule 702

$\operatorname{Int}[(d + (e*x)^m)/(a + (c*x)^2), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(d + e*x)^m, a + c*x^2, x], x] /; \operatorname{FreeQ}\{a, c, d, e, x\} \&\& \operatorname{NeQ}[$

$c*d^2 + a*e^2, 0]$  && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

### Rule 5927

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + (e\_.)\*(x\_.))^(q\_.), x\_Symbol] :> Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x]))/(e\*(q + 1)), x] - Dist[(b\*c)/(e\*(q + 1)), Int[(d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned}
 \int (e + fx)^3 (a + b \coth^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^3 (a + b \coth^{-1}(x)) dx, x, c + dx\right)}{d} \\
 &= \frac{(e + fx)^4 (a + b \coth^{-1}(c + dx))}{4f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^4}{1-x^2} dx, x, c + dx\right)}{4f} \\
 &= \frac{(e + fx)^4 (a + b \coth^{-1}(c + dx))}{4f} - \frac{b \text{Subst}\left(\int \left(-\frac{f^2(6d^2e^2 - 12cdef + (1+6c^2)f^2)}{d^4}\right) dx, x, c + dx\right)}{4f} \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4} \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4} \\
 &= \frac{bf(6d^2e^2 - 12cdef + (1 + 6c^2)f^2)x}{4d^3} + \frac{bf^2(de - cf)(c + dx)^2}{2d^4} + \frac{bf^3(c + dx)^3}{12d^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.30, size = 270, normalized size = 1.61

$$6dx(4ad^3e^3 + bf((3c^2 + 1)f^2 - 8cdef + 6d^2e^2)) + 6d^2fx^2(6ad^2e^2 + bf(2de - cf)) + 2d^3f^2x^3(12ade + bf) + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] (6\*d\*(4\*a\*d^3\*e^3 + b\*f\*(6\*d^2\*e^2 - 8\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2))\*x + 6\*d^2\*f\*(6\*a\*d^2\*e^2 + b\*f\*(2\*d\*e - c\*f))\*x^2 + 2\*d^3\*f^2\*(12\*a\*d\*e + b\*f)\*x^3 + 6\*a\*d^4\*f^3\*x^4 + 6\*b\*d^4\*x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3)\*ArcCoth[c + d\*x] - 3\*b\*(-1 + c)\*(4\*d^3\*e^3 - 6\*(-1 + c)\*d^2\*e^2\*f + 4\*(-1 + c)^2\*d\*e\*f^2 - (-1 + c)^3\*f^3)\*Log[1 - c - d\*x] - 3\*b\*(1 + c)\*(-4\*d^3\*e^3 + 6\*(1 + c)\*d^2\*e^2\*f - 4\*(1 + c)^2\*d\*e\*f^2 + (1 + c)^3\*f^3)\*Log[1 + c + d\*x])/(24\*d^4)

**fricas [B]** time = 0.55, size = 385, normalized size = 2.29

$$6ad^4f^3x^4 + 2(12ad^4ef^2 + bd^3f^3)x^3 + 6(6ad^4e^2f + 2bd^3ef^2 - bcd^2f^3)x^2 + 6(4ad^4e^3 + 6bd^3e^2f - 8bcd^2ef^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*(a+b\*arccoth(d\*x+c)),x, algorithm="fricas")

[Out] 1/24\*(6\*a\*d^4\*f^3\*x^4 + 2\*(12\*a\*d^4\*e\*f^2 + b\*d^3\*f^3)\*x^3 + 6\*(6\*a\*d^4\*e^2\*f + 2\*b\*d^3\*e\*f^2 - b\*c\*d^2\*f^3)\*x^2 + 6\*(4\*a\*d^4\*e^3 + 6\*b\*d^3\*e^2\*f - 8\*b\*c\*d^2\*e\*f^2 + (3\*b\*c^2 + b)\*d\*f^3)\*x + 3\*(4\*(b\*c + b)\*d^3\*e^3 - 6\*(b\*c^2 + 2\*b\*c + b)\*d^2\*e^2\*f + 4\*(b\*c^3 + 3\*b\*c^2 + 3\*b\*c + b)\*d\*e\*f^2 - (b\*c^4 + 4\*b\*c^3 + 6\*b\*c^2 + 4\*b\*c + b)\*f^3)\*log(d\*x + c + 1) - 3\*(4\*(b\*c - b)\*d^3\*e^3 - 6\*(b\*c^2 - 2\*b\*c + b)\*d^2\*e^2\*f + 4\*(b\*c^3 - 3\*b\*c^2 + 3\*b\*c - b)\*d\*e\*f^2 - (b\*c^4 - 4\*b\*c^3 + 6\*b\*c^2 - 4\*b\*c + b)\*f^3)\*log(d\*x + c - 1) + 3\*(b\*d^4\*f^3\*x^4 + 4\*b\*d^4\*e\*f^2\*x^3 + 6\*b\*d^4\*e^2\*f\*x^2 + 4\*b\*d^4\*e^3\*x)\*log((d\*x + c + 1)/(d\*x + c - 1)))/d^4

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)^3 (b \operatorname{arccoth}(dx + c) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*(a+b\*arccoth(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*(b\*arccoth(d\*x + c) + a), x)

**maple [B]** time = 0.04, size = 786, normalized size = 4.68

$$\frac{3bf \ln(dx+c+1)ce^2}{2d^2} - \frac{3bf \ln(dx+c+1)c^2e^2}{4d^2} - \frac{3bf \ln(dx+c-1)ce^2}{2d^2} - \frac{bf^2 \ln(dx+c-1)c^3e}{2d^3} + \frac{3bf^2 \ln(dx+c-1)c^3e}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*(a+b\*arccoth(d\*x+c)),x)

[Out] 
$$\begin{aligned} & -3/2/d^2*b*f*\ln(d*x+c+1)*c*e^{-3/4}/d^2*b*f*\ln(d*x+c+1)*c^2*e^{-3/2}/d^2*b*f* \\ & \ln(d*x+c-1)*c*e^{-1/2}/d^3*b*f^2*\ln(d*x+c-1)*c^3*e^{3/2}/d^3*b*f^2*\ln(d*x+c-1) \\ & *c^2*e^{1/2}/d^3*b*f^2*\ln(d*x+c+1)*c^3*e^{3/2}/d^3*b*f^2*\ln(d*x+c+1)*c*e^{-3/2}/d^ \\ & 3*b*f^2*\ln(d*x+c-1)*c*e^{-2*b/d^2*f^2*c*e*x+3/2}/d^3*b*f^2*\ln(d*x+c+1)*c^2*e+3 \\ & /4/d^2*b*f*\ln(d*x+c-1)*c^2*e^{2+1/2}/d*b*\ln(d*x+c-1)*e^{3+3/2*a*f*x^2*e^2+a*f^ \\ & 2*x^3*e+arccoth(d*x+c)*x*b*e^{3+1/4*b/f*arccoth(d*x+c)*e^4-1/8*b/f*\ln(d*x+c+ \\ & 1)*e^4+1/8*b/f*\ln(d*x+c-1)*e^4+1/12/d*b*f^3*x^3+1/8/d^4*b*f^3*\ln(d*x+c-1)-1 \\ & /8/d^4*b*f^3*\ln(d*x+c+1)+1/4*b/d^3*f^3*x+1/2/d*b*\ln(d*x+c+1)*e^{3+1/4*b*f^3* \\ & arccoth(d*x+c)*x^4+13/12/d^4*b*f^3*c^3+1/4/d^4*b*f^3*c+1/4*a/f*e^4+3/2/d^2* \\ & b*f*c*e^{-2-1/2}/d^4*b*f^3*\ln(d*x+c-1)*c^3+1/2/d*b*\ln(d*x+c+1)*c*e^{-3-1/2}/d*b*\ln \\ & (d*x+c-1)*c*e^{3+1/2}/d^3*b*f^2*\ln(d*x+c+1)*e^{-3/4}/d^2*b*f*\ln(d*x+c+1)*e^{2+3/ \\ & 4}/d^4*b*f^3*\ln(d*x+c-1)*c^2-1/2/d^4*b*f^3*\ln(d*x+c-1)*c^{-1/4}/d^2*b*f^3*x^2*c \\ & +1/2/d*b*f^2*e*x^2+b*f^2*arccoth(d*x+c)*e*x^3-1/2/d^4*b*f^3*\ln(d*x+c+1)*c^3 \\ & +1/2/d^3*b*f^2*\ln(d*x+c-1)*e^{-1/8}/d^4*b*f^3*\ln(d*x+c+1)*c^4-1/2/d^4*b*f^3*\ln \\ & (d*x+c+1)*c+3/4/d^2*b*f*\ln(d*x+c-1)*e^{2+3/2*b*f*arccoth(d*x+c)*e^2*x^2+3/2* \\ & b/d*f*e^2*x+3/4*b/d^3*f^3*c^2*x-3/4/d^4*b*f^3*\ln(d*x+c+1)*c^2+1/8/d^4*b*f^3 \\ & *\ln(d*x+c-1)*c^4+a*x*e^{3+1/4*a*f^3*x^4-5/2}/d^3*b*f^2*c^2*e \end{aligned}$$

**maxima [B]** time = 0.31, size = 333, normalized size = 1.98

$$\frac{1}{4}af^3x^4+ae^2fx^3+\frac{3}{2}ae^2fx^2+\frac{3}{4}\left(2x^2\operatorname{arccoth}(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*(a+b\*arccoth(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 1/4*a*f^3*x^4 + a*e*f^2*x^3 + 3/2*a*e^2*f*x^2 + 3/4*(2*x^2*arccoth(d*x + c) \\ & + d*(2*x/d^2 - (c^2 + 2*c + 1)*\log(d*x + c + 1)/d^3 + (c^2 - 2*c + 1)*\log(d*x + c - 1)/d^3)) \\ & *b*e^2*f + 1/2*(2*x^3*arccoth(d*x + c) + d*((d*x^2 - 4*c*x)/d^3 + (c^3 + 3*c^2 + 3*c + 1)*\log(d*x + c + 1)/d^4 - (c^3 - 3*c^2 + 3*c - 1)*\log(d*x + c - 1)/d^4)) \\ & *b*e*f^2 + 1/24*(6*x^4*arccoth(d*x + c) + d*(2*(d^2*x^3 - 3*c*d*x^2 + 3*(3*c^2 + 1)*x)/d^4 - 3*(c^4 + 4*c^3 + 6*c^2 + 4*c + 1)*\log(d*x + c + 1)/d^5 + 3*(c^4 - 4*c^3 + 6*c^2 - 4*c + 1)*\log(d*x + c - 1)/d^5)) \\ & *b*f^3 + a*e^3*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + \log(-(d*x + c)^2 + 1)) *b*e^3/d \end{aligned}$$

**mupad [B]** time = 2.17, size = 742, normalized size = 4.42

$$x \left( \frac{e \left( 6ac^2f^2 + 12acdef + 2ad^2e^2 + 3bdef - 6af^2 \right)}{2d^2} - \frac{(4c^2 - 4) \left( \frac{f^2(bf + 8acf + 12ade)}{4d} - \frac{2acf^3}{d} \right)}{4d^2} + \frac{2c \left( \frac{2c}{d} \right)}{4d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e + f*x)^3*(a + b*acoth(c + d*x)),x)`

[Out]  $x \left( \frac{e \left( 6a^2c^2f^2 - 6a^2f^2 + 2a^2d^2e^2 + 3b^2d^2ef + 12a^2cd^2ef \right)}{(2d^2) - \left( (4c^2 - 4) \left( \frac{f^2(bf + 8acf + 12ade)}{4d} - \frac{2acf^3}{d} \right) \right)} \right) / (4d^2) + \left( \frac{2c \left( \frac{2c}{d} \right)}{4d^2} \right) / d - \left( \frac{4a^2c^2f^3 - 4a^2f^3 + 4b^2d^2ef^2 + 12a^2d^2e^2f + 24a^2cd^2ef^2}{(4d^2) + (af^3(4c^2 - 4)) / (4d^2)} \right) / d - \log(1 - 1/(c + d*x)) \left( \frac{bf^3x^4}{8} + \frac{b^2e^3x}{2} + \frac{3b^2e^2fx^2}{4} + \frac{b^2ef^2x^3}{2} \right) - x^2 \left( \frac{c \left( \frac{f^2(bf + 8acf + 12ade)}{4d} - \frac{2acf^3}{d} \right)}{d} - \frac{4a^2c^2f^3 - 4a^2f^3 + 4b^2d^2ef^2 + 12a^2d^2e^2f + 24a^2cd^2ef^2}{(8d^2) + (af^3(4c^2 - 4)) / (8d^2)} \right) + x^3 \left( \frac{f^2(bf + 8acf + 12ade)}{12d} - \frac{2a^2c^2f^3}{(3d)} + \log(1/(c + d*x) + 1) \left( \frac{bf^3x^4}{8} + \frac{b^2e^3x}{2} + \frac{3b^2e^2fx^2}{4} + \frac{b^2ef^2x^3}{2} \right) + \frac{af^3x^4}{4} + (\log(c + d*x) - 1) \left( \frac{bf^3}{3} + \frac{6b^2c^2f^3}{4} - \frac{4b^2c^3f^3}{4} + \frac{4b^2d^3e^3}{b^2c^4f^3} - \frac{4b^2c^4f^3}{4b^2c^3f^3} + \frac{4b^2d^3ef^2}{4b^2c^3d^3e^3} + \frac{6b^2d^2e^2f}{12b^2c^2d^2e^2f} - \frac{12b^2c^2d^2e^2f}{12b^2c^2d^2e^2f} + \frac{12b^2c^2d^2ef^2}{12b^2c^2d^2ef^2} - \frac{4b^2c^3d^2ef^2}{6b^2c^2d^2e^2f} - \frac{12b^2c^3d^2ef^2}{12b^2c^2d^2e^2f} - \frac{12b^2c^2d^2ef^2}{12b^2c^2d^2ef^2} \right) \right) / (8d^4) - (\log(c + d*x) + 1) \left( \frac{bf^3}{3} + \frac{6b^2c^2f^3}{4} + \frac{4b^2c^3f^3}{4} - \frac{4b^2d^3e^3}{b^2c^4f^3} + \frac{b^2c^4f^3}{4b^2c^3f^3} + \frac{4b^2d^3ef^2}{4b^2c^3d^3e^3} - \frac{4b^2c^4f^3}{4b^2c^3d^3e^3} + \frac{6b^2d^2e^2f}{12b^2c^2d^2e^2f} + \frac{12b^2c^2d^2ef^2}{12b^2c^2d^2ef^2} - \frac{12b^2c^2d^2ef^2}{12b^2c^2d^2ef^2} - \frac{4b^2c^3d^2ef^2}{6b^2c^2d^2e^2f} + \frac{6b^2c^2d^2ef^2}{12b^2c^2d^2ef^2} - \frac{12b^2c^3d^2ef^2}{12b^2c^2d^2ef^2} \right) \right) / (8d^4)$

**sympy [A]** time = 7.46, size = 644, normalized size = 3.83

$$\left\{ \begin{array}{l} ae^3x + \frac{3ae^2fx^2}{2} + aef^2x^3 + \frac{af^3x^4}{4} - \frac{bc^4f^3 \operatorname{acoth}(c+dx)}{4d^4} + \frac{bc^3ef^2 \operatorname{acoth}(c+dx)}{d^3} - \frac{bc^3f^3 \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{d^4} + \frac{bc^3f^3 \operatorname{acoth}(c+dx)}{d^4} - \frac{3bc^2e^2}{d^4} \\ (a + b \operatorname{acoth}(c)) \left( e^3x + \frac{3e^2fx^2}{2} + e f^2 x^3 + \frac{f^3 x^4}{4} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**3*(a+b*acoth(d*x+c)),x)`

[Out] `Piecewise((a*e**3*x + 3*a*e**2*f*x**2/2 + a*e*f**2*x**3 + a*f**3*x**4/4 - b*c**4*f**3*acoth(c + d*x)/(4*d**4) + b*c**3*e*f**2*acoth(c + d*x)/d**3 - b*`

```

c**3*f**3*log(c/d + x + 1/d)/d**4 + b*c**3*f**3*acoth(c + d*x)/d**4 - 3*b*c
**2*e**2*f*acoth(c + d*x)/(2*d**2) + 3*b*c**2*e*f**2*log(c/d + x + 1/d)/d**
3 - 3*b*c**2*e*f**2*acoth(c + d*x)/d**3 + 3*b*c**2*f**3*x/(4*d**3) - 3*b*c*
*2*f**3*acoth(c + d*x)/(2*d**4) + b*c*e**3*acoth(c + d*x)/d - 3*b*c*e**2*f*
log(c/d + x + 1/d)/d**2 + 3*b*c*e**2*f*acoth(c + d*x)/d**2 - 2*b*c*e*f**2*x
/d**2 - b*c*f**3*x**2/(4*d**2) + 3*b*c*e*f**2*acoth(c + d*x)/d**3 - b*c*f**
3*log(c/d + x + 1/d)/d**4 + b*c*f**3*acoth(c + d*x)/d**4 + b*e**3*x*acoth(c
+ d*x) + 3*b*e**2*f*x**2*acoth(c + d*x)/2 + b*e*f**2*x**3*acoth(c + d*x) +
b*f**3*x**4*acoth(c + d*x)/4 + b*e**3*log(c/d + x + 1/d)/d - b*e**3*acoth(
c + d*x)/d + 3*b*e**2*f*x/(2*d) + b*e*f**2*x**2/(2*d) + b*f**3*x**3/(12*d)
- 3*b*e**2*f*acoth(c + d*x)/(2*d**2) + b*e*f**2*log(c/d + x + 1/d)/d**3 - b
*e*f**2*acoth(c + d*x)/d**3 + b*f**3*x/(4*d**3) - b*f**3*acoth(c + d*x)/(4*
d**4), Ne(d, 0)), ((a + b*acoth(c))*(e**3*x + 3*e**2*f*x**2/2 + e*f**2*x**3
+ f**3*x**4/4), True))

```



### 3.103 $\int (e + fx)^2 (a + b \coth^{-1}(c + dx)) dx$

**Optimal.** Leaf size=120

$$\frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} + \frac{b(-cf + de + f)^3 \log(-c - dx + 1)}{6d^3 f} - \frac{b(de - (c + 1)f)^3 \log(c + dx + 1)}{6d^3 f} + \frac{bf^2(c + dx + 1)}{6d^3 f}$$

[Out] b\*f\*(-c\*f+d\*e)\*x/d^2+1/6\*b\*f^2\*(d\*x+c)^2/d^3+1/3\*(f\*x+e)^3\*(a+b\*arccoth(d\*x+c))/f+1/6\*b\*(-c\*f+d\*e+f)^3\*ln(-d\*x-c+1)/d^3/f-1/6\*b\*(d\*e-(1+c)\*f)^3\*ln(d\*x+c+1)/d^3/f

**Rubi [A]** time = 0.20, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} + \frac{bfx(de - cf)}{d^2} + \frac{b(-cf + de + f)^3 \log(-c - dx + 1)}{6d^3 f} - \frac{b(de - (c + 1)f)^3 \log(c + dx + 1)}{6d^3 f}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] (b\*f\*(d\*e - c\*f)\*x)/d^2 + (b\*f^2\*(c + d\*x)^2)/(6\*d^3) + ((e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x]))/(3\*f) + (b\*(d\*e + f - c\*f)^3\*Log[1 - c - d\*x])/(6\*d^3\*f) - (b\*(d\*e - (1 + c)\*f)^3\*Log[1 + c + d\*x])/(6\*d^3\*f)

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 633

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[-(a\*c), 2]}, Dist[e/2 + (c\*d)/(2\*q), Int[1/(-q + c\*x), x], x] + Dist[e/2 - (c\*d)/(2\*q), Int[1/(q + c\*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a\*c)]

#### Rule 702

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[PolynomialDivide[(d + e\*x)^m, a + c\*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

#### Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned} \int (e + fx)^2 (a + b \coth^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2 (a + b \coth^{-1}(x)) dx, x, c + dx\right)}{d} \\ &= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^3}{1-x^2} dx, x, c + dx\right)}{3f} \\ &= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} - \frac{b \text{Subst}\left(\int \left(-\frac{3f^2(de-cf)}{d^3} - \frac{f^3x}{d^3} + \frac{(de-cf)}{d^3}\right) dx, x, c + dx\right)}{3f} \\ &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} - \frac{b \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c + dx\right)}{3f} \\ &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} - \frac{(b(de - cf) + f^2x)}{3f} \\ &= \frac{bf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2}{6d^3} + \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))}{3f} + \frac{b(de - cf)}{3f} \end{aligned}$$

**Mathematica** [A] time = 0.17, size = 174, normalized size = 1.45

$$\frac{2dx(3ad^2e^2 + bf(3de - 2cf)) + d^2fx^2(6ade + bf) + 2ad^3f^2x^3 + 2bd^3x(3e^2 + 3efx + f^2x^2) \coth^{-1}(c + dx) - b(de - cf)}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] (2\*d\*(3\*a\*d^2\*e^2 + b\*f\*(3\*d\*e - 2\*c\*f))\*x + d^2\*f\*(6\*a\*d\*e + b\*f)\*x^2 + 2\*a\*d^3\*f^2\*x^3 + 2\*b\*d^3\*x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2)\*ArcCoth[c + d\*x] - b\*(-1 + c)\*(3\*d^2\*e^2 - 3\*(-1 + c)\*d\*e\*f + (-1 + c)^2\*f^2)\*Log[1 - c - d\*x] + b\*(1 + c)\*(3\*d^2\*e^2 - 3\*(1 + c)\*d\*e\*f + (1 + c)^2\*f^2)\*Log[1 + c + d\*x])/(6\*d^3)

**fricas** [B] time = 0.43, size = 241, normalized size = 2.01

$$\frac{2ad^3f^2x^3 + (6ad^3ef + bd^2f^2)x^2 + 2(3ad^3e^2 + 3bd^2ef - 2bcd^2f^2)x + (3(bc + b)d^2e^2 - 3(bc^2 + 2bc + b)def + \dots)}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c)),x, algorithm="fricas")

[Out] 1/6\*(2\*a\*d^3\*f^2\*x^3 + (6\*a\*d^3\*e\*f + b\*d^2\*f^2)\*x^2 + 2\*(3\*a\*d^3\*e^2 + 3\*b\*d^2\*e\*f - 2\*b\*c\*d\*f^2)\*x + (3\*(b\*c + b)\*d^2\*e^2 - 3\*(b\*c^2 + 2\*b\*c + b)\*d\*e\*f + (b\*c^3 + 3\*b\*c^2 + 3\*b\*c + b)\*f^2)\*log(d\*x + c + 1) - (3\*(b\*c - b)\*d^2\*e^2 - 3\*(b\*c^2 - 2\*b\*c + b)\*d\*e\*f + (b\*c^3 - 3\*b\*c^2 + 3\*b\*c - b)\*f^2)\*log(d\*x + c - 1) + (b\*d^3\*f^2\*x^3 + 3\*b\*d^3\*e\*f\*x^2 + 3\*b\*d^3\*e^2\*x)\*log((d\*x + c + 1)/(d\*x + c - 1)))/d^3

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)^2 (b \operatorname{arccoth}(dx + c) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*(b\*arccoth(d\*x + c) + a), x)

**maple** [B] time = 0.04, size = 477, normalized size = 3.98

$$\frac{bf \ln(dx + c - 1) ce}{d^2} - \frac{bf \ln(dx + c + 1) ce}{d^2} - \frac{bf \ln(dx + c + 1) c^2 e}{2d^2} + \frac{bf \ln(dx + c - 1) c^2 e}{2d^2} + \frac{b \ln(dx + c - 1) e^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*(a+b\*arccoth(d\*x+c)),x)

[Out] -1/d^2\*b\*f\*ln(d\*x+c-1)\*c\*e-1/d^2\*b\*f\*ln(d\*x+c+1)\*c\*e-1/2/d^2\*b\*f\*ln(d\*x+c+1)\*c^2\*e+1/2/d^2\*b\*f\*ln(d\*x+c-1)\*c^2\*e+1/2/d\*b\*ln(d\*x+c-1)\*e^2-1/6\*b/f\*ln(d\*x+c+1)\*e^3+1/3\*b/f\*arccoth(d\*x+c)\*e^3+arccoth(d\*x+c)\*x\*b\*e^2+1/3\*b\*f^2\*arcc

oth(dx+c)\*x^3+1/6\*b/f\*ln(dx+c-1)\*e^3+1/6/d\*b\*f^2\*x^2+a\*f\*x^2\*e+1/6/d^3\*b\*f^2\*ln(dx+c+1)+1/6/d^3\*b\*f^2\*ln(dx+c-1)+1/2/d\*b\*ln(dx+c+1)\*e^2-5/6/d^3\*b\*f^2\*c^2+1/3\*a/f\*e^3+1/3\*a\*f^2\*x^3+a\*x\*e^2+1/6/d^3\*b\*f^2\*ln(dx+c+1)\*c^3+1/2/d^2\*b\*f\*ln(dx+c-1)\*e-1/6/d^3\*b\*f^2\*ln(dx+c-1)\*c^3+1/2/d^3\*b\*f^2\*ln(dx+c+1)\*c^2+1/2/d^3\*b\*f^2\*ln(dx+c-1)\*c^2-1/2/d^3\*b\*f^2\*ln(dx+c-1)\*c-1/2/d^2\*b\*f\*ln(dx+c+1)\*e+b\*f\*arccoth(dx+c)\*e\*x^2-1/2/d\*b\*ln(dx+c-1)\*c\*e^2-2/3\*b/d^2\*f^2\*c\*x+b/d\*f\*e\*x+1/2/d\*b\*ln(dx+c+1)\*c\*e^2+1/2/d^3\*b\*f^2\*ln(dx+c+1)\*c+1/d^2\*b\*f\*c\*e

**maxima** [A] time = 0.30, size = 207, normalized size = 1.72

$$\frac{1}{3}af^2x^3+ae^2x^2+\frac{1}{2}\left(2x^2\operatorname{arccoth}(dx+c)+d\left(\frac{2x}{d^2}-\frac{(c^2+2c+1)\log(dx+c+1)}{d^3}+\frac{(c^2-2c+1)\log(dx+c-1)}{d^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(dx+c)),x, algorithm="maxima")

[Out] 1/3\*a\*f^2\*x^3 + a\*e\*f\*x^2 + 1/2\*(2\*x^2\*arccoth(dx + c) + d\*(2\*x/d^2 - (c^2 + 2\*c + 1)\*log(dx + c + 1)/d^3 + (c^2 - 2\*c + 1)\*log(dx + c - 1)/d^3))\*b\*e\*f + 1/6\*(2\*x^3\*arccoth(dx + c) + d\*((dx^2 - 4\*c\*x)/d^3 + (c^3 + 3\*c^2 + 3\*c + 1)\*log(dx + c + 1)/d^4 - (c^3 - 3\*c^2 + 3\*c - 1)\*log(dx + c - 1)/d^4))\*b\*f^2 + a\*e^2\*x + 1/2\*(2\*(dx + c)\*arccoth(dx + c) + log(-(dx + c)^2 + 1))\*b\*e^2/d

**mupad** [B] time = 1.93, size = 386, normalized size = 3.22

$$x^2\left(\frac{f(bf+6acf+6ade)}{6d}-\frac{acf^2}{d}\right)-\ln\left(1-\frac{1}{c+dx}\right)\left(\frac{be^2x}{2}+\frac{befx^2}{2}+\frac{bf^2x^3}{6}\right)-x\left(\frac{2c\left(\frac{f(bf+6acf+6ade)}{3d}\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f\*x)^2\*(a + b\*acoth(c + d\*x)),x)

[Out] x^2\*((f\*(b\*f + 6\*a\*c\*f + 6\*a\*d\*e))/(6\*d) - (a\*c\*f^2)/d) - log(1 - 1/(c + d\*x))\*((b\*f^2\*x^3)/6 + (b\*e^2\*x)/2 + (b\*e\*f\*x^2)/2) - x\*((2\*c\*((f\*(b\*f + 6\*a\*c\*f + 6\*a\*d\*e))/(3\*d) - (2\*a\*c\*f^2)/d))/d - (3\*a\*c^2\*f^2 - 3\*a\*f^2 + 3\*a\*d^2\*e^2 + 3\*b\*d\*e\*f + 12\*a\*c\*d\*e\*f)/(3\*d^2) + (a\*f^2\*(3\*c^2 - 3))/(3\*d^2)) + log(1/(c + d\*x) + 1)\*((b\*f^2\*x^3)/6 + (b\*e^2\*x)/2 + (b\*e\*f\*x^2)/2) + (a\*f^2\*x^3)/3 + (log(c + d\*x - 1))\*((b\*f^2)/6 + d\*((b\*e\*f)/2 + (b\*c^2\*e\*f)/2 - b\*c\*e\*f) + d^2\*((b\*e^2)/2 - (b\*c\*e^2)/2) + (b\*c^2\*f^2)/2 - (b\*c^3\*f^2)/6 - (b\*c\*f^2)/2)/d^3 + (log(c + d\*x + 1))\*((b\*f^2)/6 - d\*((b\*e\*f)/2 + (b\*c^2\*e\*f)/2 + b\*c\*e\*f) + d^2\*((b\*e^2)/2 + (b\*c\*e^2)/2) + (b\*c^2\*f^2)/2 + (b\*c^3\*f^2)/6 + (b\*c\*f^2)/2)/d^3

sympy [A] time = 4.45, size = 369, normalized size = 3.08

$$\left\{ \begin{array}{l} ae^2x + aefx^2 + \frac{af^2x^3}{3} + \frac{bc^3f^2 \operatorname{acoth}(c+dx)}{3d^3} - \frac{bc^2ef \operatorname{acoth}(c+dx)}{d^2} + \frac{bc^2f^2 \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{d^3} - \frac{bc^2f^2 \operatorname{acoth}(c+dx)}{d^3} + \frac{bce^2 \operatorname{acoth}(c+dx)}{d} - \\ (a + b \operatorname{acoth}(c)) \left( e^2x + efx^2 + \frac{f^2x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*(a+b\*acoth(d\*x+c)), x)

[Out] Piecewise((a\*e\*\*2\*x + a\*e\*f\*x\*\*2 + a\*f\*\*2\*x\*\*3/3 + b\*c\*\*3\*f\*\*2\*acoth(c + d\*x)/(3\*d\*\*3) - b\*c\*\*2\*e\*f\*acoth(c + d\*x)/d\*\*2 + b\*c\*\*2\*f\*\*2\*log(c/d + x + 1/d)/d\*\*3 - b\*c\*\*2\*f\*\*2\*acoth(c + d\*x)/d\*\*3 + b\*c\*e\*\*2\*acoth(c + d\*x)/d - 2\*b\*c\*e\*f\*log(c/d + x + 1/d)/d\*\*2 + 2\*b\*c\*e\*f\*acoth(c + d\*x)/d\*\*2 - 2\*b\*c\*f\*\*2\*x/(3\*d\*\*2) + b\*c\*f\*\*2\*acoth(c + d\*x)/d\*\*3 + b\*e\*\*2\*x\*acoth(c + d\*x) + b\*e\*f\*x\*\*2\*acoth(c + d\*x) + b\*f\*\*2\*x\*\*3\*acoth(c + d\*x)/3 + b\*e\*\*2\*log(c/d + x + 1/d)/d - b\*e\*\*2\*acoth(c + d\*x)/d + b\*e\*f\*x/d + b\*f\*\*2\*x\*\*2/(6\*d) - b\*e\*f\*acoth(c + d\*x)/d\*\*2 + b\*f\*\*2\*log(c/d + x + 1/d)/(3\*d\*\*3) - b\*f\*\*2\*acoth(c + d\*x)/(3\*d\*\*3), Ne(d, 0)), ((a + b\*acoth(c))\*(e\*\*2\*x + e\*f\*x\*\*2 + f\*\*2\*x\*\*3/3), True))

### 3.104 $\int (e + fx) (a + b \coth^{-1}(c + dx)) dx$

**Optimal.** Leaf size=97

$$\frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} + \frac{b(-cf + de + f)^2 \log(-c - dx + 1)}{4d^2 f} - \frac{b(de - (c + 1)f)^2 \log(c + dx + 1)}{4d^2 f} + \frac{bfx}{2d}$$

[Out]  $1/2*b*f*x/d+1/2*(f*x+e)^2*(a+b*\operatorname{arccoth}(d*x+c))/f+1/4*b*(-c*f+d*e+f)^2*\ln(-d*x-c+1)/d^2/f-1/4*b*(d*e-(1+c)*f)^2*\ln(d*x+c+1)/d^2/f$

**Rubi [A]** time = 0.17, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6112, 5927, 702, 633, 31}

$$\frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} + \frac{b(-cf + de + f)^2 \log(-c - dx + 1)}{4d^2 f} - \frac{b(de - (c + 1)f)^2 \log(c + dx + 1)}{4d^2 f} + \frac{bfx}{2d}$$

Antiderivative was successfully verified.

[In] `Int[(e + f*x)*(a + b*ArcCoth[c + d*x]),x]`

[Out]  $(b*f*x)/(2*d) + ((e + f*x)^2*(a + b*\operatorname{ArcCoth}[c + d*x]))/(2*f) + (b*(d*e + f - c*f)^2*\operatorname{Log}[1 - c - d*x])/(4*d^2*f) - (b*(d*e - (1 + c)*f)^2*\operatorname{Log}[1 + c + d*x])/(4*d^2*f)$

#### Rule 31

`Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

#### Rule 633

`Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]`

#### Rule 702

`Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])`

#### Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx) (a + b \coth^{-1}(c + dx)) dx &= \frac{\text{Subst}\left(\int \left(\frac{de - cf}{d} + \frac{fx}{d}\right) (a + b \coth^{-1}(x)) dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)^2}{1 - x^2} dx, x, c + dx\right)}{2f} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \left(-\frac{f^2}{d^2} + \frac{d^2 e^2 - 2cdef + (1 + c^2)f^2}{d^2(1 - x^2)}\right) dx, x, c + dx\right)}{2f} \\
&= \frac{bf x}{2d} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} - \frac{b \text{Subst}\left(\int \frac{d^2 e^2 - 2cdef + (1 + c^2)f^2}{1 - x^2} dx, x, c + dx\right)}{2d^2 f} \\
&= \frac{bf x}{2d} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} - \frac{(b(de + f - cf)^2) \text{Subst}\left(\int \frac{1}{1 - x^2} dx, x, c + dx\right)}{4d^2 f} \\
&= \frac{bf x}{2d} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))}{2f} + \frac{b(de + f - cf)^2 \log(1 - c - dx)}{4d^2 f}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 138, normalized size = 1.42

$$aex + \frac{1}{2}afx^2 + \frac{b(c^2 - 2c + 1)f \log(-c - dx + 1)}{4d^2} + \frac{b(-c^2 - 2c - 1)f \log(c + dx + 1)}{4d^2} + \frac{be((c + 1) \log(c + dx + 1) - \log(1 - c - dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)\*(a + b\*ArcCoth[c + d\*x]),x]

[Out] a\*e\*x + (b\*f\*x)/(2\*d) + (a\*f\*x^2)/2 + b\*e\*x\*ArcCoth[c + d\*x] + (b\*f\*x^2\*ArcCoth[c + d\*x])/2 + (b\*(1 - 2\*c + c^2)\*f\*Log[1 - c - d\*x])/(4\*d^2) + (b\*(-1 - 2\*c - c^2)\*f\*Log[1 + c + d\*x])/(4\*d^2) + (b\*e\*(-((-1 + c)\*Log[1 - c - d\*x]) + (1 + c)\*Log[1 + c + d\*x]))/(2\*d)

**fricas** [A] time = 0.45, size = 133, normalized size = 1.37

$$\frac{2ad^2fx^2 + 2(2ad^2e + bdf)x + (2(bc + b)de - (bc^2 + 2bc + b)f)\log(dx + c + 1) - (2(bc - b)de - (bc^2 - 2bc + b)f)\log(dx + c - 1)}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c)),x, algorithm="fricas")

[Out] 1/4\*(2\*a\*d^2\*f\*x^2 + 2\*(2\*a\*d^2\*e + b\*d\*f)\*x + (2\*(b\*c + b)\*d\*e - (b\*c^2 + 2\*b\*c + b)\*f)\*log(d\*x + c + 1) - (2\*(b\*c - b)\*d\*e - (b\*c^2 - 2\*b\*c + b)\*f)\*log(d\*x + c - 1) + (b\*d^2\*f\*x^2 + 2\*b\*d^2\*e\*x)\*log((d\*x + c + 1)/(d\*x + c - 1))/d^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)(b \operatorname{arccoth}(dx + c) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*(b\*arccoth(d\*x + c) + a), x)

**maple** [B] time = 0.03, size = 184, normalized size = 1.90

$$\frac{ax^2f}{2} - \frac{ac^2f}{2d^2} + axe + \frac{ace}{d} + \frac{b \operatorname{arccoth}(dx + c)fx^2}{2} - \frac{b \operatorname{arccoth}(dx + c)fc^2}{2d^2} + \operatorname{arccoth}(dx + c)xbe + \frac{\operatorname{arccoth}(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*(a+b\*arccoth(d\*x+c)),x)

[Out] 1/2\*a\*x^2\*f - 1/2/d^2\*a\*c^2\*f + a\*x\*e + 1/d\*a\*c\*e + 1/2\*b\*arccoth(d\*x+c)\*f\*x^2 - 1/2/d^2\*b\*arccoth(d\*x+c)\*f\*c^2 + arccoth(d\*x+c)\*x\*b\*e + 1/d\*arccoth(d\*x+c)\*b\*c\*e + 1/2\*b\*f\*x/d + 1/2/d^2\*b\*c\*f - 1/2/d^2\*b\*ln(d\*x+c-1)\*c\*f + 1/2/d\*b\*ln(d\*x+c-1)\*e + 1/4/d^2\*b\*ln(d\*x+c-1)\*f - 1/2/d^2\*b\*ln(d\*x+c+1)\*c\*f + 1/2/d\*b\*ln(d\*x+c+1)\*e - 1/4/d^2\*b\*ln(d\*x+c+1)\*f



**maxima** [A] time = 0.30, size = 109, normalized size = 1.12

$$\frac{1}{2} a f x^2 + \frac{1}{4} \left( 2 x^2 \operatorname{arccoth}(d x + c) + d \left( \frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(d x + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(d x + c - 1)}{d^3} \right) \right) b f +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*a\*f\*x^2 + 1/4\*(2\*x^2\*arccoth(d\*x + c) + d\*(2\*x/d^2 - (c^2 + 2\*c + 1)\*log(d\*x + c + 1)/d^3 + (c^2 - 2\*c + 1)\*log(d\*x + c - 1)/d^3))\*b\*f + a\*e\*x + 1/2\*(2\*(d\*x + c)\*arccoth(d\*x + c) + log(-(d\*x + c)^2 + 1))\*b\*e/d

**mupad** [B] time = 2.40, size = 136, normalized size = 1.40

$$a e x + \frac{a f x^2}{2} + \frac{b e \ln(c^2 + 2 c d x + d^2 x^2 - 1)}{2 d} - \frac{b f \operatorname{acoth}(c + d x)}{2 d^2} + \frac{b f x^2 \operatorname{acoth}(c + d x)}{2} + \frac{b f x}{2 d} + b e x \operatorname{acoth}(c +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f\*x)\*(a + b\*acoth(c + d\*x)),x)

[Out] a\*e\*x + (a\*f\*x^2)/2 + (b\*e\*log(c^2 + d^2\*x^2 + 2\*c\*d\*x - 1))/(2\*d) - (b\*f\*a\*coth(c + d\*x))/(2\*d^2) + (b\*f\*x^2\*acoth(c + d\*x))/2 + (b\*f\*x)/(2\*d) + b\*e\*x\*acoth(c + d\*x) - (b\*c^2\*f\*acoth(c + d\*x))/(2\*d^2) - (b\*c\*f\*log(c^2 + d^2\*x^2 + 2\*c\*d\*x - 1))/(2\*d^2) + (b\*c\*e\*acoth(c + d\*x))/d

**sympy** [A] time = 2.27, size = 173, normalized size = 1.78

$$\left\{ \begin{array}{l} a e x + \frac{a f x^2}{2} - \frac{b c^2 f \operatorname{acoth}(c + d x)}{2 d^2} + \frac{b c e \operatorname{acoth}(c + d x)}{d} - \frac{b c f \log\left(\frac{c}{d} + x + \frac{1}{d}\right)}{d^2} + \frac{b c f \operatorname{acoth}(c + d x)}{d^2} + b e x \operatorname{acoth}(c + d x) + \frac{b f x^2 \operatorname{acoth}(c + d x)}{2} \\ (a + b \operatorname{acoth}(c)) \left( e x + \frac{f x^2}{2} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*acoth(d\*x+c)),x)

[Out] Piecewise((a\*e\*x + a\*f\*x\*\*2/2 - b\*c\*\*2\*f\*acoth(c + d\*x)/(2\*d\*\*2) + b\*c\*e\*acoth(c + d\*x)/d - b\*c\*f\*log(c/d + x + 1/d)/d\*\*2 + b\*c\*f\*acoth(c + d\*x)/d\*\*2 + b\*e\*x\*acoth(c + d\*x) + b\*f\*x\*\*2\*acoth(c + d\*x)/2 + b\*e\*log(c/d + x + 1/d)/d - b\*e\*acoth(c + d\*x)/d + b\*f\*x/(2\*d) - b\*f\*acoth(c + d\*x)/(2\*d\*\*2), Ne(d, 0)), ((a + b\*acoth(c))\*(e\*x + f\*x\*\*2/2), True))

### 3.105 $\int (a + b \coth^{-1}(c + dx)) dx$

Optimal. Leaf size=40

$$ax + \frac{b \log(1 - (c + dx)^2)}{2d} + \frac{b(c + dx) \coth^{-1}(c + dx)}{d}$$

[Out] a\*x+b\*(d\*x+c)\*arccoth(d\*x+c)/d+1/2\*b\*ln(1-(d\*x+c)^2)/d

**Rubi [A]** time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {6104, 5911, 260}

$$ax + \frac{b \log(1 - (c + dx)^2)}{2d} + \frac{b(c + dx) \coth^{-1}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcCoth[c + d\*x], x]

[Out] a\*x + (b\*(c + d\*x)\*ArcCoth[c + d\*x])/d + (b\*Log[1 - (c + d\*x)^2])/(2\*d)

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 6104

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{coth}^{-1}(c + dx)) dx &= ax + b \int \operatorname{coth}^{-1}(c + dx) dx \\
&= ax + \frac{b \operatorname{Subst}\left(\int \operatorname{coth}^{-1}(x) dx, x, c + dx\right)}{d} \\
&= ax + \frac{b(c + dx) \operatorname{coth}^{-1}(c + dx)}{d} - \frac{b \operatorname{Subst}\left(\int \frac{x}{1-x^2} dx, x, c + dx\right)}{d} \\
&= ax + \frac{b(c + dx) \operatorname{coth}^{-1}(c + dx)}{d} + \frac{b \log(1 - (c + dx)^2)}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 48, normalized size = 1.20

$$ax + \frac{b((c + 1) \log(c + dx + 1) - (c - 1) \log(-c - dx + 1))}{2d} + bx \operatorname{coth}^{-1}(c + dx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcCoth[c + d\*x], x]

[Out] a\*x + b\*x\*ArcCoth[c + d\*x] + (b\*(-((-1 + c)\*Log[1 - c - d\*x]) + (1 + c)\*Log[1 + c + d\*x]))/(2\*d)

**fricas [A]** time = 0.88, size = 60, normalized size = 1.50

$$\frac{bdx \log\left(\frac{dx+c+1}{dx+c-1}\right) + 2adx + (bc + b) \log(dx + c + 1) - (bc - b) \log(dx + c - 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccoth(d\*x+c), x, algorithm="fricas")

[Out] 1/2\*(b\*d\*x\*log((d\*x + c + 1)/(d\*x + c - 1)) + 2\*a\*d\*x + (b\*c + b)\*log(d\*x + c + 1) - (b\*c - b)\*log(d\*x + c - 1))/d

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int b \operatorname{arccoth}(dx + c) + a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccoth(d\*x+c), x, algorithm="giac")

[Out] integrate(b\*arccoth(d\*x + c) + a, x)

**maple** [A] time = 0.03, size = 42, normalized size = 1.05

$$ax + b \operatorname{arccoth}(dx + c)x + \frac{b \operatorname{arccoth}(dx + c)c}{d} + \frac{b \ln((dx + c)^2 - 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arccoth(d*x+c),x)`

[Out] `a*x+b*arccoth(d*x+c)*x+b/d*arccoth(d*x+c)*c+1/2*b/d*ln((d*x+c)^2-1)`

**maxima** [A] time = 0.30, size = 36, normalized size = 0.90

$$ax + \frac{(2(dx + c) \operatorname{arccoth}(dx + c) + \log(-(dx + c)^2 + 1))b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arccoth(d*x+c),x, algorithm="maxima")`

[Out] `a*x + 1/2*(2*(d*x + c)*arccoth(d*x + c) + log(-(d*x + c)^2 + 1))*b/d`

**mupad** [B] time = 1.75, size = 48, normalized size = 1.20

$$ax + \frac{b \ln(c^2 + 2cdx + d^2x^2 - 1)}{2} + \frac{bc \operatorname{acoth}(c + dx)}{d} + bx \operatorname{acoth}(c + dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + b*acoth(c + d*x),x)`

[Out] `a*x + ((b*log(c^2 + d^2*x^2 + 2*c*d*x - 1))/2 + b*c*acoth(c + d*x))/d + b*x*acoth(c + d*x)`

**sympy** [A] time = 0.58, size = 46, normalized size = 1.15

$$ax + b \left( \begin{cases} \frac{c \operatorname{acoth}(c+dx)}{d} + x \operatorname{acoth}(c + dx) + \frac{\log(c+dx+1)}{d} - \frac{\operatorname{acoth}(c+dx)}{d} & \text{for } d \neq 0 \\ x \operatorname{acoth}(c) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*acoth(d*x+c),x)`

[Out] `a*x + b*Piecewise((c*acoth(c + d*x)/d + x*acoth(c + d*x) + log(c + d*x + 1)/d - acoth(c + d*x)/d, Ne(d, 0)), (x*acoth(c), True))`

$$3.106 \quad \int \frac{a+b \coth^{-1}(c+dx)}{e+fx} dx$$

**Optimal.** Leaf size=130

$$\frac{(a+b \coth^{-1}(c+dx)) \log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f} - \frac{\log\left(\frac{2}{c+dx+1}\right) (a+b \coth^{-1}(c+dx))}{f} - \frac{b \operatorname{Li}_2\left(1 - \frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f}$$

[Out]  $-(a+b \operatorname{arccoth}(d*x+c))*\ln(2/(d*x+c+1))/f+(a+b \operatorname{arccoth}(d*x+c))*\ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+1/2*b*\operatorname{polylog}(2,1-2/(d*x+c+1))/f-1/2*b*\operatorname{polylog}(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f$

**Rubi [A]** time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {6112, 5921, 2402, 2315, 2447}

$$\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{2f} + \frac{(a+b \coth^{-1}(c+dx)) \log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcCoth[c + d*x])/(e + f*x), x]`

[Out]  $-\left(\left(a + b \operatorname{ArcCoth}[c + d*x]\right) * \operatorname{Log}\left[\frac{2}{(1 + c + d*x)}\right]\right) / f + \left(\left(a + b \operatorname{ArcCoth}[c + d*x]\right) * \operatorname{Log}\left[\frac{2*d*(e + f*x)}{\left(d*e + f - c*f\right)*(1 + c + d*x)}\right]\right) / f + \left(b \operatorname{PolyLog}\left[2, 1 - \frac{2}{(1 + c + d*x)}\right]\right) / (2*f) - \left(b \operatorname{PolyLog}\left[2, 1 - \frac{2*d*(e + f*x)}{\left(d*e + f - c*f\right)*(1 + c + d*x)}\right]\right) / (2*f)$

**Rule 2315**

`Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]`

**Rule 2402**

`Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]`

**Rule 2447**

`Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,`

x][[2]], Expon[Pq, x]]

### Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] :> -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

### Rubi steps

$$\int \frac{a + b \coth^{-1}(c + dx)}{e + fx} dx = \frac{\text{Subst}\left(\int \frac{a+b \coth^{-1}(x)}{\frac{de-cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}$$

$$= -\frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}$$

$$= -\frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2}{1+c+dx}\right)}{f} + \frac{(a + b \coth^{-1}(c + dx)) \log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}$$

**Mathematica [A]** time = 0.12, size = 206, normalized size = 1.58

$$\frac{a \log(e + fx)}{f} - \frac{b \text{Li}_2\left(\frac{d(e+fx)}{de-cf-f}\right)}{2f} + \frac{b \text{Li}_2\left(\frac{d(e+fx)}{de-cf+f}\right)}{2f} + \frac{b \log(e + fx) \log\left(\frac{f(-c-dx+1)}{-cf+de+f}\right)}{2f} - \frac{b \log\left(-\frac{-c-dx+1}{c+dx}\right) \log(e + fx)}{2f} - \frac{b \log\left(\frac{2d(e+fx)}{(de+f-cf)(1+c+dx)}\right)}{f}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])/(e + f\*x), x]

[Out] (a\*Log[e + f\*x])/f + (b\*Log[(f\*(1 - c - d\*x))/(d\*e + f - c\*f)]\*Log[e + f\*x])/ (2\*f) - (b\*Log[-((1 - c - d\*x)/(c + d\*x))]\*Log[e + f\*x])/ (2\*f) - (b\*Log[-((f\*(1 + c + d\*x))/(d\*e - f - c\*f))]\*Log[e + f\*x])/ (2\*f) + (b\*Log[(1 + c + d\*x)/(c + d\*x)]\*Log[e + f\*x])/ (2\*f) - (b\*PolyLog[2, (d\*(e + f\*x))/(d\*e - f - c\*f)])/ (2\*f) + (b\*PolyLog[2, (d\*(e + f\*x))/(d\*e + f - c\*f)])/ (2\*f)

**fricas** [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arccoth}(dx + c) + a}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e), x, algorithm="fricas")

[Out] integral((b\*arccoth(d\*x + c) + a)/(f\*x + e), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arccoth}(dx + c) + a}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e), x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)/(f\*x + e), x)

**maple** [A] time = 0.08, size = 202, normalized size = 1.55

$$\frac{a \ln((dx + c)f - cf + de)}{f} + \frac{b \ln((dx + c)f - cf + de) \operatorname{arccoth}(dx + c)}{f} - \frac{b \ln((dx + c)f - cf + de) \ln\left(\frac{(dx+c)f+}{cf-de+f}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))/(f\*x+e), x)

[Out] a\*ln((d\*x+c)\*f-c\*f+d\*e)/f+b\*ln((d\*x+c)\*f-c\*f+d\*e)/f\*arccoth(d\*x+c)-1/2\*b/f\*ln((d\*x+c)\*f-c\*f+d\*e)\*ln(((d\*x+c)\*f+f)/(c\*f-d\*e+f))-1/2\*b/f\*dilog(((d\*x+c)\*f+f)/(c\*f-d\*e+f))+1/2\*b/f\*ln((d\*x+c)\*f-c\*f+d\*e)\*ln(((d\*x+c)\*f-f)/(c\*f-d\*e-f))+1/2\*b/f\*dilog(((d\*x+c)\*f-f)/(c\*f-d\*e-f))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \int \frac{\log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right)}{fx + e} dx + \frac{a \log(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e),x, algorithm="maxima")

[Out] 1/2\*b\*integrate((log(1/(d\*x + c) + 1) - log(-1/(d\*x + c) + 1))/(f\*x + e), x) + a\*log(f\*x + e)/f

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acoth}(c + dx)}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c + d\*x))/(e + f\*x),x)

[Out] int((a + b\*acoth(c + d\*x))/(e + f\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acoth}(c + dx)}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))/(f\*x+e),x)

[Out] Integral((a + b\*acoth(c + d\*x))/(e + f\*x), x)



$$3.107 \quad \int \frac{a+b \coth^{-1}(c+dx)}{(e+fx)^2} dx$$

**Optimal.** Leaf size=115

$$\frac{a+b \coth^{-1}(c+dx)}{f(e+fx)} - \frac{bd \log(-c-dx+1)}{2f(-cf+de+f)} + \frac{bd \log(c+dx+1)}{2f(-cf+de-f)} - \frac{bd \log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

[Out]  $(-a-b*\operatorname{arccoth}(d*x+c))/f/(f*x+e)-1/2*b*d*\ln(-d*x-c+1)/f/(-c*f+d*e+f)+1/2*b*d*\ln(d*x+c+1)/f/(-c*f+d*e-f)-b*d*\ln(f*x+e)/(-c*f+d*e-f)/(-c*f+d*e+f)$

**Rubi [A]** time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {6110, 1982, 705, 31, 632}

$$\frac{a+b \coth^{-1}(c+dx)}{f(e+fx)} - \frac{bd \log(-c-dx+1)}{2f(-cf+de+f)} + \frac{bd \log(c+dx+1)}{2f(-cf+de-f)} - \frac{bd \log(e+fx)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])/(e + f\*x)^2, x]

[Out]  $-((a + b*\operatorname{ArcCoth}[c + d*x])/(f*(e + f*x))) - (b*d*\operatorname{Log}[1 - c - d*x])/(2*f*(d*e + f - c*f)) + (b*d*\operatorname{Log}[1 + c + d*x])/(2*f*(d*e - f - c*f)) - (b*d*\operatorname{Log}[e + f*x])/((d*e + f - c*f)*(d*e - (1 + c)*f))$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 632

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(c\*d - e\*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c\*x), x], x] - Dist[(c\*d - e\*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 705

Int[1/(((d\_.) + (e\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)), x\_Symbol] := Dist[e^2/(c\*d^2 - b\*d\*e + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 - b\*d\*e + a\*e^2), Int[(c\*d - b\*e - c\*e\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0]

2, 0] && NeQ[2\*c\*d - b\*e, 0]

### Rule 1982

Int[(u\_)^(m\_.)\*(v\_)^(p\_.), x\_Symbol] := Int[ExpandToSum[u, x]^m\*ExpandToSum[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && ! (LinearMatchQ[u, x] && QuadraticMatchQ[v, x])

### Rule 6110

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^p)/(f\*(m + 1)), x] - Dist[(b\*d\*p)/(f\*(m + 1)), Int[((e + f\*x)^(m + 1)\*(a + b\*ArcCoth[c + d\*x])^(p - 1))/(1 - (c + d\*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)^2} dx &= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{1}{(e+fx)(1-(c+dx)^2)} dx}{f} \\
 &= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{1}{(e+fx)(1-c^2-2cdx-d^2x^2)} dx}{f} \\
 &= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} + \frac{(bd) \int \frac{-d^2e+2cdf+d^2fx}{1-c^2-2cdx-d^2x^2} dx}{f(-d^2e^2 + 2cdef + (1 - c^2) f^2)} + \frac{(bdf) \int \frac{1}{e+fx} dx}{-d^2e^2 + 2cdef + (1 - c^2) f^2} \\
 &= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} - \frac{bd \log(e + fx)}{(de - f - cf)(de + f - cf)} - \frac{(bd^3) \int \frac{1}{-d-cd-d^2x} dx}{2f(de - f - cf)} + \frac{(bd^3)}{2f(de - f - cf)} \\
 &= -\frac{a + b \coth^{-1}(c + dx)}{f(e + fx)} - \frac{bd \log(1 - c - dx)}{2f(de + f - cf)} + \frac{bd \log(1 + c + dx)}{2f(de - f - cf)} - \frac{bd \log(e + fx)}{(de - f - cf)(de + f - cf)}
 \end{aligned}$$

**Mathematica** [A] time = 0.19, size = 125, normalized size = 1.09

$$\frac{1}{2} \left( -\frac{2a}{f(e + fx)} - \frac{2bd \log(e + fx)}{(c^2 - 1) f^2 - 2cdef + d^2e^2} + \frac{bd \log(-c - dx + 1)}{f((c - 1)f - de)} - \frac{bd \log(c + dx + 1)}{f(cf - de + f)} - \frac{2b \coth^{-1}(c + dx)}{f(e + fx)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])/(e + f\*x)^2, x]

[Out]  $((-2*a)/(f*(e + f*x)) - (2*b*\text{ArcCoth}[c + d*x])/(f*(e + f*x)) + (b*d*\text{Log}[1 - c - d*x])/(f*(-(d*e) + (-1 + c)*f)) - (b*d*\text{Log}[1 + c + d*x])/(f*(-(d*e) + f + c*f)) - (2*b*d*\text{Log}[e + f*x])/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2))/2$

**fricas** [B] time = 0.80, size = 262, normalized size = 2.28

$$\frac{2ad^2e^2 - 4acdef + 2(ac^2 - a)f^2 - (bd^2e^2 - (bc - b)def + (bd^2ef - (bc - b)df^2)x) \log(dx + c + 1) + (bd^2e^2 - 2(d^2e^3f - 2cde^2f^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="fricas")`

[Out]  $-1/2*(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*(a*c^2 - a)*f^2 - (b*d^2*e^2 - (b*c - b)*d*e*f + (b*d^2*e*f - (b*c - b)*d*f^2)*x)*\log(d*x + c + 1) + (b*d^2*e^2 - (b*c + b)*d*e*f + (b*d^2*e*f - (b*c + b)*d*f^2)*x)*\log(d*x + c - 1) + 2*(b*d*f^2*x + b*d*e*f)*\log(f*x + e) + (b*d^2*e^2 - 2*b*c*d*e*f + (b*c^2 - b)*f^2)*\log((d*x + c + 1)/(d*x + c - 1))/(d^2*e^3*f - 2*c*d*e^2*f^2 + (c^2 - 1)*e*f^3 + (d^2*e^2*f^2 - 2*c*d*e*f^3 + (c^2 - 1)*f^4)*x)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arccoth}(dx + c) + a}{(fx + e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="giac")`

[Out] `integrate((b*arccoth(d*x + c) + a)/(f*x + e)^2, x)`

**maple** [A] time = 0.04, size = 141, normalized size = 1.23

$$\frac{da}{(dfx + de)f} - \frac{db \operatorname{arccoth}(dx + c)}{(dfx + de)f} - \frac{db \ln((dx + c)f - cf + de)}{(cf - de - f)(cf - de + f)} + \frac{db \ln(dx + c - 1)}{f(2cf - 2de - 2f)} - \frac{db \ln(dx + c + 1)}{f(2cf - 2de + 2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccoth(d*x+c))/(f*x+e)^2,x)`

[Out]  $-d*a/(d*f*x+d*e)/f - d*b/(d*f*x+d*e)/f*\operatorname{arccoth}(d*x+c) - d*b/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e) + d*b/f/(2*c*f-2*d*e-2*f)*\ln(d*x+c-1) - d*b/f/(2*c*f-2*d*e+2*f)*\ln(d*x+c+1)$

**maxima** [A] time = 0.31, size = 121, normalized size = 1.05

$$\frac{1}{2} \left( d \left( \frac{\log(dx+c+1)}{def - (c+1)f^2} - \frac{\log(dx+c-1)}{def - (c-1)f^2} - \frac{2 \log(fx+e)}{d^2e^2 - 2cdef + (c^2-1)f^2} \right) - \frac{2 \operatorname{arccoth}(dx+c)}{f^2x+ef} \right) b - \frac{a}{f^2x+ef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^2,x, algorithm="maxima")

[Out] 1/2\*(d\*(log(d\*x + c + 1)/(d\*e\*f - (c + 1)\*f^2) - log(d\*x + c - 1)/(d\*e\*f - (c - 1)\*f^2) - 2\*log(f\*x + e)/(d^2\*e^2 - 2\*c\*d\*e\*f + (c^2 - 1)\*f^2)) - 2\*arccoth(d\*x + c)/(f^2\*x + e\*f))\*b - a/(f^2\*x + e\*f)

**mupad** [B] time = 2.08, size = 175, normalized size = 1.52

$$\ln(e+fx) \left( \frac{b(c-1)}{2e(de-f(c-1))} - \frac{b(c+1)}{2e(de-f(c+1))} \right) - \frac{a}{xf^2+ef} - \frac{b \ln\left(\frac{1}{c+dx} + 1\right)}{2f(e+fx)} - \frac{bd \ln(c+dx-1)}{2f^2-2cf^2+2def} - \frac{bd \ln(c+dx+1)}{2f^2-2cf^2+2def}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c + d\*x))/(e + f\*x)^2,x)

[Out] log(e + f\*x)\*((b\*(c - 1))/(2\*e\*(d\*e - f\*(c - 1))) - (b\*(c + 1))/(2\*e\*(d\*e - f\*(c + 1)))) - a/(e\*f + f^2\*x) - (b\*log(1/(c + d\*x) + 1))/(2\*f\*(e + f\*x)) - (b\*d\*log(c + d\*x - 1))/(2\*f^2 - 2\*c\*f^2 + 2\*d\*e\*f) - (b\*d\*log(c + d\*x + 1))/(2\*c\*f^2 + 2\*f^2 - 2\*d\*e\*f) + (b\*log(1 - 1/(c + d\*x)))/(f\*(2\*e + 2\*f\*x))

**sympy** [A] time = 9.48, size = 1658, normalized size = 14.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))/(f\*x+e)\*\*2,x)

[Out] Piecewise(((a\*x + b\*c\*acoth(c + d\*x)/d + b\*x\*acoth(c + d\*x) + b\*log(c/d + x + 1/d)/d - b\*acoth(c + d\*x)/d)/e\*\*2, Eq(f, 0)), (-2\*a\*f/(2\*e\*f\*\*2 + 2\*f\*\*3\*x) + b\*d\*e\*acoth(d\*e/f + d\*x - 1)/(2\*e\*f\*\*2 + 2\*f\*\*3\*x) + b\*d\*f\*x\*acoth(d\*e/f + d\*x - 1)/(2\*e\*f\*\*2 + 2\*f\*\*3\*x) - 2\*b\*f\*acoth(d\*e/f + d\*x - 1)/(2\*e\*f\*\*2 + 2\*f\*\*3\*x) - b\*f/(2\*e\*f\*\*2 + 2\*f\*\*3\*x), Eq(c, (d\*e - f)/f)), (-2\*a\*f/(2\*e\*f\*\*2 + 2\*f\*\*3\*x) - b\*d\*e\*acoth(d\*e/f + d\*x + 1)/(2\*e\*f\*\*2 + 2\*f\*\*3\*x) - b\*d\*f\*x\*acoth(d\*e/f + d\*x + 1)/(2\*e\*f\*\*2 + 2\*f\*\*3\*x) - 2\*b\*f\*acoth(d\*e/f + d\*x + 1)/(2\*e\*f\*\*2 + 2\*f\*\*3\*x) + b\*f/(2\*e\*f\*\*2 + 2\*f\*\*3\*x), Eq(c, (d\*e + f)/f)), (zoo\*(a\*x + b\*c\*acoth(c + d\*x)/d + b\*x\*acoth(c + d\*x) + b\*log(c/d + x + 1/d)/d - b\*acoth(c + d\*x)/d), Eq(e, -f\*x)), (-(a + b\*acoth(c))/(e\*f + f

```

*2*x), Eq(d, 0)), (-a*c**2*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**
2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + 2*
a*c*d*e*f/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d
**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - a*d**2*e**2/(c**2*e*f**3
+ c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2
*f**2*x - e*f**3 - f**4*x) + a*f**2/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2
*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x)
- b*c**2*f**2*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 -
2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*c*d
*e*f*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*
f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*c*d*f**2*x*a
coth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x
+ d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*d**2*e*f*x*acoth(c
+ d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**
2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) - b*d*e*f*log(e/f + x)/(c**2
*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d*
*2*e**2*f**2*x - e*f**3 - f**4*x) + b*d*e*f*log(c/d + x + 1/d)/(c**2*e*f**3
+ c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2
*f**2*x - e*f**3 - f**4*x) - b*d*e*f*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**
4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e
*f**3 - f**4*x) - b*d*f**2*x*log(e/f + x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*
d*e**2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f*
*4*x) + b*d*f**2*x*log(c/d + x + 1/d)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e*
*2*f**2 - 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x
) - b*d*f**2*x*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2
- 2*c*d*e*f**3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x) + b*f*
*2*acoth(c + d*x)/(c**2*e*f**3 + c**2*f**4*x - 2*c*d*e**2*f**2 - 2*c*d*e*f*
*3*x + d**2*e**3*f + d**2*e**2*f**2*x - e*f**3 - f**4*x), True))

```

$$3.108 \quad \int \frac{a+b \coth^{-1}(c+dx)}{(e+fx)^3} dx$$

**Optimal.** Leaf size=167

$$\frac{a+b \coth^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2 \log(-c-dx+1)}{4f(-cf+de+f)^2} + \frac{bd^2 \log(c+dx+1)}{4f(-cf+de-f)^2} - \frac{bd^2(de-cf) \log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2} + \frac{bd^2(de-cf) \log(e+fx)}{2(e+fx)(-cf+de+f)^2}$$

[Out]  $1/2*b*d/(-c*f+d*e-f)/(-c*f+d*e+f)/(f*x+e)+1/2*(-a-b*\operatorname{arccoth}(d*x+c))/f/(f*x+e)^2-1/4*b*d^2*\ln(-d*x-c+1)/f/(-c*f+d*e+f)^2+1/4*b*d^2*\ln(d*x+c+1)/f/(-c*f+d*e-f)^2-b*d^2*(-c*f+d*e)*\ln(f*x+e)/(-c*f+d*e+f)^2/(d*e-(1+c)*f)^2$

**Rubi [A]** time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {6110, 1982, 709, 800}

$$\frac{a+b \coth^{-1}(c+dx)}{2f(e+fx)^2} - \frac{bd^2 \log(-c-dx+1)}{4f(-cf+de+f)^2} + \frac{bd^2 \log(c+dx+1)}{4f(-cf+de-f)^2} - \frac{bd^2(de-cf) \log(e+fx)}{(-cf+de+f)^2(de-(c+1)f)^2} + \frac{bd^2(de-cf) \log(e+fx)}{2(e+fx)(-cf+de+f)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])/(e + f\*x)^3,x]

[Out]  $(b*d)/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)*(e + f*x)) - (a + b*\operatorname{ArcCoth}[c + d*x])/(2*f*(e + f*x)^2) - (b*d^2*\operatorname{Log}[1 - c - d*x])/(4*f*(d*e + f - c*f)^2) + (b*d^2*\operatorname{Log}[1 + c + d*x])/(4*f*(d*e - f - c*f)^2) - (b*d^2*(d*e - c*f)*\operatorname{Log}[e + f*x])/((d*e + f - c*f)^2*(d*e - (1 + c)*f)^2)$

### Rule 709

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1))/((m + 1)\*(c\*d^2 - b\*d\*e + a\*e^2)), x] + Dist[1/(c\*d^2 - b\*d\*e + a\*e^2), Int[((d + e\*x)^(m + 1)\*Simp[c\*d - b\*e - c\*e\*x, x])/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && NeQ[2\*c\*d - b\*e, 0] && LtQ[m, -1]

### Rule 800

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4\*a\*c, 0] && NeQ[c\*d^2 - b\*d\*e + a\*e^2, 0] && IntegerQ[m]

### Rule 1982

```
Int[(u_)^(m_)*(v_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

### Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)^3} dx &= -\frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{1}{(e+fx)^2(1-(c+dx)^2)} dx}{2f} \\ &= -\frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{1}{(e+fx)^2(1-c^2-2cdx-d^2x^2)} dx}{2f} \\ &= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \frac{-d(de-2c-dx)}{(e+fx)(1-c^2-d^2x^2)} dx}{2f} \\ &= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} + \frac{(bd) \int \left( \frac{d^2(-de+(1-c-dx))}{2(de+f-cf)(1-c^2-d^2x^2)} \right) dx}{2f} \\ &= \frac{bd}{2(de + f - cf)(de - (1 + c)f)(e + fx)} - \frac{a + b \coth^{-1}(c + dx)}{2f(e + fx)^2} - \frac{bd^2 \log(1 - c - dx)}{4f(de + f - cf)^2} \end{aligned}$$

**Mathematica [A]** time = 0.33, size = 174, normalized size = 1.04

$$\frac{1}{4} \left( -\frac{2a}{f(e + fx)^2} + \frac{2bd}{(e + fx)((c^2 - 1)f^2 - 2cdef + d^2e^2)} - \frac{4bd^2(de - cf) \log(e + fx)}{((c^2 - 1)f^2 - 2cdef + d^2e^2)^2} - \frac{bd^2 \log(-c - dx + 1)}{f(-cf + de + f)^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCoth[c + d*x])/(e + f*x)^3, x]
```

```
[Out] ((-2*a)/(f*(e + f*x)^2) + (2*b*d)/((d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)*(
e + f*x)) - (2*b*ArcCoth[c + d*x])/(f*(e + f*x)^2) - (b*d^2*Log[1 - c - d*x
```

)]/(f\*(d\*e + f - c\*f)^2) + (b\*d^2\*Log[1 + c + d\*x])/(f\*(-(d\*e) + f + c\*f)^2) - (4\*b\*d^2\*(d\*e - c\*f)\*Log[e + f\*x])/(d^2\*e^2 - 2\*c\*d\*e\*f + (-1 + c^2)\*f^2)^2)/4

**fricas** [B] time = 1.70, size = 833, normalized size = 4.99

$$2ad^4e^4 - 2(4ac + b)d^3e^3f + 4(3ac^2 + bc - a)d^2e^2f^2 - 2(4ac^3 + bc^2 - 4ac - b)def^3 + 2(ac^4 - 2ac^2 + a)f^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^3,x, algorithm="fricas")

[Out] -1/4\*(2\*a\*d^4\*e^4 - 2\*(4\*a\*c + b)\*d^3\*e^3\*f + 4\*(3\*a\*c^2 + b\*c - a)\*d^2\*e^2\*f^2 - 2\*(4\*a\*c^3 + b\*c^2 - 4\*a\*c - b)\*d\*e\*f^3 + 2\*(a\*c^4 - 2\*a\*c^2 + a)\*f^4 - 2\*(b\*d^3\*e^2\*f^2 - 2\*b\*c\*d^2\*e\*f^3 + (b\*c^2 - b)\*d\*f^4)\*x - (b\*d^4\*e^4 - 2\*(b\*c - b)\*d^3\*e^3\*f + (b\*c^2 - 2\*b\*c + b)\*d^2\*e^2\*f^2 + (b\*d^4\*e^2\*f^2 - 2\*(b\*c - b)\*d^3\*e\*f^3 + (b\*c^2 - 2\*b\*c + b)\*d^2\*f^4)\*x^2 + 2\*(b\*d^4\*e^3\*f - 2\*(b\*c - b)\*d^3\*e^2\*f^2 + (b\*c^2 - 2\*b\*c + b)\*d^2\*e\*f^3)\*x\*log(d\*x + c + 1) + (b\*d^4\*e^4 - 2\*(b\*c + b)\*d^3\*e^3\*f + (b\*c^2 + 2\*b\*c + b)\*d^2\*e^2\*f^2 + (b\*d^4\*e^2\*f^2 - 2\*(b\*c + b)\*d^3\*e\*f^3 + (b\*c^2 + 2\*b\*c + b)\*d^2\*f^4)\*x^2 + 2\*(b\*d^4\*e^3\*f - 2\*(b\*c + b)\*d^3\*e^2\*f^2 + (b\*c^2 + 2\*b\*c + b)\*d^2\*e\*f^3)\*x\*log(d\*x + c - 1) + 4\*(b\*d^3\*e^3\*f - b\*c\*d^2\*e^2\*f^2 + (b\*d^3\*e\*f^3 - b\*c\*d^2\*f^4)\*x^2 + 2\*(b\*d^3\*e^2\*f^2 - b\*c\*d^2\*e\*f^3)\*x)\*log(f\*x + e) + (b\*d^4\*e^4 - 4\*b\*c\*d^3\*e^3\*f + 2\*(3\*b\*c^2 - b)\*d^2\*e^2\*f^2 - 4\*(b\*c^3 - b\*c)\*d\*e\*f^3 + (b\*c^4 - 2\*b\*c^2 + b)\*f^4)\*log((d\*x + c + 1)/(d\*x + c - 1)))/(d^4\*e^6\*f - 4\*c\*d^3\*e^5\*f^2 + 2\*(3\*c^2 - 1)\*d^2\*e^4\*f^3 - 4\*(c^3 - c)\*d\*e^3\*f^4 + (c^4 - 2\*c^2 + 1)\*e^2\*f^5 + (d^4\*e^4\*f^3 - 4\*c\*d^3\*e^3\*f^4 + 2\*(3\*c^2 - 1)\*d^2\*e^2\*f^5 - 4\*(c^3 - c)\*d\*e\*f^6 + (c^4 - 2\*c^2 + 1)\*f^7)\*x^2 + 2\*(d^4\*e^5\*f^2 - 4\*c\*d^3\*e^4\*f^3 + 2\*(3\*c^2 - 1)\*d^2\*e^3\*f^4 - 4\*(c^3 - c)\*d\*e^2\*f^5 + (c^4 - 2\*c^2 + 1)\*e\*f^6)\*x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arccoth}(dx + c) + a}{(fx + e)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^3,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)/(f\*x + e)^3, x)



**maple [A]** time = 0.05, size = 236, normalized size = 1.41

$$\frac{d^2 a}{2(dx+de)^2 f} - \frac{d^2 b \operatorname{arccoth}(dx+c)}{2(dx+de)^2 f} + \frac{d^2 b}{2(cf-de-f)(cf-de+f)(dx+de)} + \frac{d^2 b f \ln((dx+c)f - cf + d)}{(cf-de-f)^2 (cf-de+f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))/(f\*x+e)^3,x)

[Out]  $-1/2*d^2*a/(d*f*x+d*e)^2/f - 1/2*d^2*b/(d*f*x+d*e)^2/f*\operatorname{arccoth}(d*x+c) + 1/2*d^2*b/(c*f-d*e-f)/(c*f-d*e+f)/(d*f*x+d*e) + d^2*b*f/(c*f-d*e-f)^2/(c*f-d*e+f)^2*\ln((d*x+c)*f-c*f+d*e)*c - d^3*b/(c*f-d*e-f)^2/(c*f-d*e+f)^2*\ln((d*x+c)*f-c*f+d*e)*e - 1/4*d^2*b/f/(c*f-d*e-f)^2*\ln(d*x+c-1) + 1/4*d^2*b/f/(c*f-d*e+f)^2*\ln(d*x+c+1)$

**maxima [A]** time = 0.33, size = 291, normalized size = 1.74

$$\frac{1}{4} \left( d \left( \frac{d \log(dx+c+1)}{d^2 e^2 f - 2(c+1)def^2 + (c^2+2c+1)f^3} - \frac{d \log(dx+c-1)}{d^2 e^2 f - 2(c-1)def^2 + (c^2-2c+1)f^3} - \frac{d^4 e^4 - 4cd^3 e^3 f + 2d^2 e^2 f^2}{d^4 e^4 - 4cd^3 e^3 f + 2d^2 e^2 f^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))/(f\*x+e)^3,x, algorithm="maxima")

[Out]  $1/4*(d*(d*\log(d*x+c+1)/(d^2*e^2*f-2*(c+1)*d*e*f^2+(c^2+2*c+1)*f^3)-d*\log(d*x+c-1)/(d^2*e^2*f-2*(c-1)*d*e*f^2+(c^2-2*c+1)*f^3)-4*(d^2*e-c*d*f)*\log(f*x+e)/(d^4*e^4-4*c*d^3*e^3*f+2*(3*c^2-1)*d^2*e^2*f^2-4*(c^3-c)*d*e*f^3+(c^4-2*c^2+1)*f^4)+2/(d^2*e^3-2*c*d*e^2*f+(c^2-1)*e*f^2+(d^2*e^2*f-2*c*d*e*f^2+(c^2-1)*f^3)*x))-2*\operatorname{arccoth}(d*x+c)/(f^3*x^2+2*e*f^2*x+e^2*f)*b-1/2*a/(f^3*x^2+2*e*f^2*x+e^2*f)$

**mupad [B]** time = 3.30, size = 422, normalized size = 2.53

$$\frac{b \ln\left(1 - \frac{1}{c+dx}\right)}{2f(2e^2+4efx+2f^2x^2)} - \frac{\ln(e+fx)(bd^3e-bcd^2f)}{c^4f^4-4c^3def^3+6c^2d^2e^2f^2-2c^2f^4-4cd^3e^3f+4cdef^3+d^4e^4-2d^2e^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*acoth(c+d\*x))/(e+f\*x)^3,x)

[Out]  $(b*\log(1-1/(c+d*x)))/(2*f*(2*e^2+2*f^2*x^2+4*e*f*x))- (\log(e+f*x))*(b*d^3*e-b*c*d^2*f)/(f^4-2*c^2*f^4+c^4*f^4+d^4*e^4-2*d^2*e^2*f^2+4*c*d*e*f^3+6*c^2*d^2*e^2*f^2-4*c*d^3*e^3*f-4*c^3*d*e*f^3)-((a$

$$\begin{aligned} & *f^2 - a*c^2*f^2 - a*d^2*e^2 + b*d*e*f + 2*a*c*d*e*f)/(f^2 - c^2*f^2 - d^2* \\ & e^2 + 2*c*d*e*f) + (b*d*f^2*x)/(f^2 - c^2*f^2 - d^2*e^2 + 2*c*d*e*f))/(2*e^ \\ & 2*f + 2*f^3*x^2 + 4*e*f^2*x) - (b*d^2*log(c + d*x - 1))/(4*f^3 - 8*c*f^3 + \\ & 4*c^2*f^3 + 4*d^2*e^2*f + 8*d*e*f^2 - 8*c*d*e*f^2) + (b*d^2*log(c + d*x + 1 \\ & ))/(8*c*f^3 + 4*f^3 + 4*c^2*f^3 + 4*d^2*e^2*f - 8*d*e*f^2 - 8*c*d*e*f^2) - \\ & (b*log(1/(c + d*x) + 1))/(4*f*(e^2 + f^2*x^2 + 2*e*f*x)) \end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))/(f\*x+e)\*\*3,x)

[Out] Timed out

### 3.109 $\int (e + fx)^2 (a + b \coth^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=374

$$\frac{(de - cf) \left( (c^2 + 3) f^2 - 2cdef + d^2 e^2 \right) (a + b \coth^{-1}(c + dx))^2}{3d^3 f} + \frac{\left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) (a + b \coth^{-1}(c + dx))^2}{3d^3}$$

[Out]  $\frac{1}{3} b^2 f^2 x / d^2 + 2 a b f (-c f + d e) x / d^2 + 2 b^2 f (-c f + d e) (d x + c) \operatorname{arccoth}(d x + c) / d^3 + 1/3 b f^2 (d x + c)^2 (a + b \operatorname{arccoth}(d x + c)) / d^3 - 1/3 (-c f + d e) (d^2 e^2 - 2 c d e f + (c^2 + 3) f^2) (a + b \operatorname{arccoth}(d x + c))^2 / d^3 + 1/3 (3 d^2 e^2 - 6 c d e f + (3 c^2 + 1) f^2) (a + b \operatorname{arccoth}(d x + c))^2 / d^3 + 1/3 (f x + e)^3 (a + b \operatorname{arccoth}(d x + c))^2 / f - 1/3 b^2 f^2 \operatorname{arctanh}(d x + c) / d^3 - 2/3 b (3 d^2 e^2 - 6 c d e f + (3 c^2 + 1) f^2) (a + b \operatorname{arccoth}(d x + c)) \ln(2 / (-d x - c + 1)) / d^3 + b^2 f (-c f + d e) \ln(1 - (d x + c)^2) / d^3 - 1/3 b^2 (3 d^2 e^2 - 6 c d e f + (3 c^2 + 1) f^2) \operatorname{polylog}(2, (-d x - c - 1) / (-d x - c + 1)) / d^3$

**Rubi [A]** time = 0.64, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$ , Rules used = {6112, 5929, 5911, 260, 5917, 321, 206, 6049, 5949, 5985, 5919, 2402, 2315}

$$\frac{b^2 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \operatorname{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{3d^3} - \frac{(de - cf) \left( (c^2 + 3) f^2 - 2cdef + d^2 e^2 \right) (a + b \coth^{-1}(c + dx))^2}{3d^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^2*(a + b*\operatorname{ArcCoth}[c + d*x])^2, x]$

[Out]  $\frac{b^2 f^2 x}{(3 d^2)} + \frac{(2 a b f (d e - c f) x)}{d^2} + \frac{(2 b^2 f (d e - c f) (c + d x) \operatorname{ArcCoth}[c + d x])}{d^3} + \frac{(b f^2 (c + d x)^2 (a + b \operatorname{ArcCoth}[c + d x]))}{(3 d^3)} - \frac{((d e - c f) (d^2 e^2 - 2 c d e f + (3 + c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x])^2)}{(3 d^3 f)} + \frac{((3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x])^2)}{(3 d^3)} + \frac{((e + f x)^3 (a + b \operatorname{ArcCoth}[c + d x])^2)}{(3 f)} - \frac{(b^2 f^2 \operatorname{ArcTanh}[c + d x])}{(3 d^3)} - \frac{(2 b (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) (a + b \operatorname{ArcCoth}[c + d x]) \operatorname{Log}[2 / (1 - c - d x)])}{(3 d^3)} + \frac{(b^2 f (d e - c f) \operatorname{Log}[1 - (c + d x)^2])}{d^3} - \frac{(b^2 (3 d^2 e^2 - 6 c d e f + (1 + 3 c^2) f^2) \operatorname{PolyLog}[2, -((1 + c + d x) / (1 - c - d x))])}{(3 d^3)}$

#### Rule 206

$\operatorname{Int}[(a + b x)(x^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1 * \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] x] / \operatorname{Rt}[a, 2]) / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 321

$\text{Int}[(c_.)*(x_)^{(m_)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}], x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n-1)}*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2315

$\text{Int}[\text{Log}[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}[\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

Rule 2402

$\text{Int}[\text{Log}[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x\_Symbol] \rightarrow -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}[\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$

Rule 5911

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.)^{(p_)}], x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcCoth}[c*x])^p, x] - \text{Dist}[b*c*p, \text{Int}[(x*(a + b*\text{ArcCoth}[c*x])^{(p-1)})/(1 - c^2*x^2), x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

Rule 5917

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.)^{(p_)}*((d_.)*(x_))^{(m_)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x])^{(p-1)})/(1 - c^2*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{IntegerQ}[m]) \ \&\& \ \text{NeQ}[m, -1]$

Rule 5919

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.)^{(p_)}]/((d_) + (e_.)*(x_)), x\_Symbol] \rightarrow -\text{Simp}[(a + b*\text{ArcCoth}[c*x])^p*\text{Log}[2/(1 + (e*x)/d)]/e, x] + \text{Dist}[(b*c*p)/e, \text{Int}[(a + b*\text{ArcCoth}[c*x])^{(p-1)}*\text{Log}[2/(1 + (e*x)/d)]/(1 - c^2*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[c^2*d^2 - e^2, 0]$

]

Rule 5929

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] -
  Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^(p - 1),
  (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6049

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(m_.)))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& IGtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(m_.)), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (e + fx)^2 (a + b \coth^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2 (a + b \coth^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^2}{3f} - \frac{(2b) \text{Subst}\left(\int \left(-\frac{3f^2(de-cf)(a+b \coth^{-1}(x))^2}{d^3}\right) dx, x, c + dx\right)}{d^3} \\
&= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^2}{3f} - \frac{(2b) \text{Subst}\left(\int \frac{(de-cf)(d^2e^2 - 2cdef + 3f^2x^2)}{d^3} dx, x, c + dx\right)}{d^3} \\
&= \frac{2abf(de - cf)x}{d^2} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))}{3d^3} + \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))^2}{3f} \\
&= \frac{b^2 f^2 x}{3d^2} + \frac{2abf(de - cf)x}{d^2} + \frac{2b^2 f(de - cf)(c + dx) \coth^{-1}(c + dx)}{d^3} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))^2}{3f}
\end{aligned}$$

**Mathematica [B]** time = 7.36, size = 1054, normalized size = 2.82

$$\frac{1}{3}a^2 f^2 x^3 + a^2 e f x^2 + a^2 e^2 x + \frac{1}{3}ab \left( 2x(3e^2 + 3f x e + f^2 x^2) \coth^{-1}(c + dx) + \frac{d f x(6de - 4cf + d f x) - (c - 1)(3d^2 e^2 - \dots)}{d^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x])^2,x]

[Out]  $a^2e^2x + a^2efx^2 + (a^2f^2x^3)/3 + (ab*(2x*(3e^2 + 3efx + f^2x^2)*\text{ArcCoth}[c + dx] + (dfx*(6de - 4cf + dfx) - (-1 + c)*(3d^2e^2 - 3(-1 + c)d*ef + (-1 + c)^2f^2)*\text{Log}[1 - c - dx] + (1 + c)*(3d^2e^2 - 3(1 + c)d*ef + (1 + c)^2f^2)*\text{Log}[1 + c + dx])/d^3))/3 + (b^2e^2*(1 - (c + dx)^2)*(\text{ArcCoth}[c + dx]*(\text{ArcCoth}[c + dx] - (c + dx)*\text{ArcCoth}[c + dx] + 2*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}])) - \text{PolyLog}[2, E^{(-2*\text{ArcCoth}[c + dx])}]))/(d*(c + dx)^2*(1 - (c + dx)^{-2})) - (b^2ef*(1 - (c + dx)^2)*(2*c*\text{ArcCoth}[c + dx]^2 + (c + dx)^2*(1 - (c + dx)^{-2})*\text{ArcCoth}[c + dx]^2 - 2*(c + dx)*\text{ArcCoth}[c + dx]*(-1 + c*\text{ArcCoth}[c + dx]) + 4*c*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}] - 2*\text{Log}[1/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]]]) - 2*c*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[c + dx])}]))/(d^2*(c + dx)^2*(1 - (c + dx)^{-2})) - (b^2f^2*(c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]*((1 - (c + dx)^2)*((4*\text{ArcCoth}[c + dx])/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}])) + (3*\text{ArcCoth}[c + dx]^2)/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}])) - (12*c*\text{ArcCoth}[c + dx]^2)/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (9*c^2*\text{ArcCoth}[c + dx]^2)/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (-1 + 6*c*\text{ArcCoth}[c + dx] + 3*\text{ArcCoth}[c + dx]^2 - 3*c^2*\text{ArcCoth}[c + dx]^2)/\text{Sqrt}[1 - (c + dx)^{-2}] + \text{Cosh}[3*\text{ArcCoth}[c + dx]] - 6*c*\text{ArcCoth}[c + dx]*\text{Cosh}[3*\text{ArcCoth}[c + dx]] + \text{ArcCoth}[c + dx]^2*\text{Cosh}[3*\text{ArcCoth}[c + dx]] + 3*c^2*\text{ArcCoth}[c + dx]^2*\text{Cosh}[3*\text{ArcCoth}[c + dx]] + (6*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}]))/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (18*c^2*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}]))/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) - (18*c*\text{Log}[1/(c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]])/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}]) + (4*(1 + 3*c^2)*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[c + dx])}]))/((c + dx)^3*(1 - (c + dx)^{-2}))^{(3/2)} - \text{ArcCoth}[c + dx]^2*\text{Sinh}[3*\text{ArcCoth}[c + dx]] - 3*c^2*\text{ArcCoth}[c + dx]^2*\text{Sinh}[3*\text{ArcCoth}[c + dx]] - 2*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}]]*\text{Sinh}[3*\text{ArcCoth}[c + dx]] - 6*c^2*\text{ArcCoth}[c + dx]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c + dx])}]]*\text{Sinh}[3*\text{ArcCoth}[c + dx]] + 6*c*\text{Log}[1/((c + dx)*\text{Sqrt}[1 - (c + dx)^{-2}])]*\text{Sinh}[3*\text{ArcCoth}[c + dx]])/(12*d^3)$

**fricas** [F] time = 0.58, size = 0, normalized size = 0.00

$\text{integral}(a^2f^2x^2 + 2a^2efx + a^2e^2 + (b^2f^2x^2 + 2b^2efx + b^2e^2)\text{arccoth}(dx + c)^2 + 2(abf^2x^2 + 2abefx + abe^2))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="fricas")

[Out]  $\text{integral}(a^2f^2x^2 + 2a^2efx + a^2e^2 + (b^2f^2x^2 + 2b^2efx + b^2e^2)*\text{arccoth}(d*x + c)^2 + 2*(a*b*f^2x^2 + 2*a*b*efx + a*b*e^2)*\text{arccoth}(d*x + c), x)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)^2 (b \text{arccoth}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*(b\*arccoth(d\*x + c) + a)^2, x)

maple [B] time = 0.08, size = 2694, normalized size = 7.20

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^2,x)

[Out]  $\frac{1}{3}b^2f^2x/d^2+1/2/d^3b^2f^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)*c^2+1/2/d^2*b^2*f*\ln(d*x+c+1)^2*c*e-1/3/d^3*a*b*f^2*\ln(d*x+c-1)*c^3-1/d^2*b^2*f*a$   
 $\text{rccoth}(d*x+c)*\ln(d*x+c+1)*e+1/d^2*b^2*f*\text{arccoth}(d*x+c)*\ln(d*x+c-1)*e-1/6/d^3*b^2*f^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c^3+1/2/d^3*b^2*f^2*$   
 $\ln(1/2+1/2*d*x+1/2*c)*\ln(d*x+c-1)*c+2/d^2*a*b*f*c*e+1/d^2*b^2*f*\ln(d*x+c+1)*e-1/d^3*b^2*f^2*\ln(d*x+c-1)*c-1/4/d*b^2*\ln(d*x+c-1)^2*c*e^2-1/2/d*b^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*e^2+1/2/d*b^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)*e^2-5/3/d^3*b^2*f^2*\text{arccoth}(d*x+c)*c^2+2/3*a*b*f^2*\text{arccoth}(d*x+c)*x^3+1/3*b^2/f*\text{arccoth}(d*x+c)*\ln(d*x+c-1)*e^3-1/4/d^3*b^2*f^2*\ln(d*x+c+1)^2*c+1/3/d^3*b^2*f^2*\text{arccoth}(d*x+c)*\ln(d*x+c-1)-1/12/d^3*b^2*f^2*\ln(d*x+c+1)^2*c^3+1/3/d^3*a*b*f^2*\ln(d*x+c+1)-1/d^3*b^2*f^2*\ln(d*x+c+1)*c+2*a*b*f*\text{arccoth}(d*x+c)*e*x^2+1/2/d^3*b^2*f^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)*c+1/d^3*b^2*f^2*\text{arccoth}(d*x+c)*\ln(d*x+c-1)*c^2-1/3/d^3*b^2*f^2*\text{arccoth}(d*x+c)*\ln(d*x+c-1)*c^3-1/2/d^3*b^2*f^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c^2+1/2/d^2*b^2*f*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*e-1/2/d^2*b^2*f*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)*e+2*a*b/d*f*e*x-4/3*a*b/d^2*f^2*c*x+1/2/d*b^2*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)*c*e^2+1/2/d*b^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(d*x+c-1)*c*e^2+1/d^3*a*b*f^2*\ln(d*x+c-1)*c^2+1/3/d^3*a*b*f^2*\ln(d*x+c+1)*c^3+2/d^2*b^2*f*\text{dilog}(1/2+1/2*d*x+1/2*c)*c*e+1/d^3*b^2*f^2*\text{arccoth}(d*x+c)*\ln(d*x+c+1)*c^2-1/d^3*b^2*f^2*\text{arccoth}(d*x+c)*\ln(d*x+c-1)*c+b^2*f*\text{arccoth}(d*x+c)^2*e*x^2+2*\text{arccoth}(d*x+c)*x*a*b*e^2+1/d*a*b*\ln(d*x+c+1)*e^2+1/3*a*b/f*\ln(d*x+c-1)*e^3+2/3*a*b/f*\text{arccoth}(d*x+c)*e^3-1/3*b^2/f*\text{arccoth}(d*x+c)*\ln(d*x+c+1)*e^3-1/6*b^2/f*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)*e^3+1/6*b^2/f*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*e^3-1/6*b^2/f*\ln(1/2+1/2*d*x+1/2*c)*\ln(d*x+c-1)*e^3-1/3*a*b/f*\ln(d*x+c+1)*e^3+1/3/d^3*a*b*f^2*\ln(d*x+c-1)-1/4/d^3*b^2*f^2*\ln(d*x+c+1)^2*c^2+1/4/d^2*b^2*f*\ln(d*x+c-1)^2*e-1/12/d^3*b^2*f^2*\ln(d*x+c-1)^2*c^3+1/4/d^3*b^2*f^2*\ln(d*x+c-1)^2*c^2-1/4/d^3*b^2*f^2*\ln(d*x+c-1)^2*c+1/d^2*b^2*f*\ln(d*x+c-1)*e-1/d^3*b^2*f^2*\text{dilog}(1/2+1/2*d*x+1/2*c)*c^2+1/4/d^2*b^2*f*\ln(d*x+c+1)^2*e-1/4/d*b^2*\ln(d*x+c+1)^2*c*e^2-1/2/d*b^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(d*x+c-1)*e^2-1/2/d*b^2*\ln(1/2+1/2*d*x+1/2*c)*\ln(-1/2*d*x-1/2*c+1/2)*c*e^2+1/d*b^2*\text{arccoth}(d*x+c)*\ln(d*x+c+1)*c*e^2-1/d*b^2*\text{arccoth}(d*x+c)*\ln(d*x+c-1)*c*e^2+1/3/d^3*b^2*f^2*\text{arccoth}(d*x+c)*\ln(d$



$x+c+1) * c^3 + 1/6/d^3 * b^2 * f^2 * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(d * x + c - 1) * c^3 - 1/2/d^3 * b^2 * f^2 * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(d * x + c - 1) * c^2 + 1/d^3 * b^2 * f^2 * \operatorname{arccoth}(d * x + c) * \ln(d * x + c + 1) * c + 1/4/d^2 * b^2 * f * \ln(d * x + c - 1)^2 * c^2 * e^{-1/2/d^2 * b^2 * f} * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(d * x + c - 1) * e^{-1/2/d^2 * b^2 * f} * \ln(d * x + c - 1)^2 * c * e + 1/d^3 * a * b * f^2 * \ln(d * x + c + 1) * c^2 - 1/2/d^3 * b^2 * f^2 * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(-1/2 * d * x - 1/2 * c + 1/2) * c + 1/6/d^3 * b^2 * f^2 * \ln(-1/2 * d * x - 1/2 * c + 1/2) * \ln(d * x + c + 1) * c^3 + 2/d * b^2 * f * \operatorname{arccoth}(d * x + c) * e * x + 2/d^2 * b^2 * f * \operatorname{arccoth}(d * x + c) * e * c - 4/3/d^2 * b^2 * f^2 * \operatorname{arccoth}(d * x + c) * x * c + 1/4/d^2 * b^2 * f * \ln(d * x + c + 1)^2 * c^2 * e + 1/d^3 * a * b * f^2 * \ln(d * x + c + 1) * c - 1/d^3 * a * b * f^2 * \ln(d * x + c - 1) * c + 1/d^2 * a * b * f * \ln(d * x + c - 1) * e^{-1/d^2 * a * b * f} * \ln(d * x + c + 1) * e^{-1/d^2 * b^2 * f} * \ln(-1/2 * d * x - 1/2 * c + 1/2) * \ln(d * x + c + 1) * c * e + 1/d^2 * b^2 * f * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(d * x + c - 1) * c * e + 1/d^2 * b^2 * f * \operatorname{arccoth}(d * x + c) * \ln(d * x + c - 1) * c^2 * e - 1/d^2 * a * b * f * \ln(d * x + c + 1) * c^2 * e - 2/d^2 * a * b * f * \ln(d * x + c + 1) * c * e - 2/d^2 * b^2 * f * \operatorname{arccoth}(d * x + c) * \ln(d * x + c + 1) * c * e + 1/d^2 * a * b * f * \ln(d * x + c - 1) * c^2 * e - 2/d^2 * a * b * f * \ln(d * x + c - 1) * c * e - 2/d^2 * b^2 * f * \operatorname{arccoth}(d * x + c) * \ln(d * x + c - 1) * c * e - 1/2/d^2 * b^2 * f * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(d * x + c - 1) * c^2 * e - 1/d^2 * b^2 * f * \operatorname{arccoth}(d * x + c) * \ln(d * x + c + 1) * c^2 * e + 1/2/d^2 * b^2 * f * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(-1/2 * d * x - 1/2 * c + 1/2) * c^2 * e + 1/d^2 * b^2 * f * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(-1/2 * d * x - 1/2 * c + 1/2) * c * e - 1/2/d^2 * b^2 * f * \ln(-1/2 * d * x - 1/2 * c + 1/2) * \ln(d * x + c + 1) * c^2 * e + 1/3 * a^2 * f^2 * x^3 + a^2 * x * e^2 + 1/3/d^3 * b^2 * f^2 * c + 1/3 * a^2/f * e^3 + a^2 * f * x^2 * e + 1/3 * b^2/f * \operatorname{arccoth}(d * x + c)^2 * e^3 + 1/3 * b^2 * f^2 * \operatorname{arccoth}(d * x + c)^2 * x^3 + \operatorname{arccoth}(d * x + c)^2 * x * b^2 * e^2 - 1/3/d^3 * b^2 * f^2 * \operatorname{dilog}(1/2 + 1/2 * d * x + 1/2 * c) + 1/6/d^3 * b^2 * f^2 * \ln(d * x + c - 1) - 1/4/d * b^2 * \ln(d * x + c + 1)^2 * e^2 + 1/4/d * b^2 * \ln(d * x + c - 1)^2 * e^2 - 1/d * b^2 * \operatorname{dilog}(1/2 + 1/2 * d * x + 1/2 * c) * e^2 - 1/6/d^3 * b^2 * f^2 * \ln(d * x + c + 1) + 1/12/d^3 * b^2 * f^2 * \ln(d * x + c - 1)^2 - 1/12/d^3 * b^2 * f^2 * \ln(d * x + c + 1)^2 + 1/12 * b^2/f * \ln(d * x + c + 1)^2 * e^3 + 1/12 * b^2/f * \ln(d * x + c - 1)^2 * e^3 + 1/d * b^2 * \operatorname{arccoth}(d * x + c) * \ln(d * x + c - 1) * e^2 + 1/d * b^2 * \operatorname{arccoth}(d * x + c) * \ln(d * x + c + 1) * e^2 - 1/6/d^3 * b^2 * f^2 * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(d * x + c - 1) + 1/3/d^3 * b^2 * f^2 * \operatorname{arccoth}(d * x + c) * \ln(d * x + c + 1) + 1/6/d^3 * b^2 * f^2 * \ln(-1/2 * d * x - 1/2 * c + 1/2) * \ln(d * x + c + 1) - 1/6/d^3 * b^2 * f^2 * \ln(1/2 + 1/2 * d * x + 1/2 * c) * \ln(-1/2 * d * x - 1/2 * c + 1/2) + 1/3/d * b^2 * f^2 * \operatorname{arccoth}(d * x + c) * x^2 + 1/3/d * a * b * f^2 * x^2 - 5/3/d^3 * a * b * f^2 * c^2 + 1/d * a * b * \ln(d * x + c + 1) * c * e^2 - 1/d * a * b * \ln(d * x + c - 1) * c * e^2 + 1/d * a * b * \ln(d * x + c - 1) * e^2$

**maxima [B]** time = 0.62, size = 791, normalized size = 2.11

$$\frac{1}{3} a^2 f^2 x^3 + a^2 e f x^2 + \left( 2 x^2 \operatorname{arccoth}(d x + c) + d \left( \frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(d x + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(d x + c - 1)}{d^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/3\*a^2\*f^2\*x^3 + a^2\*e\*f\*x^2 + (2\*x^2\*arccoth(d\*x + c) + d\*(2\*x/d^2 - (c^2 + 2\*c + 1)\*log(d\*x + c + 1)/d^3 + (c^2 - 2\*c + 1)\*log(d\*x + c - 1)/d^3))\*a\*b\*e\*f + 1/3\*(2\*x^3\*arccoth(d\*x + c) + d\*((d\*x^2 - 4\*c\*x)/d^3 + (c^3 + 3\*c^2 + 3\*c + 1)\*log(d\*x + c + 1)/d^4 - (c^3 - 3\*c^2 + 3\*c - 1)\*log(d\*x + c - 1)/d^4))\*a\*b\*f^2 + a^2\*e^2\*x + (2\*(d\*x + c)\*arccoth(d\*x + c) + log(-(d\*x + c

)^2 + 1))\*a\*b\*e^2/d - 1/3\*(3\*d^2\*e^2 - 6\*c\*d\*e\*f + 3\*c^2\*f^2 + f^2)\*(log(d\*x + c - 1))\*log(1/2\*d\*x + 1/2\*c + 1/2) + dilog(-1/2\*d\*x - 1/2\*c + 1/2))\*b^2/d^3 - 1/6\*(5\*c^2\*f^2 - 6\*d\*e\*f - 6\*(d\*e\*f - f^2)\*c + f^2)\*b^2\*log(d\*x + c + 1)/d^3 + 1/12\*(4\*b^2\*d\*f^2\*x + (b^2\*d^3\*f^2\*x^3 + 3\*b^2\*d^3\*e\*f\*x^2 + 3\*b^2\*d^3\*e^2\*x + (c^3\*f^2 + 3\*d^2\*e^2 - 3\*(d\*e\*f - f^2)\*c^2 - 3\*d\*e\*f + 3\*(d^2\*e^2 - 2\*d\*e\*f + f^2)\*c + f^2)\*b^2)\*log(d\*x + c + 1)^2 + (b^2\*d^3\*f^2\*x^3 + 3\*b^2\*d^3\*e\*f\*x^2 + 3\*b^2\*d^3\*e^2\*x + (c^3\*f^2 - 3\*d^2\*e^2 - 3\*(d\*e\*f + f^2)\*c^2 - 3\*d\*e\*f + 3\*(d^2\*e^2 + 2\*d\*e\*f + f^2)\*c - f^2)\*b^2)\*log(d\*x + c - 1)^2 + 2\*(b^2\*d^2\*f^2\*x^2 + 2\*(3\*d^2\*e\*f - 2\*c\*d\*f^2)\*b^2\*x - (b^2\*d^3\*f^2\*x^3 + 3\*b^2\*d^3\*e\*f\*x^2 + 3\*b^2\*d^3\*e^2\*x + (c^3\*f^2 - 3\*d^2\*e^2 - 3\*(d\*e\*f + f^2)\*c^2 - 3\*d\*e\*f + 3\*(d^2\*e^2 + 2\*d\*e\*f + f^2)\*c - f^2)\*b^2)\*log(d\*x + c - 1))\*log(d\*x + c + 1) - 2\*(b^2\*d^2\*f^2\*x^2 + 2\*(3\*d^2\*e\*f - 2\*c\*d\*f^2)\*b^2\*x - (b^2\*d^3\*f^2\*x^3 + 3\*b^2\*d^3\*e\*f\*x^2 + 6\*d\*e\*f - 6\*(d\*e\*f + f^2)\*c + f^2)\*b^2)\*log(d\*x + c - 1))/d^3

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + fx)^2 (a + b \operatorname{acoth}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f\*x)^2\*(a + b\*acoth(c + d\*x))^2,x)

[Out] int((e + f\*x)^2\*(a + b\*acoth(c + d\*x))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + dx))^2 (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*(a+b\*acoth(d\*x+c))\*\*2,x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*2\*(e + f\*x)\*\*2, x)

### 3.110 $\int (e + fx) \left( a + b \coth^{-1}(c + dx) \right)^2 dx$

**Optimal.** Leaf size=221

$$\frac{\left( (c^2 + 1)f^2 - 2cdef + d^2e^2 \right) \left( a + b \coth^{-1}(c + dx) \right)^2}{2d^2f} + \frac{(de - cf) \left( a + b \coth^{-1}(c + dx) \right)^2}{d^2} - \frac{2b(de - cf) \log\left(\frac{2}{-c-d}\right)}{d^2}$$

[Out]  $a*b*f*x/d + b^2*f*(d*x+c)*\operatorname{arccoth}(d*x+c)/d^2 + (-c*f+d*e)*(a+b*\operatorname{arccoth}(d*x+c))^2/d^2 - 1/2*(d^2*e^2 - 2*c*d*e*f + (c^2+1)*f^2)*(a+b*\operatorname{arccoth}(d*x+c))^2/d^2/f + 1/2*(f*x+e)^2*(a+b*\operatorname{arccoth}(d*x+c))^2/f - 2*b*(-c*f+d*e)*(a+b*\operatorname{arccoth}(d*x+c))*\ln(2/(-d*x-c+1))/d^2 + 1/2*b^2*f*\ln(1-(d*x+c)^2)/d^2 - b^2*(-c*f+d*e)*\operatorname{polylog}(2, (-d*x-c-1)/(-d*x-c+1))/d^2$

**Rubi [A]** time = 0.44, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {6112, 5929, 5911, 260, 6049, 5949, 5985, 5919, 2402, 2315}

$$-\frac{b^2(de - cf)\operatorname{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{d^2} + \frac{\left(-\frac{(c^2+1)f}{d} + 2ce - \frac{de^2}{f}\right) \left( a + b \coth^{-1}(c + dx) \right)^2}{2d} + \frac{(de - cf) \left( a + b \coth^{-1}(c + dx) \right)^2}{d^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)*(a + b*\operatorname{ArcCoth}[c + d*x])^2, x]$

[Out]  $(a*b*f*x)/d + (b^2*f*(c + d*x)*\operatorname{ArcCoth}[c + d*x])/d^2 + ((d*e - c*f)*(a + b*\operatorname{ArcCoth}[c + d*x])^2)/d^2 + ((2*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*\operatorname{ArcCoth}[c + d*x])^2)/(2*d) + ((e + f*x)^2*(a + b*\operatorname{ArcCoth}[c + d*x])^2)/(2*f) - (2*b*(d*e - c*f)*(a + b*\operatorname{ArcCoth}[c + d*x])*\operatorname{Log}[2/(1 - c - d*x)])/d^2 + (b^2*f*\operatorname{Log}[1 - (c + d*x)^2])/(2*d^2) - (b^2*(d*e - c*f)*\operatorname{PolyLog}[2, -((1 + c + d*x)/(1 - c - d*x))])/d^2$

#### Rule 260

$\operatorname{Int}[(x_)^m/((a_) + (b_)*(x_)^n), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \operatorname{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \operatorname{EqQ}[m, n - 1]$

#### Rule 2315

$\operatorname{Int}[\operatorname{Log}[(c_)*(x_)]/((d_) + (e_)*(x_)), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /; \operatorname{FreeQ}[\{c, d, e\}, x] \ \&\& \ \operatorname{EqQ}[e + c*d, 0]$

#### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p, x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 6049

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\*((f\_) + (g\_.)\*(x\_)^m)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I

GtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rubi steps

$$\begin{aligned}
\int (e + fx) (a + b \coth^{-1}(c + dx))^2 dx &= \frac{\text{Subst} \left( \int \left( \frac{de - cf}{d} + \frac{fx}{d} \right) (a + b \coth^{-1}(x))^2 dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst} \left( \int \left( -\frac{f^2(a + b \coth^{-1}(x))}{d^2} + \frac{(d^2 e^2 - 2cde - cf)}{d^2} \right) dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst} \left( \int \frac{(d^2 e^2 - 2cdef + (1 + c^2)f^2 + 2f(de - cf))}{1 - x^2} dx, x, c + dx \right)}{d} \\
&= \frac{abfx}{d} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^2}{2f} - \frac{b \text{Subst} \left( \int \left( \frac{d^2 e^2 \left( 1 + \frac{f(-2cde + f + c)}{d^2 e^2} \right)}{1 - x^2} \right) dx, x, c + dx \right)}{d} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^2}{2f} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \coth^{-1}(c + dx))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \coth^{-1}(c + dx))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \coth^{-1}(c + dx))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \coth^{-1}(c + dx))^2}{d^2} \\
&= \frac{abfx}{d} + \frac{b^2 f(c + dx) \coth^{-1}(c + dx)}{d^2} + \frac{(de - cf) (a + b \coth^{-1}(c + dx))^2}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.61, size = 295, normalized size = 1.33

$$-a^2 c^2 f + 2a^2 cde + 2a^2 d^2 ex + a^2 d^2 fx^2 + 2b \coth^{-1}(c + dx) \left( -((c + dx)(acf - ad(2e + fx) - bf)) - 2b(de - cf) \log \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*(a + b\*ArcCoth[c + d\*x])^2,x]

```
[Out] (2*a^2*c*d*e + 2*a*b*c*f - a^2*c^2*f + 2*a^2*d^2*e*x + 2*a*b*d*f*x + a^2*d^2*f*x^2 + b^2*(-1 + c + d*x)*(2*d*e + f - c*f + d*f*x)*ArcCoth[c + d*x]^2 + 2*b*ArcCoth[c + d*x]*(-(c + d*x)*(-(b*f) + a*c*f - a*d*(2*e + f*x))) - 2*b*(d*e - c*f)*Log[1 - E^(-2*ArcCoth[c + d*x])] + a*b*f*Log[1 - c - d*x] - a*b*f*Log[1 + c + d*x] - 4*a*b*d*e*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] - 2*b^2*f*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 4*a*b*c*f*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])] + 2*b^2*(d*e - c*f)*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/(2*d^2)
```

**fricas** [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}(a^2fx + a^2e + (b^2fx + b^2e) \operatorname{arccoth}(dx + c)^2 + 2(abfx + abe) \operatorname{arccoth}(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*f*x + a^2*e + (b^2*f*x + b^2*e)*arccoth(d*x + c)^2 + 2*(a*b*f*x + a*b*e)*arccoth(d*x + c), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)(b \operatorname{arccoth}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*arccoth(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(b*arccoth(d*x + c) + a)^2, x)
```

**maple** [B] time = 0.07, size = 857, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(a+b*arccoth(d*x+c))^2,x)
```

```
[Out] 1/4/d*b^2*ln(d*x+c-1)^2*e+1/d*a^2*c*e-1/2/d^2*a^2*c^2*f+a*b*f*x/d-1/d^2*a*b*ln(d*x+c+1)*c*f-1/d^2*a*b*ln(d*x+c-1)*c*f-1/d^2*b^2*arccoth(d*x+c)*ln(d*x+c+1)*c*f+1/2/d^2*b^2*ln(-1/2*d*x-1/2*c+1/2)*ln(1/2+1/2*d*x+1/2*c)*c*f-1/2/d^2*b^2*ln(d*x+c+1)*ln(-1/2*d*x-1/2*c+1/2)*c*f+1/2/d^2*b^2*ln(d*x+c-1)*ln(1/2+1/2*d*x+1/2*c)*c*f-1/d*b^2*dilog(1/2+1/2*d*x+1/2*c)*e-1/4/d*b^2*ln(d*x+c+1)^2*e+1/2/d^2*b^2*ln(d*x+c+1)*f+1/2/d^2*b^2*ln(d*x+c-1)*f+1/8/d^2*b^2*ln(d*x+c+1)^2*f+1/8/d^2*b^2*ln(d*x+c-1)^2*f+1/2*b^2*arccoth(d*x+c)^2*f*x^2+arccoth(d*x+c)^2*x*b^2*e-1/d^2*b^2*arccoth(d*x+c)*ln(d*x+c-1)*c*f-1/d^2*a*b*arc
```

$\coth(dx+c) \cdot c^2 \cdot f + 2/d \cdot \operatorname{arccoth}(dx+c) \cdot a \cdot b \cdot c \cdot e + 1/2 \cdot a^2 \cdot x^2 \cdot f + a^2 \cdot x \cdot e - 1/2 \cdot d \cdot b^2 \cdot \ln(dx+c-1) \cdot \ln(1/2 + 1/2 \cdot dx + 1/2 \cdot c) \cdot e + a \cdot b \cdot \operatorname{arccoth}(dx+c) \cdot f \cdot x^2 + 2 \cdot \operatorname{arccoth}(dx+c) \cdot x \cdot a \cdot b \cdot e - 1/4 \cdot d^2 \cdot b^2 \cdot \ln(dx+c+1) \cdot \ln(-1/2 \cdot dx - 1/2 \cdot c + 1/2) \cdot f - 1/2 \cdot d^2 \cdot b^2 \cdot \operatorname{arccoth}(dx+c)^2 \cdot c^2 \cdot f + 1/2 \cdot d \cdot b^2 \cdot \ln(dx+c+1) \cdot \ln(-1/2 \cdot dx - 1/2 \cdot c + 1/2) \cdot e - 1/2 \cdot d \cdot b^2 \cdot \ln(-1/2 \cdot dx - 1/2 \cdot c + 1/2) \cdot \ln(1/2 + 1/2 \cdot dx + 1/2 \cdot c) \cdot e + 1/d \cdot b^2 \cdot \operatorname{arccoth}(dx+c) \cdot \ln(dx+c-1) \cdot e + 1/d \cdot a \cdot b \cdot \ln(dx+c+1) \cdot e + 1/d \cdot a \cdot b \cdot \ln(dx+c-1) \cdot e - 1/4 \cdot d^2 \cdot b^2 \cdot \ln(dx+c-1) \cdot \ln(1/2 + 1/2 \cdot dx + 1/2 \cdot c) \cdot f + 1/4 \cdot d^2 \cdot b^2 \cdot \ln(-1/2 \cdot dx - 1/2 \cdot c + 1/2) \cdot \ln(1/2 + 1/2 \cdot dx + 1/2 \cdot c) \cdot f + 1/d^2 \cdot a \cdot b \cdot c \cdot f + 1/d \cdot b^2 \cdot \operatorname{arccoth}(dx+c) \cdot f \cdot x - 1/2 \cdot d^2 \cdot b^2 \cdot \operatorname{arccoth}(dx+c) \cdot \ln(dx+c+1) \cdot f + 1/d^2 \cdot b^2 \cdot \operatorname{arccoth}(dx+c) \cdot f \cdot c + 1/d \cdot \operatorname{arccoth}(dx+c)^2 \cdot b^2 \cdot c \cdot e + 1/d \cdot b^2 \cdot \operatorname{arccoth}(dx+c) \cdot \ln(dx+c+1) \cdot e + 1/2 \cdot d^2 \cdot b^2 \cdot \operatorname{arccoth}(dx+c) \cdot \ln(dx+c-1) \cdot f - 1/2 \cdot d^2 \cdot a \cdot b \cdot \ln(dx+c+1) \cdot f + 1/4 \cdot d^2 \cdot b^2 \cdot \ln(dx+c+1)^2 \cdot c \cdot f + 1/d^2 \cdot b^2 \cdot \operatorname{dilog}(1/2 + 1/2 \cdot dx + 1/2 \cdot c) \cdot c \cdot f - 1/4 \cdot d^2 \cdot b^2 \cdot \ln(dx+c-1)^2 \cdot c \cdot f + 1/2 \cdot d^2 \cdot a \cdot b \cdot \ln(dx+c-1) \cdot f$

**maxima** [A] time = 0.61, size = 400, normalized size = 1.81

$$\frac{1}{2} a^2 f x^2 + \frac{1}{2} \left( 2 x^2 \operatorname{arccoth}(dx+c) + d \left( \frac{2x}{d^2} - \frac{(c^2 + 2c + 1) \log(dx+c+1)}{d^3} + \frac{(c^2 - 2c + 1) \log(dx+c-1)}{d^3} \right) \right) abf + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(dx+c))^2,x, algorithm="maxima")

[Out]  $1/2 \cdot a^2 \cdot f \cdot x^2 + 1/2 \cdot (2 \cdot x^2 \cdot \operatorname{arccoth}(dx+c) + d \cdot (2 \cdot x/d^2 - (c^2 + 2 \cdot c + 1) \cdot \log(dx+c+1)/d^3 + (c^2 - 2 \cdot c + 1) \cdot \log(dx+c-1)/d^3)) \cdot a \cdot b \cdot f + a^2 \cdot e \cdot x + (2 \cdot (dx+c) \cdot \operatorname{arccoth}(dx+c) + \log(-(dx+c)^2 + 1)) \cdot a \cdot b \cdot e/d - (d \cdot e - c \cdot f) \cdot (\log(dx+c-1) \cdot \log(1/2 \cdot dx + 1/2 \cdot c + 1/2) + \operatorname{dilog}(-1/2 \cdot dx - 1/2 \cdot c + 1/2)) \cdot b^2/d^2 + 1/2 \cdot (c \cdot f + f) \cdot b^2 \cdot \log(dx+c+1)/d^2 + 1/8 \cdot ((b^2 \cdot d^2 \cdot f \cdot x^2 + 2 \cdot b^2 \cdot d^2 \cdot e \cdot x - (c^2 \cdot f - 2 \cdot (d \cdot e - f) \cdot c - 2 \cdot d \cdot e + f) \cdot b^2) \cdot \log(dx+c+1)^2 + (b^2 \cdot d^2 \cdot f \cdot x^2 + 2 \cdot b^2 \cdot d^2 \cdot e \cdot x - (c^2 \cdot f - 2 \cdot (d \cdot e + f) \cdot c + 2 \cdot d \cdot e + f) \cdot b^2) \cdot \log(dx+c-1)^2 + 2 \cdot (2 \cdot b^2 \cdot d \cdot f \cdot x - (b^2 \cdot d^2 \cdot f \cdot x^2 + 2 \cdot b^2 \cdot d^2 \cdot e \cdot x - (c^2 \cdot f - 2 \cdot (d \cdot e + f) \cdot c + 2 \cdot d \cdot e + f) \cdot b^2) \cdot \log(dx+c-1)) \cdot \log(dx+c+1) - 4 \cdot (b^2 \cdot d \cdot f \cdot x + (c \cdot f - f) \cdot b^2) \cdot \log(dx+c-1))/d^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + fx) (a + b \operatorname{acoth}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f\*x)\*(a + b\*acoth(c + d\*x))^2,x)

[Out] int((e + f\*x)\*(a + b\*acoth(c + d\*x))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + dx))^2 (e + fx) dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**2*(e + f*x), x)
```

### 3.111 $\int (a + b \coth^{-1}(c + dx))^2 dx$

**Optimal.** Leaf size=97

$$\frac{(c + dx)(a + b \coth^{-1}(c + dx))^2}{d} + \frac{(a + b \coth^{-1}(c + dx))^2}{d} - \frac{2b \log\left(\frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d} - \frac{b^2 \text{Li}_2\left(-\frac{c+dx}{-c-d}\right)}{d}$$

[Out] (a+b\*arccoth(d\*x+c))^2/d+(d\*x+c)\*(a+b\*arccoth(d\*x+c))^2/d-2\*b\*(a+b\*arccoth(d\*x+c))\*ln(2/(-d\*x-c+1))/d-b^2\*polylog(2,(-d\*x-c-1)/(-d\*x-c+1))/d

**Rubi [A]** time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6104, 5911, 5985, 5919, 2402, 2315}

$$-\frac{b^2 \text{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^2}{d} + \frac{(a + b \coth^{-1}(c + dx))^2}{d} - \frac{2b \log\left(\frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^2, x]

[Out] (a + b\*ArcCoth[c + d\*x])^2/d + ((c + d\*x)\*(a + b\*ArcCoth[c + d\*x])^2)/d - (2\*b\*(a + b\*ArcCoth[c + d\*x])\*Log[2/(1 - c - d\*x)]/d - (b^2\*PolyLog[2, -((1 + c + d\*x)/(1 - c - d\*x))])/d

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p, x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5919

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_.) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 5985

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_.) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 6104

```
Int[(((a_.) + ArcCoth[(c_.) + (d_.)*(x_)])*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + b \coth^{-1}(c + dx))^2 dx &= \frac{\text{Subst}\left(\int (a + b \coth^{-1}(x))^2 dx, x, c + dx\right)}{d} \\
&= \frac{(c + dx)(a + b \coth^{-1}(c + dx))^2}{d} - \frac{(2b) \text{Subst}\left(\int \frac{x^{(a+b \coth^{-1}(x))}}{1-x^2} dx, x, c + dx\right)}{d} \\
&= \frac{(a + b \coth^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^2}{d} - \frac{(2b) \text{Subst}\left(\int \frac{a+b}{1-x^2} dx, x, c + dx\right)}{d} \\
&= \frac{(a + b \coth^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^2}{d} - \frac{2b(a + b \coth^{-1}(c + dx))}{d} \\
&= \frac{(a + b \coth^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^2}{d} - \frac{2b(a + b \coth^{-1}(c + dx))}{d} \\
&= \frac{(a + b \coth^{-1}(c + dx))^2}{d} + \frac{(c + dx)(a + b \coth^{-1}(c + dx))^2}{d} - \frac{2b(a + b \coth^{-1}(c + dx))}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 111, normalized size = 1.14

$$a \left( ac + adx - 2b \log \left( \frac{1}{(c+dx) \sqrt{1 - \frac{1}{(c+dx)^2}}} \right) \right) + 2b \operatorname{coth}^{-1}(c + dx) \left( ac + adx - b \log \left( 1 - e^{-2 \operatorname{coth}^{-1}(c+dx)} \right) \right) + b^2 \operatorname{Li}_2 \left( e^{-2 \operatorname{coth}^{-1}(c+dx)} \right)$$


---

$d$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^2,x]

[Out] (b^2\*(-1 + c + d\*x)\*ArcCoth[c + d\*x]^2 + 2\*b\*ArcCoth[c + d\*x]\*(a\*c + a\*d\*x - b\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + a\*(a\*c + a\*d\*x - 2\*b\*Log[1/((c + d\*x)\*Sqrt[1 - (c + d\*x)^(-2)])]) + b^2\*PolyLog[2, E^(-2\*ArcCoth[c + d\*x])])/d

**fricas [F]** time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( b^2 \operatorname{arccoth}(dx + c)^2 + 2ab \operatorname{arccoth}(dx + c) + a^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2,x, algorithm="fricas")

[Out] integral(b^2\*arccoth(d\*x + c)^2 + 2\*a\*b\*arccoth(d\*x + c) + a^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(dx + c) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^2, x)

**maple [B]** time = 0.24, size = 226, normalized size = 2.33

$$\operatorname{arccoth}(dx + c)^2 x b^2 + \frac{\operatorname{arccoth}(dx + c)^2 b^2 c}{d} + 2 \operatorname{arccoth}(dx + c) x a b + \frac{b^2 \operatorname{arccoth}(dx + c)^2}{d} - \frac{2 \operatorname{arccoth}(dx + c) \ln \left( \frac{d(x+c)+1}{d(x+c)-1} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))^2,x)

[Out] arccoth(d\*x+c)^2\*x\*b^2+1/d\*arccoth(d\*x+c)^2\*b^2\*c+2\*arccoth(d\*x+c)\*x\*a\*b+1/d\*b^2\*arccoth(d\*x+c)^2-2/d\*arccoth(d\*x+c)\*ln(1+1/((d\*x+c-1)/(d\*x+c+1)))^(1/2)

))\*b^2-2/d\*arccoth(d\*x+c)\*ln(1-1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^2+2/d\*arccoth(d\*x+c)\*a\*b\*c+a^2\*x+1/d\*a\*b\*ln((d\*x+c)^2-1)-2/d\*polylog(2,-1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^2-2/d\*polylog(2,1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^2+a^2\*c/d

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2x + \frac{1}{4}b^2 \left( \frac{dx \log(dx + c - 1)^2 + (dx + c + 1) \log(dx + c + 1)^2 - 2(dx + c - 1) \log(dx + c + 1) \log(dx + c - 1)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2,x, algorithm="maxima")

[Out] a^2\*x + 1/4\*b^2\*((d\*x\*log(d\*x + c - 1)^2 + (d\*x + c + 1)\*log(d\*x + c + 1)^2 - 2\*(d\*x + c - 1)\*log(d\*x + c + 1)\*log(d\*x + c - 1))/d + integrate(2\*(c^2 + (c\*d - 3\*d)\*x - 2\*c + 1)\*log(d\*x + c - 1)/(d^2\*x^2 + 2\*c\*d\*x + c^2 - 1), x)) + (2\*(d\*x + c)\*arccoth(d\*x + c) + log(-(d\*x + c)^2 + 1))\*a\*b/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acoth}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c + d\*x))^2,x)

[Out] int((a + b\*acoth(c + d\*x))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*2,x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*2, x)

$$3.112 \quad \int \frac{(a+b \operatorname{coth}^{-1}(c+dx))^2}{e+fx} dx$$

**Optimal.** Leaf size=214

$$\frac{b(a+b \operatorname{coth}^{-1}(c+dx)) \operatorname{Li}_2\left(1 - \frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{f} + \frac{(a+b \operatorname{coth}^{-1}(c+dx))^2 \log\left(\frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f} + \frac{b \operatorname{Li}_2\left(1 - \frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{f}$$

[Out]  $-(a+b \operatorname{arccoth}(d*x+c))^2 \ln(2/(d*x+c+1))/f + (a+b \operatorname{arccoth}(d*x+c))^2 \ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f + b*(a+b \operatorname{arccoth}(d*x+c))*\operatorname{polylog}(2, 1-2/(d*x+c+1))/f - b*(a+b \operatorname{arccoth}(d*x+c))*\operatorname{polylog}(2, 1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f + 1/2*b^2*\operatorname{polylog}(3, 1-2/(d*x+c+1))/f - 1/2*b^2*\operatorname{polylog}(3, 1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f$

**Rubi [A]** time = 0.15, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {6112, 5923}

$$\frac{b(a+b \operatorname{coth}^{-1}(c+dx)) \operatorname{PolyLog}\left(2, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{f} + \frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right) (a+b \operatorname{coth}^{-1}(c+dx))}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^2/(e + f\*x), x]

[Out]  $-\left(\frac{(a+b \operatorname{ArcCoth}[c+d*x])^2 \operatorname{Log}\left[\frac{2}{1+c+d*x}\right]}{f}\right) + \left(\frac{(a+b \operatorname{ArcCoth}[c+d*x])^2 \operatorname{Log}\left[\frac{2*d*(e+f*x)}{(d*e+f-c*f)*(1+c+d*x)}\right]}{f}\right) + \left(\frac{b*(a+b \operatorname{ArcCoth}[c+d*x])*\operatorname{PolyLog}\left[2, 1-\frac{2}{1+c+d*x}\right]}{f}\right) - \left(\frac{b*(a+b \operatorname{ArcCoth}[c+d*x])*\operatorname{PolyLog}\left[2, 1-\frac{2*d*(e+f*x)}{(d*e+f-c*f)*(1+c+d*x)}\right]}{f}\right) + \left(\frac{b^2*\operatorname{PolyLog}\left[3, 1-\frac{2}{1+c+d*x}\right]}{2*f}\right) - \left(\frac{b^2*\operatorname{PolyLog}\left[3, 1-\frac{2*d*(e+f*x)}{(d*e+f-c*f)*(1+c+d*x)}\right]}{2*f}\right)$

**Rule 5923**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^2/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :>  
 $-\operatorname{Simp}\left[\frac{(a+b \operatorname{ArcCoth}[c*x])^2 \operatorname{Log}\left[\frac{2}{1+c*x}\right]}{e}, x\right] + \left(\operatorname{Simp}\left[\frac{(a+b \operatorname{ArcCoth}[c*x])^2 \operatorname{Log}\left[\frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e}, x\right] + \operatorname{Simp}\left[\frac{b*(a+b \operatorname{ArcCoth}[c*x])*\operatorname{PolyLog}\left[2, 1-\frac{2}{1+c*x}\right]}{e}, x\right] - \operatorname{Simp}\left[\frac{b*(a+b \operatorname{ArcCoth}[c*x])*\operatorname{PolyLog}\left[2, 1-\frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{e}, x\right] + \operatorname{Simp}\left[\frac{b^2*\operatorname{PolyLog}\left[3, 1-\frac{2}{1+c*x}\right]}{2*e}, x\right] - \operatorname{Simp}\left[\frac{b^2*\operatorname{PolyLog}\left[3, 1-\frac{2*c*(d+e*x)}{(c*d+e)*(1+c*x)}\right]}{2*e}, x\right]\right) /;$   
 FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 - e^2, 0]

Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

Rubi steps

$$\int \frac{(a + b \operatorname{coth}^{-1}(c + dx))^2}{e + fx} dx = \frac{\operatorname{Subst}\left(\int \frac{(a + b \operatorname{coth}^{-1}(x))^2}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \operatorname{coth}^{-1}(c + dx))^2 \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \operatorname{coth}^{-1}(c + dx))^2 \log\left(\frac{2d(e + fx)}{de + f - cf}\right)}{f}$$

Mathematica [C] time = 31.06, size = 3759, normalized size = 17.57

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^2/(e + f\*x), x]

[Out] (a^2\*Log[e + f\*x])/f + 2\*a\*b\*((ArcCoth[c + d\*x] - ArcTanh[c + d\*x])\*Log[e + f\*x])/f - (I\*(I\*ArcTanh[c + d\*x]\*(-Log[1/Sqrt[1 - (c + d\*x)^2]] + Log[I\*Sinh[ArcTanh[(d\*e - c\*f)/f] + ArcTanh[c + d\*x]]]) + ((-I)\*(I\*ArcTanh[(d\*e - c\*f)/f] + I\*ArcTanh[c + d\*x])^2 - (I/4)\*(Pi - (2\*I)\*ArcTanh[c + d\*x])^2 + 2\*(I\*ArcTanh[(d\*e - c\*f)/f] + I\*ArcTanh[c + d\*x])\*Log[1 - E^((2\*I)\*(I\*ArcTanh[(d\*e - c\*f)/f] + I\*ArcTanh[c + d\*x]))]) + (Pi - (2\*I)\*ArcTanh[c + d\*x])\*Log[1 - E^(I\*(Pi - (2\*I)\*ArcTanh[c + d\*x]))]) - (Pi - (2\*I)\*ArcTanh[c + d\*x])\*Log[2\*Sin[(Pi - (2\*I)\*ArcTanh[c + d\*x])/2]] - 2\*(I\*ArcTanh[(d\*e - c\*f)/f] + I\*ArcTanh[c + d\*x])\*Log[(2\*I)\*Sinh[ArcTanh[(d\*e - c\*f)/f] + ArcTanh[c + d\*x]]) - I\*PolyLog[2, E^((2\*I)\*(I\*ArcTanh[(d\*e - c\*f)/f] + I\*ArcTanh[c + d\*x]))] - I\*PolyLog[2, E^(I\*(Pi - (2\*I)\*ArcTanh[c + d\*x]))]/2)/f - (b^2\*(d\*e - c\*f + f\*(c + d\*x))\*(1 - (c + d\*x)^2)\*(-1/24\*(I\*f\*Pi^3 - 8\*d\*e\*ArcCoth[c + d\*x]^3 - 8\*f\*ArcCoth[c + d\*x]^3 + 8\*c\*f\*ArcCoth[c + d\*x]^3 + 24\*f\*ArcCoth[c + d\*x]^2\*Log[1 - E^(2\*ArcCoth[c + d\*x])] + 24\*f\*ArcCoth[c + d\*x]\*PolyLog[2, E^(2\*ArcCoth[c + d\*x])] - 12\*f\*PolyLog[3, E^(2\*ArcCoth[c + d\*x])])/f^2 + ((-d\*e) - f + c\*f)\*(-d\*e) + f + c\*f)\*(2\*d^2\*e^2\*ArcCoth[c + d\*x]^3 - 8\*d\*e\*f\*ArcCoth[c + d\*x]^3 - 4\*c\*d\*e\*f\*ArcCoth[c + d\*x]^3 + 4\*d\*e\*E^(2\*ArcTanh[f/(d\*e - c\*f)])\*f\*ArcCoth[c + d\*x]^3 - 10\*f^2\*ArcCoth[c + d\*x]^3 + 8\*c\*f^

$$\begin{aligned}
& 2*\text{ArcCoth}[c + d*x]^3 + 2*c^2*f^2*\text{ArcCoth}[c + d*x]^3 - 4*E^{(2*\text{ArcTanh}[f/(d*e - c*f)])}*f^2*\text{ArcCoth}[c + d*x]^3 - 4*c*E^{(2*\text{ArcTanh}[f/(d*e - c*f)])}*f^2*\text{ArcCoth}[c + d*x]^3 - (4*d^2*e^2*\text{Sqrt}[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*\text{ArcCoth}[c + d*x]^3)/E^{\text{ArcTanh}[f/(d*e - c*f)]} - (4*d*e*f*\text{Sqrt}[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*\text{ArcCoth}[c + d*x]^3)/E^{\text{ArcTanh}[f/(d*e - c*f)]} + (8*c*d*e*f*\text{Sqrt}[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*\text{ArcCoth}[c + d*x]^3)/E^{\text{ArcTanh}[f/(d*e - c*f)]} + (4*c*f^2*\text{Sqrt}[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*\text{ArcCoth}[c + d*x]^3)/E^{\text{ArcTanh}[f/(d*e - c*f)]} - (4*c^2*f^2*\text{Sqrt}[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f)^2]*\text{ArcCoth}[c + d*x]^3)/E^{\text{ArcTanh}[f/(d*e - c*f)]} + (6*I)*d*e*f*\text{Pi}*\text{ArcCoth}[c + d*x]*\text{Log}[2] + (6*I)*f^2*\text{Pi}*\text{ArcCoth}[c + d*x]*\text{Log}[2] - (6*I)*c*f^2*\text{Pi}*\text{ArcCoth}[c + d*x]*\text{Log}[2] - d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[64] - f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[64] + c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[64] - (6*I)*d*e*f*\text{Pi}*\text{ArcCoth}[c + d*x]*\text{Log}[E^{(-\text{ArcCoth}[c + d*x])} + E^{\text{ArcCoth}[c + d*x]}] - (6*I)*f^2*\text{Pi}*\text{ArcCoth}[c + d*x]*\text{Log}[E^{(-\text{ArcCoth}[c + d*x])} + E^{\text{ArcCoth}[c + d*x]}] + (6*I)*c*f^2*\text{Pi}*\text{ArcCoth}[c + d*x]*\text{Log}[E^{(-\text{ArcCoth}[c + d*x])} + E^{\text{ArcCoth}[c + d*x]}] + 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} - 6*d*e*f*E^{(2*\text{ArcTanh}[f/(d*e - c*f)])}*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 6*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} - 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 6*E^{(2*\text{ArcTanh}[f/(d*e - c*f)])}*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 6*c*E^{(2*\text{ArcTanh}[f/(d*e - c*f)])}*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 12*d*e*f*\text{ArcCoth}[c + d*x]*\text{ArcTanh}[f/(d*e - c*f)]*\text{Log}[(I/2)*E^{(-\text{ArcCoth}[c + d*x] - \text{ArcTanh}[f/(d*e - c*f)])}*(-1 + E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 12*f^2*\text{ArcCoth}[c + d*x]*\text{ArcTanh}[f/(d*e - c*f)]*\text{Log}[(I/2)*E^{(-\text{ArcCoth}[c + d*x] - \text{ArcTanh}[f/(d*e - c*f)])}*(-1 + E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} - 12*c*f^2*\text{ArcCoth}[c + d*x]*\text{ArcTanh}[f/(d*e - c*f)]*\text{Log}[(I/2)*E^{(-\text{ArcCoth}[c + d*x] - \text{ArcTanh}[f/(d*e - c*f)])}*(-1 + E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} + 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[-((d*e*(-1 + E^{(2*\text{ArcCoth}[c + d*x])) + (1 + c + E^{(2*\text{ArcCoth}[c + d*x])) - c*E^{(2*\text{ArcCoth}[c + d*x]))*f)/E^{\text{ArcCoth}[c + d*x]}] + 6*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[-((d*e*(-1 + E^{(2*\text{ArcCoth}[c + d*x])) + (1 + c + E^{(2*\text{ArcCoth}[c + d*x])) - c*E^{(2*\text{ArcCoth}[c + d*x]))*f)/E^{\text{ArcCoth}[c + d*x]}] - 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[-((d*e*(-1 + E^{(2*\text{ArcCoth}[c + d*x])) + (1 + c + E^{(2*\text{ArcCoth}[c + d*x])) - c*E^{(2*\text{ArcCoth}[c + d*x]))*f)/E^{\text{ArcCoth}[c + d*x]}] + (1 + c + E^{(2*\text{ArcCoth}[c + d*x])) - c*E^{(2*\text{ArcCoth}[c + d*x]))*f)/E^{\text{ArcCoth}[c + d*x]}] + 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 6*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] - 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 + (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 6*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 + (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] - 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 + (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e
\end{aligned}$$



$$\begin{aligned}
& - (1 + c)*f]] + (6*I)*d*e*f*Pi*ArcCoth[c + d*x]*Log[1/Sqrt[1 - (c + d*x)^{(-2)}]] + (6*I)*f^2*Pi*ArcCoth[c + d*x]*Log[1/Sqrt[1 - (c + d*x)^{(-2)}]] - (6*I)*c*f^2*Pi*ArcCoth[c + d*x]*Log[1/Sqrt[1 - (c + d*x)^{(-2)}]] - 6*d*e*f*ArcCoth[c + d*x]^2*Log[-(f/Sqrt[1 - (c + d*x)^{(-2)}]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^{(-2)})] + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^{(-2)})]] - 6*f^2*ArcCoth[c + d*x]^2*Log[-(f/Sqrt[1 - (c + d*x)^{(-2)}]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^{(-2)})] + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^{(-2)})]] + 6*c*f^2*ArcCoth[c + d*x]^2*Log[-(f/Sqrt[1 - (c + d*x)^{(-2)}]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^{(-2)})] + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^{(-2)})]] - 12*d*e*f*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] - 12*f^2*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] + 12*c*f^2*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] + 6*f*(-(d*e*(-1 + E^(2*ArcTanh[f/(d*e - c*f)]))) + (1 + E^(2*ArcTanh[f/(d*e - c*f)])) + c*(-1 + E^(2*ArcTanh[f/(d*e - c*f)]))) * ArcCoth[c + d*x]*PolyLog[2, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + 12*f*(d*e + f - c*f)*ArcCoth[c + d*x]*PolyLog[2, -(E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 12*d*e*f*ArcCoth[c + d*x]*PolyLog[2, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 12*f^2*ArcCoth[c + d*x]*PolyLog[2, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] - 12*c*f^2*ArcCoth[c + d*x]*PolyLog[2, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] - 3*d*e*f*PolyLog[3, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + 3*d*e*E^(2*ArcTanh[f/(d*e - c*f)])*f*PolyLog[3, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] - 3*f^2*PolyLog[3, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] + 3*c*f^2*PolyLog[3, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] - 3*E^(2*ArcTanh[f/(d*e - c*f)])*f^2*PolyLog[3, E^(2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))] - 12*d*e*f*PolyLog[3, -(E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] - 12*f^2*PolyLog[3, -(E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 12*c*f^2*PolyLog[3, -(E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] - 12*d*e*f*PolyLog[3, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] - 12*f^2*PolyLog[3, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 12*c*f^2*PolyLog[3, (E^ArcCoth[c + d*x]*Sqrt[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]])/(6*f^2*(d*e + f - c*f)^2*(d*e - (1 + c)*f)))/(d*(c + d*x)^2*(e + f*x)*(1 - (c + d*x)^{(-2)}))
\end{aligned}$$

**fricas** [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \operatorname{arccoth}(dx + c)^2 + 2ab \operatorname{arccoth}(dx + c) + a^2}{fx + e}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e),x, algorithm="fricas")

[Out] integral((b^2\*arccoth(d\*x + c)^2 + 2\*a\*b\*arccoth(d\*x + c) + a^2)/(f\*x + e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(dx + c) + a)^2}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e),x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^2/(f\*x + e), x)

maple [C] time = 1.50, size = 1845, normalized size = 8.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))^2/(f\*x+e),x)

[Out] 
$$\begin{aligned} & -1/2*b^2*c/(c*f-d*e-f)*\operatorname{polylog}(3, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f)) \\ & +b^2*\ln((d*x+c)*f-c*f+d*e)/f*\operatorname{arccoth}(d*x+c)^2-b^2/f*\operatorname{arccoth}(d*x+c)^2*\ln(1+1/((d*x+c-1)/(d*x+c+1))^{1/2}) \\ & -2*b^2/f*\operatorname{arccoth}(d*x+c)*\operatorname{polylog}(2, -1/((d*x+c-1)/(d*x+c+1))^{1/2})-b^2/f*\operatorname{arccoth}(d*x+c)^2*\ln(1-1/((d*x+c-1)/(d*x+c+1))^{1/2}) \\ & -2*b^2/f*\operatorname{arccoth}(d*x+c)*\operatorname{polylog}(2, 1/((d*x+c-1)/(d*x+c+1))^{1/2})+b^2/f*\operatorname{arccoth}(d*x+c)^2*\ln((d*x+c+1)/(d*x+c-1)-1) \\ & -b^2/(c*f-d*e-f)*\operatorname{arccoth}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f)) \\ & -b^2/(c*f-d*e-f)*\operatorname{arccoth}(d*x+c)*\operatorname{polylog}(2, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-b^2/f*\operatorname{arccoth}(d*x+c)^2*\ln(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f) \\ & -a*b/f*\operatorname{dilog}(((d*x+c)*f+f)/(c*f-d*e+f))+a*b/f*\operatorname{dilog}(((d*x+c)*f-f)/(c*f-d*e-f))+2*b^2/f*\operatorname{polylog}(3, -1/((d*x+c-1)/(d*x+c+1))^{1/2}) \\ & +1/2*b^2/(c*f-d*e-f)*\operatorname{polylog}(3, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+a^2*\ln((d*x+c)*f-c*f+d*e)/f+2*b^2/f*\operatorname{polylog}(3, 1/((d*x+c-1)/(d*x+c+1))^{1/2}) \\ & +1/2*I*b^2/f*Pi*\operatorname{arccoth}(d*x+c)^2*\operatorname{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\operatorname{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*\operatorname{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1)-a*b/f*\ln((d*x+c)*f-c*f+d*e)*\ln(((d*x+c)*f+f)/(c*f-d*e+f))+a*b/f*\ln((d*x+c)*f-c*f+d*e)*\ln(((d*x+c)*f-f)/(c*f-d*e-f))+b^2*c/(c*f-d*e-f)*\operatorname{arccoth}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+b^2*c/(c*f-d*e-f)*\operatorname{arccoth}(d*x+c)*\operatorname{polylog}(2, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))+2*a*b*\ln((d*x+c)*f-c*f+d*e)/f*\operatorname{arccoth}(d*x+c)-I*b^2/f*Pi*\operatorname{arccoth}(d*x+c)^2+I*b^2/f*Pi*\operatorname{arccoth}(d*x+c)^2*\operatorname{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1)^2+1/2*d*b^2/f*e/(c*f-d*e-f)*\operatorname{polylog}(3, (c* \end{aligned}$$

$f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*c$   
 $sgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/($   
 $(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3-d*b^2/f*e/(c*f-d*e-f)*arccoth(d*$   
 $x+c)^2*ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-d*b^2/f*e/(c*f-d*e$   
 $-f)*arccoth(d*x+c)*polylog(2,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-1$   
 $/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I/((d*x+c+1)/(d*x+c-1)-1))*csgn(I*((d*$   
 $x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-$   
 $1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2-1/2*I*b^2/f*Pi*arccoth(d*x+c)^2*csgn(I*((d$   
 $*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-$   
 $1)*f))*csgn(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d$   
 $*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \log(fx + e)}{f} + \int \frac{b^2 \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)^2}{4(fx + e)} + \frac{ab \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e),x, algorithm="maxima")

[Out] a^2\*log(f\*x + e)/f + integrate(1/4\*b^2\*(log(1/(d\*x + c) + 1) - log(-1/(d\*x + c) + 1))^2/(f\*x + e) + a\*b\*(log(1/(d\*x + c) + 1) - log(-1/(d\*x + c) + 1)) / (f\*x + e), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(c + dx))^2}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c + d\*x))^2/(e + f\*x),x)

[Out] int((a + b\*acoth(c + d\*x))^2/(e + f\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(c + dx))^2}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*2/(f\*x+e),x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*2/(e + f\*x), x)

$$3.113 \quad \int \frac{(a+b \coth^{-1}(c+dx))^2}{(e+fx)^2} dx$$

**Optimal.** Leaf size=480

$$\frac{2abd \log(e+fx)}{f^2 - (de-cf)^2} - \frac{abd \log(-c-dx+1)}{f(-cf+de+f)} + \frac{abd \log(c+dx+1)}{f(-cf+de-f)} - \frac{(a+b \coth^{-1}(c+dx))^2}{f(e+fx)} + \frac{b^2 d \operatorname{Li}_2\left(-\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf+de+f)} + \frac{b^2 d \operatorname{Li}_2\left(-\frac{c+dx+1}{c+dx+1}\right)}{2f(-cf+de+f)}$$

[Out]  $-(a+b \operatorname{arccoth}(d*x+c))^2/f/(f*x+e)+b^2*d*\operatorname{arccoth}(d*x+c)*\ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)-a*b*d*\ln(-d*x-c+1)/f/(-c*f+d*e+f)-b^2*d*\operatorname{arccoth}(d*x+c)*\ln(2/(d*x+c+1))/f/(-c*f+d*e-f)+2*b^2*d*\operatorname{arccoth}(d*x+c)*\ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+a*b*d*\ln(d*x+c+1)/f/(-c*f+d*e-f)+2*a*b*d*\ln(f*x+e)/(f^2-(c*f+d*e)^2)-2*b^2*d*\operatorname{arccoth}(d*x+c)*\ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+1/2*b^2*d*\operatorname{polylog}(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e+f)+1/2*b^2*d*\operatorname{polylog}(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-b^2*d*\operatorname{polylog}(2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+b^2*d*\operatorname{polylog}(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)$

**Rubi [A]** time = 1.74, antiderivative size = 485, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 19, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$ , Rules used = {6110, 1982, 705, 31, 632, 6741, 6122, 706, 633, 6688, 12, 6725, 72, 6742, 5919, 2402, 2315, 5921, 2447}

$$\frac{b^2 d \operatorname{PolyLog}\left(2, -\frac{c+dx+1}{-c-dx+1}\right)}{2f(-cf+de+f)} + \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{2f(-cf+de-f)} - \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)} + \frac{b^2 d \operatorname{PolyLog}\left(2, 1 - \frac{2}{c+dx+1}\right)}{(-cf+de+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^2/(e + f\*x)^2, x]

[Out]  $-\left(\frac{(a+b \operatorname{ArcCoth}[c+d*x])^2}{(f*(e+f*x))} + (b^2*d*\operatorname{ArcCoth}[c+d*x]*\operatorname{Log}\left[\frac{2}{(1-c-d*x)}\right])\right)/f*(d*e+f-c*f) - (a*b*d*\operatorname{Log}[1-c-d*x])/f*(d*e+f-c*f) - (b^2*d*\operatorname{ArcCoth}[c+d*x]*\operatorname{Log}\left[\frac{2}{(1+c+d*x)}\right])/f*(d*e-f-c*f) + (2*b^2*d*\operatorname{ArcCoth}[c+d*x]*\operatorname{Log}\left[\frac{2}{(1+c+d*x)}\right])/((d*e+f-c*f)*(d*e-(1+c)*f)) + (a*b*d*\operatorname{Log}[1+c+d*x])/f*(d*e-f-c*f) - (2*a*b*d*\operatorname{Log}[e+f*x])/((d*e+f-c*f)*(d*e-(1+c)*f)) - (2*b^2*d*\operatorname{ArcCoth}[c+d*x]*\operatorname{Log}\left[\frac{2*d*(e+f*x)}{(d*e+f-c*f)*(1+c+d*x)}\right])/((d*e+f-c*f)*(d*e-(1+c)*f)) + (b^2*d*\operatorname{PolyLog}[2, -((1+c+d*x)/(1-c-d*x))])/f*(d*e+f-c*f) + (b^2*d*\operatorname{PolyLog}[2, 1-2/(1+c+d*x)])/f*(d*e-f-c*f) - (b^2*d*\operatorname{PolyLog}[2, 1-2/(1+c+d*x)])/((d*e+f-c*f)*(d*e-(1+c)*f)) + (b^2*d*\operatorname{PolyLog}[2, 1-(2*d*(e+f*x))/(d*e+f-c*f)*(1+c+d*x)])/((d*e+f-c*f)*(d*e-(1+c)*f))$

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 31

```
Int[((a_) + (b_)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 72

```
Int[((e_) + (f_)*(x_))^(p_)/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rule 632

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 633

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

### Rule 705

```
Int[1/(((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol]
:= Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

### Rule 706

```
Int[1/(((d_) + (e_)*(x_))*((a_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

Rule 1982

```
Int[(u_)^(m_)*(v_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Rule 2315

```
Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 5919

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol
] := -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5921

```
Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))/((d_) + (e_)*(x_)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x)) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6110

```
Int[((a_) + ArcCoth[(c_) + (d_)*(x_)]*(b_))^(p_)*((e_) + (f_)*(x_))^(
m_), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
```

```

+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]

```

### Rule 6122

```

Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcCoth
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]

```

### Rule 6688

```

Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]

```

### Rule 6725

```

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

```

### Rule 6741

```

Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]

```

### Rule 6742

```

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(c + dx))^2}{(e + fx)^2} dx &= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)(1 - (c + dx)^2)} dx}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)(1 - c^2 - 2cdx - d^2x^2)} dx}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b) \text{Subst} \left( \int \frac{a + b \coth^{-1}(x)}{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2b) \text{Subst} \left( \int \frac{d(a + b \coth^{-1}(x))}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \text{Subst} \left( \int \frac{a + b \coth^{-1}(x)}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{(2bd) \text{Subst} \left( \int \left( -\frac{a}{(-1 + x)(1 + x)(de - cf + fx)} - \frac{b \coth^{-1}(x)}{(-1 + x)(1 + x)(de - cf + fx)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} - \frac{(2abd) \text{Subst} \left( \int \frac{1}{(-1 + x)(1 + x)(de - cf + fx)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} - \frac{(2abd) \text{Subst} \left( \int \left( \frac{1}{2(de + f - cf)(-1 + x)} + \frac{1}{2(-de + (1 + c)f)(1 + x)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} + \frac{abd \log(1 + c + dx)}{f(de - f - cf)} - \frac{2abd}{(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^2}{f(e + fx)} + \frac{b^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} - \frac{abd \log(1 - c - dx)}{f(de + f - cf)}
\end{aligned}$$



**Mathematica [C]** time = 8.90, size = 470, normalized size = 0.98

$$-\frac{a^2}{f} + \frac{2ab \left( \coth^{-1}(c+dx) \left( c^2(-f) + cd(e-fx) + d^2ex + f \right) - d(e+fx) \log \left( \frac{d(e+fx)}{(c+dx) \sqrt{1 - \frac{1}{(c+dx)^2}}} \right) \right)}{(-cf+de+f)(de-(c+1)f)} + \frac{b^2 d (1-(c+dx)^2) (e+fx)}{f \left( -\text{Li}_2 \left( \exp \left( -2 \left( \coth^{-1}(c+dx) + \text{ta} \right) \right) \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^2/(e + f\*x)^2, x]

[Out] 
$$\begin{aligned} & (-a^2/f) + (2*a*b*((f - c^2*f + d^2*e*x + c*d*(e - f*x))*\text{ArcCoth}[c + d*x] \\ & - d*(e + f*x)*\text{Log}[ -((d*(e + f*x))/((c + d*x)*\text{Sqrt}[1 - (c + d*x)^{-2}]) ) ] ) / \\ & ((d*e + f - c*f)*(d*e - (1 + c)*f)) + (b^2*d*(e + f*x)*(1 - (c + d*x)^2)*(( \\ & E^{\text{ArcTanh}[f/(-(d*e) + c*f)]*\text{ArcCoth}[c + d*x]^2})/((-d*e) + c*f)*\text{Sqrt}[1 - f^ \\ & 2/(d*e - c*f)^2]) + \text{ArcCoth}[c + d*x]^2/(d*e + d*f*x) + (f*((-I)*\text{Pi}*\text{Log}[1 + \\ & E^{2*\text{ArcCoth}[c + d*x]})] - 2*\text{ArcTanh}[f/(-(d*e) + c*f)]*\text{Log}[1 - E^{-2*(\text{ArcCoth}[c + d*x] + \\ & \text{ArcTanh}[f/(d*e - c*f)])})] + \text{ArcCoth}[c + d*x]*(I*\text{Pi} + 2*\text{ArcTanh} \\ & [f/(d*e - c*f)] + 2*\text{Log}[1 - E^{-2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f) \\ & ])])) + I*\text{Pi}*\text{Log}[1/\text{Sqrt}[1 - (c + d*x)^{-2}]] + 2*\text{ArcTanh}[f/(-(d*e) + c*f)]* \\ & \text{Log}[I*\text{Sinh}[\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)]]] - \text{PolyLog}[2, E^{-2*( \\ & \text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])})])/(d^2*e^2 - 2*c*d*e*f + (-1 + \\ & c^2)*f^2))/((c + d*x)^2*(f - f/(c + d*x)^2))/(e + f*x) \end{aligned}$$

**fricas [F]** time = 0.53, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2 \operatorname{arccoth}(dx + c)^2 + 2ab \operatorname{arccoth}(dx + c) + a^2}{f^2 x^2 + 2efx + e^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e)^2,x, algorithm="fricas")

[Out] integral((b^2\*arccoth(d\*x + c)^2 + 2\*a\*b\*arccoth(d\*x + c) + a^2)/(f^2\*x^2 + 2\*e\*f\*x + e^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(dx + c) + a)^2}{(fx + e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e)^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^2/(f\*x + e)^2, x)

**maple** [A] time = 0.08, size = 783, normalized size = 1.63

$$\frac{d a^2}{(d f x + d e) f} - \frac{d b^2 \operatorname{arccoth}(d x + c)^2}{(d f x + d e) f} - \frac{2 d b^2 \operatorname{arccoth}(d x + c) \ln((d x + c) f - c f + d e)}{(c f - d e - f)(c f - d e + f)} + \frac{2 d b^2 \operatorname{arccoth}(d x + c) \ln((d x + c) f + c f + d e)}{f(2 c f - 2 d e - f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))^2/(f\*x+e)^2,x)

[Out]  $-d*a^2/(d*f*x+d*e)/f-d*b^2/(d*f*x+d*e)/f*\operatorname{arccoth}(d*x+c)^2-2*d*b^2*\operatorname{arccoth}(d*x+c)/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)+2*d*b^2/f*\operatorname{arccoth}(d*x+c)/(2*c*f-2*d*e-2*f)*\ln(d*x+c-1)-2*d*b^2/f*\operatorname{arccoth}(d*x+c)/(2*c*f-2*d*e+2*f)*\ln(d*x+c+1)+d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)*\ln((d*x+c)*f+f)/(c*f-d*e+f)+d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{dilog}(((d*x+c)*f+f)/(c*f-d*e+f))-d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)*\ln((d*x+c)*f-f)/(c*f-d*e-f)-d*b^2/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{dilog}(((d*x+c)*f-f)/(c*f-d*e-f))+1/4*d*b^2/f/(c*f-d*e-f)*\ln(d*x+c-1)^2-1/2*d*b^2/f/(c*f-d*e-f)*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)-1/2*d*b^2/f/(c*f-d*e-f)*\ln(d*x+c-1)*\ln(1/2+1/2*d*x+1/2*c)+1/4*d*b^2/f/(c*f-d*e+f)*\ln(d*x+c+1)^2+1/2*d*b^2/f/(c*f-d*e+f)*\ln(-1/2*d*x-1/2*c+1/2)*\ln(1/2+1/2*d*x+1/2*c)-1/2*d*b^2/f/(c*f-d*e+f)*\ln(-1/2*d*x-1/2*c+1/2)*\ln(d*x+c+1)+1/2*d*b^2/f/(c*f-d*e+f)*\operatorname{dilog}(1/2+1/2*d*x+1/2*c)-2*d*a*b/(d*f*x+d*e)/f*\operatorname{arccoth}(d*x+c)-2*d*a*b/(c*f-d*e-f)/(c*f-d*e+f)*\ln((d*x+c)*f-c*f+d*e)+2*d*a*b/f/(2*c*f-2*d*e-2*f)*\ln(d*x+c-1)-2*d*a*b/f/(2*c*f-2*d*e+2*f)*\ln(d*x+c+1)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left( d \left( \frac{\log(dx+c+1)}{def-(c+1)f^2} - \frac{\log(dx+c-1)}{def-(c-1)f^2} - \frac{2 \log(fx+e)}{d^2e^2-2cdef+(c^2-1)f^2} \right) - \frac{2 \operatorname{arccoth}(dx+c)}{f^2x+ef} \right) ab - \frac{1}{4} b^2 \left( \frac{\log(dx+c+1)}{f^2x+ef} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^2/(f\*x+e)^2,x, algorithm="maxima")

[Out]  $(d*(\log(d*x + c + 1)/(d*e*f - (c + 1)*f^2) - \log(d*x + c - 1)/(d*e*f - (c - 1)*f^2) - 2*\log(f*x + e)/(d^2*e^2 - 2*c*d*e*f + (c^2 - 1)*f^2)) - 2*\operatorname{arccoth}(d*x + c)/(f^2*x + e*f))*a*b - 1/4*b^2*(\log(d*x + c + 1)^2/(f^2*x + e*f) + \operatorname{integrate}(-((d*f*x + c*f + f)*\log(d*x + c - 1)^2 + 2*(d*f*x + d*e - (d*f*x + c*f + f)*\log(d*x + c - 1))*\log(d*x + c + 1))/(d*f^3*x^3 + c*e^2*f + e^2*f + (2*d*e*f^2 + c*f^3 + f^3)*x^2 + (d*e^2*f + 2*c*e*f^2 + 2*e*f^2)*x), x) - a^2/(f^2*x + e*f)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(c + dx))^2}{(e + fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c + d\*x))^2/(e + f\*x)^2, x)

[Out] int((a + b\*acoth(c + d\*x))^2/(e + f\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(c + dx))^2}{(e + fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*2/(f\*x+e)\*\*2, x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*2/(e + f\*x)\*\*2, x)

### 3.114 $\int (e + fx)^2 \left( a + b \coth^{-1}(c + dx) \right)^3 dx$

**Optimal.** Leaf size=546

$$\frac{b^2 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \operatorname{Li}_2 \left( 1 - \frac{2}{-c-dx+1} \right) \left( a + b \coth^{-1}(c + dx) \right) - 6b^2 f (de - cf) \log \left( \frac{2}{-c-dx+1} \right) \left( a + b \coth^{-1}(c + dx) \right)}{d^3}$$

[Out] a\*b^2\*f^2\*x/d^2+b^3\*f^2\*(d\*x+c)\*arccoth(d\*x+c)/d^3-1/2\*b\*f^2\*(a+b\*arccoth(d\*x+c))^2/d^3+3\*b\*f\*(-c\*f+d\*e)\*(a+b\*arccoth(d\*x+c))^2/d^3+3\*b\*f\*(-c\*f+d\*e)\*(d\*x+c)\*(a+b\*arccoth(d\*x+c))^2/d^3+1/2\*b\*f^2\*(d\*x+c)^2\*(a+b\*arccoth(d\*x+c))^2/d^3-1/3\*(-c\*f+d\*e)\*(d^2\*e^2-2\*c\*d\*e\*f+(c^2+3)\*f^2)\*(a+b\*arccoth(d\*x+c))^3/d^3/f+1/3\*(3\*d^2\*e^2-6\*c\*d\*e\*f+(3\*c^2+1)\*f^2)\*(a+b\*arccoth(d\*x+c))^3/d^3+1/3\*(f\*x+e)^3\*(a+b\*arccoth(d\*x+c))^3/f-6\*b^2\*f\*(-c\*f+d\*e)\*(a+b\*arccoth(d\*x+c))\*ln(2/(-d\*x-c+1))/d^3-b\*(3\*d^2\*e^2-6\*c\*d\*e\*f+(3\*c^2+1)\*f^2)\*(a+b\*arccoth(d\*x+c))^2\*ln(2/(-d\*x-c+1))/d^3+1/2\*b^3\*f^2\*ln(1-(d\*x+c)^2)/d^3-3\*b^3\*f\*(-c\*f+d\*e)\*polylog(2,(-d\*x-c-1)/(-d\*x-c+1))/d^3-b^2\*(3\*d^2\*e^2-6\*c\*d\*e\*f+(3\*c^2+1)\*f^2)\*(a+b\*arccoth(d\*x+c))\*polylog(2,1-2/(-d\*x-c+1))/d^3+1/2\*b^3\*(3\*d^2\*e^2-6\*c\*d\*e\*f+(3\*c^2+1)\*f^2)\*polylog(3,1-2/(-d\*x-c+1))/d^3

**Rubi [A]** time = 1.04, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$ , Rules used = {6112, 5929, 5911, 5985, 5919, 2402, 2315, 5917, 5981, 260, 5949, 6049, 6059, 6610}

$$\frac{b^2 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right) \operatorname{PolyLog} \left( 2, 1 - \frac{2}{-c-dx+1} \right) \left( a + b \coth^{-1}(c + dx) \right) - b^3 \left( (3c^2 + 1) f^2 - 6cdef + 3d^2 e^2 \right)}{d^3} + \dots$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x])^3,x]

[Out] (a\*b^2\*f^2\*x)/d^2 + (b^3\*f^2\*(c + d\*x)\*ArcCoth[c + d\*x])/d^3 - (b\*f^2\*(a + b\*ArcCoth[c + d\*x])^2)/(2\*d^3) + (3\*b\*f\*(d\*e - c\*f)\*(a + b\*ArcCoth[c + d\*x])^2)/d^3 + (3\*b\*f\*(d\*e - c\*f)\*(c + d\*x)\*(a + b\*ArcCoth[c + d\*x])^2)/d^3 + (b\*f^2\*(c + d\*x)^2\*(a + b\*ArcCoth[c + d\*x])^2)/(2\*d^3) - ((d\*e - c\*f)\*(d^2\*e^2 - 2\*c\*d\*e\*f + (3 + c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])^3)/(3\*d^3\*f) + ((3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])^3)/(3\*d^3) + ((e + f\*x)^3\*(a + b\*ArcCoth[c + d\*x])^3)/(3\*f) - (6\*b^2\*f\*(d\*e - c\*f)\*(a + b\*ArcCoth[c + d\*x])\*Log[2/(1 - c - d\*x)])/d^3 - (b\*(3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])^2\*Log[2/(1 - c - d\*x)])/d^3 + (b^3\*f^2\*Log[1 - (c + d\*x)^2])/d^3 - (3\*b^3\*f\*(d\*e - c\*f)\*PolyLog[2, -((1 + c + d\*x)/(1 - c - d\*x))])/d^3 - (b^2\*(3\*d^2\*e^2 - 6\*c\*d\*e\*f + (1 + 3\*c^2)\*f^2)\*(a + b\*ArcCoth[c + d\*x])\*PolyLog[2, 1 - 2/(1 - c - d\*x)])/d^3 + (

$b^3*(3*d^2*e^2 - 6*c*d*e*f + (1 + 3*c^2)*f^2)*PolyLog[3, 1 - 2/(1 - c - d*x)]/(2*d^3)$

### Rule 260

$Int[(x_)^{(m_.)}/((a_) + (b_)*(x_)^{(n_)}), x\_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] \&\& EqQ[m, n - 1]$

### Rule 2315

$Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] \&\& EqQ[e + c*d, 0]$

### Rule 2402

$Int[Log[(c_)]/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] \&\& EqQ[c, 2*d] \&\& EqQ[e^2*f + d^2*g, 0]$

### Rule 5911

$Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^{(p_.)}, x\_Symbol] :> Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^{(p - 1)})/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] \&\& IGtQ[p, 0]$

### Rule 5917

$Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^{(p_.)}*((d_)*(x_))^{(m_.)}, x\_Symbol] :> Simp[((d*x)^{(m + 1)}*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^{(m + 1)}*(a + b*ArcCoth[c*x])^{(p - 1)})/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] \&\& IGtQ[p, 0] \&\& (EqQ[p, 1] || IntegerQ[m]) \&\& NeQ[m, -1]$

### Rule 5919

$Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^{(p_.)}/((d_) + (e_)*(x_)), x\_Symbol] :> -Simp[((a + b*ArcCoth[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcCoth[c*x])^{(p - 1)}*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] \&\& IGtQ[p, 0] \&\& EqQ[c^2*d^2 - e^2, 0]$

### Rule 5929

$Int[((a_) + ArcCoth[(c_)*(x_)]*(b_))^{(p_.)}*((d_) + (e_)*(x_))^{(q_.)}, x\_Symbol] :> Simp[((d + e*x)^{(q + 1)}*(a + b*ArcCoth[c*x])^p)/(e*(q + 1)), x] - Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcCoth[c*x])^{(p - 1)}], x], x]$

, (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5981

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_)^(m\_)))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 6049

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((f\_) + (g\_.)\*(x\_)^(m\_)))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

### Rule 6059

Int[(Log[u\_]\*((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_) + (d\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((e\_.) + (f\_.)\*(x\_)^(m\_)), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ

Q[p, 0]

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int (e + fx)^2 (a + b \coth^{-1}(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^2 (a + b \coth^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^3}{3f} - \frac{b \text{Subst}\left(\int \left(-\frac{3f^2(de-cf)(a+b \coth^{-1}(x))^2}{d^3}\right) dx\right)}{d^3} \\
&= \frac{(e + fx)^3 (a + b \coth^{-1}(c + dx))^3}{3f} - \frac{b \text{Subst}\left(\int \frac{((de-cf)(d^2e^2 - 2cdef + 3f^2 + c^2f))}{d^3} dx\right)}{d^3} \\
&= \frac{3bf(de - cf)(c + dx)(a + b \coth^{-1}(c + dx))^2}{d^3} + \frac{bf^2(c + dx)^2 (a + b \coth^{-1}(c + dx))^2}{2d^3} \\
&= \frac{3bf(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^3} + \frac{3bf(de - cf)(c + dx)(a + b \coth^{-1}(c + dx))^2}{d^3} \\
&= \frac{ab^2 f^2 x}{d^2} - \frac{bf^2 (a + b \coth^{-1}(c + dx))^2}{2d^3} + \frac{3bf(de - cf)(a + b \coth^{-1}(c + dx))^2}{d^3} \\
&= \frac{ab^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + dx) \coth^{-1}(c + dx)}{d^3} - \frac{bf^2 (a + b \coth^{-1}(c + dx))^2}{2d^3} + \dots \\
&= \frac{ab^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + dx) \coth^{-1}(c + dx)}{d^3} - \frac{bf^2 (a + b \coth^{-1}(c + dx))^2}{2d^3} + \dots \\
&= \frac{ab^2 f^2 x}{d^2} + \frac{b^3 f^2 (c + dx) \coth^{-1}(c + dx)}{d^3} - \frac{bf^2 (a + b \coth^{-1}(c + dx))^2}{2d^3} + \dots
\end{aligned}$$

**Mathematica [C]** time = 10.60, size = 2594, normalized size = 4.75

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)^2\*(a + b\*ArcCoth[c + d\*x])^3,x]



```

[Out] (a^2*(a*d^2*e^2 + 3*b*d*e*f - 2*b*c*f^2)*x)/d^2 + (a^2*f*(2*a*d*e + b*f)*x^
2)/(2*d) + (a^3*f^2*x^3)/3 + a^2*b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcCoth[c
+ d*x] + ((3*a^2*b*d^2*e^2 - 3*a^2*b*c*d^2*e^2 + 3*a^2*b*d*e*f - 6*a^2*b*c*
d*e*f + 3*a^2*b*c^2*d*e*f + a^2*b*f^2 - 3*a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 - a
^2*b*c^3*f^2)*Log[1 - c - d*x])/(2*d^3) + ((3*a^2*b*d^2*e^2 + 3*a^2*b*c*d^2
*e^2 - 3*a^2*b*d*e*f - 6*a^2*b*c*d*e*f - 3*a^2*b*c^2*d*e*f + a^2*b*f^2 + 3*
a^2*b*c*f^2 + 3*a^2*b*c^2*f^2 + a^2*b*c^3*f^2)*Log[1 + c + d*x])/(2*d^3) +
(3*a*b^2*e^2*(1 - (c + d*x)^2)*(ArcCoth[c + d*x]*(ArcCoth[c + d*x] - (c + d
*x)*ArcCoth[c + d*x] + 2*Log[1 - E^(-2*ArcCoth[c + d*x]))]) - PolyLog[2, E^(-
2*ArcCoth[c + d*x])])/(d*(c + d*x)^2*(1 - (c + d*x)^(-2))) - (3*a*b^2*e*f
*(1 - (c + d*x)^2)*(2*c*ArcCoth[c + d*x]^2 + (c + d*x)^2*(1 - (c + d*x)^(-2
)))*ArcCoth[c + d*x]^2 - 2*(c + d*x)*ArcCoth[c + d*x]*(-1 + c*ArcCoth[c + d*
x]) + 4*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])]) - 2*Log[1/((c +
d*x)*Sqrt[1 - (c + d*x)^(-2)])] - 2*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])])
)/(d^2*(c + d*x)^2*(1 - (c + d*x)^(-2))) + (b^3*e^2*(1 - (c + d*x)^2)*((I/8
)*Pi^3 - ArcCoth[c + d*x]^3 - (c + d*x)*ArcCoth[c + d*x]^3 + 3*ArcCoth[c +
d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])]) + 3*ArcCoth[c + d*x]*PolyLog[2, E^(2
*ArcCoth[c + d*x])]) - (3*PolyLog[3, E^(2*ArcCoth[c + d*x])])/(2))/(d*(c + d*
x)^2*(1 - (c + d*x)^(-2))) - (b^3*e*f*(1 - (c + d*x)^2)*(I*c*Pi^3 - 12*ArcC
oth[c + d*x]^2 + 12*(c + d*x)*ArcCoth[c + d*x]^2 - 8*c*ArcCoth[c + d*x]^3 -
8*c*(c + d*x)*ArcCoth[c + d*x]^3 + 4*(c + d*x)^2*(1 - (c + d*x)^(-2))*ArcC
oth[c + d*x]^3 - 24*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])]) + 24*
c*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])]) + 12*PolyLog[2, E^(-2*
ArcCoth[c + d*x])]) + 24*c*ArcCoth[c + d*x]*PolyLog[2, E^(2*ArcCoth[c + d*x]
)]) - 12*c*PolyLog[3, E^(2*ArcCoth[c + d*x])])/(4*d^2*(c + d*x)^2*(1 - (c +
d*x)^(-2))) - (a*b^2*f^2*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(1 - (c + d*x)
^2)*((4*ArcCoth[c + d*x])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (3*ArcCoth
[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (12*c*ArcCoth[c + d*x]^
2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (9*c^2*ArcCoth[c + d*x]^2)/((c +
d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (-1 + 6*c*ArcCoth[c + d*x] + 3*ArcCoth[c +
d*x]^2 - 3*c^2*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)] + Cosh[3*ArcCo
th[c + d*x]] - 6*c*ArcCoth[c + d*x]*Cosh[3*ArcCoth[c + d*x]] + ArcCoth[c +
d*x]^2*Cosh[3*ArcCoth[c + d*x]] + 3*c^2*ArcCoth[c + d*x]^2*Cosh[3*ArcCoth[c
+ d*x]] + (6*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x])])/(c + d*x)
*Sqrt[1 - (c + d*x)^(-2)]) + (18*c^2*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth
[c + d*x])])/(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (18*c*Log[1/((c + d*x)*
Sqrt[1 - (c + d*x)^(-2)])])/(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (4*(1 +
3*c^2)*PolyLog[2, E^(-2*ArcCoth[c + d*x])])/(c + d*x)^3*(1 - (c + d*x)^(-2
))^3/2) - ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 3*c^2*ArcCoth[c +
d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 2*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth
[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] - 6*c^2*ArcCoth[c + d*x]*Log[1 - E^(-2
*ArcCoth[c + d*x])]*Sinh[3*ArcCoth[c + d*x]] + 6*c*Log[1/((c + d*x)*Sqrt[1
- (c + d*x)^(-2)])]*Sinh[3*ArcCoth[c + d*x]])/(4*d^3) + (b^3*f^2*(1 - (c +
d*x)^2)*(3*c*PolyLog[2, E^(-2*ArcCoth[c + d*x])]) + ((c + d*x)^3*(1 - (c +
d*x)^(-2)))^(3/2)*((-3*I)*Pi^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - ((9*

```

```

I)*c^2*Pi^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (24*ArcCoth[c + d*x])/S
qrt[1 - (c + d*x)^(-2)] - (72*c*ArcCoth[c + d*x]^2)/Sqrt[1 - (c + d*x)^(-2)
] - (48*ArcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (216*c*A
rcCoth[c + d*x]^2)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (24*ArcCoth[c + d
*x]^3)/Sqrt[1 - (c + d*x)^(-2)] + (24*c^2*ArcCoth[c + d*x]^3)/Sqrt[1 - (c +
d*x)^(-2)] + (24*ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])
+ (96*c*ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (72*c^2*
ArcCoth[c + d*x]^3)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - 24*ArcCoth[c + d
*x]*Cosh[3*ArcCoth[c + d*x]] + 72*c*ArcCoth[c + d*x]^2*Cosh[3*ArcCoth[c + d
*x]] - 8*ArcCoth[c + d*x]^3*Cosh[3*ArcCoth[c + d*x]] - 24*c^2*ArcCoth[c + d
*x]^3*Cosh[3*ArcCoth[c + d*x]] + (432*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcC
oth[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) - (72*ArcCoth[c + d*x]
^2*Log[1 - E^(2*ArcCoth[c + d*x])])/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) -
(216*c^2*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c + d*x])])/((c + d*x)*Sqr
t[1 - (c + d*x)^(-2)]) - (72*Log[1/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)])])/((
c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (96*(1 + 3*c^2)*ArcCoth[c + d*x]*Poly
Log[2, E^(2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c + d*x)^(-2))^(3/2)) -
(48*(1 + 3*c^2)*PolyLog[3, E^(2*ArcCoth[c + d*x])])/((c + d*x)^3*(1 - (c +
d*x)^(-2))^(3/2)) + I*Pi^3*Sinh[3*ArcCoth[c + d*x]] + (3*I)*c^2*Pi^3*Sinh[3
*ArcCoth[c + d*x]] - 72*c*ArcCoth[c + d*x]^2*Sinh[3*ArcCoth[c + d*x]] - 8*A
rcCoth[c + d*x]^3*Sinh[3*ArcCoth[c + d*x]] - 24*c^2*ArcCoth[c + d*x]^3*Sinh
[3*ArcCoth[c + d*x]] - 144*c*ArcCoth[c + d*x]*Log[1 - E^(-2*ArcCoth[c + d*x
]])*Sinh[3*ArcCoth[c + d*x]] + 24*ArcCoth[c + d*x]^2*Log[1 - E^(2*ArcCoth[c
+ d*x]])*Sinh[3*ArcCoth[c + d*x]] + 72*c^2*ArcCoth[c + d*x]^2*Log[1 - E^(2
*ArcCoth[c + d*x]])*Sinh[3*ArcCoth[c + d*x]] + 24*Log[1/((c + d*x)*Sqrt[1 -
(c + d*x)^(-2)])]*Sinh[3*ArcCoth[c + d*x]])/96))/(d^3*(c + d*x)^2*(1 - (c
+ d*x)^(-2)))

```

**fricas** [F] time = 0.44, size = 0, normalized size = 0.00

$\int (a^3 f^2 x^2 + 2 a^3 e f x + a^3 e^2 + (b^3 f^2 x^2 + 2 b^3 e f x + b^3 e^2) \operatorname{arccoth}(dx + c)^3 + 3 (ab^2 f^2 x^2 + 2 ab^2 e f x + ab^2 e^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(a+b*arccoth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*f^2*x^2 + 2*a^3*e*f*x + a^3*e^2 + (b^3*f^2*x^2 + 2*b^3*e*f*x +
b^3*e^2)*arccoth(d*x + c)^3 + 3*(a*b^2*f^2*x^2 + 2*a*b^2*e*f*x + a*b^2*e^2
)*arccoth(d*x + c)^2 + 3*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x + a^2*b*e^2)*arccot
h(d*x + c), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)^2 (b \operatorname{arccoth}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*(b\*arccoth(d\*x + c) + a)^3, x)

**maple** [C] time = 16.59, size = 10477, normalized size = 19.19

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="maxima")

[Out]  $\frac{1}{3}a^3f^2x^3 + a^3e^2fx^2 + \frac{3}{2}(2x^2\operatorname{arccoth}(dx+c) + d(2x/d^2 - (c^2 + 2c + 1)\log(dx+c+1)/d^3 + (c^2 - 2c + 1)\log(dx+c-1)/d^3))a^2b^2ef + \frac{1}{2}(2x^3\operatorname{arccoth}(dx+c) + d((d^2x^2 - 4cx)/d^3 + (c^3 + 3c^2 + 3c + 1)\log(dx+c+1)/d^4 - (c^3 - 3c^2 + 3c - 1)\log(dx+c-1)/d^4))a^2b^2f^2 + a^3e^2x + \frac{3}{2}(2(dx+c)\operatorname{arccoth}(dx+c) + \log(-(dx+c)^2 + 1))a^2b^2e^2/d + \frac{1}{24}((b^3d^3f^2x^3 + 3b^3d^3e^2fx^2 + 3b^3d^3e^2x + (c^3f^2 + 3d^2e^2 - 3(d^2ef - f^2))c^2 - 3d^2ef + 3(d^2e^2 - 2d^2ef + f^2)c + f^2)b^3)\log(dx+c+1)^3 + 3(2ab^2d^3f^2x^3 + (6ab^2d^3ef + b^3d^2f^2)x^2 + 2(3ab^2d^3e^2 + (3d^2ef - 2cd^2f^2)b^3)x - (b^3d^3f^2x^3 + 3b^3d^3e^2fx^2 + 3b^3d^3e^2x + (c^3f^2 - 3d^2e^2 - 3(d^2ef + f^2))c^2 - 3d^2ef + 3(d^2e^2 + 2d^2ef + f^2)c - f^2)b^3)\log(dx+c-1))\log(dx+c+1)^2)/d^3 + \operatorname{integrate}(-1/8((b^3d^3f^2x^3 + (2d^3ef + cd^2f^2 + d^2f^2)b^3x^2 + (d^3e^2 + 2cd^2ef + 2d^2ef)b^3x + (cd^2e^2 + d^2e^2)b^3)\log(dx+c-1)^3 - 6(ab^2d^3f^2x^3 + (2d^3ef + cd^2f^2 + d^2f^2)ab^2x^2 + (d^3e^2 + 2cd^2ef + 2d^2ef)ab^2x + (cd^2e^2 + d^2e^2)ab^2)\log(dx+c-1)^2 + (4ab^2d^3f^2x^3 + 2(6ab^2d^3ef + b^3d^2f^2)x^2 - 3(b^3d^3f^2x^3 + (2d^3ef + cd^2f^2 + d^2f^2)b^3x^2 + (d^3e^2 + 2cd^2ef + 2d^2ef)b^3x + (cd^2e^2 + d^2e^2)b^3)\log(dx+c-1)^2 + 4(3ab^2d^3e^2 + (3d^2ef - 2cd^2f^2)b^3)x + 2(6(cd^2e^2 + d^2e^2)ab^2 - (c^3f^2 - 3d^2e^2 - 3(d^2ef + f^2))c^2 - 3d^2ef + 3(d^2e^2 + 2d^2ef + f^2)c - f^2)b^3 + (6ab^2d^3f^2 - b^3d^3f^2)x^3 - 3(b^3d^3ef - 2(2d^3ef + cd^2f^2 + d^2f^2)ab^2)x^2 - 3(b^3d^3e^2 - 2(d^3e^2 + 2cd^2ef + 2d^2ef)ab^2)x)\log(dx+c-1))\log(dx+c+1))/(d^3x + cd^2 + d^2), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + fx)^2 (a + b \operatorname{acoth}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e + f*x)^2*(a + b*acoth(c + d*x))^3, x)`

[Out] `int((e + f*x)^2*(a + b*acoth(c + d*x))^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + dx))^3 (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**2*(a+b*acoth(d*x+c))**3, x)`

[Out] `Integral((a + b*acoth(c + d*x))**3*(e + f*x)**2, x)`

### 3.115 $\int (e + fx) \left( a + b \coth^{-1}(c + dx) \right)^3 dx$

**Optimal.** Leaf size=326

$$\frac{3b^2(de - cf)\operatorname{Li}_2\left(1 - \frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d^2} - \frac{3b^2 f \log\left(\frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d^2} - \frac{((c^2 + 1) f^2)}{d^2}$$

[Out]  $3/2*b*f*(a+b*\operatorname{arccoth}(d*x+c))^2/d^2+3/2*b*f*(d*x+c)*(a+b*\operatorname{arccoth}(d*x+c))^2/d^2+(-c*f+d*e)*(a+b*\operatorname{arccoth}(d*x+c))^3/d^2-1/2*(d^2*e^2-2*c*d*e*f+(c^2+1)*f^2)*(a+b*\operatorname{arccoth}(d*x+c))^3/d^2/f+1/2*(f*x+e)^2*(a+b*\operatorname{arccoth}(d*x+c))^3/f-3*b^2*f*(a+b*\operatorname{arccoth}(d*x+c))*\ln(2/(-d*x-c+1))/d^2-3*b*(-c*f+d*e)*(a+b*\operatorname{arccoth}(d*x+c))^2*\ln(2/(-d*x-c+1))/d^2-3/2*b^3*f*\operatorname{polylog}(2,(-d*x-c-1)/(-d*x-c+1))/d^2-3*b^2*(-c*f+d*e)*(a+b*\operatorname{arccoth}(d*x+c))*\operatorname{polylog}(2,1-2/(-d*x-c+1))/d^2+3/2*b^3*(-c*f+d*e)*\operatorname{polylog}(3,1-2/(-d*x-c+1))/d^2$

**Rubi [A]** time = 0.72, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {6112, 5929, 5911, 5985, 5919, 2402, 2315, 6049, 5949, 6059, 6610}

$$\frac{3b^2(de - cf)\operatorname{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)(a + b \coth^{-1}(c + dx))}{d^2} + \frac{3b^3(de - cf)\operatorname{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)}{2d^2} - \frac{3b^3 f \operatorname{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)*(a + b*\operatorname{ArcCoth}[c + d*x])^3, x]$

[Out]  $(3*b*f*(a + b*\operatorname{ArcCoth}[c + d*x])^2)/(2*d^2) + (3*b*f*(c + d*x)*(a + b*\operatorname{ArcCoth}[c + d*x])^2)/(2*d^2) + ((d*e - c*f)*(a + b*\operatorname{ArcCoth}[c + d*x])^3)/d^2 + ((2*c*e - (d*e^2)/f - ((1 + c^2)*f)/d)*(a + b*\operatorname{ArcCoth}[c + d*x])^3)/(2*d) + ((e + f*x)^2*(a + b*\operatorname{ArcCoth}[c + d*x])^3)/(2*f) - (3*b^2*f*(a + b*\operatorname{ArcCoth}[c + d*x])*\operatorname{Log}[2/(1 - c - d*x)])/d^2 - (3*b*(d*e - c*f)*(a + b*\operatorname{ArcCoth}[c + d*x])^2*\operatorname{Log}[2/(1 - c - d*x)])/d^2 - (3*b^3*f*\operatorname{PolyLog}[2, -((1 + c + d*x)/(1 - c - d*x))])/(2*d^2) - (3*b^2*(d*e - c*f)*(a + b*\operatorname{ArcCoth}[c + d*x])*\operatorname{PolyLog}[2, 1 - 2/(1 - c - d*x)])/d^2 + (3*b^3*(d*e - c*f)*\operatorname{PolyLog}[3, 1 - 2/(1 - c - d*x)])/d^2$

**Rule 2315**

$\operatorname{Int}[\operatorname{Log}[(c_.)*(x_)]/((d_.) + (e_.)*(x_)), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /; \operatorname{FreeQ}\{c, d, e\}, x \ \&\& \operatorname{EqQ}[e + c*d, 0]$

**Rule 2402**

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p, x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 5919

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5929

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*ArcCoth[c\*x])^p)/(e\*(q + 1)), x] - Dist[(b\*c\*p)/(e\*(q + 1)), Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^(p - 1), (d + e\*x)^(q + 1)/(1 - c^2\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5985

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 6049

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^p\*((f\_) + (g\_.)\*(x\_))^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcCoth[c\*x])^p/(d + e\*x^2), (f + g\*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I

GtQ[p, 0] && EqQ[c^2\*d + e, 0] && IGtQ[m, 0]

### Rule 6059

Int[(Log[u\_]\*((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^((p\_.))]/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6112

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_)])\*(b\_.))^((p\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] := Dist[1/d, Subst[Int[((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned}
\int (e + fx) (a + b \coth^{-1}(c + dx))^3 dx &= \frac{\text{Subst} \left( \int \left( \frac{de - cf}{d} + \frac{fx}{d} \right) (a + b \coth^{-1}(x))^3 dx, x, c + dx \right)}{d} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \left( -\frac{f^2 (a + b \coth^{-1}(x))^2}{d^2} + \dots \right) dx \right)}{2d^2} \\
&= \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^3}{2f} - \frac{(3b) \text{Subst} \left( \int \frac{(d^2 e^2 - 2cdef + (1 + c^2) f^2 + 2f \dots)}{1 - x} dx \right)}{2d^2} \\
&= \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{(e + fx)^2 (a + b \coth^{-1}(c + dx))^3}{2f} \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \dots \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \dots \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \dots \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \dots \\
&= \frac{3bf (a + b \coth^{-1}(c + dx))^2}{2d^2} + \frac{3bf(c + dx) (a + b \coth^{-1}(c + dx))^2}{2d^2} + \dots
\end{aligned}$$

**Mathematica** [C] time = 1.46, size = 600, normalized size = 1.84

$$2a^3 f(c + dx)^2 + 2a^2(c + dx)(-2acf + 2ade + 3bf) + 3a^2 b(-2cf + 2de + f) \log(-c - dx + 1) + 3a^2 b(2de - (2c + 1))$$


---



Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*(a + b\*ArcCoth[c + d\*x])^3,x]

[Out] (2\*a^2\*(2\*a\*d\*e + 3\*b\*f - 2\*a\*c\*f)\*(c + d\*x) + 2\*a^3\*f\*(c + d\*x)^2 - 6\*a^2\*b\*(c + d\*x)\*(c\*f - d\*(2\*e + f\*x))\*ArcCoth[c + d\*x] + 3\*a^2\*b\*(2\*d\*e + f - 2\*c\*f)\*Log[1 - c - d\*x] + 3\*a^2\*b\*(2\*d\*e - (1 + 2\*c)\*f)\*Log[1 + c + d\*x] + 12\*a\*b^2\*f\*((c + d\*x)\*ArcCoth[c + d\*x] + ((-1 + (c + d\*x)^2)\*ArcCoth[c + d\*x]^2)/2 - Log[1/((c + d\*x)\*Sqrt[1 - (c + d\*x)^(-2)])]) + 12\*a\*b^2\*d\*e\*(ArcCoth[c + d\*x]\*((-1 + c + d\*x)\*ArcCoth[c + d\*x] - 2\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + PolyLog[2, E^(-2\*ArcCoth[c + d\*x])]) - 12\*a\*b^2\*c\*f\*(ArcCoth[c + d\*x]\*((-1 + c + d\*x)\*ArcCoth[c + d\*x] - 2\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + PolyLog[2, E^(-2\*ArcCoth[c + d\*x])]) + 2\*b^3\*f\*(ArcCoth[c + d\*x]\*(3\*(-1 + c + d\*x)\*ArcCoth[c + d\*x] + (-1 + c^2 + 2\*c\*d\*x + d^2\*x^2)\*ArcCoth[c + d\*x]^2 - 6\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + 3\*PolyLog[2, E^(-2\*ArcCoth[c + d\*x])]) + 4\*b^3\*d\*e\*((-1/8\*I)\*Pi^3 + ArcCoth[c + d\*x]^3 + (c + d\*x)\*ArcCoth[c + d\*x]^3 - 3\*ArcCoth[c + d\*x]^2\*Log[1 - E^(2\*ArcCoth[c + d\*x])] - 3\*ArcCoth[c + d\*x]\*PolyLog[2, E^(2\*ArcCoth[c + d\*x])] + (3\*PolyLog[3, E^(2\*ArcCoth[c + d\*x])])/2) - 4\*b^3\*c\*f\*((-1/8\*I)\*Pi^3 + ArcCoth[c + d\*x]^3 + (c + d\*x)\*ArcCoth[c + d\*x]^3 - 3\*ArcCoth[c + d\*x]^2\*Log[1 - E^(2\*ArcCoth[c + d\*x])] - 3\*ArcCoth[c + d\*x]\*PolyLog[2, E^(2\*ArcCoth[c + d\*x])] + (3\*PolyLog[3, E^(2\*ArcCoth[c + d\*x])])/2))/(4\*d^2)

**fricas** [F] time = 0.65, size = 0, normalized size = 0.00

integral(a^3fx + a^3e + (b^3fx + b^3e) arccoth(dx + c)^3 + 3(ab^2fx + ab^2e) arccoth(dx + c)^2 + 3(a^2bfx + a^2be) arccoth(dx + c), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(a^3\*f\*x + a^3\*e + (b^3\*f\*x + b^3\*e)\*arccoth(d\*x + c)^3 + 3\*(a\*b^2\*f\*x + a\*b^2\*e)\*arccoth(d\*x + c)^2 + 3\*(a^2\*b\*f\*x + a^2\*b\*e)\*arccoth(d\*x + c), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)(b \operatorname{arccoth}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)\*(b\*arccoth(d\*x + c) + a)^3, x)

**maple** [C] time = 1.42, size = 12285, normalized size = 37.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*(a+b*arccoth(d*x+c))^3,x)`

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^3 f x^2 + \frac{3}{4} \left( 2 x^2 \operatorname{arccoth}(d x + c) + d \left( \frac{2 x}{d^2} - \frac{(c^2 + 2 c + 1) \log(d x + c + 1)}{d^3} + \frac{(c^2 - 2 c + 1) \log(d x + c - 1)}{d^3} \right) \right) a^2 b f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*(a+b*arccoth(d*x+c))^3,x, algorithm="maxima")`

[Out]  $\frac{1}{2} a^3 f x^2 + \frac{3}{4} (2 x^2 \operatorname{arccoth}(d x + c) + d (2 x / d^2 - (c^2 + 2 c + 1) \log(d x + c + 1) / d^3 + (c^2 - 2 c + 1) \log(d x + c - 1) / d^3)) a^2 b f + a^3 e x + \frac{3}{2} (2 (d x + c) \operatorname{arccoth}(d x + c) + \log(-(d x + c)^2 + 1)) a^2 b e / d + \frac{1}{16} ((b^3 d^2 f x^2 + 2 b^3 d^2 e x - (c^2 f - 2 (d e - f) c - 2 d e + f) b^3) \log(d x + c + 1)^3 + 3 (2 a b^2 d^2 f x^2 + 2 (2 a b^2 d^2 e + b^3 d f) x - (b^3 d^2 f x^2 + 2 b^3 d^2 e x - (c^2 f - 2 (d e + f) c + 2 d e + f) b^3) \log(d x + c - 1)) \log(d x + c + 1)^2) / d^2 + \operatorname{integrate}(-1/8 * ((b^3 d^2 f x^2 + (d^2 e + c d f + d f) b^3 x + (c d e + d e) b^3) \log(d x + c - 1)^3 - 6 (a b^2 d^2 f x^2 + (d^2 e + c d f + d f) a b^2 x + (c d e + d e) a b^2) \log(d x + c - 1)^2 + 3 (2 a b^2 d^2 f x^2 - (b^3 d^2 f x^2 + (d^2 e + c d f + d f) b^3 x + (c d e + d e) b^3) \log(d x + c - 1)^2 + 2 (2 a b^2 d^2 e + b^3 d f) x + (4 (c d e + d e) a b^2 + (c^2 f - 2 (d e + f) c + 2 d e + f) b^3 + (4 a b^2 d^2 f - b^3 d^2 f) x^2 - 2 (b^3 d^2 e - 2 (d^2 e + c d f + d f) a b^2) x) \log(d x + c - 1)) \log(d x + c + 1)) / (d^2 x + c d + d), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e + f x) (a + b \operatorname{acoth}(c + d x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e+f*x)*(a+b*acoth(c+d*x))^3,x)`

[Out] `int((e+f*x)*(a+b*acoth(c+d*x))^3,x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + d x))^3 (e + f x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(a+b*acoth(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acoth(c + d*x))**3*(e + f*x), x)
```

$$3.116 \quad \int \left( a + b \coth^{-1}(c + dx) \right)^3 dx$$

**Optimal.** Leaf size=132

$$\frac{3b^2 \operatorname{Li}_2\left(1 - \frac{2}{-c-dx+1}\right) (a + b \coth^{-1}(c + dx))}{d} + \frac{(c + dx) (a + b \coth^{-1}(c + dx))^3}{d} + \frac{(a + b \coth^{-1}(c + dx))^3}{d} - \frac{3b \log}{d}$$

[Out] (a+b\*arccoth(d\*x+c))^3/d+(d\*x+c)\*(a+b\*arccoth(d\*x+c))^3/d-3\*b\*(a+b\*arccoth(d\*x+c))^2\*ln(2/(-d\*x-c+1))/d-3\*b^2\*(a+b\*arccoth(d\*x+c))\*polylog(2,1-2/(-d\*x-c+1))/d+3/2\*b^3\*polylog(3,1-2/(-d\*x-c+1))/d

**Rubi [A]** time = 0.23, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {6104, 5911, 5985, 5919, 5949, 6059, 6610}

$$\frac{3b^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{-c-dx+1}\right) (a + b \coth^{-1}(c + dx))}{d} + \frac{3b^3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{-c-dx+1}\right)}{2d} + \frac{(c + dx) (a + b \coth^{-1}(c + dx))^3}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^3, x]

[Out] (a + b\*ArcCoth[c + d\*x])^3/d + ((c + d\*x)\*(a + b\*ArcCoth[c + d\*x])^3)/d - (3\*b\*(a + b\*ArcCoth[c + d\*x])^2\*Log[2/(1 - c - d\*x)]/d - (3\*b^2\*(a + b\*ArcCoth[c + d\*x])\*PolyLog[2, 1 - 2/(1 - c - d\*x)]/d + (3\*b^3\*PolyLog[3, 1 - 2/(1 - c - d\*x)])/(2\*d))

**Rule 5911**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^p, x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p-1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

**Rule 5919**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^p/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2/(1 + (e\*x)/d)]/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcCoth[c\*x])^(p-1)\*Log[2/(1 + (e\*x)/d)]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

**Rule 5949**

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^p/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p+1)/(b\*c\*d\*(p+1)), x] /; FreeQ[{a, b

, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

#### Rule 5985

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.)/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcCoth[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

#### Rule 6059

Int[(Log[u\_]\*((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

#### Rule 6104

Int[((a\_.) + ArcCoth[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

#### Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{coth}^{-1}(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int (a + b \operatorname{coth}^{-1}(x))^3 dx, x, c + dx\right)}{d} \\
&= \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} - \frac{(3b) \operatorname{Subst}\left(\int \frac{x(a + b \operatorname{coth}^{-1}(x))^2}{1 - x^2} dx, x, c + dx\right)}{d} \\
&= \frac{(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} - \frac{(3b) \operatorname{Subst}\left(\int \frac{(a + b \operatorname{coth}^{-1}(x))^2}{1 - x^2} dx, x, c + dx\right)}{d} \\
&= \frac{(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} - \frac{3b(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} \\
&= \frac{(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} - \frac{3b(a + b \operatorname{coth}^{-1}(c + dx))^2}{d} \\
&= \frac{(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} + \frac{(c + dx)(a + b \operatorname{coth}^{-1}(c + dx))^3}{d} - \frac{3b(a + b \operatorname{coth}^{-1}(c + dx))^2}{d}
\end{aligned}$$

**Mathematica [C]** time = 0.33, size = 208, normalized size = 1.58

$$\frac{2a^3(c + dx) + 3a^2b \log(1 - (c + dx)^2) + 6a^2b(c + dx) \operatorname{coth}^{-1}(c + dx) + 6ab^2 \left(\operatorname{Li}_2\left(e^{-2 \operatorname{coth}^{-1}(c + dx)}\right) + \operatorname{coth}^{-1}(c + dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^3, x]

[Out] (2\*a^3\*(c + d\*x) + 6\*a^2\*b\*(c + d\*x)\*ArcCoth[c + d\*x] + 3\*a^2\*b\*Log[1 - (c + d\*x)^2] + 6\*a\*b^2\*(ArcCoth[c + d\*x]\*((-1 + c + d\*x)\*ArcCoth[c + d\*x] - 2\*Log[1 - E^(-2\*ArcCoth[c + d\*x])]) + PolyLog[2, E^(-2\*ArcCoth[c + d\*x])]) + 2\*b^3\*((-1/8\*I)\*Pi^3 + ArcCoth[c + d\*x]^3 + (c + d\*x)\*ArcCoth[c + d\*x]^3 - 3\*ArcCoth[c + d\*x]^2\*Log[1 - E^(2\*ArcCoth[c + d\*x])] - 3\*ArcCoth[c + d\*x]\*PolyLog[2, E^(2\*ArcCoth[c + d\*x])] + (3\*PolyLog[3, E^(2\*ArcCoth[c + d\*x])])/(2\*d)))/(2\*d)

**fricas [F]** time = 0.59, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^3 \operatorname{arcoth}(dx + c)^3 + 3ab^2 \operatorname{arcoth}(dx + c)^2 + 3a^2b \operatorname{arcoth}(dx + c) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(b^3\*arccoth(d\*x + c)^3 + 3\*a\*b^2\*arccoth(d\*x + c)^2 + 3\*a^2\*b\*arccoth(d\*x + c) + a^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(dx + c) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^3, x)

**maple** [B] time = 0.30, size = 485, normalized size = 3.67

$$a^3x + \frac{a^3c}{d} + \operatorname{arccoth}(dx + c)^3 x b^3 + \frac{\operatorname{arccoth}(dx + c)^3 b^3 c}{d} + \frac{b^3 \operatorname{arccoth}(dx + c)^3}{d} - \frac{3 \operatorname{arccoth}(dx + c)^2 \ln\left(1 + \frac{1}{\sqrt{\frac{dx+c}{dx+c}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(d\*x+c))^3,x)

[Out] a^3\*x+1/d\*a^3\*c+arccoth(d\*x+c)^3\*x\*b^3+1/d\*arccoth(d\*x+c)^3\*b^3\*c+1/d\*b^3\*a\*arccoth(d\*x+c)^3-3/d\*arccoth(d\*x+c)^2\*ln(1+1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^3-3/d\*arccoth(d\*x+c)^2\*ln(1-1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^3-6/d\*arccoth(d\*x+c)\*polylog(2,-1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^3-6/d\*arccoth(d\*x+c)\*polylog(2,1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^3+6/d\*polylog(3,-1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^3+6/d\*polylog(3,1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*b^3+3\*arccoth(d\*x+c)^2\*x\*a\*b^2+3/d\*arccoth(d\*x+c)^2\*a\*b^2\*c+3/d\*a\*b^2\*arccoth(d\*x+c)^2-6/d\*arccoth(d\*x+c)\*ln(1+1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*a\*b^2-6/d\*arccoth(d\*x+c)\*ln(1-1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*a\*b^2-6/d\*polylog(2,-1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*a\*b^2-6/d\*polylog(2,1/((d\*x+c-1)/(d\*x+c+1))^(1/2))\*a\*b^2+3\*arccoth(d\*x+c)\*x\*a^2\*b+3/d\*arccoth(d\*x+c)\*a^2\*b\*c+3/2/d\*a^2\*b\*ln((d\*x+c)^2-1)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3x + \frac{3\left(2(dx+c)\operatorname{arccoth}(dx+c) + \log\left(-(dx+c)^2+1\right)\right)a^2b}{2d} + \frac{\left(b^3dx + b^3(c+1)\right)\log(dx+c+1)^3 + 3\left(2ab^2dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3,x, algorithm="maxima")

[Out]  $a^3x + \frac{3}{2}(2(d*x + c)\operatorname{arccoth}(d*x + c) + \log(-(d*x + c)^2 + 1))a^2b/d + \frac{1}{8}((b^3d*x + b^3(c + 1))\log(d*x + c + 1)^3 + 3(2a*b^2d*x - (b^3d*x + b^3(c - 1))\log(d*x + c - 1))\log(d*x + c + 1)^2)/d + \operatorname{integrate}(-1/8*((b^3d*x + b^3(c + 1))\log(d*x + c - 1)^3 - 6(a*b^2d*x + a*b^2(c + 1))\log(d*x + c - 1)^2 + 3(4a*b^2d*x - (b^3d*x + b^3(c + 1))\log(d*x + c - 1)^2 + 2(2a*b^2(c + 1) - b^3(c - 1) + (2a*b^2d - b^3d)*x)\log(d*x + c - 1))\log(d*x + c + 1))/(d*x + c + 1), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acoth}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c + d\*x))^3,x)

[Out] int((a + b\*acoth(c + d\*x))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*3,x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*3, x)



$$3.117 \quad \int \frac{(a+b \coth^{-1}(c+dx))^3}{e+fx} dx$$

**Optimal.** Leaf size=308

$$\frac{3b^2 (a + b \coth^{-1}(c + dx)) \operatorname{Li}_3\left(1 - \frac{2d(e+fx)}{(de-cf+f)(c+dx+1)}\right)}{2f} + \frac{3b^2 \operatorname{Li}_3\left(1 - \frac{2}{c+dx+1}\right) (a + b \coth^{-1}(c + dx))}{2f} - \frac{3b (a + b \coth^{-1}(c + dx))}{2f}$$

```
[Out] -(a+b*arccoth(d*x+c))^3*ln(2/(d*x+c+1))/f+(a+b*arccoth(d*x+c))^3*ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/2*b*(a+b*arccoth(d*x+c))^2*polylog(2,1-2/(d*x+c+1))/f-3/2*b*(a+b*arccoth(d*x+c))^2*polylog(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/2*b^2*(a+b*arccoth(d*x+c))*polylog(3,1-2/(d*x+c+1))/f-3/2*b^2*(a+b*arccoth(d*x+c))*polylog(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f+3/4*b^3*polylog(4,1-2/(d*x+c+1))/f-3/4*b^3*polylog(4,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/f
```

**Rubi [A]** time = 0.19, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {6112, 5925}

$$\frac{3b^2 (a + b \coth^{-1}(c + dx)) \operatorname{PolyLog}\left(3, 1 - \frac{2d(e+fx)}{(c+dx+1)(-cf+de+f)}\right)}{2f} + \frac{3b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{c+dx+1}\right) (a + b \coth^{-1}(c + dx))}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCoth[c + d*x])^3/(e + f*x), x]
```

```
[Out] -(((a + b*ArcCoth[c + d*x])^3*Log[2/(1 + c + d*x)])/f) + ((a + b*ArcCoth[c + d*x])^3*Log[(2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/f + (3*b*(a + b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b*(a + b*ArcCoth[c + d*x])^2*PolyLog[2, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((2*f) + (3*b^2*(a + b*ArcCoth[c + d*x])*PolyLog[3, 1 - 2/(1 + c + d*x)])/(2*f) - (3*b^2*(a + b*ArcCoth[c + d*x])*PolyLog[3, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((2*f) + (3*b^3*PolyLog[4, 1 - 2/(1 + c + d*x)])/(4*f) - (3*b^3*PolyLog[4, 1 - (2*d*(e + f*x))/((d*e + f - c*f)*(1 + c + d*x))])/((4*f)
```

**Rule 5925**

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)^3/((d_.) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^3*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcCoth[c*x])^3*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(3*b*(a + b*ArcCoth[c*x])^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(3*b*(a + b*ArcCoth[c*x])^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])
```

```
/(2*e), x] + Simp[(3*b^2*(a + b*ArcCoth[c*x])*PolyLog[3, 1 - 2/(1 + c*x)])/
(2*e), x] - Simp[(3*b^2*(a + b*ArcCoth[c*x])*PolyLog[3, 1 - (2*c*(d + e*x))
/((c*d + e)*(1 + c*x)))]/(2*e), x] + Simp[(3*b^3*PolyLog[4, 1 - 2/(1 + c*x)
)]/(4*e), x] - Simp[(3*b^3*PolyLog[4, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c
*x)))]/(4*e), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Arc
Coth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

### Rubi steps

$$\int \frac{(a + b \operatorname{coth}^{-1}(c + dx))^3}{e + fx} dx = \frac{\operatorname{Subst}\left(\int \frac{(a + b \operatorname{coth}^{-1}(x))^3}{\frac{de - cf}{d} + \frac{fx}{d}} dx, x, c + dx\right)}{d}$$

$$= -\frac{(a + b \operatorname{coth}^{-1}(c + dx))^3 \log\left(\frac{2}{1 + c + dx}\right)}{f} + \frac{(a + b \operatorname{coth}^{-1}(c + dx))^3 \log\left(\frac{2d(e + fx)}{(de + f - cf)(1 + c + dx)}\right)}{f}$$

**Mathematica** [F] time = 28.57, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{coth}^{-1}(c + dx))^3}{e + fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x), x]

[Out] Integrate[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x), x]

**fricas** [F] time = 0.67, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^3 \operatorname{arccoth}(dx + c)^3 + 3ab^2 \operatorname{arccoth}(dx + c)^2 + 3a^2b \operatorname{arccoth}(dx + c) + a^3}{fx + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e),x, algorithm="fricas")

[Out]  $\int (b^3 \operatorname{arccoth}(dx+c)^3 + 3ab^2 \operatorname{arccoth}(dx+c)^2 + 3a^2 b \operatorname{arccoth}(dx+c) + a^3) / (fx+e), x$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(dx+c) + a)^3}{fx+e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(d*x+c))^3/(f*x+e),x, algorithm="giac")`

[Out]  $\int (b \operatorname{arccoth}(dx+c) + a)^3 / (fx+e), x$

**maple** [C] time = 1.55, size = 3796, normalized size = 12.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccoth(d*x+c))^3/(f*x+e),x)`

[Out]  $6ab^2/f \operatorname{polylog}(3, -1/((dx+c-1)/(dx+c+1))^{1/2}) + 3/2ab^2/(c-f-d*e-f) \operatorname{polylog}(3, (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e+f)) + 3/4b^3c/(c-f-d*e-f) \operatorname{polylog}(4, (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e+f)) + 6ab^2/f \operatorname{polylog}(3, 1/((dx+c-1)/(dx+c+1))^{1/2}) + 3/2b^3/(c-f-d*e-f) \operatorname{arccoth}(dx+c) \operatorname{polylog}(3, (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e+f)) - 3/2b^3/(c-f-d*e-f) \operatorname{arccoth}(dx+c)^2 \operatorname{polylog}(2, (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e+f)) - 3b^3/f \operatorname{arccoth}(dx+c)^2 \operatorname{polylog}(2, -1/((dx+c-1)/(dx+c+1))^{1/2}) - b^3/f \operatorname{arccoth}(dx+c)^3 \ln(((dx+c+1)/(dx+c-1)-1) * cf + (1-(dx+c+1)/(dx+c-1)) * e + d - (dx+c+1)/(dx+c-1) * f) + b^3/f \operatorname{arccoth}(dx+c)^3 \ln((dx+c+1)/(dx+c-1) - 1) + 6b^3/f \operatorname{arccoth}(dx+c) \operatorname{polylog}(3, -1/((dx+c-1)/(dx+c+1))^{1/2}) - b^3/f \operatorname{arccoth}(dx+c)^3 \ln(1 - 1/((dx+c-1)/(dx+c+1))^{1/2}) - b^3/f \operatorname{arccoth}(dx+c)^3 \ln(1 + 1/((dx+c-1)/(dx+c+1))^{1/2}) - 3b^3/f \operatorname{arccoth}(dx+c)^2 \operatorname{polylog}(2, 1/((dx+c-1)/(dx+c+1))^{1/2}) + 6b^3/f \operatorname{arccoth}(dx+c) \operatorname{polylog}(3, 1/((dx+c-1)/(dx+c+1))^{1/2}) + b^3 \ln((dx+c) * f - c * f + d * e) / f \operatorname{arccoth}(dx+c)^3 - b^3/(c-f-d*e-f) \operatorname{arccoth}(dx+c)^3 \ln(1 - (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e+f)) + 3/2a^2b/f \ln((dx+c) * f - c * f + d * e) * \ln(((dx+c) * f - f) / (c-f-d*e-f)) - 3/2a^2b/f \ln((dx+c) * f - c * f + d * e) * \ln(((dx+c) * f + f) / (c-f-d*e+f)) - 6b^3/f \operatorname{polylog}(4, 1/((dx+c-1)/(dx+c+1))^{1/2}) - 6b^3/f \operatorname{polylog}(4, -1/((dx+c-1)/(dx+c+1))^{1/2}) - 3/4b^3/(c-f-d*e-f) \operatorname{polylog}(4, (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e+f)) + a^3 \ln((dx+c) * f - c * f + d * e) / f + 3/2a^2b/f \operatorname{dilog}(((dx+c) * f - f) / (c-f-d*e-f)) - 3/2a^2b/f \operatorname{dilog}(((dx+c) * f + f) / (c-f-d*e+f)) + 3/2d * b^3 / f * e / (c-f-d*e-f) \operatorname{arccoth}(dx+c) \operatorname{polylog}(3, (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e+f)) - 3/2d * b^3 / f * e / (c-f-d*e-f) \operatorname{arccoth}(dx+c)^2 \operatorname{polylog}(2, (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e+f)) - d * b^3 / f * e / (c-f-d*e-f) \operatorname{arccoth}(dx+c)^3 \ln(1 - (c-f-d*e-f)(dx+c+1)/(dx+c-1)/(c-f-d*e-f))$

$$\begin{aligned}
& -d*e+f)) + 3/2*d*a*b^2/f*e/(c*f-d*e-f)*\text{polylog}(3, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) - 1/2*I*b^3/f*Pi*\text{arccoth}(d*x+c)^3*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2 - 1/2*I*b^3/f*Pi*\text{arccoth}(d*x+c)^3*\text{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2 - 3/2*b^3*c/(c*f-d*e-f)*\text{arccoth}(d*x+c)*\text{polylog}(3, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) + 3/2*b^3*c/(c*f-d*e-f)*\text{arccoth}(d*x+c)^2*\text{polylog}(2, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) + b^3*c/(c*f-d*e-f)*\text{arccoth}(d*x+c)^3*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) + 3*a*b^2*\ln((d*x+c)*f-c*f+d*e)/f*\text{arccoth}(d*x+c)^2 - I*b^3/f*Pi*\text{arccoth}(d*x+c)^3 - 3/2*a*b^2*c/(c*f-d*e-f)*\text{polylog}(3, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) - 3*a*b^2/f*\text{arccoth}(d*x+c)^2*\ln(1+1/((d*x+c-1)/(d*x+c+1))^(1/2)) - 6*a*b^2/f*\text{arccoth}(d*x+c)*\text{polylog}(2, -1/((d*x+c-1)/(d*x+c+1))^(1/2)) - 3*a*b^2/f*\text{arccoth}(d*x+c)^2*\ln(1-1/((d*x+c-1)/(d*x+c+1))^(1/2)) - 6*a*b^2/f*\text{arccoth}(d*x+c)*\text{polylog}(2, 1/((d*x+c-1)/(d*x+c+1))^(1/2)) + 3*a*b^2/f*\text{arccoth}(d*x+c)^2*\ln((d*x+c+1)/(d*x+c-1)-1) - 3*a*b^2/(c*f-d*e-f)*\text{arccoth}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) - 3*a*b^2/(c*f-d*e-f)*\text{arccoth}(d*x+c)*\text{polylog}(2, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) - 3*a*b^2/f*\text{arccoth}(d*x+c)^2*\ln(((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)) + 3*a^2*b*\ln((d*x+c)*f-c*f+d*e)/f*\text{arccoth}(d*x+c) - 3*d*a*b^2/f*e/(c*f-d*e-f)*\text{arccoth}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) - 3*d*a*b^2/f*e/(c*f-d*e-f)*\text{arccoth}(d*x+c)*\text{polylog}(2, (c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) + 1/2*I*b^3/f*Pi*\text{arccoth}(d*x+c)^3*\text{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3 - 2*I*a*b^2/f*Pi*\text{arccoth}(d*x+c)^2*\text{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2 + 3/2*I*a*b^2/f*Pi*\text{arccoth}(d*x+c)^2*\text{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3 + 3*I*a*b^2/f*Pi*\text{arccoth}(d*x+c)^2*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2 + I*b^3/f*Pi*\text{arccoth}(d*x+c)^3*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^2 + 3*a*b^2*c/(c*f-d*e-f)*\text{arccoth}(d*x+c)^2*\ln(1-(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e-f)) + 3*a*b^2*c/(c*f-d*e-f)*\text{arccoth}(d*x+c)*\text{polylog}(2, (c*f-d*e-f)*(d*x+c+1)/
\end{aligned}$$

$(d*x+c-1)/(c*f-d*e+f))-3/4*d*b^3/f*e/(c*f-d*e-f)*\text{polylog}(4,(c*f-d*e-f)*(d*x+c+1)/(d*x+c-1)/(c*f-d*e+f))-1/2*I*b^3/f*\text{Pi}*\text{arccoth}(d*x+c)^3*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3-3*I*a*b^2/f*\text{Pi}*\text{arccoth}(d*x+c)^2$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \log(fx + e)}{f} + \int \frac{b^3 \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)^3}{8(fx + e)} + \frac{3ab^2 \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)^2}{4(fx + e)} + \frac{3a^2b \left( \log\left(\frac{1}{dx+c} + 1\right) - \log\left(-\frac{1}{dx+c} + 1\right) \right)}{4(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e),x, algorithm="maxima")

[Out] a^3\*log(f\*x + e)/f + integrate(1/8\*b^3\*(log(1/(d\*x + c) + 1) - log(-1/(d\*x + c) + 1))^3/(f\*x + e) + 3/4\*a\*b^2\*(log(1/(d\*x + c) + 1) - log(-1/(d\*x + c) + 1))^2/(f\*x + e) + 3/2\*a^2\*b\*(log(1/(d\*x + c) + 1) - log(-1/(d\*x + c) + 1)))/(f\*x + e), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(c + dx))^3}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c + d\*x))^3/(e + f\*x),x)

[Out] int((a + b\*acoth(c + d\*x))^3/(e + f\*x), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(c + dx))^3}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*3/(f\*x+e),x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*3/(e + f\*x), x)

$$3.118 \quad \int \frac{(a+b \coth^{-1}(c+dx))^3}{(e+fx)^2} dx$$

**Optimal.** Leaf size=1089

$$\frac{3d \coth^{-1}(c+dx)^2 \log\left(\frac{2}{-c-dx+1}\right) b^3}{2f(de-cf+f)} - \frac{3d \coth^{-1}(c+dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{2f(de-cf-f)} + \frac{3d \coth^{-1}(c+dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{(de-cf+f)(de-(c+1)f)} - \frac{3d}{(de-cf+f)(de-(c+1)f)}$$

[Out]  $-(a+b \operatorname{arccoth}(d*x+c))^3/f/(f*x+e)+3*a*b^2*d*\operatorname{arccoth}(d*x+c)*\ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/2*b^3*d*\operatorname{arccoth}(d*x+c)^2*\ln(2/(-d*x-c+1))/f/(-c*f+d*e+f)-3/2*a^2*b*d*\ln(-d*x-c+1)/f/(-c*f+d*e+f)-3*a*b^2*d*\operatorname{arccoth}(d*x+c)*\ln(2/(d*x+c+1))/f/(-c*f+d*e-f)+6*a*b^2*d*\operatorname{arccoth}(d*x+c)*\ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3/2*b^3*d*\operatorname{arccoth}(d*x+c)^2*\ln(2/(d*x+c+1))/f/(-c*f+d*e-f)+3*b^3*d*\operatorname{arccoth}(d*x+c)^2*\ln(2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a^2*b*d*\ln(d*x+c+1)/f/(-c*f+d*e-f)+3*a^2*b*d*\ln(f*x+e)/(f^2-(-c*f+d*e)^2)-6*a*b^2*d*\operatorname{arccoth}(d*x+c)*\ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3*b^3*d*\operatorname{arccoth}(d*x+c)^2*\ln(2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*a*b^2*d*\operatorname{polylog}(2,(-d*x-c-1)/(-d*x-c+1))/f/(-c*f+d*e+f)+3/2*b^3*d*\operatorname{arccoth}(d*x+c)*\operatorname{polylog}(2,1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/2*a*b^2*d*\operatorname{polylog}(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*a*b^2*d*\operatorname{polylog}(2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*d*\operatorname{arccoth}(d*x+c)*\operatorname{polylog}(2,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3*b^3*d*\operatorname{arccoth}(d*x+c)*\operatorname{polylog}(2,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*a*b^2*d*\operatorname{polylog}(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3*b^3*d*\operatorname{arccoth}(d*x+c)*\operatorname{polylog}(2,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)-3/4*b^3*d*\operatorname{polylog}(3,1-2/(-d*x-c+1))/f/(-c*f+d*e+f)+3/4*b^3*d*\operatorname{polylog}(3,1-2/(d*x+c+1))/f/(-c*f+d*e-f)-3/2*b^3*d*\operatorname{polylog}(3,1-2/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)+3/2*b^3*d*\operatorname{polylog}(3,1-2*d*(f*x+e)/(-c*f+d*e+f)/(d*x+c+1))/(-c*f+d*e-f)/(-c*f+d*e+f)$

**Rubi [A]** time = 2.78, antiderivative size = 1094, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$ , Rules used = {6110, 6741, 6122, 6688, 12, 6725, 72, 6742, 5919, 2402, 2315, 5921, 2447, 5949, 6059, 6610, 6057, 5923}

$$\frac{3d \coth^{-1}(c+dx)^2 \log\left(\frac{2}{-c-dx+1}\right) b^3}{2f(de-cf+f)} - \frac{3d \coth^{-1}(c+dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{2f(de-cf-f)} + \frac{3d \coth^{-1}(c+dx)^2 \log\left(\frac{2}{c+dx+1}\right) b^3}{(de-cf+f)(de-(c+1)f)} - \frac{3d}{(de-cf+f)(de-(c+1)f)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x)^2, x]

[Out]  $-(a + b \operatorname{ArcCoth}[c + d*x])^3/(f*(e + f*x)) + (3*a*b^2*d*\operatorname{ArcCoth}[c + d*x]*\operatorname{Log}[2/(1 - c - d*x)])/f*(d*e + f - c*f) + (3*b^3*d*\operatorname{ArcCoth}[c + d*x]^2*\operatorname{Log}[$

$$\begin{aligned} & 2/(1 - c - d*x)]/(2*f*(d*e + f - c*f)) - (3*a^2*b*d*Log[1 - c - d*x])/(2*f \\ & *(d*e + f - c*f)) - (3*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/(f*(d \\ & *e - f - c*f)) + (6*a*b^2*d*ArcCoth[c + d*x]*Log[2/(1 + c + d*x)])/((d*e + \\ & f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d* \\ & x)])/(2*f*(d*e - f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]^2*Log[2/(1 + c + d*x \\ & )])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a^2*b*d*Log[1 + c + d*x])/(2*f \\ & *(d*e - f - c*f)) - (3*a^2*b*d*Log[e + f*x])/((d*e + f - c*f)*(d*e - (1 + c \\ & )*f)) - (6*a*b^2*d*ArcCoth[c + d*x]*Log[(2*d*(e + f*x))]/((d*e + f - c*f)*(1 \\ & + c + d*x)))/((d*e + f - c*f)*(d*e - (1 + c)*f)) - (3*b^3*d*ArcCoth[c + d \\ & *x]^2*Log[(2*d*(e + f*x))]/((d*e + f - c*f)*(1 + c + d*x)))/((d*e + f - c*f \\ & )*(d*e - (1 + c)*f)) + (3*a*b^2*d*PolyLog[2, -((1 + c + d*x)/(1 - c - d*x)) \\ & ])/(2*f*(d*e + f - c*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - 2/(1 - \\ & c - d*x)])/(2*f*(d*e + f - c*f)) + (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x \\ & )])/(2*f*(d*e - f - c*f)) - (3*a*b^2*d*PolyLog[2, 1 - 2/(1 + c + d*x)])/((d \\ & *e + f - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - \\ & 2/(1 + c + d*x)])/(2*f*(d*e - f - c*f)) - (3*b^3*d*ArcCoth[c + d*x]*PolyLo \\ & g[2, 1 - 2/(1 + c + d*x)])/((d*e + f - c*f)*(d*e - (1 + c)*f)) + (3*a*b^2*d \\ & *PolyLog[2, 1 - (2*d*(e + f*x))]/((d*e + f - c*f)*(1 + c + d*x)))/((d*e + f \\ & - c*f)*(d*e - (1 + c)*f)) + (3*b^3*d*ArcCoth[c + d*x]*PolyLog[2, 1 - (2*d* \\ & (e + f*x))]/((d*e + f - c*f)*(1 + c + d*x)))/((d*e + f - c*f)*(d*e - (1 + c \\ & )*f)) - (3*b^3*d*PolyLog[3, 1 - 2/(1 - c - d*x)])/(4*f*(d*e + f - c*f)) + ( \\ & 3*b^3*d*PolyLog[3, 1 - 2/(1 + c + d*x)])/(4*f*(d*e - f - c*f)) - (3*b^3*d*P \\ & olyLog[3, 1 - 2/(1 + c + d*x)])/(2*(d*e + f - c*f)*(d*e - (1 + c)*f)) + (3* \\ & b^3*d*PolyLog[3, 1 - (2*d*(e + f*x))]/((d*e + f - c*f)*(1 + c + d*x)))/(2*( \\ & d*e + f - c*f)*(d*e - (1 + c)*f)) \end{aligned}$$

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

$c, d, e, f, g\}, x] \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

### Rule 2447

$\text{Int}[\text{Log}[u_]*(Pq_)^{(m_.)}, x\_Symbol] \text{ :> With}[\{C = \text{FullSimplify}[(Pq^m*(1 - u)) / D[u, x]]\}, \text{Simp}[C*\text{PolyLog}[2, 1 - u], x] /; \text{FreeQ}[C, x]] /; \text{IntegerQ}[m] \&\& \text{PolyQ}[Pq, x] \&\& \text{RationalFunctionQ}[u, x] \&\& \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]$

### Rule 5919

$\text{Int}[\{(a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.)\}^{(p_.)} / \{(d_.) + (e_.)*(x_.)\}, x\_Symbol] \text{ :> -Simp}[\{(a + b*\text{ArcCoth}[c*x])^p*\text{Log}[2/(1 + (e*x)/d)]\} / e, x] + \text{Dist}[(b*c*p) / e, \text{Int}[\{(a + b*\text{ArcCoth}[c*x])^{(p - 1)}*\text{Log}[2/(1 + (e*x)/d)]\} / (1 - c^2*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[c^2*d^2 - e^2, 0]$

### Rule 5921

$\text{Int}[\{(a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.)\} / \{(d_.) + (e_.)*(x_.)\}, x\_Symbol] \text{ :> -Simp}[\{(a + b*\text{ArcCoth}[c*x])*\text{Log}[2/(1 + c*x)]\} / e, x] + (\text{Dist}[(b*c) / e, \text{Int}[\text{Log}[2/(1 + c*x)] / (1 - c^2*x^2), x], x] - \text{Dist}[(b*c) / e, \text{Int}[\text{Log}[(2*c*(d + e*x)) / ((c*d + e)*(1 + c*x))] / (1 - c^2*x^2), x], x] + \text{Simp}[\{(a + b*\text{ArcCoth}[c*x])*\text{Log}[(2*c*(d + e*x)) / ((c*d + e)*(1 + c*x))] / e, x]) /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[c^2*d^2 - e^2, 0]$

### Rule 5923

$\text{Int}[\{(a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.)\}^2 / \{(d_.) + (e_.)*(x_.)\}, x\_Symbol] \text{ :> -Simp}[\{(a + b*\text{ArcCoth}[c*x])^2*\text{Log}[2/(1 + c*x)]\} / e, x] + (\text{Simp}[\{(a + b*\text{ArcCoth}[c*x])^2*\text{Log}[(2*c*(d + e*x)) / ((c*d + e)*(1 + c*x))] / e, x] + \text{Simp}[(b*(a + b*\text{ArcCoth}[c*x])*\text{PolyLog}[2, 1 - 2/(1 + c*x)]) / e, x] - \text{Simp}[(b*(a + b*\text{ArcCoth}[c*x])*\text{PolyLog}[2, 1 - (2*c*(d + e*x)) / ((c*d + e)*(1 + c*x))] / e, x] + \text{Simp}[(b^2*\text{PolyLog}[3, 1 - 2/(1 + c*x)]) / (2*e), x] - \text{Simp}[(b^2*\text{PolyLog}[3, 1 - (2*c*(d + e*x)) / ((c*d + e)*(1 + c*x))] / (2*e), x]) /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[c^2*d^2 - e^2, 0]$

### Rule 5949

$\text{Int}[\{(a_.) + \text{ArcCoth}[(c_.)*(x_.)]*(b_.)\}^{(p_.)} / \{(d_.) + (e_.)*(x_.)^2\}, x\_Symbol] \text{ :> Simp}[(a + b*\text{ArcCoth}[c*x])^{(p + 1)} / (b*c*d*(p + 1)), x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{EqQ}[c^2*d + e, 0] \&\& \text{NeQ}[p, -1]$

### Rule 6057



```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

### Rule 6059

```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcCoth[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6110

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(
m_)), x_Symbol] := Simp[((e + f*x)^(m + 1)*(a + b*ArcCoth[c + d*x])^p)/(f*(m
+ 1)), x] - Dist[(b*d*p)/(f*(m + 1)), Int[((e + f*x)^(m + 1)*(a + b*ArcCot
h[c + d*x])^(p - 1))/(1 - (c + d*x)^2), x], x] /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[p, 0] && ILtQ[m, -1]
```

### Rule 6122

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_)^(
m_.))*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] := Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcCoth
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

### Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rule 6741

Int[u\_, x\_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 6742

Int[u\_, x\_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(c + dx))^3}{(e + fx)^2} dx &= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \int \frac{(a + b \coth^{-1}(c + dx))^2}{(e + fx)(1 - (c + dx)^2)} dx}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \int \frac{(a + b \coth^{-1}(c + dx))^2}{(e + fx)(1 - c^2 - 2cdx - d^2x^2)} dx}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3b) \text{Subst} \left( \int \frac{(a + b \coth^{-1}(x))^2}{\left(\frac{de - cf}{d} + \frac{fx}{d}\right)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3b) \text{Subst} \left( \int \frac{d(a + b \coth^{-1}(x))^2}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \text{Subst} \left( \int \frac{(a + b \coth^{-1}(x))^2}{(de - cf + fx)(1 - x^2)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{(3bd) \text{Subst} \left( \int \left( -\frac{a^2}{(-1+x)(1+x)(de - cf + fx)} - \frac{2ab \coth^{-1}(x)}{(-1+x)(1+x)(de - cf + fx)} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} - \frac{(3a^2bd) \text{Subst} \left( \int \frac{1}{(-1+x)(1+x)(de - cf + fx)} dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} - \frac{(3a^2bd) \text{Subst} \left( \int \left( \frac{1}{2(de + f - cf)(-1+x)} + \frac{1}{2(-de + (1+c)f(1+x))} \right) dx, x, c + dx \right)}{f} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} - \frac{3a^2bd \log(1 - c - dx)}{2f(de + f - cf)} + \frac{3a^2bd \log(1 + c + dx)}{2f(de - f - cf)} - \frac{3ab^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} + \frac{3b^3d \coth^{-1}(c + dx)}{2f(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{3ab^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} + \frac{3b^3d \coth^{-1}(c + dx)}{2f(de + f - cf)} \\
&= -\frac{(a + b \coth^{-1}(c + dx))^3}{f(e + fx)} + \frac{3ab^2d \coth^{-1}(c + dx) \log\left(\frac{2}{1 - c - dx}\right)}{f(de + f - cf)} + \frac{3b^3d \coth^{-1}(c + dx)}{2f(de + f - cf)}
\end{aligned}$$

**Mathematica** [C] time = 31.31, size = 3937, normalized size = 3.62

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[c + d\*x])^3/(e + f\*x)^2,x]

[Out] 
$$-(a^3/(f*(e + f*x))) - (3*a^2*b*ArcCoth[c + d*x])/(f*(e + f*x)) + (3*a^2*b*d*Log[1 - c - d*x])/(2*f*(-(d*e) - f + c*f)) - (3*a^2*b*d*Log[1 + c + d*x])/(2*f*(-(d*e) + f + c*f)) - (3*a^2*b*d*Log[e + f*x])/(d^2*e^2 - 2*c*d*e*f - f^2 + c^2*f^2) + (3*a*b^2*(1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))^2*((E^ArcTanh[f/(-(d*e) + c*f)]*ArcCoth[c + d*x]^2)/((-d*e) + c*f)*Sqrt[1 - f^2/(d*e - c*f)^2] + ArcCoth[c + d*x]^2/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) + (f*(I*Pi*ArcCoth[c + d*x] + 2*ArcCoth[c + d*x]*ArcTanh[f/(d*e - c*f)] - I*Pi*Log[1 + E^(2*ArcCoth[c + d*x])]) + 2*ArcCoth[c + d*x]*Log[1 - E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])]) - 2*ArcTanh[f/(-(d*e) + c*f)]*Log[1 - E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)])]) + I*Pi*Log[1/Sqrt[1 - (c + d*x)^(-2)]] + 2*ArcTanh[f/(-(d*e) + c*f)]*Log[I*Sinh[ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]]] - PolyLog[2, E^(-2*(ArcCoth[c + d*x] + ArcTanh[f/(d*e - c*f)]))])))/(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)))/(d*f*(e + f*x)^2) - (b^3*(1 - (c + d*x)^2)*(f/Sqrt[1 - (c + d*x)^(-2)] + (d*e - c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))^2*((d*ArcCoth[c + d*x]^3)/(f*(c + d*x)*Sqrt[1 - (c + d*x)^(-2)]*(-f/Sqrt[1 - (c + d*x)^(-2)]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]) + (c*f)/((c + d*x)*Sqrt[1 - (c + d*x)^(-2)]))) - (d*(2*d^2*e^2*ArcCoth[c + d*x]^3 - 8*d*e*f*ArcCoth[c + d*x]^3 - 4*c*d*e*f*ArcCoth[c + d*x]^3 + 4*d*e*E^(2*ArcTanh[f/(d*e - c*f)])*f*ArcCoth[c + d*x]^3 - 10*f^2*ArcCoth[c + d*x]^3 + 8*c*f^2*ArcCoth[c + d*x]^3 + 2*c^2*f^2*ArcCoth[c + d*x]^3 - 4*E^(2*ArcTanh[f/(d*e - c*f)])*f^2*ArcCoth[c + d*x]^3 - 4*c*E^(2*ArcTanh[f/(d*e - c*f)])*f^2*ArcCoth[c + d*x]^3 - (4*d^2*e^2*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f]^2)*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] - (4*d*e*f*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f]^2)*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (8*c*d*e*f*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f]^2)*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (4*c*f^2*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f]^2)*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] - (4*c^2*f^2*Sqrt[(d^2*e^2 - 2*c*d*e*f + (-1 + c^2)*f^2)/(d*e - c*f]^2)*ArcCoth[c + d*x]^3)/E^ArcTanh[f/(d*e - c*f)] + (6*I)*d*e*f*Pi*ArcCoth[c + d*x]*Log[2] + (6*I)*f^2*Pi*ArcCoth[c + d*x]*Log[2] - (6*I)*c*f^2*Pi*ArcCoth[c + d*x]*Log[2] - d*e*f*ArcCoth[c + d*x]^2*Log[64] - f^2*ArcCoth[c + d*x]^2*Log[64] + c*f^2*ArcCoth[c + d*x]^2*Log[64] - (6*I)*d*e*f*Pi*ArcCoth[c + d*x]*Log[E^(-ArcCoth[c + d*x])] + E^ArcCoth[c + d*x] - (6*I)*f^2*Pi*ArcCoth[c + d*x]*Log[E^(-ArcCoth[c + d*x])] + E^ArcCoth[c + d*x] + (6*I)*c*f^2*Pi*ArcCoth[c + d*x]*Log[E^(-Ar$$

$$\begin{aligned}
& c\text{Coth}[c + d*x]) + E^{\text{ArcCoth}[c + d*x]} + 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - \\
& E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])}] - 6*d*e*E^{(2*\text{ArcTanh}[f/(d*e - c*f)])} \\
& *f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} \\
& )}] + 6*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} \\
& )}] - 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} \\
& )}] + 6*E^{(2*\text{ArcTanh}[f/(d*e - c*f)])} *f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} \\
& )}] + 6*c*E^{(2*\text{ArcTanh}[f/(d*e - c*f)])} *f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} \\
& )}] + 12*d*e*f*\text{ArcCoth}[c + d*x] * \text{ArcTanh}[f/(d*e - c*f)] * \text{Log}[(I/2)*E^{(-\text{ArcCoth}[c + d*x] - \text{ArcTanh}[f/(d*e - c*f)])} \\
& )*(-1 + E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} )}] + 12*f^2*A \\
& \text{rcCoth}[c + d*x]*\text{ArcTanh}[f/(d*e - c*f)] * \text{Log}[(I/2)*E^{(-\text{ArcCoth}[c + d*x] - \text{Arc} \\
& \text{Tanh}[f/(d*e - c*f)])} )*(-1 + E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)])} \\
& )}] - 12*c*f^2*\text{ArcCoth}[c + d*x]*\text{ArcTanh}[f/(d*e - c*f)] * \text{Log}[(I/2)*E^{(-\text{ArcCot} \\
& \text{h}[c + d*x] - \text{ArcTanh}[f/(d*e - c*f)])} )*(-1 + E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh} \\
& [f/(d*e - c*f)])} )}] + 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[-((d*e*(-1 + E^{(2*\text{ArcC} \\
& \text{oth}[c + d*x])) + (1 + c + E^{(2*\text{ArcCoth}[c + d*x])} - c*E^{(2*\text{ArcCoth}[c + d*x])} \\
& )*f)/E^{\text{ArcCoth}[c + d*x]})] + 6*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[-((d*e*(-1 + E^{(2* \\
& \text{ArcCoth}[c + d*x])) + (1 + c + E^{(2*\text{ArcCoth}[c + d*x])} - c*E^{(2*\text{ArcCoth}[c + d \\
& *x]))*f)/E^{\text{ArcCoth}[c + d*x]})] - 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[-((d*e*(-1 + \\
& E^{(2*\text{ArcCoth}[c + d*x])) + (1 + c + E^{(2*\text{ArcCoth}[c + d*x])} - c*E^{(2*\text{ArcCoth} \\
& [c + d*x]))*f)/E^{\text{ArcCoth}[c + d*x]})] + 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - (E \\
& ^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 6*f^2*\text{ArcCo} \\
& \text{th}[c + d*x]^2*\text{Log}[1 - (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - ( \\
& 1 + c)*f]] - 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 - (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d* \\
& e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 + ( \\
& E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + 6*f^2*\text{ArcC} \\
& \text{oth}[c + d*x]^2*\text{Log}[1 + (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d*e + f - c*f])/Sqrt[d*e - \\
& (1 + c)*f]] - 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[1 + (E^{\text{ArcCoth}[c + d*x]}*\text{Sqrt}[d \\
& *e + f - c*f])/Sqrt[d*e - (1 + c)*f]] + (6*I)*d*e*f*Pi*\text{ArcCoth}[c + d*x]*\text{Log} \\
& [1/Sqrt[1 - (c + d*x)^{-2}]] + (6*I)*f^2*Pi*\text{ArcCoth}[c + d*x]*\text{Log}[1/Sqrt[1 - \\
& (c + d*x)^{-2}]] - (6*I)*c*f^2*Pi*\text{ArcCoth}[c + d*x]*\text{Log}[1/Sqrt[1 - (c + d*x) \\
& )^{-2}]] - 6*d*e*f*\text{ArcCoth}[c + d*x]^2*\text{Log}[-(f/Sqrt[1 - (c + d*x)^{-2}])] - ( \\
& d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^{-2}]) + (c*f)/((c + d*x)*Sqrt[1 - (c + \\
& d*x)^{-2}])] - 6*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[-(f/Sqrt[1 - (c + d*x)^{-2}])] - \\
& (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^{-2}]) + (c*f)/((c + d*x)*Sqrt[1 - (c \\
& + d*x)^{-2}])] + 6*c*f^2*\text{ArcCoth}[c + d*x]^2*\text{Log}[-(f/Sqrt[1 - (c + d*x)^{-2} \\
& ]) - (d*e)/((c + d*x)*Sqrt[1 - (c + d*x)^{-2}]) + (c*f)/((c + d*x)*Sqrt[1 - \\
& (c + d*x)^{-2}])] - 12*d*e*f*\text{ArcCoth}[c + d*x]*\text{ArcTanh}[f/(d*e - c*f)] * \text{Log}[I \\
& *\text{Sinh}[\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f)]]] - 12*f^2*\text{ArcCoth}[c + d*x] \\
& *\text{ArcTanh}[f/(d*e - c*f)] * \text{Log}[I*\text{Sinh}[\text{ArcCoth}[c + d*x] + \text{ArcTanh}[f/(d*e - c*f) \\
& ]]] + 12*c*f^2*\text{ArcCoth}[c + d*x]*\text{ArcTanh}[f/(d*e - c*f)] * \text{Log}[I*\text{Sinh}[\text{ArcCoth}[c \\
& + d*x] + \text{ArcTanh}[f/(d*e - c*f)]]] + 6*f*(-(d*e*(-1 + E^{(2*\text{ArcTanh}[f/(d*e - \\
& c*f)])} ) + (1 + E^{(2*\text{ArcTanh}[f/(d*e - c*f)])} ) + c*(-1 + E^{(2*\text{ArcTanh}[f/(d*e \\
& - c*f)])} ))*f)*\text{ArcCoth}[c + d*x]*\text{PolyLog}[2, E^{(2*(\text{ArcCoth}[c + d*x] + \text{ArcTanh}
\end{aligned}$$

$$\left[ \frac{f}{(d \cdot e - c \cdot f)} \right] + 12 \cdot f \cdot (d \cdot e + f - c \cdot f) \cdot \text{ArcCoth}[c + d \cdot x] \cdot \text{PolyLog}[2, -((E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f])] + 12 \cdot d \cdot e \cdot f \cdot \text{ArcCoth}[c + d \cdot x] \cdot \text{PolyLog}[2, (E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f]] + 12 \cdot f^2 \cdot \text{ArcCoth}[c + d \cdot x] \cdot \text{PolyLog}[2, (E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f]] - 12 \cdot c \cdot f^2 \cdot \text{ArcCoth}[c + d \cdot x] \cdot \text{PolyLog}[2, (E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f]] - 3 \cdot d \cdot e \cdot f \cdot \text{PolyLog}[3, E^{(2 \cdot (\text{ArcCoth}[c + d \cdot x] + \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} + 3 \cdot d \cdot e \cdot E^{(2 \cdot \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} \cdot f \cdot \text{PolyLog}[3, E^{(2 \cdot (\text{ArcCoth}[c + d \cdot x] + \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} - 3 \cdot f^2 \cdot \text{PolyLog}[3, E^{(2 \cdot (\text{ArcCoth}[c + d \cdot x] + \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} + 3 \cdot c \cdot f^2 \cdot \text{PolyLog}[3, E^{(2 \cdot (\text{ArcCoth}[c + d \cdot x] + \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} - 3 \cdot E^{(2 \cdot \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} \cdot f^2 \cdot \text{PolyLog}[3, E^{(2 \cdot (\text{ArcCoth}[c + d \cdot x] + \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} - 3 \cdot c \cdot E^{(2 \cdot \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} \cdot f^2 \cdot \text{PolyLog}[3, E^{(2 \cdot (\text{ArcCoth}[c + d \cdot x] + \text{ArcTanh}[f / (d \cdot e - c \cdot f)])} - 12 \cdot d \cdot e \cdot f \cdot \text{PolyLog}[3, -((E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f])] - 12 \cdot f^2 \cdot \text{PolyLog}[3, -((E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f])] + 12 \cdot c \cdot f^2 \cdot \text{PolyLog}[3, -((E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f])] - 12 \cdot d \cdot e \cdot f \cdot \text{PolyLog}[3, (E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f]] - 12 \cdot f^2 \cdot \text{PolyLog}[3, (E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f]] + 12 \cdot c \cdot f^2 \cdot \text{PolyLog}[3, (E^{\text{ArcCoth}[c + d \cdot x]} \cdot \text{Sqrt}[d \cdot e + f - c \cdot f]) / \text{Sqrt}[d \cdot e - (1 + c) \cdot f]]) / (2 \cdot f \cdot (d \cdot e + f - c \cdot f)^2 \cdot (d \cdot e - (1 + c) \cdot f))] / (d^2 \cdot (e + f \cdot x)^2)$$

**fricas** [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^3 \operatorname{arccoth}(dx + c)^3 + 3ab^2 \operatorname{arccoth}(dx + c)^2 + 3a^2b \operatorname{arccoth}(dx + c) + a^3}{f^2x^2 + 2efx + e^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e)^2,x, algorithm="fricas")

[Out] integral((b^3\*arccoth(d\*x + c)^3 + 3\*a\*b^2\*arccoth(d\*x + c)^2 + 3\*a^2\*b\*arccoth(d\*x + c) + a^3)/(f^2\*x^2 + 2\*e\*f\*x + e^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(dx + c) + a)^3}{(fx + e)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e)^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^3/(f\*x + e)^2, x)

maple [C] time = 1.32, size = 4619, normalized size = 4.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\text{arccoth}(d*x+c))^3/(f*x+e)^2, x)$

[Out] 
$$\begin{aligned} & -3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I*(d*x+c+1) \\ & / (d*x+c-1))*\text{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d* \\ & x+c+1)/(d*x+c-1)-1))-3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c) \\ & ^2*\text{Pi}*e*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))*\text{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/( \\ & d*x+c-1)-1))^2-3/2*I*d^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}* \\ & e*\text{csgn}(I/((d*x+c-1)/(d*x+c+1))^{(1/2)})*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))^2-3/4*I*d \\ & ^2*b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*e*\text{csgn}(I/((d*x+c+1)/(d \\ & *x+c-1)-1))*\text{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^2+3/4*I*d^2 \\ & *b^3/f/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*e*\text{csgn}(I/((d*x+c-1)/(d*x \\ & +c+1))^{(1/2)})^2*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))-d*a^3/(d*f*x+d*e)/f+3/4*I*d*b^3 \\ & / (c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I/((d*x+c-1)/(d*x+c+1))^{( \\ & 1/2)})^2*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))+3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arc} \\ & \text{coth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*((d*x+c+1)/(d*x+c- \\ & 1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1) \\ & )/(d*x+c-1)-1))^2-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c \\ & \text{sgn}(I*(d*x+c+1)/(d*x+c-1))*\text{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)- \\ & 1))^2-3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I/((d*x+ \\ & c-1)/(d*x+c+1))^{(1/2)})*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))^2-3/4*I*d*b^3/(c*f-d*e-f) \\ & )/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*(d \\ & *x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^2+3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d \\ & *e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/ \\ & (d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f))*\text{csgn}(I*((d*x+c+1)/(d*x+c-1)-1) \\ & *c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d* \\ & x+c-1)-1))^2-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn} \\ & (I*(d*x+c+1)/(d*x+c-1))^3-3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c) \\ & )^2*\text{Pi}*c*\text{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^3+3/4*I*d*b^3/ \\ & (c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))*\text{csg} \\ & n(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1) \\ & )-1))+3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I*(d*x \\ & +c+1)/(d*x+c-1))*\text{csgn}(I*(d*x+c+1)/(d*x+c-1)/((d*x+c+1)/(d*x+c-1)-1))^2-3/4* \\ & I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I/((d*x+c-1)/(d* \\ & x+c+1))^{(1/2)})^2*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))+3/4*I*d^2*b^3/f/(c*f-d*e-f)/(c \\ & *f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*e*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))^3+3/2*I*d*b^3/( \\ & c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I/((d*x+c-1)/(d*x+c+1))^{( \\ & 1/2)})*\text{csgn}(I*(d*x+c+1)/(d*x+c-1))^2+3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arc} \\ & \text{coth}(d*x+c)^2*\text{Pi}*c*\text{csgn}(I/((d*x+c+1)/(d*x+c-1)-1))*\text{csgn}(I*(d*x+c+1)/(d*x+c- \\ & 1)/((d*x+c+1)/(d*x+c-1)-1))^2-3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\text{arccoth}(d \end{aligned}$$

$$\begin{aligned}
& *x+c)^{2\pi} * \operatorname{csgn}(I / ((d*x+c+1) / (d*x+c-1) - 1)) * \operatorname{csgn}(I * (((d*x+c+1) / (d*x+c-1) - 1) * \\
& c*f + (1 - (d*x+c+1) / (d*x+c-1)) * e*d + (- (d*x+c+1) / (d*x+c-1) - 1) * f)) * \operatorname{csgn}(I * (((d*x+ \\
& c+1) / (d*x+c-1) - 1) * c*f + (1 - (d*x+c+1) / (d*x+c-1)) * e*d + (- (d*x+c+1) / (d*x+c-1) - 1) * \\
& f) / ((d*x+c+1) / (d*x+c-1) - 1)) + 3/4 * I * d^2 * b^3 / f / (c*f - d*e - f) / (c*f - d*e + f) * \operatorname{arccoth} \\
& (d*x+c)^{2\pi} * e * \operatorname{csgn}(I * (d*x+c+1) / (d*x+c-1) / ((d*x+c+1) / (d*x+c-1) - 1))^{-3} * d * a * \\
& b^2 / (c*f - d*e - f) / (c*f - d*e + f) * \operatorname{dilog}(((d*x+c) * f - f) / (c*f - d*e - f)) + 3/4 * d * a * b^2 / f / \\
& (c*f - d*e - f) * \ln(d*x+c-1)^{-2} * 3/2 * d * a * b^2 / f / (c*f - d*e - f) * \operatorname{dilog}(1/2 + 1/2 * d*x + 1/2 * c \\
& ) + 3/4 * d * a * b^2 / f / (c*f - d*e + f) * \ln(d*x+c+1)^{-2} * 3/2 * d * a * b^2 / f / (c*f - d*e + f) * \operatorname{dilog}(1 \\
& / 2 + 1/2 * d*x + 1/2 * c) - 3/2 * d * b^3 * f / (c*f - d*e - f)^2 / (c*f - d*e + f) * \operatorname{polylog}(3, (c*f - d*e - \\
& f) * (d*x+c+1) / (d*x+c-1) / (c*f - d*e + f)) + 3/4 * I * d^2 * b^3 / f / (c*f - d*e - f) / (c*f - d*e + f) \\
& * \operatorname{arccoth}(d*x+c)^{2\pi} * e * \operatorname{csgn}(I * (d*x+c+1) / (d*x+c-1)) * \operatorname{csgn}(I / ((d*x+c+1) / (d*x+c \\
& -1) - 1)) * \operatorname{csgn}(I * (d*x+c+1) / (d*x+c-1) / ((d*x+c+1) / (d*x+c-1) - 1))^{-3} * 2 * d * a * b^2 / f / ( \\
& c*f - d*e - f) * \ln(d*x+c-1) * \ln(1/2 + 1/2 * d*x + 1/2 * c) - 3/2 * d * a * b^2 / f / (c*f - d*e + f) * \ln(- \\
& 1/2 * d*x - 1/2 * c + 1/2) * \ln(d*x+c+1) + 3/2 * d * a * b^2 / f / (c*f - d*e + f) * \ln(-1/2 * d*x - 1/2 * c + \\
& 1/2) * \ln(1/2 + 1/2 * d*x + 1/2 * c) + 3 * d^2 * b^3 / (c*f - d*e - f)^2 / (c*f - d*e + f) * e * \operatorname{arccoth}(d* \\
& x+c)^2 * \ln(1 - (c*f - d*e - f) * (d*x+c+1) / (d*x+c-1) / (c*f - d*e + f)) + 3 * d^2 * b^3 / (c*f - d*e \\
& - f)^2 / (c*f - d*e + f) * e * \operatorname{arccoth}(d*x+c) * \operatorname{polylog}(2, (c*f - d*e - f) * (d*x+c+1) / (d*x+c-1 \\
& )) / (c*f - d*e + f)) + 3 * d * b^3 * f / (c*f - d*e - f)^2 / (c*f - d*e + f) * \operatorname{arccoth}(d*x+c)^2 * \ln(1 - (c \\
& *f - d*e - f) * (d*x+c+1) / (d*x+c-1) / (c*f - d*e + f)) + 3 * d * b^3 * f / (c*f - d*e - f)^2 / (c*f - d*e \\
& + f) * \operatorname{arccoth}(d*x+c) * \operatorname{polylog}(2, (c*f - d*e - f) * (d*x+c+1) / (d*x+c-1) / (c*f - d*e + f)) + 3 \\
& * I * d * b^3 / (c*f - d*e - f) / (c*f - d*e + f) * \operatorname{arccoth}(d*x+c)^{2\pi} * I + 3/2 * d * b^3 * f / (c*f - d*e - f \\
& )^2 / (c*f - d*e + f) * c * \operatorname{polylog}(3, (c*f - d*e - f) * (d*x+c+1) / (d*x+c-1) / (c*f - d*e + f)) - 6 * \\
& d * a * b^2 * \operatorname{arccoth}(d*x+c) / (c*f - d*e - f) / (c*f - d*e + f) * \ln((d*x+c) * f - c*f + d*e) - 3/2 * d^2 \\
& * b^3 / (c*f - d*e - f)^2 / (c*f - d*e + f) * e * \operatorname{polylog}(3, (c*f - d*e - f) * (d*x+c+1) / (d*x+c-1) \\
& / (c*f - d*e + f)) - 3 * d * b^3 / (c*f - d*e - f) / (c*f - d*e + f) * \operatorname{arccoth}(d*x+c)^2 * \ln(2) - 3 * d * a^2 \\
& * b / (c*f - d*e - f) / (c*f - d*e + f) * \ln((d*x+c) * f - c*f + d*e) + 3 * d * a^2 * b / f / (2 * c * f - 2 * d * e - \\
& 2 * f) * \ln(d*x+c-1) - 3 * d * a^2 * b / f / (2 * c * f - 2 * d * e + 2 * f) * \ln(d*x+c+1) + 3 * d * b^3 / (c*f - d*e \\
& - f) / (c*f - d*e + f) * \operatorname{arccoth}(d*x+c)^2 * \ln(((d*x+c+1) / (d*x+c-1) - 1) * c*f + (1 - (d*x+c+1 \\
& )) / (d*x+c-1)) * e*d + (- (d*x+c+1) / (d*x+c-1) - 1) * f) - 3/2 * d * b^3 / f * \operatorname{arccoth}(d*x+c)^2 / ( \\
& c*f - d*e + f) * \ln(((d*x+c-1) / (d*x+c+1)) + 3 * d * b^3 / f * \operatorname{arccoth}(d*x+c)^2 / (2 * c * f - 2 * d * e - \\
& 2 * f) * \ln(d*x+c-1) - 3 * d * b^3 / f * \operatorname{arccoth}(d*x+c)^2 / (2 * c * f - 2 * d * e + 2 * f) * \ln(d*x+c+1) - 3 \\
& * d * b^3 * \operatorname{arccoth}(d*x+c)^2 / (c*f - d*e - f) / (c*f - d*e + f) * \ln((d*x+c) * f - c*f + d*e) - 3 * d * a \\
& * b^2 / (d*f * x + d*e) / f * \operatorname{arccoth}(d*x+c)^2 - 3 * d * a^2 * b / (d*f * x + d*e) / f * \operatorname{arccoth}(d*x+c) + \\
& 3 * d * a * b^2 / (c*f - d*e - f) / (c*f - d*e + f) * \operatorname{dilog}(((d*x+c) * f + f) / (c*f - d*e + f)) + 6 * d * a * b^2 \\
& / f * \operatorname{arccoth}(d*x+c) / (2 * c * f - 2 * d * e - 2 * f) * \ln(d*x+c-1) - 6 * d * a * b^2 / f * \operatorname{arccoth}(d*x+c) \\
& / (2 * c * f - 2 * d * e + 2 * f) * \ln(d*x+c+1) + 3 * d * a * b^2 / (c*f - d*e - f) / (c*f - d*e + f) * \ln((d*x+c) \\
& * f - c*f + d*e) * \ln(((d*x+c) * f + f) / (c*f - d*e + f)) - 3 * d * a * b^2 / (c*f - d*e - f) / (c*f - d*e + f) \\
& * \ln((d*x+c) * f - c*f + d*e) * \ln(((d*x+c) * f - f) / (c*f - d*e - f)) - d * b^3 / (d*f * x + d*e) / f * \operatorname{ar} \\
& \operatorname{ccoth}(d*x+c)^3 - d * b^3 / f * \operatorname{arccoth}(d*x+c)^3 / (c*f - d*e + f) - 3 * d * b^3 * f / (c*f - d*e - f)^2 \\
& / (c*f - d*e + f) * c * \operatorname{arccoth}(d*x+c)^2 * \ln(1 - (c*f - d*e - f) * (d*x+c+1) / (d*x+c-1) / (c*f - d \\
& *e + f)) - 3 * d * b^3 * f / (c*f - d*e - f)^2 / (c*f - d*e + f) * c * \operatorname{arccoth}(d*x+c) * \operatorname{polylog}(2, (c*f - \\
& d*e - f) * (d*x+c+1) / (d*x+c-1) / (c*f - d*e + f)) + 3/4 * I * d * b^3 / (c*f - d*e - f) / (c*f - d*e + f) \\
& * \operatorname{arccoth}(d*x+c)^{2\pi} * I * \operatorname{csgn}(I * (d*x+c+1) / (d*x+c-1) / ((d*x+c+1) / (d*x+c-1) - 1))^{-3} - \\
& 3 * I * d * b^3 / (c*f - d*e - f) / (c*f - d*e + f) * \operatorname{arccoth}(d*x+c)^{2\pi} * I * \operatorname{csgn}(I * (((d*x+c+1) / (d \\
& *x+c-1) - 1) * c*f + (1 - (d*x+c+1) / (d*x+c-1)) * e*d + (- (d*x+c+1) / (d*x+c-1) - 1) * f) / ((d*
\end{aligned}$$



$x+c+1)/((d*x+c-1)-1))^2+3/2*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2$   
 $*\operatorname{Pi}*c\operatorname{sgn}(I*((d*x+c+1)/(d*x+c-1)-1)*c*f+(1-(d*x+c+1)/(d*x+c-1))*e*d+(-(d*x+c+1)/(d*x+c-1)-1)*f)/((d*x+c+1)/(d*x+c-1)-1))^3+3/4*I*d*b^3/(c*f-d*e-f)/(c*f-d*e+f)*\operatorname{arccoth}(d*x+c)^2*\operatorname{Pi}*c\operatorname{sgn}(I*(d*x+c+1)/(d*x+c-1))^3$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3}{2} \left( d \left( \frac{\log(dx+c+1)}{def-(c+1)f^2} - \frac{\log(dx+c-1)}{def-(c-1)f^2} - \frac{2 \log(fx+e)}{d^2e^2-2cdef+(c^2-1)f^2} \right) - \frac{2 \operatorname{arccoth}(dx+c)}{f^2x+ef} \right) a^2 b - \frac{a^3}{f^2x+ef} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(d\*x+c))^3/(f\*x+e)^2,x, algorithm="maxima")

[Out]  $3/2*(d*(\log(d*x+c+1)/(d*e*f-(c+1)*f^2)-\log(d*x+c-1)/(d*e*f-(c-1)*f^2)-2*\log(f*x+e)/(d^2*e^2-2*c*d*e*f+(c^2-1)*f^2))-2*\operatorname{arccoth}(d*x+c)/(f^2*x+e*f))*a^2*b-a^3/(f^2*x+e*f)+1/8*((d^2*e*f-c*d*f^2+d*f^2)*b^3*x+(c*d*e*f-c^2*f^2+d*e*f+f^2)*b^3)*\log(d*x+c+1)^3-3*(2*(d^2*e^2-2*c*d*e*f+c^2*f^2-f^2)*a*b^2+((d^2*e*f-c*d*f^2-d*f^2)*b^3*x+(c*d*e*f-c^2*f^2-d*e*f+f^2)*b^3)*\log(d*x+c-1))*\log(d*x+c+1)^2/(d^2*e^3*f-2*c*d*e^2*f^2+c^2*e*f^3-e*f^3+(d^2*e^2*f^2-2*c*d*e*f^3+c^2*f^4-f^4)*x)+\operatorname{integrate}(-1/8*((d^2*e*f-c*d*f^2+d*f^2)*b^3*x+(c*d*e*f-c^2*f^2+d*e*f+f^2)*b^3)*\log(d*x+c-1)^3-6*((d^2*e*f-c*d*f^2+d*f^2)*a*b^2*x+(c*d*e*f-c^2*f^2+d*e*f+f^2)*a*b^2)*\log(d*x+c-1)^2-3*(4*(d^2*e*f-c*d*f^2+d*f^2)*a*b^2*x+4*(d^2*e^2-c*d*e*f+d*e*f)*a*b^2+((d^2*e*f-c*d*f^2+d*f^2)*b^3*x+(c*d*e*f-c^2*f^2+d*e*f+f^2)*b^3)*\log(d*x+c-1)^2+2*(b^3*d^2*f^2*x^2-2*(c*d*e*f-c^2*f^2+d*e*f+f^2)*a*b^2+(c*d*e*f-d*e*f)*b^3-(2*(d^2*e*f-c*d*f^2+d*f^2)*a*b^2-(d^2*e*f+c*d*f^2-d*f^2)*b^3)*x)*\log(d*x+c-1))*\log(d*x+c+1))/(c*d*e^3*f-c^2*e^2*f^2+d*e^3*f+e^2*f^2+(d^2*e*f^3-c*d*f^4+d*f^4)*x^3+(2*d^2*e^2*f^2-c*d*e*f^3-c^2*f^4+3*d*e*f^3+f^4)*x^2+(d^2*e^3*f+c*d*e^2*f^2-2*c^2*e*f^3+3*d*e^2*f^2+2*e*f^3)*x), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(c + dx))^3}{(e + fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c + d\*x))^3/(e + f\*x)^2,x)

[Out] int((a + b\*acoth(c + d\*x))^3/(e + f\*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(c + dx))^3}{(e + fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(d\*x+c))\*\*3/(f\*x+e)\*\*2,x)

[Out] Integral((a + b\*acoth(c + d\*x))\*\*3/(e + f\*x)\*\*2, x)

### 3.119 $\int (e + fx)^m \left( a + b \coth^{-1}(c + dx) \right) dx$

**Optimal.** Leaf size=162

$$\frac{(e + fx)^{m+1} \left( a + b \coth^{-1}(c + dx) \right)}{f(m+1)} + \frac{bd(e + fx)^{m+2} {}_2F_1 \left( 1, m+2; m+3; \frac{d(e+fx)}{de-cf-f} \right)}{2f(m+1)(m+2)(de - (c+1)f)} - \frac{bd(e + fx)^{m+2} {}_2F_1 \left( 1, m+2; \right)}{2f(m+1)(m+2)(-cf)}$$

[Out] (f\*x+e)^(1+m)\*(a+b\*arccoth(d\*x+c))/f/(1+m)+1/2\*b\*d\*(f\*x+e)^(2+m)\*hypergeom([1, 2+m], [3+m], d\*(f\*x+e)/(-c\*f+d\*e-f))/f/(d\*e-(1+c)\*f)/(1+m)/(2+m)-1/2\*b\*d\*(f\*x+e)^(2+m)\*hypergeom([1, 2+m], [3+m], d\*(f\*x+e)/(-c\*f+d\*e+f))/f/(-c\*f+d\*e+f)/(1+m)/(2+m)

**Rubi [A]** time = 0.25, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {6112, 5927, 712, 68}

$$\frac{(e + fx)^{m+1} \left( a + b \coth^{-1}(c + dx) \right)}{f(m+1)} + \frac{bd(e + fx)^{m+2} {}_2F_1 \left( 1, m+2; m+3; \frac{d(e+fx)}{de-cf-f} \right)}{2f(m+1)(m+2)(de - (c+1)f)} - \frac{bd(e + fx)^{m+2} {}_2F_1 \left( 1, m+2; \right)}{2f(m+1)(m+2)(-cf)}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x]), x]

[Out] ((e + f\*x)^(1 + m)\*(a + b\*ArcCoth[c + d\*x]))/(f\*(1 + m)) + (b\*d\*(e + f\*x)^(2 + m)\*Hypergeometric2F1[1, 2 + m, 3 + m, (d\*(e + f\*x))/(d\*e - f - c\*f)])/((2\*f\*(d\*e - (1 + c)\*f)\*(1 + m)\*(2 + m)) - (b\*d\*(e + f\*x)^(2 + m)\*Hypergeometric2F1[1, 2 + m, 3 + m, (d\*(e + f\*x))/(d\*e + f - c\*f)])/((2\*f\*(d\*e + f - c\*f)\*(1 + m)\*(2 + m)))

#### Rule 68

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(((b\*c - a\*d)^n\*(a + b\*x)^(m + 1)\*Hypergeometric2F1[-n, m + 1, m + 2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b^(n + 1)\*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b\*c - a\*d, 0] && !IntegerQ[m] && IntegerQ[n]

#### Rule 712

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m, 1/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IntegerQ[m]

#### Rule 5927

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(q_.), x_Symbol]
:> Simp[((d + e*x)^(q + 1)*(a + b*ArcCoth[c*x]))/(e*(q + 1)), x] - Dist[(b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_.) + (d_.)*(x_.)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx)) dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \coth^{-1}(x)) dx, x, c + dx\right)}{d}$$

$$= \frac{(e + fx)^{1+m} (a + b \coth^{-1}(c + dx))}{f(1 + m)} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{1-x^2} dx, x, c + dx\right)}{f(1 + m)}$$

$$= \frac{(e + fx)^{1+m} (a + b \coth^{-1}(c + dx))}{f(1 + m)} - \frac{b \text{Subst}\left(\int \left(\frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{2(1-x)} + \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{2(1+x)}\right) dx, x, c + dx\right)}{f(1 + m)}$$

$$= \frac{(e + fx)^{1+m} (a + b \coth^{-1}(c + dx))}{f(1 + m)} - \frac{b \text{Subst}\left(\int \frac{\left(\frac{de-cf}{d} + \frac{fx}{d}\right)^{1+m}}{1-x} dx, x, c + dx\right)}{2f(1 + m)}$$

$$= \frac{(e + fx)^{1+m} (a + b \coth^{-1}(c + dx))}{f(1 + m)} + \frac{bd(e + fx)^{2+m} {}_2F_1\left(1, 2 + m; 3 + m; \frac{de - (1 + c)f}{2f}\right)}{2f(de - (1 + c)f)(1 + m)(2 - m)}$$

**Mathematica** [F] time = 2.53, size = 0, normalized size = 0.00

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx)) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e + f*x)^m*(a + b*ArcCoth[c + d*x]), x]
```

[Out] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x]), x]

**fricas** [F] time = 0.39, size = 0, normalized size = 0.00

$$\text{integral}\left((b \operatorname{arccoth}(dx + c) + a)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x, algorithm="fricas")

[Out] integral((b\*arccoth(d\*x + c) + a)\*(f\*x + e)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(dx + c) + a)(fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)\*(f\*x + e)^m, x)

**maple** [F] time = 1.93, size = 0, normalized size = 0.00

$$\int (fx + e)^m (a + b \operatorname{arccoth}(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x)

[Out] int((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \left( \frac{(fx + e)(fx + e)^m \log(dx + c + 1)}{f(m + 1)} - \int \frac{(dfx + de + (df(m + 1)x + cf(m + 1) + f(m + 1)) \log(dx + c - 1))}{df(m + 1)x + cf(m + 1) + f(m + 1)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*b\*((f\*x + e)\*(f\*x + e)^m\*log(d\*x + c + 1)/(f\*(m + 1)) - integrate((d\*f\*x + d\*e + (d\*f\*(m + 1)\*x + c\*f\*(m + 1) + f\*(m + 1))\*log(d\*x + c - 1))\*(f\*x + e)^m/(d\*f\*(m + 1)\*x + c\*f\*(m + 1) + f\*(m + 1)), x)) + (f\*x + e)^(m + 1)\*a/(f\*(m + 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e + f x)^m (a + b \operatorname{acoth}(c + d x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e + f*x)^m*(a + b*acoth(c + d*x)), x)`

[Out] `int((e + f*x)^m*(a + b*acoth(c + d*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acoth}(c + d x)) (e + f x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**m*(a+b*acoth(d*x+c)), x)`

[Out] `Integral((a + b*acoth(c + d*x))*(e + f*x)**m, x)`

$$3.120 \quad \int (e + fx)^m (a + b \coth^{-1}(c + dx))^2 dx$$

Optimal. Leaf size=23

$$\text{Int}\left((e + fx)^m (a + b \coth^{-1}(c + dx))^2, x\right)$$

[Out] Unintegrable((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^2,x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^2 dx$$

Verification is Not applicable to the result.

[In] Int[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^2,x]

[Out] Defer[Subst][Defer[Int][((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^2, x], x, c + d\*x]/d

Rubi steps

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^2 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \coth^{-1}(x))^2 dx, x, c + dx\right)}{d}$$

Mathematica [A] time = 2.65, size = 0, normalized size = 0.00

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^2,x]

[Out] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^2, x]

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{arccoth}(dx + c)^2 + 2ab \operatorname{arccoth}(dx + c) + a^2\right)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="fricas")

[Out] integral((b^2\*arccoth(d\*x + c)^2 + 2\*a\*b\*arccoth(d\*x + c) + a^2)\*(f\*x + e)^m, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(dx + c) + a)^2 (fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^2\*(f\*x + e)^m, x)

**maple** [A] time = 1.79, size = 0, normalized size = 0.00

$$\int (fx + e)^m (a + b \operatorname{arccoth}(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^2,x)

[Out] int((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(b^2 fx + b^2 e)(fx + e)^m \log(dx + c + 1)^2}{4 f(m + 1)} + \frac{(fx + e)^{m+1} a^2}{f(m + 1)} - \int \frac{((b^2 df(m + 1)x + (cf(m + 1) + f(m + 1))b^2) \log(dx + c + 1) + (b^2 df(m + 1)x + (cf(m + 1) + f(m + 1))b^2) \log(dx + c - 1) - 2*(b^2*d*e - 2*(c*f*(m + 1) + f*(m + 1))*a*b - (2*a*b*d*f*(m + 1) - b^2*d*f)*x + (b^2*d*f*(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*b^2)*\log(dx + c - 1))*\log(dx + c + 1) - 4*(a*b*d*f*(m + 1)*x + (c*f*(m + 1) + f*(m + 1))*a*b)*\log(dx + c - 1))*(f*x + e)^m/(d*f*(m + 1)*x + c*f*(m + 1) + f*(m + 1))}{f(m + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^2,x, algorithm="maxima")

[Out] 1/4\*(b^2\*f\*x + b^2\*e)\*(f\*x + e)^m\*log(d\*x + c + 1)^2/(f\*(m + 1)) + (f\*x + e)^(m + 1)\*a^2/(f\*(m + 1)) - integrate(-1/4\*((b^2\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*b^2)\*log(d\*x + c - 1)^2 - 2\*(b^2\*d\*e - 2\*(c\*f\*(m + 1) + f\*(m + 1))\*a\*b - (2\*a\*b\*d\*f\*(m + 1) - b^2\*d\*f)\*x + (b^2\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*b^2)\*log(d\*x + c - 1))\*log(d\*x + c + 1) - 4\*(a\*b\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*a\*b)\*log(d\*x + c - 1))\*(f\*x + e)^m/(d\*f\*(m + 1)\*x + c\*f\*(m + 1) + f\*(m + 1)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e + fx)^m (a + b \operatorname{acoth}(c + dx))^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^m*(a + b*acoth(c + d*x))^2,x)
```

```
[Out] int((e + f*x)^m*(a + b*acoth(c + d*x))^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**m*(a+b*acoth(d*x+c))**2,x)
```

```
[Out] Timed out
```

$$3.121 \quad \int (e + fx)^m (a + b \coth^{-1}(c + dx))^3 dx$$

**Optimal.** Leaf size=23

$$\text{Int}\left((e + fx)^m (a + b \coth^{-1}(c + dx))^3, x\right)$$

[Out] Unintegrable((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^3,x)

**Rubi [A]** time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^3 dx$$

Verification is Not applicable to the result.

[In] Int[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^3,x]

[Out] Defer[Subst][Defer[Int][((d\*e - c\*f)/d + (f\*x)/d)^m\*(a + b\*ArcCoth[x])^3, x], x, c + d\*x]/d

Rubi steps

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^3 dx = \frac{\text{Subst}\left(\int \left(\frac{de-cf}{d} + \frac{fx}{d}\right)^m (a + b \coth^{-1}(x))^3 dx, x, c + dx\right)}{d}$$

**Mathematica [A]** time = 0.35, size = 0, normalized size = 0.00

$$\int (e + fx)^m (a + b \coth^{-1}(c + dx))^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^3,x]

[Out] Integrate[(e + f\*x)^m\*(a + b\*ArcCoth[c + d\*x])^3, x]

**fricas [A]** time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3 \operatorname{arcoth}(dx + c)^3 + 3ab^2 \operatorname{arcoth}(dx + c)^2 + 3a^2b \operatorname{arcoth}(dx + c) + a^3\right)(fx + e)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="fricas")

[Out] integral((b^3\*arccoth(d\*x + c)^3 + 3\*a\*b^2\*arccoth(d\*x + c)^2 + 3\*a^2\*b\*arccoth(d\*x + c) + a^3)\*(f\*x + e)^m, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(dx + c) + a)^3 (fx + e)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((b\*arccoth(d\*x + c) + a)^3\*(f\*x + e)^m, x)

**maple** [A] time = 2.06, size = 0, normalized size = 0.00

$$\int (fx + e)^m (a + b \operatorname{arccoth}(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^3,x)

[Out] int((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(b^3fx + b^3e)(fx + e)^m \log(dx + c + 1)^3}{8f(m + 1)} + \frac{(fx + e)^{m+1} a^3}{f(m + 1)} - \int \frac{((b^3df(m + 1)x + (cf(m + 1) + f(m + 1))b^3) \log(dx + c + 1) + (b^3d^2f(m + 1)x + (c^2f(m + 1) + f(m + 1))b^3) \log(dx + c - 1) + (b^3d^2e - 2(c^2f(m + 1) + f(m + 1))b^3) \log(dx + c + 1) + (b^3d^2e - 2(c^2f(m + 1) + f(m + 1))b^3) \log(dx + c - 1) + 6(a*b^2*d*f(m + 1)*x + (c*f(m + 1) + f(m + 1))*a*b^2)*\log(dx + c - 1)^2 - 3*(4*a^2*b*d*f(m + 1)*x + 4*(c*f(m + 1) + f(m + 1))*a^2*b + (b^3*d*f(m + 1)*x + (c*f(m + 1) + f(m + 1))*b^3)*\log(dx + c - 1)^2 - 4*(a*b^2*d*f(m + 1)*x + (c*f(m + 1) + f(m + 1))*a*b^2)*\log(dx + c - 1))*\log(dx + c + 1)}{f(m + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^m\*(a+b\*arccoth(d\*x+c))^3,x, algorithm="maxima")

[Out] 1/8\*(b^3\*f\*x + b^3\*e)\*(f\*x + e)^m\*log(dx + c + 1)^3/(f\*(m + 1)) + (f\*x + e)^(m + 1)\*a^3/(f\*(m + 1)) - integrate(1/8\*((b^3\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*b^3)\*log(dx + c - 1)^3 + 3\*(b^3\*d\*e - 2\*(c\*f\*(m + 1) + f\*(m + 1))\*a\*b^2 - (2\*a\*b^2\*d\*f\*(m + 1) - b^3\*d\*f)\*x + (b^3\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*b^3)\*log(dx + c - 1))\*log(dx + c + 1)^2 - 6\*(a\*b^2\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*a\*b^2)\*log(dx + c - 1)^2 - 3\*(4\*a^2\*b\*d\*f\*(m + 1)\*x + 4\*(c\*f\*(m + 1) + f\*(m + 1))\*a^2\*b + (b^3\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*b^3)\*log(dx + c - 1)^2 - 4\*(a\*b^2\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*a\*b^2)\*log(dx + c - 1))\*log(dx + c + 1)

+ 12\*(a^2\*b\*d\*f\*(m + 1)\*x + (c\*f\*(m + 1) + f\*(m + 1))\*a^2\*b)\*log(d\*x + c - 1)\*(f\*x + e)^m/(d\*f\*(m + 1)\*x + c\*f\*(m + 1) + f\*(m + 1)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e + f x)^m (a + b \operatorname{acoth}(c + d x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f\*x)^m\*(a + b\*acoth(c + d\*x))^3,x)

[Out] int((e + f\*x)^m\*(a + b\*acoth(c + d\*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*m\*(a+b\*acoth(d\*x+c))\*\*3,x)

[Out] Timed out

$$3.122 \quad \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Optimal. Leaf size=43

$$\text{Int}\left[\frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}, x\right]$$

[Out] Unintegrable((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1), x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

[Out] Defer[Int][(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

Rubi steps

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx = \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^n/(1 - c^2\*x^2), x]

**fricas** [A] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral} \left( -\frac{\left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)^n}{c^2 x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1),x, algorithm="fricas")

[Out] integral(-(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^n/(c^2\*x^2 - 1), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)^n}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1),x, algorithm="giac")

[Out] integrate(-(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^n/(c^2\*x^2 - 1), x)

**maple** [A] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{\left( a + b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) \right)^n}{-c^2 x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1),x)

[Out] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)^n}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^n/(-c^2\*x^2+1),x, algorithm="maxima")

[Out] -integrate((b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^n/(c^2\*x^2 - 1), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a + b \operatorname{acoth}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))^n/(c^2\*x^2 - 1),x)

[Out] -int((a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))^n/(c^2\*x^2 - 1), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a + b \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*n/(-c\*\*2\*x\*\*2+1),x)

[Out] -Integral((a + b\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)))\*\*n/(c\*\*2\*x\*\*2 - 1), x)

$$3.123 \quad \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2} dx$$

**Optimal.** Leaf size=460

$$\frac{3b^2 \operatorname{Li}_3\left(1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} + \frac{3b^2 \operatorname{Li}_3\left(1 - \frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1\right)}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} - 3b \operatorname{Li}_2\left(1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)$$

[Out]  $-2*\operatorname{arccoth}(1-2/((1-(-c*x+1)^{(1/2))/(c*x+1)^{(1/2)})))*(a+b*\operatorname{arccoth}((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}))^3/c-3/2*b*(a+b*\operatorname{arccoth}((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}))^2*\operatorname{polylog}(2,1-2/((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}+1))/c+3/2*b*(a+b*\operatorname{arccoth}((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}))^2*\operatorname{polylog}(2,1-2*(-c*x+1)^{(1/2)/((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}+1)/(c*x+1)^{(1/2)})/c-3/2*b^2*(a+b*\operatorname{arccoth}((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}))*\operatorname{polylog}(3,1-2/((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}+1))/c+3/2*b^2*(a+b*\operatorname{arccoth}((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}))*\operatorname{polylog}(3,1-2*(-c*x+1)^{(1/2)/((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}+1)/(c*x+1)^{(1/2)})/c-3/4*b^3*\operatorname{polylog}(4,1-2/((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}+1))/c+3/4*b^3*\operatorname{polylog}(4,1-2*(-c*x+1)^{(1/2)/((-c*x+1)^{(1/2))/(c*x+1)^{(1/2)}+1)/(c*x+1)^{(1/2)})/c$

**Rubi [A]** time = 0.60, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$ , Rules used = {6681, 5915, 6053, 5949, 6057, 6061, 6610}

$$\frac{3b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} + \frac{3b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1\right)}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c} - 3b \operatorname{Li}_2\left(1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])^3/(1 - c^2*x^2), x]$

[Out]  $(-2*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])^3*\operatorname{ArcCoth}[1 - 2/(1 - \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/c - (3*b*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])^2*\operatorname{PolyLog}[2, 1 - 2/(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/(2*c) + (3*b*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])^2*\operatorname{PolyLog}[2, 1 - (2*\operatorname{Sqrt}[1 - c*x])/(1 + \operatorname{Sqrt}[1 + c*x]*(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]))])/(2*c) - (3*b^2*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])*\operatorname{PolyLog}[3, 1 - 2/(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/(2*c) + (3*b^2*(a + b*\operatorname{ArcCoth}[\operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]])*\operatorname{PolyLog}[3, 1 - (2*\operatorname{Sqrt}[1 - c*x])/(1 + \operatorname{Sqrt}[1 + c*x]*(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x]))])/(2*c) - (3*b^3*\operatorname{PolyLog}[4, 1 - 2/(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/(2*c) - (3*b^3*\operatorname{PolyLog}[4, 1 - 2/(1 + \operatorname{Sqrt}[1 - c*x]/\operatorname{Sqrt}[1 + c*x])])/(2*c)$



x]])]/(4\*c) + (3\*b^3\*PolyLog[4, 1 - (2\*Sqrt[1 - c\*x])/(Sqrt[1 + c\*x]\*(1 + Sqrt[1 - c\*x]/Sqrt[1 + c\*x])))]/(4\*c)

### Rule 5915

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_)/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcCoth[c\*x])^p\*ArcCoth[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*ArcCoth[1 - 2/(1 - c\*x))]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 6053

Int[(ArcCoth[u\_] \* ((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_)) / ((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]]\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegrand[1 - 1/u, x]]\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6057

Int[(Log[u\_] \* ((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_)) / ((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u]) / (2\*c\*d), x] - Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u]) / (d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c\*x))^2, 0]

### Rule 6061

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_)\*PolyLog[k\_, u\_]) / ((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[k + 1, u]) / (2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[k + 1, u]) / (d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

## Rule 6681

Int[((a\_.) + (b\_.)\*(F\_)[((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^(n\_.)/((A\_.) + (C\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F[c\*x])^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

## Rubi steps

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1 - c^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{(a+b \coth^{-1}(x))^3}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6b) \text{Subst}\left(\int \frac{\coth^{-1}\left(1 - \frac{2}{1-x}\right)(a+b \coth^{-1}(x))^2}{1-x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(3b) \text{Subst}\left(\int \frac{(a+b \coth^{-1}(x))^2}{1-x^2} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2c}$$

$$= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2c}$$

$$= -\frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{3b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2c}$$

**Mathematica** [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1 - c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^3/(1 - c^2\*x^2), x]

**fricas** [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left( -\frac{b^3 \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^3 + 3ab^2 \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 + 3a^2b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a^3}{c^2x^2 - 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1), x, algorithm="fricas")

[Out] integral(-(b^3\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^3 + 3\*a\*b^2\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + 3\*a^2\*b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a^3)/(c^2\*x^2 - 1), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a\right)^3}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1), x, algorithm="giac")

[Out] integrate(-(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^3/(c^2\*x^2 - 1), x)

**maple** [B] time = 1.67, size = 1492, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^3/(-c^2\*x^2+1), x)

[Out] 3\*a\*b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^2\*ln(1+1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1)))^2+6\*a\*b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))\*polylog(2,-1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1)))^2)-3/4\*b^3/c\*polylog(4,-1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1)))^2)+6\*b^3/c\*polylog(4,1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1)))^2)

```

)))+6*b^3/c*polylog(4,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)+3*a*b^2/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+6*a*b^2/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-3*a*b^2/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))-3*a*b^2/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))+3/2*a*b^2/c*polylog(3,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))-6*a*b^2/c*polylog(3,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-6*a*b^2/c*polylog(3,1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))-3/2*b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))+3/2*b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)*((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)))+b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+3*a^2*b/c*dilog(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))-3/4*a^2*b/c*dilog(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^2/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2)+3*b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-6*b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))+3*b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))-6*b^3/c*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-1/(((c*x+1)^(1/2)/(c*x+1)^(1/2)-1)/((c*x+1)^(1/2)/(c*x+1)^(1/2)+1))^(1/2))

```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^3 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) - \frac{(b^3 \log(cx+1) - b^3 \log(-cx+1)) \log(-\sqrt{cx+1} + \sqrt{-cx+1})^3}{16c} - \int \frac{4(\sqrt{cx} - \dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth((c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*log(-c*x + 1))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1))^3/c - integrate(1/32*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
```

```

^3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1) + sq
rt(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*
x + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) + (b^3*c
*x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*log(c*x
+ 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(-sqrt(c*x + 1) +
sqrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*b)*log(sq
rt(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqrt(-c*x + 1
)*a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt
(-c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x
+ 1) + sqrt(-c*x + 1)))*log(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/((c^2*x^2 -
1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)

```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a + b \operatorname{acoth}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))^3/(c^2\*x^2 - 1), x)

[Out] int(-(a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))^3/(c^2\*x^2 - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^3}{c^2 x^2 - 1} dx - \int \frac{b^3 \operatorname{acoth}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2 x^2 - 1} dx - \int \frac{3ab^2 \operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2 x^2 - 1} dx - \int \frac{3a^2b \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*3/(-c\*\*2\*x\*\*2+1), x)

[Out] -Integral(a\*\*3/(c\*\*2\*x\*\*2 - 1), x) - Integral(b\*\*3\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*3/(c\*\*2\*x\*\*2 - 1), x) - Integral(3\*a\*b\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*2/(c\*\*2\*x\*\*2 - 1), x) - Integral(3\*a\*\*2\*b\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))/(c\*\*2\*x\*\*2 - 1), x)

$$3.124 \quad \int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2} dx$$

**Optimal.** Leaf size=302

$$\frac{b\text{Li}_2\left(1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} + \frac{b\text{Li}_2\left(1 - \frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1\right)}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} - \frac{2 \coth^{-1}\left(1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

[Out]  $-2*\text{arccoth}(1-2/(1-(-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)}))*(a+b*\text{arccoth}((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)}))^2/c - b*(a+b*\text{arccoth}((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)}))*\text{polylog}(2,1-2/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)}+1))/c + b*(a+b*\text{arccoth}((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)}))*\text{polylog}(2,1-2*(-c*x+1)^{(1/2)}/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)}+1)/(c*x+1)^{(1/2)})/c - 1/2*b^2*\text{polylog}(3,1-2/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)}+1))/c + 1/2*b^2*\text{polylog}(3,1-2*(-c*x+1)^{(1/2)}/((-c*x+1)^{(1/2)}/(c*x+1)^{(1/2)}+1)/(c*x+1)^{(1/2)})/c$

**Rubi [A]** time = 0.35, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {6681, 5915, 6053, 5949, 6057, 6610}

$$\frac{b\text{PolyLog}\left(2,1 - \frac{2}{\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c} + \frac{b\text{PolyLog}\left(2,1 - \frac{2\sqrt{1-cx}}{\sqrt{cx+1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}} + 1\right)}\right)\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcCoth}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]])^2/(1 - c^2*x^2), x]$

[Out]  $(-2*(a + b*\text{ArcCoth}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]])^2*\text{ArcCoth}[1 - 2/(1 - \text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x])])/c - (b*(a + b*\text{ArcCoth}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]])*\text{PolyLog}[2, 1 - 2/(1 + \text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x])])/c + (b*(a + b*\text{ArcCoth}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]])*\text{PolyLog}[2, 1 - (2*\text{Sqrt}[1 - c*x])/(\text{Sqrt}[1 + c*x]*(1 + \text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]))])/c - (b^2*\text{PolyLog}[3, 1 - 2/(1 + \text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x])]/(2*c) + (b^2*\text{PolyLog}[3, 1 - (2*\text{Sqrt}[1 - c*x])/(\text{Sqrt}[1 + c*x]*(1 + \text{Sqrt}[1 - c*x]/\text{Sqrt}[1 + c*x]))]/(2*c)$

**Rule 5915**

$\text{Int}[(a_. + \text{ArcCoth}[c_.*(x_.)]*(b_.))^p/(x_), x\_Symbol] \rightarrow \text{Simp}[2*(a + b*\text{ArcCoth}[c*x])^p*\text{ArcCoth}[1 - 2/(1 - c*x)], x] - \text{Dist}[2*b*c*p, \text{Int}[(a + b*\text{ArcCoth}[c*x])^{p-1}*\text{ArcCoth}[1 - 2/(1 - c*x)]/(1 - c^2*x^2), x], x] /; F$

reeQ[{a, b, c}, x] && IGtQ[p, 1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 6053

Int[(ArcCoth[u\_]\*((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Dist[1/2, Int[(Log[SimplifyIntegrand[1 + 1/u, x]]\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] - Dist[1/2, Int[(Log[SimplifyIntegrand[1 - 1/u, x]]\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6057

Int[(Log[u\_]\*((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Simp[((a + b\*ArcCoth[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] - Dist[(b\*p)/2, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rule 6681

Int[((a\_.) + (b\_.)\*(F\_)[((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_.)])/Sqrt[(f\_.) + (g\_.)\*(x\_.)])^(n\_.)/((A\_.) + (C\_.)\*(x\_.)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F[c\*x])^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && EqQ[e\*f + d\*g, 0] && IGtQ[n, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \coth^{-1}(x))^2}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} \\
&= \frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b) \text{Subst}\left(\int \frac{\coth^{-1}\left(1 - \frac{2}{1-x}\right)(a)}{1-x} dx\right)}{c} \\
&= \frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{(2b) \text{Subst}\left(\int \frac{(a+b \coth^{-1}(x))}{1-x^2} dx\right)}{c} \\
&= \frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \text{Li}}{c} \\
&= \frac{2\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2 \coth^{-1}\left(1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{b\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right) \text{Li}}{c}
\end{aligned}$$

**Mathematica** [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2/(1 - c^2\*x^2), x]

[Out] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2/(1 - c^2\*x^2), x]

**fricas** [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b^2 \operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2ab \operatorname{arcoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^2}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1),x, algorithm="fricas")

[Out] integral(-(b^2\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 + 2\*a\*b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a^2)/(c^2\*x^2 - 1), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1),x, algorithm="giac")

[Out] integrate(-(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)^2/(c^2\*x^2 - 1), x)

**maple** [B] time = 0.73, size = 696, normalized size = 2.30

$$\frac{a^2 \ln(cx+1)}{2c} - \frac{a^2 \ln(cx-1)}{2c} + \frac{b^2 \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1 + \frac{1}{\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1}}\right)}{c} + \frac{2b^2 \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) \operatorname{polylog}\left(2, -\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} + 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1),x)

[Out] 1/2\*a^2/c\*ln(c\*x+1)-1/2\*a^2/c\*ln(c\*x-1)+b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^2\*ln(1+1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))^(1/2))+2\*b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))\*polylog(2,-1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))^(1/2))-2\*b^2/c\*polylog(3,-1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))^(1/2))+b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^2\*ln(1-1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))^(1/2))+2\*b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))\*polylog(2,1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))^(1/2))-2\*b^2/c\*polylog(3,1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))^(1/2))-b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^2\*ln(1+1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))-b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))\*polylog(2,-1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))^(1/2))-b^2/c\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))^2\*ln(1-1/(((c\*x+1)^(1/2)/(c\*x+1)^(1/2)-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))^(1/2))

$$\frac{1}{2} \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) + \frac{(b^2 \log(cx+1) - b^2 \log(-cx+1)) \log(-\sqrt{cx+1} + \sqrt{-cx+1})^2}{8c} + \int -\frac{2(\sqrt{cx+1})}{c^2 x^2 - 1} dx$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^2 \left( \frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c} \right) + \frac{(b^2 \log(cx+1) - b^2 \log(-cx+1)) \log(-\sqrt{cx+1} + \sqrt{-cx+1})^2}{8c} + \int -\frac{2(\sqrt{cx+1})}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2/(-c^2\*x^2+1),x, alg orithm="maxima")

[Out] 1/2\*a^2\*(log(c\*x + 1)/c - log(c\*x - 1)/c) + 1/8\*(b^2\*log(c\*x + 1) - b^2\*log(-c\*x + 1))\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1))^2/c + integrate(-1/8\*(2\*(sqrt(c\*x + 1)\*b^2 - sqrt(-c\*x + 1)\*b^2)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1))^2 + 8\*(sqrt(c\*x + 1)\*a\*b - sqrt(-c\*x + 1)\*a\*b)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) - (4\*(sqrt(c\*x + 1)\*b^2 - sqrt(-c\*x + 1)\*b^2)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) + (8\*a\*b - (b^2\*c\*x - b^2)\*log(c\*x + 1) + (b^2\*c\*x - b^2)\*log(-c\*x + 1))\*sqrt(c\*x + 1) - (8\*a\*b - (b^2\*c\*x + b^2)\*log(c\*x + 1) + (b^2\*c\*x + b^2)\*log(-c\*x + 1))\*sqrt(-c\*x + 1))\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1)))/((c^2\*x^2 - 1)\*sqrt(c\*x + 1) - (c^2\*x^2 - 1)\*sqrt(-c\*x + 1)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a + b \operatorname{acoth}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))^2/(c^2\*x^2 - 1),x)

[Out] int(-(a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))^2/(c^2\*x^2 - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2}{c^2 x^2 - 1} dx - \int \frac{b^2 \operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2 x^2 - 1} dx - \int \frac{2ab \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*2/(-c\*\*2\*x\*\*2+1),x)

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

$$3.125 \quad \int \frac{a+b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2} dx$$

Optimal. Leaf size=89

$$-\frac{a \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c} - \frac{b \operatorname{Li}_2\left(-\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c} + \frac{b \operatorname{Li}_2\left(\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c}$$

[Out]  $-a \ln((-c*x+1)^{(1/2)/(c*x+1)^{(1/2)})/c-1/2*b*\operatorname{polylog}(2,-(c*x+1)^{(1/2)/(-c*x+1)^{(1/2)})/c+1/2*b*\operatorname{polylog}(2,(c*x+1)^{(1/2)/(-c*x+1)^{(1/2)})/c$

**Rubi [A]** time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$ , Rules used = {206, 6681, 5913}

$$-\frac{b \operatorname{PolyLog}\left(2, -\frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c} + \frac{b \operatorname{PolyLog}\left(2, \frac{\sqrt{cx+1}}{\sqrt{1-cx}}\right)}{2c} - \frac{a \log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/(1 - c^2\*x^2), x]

[Out]  $-\left(\frac{a \operatorname{Log}\left[\frac{\operatorname{Sqrt}[1 - c*x]}{\operatorname{Sqrt}[1 + c*x]}\right]}{c}\right) - \left(\frac{b \operatorname{PolyLog}\left[2, -\left(\frac{\operatorname{Sqrt}[1 + c*x]}{\operatorname{Sqrt}[1 - c*x]}\right)\right]}{2c}\right) + \left(\frac{b \operatorname{PolyLog}\left[2, \frac{\operatorname{Sqrt}[1 + c*x]}{\operatorname{Sqrt}[1 - c*x]}\right]}{2c}\right)$

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 5913

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (Simp[(b\*PolyLog[2, -(c\*x)^(-1)])/2, x] - Simp[(b\*PolyLog[2, 1/(c\*x)])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rule 6681

Int[((a\_.) + (b\_.)\*(F\_)[((c\_.)\*Sqrt[(d\_.) + (e\_.)\*(x\_)])/Sqrt[(f\_.) + (g\_.)\*(x\_)])^n\_)/((A\_.) + (C\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*e\*g)/(C\*(e\*f - d\*g)), Subst[Int[(a + b\*F[c\*x])^n/x, x], x, Sqrt[d + e\*x]/Sqrt[f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C\*d\*f - A\*e\*g, 0] && E

qQ[e\*f + d\*g, 0] && IGtQ[n, 0]

### Rubi steps

$$\int \frac{a + b \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1 - c^2x^2} dx = -\frac{\text{Subst}\left(\int \frac{a+b \coth^{-1}(x)}{x} dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{a \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c} - \frac{b \text{Li}_2\left(-\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c} + \frac{b \text{Li}_2\left(\frac{\sqrt{1+cx}}{\sqrt{1-cx}}\right)}{2c}$$

**Mathematica [A]** time = 0.40, size = 98, normalized size = 1.10

$$\frac{a \tanh^{-1}(cx)}{c} + \frac{b \left( \text{Li}_2\left(-e^{-\tanh^{-1}(cx)}\right) - \text{Li}_2\left(e^{-\tanh^{-1}(cx)}\right) + \tanh^{-1}(cx) \left( 2 \coth^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right) + \log\left(1 - e^{-\tanh^{-1}(cx)}\right) \right) \right)}{2c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])/(1 - c^2\*x^2), x]

[Out] (a\*ArcTanh[c\*x])/c + (b\*(ArcTanh[c\*x]\*(2\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]) + Log[1 - E^(-ArcTanh[c\*x])] - Log[1 + E^(-ArcTanh[c\*x])]) + PolyLog[2, -E^(-ArcTanh[c\*x])] - PolyLog[2, E^(-ArcTanh[c\*x])]))/(2\*c)

**fricas [F]** time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1), x, algorithm="fricas")

[Out] integral(-(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)/(c^2\*x^2 - 1), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b \operatorname{arccoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorithm="giac")

[Out] integrate(-(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)/(c^2\*x^2 - 1), x)

**maple** [A] time = 0.72, size = 119, normalized size = 1.34

$$\frac{a \ln(cx + 1)}{2c} - \frac{a \ln(cx - 1)}{2c} + \frac{b \operatorname{dilog}\left(\frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1}{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} + 1}\right)}{c} - \frac{b \operatorname{dilog}\left(\frac{\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1\right)^2}{\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} + 1\right)^2}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x)

[Out] 1/2\*a/c\*ln(c\*x+1)-1/2\*a/c\*ln(c\*x-1)+b/c\*dilog(((c\*x+1)^(1/2)/(c\*x+1)^(1/2))-1)/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1))-1/4\*b/c\*dilog(((c\*x+1)^(1/2)/(c\*x+1)^(1/2))-1)^2/((c\*x+1)^(1/2)/(c\*x+1)^(1/2)+1)^2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} b \left( \frac{(\log(cx + 1) - \log(-cx + 1)) \log(\sqrt{cx + 1} + \sqrt{-cx + 1}) - (\log(cx + 1) - \log(-cx + 1)) \log(-\sqrt{cx + 1} + \sqrt{-cx + 1})}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))/(-c^2\*x^2+1),x, algorithm="maxima")

[Out] 1/4\*b\*((log(c\*x + 1) - log(-c\*x + 1))\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) - (log(c\*x + 1) - log(-c\*x + 1))\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1)))/c - 2\*integrate(-1/2\*sqrt(c\*x + 1)\*(log(c\*x + 1) - log(-c\*x + 1))/((c^2\*x^2 - 1)\*sqrt(c\*x + 1) + (c^2\*x^2 - 1)\*sqrt(-c\*x + 1)), x) - 2\*integrate(1/2\*sqrt(c\*x + 1)\*(log(c\*x + 1) - log(-c\*x + 1))/((c^2\*x^2 - 1)\*sqrt(c\*x + 1) - (c^2\*x^2 - 1)\*sqrt(-c\*x + 1)), x) + 1/2\*a\*(log(c\*x + 1)/c - log(c\*x - 1)/c)

**mapad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{acoth}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c^2 x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)`

[Out] `int(-(a + b*acoth((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2 - 1} dx - \int \frac{b \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acoth((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1), x)`

[Out] `-Integral(a/(c**2*x**2 - 1), x) - Integral(b*acoth(sqrt(-c*x + 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)`

$$3.126 \quad \int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)} dx$$

Optimal. Leaf size=43

$$\text{Int} \left( \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)}, x \right)$$

[Out] Unintegrable(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])),x]

[Out] Defer[Int][1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])),x]

Rubi steps

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)} dx = \int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)} dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])),x]



[Out] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])), x  
]

**fricas** [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral} \left( -\frac{1}{ac^2x^2 + (bc^2x^2 - b) \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x, alg  
orithm="fricas")

[Out] integral(-1/(a\*c^2\*x^2 + (b\*c^2\*x^2 - b)\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x +  
1)) - a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(c^2x^2 - 1) \left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x, alg  
orithm="giac")

[Out] integrate(-1/((c^2\*x^2 - 1)\*(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)),  
x)

**maple** [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{(-c^2x^2 + 1) \left( a + b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

[Out] int(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(c^2x^2 - 1) \left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2))),x, algorithm="maxima")

[Out] -integrate(1/((c^2\*x^2 - 1)\*(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a + b \operatorname{acoth}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right) (c^2 x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))\*(c^2\*x^2 - 1)),x)

[Out] -int(1/((a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))\*(c^2\*x^2 - 1)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ac^2x^2 - a + bc^2x^2 \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c\*\*2\*x\*\*2+1)/(a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2))),x)

[Out] -Integral(1/(a\*c\*\*2\*x\*\*2 - a + b\*c\*\*2\*x\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) - b\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))), x)

$$3.127 \quad \int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx$$

**Optimal.** Leaf size=43

$$\text{Int} \left( \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2}, x \right)$$

[Out] Unintegrable(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2, x)

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

[Out] Defer[Int][1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]])^2), x]

Rubi steps

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx = \int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx$$

**Mathematica [A]** time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2) \left( a + b \coth^{-1} \left( \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]))^2, x]

[Out] Integrate[1/((1 - c^2\*x^2)\*(a + b\*ArcCoth[Sqrt[1 - c\*x]/Sqrt[1 + c\*x]]))^2, x]

**fricas** [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left( -\frac{1}{a^2 c^2 x^2 + (b^2 c^2 x^2 - b^2) \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)^2 - a^2 + 2 (abc^2 x^2 - ab) \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="fricas")

[Out] integral(-1/(a^2\*c^2\*x^2 + (b^2\*c^2\*x^2 - b^2)\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))^2 - a^2 + 2\*(a\*b\*c^2\*x^2 - a\*b)\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c^2 x^2 - 1) \left( b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) + a \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="giac")

[Out] integrate(-1/((c^2\*x^2 - 1)\*(b\*arccoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + a))^2), x)

**maple** [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(-c^2 x^2 + 1) \left( a + b \operatorname{arccoth} \left( \frac{\sqrt{-cx+1}}{\sqrt{cx+1}} \right) \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x)

[Out] int(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4cx}{\sqrt{cx+1}\sqrt{-cx+1}b^2c\log(\sqrt{cx+1}+\sqrt{-cx+1})-\sqrt{cx+1}\sqrt{-cx+1}b^2c\log(-\sqrt{cx+1}+\sqrt{-cx+1})+2\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c^2\*x^2+1)/(a+b\*arccoth((-c\*x+1)^(1/2)/(c\*x+1)^(1/2)))^2,x, algorithm="maxima")

[Out] 4\*c\*x/(sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b^2\*c\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) - sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*b^2\*c\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1)) + 2\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*a\*b\*c) - integrate(-4/((b^2\*c^2\*x^2 - b^2)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*log(sqrt(c\*x + 1) + sqrt(-c\*x + 1)) - (b^2\*c^2\*x^2 - b^2)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*log(-sqrt(c\*x + 1) + sqrt(-c\*x + 1))) + 2\*(a\*b\*c^2\*x^2 - a\*b)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a + b \operatorname{acoth}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2 (c^2 x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))^2\*(c^2\*x^2 - 1)),x)

[Out] -int(1/((a + b\*acoth((1 - c\*x)^(1/2)/(c\*x + 1)^(1/2)))^2\*(c^2\*x^2 - 1)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2c^2x^2 - a^2 + 2abc^2x^2 \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - 2ab \operatorname{acoth}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + b^2c^2x^2 \operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b^2 \operatorname{acoth}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-c\*\*2\*x\*\*2+1)/(a+b\*acoth((-c\*x+1)\*\*(1/2)/(c\*x+1)\*\*(1/2)))\*\*2,x)

[Out] -Integral(1/(a\*\*2\*c\*\*2\*x\*\*2 - a\*\*2 + 2\*a\*b\*c\*\*2\*x\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) - 2\*a\*b\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1)) + b\*\*2\*c\*\*2\*x\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*2 - b\*\*2\*acoth(sqrt(-c\*x + 1)/sqrt(c\*x + 1))\*\*2), x)

### 3.128 $\int x^m \coth^{-1}(\tanh(a + bx)) dx$

Optimal. Leaf size=37

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))}{m + 1} - \frac{bx^{m+2}}{m^2 + 3m + 2}$$

[Out]  $-b*x^{(2+m)}/(m^2+3*m+2)+x^{(1+m)*\operatorname{arccoth}(\tanh(b*x+a))/(1+m)$

**Rubi [A]** time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))}{m + 1} - \frac{bx^{m+2}}{m^2 + 3m + 2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^m * \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]], x]$

[Out]  $-((b*x^{(2 + m)})/(2 + 3*m + m^2)) + (x^{(1 + m)*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/(1 + m)$

#### Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[x^{(m + 1)}/(m + 1), x] /; \operatorname{FreeQ}[m, x] \ \&\& \operatorname{NeQ}[m, -1]$

#### Rule 2168

$\operatorname{Int}[(u_)^{(m_.)} * (v_)^{(n_.)}, x\_Symbol] :> \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^{(m + 1)} * v^{(n)}) / (a * (m + 1)), x] - \operatorname{Dist}[(b * n) / (a * (m + 1)), \operatorname{Int}[u^{(m + 1)} * v^{(n - 1)}, x], x] /; \operatorname{NeQ}[b * u - a * v, 0] /; \operatorname{FreeQ}[\{m, n\}, x] \ \&\& \operatorname{PiecewiseLinearQ}[u, v, x] \ \&\& \operatorname{NeQ}[m, -1] \ \&\& ((\operatorname{LtQ}[m, -1] \ \&\& \operatorname{GtQ}[n, 0] \ \&\& !(\operatorname{ILtQ}[m + n, -2] \ \&\& (\operatorname{FractionQ}[m] \ || \operatorname{GeQ}[2 * n + m + 1, 0]))) \ || (\operatorname{IGtQ}[n, 0] \ \&\& \operatorname{IGtQ}[m, 0] \ \&\& \operatorname{LeQ}[n, m]) \ || (\operatorname{IGtQ}[n, 0] \ \&\& !\operatorname{IntegerQ}[m]) \ || (\operatorname{ILtQ}[m, 0] \ \&\& !\operatorname{IntegerQ}[n]))$

#### Rubi steps

$$\begin{aligned} \int x^m \coth^{-1}(\tanh(a + bx)) dx &= \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))}{1 + m} - \frac{b \int x^{1+m} dx}{1 + m} \\ &= -\frac{bx^{2+m}}{2 + 3m + m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))}{1 + m} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 34, normalized size = 0.92

$$x^m \left( \frac{x \left( \coth^{-1}(\tanh(a + bx)) - bx \right)}{m + 1} + \frac{bx^2}{m + 2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]], x]

[Out] x^m\*((b\*x^2)/(2 + m) + (x\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])))/(1 + m)

**fricas [A]** time = 0.54, size = 33, normalized size = 0.89

$$\frac{((bm + b)x^2 + (am + 2a)x)x^m}{m^2 + 3m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a)), x, algorithm="fricas")

[Out] ((b\*m + b)\*x^2 + (a\*m + 2\*a)\*x)\*x^m/(m^2 + 3\*m + 2)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(x^m\*arccoth(tanh(b\*x + a)), x)

**maple [C]** time = 0.16, size = 676, normalized size = 18.27

$$\frac{x x^m \ln(e^{bx+a})}{1+m} \frac{x \left( -2i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2 - i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2 m + 4i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right) \operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2 \right)}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*arccoth(tanh(b\*x+a)), x)

[Out] 1/(1+m)\*x\*x^m\*ln(exp(b\*x+a))-1/4\*x\*(-2\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2-I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2+m+4\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^3+I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2

$$+2*a)/(\exp(2*b*x+2*a)+1))^m-2*I*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2*m+4*I*Pi+2*I*Pi*m+2*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^3*m+I*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3*m-I*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2*m+I*Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))^m-4*I*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2+I*Pi*csgn(I*\exp(2*b*x+2*a))^3*m-4*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-2*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^2*m+2*I*Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))+4*b*x+2*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-2*I*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+2*I*Pi*csgn(I*\exp(2*b*x+2*a))^3)/(1+m)/(2+m)*x^m$$

**maxima** [A] time = 0.34, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1} \operatorname{arccoth}(\tanh(bx+a))}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] -b\*x^2\*x^m/((m+2)\*(m+1)) + x^(m+1)\*arccoth(tanh(b\*x+a))/(m+1)

**mupad** [B] time = 1.61, size = 96, normalized size = 2.59

$$\frac{2bx^m x^2 (m+1)}{2m^2 + 6m + 4} - \frac{xx^m (m+2) \left( \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx \right)}{2m^2 + 6m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*acoth(tanh(a+b\*x)),x)

[Out] (2\*b\*x^m\*x^2\*(m+1))/(6\*m+2\*m^2+4) - (x\*x^m\*(m+2)\*(log(-2/(exp(2\*a)\*exp(2\*b\*x)-1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x)-1)) + 2\*b\*x))/(6\*m+2\*m^2+4)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b \log(x) - \frac{\operatorname{acoth}(\tanh(a+bx))}{x} & \text{for } m = -2 \\ \int \frac{\operatorname{acoth}(\tanh(a+bx))}{x} dx & \text{for } m = -1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} + \frac{2xx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(tanh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2), True))
```

### 3.129 $\int x^2 \coth^{-1}(\tanh(a + bx)) dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx)) - \frac{bx^4}{12}$$

[Out]  $-1/12*b*x^4+1/3*x^3*\operatorname{arccoth}(\tanh(b*x+a))$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$\frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx)) - \frac{bx^4}{12}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]], x]$

[Out]  $-(b*x^4)/12 + (x^3*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/3$

#### Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[x^{(m + 1)}/(m + 1), x] /;$  FreeQ[m, x] && NeQ[m, -1]

#### Rule 2168

$\operatorname{Int}[(u_)^{(m_.)}*(v_)^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^{(m + 1)}*v^{(n)})/(a*(m + 1)), x] - \operatorname{Dist}[(b*n)/(a*(m + 1)), \operatorname{Int}[u^{(m + 1)}*v^{(n - 1)}, x], x] /;$  NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

#### Rubi steps

$$\begin{aligned} \int x^2 \coth^{-1}(\tanh(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx)) - \frac{1}{3}b \int x^3 dx \\ &= -\frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx)) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3 (bx - 4 \operatorname{coth}^{-1}(\tanh(a + bx)))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Tanh[a + b\*x]],x]

[Out] -1/12\*(x^3\*(b\*x - 4\*ArcCoth[Tanh[a + b\*x]]))

**fricas [A]** time = 0.64, size = 13, normalized size = 0.57

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out] 1/4\*b\*x^4 + 1/3\*a\*x^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(tanh(b\*x + a)), x)

**maple [B]** time = 0.38, size = 59, normalized size = 2.57

$$\frac{x^3 \operatorname{arccoth}(\tanh(bx + a))}{3} + \frac{-\frac{(bx+a)^4}{4} + (bx+a)^3 a - \frac{3a^2(bx+a)^2}{2} + (bx+a)a^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(tanh(b\*x+a)),x)

[Out] 1/3\*x^3\*arccoth(tanh(b\*x+a))+1/3/b^3\*(-1/4\*(b\*x+a)^4+(b\*x+a)^3\*a-3/2\*a^2\*(b\*x+a)^2+(b\*x+a)\*a^3)

**maxima [A]** time = 0.38, size = 19, normalized size = 0.83

$$-\frac{1}{12}bx^4 + \frac{1}{3}x^3 \operatorname{arccoth}(\tanh(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] -1/12\*b\*x^4 + 1/3\*x^3\*arccoth(tanh(b\*x + a))

mupad [B] time = 0.09, size = 19, normalized size = 0.83

$$\frac{x^3 \operatorname{acoth}(\tanh(a + bx))}{3} - \frac{bx^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*acoth(tanh(a + b\*x)),x)

[Out] (x^3\*acoth(tanh(a + b\*x)))/3 - (b\*x^4)/12

sympy [A] time = 0.40, size = 19, normalized size = 0.83

$$-\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}(\tanh(a + bx))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(tanh(b\*x+a)),x)

[Out] -b\*x\*\*4/12 + x\*\*3\*acoth(tanh(a + b\*x))/3

### 3.130 $\int x \coth^{-1}(\tanh(a + bx)) dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2 \coth^{-1}(\tanh(a + bx)) - \frac{bx^3}{6}$$

[Out]  $-1/6*b*x^3+1/2*x^2*\operatorname{arccoth}(\tanh(b*x+a))$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {6240, 30}

$$\frac{1}{2}x^2 \coth^{-1}(\tanh(a + bx)) - \frac{bx^3}{6}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]], x]$

[Out]  $-(b*x^3)/6 + (x^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/2$

#### Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[x^{(m + 1)}/(m + 1), x] /;$  FreeQ[m, x] && NeQ[m, -1]

#### Rule 6240

$\operatorname{Int}[\operatorname{ArcCoth}[(c_.) + (d_.)*\operatorname{Tanh}[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e + f*x)^{(m + 1)}*\operatorname{ArcCoth}[c + d*\operatorname{Tanh}[a + b*x]]/(f*(m + 1)), x] + \operatorname{Dist}[b/(f*(m + 1)), \operatorname{Int}[(e + f*x)^{(m + 1)}/(c - d + c*E^{(2*a + 2*b*x)}), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(\tanh(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(\tanh(a + bx)) - \frac{1}{2}b \int x^2 dx \\ &= -\frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(\tanh(a + bx)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2 (bx - 3 \coth^{-1}(\tanh(a + bx)))$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Tanh[a + b\*x]], x]

[Out] -1/6\*(x^2\*(b\*x - 3\*ArcCoth[Tanh[a + b\*x]]))

**fricas** [A] time = 0.45, size = 13, normalized size = 0.57

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a)), x, algorithm="fricas")

[Out] 1/3\*b\*x^3 + 1/2\*a\*x^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(x\*arccoth(tanh(b\*x + a)), x)

**maple** [B] time = 0.38, size = 48, normalized size = 2.09

$$\frac{x^2 \operatorname{arccoth}(\tanh(bx + a))}{2} + \frac{-\frac{(bx+a)^3}{3} + (bx+a)^2 a - a^2 (bx+a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(tanh(b\*x+a)), x)

[Out] 1/2\*x^2\*arccoth(tanh(b\*x+a))+1/2/b^2\*(-1/3\*(b\*x+a)^3+(b\*x+a)^2\*a-a^2\*(b\*x+a))

**maxima** [A] time = 0.38, size = 19, normalized size = 0.83

$$-\frac{1}{6}bx^3 + \frac{1}{2}x^2 \operatorname{arccoth}(\tanh(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a)), x, algorithm="maxima")

[Out]  $-1/6*b*x^3 + 1/2*x^2*\operatorname{arccoth}(\tanh(b*x + a))$

**mupad** [B] time = 1.13, size = 19, normalized size = 0.83

$$\frac{x^2 \operatorname{acoth}(\tanh(a + bx))}{2} - \frac{bx^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(tanh(a + b*x)),x)`

[Out]  $(x^2*\operatorname{acoth}(\tanh(a + b*x)))/2 - (b*x^3)/6$

**sympy** [A] time = 0.23, size = 19, normalized size = 0.83

$$-\frac{bx^3}{6} + \frac{x^2 \operatorname{acoth}(\tanh(a + bx))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(tanh(b*x+a)),x)`

[Out]  $-b*x**3/6 + x**2*\operatorname{acoth}(\tanh(a + b*x))/2$

### 3.131 $\int \coth^{-1}(\tanh(a + bx)) dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a + bx))^2}{2b}$$

[Out] 1/2\*arccoth(tanh(b\*x+a))^2/b

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]], x]

[Out] ArcCoth[Tanh[a + b\*x]]^2/(2\*b)

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tanh(a + bx)) dx &= \frac{\text{Subst}\left(\int x dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\tanh(a + bx))^2}{2b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.12

$$x \coth^{-1}(\tanh(a + bx)) - \frac{bx^2}{2}$$



Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]], x]

[Out]  $-1/2*(b*x^2) + x*\text{ArcCoth}[\text{Tanh}[a + b*x]]$

**fricas** [A] time = 0.43, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a)), x, algorithm="fricas")

[Out]  $1/2*b*x^2 + a*x$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a)), x)

**maple** [B] time = 0.06, size = 32, normalized size = 2.00

$$\frac{\text{arctanh}(\tanh(bx + a)) \text{arccoth}(\tanh(bx + a)) - \frac{\text{arctanh}(\tanh(bx+a))^2}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a)), x)

[Out]  $1/b*(\text{arctanh}(\tanh(b*x+a))*\text{arccoth}(\tanh(b*x+a))-1/2*\text{arctanh}(\tanh(b*x+a))^2)$

**maxima** [A] time = 0.38, size = 16, normalized size = 1.00

$$-\frac{1}{2}bx^2 + x \text{arccoth}(\tanh(bx + a))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a)), x, algorithm="maxima")

[Out]  $-1/2*b*x^2 + x*\text{arccoth}(\tanh(b*x + a))$

**mupad** [B] time = 1.12, size = 16, normalized size = 1.00

$$x \operatorname{acoth}(\tanh(a + bx)) - \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x)),x)`

[Out] `x*acoth(tanh(a + b*x)) - (b*x^2)/2`

**sympy** [A] time = 0.16, size = 19, normalized size = 1.19

$$\begin{cases} \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2b} & \text{for } b \neq 0 \\ x \operatorname{acoth}(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a)),x)`

[Out] `Piecewise((acoth(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*acoth(tanh(a)), True))`

$$3.132 \quad \int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx$$

Optimal. Leaf size=21

$$bx - \log(x) (bx - \coth^{-1}(\tanh(a + bx)))$$

[Out] b\*x-(b\*x-arcCoth(tanh(b\*x+a)))\*ln(x)

**Rubi [A]** time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2158, 29}

$$bx - \log(x) (bx - \coth^{-1}(\tanh(a + bx)))$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]/x,x]

[Out] b\*x - (b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a + bx))}{x} dx &= bx - (bx - \coth^{-1}(\tanh(a + bx))) \int \frac{1}{x} dx \\ &= bx - (bx - \coth^{-1}(\tanh(a + bx))) \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 19, normalized size = 0.90

$$\log(x) (\coth^{-1}(\tanh(a + bx)) - bx) + bx$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]/x,x]

[Out] b\*x + (-(b\*x) + ArcCoth[Tanh[a + b\*x]])\*Log[x]

**fricas** [A] time = 0.48, size = 8, normalized size = 0.38

$$bx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x,x, algorithm="fricas")

[Out] b\*x + a\*log(x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))/x, x)

**maple** [C] time = 0.42, size = 354, normalized size = 16.86

$$\ln(x) \ln(e^{bx+a}) - \ln(x)xb + bx - \frac{i \ln(x) \pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a+1}}\right)^3}{2} + \frac{i \ln(x) \pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a+1}}\right) \operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a+1}}\right)^2}{4} + \frac{i \ln(x) \pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))/x,x)

[Out] ln(x)\*ln(exp(b\*x+a))-ln(x)\*x\*b+b\*x-1/2\*I\*ln(x)\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))^3+1/4\*I\*ln(x)\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2+1/4\*I\*ln(x)\*Pi\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2-1/4\*I\*ln(x)\*Pi\*csgn(I\*exp(b\*x+a))^2\*csgn(I\*exp(2\*b\*x+2\*a))-1/2\*I\*Pi\*ln(x)-1/4\*I\*ln(x)\*Pi\*csgn(I\*exp(2\*b\*x+2\*a))^3+1/2\*I\*ln(x)\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))^2+1/2\*I\*ln(x)\*Pi\*csgn(I\*exp(b\*x+a))\*csgn(I\*exp(2\*b\*x+2\*a))^2-1/4\*I\*ln(x)\*Pi\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^3-1/4\*I\*ln(x)\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))

**maxima** [A] time = 0.33, size = 34, normalized size = 1.62

$$-b\left(x + \frac{a}{b}\right) \log(x) + b\left(x + \frac{a \log(x)}{b}\right) + \operatorname{arccoth}(\tanh(bx + a)) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x,x, algorithm="maxima")

[Out] -b\*(x + a/b)\*log(x) + b\*(x + a\*log(x)/b) + arccoth(tanh(b\*x + a))\*log(x)

mupad [B] time = 0.18, size = 59, normalized size = 2.81

$$bx - \ln(x) \left( \frac{\ln\left(-\frac{2}{e^{2a} e^{2bx} - 1}\right)}{2} - \frac{\ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx} - 1}\right)}{2} + bx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))/x,x)

[Out] b\*x - log(x)\*(log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))/2 - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))/2 + b\*x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(\tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))/x,x)

[Out] Integral(acoth(tanh(a + b\*x))/x, x)

$$3.133 \quad \int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx$$

Optimal. Leaf size=17

$$b \log(x) - \frac{\coth^{-1}(\tanh(a+bx))}{x}$$

[Out] -arccoth(tanh(b\*x+a))/x+b\*ln(x)

**Rubi [A]** time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 29}

$$b \log(x) - \frac{\coth^{-1}(\tanh(a+bx))}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]/x^2, x]

[Out] -(ArcCoth[Tanh[a + b\*x]]/x) + b\*Log[x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2\*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{x} + b \int \frac{1}{x} dx \\ &= -\frac{\coth^{-1}(\tanh(a+bx))}{x} + b \log(x) \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 18, normalized size = 1.06

$$-\frac{\coth^{-1}(\tanh(a + bx))}{x} + b \log(x) + b$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]/x^2,x]

[Out] b - ArcCoth[Tanh[a + b\*x]]/x + b\*Log[x]

**fricas** [A] time = 0.94, size = 13, normalized size = 0.76

$$\frac{bx \log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^2,x, algorithm="fricas")

[Out] (b\*x\*log(x) - a)/x

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^2,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))/x^2, x)

**maple** [A] time = 0.38, size = 20, normalized size = 1.18

$$-\frac{\operatorname{arccoth}(\tanh(bx + a))}{x} + b \ln(bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))/x^2,x)

[Out] -arccoth(tanh(b\*x+a))/x+b\*ln(b\*x)

**maxima** [A] time = 0.39, size = 17, normalized size = 1.00

$$b \log(x) - \frac{\operatorname{arccoth}(\tanh(bx + a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^2,x, algorithm="maxima")

[Out] b\*log(x) - arccoth(tanh(b\*x + a))/x

mupad [B] time = 0.09, size = 17, normalized size = 1.00

$$b \ln(x) - \frac{\operatorname{arccoth}(\tanh(a + bx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))/x^2,x)

[Out] b\*log(x) - acoth(tanh(a + b\*x))/x

sympy [A] time = 0.25, size = 14, normalized size = 0.82

$$b \log(x) - \frac{\operatorname{acoth}(\tanh(a + bx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))/x\*\*2,x)

[Out] b\*log(x) - acoth(tanh(a + b\*x))/x



$$3.134 \quad \int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx$$

Optimal. Leaf size=23

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2} - \frac{b}{2x}$$

[Out]  $-1/2*b/x-1/2*\operatorname{arccoth}(\tanh(b*x+a))/x^2$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\tanh(a+bx))}{2x^2} - \frac{b}{2x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Tanh[a + b*x]]/x^3,x]`

[Out]  $-b/(2*x) - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]/(2*x^2)$

Rule 30

`Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 2168

`Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a+bx))}{x^3} dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{2x^2} + \frac{1}{2}b \int \frac{1}{x^2} dx \\ &= -\frac{b}{2x} - \frac{\coth^{-1}(\tanh(a+bx))}{2x^2} \end{aligned}$$

**Mathematica** [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{\coth^{-1}(\tanh(a + bx)) + bx}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]/x^3,x]

[Out] -1/2\*(b\*x + ArcCoth[Tanh[a + b\*x]])/x^2

**fricas** [A] time = 1.21, size = 11, normalized size = 0.48

$$-\frac{2bx + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^3,x, algorithm="fricas")

[Out] -1/2\*(2\*b\*x + a)/x^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))/x^3, x)

**maple** [A] time = 0.39, size = 20, normalized size = 0.87

$$-\frac{b}{2x} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))/x^3,x)

[Out] -1/2\*b/x-1/2\*arccoth(tanh(b\*x+a))/x^2

**maxima** [A] time = 0.38, size = 19, normalized size = 0.83

$$-\frac{b}{2x} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^3,x, algorithm="maxima")

[Out]  $-1/2*b/x - 1/2*\operatorname{arccoth}(\tanh(b*x + a))/x^2$

mupad [B] time = 1.14, size = 16, normalized size = 0.70

$$-\frac{\operatorname{acoth}(\tanh(a + bx)) + bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))/x^3,x)

[Out]  $-(\operatorname{acoth}(\tanh(a + b*x)) + b*x)/(2*x^2)$

sympy [A] time = 0.51, size = 19, normalized size = 0.83

$$\frac{b}{2x} - \frac{\operatorname{acoth}(\tanh(a + bx))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))/x\*\*3,x)

[Out]  $-b/(2*x) - \operatorname{acoth}(\tanh(a + b*x))/(2*x**2)$

$$3.135 \quad \int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx$$

Optimal. Leaf size=23

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3} - \frac{b}{6x^2}$$

[Out]  $-1/6*b/x^2-1/3*\operatorname{arccoth}(\tanh(b*x+a))/x^3$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\tanh(a+bx))}{3x^3} - \frac{b}{6x^2}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Tanh[a + b*x]]/x^4, x]`

[Out]  $-b/(6*x^2) - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]/(3*x^3)$

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a+bx))}{x^4} dx &= -\frac{\coth^{-1}(\tanh(a+bx))}{3x^3} + \frac{1}{3}b \int \frac{1}{x^3} dx \\ &= -\frac{b}{6x^2} - \frac{\coth^{-1}(\tanh(a+bx))}{3x^3} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 20, normalized size = 0.87

$$\frac{2 \operatorname{coth}^{-1}(\tanh(a + bx)) + bx}{6x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]/x^4,x]

[Out] -1/6\*(b\*x + 2\*ArcCoth[Tanh[a + b\*x]])/x^3

**fricas** [A] time = 0.57, size = 13, normalized size = 0.57

$$\frac{3bx + 2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^4,x, algorithm="fricas")

[Out] -1/6\*(3\*b\*x + 2\*a)/x^3

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^4,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))/x^4, x)

**maple** [A] time = 0.40, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))/x^4,x)

[Out] -1/6\*b/x^2-1/3\*arccoth(tanh(b\*x+a))/x^3

**maxima** [A] time = 0.38, size = 19, normalized size = 0.83

$$-\frac{b}{6x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))/x^4,x, algorithm="maxima")

[Out] -1/6\*b/x^2 - 1/3\*arccoth(tanh(b\*x + a))/x^3

mupad [B] time = 1.12, size = 19, normalized size = 0.83

$$-\frac{\operatorname{acoth}(\tanh(a + bx))}{3x^3} - \frac{b}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))/x^4,x)

[Out] - acoth(tanh(a + b\*x))/(3\*x^3) - b/(6\*x^2)

sympy [A] time = 0.79, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2} - \frac{\operatorname{acoth}(\tanh(a + bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))/x\*\*4,x)

[Out] -b/(6\*x\*\*2) - acoth(tanh(a + b\*x))/(3\*x\*\*3)

### 3.136 $\int x^m \coth^{-1}(\tanh(a + bx))^2 dx$

Optimal. Leaf size=71

$$-\frac{2bx^{m+2} \coth^{-1}(\tanh(a + bx))}{m^2 + 3m + 2} + \frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^2}{m + 1} + \frac{2b^2x^{m+3}}{m^3 + 6m^2 + 11m + 6}$$

[Out]  $2*b^2*x^{(3+m)}/(m^3+6*m^2+11*m+6)-2*b*x^{(2+m)*\operatorname{arccoth}(\tanh(b*x+a))}/(m^2+3*m+2)+x^{(1+m)*\operatorname{arccoth}(\tanh(b*x+a))^2}/(1+m)$

**Rubi [A]** time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$-\frac{2bx^{m+2} \coth^{-1}(\tanh(a + bx))}{m^2 + 3m + 2} + \frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^2}{m + 1} + \frac{2b^2x^{m+3}}{m^3 + 6m^2 + 11m + 6}$$

Antiderivative was successfully verified.

[In] Int[x^m\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out]  $(2*b^2*x^{(3 + m)})/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^{(2 + m)*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/(2 + 3*m + m^2) + (x^{(1 + m)*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2)/(1 + m)$

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)\*v^n)/(a\*(m + 1)), x] - Dist[(b\*n)/(a\*(m + 1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

#### Rubi steps

$$\begin{aligned} \int x^m \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^2}{1+m} - \frac{(2b) \int x^{1+m} \coth^{-1}(\tanh(a + bx)) dx}{1+m} \\ &= -\frac{2bx^{2+m} \coth^{-1}(\tanh(a + bx))}{2+3m+m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^2}{1+m} + \frac{(2b^2) \int x^{2+m} dx}{2+3m+m^2} \\ &= \frac{2b^2 x^{3+m}}{6+11m+6m^2+m^3} - \frac{2bx^{2+m} \coth^{-1}(\tanh(a + bx))}{2+3m+m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^2}{1+m} \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 62, normalized size = 0.87

$$\frac{x^{m+1} \left( (m^2 + 5m + 6) \coth^{-1}(\tanh(a + bx))^2 - 2b(m + 3)x \coth^{-1}(\tanh(a + bx)) + 2b^2 x^2 \right)}{(m + 1)(m + 2)(m + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (x^(1 + m)\*(2\*b^2\*x^2 - 2\*b\*(3 + m)\*x\*ArcCoth[Tanh[a + b\*x]] + (6 + 5\*m + m^2)\*ArcCoth[Tanh[a + b\*x]]^2))/((1 + m)\*(2 + m)\*(3 + m))

**fricas [A]** time = 0.50, size = 101, normalized size = 1.42

$$\frac{(4(b^2 m^2 + 3b^2 m + 2b^2)x^3 + 8(abm^2 + 4abm + 3ab)x^2 + (4a^2 m^2 - \pi^2(m^2 + 5m + 6) + 20a^2 m + 24a^2)x)x^m}{4(m^3 + 6m^2 + 11m + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 1/4\*(4\*(b^2\*m^2 + 3\*b^2\*m + 2\*b^2)\*x^3 + 8\*(a\*b\*m^2 + 4\*a\*b\*m + 3\*a\*b)\*x^2 + (4\*a^2\*m^2 - pi^2\*(m^2 + 5\*m + 6) + 20\*a^2\*m + 24\*a^2)\*x)\*x^m/(m^3 + 6\*m^2 + 11\*m + 6)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x^m\*arccoth(tanh(b\*x + a))^2, x)



**maple** [C] time = 1.00, size = 9175, normalized size = 129.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (x^m \operatorname{arccoth}(\tanh(bx+a)))^2 dx$

[Out] result too large to display

**maxima** [A] time = 0.39, size = 73, normalized size = 1.03

$$\frac{2b^2x^3x^m}{(m+3)(m+2)(m+1)} - \frac{2bx^2x^m \operatorname{arccoth}(\tanh(bx+a))}{(m+2)(m+1)} + \frac{x^{m+1} \operatorname{arccoth}(\tanh(bx+a))^2}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(x^m \operatorname{arccoth}(\tanh(bx+a))^2, x, \text{algorithm}="maxima")$

[Out]  $2b^2x^3x^m/((m+3)(m+2)(m+1)) - 2bx^2x^m \operatorname{arccoth}(\tanh(bx+a))/((m+2)(m+1)) + x^{m+1} \operatorname{arccoth}(\tanh(bx+a))^2/(m+1)$

**mupad** [B] time = 1.32, size = 203, normalized size = 2.86

$$\frac{4b^2x^m x^3 (m^2 + 3m + 2)}{4m^3 + 24m^2 + 44m + 24} + \frac{xx^m \left( \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx \right)^2 (m^2 + 5m + 6)}{4m^3 + 24m^2 + 44m + 24} - \frac{4bx^m x^2 \left( \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx \right)}{4m^3 + 24m^2 + 44m + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (x^m \operatorname{acoth}(\tanh(a+bx)))^2 dx$

[Out]  $(4b^2x^m x^3(3m+m^2+2))/(44m+24m^2+4m^3+24) + (xx^m(\log(-2/(\exp(2a)\exp(2bx)-1)) - \log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx)-1)) + 2bx)^2(5m+m^2+6))/(44m+24m^2+4m^3+24) - (4bx^m x^2(\log(-2/(\exp(2a)\exp(2bx)-1)) - \log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx)-1)) + 2bx)(4m+m^2+3))/(44m+24m^2+4m^3+24)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left\{ \begin{array}{l} b^2 \log(x) - \frac{b \operatorname{acoth}(\tanh(a+bx))}{x} - \frac{\operatorname{acoth}^2(\tanh(a+bx))}{2x^2} \\ \int \frac{\operatorname{acoth}^2(\tanh(a+bx))}{x^2} dx \\ \int \frac{\operatorname{acoth}^2(\tanh(a+bx))}{x} dx \\ \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} - \frac{2bmx^2x^m \operatorname{acoth}(\tanh(a+bx))}{m^3+6m^2+11m+6} - \frac{6bx^2x^m \operatorname{acoth}(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{m^2xx^m \operatorname{acoth}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{5mxx^m \operatorname{acoth}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((b**2*log(x) - b*acoth(tanh(a + b*x))/x - acoth(tanh(a + b*x))**2
/(2*x**2), Eq(m, -3)), (Integral(acoth(tanh(a + b*x))**2/x**2, x), Eq(m, -2
)), (Integral(acoth(tanh(a + b*x))**2/x, x), Eq(m, -1)), (2*b**2*x**3*x**m/
(m**3 + 6*m**2 + 11*m + 6) - 2*b*m*x**2*x**m*acoth(tanh(a + b*x))/(m**3 + 6
*m**2 + 11*m + 6) - 6*b*x**2*x**m*acoth(tanh(a + b*x))/(m**3 + 6*m**2 + 11*
m + 6) + m**2*x*x**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 5
*m*x*x**m*acoth(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 6*x*x**m*aco
th(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6), True))
```

### 3.137 $\int x^3 \coth^{-1}(\tanh(a + bx))^2 dx$

Optimal. Leaf size=42

$$-\frac{1}{10}bx^5 \coth^{-1}(\tanh(a + bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2 + \frac{b^2x^6}{60}$$

[Out]  $1/60*b^2*x^6-1/10*b*x^5*\operatorname{arccoth}(\tanh(b*x+a))+1/4*x^4*\operatorname{arccoth}(\tanh(b*x+a))^2$

**Rubi [A]** time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$-\frac{1}{10}bx^5 \coth^{-1}(\tanh(a + bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2 + \frac{b^2x^6}{60}$$

Antiderivative was successfully verified.

[In] `Int[x^3*ArcCoth[Tanh[a + b*x]]^2,x]`

[Out]  $(b^2*x^6)/60 - (b*x^5*ArcCoth[Tanh[a + b*x]])/10 + (x^4*ArcCoth[Tanh[a + b*x]]^2)/4$

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2 - \frac{1}{2}b \int x^4 \coth^{-1}(\tanh(a + bx)) dx \\
&= -\frac{1}{10}bx^5 \coth^{-1}(\tanh(a + bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{10}b^2 \int x^5 dx \\
&= \frac{b^2x^6}{60} - \frac{1}{10}bx^5 \coth^{-1}(\tanh(a + bx)) + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^2
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 37, normalized size = 0.88

$$\frac{1}{60}x^4 \left( -6bx \coth^{-1}(\tanh(a + bx)) + 15 \coth^{-1}(\tanh(a + bx))^2 + b^2x^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (x^4\*(b^2\*x^2 - 6\*b\*x\*ArcCoth[Tanh[a + b\*x]] + 15\*ArcCoth[Tanh[a + b\*x]]^2)/60)

**fricas** [A] time = 0.58, size = 30, normalized size = 0.71

$$\frac{1}{6}b^2x^6 + \frac{2}{5}abx^5 - \frac{1}{16}(\pi^2 - 4a^2)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 1/6\*b^2\*x^6 + 2/5\*a\*b\*x^5 - 1/16\*(pi^2 - 4\*a^2)\*x^4

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x^3\*arccoth(tanh(b\*x + a))^2, x)

**maple** [C] time = 0.39, size = 3418, normalized size = 81.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.



$$\begin{aligned}
& +1))^{3*} \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) + 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{2*} * \text{csgn}(I*\exp(b*x+a))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)) - 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+2*a))^{3*} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} - 1/20 * I * \text{Pi} * b * x^5 * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{2*} + 1/20 * I * \text{Pi} * b * x^5 * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{3*} - 1/16 * \text{Pi}^{2*x^4} - 1/64 * \text{Pi}^{2*x^4} * \text{csgn}(I*\exp(2*b*x+2*a))^{6*} - 1/64 * \text{Pi}^{2*x^4} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{6*} + 1/4 * x^4 * \ln(\exp(b*x+a))^{2*} - 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1)) * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) + 1/20 * I * \text{Pi} * b * x^5 - 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{4*} * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) + 1/32 * \text{Pi}^{2*x^4} * \text{csgn}(I*\exp(b*x+a))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} - 1/40 * I * \text{Pi} * b * x^5 * \text{csgn}(I/(\exp(2*b*x+2*a)+1)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} + 1/40 * I * \text{Pi} * b * x^5 * \text{csgn}(I*\exp(b*x+a))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)) - 1/20 * I * \text{Pi} * b * x^5 * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+2*a))^{2*} - 1/40 * I * \text{Pi} * b * x^5 * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} - 1/16 * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{4*} * \text{Pi}^{2*x^4} - 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{3*} + 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} - 1/64 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{4*} + 1/60 * b^{2*x^6} + 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+2*a))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{3*} + 1/8 * \text{Pi}^{2*x^4} * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+2*a))^{2*} + 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} - 1/8 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{2*} * \text{csgn}(I*\exp(b*x+a)) * \text{csgn}(I*\exp(2*b*x+2*a))^{2*} - 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} - 1/64 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} + 1/32 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)) * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{3*} - 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{3*} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2*} + 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{3*} - 1/16 * \text{Pi}^{2*x^4} * \text{csgn}(I/(\exp(2*b*x+2*a)+1))^{3*} * \text{csgn}(I*\exp(b*x+a))^{2*} * \text{csgn}(I*\exp(2*b*x+2*a)) + 1/40 * I * \text{Pi} * b * x^5 * \text{csgn}(I*\exp(2*b*x+2*a))^{3*} + 1/40 * I * \text{Pi} * b * x^5 * \text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{3*}
\end{aligned}$$

**maxima** [A] time = 0.45, size = 36, normalized size = 0.86

$$\frac{1}{60} b^2 x^6 - \frac{1}{10} b x^5 \operatorname{arccoth}(\tanh(bx + a)) + \frac{1}{4} x^4 \operatorname{arccoth}(\tanh(bx + a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 1/60\*b^2\*x^6 - 1/10\*b\*x^5\*arccoth(tanh(b\*x + a)) + 1/4\*x^4\*arccoth(tanh(b\*x + a))^2

**mupad [B]** time = 1.18, size = 36, normalized size = 0.86

$$\frac{b^2 x^6}{60} - \frac{b x^5 \operatorname{acoth}(\tanh(a + b x))}{10} + \frac{x^4 \operatorname{acoth}(\tanh(a + b x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*acoth(tanh(a + b*x))^2,x)`

[Out] `(x^4*acoth(tanh(a + b*x))^2)/4 + (b^2*x^6)/60 - (b*x^5*acoth(tanh(a + b*x)))/10`

**sympy [A]** time = 2.62, size = 78, normalized size = 1.86

$$\begin{cases} \frac{x^3 \operatorname{acoth}^3(\tanh(a+bx))}{3b} - \frac{x^2 \operatorname{acoth}^4(\tanh(a+bx))}{4b^2} + \frac{x \operatorname{acoth}^5(\tanh(a+bx))}{10b^3} - \frac{\operatorname{acoth}^6(\tanh(a+bx))}{60b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}^2(\tanh(a))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(tanh(b*x+a))**2,x)`

[Out] `Piecewise((x**3*acoth(tanh(a + b*x))**3/(3*b) - x**2*acoth(tanh(a + b*x))**4/(4*b**2) + x*acoth(tanh(a + b*x))**5/(10*b**3) - acoth(tanh(a + b*x))**6/(60*b**4), Ne(b, 0)), (x**4*acoth(tanh(a))**2/4, True))`

### 3.138 $\int x^2 \coth^{-1}(\tanh(a + bx))^2 dx$

Optimal. Leaf size=42

$$-\frac{1}{6}bx^4 \coth^{-1}(\tanh(a + bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2 + \frac{b^2x^5}{30}$$

[Out]  $1/30*b^2*x^5-1/6*b*x^4*\operatorname{arccoth}(\tanh(b*x+a))+1/3*x^3*\operatorname{arccoth}(\tanh(b*x+a))^2$

**Rubi [A]** time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$-\frac{1}{6}bx^4 \coth^{-1}(\tanh(a + bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2 + \frac{b^2x^5}{30}$$

Antiderivative was successfully verified.

[In] `Int[x^2*ArcCoth[Tanh[a + b*x]]^2,x]`

[Out]  $(b^2*x^5)/30 - (b*x^4*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/6 + (x^3*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2)/3$

#### Rule 30

`Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps



$$\begin{aligned}
\int x^2 \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2 - \frac{1}{3}(2b) \int x^3 \coth^{-1}(\tanh(a + bx)) dx \\
&= -\frac{1}{6}bx^4 \coth^{-1}(\tanh(a + bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{6}b^2 \int x^4 dx \\
&= \frac{b^2x^5}{30} - \frac{1}{6}bx^4 \coth^{-1}(\tanh(a + bx)) + \frac{1}{3}x^3 \coth^{-1}(\tanh(a + bx))^2
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 37, normalized size = 0.88

$$\frac{1}{30}x^3 \left( -5bx \coth^{-1}(\tanh(a + bx)) + 10 \coth^{-1}(\tanh(a + bx))^2 + b^2x^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (x^3\*(b^2\*x^2 - 5\*b\*x\*ArcCoth[Tanh[a + b\*x]] + 10\*ArcCoth[Tanh[a + b\*x]]^2)/30)

**fricas [A]** time = 0.53, size = 30, normalized size = 0.71

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 - \frac{1}{12}(\pi^2 - 4a^2)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 1/5\*b^2\*x^5 + 1/2\*a\*b\*x^4 - 1/12\*(pi^2 - 4\*a^2)\*x^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x^2\*arccoth(tanh(b\*x + a))^2, x)

**maple [C]** time = 0.39, size = 3418, normalized size = 81.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.



```

x+2*a)/(exp(2*b*x+2*a)+1))^4+1/24*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^4*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^3
*csgn(I/(exp(2*b*x+2*a)+1))^3-1/48*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))^2*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+1/24*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^5-1/48*Pi^2*x^3*csgn(I*exp(b*x+a))^4*csgn(I*exp(2*b*x+2*a))^2+1/12*Pi^2*x^3*csgn(I*exp(b*x+a))^3*csgn(I*exp(2*b*x+2*a))^3-1/8*Pi^2*x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))^4+1/12*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^5+1/12*Pi^2*x^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/48*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+1/6*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12*Pi^2*x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/24*I*Pi*b*x^4*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/12*Pi^2*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/24*Pi^2*x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/12*Pi^2*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/12*Pi^2*x^3*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)+1))^2-1/12*Pi^2*x^3+1/3*x^3*ln(exp(b*x+a))^2+1/24*I*Pi*b*x^4*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+(-1/6*b*x^4+1/3*I*Pi*x^3*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/6*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/3*I*Pi*x^3+1/6*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/3*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))^3+1/6*I*Pi*x^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/6*I*Pi*x^3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/3*I*Pi*x^3*csgn(I/(exp(2*b*x+2*a)+1))^2-1/6*I*Pi*x^3*csgn(I*exp(2*b*x+2*a))^3)*ln(exp(b*x+a))

```

**maxima [A]** time = 0.45, size = 36, normalized size = 0.86

$$\frac{1}{30} b^2 x^5 - \frac{1}{6} b x^4 \operatorname{arccoth}(\tanh(bx + a)) + \frac{1}{3} x^3 \operatorname{arccoth}(\tanh(bx + a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 1/30\*b^2\*x^5 - 1/6\*b\*x^4\*arccoth(tanh(b\*x + a)) + 1/3\*x^3\*arccoth(tanh(b\*x + a))^2

**mupad [B]** time = 1.18, size = 36, normalized size = 0.86

$$\frac{b^2 x^5}{30} - \frac{b x^4 \operatorname{acoth}(\tanh(a + b x))}{6} + \frac{x^3 \operatorname{acoth}(\tanh(a + b x))^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(tanh(a + b*x))^2,x)`

[Out] `(x^3*acoth(tanh(a + b*x))^2)/3 + (b^2*x^5)/30 - (b*x^4*acoth(tanh(a + b*x)))/6`

**sympy [A]** time = 1.26, size = 60, normalized size = 1.43

$$\begin{cases} \frac{x^2 \operatorname{acoth}^3(\tanh(a+bx))}{3b} - \frac{x \operatorname{acoth}^4(\tanh(a+bx))}{6b^2} + \frac{\operatorname{acoth}^5(\tanh(a+bx))}{30b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acoth}^2(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(tanh(b*x+a))**2,x)`

[Out] `Piecewise((x**2*acoth(tanh(a + b*x))**3/(3*b) - x*acoth(tanh(a + b*x))**4/(6*b**2) + acoth(tanh(a + b*x))**5/(30*b**3), Ne(b, 0)), (x**3*acoth(tanh(a))**2/3, True))`

### 3.139 $\int x \coth^{-1}(\tanh(a + bx))^2 dx$

Optimal. Leaf size=34

$$\frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a + bx))^4}{12b^2}$$

[Out]  $1/3*x*\operatorname{arccoth}(\tanh(b*x+a))^3/b-1/12*\operatorname{arccoth}(\tanh(b*x+a))^4/b^2$

**Rubi** [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a + bx))^4}{12b^2}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[Tanh[a + b*x]]^2,x]`

[Out] `(x*ArcCoth[Tanh[a + b*x]]^3)/(3*b) - ArcCoth[Tanh[a + b*x]]^4/(12*b^2)`

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2157

`Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\int \coth^{-1}(\tanh(a + bx))^3 dx}{3b} \\ &= \frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\text{Subst}\left(\int x^3 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{3b^2} \\ &= \frac{x \coth^{-1}(\tanh(a + bx))^3}{3b} - \frac{\coth^{-1}(\tanh(a + bx))^4}{12b^2} \end{aligned}$$

**Mathematica [B]** time = 0.08, size = 74, normalized size = 2.18

$$\frac{(a + bx) \left( 4(2a^2 + abx - b^2x^2) \coth^{-1}(\tanh(a + bx)) - ((3a - bx)(a + bx)^2) - 6(a - bx) \coth^{-1}(\tanh(a + bx))^2 \right)}{12b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] ((a + b\*x)\*(-(3\*a - b\*x)\*(a + b\*x)^2) + 4\*(2\*a^2 + a\*b\*x - b^2\*x^2)\*ArcCoth[Tanh[a + b\*x]] - 6\*(a - b\*x)\*ArcCoth[Tanh[a + b\*x]]^2)/(12\*b^2)

**fricas [A]** time = 0.67, size = 30, normalized size = 0.88

$$\frac{1}{4}b^2x^4 + \frac{2}{3}abx^3 - \frac{1}{8}(\pi^2 - 4a^2)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 1/4\*b^2\*x^4 + 2/3\*a\*b\*x^3 - 1/8\*(pi^2 - 4\*a^2)\*x^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x\*arccoth(tanh(b\*x + a))^2, x)

**maple [C]** time = 0.40, size = 3418, normalized size = 100.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x \cdot \text{arccoth}(\tanh(b \cdot x + a))^2, x)$

[Out]  $\frac{1}{12} I \pi b^3 x^3 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right) - \frac{1}{8} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^4 + \frac{1}{16} \pi^2 x^2 \text{csgn}(I \exp(bx+a))^2 \text{csgn}(I \exp(2bx+2a))^2 \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 + \frac{1}{16} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \text{csgn}(I \exp(2bx+2a))^2 \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 + \frac{1}{16} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{16} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \text{csgn}(I \exp(2bx+2a))^4 \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right) + \frac{1}{16} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \text{csgn}(I \exp(2bx+2a))^3 \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 + \frac{1}{4} \pi^2 x^2 \text{csgn}(I \exp(bx+a)) \text{csgn}(I \exp(2bx+2a))^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{8} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 + \frac{1}{12} I \pi b^3 x^3 \text{csgn}(I \exp(2bx+2a))^3 - \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^3 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^3 + \frac{1}{8} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 - \frac{1}{8} \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^4 \pi^2 x^2 - \frac{1}{16} \pi^2 x^2 \text{csgn}(I \exp(bx+a))^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(bx+a))^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a))^3 \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 + \frac{1}{16} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^5 - \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a))^3 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{32} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a))^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^4 - \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^3 + \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a))^3 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^2 + \frac{1}{12} I \pi b^3 x^3 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 + \frac{1}{8} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^4 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 + \frac{1}{16} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^5 + \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(bx+a)) \text{csgn}(I \exp(2bx+2a))^5 + \frac{1}{16} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a))^4 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 - \frac{1}{16} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a))^3 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{32} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^4 + \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^3 + \frac{1}{8} \pi^2 x^2 \text{csgn}(I \exp(bx+a)) \text{csgn}(I \exp(2bx+2a))^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 - \frac{1}{6} I \pi b^3 x^3 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^2 + \frac{1}{4} \pi^2 x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right) \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^2 - \frac{1}{4} I \pi x^2 \text{csgn}(I \exp(2bx+2a)) \text{csgn}\left(\frac{I \exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^3 + \frac{1}{2} I \pi x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^2 - \frac{1}{2} I \pi x^2 \text{csgn}\left(\frac{I}{\exp(2bx+2a)+1}\right)^3 + \frac{1}{2} I \pi x^2 \text{csgn}(I \exp(bx+a)) \text{csgn}(I \exp(2bx+2a))^2 - \frac{1}{4} I \pi x^2 \text{cs}$





**mupad [B]** time = 1.15, size = 36, normalized size = 1.06

$$\frac{b^2 x^4}{12} - \frac{b x^3 \operatorname{acoth}(\tanh(a + b x))}{3} + \frac{x^2 \operatorname{acoth}(\tanh(a + b x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(tanh(a + b*x))^2,x)`

[Out] `(x^2*acoth(tanh(a + b*x))^2)/2 + (b^2*x^4)/12 - (b*x^3*acoth(tanh(a + b*x)))/3`

**sympy [A]** time = 0.67, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x \operatorname{acoth}^3(\tanh(a+bx))}{3b} - \frac{\operatorname{acoth}^4(\tanh(a+bx))}{12b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acoth}^2(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(tanh(b*x+a))**2,x)`

[Out] `Piecewise((x*acoth(tanh(a + b*x))**3/(3*b) - acoth(tanh(a + b*x))**4/(12*b**2), Ne(b, 0)), (x**2*acoth(tanh(a))**2/2, True))`

### 3.140 $\int \coth^{-1}(\tanh(a + bx))^2 dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a + bx))^3}{3b}$$

[Out] 1/3\*arccoth(tanh(b\*x+a))^3/b

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] ArcCoth[Tanh[a + b\*x]]^3/(3\*b)

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tanh(a + bx))^2 dx &= \frac{\text{Subst}\left(\int x^2 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\tanh(a + bx))^3}{3b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\coth^{-1}(\tanh(a + bx))^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] ArcCoth[Tanh[a + b\*x]]^3/(3\*b)

**fricas** [A] time = 0.63, size = 27, normalized size = 1.69

$$\frac{1}{3} b^2 x^3 + a b x^2 - \frac{1}{4} (\pi^2 - 4 a^2) x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 1/3\*b^2\*x^3 + a\*b\*x^2 - 1/4\*(pi^2 - 4\*a^2)\*x

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^2, x)

**maple** [A] time = 0.09, size = 15, normalized size = 0.94

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^2,x)

[Out] 1/3\*arccoth(tanh(b\*x+a))^3/b

**maxima** [B] time = 0.44, size = 33, normalized size = 2.06

$$\frac{1}{3} b^2 x^3 - b x^2 \operatorname{arccoth}(\tanh(bx + a)) + x \operatorname{arccoth}(\tanh(bx + a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 1/3\*b^2\*x^3 - b\*x^2\*arccoth(tanh(b\*x + a)) + x\*arccoth(tanh(b\*x + a))^2

**mupad** [B] time = 1.12, size = 33, normalized size = 2.06

$$\frac{b^2 x^3}{3} - b x^2 \operatorname{acoth}(\tanh(a + b x)) + x \operatorname{acoth}(\tanh(a + b x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^2,x)`

[Out] `x*acoth(tanh(a + b*x))^2 + (b^2*x^3)/3 - b*x^2*acoth(tanh(a + b*x))`

**sympy** [A] time = 0.30, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3b} & \text{for } b \neq 0 \\ x \operatorname{acoth}^2(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**2,x)`

[Out] `Piecewise((acoth(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*acoth(tanh(a))**2, True))`

$$3.141 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x} dx$$

Optimal. Leaf size=49

$$-bx (bx - \coth^{-1}(\tanh(a + bx))) + \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 + \log(x) (bx - \coth^{-1}(\tanh(a + bx)))^2$$

[Out]  $-b*x*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))+1/2*\operatorname{arccoth}(\tanh(b*x+a))^2+(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2*\ln(x)$

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2159, 2158, 29}

$$-bx (bx - \coth^{-1}(\tanh(a + bx))) + \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 + \log(x) (bx - \coth^{-1}(\tanh(a + bx)))^2$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2/x, x]$

[Out]  $-(b*x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])) + \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2/2 + (b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2*\operatorname{Log}[x]$

Rule 29

$\operatorname{Int}[(x_)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[x], x]$

Rule 2158

$\operatorname{Int}[(v_)/(u_), x\_Symbol] \rightarrow \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(b*x)/a, x] - \operatorname{Dist}[(b*u - a*v)/a, \operatorname{Int}[1/u, x], x] /; \operatorname{NeQ}[b*u - a*v, 0]] /; \operatorname{PiecewiseLinearQ}[u, v, x]$

Rule 2159

$\operatorname{Int}[(v_)^{(n_)}(u_), x\_Symbol] \rightarrow \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[v^n/(a*n), x] - \operatorname{Dist}[(b*u - a*v)/a, \operatorname{Int}[v^{(n-1)}/u, x], x] /; \operatorname{NeQ}[b*u - a*v, 0]] /; \operatorname{PiecewiseLinearQ}[u, v, x] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[n, 1]$

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x} dx &= \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 - (bx - \coth^{-1}(\tanh(a+bx))) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} \\ &= -bx (bx - \coth^{-1}(\tanh(a+bx))) + \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 - ((bx - \coth^{-1}(\tanh(a+bx))) \\ &= -bx (bx - \coth^{-1}(\tanh(a+bx))) + \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 + (bx - \coth^{-1}(\tanh(a+bx))) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 53, normalized size = 1.08

$$\frac{1}{2}(a+bx)^2 - (a+bx) \left( -2 \coth^{-1}(\tanh(a+bx)) + a + 2bx \right) + \log(bx) \left( \coth^{-1}(\tanh(a+bx)) - bx \right)^2$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x,x]

[Out] (a + b\*x)^2/2 - (a + b\*x)\*(a + 2\*b\*x - 2\*ArcCoth[Tanh[a + b\*x]]) + (-b\*x) + ArcCoth[Tanh[a + b\*x]]^2\*Log[b\*x]

**fricas [A]** time = 0.62, size = 27, normalized size = 0.55

$$\frac{1}{2} b^2 x^2 + 2 a b x - \frac{1}{4} (\pi^2 - 4 a^2) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x,x, algorithm="fricas")

[Out] 1/2\*b^2\*x^2 + 2\*a\*b\*x - 1/4\*(pi^2 - 4\*a^2)\*log(x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^2/x, x)

**maple [C]** time = 0.33, size = 3774, normalized size = 77.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\text{arccoth}(\tanh(b*x+a))^2/x, x)$

[Out] 
$$\begin{aligned} & -3/2*b^2*x^2+\ln(x)*\ln(\exp(b*x+a))^2-1/2*I*\text{Pi}*b*x*\text{csgn}(I/(\exp(2*b*x+2*a)+1)) \\ & * \text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-1/8*\text{Pi}^2* \\ & \ln(x)*\text{csgn}(I*\exp(2*b*x+2*a))^3*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+ \\ & 1/4*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^4*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b* \\ & x+2*a)+1))^2+1/8*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a) \\ & )/(\exp(2*b*x+2*a)+1))^5-1/4*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3*\text{csgn}(I* \\ & \exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/16*\text{Pi}^2*\ln(x)*\text{csgn}(I*\exp(2*b*x+2*a)) \\ & ^2*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4+1/8*\text{Pi}^2*\ln(x)*\text{csgn}(I*\exp(2* \\ & b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^5-1/4*\text{Pi}^2*\ln(x)*\text{csgn}(I \\ & /(\exp(2*b*x+2*a)+1))^3*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-1/4*\text{Pi}^2 \\ & *\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3*\text{csgn}(I*\exp(2*b*x+2*a))^3+1/4*\text{Pi}^2*\ln(x) \\ & *\text{csgn}(I*\exp(2*b*x+2*a))^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2+1/4*\text{Pi}^2*\ln(x)*\text{csgn}( \\ & I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2+1/4*\text{Pi}^2 \\ & *\ln(x)*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^5+1/8*\text{Pi}^2*\ln(x)*\text{csgn}(I*e \\ & xp(2*b*x+2*a))^4*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-3/8*\text{Pi}^2*\ln(x) \\ & *\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))^4-1/16*\text{Pi}^2*\ln(x)*\text{csgn}(I*\exp(b \\ & *x+a))^4*\text{csgn}(I*\exp(2*b*x+2*a))^2+1/4*\text{Pi}^2*\ln(x)*\text{csgn}(I*\exp(b*x+a))^3*\text{csgn}( \\ & I*\exp(2*b*x+2*a))^3-1/16*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2*\text{csgn}(I*\exp \\ & (2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4+1/4*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1)) \\ & *\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/2*I*\text{Pi}*x*b*\text{csgn}(I*\exp(2*b*x+ \\ & 2*a)/(\exp(2*b*x+2*a)+1))^3+1/8*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I \\ & *\exp(2*b*x+2*a))^2*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-1/4*\text{Pi}^2*\ln(x) \\ & *\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3 \\ & -1/4*\text{Pi}^2*\ln(x)*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^3*\text{csgn}(I*\exp(2*b* \\ & x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/4*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(b*x+a))^ \\ & 2*\text{Pi}^2*\ln(x)+1/2*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^5-I*\text{Pi}*\ln(x)*\ln(\exp( \\ & b*x+a))-I*\text{Pi}*x*b-1/2*I*\text{Pi}*\ln(x)*\ln(\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^3-1/2 \\ & *I*\text{Pi}*\ln(x)*\ln(\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-1/8* \\ & \text{Pi}^2*\ln(x)*\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a) \\ & )/(\exp(2*b*x+2*a)+1))^3-1/16*\text{Pi}^2*\ln(x)*\text{csgn}(I*\exp(2*b*x+2*a))^6-1/16*\text{Pi}^2* \\ & \ln(x)*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^6-1/4*\text{csgn}(I*\exp(2*b*x+2*a) \\ & )^3*\text{Pi}^2*\ln(x)+1/4*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2*\text{csgn}(I*\exp(2 \\ & *b*x+2*a))*\text{Pi}^2*\ln(x)+1/2*\text{csgn}(I*\exp(2*b*x+2*a))^2*\text{csgn}(I*\exp(b*x+a))*\text{Pi}^2* \\ & \ln(x)-1/4*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^4*\text{csgn}(I*\exp(2*b*x+2*a))*\text{cs \\ & gn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))+1/2*I*\text{Pi}*\ln(x)*\ln(\exp(b*x+a))*\text{csgn}( \\ & I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/2*I*\text{Pi} \\ & \ln(x)*\ln(\exp(b*x+a))*\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))-1/4*\text{Pi}^2*\ln \\ & (x)*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+1/2*\text{csgn}(I/(\exp(2*b*x+2*a) \\ & +1))^2*\text{Pi}^2*\ln(x)-1/4*\text{Pi}^2*\ln(x)-2*b*\ln(x)*\ln(\exp(b*x+a))*x-1/2*\text{Pi}^2*\ln(x)* \\ & \text{csgn}(I/(\exp(2*b*x+2*a)+1))^3-1/4*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^4-1/ \\ & 4*\text{Pi}^2*\ln(x)*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^6+b^2*x^2*\ln(x)+2*b*\ln(\exp(b*x+a))* \end{aligned}$$

$$\begin{aligned}
& x+1/2*I*Pi*ln(x)*x*b*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn \\
& (I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/2*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a) \\
& +1))^2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+I*Pi*ln(x)*x*b-1/4*Pi^2* \\
& ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2* \\
& a)/(exp(2*b*x+2*a)+1))+1/8*Pi^2*ln(x)*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x \\
& +2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-I*Pi*x*b*csgn(I/(exp(2 \\
& *b*x+2*a)+1))^3-I*Pi*ln(x)*x*b*csgn(I/(exp(2*b*x+2*a)+1))^2+1/4*Pi^2*ln(x)* \\
& csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^3*csgn \\
& (I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a) \\
& +1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp \\
& (2*b*x+2*a)+1))^2+1/4*Pi^2*ln(x)*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^ \\
& 2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I*Pi*ln(x)*x*b*csgn(I/(exp(2* \\
& b*x+2*a)+1))^3-I*Pi*ln(x)*x*b*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+I \\
& *Pi*ln(x)*ln(exp(b*x+a))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+I*Pi*x \\
& *b*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/2*I*Pi*ln(x)*x*b*csgn(I*exp \\
& (2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/2*I*Pi*x*b*csgn(I*exp(b*x+a))^2*csgn(I \\
& *exp(2*b*x+2*a))+1/2*Pi^2*ln(x)*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2 \\
& *csgn(I/(exp(2*b*x+2*a)+1))^3-1/8*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csg \\
& n(I*exp(2*b*x+2*a))^4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/4*Pi^2*ln \\
& (x)*csgn(I/(exp(2*b*x+2*a)+1))^3*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2* \\
& a)/(exp(2*b*x+2*a)+1))+1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2*csgn(I*exp \\
& (b*x+a))^2*csgn(I*exp(2*b*x+2*a))-I*Pi*ln(x)*ln(exp(b*x+a))*csgn(I/(exp(2 \\
& *b*x+2*a)+1))^3+I*Pi*ln(x)*ln(exp(b*x+a))*csgn(I/(exp(2*b*x+2*a)+1))^2+I*Pi \\
& *x*b*csgn(I/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi*x*b*csgn(I*exp(2*b*x+2*a))^3+1/2 \\
& *I*Pi*ln(x)*ln(exp(b*x+a))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp \\
& (2*b*x+2*a)+1))^2+1/2*I*Pi*x*b*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a) \\
& )/(exp(2*b*x+2*a)+1))^2+1/2*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp( \\
& 2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^ \\
& 2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/4*Pi \\
& ^2*ln(x)*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2 \\
& *csgn(I/(exp(2*b*x+2*a)+1))^3+1/8*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2*c \\
& sgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/8*Pi^2* \\
& ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))^3*csgn(I*exp(2*b*x+ \\
& 2*a)/(exp(2*b*x+2*a)+1))^2-1/16*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))^2*csg \\
& n(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/2*I*Pi* \\
& ln(x)*ln(exp(b*x+a))*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn \\
& (I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+1/2*I*Pi*ln(x)*x*b*csgn(I*exp(b*x+a)) \\
& ^2*csgn(I*exp(2*b*x+2*a))-1/4*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I* \\
& exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^4+1/2*I*Pi*ln(x)* \\
& x*b*csgn(I*exp(2*b*x+2*a))^3+1/8*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn \\
& (I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2* \\
& a)+1))^2-1/8*Pi^2*ln(x)*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))^2*csg \\
& n(I*exp(2*b*x+2*a))^2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/2*I*Pi*ln \\
& (x)*x*b*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1) \\
& )^2-1/2*I*Pi*ln(x)*x*b*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*
\end{aligned}$$



$b*x+2*a)+1))^2$

**maxima** [C] time = 0.73, size = 38, normalized size = 0.78

$$\frac{1}{2}b^2x^2 + \frac{1}{8}(-8i\pi b + 16ab)x - \frac{1}{4}(\pi^2 + 4i\pi a - 4a^2)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x,x, algorithm="maxima")

[Out] 1/2\*b^2\*x^2 + 1/8\*(-8\*I\*pi\*b + 16\*a\*b)\*x - 1/4\*(pi^2 + 4\*I\*pi\*a - 4\*a^2)\*log(x)

**mupad** [B] time = 0.29, size = 183, normalized size = 3.73

$$\ln(x) \left( \frac{\left( 2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx \right)^2}{4} - a \left( 2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2b \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))^2/x,x)

[Out] log(x)\*((2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^2/4 - a\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x) + a^2) + (b^2\*x^2)/2 - b\*x\*(log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(\tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*2/x,x)

[Out] Integral(acoth(tanh(a + b\*x))\*\*2/x, x)

$$3.142 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx$$

Optimal. Leaf size=39

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b \log(x) (bx - \coth^{-1}(\tanh(a+bx))) + 2b^2x$$

[Out] 2\*b^2\*x-arcCoth(tanh(b\*x+a))^2/x-2\*b\*(b\*x-arcCoth(tanh(b\*x+a)))\*ln(x)

**Rubi [A]** time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2158, 29}

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{x} - 2b \log(x) (bx - \coth^{-1}(\tanh(a+bx))) + 2b^2x$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2/x^2, x]

[Out] 2\*b^2\*x - ArcCoth[Tanh[a + b\*x]]^2/x - 2\*b\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2\*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a + bx))^2}{x^2} dx &= -\frac{\coth^{-1}(\tanh(a + bx))^2}{x} + (2b) \int \frac{\coth^{-1}(\tanh(a + bx))}{x} dx \\
&= 2b^2x - \frac{\coth^{-1}(\tanh(a + bx))^2}{x} - (2b(bx - \coth^{-1}(\tanh(a + bx)))) \int \frac{1}{x} dx \\
&= 2b^2x - \frac{\coth^{-1}(\tanh(a + bx))^2}{x} - 2b(bx - \coth^{-1}(\tanh(a + bx))) \log(x)
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 37, normalized size = 0.95

$$-\frac{\coth^{-1}(\tanh(a + bx))^2}{x} + 2b(\log(x) + 1) \coth^{-1}(\tanh(a + bx)) - 2b^2x \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x^2,x]

[Out] -(ArcCoth[Tanh[a + b\*x]]^2/x) - 2\*b^2\*x\*Log[x] + 2\*b\*ArcCoth[Tanh[a + b\*x]]\*(1 + Log[x])

**fricas [A]** time = 0.58, size = 29, normalized size = 0.74

$$\frac{4b^2x^2 + 8abx \log(x) + \pi^2 - 4a^2}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^2,x, algorithm="fricas")

[Out] 1/4\*(4\*b^2\*x^2 + 8\*a\*b\*x\*log(x) + pi^2 - 4\*a^2)/x

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^2,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^2/x^2, x)

**maple [C]** time = 0.27, size = 1095, normalized size = 28.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(tanh(b*x+a))^2/x^2,x)`

[Out]  $2*b^2*x-I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2+1/2*I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))+I*\text{Pi}*\ln(x)*b*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2+I*\text{Pi}*\ln(\exp(b*x+a))/x-1/x*\ln(\exp(b*x+a))^2+1/2*I*\text{Pi}*\ln(x)*b*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/2*I*\text{Pi}*\ln(x)*b*\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))-1/2*I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-2*\ln(x)*x*b^2+2*\ln(x)*\ln(\exp(b*x+a))*b+1/2*I*\text{Pi}*\ln(x)*b*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/2*I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-1/2*I*\text{Pi}*\ln(x)*b*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-I*\text{Pi}*\ln(x)*b+1/2*I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))+1/16*\text{Pi}^2*(-2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2+\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-\text{csgn}(I/(\exp(2*b*x+2*a)+1))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+2*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3+\text{csgn}(I*\exp(b*x+a))^2*\text{csgn}(I*\exp(2*b*x+2*a))-2*\text{csgn}(I*\exp(b*x+a))*\text{csgn}(I*\exp(2*b*x+2*a))^2+\text{csgn}(I*\exp(2*b*x+2*a))^3-\text{csgn}(I*\exp(2*b*x+2*a))*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+2)^2/x+1/2*I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I*\exp(2*b*x+2*a))^3+1/2*I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-I*\text{Pi}*\ln(x)*b*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3+I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^3-1/2*I*\text{Pi}*\ln(x)*b*\text{csgn}(I*\exp(2*b*x+2*a))^3-I*\text{Pi}*\ln(\exp(b*x+a))/x*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2+I*\text{Pi}*\ln(x)*b*\text{csgn}(I/(\exp(2*b*x+2*a)+1))^2-1/2*I*\text{Pi}*\ln(x)*b*\text{csgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3$

**maxima** [A] time = 0.38, size = 54, normalized size = 1.38

$$2b \operatorname{arccoth}(\tanh(bx+a)) \log(x) - 2 \left( b \left( x + \frac{a}{b} \right) \log(x) - b \left( x + \frac{a \log(x)}{b} \right) \right) b - \frac{\operatorname{arccoth}(\tanh(bx+a))^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="maxima")`

[Out]  $2*b*\operatorname{arccoth}(\tanh(b*x+a))*\log(x) - 2*(b*(x+a/b)*\log(x) - b*(x+a*\log(x)/b))*b - \operatorname{arccoth}(\tanh(b*x+a))^2/x$

**mupad** [B] time = 1.25, size = 207, normalized size = 5.31

$$b \ln\left(\frac{e^{2bx}}{e^{2a}e^{2bx}-1}\right) - \frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)^2}{4x} - b \ln\left(\frac{1}{e^{2a}e^{2bx}-1}\right) - \frac{\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)^2}{4x} + b \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) \ln(x) - b \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^2/x^2,x)`

[Out] `b*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2/(4*x) - b*log(1/(exp(2*a)*exp(2*b*x) - 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1))^2/(4*x) + b*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(x) - b*log(-2/(exp(2*a)*exp(2*b*x) - 1))*log(x) + (log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-2/(exp(2*a)*exp(2*b*x) - 1)))/(2*x) - 2*b^2*x*log(x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^2(\tanh(a + bx))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**2/x**2,x)`

[Out] `Integral(acoth(tanh(a + b*x))**2/x**2, x)`

$$3.143 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx$$

Optimal. Leaf size=36

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b \coth^{-1}(\tanh(a+bx))}{x} + b^2 \log(x)$$

[Out]  $-b \operatorname{arccoth}(\tanh(bx+a))/x - 1/2 \operatorname{arccoth}(\tanh(bx+a))^2/x^2 + b^2 \ln(x)$

**Rubi [A]** time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 29}

$$-\frac{\coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{b \coth^{-1}(\tanh(a+bx))}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2/x^3, x]

[Out]  $-((b \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/x) - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2/(2*x^2) + b^2 \operatorname{Log}[x]$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2\*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a + bx))^2}{x^3} dx &= -\frac{\coth^{-1}(\tanh(a + bx))^2}{2x^2} + b \int \frac{\coth^{-1}(\tanh(a + bx))}{x^2} dx \\ &= -\frac{b \coth^{-1}(\tanh(a + bx))}{x} - \frac{\coth^{-1}(\tanh(a + bx))^2}{2x^2} + b^2 \int \frac{1}{x} dx \\ &= -\frac{b \coth^{-1}(\tanh(a + bx))}{x} - \frac{\coth^{-1}(\tanh(a + bx))^2}{2x^2} + b^2 \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 42, normalized size = 1.17

$$\frac{2bx \coth^{-1}(\tanh(a + bx)) + \coth^{-1}(\tanh(a + bx))^2 - b^2 x^2 (2 \log(x) + 3)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x^3,x]

[Out] -1/2\*(2\*b\*x\*ArcCoth[Tanh[a + b\*x]] + ArcCoth[Tanh[a + b\*x]]^2 - b^2\*x^2\*(3 + 2\*Log[x]))/x^2

**fricas [A]** time = 0.42, size = 29, normalized size = 0.81

$$\frac{8b^2x^2 \log(x) - 16abx + \pi^2 - 4a^2}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^3,x, algorithm="fricas")

[Out] 1/8\*(8\*b^2\*x^2\*log(x) - 16\*a\*b\*x + pi^2 - 4\*a^2)/x^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^2/x^3, x)

**maple [C]** time = 0.43, size = 3213, normalized size = 89.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\text{arccoth}(\tanh(b*x+a))^2/x^3, x)$

[Out] 
$$\begin{aligned} & -1/2/x^2*\ln(\exp(b*x+a))^2-1/4*(4*b*x+2*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1)))^2-I* \\ & Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))+2*I*Pi*csgn(I*\exp(b*x+a))*csgn \\ & (I*\exp(2*b*x+2*a))^2-2*I*Pi-I*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1) \\ & )^3+I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1) \\ & )^2-I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b* \\ & x+2*a)/(\exp(2*b*x+2*a)+1))+I*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a) \\ & )/(\exp(2*b*x+2*a)+1))^2-I*Pi*csgn(I*\exp(2*b*x+2*a))^3-2*I*Pi*csgn(I/(\exp(2* \\ & b*x+2*a)+1))^3)/x^2*\ln(\exp(b*x+a))+1/32*(4*Pi^2-16*I*Pi*b*x*csgn(I/(\exp(2*b \\ & *x+2*a)+1))^2+16*I*Pi*b*x*csgn(I/(\exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I*\exp(b* \\ & x+a))*csgn(I*\exp(2*b*x+2*a))^5-8*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))^5-4*Pi^2*c \\ & sgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+Pi^2*csgn \\ & (I*\exp(b*x+a))^4*csgn(I*\exp(2*b*x+2*a))^2-4*Pi^2*csgn(I*\exp(b*x+a))^3*csgn( \\ & I*\exp(2*b*x+2*a))^3+4*csgn(I*\exp(2*b*x+2*a))^3*Pi^2+4*Pi^2*csgn(I/(\exp(2*b* \\ & x+2*a)+1))^6-8*csgn(I/(\exp(2*b*x+2*a)+1))^2*Pi^2+4*csgn(I*\exp(2*b*x+2*a)/( \\ & \exp(2*b*x+2*a)+1))^3*Pi^2+2*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(b*x+a) \\ & )^2*csgn(I*\exp(2*b*x+2*a))^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-4*P \\ & i^2*csgn(I/(\exp(2*b*x+2*a)+1))^3*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2* \\ & a)/(\exp(2*b*x+2*a)+1))+Pi^2*csgn(I*\exp(2*b*x+2*a))^6+Pi^2*csgn(I*\exp(2*b*x+ \\ & 2*a)/(\exp(2*b*x+2*a)+1))^6-8*Pi^2*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a) \\ & )^2-4*Pi^2*csgn(I*\exp(2*b*x+2*a))^3*csgn(I/(\exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn \\ & (I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+8*I*Pi*x \\ & *b*csgn(I*\exp(2*b*x+2*a))^3+8*I*Pi*x*b*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a) \\ & +1))^3-2*Pi^2*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a) \\ & +1))^5+4*Pi^2*csgn(I*\exp(2*b*x+2*a))^3*csgn(I/(\exp(2*b*x+2*a)+1))^3+Pi^2*csgn \\ & (I*\exp(2*b*x+2*a))^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4+32*b^2*x \\ & ^2*\ln(x)-2*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x \\ & +2*a)+1))^5+4*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))^4+4*Pi^2*csgn(I*\exp(2*b*x+2*a) \\ & )/(\exp(2*b*x+2*a)+1))^2*csgn(I/(\exp(2*b*x+2*a)+1))^3-4*Pi^2*csgn(I*\exp(2*b* \\ & x+2*a)/(\exp(2*b*x+2*a)+1))^3*csgn(I/(\exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I/(\exp \\ & (2*b*x+2*a)+1))^4*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-8*Pi^2*csgn( \\ & I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2*csgn(I/(\exp(2*b*x+2*a)+1))^3-2*Pi^2* \\ & csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))*csgn \\ & (I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-4*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*c \\ & sgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^3*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x \\ & +2*a)+1))+4*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I* \\ & \exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4+4*Pi^2*csgn(I*\exp(b*x+a))^2*csgn(I*\exp( \\ & 2*b*x+2*a))+6*Pi^2*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))^4+2*Pi^2*csgn \\ & (I*\exp(2*b*x+2*a))^3*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+Pi^2*csgn \\ & (I/(\exp(2*b*x+2*a)+1))^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4-2*Pi^2 \\ & *csgn(I*\exp(2*b*x+2*a))^4*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+4*Pi^2 \\ & *csgn(I/(\exp(2*b*x+2*a)+1))^3*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3+ \end{aligned}$$



$$8 * i * \pi * b * x * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1)) + 4 * \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right)^3 * \operatorname{csgn}(i * \exp(b * x + a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) + 16 * i * \pi * x * b + 2 * \pi^2 * \operatorname{csgn}(i * \exp(b * x + a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^3 - 4 * \pi^2 * \operatorname{csgn}(i * \exp(b * x + a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^3 + 4 * \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right)^4 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1)) - 8 * i * \pi * b * x * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 + 8 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right)^3 * \pi^2 - 4 * \pi^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^3 + \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right)^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 - 2 * \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^3 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 - 2 * \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^3 + 4 * \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1)) - 4 * \pi^2 * \operatorname{csgn}(i * \exp(b * x + a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right)^2 + 4 * \pi^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right)^2 + 8 * \pi^2 * \operatorname{csgn}(i * \exp(b * x + a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right)^2 - 2 * \pi^2 * \operatorname{csgn}(i * \exp(b * x + a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 + 4 * \pi^2 * \operatorname{csgn}(i * \exp(b * x + a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^3 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 - 2 * \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right)^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^3 + 2 * \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^4 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1)) + 8 * i * \pi * b * x * \operatorname{csgn}(i * \exp(b * x + a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) + 4 * \pi^2 * \operatorname{csgn}\left(\frac{1}{\exp(2 * b * x + 2 * a) + 1}\right) * \operatorname{csgn}(i * \exp(b * x + a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^2 * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 - 8 * i * \pi * b * x * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a) / (\exp(2 * b * x + 2 * a) + 1))^2 - 16 * i * \pi * b * x * \operatorname{csgn}(i * \exp(b * x + a)) * \operatorname{csgn}(i * \exp(2 * b * x + 2 * a))^2) / x^2$$

**maxima** [A] time = 0.46, size = 34, normalized size = 0.94

$$b^2 \log(x) - \frac{b \operatorname{arccoth}(\tanh(bx + a))}{x} - \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^3,x, algorithm="maxima")

[Out] b^2\*log(x) - b\*arccoth(tanh(b\*x + a))/x - 1/2\*arccoth(tanh(b\*x + a))^2/x^2

**mupad** [B] time = 1.16, size = 34, normalized size = 0.94

$$b^2 \ln(x) - \frac{\frac{\operatorname{arccoth}(\tanh(a+bx))^2}{2} + bx \operatorname{arccoth}(\tanh(a + bx))}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^2/x^3,x)`

[Out]  $b^2 \log(x) - (\operatorname{acoth}(\tanh(a + bx))^2/2 + b*x*\operatorname{acoth}(\tanh(a + bx)))/x^2$

sympy [A] time = 0.57, size = 32, normalized size = 0.89

$$b^2 \log(x) - \frac{b \operatorname{acoth}(\tanh(a + bx))}{x} - \frac{\operatorname{acoth}^2(\tanh(a + bx))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**2/x**3,x)`

[Out]  $b**2*\log(x) - b*\operatorname{acoth}(\tanh(a + b*x))/x - \operatorname{acoth}(\tanh(a + b*x))**2/(2*x**2)$

$$3.144 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx$$

Optimal. Leaf size=31

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3 (bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out]  $1/3*\operatorname{arccoth}(\tanh(b*x+a))^3/x^3/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))$

**Rubi** [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3 (bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2/x^4, x]

[Out] ArcCoth[Tanh[a + b\*x]]^3/(3\*x^3\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

Rule 2167

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx = \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3 (bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica** [A] time = 0.05, size = 34, normalized size = 1.10

$$\frac{bx \coth^{-1}(\tanh(a+bx)) + \coth^{-1}(\tanh(a+bx))^2 + b^2 x^2}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x^4, x]

[Out]  $-1/3*(b^2*x^2 + b*x*\text{ArcCoth}[\text{Tanh}[a + b*x]] + \text{ArcCoth}[\text{Tanh}[a + b*x]]^2)/x^3$   
**fricas** [A] time = 1.44, size = 29, normalized size = 0.94

$$\frac{12b^2x^2 + 12abx - \pi^2 + 4a^2}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="fricas")`

[Out]  $-1/12*(12*b^2*x^2 + 12*a*b*x - \pi^2 + 4*a^2)/x^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(\tanh(bx + a))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="giac")`

[Out] `integrate(arccoth(tanh(b*x + a))^2/x^4, x)`

**maple** [C] time = 0.39, size = 3217, normalized size = 103.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(tanh(b*x+a))^2/x^4,x)`

[Out]  $-1/3/x^3*\ln(\exp(b*x+a))^2 - 1/6*(2*b*x - I*\text{Pi}*c\text{sgn}(I*\exp(b*x+a))^2*c\text{sgn}(I*\exp(2*b*x+2*a)) + 2*I*\text{Pi}*c\text{sgn}(I*\exp(b*x+a))*c\text{sgn}(I*\exp(2*b*x+2*a))^2 + 2*I*\text{Pi}*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^2 - 2*I*\text{Pi} - I*\text{Pi}*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3 + I*\text{Pi}*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 - 2*I*\text{Pi}*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^3 - I*\text{Pi}*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1)) + I*\text{Pi}*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 - I*\text{Pi}*c\text{sgn}(I*\exp(2*b*x+2*a))^3)/x^3*\ln(\exp(b*x+a)) - 1/48*(-4*\text{Pi}^2 + 16*b^2*x^2 + 4*\text{Pi}^2*c\text{sgn}(I*\exp(b*x+a))*c\text{sgn}(I*\exp(2*b*x+2*a))^5 + 8*\text{Pi}^2*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^5 + 4*\text{Pi}^2*c\text{sgn}(I*\exp(2*b*x+2*a))*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2 - \text{Pi}^2*c\text{sgn}(I*\exp(b*x+a))^4*c\text{sgn}(I*\exp(2*b*x+2*a))^2 + 4*\text{Pi}^2*c\text{sgn}(I*\exp(b*x+a))^3*c\text{sgn}(I*\exp(2*b*x+2*a))^3 - 4*c\text{sgn}(I*\exp(2*b*x+2*a))^3*\text{Pi}^2 - 4*\text{Pi}^2*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^6 + 8*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))^2*\text{Pi}^2 - 4*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3*\text{Pi}^2 - 2*\text{Pi}^2*c\text{sgn}(I/(\exp(2*b*x+2*a)+1))*c\text{sgn}(I*\exp(b*x+a))^2*c\text{sgn}(I*\exp(2*b*x+2*a))^2*c\text{sgn}(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))$

$$\begin{aligned}
&)+4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^3\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))-\pi^2\text{csgn}(I\exp(2bx+2a))^6-\pi^2\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^6+8\pi^2\text{csgn}(I\exp(bx+a))*\text{csgn}(I\exp(2bx+2a))^2+4\pi^2\text{csgn}(I\exp(2bx+2a))^3\text{csgn}(I/(\exp(2bx+2a)+1))^2+4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+2\pi^2\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^5-4\pi^2\text{csgn}(I\exp(2bx+2a))^3\text{csgn}(I/(\exp(2bx+2a)+1))^3-\pi^2\text{csgn}(I\exp(2bx+2a))^2\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^4-8I\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^5-4I\pi^2\text{csgn}(I\exp(2bx+2a))^3-4I\pi^2\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-4I\pi^2\text{csgn}(I\exp(bx+a))^2\text{csgn}(I\exp(2bx+2a))+8I\pi^2\text{csgn}(I\exp(bx+a))*\text{csgn}(I\exp(2bx+2a))^2-4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^4-4\pi^2\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2\text{csgn}(I/(\exp(2bx+2a)+1))^3+4\pi^2\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3\text{csgn}(I/(\exp(2bx+2a)+1))^2+4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^4\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+8\pi^2\text{csgn}(I\exp(bx+a))*\text{csgn}(I\exp(2bx+2a))^2\text{csgn}(I/(\exp(2bx+2a)+1))^3+2\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(bx+a))^2\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(bx+a))*\text{csgn}(I\exp(2bx+2a))^3\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))+4I\pi^2\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+4I\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^4+8I\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^2-8I\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^3-4\pi^2\text{csgn}(I\exp(bx+a))^2\text{csgn}(I\exp(2bx+2a))-6\pi^2\text{csgn}(I\exp(bx+a))^2\text{csgn}(I\exp(2bx+2a))^4-2\pi^2\text{csgn}(I\exp(2bx+2a))^3\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-4I\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))-\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^2\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^4+2\pi^2\text{csgn}(I\exp(2bx+2a))^4\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^3\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2-4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^3\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-4\pi^2\text{csgn}(I\exp(bx+a))^2\text{csgn}(I\exp(2bx+2a))-2\pi^2\text{csgn}(I\exp(bx+a))^2\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3+4\pi^2\text{csgn}(I\exp(bx+a))*\text{csgn}(I\exp(2bx+2a))^2\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^4\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))-8\text{csgn}(I/(\exp(2bx+2a)+1))^3\pi^2+4\pi^2\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2\text{csgn}(I/(\exp(2bx+2a)+1))^3-\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))^2\text{csgn}(I\exp(2bx+2a))^2\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+2\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(2bx+2a))^3\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2+2\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3-4\pi^2\text{csgn}(I/(\exp(2bx+2a)+1))*\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))+4\pi^2\text{csgn}(I\exp(bx+a))^2\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I/(\exp(2bx+2a)+1))^2-4\pi^2\text{csgn}(I\exp(2bx+2a))*\text{csgn}(I\exp(2bx+2a)
\end{aligned}$$

)/(exp(2\*b\*x+2\*a)+1))^2\*csgn(I/(exp(2\*b\*x+2\*a)+1))^2-8\*Pi^2\*csgn(I\*exp(b\*x+a))\*csgn(I\*exp(2\*b\*x+2\*a))^2\*csgn(I/(exp(2\*b\*x+2\*a)+1))^2+2\*Pi^2\*csgn(I\*exp(b\*x+a))^2\*csgn(I\*exp(2\*b\*x+2\*a))^2\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2-4\*Pi^2\*csgn(I\*exp(b\*x+a))\*csgn(I\*exp(2\*b\*x+2\*a))^3\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2+2\*Pi^2\*csgn(I/(exp(2\*b\*x+2\*a)+1))^2\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^3-2\*Pi^2\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))^4\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))-4\*Pi^2\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(b\*x+a))\*csgn(I\*exp(2\*b\*x+2\*a))^2\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2)/x^3

**maxima** [A] time = 0.45, size = 36, normalized size = 1.16

$$-\frac{b^2}{3x} - \frac{b \operatorname{arccoth}(\tanh(bx + a))}{3x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^4,x, algorithm="maxima")

[Out] -1/3\*b^2/x - 1/3\*b\*arccoth(tanh(b\*x + a))/x^2 - 1/3\*arccoth(tanh(b\*x + a))^2/x^3

**mupad** [B] time = 1.12, size = 32, normalized size = 1.03

$$-\frac{b^2 x^2 + b x \operatorname{acoth}(\tanh(a + b x)) + \operatorname{acoth}(\tanh(a + b x))^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))^2/x^4,x)

[Out] -(acoth(tanh(a + b\*x))^2 + b^2\*x^2 + b\*x\*acoth(tanh(a + b\*x)))/(3\*x^3)

**sympy** [A] time = 0.87, size = 37, normalized size = 1.19

$$-\frac{b^2}{3x} - \frac{b \operatorname{acoth}(\tanh(a + b x))}{3x^2} - \frac{\operatorname{acoth}^2(\tanh(a + b x))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*2/x\*\*4,x)

[Out] -b\*\*2/(3\*x) - b\*acoth(tanh(a + b\*x))/(3\*x\*\*2) - acoth(tanh(a + b\*x))\*\*2/(3\*x\*\*3)

$$3.145 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} dx$$

Optimal. Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4(bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \coth^{-1}(\tanh(a+bx))^3}{12x^3(bx - \coth^{-1}(\tanh(a+bx)))^2}$$

[Out] 1/12\*b\*arccoth(tanh(b\*x+a))^3/x^3/(b\*x-arccoth(tanh(b\*x+a)))^2+1/4\*arccoth(tanh(b\*x+a))^3/x^4/(b\*x-arccoth(tanh(b\*x+a)))

**Rubi** [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2171, 2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4(bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \coth^{-1}(\tanh(a+bx))^3}{12x^3(bx - \coth^{-1}(\tanh(a+bx)))^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^2/x^5,x]

[Out] (b\*ArcCoth[Tanh[a + b\*x]]^3)/(12\*x^3\*(b\*x - ArcCoth[Tanh[a + b\*x]]))^2 + ArcCoth[Tanh[a + b\*x]]^3/(4\*x^4\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

#### Rule 2167

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]

#### Rule 2171

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] + Dist[(b\*(m + n + 2))/((m + 1)\*(b\*u - a\*v)), Int[u^(m + 1)\*v^n, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -1]

#### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^5} dx = \frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^4} dx}{4 (bx - \coth^{-1}(\tanh(a+bx)))}$$

$$= \frac{b \coth^{-1}(\tanh(a+bx))^3}{12x^3 (bx - \coth^{-1}(\tanh(a+bx)))^2} + \frac{\coth^{-1}(\tanh(a+bx))^3}{4x^4 (bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica** [A] time = 0.03, size = 37, normalized size = 0.58

$$-\frac{2bx \coth^{-1}(\tanh(a+bx)) + 3 \coth^{-1}(\tanh(a+bx))^2 + b^2 x^2}{12x^4}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^2/x^5,x]

[Out] -1/12\*(b^2\*x^2 + 2\*b\*x\*ArcCoth[Tanh[a + b\*x]] + 3\*ArcCoth[Tanh[a + b\*x]]^2)/x^4

**fricas** [A] time = 0.61, size = 29, normalized size = 0.45

$$-\frac{24 b^2 x^2 + 32 a b x - 3 \pi^2 + 12 a^2}{48 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^5,x, algorithm="fricas")

[Out] -1/48\*(24\*b^2\*x^2 + 32\*a\*b\*x - 3\*pi^2 + 12\*a^2)/x^4

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^2/x^5,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^2/x^5, x)

**maple** [C] time = 0.39, size = 3217, normalized size = 50.27

output too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\text{arccoth}(\tanh(b*x+a))^2/x^5, x)$

[Out] 
$$\begin{aligned} & -1/4/x^4*\ln(\exp(b*x+a))^2-1/24*(4*b*x-3*I*Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))+6*I*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2-6*I*Pi-3*I \\ & *Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-6*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^3+3*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-3*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I \\ & *\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))+6*I*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^2+3*I*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-3*I* \\ & Pi*csgn(I*\exp(2*b*x+2*a))^3)/x^4*\ln(\exp(b*x+a))-1/192*(-12*Pi^2+16*b^2*x^2+12*Pi^2*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^5+24*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))^5+12*Pi^2*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-3*Pi^2*csgn(I*\exp(b*x+a))^4*csgn(I*\exp(2*b*x+2*a))^2+12*Pi^2*csgn(I*\exp(b*x+a))^3*csgn(I*\exp(2*b*x+2*a))^3-12*csgn(I*\exp(2*b*x+2*a))^3*Pi^2-12*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))^6+24*csgn(I/(\exp(2*b*x+2*a)+1))^2*Pi^2-12*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3*Pi^2-6*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))+12*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))^3*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-3*Pi^2*csgn(I*\exp(2*b*x+2*a))^6-3*Pi^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^6+16*I*Pi*b*x*csgn(I/(\exp(2*b*x+2*a)+1))^2-16*I*Pi*b*x*csgn(I/(\exp(2*b*x+2*a)+1))^3+24*Pi^2*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2+12*Pi^2*csgn(I*\exp(2*b*x+2*a))^3*csgn(I/(\exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+6*Pi^2*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^5-12*Pi^2*csgn(I*\exp(2*b*x+2*a))^3*csgn(I/(\exp(2*b*x+2*a)+1))^3-3*Pi^2*csgn(I*\exp(2*b*x+2*a))^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4-16*I*Pi*x*b+6*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^5-12*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))^4-12*Pi^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2*csgn(I/(\exp(2*b*x+2*a)+1))^3+12*Pi^2*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3*csgn(I/(\exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))^4*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+24*Pi^2*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2*csgn(I/(\exp(2*b*x+2*a)+1))^3+6*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+12*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^3*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-8*I*Pi*b*x*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))+16*I*Pi*b*x*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2+8*I*Pi*b*x*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2+8*I*Pi*b*x*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^2-12*Pi^2*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^4-8*I*Pi*x*b*csgn(I*\exp(2*b*x+2*a))^3-8*I*Pi*x*b*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^3-12*Pi^2*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))-18*Pi^2*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2$$

```

*x+2*a))4-6*Pi2*csgn(I*exp(2*b*x+2*a))3*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))3-8*I*Pi*b*x*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-3*Pi2*csgn(I/(exp(2*b*x+2*a)+1))
2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))4+6*Pi2*csgn(I*exp(2*b*x+2*a)
)4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))2-12*Pi2*csgn(I/(exp(2*b*x+2
*a)+1))3*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))3-12*Pi2*csgn(I/(exp(2
*b*x+2*a)+1))3*csgn(I*exp(b*x+a))2*csgn(I*exp(2*b*x+2*a))-6*Pi2*csgn(I*exp
(b*x+a))2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))3+12*Pi2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))2*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))3-12*Pi2*csgn(I/(exp(2*b*x+2*a)+1))4*csgn(I*exp(2
*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-24*csgn(I/(exp(2*b*x+2
*a)+1))3*Pi2+12*Pi2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)+1))2*csgn(I/(exp(2*b*x+2*a)+1))3-3*Pi2*csgn(I/(exp(2*b*x+2*a)+1
))2*csgn(I*exp(2*b*x+2*a))2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))2+6
*Pi2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))3*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))2+6*Pi2*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b
*x+2*a))2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))3-12*Pi2*csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))+12*Pi2*csgn(I*exp(b*x+a))2*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+
2*a)+1))2-12*Pi2*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+
2*a)+1))2*csgn(I/(exp(2*b*x+2*a)+1))2-24*Pi2*csgn(I*exp(b*x+a))*csgn(I*exp
(2*b*x+2*a))2*csgn(I/(exp(2*b*x+2*a)+1))2+6*Pi2*csgn(I*exp(b*x+a))2*c
sgn(I*exp(2*b*x+2*a))2*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))2-12*Pi2
*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))3*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))2+6*Pi2*csgn(I/(exp(2*b*x+2*a)+1))2*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))3-6*Pi2*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))4*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-12*Pi2
*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))2*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))2/x4

```

**maxima** [A] time = 0.45, size = 36, normalized size = 0.56

$$-\frac{b^2}{12x^2} - \frac{b \operatorname{arccoth}(\tanh(bx+a))}{6x^3} - \frac{\operatorname{arccoth}(\tanh(bx+a))^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))<sup>2</sup>/x<sup>5</sup>,x, algorithm="maxima")

[Out] -1/12\*b<sup>2</sup>/x<sup>2</sup> - 1/6\*b\*arccoth(tanh(b\*x + a))/x<sup>3</sup> - 1/4\*arccoth(tanh(b\*x + a))<sup>2</sup>/x<sup>4</sup>

**mupad** [B] time = 1.17, size = 36, normalized size = 0.56

$$-\frac{\operatorname{acoth}(\tanh(a+bx))^2}{4x^4} - \frac{b^2}{12x^2} - \frac{b \operatorname{acoth}(\tanh(a+bx))}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^2/x^5,x)`

[Out] `- acoth(tanh(a + b*x))^2/(4*x^4) - b^2/(12*x^2) - (b*acoth(tanh(a + b*x)))/(6*x^3)`

**sympy** [A] time = 1.29, size = 39, normalized size = 0.61

$$-\frac{b^2}{12x^2} - \frac{b \operatorname{acoth}(\tanh(a + bx))}{6x^3} - \frac{\operatorname{acoth}^2(\tanh(a + bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**2/x**5,x)`

[Out] `-b**2/(12*x**2) - b*acoth(tanh(a + b*x))/(6*x**3) - acoth(tanh(a + b*x))**2/(4*x**4)`

### 3.146 $\int x^m \coth^{-1}(\tanh(a + bx))^3 dx$

**Optimal.** Leaf size=110

$$\frac{6b^2x^{m+3} \coth^{-1}(\tanh(a + bx))}{m^3 + 6m^2 + 11m + 6} - \frac{3bx^{m+2} \coth^{-1}(\tanh(a + bx))^2}{m^2 + 3m + 2} + \frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^3}{m + 1} - \frac{6b^3x^m}{(m + 1)(m^3 + 9m^2 + 6m + 6)}$$

[Out]  $-6*b^3*x^{(4+m)/(1+m)/(m^3+9*m^2+26*m+24)+6*b^2*x^{(3+m)*\operatorname{arccoth}(\tanh(b*x+a))}/(m^3+6*m^2+11*m+6)-3*b*x^{(2+m)*\operatorname{arccoth}(\tanh(b*x+a))^2/(m^2+3*m+2)+x^{(1+m)*\operatorname{arccoth}(\tanh(b*x+a))^3/(1+m)}$

**Rubi [A]** time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$\frac{6b^2x^{m+3} \coth^{-1}(\tanh(a + bx))}{m^3 + 6m^2 + 11m + 6} - \frac{3bx^{m+2} \coth^{-1}(\tanh(a + bx))^2}{m^2 + 3m + 2} + \frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^3}{m + 1} - \frac{6b^3x^m}{(m + 1)(m^3 + 9m^2 + 6m + 6)}$$

Antiderivative was successfully verified.

[In] Int[x^m\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out]  $(-6*b^3*x^{(4 + m)})/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^{(3 + m)*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]}/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^{(2 + m)*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2}/(2 + 3*m + m^2) + (x^{(1 + m)*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^3}/(1 + m)$

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2168

Int[(u\_)^(m\_.)\*(v\_)^(n\_.), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)\*v^n)/(a\*(m + 1)), x] - Dist[(b\*n)/(a\*(m + 1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

#### Rubi steps

$$\begin{aligned}
\int x^m \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^3}{1 + m} - \frac{(3b) \int x^{1+m} \coth^{-1}(\tanh(a + bx))^2 dx}{1 + m} \\
&= -\frac{3bx^{2+m} \coth^{-1}(\tanh(a + bx))^2}{2 + 3m + m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^3}{1 + m} + \frac{(6b^2) \int x^2 dx}{1 + m} \\
&= \frac{6b^2 x^{3+m} \coth^{-1}(\tanh(a + bx))}{6 + 11m + 6m^2 + m^3} - \frac{3bx^{2+m} \coth^{-1}(\tanh(a + bx))^2}{2 + 3m + m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))^3}{1 + m} \\
&= -\frac{6b^3 x^{4+m}}{(4 + m)(6 + 11m + 6m^2 + m^3)} + \frac{6b^2 x^{3+m} \coth^{-1}(\tanh(a + bx))}{6 + 11m + 6m^2 + m^3} - \frac{3bx^{2+m}}{(4 + m)(6 + 11m + 6m^2 + m^3)}
\end{aligned}$$

**Mathematica [A]** time = 0.28, size = 97, normalized size = 0.88

$$\frac{x^{m+1} (6b^2(m+4)x^2 \coth^{-1}(\tanh(a+bx)) - 3b(m^2+7m+12)x \coth^{-1}(\tanh(a+bx))^2 + (m^3+9m^2+26m+m+1)(m+2)(m+3)(m+4))}{(m+1)(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] (x^(1 + m)\*(-6\*b^3\*x^3 + 6\*b^2\*(4 + m)\*x^2\*ArcCoth[Tanh[a + b\*x]] - 3\*b\*(12 + 7\*m + m^2)\*x\*ArcCoth[Tanh[a + b\*x]]^2 + (24 + 26\*m + 9\*m^2 + m^3)\*ArcCoth[Tanh[a + b\*x]]^3)/((1 + m)\*(2 + m)\*(3 + m)\*(4 + m))

**fricas [A]** time = 0.62, size = 209, normalized size = 1.90

$$\frac{4(b^3 m^3 + 6b^3 m^2 + 11b^3 m + 6b^3)x^4 + 12(ab^2 m^3 + 7ab^2 m^2 + 14ab^2 m + 8ab^2)x^3 + 3(4a^2 b m^3 + 32a^2 b m^2 + 4a^2 b m + 12a^2 b)x^2 + 3(4a^2 b m^3 + 32a^2 b m^2 + 4a^2 b m + 12a^2 b)x + 96a^2 b}{4(m^4 + 10m^3 + 35m^2 + 50m + 24)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/4\*(4\*(b^3\*m^3 + 6\*b^3\*m^2 + 11\*b^3\*m + 6\*b^3)\*x^4 + 12\*(a\*b^2\*m^3 + 7\*a\*b^2\*m^2 + 14\*a\*b^2\*m + 8\*a\*b^2)\*x^3 + 3\*(4\*a^2\*b\*m^3 + 32\*a^2\*b\*m^2 + 76\*a^2\*b\*m - pi^2\*(b\*m^3 + 8\*b\*m^2 + 19\*b\*m + 12\*b) + 48\*a^2\*b)\*x^2 + (4\*a^3\*m^3 + 36\*a^3\*m^2 + 104\*a^3\*m - 3\*pi^2\*(a\*m^3 + 9\*a\*m^2 + 26\*a\*m + 24\*a) + 96\*a^3)\*x)\*x^m/(m^4 + 10\*m^3 + 35\*m^2 + 50\*m + 24)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^m\*arccoth(tanh(b\*x + a))^3, x)

maple [C] time = 9.36, size = 63382, normalized size = 576.20

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*arccoth(tanh(b\*x+a))^3,x)

[Out] result too large to display

maxima [A] time = 0.46, size = 109, normalized size = 0.99

$$-\frac{3bx^2x^m \operatorname{arccoth}(\tanh(bx+a))^2}{(m+2)(m+1)} + \frac{x^{m+1} \operatorname{arccoth}(\tanh(bx+a))^3}{m+1} - \frac{6\left(\frac{b^2x^4x^m}{(m+4)(m+3)(m+2)} - \frac{bx^3x^m \operatorname{arccoth}(\tanh(bx+a))}{(m+3)(m+2)}\right)b}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $-3*b*x^2*x^m*\operatorname{arccoth}(\tanh(b*x + a))^2/((m + 2)*(m + 1)) + x^{(m + 1)}*\operatorname{arccoth}(\tanh(b*x + a))^3/(m + 1) - 6*(b^2*x^4*x^m/((m + 4)*(m + 3)*(m + 2)) - b*x^3*x^m*\operatorname{arccoth}(\tanh(b*x + a))/((m + 3)*(m + 2)))*b/(m + 1)$

mupad [B] time = 1.45, size = 332, normalized size = 3.02

$$\frac{8b^3x^m x^4 (m^3 + 6m^2 + 11m + 6)}{8m^4 + 80m^3 + 280m^2 + 400m + 192} - \frac{xx^m \left( \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx \right)^3 (m^3 + 9m^2 + 26m + 24)}{8m^4 + 80m^3 + 280m^2 + 400m + 192}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*acoth(tanh(a + b\*x))^3,x)

[Out]  $(8*b^3*x^m*x^4*(11*m + 6*m^2 + m^3 + 6))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) - (x*x^m*(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^3*(26*m + 9*m^2 + m^3 + 24))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) - (12*b^2*x^m*x^3*(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)*(14*m + 7*m^2 + m^3 + 8))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) + (6*b*x^m*x^2*(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2*(19*m + 8*m^2 + m^3 + 12))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\left\{ \begin{array}{l} b^3 \log(x) - \frac{b^2 \operatorname{acoth}(\tanh(a+bx))}{x} - \frac{b \operatorname{acoth}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{acoth}^3(\tanh(a+bx))}{3x^3} \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^3} dx \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x^2} dx \\ \int \frac{\operatorname{acoth}^3(\tanh(a+bx))}{x} dx \\ -\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^2mx^3x^m \operatorname{acoth}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24b^2x^3x^m \operatorname{acoth}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{3bm^2x^2x^m \operatorname{acoth}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{21bm^2x^2x^m \operatorname{acoth}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*acoth(tanh(b\*x+a))\*\*3,x)

[Out] Piecewise((b\*\*3\*log(x) - b\*\*2\*acoth(tanh(a + b\*x))/x - b\*acoth(tanh(a + b\*x))\*\*2/(2\*x\*\*2) - acoth(tanh(a + b\*x))\*\*3/(3\*x\*\*3), Eq(m, -4)), (Integral(acoth(tanh(a + b\*x))\*\*3/x\*\*3, x), Eq(m, -3)), (Integral(acoth(tanh(a + b\*x))\*\*3/x\*\*2, x), Eq(m, -2)), (Integral(acoth(tanh(a + b\*x))\*\*3/x, x), Eq(m, -1)), (-6\*b\*\*3\*x\*\*4\*x\*\*m/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) + 6\*b\*\*2\*m\*x\*\*3\*x\*\*m\*acoth(tanh(a + b\*x))/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) + 24\*b\*\*2\*x\*\*3\*x\*\*m\*acoth(tanh(a + b\*x))/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) - 3\*b\*m\*\*2\*x\*\*2\*x\*\*m\*acoth(tanh(a + b\*x))\*\*2/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) - 21\*b\*m\*x\*\*2\*x\*\*m\*acoth(tanh(a + b\*x))\*\*2/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) - 36\*b\*x\*\*2\*x\*\*m\*acoth(tanh(a + b\*x))\*\*2/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) + m\*\*3\*x\*x\*\*m\*acoth(tanh(a + b\*x))\*\*3/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) + 9\*m\*\*2\*x\*x\*\*m\*acoth(tanh(a + b\*x))\*\*3/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) + 26\*m\*x\*x\*\*m\*acoth(tanh(a + b\*x))\*\*3/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24) + 24\*x\*x\*\*m\*acoth(tanh(a + b\*x))\*\*3/(m\*\*4 + 10\*m\*\*3 + 35\*m\*\*2 + 50\*m + 24), True))

### 3.147 $\int x^4 \coth^{-1}(\tanh(a + bx))^3 dx$

Optimal. Leaf size=61

$$\frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a+bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a+bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^3 - \frac{1}{280}b^3x^8$$

[Out]  $-1/280*b^3*x^8+1/35*b^2*x^7*\operatorname{arccoth}(\tanh(b*x+a))-1/10*b*x^6*\operatorname{arccoth}(\tanh(b*x+a))^2+1/5*x^5*\operatorname{arccoth}(\tanh(b*x+a))^3$

**Rubi [A]** time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$\frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a+bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a+bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a+bx))^3 - \frac{1}{280}b^3x^8$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^4*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^3, x]$

[Out]  $-(b^3*x^8)/280 + (b^2*x^7*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/35 - (b*x^6*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2)/10 + (x^5*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^3)/5$

#### Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[x^{(m + 1)}/(m + 1), x] \text{ /; } \operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{NeQ}[m, -1]$

#### Rule 2168

$\operatorname{Int}[(u_)^{(m_.)}*(v_)^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^{(m + 1)}*v^{(n)})/(a*(m + 1)), x] - \operatorname{Dist}[(b*n)/(a*(m + 1)), \operatorname{Int}[u^{(m + 1)}*v^{(n - 1)}, x], x] \text{ /; } \operatorname{NeQ}[b*u - a*v, 0] \text{ /; } \operatorname{FreeQ}[\{m, n\}, x] \ \&\& \ \operatorname{PiecewiseLinearQ}[u, v, x] \ \&\& \ \operatorname{NeQ}[m, -1] \ \&\& \ ((\operatorname{LtQ}[m, -1] \ \&\& \ \operatorname{GtQ}[n, 0] \ \&\& \ !(\operatorname{ILtQ}[m + n, -2] \ \&\& \ (\operatorname{FractionQ}[m] \ || \ \operatorname{GeQ}[2*n + m + 1, 0]))) \ || \ (\operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ \operatorname{LeQ}[n, m]) \ || \ (\operatorname{IGtQ}[n, 0] \ \&\& \ !\operatorname{IntegerQ}[m]) \ || \ (\operatorname{ILtQ}[m, 0] \ \&\& \ !\operatorname{IntegerQ}[n]))$

#### Rubi steps



$$\begin{aligned}
\int x^4 \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3 - \frac{1}{5}(3b) \int x^5 \coth^{-1}(\tanh(a + bx))^2 dx \\
&= -\frac{1}{10}bx^6 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3 + \frac{1}{5}b^2 \int x^6 \coth^{-1}(\tanh(a + bx)) dx \\
&= \frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a + bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3 \\
&= -\frac{1}{280}b^3x^8 + \frac{1}{35}b^2x^7 \coth^{-1}(\tanh(a + bx)) - \frac{1}{10}bx^6 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{5}x^5 \coth^{-1}(\tanh(a + bx))^3
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 54, normalized size = 0.89

$$-\frac{1}{280}x^5 \left( -8b^2x^2 \coth^{-1}(\tanh(a + bx)) + 28bx \coth^{-1}(\tanh(a + bx))^2 - 56 \coth^{-1}(\tanh(a + bx))^3 + b^3x^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] -1/280\*(x^5\*(b^3\*x^3 - 8\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + 28\*b\*x\*ArcCoth[Tanh[a + b\*x]]^2 - 56\*ArcCoth[Tanh[a + b\*x]]^3))

**fricas [A]** time = 0.62, size = 52, normalized size = 0.85

$$\frac{1}{8}b^3x^8 + \frac{3}{7}ab^2x^7 - \frac{1}{8}(\pi^2b - 4a^2b)x^6 - \frac{1}{20}(3\pi^2a - 4a^3)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/8\*b^3\*x^8 + 3/7\*a\*b^2\*x^7 - 1/8\*(pi^2\*b - 4\*a^2\*b)\*x^6 - 1/20\*(3\*pi^2\*a - 4\*a^3)\*x^5

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^4\*arccoth(tanh(b\*x + a))^3, x)

maple [C] time = 1.14, size = 18111, normalized size = 296.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*arccoth(tanh(b*x+a))^3,x)`

[Out] result too large to display

maxima [A] time = 0.52, size = 54, normalized size = 0.89

$$-\frac{1}{10}bx^6 \operatorname{arccoth}(\tanh(bx+a))^2 + \frac{1}{5}x^5 \operatorname{arccoth}(\tanh(bx+a))^3 - \frac{1}{280}(b^2x^8 - 8bx^7 \operatorname{arccoth}(\tanh(bx+a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")`

[Out]  $-1/10*b*x^6*\operatorname{arccoth}(\tanh(b*x+a))^2 + 1/5*x^5*\operatorname{arccoth}(\tanh(b*x+a))^3 - 1/280*(b^2*x^8 - 8*b*x^7*\operatorname{arccoth}(\tanh(b*x+a)))*b$

mupad [B] time = 1.25, size = 53, normalized size = 0.87

$$-\frac{b^3x^8}{280} + \frac{b^2x^7 \operatorname{acoth}(\tanh(a+bx))}{35} - \frac{bx^6 \operatorname{acoth}(\tanh(a+bx))^2}{10} + \frac{x^5 \operatorname{acoth}(\tanh(a+bx))^3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*acoth(tanh(a+b*x))^3,x)`

[Out]  $(x^5*\operatorname{acoth}(\tanh(a+b*x))^3)/5 - (b^3*x^8)/280 - (b*x^6*\operatorname{acoth}(\tanh(a+b*x))^2)/10 + (b^2*x^7*\operatorname{acoth}(\tanh(a+b*x)))/35$

sympy [A] time = 6.88, size = 97, normalized size = 1.59

$$\begin{cases} \frac{x^4 \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{x^3 \operatorname{acoth}^5(\tanh(a+bx))}{5b^2} + \frac{x^2 \operatorname{acoth}^6(\tanh(a+bx))}{10b^3} - \frac{x \operatorname{acoth}^7(\tanh(a+bx))}{35b^4} + \frac{\operatorname{acoth}^8(\tanh(a+bx))}{280b^5} & \text{for } b \neq 0 \\ \frac{x^5 \operatorname{acoth}^3(\tanh(a))}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*acoth(tanh(b*x+a))**3,x)`

[Out] `Piecewise((x**4*acoth(tanh(a+b*x))**4/(4*b) - x**3*acoth(tanh(a+b*x))**5/(5*b**2) + x**2*acoth(tanh(a+b*x))**6/(10*b**3) - x*acoth(tanh(a+b*x))**7/(35*b**4) + acoth(tanh(a+b*x))**8/(280*b**5), Ne(b, 0)), (x**5*acoth(tanh(a))**3/5, True))`

### 3.148 $\int x^3 \coth^{-1}(\tanh(a + bx))^3 dx$

Optimal. Leaf size=61

$$\frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^3 - \frac{1}{140}b^3x^7$$

[Out]  $-1/140*b^3*x^7+1/20*b^2*x^6*\operatorname{arccoth}(\tanh(b*x+a))-3/20*b*x^5*\operatorname{arccoth}(\tanh(b*x+a))^2+1/4*x^4*\operatorname{arccoth}(\tanh(b*x+a))^3$

**Rubi [A]** time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 30}

$$\frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a+bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a+bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a+bx))^3 - \frac{1}{140}b^3x^7$$

Antiderivative was successfully verified.

[In] `Int[x^3*ArcCoth[Tanh[a + b*x]]^3,x]`

[Out]  $-(b^3*x^7)/140 + (b^2*x^6*ArcCoth[Tanh[a + b*x]])/20 - (3*b*x^5*ArcCoth[Tanh[a + b*x]]^2)/20 + (x^4*ArcCoth[Tanh[a + b*x]]^3)/4$

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3 - \frac{1}{4}(3b) \int x^4 \coth^{-1}(\tanh(a + bx))^2 dx \\
&= -\frac{3}{20}bx^5 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3 + \frac{1}{10}(3b^2) \int x^5 \coth^{-1}(\tanh(a + bx)) dx \\
&= \frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a + bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3 \\
&= -\frac{1}{140}b^3x^7 + \frac{1}{20}b^2x^6 \coth^{-1}(\tanh(a + bx)) - \frac{3}{20}bx^5 \coth^{-1}(\tanh(a + bx))^2 + \frac{1}{4}x^4 \coth^{-1}(\tanh(a + bx))^3
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 54, normalized size = 0.89

$$-\frac{1}{140}x^4 \left( -7b^2x^2 \coth^{-1}(\tanh(a + bx)) + 21bx \coth^{-1}(\tanh(a + bx))^2 - 35 \coth^{-1}(\tanh(a + bx))^3 + b^3x^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] -1/140\*(x^4\*(b^3\*x^3 - 7\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + 21\*b\*x\*ArcCoth[Tanh[a + b\*x]]^2 - 35\*ArcCoth[Tanh[a + b\*x]]^3))

**fricas** [A] time = 0.46, size = 52, normalized size = 0.85

$$\frac{1}{7}b^3x^7 + \frac{1}{2}ab^2x^6 - \frac{3}{20}(\pi^2b - 4a^2b)x^5 - \frac{1}{16}(3\pi^2a - 4a^3)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/7\*b^3\*x^7 + 1/2\*a\*b^2\*x^6 - 3/20\*(pi^2\*b - 4\*a^2\*b)\*x^5 - 1/16\*(3\*pi^2\*a - 4\*a^3)\*x^4

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^3\*arccoth(tanh(b\*x + a))^3, x)

**maple** [C] time = 1.28, size = 18111, normalized size = 296.90

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*arccoth(tanh(b*x+a))^3,x)`

[Out] result too large to display

**maxima** [A] time = 0.52, size = 54, normalized size = 0.89

$$-\frac{3}{20}bx^5 \operatorname{arccoth}(\tanh(bx+a))^2 + \frac{1}{4}x^4 \operatorname{arccoth}(\tanh(bx+a))^3 - \frac{1}{140}(b^2x^7 - 7bx^6 \operatorname{arccoth}(\tanh(bx+a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")`

[Out]  $-3/20*b*x^5*\operatorname{arccoth}(\tanh(b*x+a))^2 + 1/4*x^4*\operatorname{arccoth}(\tanh(b*x+a))^3 - 1/140*(b^2*x^7 - 7*b*x^6*\operatorname{arccoth}(\tanh(b*x+a)))*b$

**mupad** [B] time = 1.23, size = 53, normalized size = 0.87

$$-\frac{b^3x^7}{140} + \frac{b^2x^6 \operatorname{acoth}(\tanh(a+bx))}{20} - \frac{3bx^5 \operatorname{acoth}(\tanh(a+bx))^2}{20} + \frac{x^4 \operatorname{acoth}(\tanh(a+bx))^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*acoth(tanh(a+b*x))^3,x)`

[Out]  $(x^4*\operatorname{acoth}(\tanh(a+b*x))^3)/4 - (b^3*x^7)/140 - (3*b*x^5*\operatorname{acoth}(\tanh(a+b*x))^2)/20 + (b^2*x^6*\operatorname{acoth}(\tanh(a+b*x)))/20$

**sympy** [A] time = 4.09, size = 80, normalized size = 1.31

$$\begin{cases} \frac{x^3 \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{3x^2 \operatorname{acoth}^5(\tanh(a+bx))}{20b^2} + \frac{x \operatorname{acoth}^6(\tanh(a+bx))}{20b^3} - \frac{\operatorname{acoth}^7(\tanh(a+bx))}{140b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}^3(\tanh(a))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(tanh(b*x+a))**3,x)`

[Out] `Piecewise((x**3*acoth(tanh(a+b*x))**4/(4*b) - 3*x**2*acoth(tanh(a+b*x))**5/(20*b**2) + x*acoth(tanh(a+b*x))**6/(20*b**3) - acoth(tanh(a+b*x))**7/(140*b**4), Ne(b, 0)), (x**4*acoth(tanh(a))**3/4, True))`

### 3.149 $\int x^2 \coth^{-1}(\tanh(a + bx))^3 dx$

Optimal. Leaf size=53

$$\frac{\coth^{-1}(\tanh(a + bx))^6}{60b^3} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b}$$

[Out]  $1/4*x^2*\operatorname{arccoth}(\tanh(b*x+a))^4/b-1/10*x*\operatorname{arccoth}(\tanh(b*x+a))^5/b^2+1/60*\operatorname{arccoth}(\tanh(b*x+a))^6/b^3$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^6}{60b^3} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b}$$

Antiderivative was successfully verified.

[In] `Int[x^2*ArcCoth[Tanh[a + b*x]]^3,x]`

[Out]  $(x^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^4)/(4*b) - (x*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^5)/(10*b^2) + \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^6/(60*b^3)$

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2157

`Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\int x \coth^{-1}(\tanh(a + bx))^4 dx}{2b} \\
&= \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{\int \coth^{-1}(\tanh(a + bx))^4 dx}{10b^2} \\
&= \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{\text{Subst}\left(\int x^5 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{10b^2} \\
&= \frac{x^2 \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{x \coth^{-1}(\tanh(a + bx))^5}{10b^2} + \frac{\coth^{-1}(\tanh(a + bx))^6}{60b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 54, normalized size = 1.02

$$-\frac{1}{60}x^3 \left(-6b^2x^2 \coth^{-1}(\tanh(a + bx)) + 15bx \coth^{-1}(\tanh(a + bx))^2 - 20 \coth^{-1}(\tanh(a + bx))^3 + b^3x^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] -1/60\*(x^3\*(b^3\*x^3 - 6\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + 15\*b\*x\*ArcCoth[Tanh[a + b\*x]]^2 - 20\*ArcCoth[Tanh[a + b\*x]]^3))

**fricas [A]** time = 0.71, size = 52, normalized size = 0.98

$$\frac{1}{6}b^3x^6 + \frac{3}{5}ab^2x^5 - \frac{3}{16}(\pi^2b - 4a^2b)x^4 - \frac{1}{12}(3\pi^2a - 4a^3)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/6\*b^3\*x^6 + 3/5\*a\*b^2\*x^5 - 3/16\*(pi^2\*b - 4\*a^2\*b)\*x^4 - 1/12\*(3\*pi^2\*a - 4\*a^3)\*x^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^2\*arccoth(tanh(b\*x + a))^3, x)

**maple** [C] time = 1.13, size = 18111, normalized size = 341.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(tanh(b*x+a))^3,x)`

[Out] result too large to display

**maxima** [A] time = 0.52, size = 54, normalized size = 1.02

$$-\frac{1}{4}bx^4 \operatorname{arccoth}(\tanh(bx+a))^2 + \frac{1}{3}x^3 \operatorname{arccoth}(\tanh(bx+a))^3 - \frac{1}{60}(b^2x^6 - 6bx^5 \operatorname{arccoth}(\tanh(bx+a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")`

[Out]  $-1/4*b*x^4*\operatorname{arccoth}(\tanh(b*x + a))^2 + 1/3*x^3*\operatorname{arccoth}(\tanh(b*x + a))^3 - 1/60*(b^2*x^6 - 6*b*x^5*\operatorname{arccoth}(\tanh(b*x + a)))*b$

**mupad** [B] time = 1.21, size = 53, normalized size = 1.00

$$-\frac{b^3 x^6}{60} + \frac{b^2 x^5 \operatorname{acoth}(\tanh(a + bx))}{10} - \frac{b x^4 \operatorname{acoth}(\tanh(a + bx))^2}{4} + \frac{x^3 \operatorname{acoth}(\tanh(a + bx))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(tanh(a + b*x))^3,x)`

[Out]  $(x^3*\operatorname{acoth}(\tanh(a + b*x))^3)/3 - (b^3*x^6)/60 - (b*x^4*\operatorname{acoth}(\tanh(a + b*x))^2)/4 + (b^2*x^5*\operatorname{acoth}(\tanh(a + b*x)))/10$

**sympy** [A] time = 2.38, size = 60, normalized size = 1.13

$$\begin{cases} \frac{x^2 \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{x \operatorname{acoth}^5(\tanh(a+bx))}{10b^2} + \frac{\operatorname{acoth}^6(\tanh(a+bx))}{60b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{acoth}^3(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(tanh(b*x+a))**3,x)`

[Out] `Piecewise((x**2*acoth(tanh(a + b*x))**4/(4*b) - x*acoth(tanh(a + b*x))**5/(10*b**2) + acoth(tanh(a + b*x))**6/(60*b**3), Ne(b, 0)), (x**3*acoth(tanh(a))**3/3, True))`



### 3.150 $\int x \coth^{-1}(\tanh(a + bx))^3 dx$

Optimal. Leaf size=34

$$\frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a + bx))^5}{20b^2}$$

[Out]  $1/4*x*\operatorname{arccoth}(\tanh(b*x+a))^4/b-1/20*\operatorname{arccoth}(\tanh(b*x+a))^5/b^2$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a + bx))^5}{20b^2}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[Tanh[a + b*x]]^3,x]`

[Out]  $(x*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^4)/(4*b) - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^5/(20*b^2)$

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2157

`Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\int \coth^{-1}(\tanh(a + bx))^4 dx}{4b} \\
&= \frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\text{Subst}\left(\int x^4 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{4b^2} \\
&= \frac{x \coth^{-1}(\tanh(a + bx))^4}{4b} - \frac{\coth^{-1}(\tanh(a + bx))^5}{20b^2}
\end{aligned}$$

**Mathematica [B]** time = 0.07, size = 99, normalized size = 2.91

$$\frac{(a + bx) \left( 10(2a^2 + abx - b^2x^2) \coth^{-1}(\tanh(a + bx))^2 + (4a - bx)(a + bx)^3 - 5(3a - bx)(a + bx)^2 \coth^{-1}(\tanh(a + bx)) \right)}{20b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] ((a + b\*x)\*((4\*a - b\*x)\*(a + b\*x)^3 - 5\*(3\*a - b\*x)\*(a + b\*x)^2\*ArcCoth[Tanh[a + b\*x]] + 10\*(2\*a^2 + a\*b\*x - b^2\*x^2)\*ArcCoth[Tanh[a + b\*x]]^2 - 10\*(a - b\*x)\*ArcCoth[Tanh[a + b\*x]]^3))/(20\*b^2)

**fricas [A]** time = 0.55, size = 52, normalized size = 1.53

$$\frac{1}{5} b^3 x^5 + \frac{3}{4} ab^2 x^4 - \frac{1}{4} (\pi^2 b - 4 a^2 b) x^3 - \frac{1}{8} (3 \pi^2 a - 4 a^3) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/5\*b^3\*x^5 + 3/4\*a\*b^2\*x^4 - 1/4\*(pi^2\*b - 4\*a^2\*b)\*x^3 - 1/8\*(3\*pi^2\*a - 4\*a^3)\*x^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x\*arccoth(tanh(b\*x + a))^3, x)

**maple** [C] time = 1.13, size = 18111, normalized size = 532.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(tanh(b*x+a))^3,x)`

[Out] result too large to display

**maxima** [A] time = 0.52, size = 54, normalized size = 1.59

$$-\frac{1}{2}bx^3 \operatorname{arccoth}(\tanh(bx+a))^2 + \frac{1}{2}x^2 \operatorname{arccoth}(\tanh(bx+a))^3 - \frac{1}{20}(b^2x^5 - 5bx^4 \operatorname{arccoth}(\tanh(bx+a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="maxima")`

[Out]  $-1/2*b*x^3*\operatorname{arccoth}(\tanh(b*x+a))^2 + 1/2*x^2*\operatorname{arccoth}(\tanh(b*x+a))^3 - 1/20*(b^2*x^5 - 5*b*x^4*\operatorname{arccoth}(\tanh(b*x+a)))*b$

**mupad** [B] time = 0.12, size = 53, normalized size = 1.56

$$-\frac{b^3 x^5}{20} + \frac{b^2 x^4 \operatorname{acoth}(\tanh(a+bx))}{4} - \frac{b x^3 \operatorname{acoth}(\tanh(a+bx))^2}{2} + \frac{x^2 \operatorname{acoth}(\tanh(a+bx))^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(tanh(a+b*x))^3,x)`

[Out]  $(x^2*\operatorname{acoth}(\tanh(a+b*x))^3)/2 - (b^3*x^5)/20 - (b*x^3*\operatorname{acoth}(\tanh(a+b*x))^2)/2 + (b^2*x^4*\operatorname{acoth}(\tanh(a+b*x)))/4$

**sympy** [A] time = 1.21, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x \operatorname{acoth}^4(\tanh(a+bx))}{4b} - \frac{\operatorname{acoth}^5(\tanh(a+bx))}{20b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{acoth}^3(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(tanh(b*x+a))**3,x)`

[Out] `Piecewise((x*acoth(tanh(a+b*x))**4/(4*b) - acoth(tanh(a+b*x))**5/(20*b*2), Ne(b, 0)), (x**2*acoth(tanh(a))**3/2, True))`

### 3.151 $\int \coth^{-1}(\tanh(a + bx))^3 dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\tanh(a + bx))^4}{4b}$$

[Out] 1/4\*arccoth(tanh(b\*x+a))^4/b

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^4}{4b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] ArcCoth[Tanh[a + b\*x]]^4/(4\*b)

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tanh(a + bx))^3 dx &= \frac{\text{Subst}\left(\int x^3 dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\tanh(a + bx))^4}{4b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\coth^{-1}(\tanh(a + bx))^4}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] ArcCoth[Tanh[a + b\*x]]^4/(4\*b)

**fricas** [B] time = 0.93, size = 49, normalized size = 3.06

$$\frac{1}{4}b^3x^4 + ab^2x^3 - \frac{3}{8}(\pi^2b - 4a^2b)x^2 - \frac{1}{4}(3\pi^2a - 4a^3)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/4\*b^3\*x^4 + a\*b^2\*x^3 - 3/8\*(pi^2\*b - 4\*a^2\*b)\*x^2 - 1/4\*(3\*pi^2\*a - 4\*a^3)\*x

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^3, x)

**maple** [A] time = 0.09, size = 15, normalized size = 0.94

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3,x)

[Out] 1/4\*arccoth(tanh(b\*x+a))^4/b

**maxima** [B] time = 0.52, size = 51, normalized size = 3.19

$$-\frac{3}{2}bx^2 \operatorname{arccoth}(\tanh(bx + a))^2 + x \operatorname{arccoth}(\tanh(bx + a))^3 - \frac{1}{4}(b^2x^4 - 4bx^3 \operatorname{arccoth}(\tanh(bx + a)))b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $-3/2*b*x^2*arccoth(\tanh(b*x + a))^2 + x*arccoth(\tanh(b*x + a))^3 - 1/4*(b^2*x^4 - 4*b*x^3*arccoth(\tanh(b*x + a)))*b$

**mupad [B]** time = 1.18, size = 47, normalized size = 2.94

$$\frac{x (2 \operatorname{acoth}(\tanh(a + bx)) - bx) (b^2 x^2 - 2 b x \operatorname{acoth}(\tanh(a + bx)) + 2 \operatorname{acoth}(\tanh(a + bx))^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^3,x)`

[Out]  $(x*(2*acoth(\tanh(a + b*x)) - b*x)*(2*acoth(\tanh(a + b*x))^2 + b^2*x^2 - 2*b*x*acoth(\tanh(a + b*x))))/4$

**sympy [A]** time = 0.61, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{acoth}^4(\tanh(a+bx))}{4b} & \text{for } b \neq 0 \\ x \operatorname{acoth}^3(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**3,x)`

[Out] `Piecewise((acoth(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*acoth(tanh(a))**3, True))`

$$3.152 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x} dx$$

Optimal. Leaf size=77

$$bx \left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2 - \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 \left( bx - \coth^{-1}(\tanh(a+bx)) \right) + \frac{1}{3} \coth^{-1}(\tanh(a+bx))^3$$

[Out] b\*x\*(b\*x-arcCoth(tanh(b\*x+a)))^2-1/2\*(b\*x-arcCoth(tanh(b\*x+a)))\*arcCoth(tanh(b\*x+a))^2+1/3\*arcCoth(tanh(b\*x+a))^3-(b\*x-arcCoth(tanh(b\*x+a)))^3\*ln(x)

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2159, 2158, 29}

$$bx \left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2 - \frac{1}{2} \coth^{-1}(\tanh(a+bx))^2 \left( bx - \coth^{-1}(\tanh(a+bx)) \right) + \frac{1}{3} \coth^{-1}(\tanh(a+bx))^3$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x, x]

[Out] b\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2 - ((b\*x - ArcCoth[Tanh[a + b\*x]])\*ArcCoth[Tanh[a + b\*x]]^2)/2 + ArcCoth[Tanh[a + b\*x]]^3/3 - (b\*x - ArcCoth[Tanh[a + b\*x]])^3\*Log[x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 2158

Int[(v\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

Rule 2159

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a\*n), x] - Dist[(b\*u - a\*v)/a, Int[v^(n-1)/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x} dx &= \frac{1}{3} \coth^{-1}(\tanh(a+bx))^3 - (bx - \coth^{-1}(\tanh(a+bx))) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} \\
&= -\frac{1}{2} (bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^2 + \frac{1}{3} \coth^{-1}(\tanh(a+bx)) \\
&= bx (bx - \coth^{-1}(\tanh(a+bx)))^2 - \frac{1}{2} (bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx)) \\
&= bx (bx - \coth^{-1}(\tanh(a+bx)))^2 - \frac{1}{2} (bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 104, normalized size = 1.35

$$(a+bx) \left( a^2 - 3a(-\coth^{-1}(\tanh(a+bx)) + a + bx) + 3(-\coth^{-1}(\tanh(a+bx)) + a + bx)^2 \right) + \frac{1}{3}(a+bx)^3 - \frac{1}{2}(a+bx)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x,x]

[Out] (a + b\*x)^3/3 + (a + b\*x)\*(a^2 - 3\*a\*(a + b\*x - ArcCoth[Tanh[a + b\*x]])) + 3\*(a + b\*x - ArcCoth[Tanh[a + b\*x]])^2) - ((a + b\*x)^2\*(2\*a + 3\*b\*x - 3\*ArcCoth[Tanh[a + b\*x]]))/2 + (-b\*x) + ArcCoth[Tanh[a + b\*x]]^3\*Log[b\*x]

**fricas [A]** time = 0.67, size = 49, normalized size = 0.64

$$\frac{1}{3}b^3x^3 + \frac{3}{2}ab^2x^2 - \frac{3}{4}(\pi^2b - 4a^2b)x - \frac{1}{4}(3\pi^2a - 4a^3)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x,x, algorithm="fricas")

[Out] 1/3\*b^3\*x^3 + 3/2\*a\*b^2\*x^2 - 3/4\*(pi^2\*b - 4\*a^2\*b)\*x - 1/4\*(3\*pi^2\*a - 4\*a^3)\*log(x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^3/x, x)



**maple** [C] time = 0.90, size = 21848, normalized size = 283.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(tanh(b*x+a))^3/x,x)`

[Out] result too large to display

**maxima** [C] time = 0.76, size = 75, normalized size = 0.97

$$\frac{1}{3} b^3 x^3 + \frac{1}{24} (-18i \pi b^2 + 36 a b^2) x^2 - \frac{1}{24} (18 \pi^2 b + 72i \pi a b - 72 a^2 b) x + \frac{1}{8} (i \pi^3 - 6 \pi^2 a - 12i \pi a^2 + 8 a^3) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="maxima")`

[Out]  $\frac{1}{3} b^3 x^3 + \frac{1}{24} (-18i \pi b^2 + 36 a b^2) x^2 - \frac{1}{24} (18 \pi^2 b + 72i \pi a b - 72 a^2 b) x + \frac{1}{8} (i \pi^3 - 6 \pi^2 a - 12i \pi a^2 + 8 a^3) \log(x)$

**mupad** [B] time = 0.14, size = 306, normalized size = 3.97

$$\frac{b^3 x^3}{3} - \ln(x) \left( \frac{\left( 2a - \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + \ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) + 2bx \right)^3}{8} - a^3 - \frac{3a \left( 2a - \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + \ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) \right)}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^3/x,x)`

[Out]  $(b^3 x^3)/3 - \log(x) \left( (2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx \right)^3 / 8 - a^3 - (3a(2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx)^2 / 4 + (3a^2(2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx) / 2 - (3b^2 x^2 (\log(-2 / (\exp(2a) \exp(2bx) - 1)) - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1)) + 2bx)) / 4 + (3bx (\log(-2 / (\exp(2a) \exp(2bx) - 1)) - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1)) + 2bx)^2) / 4$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(\tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x, x)
```

$$3.153 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^2} dx$$

Optimal. Leaf size=68

$$-3b^2x \left( bx - \coth^{-1}(\tanh(a+bx)) \right) - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} + \frac{3}{2}b \coth^{-1}(\tanh(a+bx))^2 + 3b \log(x) \left( bx - \coth^{-1}(\tanh(a+bx)) \right)$$

[Out]  $-3*b^2*x*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))+3/2*b*\operatorname{arccoth}(\tanh(b*x+a))^2-\operatorname{arccoth}(\tanh(b*x+a))^3/x+3*b*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2*\ln(x)$

**Rubi** [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2168, 2159, 2158, 29}

$$-3b^2x \left( bx - \coth^{-1}(\tanh(a+bx)) \right) - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} + \frac{3}{2}b \coth^{-1}(\tanh(a+bx))^2 + 3b \log(x) \left( bx - \coth^{-1}(\tanh(a+bx)) \right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x^2,x]

[Out]  $-3*b^2*x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + (3*b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2)/2 - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^3/x + 3*b*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2*\operatorname{Log}[x]$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

Rule 2159

Int[(v\_)^(n\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a\*n), x] - Dist[(b\*u - a\*v)/a, Int[v^(n-1)/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n, 1]

Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[v^n/u, x], x] /; NeQ[b\*n, 0] && GtQ[m, 0] && NeQ[n, 0]

```

1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
&& !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a + bx))^3}{x^2} dx &= -\frac{\coth^{-1}(\tanh(a + bx))^3}{x} + (3b) \int \frac{\coth^{-1}(\tanh(a + bx))^2}{x} dx \\
&= \frac{3}{2}b \coth^{-1}(\tanh(a + bx))^2 - \frac{\coth^{-1}(\tanh(a + bx))^3}{x} - (3b) (bx - \coth^{-1}(\tanh(a + bx))) \\
&= -3b^2x (bx - \coth^{-1}(\tanh(a + bx))) + \frac{3}{2}b \coth^{-1}(\tanh(a + bx))^2 - \frac{\coth^{-1}(\tanh(a + bx))^3}{x} \\
&= -3b^2x (bx - \coth^{-1}(\tanh(a + bx))) + \frac{3}{2}b \coth^{-1}(\tanh(a + bx))^2 - \frac{\coth^{-1}(\tanh(a + bx))^3}{x}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 62, normalized size = 0.91

$$-6b^2x \log(x) \coth^{-1}(\tanh(a+bx)) - \frac{\coth^{-1}(\tanh(a+bx))^3}{x} + 3b(\log(x)+1) \coth^{-1}(\tanh(a+bx))^2 + \frac{3}{2}b^3x^2(2 \log(x) - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCoth[Tanh[a + b*x]]^3/x^2, x]
```

```
[Out] -(ArcCoth[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcCoth[Tanh[a + b*x]]*Log[x] + 3*b
*ArcCoth[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

**fricas** [A] time = 0.46, size = 51, normalized size = 0.75

$$\frac{2b^3x^3 + 12ab^2x^2 + 3\pi^2a - 4a^3 - 3(\pi^2b - 4a^2b)x \log(x)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tanh(b*x+a))^3/x^2, x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^3 + 12*a*b^2*x^2 + 3*pi^2*a - 4*a^3 - 3*(pi^2*b - 4*a^2*b)*x*log(x))/x
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^2,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^3/x^2, x)

**maple** [C] time = 0.46, size = 7683, normalized size = 112.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3/x^2,x)

[Out] result too large to display

**maxima** [C] time = 0.61, size = 124, normalized size = 1.82

$$3b \operatorname{arccoth}(\tanh(bx + a))^2 \log(x) - \frac{3}{2} \left( 2 \operatorname{arccoth}(\tanh(bx + a))^2 \log(x) - \left( bx^2 - 2(-i\pi - 2a)x + 2 \left( -\frac{i\pi(bx + a)}{b} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^2,x, algorithm="maxima")

[Out] 3\*b\*arccoth(tanh(b\*x + a))^2\*log(x) - 3/2\*(2\*arccoth(tanh(b\*x + a))^2\*log(x) - (b\*x^2 - 2\*(-I\*pi - 2\*a)\*x + 2\*(-I\*pi\*(b\*x + a)/b - (b\*x + a)^2/b)\*log(x) + 2\*arccoth(tanh(b\*x + a))^2\*log(x)/b + 2\*(I\*pi\*a + a^2)\*log(x)/b)\*b - arccoth(tanh(b\*x + a))^3/x

**mupad** [B] time = 1.20, size = 372, normalized size = 5.47

$$\ln(x) \left( 3a^2b + \frac{3b \left( 2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx \right)^2}{4} - 3ab \left( 2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoht(tanh(a + b\*x))^3/x^2,x)

```
[Out] log(x)*(3*a^2*b + (3*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/4 - 3*a*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) + ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(8*x) + (b^3*x^2)/2 - (3*b^2*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/2
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(\tanh(a + bx))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**2,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x**2, x)
```

$$3.154 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3} dx$$

Optimal. Leaf size=60

$$-3b^2 \log(x) (bx - \coth^{-1}(\tanh(a+bx))) - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - \frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} + 3b^3x$$

[Out] 3\*b^3\*x-3/2\*b\*arccoth(tanh(b\*x+a))^2/x-1/2\*arccoth(tanh(b\*x+a))^3/x^2-3\*b^2\*(b\*x-arccoth(tanh(b\*x+a)))\*ln(x)

**Rubi [A]** time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2158, 29}

$$-3b^2 \log(x) (bx - \coth^{-1}(\tanh(a+bx))) - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - \frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} + 3b^3x$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x^3,x]

[Out] 3\*b^3\*x - (3\*b\*ArcCoth[Tanh[a + b\*x]]^2)/(2\*x) - ArcCoth[Tanh[a + b\*x]]^3/(2\*x^2) - 3\*b^2\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 2158

Int[(v\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2\*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^3} dx &= -\frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} + \frac{1}{2}(3b) \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^2} dx \\
&= -\frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} + (3b^2) \int \frac{\coth^{-1}(\tanh(a+bx))}{x} dx \\
&= 3b^3x - \frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - (3b^2)(bx - \coth^{-1}(\tanh(a+bx))) \log(x) \\
&= 3b^3x - \frac{3b \coth^{-1}(\tanh(a+bx))^2}{2x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{2x^2} - 3b^2(bx - \coth^{-1}(\tanh(a+bx))) \log(x)
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 66, normalized size = 1.10

$$3b^2 \log(x) (\coth^{-1}(\tanh(a+bx)) - bx) - \frac{(\coth^{-1}(\tanh(a+bx)) - bx)^3}{2x^2} - \frac{3b (\coth^{-1}(\tanh(a+bx)) - bx)^2}{x} + b^3x$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x^3,x]

[Out] b^3\*x - (3\*b\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^2)/x - (-(b\*x) + ArcCoth[Tanh[a + b\*x]])^3/(2\*x^2) + 3\*b^2\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])\*Log[x]

**fricas** [A] time = 0.51, size = 51, normalized size = 0.85

$$\frac{8b^3x^3 + 24ab^2x^2 \log(x) + 3\pi^2a - 4a^3 + 6(\pi^2b - 4a^2b)x}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^3,x, algorithm="fricas")

[Out] 1/8\*(8\*b^3\*x^3 + 24\*a\*b^2\*x^2\*log(x) + 3\*pi^2\*a - 4\*a^3 + 6\*(pi^2\*b - 4\*a^2\*b)\*x)/x^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^3/x^3, x)



**maple [C]** time = 0.55, size = 7366, normalized size = 122.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^3/x^3,x)

[Out] result too large to display

**maxima [A]** time = 0.46, size = 72, normalized size = 1.20

$$3 \left( b \operatorname{arccoth}(\tanh(bx + a)) \log(x) - \left( b \left( x + \frac{a}{b} \right) \log(x) - b \left( x + \frac{a \log(x)}{b} \right) \right) b \right) b - \frac{3 b \operatorname{arccoth}(\tanh(bx + a))^2}{2x} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^3,x, algorithm="maxima")

[Out] 3\*(b\*arccoth(tanh(b\*x + a))\*log(x) - (b\*(x + a/b)\*log(x) - b\*(x + a\*log(x)/b))\*b)\*b - 3/2\*b\*arccoth(tanh(b\*x + a))^2/x - 1/2\*arccoth(tanh(b\*x + a))^3/x^2

**mupad [B]** time = 1.33, size = 383, normalized size = 6.38

$$\frac{\ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right)^3}{16x^2} - \frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right)^3}{16x^2} + \frac{9b^2 \ln\left(\frac{e^{2bx}}{e^{2a}e^{2bx-1}}\right)}{4} - \frac{9b^2 \ln\left(\frac{1}{e^{2a}e^{2bx-1}}\right)}{4} - \frac{3b^3 x}{2} - \frac{3b \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right)^2}{8x} + \frac{3b^2 \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right)}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))^3/x^3,x)

[Out] log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))^3/(16\*x^2) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))^3/(16\*x^2) + (9\*b^2\*log(exp(2\*b\*x)/(exp(2\*a)\*exp(2\*b\*x) - 1)))/4 - (9\*b^2\*log(1/(exp(2\*a)\*exp(2\*b\*x) - 1)))/4 - (3\*b^3\*x)/2 - (3\*b\*log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))^2)/(8\*x) + (3\*b^2\*log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))\*log(x))/2 - 3\*b^3\*x\*log(x) - (3\*b\*log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))^2)/(8\*x) - (3\*b^2\*log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))\*log(x))/2 - (3\*log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))\*log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))^2)/(16\*x^2) + (3\*log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))^2\*log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)))/(16\*x^2) + (3\*b\*log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))\*log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)))/(4\*x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^3(\tanh(a + bx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(tanh(b*x+a))**3/x**3,x)
```

```
[Out] Integral(acoth(tanh(a + b*x))**3/x**3, x)
```

$$3.155 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx$$

Optimal. Leaf size=55

$$-\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} - \frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2} + b^3 \log(x)$$

[Out]  $-b^2 \operatorname{arccoth}(\tanh(bx+a))/x - 1/2 b \operatorname{arccoth}(\tanh(bx+a))^2/x^2 - 1/3 \operatorname{arccoth}(\tanh(bx+a))^3/x^3 + b^3 \ln(x)$

**Rubi [A]** time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 29}

$$-\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x} - \frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b^3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x^4,x]

[Out]  $-((b^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/x) - (b \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2)/(2*x^2) - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^3/(3*x^3) + b^3 \operatorname{Log}[x]$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_.), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2\*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^4} dx &= -\frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b \int \frac{\coth^{-1}(\tanh(a+bx))^2}{x^3} dx \\
&= -\frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + b^2 \int \frac{\coth^{-1}(\tanh(a+bx))}{x^2} dx \\
&= -\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x} - \frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + \dots \\
&= -\frac{b^2 \coth^{-1}(\tanh(a+bx))}{x} - \frac{b \coth^{-1}(\tanh(a+bx))^2}{2x^2} - \frac{\coth^{-1}(\tanh(a+bx))^3}{3x^3} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 1.09

$$\frac{-6b^2x^2 \coth^{-1}(\tanh(a+bx)) - 3bx \coth^{-1}(\tanh(a+bx))^2 - 2 \coth^{-1}(\tanh(a+bx))^3 + b^3x^3(6 \log(x) + 11)}{6x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x^4, x]

[Out]  $(-6*b^2*x^2*ArcCoth[Tanh[a + b*x]] - 3*b*x*ArcCoth[Tanh[a + b*x]]^2 - 2*ArcCoth[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)$

**fricas [A]** time = 0.92, size = 51, normalized size = 0.93

$$\frac{24b^3x^3 \log(x) - 72ab^2x^2 + 6\pi^2a - 8a^3 + 9(\pi^2b - 4a^2b)x}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^4, x, algorithm="fricas")

[Out]  $1/24*(24*b^3*x^3*\log(x) - 72*a*b^2*x^2 + 6*\pi^2*a - 8*a^3 + 9*(\pi^2*b - 4*a^2*b)*x)/x^3$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^4, x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^3/x^4, x)

**maple** [C] time = 1.59, size = 17237, normalized size = 313.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(tanh(b*x+a))^3/x^4,x)`

[Out] result too large to display

**maxima** [A] time = 0.59, size = 52, normalized size = 0.95

$$\left(b^2 \log(x) - \frac{b \operatorname{arccoth}(\tanh(bx + a))}{x}\right)b - \frac{b \operatorname{arccoth}(\tanh(bx + a))^2}{2x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="maxima")`

[Out]  $(b^2 \log(x) - b \operatorname{arccoth}(\tanh(bx + a))/x) * b - 1/2 * b * \operatorname{arccoth}(\tanh(bx + a))^2 / x^2 - 1/3 * \operatorname{arccoth}(\tanh(bx + a))^3 / x^3$

**mupad** [B] time = 1.18, size = 51, normalized size = 0.93

$$b^3 \ln(x) - \frac{b^2 x^2 \operatorname{acoth}(\tanh(a + bx)) + \frac{bx \operatorname{acoth}(\tanh(a + bx))^2}{2} + \frac{\operatorname{acoth}(\tanh(a + bx))^3}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^3/x^4,x)`

[Out]  $b^3 \log(x) - (\operatorname{acoth}(\tanh(a + b*x))^3 / 3 + (b*x * \operatorname{acoth}(\tanh(a + b*x))^2) / 2 + b^2 * x^2 * \operatorname{acoth}(\tanh(a + b*x))) / x^3$

**sympy** [A] time = 0.85, size = 51, normalized size = 0.93

$$b^3 \log(x) - \frac{b^2 \operatorname{acoth}(\tanh(a + bx))}{x} - \frac{b \operatorname{acoth}^2(\tanh(a + bx))}{2x^2} - \frac{\operatorname{acoth}^3(\tanh(a + bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**3/x**4,x)`

[Out]  $b**3 * \log(x) - b**2 * \operatorname{acoth}(\tanh(a + b*x)) / x - b * \operatorname{acoth}(\tanh(a + b*x))**2 / (2*x**2) - \operatorname{acoth}(\tanh(a + b*x))**3 / (3*x**3)$

$$3.156 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx$$

Optimal. Leaf size=31

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4(bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out]  $1/4*\operatorname{arccoth}(\tanh(b*x+a))^4/x^4/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))$

**Rubi [A]** time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4(bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Tanh[a + b*x]]^3/x^5,x]`

[Out] `ArcCoth[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcCoth[Tanh[a + b*x]]))`

Rule 2167

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]`

Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{4x^4(bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 1.61

$$\frac{b^2x^2 \coth^{-1}(\tanh(a+bx)) + bx \coth^{-1}(\tanh(a+bx))^2 + \coth^{-1}(\tanh(a+bx))^3 + b^3x^3}{4x^4}$$

Antiderivative was successfully verified.

[In] `Integrate[ArcCoth[Tanh[a + b*x]]^3/x^5,x]`

[Out]  $-1/4*(b^3*x^3 + b^2*x^2*\text{ArcCoth}[\text{Tanh}[a + b*x]] + b*x*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2 + \text{ArcCoth}[\text{Tanh}[a + b*x]]^3)/x^4$

**fricas** [A] time = 0.47, size = 49, normalized size = 1.58

$$\frac{16b^3x^3 + 24ab^2x^2 - 3\pi^2a + 4a^3 - 4(\pi^2b - 4a^2b)x}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="fricas")`

[Out]  $-1/16*(16*b^3*x^3 + 24*a*b^2*x^2 - 3*\pi^2*a + 4*a^3 - 4*(\pi^2*b - 4*a^2*b)*x)/x^4$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(\tanh(bx+a))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="giac")`

[Out] `integrate(arccoth(tanh(b*x + a))^3/x^5, x)`

**maple** [C] time = 1.47, size = 17235, normalized size = 555.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(tanh(b*x+a))^3/x^5,x)`

[Out] result too large to display

**maxima** [A] time = 0.52, size = 53, normalized size = 1.71

$$-\frac{1}{4}b\left(\frac{b^2}{x} + \frac{b \text{arccoth}(\tanh(bx+a))}{x^2}\right) - \frac{b \text{arccoth}(\tanh(bx+a))^2}{4x^3} - \frac{\text{arccoth}(\tanh(bx+a))^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="maxima")`

[Out]  $-1/4*b*(b^2/x + b*\text{arccoth}(\tanh(b*x + a))/x^2) - 1/4*b*\text{arccoth}(\tanh(b*x + a))^2/x^3 - 1/4*\text{arccoth}(\tanh(b*x + a))^3/x^4$

**mupad [B]** time = 1.19, size = 48, normalized size = 1.55

$$\frac{b^3 x^3 + b^2 x^2 \operatorname{acoth}(\tanh(a + bx)) + b x \operatorname{acoth}(\tanh(a + bx))^2 + \operatorname{acoth}(\tanh(a + bx))^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^3/x^5, x)`

[Out]  $-(\operatorname{acoth}(\tanh(a + b*x))^3 + b^3*x^3 + b*x*\operatorname{acoth}(\tanh(a + b*x))^2 + b^2*x^2*\operatorname{acoth}(\tanh(a + b*x)))/(4*x^4)$

**sympy [B]** time = 1.32, size = 56, normalized size = 1.81

$$-\frac{b^3}{4x} - \frac{b^2 \operatorname{acoth}(\tanh(a + bx))}{4x^2} - \frac{b \operatorname{acoth}^2(\tanh(a + bx))}{4x^3} - \frac{\operatorname{acoth}^3(\tanh(a + bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**3/x**5, x)`

[Out]  $-b**3/(4*x) - b**2*\operatorname{acoth}(\tanh(a + b*x))/(4*x**2) - b*\operatorname{acoth}(\tanh(a + b*x))**2/(4*x**3) - \operatorname{acoth}(\tanh(a + b*x))**3/(4*x**4)$



$$3.157 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx$$

Optimal. Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5(bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \coth^{-1}(\tanh(a+bx))^4}{20x^4(bx - \coth^{-1}(\tanh(a+bx)))^2}$$

[Out] 1/20\*b\*arccoth(tanh(b\*x+a))^4/x^4/(b\*x-arccoth(tanh(b\*x+a)))^2+1/5\*arccoth(tanh(b\*x+a))^4/x^5/(b\*x-arccoth(tanh(b\*x+a)))

**Rubi** [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2171, 2167}

$$\frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5(bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \coth^{-1}(\tanh(a+bx))^4}{20x^4(bx - \coth^{-1}(\tanh(a+bx)))^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^3/x^6,x]

[Out] (b\*ArcCoth[Tanh[a + b\*x]]^4)/(20\*x^4\*(b\*x - ArcCoth[Tanh[a + b\*x]]^2) + ArcCoth[Tanh[a + b\*x]]^4/(5\*x^5\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

#### Rule 2167

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]

#### Rule 2171

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] + Dist[(b\*(m + n + 2))/((m + 1)\*(b\*u - a\*v)), Int[u^(m + 1)\*v^n, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -1]

#### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^6} dx = \frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5 (bx - \coth^{-1}(\tanh(a+bx)))} + \frac{b \int \frac{\coth^{-1}(\tanh(a+bx))^3}{x^5} dx}{5 (bx - \coth^{-1}(\tanh(a+bx)))}$$

$$= \frac{b \coth^{-1}(\tanh(a+bx))^4}{20x^4 (bx - \coth^{-1}(\tanh(a+bx)))^2} + \frac{\coth^{-1}(\tanh(a+bx))^4}{5x^5 (bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.04, size = 54, normalized size = 0.84

$$\frac{2b^2x^2 \coth^{-1}(\tanh(a+bx)) + 3bx \coth^{-1}(\tanh(a+bx))^2 + 4 \coth^{-1}(\tanh(a+bx))^3 + b^3x^3}{20x^5}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^3/x^6,x]

[Out] -1/20\*(b^3\*x^3 + 2\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + 3\*b\*x\*ArcCoth[Tanh[a + b\*x]]^2 + 4\*ArcCoth[Tanh[a + b\*x]]^3)/x^5

**fricas [A]** time = 0.51, size = 49, normalized size = 0.77

$$\frac{40b^3x^3 + 80ab^2x^2 - 12\pi^2a + 16a^3 - 15(\pi^2b - 4a^2b)x}{80x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^6,x, algorithm="fricas")

[Out] -1/80\*(40\*b^3\*x^3 + 80\*a\*b^2\*x^2 - 12\*pi^2\*a + 16\*a^3 - 15\*(pi^2\*b - 4\*a^2\*b)\*x)/x^5

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^3/x^6,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^3/x^6, x)

**maple [C]** time = 1.42, size = 17234, normalized size = 269.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(tanh(b*x+a))^3/x^6,x)`

[Out] result too large to display

**maxima** [A] time = 0.54, size = 54, normalized size = 0.84

$$-\frac{1}{20}b\left(\frac{b^2}{x^2} + \frac{2b \operatorname{arccoth}(\tanh(bx+a))}{x^3}\right) - \frac{3b \operatorname{arccoth}(\tanh(bx+a))^2}{20x^4} - \frac{\operatorname{arccoth}(\tanh(bx+a))^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="maxima")`

[Out] `-1/20*b*(b^2/x^2 + 2*b*arccoth(tanh(b*x + a))/x^3) - 3/20*b*arccoth(tanh(b*x + a))^2/x^4 - 1/5*arccoth(tanh(b*x + a))^3/x^5`

**mupad** [B] time = 0.12, size = 53, normalized size = 0.83

$$-\frac{\operatorname{acoth}(\tanh(a+bx))^3}{5x^5} - \frac{b^3}{20x^2} - \frac{b^2 \operatorname{acoth}(\tanh(a+bx))}{10x^3} - \frac{3b \operatorname{acoth}(\tanh(a+bx))^2}{20x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^3/x^6,x)`

[Out] `-acoth(tanh(a + b*x))^3/(5*x^5) - b^3/(20*x^2) - (b^2*acoth(tanh(a + b*x)))/(10*x^3) - (3*b*acoth(tanh(a + b*x))^2)/(20*x^4)`

**sympy** [A] time = 2.12, size = 60, normalized size = 0.94

$$-\frac{b^3}{20x^2} - \frac{b^2 \operatorname{acoth}(\tanh(a+bx))}{10x^3} - \frac{3b \operatorname{acoth}^2(\tanh(a+bx))}{20x^4} - \frac{\operatorname{acoth}^3(\tanh(a+bx))}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**3/x**6,x)`

[Out] `-b**3/(20*x**2) - b**2*acoth(tanh(a + b*x))/(10*x**3) - 3*b*acoth(tanh(a + b*x))**2/(20*x**4) - acoth(tanh(a + b*x))**3/(5*x**5)`

$$3.158 \quad \int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=53

$$-\frac{x^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(m+1)(bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out]  $-x^{(1+m)}*\text{hypergeom}([1, 1+m], [2+m], b*x/(b*x-\text{arccoth}(\tanh(b*x+a))))/(1+m)/(b*x-\text{arccoth}(\tanh(b*x+a)))$

**Rubi [A]** time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2164}

$$-\frac{x^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(m+1)(bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^m/\text{ArcCoth}[\text{Tanh}[a + b*x]], x]$

[Out]  $-((x^{(1+m)}*\text{Hypergeometric2F1}[1, 1+m, 2+m, (b*x)/(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]])])/((1+m)*(b*x - \text{ArcCoth}[\text{Tanh}[a + b*x]]))$

Rule 2164

$\text{Int}[(v_)^{(n_)}/(u_), x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[D[u, x]], b = \text{Simplify}[D[v, x]]\}, \text{Simp}[(v^{(n+1)}*\text{Hypergeometric2F1}[1, n+1, n+2, -((a*v)/(b*u - a*v))])/((n+1)*(b*u - a*v)), x] /; \text{NeQ}[b*u - a*v, 0] /; \text{PiecewiseLinearQ}[u, v, x] \&\& \text{!IntegerQ}[n]$

Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))} dx = -\frac{x^{1+m} {}_2F_1\left(1, 1+m; 2+m; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(1+m)(bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica [A]** time = 0.09, size = 51, normalized size = 0.96

$$\frac{x^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)(\coth^{-1}(\tanh(a+bx))-bx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/ArcCoth[Tanh[a + b\*x]],x]

[Out] (x^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -((b\*x)/(-b\*x) + ArcCoth[Tanh[a + b\*x]])])/((1 + m)\*(-b\*x) + ArcCoth[Tanh[a + b\*x]])

**fricas** [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\text{arcoth}(\tanh(bx + a))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out] integral(x^m/arccoth(tanh(b\*x + a)), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arcoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^m/arccoth(tanh(b\*x + a)), x)

**maple** [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/arccoth(tanh(b\*x+a)),x)

[Out] int(x^m/arccoth(tanh(b\*x+a)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arcoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] integrate(x<sup>m</sup>/arccoth(tanh(b\*x + a)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>m</sup>/acoth(tanh(a + b\*x)), x)

[Out] int(x<sup>m</sup>/acoth(tanh(a + b\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/acoth(tanh(b\*x+a)), x)

[Out] Integral(x\*\*m/acoth(tanh(a + b\*x)), x)

$$3.159 \quad \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))} dx$$

**Optimal.** Leaf size=81

$$\frac{(bx - \coth^{-1}(\tanh(a + bx)))^3 \log(\coth^{-1}(\tanh(a + bx)))}{b^4} + \frac{x(bx - \coth^{-1}(\tanh(a + bx)))^2}{b^3} + \frac{x^2(bx - \coth^{-1}(\tanh(a + bx)))}{2b^2}$$

[Out]  $1/3*x^3/b+1/2*x^2*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/b^2+x*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2/b^3+(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^3*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/b^4$

**Rubi [A]** time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2159, 2158, 2157, 29}

$$\frac{x^2(bx - \coth^{-1}(\tanh(a + bx)))}{2b^2} + \frac{x(bx - \coth^{-1}(\tanh(a + bx)))^2}{b^3} + \frac{(bx - \coth^{-1}(\tanh(a + bx)))^3 \log(\coth^{-1}(\tanh(a + bx)))}{b^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcCoth[Tanh[a + b\*x]], x]

[Out]  $x^3/(3*b) + (x^2*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]))/(2*b^2) + (x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2)/b^3 + ((b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^3*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/b^4$

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

#### Rule 2159

Int[(v\_)^(n\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a\*n), x] - Dist[(b\*u - a\*v)/a, Int[v^(n - 1)/u, x], x]

]; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\coth^{-1}(\tanh(a + bx))} dx &= \frac{x^3}{3b} - \frac{(-bx + \coth^{-1}(\tanh(a + bx))) \int \frac{x^2}{\coth^{-1}(\tanh(a + bx))} dx}{b} \\
 &= \frac{x^3}{3b} + \frac{x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{2b^2} + \frac{(-bx + \coth^{-1}(\tanh(a + bx)))^2 \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{b^2} \\
 &= \frac{x^3}{3b} + \frac{x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{2b^2} + \frac{x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^3} - \frac{(-bx + \coth^{-1}(\tanh(a + bx)))^2}{b^2} \\
 &= \frac{x^3}{3b} + \frac{x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{2b^2} + \frac{x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^3} - \frac{(-bx + \coth^{-1}(\tanh(a + bx)))^2}{b^2} \\
 &= \frac{x^3}{3b} + \frac{x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{2b^2} + \frac{x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^3} + \frac{(bx - \coth^{-1}(\tanh(a + bx)))^2}{b^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 79, normalized size = 0.98

$$\frac{(\coth^{-1}(\tanh(a + bx)) - bx)^3 \log(\coth^{-1}(\tanh(a + bx)))}{b^4} + \frac{x (\coth^{-1}(\tanh(a + bx)) - bx)^2}{b^3} - \frac{x^2 (\coth^{-1}(\tanh(a + bx)))^2}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCoth[Tanh[a + b\*x]],x]

[Out] x^3/(3\*b) - (x^2\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))/(2\*b^2) + (x\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^2)/b^3 - ((-(b\*x) + ArcCoth[Tanh[a + b\*x]])^3\*Log[ArcCoth[Tanh[a + b\*x]]])/b^4

**fricas [A]** time = 0.59, size = 127, normalized size = 1.57

$$\frac{8b^3x^3 - 12ab^2x^2 - 6(\pi^2b - 4a^2b)x - 6(\pi^3 - 12\pi a^2) \arctan\left(-\frac{2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right) + 3(3\pi^2a - 4a^3)}{24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a)),x, algorithm="fricas")



[Out]  $\frac{1}{24}(8b^3x^3 - 12ab^2x^2 - 6(\pi^2b - 4a^2b)x - 6(\pi^3 - 12\pi a^2)\arctan(-\frac{2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}) + 3(3\pi^2a - 4a^3)\log(4b^2x^2 + 8abx + \pi^2 + 4a^2))/b^4$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")`

[Out] `integrate(x^3/arccoth(tanh(b*x + a)), x)`

**maple** [C] time = 4.80, size = 130774, normalized size = 1614.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/arccoth(tanh(b*x+a)),x)`

[Out] result too large to display

**maxima** [C] time = 0.53, size = 86, normalized size = 1.06

$$\frac{4b^2x^3 + (3i\pi b - 6ab)x^2 - (3\pi^2 + 12i\pi a - 12a^2)x}{12b^3} - \frac{(i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3)\log(-i\pi + 2bx + 2a)}{8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="maxima")`

[Out]  $\frac{1}{12}(4b^2x^3 + (3I\pi b - 6a^2b)x^2 - (3\pi^2 + 12I\pi a - 12a^2)x)/b^3 - \frac{1}{8}(I\pi^3 - 6\pi^2a - 12I\pi a^2 + 8a^3)\log(-I\pi + 2bx + 2a)/b^4$

**mupad** [B] time = 0.13, size = 354, normalized size = 4.37

$$\frac{x^3}{3b} + \frac{x^2 \left( \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx \right)}{4b^2} + \frac{x \left( \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx \right)^2}{4b^3} + \frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right)\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/acoth(tanh(a + b*x)),x)`

```
[Out] x^3/(3*b) + (x^2*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) - 1) + 2*b*x))/(4*b^2) + (x*(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x)^2)/(4*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
- 1)) - log(-2/(exp(2*a)*exp(2*b*x) - 1)))*((2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)
^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) -
1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x
) - 1)) + 2*b*x)))/(8*b^4)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acoth(tanh(b*x+a)),x)
```

```
[Out] Integral(x**3/acoth(tanh(a + b*x)), x)
```

$$3.160 \quad \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=56

$$\frac{(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{x(bx - \coth^{-1}(\tanh(a+bx)))}{b^2} + \frac{x^2}{2b}$$

[Out]  $1/2*x^2/b+x*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/b^2+(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/b^3$

**Rubi [A]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2159, 2158, 2157, 29}

$$\frac{x(bx - \coth^{-1}(\tanh(a+bx)))}{b^2} + \frac{(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcCoth[Tanh[a + b\*x]], x]

[Out]  $x^2/(2*b) + (x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]))/b^2 + ((b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/b^3$

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rule 2157

Int[(u\_)^(m\_), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

#### Rule 2159

Int[(v\_)^(n\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a\*n), x] - Dist[(b\*u - a\*v)/a, Int[v^(n-1)/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,

1]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\coth^{-1}(\tanh(a + bx))} dx &= \frac{x^2}{2b} - \frac{(-bx + \coth^{-1}(\tanh(a + bx))) \int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx}{b} \\
&= \frac{x^2}{2b} + \frac{x(bx - \coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{(-bx + \coth^{-1}(\tanh(a + bx)))^2 \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{b^2} \\
&= \frac{x^2}{2b} + \frac{x(bx - \coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{(-bx + \coth^{-1}(\tanh(a + bx)))^2 \text{Subst}\left(\int \frac{1}{x}\right)}{b^3} \\
&= \frac{x^2}{2b} + \frac{x(bx - \coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{(bx - \coth^{-1}(\tanh(a + bx)))^2 \log(\coth^{-1}(\tanh(a + bx)))}{b^3}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 55, normalized size = 0.98

$$\frac{(\coth^{-1}(\tanh(a + bx)) - bx)^2 \log(\coth^{-1}(\tanh(a + bx)))}{b^3} - \frac{x(\coth^{-1}(\tanh(a + bx)) - bx)}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

`[In] Integrate[x^2/ArcCoth[Tanh[a + b*x]],x]``[Out] x^2/(2*b) - (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcCoth[Tanh[a + b*x]])^2*Log[ArcCoth[Tanh[a + b*x]]])/b^3`**fricas** [A] time = 0.48, size = 97, normalized size = 1.73

$$\frac{4b^2x^2 - 8abx - 16\pi a \arctan\left(-\frac{2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right) - (\pi^2 - 4a^2) \log(4b^2x^2 + 8abx + \pi^2 + 4a^2)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="fricas")``[Out] 1/8*(4*b^2*x^2 - 8*a*b*x - 16*pi*a*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/pi) - (pi^2 - 4*a^2)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/b^3`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2/arccoth(tanh(b\*x + a)), x)

**maple** [C] time = 1.18, size = 28786, normalized size = 514.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arccoth(tanh(b\*x+a)),x)

[Out] result too large to display

**maxima** [C] time = 0.53, size = 51, normalized size = 0.91

$$\frac{bx^2 + (i\pi - 2a)x}{2b^2} - \frac{(\pi^2 + 4i\pi a - 4a^2)\log(-i\pi + 2bx + 2a)}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] 1/2\*(b\*x^2 + (I\*pi - 2\*a)\*x)/b^2 - 1/4\*(pi^2 + 4\*I\*pi\*a - 4\*a^2)\*log(-I\*pi + 2\*b\*x + 2\*a)/b^3

**mupad** [B] time = 1.33, size = 234, normalized size = 4.18

$$\frac{x^2}{2b} + \frac{x \left( \ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + 2bx \right)}{2b^2} + \frac{\ln\left(\ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) - \ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right)\right) \left(2a - \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right)\right)}{2b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/acoth(tanh(a + b\*x)),x)

[Out] x^2/(2\*b) + (x\*(log(-2/(exp(2\*a)\*exp(2\*b\*x)) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)/(2\*b^2) + (log(log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))))\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))))/2\*b^2 + ...

```
(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 4
*a^2))/(4*b^3)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/acoth(tanh(b\*x+a)),x)

[Out] Integral(x\*\*2/acoth(tanh(a + b\*x)), x)

$$3.161 \quad \int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=31

$$\frac{(bx - \coth^{-1}(\tanh(a + bx))) \log(\coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{x}{b}$$

[Out] x/b+(b\*x-arccoth(tanh(b\*x+a)))\*ln(arccoth(tanh(b\*x+a)))/b^2

**Rubi [A]** time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2158, 2157, 29}

$$\frac{(bx - \coth^{-1}(\tanh(a + bx))) \log(\coth^{-1}(\tanh(a + bx)))}{b^2} + \frac{x}{b}$$

Antiderivative was successfully verified.

[In] Int[x/ArcCoth[Tanh[a + b\*x]], x]

[Out] x/b + ((b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^2

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx &= \frac{x}{b} - \frac{(-bx + \coth^{-1}(\tanh(a + bx))) \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{b} \\
&= \frac{x}{b} - \frac{(-bx + \coth^{-1}(\tanh(a + bx))) \text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b^2} \\
&= \frac{x}{b} + \frac{(bx - \coth^{-1}(\tanh(a + bx))) \log(\coth^{-1}(\tanh(a + bx)))}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 31, normalized size = 1.00

$$\frac{x}{b} - \frac{(\coth^{-1}(\tanh(a + bx)) - bx) \log(\coth^{-1}(\tanh(a + bx)))}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcCoth[Tanh[a + b\*x]], x]

[Out] x/b - (((-b\*x) + ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]])/b^2

**fricas [B]** time = 0.60, size = 79, normalized size = 2.55

$$\frac{2bx + 2\pi \arctan\left(-\frac{2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right) - a \log(4b^2x^2 + 8abx + \pi^2 + 4a^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a)), x, algorithm="fricas")

[Out] 1/2\*(2\*b\*x + 2\*pi\*arctan(-(2\*b\*x + 2\*a - sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/pi) - a\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/b^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(x/arccoth(tanh(b\*x + a)), x)







$*b*x+2*a))^{-2} \pi \operatorname{csgn}(I \exp(b*x+a)) \operatorname{csgn}(I \exp(2*b*x+2*a))^{2+\pi} \operatorname{csgn}(I \exp(2*b*x+2*a))^{-3-\pi} \operatorname{csgn}(I \exp(2*b*x+2*a)) \operatorname{csgn}(I \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^{2+\pi} \operatorname{csgn}(I \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^{-3+4} I * (\ln(\exp(b*x+a)) - b*x-a) + 4 * I * b*x + 4 * I * a + 2 * \pi) * x - 1 / b^2 * \ln(-2 * \pi \operatorname{csgn}(I / (\exp(2*b*x+2*a)+1))^{2+\pi} \operatorname{csgn}(I / (\exp(2*b*x+2*a)+1)) * \operatorname{csgn}(I \exp(2*b*x+2*a)) \operatorname{csgn}(I \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^{-\pi} \operatorname{csgn}(I / (\exp(2*b*x+2*a)+1)) * \operatorname{csgn}(I \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^{2+2 * \pi} \operatorname{csgn}(I / (\exp(2*b*x+2*a)+1))^{-3+\pi} \operatorname{csgn}(I \exp(b*x+a))^{2 * \pi} \operatorname{csgn}(I \exp(2*b*x+2*a))^{-2 * \pi} \operatorname{csgn}(I \exp(b*x+a)) \operatorname{csgn}(I \exp(2*b*x+2*a))^{2+\pi} \operatorname{csgn}(I \exp(2*b*x+2*a))^{-3-\pi} \operatorname{csgn}(I \exp(2*b*x+2*a)) \operatorname{csgn}(I \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^{2+\pi} \operatorname{csgn}(I \exp(2*b*x+2*a) / (\exp(2*b*x+2*a)+1))^{-3+4} I * (\ln(\exp(b*x+a)) - b*x-a) + 4 * I * b*x + 4 * I * a + 2 * \pi) * \ln(\exp(b*x+a))$

**maxima** [C] time = 0.52, size = 30, normalized size = 0.97

$$\frac{x}{b} - \frac{(-i\pi + 2a) \log(-i\pi + 2bx + 2a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] x/b - 1/2\*(-I\*pi + 2\*a)\*log(-I\*pi + 2\*b\*x + 2\*a)/b^2

**mupad** [B] time = 0.14, size = 108, normalized size = 3.48

$$\frac{x}{b} + \frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) - \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)\right) \left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acoth(tanh(a + b\*x)),x)

[Out] x/b + (log(log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))))\*(log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1) + 2\*b\*x))/(2\*b^2)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/acoth(tanh(b\*x+a)),x)

[Out] Integral(x/acoth(tanh(a + b\*x)), x)

$$3.162 \quad \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=12

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b}$$

[Out] ln(arccoth(tanh(b\*x+a)))/b

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 29}

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^(-1), x]

[Out] Log[ArcCoth[Tanh[a + b\*x]]]/b

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\ &= \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b} \end{aligned}$$

Mathematica [A] time = 0.05, size = 12, normalized size = 1.00

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^(-1), x]

[Out] Log[ArcCoth[Tanh[a + b\*x]]]/b

**fricas** [B] time = 0.68, size = 28, normalized size = 2.33

$$\frac{\log(4b^2x^2 + 8abx + \pi^2 + 4a^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a)), x, algorithm="fricas")

[Out] 1/2\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(1/arccoth(tanh(b\*x + a)), x)

**maple** [A] time = 0.07, size = 13, normalized size = 1.08

$$\frac{\ln(\operatorname{arccoth}(\tanh(bx + a)))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccoth(tanh(b\*x+a)), x)

[Out] ln(arccoth(tanh(b\*x+a)))/b

**maxima** [C] time = 0.41, size = 16, normalized size = 1.33

$$\frac{\log\left(-\frac{1}{2}i\pi - bx - a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a)), x, algorithm="maxima")

[Out]  $\log(-1/2*I*pi - b*x - a)/b$

**mupad [B]** time = 1.18, size = 12, normalized size = 1.00

$$\frac{\ln(\operatorname{acoth}(\tanh(a + b x)))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/acoth(tanh(a + b*x)),x)`

[Out]  $\log(\operatorname{acoth}(\tanh(a + b*x)))/b$

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/acoth(tanh(b*x+a)),x)`

[Out] Exception raised: TypeError

$$3.163 \quad \int \frac{1}{x \coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=44

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\log(x)}{bx - \coth^{-1}(\tanh(a+bx))}$$

[Out]  $-\ln(x)/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))+\ln(\operatorname{arccoth}(\tanh(b*x+a)))/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2160, 2157, 29}

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\log(x)}{bx - \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*ArcCoth[Tanh[a + b\*x]]), x]

[Out]  $-(\operatorname{Log}[x]/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])) + \operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]]/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx &= -\frac{\int \frac{1}{x} dx}{bx - \coth^{-1}(\tanh(a + bx))} + \frac{b \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{bx - \coth^{-1}(\tanh(a + bx))} \\
&= -\frac{\log(x)}{bx - \coth^{-1}(\tanh(a + bx))} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a + bx))\right)}{bx - \coth^{-1}(\tanh(a + bx))} \\
&= -\frac{\log(x)}{bx - \coth^{-1}(\tanh(a + bx))} + \frac{\log(\coth^{-1}(\tanh(a + bx)))}{bx - \coth^{-1}(\tanh(a + bx))}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 29, normalized size = 0.66

$$\frac{\log(\coth^{-1}(\tanh(a + bx))) - \log(x)}{bx - \coth^{-1}(\tanh(a + bx))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*ArcCoth[Tanh[a + b\*x]]), x]

[Out] (-Log[x] + Log[ArcCoth[Tanh[a + b\*x]]])/(b\*x - ArcCoth[Tanh[a + b\*x]])

**fricas [A]** time = 0.52, size = 87, normalized size = 1.98

$$\frac{2 \left( 2 \pi \arctan \left( -\frac{2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi} \right) + a \log(4b^2x^2 + 8abx + \pi^2 + 4a^2) - 2a \log(x) \right)}{\pi^2 + 4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a)), x, algorithm="fricas")

[Out] -2\*(2\*pi\*arctan(-(2\*b\*x + 2\*a - sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/pi) + a\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2) - 2\*a\*log(x))/(pi^2 + 4\*a^2)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(1/(x\*arccoth(tanh(b\*x + a))), x)



**maple [C]** time = 10.10, size = 972, normalized size = 22.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arccoth(tanh(b\*x+a)),x)

[Out]  $4 \cdot I / (2 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)))^3 + \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) - \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^3 - 2 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^2 - \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a))^2 \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) + \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^3 - 4 \cdot I \cdot b \cdot x - 2 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + 4 \cdot I \cdot \ln(\exp(b \cdot x + a)) + 2 \cdot \text{Pi} \cdot \ln(x) - 4 \cdot I / (2 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)))^3 + \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) - \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^3 - 2 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^2 - \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a))^2 \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) + \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^3 - 4 \cdot I \cdot b \cdot x - 2 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + 4 \cdot I \cdot \ln(\exp(b \cdot x + a)) + 2 \cdot \text{Pi} \cdot \ln(-2 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) - \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + 2 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^3 + \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a))^2 \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) - 2 \cdot \text{Pi} \cdot \text{csgn}(I \cdot \exp(b \cdot x + a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^2 + \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a))^3 - \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a)) \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^2 + \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot b \cdot x + 2 \cdot a) / (\exp(2 \cdot b \cdot x + 2 \cdot a) + 1))^3 + 4 \cdot I \cdot (\ln(\exp(b \cdot x + a)) - b \cdot x - a) + 4 \cdot I \cdot b \cdot x + 4 \cdot I \cdot a + 2 \cdot \text{Pi}$

**maxima [C]** time = 0.52, size = 37, normalized size = 0.84

$$\frac{2 \log(-i \pi + 2 b x + 2 a)}{i \pi - 2 a} - \frac{2 \log(x)}{i \pi - 2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $2 \cdot \log(-I \cdot \text{pi} + 2 \cdot b \cdot x + 2 \cdot a) / (I \cdot \text{pi} - 2 \cdot a) - 2 \cdot \log(x) / (I \cdot \text{pi} - 2 \cdot a)$

**mupad [B]** time = 2.91, size = 113, normalized size = 2.57

$$\frac{4 \operatorname{atanh}\left(\frac{4 b x}{\ln\left(-\frac{2}{e^{2 a} e^{2 b x-1}}\right) - \ln\left(\frac{2 e^{2 a} e^{2 b x}}{e^{2 a} e^{2 b x-1}}\right)} + 2 b x} - 1\right)}{\ln\left(-\frac{2}{e^{2 a} e^{2 b x-1}}\right) - \ln\left(\frac{2 e^{2 a} e^{2 b x}}{e^{2 a} e^{2 b x-1}}\right) + 2 b x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*acoth(tanh(a + b*x))),x)`

[Out]  $-4*\operatorname{atanh}\left(\frac{4*b*x}{\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)}\right)/(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/acoth(tanh(b*x+a)),x)`

[Out] `Integral(1/(x*acoth(tanh(a + b*x))), x)`

$$3.164 \quad \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=65

$$\frac{1}{x(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{b \log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^2} + \frac{b \log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))^2}$$

[Out] 1/x/(b\*x-arccoth(tanh(b\*x+a)))-b\*ln(x)/(b\*x-arccoth(tanh(b\*x+a)))^2+b\*ln(arccoth(tanh(b\*x+a)))/(b\*x-arccoth(tanh(b\*x+a)))^2

**Rubi** [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{x(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{b \log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^2} + \frac{b \log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*ArcCoth[Tanh[a + b\*x]]), x]

[Out] 1/(x\*(b\*x - ArcCoth[Tanh[a + b\*x]])) - (b\*Log[x])/(b\*x - ArcCoth[Tanh[a + b\*x]])^2 + (b\*Log[ArcCoth[Tanh[a + b\*x]]])/(b\*x - ArcCoth[Tanh[a + b\*x]])^2

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

Rule 2163

Int[(v\_)^(n\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n + 1)/((n + 1)\*(b\*u - a\*v)), x] - Dist[(a\*(n + 1))/((n

+ 1)\*(b\*u - a\*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))} dx &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} + \frac{b \int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx}{bx - \coth^{-1}(\tanh(a + bx))} \\
 &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b \int \frac{1}{x} dx}{(bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{b^2 \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{(bx - \coth^{-1}(\tanh(a + bx)))^2} \\
 &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{b \operatorname{Subst}\left(\int \frac{1}{x} dx, bx - \coth^{-1}(\tanh(a + bx)), x\right)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} \\
 &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{b \log(\coth^{-1}(\tanh(a + bx)))}{(bx - \coth^{-1}(\tanh(a + bx)))^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 45, normalized size = 0.69

$$\frac{bx (\log(\coth^{-1}(\tanh(a + bx))) - \log(x) + 1) - \coth^{-1}(\tanh(a + bx))}{x (\coth^{-1}(\tanh(a + bx)) - bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*ArcCoth[Tanh[a + b\*x]]),x]

[Out] (-ArcCoth[Tanh[a + b\*x]] + b\*x\*(1 - Log[x] + Log[ArcCoth[Tanh[a + b\*x]]]))/(x\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^2)

**fricas [B]** time = 0.59, size = 137, normalized size = 2.11

$$\frac{2 \left( 16 \pi abx \arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right) - 2\pi^2a - 8a^3 - (\pi^2b - 4a^2b)x \log(4b^2x^2 + 8abx + \pi^2 + 4a^2) \right)}{(\pi^4 + 8\pi^2a^2 + 16a^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out]  $2*(16*\pi*a*b*x*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi) - 2*\pi^2*a - 8*a^3 - (\pi^2*b - 4*a^2*b)*x*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2) + 2*(\pi^2*b - 4*a^2*b)*x*\log(x))/((\pi^4 + 8*\pi^2*a^2 + 16*a^4)*x)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")`

[Out] `integrate(1/(x^2*arccoth(tanh(b*x + a))), x)`

**maple** [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/arccoth(tanh(b*x+a)),x)`

[Out] `int(1/x^2/arccoth(tanh(b*x+a)),x)`

**maxima** [C] time = 0.52, size = 65, normalized size = 1.00

$$-\frac{4b \log(-i\pi + 2bx + 2a)}{\pi^2 + 4i\pi a - 4a^2} + \frac{4b \log(x)}{\pi^2 + 4i\pi a - 4a^2} + \frac{2}{(i\pi - 2a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="maxima")`

[Out]  $-4*b*\log(-I*\pi + 2*b*x + 2*a)/(\pi^2 + 4*I*\pi*a - 4*a^2) + 4*b*\log(x)/(\pi^2 + 4*I*\pi*a - 4*a^2) + 2/((I*\pi - 2*a)*x)$

**mupad** [B] time = 3.12, size = 220, normalized size = 3.38

$$\frac{2 \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right) - 2 \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + 4bx + bx \operatorname{atan}\left(\frac{\ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) 1i - \ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) 1i + bx 2i}{\ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + 2bx}\right)}{x \left(\ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right) - \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + 2bx\right)^2} 8i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*acoth(tanh(a + b\*x))),x)

[Out] (2\*log(-1/(exp(2\*a)\*exp(2\*b\*x) - 1)) - 2\*log((exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 4\*b\*x + b\*x\*atan((log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))\*1i - log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))\*1i + b\*x\*2i)/(log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x))\*8i)/(x\*(log(-1/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/acoth(tanh(b\*x+a)),x)

[Out] Integral(1/(x\*\*2\*acoth(tanh(a + b\*x))), x)

$$3.165 \quad \int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))} dx$$

Optimal. Leaf size=92

$$-\frac{b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^3} + \frac{b^2 \log(\coth^{-1}(\tanh(a + bx)))}{(bx - \coth^{-1}(\tanh(a + bx)))^3} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))}$$

[Out] b/x/(b\*x-arccoth(tanh(b\*x+a)))^2+1/2/x^2/(b\*x-arccoth(tanh(b\*x+a)))-b^2\*ln(x)/(b\*x-arccoth(tanh(b\*x+a)))^3+b^2\*ln(arccoth(tanh(b\*x+a)))/(b\*x-arccoth(tanh(b\*x+a)))^3

**Rubi [A]** time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2163, 2160, 2157, 29}

$$-\frac{b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^3} + \frac{b^2 \log(\coth^{-1}(\tanh(a + bx)))}{(bx - \coth^{-1}(\tanh(a + bx)))^3} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*ArcCoth[Tanh[a + b\*x]]), x]

[Out] b/(x\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2) + 1/(2\*x^2\*(b\*x - ArcCoth[Tanh[a + b\*x]])) - (b^2\*Log[x])/(b\*x - ArcCoth[Tanh[a + b\*x]])^3 + (b^2\*Log[ArcCoth[Tanh[a + b\*x]]])/(b\*x - ArcCoth[Tanh[a + b\*x]])^3

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \coth^{-1}(\tanh(a + bx))} dx &= \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{b \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))} dx}{-bx + \coth^{-1}(\tanh(a + bx))} \\ &= \frac{b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= \frac{b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= \frac{b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= \frac{b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} - \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 0.72

$$\frac{b^2 x^2 (2 \log(\coth^{-1}(\tanh(a + bx))) - 2 \log(x) + 3) - 4bx \coth^{-1}(\tanh(a + bx)) + \coth^{-1}(\tanh(a + bx))^2}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*ArcCoth[Tanh[a + b\*x]]), x]

[Out] (-4\*b\*x\*ArcCoth[Tanh[a + b\*x]] + ArcCoth[Tanh[a + b\*x]]^2 + b^2\*x^2\*(3 - 2\*Log[x] + 2\*Log[ArcCoth[Tanh[a + b\*x]]]))/(2\*x^2\*(b\*x - ArcCoth[Tanh[a + b\*x]]))^3

**fricas [B]** time = 0.68, size = 199, normalized size = 2.16

$$\frac{2 \left( \pi^4 a + 8 \pi^2 a^3 + 16 a^5 - 8 (\pi^3 b^2 - 12 \pi a^2 b^2) x^2 \arctan \left( -\frac{2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi} \right) - 4 (3 \pi^2 ab^2 - 4 a^3 b^2) x^2 \right)}{(\pi^6 + 12 \pi^4 a^2 + 48 \pi^2 a^4 + 64 a^6)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a)),x, algorithm="fricas")

[Out]  $-2*(\pi^4*a + 8*\pi^2*a^3 + 16*a^5 - 8*(\pi^3*b^2 - 12*\pi*a^2*b^2)*x^2*\arctan(-\frac{2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}}{\pi}) - 4*(3*\pi^2*a*b^2 - 4*a^3*b^2)*x^2*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2) + 8*(3*\pi^2*a*b^2 - 4*a^3*b^2)*x^2*\log(x) + 2*(\pi^4*b - 16*a^4*b)*x)/((\pi^6 + 12*\pi^4*a^2 + 48*\pi^2*a^4 + 64*a^6)*x^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(1/(x^3\*arccoth(tanh(b\*x + a))), x)

**maple** [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/arccoth(tanh(b\*x+a)),x)

[Out] int(1/x^3/arccoth(tanh(b\*x+a)),x)

**maxima** [C] time = 0.55, size = 108, normalized size = 1.17

$$\frac{8b^2 \log(-i\pi + 2bx + 2a)}{-i\pi^3 + 6\pi^2a + 12i\pi a^2 - 8a^3} - \frac{8b^2 \log(x)}{-i\pi^3 + 6\pi^2a + 12i\pi a^2 - 8a^3} - \frac{2(i\pi + 4bx - 2a)}{(2\pi^2 + 8i\pi a - 8a^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $8*b^2*\log(-I*\pi + 2*b*x + 2*a)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3) - 8*b^2*\log(x)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3) - 2*(I*\pi + 4*b*x - 2*a)/((2*\pi^2 + 8*I*\pi*a - 8*a^2)*x^2)$

mupad [B] time = 3.60, size = 300, normalized size = 3.26

$$\frac{\ln\left(-\frac{1}{e^{2a}e^{2bx}-1}\right)^2 - \ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)\left(2\ln\left(-\frac{1}{e^{2a}e^{2bx}-1}\right) + 8bx\right) + \ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)^2 + 12b^2x^2 + 8bx\ln\left(-\frac{1}{e^{2a}e^{2bx}-1}\right)}{x^2\left(\ln\left(-\frac{1}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3\*acoth(tanh(a + b\*x))),x)

[Out] (log(-1/(exp(2\*a)\*exp(2\*b\*x) - 1))^2 - log((exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))\*(2\*log(-1/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 8\*b\*x) + log((exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))^2 + 12\*b^2\*x^2 + b^2\*x^2\*atan(log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))\*1i - log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))\*1i + b\*x\*2i)/(log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x))\*16i + 8\*b\*x\*log(-1/(exp(2\*a)\*exp(2\*b\*x) - 1)))/(x^2\*(log(-1/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{acoth}(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/acoth(tanh(b\*x+a)),x)

[Out] Integral(1/(x\*\*3\*acoth(tanh(a + b\*x))), x)

$$3.166 \quad \int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2} dx$$

Optimal. Leaf size=65

$$-\frac{x^m {}_2F_1\left(1, m; m+1; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{b(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{x^m}{b \coth^{-1}(\tanh(a+bx))}$$

[Out]  $-x^m/b/\operatorname{arccoth}(\tanh(b*x+a))-x^m*\operatorname{hypergeom}([1, m], [1+m], b*x/(b*x-\operatorname{arccoth}(\tanh(b*x+a))))/b/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))$

**Rubi [A]** time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 2164}

$$-\frac{x^m {}_2F_1\left(1, m; m+1; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{b(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{x^m}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^m/\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2, x]$

[Out]  $-(x^m/(b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])) - (x^m*\operatorname{Hypergeometric2F1}[1, m, 1 + m, (b*x)/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])]/(b*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]))$

#### Rule 2164

$\operatorname{Int}[(v_)^(n_)/(u_), x\_Symbol] := \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(v^(n+1)*\operatorname{Hypergeometric2F1}[1, n+1, n+2, -((a*v)/(b*u - a*v))]/((n+1)*(b*u - a*v)), x] /; \operatorname{NeQ}[b*u - a*v, 0]] /; \operatorname{PiecewiseLinearQ}[u, v, x] \&\& \operatorname{IntegerQ}[n]$

#### Rule 2168

$\operatorname{Int}[(u_)^(m_)*(v_)^(n_), x\_Symbol] := \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^(m+1)*v^n)/(a*(m+1)), x] - \operatorname{Dist}[(b*n)/(a*(m+1)), \operatorname{Int}[u^(m+1)*v^(n-1), x], x] /; \operatorname{NeQ}[b*u - a*v, 0]] /; \operatorname{FreeQ}[\{m, n\}, x] \&\& \operatorname{PiecewiseLinearQ}[u, v, x] \&\& \operatorname{NeQ}[m, -1] \&\& ((\operatorname{LtQ}[m, -1] \&\& \operatorname{GtQ}[n, 0] \&\& !(\operatorname{ILtQ}[m+n, -2] \&\& (\operatorname{FractionQ}[m] || \operatorname{GeQ}[2*n+m+1, 0]))) || (\operatorname{IGtQ}[n, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{LeQ}[n, m]) || (\operatorname{IGtQ}[n, 0] \&\& !\operatorname{IntegerQ}[m]) || (\operatorname{ILtQ}[m, 0] \&\& !\operatorname{IntegerQ}[n]))$

#### Rubi steps

$$\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^2} dx = -\frac{x^m}{b \coth^{-1}(\tanh(a+bx))} + \frac{m \int \frac{x^{-1+m}}{\coth^{-1}(\tanh(a+bx))} dx}{b}$$

$$= -\frac{x^m}{b \coth^{-1}(\tanh(a+bx))} - \frac{x^m {}_2F_1\left(1, m; 1+m; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{b (bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica** [A] time = 0.54, size = 51, normalized size = 0.78

$$\frac{x^{m+1} {}_2F_1\left(2, m+1; m+2; -\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1) (\coth^{-1}(\tanh(a+bx)) - bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/ArcCoth[Tanh[a + b\*x]]^2, x]

[Out] (x^(1 + m)\*Hypergeometric2F1[2, 1 + m, 2 + m, -((b\*x)/(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))])/((1 + m)\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^2)

**fricas** [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\text{arccoth}(\tanh(bx+a))^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^2, x, algorithm="fricas")

[Out] integral(x^m/arccoth(tanh(b\*x + a))^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arccoth}(\tanh(bx+a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^2, x, algorithm="giac")

[Out] integrate(x^m/arccoth(tanh(b\*x + a))^2, x)

**maple** [F] time = 2.95, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/arccoth(tanh(b\*x+a))^2,x)

[Out] int(x^m/arccoth(tanh(b\*x+a))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arcoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] integrate(x^m/arccoth(tanh(b\*x + a))^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{acoth}(\tanh(a + bx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/acoth(tanh(a + b\*x))^2,x)

[Out] int(x^m/acoth(tanh(a + b\*x))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(x\*\*m/acoth(tanh(a + b\*x))\*\*2, x)

$$3.167 \quad \int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{4 \left( bx - \coth^{-1}(\tanh(a+bx)) \right)^3 \log \left( \coth^{-1}(\tanh(a+bx)) \right)}{b^5} + \frac{4x \left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2}{b^4} + \frac{2x^2 \left( bx - \coth^{-1}(\tanh(a+bx)) \right)}{b^3}$$

[Out]  $4/3*x^3/b^2+2*x^2*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/b^3+4*x*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2/b^4-x^4/b/\operatorname{arccoth}(\tanh(b*x+a))+4*(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^3*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/b^5$

**Rubi [A]** time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{2x^2 \left( bx - \coth^{-1}(\tanh(a+bx)) \right)}{b^3} + \frac{4x \left( bx - \coth^{-1}(\tanh(a+bx)) \right)^2}{b^4} + \frac{4 \left( bx - \coth^{-1}(\tanh(a+bx)) \right)^3 \log \left( \coth^{-1}(\tanh(a+bx)) \right)}{b^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out]  $(4*x^3)/(3*b^2) + (2*x^2*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]))/b^3 + (4*x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2)/b^4 - x^4/(b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + (4*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^3*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/b^5$

**Rule 29**

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

**Rule 2157**

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

**Rule 2158**

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

**Rule 2159**

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
;/; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n},
x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
&& !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\coth^{-1}(\tanh(a + bx))^2} dx &= -\frac{x^4}{b \coth^{-1}(\tanh(a + bx))} + \frac{4 \int \frac{x^3}{\coth^{-1}(\tanh(a + bx))} dx}{b} \\
&= \frac{4x^3}{3b^2} - \frac{x^4}{b \coth^{-1}(\tanh(a + bx))} - \frac{(4(-bx + \coth^{-1}(\tanh(a + bx)))) \int \frac{x^2}{\coth^{-1}(\tanh(a + bx))} dx}{b^2} \\
&= \frac{4x^3}{3b^2} + \frac{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{b^3} - \frac{x^4}{b \coth^{-1}(\tanh(a + bx))} + \frac{(4(-bx + \coth^{-1}(\tanh(a + bx))))^2 \int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx}{b^4} \\
&= \frac{4x^3}{3b^2} + \frac{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{b^3} + \frac{4x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^4} - \frac{x^4}{b \coth^{-1}(\tanh(a + bx))} \\
&= \frac{4x^3}{3b^2} + \frac{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{b^3} + \frac{4x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^4} - \frac{x^4}{b \coth^{-1}(\tanh(a + bx))} \\
&= \frac{4x^3}{3b^2} + \frac{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))}{b^3} + \frac{4x (bx - \coth^{-1}(\tanh(a + bx)))^2}{b^4} - \frac{x^4}{b \coth^{-1}(\tanh(a + bx))}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 106, normalized size = 1.08

$$\frac{(\coth^{-1}(\tanh(a + bx)) - bx)^4}{b^5 \coth^{-1}(\tanh(a + bx))} - \frac{4(\coth^{-1}(\tanh(a + bx)) - bx)^3 \log(\coth^{-1}(\tanh(a + bx)))}{b^5} + \frac{3x(\coth^{-1}(\tanh(a + bx)) - bx)^2}{b^4} - \frac{x^4}{b \coth^{-1}(\tanh(a + bx))}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out]  $x^3/(3*b^2) - (x^2*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]]))/b^3 + (3*x*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^2)/b^4 - (-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^4/(b^5*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - (4*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^3*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/b^5$

**fricas** [B] time = 0.52, size = 326, normalized size = 3.33

$16b^5x^5 - 16ab^4x^4 + 9\pi^4a + 24\pi^2a^3 - 48a^5 - 32(\pi^2b^3 - 2a^2b^3)x^3 - 12(7\pi^2ab^2 - 20a^3b^2)x^2 - 12(\pi^4b - 6\pi^2$

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out]  $1/12*(16*b^5*x^5 - 16*a*b^4*x^4 + 9*\pi^4*a + 24*\pi^2*a^3 - 48*a^5 - 32*(\pi^2*b^3 - 2*a^2*b^3)*x^3 - 12*(7*\pi^2*a*b^2 - 20*a^3*b^2)*x^2 - 12*(\pi^4*b - 6*\pi^2*a^2*b - 8*a^4*b)*x - 12*(\pi^5 - 8*\pi^3*a^2 - 48*\pi*a^4 + 4*(\pi^3*b^2 - 12*\pi*a^2*b^2)*x^2 + 8*(\pi^3*a*b - 12*\pi*a^3*b)*x)*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi) + 6*(3*\pi^4*a + 8*\pi^2*a^3 - 16*a^5 + 4*(3*\pi^2*a*b^2 - 4*a^3*b^2)*x^2 + 8*(3*\pi^2*a^2*b - 4*a^4*b)*x)*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2)/(4*b^7*x^2 + 8*a*b^6*x + \pi^2*b^5 + 4*a^2*b^5)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\text{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x^4/arccoth(tanh(b\*x + a))^2, x)

**maple** [C] time = 5.95, size = 131085, normalized size = 1337.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/arccoth(tanh(b\*x+a))^2,x)



[Out] result too large to display

**maxima [C]** time = 0.77, size = 178, normalized size = 1.82

$$\frac{4(16b^4x^4 - 3\pi^4 - 24i\pi^3a + 72\pi^2a^2 + 96i\pi a^3 - 48a^4 + (16i\pi b^3 - 32ab^3)x^3 - (24\pi^2b^2 + 96i\pi ab^2 - 96a^2b^2)x^2 + (18i\pi^3b - 108\pi^2a*b - 216i\pi a^2b + 144a^3b)x)/(192b^6x - 96i\pi b^5 + 192ab^5) - 1/2*(i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3)*\log(-i\pi + 2b*x + 2a)/b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 4\*(16\*b^4\*x^4 - 3\*pi^4 - 24\*I\*pi^3\*a + 72\*pi^2\*a^2 + 96\*I\*pi\*a^3 - 48\*a^4 + (16\*I\*pi\*b^3 - 32\*a\*b^3)\*x^3 - (24\*pi^2\*b^2 + 96\*I\*pi\*a\*b^2 - 96\*a^2\*b^2)\*x^2 + (18\*I\*pi^3\*b - 108\*pi^2\*a\*b - 216\*I\*pi\*a^2\*b + 144\*a^3\*b)\*x)/(192\*b^6\*x - 96\*I\*pi\*b^5 + 192\*a\*b^5) - 1/2\*(I\*pi^3 - 6\*pi^2\*a - 12\*I\*pi\*a^2 + 8\*a^3)\*log(-I\*pi + 2\*b\*x + 2\*a)/b^5

**mupad [B]** time = 1.29, size = 669, normalized size = 6.83

$$\frac{x^3}{3b^2} \frac{\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) + 2bx\right)^4 + 24a^2 \left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) + 2bx\right)^2}{2b \left(8ab^4 + 8b^5x - 4b^4 \left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) + 2bx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/acoth(tanh(a + b\*x))^2,x)

[Out] x^3/(3\*b^2) - ((2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^4 + 24\*a^2\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^2 + 16\*a^4 - 8\*a\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^3 - 32\*a^3\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x))/(2\*b\*(8\*a\*b^4 + 8\*b^5\*x - 4\*b^4\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x))) + (x^2\*(log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x))/(2\*b^3) + (3\*x\*(log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^2)/(4\*b^4) + (log(log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1))))\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^3 - 8\*a^3 - 6\*a\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^2 + 12\*a^2\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x))/(2\*b^5)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(x\*\*4/acoth(tanh(a + b\*x))\*\*2, x)

$$3.168 \quad \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=75

$$\frac{3(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^4} + \frac{3x(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^3}{b \coth^{-1}(\tanh(a+bx))}$$

[Out] 3/2\*x^2/b^2+3\*x\*(b\*x-arccoth(tanh(b\*x+a)))/b^3-x^3/b/arccoth(tanh(b\*x+a))+3\*(b\*x-arccoth(tanh(b\*x+a)))^2\*ln(arccoth(tanh(b\*x+a)))/b^4

**Rubi [A]** time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{3x(bx - \coth^{-1}(\tanh(a+bx)))}{b^3} + \frac{3(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^3}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Int[x^3/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (3\*x^2)/(2\*b^2) + (3\*x\*(b\*x - ArcCoth[Tanh[a + b\*x]]))/b^3 - x^3/(b\*ArcCoth[Tanh[a + b\*x]]) + (3\*(b\*x - ArcCoth[Tanh[a + b\*x]])^2\*Log[ArcCoth[Tanh[a + b\*x]]])/b^4

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rule 2157

Int[(u\_)^(m\_), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

#### Rule 2159

Int[(v\_)^(n\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^n/(a\*n), x] - Dist[(b\*u - a\*v)/a, Int[v^(n-1)/u, x], x]

;/; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n, 1]

### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] :=> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)\*v^n)/(a\*(m + 1)), x] - Dist[(b\*n)/(a\*(m + 1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

### Rubi steps

$$\begin{aligned} \int \frac{x^3}{\coth^{-1}(\tanh(a + bx))^2} dx &= -\frac{x^3}{b \coth^{-1}(\tanh(a + bx))} + \frac{3 \int \frac{x^2}{\coth^{-1}(\tanh(a + bx))} dx}{b} \\ &= \frac{3x^2}{2b^2} - \frac{x^3}{b \coth^{-1}(\tanh(a + bx))} - \frac{(3(-bx + \coth^{-1}(\tanh(a + bx)))) \int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx}{b^2} \\ &= \frac{3x^2}{2b^2} + \frac{3x(bx - \coth^{-1}(\tanh(a + bx)))}{b^3} - \frac{x^3}{b \coth^{-1}(\tanh(a + bx))} + \frac{(3(-bx + \coth^{-1}(\tanh(a + bx)))) \int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx}{b^2} \\ &= \frac{3x^2}{2b^2} + \frac{3x(bx - \coth^{-1}(\tanh(a + bx)))}{b^3} - \frac{x^3}{b \coth^{-1}(\tanh(a + bx))} + \frac{(3(-bx + \coth^{-1}(\tanh(a + bx)))) \int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx}{b^2} \\ &= \frac{3x^2}{2b^2} + \frac{3x(bx - \coth^{-1}(\tanh(a + bx)))}{b^3} - \frac{x^3}{b \coth^{-1}(\tanh(a + bx))} + \frac{3(bx - \coth^{-1}(\tanh(a + bx))) \int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx}{b^3} \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 83, normalized size = 1.11

$$\frac{(\coth^{-1}(\tanh(a + bx)) - bx)^3}{b^4 \coth^{-1}(\tanh(a + bx))} + \frac{3(\coth^{-1}(\tanh(a + bx)) - bx)^2 \log(\coth^{-1}(\tanh(a + bx)))}{b^4} - \frac{2x(\coth^{-1}(\tanh(a + bx)) - bx)}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCoth[Tanh[a + b\*x]]^2, x]

[Out]  $x^2/(2*b^2) - (2*x*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]]))/b^3 + (-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^3/(b^4*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + (3*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/b^4$

**fricas** [B] time = 0.50, size = 244, normalized size = 3.25

$$\frac{16b^4x^4 - 32ab^3x^3 - 2\pi^4 + 32a^4 + 4(\pi^2b^2 - 28a^2b^2)x^2 - 8(5\pi^2ab + 4a^3b)x - 48(4\pi ab^2x^2 + 8\pi a^2bx + \pi^3a)}{8(4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")`

[Out]  $1/8*(16*b^4*x^4 - 32*a*b^3*x^3 - 2*\pi^4 + 32*a^4 + 4*(\pi^2*b^2 - 28*a^2*b^2)*x^2 - 8*(5*\pi^2*a*b + 4*a^3*b)*x - 48*(4*\pi*a*b^2*x^2 + 8*\pi*a^2*b*x + \pi^3*a + 4*\pi*a^3)*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi) - 3*(\pi^4 - 16*a^4 + 4*(\pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*(\pi^2*a*b - 4*a^3*b)*x)*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2)/(4*b^6*x^2 + 8*a*b^5*x + \pi^2*b^4 + 4*a^2*b^4)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\text{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

[Out] `integrate(x^3/arccoth(tanh(b*x + a))^2, x)`

**maple** [C] time = 1.48, size = 29109, normalized size = 388.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/arccoth(tanh(b*x+a))^2,x)`

[Out] result too large to display

**maxima** [C] time = 0.76, size = 124, normalized size = 1.65

$$\frac{4(4b^3x^3 + i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3 + (6i\pi b^2 - 12ab^2)x^2 + (4\pi^2b + 16i\pi ab - 16a^2b)x)(3\pi^2 + 12i\pi a - 12i\pi b)}{32b^5x - 16i\pi b^4 + 32ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out]  $4*(4*b^3*x^3 + I*pi^3 - 6*pi^2*a - 12*I*pi*a^2 + 8*a^3 + (6*I*pi*b^2 - 12*a*b^2)*x^2 + (4*pi^2*b + 16*I*pi*a*b - 16*a^2*b)*x)/(32*b^5*x - 16*I*pi*b^4 + 32*a*b^4) - 1/4*(3*pi^2 + 12*I*pi*a - 12*a^2)*\log(-I*pi + 2*b*x + 2*a)/b^4$

**mupad [B]** time = 0.17, size = 490, normalized size = 6.53

$$\frac{x^2}{2b^2} + \frac{\ln\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) - \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)\right) \left(3\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^2 - 12a\left(2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) + 2bx\right)\right)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/acoth(tanh(a + b\*x))^2,x)

[Out]  $x^2/(2*b^2) + (\log(\log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))) - \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)))*(3*(2*a - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 - 12*a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x + 12*a^2))/ (4*b^4) - ((2*a - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))/ (4*b*(2*a*b^3 + 2*b^4*x - b^3*(2*a - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))) + (x*(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1))) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))/b^3$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(x\*\*3/acoth(tanh(a + b\*x))\*\*2, x)

$$3.169 \quad \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=50

$$\frac{2(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} + \frac{2x}{b^2}$$

[Out]  $2*x/b^2 - x^2/b/\operatorname{arccoth}(\tanh(b*x+a)) + 2*(b*x - \operatorname{arccoth}(\tanh(b*x+a)))*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/b^3$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2168, 2158, 2157, 29}

$$\frac{2(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} + \frac{2x}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out]  $(2*x)/b^2 - x^2/(b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + (2*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/b^3$

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

#### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_.), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n},

x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} dx &= -\frac{x^2}{b \coth^{-1}(\tanh(a+bx))} + \frac{2 \int \frac{x}{\coth^{-1}(\tanh(a+bx))} dx}{b} \\ &= \frac{2x}{b^2} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} - \frac{(2(-bx + \coth^{-1}(\tanh(a+bx)))) \int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{b^2} \\ &= \frac{2x}{b^2} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} - \frac{(2(-bx + \coth^{-1}(\tanh(a+bx)))) \text{Subst}\left(\int \frac{1}{x} dx, x, -bx + \coth^{-1}(\tanh(a+bx))\right)}{b^3} \\ &= \frac{2x}{b^2} - \frac{x^2}{b \coth^{-1}(\tanh(a+bx))} + \frac{2(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^3} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 56, normalized size = 1.12

$$\frac{\frac{(\coth^{-1}(\tanh(a+bx))-bx)^2}{\coth^{-1}(\tanh(a+bx))} + 2(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx))) + bx}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (b\*x - (-(b\*x) + ArcCoth[Tanh[a + b\*x]]))^2/ArcCoth[Tanh[a + b\*x]] + 2\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*Log[ArcCoth[Tanh[a + b\*x]]]/b^3

**fricas [B]** time = 0.46, size = 189, normalized size = 3.78

$$\frac{4b^3x^3 + 8ab^2x^2 + 2\pi^2bx - \pi^2a - 4a^3 + 2(4\pi b^2x^2 + 8\pi abx + \pi^3 + 4\pi a^2) \arctan\left(-\frac{2bx+2a-\sqrt{4b^2x^2+8abx+\pi^2+4a^2}}{\pi}\right)}{4b^5x^2 + 8ab^4x + \pi^2b^3 + 4a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")



```
[Out] (4*b^3*x^3 + 8*a*b^2*x^2 + 2*pi^2*b*x - pi^2*a - 4*a^3 + 2*(4*pi*b^2*x^2 +
8*pi*a*b*x + pi^3 + 4*pi*a^2)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b
*x + pi^2 + 4*a^2))/pi) - (4*a*b^2*x^2 + 8*a^2*b*x + pi^2*a + 4*a^3)*log(4*
b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2))/(4*b^5*x^2 + 8*a*b^4*x + pi^2*b^3 + 4*a^
2*b^3)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/arccoth(tanh(b*x + a))^2, x)
```

**maple** [C] time = 0.48, size = 4626, normalized size = 92.52

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] -4*I*x^2/b/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1)
)*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn
(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*P
i*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-P
i*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a))+2*x/b^2-I
/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(e
xp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(
exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*cs
gn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*b*x+4*I
*a+2*Pi)*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+1/2*I/b^3*ln(-2*Pi*csgn(I/(exp(2*b
*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(
I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2
*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
```

$$\begin{aligned}
& )+1))^{3+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*b*x+4*I*a+2*Pi)*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))- \\
& 1/2*I/b^3*\ln(-2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{2+Pi*csgn(I/(\exp(2*b*x+2*a)+1))})*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-Pi*csgn \\
& (I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2+2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{3+Pi*csgn(I*\exp(b*x+a))^{2*csgn(I*\exp(2*b*x+2*a))-2* \\
& Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^{2+Pi*csgn(I*\exp(2*b*x+2*a))^{3- \\
& Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2+Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{3+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*b* \\
& x+4*I*a+2*Pi)*Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b* \\
& x+2*a)+1))^{2+I/b^3*\ln(-2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{2+Pi*csgn(I/(\exp(2*b \\
& *x+2*a)+1))})*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1) \\
& )-Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2 \\
& +2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{3+Pi*csgn(I*\exp(b*x+a))^{2*csgn(I*\exp(2*b*x \\
& +2*a))-2*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^{2+Pi*csgn(I*\exp(2*b*x \\
& +2*a))^{3-Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1) \\
& )^{2+Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{3+4*I*(\ln(\exp(b*x+a))-b*x- \\
& a)+4*I*b*x+4*I*a+2*Pi)*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{3+1/2*I/b^3*\ln(-2*Pi*c \\
& sgn(I/(\exp(2*b*x+2*a)+1))^{2+Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+ \\
& 2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-Pi*csgn(I/(\exp(2*b*x+2*a)+1 \\
& ))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2+2*Pi*csgn(I/(\exp(2*b*x+2*a)+ \\
& 1))^{3+Pi*csgn(I*\exp(b*x+a))^{2*csgn(I*\exp(2*b*x+2*a))-2*Pi*csgn(I*\exp(b*x+a) \\
& )}*csgn(I*\exp(2*b*x+2*a))^{2+Pi*csgn(I*\exp(2*b*x+2*a))^{3-Pi*csgn(I*\exp(2*b*x+ \\
& 2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2+Pi*csgn(I*\exp(2*b*x+2*a)/ \\
& (\exp(2*b*x+2*a)+1))^{3+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*b*x+4*I*a+2*Pi)*Pi*csgn \\
& (I*\exp(b*x+a))^{2*csgn(I*\exp(2*b*x+2*a))-I/b^3*\ln(-2*Pi*csgn(I/(\exp(2*b*x+2 \\
& *a)+1))^{2+Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2 \\
& *b*x+2*a)/(\exp(2*b*x+2*a)+1))-Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b* \\
& x+2*a)/(\exp(2*b*x+2*a)+1))^{2+2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{3+Pi*csgn(I*ex \\
& p(b*x+a))^{2*csgn(I*\exp(2*b*x+2*a))-2*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x \\
& +2*a))^{2+Pi*csgn(I*\exp(2*b*x+2*a))^{3-Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2 \\
& *b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2+Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1) \\
& )^{3+4*I*(\ln(\exp(b*x+a))-b*x-a)+4*I*b*x+4*I*a+2*Pi)*Pi*csgn(I*\exp(b*x+a))*cs \\
& gn(I*\exp(2*b*x+2*a))^{2+1/2*I/b^3*\ln(-2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{2+Pi*c \\
& sgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp \\
& (2*b*x+2*a)+1))-Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2* \\
& b*x+2*a)+1))^{2+2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{3+Pi*csgn(I*\exp(b*x+a))^{2*cs \\
& gn(I*\exp(2*b*x+2*a))-2*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^{2+Pi*cs \\
& gn(I*\exp(2*b*x+2*a))^{3-Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp \\
& (2*b*x+2*a)+1))^{2+Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{3+4*I*(\ln(ex \\
& p(b*x+a))-b*x-a)+4*I*b*x+4*I*a+2*Pi)*Pi*csgn(I*\exp(2*b*x+2*a))^{3-1/2*I/b^3* \\
& \ln(-2*Pi*csgn(I/(\exp(2*b*x+2*a)+1))^{2+Pi*csgn(I/(\exp(2*b*x+2*a)+1))*csgn(I* \\
& \exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))-Pi*csgn(I/(\exp(2* \\
& b*x+2*a)+1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)+1))^{2+2*Pi*csgn(I/(\exp(2 \\
& *b*x+2*a)+1))^{3+Pi*csgn(I*\exp(b*x+a))^{2*csgn(I*\exp(2*b*x+2*a))-2*Pi*csgn(I*
\end{aligned}$$

```

exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2
*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*b*x+4*I*a+2*
Pi)*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1
/2*I/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1)
))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(
I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn
(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*P
i*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-P
i*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*b*x
+4*I*a+2*Pi)*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I/b^3*Pi*ln(-2*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2
*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*
x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*
b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*b*x+4*I*a+2*Pi)+2/
b^2*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(ex
p(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(e
xp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csg
n(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x-a)+4*I*b*x+4*I*
a+2*Pi)*x-2/b^3*ln(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+
2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-P
i*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*
Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*
a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2
+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+4*I*(ln(exp(b*x+a))-b*x-a)+
4*I*b*x+4*I*a+2*Pi)*ln(exp(b*x+a))

```

**maxima** [C] time = 0.75, size = 80, normalized size = 1.60

$$\frac{4(4b^2x^2 + \pi^2 + 4i\pi a - 4a^2 + (-2i\pi b + 4ab)x)}{16b^4x - 8i\pi b^3 + 16ab^3} - \frac{(-i\pi + 2a)\log(-i\pi + 2bx + 2a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out] 4\*(4\*b^2\*x^2 + pi^2 + 4\*I\*pi\*a - 4\*a^2 + (-2\*I\*pi\*b + 4\*a\*b)\*x)/(16\*b^4\*x - 8\*I\*pi\*b^3 + 16\*a\*b^3) - (-I\*pi + 2\*a)\*log(-I\*pi + 2\*b\*x + 2\*a)/b^3

**mupad [B]** time = 1.29, size = 302, normalized size = 6.04

$$\frac{x \left( 2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) + 2bx \right)^2 - 4a \left( 2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) + 2bx \right) + 4a^2}{b^2 \left( 2ab^2 + 2b^3x - b^2 \left( 2a - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) + 2bx \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/acoth(tanh(a + b*x))^2,x)`

[Out]  $x/b^2 - ((2a - \log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx) - 1))) + \log(-2/(\exp(2a)\exp(2bx) - 1)) + 2bx)^2 - 4a(2a - \log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx) - 1))) + \log(-2/(\exp(2a)\exp(2bx) - 1)) + 2bx + 4a^2)/(2b(2ab^2 + 2b^3x - b^2(2a - \log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx) - 1))) + \log(-2/(\exp(2a)\exp(2bx) - 1)) + 2bx)) + (\log(\log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx) - 1))) - \log(-2/(\exp(2a)\exp(2bx) - 1)))(\log(-2/(\exp(2a)\exp(2bx) - 1)) - \log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx) - 1)) + 2bx)/b^3$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/acoth(tanh(b*x+a))**2,x)`

[Out] `Integral(x**2/acoth(tanh(a + b*x))**2, x)`

$$3.170 \quad \int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx$$

Optimal. Leaf size=28

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^2} - \frac{x}{b \coth^{-1}(\tanh(a+bx))}$$

[Out]  $-x/b/\operatorname{arccoth}(\tanh(b*x+a))+\ln(\operatorname{arccoth}(\tanh(b*x+a)))/b^2$

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 29}

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^2} - \frac{x}{b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] `Int[x/ArcCoth[Tanh[a + b*x]]^2,x]`

[Out]  $-(x/(b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])) + \operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]]/b^2$

Rule 29

`Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]`

Rule 2157

`Int[(u_)^(m_), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[  
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]`

Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim-  
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +  
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n},  
x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]  
&& !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[  
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ  
[m, 0] && !IntegerQ[n]))`

Rubi steps

$$\begin{aligned}
\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx &= -\frac{x}{b \coth^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{b} \\
&= -\frac{x}{b \coth^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^2} \\
&= -\frac{x}{b \coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 27, normalized size = 0.96

$$-\frac{bx}{\coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx))) + 1}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcCoth[Tanh[a + b\*x]]^2,x]

[Out] (1 - (b\*x)/ArcCoth[Tanh[a + b\*x]] + Log[ArcCoth[Tanh[a + b\*x]]])/b^2

**fricas [B]** time = 0.75, size = 97, normalized size = 3.46

$$\frac{8abx + 2\pi^2 + 8a^2 + (4b^2x^2 + 8abx + \pi^2 + 4a^2)\log(4b^2x^2 + 8abx + \pi^2 + 4a^2)}{2(4b^4x^2 + 8ab^3x + \pi^2b^2 + 4a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 1/2\*(8\*a\*b\*x + 2\*pi^2 + 8\*a^2 + (4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2))/(4\*b^4\*x^2 + 8\*a\*b^3\*x + pi^2\*b^2 + 4\*a^2\*b^2)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arccoth}(\tanh(bx+a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(x/arccoth(tanh(b\*x + a))^2, x)

**maple** [C] time = 0.27, size = 625, normalized size = 22.32

$$b \left( -2\pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a}+1} \right)^2 + \pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a}+1} \right) \operatorname{csgn} \left( i e^{2bx+2a} \right) \operatorname{csgn} \left( \frac{i e^{2bx+2a}}{e^{2bx+2a}+1} \right) - \pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a}+1} \right) \operatorname{csgn} \left( \frac{i e^{2bx+2a}}{e^{2bx+2a}+1} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccoth(tanh(b\*x+a))^2,x)

[Out]  $-4*I*x/b/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))+1/b^2*ln(ln(exp(b*x+a))-1/4*I*Pi*(-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*csgn(I/(exp(2*b*x+2*a)+1))^3+csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a))^3-csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2))$

**maxima** [C] time = 0.76, size = 47, normalized size = 1.68

$$\frac{4(-i\pi + 2a)}{8b^3x - 4i\pi b^2 + 8ab^2} + \frac{\log(-i\pi + 2bx + 2a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out]  $4*(-I*pi + 2*a)/(8*b^3*x - 4*I*pi*b^2 + 8*a*b^2) + \log(-I*pi + 2*b*x + 2*a)/b^2$

**mupad** [B] time = 0.09, size = 28, normalized size = 1.00

$$\frac{\ln(\operatorname{acoth}(\tanh(a + bx)))}{b^2} - \frac{x}{b \operatorname{acoth}(\tanh(a + bx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/acoth(tanh(a + b*x))^2,x)
```

```
[Out] log(acoth(tanh(a + b*x)))/b^2 - x/(b*acoth(tanh(a + b*x)))
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Exception raised: TypeError
```



$$3.171 \quad \int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} dx$$

Optimal. Leaf size=14

$$-\frac{1}{b \coth^{-1}(\tanh(a + bx))}$$

[Out] -1/b/arccoth(tanh(b\*x+a))

**Rubi [A]** time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$-\frac{1}{b \coth^{-1}(\tanh(a + bx))}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^(-2), x]

[Out] -(1/(b\*ArcCoth[Tanh[a + b\*x]]))

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2157

Int[(u\_)^(m\_), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a + bx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= -\frac{1}{b \coth^{-1}(\tanh(a + bx))} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{b \coth^{-1}(\tanh(a + bx))}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^(-2), x]

[Out] -(1/(b\*ArcCoth[Tanh[a + b\*x]]))

**fricas** [B] time = 0.65, size = 36, normalized size = 2.57

$$-\frac{4(bx + a)}{4b^3x^2 + 8ab^2x + \pi^2b + 4a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^2, x, algorithm="fricas")

[Out] -4\*(b\*x + a)/(4\*b^3\*x^2 + 8\*a\*b^2\*x + pi^2\*b + 4\*a^2\*b)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^2, x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^(-2), x)

**maple** [A] time = 0.08, size = 15, normalized size = 1.07

$$-\frac{1}{b \operatorname{arccoth}(\tanh(bx + a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccoth(tanh(b\*x+a))^2, x)

[Out] -1/b/arccoth(tanh(b\*x+a))

**maxima** [C] time = 0.41, size = 18, normalized size = 1.29

$$\frac{4}{-2(i\pi + 2bx + 2a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^2, x, algorithm="maxima")

[Out] 4/((-2\*I\*pi - 4\*b\*x - 4\*a)\*b)

mupad [B] time = 1.14, size = 14, normalized size = 1.00

$$\frac{1}{b \operatorname{acoth}(\tanh(a + b x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/acoth(tanh(a + b*x))^2,x)`

[Out] `-1/(b*acoth(tanh(a + b*x)))`

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/acoth(tanh(b*x+a))**2,x)`

[Out] Exception raised: TypeError

$$3.172 \quad \int \frac{1}{x \coth^{-1}(\tanh(a+bx))^2} dx$$

**Optimal.** Leaf size=70

$$-\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{\log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} - \frac{\log(\coth^{-1}(\tanh(a + bx)))}{(bx - \coth^{-1}(\tanh(a + bx)))}$$

[Out]  $-1/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/\operatorname{arccoth}(\tanh(b*x+a))+\ln(x)/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2-\ln(\operatorname{arccoth}(\tanh(b*x+a)))/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2$

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2163, 2160, 2157, 29}

$$-\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{\log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))^2} - \frac{\log(\coth^{-1}(\tanh(a + bx)))}{(bx - \coth^{-1}(\tanh(a + bx)))}$$

Antiderivative was successfully verified.

[In] `Int[1/(x*ArcCoth[Tanh[a + b*x]]^2),x]`

[Out]  $-(1/((b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])) + \operatorname{Log}[x]/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2 - \operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]]/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2$

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 2157

`Int[(u_)^(m_.), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]`

Rule 2160

`Int[1/((u_)*(v_)), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]`

Rule 2163

`Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/(n`

+ 1)\*(b\*u - a\*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]

### Rubi steps

$$\begin{aligned} \int \frac{1}{x \coth^{-1}(\tanh(a + bx))^2} dx &= -\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{\int \frac{1}{x \coth^{-1}(\tanh(a + bx))}}{-bx + \coth^{-1}(\tanh(a + bx))} \\ &= -\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} - \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{\log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{(bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{\log(x)}{(bx - \coth^{-1}(\tanh(a + bx)))} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 53, normalized size = 0.76

$$\frac{\coth^{-1}(\tanh(a + bx)) \left( -\log(\coth^{-1}(\tanh(a + bx))) + \log(bx) + 1 \right) - bx}{\coth^{-1}(\tanh(a + bx)) \left( \coth^{-1}(\tanh(a + bx)) - bx \right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*ArcCoth[Tanh[a + b\*x]])^2, x]

[Out] (-(b\*x) + ArcCoth[Tanh[a + b\*x]])\*(1 + Log[b\*x] - Log[ArcCoth[Tanh[a + b\*x]]])/(ArcCoth[Tanh[a + b\*x]]\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^2)

**fricas [B]** time = 0.46, size = 306, normalized size = 4.37

$$\frac{2 \left( 2 \pi^4 - 32 a^4 - 8 (\pi^2 ab + 4 a^3 b) x + 16 (4 \pi ab^2 x^2 + 8 \pi a^2 bx + \pi^3 a + 4 \pi a^3) \arctan \left( -\frac{2 bx + 2 a - \sqrt{4 b^2 x^2 + 8 abx + \pi^2}}{\pi} \right) \right)}{\pi^6 + 12 \pi^4 a^2 + 48 \pi^2 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a))^2, x, algorithm="fricas")

```
[Out] -2*(2*pi^4 - 32*a^4 - 8*(pi^2*a*b + 4*a^3*b)*x + 16*(4*pi*a*b^2*x^2 + 8*pi*
a^2*b*x + pi^3*a + 4*pi*a^3)*arctan(-(2*b*x + 2*a - sqrt(4*b^2*x^2 + 8*a*b*
x + pi^2 + 4*a^2))/pi) - (pi^4 - 16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*
(pi^2*a*b - 4*a^3*b)*x)*log(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2) + 2*(pi^4 -
16*a^4 + 4*(pi^2*b^2 - 4*a^2*b^2)*x^2 + 8*(pi^2*a*b - 4*a^3*b)*x)*log(x))/
(pi^6 + 12*pi^4*a^2 + 48*pi^2*a^4 + 64*a^6 + 4*(pi^4*b^2 + 8*pi^2*a^2*b^2 +
16*a^4*b^2)*x^2 + 8*(pi^4*a*b + 8*pi^2*a^3*b + 16*a^5*b)*x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccoth(tanh(b*x + a))^2), x)
```

**maple** [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccoth(tanh(b*x+a))^2,x)
```

```
[Out] int(1/x/arccoth(tanh(b*x+a))^2,x)
```

**maxima** [C] time = 0.77, size = 77, normalized size = 1.10

$$\frac{4 \log(-i\pi + 2bx + 2a)}{\pi^2 + 4i\pi a - 4a^2} - \frac{4 \log(x)}{\pi^2 + 4i\pi a - 4a^2} - \frac{4}{\pi^2 + 4i\pi a - 4a^2 + (2i\pi b - 4ab)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 4*log(-I*pi + 2*b*x + 2*a)/(pi^2 + 4*I*pi*a - 4*a^2) - 4*log(x)/(pi^2 + 4*I
*pi*a - 4*a^2) - 4/(pi^2 + 4*I*pi*a - 4*a^2 + (2*I*pi*b - 4*a*b)*x)
```

**mupad** [B] time = 4.03, size = 421, normalized size = 6.01

$$4 \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right) - 4 \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + 8bx + \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) \operatorname{atan}\left(\frac{\ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) - \ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) + 2bx}{\ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + 2bx}\right) 8i - \operatorname{ata}$$


---


$$\left(\ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) - \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right)\right) \left(\ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right) - \ln\left(\frac{e^2}{e^{2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*acoth(tanh(a + b*x))^2),x)`

[Out] 
$$-(4*\log(-1/(\exp(2*a)*\exp(2*b*x) - 1))) - 4*\log((\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 8*b*x + \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))*\operatorname{atan}((\log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))*1i - \log(-2/(\exp(2*a)*\exp(2*b*x) - 1))*1i + b*x*2i)/(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))*8i - \operatorname{atan}((\log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))*1i - \log(-2/(\exp(2*a)*\exp(2*b*x) - 1))*1i + b*x*2i)/(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))*(\log(2) + \log(-1/(\exp(2*a)*\exp(2*b*x) - 1)))*8i)/((\log((\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) - \log(-1/(\exp(2*a)*\exp(2*b*x) - 1)))*(\log(-1/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2)$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/acoth(tanh(b*x+a))**2,x)`

[Out] `Integral(1/(x*acoth(tanh(a + b*x))**2), x)`

$$3.173 \quad \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^2} dx$$

Optimal. Leaf size=102

$$-\frac{2b}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} + \frac{1}{x(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))} + \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))}$$

[Out]  $-2*b/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2/\operatorname{arccoth}(\tanh(b*x+a))+1/x/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/\operatorname{arccoth}(\tanh(b*x+a))+2*b*\ln(x)/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^3-2*b*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^3$

**Rubi [A]** time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2171, 2163, 2160, 2157, 29}

$$-\frac{2b}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} + \frac{1}{x(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))} + \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*ArcCoth[Tanh[a + b\*x]]^2),x]

[Out]  $(-2*b)/((b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + 1/(x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + (2*b*\operatorname{Log}[x])/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^3 - (2*b*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^3$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

Rule 2163



```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

### Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))^2} dx &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} - \frac{(2b) \int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx}{-bx + \coth^{-1}(\tanh(a + bx))} \\
&= -\frac{2b}{(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{2b}{(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{2b}{(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{2b}{(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 70, normalized size = 0.69

$$\frac{\coth^{-1}(\tanh(a + bx))^2 + 2bx \coth^{-1}(\tanh(a + bx)) (\log(x) - \log(\coth^{-1}(\tanh(a + bx)))) - b^2 x^2}{x (bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcCoth[Tanh[a + b*x]]^2), x]
```

[Out]  $(-(b^2x^2) + \text{ArcCoth}[\text{Tanh}[a + bx]]^2 + 2bx\text{ArcCoth}[\text{Tanh}[a + bx]])(\text{Log}[x] - \text{Log}[\text{ArcCoth}[\text{Tanh}[a + bx]]]) / (x(bx - \text{ArcCoth}[\text{Tanh}[a + bx]])^3 \text{ArcCoth}[\text{Tanh}[a + bx]])$

**fricas** [B] time = 0.68, size = 480, normalized size = 4.71

$$4 \left( \pi^6 + 4\pi^4 a^2 - 16\pi^2 a^4 - 64a^6 + 8(\pi^4 b^2 - 16a^4 b^2)x^2 + 4(5\pi^4 ab + 8\pi^2 a^3 b - 48a^5 b)x - 8(4(\pi^3 b^3 - 12\pi a^2 b^3) \right.$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="fricas")`

[Out]  $4(\pi^6 + 4\pi^4 a^2 - 16\pi^2 a^4 - 64a^6 + 8(\pi^4 b^2 - 16a^4 b^2)x^2 + 4(5\pi^4 ab + 8\pi^2 a^3 b - 48a^5 b)x - 8(4(\pi^3 b^3 - 12\pi a^2 b^3)x^3 + 8(\pi^3 a b^2 - 12\pi a^3 b^2)x^2 + (\pi^5 b - 8\pi^3 a^2 b - 48\pi a^4 b)x) \arctan(-\frac{2bx + 2a - \sqrt{4b^2 x^2 + 8abx + \pi^2 + 4a^2}}{\pi}) - 4(4(3\pi^2 a b^3 - 4a^3 b^3)x^3 + 8(3\pi^2 a^2 b^2 - 4a^4 b^2)x^2 + (3\pi^4 a b + 8\pi^2 a^3 b - 16a^5 b)x) \log(4b^2 x^2 + 8abx + \pi^2 + 4a^2) + 8(4(3\pi^2 a b^3 - 4a^3 b^3)x^3 + 8(3\pi^2 a^2 b^2 - 4a^4 b^2)x^2 + (3\pi^4 a b + 8\pi^2 a^3 b - 16a^5 b)x) \log(x) / (4(\pi^6 b^2 + 12\pi^4 a^2 b^2 + 48\pi^2 a^4 b^2 + 64a^6 b^2)x^3 + 8(\pi^6 a b + 12\pi^4 a^3 b + 48\pi^2 a^5 b + 64a^7 b)x^2 + (\pi^8 + 16\pi^6 a^2 + 96\pi^4 a^4 + 256\pi^2 a^6 + 256a^8)x)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

[Out] `integrate(1/(x^2*arccoth(tanh(b*x + a))^2), x)`

**maple** [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/arccoth(tanh(b*x+a))^2,x)`

[Out] `int(1/x^2/arccoth(tanh(b*x+a))^2,x)`

**maxima** [C] time = 0.78, size = 135, normalized size = 1.32

$$\frac{16b \log(-i\pi + 2bx + 2a)}{-i\pi^3 + 6\pi^2a + 12i\pi a^2 - 8a^3} + \frac{16b \log(x)}{-i\pi^3 + 6\pi^2a + 12i\pi a^2 - 8a^3} - \frac{4(i\pi - 4bx - 2a)}{(2\pi^2b + 8i\pi ab - 8a^2b)x^2 - (i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^2,x, algorithm="maxima")

[Out]  $-16*b*\log(-I*\pi + 2*b*x + 2*a)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3) + 16*b*\log(x)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3) - 4*(I*\pi - 4*b*x - 2*a)/((2*\pi^2*b + 8*I*\pi*a*b - 8*a^2*b)*x^2 - (I*\pi^3 - 6*\pi^2*a - 12*I*\pi*a^2 + 8*a^3)*x)$

**mupad** [B] time = 3.80, size = 453, normalized size = 4.44

$$\frac{4 \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right)^2 - \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) \left(8 \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right) + bx \operatorname{atan}\left(\frac{\ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) i - \ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) i + bx 2i}{\ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + 2bx}\right)}{32i}\right) + 4}{x \left(\ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) - \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right)\right) \left(\ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*acoth(tanh(a + b\*x)))^2,x)

[Out]  $(4*\log(-1/(\exp(2*a)*\exp(2*b*x) - 1)))^2 - \log((\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))*(8*\log(-1/(\exp(2*a)*\exp(2*b*x) - 1)) + b*x*\operatorname{atan}((\log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))*i - \log(-2/(\exp(2*a)*\exp(2*b*x) - 1))*i + b*x*2i)/(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))*32i) + 4*\log((\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))^2 - 16*b^2*x^2 + b*x*\log(-1/(\exp(2*a)*\exp(2*b*x) - 1))*\operatorname{atan}((\log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1))*i - \log(-2/(\exp(2*a)*\exp(2*b*x) - 1))*i + b*x*2i)/(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))*32i)/(x*(\log((\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) - \log(-1/(\exp(2*a)*\exp(2*b*x) - 1)))*(\log(-1/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^3)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acoth(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**2*acoth(tanh(a + b*x))**2), x)
```

$$3.174 \quad \int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^2} dx$$

Optimal. Leaf size=143

$$\frac{3b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))} + \frac{3b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^4} - \frac{3b^2 \log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))^4}$$

[Out]  $-3*b^2/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^3/\operatorname{arccoth}(\tanh(b*x+a))+3/2*b/x/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2/\operatorname{arccoth}(\tanh(b*x+a))+1/2/x^2/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/\operatorname{arccoth}(\tanh(b*x+a))+3*b^2*\ln(x)/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^4-3*b^2*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^4$

**Rubi [A]** time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{3b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))} + \frac{3b^2 \log(x)}{(bx - \coth^{-1}(\tanh(a+bx)))^4} - \frac{3b^2 \log(\coth^{-1}(\tanh(a+bx)))}{(bx - \coth^{-1}(\tanh(a+bx)))^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*ArcCoth[Tanh[a + b\*x]]^2), x]

[Out]  $(-3*b^2)/((b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^3*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + (3*b)/(2*x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + 1/(2*x^2*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + (3*b^2*\operatorname{Log}[x])/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^4 - (3*b^2*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^4$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \coth^{-1}(\tanh(a + bx))^2} dx &= \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))} + \frac{(3b) \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))} dx}{2 (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= \frac{3b}{2x (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))} + \frac{(3b) \int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx}{2x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} + \frac{(3b) \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{2x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} + \frac{(3b) \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{2x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} + \frac{(3b) \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{2x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} + \frac{(3b) \int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx}{2x (bx - \coth^{-1}(\tanh(a + bx)))}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 0.64

$$\frac{-3b^2 x^2 \coth^{-1}(\tanh(a + bx)) (-2 \log(\coth^{-1}(\tanh(a + bx))) + 2 \log(x) - 1) - 6bx \coth^{-1}(\tanh(a + bx))^2 + \coth^{-1}(\tanh(a + bx))}{2x^2 \coth^{-1}(\tanh(a + bx)) (\coth^{-1}(\tanh(a + bx)) - bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*ArcCoth[Tanh[a + b\*x]]^2),x]

[Out] 
$$-1/2*(2*b^3*x^3 - 6*b*x*ArcCoth[Tanh[a + b*x]]^2 + ArcCoth[Tanh[a + b*x]]^3 - 3*b^2*x^2*ArcCoth[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a + b*x]]]))/(x^2*ArcCoth[Tanh[a + b*x]]*(-(b*x) + ArcCoth[Tanh[a + b*x]]))^4)$$

**fricas** [B] time = 0.55, size = 644, normalized size = 4.50

$$2 \left( \pi^8 + 8 \pi^6 a^2 - 128 \pi^2 a^6 - 256 a^8 - 96 (3 \pi^4 a b^3 + 8 \pi^2 a^3 b^3 - 16 a^5 b^3) x^3 + 12 (\pi^6 b^2 - 44 \pi^4 a^2 b^2 - 144 \pi^2 a^4 b^2 \right.$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^2,x, algorithm="fricas")

[Out] 
$$2*(\pi^8 + 8\pi^6 a^2 - 128\pi^2 a^6 - 256a^8 - 96(3\pi^4 a b^3 + 8\pi^2 a^3 b^3 - 16a^5 b^3)x^3 + 12(\pi^6 b^2 - 44\pi^4 a^2 b^2 - 144\pi^2 a^4 b^2 + 192a^6 b^2)x^2 - 8(5\pi^6 a b + 36\pi^4 a^3 b + 48\pi^2 a^5 b - 64a^7 b)x + 384(4(\pi^3 a b^4 - 4\pi a^3 b^4)x^4 + 8(\pi^3 a^2 b^3 - 4\pi a^4 b^3)x^3 + (\pi^5 a b^2 - 16\pi a^5 b^2)x^2) \arctan(-\frac{2bx + 2a - \sqrt{4b^2 x^2 + 8abx + \pi^2 + 4a^2}}{\pi}) - 12(4(\pi^4 b^4 - 24\pi^2 a^2 b^4 + 16a^4 b^4)x^4 + 8(\pi^4 a b^3 - 24\pi^2 a^3 b^3 + 16a^5 b^3)x^3 + (\pi^6 b^2 - 20\pi^4 a^2 b^2 - 80\pi^2 a^4 b^2 + 64a^6 b^2)x^2) \log(4b^2 x^2 + 8abx + \pi^2 + 4a^2) + 24(4(\pi^4 b^4 - 24\pi^2 a^2 b^4 + 16a^4 b^4)x^4 + 8(\pi^4 a b^3 - 24\pi^2 a^3 b^3 + 16a^5 b^3)x^3 + (\pi^6 b^2 - 20\pi^4 a^2 b^2 - 80\pi^2 a^4 b^2 + 64a^6 b^2)x^2) \log(x) / (4(\pi^8 b^2 + 16\pi^6 a^2 b^2 + 96\pi^4 a^4 b^2 + 256\pi^2 a^6 b^2 + 256a^8 b^2)x^4 + 8(\pi^8 a b + 16\pi^6 a^3 b + 96\pi^4 a^5 b + 256\pi^2 a^7 b + 256a^9 b)x^3 + (\pi^{10} + 20\pi^8 a^2 + 160\pi^6 a^4 + 640\pi^4 a^6 + 1280\pi^2 a^8 + 1024a^{10})x^2)$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^2,x, algorithm="giac")

[Out] integrate(1/(x^3\*arccoth(tanh(b\*x + a))^2), x)

**maple** [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/arccoth(tanh(b*x+a))^2,x)`

[Out] `int(1/x^3/arccoth(tanh(b*x+a))^2,x)`

**maxima** [C] time = 0.76, size = 191, normalized size = 1.34

$$\frac{48b^2 \log(-i\pi + 2bx + 2a)}{\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4} + \frac{48b^2 \log(x)}{\pi^4 + 8i\pi^3a - 24\pi^2a^2 - 32i\pi a^3 + 16a^4} - \frac{4(24b^2 \log(x) - 48b^2 \log(-i\pi + 2bx + 2a))}{(-4i\pi^3b + 24\pi^2ab + 48i\pi a^3 - 16a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="maxima")`

[Out] `-48*b^2*log(-I*pi + 2*b*x + 2*a)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 + 16*a^4) + 48*b^2*log(x)/(pi^4 + 8*I*pi^3*a - 24*pi^2*a^2 - 32*I*pi*a^3 + 16*a^4) - 4*(24*b^2*x^2 + pi^2 + 4*I*pi*a - 4*a^2 + (-6*I*pi*b + 12*a*b)*x)/((-4*I*pi^3*b + 24*pi^2*a*b + 48*I*pi*a^2*b - 32*a^3*b)*x^3 - (2*pi^4 + 16*I*pi^3*a - 48*pi^2*a^2 - 64*I*pi*a^3 + 32*a^4)*x^2)`

**mupad** [B] time = 4.90, size = 689, normalized size = 4.82

$$2 \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right)^3 - 2 \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right)^3 - 6 \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right)^2 + 6 \ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right)^2 \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right) - 3 \ln\left(-\frac{1}{e^{2a} e^{2bx-1}}\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*acoth(tanh(a + b*x))^2),x)`

[Out] `(2*log(-1/(exp(2*a)*exp(2*b*x) - 1))^3 - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^3 - 6*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 + 6*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - 32*b^3*x^3 + 24*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))^2 + 24*b*x*log(-1/(exp(2*a)*exp(2*b*x) - 1))^2 - 24*b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 24*b^2*x^2*log(-1/(exp(2*a)*exp(2*b*x) - 1)) - b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))*96i - 48*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*log(-1/(exp(2*a)*exp(2*b*x) - 1)) + b^2*x^2*log(-1/(exp(2*a)*exp(2*b*x) - 1))*atan((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))`



```
(exp(2*a)*exp(2*b*x) - 1))*1i - log(-2/(exp(2*a)*exp(2*b*x) - 1))*1i + b*x*
2i)/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1) + 2*b*x))*96i)/(x^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) - log(-1/(exp(2*a)*exp(2*b*x) - 1)))*(log(-1/(exp(2*a)*
exp(2*b*x) - 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1) + 2
*b*x)^4)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{acoth}^2(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/acoth(tanh(b\*x+a))\*\*2,x)

[Out] Integral(1/(x\*\*3\*acoth(tanh(a + b\*x))\*\*2), x)

$$3.175 \quad \int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=94

$$\frac{mx^{m-1} {}_2F_1\left(1, m-1; m; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{2b^2 (bx - \coth^{-1}(\tanh(a+bx)))} - \frac{mx^{m-1}}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out]  $-1/2*x^m/b/\operatorname{arccoth}(\tanh(b*x+a))^{-2}-1/2*m*x^{(-1+m)}/b^2/\operatorname{arccoth}(\tanh(b*x+a))-1/2*m*x^{(-1+m)}*\operatorname{hypergeom}([1, -1+m], [m], b*x/(b*x-\operatorname{arccoth}(\tanh(b*x+a))))/b^2/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))$

**Rubi [A]** time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 2164}

$$\frac{mx^{m-1} {}_2F_1\left(1, m-1; m; \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{2b^2 (bx - \coth^{-1}(\tanh(a+bx)))} - \frac{mx^{m-1}}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^m/\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^3, x]$

[Out]  $-x^m/(2*b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) - (m*x^{(-1 + m)})/(2*b^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) - (m*x^{(-1 + m)}*\operatorname{Hypergeometric2F1}[1, -1 + m, m, (b*x)/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])])/(2*b^2*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]))$

#### Rule 2164

$\operatorname{Int}[(v_)^{(n_)}/(u_), x\_Symbol] := \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(v^{(n+1)}*\operatorname{Hypergeometric2F1}[1, n+1, n+2, -((a*v)/(b*u - a*v))])]/((n+1)*(b*u - a*v)), x] /; \operatorname{NeQ}[b*u - a*v, 0] /; \operatorname{PiecewiseLinearQ}[u, v, x] \&\& !\operatorname{IntegerQ}[n]$

#### Rule 2168

$\operatorname{Int}[(u_)^{(m_*)}*(v_)^{(n_*)}, x\_Symbol] := \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^{(m+1)}*v^n)/(a*(m+1)), x] - \operatorname{Dist}[(b^n)/(a*(m+1)), \operatorname{Int}[u^{(m+1)}*v^{(n-1)}, x], x] /; \operatorname{NeQ}[b*u - a*v, 0] /; \operatorname{FreeQ}[\{m, n\}, x] \&\& \operatorname{PiecewiseLinearQ}[u, v, x] \&\& \operatorname{NeQ}[m, -1] \&\& ((\operatorname{LtQ}[m, -1] \&\& \operatorname{GtQ}[n, 0] \&\& !(\operatorname{ILtQ}[m+n, -2] \&\& (\operatorname{FractionQ}[m] \mid \operatorname{GeQ}[2*n+m+1, 0]))) \mid (\operatorname{IGtQ}[n, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{LeQ}[n, m]) \mid (\operatorname{IGtQ}[n, 0] \&\& !\operatorname{IntegerQ}[m]) \mid (\operatorname{ILtQ}[m, 0] \&\& !\operatorname{IntegerQ}[n]))$

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{m \int \frac{x^{-1+m}}{\coth^{-1}(\tanh(a+bx))^2} dx}{2b} \\
&= -\frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{((1-m)m) \int \frac{x^{-2+m}}{\coth^{-1}(\tanh(a+bx))} dx}{2b^2} \\
&= -\frac{x^m}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{mx^{-1+m}}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{mx^{-1+m} {}_2F_1\left(1, -1 + \frac{m}{2}, \frac{m}{2}, -\frac{bx}{\coth^{-1}(\tanh(a+bx))}\right)}{2b^2 (bx - \coth^{-1}(\tanh(a+bx)))}
\end{aligned}$$

**Mathematica [A]** time = 0.54, size = 51, normalized size = 0.54

$$\frac{x^{m+1} {}_2F_1\left(3, m+1; m+2; -\frac{bx}{\coth^{-1}(\tanh(a+bx))-bx}\right)}{(m+1) \left(\coth^{-1}(\tanh(a+bx)) - bx\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/ArcCoth[Tanh[a + b\*x]]^3, x]

[Out] (x^(1+m)\*Hypergeometric2F1[3, 1+m, 2+m, -(b\*x)/(-(b\*x) + ArcCoth[Tanh[a + b\*x]])])/((1+m)\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])^3)

**fricas [F]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\text{arcoth}(\tanh(bx+a))^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^3, x, algorithm="fricas")

[Out] integral(x^m/arccoth(tanh(b\*x + a))^3, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\text{arcoth}(\tanh(bx+a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^m/arccoth(tanh(b\*x + a))^3, x)

**maple** [F] time = 3.50, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/arccoth(tanh(b\*x+a))^3,x)

[Out] int(x^m/arccoth(tanh(b\*x+a))^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] integrate(x^m/arccoth(tanh(b\*x + a))^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\operatorname{acoth}(\tanh(a + bx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/acoth(tanh(a + b\*x))^3,x)

[Out] int(x^m/acoth(tanh(a + b\*x))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(x\*\*m/acoth(tanh(a + b\*x))\*\*3, x)

$$3.176 \quad \int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=92

$$\frac{6(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^5} + \frac{6x(bx - \coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))}$$

[Out]  $3x^2/b^3 + 6x*(bx - \operatorname{arccoth}(\tanh(bx+a)))/b^4 - 1/2x^4/b/\operatorname{arccoth}(\tanh(bx+a))$   
 $^2 - 2x^3/b^2/\operatorname{arccoth}(\tanh(bx+a)) + 6*(bx - \operatorname{arccoth}(\tanh(bx+a)))^2 * \ln(\operatorname{arccoth}(\tanh(bx+a)))/b^5$

**Rubi [A]** time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00,  
 number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} =$   
 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$-\frac{2x^3}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{6x(bx - \coth^{-1}(\tanh(a+bx)))}{b^4} + \frac{6(bx - \coth^{-1}(\tanh(a+bx)))^2 \log(\coth^{-1}(\tanh(a+bx)))}{b^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/ArcCoth[Tanh[a + b\*x]]^3, x]

[Out]  $(3x^2)/b^3 + (6x*(bx - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]))/b^4 - x^4/(2*b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) - (2x^3)/(b^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + (6*(bx - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2 * \operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/b^5$

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rule 2157

Int[(u\_)^(m\_), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x]

#### Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
/; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

### Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n},
x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
&& !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\coth^{-1}(\tanh(a + bx))^3} dx &= -\frac{x^4}{2b \coth^{-1}(\tanh(a + bx))^2} + \frac{2 \int \frac{x^3}{\coth^{-1}(\tanh(a + bx))^2} dx}{b} \\
&= -\frac{x^4}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{2x^3}{b^2 \coth^{-1}(\tanh(a + bx))} + \frac{6 \int \frac{x^2}{\coth^{-1}(\tanh(a + bx))} dx}{b^2} \\
&= \frac{3x^2}{b^3} - \frac{x^4}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{2x^3}{b^2 \coth^{-1}(\tanh(a + bx))} - \frac{6(-bx + \coth^{-1}(\tanh(a + bx)))}{b^2 \coth^{-1}(\tanh(a + bx))} \\
&= \frac{3x^2}{b^3} + \frac{6x(bx - \coth^{-1}(\tanh(a + bx)))}{b^4} - \frac{x^4}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{6(-bx + \coth^{-1}(\tanh(a + bx)))}{b^2 \coth^{-1}(\tanh(a + bx))} \\
&= \frac{3x^2}{b^3} + \frac{6x(bx - \coth^{-1}(\tanh(a + bx)))}{b^4} - \frac{x^4}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{6(-bx + \coth^{-1}(\tanh(a + bx)))}{b^2 \coth^{-1}(\tanh(a + bx))} \\
&= \frac{3x^2}{b^3} + \frac{6x(bx - \coth^{-1}(\tanh(a + bx)))}{b^4} - \frac{x^4}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{6(-bx + \coth^{-1}(\tanh(a + bx)))}{b^2 \coth^{-1}(\tanh(a + bx))}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 114, normalized size = 1.24

$$-\frac{(\coth^{-1}(\tanh(a + bx)) - bx)^4}{2b^5 \coth^{-1}(\tanh(a + bx))^2} + \frac{4(\coth^{-1}(\tanh(a + bx)) - bx)^3}{b^5 \coth^{-1}(\tanh(a + bx))} + \frac{6(\coth^{-1}(\tanh(a + bx)) - bx)^2 \log(\coth^{-1}(\tanh(a + bx)))}{b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out]  $x^2/(2*b^3) - (3*x*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]]))/b^4 + (4*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^3)/(b^5*\text{ArcCoth}[\text{Tanh}[a + b*x]]) - (-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^4/(2*b^5*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) + (6*(-(b*x) + \text{ArcCoth}[\text{Tanh}[a + b*x]])^2*\text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]])/b^5$

**fricas** [B] time = 1.14, size = 493, normalized size = 5.36

$64 b^6 x^6 - 128 a b^5 x^5 - 7 \pi^6 - 28 \pi^4 a^2 + 112 \pi^2 a^4 + 448 a^6 + 32 (\pi^2 b^4 - 36 a^2 b^4) x^4 - 512 (\pi^2 a b^3 + 3 a^3 b^3) x^3 - 3$

---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $1/8*(64*b^6*x^6 - 128*a*b^5*x^5 - 7*\pi^6 - 28*\pi^4*a^2 + 112*\pi^2*a^4 + 448*a^6 + 32*(\pi^2*b^4 - 36*a^2*b^4)*x^4 - 512*(\pi^2*a*b^3 + 3*a^3*b^3)*x^3 - 32*(\pi^4*b^2 + 32*\pi^2*a^2*b^2)*x^2 - 32*(5*\pi^4*a*b + 12*\pi^2*a^3*b - 32*a^5*b)*x - 96*(16*\pi*a*b^4*x^4 + 64*\pi*a^2*b^3*x^3 + \pi^5*a + 8*\pi^3*a^3 + 16*\pi*a^5 + 8*(\pi^3*a*b^2 + 12*\pi*a^3*b^2)*x^2 + 16*(\pi^3*a^2*b + 4*\pi*a^4*b)*x)*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi - 6*(\pi^6 + 4*\pi^4*a^2 - 16*\pi^2*a^4 - 64*a^6 + 16*(\pi^2*b^4 - 4*a^2*b^4)*x^4 + 64*(\pi^2*a*b^3 - 4*a^3*b^3)*x^3 + 8*(\pi^4*b^2 + 8*\pi^2*a^2*b^2 - 48*a^4*b^2)*x^2 + 16*(\pi^4*a*b - 16*a^5*b)*x)*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2))/(16*b^9*x^4 + 64*a*b^8*x^3 + \pi^4*b^5 + 8*\pi^2*a^2*b^5 + 16*a^4*b^5 + 8*(\pi^2*b^7 + 12*a^2*b^7)*x^2 + 16*(\pi^2*a*b^6 + 4*a^3*b^6)*x$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{arccoth}(\tanh(bx+a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^4/arccoth(tanh(b\*x + a))^3, x)

**maple** [C] time = 1.51, size = 29456, normalized size = 320.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^4/\text{arccoth}(\tanh(b*x+a))^3, x)$

[Out] result too large to display

**maxima** [C] time = 1.15, size = 197, normalized size = 2.14

$$\frac{8(16b^4x^4 + 7\pi^4 + 56i\pi^3a - 168\pi^2a^2 - 224i\pi a^3 + 112a^4 + (32i\pi b^3 - 64ab^3)x^3 + (44\pi^2b^2 + 176i\pi ab^2 - 176i\pi b^2a^2)x^2 + (4i\pi^3b - 24\pi^2ab - 48i\pi a^2b + 32a^3b)x + (-256i\pi b^6 + 512a^2b^5 - 256i\pi ab^5 + 256a^2b^5 + (-256i\pi b^6 + 512a^2b^5)x) - 1/2(3\pi^2 + 12i\pi a - 12a^2)\log(-i\pi + 2bx + 2a)/b^5}{256b^7x^2 - 64\pi^2b^5 - 256i\pi ab^5 + 256a^2b^5 + (-256i\pi b^6 + 512a^2b^5)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^4/\text{arccoth}(\tanh(b*x+a))^3, x, \text{algorithm}="maxima")$

[Out]  $8*(16*b^4*x^4 + 7*\pi^4 + 56*I*\pi^3*a - 168*\pi^2*a^2 - 224*I*\pi*a^3 + 112*a^4 + (32*I*\pi*b^3 - 64*a*b^3)*x^3 + (44*\pi^2*b^2 + 176*I*\pi*a*b^2 - 176*a^2*b^2)*x^2 + (4*I*\pi^3*b - 24*\pi^2*a*b - 48*I*\pi*a^2*b + 32*a^3*b)*x)/(256*b^7*x^2 - 64*\pi^2*b^5 - 256*I*\pi*a*b^5 + 256*a^2*b^5 + (-256*I*\pi*b^6 + 512*a*b^6)*x) - 1/2*(3*\pi^2 + 12*I*\pi*a - 12*a^2)*\log(-I*\pi + 2*b*x + 2*a)/b^5$

**mupad** [B] time = 1.38, size = 867, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^4/\text{acoth}(\tanh(a + b*x))^3, x)$

[Out]  $((7*((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^4 + 24*a^2*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^3 - 32*a^3*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)))/(4*b) - x*(4*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^3 - 32*a^3 - 24*a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 + 48*a^2*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)))/(2*b^4*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 + x*(16*a*b^5 - 8*b^5*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)) + 8*a^2*b^4 + 8*b^6*x^2 - 8*a*b^4*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)) + x^2/(2*b^3) + (\log(\log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) - \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)))*(3*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)))$



```

xp(2*b*x) - 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x) + 12*a^2
)/(2*b^5) + (3*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b^4)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(x\*\*4/acoth(tanh(a + b\*x))\*\*3, x)

$$3.177 \quad \int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=71

$$\frac{3(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x^3}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out]  $3*x/b^3 - 1/2*x^3/b/\operatorname{arccoth}(\tanh(b*x+a))^2 - 3/2*x^2/b^2/\operatorname{arccoth}(\tanh(b*x+a)) + 3*(b*x - \operatorname{arccoth}(\tanh(b*x+a)))*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/b^4$

**Rubi [A]** time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2168, 2158, 2157, 29}

$$-\frac{3x^2}{2b^2 \coth^{-1}(\tanh(a+bx))} + \frac{3(bx - \coth^{-1}(\tanh(a+bx))) \log(\coth^{-1}(\tanh(a+bx)))}{b^4} - \frac{x^3}{2b \coth^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^3, x]$

[Out]  $(3*x)/b^3 - x^3/(2*b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) - (3*x^2)/(2*b^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) + (3*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/b^4$

Rule 29

$\operatorname{Int}[(x_)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[x], x]$

Rule 2157

$\operatorname{Int}[(u_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{c = \operatorname{Simplify}[D[u, x]]\}, \operatorname{Dist}[1/c, \operatorname{Subst}[\operatorname{Int}[x^m, x], x, u], x]] /; \operatorname{FreeQ}[m, x] \ \&\& \operatorname{PiecewiseLinearQ}[u, x]$

Rule 2158

$\operatorname{Int}[(v_)/(u_), x\_Symbol] \rightarrow \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(b*x)/a, x] - \operatorname{Dist}[(b*u - a*v)/a, \operatorname{Int}[1/u, x], x] /; \operatorname{NeQ}[b*u - a*v, 0]] /; \operatorname{PiecewiseLinearQ}[u, v, x]$

Rule 2168

$\operatorname{Int}[(u_)^{(m_)}*(v_)^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^{(m+1)}*v^n)/(a*(m+1)), x] - \operatorname{Dist}[(b*n)/(a*(m+1)), x]$

1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\coth^{-1}(\tanh(a + bx))^3} dx &= -\frac{x^3}{2b \coth^{-1}(\tanh(a + bx))^2} + \frac{3 \int \frac{x^2}{\coth^{-1}(\tanh(a + bx))^2} dx}{2b} \\
 &= -\frac{x^3}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a + bx))} + \frac{3 \int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx}{b^2} \\
 &= \frac{3x}{b^3} - \frac{x^3}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a + bx))} - \frac{3(-bx + \coth^{-1}(\tanh(a + bx)))}{b^2} \\
 &= \frac{3x}{b^3} - \frac{x^3}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a + bx))} - \frac{3(-bx + \coth^{-1}(\tanh(a + bx)))}{b^2} \\
 &= \frac{3x}{b^3} - \frac{x^3}{2b \coth^{-1}(\tanh(a + bx))^2} - \frac{3x^2}{2b^2 \coth^{-1}(\tanh(a + bx))} + \frac{3(bx - \coth^{-1}(\tanh(a + bx)))}{b^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 86, normalized size = 1.21

$$\frac{3b^2x^2 \coth^{-1}(\tanh(a + bx)) - bx \coth^{-1}(\tanh(a + bx))^2 (6 \log(\coth^{-1}(\tanh(a + bx))) + 11) + \coth^{-1}(\tanh(a + bx))}{2b^4 \coth^{-1}(\tanh(a + bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/ArcCoth[Tanh[a + b\*x]]^3, x]

[Out] -1/2\*(b^3\*x^3 + 3\*b^2\*x^2\*ArcCoth[Tanh[a + b\*x]] + ArcCoth[Tanh[a + b\*x]]^3 \* (5 + 6\*Log[ArcCoth[Tanh[a + b\*x]]]) - b\*x\*ArcCoth[Tanh[a + b\*x]]^2\*(11 + 6\*Log[ArcCoth[Tanh[a + b\*x]]]))/(b^4\*ArcCoth[Tanh[a + b\*x]]^2)

**fricas [B]** time = 0.65, size = 418, normalized size = 5.89

$$32b^5x^5 + 128ab^4x^4 - 5\pi^4a - 40\pi^2a^3 - 80a^5 + 8(5\pi^2b^3 + 12a^2b^3)x^3 + 4(11\pi^2ab^2 - 36a^3b^2)x^2 + 2(3\pi^4b -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $\frac{1}{2}(32b^5x^5 + 128ab^4x^4 - 5\pi^4a - 40\pi^2a^3 - 80a^5 + 8(5\pi^2b^3 + 12a^2b^3)x^3 + 4(11\pi^2ab^2 - 36a^3b^2)x^2 + 2(3\pi^4b - 16\pi^2a^2b - 112a^4b)x + 6(16\pi b^4x^4 + 64\pi ab^3x^3 + \pi^5 + 8\pi^3a^2 + 16\pi a^4 + 8(\pi^3b^2 + 12\pi a^2b^2)x^2 + 16(\pi^3ab + 4\pi a^3b)x) \arctan\left(\frac{-(2bx + 2a - \sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2})}{\pi}\right) - 3(16ab^4x^4 + 64a^2b^3x^3 + \pi^4a + 8\pi^2a^3 + 16a^5 + 8(\pi^2ab^2 + 12a^3b^2)x^2 + 16(\pi^2a^2b + 4a^4b)x) \log(4b^2x^2 + 8abx + \pi^2 + 4a^2) / (16b^8x^4 + 64ab^7x^3 + \pi^4b^4 + 8\pi^2a^2b^4 + 16a^4b^4 + 8(\pi^2b^6 + 12a^2b^6)x^2 + 16(\pi^2ab^5 + 4a^3b^5)x)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{arccoth}(\tanh(bx+a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^3/arccoth(tanh(b\*x + a))^3, x)

**maple** [C] time = 0.78, size = 4977, normalized size = 70.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/arccoth(tanh(b\*x+a))^3,x)

[Out]  $-2I(-6\pi x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^2 + 3\pi x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1)) - 3\pi x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^2 + 6\pi x^2 \operatorname{csgn}(I/(\exp(2bx+2a)+1))^3 + 3\pi x^2 \operatorname{csgn}(I \exp(bx+a))^2 \operatorname{csgn}(I \exp(2bx+2a)) - 6\pi x^2 \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^2 + 3\pi x^2 \operatorname{csgn}(I \exp(2bx+2a))^3 - 3\pi x^2 \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^2 + 3\pi x^2 \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^3 + 4I x^3 b + 6\pi x^2 + 12I x^2 \ln(\exp(bx+a)) / b^2 / (-2\pi \operatorname{csgn}(I/(\exp(2bx+2a)+1))^2 + \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1)) - \pi \operatorname{csgn}(I/(\exp(2bx+2a)+1)) \operatorname{csgn}(I \exp(2bx+2a)/(\exp(2bx+2a)+1))^2 + 2\pi \operatorname{csgn}(I/(\exp(2bx+2a)+1))^3 + \pi \operatorname{csgn}(I \exp(bx+a))^2 \operatorname{csgn}(I \exp(2bx+2a)) - 2\pi \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^2 + \pi \operatorname{csgn}(I \exp(2bx+2a))^3 - \pi \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}($





$$\frac{x^3 \left( (2bx+2a+1)^3 \operatorname{csgn}(I \exp(bx+a))^2 \operatorname{csgn}(I \exp(2bx+2a)) - 2\pi \operatorname{csgn}(I \exp(bx+a)) \operatorname{csgn}(I \exp(2bx+2a))^2 + \pi \operatorname{csgn}(I \exp(2bx+2a))^3 - \pi \operatorname{csgn}(I \exp(2bx+2a)) \operatorname{csgn}(I \exp(2bx+2a)) / (\exp(2bx+2a)+1) \right)^2 + \pi \operatorname{csgn}(I \exp(2bx+2a)) / (\exp(2bx+2a)+1) \right)^3 + 4I (\ln(\exp(bx+a)) - bx - a) + 4Ibx + 4I(a + 2\pi) \ln(\exp(bx+a))$$

**maxima** [C] time = 1.15, size = 144, normalized size = 2.03

$$\frac{8 \left( 16b^3x^3 - 5i\pi^3 + 30\pi^2a + 60i\pi a^2 - 40a^3 + (-16i\pi b^2 + 32ab^2)x^2 + (8\pi^2b + 32i\pi ab - 32a^2b)x \right) \cdot 3(-i\pi)}{128b^6x^2 - 32\pi^2b^4 - 128i\pi ab^4 + 128a^2b^4 + (-128i\pi b^5 + 256ab^5)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $8 \cdot (16b^3x^3 - 5I\pi^3 + 30\pi^2a + 60I\pi a^2 - 40a^3 + (-16I\pi b^2 + 32a \cdot b^2) \cdot x^2 + (8\pi^2b + 32I\pi a \cdot b - 32a^2b) \cdot x) / (128b^6x^2 - 32\pi^2b^4 - 128I\pi a \cdot b^4 + 128a^2b^4 + (-128I\pi b^5 + 256a \cdot b^5) \cdot x) - 3/2 \cdot (-I\pi + 2a) \cdot \log(-I\pi + 2bx + 2a) / b^4$

**mupad** [B] time = 1.43, size = 620, normalized size = 8.73

$$\frac{x \left( 3 \left( 2a - \ln \left( \frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}} \right) + \ln \left( -\frac{2}{e^{2a} e^{2bx-1}} \right) + 2bx \right)^2 - 12a \left( 2a - \ln \left( \frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}} \right) + \ln \left( -\frac{2}{e^{2a} e^{2bx-1}} \right) + 2bx \right) \right)}{b^3} \cdot \frac{b^3 \left( 2a - \ln \left( \frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}} \right) + \ln \left( -\frac{2}{e^{2a} e^{2bx-1}} \right) + 2bx \right)^2 + x \left( 8ab^4 - 4b^4 \left( 2a - \ln \left( \frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}} \right) + \ln \left( -\frac{2}{e^{2a} e^{2bx-1}} \right) + 2bx \right) \right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/acoth(tanh(a + b\*x))^3,x)

[Out]  $x/b^3 - (x \cdot (3 \cdot (2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx)^2 - 12a \cdot (2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx + 12a^2) - (5 \cdot ((2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx)^3 - 8a^3 - 6a \cdot (2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx)^2 + 12a^2 \cdot (2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx) / (4b)) / (b^3 \cdot (2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx)^2 + x \cdot (8a \cdot b^4 - 4b^4 \cdot (2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx)) + 4a^2 \cdot b^3 + 4b^5 \cdot x^2 - 4a \cdot b^3 \cdot (2a - \log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) + \log(-2 / (\exp(2a) \exp(2bx) - 1)) + 2bx) + (\log(\log((2 \exp(2a) \exp(2bx)) / (\exp(2a) \exp(2bx) - 1))) / (\exp(2a) \exp(2bx) - 1))$

- 1)) - log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)))\*(3\*log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - 3\*log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1) + 6\*b\*x))/(2\*b^4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(x\*\*3/acoth(tanh(a + b\*x))\*\*3, x)



$$3.178 \quad \int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} dx$$

Optimal. Leaf size=47

$$\frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out]  $-1/2*x^2/b/\operatorname{arccoth}(\tanh(b*x+a))^{-2}-x/b^2/\operatorname{arccoth}(\tanh(b*x+a))+\ln(\operatorname{arccoth}(\tanh(b*x+a)))/b^3$

**Rubi [A]** time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 29}

$$-\frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^3} - \frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/\text{ArcCoth}[\text{Tanh}[a + b*x]]^3, x]$

[Out]  $-x^2/(2*b*\text{ArcCoth}[\text{Tanh}[a + b*x]]^2) - x/(b^2*\text{ArcCoth}[\text{Tanh}[a + b*x]]) + \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]/b^3$

Rule 29

$\text{Int}[(x_)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

Rule 2157

$\text{Int}[(u_)^{(m_.)}, x\_Symbol] \rightarrow \text{With}[\{c = \text{Simplify}[\text{D}[u, x]]\}, \text{Dist}[1/c, \text{Subst}[\text{Int}[x^m, x], x, u], x]] /;$   $\text{FreeQ}[m, x] \ \&\& \ \text{PiecewiseLinearQ}[u, x]$

Rule 2168

$\text{Int}[(u_)^{(m_.)}*(v_)^{(n_.)}, x\_Symbol] \rightarrow \text{With}[\{a = \text{Simplify}[\text{D}[u, x]], b = \text{Simplify}[\text{D}[v, x]]\}, \text{Simp}[(u^{(m+1)}*v^n)/(a*(m+1)), x] - \text{Dist}[(b*n)/(a*(m+1)), \text{Int}[u^{(m+1)}*v^{(n-1)}, x], x] /;$   $\text{NeQ}[b*u - a*v, 0] /;$   $\text{FreeQ}[\{m, n\}, x] \ \&\& \ \text{PiecewiseLinearQ}[u, v, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ ((\text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ !(\text{ILtQ}[m+n, -2] \ \&\& \ (\text{FractionQ}[m] \ || \ \text{GeQ}[2*n+m+1, 0]))) \ || \ (\text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{LeQ}[n, m]) \ || \ (\text{IGtQ}[n, 0] \ \&\& \ !\text{IntegerQ}[m]) \ || \ (\text{ILtQ}[m, 0] \ \&\& \ !\text{IntegerQ}[n]))$

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{x}{\coth^{-1}(\tanh(a+bx))^2} dx}{b} \\
&= -\frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))} dx}{b^2} \\
&= -\frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b^3} \\
&= -\frac{x^2}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{x}{b^2 \coth^{-1}(\tanh(a+bx))} + \frac{\log(\coth^{-1}(\tanh(a+bx)))}{b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 49, normalized size = 1.04

$$\frac{-\frac{b^2 x^2}{\coth^{-1}(\tanh(a+bx))^2} - \frac{2bx}{\coth^{-1}(\tanh(a+bx))} + 2 \log(\coth^{-1}(\tanh(a+bx))) + 3}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/ArcCoth[Tanh[a + b\*x]]^3,x]

[Out] (3 - (b^2\*x^2)/ArcCoth[Tanh[a + b\*x]]^2 - (2\*b\*x)/ArcCoth[Tanh[a + b\*x]] + 2\*Log[ArcCoth[Tanh[a + b\*x]]])/(2\*b^3)

**fricas [B]** time = 0.51, size = 250, normalized size = 5.32

$$\frac{64 ab^3 x^3 + 3 \pi^4 + 24 \pi^2 a^2 + 48 a^4 + 4 (5 \pi^2 b^2 + 44 a^2 b^2) x^2 + 40 (\pi^2 ab + 4 a^3 b) x + (16 b^4 x^4 + 64 ab^3 x^3 + \pi^4 + 8 \pi^2 a^2 b^2)}{2 (16 b^7 x^4 + 64 ab^6 x^3 + \pi^4 b^3 + 8 \pi^2 a^2 b^3 + 16 a^4 b^3 + 8 (\pi^2 b^2 a^2 + 4 a^3 b) x + 3 \pi^4 + 24 \pi^2 a^2 + 48 a^4 + 4 (5 \pi^2 b^2 + 44 a^2 b^2) x^2 + 40 (\pi^2 ab + 4 a^3 b) x + 16 b^4 x^4 + 64 ab^3 x^3 + \pi^4 + 8 \pi^2 a^2 b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 1/2\*(64\*a\*b^3\*x^3 + 3\*pi^4 + 24\*pi^2\*a^2 + 48\*a^4 + 4\*(5\*pi^2\*b^2 + 44\*a^2\*b^2)\*x^2 + 40\*(pi^2\*a\*b + 4\*a^3\*b)\*x + (16\*b^4\*x^4 + 64\*a\*b^3\*x^3 + pi^4 + 8\*pi^2\*a^2 + 16\*a^4 + 8\*(pi^2\*b^2 + 12\*a^2\*b^2)\*x^2 + 16\*(pi^2\*a\*b + 4\*a^3\*b)\*x)\*log(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/(16\*b^7\*x^4 + 64\*a\*b^6\*x^3 + pi^4\*b^3 + 8\*pi^2\*a^2\*b^3 + 16\*a^4\*b^3 + 8\*(pi^2\*b^5 + 12\*a^2\*b^5)\*x^2 + 16\*(pi^2\*a\*b^4 + 4\*a^3\*b^4)\*x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(x^2/arccoth(tanh(b\*x + a))^3, x)

**maple** [C] time = 0.29, size = 952, normalized size = 20.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/arccoth(tanh(b\*x+a))^3,x)

[Out] 
$$\begin{aligned} & -4*I*(-2*Pi*x*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*x*csgn(I/(exp(2*b*x+2*a)+1)))* \\ & csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*x*csgn( \\ & I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*x*csgn( \\ & I/(exp(2*b*x+2*a)+1))^3+Pi*x*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a)) \\ & -2*Pi*x*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*x*csgn(I*exp(2*b*x+2*a)) \\ & ^3-Pi*x*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)) \\ & ^2+Pi*x*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*I*x^2*b+2*Pi*x+4*I*x \\ & *ln(exp(b*x+a))/b^2/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1)) \\ & )*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)) \\ & -Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2 \\ & +2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a)) \\ & -2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a)) \\ & ^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1)) \\ & ^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)) \\ & ^2+1/b^3*ln(ln(exp(b*x+a))-1/4*I*Pi*(-2*csgn(I/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1)) \\ & )*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1)) \\ & )*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*csgn(I/(exp(2*b*x+2*a)+1))^3+csgn(I*exp(b*x+a))^2 \\ & *csgn(I*exp(2*b*x+2*a))-2*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+csgn(I*exp(2*b*x+2*a)) \\ & ^3-csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2)) \end{aligned}$$

**maxima** [C] time = 1.12, size = 94, normalized size = 2.00

$$\frac{8(3\pi^2 + 12i\pi a - 12a^2 + (8i\pi b - 16ab)x)}{64b^5x^2 - 16\pi^2b^3 - 64i\pi ab^3 + 64a^2b^3 + (-64i\pi b^4 + 128ab^4)x} + \frac{\log(-i\pi + 2bx + 2a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] 
$$\frac{-8(3\pi^2 + 12I\pi a - 12a^2 + (8I\pi b - 16ab)x)/(64b^5x^2 - 16\pi i^2b^3 - 64I\pi ab^3 + 64a^2b^3 + (-64I\pi b^4 + 128ab^4)x) + \log(-I\pi + 2bx + 2a)}{b^3}$$

**mupad [B]** time = 1.22, size = 46, normalized size = 0.98

$$\frac{\ln(\operatorname{acoth}(\tanh(a + bx)))}{b^3} - \frac{\frac{b^2x^2}{2} + bx \operatorname{acoth}(\tanh(a + bx))}{b^3 \operatorname{acoth}(\tanh(a + bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/acoth(tanh(a + b\*x))^3,x)

[Out] 
$$\frac{\log(\operatorname{acoth}(\tanh(a + bx)))}{b^3} - \frac{((b^2x^2)/2 + bx \operatorname{acoth}(\tanh(a + bx)))}{b^3 \operatorname{acoth}(\tanh(a + bx))^2}$$

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Exception raised: TypeError

$$3.179 \quad \int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx$$

Optimal. Leaf size=34

$$-\frac{1}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out]  $-1/2*x/b/\operatorname{arccoth}(\tanh(b*x+a))^2-1/2/b^2/\operatorname{arccoth}(\tanh(b*x+a))$

**Rubi [A]** time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 30}

$$-\frac{1}{2b^2 \coth^{-1}(\tanh(a+bx))} - \frac{x}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] `Int[x/ArcCoth[Tanh[a + b*x]]^3,x]`

[Out]  $-x/(2*b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) - 1/(2*b^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])$

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

Rule 2157

`Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]`

Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

Rubi steps

$$\begin{aligned} \int \frac{x}{\coth^{-1}(\tanh(a+bx))^3} dx &= -\frac{x}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{1}{\coth^{-1}(\tanh(a+bx))^2} dx}{2b} \\ &= -\frac{x}{2b \coth^{-1}(\tanh(a+bx))^2} + \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{2b^2} \\ &= -\frac{x}{2b \coth^{-1}(\tanh(a+bx))^2} - \frac{1}{2b^2 \coth^{-1}(\tanh(a+bx))} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 27, normalized size = 0.79

$$-\frac{\coth^{-1}(\tanh(a+bx)) + bx}{2b^2 \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/ArcCoth[Tanh[a + b\*x]]^3, x]

[Out] -1/2\*(b\*x + ArcCoth[Tanh[a + b\*x]])/(b^2\*ArcCoth[Tanh[a + b\*x]]^2)

**fricas [B]** time = 0.84, size = 124, normalized size = 3.65

$$\frac{2(8b^3x^3 + 20ab^2x^2 + 16a^2bx + \pi^2a + 4a^3)}{16b^6x^4 + 64ab^5x^3 + \pi^4b^2 + 8\pi^2a^2b^2 + 16a^4b^2 + 8(\pi^2b^4 + 12a^2b^4)x^2 + 16(\pi^2ab^3 + 4a^3b^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^3, x, algorithm="fricas")

[Out] -2\*(8\*b^3\*x^3 + 20\*a\*b^2\*x^2 + 16\*a^2\*b\*x + pi^2\*a + 4\*a^3)/(16\*b^6\*x^4 + 64\*a\*b^5\*x^3 + pi^4\*b^2 + 8\*pi^2\*a^2\*b^2 + 16\*a^4\*b^2 + 8\*(pi^2\*b^4 + 12\*a^2\*b^4)\*x^2 + 16\*(pi^2\*a\*b^3 + 4\*a^3\*b^3)\*x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{arccoth}(\tanh(bx+a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^3, x, algorithm="giac")

[Out] integrate(x/arccoth(tanh(b\*x + a))^3, x)

**maple** [C] time = 0.27, size = 634, normalized size = 18.65

$$\frac{2i \left( -2\pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a+1}} \right)^2 + \pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a+1}} \right) \operatorname{csgn} (ie^{2bx+2a}) \operatorname{csgn} \left( \frac{ie^{2bx+2a}}{e^{2bx+2a+1}} \right) - \pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a+1}} \right) \operatorname{csgn} \left( \frac{ie^{2bx+2a}}{e^{2bx+2a+1}} \right) \right)}{b^2 \left( -2\pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a+1}} \right)^2 + \pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a+1}} \right) \operatorname{csgn} (ie^{2bx+2a}) \operatorname{csgn} \left( \frac{ie^{2bx+2a}}{e^{2bx+2a+1}} \right) - \pi \operatorname{csgn} \left( \frac{i}{e^{2bx+2a+1}} \right) \operatorname{csgn} \left( \frac{ie^{2bx+2a}}{e^{2bx+2a+1}} \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/arccoth(tanh(b\*x+a))^3,x)

[Out]  $-2*I*(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a))+4*I*b*x)/b^2/(-2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+2*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3+Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+Pi*csgn(I*exp(2*b*x+2*a))^3-Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+2*Pi+4*I*ln(exp(b*x+a)))^2$

**maxima** [C] time = 1.11, size = 62, normalized size = 1.82

$$\frac{8(-i\pi + 4bx + 2a)}{32b^4x^2 - 8\pi^2b^2 - 32i\pi ab^2 + 32a^2b^2 + (-32i\pi b^3 + 64ab^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $-8*(-I*pi + 4*b*x + 2*a)/(32*b^4*x^2 - 8*pi^2*b^2 - 32*I*pi*a*b^2 + 32*a^2*b^2 + (-32*I*pi*b^3 + 64*a*b^3)*x)$

**mupad** [B] time = 0.09, size = 25, normalized size = 0.74

$$\frac{\operatorname{acoth}(\tanh(a + bx)) + bx}{2b^2 \operatorname{acoth}(\tanh(a + bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/acoth(tanh(a + b\*x))^3,x)

```
[Out] -(acoth(tanh(a + b*x)) + b*x)/(2*b^2*acoth(tanh(a + b*x))^2)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: TypeError
```



$$3.180 \quad \int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} dx$$

Optimal. Leaf size=16

$$-\frac{1}{2b \coth^{-1}(\tanh(a+bx))^2}$$

[Out] -1/2/b/arccoth(tanh(b\*x+a))^2

**Rubi [A]** time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$-\frac{1}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^(-3), x]

[Out] -1/(2\*b\*ArcCoth[Tanh[a + b\*x]]^2)

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2157

Int[(u\_)^(m\_), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\coth^{-1}(\tanh(a+bx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, \coth^{-1}(\tanh(a+bx))\right)}{b} \\ &= -\frac{1}{2b \coth^{-1}(\tanh(a+bx))^2} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 16, normalized size = 1.00

$$-\frac{1}{2b \coth^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^(-3), x]

[Out] -1/2\*1/(b\*ArcCoth[Tanh[a + b\*x]]^2)

**fricas** [B] time = 0.57, size = 107, normalized size = 6.69

$$\frac{2(4b^2x^2 + 8abx - \pi^2 + 4a^2)}{16b^5x^4 + 64ab^4x^3 + \pi^4b + 8\pi^2a^2b + 16a^4b + 8(\pi^2b^3 + 12a^2b^3)x^2 + 16(\pi^2ab^2 + 4a^3b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] -2\*(4\*b^2\*x^2 + 8\*a\*b\*x - pi^2 + 4\*a^2)/(16\*b^5\*x^4 + 64\*a\*b^4\*x^3 + pi^4\*b + 8\*pi^2\*a^2\*b + 16\*a^4\*b + 8\*(pi^2\*b^3 + 12\*a^2\*b^3)\*x^2 + 16\*(pi^2\*a\*b^2 + 4\*a^3\*b^2)\*x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^(-3), x)

**maple** [A] time = 0.08, size = 15, normalized size = 0.94

$$\frac{1}{2b\operatorname{arccoth}(\tanh(bx + a))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/arccoth(tanh(b\*x+a))^3,x)

[Out] -1/2/b/arccoth(tanh(b\*x+a))^2

**maxima** [C] time = 0.43, size = 30, normalized size = 1.88

$$\frac{8}{(4\pi^2 - 16i\pi(bx + a) - 16(bx + a)^2)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $8/((4\pi^2 - 16I\pi(bx + a) - 16(bx + a)^2)*b)$

**mupad [B]** time = 0.07, size = 14, normalized size = 0.88

$$\frac{1}{2b \operatorname{acoth}(\tanh(a + bx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/acoth(tanh(a + b\*x))^3,x)

[Out]  $-1/(2*b*acoth(\tanh(a + b*x))^2)$

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Exception raised: TypeError

$$3.181 \quad \int \frac{1}{x \coth^{-1}(\tanh(a+bx))^3} dx$$

Optimal. Leaf size=97

$$\frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} - \frac{1}{2(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^2} - \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^3}$$

[Out] -1/2/(b\*x-arccoth(tanh(b\*x+a)))/arccoth(tanh(b\*x+a))^2+1/(b\*x-arccoth(tanh(b\*x+a)))^2/arccoth(tanh(b\*x+a))-ln(x)/(b\*x-arccoth(tanh(b\*x+a)))^3+ln(arccoth(tanh(b\*x+a)))/(b\*x-arccoth(tanh(b\*x+a)))^3

**Rubi [A]** time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^2 \coth^{-1}(\tanh(a+bx))} - \frac{1}{2(bx - \coth^{-1}(\tanh(a+bx))) \coth^{-1}(\tanh(a+bx))^2} - \frac{1}{(bx - \coth^{-1}(\tanh(a+bx)))^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out] -1/(2\*(b\*x - ArcCoth[Tanh[a + b\*x]])\*ArcCoth[Tanh[a + b\*x]]^2) + 1/((b\*x - ArcCoth[Tanh[a + b\*x]])^2\*ArcCoth[Tanh[a + b\*x]]) - Log[x]/(b\*x - ArcCoth[Tanh[a + b\*x]])^3 + Log[ArcCoth[Tanh[a + b\*x]]]/(b\*x - ArcCoth[Tanh[a + b\*x]])^3

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x \coth^{-1}(\tanh(a + bx))^3} dx &= -\frac{1}{2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} - \frac{\int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx}{bx - \coth^{-1}(\tanh(a + bx))} \\ &= -\frac{1}{2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{1}{2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{(bx - \coth^{-1}(\tanh(a + bx)))} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 74, normalized size = 0.76

$$\frac{-4bx \coth^{-1}(\tanh(a + bx)) + \coth^{-1}(\tanh(a + bx))^2 (-2 \log(\coth^{-1}(\tanh(a + bx))) + 2 \log(bx) + 3) + b^2 x^2}{2 \coth^{-1}(\tanh(a + bx))^2 (\coth^{-1}(\tanh(a + bx)) - bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*ArcCoth[Tanh[a + b\*x]]^3), x]

[Out] (b^2\*x^2 - 4\*b\*x\*ArcCoth[Tanh[a + b\*x]] + ArcCoth[Tanh[a + b\*x]]^2\*(3 + 2\*Log[b\*x] - 2\*Log[ArcCoth[Tanh[a + b\*x]]]))/(2\*ArcCoth[Tanh[a + b\*x]]^2\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]]^3)

**fricas [B]** time = 0.61, size = 811, normalized size = 8.36

$$8 \left( 9 \pi^6 a + 60 \pi^4 a^3 + 48 \pi^2 a^5 - 192 a^7 + 8 (\pi^4 b^3 - 16 a^4 b^3) x^3 + 4 (9 \pi^4 a b^2 + 8 \pi^2 a^3 b^2 - 112 a^5 b^2) x^2 + 4 (\pi^6 b^3 - 12 a^4 b^3) x + 4 \pi^6 a^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $-8*(9*\pi^6*a + 60*\pi^4*a^3 + 48*\pi^2*a^5 - 192*a^7 + 8*(\pi^4*b^3 - 16*a^4*b^3)*x^3 + 4*(9*\pi^4*a*b^2 + 8*\pi^2*a^3*b^2 - 112*a^5*b^2)*x^2 + 4*(\pi^6*b + 16*\pi^4*a^2*b + 16*\pi^2*a^4*b - 128*a^6*b)*x - 2*(\pi^7 - 4*\pi^5*a^2 - 80*\pi^3*a^4 - 192*\pi*a^6 + 16*(\pi^3*b^4 - 12*\pi*a^2*b^4)*x^4 + 64*(\pi^3*a*b^3 - 12*\pi*a^3*b^3)*x^3 + 8*(\pi^5*b^2 - 144*\pi*a^4*b^2)*x^2 + 16*(\pi^5*a*b - 8*\pi^3*a^3*b - 48*\pi*a^5*b)*x)*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi) - (3*\pi^6*a + 20*\pi^4*a^3 + 16*\pi^2*a^5 - 64*a^7 + 16*(3*\pi^2*a*b^4 - 4*a^3*b^4)*x^4 + 64*(3*\pi^2*a^2*b^3 - 4*a^4*b^3)*x^3 + 8*(3*\pi^4*a*b^2 + 32*\pi^2*a^3*b^2 - 48*a^5*b^2)*x^2 + 16*(3*\pi^4*a^2*b + 8*\pi^2*a^4*b - 16*a^6*b)*x)*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2) + 2*(3*\pi^6*a + 20*\pi^4*a^3 + 16*\pi^2*a^5 - 64*a^7 + 16*(3*\pi^2*a*b^4 - 4*a^3*b^4)*x^4 + 64*(3*\pi^2*a^2*b^3 - 4*a^4*b^3)*x^3 + 8*(3*\pi^4*a*b^2 + 32*\pi^2*a^3*b^2 - 48*a^5*b^2)*x^2 + 16*(3*\pi^4*a^2*b + 8*\pi^2*a^4*b - 16*a^6*b)*x)*\log(x))/(\pi^{10} + 20*\pi^8*a^2 + 160*\pi^6*a^4 + 640*\pi^4*a^6 + 1280*\pi^2*a^8 + 1024*a^{10} + 16*(\pi^6*b^4 + 12*\pi^4*a^2*b^4 + 48*\pi^2*a^4*b^4 + 64*a^6*b^4)*x^4 + 64*(\pi^6*a*b^3 + 12*\pi^4*a^3*b^3 + 48*\pi^2*a^5*b^3 + 64*a^7*b^3)*x^3 + 8*(\pi^8*b^2 + 24*\pi^6*a^2*b^2 + 192*\pi^4*a^4*b^2 + 640*\pi^2*a^6*b^2 + 768*a^8*b^2)*x^2 + 16*(\pi^8*a*b + 16*\pi^6*a^3*b + 96*\pi^4*a^5*b + 256*\pi^2*a^7*b + 256*a^9*b)*x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(1/(x\*arccoth(tanh(b\*x + a))^3), x)

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/arccoth(tanh(b\*x+a))^3,x)

[Out] int(1/x/arccoth(tanh(b\*x+a))^3,x)

**maxima [C]** time = 1.13, size = 173, normalized size = 1.78

$$\frac{8(-3i\pi + 4bx + 6a)}{2\pi^4 + 16i\pi^3a - 48\pi^2a^2 - 64i\pi a^3 + 32a^4 - (8\pi^2b^2 + 32i\pi ab^2 - 32a^2b^2)x^2 + (8i\pi^3b - 48\pi^2ab - 96i\pi a^2b +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out]  $8*(-3*I*\pi + 4*b*x + 6*a)/(2*\pi^4 + 16*I*\pi^3*a - 48*\pi^2*a^2 - 64*I*\pi*a^3 + 32*a^4 - (8*\pi^2*b^2 + 32*I*\pi*a*b^2 - 32*a^2*b^2)*x^2 + (8*I*\pi^3*b - 48*\pi^2*a*b - 96*I*\pi*a^2*b + 64*a^3*b)*x) + 8*\log(-I*\pi + 2*b*x + 2*a)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3) - 8*\log(x)/(-I*\pi^3 + 6*\pi^2*a + 12*I*\pi*a^2 - 8*a^3)$

**mupad [B]** time = 6.49, size = 902, normalized size = 9.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*acoth(tanh(a + b\*x))^3),x)

[Out]  $- (16*\operatorname{atanh}((16*(4*b*x - ((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1))) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 + 12*a^2*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))/((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x) + 4*a^2))*((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2/16 - (a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))/4 + a^2/4))/(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^3)/(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x) - (12/(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x) - (16*b*x)/((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x) + 4*a^2))/((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x) + 4*a^2) + x*(8*a*b - 4$

```
*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1) + 2*b*x)) + 4*b^2*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acoth(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x*acoth(tanh(a + b*x))**3), x)
```



$$3.182 \quad \int \frac{1}{x^2 \coth^{-1}(\tanh(a+bx))^3} dx$$

Optimal. Leaf size=131

$$\frac{3b}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} - \frac{3b}{2(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \dots$$

[Out]  $-3/2*b/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2/\operatorname{arccoth}(\tanh(b*x+a))^2+1/x/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/\operatorname{arccoth}(\tanh(b*x+a))^2+3*b/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^3/\operatorname{arccoth}(\tanh(b*x+a))-3*b*\ln(x)/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^4+3*b*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^4$

**Rubi [A]** time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{3b}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))} - \frac{3b}{2(bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \dots$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*ArcCoth[Tanh[a + b\*x]]^3), x]

[Out]  $(-3*b)/(2*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) + 1/(x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) + (3*b)/((b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^3*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) - (3*b*\operatorname{Log}[x])/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^4 + (3*b*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^4$

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))^3} dx &= \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} - \frac{(3b) \int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx}{-bx + \coth^{-1}(\tanh(a + bx))} \\ &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\ &= -\frac{3b}{2 (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 93, normalized size = 0.71

$$\frac{-6b^2x^2 \coth^{-1}(\tanh(a + bx)) + 2 \coth^{-1}(\tanh(a + bx))^3 + 3bx \coth^{-1}(\tanh(a + bx))^2 (-2 \log(\coth^{-1}(\tanh(a + bx))) - bx)}{2x \coth^{-1}(\tanh(a + bx))^2 (\coth^{-1}(\tanh(a + bx)) - bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out] 
$$-1/2*(b^3*x^3 - 6*b^2*x^2*ArcCoth[Tanh[a + b*x]] + 2*ArcCoth[Tanh[a + b*x]]^3 + 3*b*x*ArcCoth[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcCoth[Tanh[a + b*x]]]))/(x*ArcCoth[Tanh[a + b*x]]^2*(-(b*x) + ArcCoth[Tanh[a + b*x]]))^4$$

**fricas** [B] time = 0.54, size = 1078, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out] 
$$8*(6*\pi^8*a + 64*\pi^6*a^3 + 192*\pi^4*a^5 - 512*a^9 + 96*(3*\pi^4*a*b^4 + 8*\pi^2*a^3*b^4 - 16*a^5*b^4))*x^4 - 12*(\pi^6*b^3 - 92*\pi^4*a^2*b^3 - 272*\pi^2*a^4*b^3 + 448*a^6*b^3)*x^3 + 8*(11*\pi^6*a*b^2 + 228*\pi^4*a^3*b^2 + 528*\pi^2*a^5*b^2 - 832*a^7*b^2)*x^2 - (5*\pi^8*b - 176*\pi^6*a^2*b - 1440*\pi^4*a^4*b - 1792*\pi^2*a^6*b + 3328*a^8*b)*x - 96*(16*(\pi^3*a*b^5 - 4*\pi*a^3*b^5))*x^5 + 64*(\pi^3*a^2*b^4 - 4*\pi*a^4*b^4)*x^4 + 8*(\pi^5*a*b^3 + 8*\pi^3*a^3*b^3 - 48*\pi*a^5*b^3)*x^3 + 16*(\pi^5*a^2*b^2 - 16*\pi*a^6*b^2)*x^2 + (\pi^7*a*b + 4*\pi^5*a^3*b - 16*\pi^3*a^5*b - 64*\pi*a^7*b)*x*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi) + 3*(16*(\pi^4*b^5 - 24*\pi^2*a^2*b^5 + 16*a^4*b^5)*x^5 + 64*(\pi^4*a*b^4 - 24*\pi^2*a^3*b^4 + 16*a^5*b^4)*x^4 + 8*(\pi^6*b^3 - 12*\pi^4*a^2*b^3 - 272*\pi^2*a^4*b^3 + 192*a^6*b^3)*x^3 + 16*(\pi^6*a*b^2 - 20*\pi^4*a^3*b^2 - 80*\pi^2*a^5*b^2 + 64*a^7*b^2)*x^2 + (\pi^8*b - 16*\pi^6*a^2*b - 160*\pi^4*a^4*b - 256*\pi^2*a^6*b + 256*a^8*b)*x*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2) - 6*(16*(\pi^4*b^5 - 24*\pi^2*a^2*b^5 + 16*a^4*b^5)*x^5 + 64*(\pi^4*a*b^4 - 24*\pi^2*a^3*b^4 + 16*a^5*b^4)*x^4 + 8*(\pi^6*b^3 - 12*\pi^4*a^2*b^3 - 272*\pi^2*a^4*b^3 + 192*a^6*b^3)*x^3 + 16*(\pi^6*a*b^2 - 20*\pi^4*a^3*b^2 - 80*\pi^2*a^5*b^2 + 64*a^7*b^2)*x^2 + (\pi^8*b - 16*\pi^6*a^2*b - 160*\pi^4*a^4*b - 256*\pi^2*a^6*b + 256*a^8*b)*x*\log(x))/(16*(\pi^8*b^4 + 16*\pi^6*a^2*b^4 + 96*\pi^4*a^4*b^4 + 256*\pi^2*a^6*b^4 + 256*a^8*b^4)*x^5 + 64*(\pi^8*a*b^3 + 16*\pi^6*a^3*b^3 + 96*\pi^4*a^5*b^3 + 256*\pi^2*a^7*b^3 + 256*a^9*b^3)*x^4 + 8*(\pi^10*b^2 + 28*\pi^8*a^2*b^2 + 288*\pi^6*a^4*b^2 + 1408*\pi^4*a^6*b^2 + 3328*\pi^2*a^8*b^2 + 3072*a^10*b^2)*x^3 + 16*(\pi^10*a*b + 20*\pi^8*a^3*b + 160*\pi^6*a^5*b + 640*\pi^4*a^7*b + 1280*\pi^2*a^9*b + 1024*a^11*b)*x^2 + (\pi^12 + 24*\pi^10*a^2 + 240*\pi^8*a^4 + 1280*\pi^6*a^6 + 3840*\pi^4*a^8 + 6144*\pi^2*a^10 + 4096*a^12)*x)$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(1/(x^2\*arccoth(tanh(b\*x + a))^3), x)

**maple** [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/arccoth(tanh(b\*x+a))^3,x)

[Out] int(1/x^2/arccoth(tanh(b\*x+a))^3,x)

**maxima** [C] time = 1.15, size = 243, normalized size = 1.85

$$\frac{48 b \log(-i \pi + 2 b x + 2 a)}{\pi^4 + 8 i \pi^3 a - 24 \pi^2 a^2 - 32 i \pi a^3 + 16 a^4} - \frac{48 b \log(x)}{\pi^4 + 8 i \pi^3 a - 24 \pi^2 a^2 - 32 i \pi a^3 + 16 a^4} + \frac{8 i \pi^3 b^2 + 24 \pi^2 a b^2 + 48 i \pi a^3 b}{\pi^4 + 8 i \pi^3 a - 24 \pi^2 a^2 - 32 i \pi a^3 + 16 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/arccoth(tanh(b\*x+a))^3,x, algorithm="maxima")

[Out] 48\*b\*log(-I\*pi + 2\*b\*x + 2\*a)/(pi^4 + 8\*I\*pi^3\*a - 24\*pi^2\*a^2 - 32\*I\*pi\*a^3 + 16\*a^4) - 48\*b\*log(x)/(pi^4 + 8\*I\*pi^3\*a - 24\*pi^2\*a^2 - 32\*I\*pi\*a^3 + 16\*a^4) + 8\*(12\*b^2\*x^2 - pi^2 - 4\*I\*pi\*a + 4\*a^2 + (-9\*I\*pi\*b + 18\*a\*b)\*x)/((-4\*I\*pi^3\*b^2 + 24\*pi^2\*a\*b^2 + 48\*I\*pi\*a^2\*b^2 - 32\*a^3\*b^2)\*x^3 - (4\*pi^4\*b + 32\*I\*pi^3\*a\*b - 96\*pi^2\*a^2\*b - 128\*I\*pi\*a^3\*b + 64\*a^4\*b)\*x^2 + (I\*pi^5 - 10\*pi^4\*a - 40\*I\*pi^3\*a^2 + 80\*pi^2\*a^3 + 80\*I\*pi\*a^4 - 32\*a^5)\*x)

**mupad** [B] time = 5.26, size = 1074, normalized size = 8.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*acoth(tanh(a + b\*x))^3),x)

[Out] (8/(log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x) - (72\*b\*x)/((2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x)^2 - 4\*a\*(2\*a - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1)) + log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) + 2\*b\*x) + 4\*a^2) + (96\*b^2\*x^2)/((log(-2/(exp(2\*a)\*exp(2\*b\*x) - 1)) - log((2\*exp(2\*a)\*exp(2\*b\*x))/(exp(2\*a)\*exp(2\*b\*x) - 1))

$$\begin{aligned}
& 1)) + 2*b*x)*((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1) \\
& ) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - \log((2*\exp(2* \\
& a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1 \\
& )) + 2*b*x) + 4*a^2)))/(x*((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp \\
& (2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - 1 \\
& \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*ex \\
& p(2*b*x) - 1)) + 2*b*x) + 4*a^2) + x^2*(8*a*b - 4*b*(2*a - \log((2*\exp(2*a)* \\
& \exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) \\
& + 2*b*x)) + 4*b^2*x^3) + (96*b*atanh(((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(e \\
& xp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^4 + 2 \\
& 4*a^2*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(- \\
& 2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - \log((2*\exp(2* \\
& a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1 \\
& )) + 2*b*x)^3 - 32*a^3*(2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b \\
& *x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))/(log(-2/(\exp(2*a)*e \\
& xp(2*b*x) - 1)) - log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \\
& 2*b*x)^4 - (4*b*x*((2*a - \log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) \\
& - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - \log((2*ex \\
& p(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \log(-2/(\exp(2*a)*\exp(2*b*x) \\
& - 1)) + 2*b*x) + 4*a^2))/(log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - log((2*\exp(2 \\
& *a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^3))/(log(-2/(\exp(2*a)*e \\
& xp(2*b*x) - 1)) - log((2*\exp(2*a)*\exp(2*b*x)))/(\exp(2*a)*\exp(2*b*x) - 1)) + \\
& 2*b*x)^4
\end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(1/(x\*\*2\*acoth(tanh(a + b\*x))\*\*3), x)

$$3.183 \quad \int \frac{1}{x^3 \coth^{-1}(\tanh(a+bx))^3} dx$$

**Optimal.** Leaf size=170

$$\frac{6b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^4 \coth^{-1}(\tanh(a+bx))} - \frac{3b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))^2} (bx - \coth^{-1}(\tanh(a+bx)))^2$$

[Out]  $-3*b^2/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^3/\operatorname{arccoth}(\tanh(b*x+a))^2+2*b/x/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^2/\operatorname{arccoth}(\tanh(b*x+a))^2+1/2/x^2/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/\operatorname{arccoth}(\tanh(b*x+a))^2+6*b^2/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^4/\operatorname{arccoth}(\tanh(b*x+a))-6*b^2*\ln(x)/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^5+6*b^2*\ln(\operatorname{arccoth}(\tanh(b*x+a)))/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))^5$

**Rubi [A]** time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{6b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^4 \coth^{-1}(\tanh(a+bx))} - \frac{3b^2}{(bx - \coth^{-1}(\tanh(a+bx)))^3 \coth^{-1}(\tanh(a+bx))^2} (bx - \coth^{-1}(\tanh(a+bx)))^2$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out]  $(-3*b^2)/((b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^3*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) + (2*b)/(x*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^2*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) + 1/(2*x^2*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^2) + (6*b^2)/((b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^4*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]) - (6*b^2*\operatorname{Log}[x])/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^5 + (6*b^2*\operatorname{Log}[\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]])/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])^5$

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

### Rule 2160

Int[1/((u\_)\*(v\_)), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Dist[b/(b\*u - a\*v), Int[1/v, x], x] - Dist[a/(b\*u - a\*v), Int[1/u

, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

### Rule 2163

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[v^(n + 1)/((n + 1)\*(b\*u - a\*v)), x] - Dist[(a\*(n + 1))/((n + 1)\*(b\*u - a\*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && LtQ[n, -1]

### Rule 2171

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, -Simp[(u^(m + 1)\*v^(n + 1))/((m + 1)\*(b\*u - a\*v)), x] + Dist[(b\*(m + n + 2))/((m + 1)\*(b\*u - a\*v)), Int[u^(m + 1)\*v^n, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \coth^{-1}(\tanh(a + bx))^3} dx &= \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^2} + \frac{(2b) \int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))}}{bx - \coth^{-1}(\tanh(a + bx))} \\
&= \frac{2b}{x (bx - \coth^{-1}(\tanh(a + bx)))^2 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))} \\
&= -\frac{3b^2}{(bx - \coth^{-1}(\tanh(a + bx)))^3 \coth^{-1}(\tanh(a + bx))^2} + \frac{1}{x (bx - \coth^{-1}(\tanh(a + bx)))}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 107, normalized size = 0.63

$$\frac{8b^3x^3 \coth^{-1}(\tanh(a + bx)) - 12b^2x^2 \coth^{-1}(\tanh(a + bx))^2 (\log(x) - \log(\coth^{-1}(\tanh(a + bx)))) - 8bx \coth^{-1}(\tanh(a + bx))}{2x^2 (bx - \coth^{-1}(\tanh(a + bx)))^5 \coth^{-1}(\tanh(a + bx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*ArcCoth[Tanh[a + b\*x]]^3),x]

[Out]  $(-b^4x^4) + 8b^3x^3 \text{ArcCoth}[\text{Tanh}[a + b*x]] - 8b^2x^2 \text{ArcCoth}[\text{Tanh}[a + b*x]]^2 + \text{ArcCoth}[\text{Tanh}[a + b*x]]^3 - 12b^2x^2 \text{ArcCoth}[\text{Tanh}[a + b*x]]^2 (\text{Log}[x] - \text{Log}[\text{ArcCoth}[\text{Tanh}[a + b*x]]]) / (2x^2 (bx - \text{ArcCoth}[\text{Tanh}[a + b*x]])^5 \text{ArcCoth}[\text{Tanh}[a + b*x]]^2)$

**fricas [B]** time = 0.62, size = 1316, normalized size = 7.74

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="fricas")

[Out]  $8*(3*\pi^{10}*a + 44*\pi^8*a^3 + 224*\pi^6*a^5 + 384*\pi^4*a^7 - 256*\pi^2*a^9 - 1024*a^{11} + 192*(\pi^6*b^5 - 20*\pi^4*a^2*b^5 - 80*\pi^2*a^4*b^5 + 64*a^6*b^5)*x^5 + 96*(11*\pi^6*a*b^4 - 140*\pi^4*a^3*b^4 - 624*\pi^2*a^5*b^4 + 448*a^7*b^4)*x^4 + 16*(5*\pi^8*b^3 + 16*\pi^6*a^2*b^3 - 1440*\pi^4*a^4*b^3 - 4864*\pi^2*a^6*b^3 + 3328*a^8*b^3)*x^3 + 4*(65*\pi^8*a*b^2 - 272*\pi^6*a^3*b^2 - 4896*\pi^4*a^5*b^2 - 9472*\pi^2*a^7*b^2 + 6400*a^9*b^2)*x^2 + 2*(3*\pi^{10}*b - 12*\pi^8*a^2*b - 416*\pi^6*a^4*b - 1920*\pi^4*a^6*b - 2304*\pi^2*a^8*b + 1024*a^{10}*b)*x - 48*(16*(\pi^5*b^6 - 40*\pi^3*a^2*b^6 + 80*\pi*a^4*b^6)*x^6 + 64*(\pi^5*a*b^5 - 40*\pi^3*a^3*b^5 + 80*\pi*a^5*b^5)*x^5 + 8*(\pi^7*b^4 - 28*\pi^5*a^2*b^4 - 400*\pi^3*a^4*b^4 + 960*\pi*a^6*b^4)*x^4 + 16*(\pi^7*a*b^3 - 36*\pi^5*a^3*b^3 - 800*\pi^3*a^5*b^3 + 320*\pi*a^7*b^3)*x^3 + (\pi^9*b^2 - 32*\pi^7*a^2*b^2 - 224*\pi^5*a^4*b^2 + 1280*\pi*a^8*b^2)*x^2)*\arctan(-(2*b*x + 2*a - \sqrt{4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2}))/\pi - 24*(16*(5*\pi^4*a*b^6 - 40*\pi^2*a^3*b^6 + 16*a^5*b^6)*x^6 + 64*(5*\pi^4*a^2*b^5 - 40*\pi^2*a^4*b^5 + 16*a^6*b^5)*x^5 + 8*(5*\pi^6*a*b^4 + 20*\pi^4*a^3*b^4 - 464*\pi^2*a^5*b^4 + 192*a^7*b^4)*x^4 + 16*(5*\pi^6*a^2*b^3 - 20*\pi^4*a^4*b^3 - 144*\pi^2*a^6*b^3 + 64*a^8*b^3)*x^3 + (5*\pi^8*a*b^2 - 224*\pi^4*a^5*b^2 - 512*\pi^2*a^7*b^2 + 256*a^9*b^2)*x^2)*\log(4*b^2*x^2 + 8*a*b*x + \pi^2 + 4*a^2) + 48*(16*(5*\pi^4*a*b^6 - 40*\pi^2*a^3*b^6 + 16*a^5*b^6)*x^6 + 64*(5*\pi^4*a^2*b^5 - 40*\pi^2*a^4*b^5 + 16*a^6*b^5)*x^5 + 8*(5*\pi^6*a*b^4 + 20*\pi^4*a^3*b^4 - 464*\pi^2*a^5*b^4 + 192*a^7*b^4)*x^4 + 16*(5*\pi^6*a^2*b^3 - 20*\pi^4*a^4*b^3 - 144*\pi^2*a^6*b^3 + 64*a^8*b^3)*x^3 + (5*\pi^8*a*b^2 - 224*\pi^4*a^5*b^2 - 512*\pi^2*a^7*b^2 + 256*a^9*b^2)*x^2)*\log(x))/(16*(\pi^{10}*b^4 + 20*\pi^8*a^2*b^4 + 160*\pi^6*a^4*b^4 + 640*\pi^4*a^6*b^4 + 1280*\pi^2*a^8*b^4 + 1024*a^{10}*b^4)*x^6 + 64*(\pi^{10}*a*b^3 + 20*\pi^8*a^3*b^3 + 160*\pi^6*a^5*b^3 + 640*\pi^4*a^7*b^3 + 1280*\pi^2*a^9*b^3 + 1024*a^{11}*b^3)*x^5 + 8*(\pi^{12}*b^2 + 32*\pi^{10}*a^2*b^2 + 400*\pi^8*a^4*b^2 + 2560*\pi^6*a^6*b^2 + 8960*\pi^4*a^8*b^2 + 16384*\pi^2*a^{10}*b^2 + 12288*a^{12}*b^2)*x^4 + 16*(\pi^{12}*a*b + 24*\pi^{10}*a^3*b + 240*\pi^8*a^5*b + 1280*\pi^6*a^7*b + 3840*\pi^4*a^9*b + 6144*\pi^2*a^{11}*b + 4096*a^{13}*b)*x^3 + (\pi^{14} + 28*\pi^{12}*a^2 + 336*\pi^{10}*a^4 + 2240*\pi^8*a^6 + 8960*\pi^6*a^8 + 21504*\pi^4*a^{10} + 28672*\pi^2*a^{12} + 16384*a^{14})*x^2)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/arccoth(tanh(b\*x+a))^3,x, algorithm="giac")

[Out] integrate(1/(x^3\*arccoth(tanh(b\*x + a))^3), x)



```

*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x)^2*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1
)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1
)) + 2*b*x + 4*a^2)))/(x^3*(8*a*b - 4*b*(2*a - log((2*exp(2*a)*exp(2*b*x))
)/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)) +
x^2*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-
2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*
x) + 4*a^2) + 4*b^2*x^4) - (384*b^2*atanh((4*b*x*((2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1
)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x + 4*a^2)))/(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
2*b*x)^3 - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
+ log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) -
1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(
2*a)*exp(2*b*x) - 1)) + 2*b*x)^4 + 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x)
))/(exp(2*a)*exp(2*b*x) - 1)) + log(-2/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/
(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) - 1)) + 2*b*x)^5))/(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^5

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{acoth}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/acoth(tanh(b\*x+a))\*\*3,x)

[Out] Integral(1/(x\*\*3\*acoth(tanh(a + b\*x))\*\*3), x)

### 3.184 $\int x^m \coth^{-1}(\tanh(a + bx))^n dx$

**Optimal.** Leaf size=79

$$\frac{x^m \left( \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))} \right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} {}_2F_1 \left( -m, n+1; n+2; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))} \right)}{b(n+1)}$$

[Out]  $x^m \operatorname{arccoth}(\tanh(bx+a))^{1+n} \operatorname{hypergeom}([-m, 1+n], [2+n], -\operatorname{arccoth}(\tanh(bx+a)) / (bx - \operatorname{arccoth}(\tanh(bx+a)))) / b / (1+n) / ((bx / (bx - \operatorname{arccoth}(\tanh(bx+a))))^m)$

**Rubi [A]** time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2173}

$$\frac{x^m \left( \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))} \right)^{-m} \coth^{-1}(\tanh(a+bx))^{n+1} {}_2F_1 \left( -m, n+1; n+2; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))} \right)}{b(n+1)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^m \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^n, x]$

[Out]  $(x^m \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]^{1+n} \operatorname{Hypergeometric2F1}[-m, 1+n, 2+n, -(\operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]] / (bx - \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]])))] / (b(1+n) * ((bx) / (bx - \operatorname{ArcCoth}[\operatorname{Tanh}[a + bx]]))^m)$

#### Rule 2173

$\operatorname{Int}[(u_)^m (v_)^n, x\_Symbol] := \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^m v^{n+1} \operatorname{Hypergeometric2F1}[-m, n+1, n+2, -(a*v)/(b*u - a*v)]) / (b*(n+1) * ((b*u)/(b*u - a*v))^m), x] /; \operatorname{NeQ}[b*u - a*v, 0] /; \operatorname{PiecewiseLinearQ}[u, v, x] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegerQ}[n]$

#### Rubi steps

$$\int x^m \coth^{-1}(\tanh(a + bx))^n dx = \frac{x^m \left( \frac{bx}{bx - \coth^{-1}(\tanh(a+bx))} \right)^{-m} \coth^{-1}(\tanh(a + bx))^{1+n} {}_2F_1 \left( -m, 1+n; 2+n; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))} \right)}{b(1+n)}$$

**Mathematica [A]** time = 0.14, size = 71, normalized size = 0.90

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))^n \left( \frac{bx}{\coth^{-1}(\tanh(a+bx)) - bx} + 1 \right)^{-n} {}_2F_1 \left( m+1, -n; m+2; -\frac{bx}{\coth^{-1}(\tanh(a+bx)) - bx} \right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] (x^(1 + m)\*ArcCoth[Tanh[a + b\*x]]^n\*Hypergeometric2F1[1 + m, -n, 2 + m, -((b\*x)/(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))])/((1 + m)\*(1 + (b\*x)/(-(b\*x) + ArcCoth[Tanh[a + b\*x]]))^n)

**fricas** [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \operatorname{arccoth}(\tanh(bx + a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^n,x, algorithm="fricas")

[Out] integral(x^m\*arccoth(tanh(b\*x + a))^n, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^n,x, algorithm="giac")

[Out] integrate(x^m\*arccoth(tanh(b\*x + a))^n, x)

**maple** [F] time = 14.58, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*arccoth(tanh(b\*x+a))^n,x)

[Out] int(x^m\*arccoth(tanh(b\*x+a))^n,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

[Out] integrate(x^m\*arccoth(tanh(b\*x + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \operatorname{acoth}(\tanh(a + bx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*acoth(tanh(a + b*x))^n,x)`

[Out] `int(x^m*acoth(tanh(a + b*x))^n, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{acoth}^n(\tanh(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*acoth(tanh(b*x+a))**n,x)`

[Out] `Integral(x**m*acoth(tanh(a + b*x))**n, x)`

### 3.185 $\int x^4 \coth^{-1}(\tanh(a + bx))^n dx$

**Optimal.** Leaf size=165

$$\frac{24 \coth^{-1}(\tanh(a + bx))^{n+5}}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)} - \frac{24x \coth^{-1}(\tanh(a + bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{4x^4 \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

[Out]  $x^4 \operatorname{arccoth}(\tanh(bx+a))^{(1+n)}/b/(1+n) - 4x^3 \operatorname{arccoth}(\tanh(bx+a))^{(2+n)}/b^2/(1+n)/(2+n) + 12x^2 \operatorname{arccoth}(\tanh(bx+a))^{(3+n)}/b^3/(3+n)/(n^2+3n+2) - 24x \operatorname{arccoth}(\tanh(bx+a))^{(4+n)}/b^4/(n^2+5n+4)/(n^2+5n+6) + 24 \operatorname{arccoth}(\tanh(bx+a))^{(5+n)}/b^5/(n^2+7n+12)/(n^3+8n^2+17n+10)$

**Rubi [A]** time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 30}

$$-\frac{4x^3 \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{24x \coth^{-1}(\tanh(a + bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{24 \coth^{-1}(\tanh(a + bx))^{n+5}}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)}$$

Antiderivative was successfully verified.

[In] Int[x^4\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out]  $(x^4 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(1+n)})/(b*(1+n)) - (4*x^3 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(2+n)})/(b^2*(1+n)*(2+n)) + (12*x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(3+n)})/(b^3*(1+n)*(2+n)*(3+n)) - (24*x \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(4+n)})/(b^4*(1+n)*(2+n)*(3+n)*(4+n)) + (24 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(5+n)})/(b^5*(1+n)*(2+n)*(3+n)*(4+n)*(5+n))$

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2157

Int[(u\_)^(m\_), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0])

`&& !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

### Rubi steps

$$\begin{aligned}
 \int x^4 \coth^{-1}(\tanh(a + bx))^n dx &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4 \int x^3 \coth^{-1}(\tanh(a + bx))^{1+n} dx}{b(1+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12 \int x^2 \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^2(1+n)(2+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)} - \frac{24 \int x \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^3(1+n)(2+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)} - \frac{24x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^4(1+n)(2+n)} + \frac{24 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^4(1+n)(2+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)} - \frac{24x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^4(1+n)(2+n)} + \frac{24 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^4(1+n)(2+n)} \\
 &= \frac{x^4 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{4x^3 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)} - \frac{24x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^4(1+n)(2+n)} + \frac{24 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^4(1+n)(2+n)}
 \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 146, normalized size = 0.88

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1} \left( -4b^3 (n^3 + 12n^2 + 47n + 60) x^3 \coth^{-1}(\tanh(a + bx)) + 12b^2 (n^2 + 9n + 20) x^2 \coth^{-1}(\tanh(a + bx)) - 24b (n + 1) x \coth^{-1}(\tanh(a + bx)) + 24 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx \right)}{b^5(n+1)(n+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] (ArcCoth[Tanh[a + b\*x]]^(1 + n)\*(b^4\*(120 + 154\*n + 71\*n^2 + 14\*n^3 + n^4)\*x^4 - 4\*b^3\*(60 + 47\*n + 12\*n^2 + n^3)\*x^3\*ArcCoth[Tanh[a + b\*x]] + 12\*b^2\*(20 + 9\*n + n^2)\*x^2\*ArcCoth[Tanh[a + b\*x]]^2 - 24\*b\*(5 + n)\*x\*ArcCoth[Tanh[a + b\*x]]^3 + 24\*ArcCoth[Tanh[a + b\*x]]^4)/(b^5\*(1 + n)\*(2 + n)\*(3 + n)\*(4 + n)\*(5 + n))

**fricas [B]** time = 0.72, size = 583, normalized size = 3.53

$$2 \left( 2 \left( b^5 n^4 + 10 b^5 n^3 + 35 b^5 n^2 + 50 b^5 n + 24 b^5 \right) x^5 + 15 \pi^4 a - 120 \pi^2 a^3 + 48 a^5 + 2 \left( a b^4 n^4 + 6 a b^4 n^3 + 11 a b^4 n^2 \right) \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(tanh(b\*x+a))^n,x, algorithm="fricas")

[Out]  $\frac{1}{4} \cdot (2 \cdot (2 \cdot (b^5 \cdot n^4 + 10 \cdot b^5 \cdot n^3 + 35 \cdot b^5 \cdot n^2 + 50 \cdot b^5 \cdot n + 24 \cdot b^5)) \cdot x^5 + 15 \cdot \pi^4 \cdot a - 120 \cdot \pi^2 \cdot a^3 + 48 \cdot a^5 + 2 \cdot (a \cdot b^4 \cdot n^4 + 6 \cdot a \cdot b^4 \cdot n^3 + 11 \cdot a \cdot b^4 \cdot n^2 + 6 \cdot a \cdot b^4 \cdot n)) \cdot x^4 - 2 \cdot (4 \cdot a^2 \cdot b^3 \cdot n^3 + 12 \cdot a^2 \cdot b^3 \cdot n^2 + 8 \cdot a^2 \cdot b^3 \cdot n - \pi^2 \cdot (b^3 \cdot n^3 + 3 \cdot b^3 \cdot n^2 + 2 \cdot b^3 \cdot n)) \cdot x^3 + 6 \cdot (4 \cdot a^3 \cdot b^2 \cdot n^2 + 4 \cdot a^3 \cdot b^2 \cdot n - 3 \cdot \pi^2 \cdot (a \cdot b^2 \cdot n^2 + a \cdot b^2 \cdot n)) \cdot x^2 - 3 \cdot (\pi^4 \cdot b \cdot n - 24 \cdot \pi^2 \cdot a^2 \cdot b \cdot n + 16 \cdot a^4 \cdot b \cdot n) \cdot x \cdot (b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + \frac{1}{4} \cdot \pi^2 + a^2)^{\frac{1}{2} \cdot n} \cdot \cos(2 \cdot n \cdot \arctan(\frac{-2 \cdot b \cdot x}{\pi - 2 \cdot a/\pi + \sqrt{4 \cdot b^2 \cdot x^2 + 8 \cdot a \cdot b \cdot x + \pi^2 + 4 \cdot a^2}}{\pi})) - (2 \cdot \pi \cdot (b^4 \cdot n^4 + 6 \cdot b^4 \cdot n^3 + 11 \cdot b^4 \cdot n^2 + 6 \cdot b^4 \cdot n)) \cdot x^4 + 3 \cdot \pi^5 - 120 \cdot \pi^3 \cdot a^2 + 240 \cdot \pi \cdot a^4 - 16 \cdot \pi \cdot (a \cdot b^3 \cdot n^3 + 3 \cdot a \cdot b^3 \cdot n^2 + 2 \cdot a \cdot b^3 \cdot n)) \cdot x^3 - 6 \cdot (\pi^3 \cdot (b^2 \cdot n^2 + b^2 \cdot n) - 12 \cdot \pi \cdot (a^2 \cdot b^2 \cdot n^2 + a^2 \cdot b^2 \cdot n)) \cdot x^2 + 48 \cdot (\pi^3 \cdot a \cdot b \cdot n - 4 \cdot \pi \cdot a^3 \cdot b \cdot n) \cdot x \cdot (b^2 \cdot x^2 + 2 \cdot a \cdot b \cdot x + \frac{1}{4} \cdot \pi^2 + a^2)^{\frac{1}{2} \cdot n} \cdot \sin(2 \cdot n \cdot \arctan(\frac{-2 \cdot b \cdot x}{\pi - 2 \cdot a/\pi + \sqrt{4 \cdot b^2 \cdot x^2 + 8 \cdot a \cdot b \cdot x + \pi^2 + 4 \cdot a^2}}{\pi}))) / (b^5 \cdot n^5 + 15 \cdot b^5 \cdot n^4 + 85 \cdot b^5 \cdot n^3 + 225 \cdot b^5 \cdot n^2 + 274 \cdot b^5 \cdot n + 120 \cdot b^5)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(tanh(b\*x+a))^n,x, algorithm="giac")

[Out] integrate(x^4\*arccoth(tanh(b\*x + a))^n, x)

**maple** [B] time = 33.06, size = 504228, normalized size = 3055.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*arccoth(tanh(b\*x+a))^n,x)

[Out] result too large to display

**maxima** [C] time = 0.54, size = 380, normalized size = 2.30

---


$$(4(n^4 + 10n^3 + 35n^2 + 50n + 24)b^5x^5 - 3i\pi^5 + 30\pi^4a + 120i\pi^3a^2 - 240\pi^2a^3 - 240i\pi a^4 + 96a^5 - 2(i\pi(n^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

```
[Out] (4*(n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 - 3*I*pi^5 + 30*pi^4*a + 120
*I*pi^3*a^2 - 240*pi^2*a^3 - 240*I*pi*a^4 + 96*a^5 - 2*(I*pi*(n^4 + 6*n^3 +
11*n^2 + 6*n)*b^4 - 2*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4)*x^4 + (4*pi^2*(n
^3 + 3*n^2 + 2*n)*b^3 + 16*I*pi*(n^3 + 3*n^2 + 2*n)*a*b^3 - 16*(n^3 + 3*n^2
+ 2*n)*a^2*b^3)*x^3 + (6*I*pi^3*(n^2 + n)*b^2 - 36*pi^2*(n^2 + n)*a*b^2 -
72*I*pi*(n^2 + n)*a^2*b^2 + 48*(n^2 + n)*a^3*b^2)*x^2 - (6*pi^4*b*n + 48*I*
pi^3*a*b*n - 144*pi^2*a^2*b*n - 192*I*pi*a^3*b*n + 96*a^4*b*n)*x*(cosh(-n*
log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x + 2*a)))/((2^(n + 2)*
n^5 + 15*2^(n + 2)*n^4 + 85*2^(n + 2)*n^3 + 225*2^(n + 2)*n^2 + 137*2^(n +
3)*n + 15*2^(n + 5))*b^5)
```

**mupad [B]** time = 2.19, size = 546, normalized size = 3.31

$$-\left(\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right)}{2} - \frac{\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right)}{2}\right)^n \left(\frac{3\left(\ln\left(-\frac{2}{e^{2a}e^{2bx}-1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}-1}\right) + 2bx\right)^5}{4b^5(n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)} - \frac{x^5(n^4 + 10n^3 + 35n^2 + 50n + 24)}{n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*acoth(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))/2 - log(-2/(exp(2*
a)*exp(2*b*x) - 1))/2)^n*((3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^5)/(4*b^5*(274*n + 2
25*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (x^5*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (3*n*x*(log(-2/(e
xp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
- 1)) + 2*b*x)^4)/(2*b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
+ (n*x^4*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)*(11*n + 6*n^2 + n^3 + 6))/(2*b*(274*n +
225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (3*n*x^2*(n + 1)*(log(-2/(exp(2*a
)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1))
+ 2*b*x)^3)/(2*b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (n*x
^3*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) - 1)) + 2*b*x)^2*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*
n^3 + 15*n^4 + n^5 + 120)))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**5*acoth(tanh(a))**n/5, Eq(b, 0)), (-x**4/(4*b*acoth(tanh(a +
b*x))**4) - x**3/(3*b**2*acoth(tanh(a + b*x))**3) - x**2/(2*b**3*acoth(tanh
(a + b*x))**2) - x/(b**4*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/
b**5, Eq(n, -5)), (Integral(x**4/acoth(tanh(a + b*x))**4, x), Eq(n, -4)), (
Integral(x**4/acoth(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**4/acoth
(tanh(a + b*x))**2, x), Eq(n, -2)), (Integral(x**4/acoth(tanh(a + b*x)), x)
, Eq(n, -1)), (b**4*n**4*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120
*b**5) + 14*b**4*n**3*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b*
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b*
**5) + 71*b**4*n**2*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**5*
n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
+ 154*b**4*n*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12
0*b**4*x**4*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b*
**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*b**3*n*
**3*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 48*b**3*n*
**2*x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 188*b**3*n
**x**3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 240*b**3*x**
3*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*b**2*n**2*x**
2*acoth(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 108*b**2*n*x**2*
acoth(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 +
85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 240*b**2*x**2*acot
h(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*b*n*x*acoth(tanh(a
+ b*x))**4*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
+ 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 120*b*x*acoth(tanh(a + b*x))**4
*acoth(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**
5*n**2 + 274*b**5*n + 120*b**5) + 24*acoth(tanh(a + b*x))**5*acoth(tanh(a +
b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b*
**5*n + 120*b**5), True))
```

### 3.186 $\int x^3 \coth^{-1}(\tanh(a + bx))^n dx$

**Optimal.** Leaf size=121

$$-\frac{6 \coth^{-1}(\tanh(a + bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{6x \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{x^3 \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

[Out]  $x^3 \operatorname{arccoth}(\tanh(bx+a))^{(1+n)}/b/(1+n) - 3x^2 \operatorname{arccoth}(\tanh(bx+a))^{(2+n)}/b^2/(1+n)/(2+n) + 6x \operatorname{arccoth}(\tanh(bx+a))^{(3+n)}/b^3/(3+n)/(n^2+3n+2) - 6 \operatorname{arccoth}(\tanh(bx+a))^{(4+n)}/b^4/(n^2+5n+4)/(n^2+5n+6)$

**Rubi [A]** time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 30}

$$-\frac{3x^2 \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{6x \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{6 \coth^{-1}(\tanh(a + bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)} + \frac{x^3 \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out]  $(x^3 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(1+n)})/(b*(1+n)) - (3*x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(2+n)})/(b^2*(1+n)*(2+n)) + (6*x \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(3+n)})/(b^3*(1+n)*(2+n)*(3+n)) - (6 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(4+n)})/(b^4*(1+n)*(2+n)*(3+n)*(4+n))$

#### Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2157

Int[(u\_)^(m\_), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b^n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2\*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[m]))

[m, 0] && !IntegerQ[n]))

### Rubi steps

$$\begin{aligned}
 \int x^3 \coth^{-1}(\tanh(a + bx))^n dx &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3 \int x^2 \coth^{-1}(\tanh(a + bx))^{1+n} dx}{b(1+n)} \\
 &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6 \int x \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^2(1+n)(2+n)} \\
 &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)} - \frac{6 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^3(1+n)(2+n)} \\
 &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)} - \frac{6 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^3(1+n)(2+n)} \\
 &= \frac{x^3 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{3x^2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)} - \frac{6 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^3(1+n)(2+n)}
 \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 106, normalized size = 0.88

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1} \left( -3b^2 (n^2 + 7n + 12) x^2 \coth^{-1}(\tanh(a + bx)) + 6b(n + 4)x \coth^{-1}(\tanh(a + bx))^2 - 6 \right)}{b^4(n + 1)(n + 2)(n + 3)(n + 4)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] (ArcCoth[Tanh[a + b\*x]]^(1 + n)\*(b^3\*(24 + 26\*n + 9\*n^2 + n^3)\*x^3 - 3\*b^2\*(12 + 7\*n + n^2)\*x^2\*ArcCoth[Tanh[a + b\*x]] + 6\*b\*(4 + n)\*x\*ArcCoth[Tanh[a + b\*x]]^2 - 6\*ArcCoth[Tanh[a + b\*x]]^3))/(b^4\*(1 + n)\*(2 + n)\*(3 + n)\*(4 + n))

**fricas [B]** time = 1.48, size = 411, normalized size = 3.40

$$\frac{(8(b^4 n^3 + 6b^4 n^2 + 11b^4 n + 6b^4)x^4 - 3\pi^4 + 72\pi^2 a^2 - 48a^4 + 8(ab^3 n^3 + 3ab^3 n^2 + 2ab^3 n)x^3 - 6(4a^2 b^2 n^2 + \dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(tanh(b\*x+a))^n,x, algorithm="fricas")

```
[Out] 1/8*((8*(b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 3*pi^4 + 72*pi^2*a^2
- 48*a^4 + 8*(a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 6*(4*a^2*b^2*n^2
+ 4*a^2*b^2*n - pi^2*(b^2*n^2 + b^2*n))*x^2 - 12*(3*pi^2*a*b*n - 4*a^3*b*n)
*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 + a^2)^(1/2*n)*cos(2*n*arctan(-2*b*x/pi -
2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x + pi^2 + 4*a^2)/pi)) - 2*(2*pi*(b^3*n^3
+ 3*b^3*n^2 + 2*b^3*n)*x^3 + 12*pi^3*a - 48*pi*a^3 - 12*pi*(a*b^2*n^2 + a*b
^2*n)*x^2 - 3*(pi^3*b*n - 12*pi*a^2*b*n)*x)*(b^2*x^2 + 2*a*b*x + 1/4*pi^2 +
a^2)^(1/2*n)*sin(2*n*arctan(-2*b*x/pi - 2*a/pi + sqrt(4*b^2*x^2 + 8*a*b*x
+ pi^2 + 4*a^2)/pi)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^
4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate(x^3*arccoth(tanh(b*x + a))^n, x)
```

**maple** [C] time = 18.41, size = 129477, normalized size = 1070.06

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccoth(tanh(b*x+a))^n,x)
```

```
[Out] result too large to display
```

**maxima** [C] time = 0.53, size = 255, normalized size = 2.11

$$\frac{(8(n^3 + 6n^2 + 11n + 6)b^4x^4 - 3\pi^4 - 24i\pi^3a + 72\pi^2a^2 + 96i\pi a^3 - 48a^4 + (-4i\pi(n^3 + 3n^2 + 2n)b^3 + 8(n^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccoth(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (8*(n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 - 3*pi^4 - 24*I*pi^3*a + 72*pi^2*a^2 +
96*I*pi*a^3 - 48*a^4 + (-4*I*pi*(n^3 + 3*n^2 + 2*n)*b^3 + 8*(n^3 + 3*n^2 +
2*n)*a*b^3)*x^3 + (6*pi^2*(n^2 + n)*b^2 + 24*I*pi*(n^2 + n)*a*b^2 - 24*(n^2
+ n)*a^2*b^2)*x^2 + (6*I*pi^3*b*n - 36*pi^2*a*b*n - 72*I*pi*a^2*b*n + 48*a
^3*b*n)*x)*(cosh(-n*log(-I*pi + 2*b*x + 2*a)) - sinh(-n*log(-I*pi + 2*b*x +
```

$2*a)))/((2^{n+3}*n^4 + 5*2^{n+4}*n^3 + 35*2^{n+3}*n^2 + 25*2^{n+4}*n + 3*2^{n+6})*b^4)$

**mupad** [B] time = 1.50, size = 418, normalized size = 3.45

$$-\left(\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right)}{2} - \frac{\ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right)}{2}\right)^n \left(\frac{3\left(\ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx\right)^4}{8b^4(n^4 + 10n^3 + 35n^2 + 50n + 24)} - \frac{x^4(n^3 + 6n^2 + 11n + 6)}{n^4 + 10n^3 + 35n^2 + 50n + 6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*acoth(tanh(a + b*x))^n,x)`

[Out]  $-\left(\log\left(\frac{2\exp(2a)\exp(2bx)}{\exp(2a)\exp(2bx)-1}\right)/2 - \log\left(-\frac{2}{\exp(2a)\exp(2bx)-1}\right)/2\right)^n \left(\frac{3\left(\log\left(-\frac{2}{\exp(2a)\exp(2bx)-1}\right) - \log\left(\frac{2\exp(2a)\exp(2bx)}{\exp(2a)\exp(2bx)-1}\right) + 2bx\right)^4}{8b^4(50n+35n^2+10n^3+n^4+24)} - \frac{x^4(11n+6n^2+n^3+6)}{(50n+35n^2+10n^3+n^4+24)} + \frac{3n\left(\log\left(-\frac{2}{\exp(2a)\exp(2bx)-1}\right) - \log\left(\frac{2\exp(2a)\exp(2bx)}{\exp(2a)\exp(2bx)-1}\right) + 2bx\right)^3}{4b^3(50n+35n^2+10n^3+n^4+24)} + \frac{n^3\left(\log\left(-\frac{2}{\exp(2a)\exp(2bx)-1}\right) - \log\left(\frac{2\exp(2a)\exp(2bx)}{\exp(2a)\exp(2bx)-1}\right) + 2bx\right)^2}{2b(50n+35n^2+10n^3+n^4+24)} + \frac{3n^2\left(\log\left(-\frac{2}{\exp(2a)\exp(2bx)-1}\right) - \log\left(\frac{2\exp(2a)\exp(2bx)}{\exp(2a)\exp(2bx)-1}\right) + 2bx\right)}{4b^2(50n+35n^2+10n^3+n^4+24)}\right)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(tanh(b*x+a))**n,x)`

[Out]  $\text{Piecewise}\left(\left(x^4\text{acoth}(\tanh(a))\right)^n/4, \text{Eq}(b, 0)\right), \left(-x^3/(3b\text{acoth}(\tanh(a + bx)))^3 - x^2/(2b^2\text{acoth}(\tanh(a + bx)))^2 - x/(b^3\text{acoth}(\tanh(a + bx))) + \log(\text{acoth}(\tanh(a + bx)))/b^4, \text{Eq}(n, -4)\right), \left(\text{Integral}(x^3/\text{acoth}(\tanh(a + bx))^3, x), \text{Eq}(n, -3)\right), \left(\text{Integral}(x^3/\text{acoth}(\tanh(a + bx))^2, x), \text{Eq}(n, -2)\right), \left(\text{Integral}(x^3/\text{acoth}(\tanh(a + bx)), x), \text{Eq}(n, -1)\right), \left(b^3n^3x^3\text{acoth}(\tanh(a + bx))\text{acoth}(\tanh(a + bx))\right)^n/(b^4n^4 + 10b^4n^3 + 35b^4n^2 + 50b^4n + 24b^4) + 9b^3n^2x^3\text{acoth}(\tanh(a + bx))\text{acoth}(\tanh(a + bx))\right)^n/(b^4n^4 + 10b^4n^3 + 35b^4n^2 + 50b^4n + 24b^4) + 26b^3nx^3\text{acoth}(\tanh(a + bx))\text{acoth}(\tanh(a + bx))\right)^n/(b^4n^4 + 10b^4n^3 + 35b^4n^2 + 50b^4n + 24b^4) + 24b^3x^3\text{acoth}(\tanh(a + bx))\text{acoth}(\tanh(a + bx))\right)^n/(b^4n^4 + 10b^4n^3 + 35b^4n^2 + 50b^4n + 24b^4) - 3b^2n^2x^2\text{acoth}(\tanh(a + bx))\text{acoth}(\tanh(a + bx))\right)^n/(b^4n^4 + 10b^4n^3 + 35b^4n^2 + 50b^4n + 24b^4)$

```

(a + b*x)**2*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) - 21*b**2*n*x**2*acoth(tanh(a + b*x))**2*acoth(t
anh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 36*b**2*x**2*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b*n*x*acoth(t
anh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**
4*n**2 + 50*b**4*n + 24*b**4) + 24*b*x*acoth(tanh(a + b*x))**3*acoth(tanh(a
+ b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
- 6*acoth(tanh(a + b*x))**4*acoth(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n
**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))

```



### 3.187 $\int x^2 \coth^{-1}(\tanh(a + bx))^n dx$

Optimal. Leaf size=82

$$\frac{2 \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} - \frac{2x \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{x^2 \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

[Out]  $x^2 \operatorname{arccoth}(\tanh(bx+a))^{(1+n)}/b/(1+n) - 2x \operatorname{arccoth}(\tanh(bx+a))^{(2+n)}/b^2/(1+n)/(2+n) + 2 \operatorname{arccoth}(\tanh(bx+a))^{(3+n)}/b^3/(3+n)/(n^2+3n+2)$

**Rubi [A]** time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2168, 2157, 30}

$$-\frac{2x \coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)} + \frac{2 \coth^{-1}(\tanh(a + bx))^{n+3}}{b^3(n+1)(n+2)(n+3)} + \frac{x^2 \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] `Int[x^2*ArcCoth[Tanh[a + b*x]]^n,x]`

[Out]  $(x^2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(1+n)})/(b*(1+n)) - (2*x \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(2+n)})/(b^2*(1+n)*(2+n)) + (2 \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(3+n)})/(b^3*(1+n)*(2+n)*(3+n))$

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2157

`Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)*v^n)/(a*(m+1)), x] - Dist[(b*n)/(a*(m+1)), Int[u^(m+1)*v^(n-1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(\tanh(a + bx))^n dx &= \frac{x^2 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{2 \int x \coth^{-1}(\tanh(a + bx))^{1+n} dx}{b(1+n)} \\
&= \frac{x^2 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{2x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \int \coth^{-1}(\tanh(a + bx))^{2+n} dx}{b^2(1+n)} \\
&= \frac{x^2 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{2x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \operatorname{Subst}\left(\int x^{2+n} \coth^{-1}(\tanh(a + bx))^{2+n} dx\right)}{b^3(1+n)} \\
&= \frac{x^2 \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{2x \coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{2 \coth^{-1}(\tanh(a + bx))^{2+n}}{b^3(1+n)(2+n)}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 71, normalized size = 0.87

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1} \left(-2b(n+3)x \coth^{-1}(\tanh(a + bx)) + 2 \coth^{-1}(\tanh(a + bx))^2 + b^2(n^2 + 5n + 6)x^2\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] (ArcCoth[Tanh[a + b\*x]]^(1 + n)\*(b^2\*(6 + 5\*n + n^2)\*x^2 - 2\*b\*(3 + n)\*x\*ArcCoth[Tanh[a + b\*x]] + 2\*ArcCoth[Tanh[a + b\*x]]^2))/(b^3\*(1 + n)\*(2 + n)\*(3 + n))

**fricas [B]** time = 0.46, size = 287, normalized size = 3.50

$$\frac{2 \left( 2 \left( b^3 n^2 + 3 b^3 n + 2 b^3 \right) x^3 - 3 \pi^2 a + 4 a^3 + 2 \left( a b^2 n^2 + a b^2 n \right) x^2 + \left( \pi^2 b n - 4 a^2 b n \right) x \right) \left( b^2 x^2 + 2 a b x + \frac{1}{4} \pi^2 + a^2 \right)}{b^3(n+1)(n+2)(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^n,x, algorithm="fricas")

[Out] 1/4\*(2\*(2\*(b^3\*n^2 + 3\*b^3\*n + 2\*b^3)\*x^3 - 3\*pi^2\*a + 4\*a^3 + 2\*(a\*b^2\*n^2 + a\*b^2\*n)\*x^2 + (pi^2\*b\*n - 4\*a^2\*b\*n)\*x)\*(b^2\*x^2 + 2\*a\*b\*x + 1/4\*pi^2 + a^2)^(1/2\*n)\*cos(2\*n\*arctan(-2\*b\*x/pi - 2\*a/pi + sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/pi)) + (8\*pi\*a\*b\*n\*x - 2\*pi\*(b^2\*n^2 + b^2\*n)\*x^2 + pi^3 - 12\*pi\*a^2)\*(b^2\*x^2 + 2\*a\*b\*x + 1/4\*pi^2 + a^2)^(1/2\*n)\*sin(2\*n\*arctan(-2\*b\*x/pi - 2\*a/pi + sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/pi)))/(b^3\*n^3 + 6\*b^3\*n^2 + 11\*b^3\*n + 6\*b^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^n,x, algorithm="giac")

[Out] integrate(x^2\*arccoth(tanh(b\*x + a))^n, x)

**maple** [C] time = 14.73, size = 25561, normalized size = 311.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(tanh(b\*x+a))^n,x)

[Out] result too large to display

**maxima** [C] time = 0.54, size = 166, normalized size = 2.02

$$\frac{(4(n^2 + 3n + 2)b^3x^3 + i\pi^3 - 6\pi^2a - 12i\pi a^2 + 8a^3 - 2(i\pi(n^2 + n)b^2 - 2(n^2 + n)ab^2)x^2 + (2\pi^2bn + 8i\pi abn - 8a^2bn)x + (2\pi^2bn + 8i\pi abn - 8a^2bn)x)}{(2^{n+2}n^3 + 3 \cdot 2^{n+3}n^2 + 11 \cdot 2^{n+2}n + 3 \cdot 2^{n+3})b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

[Out] (4\*(n^2 + 3\*n + 2)\*b^3\*x^3 + I\*pi^3 - 6\*pi^2\*a - 12\*I\*pi\*a^2 + 8\*a^3 - 2\*(I\*pi\*(n^2 + n)\*b^2 - 2\*(n^2 + n)\*a\*b^2)\*x^2 + (2\*pi^2\*b\*n + 8\*I\*pi\*a\*b\*n - 8\*a^2\*b\*n)\*x\*(cosh(-n\*log(-I\*pi + 2\*b\*x + 2\*a)) - sinh(-n\*log(-I\*pi + 2\*b\*x + 2\*a)))/((2^(n + 2)\*n^3 + 3\*2^(n + 3)\*n^2 + 11\*2^(n + 2)\*n + 3\*2^(n + 3))\*b^3)

**mupad** [B] time = 1.36, size = 304, normalized size = 3.71

$$-\left(\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right)}{2} - \frac{\ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right)}{2}\right)^n \left(\frac{\left(\ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx\right)^3}{4b^3(n^3 + 6n^2 + 11n + 6)} - \frac{x^3(n^2 + 3n + 2)}{n^3 + 6n^2 + 11n + 6} + \frac{nx}{n^3 + 6n^2 + 11n + 6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*acoth(tanh(a + b\*x))^n,x)

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)))/2 - log(-2/(exp(2*a)*exp(2*b*x) - 1))/2)^n*((log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^3/(4*b^3*(11*n + 6*n^2 + n^3 + 6)) - (x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (n*x*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x)^2)/(2*b^2*(11*n + 6*n^2 + n^3 + 6)) + (n*x^2*(n + 1)*(log(-2/(exp(2*a)*exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) + 2*b*x))/(2*b*(11*n + 6*n^2 + n^3 + 6)))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left\{ \begin{array}{l} \frac{x^3 \operatorname{acoth}^n(\tanh(a))}{3} \\ - \frac{x^2}{2b \operatorname{acoth}^2(\tanh(a+bx))} - \frac{x}{b^2 \operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^3} \\ \int \frac{x^2}{\operatorname{acoth}^2(\tanh(a+bx))} dx \\ \int \frac{x^2}{\operatorname{acoth}(\tanh(a+bx))} dx \\ \frac{b^2 n^2 x^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^3 n^3 + 6b^3 n^2 + 11b^3 n + 6b^3} + \frac{5b^2 n x^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^3 n^3 + 6b^3 n^2 + 11b^3 n + 6b^3} + \frac{6b^2 x^2 \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^3 n^3 + 6b^3 n^2 + 11b^3 n + 6b^3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**3*acoth(tanh(a))**n/3, Eq(b, 0)), (-x**2/(2*b*acoth(tanh(a + b*x))**2) - x/(b**2*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**3, Eq(n, -3)), (Integral(x**2/acoth(tanh(a + b*x))**2, x), Eq(n, -2)), (Integral(x**2/acoth(tanh(a + b*x)), x), Eq(n, -1)), (b**2*n**2*x**2*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 5*b**2*n*x**2*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**2*x**2*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*b*n*x*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 6*b*x*acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*acoth(tanh(a + b*x))**3*acoth(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True))
```

### 3.188 $\int x \coth^{-1}(\tanh(a + bx))^n dx$

Optimal. Leaf size=48

$$\frac{x \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)} - \frac{\coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)}$$

[Out]  $x \operatorname{arccoth}(\tanh(bx+a))^{(1+n)}/b/(1+n) - \operatorname{arccoth}(\tanh(bx+a))^{(2+n)}/b^2/(1+n)/(2+n)$

**Rubi [A]** time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {2168, 2157, 30}

$$\frac{x \coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)} - \frac{\coth^{-1}(\tanh(a + bx))^{n+2}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[Tanh[a + b*x]]^n,x]`

[Out]  $(x \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(1+n)})/(b*(1+n)) - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(2+n)}/(b^2*(1+n)*(2+n))$

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2157

`Int[(u_)^(m_), x_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)*v^n)/(a*(m+1)), x] - Dist[(b*n)/(a*(m+1)), Int[u^(m+1)*v^(n-1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(\tanh(a + bx))^n dx &= \frac{x \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{\int \coth^{-1}(\tanh(a + bx))^{1+n} dx}{b(1+n)} \\
&= \frac{x \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{\text{Subst}\left(\int x^{1+n} dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b^2(1+n)} \\
&= \frac{x \coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} - \frac{\coth^{-1}(\tanh(a + bx))^{2+n}}{b^2(1+n)(2+n)}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 41, normalized size = 0.85

$$\frac{(b(n+2)x - \coth^{-1}(\tanh(a + bx))) \coth^{-1}(\tanh(a + bx))^{n+1}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Tanh[a + b\*x]]^n,x]

[Out] ((b\*(2 + n)\*x - ArcCoth[Tanh[a + b\*x]])\*ArcCoth[Tanh[a + b\*x]]^(1 + n))/(b^2\*(1 + n)\*(2 + n))

**fricas [B]** time = 0.57, size = 210, normalized size = 4.38

$$\frac{(4 abnx + 4(b^2n + b^2)x^2 + \pi^2 - 4a^2)\left(b^2x^2 + 2abx + \frac{1}{4}\pi^2 + a^2\right)^{\frac{1}{2}n} \cos\left(2n \arctan\left(-\frac{2bx}{\pi} - \frac{2a}{\pi} + \frac{\sqrt{4b^2x^2 + 8abx + \pi^2}}{\pi}\right)\right)}{4(b^2n^2 + 3b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^n,x, algorithm="fricas")

[Out] 1/4\*((4\*a\*b\*n\*x + 4\*(b^2\*n + b^2)\*x^2 + pi^2 - 4\*a^2)\*(b^2\*x^2 + 2\*a\*b\*x + 1/4\*pi^2 + a^2)^(1/2\*n)\*cos(2\*n\*arctan(-2\*b\*x/pi - 2\*a/pi + sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/pi)) - 2\*(pi\*b\*n\*x - 2\*pi\*a)\*(b^2\*x^2 + 2\*a\*b\*x + 1/4\*pi^2 + a^2)^(1/2\*n)\*sin(2\*n\*arctan(-2\*b\*x/pi - 2\*a/pi + sqrt(4\*b^2\*x^2 + 8\*a\*b\*x + pi^2 + 4\*a^2)/pi)))/(b^2\*n^2 + 3\*b^2\*n + 2\*b^2)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^n,x, algorithm="giac")

[Out] integrate(x\*arccoth(tanh(b\*x + a))^n, x)

**maple** [C] time = 13.67, size = 480, normalized size = 10.00

$$\frac{\left(\frac{1}{2}\right)^n \left( 2 \ln(e^{bx+a}) - \frac{i\pi \operatorname{csgn}(ie^{2bx+2a})(-\operatorname{csgn}(ie^{2bx+2a}) + \operatorname{csgn}(ie^{bx+a}))^2}{2} - \frac{i\pi \operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a+1}}\right)\left(-\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a+1}}\right) + \operatorname{csgn}(ie^{2bx+2a})\right)\left(-\operatorname{csgn}\right)}{2} \right)}{2b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(tanh(b\*x+a))^n,x)

[Out]  $\frac{1}{2} \frac{1}{b} \left(\frac{1}{2}\right)^n \left( 2 \ln(\exp(b*x+a)) - \frac{1}{2} I \pi \operatorname{csgn}(I \exp(2*b*x+2*a)) * (-\operatorname{csgn}(I \exp(2*b*x+2*a)) + \operatorname{csgn}(I \exp(b*x+a)))^2 - \frac{1}{2} I \pi \operatorname{csgn}(I \exp(2*b*x+2*a)) / (\exp(2*b*x+2*a)+1) * (-\operatorname{csgn}(I \exp(2*b*x+2*a)) / (\exp(2*b*x+2*a)+1)) + \operatorname{csgn}(I \exp(2*b*x+2*a)) \right) * (-\operatorname{csgn}(I \exp(2*b*x+2*a)) / (\exp(2*b*x+2*a)+1)) + \operatorname{csgn}(I / (\exp(2*b*x+2*a)+1)) - I \pi - I \pi \operatorname{csgn}(I / (\exp(2*b*x+2*a)+1))^2 * (\operatorname{csgn}(I / (\exp(2*b*x+2*a)+1)) - 1) \right)^{(1+n)} / (1+n) * x - \frac{1}{4} \frac{1}{b^2} \left(\frac{1}{2}\right)^n / (1+n) * \left( 2 \ln(\exp(b*x+a)) - \frac{1}{2} I \pi \operatorname{csgn}(I \exp(2*b*x+2*a)) * (-\operatorname{csgn}(I \exp(2*b*x+2*a)) + \operatorname{csgn}(I \exp(b*x+a)))^2 - \frac{1}{2} I \pi \operatorname{csgn}(I \exp(2*b*x+2*a)) / (\exp(2*b*x+2*a)+1) * (-\operatorname{csgn}(I \exp(2*b*x+2*a)) / (\exp(2*b*x+2*a)+1)) + \operatorname{csgn}(I \exp(2*b*x+2*a)) \right) * (-\operatorname{csgn}(I \exp(2*b*x+2*a)) / (\exp(2*b*x+2*a)+1)) + \operatorname{csgn}(I / (\exp(2*b*x+2*a)+1)) - I \pi - I \pi \operatorname{csgn}(I / (\exp(2*b*x+2*a)+1))^2 * (\operatorname{csgn}(I / (\exp(2*b*x+2*a)+1)) - 1) \right)^{(2+n)} / (2+n)$

**maxima** [C] time = 0.54, size = 101, normalized size = 2.10

$$\frac{(4b^2(n+1)x^2 + \pi^2 + 4i\pi a - 4a^2 + (-2i\pi bn + 4abn)x) \left( \cosh(-n \log(-i\pi + 2bx + 2a)) - \sinh(-n \log(-i\pi + 2bx + 2a)) \right)}{(2^{n+2}n^2 + 3 \cdot 2^{n+2}n + 2^{n+3})b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

[Out]  $(4*b^2*(n+1)*x^2 + \pi^2 + 4*I*\pi*a - 4*a^2 + (-2*I*\pi*b*n + 4*a*b*n)*x) * (\cosh(-n*\log(-I*\pi + 2*b*x + 2*a)) - \sinh(-n*\log(-I*\pi + 2*b*x + 2*a))) / ((2^{(n+2)}*n^2 + 3*2^{(n+2)}*n + 2^{(n+3)})*b^2)$

**mupad** [B] time = 1.31, size = 205, normalized size = 4.27

$$\frac{\left( \frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right)}{2} - \frac{\ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right)}{2} \right)^n \left( \frac{\left( \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx \right)^2}{4b^2(n^2 + 3n + 2)} - \frac{x^2(n+1)}{n^2 + 3n + 2} + \frac{nx \left( \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) + 2bx \right)}{n^2 + 3n + 2} \right)}{4b^2(n^2 + 3n + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(tanh(a + b*x))^n,x)`

[Out]  $-(\log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)))/2 - \log(-2/(\exp(2*a)*\exp(2*b*x) - 1))/2)^n * ((\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x)^2 / (4*b^2*(3*n + n^2 + 2)) - (x^2*(n + 1))/(3*n + n^2 + 2) + (n*x*(\log(-2/(\exp(2*a)*\exp(2*b*x) - 1)) - \log((2*\exp(2*a)*\exp(2*b*x))/(\exp(2*a)*\exp(2*b*x) - 1)) + 2*b*x))/(2*b*(3*n + n^2 + 2)))$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left\{ \begin{array}{l} \frac{x^2 \operatorname{acoth}^n(\tanh(a))}{2} \\ -\frac{x}{b \operatorname{acoth}(\tanh(a+bx))} + \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b^2} \\ \int \frac{x}{\operatorname{acoth}(\tanh(a+bx))} dx \\ \frac{bnx \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^2 n^2 + 3b^2 n + 2b^2} + \frac{2bx \operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^2 n^2 + 3b^2 n + 2b^2} - \frac{\operatorname{acoth}^2(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{b^2 n^2 + 3b^2 n + 2b^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(tanh(b*x+a))**n,x)`

[Out] `Piecewise((x**2*acoth(tanh(a))**n/2, Eq(b, 0)), (-x/(b*acoth(tanh(a + b*x))) + log(acoth(tanh(a + b*x)))/b**2, Eq(n, -2)), (Integral(x/acoth(tanh(a + b*x)), x), Eq(n, -1)), (b*n*x*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + 2*b*x*acoth(tanh(a + b*x))*acoth(tanh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2) - acoth(tanh(a + b*x))**2*acoth(tanh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True))`



$$3.189 \quad \int \coth^{-1}(\tanh(a + bx))^n dx$$

Optimal. Leaf size=20

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

[Out] arccoth(tanh(b\*x+a))^(1+n)/b/(1+n)

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^n, x]

[Out] ArcCoth[Tanh[a + b\*x]]^(1 + n)/(b\*(1 + n))

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2157

Int[(u\_)^(m\_), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

Rubi steps

$$\begin{aligned} \int \coth^{-1}(\tanh(a + bx))^n dx &= \frac{\text{Subst}\left(\int x^n dx, x, \coth^{-1}(\tanh(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\tanh(a + bx))^{1+n}}{b(1+n)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{\coth^{-1}(\tanh(a + bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^n, x]

[Out] ArcCoth[Tanh[a + b\*x]]^(1 + n)/(b\*(1 + n))

**fricas** [B] time = 0.57, size = 164, normalized size = 8.20

$$\frac{2(bx + a)\left(b^2x^2 + 2abx + \frac{1}{4}\pi^2 + a^2\right)^{\frac{1}{2}n} \cos\left(2n \arctan\left(-\frac{2bx}{\pi} - \frac{2a}{\pi} + \frac{\sqrt{4b^2x^2 + 8abx + \pi^2 + 4a^2}}{\pi}\right)\right) - \pi\left(b^2x^2 + 2abx + \frac{1}{4}\pi^2 + a^2\right)^{\frac{1}{2}n}}{2(bn + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n, x, algorithm="fricas")

[Out]  $\frac{1}{2} * (2 * (b * x + a) * (b^2 * x^2 + 2 * a * b * x + \frac{1}{4} * \pi^2 + a^2)^{(1/2 * n)} * \cos(2 * n * \arctan(-2 * b * x / \pi - 2 * a / \pi + \sqrt{4 * b^2 * x^2 + 8 * a * b * x + \pi^2 + 4 * a^2} / \pi)) - \pi * (b^2 * x^2 + 2 * a * b * x + \frac{1}{4} * \pi^2 + a^2)^{(1/2 * n)} * \sin(2 * n * \arctan(-2 * b * x / \pi - 2 * a / \pi + \sqrt{4 * b^2 * x^2 + 8 * a * b * x + \pi^2 + 4 * a^2} / \pi))) / (b * n + b)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\tanh(bx + a))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n, x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^n, x)

**maple** [A] time = 0.08, size = 21, normalized size = 1.05

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^{1+n}}{b(1 + n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^n, x)

[Out] arccoth(tanh(b\*x+a))^(1+n)/b/(1+n)

**maxima** [C] time = 0.53, size = 65, normalized size = 3.25

$$\frac{(-i\pi + 2bx + 2a)\left(\cosh(-n \log(-i\pi + 2bx + 2a)) - \sinh(-n \log(-i\pi + 2bx + 2a))\right)}{(2^{n+1}n + 2^{n+1})b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n,x, algorithm="maxima")

[Out]  $(-I\pi + 2bx + 2a) \cdot (\cosh(-n \log(-I\pi + 2bx + 2a)) - \sinh(-n \log(-I\pi + 2bx + 2a))) / ((2^{n+1})^n + 2^{n+1}) \cdot b$

mupad [B] time = 1.27, size = 121, normalized size = 6.05

$$\left(\frac{1}{2}\right)^n \left( \frac{x}{n+1} - \frac{\ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) - \frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right)}{2} + bx}{b(n+1)} \right) \left( \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx-1}}\right) - \ln\left(-\frac{2}{e^{2a}e^{2bx-1}}\right) \right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))^n,x)

[Out]  $(1/2)^n \cdot (x/(n+1) - (\log(-2/(\exp(2a)\exp(2bx) - 1))/2 - \log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx) - 1))/2 + bx)/(b(n+1))) \cdot (\log((2\exp(2a)\exp(2bx))/(\exp(2a)\exp(2bx) - 1)) - \log(-2/(\exp(2a)\exp(2bx) - 1)))^n$

sympy [A] time = 0.74, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x}{\operatorname{acoth}(\tanh(a))} & \text{for } b = 0 \wedge n = -1 \\ x \operatorname{acoth}^n(\tanh(a)) & \text{for } b = 0 \\ \frac{\log(\operatorname{acoth}(\tanh(a+bx)))}{b} & \text{for } n = -1 \\ \frac{\operatorname{acoth}(\tanh(a+bx)) \operatorname{acoth}^n(\tanh(a+bx))}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*n,x)

[Out] Piecewise((x/acoth(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x\*acoth(tanh(a))\*\*n, Eq(b, 0)), (log(acoth(tanh(a + b\*x)))/b, Eq(n, -1)), (acoth(tanh(a + b\*x))\*\*n, Eq(n, -1)), (acoth(tanh(a + b\*x))\*\*n/(b\*n + b), True))

$$3.190 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} dx$$

Optimal. Leaf size=64

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1} {}_2F_1\left(1, n+1; n+2; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(n+1)(bx - \coth^{-1}(\tanh(a+bx)))}$$

[Out] arccoth(tanh(b\*x+a))^(1+n)\*hypergeom([1, 1+n], [2+n], -arccoth(tanh(b\*x+a))/(b\*x-arccoth(tanh(b\*x+a))))/(1+n)/(b\*x-arccoth(tanh(b\*x+a)))

**Rubi [A]** time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {2164}

$$\frac{\coth^{-1}(\tanh(a+bx))^{n+1} {}_2F_1\left(1, n+1; n+2; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(n+1)(bx - \coth^{-1}(\tanh(a+bx)))}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^n/x, x]

[Out] (ArcCoth[Tanh[a + b\*x]]^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcCot h[Tanh[a + b\*x]]/(b\*x - ArcCoth[Tanh[a + b\*x]])))]/((1 + n)\*(b\*x - ArcCoth[Tanh[a + b\*x]]))

### Rule 2164

Int[(v\_)^(n\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(v^(n + 1)\*Hypergeometric2F1[1, n + 1, n + 2, -((a\*v)/(b\*u - a\*v)))]/((n + 1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[n]

### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a+bx))^n}{x} dx = \frac{\coth^{-1}(\tanh(a+bx))^{1+n} {}_2F_1\left(1, 1+n; 2+n; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{(1+n)(bx - \coth^{-1}(\tanh(a+bx)))}$$

**Mathematica** [A] time = 0.09, size = 60, normalized size = 0.94

$$\frac{\coth^{-1}(\tanh(a + bx))^n \left( \frac{\coth^{-1}(\tanh(a+bx))}{bx} \right)^{-n} {}_2F_1 \left( -n, -n; 1 - n; 1 - \frac{\coth^{-1}(\tanh(a+bx))}{bx} \right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^n/x, x]

[Out] (ArcCoth[Tanh[a + b\*x]]^n\*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcCoth[Tanh[a + b\*x]]/(b\*x)]/(b\*x))/(n\*(ArcCoth[Tanh[a + b\*x]]/(b\*x))^n)

**fricas** [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x, x, algorithm="fricas")

[Out] integral(arccoth(tanh(b\*x + a))^n/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x, x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x, x)

**maple** [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^n/x, x)

[Out] int(arccoth(tanh(b\*x+a))^n/x, x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x,x, algorithm="maxima")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{arccoth}(\tanh(a + bx))^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))^n/x,x)

[Out] int(acoth(tanh(a + b\*x))^n/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^n(\tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*n/x,x)

[Out] Integral(acoth(tanh(a + b\*x))\*\*n/x, x)

$$3.191 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^2} dx$$

**Optimal.** Leaf size=71

$$\frac{b \coth^{-1}(\tanh(a+bx))^n {}_2F_1\left(1, n; n+1; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

[Out]  $-\operatorname{arccoth}(\tanh(b*x+a))^n/x + b*\operatorname{arccoth}(\tanh(b*x+a))^n*\operatorname{hypergeom}([1, n], [1+n], -\operatorname{arccoth}(\tanh(b*x+a))/(b*x - \operatorname{arccoth}(\tanh(b*x+a))))/(b*x - \operatorname{arccoth}(\tanh(b*x+a)))$

**Rubi [A]** time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 2164}

$$\frac{b \coth^{-1}(\tanh(a+bx))^n {}_2F_1\left(1, n; n+1; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{bx - \coth^{-1}(\tanh(a+bx))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Tanh[a + b*x]]^n/x^2, x]`

[Out]  $-(\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^n/x) + (b*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^n*\operatorname{Hypergeometric2F1}[1, n, 1 + n, -(\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])))]/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])$

#### Rule 2164

`Int[(v_)^(n_)/(u_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u - a*v))])]/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[n]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps

$$\int \frac{\coth^{-1}(\tanh(a + bx))^n}{x^2} dx = -\frac{\coth^{-1}(\tanh(a + bx))^n}{x} + (bn) \int \frac{\coth^{-1}(\tanh(a + bx))^{-1+n}}{x} dx$$

$$= -\frac{\coth^{-1}(\tanh(a + bx))^n}{x} + \frac{b \coth^{-1}(\tanh(a + bx))^n {}_2F_1\left(1, n; 1 + n; -\frac{\coth^{-1}(\tanh(a + bx))}{bx - \coth^{-1}(\tanh(a + bx))}\right)}{bx - \coth^{-1}(\tanh(a + bx))}$$

**Mathematica** [A] time = 0.04, size = 67, normalized size = 0.94

$$\frac{\coth^{-1}(\tanh(a + bx))^n \left(\frac{\coth^{-1}(\tanh(a + bx))}{bx}\right)^{-n} {}_2F_1\left(1 - n, -n; 2 - n; 1 - \frac{\coth^{-1}(\tanh(a + bx))}{bx}\right)}{(n - 1)x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^n/x^2,x]

[Out] (ArcCoth[Tanh[a + b\*x]]^n\*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcCoth[Tanh[a + b\*x]]/(b\*x)]/((-1 + n)\*x\*(ArcCoth[Tanh[a + b\*x]]/(b\*x))^n)

**fricas** [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(\tanh(bx + a))^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^2,x, algorithm="fricas")

[Out] integral(arccoth(tanh(b\*x + a))^n/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(\tanh(bx + a))^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^2,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x^2, x)

**maple** [F] time = 13.99, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(\tanh(bx + a))^n}{x^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(tanh(b*x+a))^n/x^2,x)`

[Out] `int(arccoth(tanh(b*x+a))^n/x^2,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx+a))^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(tanh(b*x+a))^n/x^2,x, algorithm="maxima")`

[Out] `integrate(arccoth(tanh(b*x + a))^n/x^2, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(\tanh(a+bx))^n}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tanh(a + b*x))^n/x^2,x)`

[Out] `int(acoth(tanh(a + b*x))^n/x^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^n(\tanh(a+bx))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(tanh(b*x+a))**n/x**2,x)`

[Out] `Integral(acoth(tanh(a + b*x))**n/x**2, x)`

$$3.192 \quad \int \frac{\coth^{-1}(\tanh(a+bx))^n}{x^3} dx$$

**Optimal.** Leaf size=101

$$\frac{b^2 n \coth^{-1}(\tanh(a+bx))^{n-1} {}_2F_1\left(1, n-1; n; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{2(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{bn \coth^{-1}(\tanh(a+bx))}{2x}$$

[Out]  $-1/2*b*n*\operatorname{arccoth}(\tanh(b*x+a))^{(-1+n)}/x-1/2*\operatorname{arccoth}(\tanh(b*x+a))^n/x^2+1/2*b^2*n*\operatorname{arccoth}(\tanh(b*x+a))^{(-1+n)}*\operatorname{hypergeom}([1, -1+n], [n], -\operatorname{arccoth}(\tanh(b*x+a)))/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))/(b*x-\operatorname{arccoth}(\tanh(b*x+a)))$

**Rubi [A]** time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2168, 2164}

$$\frac{b^2 n \coth^{-1}(\tanh(a+bx))^{n-1} {}_2F_1\left(1, n-1; n; -\frac{\coth^{-1}(\tanh(a+bx))}{bx - \coth^{-1}(\tanh(a+bx))}\right)}{2(bx - \coth^{-1}(\tanh(a+bx)))} - \frac{\coth^{-1}(\tanh(a+bx))^n}{2x^2} - \frac{bn \coth^{-1}(\tanh(a+bx))}{2x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tanh[a + b\*x]]^n/x^3, x]

[Out]  $-(b*n*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(-1 + n)})/(2*x) - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^n/(2*x^2) + (b^2*n*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]^{(-1 + n)}*\operatorname{Hypergeometric2F1}[1, -1 + n, n, -(\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]/(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])))]/(2*(b*x - \operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]]))$

#### Rule 2164

Int[(v\_)^(n\_)/(u\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(v^(n+1)\*Hypergeometric2F1[1, n+1, n+2, -((a\*v)/(b\*u - a\*v))])]/((n+1)\*(b\*u - a\*v)), x] /; NeQ[b\*u - a\*v, 0] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[n]

#### Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m+1)\*v^n)/(a\*(m+1)), x] - Dist[(b\*n)/(a\*(m+1)), Int[u^(m+1)\*v^(n-1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m+n, -2] && (FractionQ[m] || GeQ[2\*n+m+1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ

[m, 0] && !IntegerQ[n]))

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\tanh(a + bx))^n}{x^3} dx &= -\frac{\coth^{-1}(\tanh(a + bx))^n}{2x^2} + \frac{1}{2}(bn) \int \frac{\coth^{-1}(\tanh(a + bx))^{-1+n}}{x^2} dx \\ &= -\frac{bn \coth^{-1}(\tanh(a + bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a + bx))^n}{2x^2} - \frac{1}{2}(b^2(1-n)n) \int \frac{\coth^{-1}(\tanh(a + bx))^{-2+n}}{x} dx \\ &= -\frac{bn \coth^{-1}(\tanh(a + bx))^{-1+n}}{2x} - \frac{\coth^{-1}(\tanh(a + bx))^n}{2x^2} + \frac{b^2n \coth^{-1}(\tanh(a + bx))^{-2+n}}{2} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 67, normalized size = 0.66

$$\frac{\coth^{-1}(\tanh(a + bx))^n \left( \frac{\coth^{-1}(\tanh(a + bx))}{bx} \right)^{-n} {}_2F_1 \left( 2 - n, -n; 3 - n; 1 - \frac{\coth^{-1}(\tanh(a + bx))}{bx} \right)}{(n - 2)x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tanh[a + b\*x]]^n/x^3, x]

[Out] (ArcCoth[Tanh[a + b\*x]]^n\*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcCoth[Tanh[a + b\*x]]/(b\*x)]/((-2 + n)\*x^2\*(ArcCoth[Tanh[a + b\*x]]/(b\*x))^n)

**fricas [F]** time = 0.59, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\text{arccoth}(\tanh(bx + a))^n}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^3, x, algorithm="fricas")

[Out] integral(arccoth(tanh(b\*x + a))^n/x^3, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arccoth}(\tanh(bx + a))^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^3,x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x^3, x)

**maple** [F] time = 13.91, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tanh(b\*x+a))^n/x^3,x)

[Out] int(arccoth(tanh(b\*x+a))^n/x^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tanh(b\*x+a))^n/x^3,x, algorithm="maxima")

[Out] integrate(arccoth(tanh(b\*x + a))^n/x^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acoth}(\tanh(a + bx))^n}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tanh(a + b\*x))^n/x^3,x)

[Out] int(acoth(tanh(a + b\*x))^n/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}^n(\tanh(a + bx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tanh(b\*x+a))\*\*n/x\*\*3,x)

[Out] Integral(acoth(tanh(a + b\*x))\*\*n/x\*\*3, x)

### 3.193 $\int x^m \coth^{-1}(\tanh(a + bx)) dx$

Optimal. Leaf size=37

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))}{m + 1} - \frac{bx^{m+2}}{m^2 + 3m + 2}$$

[Out]  $-b*x^{(2+m)}/(m^2+3*m+2)+x^{(1+m)*\operatorname{arccoth}(\tanh(b*x+a))}/(1+m)$

**Rubi [A]** time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$\frac{x^{m+1} \coth^{-1}(\tanh(a + bx))}{m + 1} - \frac{bx^{m+2}}{m^2 + 3m + 2}$$

Antiderivative was successfully verified.

[In] `Int[x^m*ArcCoth[Tanh[a + b*x]], x]`

[Out]  $-((b*x^{(2 + m)})/(2 + 3*m + m^2)) + (x^{(1 + m)*\operatorname{ArcCoth}[\operatorname{Tanh}[a + b*x]])/(1 + m)$

#### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

#### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

#### Rubi steps

$$\begin{aligned} \int x^m \coth^{-1}(\tanh(a + bx)) dx &= \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))}{1 + m} - \frac{b \int x^{1+m} dx}{1 + m} \\ &= -\frac{bx^{2+m}}{2 + 3m + m^2} + \frac{x^{1+m} \coth^{-1}(\tanh(a + bx))}{1 + m} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 34, normalized size = 0.92

$$x^m \left( \frac{x \left( \coth^{-1}(\tanh(a + bx)) - bx \right)}{m + 1} + \frac{bx^2}{m + 2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*ArcCoth[Tanh[a + b\*x]], x]

[Out] x^m\*((b\*x^2)/(2 + m) + (x\*(-(b\*x) + ArcCoth[Tanh[a + b\*x]])))/(1 + m)

**fricas [A]** time = 0.47, size = 33, normalized size = 0.89

$$\frac{((bm + b)x^2 + (am + 2a)x)x^m}{m^2 + 3m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a)), x, algorithm="fricas")

[Out] ((b\*m + b)\*x^2 + (a\*m + 2\*a)\*x)\*x^m/(m^2 + 3\*m + 2)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arccoth}(\tanh(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(x^m\*arccoth(tanh(b\*x + a)), x)

**maple [C]** time = 0.16, size = 676, normalized size = 18.27

$$\frac{x x^m \ln(e^{bx+a})}{1+m} - \frac{x \left( 2i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a+1}}\right) \operatorname{csgn}(ie^{2bx+2a}) \operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a+1}}\right) - i\pi \operatorname{csgn}\left(\frac{i}{e^{2bx+2a+1}}\right) \operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a+1}}\right) \right)^2}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*arccoth(tanh(b\*x+a)), x)

[Out] 1/(1+m)\*x\*x^m\*ln(exp(b\*x+a))-1/4\*x\*(2\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))-I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))^2\*m-2\*I\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))^2\*m-I\*Pi\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a))

$$\frac{1}{(\exp(2bx+2a)+1)^{2m+4}} \frac{I\pi - 2I\pi \operatorname{csgn}\left(\frac{1}{\exp(2bx+2a)+1}\right) \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{2+4I\pi} \operatorname{csgn}\left(\frac{1}{\exp(2bx+2a)+1}\right)^{3-2I\pi} \operatorname{csgn}\left(\frac{\exp(bx+a)}{\exp(2bx+2a)+1}\right) \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{2m-4I\pi} \operatorname{csgn}\left(\frac{1}{\exp(2bx+2a)+1}\right)^{2-4I\pi} \operatorname{csgn}\left(\frac{\exp(bx+a)}{\exp(2bx+2a)+1}\right) \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{2+I\pi} \operatorname{csgn}\left(\frac{1}{\exp(2bx+2a)+1}\right) \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right) \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{m+2I\pi} \operatorname{csgn}\left(\frac{\exp(bx+a)}{\exp(2bx+2a)+1}\right)^{2} \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{3m+I\pi} \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{3m+2I\pi} \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{3+2I\pi} \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{m+4I\pi} \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{2+2I\pi} \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{3+I\pi} \operatorname{csgn}\left(\frac{\exp(2bx+2a)}{\exp(2bx+2a)+1}\right)^{3m}}{(1+m)/(2+m)x^m$$

**maxima** [A] time = 0.32, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1} \operatorname{arccoth}(\tanh(bx+a))}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*arccoth(tanh(b\*x+a)),x, algorithm="maxima")

[Out] -b\*x^2\*x^m/((m+2)\*(m+1)) + x^(m+1)\*arccoth(tanh(b\*x+a))/(m+1)

**mupad** [B] time = 0.00, size = 96, normalized size = 2.59

$$\frac{2bx^m x^2 (m+1)}{2m^2 + 6m + 4} - \frac{xx^m (m+2) \left( \ln\left(-\frac{2}{e^{2a} e^{2bx-1}}\right) - \ln\left(\frac{2e^{2a} e^{2bx}}{e^{2a} e^{2bx-1}}\right) + 2bx \right)}{2m^2 + 6m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*acoth(tanh(a+b\*x)),x)

[Out]  $(2bx^m x^2 (m+1))/(6m+2m^2+4) - (xx^m (m+2) * (\log(-2/(\exp(2a)*\exp(2bx)-1)) - \log((2*\exp(2a)*\exp(2bx))/(\exp(2a)*\exp(2bx)-1)) + 2bx))/(6m+2m^2+4)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b \log(x) - \frac{\operatorname{acoth}(\tanh(a+bx))}{x} & \text{for } m = -2 \\ \int \frac{\operatorname{acoth}(\tanh(a+bx))}{x} dx & \text{for } m = -1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} + \frac{2xx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(tanh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m**2 + 3*m + 2), True))
```



### 3.194 $\int x^2 \coth^{-1}(\coth(a + bx)) dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3 \coth^{-1}(\coth(a + bx)) - \frac{bx^4}{12}$$

[Out]  $-1/12*b*x^4+1/3*x^3*\operatorname{arccoth}(\coth(b*x+a))$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$\frac{1}{3}x^3 \coth^{-1}(\coth(a + bx)) - \frac{bx^4}{12}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{ArcCoth}[\operatorname{Coth}[a + b*x]], x]$

[Out]  $-(b*x^4)/12 + (x^3*\operatorname{ArcCoth}[\operatorname{Coth}[a + b*x]])/3$

#### Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[x^{(m + 1)}/(m + 1), x] /;$  FreeQ[m, x] && NeQ[m, -1]

#### Rule 2168

$\operatorname{Int}[(u_)^{(m_.)}*(v_)^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}[\{a = \operatorname{Simplify}[D[u, x]], b = \operatorname{Simplify}[D[v, x]]\}, \operatorname{Simp}[(u^{(m + 1)}*v^{(n)})/(a*(m + 1)), x] - \operatorname{Dist}[(b*n)/(a*(m + 1)), \operatorname{Int}[u^{(m + 1)}*v^{(n - 1)}, x], x] /;$  NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

#### Rubi steps

$$\begin{aligned} \int x^2 \coth^{-1}(\coth(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(\coth(a + bx)) - \frac{1}{3}b \int x^3 dx \\ &= -\frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(\coth(a + bx)) \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3 (bx - 4 \coth^{-1}(\coth(a + bx)))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Coth[a + b\*x]],x]

[Out] -1/12\*(x^3\*(b\*x - 4\*ArcCoth[Coth[a + b\*x]]))

**fricas** [A] time = 0.39, size = 13, normalized size = 0.57

$$\frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/4\*x^4\*b + 1/3\*x^3\*a

**giac** [A] time = 0.12, size = 13, normalized size = 0.57

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(coth(b\*x+a)),x, algorithm="giac")

[Out] 1/4\*b\*x^4 + 1/3\*a\*x^3

**maple** [B] time = 0.44, size = 59, normalized size = 2.57

$$\frac{x^3 \operatorname{arccoth}(\coth(bx + a))}{3} + \frac{-\frac{(bx+a)^4}{4} + (bx+a)^3 a - \frac{3a^2(bx+a)^2}{2} + (bx+a)a^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(coth(b\*x+a)),x)

[Out] 1/3\*x^3\*arccoth(coth(b\*x+a))+1/3/b^3\*(-1/4\*(b\*x+a)^4+(b\*x+a)^3\*a-3/2\*a^2\*(b\*x+a)^2+(b\*x+a)\*a^3)

**maxima** [A] time = 0.31, size = 13, normalized size = 0.57

$$\frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(coth(b*x+a)),x, algorithm="maxima")`

[Out]  $1/4*b*x^4 + 1/3*a*x^3$

**mupad** [B] time = 0.08, size = 19, normalized size = 0.83

$$\frac{x^3 \operatorname{acoth}(\operatorname{coth}(a + bx))}{3} - \frac{bx^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(coth(a + b*x)),x)`

[Out]  $(x^3*\operatorname{acoth}(\operatorname{coth}(a + b*x)))/3 - (b*x^4)/12$

**sympy** [A] time = 11.34, size = 39, normalized size = 1.70

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ -\frac{bx^4}{12} + \frac{x^3 \operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(coth(b*x+a)),x)`

[Out] `Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x))))), (-b*x**4/12 + x**3*acoth(1/tanh(a + b*x))/3, True))`

### 3.195 $\int x \coth^{-1}(\coth(a + bx)) dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{bx^3}{6}$$

[Out]  $-1/6*b*x^3+1/2*x^2*\operatorname{arccoth}(\coth(b*x+a))$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {6242, 30}

$$\frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{bx^3}{6}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{ArcCoth}[\operatorname{Coth}[a + b*x]], x]$

[Out]  $-(b*x^3)/6 + (x^2*\operatorname{ArcCoth}[\operatorname{Coth}[a + b*x]])/2$

#### Rule 30

$\operatorname{Int}[(x_)^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[x^{(m + 1)}/(m + 1), x] /;$  FreeQ[m, x] && NeQ[m, -1]

#### Rule 6242

$\operatorname{Int}[\operatorname{ArcCoth}[(c_.) + \operatorname{Coth}[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(e + f*x)^{(m + 1)}*\operatorname{ArcCoth}[c + d*\operatorname{Coth}[a + b*x]]/(f*(m + 1)), x] + \operatorname{Dist}[b/(f*(m + 1)), \operatorname{Int}[(e + f*x)^{(m + 1)}/(c - d - c*E^{(2*a + 2*b*x)}), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]

#### Rubi steps

$$\begin{aligned} \int x \coth^{-1}(\coth(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) - \frac{1}{2}b \int x^2 dx \\ &= -\frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(\coth(a + bx)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2 (bx - 3 \coth^{-1}(\coth(a + bx)))$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Coth[a + b\*x]],x]

[Out] -1/6\*(x^2\*(b\*x - 3\*ArcCoth[Coth[a + b\*x]]))

**fricas** [A] time = 0.36, size = 13, normalized size = 0.57

$$\frac{1}{3}x^3b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/3\*x^3\*b + 1/2\*x^2\*a

**giac** [A] time = 0.13, size = 13, normalized size = 0.57

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(coth(b\*x+a)),x, algorithm="giac")

[Out] 1/3\*b\*x^3 + 1/2\*a\*x^2

**maple** [B] time = 0.39, size = 48, normalized size = 2.09

$$\frac{x^2 \operatorname{arccoth}(\coth(bx+a))}{2} + \frac{-\frac{(bx+a)^3}{3} + (bx+a)^2 a - a^2 (bx+a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(coth(b\*x+a)),x)

[Out] 1/2\*x^2\*arccoth(coth(b\*x+a))+1/2/b^2\*(-1/3\*(b\*x+a)^3+(b\*x+a)^2\*a-a^2\*(b\*x+a))

**maxima** [A] time = 0.31, size = 13, normalized size = 0.57

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(coth(b\*x+a)),x, algorithm="maxima")

[Out]  $1/3*b*x^3 + 1/2*a*x^2$

**mupad** [B] time = 0.06, size = 19, normalized size = 0.83

$$\frac{x^2 \operatorname{acoth}(\operatorname{coth}(a + b x))}{2} - \frac{b x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(coth(a + b*x)),x)`

[Out]  $(x^2*\operatorname{acoth}(\operatorname{coth}(a + b*x)))/2 - (b*x^3)/6$

**sympy** [A] time = 5.60, size = 39, normalized size = 1.70

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ -\frac{bx^3}{6} + \frac{x^2 \operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(coth(b*x+a)),x)`

[Out] `Piecewise((0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x))))), (-b*x**3/6 + x**2*acoth(1/tanh(a + b*x))/2, True))`

### 3.196 $\int \coth^{-1}(\coth(a + bx)) dx$

Optimal. Leaf size=16

$$\frac{\coth^{-1}(\coth(a + bx))^2}{2b}$$

[Out] 1/2\*arccoth(coth(b\*x+a))^2/b

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2157, 30}

$$\frac{\coth^{-1}(\coth(a + bx))^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Coth[a + b\*x]], x]

[Out] ArcCoth[Coth[a + b\*x]]^2/(2\*b)

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2157

Int[(u\_)^(m\_.), x\_Symbol] := With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(\coth(a + bx)) dx &= \frac{\text{Subst}\left(\int x dx, x, \coth^{-1}(\coth(a + bx))\right)}{b} \\ &= \frac{\coth^{-1}(\coth(a + bx))^2}{2b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.12

$$x \coth^{-1}(\coth(a + bx)) - \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Coth[a + b\*x]], x]

[Out] -1/2\*(b\*x^2) + x\*ArcCoth[Coth[a + b\*x]]

**fricas** [A] time = 0.51, size = 10, normalized size = 0.62

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a)), x, algorithm="fricas")

[Out] 1/2\*x^2\*b + x\*a

**giac** [A] time = 0.12, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a)), x, algorithm="giac")

[Out] 1/2\*b\*x^2 + a\*x

**maple** [B] time = 0.06, size = 32, normalized size = 2.00

$$\frac{\operatorname{arctanh}(\operatorname{coth}(bx+a)) \operatorname{arccoth}(\operatorname{coth}(bx+a)) - \frac{\operatorname{arctanh}(\operatorname{coth}(bx+a))^2}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(coth(b\*x+a)), x)

[Out] 1/b\*(arctanh(coth(b\*x+a))\*arccoth(coth(b\*x+a))-1/2\*arctanh(coth(b\*x+a))^2)

**maxima** [A] time = 0.31, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a)), x, algorithm="maxima")

[Out] 1/2\*b\*x^2 + a\*x



mupad [B] time = 1.21, size = 16, normalized size = 1.00

$$x \operatorname{acoth}(\operatorname{coth}(a + b x)) - \frac{b x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(coth(a + b*x)), x)`

[Out] `x*acoth(coth(a + b*x)) - (b*x^2)/2`

sympy [A] time = 2.93, size = 37, normalized size = 2.31

$$\begin{cases} x \operatorname{acoth}(\operatorname{coth}(a)) & \text{for } b = 0 \\ 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ \frac{\operatorname{acoth}^2\left(\frac{1}{\tanh(a+bx)}\right)}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(coth(b*x+a)), x)`

[Out] `Piecewise((x*acoth(coth(a)), Eq(b, 0)), (0, Eq(a, log(exp(-b*x))) | Eq(a, log(-exp(-b*x))))) , (acoth(1/tanh(a + b*x))**2/(2*b), True))`

$$3.197 \quad \int \frac{\coth^{-1}(\coth(a+bx))}{x} dx$$

Optimal. Leaf size=21

$$bx - \log(x) (bx - \coth^{-1}(\coth(a + bx)))$$

[Out] b\*x-(b\*x-arccoth(coth(b\*x+a)))\*ln(x)

**Rubi [A]** time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2158, 29}

$$bx - \log(x) (bx - \coth^{-1}(\coth(a + bx)))$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Coth[a + b\*x]]/x,x]

[Out] b\*x - (b\*x - ArcCoth[Coth[a + b\*x]])\*Log[x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2158

Int[(v\_)/(u\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b\*x)/a, x] - Dist[(b\*u - a\*v)/a, Int[1/u, x], x] /; NeQ[b\*u - a\*v, 0]] /; PiecewiseLinearQ[u, v, x]

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\coth(a + bx))}{x} dx &= bx - (bx - \coth^{-1}(\coth(a + bx))) \int \frac{1}{x} dx \\ &= bx - (bx - \coth^{-1}(\coth(a + bx))) \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 19, normalized size = 0.90

$$\log(x) (\coth^{-1}(\coth(a + bx)) - bx) + bx$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Coth[a + b\*x]]/x,x]

[Out] b\*x + (-(b\*x) + ArcCoth[Coth[a + b\*x]])\*Log[x]

**fricas** [A] time = 0.50, size = 8, normalized size = 0.38

$$bx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x,x, algorithm="fricas")

[Out] b\*x + a\*log(x)

**giac** [A] time = 0.13, size = 9, normalized size = 0.43

$$bx + a \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x,x, algorithm="giac")

[Out] b\*x + a\*log(abs(x))

**maple** [A] time = 0.31, size = 27, normalized size = 1.29

$$bx + a \ln(x) + \ln(x) (\operatorname{arccoth}(\coth(bx + a)) - bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(coth(b\*x+a))/x,x)

[Out] b\*x+a\*ln(x)+ln(x)\*(arccoth(coth(b\*x+a))-b\*x-a)

**maxima** [A] time = 0.31, size = 8, normalized size = 0.38

$$bx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x,x, algorithm="maxima")

[Out] b\*x + a\*log(x)

**mupad** [B] time = 0.55, size = 58, normalized size = 2.76

$$bx - \frac{\ln\left(\frac{1}{e^{2a} e^{2bx} + 1}\right) \ln(x)}{2} + \frac{\ln\left(\frac{e^{2a} e^{2bx}}{e^{2a} e^{2bx} + 1}\right) \ln(x)}{2} - bx \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(coth(a + b*x))/x,x)`

[Out] `b*x - (log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 + (log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - b*x*log(x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(\operatorname{coth}(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(coth(b*x+a))/x,x)`

[Out] `Integral(acoth(coth(a + b*x))/x, x)`

$$3.198 \quad \int \frac{\coth^{-1}(\coth(a+bx))}{x^2} dx$$

Optimal. Leaf size=17

$$b \log(x) - \frac{\coth^{-1}(\coth(a + bx))}{x}$$

[Out] -arccoth(coth(b\*x+a))/x+b\*ln(x)

**Rubi [A]** time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 29}

$$b \log(x) - \frac{\coth^{-1}(\coth(a + bx))}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Coth[a + b\*x]]/x^2,x]

[Out] -(ArcCoth[Coth[a + b\*x]]/x) + b\*Log[x]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 2168

Int[(u\_)^(m\_)\*(v\_)^(n\_), x\_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)\*v^n)/(a\*(m + 1)), x] - Dist[(b\*n)/(a\*(m + 1)), Int[u^(m + 1)\*v^(n - 1), x], x] /; NeQ[b\*u - a\*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2\*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))

Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\coth(a + bx))}{x^2} dx &= -\frac{\coth^{-1}(\coth(a + bx))}{x} + b \int \frac{1}{x} dx \\ &= -\frac{\coth^{-1}(\coth(a + bx))}{x} + b \log(x) \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 18, normalized size = 1.06

$$-\frac{\coth^{-1}(\coth(a + bx))}{x} + b \log(x) + b$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Coth[a + b\*x]]/x^2,x]

[Out] b - ArcCoth[Coth[a + b\*x]]/x + b\*Log[x]

**fricas** [A] time = 0.51, size = 13, normalized size = 0.76

$$\frac{bx \log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x^2,x, algorithm="fricas")

[Out] (b\*x\*log(x) - a)/x

**giac** [A] time = 0.14, size = 12, normalized size = 0.71

$$b \log(|x|) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x^2,x, algorithm="giac")

[Out] b\*log(abs(x)) - a/x

**maple** [A] time = 0.39, size = 20, normalized size = 1.18

$$-\frac{\operatorname{arccoth}(\coth(bx + a))}{x} + b \ln(bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(coth(b\*x+a))/x^2,x)

[Out] -arccoth(coth(b\*x+a))/x+b\*ln(b\*x)

**maxima** [A] time = 0.31, size = 11, normalized size = 0.65

$$b \log(x) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x^2,x, algorithm="maxima")

[Out] b\*log(x) - a/x

**mupad [B]** time = 0.07, size = 17, normalized size = 1.00

$$b \ln(x) - \frac{\operatorname{acoth}(\operatorname{coth}(a + bx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(coth(a + b\*x))/x^2,x)

[Out] b\*log(x) - acoth(coth(a + b\*x))/x

**sympy [A]** time = 5.74, size = 34, normalized size = 2.00

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ b \log(x) - \frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(coth(b\*x+a))/x\*\*2,x)

[Out] Piecewise((0, Eq(a, log(exp(-b\*x))) | Eq(a, log(-exp(-b\*x))))), (b\*log(x) - acoth(1/tanh(a + b\*x))/x, True))

$$3.199 \quad \int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx$$

Optimal. Leaf size=23

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2} - \frac{b}{2x}$$

[Out]  $-1/2*b/x - 1/2*\operatorname{arccoth}(\coth(b*x+a))/x^2$

**Rubi [A]** time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {2168, 30}

$$-\frac{\coth^{-1}(\coth(a+bx))}{2x^2} - \frac{b}{2x}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Coth[a + b*x]]/x^3, x]`

[Out]  $-b/(2*x) - \operatorname{ArcCoth}[\operatorname{Coth}[a + b*x]]/(2*x^2)$

### Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

### Rule 2168

`Int[(u_)^(m_)*(v_)^(n_), x_Symbol] := With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))`

### Rubi steps

$$\begin{aligned} \int \frac{\coth^{-1}(\coth(a+bx))}{x^3} dx &= -\frac{\coth^{-1}(\coth(a+bx))}{2x^2} + \frac{1}{2}b \int \frac{1}{x^2} dx \\ &= -\frac{b}{2x} - \frac{\coth^{-1}(\coth(a+bx))}{2x^2} \end{aligned}$$



**Mathematica** [A] time = 0.02, size = 18, normalized size = 0.78

$$\frac{\coth^{-1}(\coth(a + bx)) + bx}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Coth[a + b\*x]]/x^3,x]

[Out] -1/2\*(b\*x + ArcCoth[Coth[a + b\*x]])/x^2

**fricas** [A] time = 0.54, size = 11, normalized size = 0.48

$$\frac{2bx + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x^3,x, algorithm="fricas")

[Out] -1/2\*(2\*b\*x + a)/x^2

**giac** [A] time = 0.12, size = 11, normalized size = 0.48

$$\frac{2bx + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x^3,x, algorithm="giac")

[Out] -1/2\*(2\*b\*x + a)/x^2

**maple** [A] time = 0.38, size = 20, normalized size = 0.87

$$\frac{b}{2x} - \frac{\operatorname{arccoth}(\coth(bx + a))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(coth(b\*x+a))/x^3,x)

[Out] -1/2\*b/x-1/2\*arccoth(coth(b\*x+a))/x^2

**maxima** [A] time = 0.31, size = 11, normalized size = 0.48

$$\frac{2bx + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(coth(b\*x+a))/x^3,x, algorithm="maxima")

[Out] -1/2\*(2\*b\*x + a)/x^2

mupad [B] time = 1.14, size = 16, normalized size = 0.70

$$-\frac{\operatorname{acoth}(\operatorname{coth}(a + bx)) + bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(coth(a + b\*x))/x^3,x)

[Out] -(acoth(coth(a + b\*x)) + b\*x)/(2\*x^2)

sympy [A] time = 10.80, size = 39, normalized size = 1.70

$$\begin{cases} 0 & \text{for } a = \log(-e^{-bx}) \vee a = \log(e^{-bx}) \\ -\frac{b}{2x} - \frac{\operatorname{acoth}\left(\frac{1}{\tanh(a+bx)}\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(coth(b\*x+a))/x\*\*3,x)

[Out] Piecewise((0, Eq(a, log(exp(-b\*x))) | Eq(a, log(-exp(-b\*x))))), (-b/(2\*x) - acoth(1/tanh(a + b\*x))/(2\*x\*\*2), True))

### 3.200 $\int \coth^{-1}(\cosh(x)) dx$

Optimal. Leaf size=27

$$-\text{Li}_2(-e^x) + \text{Li}_2(e^x) - 2x \tanh^{-1}(e^x) + x \coth^{-1}(\cosh(x))$$

[Out]  $x \cdot \text{arccoth}(\cosh(x)) - 2x \cdot \text{arctanh}(\exp(x)) - \text{polylog}(2, -\exp(x)) + \text{polylog}(2, \exp(x))$

**Rubi [A]** time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$ , Rules used = {6272, 4182, 2279, 2391}

$$-\text{PolyLog}(2, -e^x) + \text{PolyLog}(2, e^x) - 2x \tanh^{-1}(e^x) + x \coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Cosh[x]], x]`

[Out]  $x \cdot \text{ArcCoth}[\text{Cosh}[x]] - 2x \cdot \text{ArcTanh}[E^x] - \text{PolyLog}[2, -E^x] + \text{PolyLog}[2, E^x]$

#### Rule 2279

`Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]`  
`:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /;` `FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

#### Rule 2391

`Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /;` `FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

#### Rule 4182

`Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.)), x_Symbol]`  
`:= Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/ (f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /;` `FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]`

#### Rule 6272

`Int[ArcCoth[u_], x_Symbol] := Simp[x*ArcCoth[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 - u^2), x], x] /;` `InverseFunctionFreeQ[u, x]`

#### Rubi steps

$$\begin{aligned}
\int \coth^{-1}(\cosh(x)) dx &= x \coth^{-1}(\cosh(x)) + \int x \operatorname{csch}(x) dx \\
&= x \coth^{-1}(\cosh(x)) - 2x \tanh^{-1}(e^x) - \int \log(1 - e^x) dx + \int \log(1 + e^x) dx \\
&= x \coth^{-1}(\cosh(x)) - 2x \tanh^{-1}(e^x) - \operatorname{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^x\right) + \operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^x\right) \\
&= x \coth^{-1}(\cosh(x)) - 2x \tanh^{-1}(e^x) - \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 47, normalized size = 1.74

$$\operatorname{Li}_2(-e^{-x}) - \operatorname{Li}_2(e^{-x}) + x(\log(1 - e^{-x}) - \log(e^{-x} + 1)) + x \coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Cosh[x]], x]

[Out] x\*ArcCoth[Cosh[x]] + x\*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^(-x)] - PolyLog[2, E^(-x)]

**fricas [B]** time = 1.39, size = 57, normalized size = 2.11

$$\frac{1}{2} x \log\left(\frac{\cosh(x) + 1}{\cosh(x) - 1}\right) - x \log(\cosh(x) + \sinh(x) + 1) + x \log(-\cosh(x) - \sinh(x) + 1) + \operatorname{Li}_2(\cosh(x) + \sinh(x)) - \operatorname{Li}_2(-\cosh(x) - \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cosh(x)), x, algorithm="fricas")

[Out] 1/2\*x\*log((cosh(x) + 1)/(cosh(x) - 1)) - x\*log(cosh(x) + sinh(x) + 1) + x\*log(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - sinh(x))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cosh(x)), x, algorithm="giac")

[Out] integrate(arccoth(cosh(x)), x)

**maple** [A] time = 0.37, size = 21, normalized size = 0.78

$$x \operatorname{arccoth}(\cosh(x)) + 2 \operatorname{dilog}(e^{-x}) - \frac{\operatorname{dilog}(e^{-2x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(cosh(x)), x)`

[Out] `x*arccoth(cosh(x))+2*dilog(exp(-x))-1/2*dilog(exp(-2*x))`

**maxima** [A] time = 0.37, size = 33, normalized size = 1.22

$$x \operatorname{arccoth}(\cosh(x)) - x \log(e^x + 1) + x \log(-e^x + 1) - \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(cosh(x)), x, algorithm="maxima")`

[Out] `x*arccoth(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilog(e^x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(cosh(x)), x)`

[Out] `int(acoth(cosh(x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(cosh(x)), x)`

[Out] `Integral(acoth(cosh(x)), x)`

### 3.201 $\int x \coth^{-1}(\cosh(x)) dx$

Optimal. Leaf size=51

$$-x\text{Li}_2(-e^x) + x\text{Li}_2(e^x) + \text{Li}_3(-e^x) - \text{Li}_3(e^x) - x^2 \tanh^{-1}(e^x) + \frac{1}{2}x^2 \coth^{-1}(\cosh(x))$$

[Out]  $1/2*x^2*\text{arccoth}(\cosh(x))-x^2*\text{arctanh}(\exp(x))-x*\text{polylog}(2,-\exp(x))+x*\text{polylog}(2,\exp(x))+\text{polylog}(3,-\exp(x))-\text{polylog}(3,\exp(x))$

**Rubi [A]** time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$ , Rules used = {6274, 4182, 2531, 2282, 6589}

$$-x\text{PolyLog}(2,-e^x)+x\text{PolyLog}(2,e^x)+\text{PolyLog}(3,-e^x)-\text{PolyLog}(3,e^x)-x^2 \tanh^{-1}(e^x)+\frac{1}{2}x^2 \coth^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[Cosh[x]],x]

[Out]  $(x^2*\text{ArcCoth}[\text{Cosh}[x]])/2 - x^2*\text{ArcTanh}[E^x] - x*\text{PolyLog}[2, -E^x] + x*\text{PolyLog}[2, E^x] + \text{PolyLog}[3, -E^x] - \text{PolyLog}[3, E^x]$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

#### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 + E^(-(I*e) +
```

f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 6274

Int[((a\_.) + ArcCoth[u\_]\*(b\_.))\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(a + b\*ArcCoth[u]))/(d\*(m + 1)), x] - Dist[b/(d\*(m + 1)), Int[SimplifyIntegrand[((c + d\*x)^(m + 1)\*D[u, x])/(1 - u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d\*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(\cosh(x)) dx &= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) + \frac{1}{2} \int x^2 \operatorname{csch}(x) dx \\
 &= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - x^2 \tanh^{-1}(e^x) - \int x \log(1 - e^x) dx + \int x \log(1 + e^x) dx \\
 &= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - x^2 \tanh^{-1}(e^x) - x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2(e^x) + \int \operatorname{Li}_2(-e^x) dx - \int \operatorname{Li}_2(e^x) dx \\
 &= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - x^2 \tanh^{-1}(e^x) - x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2(e^x) + \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2(-x)}{x} dx, e^x, x\right) \\
 &= \frac{1}{2}x^2 \coth^{-1}(\cosh(x)) - x^2 \tanh^{-1}(e^x) - x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2(e^x) + \operatorname{Li}_3(-e^x) - \operatorname{Li}_3(e^x)
 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 81, normalized size = 1.59

$$\frac{1}{2} \left( 2x \operatorname{Li}_2(-e^{-x}) - 2x \operatorname{Li}_2(e^{-x}) + 2 \operatorname{Li}_3(-e^{-x}) - 2 \operatorname{Li}_3(e^{-x}) + x^2 \log(1 - e^{-x}) - x^2 \log(e^{-x} + 1) + x^2 \coth^{-1}(\cosh(x)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[Cosh[x]], x]

[Out] (x^2\*ArcCoth[Cosh[x]] + x^2\*Log[1 - E^(-x)] - x^2\*Log[1 + E^(-x)] + 2\*x\*PolyLog[2, -E^(-x)] - 2\*x\*PolyLog[2, E^(-x)] + 2\*PolyLog[3, -E^(-x)] - 2\*PolyLog[3, E^(-x)])/2

**fricas** [C] time = 2.01, size = 87, normalized size = 1.71

$$\frac{1}{4} x^2 \log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{2} x^2 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{2} x^2 \log(-\cosh(x) - \sinh(x) + 1) + x \operatorname{Li}_2(\cosh(x) + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(cosh(x)),x, algorithm="fricas")

[Out] 1/4\*x^2\*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/2\*x^2\*log(cosh(x) + sinh(x) + 1) + 1/2\*x^2\*log(-cosh(x) - sinh(x) + 1) + x\*dilog(cosh(x) + sinh(x)) - x\*dilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -cosh(x) - sinh(x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(cosh(x)),x, algorithm="giac")

[Out] integrate(x\*arccoth(cosh(x)), x)

**maple** [C] time = 0.52, size = 449, normalized size = 8.80

$$\frac{i\pi \operatorname{csgn}(ie^{-x}(e^x+1)^2)^3 x^2}{8} - \frac{i\pi \operatorname{csgn}(i(e^x+1)^2) \operatorname{csgn}(ie^{-x}) \operatorname{csgn}(ie^{-x}(e^x+1)^2) x^2}{8} - \frac{i\pi \operatorname{csgn}(i(e^x-1)^2) \operatorname{csgn}(ie^{-x}) \operatorname{csgn}(ie^{-x}(e^x-1)^2) x^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(cosh(x)),x)

[Out] -1/8\*I\*Pi\*csgn(I\*exp(-x)\*(exp(x)+1)^2)^3\*x^2-1/8\*I\*Pi\*csgn(I\*(exp(x)+1)^2)\*csgn(I\*exp(-x))\*csgn(I\*exp(-x)\*(exp(x)+1)^2)\*x^2-1/8\*I\*Pi\*csgn(I\*(exp(x)-1)^2)\*csgn(I\*exp(-x)\*(exp(x)-1)^2)^2\*x^2-1/8\*I\*Pi\*csgn(I\*(exp(x)+1)^2)^3\*x^2+1/8\*I\*Pi\*csgn(I\*(exp(x)-1))^2\*csgn(I\*(exp(x)-1)^2)\*x^2+1/8\*I\*Pi\*csgn(I\*exp(-x)\*(exp(x)-1)^2)^3\*x^2+1/8\*I\*Pi\*csgn(I\*exp(-x))\*csgn(I\*exp(-x)\*(exp(x)+1)^2)^2\*x^2-1/4\*I\*Pi\*csgn(I\*(exp(x)-1))\*csgn(I\*(exp(x)-1)^2)^2\*x^2-1/8\*I\*Pi\*csgn(I\*exp(-x))\*csgn(I\*exp(-x)\*(exp(x)-1)^2)^2\*x^2-1/8\*I\*Pi\*csgn(I\*(exp(x)+1))^2\*csgn(I\*(exp(x)+1)^2)\*x^2+1/4\*I\*Pi\*csgn(I\*(exp(x)+1))\*csgn(I\*(exp(x)+1)^2)^2\*x^2+1/8\*I\*Pi\*csgn(I\*(exp(x)-1)^2)\*csgn(I\*exp(-x))\*csgn(I\*exp(-x)\*(exp(x)-1)^2)\*x^2+1/8\*I\*Pi\*csgn(I\*(exp(x)+1)^2)\*csgn(I\*exp(-x)\*(exp(x)+1)^2)^2\*x^2+polylog(3,-exp(x))-polylog(3,exp(x))+1/8\*I\*Pi\*csgn(I\*(exp(x)-1)^2)^3\*x^2-1/2\*x^2\*ln(exp(x)-1)+1/2\*x^2\*ln(1-exp(x))+x\*polylog(2,exp(x))-x\*polylog(2,-exp(x))



**maxima** [A] time = 0.38, size = 56, normalized size = 1.10

$$\frac{1}{2} x^2 \operatorname{arccoth}(\cosh(x)) - \frac{1}{2} x^2 \log(e^x + 1) + \frac{1}{2} x^2 \log(-e^x + 1) - x \operatorname{Li}_2(-e^x) + x \operatorname{Li}_2(e^x) + \operatorname{Li}_3(-e^x) - \operatorname{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(cosh(x)),x, algorithm="maxima")

[Out] 1/2\*x^2\*arccoth(cosh(x)) - 1/2\*x^2\*log(e^x + 1) + 1/2\*x^2\*log(-e^x + 1) - x\*dilog(-e^x) + x\*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(cosh(x)),x)

[Out] int(x\*acoth(cosh(x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(cosh(x)),x)

[Out] Integral(x\*acoth(cosh(x)), x)

### 3.202 $\int x^2 \coth^{-1}(\cosh(x)) dx$

Optimal. Leaf size=77

$$-x^2 \text{Li}_2(-e^x) + x^2 \text{Li}_2(e^x) + 2x \text{Li}_3(-e^x) - 2x \text{Li}_3(e^x) - 2 \text{Li}_4(-e^x) + 2 \text{Li}_4(e^x) - \frac{2}{3} x^3 \tanh^{-1}(e^x) + \frac{1}{3} x^3 \coth^{-1}(\cosh(x))$$

[Out]  $1/3*x^3*\text{arccoth}(\cosh(x))-2/3*x^3*\text{arctanh}(\exp(x))-x^2*\text{polylog}(2,-\exp(x))+x^2*\text{polylog}(2,\exp(x))+2*x*\text{polylog}(3,-\exp(x))-2*x*\text{polylog}(3,\exp(x))-2*\text{polylog}(4,-\exp(x))+2*\text{polylog}(4,\exp(x))$

**Rubi [A]** time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$ , Rules used = {6274, 4182, 2531, 6609, 2282, 6589}

$$-x^2 \text{PolyLog}(2, -e^x) + x^2 \text{PolyLog}(2, e^x) + 2x \text{PolyLog}(3, -e^x) - 2x \text{PolyLog}(3, e^x) - 2 \text{PolyLog}(4, -e^x) + 2 \text{PolyLog}(4, e^x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{ArcCoth}[\text{Cosh}[x]], x]$

[Out]  $(x^3*\text{ArcCoth}[\text{Cosh}[x]])/3 - (2*x^3*\text{ArcTanh}[E^x])/3 - x^2*\text{PolyLog}[2, -E^x] + x^2*\text{PolyLog}[2, E^x] + 2*x*\text{PolyLog}[3, -E^x] - 2*x*\text{PolyLog}[3, E^x] - 2*\text{PolyLog}[4, -E^x] + 2*\text{PolyLog}[4, E^x]$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] \text{ :> With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] \text{ /; FunctionOfExponentialQ}[u, x] \ \&\& \ \text{!MatchQ}[u, (w\_)*((a\_)*(v\_))^{(n\_)}] \text{ /; FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ \text{!MatchQ}[u, E^{((c\_)*((a\_)+(b\_)*x))}] * (F\_)[v_] \text{ /; FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\text{Int}[\text{Log}[1 + (e\_)*((F\_)^{((c\_)*((a\_)+(b\_)*x))})^{(n\_)}] * ((f\_)+(g\_)*(x_))^{(m_)}, x\_Symbol] \text{ :> -Simp}[\text{((f + g*x)^m * PolyLog[2, -(e*(F^(c*(a + b*x))))^n])} / (b*c*n*Log[F]), x] + \text{Dist}[(g*m) / (b*c*n*Log[F]), \text{Int}[(f + g*x)^{m-1} * PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] \text{ /; FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 4182

$\text{Int}[\text{csc}[(e\_)+(Complex[0, fz\_])*(f\_)*(x_)] * ((c\_)+(d\_)*(x_))^{(m_)}, x\_Symbol] \text{ :> Simp}[\text{(-2*(c + d*x)^m * ArcTanh[E^{-(I*e) + f*fz*x}])} / (f*fz*I), x]$

+ (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-I\*e) + f\*fz\*x], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-I\*e) + f\*fz\*x], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 6274

Int[((a\_.) + ArcCoth[u\_]\*(b\_.))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(a + b\*ArcCoth[u]))/(d\*(m + 1)), x] - Dist[b/(d\*(m + 1)), Int[SimplifyIntegrand[((c + d\*x)^(m + 1)\*D[u, x])/(1 - u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d\*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int x^2 \coth^{-1}(\cosh(x)) dx &= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) + \frac{1}{3} \int x^3 \operatorname{csch}(x) dx \\
 &= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - \int x^2 \log(1 - e^x) dx + \int x^2 \log(1 + e^x) dx \\
 &= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2 \int x \operatorname{Li}_2(-e^x) dx \\
 &= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x \operatorname{Li}_3(-e^x) - 2 \int x \operatorname{Li}_3(-e^x) dx \\
 &= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x \operatorname{Li}_3(-e^x) - 2 \int x \operatorname{Li}_3(-e^x) dx \\
 &= \frac{1}{3}x^3 \coth^{-1}(\cosh(x)) - \frac{2}{3}x^3 \tanh^{-1}(e^x) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x \operatorname{Li}_3(-e^x) - 2 \int x \operatorname{Li}_3(-e^x) dx
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 109, normalized size = 1.42

$$\frac{1}{24} \left( 24x^2 \text{Li}_2(-e^{-x}) + 24x^2 \text{Li}_2(e^x) + 48x \text{Li}_3(-e^{-x}) - 48x \text{Li}_3(e^x) + 48 \text{Li}_4(-e^{-x}) + 48 \text{Li}_4(e^x) - 2x^4 - 8x^3 \log(e^{-x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[Cosh[x]],x]

[Out] (Pi^4 - 2\*x^4 + 8\*x^3\*ArcCoth[Cosh[x]] - 8\*x^3\*Log[1 + E^(-x)] + 8\*x^3\*Log[1 - E^x] + 24\*x^2\*PolyLog[2, -E^(-x)] + 24\*x^2\*PolyLog[2, E^x] + 48\*x\*PolyLog[3, -E^(-x)] - 48\*x\*PolyLog[3, E^x] + 48\*PolyLog[4, -E^(-x)] + 48\*PolyLog[4, E^x])/24

**fricas [C]** time = 1.56, size = 117, normalized size = 1.52

$$\frac{1}{6} x^3 \log\left(\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{3} x^3 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{3} x^3 \log(-\cosh(x) - \sinh(x) + 1) + x^2 \text{Li}_2(\cosh(x) + \sinh(x)) - x^2 \text{Li}_2(-\cosh(x) - \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(cosh(x)),x, algorithm="fricas")

[Out] 1/6\*x^3\*log((cosh(x) + 1)/(cosh(x) - 1)) - 1/3\*x^3\*log(cosh(x) + sinh(x) + 1) + 1/3\*x^3\*log(-cosh(x) - sinh(x) + 1) + x^2\*dilog(cosh(x) + sinh(x)) - x^2\*dilog(-cosh(x) - sinh(x)) - 2\*x\*polylog(3, cosh(x) + sinh(x)) + 2\*x\*polylog(3, -cosh(x) - sinh(x)) + 2\*polylog(4, cosh(x) + sinh(x)) - 2\*polylog(4, -cosh(x) - sinh(x))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(cosh(x)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(cosh(x)), x)

**maple [C]** time = 0.50, size = 471, normalized size = 6.12

$$\frac{i\pi \operatorname{csgn}(i(e^x - 1)^2) \operatorname{csgn}(ie^{-x}(e^x - 1)^2)^2 x^3}{12} + \frac{i\pi \operatorname{csgn}(ie^{-x}(e^x - 1)^2)^3 x^3}{12} + \frac{i\pi \operatorname{csgn}(i(e^x - 1)^2) \operatorname{csgn}(ie^{-x}) \operatorname{csgn}(ie^{-x}(e^x - 1)^2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(cosh(x)),x)

```
[Out] -1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3+1/12*I*P
i*csgn(I*exp(-x)*(exp(x)-1)^2)^3*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*
exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)*x^3-1/12*I*Pi*csgn(I*(exp(x)+1))^2*cs
gn(I*(exp(x)+1)^2)*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x))*csgn(
I*exp(-x)*(exp(x)+1)^2)*x^3+1/6*I*Pi*csgn(I*(exp(x)+1))*csgn(I*(exp(x)+1)^2
)^2*x^3-1/12*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^3+1/12*I*Pi*csgn(I*(exp(
x)+1)^2)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3-1/6*I*Pi*csgn(I*(exp(x)-1))*csg
n(I*(exp(x)-1)^2)^2*x^3+1/3*x^3*ln(1-exp(x))-1/3*x^3*ln(exp(x)-1)+2*x*polylog
(3,-exp(x))-2*x*polylog(3,exp(x))-2*polylog(4,-exp(x))+2*polylog(4,exp(x)
)+1/12*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3-1/12*I*Pi*cs
gn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3+1/12*I*Pi*csgn(I*(exp(x)-1
))^2*csgn(I*(exp(x)-1)^2)*x^3+x^2*polylog(2,exp(x))-x^2*polylog(2,-exp(x))+
1/12*I*Pi*csgn(I*(exp(x)-1)^2)^3*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)^3*x^3
```

**maxima** [A] time = 0.37, size = 78, normalized size = 1.01

$$\frac{1}{3} x^3 \operatorname{arccoth}(\cosh(x)) - \frac{1}{3} x^3 \log(e^x + 1) + \frac{1}{3} x^3 \log(-e^x + 1) - x^2 \operatorname{Li}_2(-e^x) + x^2 \operatorname{Li}_2(e^x) + 2x \operatorname{Li}_3(-e^x) - 2x \operatorname{Li}_3(e^x) - 2 \operatorname{Li}_4(-e^x) + 2 \operatorname{Li}_4(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) - x
^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e^x
) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(cosh(x)),x)
```

```
[Out] int(x^2*acoth(cosh(x)), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(\cosh(x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(cosh(x)),x)
```

```
[Out] Integral(x**2*acoth(cosh(x)), x)
```

### 3.203 $\int x^2 \coth^{-1}(c + d \tanh(a + bx)) dx$

Optimal. Leaf size=307

$$\frac{\operatorname{Li}_4\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3} - \frac{\operatorname{Li}_4\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3} - \frac{x\operatorname{Li}_3\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2} + \frac{x\operatorname{Li}_3\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2} + \frac{x^2\operatorname{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}$$

[Out]  $1/3*x^3*\operatorname{arccoth}(c+d*\tanh(b*x+a))+1/6*x^3*\ln(1+(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))-1/6*x^3*\ln(1+(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*\operatorname{polylog}(2,-(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*\operatorname{polylog}(2,-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))/b-1/4*x*\operatorname{polylog}(3,-(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*\operatorname{polylog}(3,-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*\operatorname{polylog}(4,-(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))/b^3-1/8*\operatorname{polylog}(4,-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))/b^3$

**Rubi [A]** time = 0.46, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6244, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\operatorname{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2} + \frac{x\operatorname{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3} - \frac{\operatorname{PolyLog}\left(4,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{ArcCoth}[c + d*\operatorname{Tanh}[a + b*x]], x]$

[Out]  $(x^3*\operatorname{ArcCoth}[c + d*\operatorname{Tanh}[a + b*x]])/3 + (x^3*\operatorname{Log}[1 + ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)])/6 - (x^3*\operatorname{Log}[1 + ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)])/6 + (x^2*\operatorname{PolyLog}[2, -(((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d))])/(4*b) - (x^2*\operatorname{PolyLog}[2, -(((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d))])/(4*b) - (x*\operatorname{PolyLog}[3, -(((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d))])/(4*b^2) + (x*\operatorname{PolyLog}[3, -(((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d))])/(4*b^2) + \operatorname{PolyLog}[4, -(((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d))]/(8*b^3) - \operatorname{PolyLog}[4, -(((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d))]/(8*b^3)$

#### Rule 2190

$\operatorname{Int}[(((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)*((c_) + (d_)*(x_))^\wedge(m_))/((a_) + (b_)*((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)), x\_Symbol] := \operatorname{Simp}[\frac{(c + d*x)^\wedge m * \operatorname{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n]/a]}{(b*f*g*n*\operatorname{Log}[F])}, x] - \operatorname{Dist}[\frac{(d*m)}{(b*f*g*n*\operatorname{Log}[F])}, \operatorname{Int}[(c + d*x)^\wedge(m - 1)*\operatorname{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n]/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6244

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_)^(m
_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*
(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.), x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(c + d \tanh(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{3}(b(1 - c - d)) \int \frac{e^{2a+2bx} x^3}{1 - c + d + (1 - c - d)e^{2a+2bx}} dx \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6}x^3 \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.50, size = 345, normalized size = 1.12

$$4b^3 x^3 \log \left( \frac{(c-d-1)(\cosh(2(a+bx)) - \sinh(2(a+bx)))}{c+d-1} + 1 \right) - 4b^3 x^3 \log \left( \frac{(c-d+1)(\cosh(2(a+bx)) - \sinh(2(a+bx)))}{c+d+1} + 1 \right) - 6b^2 x^2 \text{Li}_2 \left( \frac{(c-d-1)(\cosh(2(a+bx)) - \sinh(2(a+bx)))}{c+d-1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[c + d\*Tanh[a + b\*x]],x]

[Out] (x^3\*ArcCoth[c + d\*Tanh[a + b\*x]])/3 + (4\*b^3\*x^3\*Log[1 + ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])))/(-1 + c + d) - 4\*b^3\*x^3\*Log[1 + ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])))/(1 + c + d) - 6\*b^2\*x^2\*PolyLog[2, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])))/(-1 + c + d) + 6\*b^2\*x^2\*PolyLog[2, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])))/(1 + c + d) - 6\*b\*x\*PolyLog[3, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])))/(-1 + c + d) + 6\*b\*x\*PolyLog[3, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])))/(1 + c + d) - 3\*PolyLog[4, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])))/(-1 + c + d) + 3\*PolyLog[4, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])))/(1 + c + d)]/(24\*b^3)

**fricas [C]** time = 0.80, size = 899, normalized size = 2.93

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")`

[Out]  $\frac{1}{6}(b^3x^3 \log(((c+1)\cosh(bx+a) + d\sinh(bx+a))/((c-1)\cosh(bx+a) + d\sinh(bx+a))) - 3b^2x^2 \operatorname{dilog}(\sqrt{-(c+d+1)/(c-d+1)}) * (\cosh(bx+a) + \sinh(bx+a))) - 3b^2x^2 \operatorname{dilog}(-\sqrt{-(c+d+1)/(c-d+1)}) * (\cosh(bx+a) + \sinh(bx+a))) + 3b^2x^2 \operatorname{dilog}(\sqrt{-(c+d-1)/(c-d-1)}) * (\cosh(bx+a) + \sinh(bx+a))) + 3b^2x^2 \operatorname{dilog}(-\sqrt{-(c+d-1)/(c-d-1)}) * (\cosh(bx+a) + \sinh(bx+a))) + a^3 \log(2*(c+d+1)\cosh(bx+a) + 2*(c+d+1)\sinh(bx+a) + 2*(c-d+1)\sqrt{-(c+d+1)/(c-d+1)}) + a^3 \log(2*(c+d+1)\cosh(bx+a) + 2*(c+d+1)\sinh(bx+a) - 2*(c-d+1)\sqrt{-(c+d+1)/(c-d+1)}) - a^3 \log(2*(c+d-1)\cosh(bx+a) + 2*(c+d-1)\sinh(bx+a) + 2*(c-d-1)\sqrt{-(c+d-1)/(c-d-1)}) - a^3 \log(2*(c+d-1)\cosh(bx+a) + 2*(c+d-1)\sinh(bx+a) - 2*(c-d-1)\sqrt{-(c+d-1)/(c-d-1)}) + 6bx \operatorname{polylog}(3, \sqrt{-(c+d+1)/(c-d+1)}) * (\cosh(bx+a) + \sinh(bx+a))) + 6bx \operatorname{polylog}(3, -\sqrt{-(c+d+1)/(c-d+1)}) * (\cosh(bx+a) + \sinh(bx+a))) - 6bx \operatorname{polylog}(3, \sqrt{-(c+d-1)/(c-d-1)}) * (\cosh(bx+a) + \sinh(bx+a))) - 6bx \operatorname{polylog}(3, -\sqrt{-(c+d-1)/(c-d-1)}) * (\cosh(bx+a) + \sinh(bx+a))) - (b^3x^3 + a^3) \log(\sqrt{-(c+d+1)/(c-d+1)}) * (\cosh(bx+a) + \sinh(bx+a)) + 1) - (b^3x^3 + a^3) \log(-\sqrt{-(c+d+1)/(c-d+1)}) * (\cosh(bx+a) + \sinh(bx+a)) + 1) + (b^3x^3 + a^3) \log(\sqrt{-(c+d-1)/(c-d-1)}) * (\cosh(bx+a) + \sinh(bx+a)) + 1) + (b^3x^3 + a^3) \log(-\sqrt{-(c+d-1)/(c-d-1)}) * (\cosh(bx+a) + \sinh(bx+a)) + 1) - 6 \operatorname{polylog}(4, \sqrt{-(c+d+1)/(c-d+1)}) * (\cosh(bx+a) + \sinh(bx+a))) - 6 \operatorname{polylog}(4, -\sqrt{-(c+d+1)/(c-d+1)}) * (\cosh(bx+a) + \sinh(bx+a))) + 6 \operatorname{polylog}(4, \sqrt{-(c+d-1)/(c-d-1)}) * (\cosh(bx+a) + \sinh(bx+a))) + 6 \operatorname{polylog}(4, -\sqrt{-(c+d-1)/(c-d-1)}) * (\cosh(bx+a) + \sinh(bx+a))))/b^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(d \tanh(bx+a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(d*tanh(b*x+a)),x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(d*tanh(b*x+a)+c),x)`

**maple** [C] time = 10.11, size = 5294, normalized size = 17.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(c+d*tanh(b*x+a)),x)`

[Out] result too large to display

**maxima** [A] time = 0.71, size = 281, normalized size = 0.92

$$\frac{1}{3}x^3 \operatorname{arccoth}(d \tanh(bx + a) + c) - \frac{1}{18}bd \left( \frac{4b^3x^3 \log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1} + 1\right) + 6b^2x^2 \operatorname{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) - 6bx \operatorname{Li}_3\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^4d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")`

[Out]  $\frac{1}{3}x^3 \operatorname{arccoth}(d \tanh(bx + a) + c) - \frac{1}{18}b^3d \left( \frac{4b^3x^3 \log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1} + 1\right) + 6b^2x^2 \operatorname{dilog}\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) - 6bx \operatorname{polylog}\left(3, -\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) + 3 \operatorname{polylog}\left(4, -\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^4d} - \left( 4b^3x^3 \log\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1} + 1\right) + 6b^2x^2 \operatorname{dilog}\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right) - 6bx \operatorname{polylog}\left(3, -\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right) + 3 \operatorname{polylog}\left(4, -\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)}{b^4d} \right) \right)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(c + d \tanh(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(c + d*tanh(a + b*x)),x)`

[Out] `int(x^2*acoth(c + d*tanh(a + b*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(c + d \tanh(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(c+d*tanh(b*x+a)),x)`

[Out] `Integral(x**2*acoth(c + d*tanh(a + b*x)), x)`

### 3.204 $\int x \coth^{-1}(c + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=231

$$-\frac{\operatorname{Li}_3\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\operatorname{Li}_3\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x\operatorname{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x\operatorname{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{4}x^2 \log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)$$

[Out]  $1/2*x^2*\operatorname{arccoth}(c+d*\tanh(b*x+a))+1/4*x^2*\ln(1+(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))-1/4*x^2*\ln(1+(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))+1/4*x*\operatorname{polylog}(2,-(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))/b-1/4*x*\operatorname{polylog}(2,-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))/b-1/8*\operatorname{polylog}(3,-(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*\operatorname{polylog}(3,-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))/b^2$

**Rubi [A]** time = 0.38, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6244, 2190, 2531, 2282, 6589}

$$-\frac{\operatorname{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\operatorname{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x\operatorname{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x\operatorname{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{ArcCoth}[c + d*\operatorname{Tanh}[a + b*x]], x]$

[Out]  $(x^2*\operatorname{ArcCoth}[c + d*\operatorname{Tanh}[a + b*x]])/2 + (x^2*\operatorname{Log}[1 + ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)])/4 - (x^2*\operatorname{Log}[1 + ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)])/4 + (x*\operatorname{PolyLog}[2, -(((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d))])/(4*b) - (x*\operatorname{PolyLog}[2, -(((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d))])/(4*b) - \operatorname{PolyLog}[3, -(((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d))]/(8*b^2) + \operatorname{PolyLog}[3, -(((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d))]/(8*b^2)$

#### Rule 2190

$\operatorname{Int}[(((F_)^{((g_.)*((e_.) + (f_.)*(x_)))})^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)})/((a_.) + (b_.)*((F_)^{((g_.)*((e_.) + (f_.)*(x_)))})^{(n_.)}), x\_Symbol] :> \operatorname{Simp} [((c + d*x)^m*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a])/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2282

$\operatorname{Int}[u, x\_Symbol] :> \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& !\operatorname{MatchQ}[u, (w_)*((a_.)*(v_)^{(n_)})^{(m_)} /; \operatorname{FreeQ}[$

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*x_)))]^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6244

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*x_]]*((e_.) + (f_.)*x_)^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*
(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*x_)^(p_.)]/((d_.) + (e_.)*x_), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(c + d \tanh(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{2}(b(1 - c - d)) \int \frac{e^{2a+2bx}x^2}{1 - c + d + (1 - c - d)e^{2a+2bx}} dx \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 259, normalized size = 1.12

$$2b^2x^2 \log\left(\frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1} + 1\right) - 2b^2x^2 \log\left(\frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1} + 1\right) - 2bx\text{Li}_2\left(\frac{(c-d-1)e^{2a+2bx}}{1-c+d}\right) + 2bx\text{Li}_2\left(\frac{(c-d+1)e^{2a+2bx}}{1-c+d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[c + d\*Tanh[a + b\*x]], x]

[Out] (x^2\*ArcCoth[c + d\*Tanh[a + b\*x]])/2 + (2\*b^2\*x^2\*Log[1 + ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)]))/(-1 + c + d)] - 2\*b^2\*x^2\*Log[1 + ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)]))/(1 + c + d)] - 2\*b\*x\*PolyLog[2, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)]))/(-1 + c + d)] + 2\*b\*x\*PolyLog[2, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)]))/(1 + c + d)] - PolyLog[3, ((-1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)]))/(-1 + c + d)] + PolyLog[3, ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)]))/(1 + c + d)]/(8\*b^2)

**fricas [C]** time = 0.75, size = 745, normalized size = 3.23

$$b^2x^2 \log\left(\frac{(c+1)\cosh(bx+a)+d\sinh(bx+a)}{(c-1)\cosh(bx+a)+d\sinh(bx+a)}\right) - 2bx\text{Li}_2\left(\sqrt{\frac{c+d+1}{c-d+1}}(\cosh(bx+a) + \sinh(bx+a))\right) - 2bx\text{Li}_2\left(-\sqrt{\frac{c+d+1}{c-d+1}}(\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tanh(b\*x+a)), x, algorithm="fricas")

```
[Out] 1/4*(b^2*x^2*log(((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*log(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(d \tanh(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*tanh(b*x + a) + c), x)
```

**maple** [C] time = 3.58, size = 4990, normalized size = 21.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*tanh(b*x+a)),x)
```

```
[Out] 1/4*x^2*ln((exp(2*b*x+2*a)+1)*c+(exp(2*b*x+2*a)-1)*d+exp(2*b*x+2*a)+1)-1/2/b/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*x*a-1/4/b^2*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a^2-1/4/b^2*c/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a-1/4/b^2*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a^2-1/4/b^2*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(-1-c+d))*a+1/2/b^2*a^2*c/(1+c+d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+(-(1+c-d)*(1+c+d))^(1/2))
```

$$\begin{aligned}
& -\exp(b*x+a)/(-1+c-d)*(1+c+d))^{(1/2)}+1/8/b^2/(c+d-1)*\text{polylog}(3,(c+d-1)*\exp(2*b*x+2*a)/(1-c+d))-1/4/(c+d-1)*\ln(1-(c+d-1)*\exp(2*b*x+2*a)/(1-c+d))*x^2- \\
& 1/4/b^2*a^2/(1+c+d)*\ln(c*\exp(2*b*x+2*a)+d*\exp(2*b*x+2*a)+\exp(2*b*x+2*a)+c-d \\
& +1)+1/4/b^2*c*a^2/(c+d-1)*\ln(c*\exp(2*b*x+2*a)+d*\exp(2*b*x+2*a)-\exp(2*b*x+2* \\
& a)+c-d-1)+1/4/b^2*d*a^2/(c+d-1)*\ln(c*\exp(2*b*x+2*a)+d*\exp(2*b*x+2*a)-\exp(2* \\
& b*x+2*a)+c-d-1)+1/8*I*Pi*x^2*c\text{sgn}(I*((\exp(2*b*x+2*a)+1)*c+(\exp(2*b*x+2*a)- \\
& 1)*d-\exp(2*b*x+2*a)-1)/(\exp(2*b*x+2*a)+1))^{3-1/4}*x^2*\ln((\exp(2*b*x+2*a)+1)*c \\
& +(\exp(2*b*x+2*a)-1)*d-\exp(2*b*x+2*a)-1)-1/4/b^2*a^2/(c+d-1)*\ln(c*\exp(2*b*x+ \\
& 2*a)+d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)+c-d-1)+1/2/b^2*a^2*c/(1+c+d)*\ln((c*\exp \\
& (b*x+a)+\exp(b*x+a)*d+(-1+c-d)*(1+c+d))^{(1/2)}+\exp(b*x+a)/(-1+c-d)*(1+c+d) \\
& )^{(1/2)}+1/8/b^2/(1+c+d)*\text{polylog}(3,(1+c+d)*\exp(2*b*x+2*a)/(-1-c+d))-1/4/(1+ \\
& c+d)*\ln(1-(1+c+d)*\exp(2*b*x+2*a)/(-1-c+d))*x^2-1/8*I*Pi*x^2*c\text{sgn}(I*((\exp(2* \\
& b*x+2*a)+1)*c+(\exp(2*b*x+2*a)-1)*d+\exp(2*b*x+2*a)+1)/(\exp(2*b*x+2*a)+1))^{3- \\
& 1/4}*c/(1+c+d)*\ln(1-(1+c+d)*\exp(2*b*x+2*a)/(-1-c+d))*x^2-1/4*d/(1+c+d)*\ln(1- \\
& (1+c+d)*\exp(2*b*x+2*a)/(-1-c+d))*x^2+1/2/b^2*a/(1+c+d)*\text{dilog}((-c*\exp(b*x+a) \\
& -\exp(b*x+a)*d+(-1+c-d)*(1+c+d))^{(1/2)}-\exp(b*x+a)/(-1+c-d)*(1+c+d))^{(1/2)} \\
& )+1/2/b^2*a/(1+c+d)*\text{dilog}((c*\exp(b*x+a)+\exp(b*x+a)*d+(-1+c-d)*(1+c+d))^{(1/ \\
& 2)}+\exp(b*x+a)/(-1+c-d)*(1+c+d))^{(1/2)}+1/8/b^2*d/(1+c+d)*\text{polylog}(3,(1+c+d) \\
& )*\exp(2*b*x+2*a)/(-1-c+d))+1/8/b^2*c/(1+c+d)*\text{polylog}(3,(1+c+d)*\exp(2*b*x+2* \\
& a)/(-1-c+d))+1/2/b^2*a^2/(1+c+d)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+(-1+c-d)*( \\
& 1+c+d))^{(1/2)}-\exp(b*x+a)/(-1+c-d)*(1+c+d))^{(1/2)}+1/2/b^2*a^2/(1+c+d)*\ln( \\
& (c*\exp(b*x+a)+\exp(b*x+a)*d+(-1+c-d)*(1+c+d))^{(1/2)}+\exp(b*x+a)/(-1+c-d)*( \\
& 1+c+d))^{(1/2)}-1/4/b^2/(1+c+d)*\ln(1-(1+c+d)*\exp(2*b*x+2*a)/(-1-c+d))*a^2-1/ \\
& 4/b^2/(1+c+d)*\text{polylog}(2,(1+c+d)*\exp(2*b*x+2*a)/(-1-c+d))*a-1/4/b/(1+c+d)*\text{po \\
& lylog}(2,(1+c+d)*\exp(2*b*x+2*a)/(-1-c+d))*x+1/4*c/(c+d-1)*\ln(1-(c+d-1)*\exp(2 \\
& *b*x+2*a)/(1-c+d))*x^2+1/4*d/(c+d-1)*\ln(1-(c+d-1)*\exp(2*b*x+2*a)/(1-c+d))*x \\
& ^2-1/8/b^2*c/(c+d-1)*\text{polylog}(3,(c+d-1)*\exp(2*b*x+2*a)/(1-c+d))-1/8/b^2*d/(c \\
& +d-1)*\text{polylog}(3,(c+d-1)*\exp(2*b*x+2*a)/(1-c+d))+1/2/b^2*a/(c+d-1)*\text{dilog}((-c \\
& *\exp(b*x+a)-\exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^{(1/2)}+\exp(b*x+a)/(-(c-d-1)*(c+ \\
& d-1))^{(1/2)}+1/2/b^2*a/(c+d-1)*\text{dilog}((c*\exp(b*x+a)+\exp(b*x+a)*d+(-(c-d-1)*( \\
& c+d-1))^{(1/2)}-\exp(b*x+a)/(-(c-d-1)*(c+d-1))^{(1/2)}-1/4/b^2/(c+d-1)*\ln(1-(c \\
& +d-1)*\exp(2*b*x+2*a)/(1-c+d))*a^2-1/4/b/(c+d-1)*\text{polylog}(2,(c+d-1)*\exp(2*b*x \\
& +2*a)/(1-c+d))*x-1/4/b^2/(c+d-1)*\text{polylog}(2,(c+d-1)*\exp(2*b*x+2*a)/(1-c+d))* \\
& a+1/2/b^2*a^2/(c+d-1)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^{(1/ \\
& 2)}+\exp(b*x+a)/(-(c-d-1)*(c+d-1))^{(1/2)}+1/2/b^2*a^2/(c+d-1)*\ln((c*\exp(b*x+ \\
& a)+\exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^{(1/2)}-\exp(b*x+a)/(-(c-d-1)*(c+d-1))^{(1/ \\
& 2)}))-1/2/b/(c+d-1)*\ln(1-(c+d-1)*\exp(2*b*x+2*a)/(1-c+d))*x*a+1/2/b*a/(c+d-1)* \\
& \ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^{(1/2)}+\exp(b*x+a)/(-(c-d- \\
& 1)*(c+d-1))^{(1/2)})*x+1/2/b*a/(c+d-1)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+(-(c-d-1) \\
& )*(c+d-1))^{(1/2)}-\exp(b*x+a)/(-(c-d-1)*(c+d-1))^{(1/2)})*x-1/2/b^2*c*a^2/(c+d \\
& -1)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^{(1/2)}+\exp(b*x+a)/(-( \\
& c-d-1)*(c+d-1))^{(1/2)}))-1/2/b^2*c*a^2/(c+d-1)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+ \\
& (-c-d-1)*(c+d-1))^{(1/2)}-\exp(b*x+a)/(-(c-d-1)*(c+d-1))^{(1/2)}))-1/2/b^2*d*a^ \\
& 2/(c+d-1)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+(-(c-d-1)*(c+d-1))^{(1/2)}+\exp(b*x+a) \\
& ))/(-(c-d-1)*(c+d-1))^{(1/2)}-1/2/b^2*d*a^2/(c+d-1)*\ln((c*\exp(b*x+a)+\exp(b*x
\end{aligned}$$

$$\begin{aligned}
& +a) * d + (- (c-d-1) * (c+d-1))^{(1/2)} - \exp(b*x+a) / (- (c-d-1) * (c+d-1))^{(1/2)} - 1/2/b^2 * c * a / (c+d-1) * \operatorname{dilog}((-c * \exp(b*x+a) - \exp(b*x+a) * d + (- (c-d-1) * (c+d-1))^{(1/2)} + \exp(b*x+a)) / (- (c-d-1) * (c+d-1))^{(1/2)}) - 1/2/b^2 * c * a / (c+d-1) * \operatorname{dilog}((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (c-d-1) * (c+d-1))^{(1/2)} - \exp(b*x+a)) / (- (c-d-1) * (c+d-1))^{(1/2)}) - 1/2/b^2 * d * a / (c+d-1) * \operatorname{dilog}((-c * \exp(b*x+a) - \exp(b*x+a) * d + (- (c-d-1) * (c+d-1))^{(1/2)} + \exp(b*x+a)) / (- (c-d-1) * (c+d-1))^{(1/2)}) - 1/2/b^2 * d * a / (c+d-1) * \operatorname{dilog}((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (c-d-1) * (c+d-1))^{(1/2)} - \exp(b*x+a)) / (- (c-d-1) * (c+d-1))^{(1/2)}) + 1/4/b^2 * c / (c+d-1) * \ln(1 - (c+d-1) * \exp(2*b*x+2*a) / (1-c+d)) * a^2 + 1/4/b * c / (c+d-1) * \operatorname{polylog}(2, (c+d-1) * \exp(2*b*x+2*a) / (1-c+d)) * x + 1/4/b^2 * c / (c+d-1) * \operatorname{polylog}(2, (c+d-1) * \exp(2*b*x+2*a) / (1-c+d)) * a + 1/4/b^2 * d / (c+d-1) * \ln(1 - (c+d-1) * \exp(2*b*x+2*a) / (1-c+d)) * a^2 + 1/4/b * d / (c+d-1) * \operatorname{polylog}(2, (c+d-1) * \exp(2*b*x+2*a) / (1-c+d)) * x + 1/4/b^2 * d / (c+d-1) * \operatorname{polylog}(2, (c+d-1) * \exp(2*b*x+2*a) / (1-c+d)) * a - 1/4/b^2 * a^2 * c / (1+c+d) * \ln(c * \exp(2*b*x+2*a) + d * \exp(2*b*x+2*a) + \exp(2*b*x+2*a) + c - d + 1) - 1/4/b^2 * a^2 * d / (1+c+d) * \ln(c * \exp(2*b*x+2*a) + d * \exp(2*b*x+2*a) + \exp(2*b*x+2*a) + c - d + 1) + 1/8 * I * \pi * x^2 * c * \operatorname{sgn}(I / (\exp(2*b*x+2*a) + 1)) * c * \operatorname{sgn}(I * ((\exp(2*b*x+2*a) + 1) * c + (\exp(2*b*x+2*a) - 1) * d + \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) + 1))^{(1/2)} - 1/8 * I * \pi * x^2 * c * \operatorname{sgn}(I * ((\exp(2*b*x+2*a) + 1) * c + (\exp(2*b*x+2*a) - 1) * d - \exp(2*b*x+2*a) - 1)) * c * \operatorname{sgn}(I * ((\exp(2*b*x+2*a) + 1) * c + (\exp(2*b*x+2*a) - 1) * d - \exp(2*b*x+2*a) - 1) / (\exp(2*b*x+2*a) + 1))^{(1/2)} + 1/8 * I * \pi * x^2 * c * \operatorname{sgn}(I * ((\exp(2*b*x+2*a) + 1) * c + (\exp(2*b*x+2*a) - 1) * d + \exp(2*b*x+2*a) + 1)) * c * \operatorname{sgn}(I * ((\exp(2*b*x+2*a) + 1) * c + (\exp(2*b*x+2*a) - 1) * d + \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) + 1))^{(1/2)} + 1/2/b^2 * a * c / (1+c+d) * \operatorname{dilog}((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} + \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) + 1/2/b^2 * a * d / (1+c+d) * \operatorname{dilog}((-c * \exp(b*x+a) - \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} - \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) + 1/2/b^2 * a * d / (1+c+d) * \operatorname{dilog}((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} + \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) + 1/2/b^2 * a * c / (1+c+d) * \operatorname{dilog}((-c * \exp(b*x+a) - \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} - \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) + 1/2/b^2 * a^2 * d / (1+c+d) * \ln((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} + \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) + 1/2/b^2 * a^2 * d / (1+c+d) * \ln((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} - \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) + 1/2/b * c / (1+c+d) * \operatorname{polylog}(2, (1+c+d) * \exp(2*b*x+2*a) / (-1-c+d)) * x - 1/4/b * d / (1+c+d) * \operatorname{polylog}(2, (1+c+d) * \exp(2*b*x+2*a) / (-1-c+d)) * x + 1/2/b * a / (1+c+d) * \ln((-c * \exp(b*x+a) - \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} - \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) * x + 1/2/b * a / (1+c+d) * \ln((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} + \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) * x - 1/8 * I * \pi * x^2 * c * \operatorname{sgn}(I / (\exp(2*b*x+2*a) + 1)) * c * \operatorname{sgn}(I * ((\exp(2*b*x+2*a) + 1) * c + (\exp(2*b*x+2*a) - 1) * d - \exp(2*b*x+2*a) - 1) / (\exp(2*b*x+2*a) + 1))^{(1/2)} - 1/2/b * c / (1+c+d) * \ln(1 - (1+c+d) * \exp(2*b*x+2*a) / (-1-c+d)) * x * a - 1/2/b * d / (1+c+d) * \ln(1 - (1+c+d) * \exp(2*b*x+2*a) / (-1-c+d)) * x * a + 1/2/b * a * c / (1+c+d) * \ln((-c * \exp(b*x+a) - \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} - \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) * x + 1/2/b * a * c / (1+c+d) * \ln((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} + \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) * x + 1/2/b * a * d / (1+c+d) * \ln((-c * \exp(b*x+a) - \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} - \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) * x + 1/2/b * a * d / (1+c+d) * \ln((c * \exp(b*x+a) + \exp(b*x+a) * d + (- (1+c-d) * (1+c+d))^{(1/2)} + \exp(b*x+a)) / (- (1+c-d) * (1+c+d))^{(1/2)}) * x + 1/8 * I * \pi * x^2 * c * \operatorname{sgn}(I / (\exp(2*b*x+2*a) + 1)) * c * \operatorname{sgn}(I * ((\exp(2*b*x+2*a) + 1) * c + (\exp(2*b*x+2*a) - 1) * d - \exp(2*b*x+
\end{aligned}$$



$2*a)-1)) * \text{csgn}(I * ((\exp(2*b*x+2*a)+1)*c + (\exp(2*b*x+2*a)-1)*d - \exp(2*b*x+2*a)-1) / (\exp(2*b*x+2*a)+1)) - 1/8 * I * \text{Pi} * x^2 * \text{csgn}(I / (\exp(2*b*x+2*a)+1)) * \text{csgn}(I * ((\exp(2*b*x+2*a)+1)*c + (\exp(2*b*x+2*a)-1)*d + \exp(2*b*x+2*a)+1)) * \text{csgn}(I * ((\exp(2*b*x+2*a)+1)*c + (\exp(2*b*x+2*a)-1)*d + \exp(2*b*x+2*a)+1) / (\exp(2*b*x+2*a)+1)) + 1/2/b * c / (c+d-1) * \ln(1 - (c+d-1) * \exp(2*b*x+2*a) / (1-c+d)) * x * a + 1/2/b * d / (c+d-1) * \ln(1 - (c+d-1) * \exp(2*b*x+2*a) / (1-c+d)) * x * a - 1/2/b * c * a / (c+d-1) * \ln((-c * \exp(b*x+a) - \exp(b*x+a) * d + (-c-d-1) * (c+d-1))^{1/2} + \exp(b*x+a)) / (-c-d-1) * (c+d-1)^{1/2}) * x - 1/2/b * c * a / (c+d-1) * \ln((c * \exp(b*x+a) + \exp(b*x+a) * d + (-c-d-1) * (c+d-1))^{1/2} - \exp(b*x+a)) / (-c-d-1) * (c+d-1)^{1/2}) * x - 1/2/b * d * a / (c+d-1) * \ln((-c * \exp(b*x+a) - \exp(b*x+a) * d + (-c-d-1) * (c+d-1))^{1/2} + \exp(b*x+a)) / (-c-d-1) * (c+d-1)^{1/2}) * x - 1/2/b * d * a / (c+d-1) * \ln((c * \exp(b*x+a) + \exp(b*x+a) * d + (-c-d-1) * (c+d-1))^{1/2} - \exp(b*x+a)) / (-c-d-1) * (c+d-1)^{1/2}) * x$

**maxima** [A] time = 0.71, size = 215, normalized size = 0.93

$$-\frac{1}{8}bd \left( \frac{2b^2x^2 \log\left(\frac{(c+d+1)e^{2bx+2a}}{c-d+1} + 1\right) + 2bx \text{Li}_2\left(-\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right) - \text{Li}_3\left(-\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right)}{b^3d} - \frac{2b^2x^2 \log\left(\frac{(c+d-1)e^{2bx+2a}}{c-d-1}\right)}{b^3d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/8*b*d*((2*b^2*x^2*\log((c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}+1)+2*b*x*d*\text{dilog}(-(c+d+1)*e^{(2*b*x+2*a)/(c-d+1)})-\text{polylog}(3,-(c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}))/b^3*d)-(2*b^2*x^2*\log((c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}+1)+2*b*x*d*\text{dilog}(-(c+d-1)*e^{(2*b*x+2*a)/(c-d-1)})-\text{polylog}(3,-(c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}))/b^3*d)+1/2*x^2*\text{arccoth}(d*\text{tanh}(b*x+a)+c)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acoth}(c + d \tanh(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(c + d\*tanh(a + b\*x)),x)

[Out] int(x\*acoth(c + d\*tanh(a + b\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(c + d \tanh(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x*acoth(c + d*tanh(a + b*x)), x)
```

### 3.205 $\int \coth^{-1}(c + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=150

$$\frac{\operatorname{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{\operatorname{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{2}x \log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1} + 1\right) - \frac{1}{2}x \log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1} + 1\right) +$$

[Out]  $x \operatorname{arccoth}(c+d \tanh(bx+a)) + 1/2 * x * \ln(1 + ((1-c-d) \exp(2bx+2a))/(1-c+d)) - 1/2 * x * \ln(1 + ((1+c+d) \exp(2bx+2a))/(1+c-d)) + 1/4 * \operatorname{polylog}(2, -(1-c-d) \exp(2bx+2a)/(1-c+d)) / b - 1/4 * \operatorname{polylog}(2, -(1+c+d) \exp(2bx+2a)/(1+c-d)) / b$

**Rubi [A]** time = 0.23, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6236, 2190, 2279, 2391}

$$\frac{\operatorname{PolyLog}\left(2, -\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{\operatorname{PolyLog}\left(2, -\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{2}x \log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1} + 1\right) - \frac{1}{2}x \log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1} + 1\right) +$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{ArcCoth}[c + d \operatorname{Tanh}[a + bx]], x]$

[Out]  $x \operatorname{ArcCoth}[c + d \operatorname{Tanh}[a + bx]] + (x \operatorname{Log}[1 + ((1 - c - d) E^{(2a + 2bx)}) / (1 - c + d)]) / 2 - (x \operatorname{Log}[1 + ((1 + c + d) E^{(2a + 2bx)}) / (1 + c - d)]) / 2 + \operatorname{PolyLog}[2, -(((1 - c - d) E^{(2a + 2bx)}) / (1 - c + d))] / (4b) - \operatorname{PolyLog}[2, -(((1 + c + d) E^{(2a + 2bx)}) / (1 + c - d))] / (4b)$

#### Rule 2190

$\operatorname{Int}[(((F_)^((g_.) * ((e_.) + (f_.) * (x_))))^{(n_.)} * ((c_.) + (d_.) * (x_)))^{(m_.)} / ((a_.) + (b_.) * ((F_)^((g_.) * ((e_.) + (f_.) * (x_))))^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Simp} [((c + dx)^m \operatorname{Log}[1 + (b(F^g(e + fx))^n)/a]) / (bfgn \operatorname{Log}[F]), x] - \operatorname{Dist}[(d^m) / (bfgn \operatorname{Log}[F]), \operatorname{Int}[(c + dx)^{(m-1)} \operatorname{Log}[1 + (b(F^g(e + fx))^n)/a], x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \} \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_.) + (b_.) * ((F_)^((e_.) * ((c_.) + (d_.) * (x_))))^{(n_.)}], x\_Symbol] \rightarrow \operatorname{Dist}[1 / (de^n \operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + bx] / x, x], x, (F^{e(c + dx)})^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x \} \ \&\& \ \operatorname{GtQ}[a, 0]$

#### Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 6236

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] := Simp[x*ArcCoth[c + d*Tanh[a + b*x]], x] + (Dist[b*(1 - c - d), Int[(x*E^(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[b*(1 + c + d), Int[(x*E^(2*a + 2*b*x))/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(c + d \tanh(a + bx)) dx &= x \coth^{-1}(c + d \tanh(a + bx)) + (b(1 - c - d)) \int \frac{e^{2a+2bx} x}{1 - c + d + (1 - c - d)e^{2a+2bx}} \\ &= x \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c + d)e^{2a+2bx}}{1 + c - d} \right) \\ &= x \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c + d)e^{2a+2bx}}{1 + c - d} \right) \\ &= x \coth^{-1}(c + d \tanh(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 + c + d)e^{2a+2bx}}{1 + c - d} \right) \end{aligned}$$

**Mathematica [A]** time = 1.42, size = 131, normalized size = 0.87

$$\frac{\text{Li}_2\left(-\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) - \text{Li}_2\left(-\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right) + 2bx \left( \log\left(\frac{(c+d-1)e^{2(a+bx)}}{c-d-1} + 1\right) - \log\left(\frac{(c+d+1)e^{2(a+bx)}}{c-d+1} + 1\right) \right)}{4b} + x \coth^{-1}(d \tanh(a + bx))$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[c + d\*Tanh[a + b\*x]], x]

[Out] x\*ArcCoth[c + d\*Tanh[a + b\*x]] + (2\*b\*x\*(Log[1 + ((-1 + c + d)\*E^(2\*(a + b\*x)))/(-1 + c - d)] - Log[1 + ((1 + c + d)\*E^(2\*(a + b\*x)))/(1 + c - d)]) + PolyLog[2, -(((1 + c + d)\*E^(2\*(a + b\*x)))/(-1 + c - d))] - PolyLog[2, -(((1 + c + d)\*E^(2\*(a + b\*x)))/(1 + c - d))]/(4\*b)

**fricas [B]** time = 0.85, size = 551, normalized size = 3.67

$$bx \log \left( \frac{(c+1) \cosh(bx+a) + d \sinh(bx+a)}{(c-1) \cosh(bx+a) + d \sinh(bx+a)} \right) + a \log \left( 2(c+d+1) \cosh(bx+a) + 2(c+d+1) \sinh(bx+a) + 2(c-d+1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="fricas")`

[Out] 
$$\frac{1}{2}*(b*x*\log(((c + 1)*\cosh(b*x + a) + d*\sinh(b*x + a))/((c - 1)*\cosh(b*x + a) + d*\sinh(b*x + a))) + a*\log(2*(c + d + 1)*\cosh(b*x + a) + 2*(c + d + 1)*\sinh(b*x + a) + 2*(c - d + 1)*\sqrt{-(c + d + 1)/(c - d + 1)}) + a*\log(2*(c + d + 1)*\cosh(b*x + a) + 2*(c + d + 1)*\sinh(b*x + a) - 2*(c - d + 1)*\sqrt{-(c + d + 1)/(c - d + 1)}) - a*\log(2*(c + d - 1)*\cosh(b*x + a) + 2*(c + d - 1)*\sinh(b*x + a) + 2*(c - d - 1)*\sqrt{-(c + d - 1)/(c - d - 1)}) - a*\log(2*(c + d - 1)*\cosh(b*x + a) + 2*(c + d - 1)*\sinh(b*x + a) - 2*(c - d - 1)*\sqrt{-(c + d - 1)/(c - d - 1)}) - (b*x + a)*\log(\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - (b*x + a)*\log(-\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + (b*x + a)*\log(\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + (b*x + a)*\log(-\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - \operatorname{dilog}(\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) - \operatorname{dilog}(-\sqrt{-(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + \operatorname{dilog}(\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + \operatorname{dilog}(-\sqrt{-(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))))/b$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(d \tanh(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="giac")`

[Out] `integrate(arccoth(d*tanh(b*x + a) + c), x)`

**maple** [B] time = 0.35, size = 306, normalized size = 2.04

$$-\frac{\operatorname{arccoth}(c + d \tanh(bx + a)) \ln(d \tanh(bx + a) - d)}{2b} + \frac{\operatorname{arccoth}(c + d \tanh(bx + a)) \ln(d \tanh(bx + a) + d)}{2b} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(c+d*tanh(b*x+a)),x)`

[Out] 
$$-1/2/b*\operatorname{arccoth}(c+d*\tanh(b*x+a))*\ln(d*\tanh(b*x+a)-d)+1/2/b*\operatorname{arccoth}(c+d*\tanh(b*x+a))*\ln(d*\tanh(b*x+a)+d)+1/4/b*\operatorname{dilog}((d*\tanh(b*x+a)+c+1)/(1+c+d))+1/4/b*\ln(d*\tanh(b*x+a)-d)*\ln((d*\tanh(b*x+a)+c+1)/(1+c+d))-1/4/b*\operatorname{dilog}((d*\tanh(b*x+a)+c-1)/(c+d-1))-1/4/b*\ln(d*\tanh(b*x+a)-d)*\ln((d*\tanh(b*x+a)+c-1)/(c+d-1))+1/4/b*\operatorname{dilog}((d*\tanh(b*x+a)+c-1)/(c-d-1))+1/4/b*\ln(d*\tanh(b*x+a)+d)*\ln((d*$$

$\operatorname{anh}(b*x+a)+c-1)/(c-d-1))-1/4/b*d\operatorname{dilog}((d*\tanh(b*x+a)+c+1)/(1+c-d))-1/4/b*\ln(d*\tanh(b*x+a)+d)*\ln((d*\tanh(b*x+a)+c+1)/(1+c-d))$

**maxima** [A] time = 0.69, size = 142, normalized size = 0.95

$$-\frac{1}{4}bd \left( \frac{2bx \log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1} + 1\right) + \operatorname{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^2d} - \frac{2bx \log\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1} + 1\right) + \operatorname{Li}_2\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)}{b^2d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*tanh(b*x+a)),x, algorithm="maxima")`

[Out]  $-1/4*b*d*((2*b*x*\log((c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}+1)+\operatorname{dilog}(-(c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}))/(b^2*d)-(2*b*x*\log((c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}+1)+\operatorname{dilog}(-(c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}))/(b^2*d))+x*\operatorname{arccoth}(d*\tanh(b*x+a)+c)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(c+d \tanh(a+bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(c+d*tanh(a+b*x)),x)`

[Out] `int(acoth(c+d*tanh(a+b*x)),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(c+d \tanh(a+bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(c+d*tanh(b*x+a)),x)`

[Out] `Integral(acoth(c+d*tanh(a+b*x)),x)`

$$3.206 \quad \int \frac{\coth^{-1}(c+d \tanh(a+bx))}{x} dx$$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(d \tanh(a + bx) + c)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(c+d\*tanh(b\*x+a))/x, x)

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(c + d \tanh(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[c + d\*Tanh[a + b\*x]]/x, x]

[Out] Defer[Int][ArcCoth[c + d\*Tanh[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c + d \tanh(a + bx))}{x} dx = \int \frac{\coth^{-1}(c + d \tanh(a + bx))}{x} dx$$

Mathematica [A] time = 9.54, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(c + d \tanh(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[c + d\*Tanh[a + b\*x]]/x, x]

[Out] Integrate[ArcCoth[c + d\*Tanh[a + b\*x]]/x, x]

fricas [A] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcoth}(d \tanh(bx + a) + c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tanh(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*tanh(b\*x + a) + c)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \tanh(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tanh(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*tanh(b\*x + a) + c)/x, x)

**maple** [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(c + d \tanh(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*tanh(b\*x+a))/x,x)

[Out] int(arccoth(c+d\*tanh(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \tanh(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tanh(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*tanh(b\*x + a) + c)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(c + d \tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(c + d\*tanh(a + b\*x))/x,x)

[Out] int(acoth(c + d\*tanh(a + b\*x))/x, x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(c + d \tanh(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c+d\*tanh(b\*x+a))/x,x)

[Out] Integral(acoth(c + d\*tanh(a + b\*x))/x, x)

### 3.207 $\int x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=155

$$\frac{3\text{Li}_5\left(-\left((d+1)e^{2a+2bx}\right)\right)}{16b^4} - \frac{3x\text{Li}_4\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^3} + \frac{3x^2\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^2} - \frac{x^3\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b} - \frac{1}{8}x$$

[Out] 1/20\*b\*x^5+1/4\*x^4\*arccoth(1+d+d\*tanh(b\*x+a))-1/8\*x^4\*ln(1+(1+d)\*exp(2\*b\*x+2\*a))-1/4\*x^3\*polylog(2,-(1+d)\*exp(2\*b\*x+2\*a))/b+3/8\*x^2\*polylog(3,-(1+d)\*exp(2\*b\*x+2\*a))/b^2-3/8\*x\*polylog(4,-(1+d)\*exp(2\*b\*x+2\*a))/b^3+3/16\*polylog(5,-(1+d)\*exp(2\*b\*x+2\*a))/b^4

**Rubi [A]** time = 0.31, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2} - \frac{3x\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3} + \frac{3\text{PolyLog}\left(5,-(d+1)e^{2a+2bx}\right)}{16b^4} - \frac{x^3\text{PolyLog}\left(5,-(d+1)e^{2a+2bx}\right)}{8b^4}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[1 + d + d\*Tanh[a + b\*x]],x]

[Out] (b\*x^5)/20 + (x^4\*ArcCoth[1 + d + d\*Tanh[a + b\*x]])/4 - (x^4\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/8 - (x^3\*PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (3\*x^2\*PolyLog[3, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(8\*b^2) - (3\*x\*PolyLog[4, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(8\*b^3) + (3\*PolyLog[5, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(16\*b^4)

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x))))^n], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_)^(m
_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) dx &= \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) + \frac{1}{4}b \int \frac{x^4}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4}(b(1 + d)) \int \frac{e^{2a+2bx}}{1 + (1 + d)} \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8}x^4 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8}x^4 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8}x^4 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8}x^4 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8}x^4 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8}x^4 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8}x^4 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{8}x^4 \log(1 + (1 + d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 144, normalized size = 0.93

$$\frac{1}{16} \left( \frac{3\text{Li}_5\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{6x\text{Li}_4\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x^2\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{4x^3\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 2x^4 \log\left(\frac{e^{-2(a+bx)}}{d+1} + 1\right) + 4x^5 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (4\*x^4\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - 2\*x^4\*Log[1 + 1/((1 + d)\*E^(2\*(a + b\*x)))] + (4\*x^3\*PolyLog[2, -(1/((1 + d)\*E^(2\*(a + b\*x)))]])/b + (6\*x^2\*PolyLog[3, -(1/((1 + d)\*E^(2\*(a + b\*x)))]])/b^2 + (6\*x\*PolyLog[4, -(1/((1 + d)\*E^(2\*(a + b\*x)))]])/b^3 + (3\*PolyLog[5, -(1/((1 + d)\*E^(2\*(a + b\*x)))]])/b^4)/16

**fricas [C]** time = 1.75, size = 450, normalized size = 2.90

$$\frac{2b^5x^5 + 5b^4x^4 \log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20b^3x^3\text{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a)+\sinh(bx+a))\right) - 20b^3x^4 \log(1 + (1 + d)e^{2a+2bx})}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d\*d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out] 1/40\*(2\*b^5\*x^5 + 5\*b^4\*x^4\*log(((d + 2)\*cosh(b\*x + a) + d\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(-1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*a^4\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) + sqrt(-4\*d - 4)) - 5\*a^4\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) - sqrt(-4\*d - 4)) + 60\*b^2\*x^2\*polylog(3, 1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 60\*b^2\*x^2\*polylog(3, -1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, 1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, -1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*(b^4\*x^4 - a^4)\*log(1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 5\*(b^4\*x^4 - a^4)\*log(-1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 120\*polylog(5, 1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 120\*polylog(5, -1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^4

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(d \tanh(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d\*d\*tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^3\*arccoth(d\*tanh(b\*x + a) + d + 1), x)

**maple** [C] time = 5.74, size = 1726, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(1+d\*d\*tanh(b\*x+a)),x)

[Out] 1/16\*I\*x^4\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))-1/16\*I\*x^4\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*(d\*exp(2\*b\*x+2\*a)+exp(2\*b\*x+2\*a)+1))\*csgn(I/(exp(2\*b\*x+2\*a)+1)\*(d\*exp(2\*b\*x+2\*a)+exp(2\*b\*x+2\*a)+1))+1/16\*I\*x^4\*Pi\*csgn(I\*d)\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*d/(exp(2\*b\*x+2\*a)+1)\*exp(2\*b\*x+2\*a))+3/16/b^4/(1+d)\*polylog(5, -(1+d)\*exp(2\*b\*x+2\*a))-1/8/(1+d)\*ln(1+(1+d)\*exp(2\*b\*x+2\*a))\*x^4-3/8/b^3/(1+d)\*polylog(4, -(1+d)\*exp(2\*b\*x+2\*a))\*x+1/2/b^4\*a^3/(1+d)\*dilog(1+exp(b\*x+a)\*(-d-1)^(1/2))-1/8\*d/(1+d)\*ln(1+(1+d)\*exp(2\*b\*x+2\*a))\*x^4+1/2/b^4\*a^3/(1+d)\*dilog(1-exp(b\*x+a)\*(-d-1)^(1/2))+3/16/b^4\*d/(1+d)\*polylog(5, -(1+d)\*exp(2\*b\*x+2\*a))+1/2/b^4\*a^4/(1+d)\*ln(1+exp(b\*x+a)\*(-d-1)^(1/2))+1/2/b^4\*a^4/(1+d)\*ln(1-exp(b\*x+a)\*(-d-1)^(1/2))-3/8/b^4\*a^4/(1+d)\*ln(1+(1+d)\*exp

$(2bx+2a)) - 1/4/b^4*a^3/(1+d)*polylog(2, -(1+d)*exp(2bx+2a)) - 1/4/b/(1+d)$   
 $*polylog(2, -(1+d)*exp(2bx+2a))*x^3+3/8/b^2/(1+d)*polylog(3, -(1+d)*exp(2$   
 $bx+2a))*x^2+1/2/b^4*d*a^4/(1+d)*ln(1-exp(bx+a))*(-d-1)^(1/2))+1/2/b^3*a^3$   
 $/(1+d)*ln(1+exp(bx+a))*(-d-1)^(1/2))*x-1/8/b^4*a^4/(1+d)*ln(d*exp(2bx+2a$   
 $) + exp(2bx+2a)+1)+1/20*b*x^5+1/16*I*x^4*Pi*csgn(I*d/(exp(2bx+2a)+1)*exp$   
 $(2bx+2a))^3+1/2/b^4*d*a^4/(1+d)*ln(1+exp(bx+a))*(-d-1)^(1/2))+1/8*x^4*ln$   
 $(d*exp(2bx+2a)+exp(2bx+2a)+1)+1/2/b^3*d*a^3/(1+d)*ln(1-exp(bx+a))*(-$   
 $d-1)^(1/2))*x-1/2/b^3*d/(1+d)*ln(1+(1+d)*exp(2bx+2a))*x*a^3+1/2/b^3*d*a^$   
 $3/(1+d)*ln(1+exp(bx+a))*(-d-1)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1-exp(bx+a))*$   
 $(-d-1)^(1/2))*x-3/8/b^4*d/(1+d)*ln(1+(1+d)*exp(2bx+2a))*a^4-1/4/b*d/(1+d)$   
 $*polylog(2, -(1+d)*exp(2bx+2a))*x^3-1/4/b^4*d/(1+d)*polylog(2, -(1+d)*exp($   
 $2bx+2a))*a^3+3/8/b^2*d/(1+d)*polylog(3, -(1+d)*exp(2bx+2a))*x^2-3/8/b^$   
 $3*d/(1+d)*polylog(4, -(1+d)*exp(2bx+2a))*x-1/2/b^3*a^3/(1+d)*ln(1+(1+d)*exp$   
 $(2bx+2a))*x+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(bx+a))*(-d-1)^(1/2))+1/2/b$   
 $^4*d*a^3/(1+d)*dilog(1-exp(bx+a))*(-d-1)^(1/2))-1/4*x^4*ln(exp(bx+a))-1/8*$   
 $x^4*ln(d)+1/16*I*x^4*Pi*csgn(I/(exp(2bx+2a)+1))*csgn(I/(exp(2bx+2a)+1$   
 $))*(d*exp(2bx+2a)+exp(2bx+2a)+1))^2+1/16*I*x^4*Pi*csgn(I*exp(2bx+2a$   
 $))^3+1/16*I*x^4*Pi*csgn(I*exp(2bx+2a)/(exp(2bx+2a)+1))^3-1/16*I*x^4*P$   
 $i*csgn(I/(exp(2bx+2a)+1)*(d*exp(2bx+2a)+exp(2bx+2a)+1))^3+1/16*I*x$   
 $^4*Pi*csgn(I*(d*exp(2bx+2a)+exp(2bx+2a)+1))*csgn(I/(exp(2bx+2a)+1$   
 $))*(d*exp(2bx+2a)+exp(2bx+2a)+1))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(e$   
 $xp(2bx+2a)+1)*exp(2bx+2a))^2-1/16*I*x^4*Pi*csgn(I/(exp(2bx+2a)+1))$   
 $*csgn(I*exp(2bx+2a)/(exp(2bx+2a)+1))^2-1/16*I*x^4*Pi*csgn(I*exp(2bx$   
 $+2a))*csgn(I*exp(2bx+2a)/(exp(2bx+2a)+1))^2-1/16*I*x^4*Pi*csgn(I*exp$   
 $(2bx+2a)/(exp(2bx+2a)+1))*csgn(I*d/(exp(2bx+2a)+1)*exp(2bx+2a))$   
 $^2-1/8/b^4*d*a^4/(1+d)*ln(d*exp(2bx+2a)+exp(2bx+2a)+1)+1/16*I*x^4*Pi*$   
 $csgn(I*exp(bx+a))^2*csgn(I*exp(2bx+2a))-1/8*I*x^4*Pi*csgn(I*exp(bx+a))$   
 $*csgn(I*exp(2bx+2a))^2$

**maxima** [A] time = 1.12, size = 149, normalized size = 0.96

$$\frac{1}{4}x^4 \operatorname{arccoth}(d \tanh(bx+a) + d + 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4x^4 \log((d+1)e^{2bx+2a}) + 1) + 4b^3x^3 \operatorname{Li}_2(-(d+1)e^{2bx+2a})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d\*d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(d\*tanh(b\*x + a) + d + 1) + 1/40\*(2\*x^5/d - 5\*(2\*b^4\*x^4\*log((d + 1)\*e^(2\*b\*x + 2\*a) + 1) + 4\*b^3\*x^3\*dilog(-(d + 1)\*e^(2\*b\*x + 2\*a)) - 6\*b^2\*x^2\*polylog(3, -(d + 1)\*e^(2\*b\*x + 2\*a)) + 6\*b\*x\*polylog(4, -(d + 1)\*e^(2\*b\*x + 2\*a)) - 3\*polylog(5, -(d + 1)\*e^(2\*b\*x + 2\*a))))/(b^5\*d))\*b\*d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acoth}(d + d \tanh(a + bx) + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*acoth(d + d*tanh(a + b*x) + 1), x)`

[Out] `int(x^3*acoth(d + d*tanh(a + b*x) + 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}(d \tanh(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(1+d+d*tanh(b*x+a)), x)`

[Out] `Integral(x**3*acoth(d*tanh(a + b*x) + d + 1), x)`

### 3.208 $\int x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) dx$

Optimal. Leaf size=128

$$-\frac{\text{Li}_4\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^3} + \frac{x\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2} - \frac{x^2\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b} - \frac{1}{6}x^3 \log\left(\left((d+1)e^{2a+2bx} + 1\right)\right) + \frac{1}{3}x$$

[Out] 1/12\*b\*x^4+1/3\*x^3\*arccoth(1+d\*d\*tanh(b\*x+a))-1/6\*x^3\*ln(1+(1+d)\*exp(2\*b\*x+2\*a))-1/4\*x^2\*polylog(2,-(1+d)\*exp(2\*b\*x+2\*a))/b+1/4\*x\*polylog(3,-(1+d)\*exp(2\*b\*x+2\*a))/b^2-1/8\*polylog(4,-(1+d)\*exp(2\*b\*x+2\*a))/b^3

**Rubi [A]** time = 0.27, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{4b^2} - \frac{\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3} - \frac{x^2\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{6}x^3 \log\left(\left((d+1)e^{2a+2bx} + 1\right)\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 + d + d\*Tanh[a + b\*x]],x]

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 + d + d\*Tanh[a + b\*x]])/3 - (x^3\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (x\*PolyLog[3, -((1 + d)\*E^(2\*a + 2\*b\*x))])/(4\*b^2) - PolyLog[4, -((1 + d)\*E^(2\*a + 2\*b\*x))]/(8\*b^3)

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x))))^n], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi



```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_)^(m
_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) + \frac{1}{3} b \int \frac{x^3}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{3} (b(1 + d)) \int \frac{e^{2a+2bx}}{1 + (1 + d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 118, normalized size = 0.92

$$\frac{1}{24} \left( \frac{3 \operatorname{Li}_4\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x \operatorname{Li}_3\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{6x^2 \operatorname{Li}_2\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 4x^3 \log\left(\frac{e^{-2(a+bx)}}{d+1} + 1\right) + 8x^3 \coth^{-1}(d \tanh(a + bx)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (8\*x^3\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - 4\*x^3\*Log[1 + 1/((1 + d)\*E^(2\*(a + b\*x)))] + (6\*x^2\*PolyLog[2, -(1/((1 + d)\*E^(2\*(a + b\*x)))]])/b + (6\*x\*PolyLog[3, -(1/((1 + d)\*E^(2\*(a + b\*x)))]])/b^2 + (3\*PolyLog[4, -(1/((1 + d)\*E^(2\*(a + b\*x)))]])/b^3)/24

**fricas [C]** time = 3.99, size = 381, normalized size = 2.98

$$\frac{b^4 x^4 + 2 b^3 x^3 \log\left(\frac{(d+2) \cosh(bx+a) + d \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)}\right) - 6 b^2 x^2 \operatorname{Li}_2\left(\frac{1}{2} \sqrt{-4d-4} (\cosh(bx+a) + \sinh(bx+a))\right) - 6 b^2 x^2 \operatorname{Li}_2\left(\frac{1}{2} \sqrt{-4d-4} (\cosh(bx+a) - \sinh(bx+a))\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+d\*d\*tanh(b\*x+a)), x, algorithm="fricas")

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*
x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d -
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d - 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d - 4)*(cosh
(b*x + a) + sinh(b*x + a))))/b^3
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(d \tanh(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tanh(b*x + a) + d + 1), x)
```

**maple** [C] time = 5.32, size = 1667, normalized size = 13.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^3-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/
2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/3/b^3/(1+d)*ln(1+(1+d)*e
xp(2*b*x+2*a))*a^3-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3
/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,-(1+d)*
exp(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a
^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/12*I*x^3*Pi*csgn(I*(d*exp(2*b*x+2*
a)+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x
+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2
*b*x+2*a)+1))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)+1))^2+1/12*b*x^4+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^2+1/12*I*
x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/6*I*x^3*Pi*csgn(I*exp(
b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/6/b^3*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(
2*b*x+2*a)+1)-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)+1/12*I*x^3*Pi*csgn(I*d/(
```

```

exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/6*x^3*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/6*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))-1/2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/6/b^3*d*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/6/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))+1/2/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^2+1/3/b^3*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))

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**maxima** [A] time = 1.09, size = 125, normalized size = 0.98

$$\frac{1}{3}x^3 \operatorname{arccoth}(d \tanh(bx + a) + d + 1) + \frac{1}{36} \left( \frac{3x^4}{d} - \frac{2(4b^3x^3 \log((d+1)e^{2bx+2a} + 1) + 6b^2x^2 \operatorname{Li}_2(-(d+1)e^{2bx+2a}))}{b^4d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+d\*d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(d\*tanh(b\*x + a) + d + 1) + 1/36\*(3\*x^4/d - 2\*(4\*b^3\*x^3\*log((d + 1)\*e^(2\*b\*x + 2\*a) + 1) + 6\*b^2\*x^2\*dilog(-(d + 1)\*e^(2\*b\*x + 2\*a)) - 6\*b\*x\*polylog(3, -(d + 1)\*e^(2\*b\*x + 2\*a)) + 3\*polylog(4, -(d + 1)\*e^(2\*b\*x + 2\*a)))/(b^4\*d))\*b\*d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(d + d \tanh(a + bx) + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*acoth(d + d\*tanh(a + b\*x) + 1),x)

[Out] int(x^2\*acoth(d + d\*tanh(a + b\*x) + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(d \tanh(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(1+d*d*tanh(b*x+a)),x)`

[Out] `Integral(x**2*acoth(d*tanh(a + b*x) + d + 1), x)`

### 3.209 $\int x \coth^{-1}(1 + d + d \tanh(a + bx)) dx$

Optimal. Leaf size=101

$$\frac{\operatorname{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^2} - \frac{x\operatorname{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b} - \frac{1}{4}x^2 \log\left(\left(d+1\right)e^{2a+2bx} + 1\right) + \frac{1}{2}x^2 \coth^{-1}\left(d \tanh(a+bx) + d + 1\right)$$

[Out]  $1/6*b*x^3 + 1/2*x^2*\operatorname{arccoth}(1+d+d*\tanh(b*x+a)) - 1/4*x^2*\ln(1+(1+d)*\exp(2*b*x+2*a)) - 1/4*x*\operatorname{polylog}(2, -(1+d)*\exp(2*b*x+2*a))/b + 1/8*\operatorname{polylog}(3, -(1+d)*\exp(2*b*x+2*a))/b^2$

**Rubi [A]** time = 0.23, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6240, 2184, 2190, 2531, 2282, 6589}

$$\frac{\operatorname{PolyLog}\left(3, -(d+1)e^{2a+2bx}\right)}{8b^2} - \frac{x\operatorname{PolyLog}\left(2, -(d+1)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left(\left(d+1\right)e^{2a+2bx} + 1\right) + \frac{1}{2}x^2 \coth^{-1}\left(d \tanh(a+bx) + d + 1\right)$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[1 + d + d*Tanh[a + b*x]], x]`

[Out]  $(b*x^3)/6 + (x^2*\operatorname{ArcCoth}[1 + d + d*\operatorname{Tanh}[a + b*x]])/2 - (x^2*\operatorname{Log}[1 + (1 + d)*E^{(2*a + 2*b*x)}])/4 - (x*\operatorname{PolyLog}[2, -((1 + d)*E^{(2*a + 2*b*x)})])/(4*b) + \operatorname{PolyLog}[3, -((1 + d)*E^{(2*a + 2*b*x)})]/(8*b^2)$

#### Rule 2184

`Int[(((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(((c + d*x)^m*(F^(g*(e + f*x))))^n)/(a + b*(F^(g*(e + f*x))))^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[`

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
)))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6240

Int[ArcCoth[(c\_.) + (d\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_)^(m  
\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tanh[a + b\*x]])/(f\*  
(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d + c\*E^(2\*a  
+ 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -  
d)^2, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(1 + d + d \tanh(a + bx)) dx &= \frac{1}{2} x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) + \frac{1}{2} b \int \frac{x^2}{1 + (1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{2} (b(1 + d)) \int \frac{e^{2a+2bx}}{1 + (1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 + d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 + d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 + d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 + d)e^{2a+2bx})
 \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 91, normalized size = 0.90

$$\frac{2b^2x^2 \left( 2 \coth^{-1}(d \tanh(a + bx) + d + 1) - \log\left(\frac{e^{-2(a+bx)}}{d+1} + 1\right) \right) + 2bx \operatorname{Li}_2\left(-\frac{e^{-2(a+bx)}}{d+1}\right) + \operatorname{Li}_3\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (2\*b^2\*x^2\*(2\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - Log[1 + 1/((1 + d)\*E^(2\*(a + b\*x))])) + 2\*b\*x\*PolyLog[2, -(1/((1 + d)\*E^(2\*(a + b\*x)))] + PolyLog[3, -(1/((1 + d)\*E^(2\*(a + b\*x)))])/(8\*b^2)

**fricas** [C] time = 0.75, size = 322, normalized size = 3.19

$$\frac{2b^3x^3 + 3b^2x^2 \log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6bx \operatorname{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a) + \sinh(bx+a))\right) - 6bx \operatorname{Li}_2\left(-\frac{1}{2}\sqrt{-4d-4}(\cosh(bx+a) + \sinh(bx+a))\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+d\*d\*tanh(b\*x+a)), x, algorithm="fricas")

[Out] 1/12\*(2\*b^3\*x^3 + 3\*b^2\*x^2\*log(((d + 2)\*cosh(b\*x + a) + d\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 6\*b\*x\*dilog(1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 6\*b\*x\*dilog(-1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 3\*a^2\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) + sqrt(-4\*d - 4)) - 3\*a^2\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) - sqrt(-4\*d - 4)) - 3\*(b^2\*x^2 - a^2)\*log(1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 3\*(b^2\*x^2 - a^2)\*log(-1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 6\*polylog(3, 1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 6\*polylog(3, -1/2\*sqrt(-4\*d - 4)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(d \tanh(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+d\*d\*tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*tanh(b\*x + a) + d + 1), x)

**maple** [C] time = 5.09, size = 1584, normalized size = 15.68

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(1+d+d*tanh(b*x+a)),x)`

[Out]  $\frac{1}{8}I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3-1/2*x^2*\ln(exp(b*x+a))-1/4*x^2*\ln(d)+1/6*b*x^3-1/4/b^2*d*a^2/(1+d)*\ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/4*d/(1+d)*\ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/4/b^2*a^2/(1+d)*\ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/2/b^2*d*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b*d/(1+d)*\ln(1+(1+d)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*\ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b*d*a/(1+d)*\ln(1-exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b/(1+d)*\ln(1+(1+d)*exp(2*b*x+2*a))*x*a-1/4/b^2*d/(1+d)*\ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*\ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b*a/(1+d)*\ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^2*d*a^2/(1+d)*\ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d*a^2/(1+d)*\ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/4/b^2/(1+d)*\ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a+1/2/b^2*a^2/(1+d)*\ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a^2/(1+d)*\ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/8/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))-1/4/(1+d)*\ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^3+1/4*x^2*\ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))$

**maxima** [A] time = 1.10, size = 101, normalized size = 1.00

$$\frac{1}{24} \left( \frac{4x^3}{d} - \frac{3(2b^2x^2 \log((d+1)e^{2bx+2a}) + 1) + 2bx \operatorname{Li}_2(-(d+1)e^{2bx+2a}) - \operatorname{Li}_3(-(d+1)e^{2bx+2a}))}{b^3d} \right) bd + \frac{1}{2} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+d\*d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{24} \cdot \left( \frac{4x^3}{d} - 3 \cdot (2b^2x^2 \cdot \log((d+1)e^{2bx+2a}) + 1) + 2bx \cdot \operatorname{dilog}(- (d+1)e^{2bx+2a}) - \operatorname{polylog}(3, -(d+1)e^{2bx+2a})) \right) / (b^3d) + b \cdot d + \frac{1}{2}x^2 \cdot \operatorname{arccoth}(d \cdot \tanh(bx+a) + d + 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(d + d \tanh(a + bx) + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(d + d\*tanh(a + b\*x) + 1),x)

[Out] int(x\*acoth(d + d\*tanh(a + b\*x) + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(d \tanh(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1+d\*d\*tanh(b\*x+a)),x)

[Out] Integral(x\*acoth(d\*tanh(a + b\*x) + d + 1), x)

### 3.210 $\int \coth^{-1}(1 + d + d \tanh(a + bx)) dx$

**Optimal.** Leaf size=69

$$-\frac{\operatorname{Li}_2\left(-\left(d+1\right)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(\left(d+1\right)e^{2a+2bx} + 1\right) + x \coth^{-1}\left(d \tanh(a+bx) + d+1\right) + \frac{bx^2}{2}$$

[Out] 1/2\*b\*x^2+x\*arccoth(1+d+d\*tanh(b\*x+a))-1/2\*x\*ln(1+(1+d)\*exp(2\*b\*x+2\*a))-1/4\*polylog(2,-(1+d)\*exp(2\*b\*x+2\*a))/b

**Rubi [A]** time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6232, 2184, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2,-\left(d+1\right)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(\left(d+1\right)e^{2a+2bx} + 1\right) + x \coth^{-1}\left(d \tanh(a+bx) + d+1\right) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 + d + d\*Tanh[a + b\*x]], x]

[Out] (b\*x^2)/2 + x\*ArcCoth[1 + d + d\*Tanh[a + b\*x]] - (x\*Log[1 + (1 + d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, -((1 + d)\*E^(2\*a + 2\*b\*x))]/(4\*b)

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6232

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] := Simp[x*ArcCoth[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(1 + d + d \tanh(a + bx)) dx &= x \coth^{-1}(1 + d + d \tanh(a + bx)) + b \int \frac{x}{1 + (1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \tanh(a + bx)) - (b(1 + d)) \int \frac{e^{2a+2bx}x}{1 + (1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{2}x \log(1 + (1 + d)e^{2a+2bx}) + \frac{1}{2} \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{2}x \log(1 + (1 + d)e^{2a+2bx}) + \frac{1}{2} \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \tanh(a + bx)) - \frac{1}{2}x \log(1 + (1 + d)e^{2a+2bx}) - \frac{1}{2}
 \end{aligned}$$

**Mathematica [B]** time = 0.91, size = 201, normalized size = 2.91

---


$$-2\text{Li}_2(-\sqrt{-d-1}e^{a+bx}) - 2\text{Li}_2(\sqrt{-d-1}e^{a+bx}) - 2\log(e^{a+bx})\log(1 - \sqrt{-d-1}e^{a+bx}) - 2\log(e^{a+bx})\log(\sqrt{-d-1}e^{a+bx})$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 + d + d*Tanh[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 + d + d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[E^(a + b*x)]*Log[1 - Sqrt[-1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 + Sqrt[-1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x) + (1 + d)*E^(a + b*x)] - 2*b*x*Log[(2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]] - 2*PolyLog[2, -(Sqrt[-1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 - d]*E^(a + b*x)])/(4*b)
```

**fricas** [B] time = 0.97, size = 238, normalized size = 3.45

$$\frac{b^2 x^2 + b x \log\left(\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a \log\left(2(d+1)\cosh(bx+a) + 2(d+1)\sinh(bx+a) + \sqrt{-4d-4}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(b^2*x^2 + b*x*\log(((d + 2)*\cosh(b*x + a) + d*\sinh(b*x + a))/(d*\cosh(b*x + a) + d*\sinh(b*x + a))) + a*\log(2*(d + 1)*\cosh(b*x + a) + 2*(d + 1)*\sinh(b*x + a) + \sqrt{-4*d - 4}) + a*\log(2*(d + 1)*\cosh(b*x + a) + 2*(d + 1)*\sinh(b*x + a) - \sqrt{-4*d - 4}) - (b*x + a)*\log(1/2*\sqrt{-4*d - 4}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - (b*x + a)*\log(-1/2*\sqrt{-4*d - 4}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - \operatorname{dilog}(1/2*\sqrt{-4*d - 4}*(\cosh(b*x + a) + \sinh(b*x + a))) - \operatorname{dilog}(-1/2*\sqrt{-4*d - 4}*(\cosh(b*x + a) + \sinh(b*x + a))))/b$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(d \tanh(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*tanh(b\*x + a) + d + 1), x)

**maple** [B] time = 0.46, size = 247, normalized size = 3.58

$$\frac{\operatorname{arccoth}(1 + d + d \tanh(bx + a)) \ln(d \tanh(bx + a) - d)}{2b} + \frac{\operatorname{arccoth}(1 + d + d \tanh(bx + a)) \ln(d \tanh(bx + a) + d)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+d+d\*tanh(b\*x+a)),x)

[Out]  $-\frac{1}{2}/b*\operatorname{arccoth}(1+d+d*\tanh(b*x+a))*\ln(d*\tanh(b*x+a)-d)+\frac{1}{2}/b*\operatorname{arccoth}(1+d+d*\tanh(b*x+a))*\ln(d*\tanh(b*x+a)+d)-\frac{1}{4}/b*\operatorname{dilog}(1/2*(d*\tanh(b*x+a)+d)/d)-\frac{1}{4}/b*\ln(d*\tanh(b*x+a)-d)*\ln(1/2*(d*\tanh(b*x+a)+d)/d)+\frac{1}{4}/b*\operatorname{dilog}((d*\tanh(b*x+a)+d+2)/(2*d+2))+\frac{1}{4}/b*\ln(d*\tanh(b*x+a)-d)*\ln((d*\tanh(b*x+a)+d+2)/(2*d+2))+\frac{1}{8}/b*\ln(d*\tanh(b*x+a)+d)^2-\frac{1}{4}/b*\operatorname{dilog}(1+1/2*d*\tanh(b*x+a)+1/2*d)-\frac{1}{4}/b*\ln(d*\tanh(b*x+a)+d)*\ln(1+1/2*d*\tanh(b*x+a)+1/2*d)$

**maxima** [A] time = 1.10, size = 72, normalized size = 1.04

$$\frac{1}{4} b d \left( \frac{2x^2}{d} - \frac{2bx \log\left((d+1)e^{2bx+2a} + 1\right) + \operatorname{Li}_2\left(- (d+1)e^{2bx+2a}\right)}{b^2 d} \right) + x \operatorname{arccoth}(d \tanh(bx + a) + d + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d\*d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $1/4*b*d*(2*x^2/d - (2*b*x*\log((d + 1)*e^{(2*b*x + 2*a) + 1)} + \operatorname{dilog}(-(d + 1)*e^{(2*b*x + 2*a)})))/(b^2*d) + x*\operatorname{arccoth}(d*\tanh(b*x + a) + d + 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(d + d \tanh(a + b x) + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(d + d\*tanh(a + b\*x) + 1),x)

[Out] int(acoth(d + d\*tanh(a + b\*x) + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(d \tanh(a + b x) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+d\*d\*tanh(b\*x+a)),x)

[Out] Integral(acoth(d\*tanh(a + b\*x) + d + 1), x)

$$3.211 \quad \int \frac{\coth^{-1}(1+d+d \tanh(a+bx))}{x} dx$$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\coth^{-1}(d \tanh(a + bx) + d + 1)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(1+d+d\*tanh(b\*x+a))/x,x)

**Rubi** [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(1 + d + d \tanh(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1 + d + d \tanh(a + bx))}{x} dx = \int \frac{\coth^{-1}(1 + d + d \tanh(a + bx))}{x} dx$$

**Mathematica** [A] time = 3.35, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1 + d + d \tanh(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 + d + d\*Tanh[a + b\*x]]/x, x]

**fricas** [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(d \tanh(bx + a) + d + 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d\*d\*tanh(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*tanh(b\*x + a) + d + 1)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \tanh(bx + a) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d\*d\*tanh(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*tanh(b\*x + a) + d + 1)/x, x)

**maple** [A] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1 + d + d \tanh(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+d\*d\*tanh(b\*x+a))/x,x)

[Out] int(arccoth(1+d\*d\*tanh(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \tanh(bx + a) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d\*d\*tanh(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*tanh(b\*x + a) + d + 1)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acoth}(d + d \tanh(a + bx) + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(d + d\*tanh(a + b\*x) + 1)/x,x)

[Out] int(acoth(d + d\*tanh(a + b\*x) + 1)/x, x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(d \tanh(a + bx) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+d\*d\*tanh(b\*x+a))/x,x)

[Out] Integral(acoth(d\*tanh(a + b\*x) + d + 1)/x, x)

### 3.212 $\int x^3 \coth^{-1}(1 - d - d \tanh(a + bx)) dx$

**Optimal.** Leaf size=168

$$\frac{3\text{Li}_5\left(-\left(1-d\right)e^{2a+2bx}\right)}{16b^4} - \frac{3x\text{Li}_4\left(-\left(1-d\right)e^{2a+2bx}\right)}{8b^3} + \frac{3x^2\text{Li}_3\left(-\left(1-d\right)e^{2a+2bx}\right)}{8b^2} - \frac{x^3\text{Li}_2\left(-\left(1-d\right)e^{2a+2bx}\right)}{4b} - \frac{1}{8}x^4$$

[Out] 1/20\*b\*x^5+1/4\*x^4\*arccoth(1-d-d\*tanh(b\*x+a))-1/8\*x^4\*ln(1+(1-d)\*exp(2\*b\*x+2\*a))-1/4\*x^3\*polylog(2,-(1-d)\*exp(2\*b\*x+2\*a))/b+3/8\*x^2\*polylog(3,-(1-d)\*exp(2\*b\*x+2\*a))/b^2-3/8\*x\*polylog(4,-(1-d)\*exp(2\*b\*x+2\*a))/b^3+3/16\*polylog(5,-(1-d)\*exp(2\*b\*x+2\*a))/b^4

**Rubi [A]** time = 0.30, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-\left(1-d\right)e^{2a+2bx}\right)}{8b^2} - \frac{3x\text{PolyLog}\left(4,-\left(1-d\right)e^{2a+2bx}\right)}{8b^3} + \frac{3\text{PolyLog}\left(5,-\left(1-d\right)e^{2a+2bx}\right)}{16b^4} - \frac{x^3\text{PolyLog}\left(2,-\left(1-d\right)e^{2a+2bx}\right)}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[1 - d - d\*Tanh[a + b\*x]],x]

[Out] (b\*x^5)/20 + (x^4\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/4 - (x^4\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/8 - (x^3\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (3\*x^2\*PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(8\*b^2) - (3\*x\*PolyLog[4, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(8\*b^3) + (3\*PolyLog[5, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(16\*b^4)

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(1-d-d \tanh(a+bx)) dx &= \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) + \frac{1}{4}b \int \frac{x^4}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{4}(b(1-d)) \int \frac{e^{2a+2bx}}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{8}x^4 \log(1+(1-d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 144, normalized size = 0.86

$$\frac{1}{16} \left( \frac{3\text{Li}_5\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{6x\text{Li}_4\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2\text{Li}_3\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{4x^3\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 2x^4 \log\left(1 - \frac{e^{-2(a+bx)}}{d-1}\right) + 4x^4 \coth^{-1}\left(\frac{1-d-d \tanh(a+bx)}{1-d}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[1-d-d\*Tanh[a+b\*x]],x]

[Out] (4\*x^4\*ArcCoth[1-d-d\*Tanh[a+b\*x]] - 2\*x^4\*Log[1-1/((-1+d)\*E^(2\*(a+b\*x))]) + (4\*x^3\*PolyLog[2,1/((-1+d)\*E^(2\*(a+b\*x))])/b + (6\*x^2\*PolyLog[3,1/((-1+d)\*E^(2\*(a+b\*x))])/b^2 + (6\*x\*PolyLog[4,1/((-1+d)\*E^(2\*(a+b\*x))])/b^3 + (3\*PolyLog[5,1/((-1+d)\*E^(2\*(a+b\*x))])/b^4)/6

**fricas [C]** time = 0.69, size = 423, normalized size = 2.52

$$\frac{2b^5x^5 - 5b^4x^4 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) - 20b^3x^3 \text{Li}_2\left(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right) - 20b^3x^3 \text{Li}_2\left(\frac{1-d-d \tanh(a+bx)}{1-d}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out] 1/40\*(2\*b^5\*x^5 - 5\*b^4\*x^4\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/((d - 2)\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(-sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*a^4\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + 2\*sqrt(d - 1)) - 5\*a^4\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - 2\*sqrt(d - 1)) + 60\*b^2\*x^2\*polylog(3, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 60\*b^2\*x^2\*polylog(3, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*(b^4\*x^4 - a^4)\*log(sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 5\*(b^4\*x^4 - a^4)\*log(-sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 120\*polylog(5, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 120\*polylog(5, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^4

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(-d \tanh(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^3\*arccoth(-d\*tanh(b\*x + a) - d + 1), x)

maple [C] time = 5.43, size = 1802, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(1-d-d\*tanh(b\*x+a)),x)

[Out] 1/16\*I\*x^4\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*exp(2\*b\*x+2\*a))\*csgn(I\*exp(2\*b\*x+2\*a)/(exp(2\*b\*x+2\*a)+1))+1/16\*I\*x^4\*Pi\*csgn(I\*d)\*csgn(I\*exp(2\*b\*x+2\*a))/(exp(2\*b\*x+2\*a)+1))\*csgn(I\*d/(exp(2\*b\*x+2\*a)+1)\*exp(2\*b\*x+2\*a))-1/4/b^4\*d/(d-1)\*polylog(2,(d-1)\*exp(2\*b\*x+2\*a))\*a^3+1/2/b^4\*d\*a^4/(d-1)\*ln(1-exp(b\*x+a)\*(d-1)^(1/2))+1/2/b^4\*d\*a^4/(d-1)\*ln(1+exp(b\*x+a)\*(d-1)^(1/2))-1/8\*I\*x^4\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)+1)\*(d\*exp(2\*b\*x+2\*a)-exp(2\*b\*x+2\*a)-1))^2-1/8\*d/(d-1)\*ln(1-(d-1)\*exp(2\*b\*x+2\*a))\*x^4-3/8/b^2/(d-1)\*polylog(3,(d-1)\*exp(2\*b\*x+2\*a))\*x^2+3/8/b^3/(d-1)\*polylog(4,(d-1)\*exp(2\*b\*x+2\*a))\*x-1/2/b^4\*a^4/(d-1)\*ln(1-exp(b\*x+a)\*(d-1)^(1/2))-1/2/b^4\*a^4/(d-1)\*ln(1+exp(b\*x+a)\*(d-1)^(1/2))+3/16/b^4\*d/(d-1)\*polylog(5,(d-1)\*exp(2\*b\*x+2\*a))-1/2/b^4\*a^3/(d-1)\*dilog(1-exp(b\*x+a)\*(d-1)^(1/2))-1/2/b^4\*a^3/(d-1)\*dilog(1+exp(b\*x+a)\*(d-1)^(1/2))+3/8/b^4/(d-1)\*ln(1-(d-1)\*exp(2\*b\*x+2\*a))\*a^4+1/4/b/(d-1)\*polylog(2,(d-1)

```

*exp(2*b*x+2*a))*x^3+1/4/b^4/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^3+1/20
*b*x^5+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+
2*a)-1))^3-1/8/b^4*d*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)-1/16*I
*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3+1/8/b^4*a^4/(d-1)*ln(
d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+1/8/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^4
-3/16/b^4/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*a))-1/2/b^3*a^3/(d-1)*ln(1+exp(
b*x+a)*(d-1)^(1/2))*x-3/8/b^4*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^4-1/4/b*
d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*
ln(d)+1/8*x^4*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+1/2/b^3/(d-1)*ln(1-(d-1
)*exp(2*b*x+2*a))*x*a^3+1/2/b^4*d*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))
+3/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(d-1)*polylo
g(4,(d-1)*exp(2*b*x+2*a))*x+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1
/2))-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x+1/16*I*x^4*Pi*csgn(I*
exp(2*b*x+2*a))^3+1/8*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^
2+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/16*I*x^4*Pi*c
sgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp
(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2-1/16*I*x^4*
Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I
*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-
1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x
+2*a)+1)*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*
b*x+2*a))-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/16*I*x
^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)
)*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))-1/2/b^3*d/
(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*
(d-1)^(1/2))*x+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x

```

**maxima** [A] time = 1.10, size = 146, normalized size = 0.87

$$-\frac{1}{4}x^4 \operatorname{arccoth}(d \tanh(bx+a) + d - 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4x^4 \log(-(d-1)e^{(2bx+2a)} + 1) + 4b^3x^3 \operatorname{Li}_2((d-1)e^{(2bx+2a)}))}{(b^5d)} \right) * b * d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d\*d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out] -1/4\*x^4\*arccoth(d\*tanh(b\*x + a) + d - 1) + 1/40\*(2\*x^5/d - 5\*(2\*b^4\*x^4\*log(-(d - 1)\*e^(2\*b\*x + 2\*a) + 1) + 4\*b^3\*x^3\*dilog((d - 1)\*e^(2\*b\*x + 2\*a)) - 6\*b^2\*x^2\*polylog(3, (d - 1)\*e^(2\*b\*x + 2\*a)) + 6\*b\*x\*polylog(4, (d - 1)\*e^(2\*b\*x + 2\*a)) - 3\*polylog(5, (d - 1)\*e^(2\*b\*x + 2\*a)))/(b^5\*d))\*b\*d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^3 \operatorname{acoth}(d + d \tanh(a + bx) - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x^3*acoth(d + d*tanh(a + b*x) - 1), x)`

[Out] `int(-x^3*acoth(d + d*tanh(a + b*x) - 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}(-d \tanh(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(1-d-d*tanh(b*x+a)), x)`

[Out] `Integral(x**3*acoth(-d*tanh(a + b*x) - d + 1), x)`

### 3.213 $\int x^2 \coth^{-1}(1 - d - d \tanh(a + bx)) dx$

Optimal. Leaf size=139

$$-\frac{\text{Li}_4\left(-\left(1-d\right)e^{2a+2bx}\right)}{8b^3} + \frac{x\text{Li}_3\left(-\left(1-d\right)e^{2a+2bx}\right)}{4b^2} - \frac{x^2\text{Li}_2\left(-\left(1-d\right)e^{2a+2bx}\right)}{4b} - \frac{1}{6}x^3 \log\left(\left(1-d\right)e^{2a+2bx} + 1\right) + \frac{1}{3}x^3$$

[Out] 1/12\*b\*x^4+1/3\*x^3\*arccoth(1-d-d\*tanh(b\*x+a))-1/6\*x^3\*ln(1+(1-d)\*exp(2\*b\*x+2\*a))-1/4\*x^2\*polylog(2,-(1-d)\*exp(2\*b\*x+2\*a))/b+1/4\*x\*polylog(3,-(1-d)\*exp(2\*b\*x+2\*a))/b^2-1/8\*polylog(4,-(1-d)\*exp(2\*b\*x+2\*a))/b^3

**Rubi [A]** time = 0.27, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6240, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-\left(1-d\right)e^{2a+2bx}\right)}{4b^2} - \frac{\text{PolyLog}\left(4,-\left(1-d\right)e^{2a+2bx}\right)}{8b^3} - \frac{x^2\text{PolyLog}\left(2,-\left(1-d\right)e^{2a+2bx}\right)}{4b} - \frac{1}{6}x^3 \log\left(\left(1-d\right)e^{2a+2bx} + 1\right) + \frac{1}{3}x^3$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]],x]

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/3 - (x^3\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + (x\*PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b^2) - PolyLog[4, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(8\*b^3)

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi



```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6240

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_)^(m
_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1-d-d \tanh(a+bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) + \frac{1}{3}b \int \frac{x^3}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{3}(b(1-d)) \int \frac{e^{2a+2bx}}{1+(1-d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \tanh(a+bx)) - \frac{1}{6}x^3 \log(1+(1-d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 119, normalized size = 0.86

$$\frac{1}{24} \left( \frac{3\text{Li}_4\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x\text{Li}_3\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{6x^2\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3 \log\left(1 - \frac{e^{-2(a+bx)}}{d-1}\right) + 8x^3 \coth^{-1}(d(-\tanh(a+bx))) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]], x]

[Out] (8\*x^3\*ArcCoth[1 - d - d\*Tanh[a + b\*x]] - 4\*x^3\*Log[1 - 1/((-1 + d)\*E^(2\*(a + b\*x))]) + (6\*x^2\*PolyLog[2, 1/((-1 + d)\*E^(2\*(a + b\*x))])/b + (6\*x\*PolyLog[3, 1/((-1 + d)\*E^(2\*(a + b\*x))])/b^2 + (3\*PolyLog[4, 1/((-1 + d)\*E^(2\*(a + b\*x))])/b^3)/24

**fricas [C]** time = 1.86, size = 359, normalized size = 2.58

$$\frac{b^4 x^4 - 2 b^3 x^3 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) - 6 b^2 x^2 \text{Li}_2\left(\sqrt{d-1} (\cosh(bx+a) + \sinh(bx+a))\right) - 6 b^2 x^2 \text{Li}_2\left(-\sqrt{d-1} (\cosh(bx+a) + \sinh(bx+a))\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-d-d\*tanh(b\*x+a)), x, algorithm="fricas")

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*
cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(
b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) +
2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a
) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
+ a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)))
- 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 1
2*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -
sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(-d \tanh(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*tanh(b*x + a) - d + 1), x)
```

**maple** [C] time = 5.31, size = 1745, normalized size = 12.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-d-d*tanh(b*x+a)),x)
```

```
[Out] -1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3+1/12*I*x^3*Pi*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/6*x^3*ln(d*exp(2*b*x+2*a)-exp
(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2
*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+1/8/b^3/(d-1)*polylog(4,(d-1)
*exp(2*b*x+2*a))+1/6/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3+1/2/b^3*a^2/(d-1)
*dilog(1-exp(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)
^(1/2))-1/3/b^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(2,
(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^2
-1/4/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1-exp
(b*x+a)*(d-1)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/8/b^3
*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))-1/6*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2
*a))*x^3+1/6*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/6*I*x
^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+1/12
*b*x^4-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^2/(d
-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/3/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2
a))*a^3-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*p
```

$$\text{polylog}(2, (d-1) \exp(2bx+2a)) a^{2+1/4} / b^{2d} / (d-1) \text{polylog}(3, (d-1) \exp(2bx+2a)) x^{-1/2} / b^{2d} / (d-1) \ln(1 - (d-1) \exp(2bx+2a)) x a^{2+1/2} / b^{2a} / (d-1) \ln(1 - \exp(bx+a) (d-1)^{1/2}) x + 1/2 / b^{2a} / (d-1) \ln(1 + \exp(bx+a) (d-1)^{1/2}) x - 1/2 / b^{3d} a^3 / (d-1) \ln(1 - \exp(bx+a) (d-1)^{1/2}) - 1/2 / b^{3d} a^3 / (d-1) \ln(1 + \exp(bx+a) (d-1)^{1/2}) - 1/12 I x^3 \text{Pisgn}(I d) \text{csgn}(I d / (\exp(2bx+2a) + 1)) \exp(2bx+2a)^{-2} - 1/12 I x^3 \text{Pisgn}(I / (\exp(2bx+2a) + 1)) \text{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a) + 1))^{-2} + 1/12 I x^3 \text{Pisgn}(I / (\exp(2bx+2a) + 1)) (d \exp(2bx+2a) - \exp(2bx+2a) - 1)^{-3} - 1/3 x^3 \ln(\exp(bx+a)) - 1/6 x^3 \ln(d) + 1/12 I x^3 \text{Pisgn}(I \exp(2bx+2a))^{-3} + 1/12 I x^3 \text{Pisgn}(I \exp(bx+a))^{-2} \text{csgn}(I \exp(2bx+2a)) - 1/6 I x^3 \text{Pisgn}(I \exp(bx+a)) \text{csgn}(I \exp(2bx+2a))^{-2} - 1/12 I x^3 \text{Pisgn}(I \exp(2bx+2a) / (\exp(2bx+2a) + 1)) \text{csgn}(I d / (\exp(2bx+2a) + 1)) \exp(2bx+2a)^{-2} - 1/12 I x^3 \text{Pisgn}(I \exp(2bx+2a)) \text{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a) + 1))^{-2} - 1/6 / b^{3a} a^3 / (d-1) \ln(d \exp(2bx+2a) - \exp(2bx+2a) - 1) + 1/6 / b^{3d} a^3 / (d-1) \ln(d \exp(2bx+2a) - \exp(2bx+2a) - 1) + 1/12 I x^3 \text{Pisgn}(I (d \exp(2bx+2a) - \exp(2bx+2a) - 1)) \text{csgn}(I / (\exp(2bx+2a) + 1)) (d \exp(2bx+2a) - \exp(2bx+2a) - 1))^{-2} + 1/12 I x^3 \text{Pisgn}(I d) \text{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a) + 1)) \text{csgn}(I d / (\exp(2bx+2a) + 1)) \exp(2bx+2a) - 1/2 / b^{2d} a^2 / (d-1) \ln(1 + \exp(bx+a) (d-1)^{1/2}) x + 1/2 / b^{2d} / (d-1) \ln(1 - (d-1) \exp(2bx+2a)) x a^{2-1/2} / b^{2d} a^2 / (d-1) \ln(1 - \exp(bx+a) (d-1)^{1/2}) x + 1/12 I x^3 \text{Pisgn}(I / (\exp(2bx+2a) + 1)) \text{csgn}(I \exp(2bx+2a)) \text{csgn}(I \exp(2bx+2a) / (\exp(2bx+2a) + 1)) - 1/12 I x^3 \text{Pisgn}(I / (\exp(2bx+2a) + 1)) \text{csgn}(I (d \exp(2bx+2a) - \exp(2bx+2a) - 1)) \text{csgn}(I / (\exp(2bx+2a) + 1)) (d \exp(2bx+2a) - \exp(2bx+2a) - 1))$$

**maxima** [A] time = 1.11, size = 123, normalized size = 0.88

$$-\frac{1}{3} x^3 \operatorname{arccoth}(d \tanh(bx + a) + d - 1) + \frac{1}{36} \left( \frac{3x^4}{d} - \frac{2(4b^3x^3 \log(-(d-1)e^{2bx+2a}) + 1) + 6b^2x^2 \operatorname{Li}_2((d-1)e^{2bx+2a})}{b^4d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-d\*d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out] -1/3\*x^3\*arccoth(d\*tanh(b\*x + a) + d - 1) + 1/36\*(3\*x^4/d - 2\*(4\*b^3\*x^3\*log(-(d - 1)\*e^(2\*b\*x + 2\*a) + 1) + 6\*b^2\*x^2\*dilog((d - 1)\*e^(2\*b\*x + 2\*a)) - 6\*b\*x\*polylog(3, (d - 1)\*e^(2\*b\*x + 2\*a)) + 3\*polylog(4, (d - 1)\*e^(2\*b\*x + 2\*a)))/(b^4\*d))\*b\*d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2 \operatorname{acoth}(d + d \tanh(a + bx) - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^2\*acoth(d + d\*tanh(a + b\*x) - 1),x)

[Out] `int(-x^2*acoth(d + d*tanh(a + b*x) - 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(-d \tanh(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(1-d-d*tanh(b*x+a)), x)`

[Out] `Integral(x**2*acoth(-d*tanh(a + b*x) - d + 1), x)`

### 3.214 $\int x \coth^{-1}(1 - d - d \tanh(a + bx)) dx$

**Optimal.** Leaf size=110

$$\frac{\text{Li}_3\left(-\left(1-d\right)e^{2a+2bx}\right)}{8b^2} - \frac{x\text{Li}_2\left(-\left(1-d\right)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left(\left(1-d\right)e^{2a+2bx} + 1\right) + \frac{1}{2}x^2 \coth^{-1}\left(d(-\tanh(a+bx)) - d\right)$$

[Out] 1/6\*b\*x^3+1/2\*x^2\*arccoth(1-d-d\*tanh(b\*x+a))-1/4\*x^2\*ln(1+(1-d)\*exp(2\*b\*x+2\*a))-1/4\*x\*polylog(2,-(1-d)\*exp(2\*b\*x+2\*a))/b+1/8\*polylog(3,-(1-d)\*exp(2\*b\*x+2\*a))/b^2

**Rubi [A]** time = 0.24, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {6240, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-\left(1-d\right)e^{2a+2bx}\right)}{8b^2} - \frac{x\text{PolyLog}\left(2,-\left(1-d\right)e^{2a+2bx}\right)}{4b} - \frac{1}{4}x^2 \log\left(\left(1-d\right)e^{2a+2bx} + 1\right) + \frac{1}{2}x^2 \coth^{-1}\left(d(-\tanh(a+bx)) - d\right)$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 - d - d\*Tanh[a + b\*x]],x]

[Out] (b\*x^3)/6 + (x^2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]])/2 - (x^2\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/4 - (x\*PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))])/(4\*b) + PolyLog[3, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(8\*b^2)

#### Rule 2184

Int[(((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x))))^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))  
^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6240

Int[ArcCoth[(c\_.) + (d\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_)^(m  
\_.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Tanh[a + b\*x]])/(f\*  
(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d + c\*E^(2\*a  
+ 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -  
d)^2, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(1 - d - d \tanh(a + bx)) dx &= \frac{1}{2} x^2 \coth^{-1}(1 - d - d \tanh(a + bx)) + \frac{1}{2} b \int \frac{x^2}{1 + (1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{2} (b(1 - d)) \int \frac{e^{2a+2bx}}{1 + (1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 - d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 - d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 - d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 - d)e^{2a+2bx})
 \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 93, normalized size = 0.85

$$\frac{2b^2x^2 \left( 2 \coth^{-1}(d(-\tanh(a+bx)) - d + 1) - \log\left(1 - \frac{e^{-2(a+bx)}}{d-1}\right) \right) + 2bx\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d-1}\right) + \text{Li}_3\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 - d - d\*Tanh[a + b\*x]], x]

[Out] (2\*b^2\*x^2\*(2\*ArcCoth[1 - d - d\*Tanh[a + b\*x]] - Log[1 - 1/((-1 + d)\*E^(2\*(a + b\*x))])) + 2\*b\*x\*PolyLog[2, 1/((-1 + d)\*E^(2\*(a + b\*x))]] + PolyLog[3, 1/((-1 + d)\*E^(2\*(a + b\*x))]])/(8\*b^2)

**fricas** [C] time = 1.01, size = 305, normalized size = 2.77

$$\frac{2b^3x^3 - 3b^2x^2 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) - 6bx\text{Li}_2\left(\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right) - 6bx\text{Li}_2\left(-\sqrt{d-1}(\cosh(bx+a) + \sinh(bx+a))\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*tanh(b\*x+a)), x, algorithm="fricas")

[Out] 1/12\*(2\*b^3\*x^3 - 3\*b^2\*x^2\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/((d - 2)\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 6\*b\*x\*dilog(sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 6\*b\*x\*dilog(-sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 3\*a^2\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + 2\*sqrt(d - 1)) - 3\*a^2\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - 2\*sqrt(d - 1)) - 3\*(b^2\*x^2 - a^2)\*log(sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 3\*(b^2\*x^2 - a^2)\*log(-sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 6\*polylog(3, sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 6\*polylog(3, -sqrt(d - 1)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(-d \tanh(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*tanh(b\*x+a)), x, algorithm="giac")

[Out] integrate(x\*arccoth(-d\*tanh(b\*x + a) - d + 1), x)

**maple** [C] time = 4.90, size = 1664, normalized size = 15.13

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(1-d-d*tanh(b*x+a)),x)`

[Out]  $\frac{1}{4}b^{-2}(d-1)\text{polylog}(2,(d-1)\exp(2bx+2a))^a - \frac{1}{2}b^{-2}a^2(d-1)\ln(1-\exp(bx+a)(d-1)^{1/2}) - \frac{1}{2}b^{-2}a^2(d-1)\ln(1+\exp(bx+a)(d-1)^{1/2}) + \frac{1}{8}b^{-2}d(d-1)\text{polylog}(3,(d-1)\exp(2bx+2a)) - \frac{1}{2}b^{-2}a(d-1)\text{dilog}(1-\exp(bx+a)(d-1)^{1/2}) - \frac{1}{2}b^{-2}a(d-1)\text{dilog}(1+\exp(bx+a)(d-1)^{1/2}) + \frac{1}{4}b^{-2}(d-1)\ln(1-(d-1)\exp(2bx+2a))^a + \frac{1}{4}b(d-1)\text{polylog}(2,(d-1)\exp(2bx+2a))^x - \frac{1}{4}I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)+1)(d\exp(2bx+2a)-\exp(2bx+2a)-1))^2 - \frac{1}{2}x^2\ln(\exp(bx+a)) - \frac{1}{4}x^2\ln(d) - \frac{1}{8}I^2x^2\text{Picsgn}(I^2d/(\exp(2bx+2a)+1)\exp(2bx+2a))^3 + \frac{1}{8}I^2x^2\text{Picsgn}(I\exp(2bx+2a))^3 + \frac{1}{8}I^2x^2\text{Picsgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^3 + \frac{1}{4}(d-1)\ln(1-(d-1)\exp(2bx+2a))^x - \frac{1}{8}b^{-2}(d-1)\text{polylog}(3,(d-1)\exp(2bx+2a)) + \frac{1}{4}b^{-2}a^2(d-1)\ln(d\exp(2bx+2a)-\exp(2bx+2a)-1) + \frac{1}{8}I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)+1)(d\exp(2bx+2a)-\exp(2bx+2a)-1))^3 + \frac{1}{6}b^2x^3 + \frac{1}{2}b^{-2}d^2a^2(d-1)\ln(1-\exp(bx+a)(d-1)^{1/2}) + \frac{1}{2}b^{-2}d^2a^2(d-1)\ln(1+\exp(bx+a)(d-1)^{1/2}) + \frac{1}{2}b^{-2}d^2a(d-1)\text{dilog}(1-\exp(bx+a)(d-1)^{1/2}) + \frac{1}{2}b^{-2}d^2a(d-1)\text{dilog}(1+\exp(bx+a)(d-1)^{1/2}) - \frac{1}{4}b^{-2}d(d-1)\ln(1-(d-1)\exp(2bx+2a))^a - \frac{1}{4}b^{-2}d(d-1)\text{polylog}(2,(d-1)\exp(2bx+2a))^x - \frac{1}{4}b^{-2}d(d-1)\text{polylog}(2,(d-1)\exp(2bx+2a))^a + \frac{1}{2}b(d-1)\ln(1-(d-1)\exp(2bx+2a))^x - \frac{1}{2}b^2a(d-1)\ln(1-\exp(bx+a)(d-1)^{1/2})^x - \frac{1}{2}b^2a(d-1)\ln(1+\exp(bx+a)(d-1)^{1/2})^x - \frac{1}{8}I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)+1))^2 \text{csign}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 - \frac{1}{4}d(d-1)\ln(1-(d-1)\exp(2bx+2a))^x + \frac{1}{4}I^2x^2\text{Picsgn}(I^2d/(\exp(2bx+2a)+1)\exp(2bx+2a))^2 + \frac{1}{8}I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)+1))^2 \text{csign}(I\exp(2bx+2a)/(\exp(2bx+2a)+1)) - \frac{1}{8}I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)+1))^2 \text{csign}(I(d\exp(2bx+2a)-\exp(2bx+2a)-1)) \text{csign}(I/(\exp(2bx+2a)+1)(d\exp(2bx+2a)-\exp(2bx+2a)-1)) - \frac{1}{8}I^2x^2\text{Picsgn}(I\exp(2bx+2a))^2 \text{csign}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 + \frac{1}{4}x^2\ln(d\exp(2bx+2a)-\exp(2bx+2a)-1) + \frac{1}{8}I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)+1)(d\exp(2bx+2a)-\exp(2bx+2a)-1))^2 + \frac{1}{8}I^2x^2\text{Picsgn}(I(d\exp(2bx+2a)-\exp(2bx+2a)-1)) \text{csign}(I/(\exp(2bx+2a)+1)(d\exp(2bx+2a)-\exp(2bx+2a)-1))^2 + \frac{1}{2}b^2d^2a(d-1)\ln(1+\exp(bx+a)(d-1)^{1/2})^x - \frac{1}{4}b^{-2}d^2a^2(d-1)\ln(d\exp(2bx+2a)-\exp(2bx+2a)-1) - \frac{1}{8}I^2x^2\text{Picsgn}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 \text{csign}(I^2d/(\exp(2bx+2a)+1)\exp(2bx+2a))^2 - \frac{1}{8}I^2x^2\text{Picsgn}(I^2d)\text{csign}(I^2d/(\exp(2bx+2a)+1)\exp(2bx+2a))^2 + \frac{1}{8}I^2x^2\text{Picsgn}(I\exp(bx+a))^2 \text{csign}(I\exp(2bx+2a)) - \frac{1}{4}I^2x^2\text{Picsgn}(I\exp(bx+a))^2 \text{csign}(I\exp(2bx+2a))^2 + \frac{1}{8}I^2x^2\text{Picsgn}(I^2d)\text{csign}(I\exp(2bx+2a)/(\exp(2bx+2a)+1))^2 \text{csign}(I^2d/(\exp(2bx+2a)+1)\exp(2bx+2a)) - \frac{1}{2}b^2d(d-1)\ln(1-(d-1)\exp(2bx+2a))^x + \frac{1}{2}b^2d^2a(d-1)\ln(1-\exp(bx+a)(d-1)^{1/2})^x$

**maxima** [A] time = 1.10, size = 100, normalized size = 0.91

$$\frac{1}{24} \left( \frac{4x^3}{d} - \frac{3(2b^2x^2 \log(-(d-1)e^{(2bx+2a)} + 1) + 2bx\text{Li}_2((d-1)e^{(2bx+2a)}) - \text{Li}_3((d-1)e^{(2bx+2a)}))}{b^3d} \right) bd - \frac{1}{2} x^2 \ar$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{24} \cdot \left( \frac{4x^3}{d} - 3 \cdot (2b^2x^2 \log(-(d-1)e^{(2bx+2a)} + 1) + 2bx \operatorname{dilog}((d-1)e^{(2bx+2a)}) - \operatorname{polylog}(3, (d-1)e^{(2bx+2a)})) / (b^3d) \right) \cdot b \cdot d - \frac{1}{2} x^2 \operatorname{arccoth}(d \tanh(bx+a) + d - 1)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x \operatorname{acoth}(d + d \tanh(a + bx) - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x\*acoth(d + d\*tanh(a + b\*x) - 1),x)

[Out] int(-x\*acoth(d + d\*tanh(a + b\*x) - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(-d \tanh(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1-d-d\*tanh(b\*x+a)),x)

[Out] Integral(x\*acoth(-d\*tanh(a + b\*x) - d + 1), x)

### 3.215 $\int \coth^{-1}(1 - d - d \tanh(a + bx)) dx$

**Optimal.** Leaf size=76

$$-\frac{\operatorname{Li}_2\left(-\left(1-d\right)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(\left(1-d\right)e^{2a+2bx} + 1\right) + x \coth^{-1}\left(d(-\tanh(a+bx)) - d + 1\right) + \frac{bx^2}{2}$$

[Out] 1/2\*b\*x^2+x\*arccoth(1-d-d\*tanh(b\*x+a))-1/2\*x\*ln(1+(1-d)\*exp(2\*b\*x+2\*a))-1/4\*polylog(2,-(1-d)\*exp(2\*b\*x+2\*a))/b

**Rubi [A]** time = 0.15, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6232, 2184, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2, -\left(1-d\right)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(\left(1-d\right)e^{2a+2bx} + 1\right) + x \coth^{-1}\left(d(-\tanh(a+bx)) - d + 1\right) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 - d - d\*Tanh[a + b\*x]], x]

[Out] (b\*x^2)/2 + x\*ArcCoth[1 - d - d\*Tanh[a + b\*x]] - (x\*Log[1 + (1 - d)\*E^(2\*a + 2\*b\*x)])/2 - PolyLog[2, -((1 - d)\*E^(2\*a + 2\*b\*x))]/(4\*b)

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6232

```
Int[ArcCoth[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcCoth[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(1 - d - d \tanh(a + bx)) dx &= x \coth^{-1}(1 - d - d \tanh(a + bx)) + b \int \frac{x}{1 + (1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \tanh(a + bx)) - (b(1 - d)) \int \frac{e^{2a+2bx} x}{1 + (1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{2} x \log(1 + (1 - d)e^{2a+2bx}) + \frac{1}{2} \operatorname{Li}_2(-\sqrt{d-1} e^{a+bx}) \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{2} x \log(1 + (1 - d)e^{2a+2bx}) + \frac{1}{2} \operatorname{Li}_2(\sqrt{d-1} e^{a+bx}) \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \tanh(a + bx)) - \frac{1}{2} x \log(1 + (1 - d)e^{2a+2bx}) - \frac{1}{2} \operatorname{Li}_2(-\sqrt{d-1} e^{a+bx})
 \end{aligned}$$

**Mathematica [B]** time = 0.98, size = 200, normalized size = 2.63

---


$$-2\operatorname{Li}_2(-\sqrt{d-1} e^{a+bx}) - 2\operatorname{Li}_2(\sqrt{d-1} e^{a+bx}) - 2 \log(e^{a+bx}) \log(1 - \sqrt{d-1} e^{a+bx}) - 2 \log(e^{a+bx}) \log(\sqrt{d-1} e^{a+bx})$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[1 - d - d*Tanh[a + b*x]], x]
```

```
[Out] x*ArcCoth[1 - d - d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[E^(a + b*x)]*Log[1 - Sqrt[-1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 + Sqrt[-1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (-1 + d)*E^(2*(a + b*x)))] - 2*b*x*Log[(-2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]] - 2*PolyLog[2, -(Sqrt[-1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 + d]*E^(a + b*x)])/(4*b)
```

**fricas** [B] time = 0.80, size = 227, normalized size = 2.99

$$\frac{b^2 x^2 - bx \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{(d-2) \cosh(bx+a) + d \sinh(bx+a)}\right) + a \log\left(2(d-1) \cosh(bx+a) + 2(d-1) \sinh(bx+a) + 2\sqrt{d-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(b^2*x^2 - b*x*\log((d*\cosh(b*x + a) + d*\sinh(b*x + a))/((d - 2)*\cosh(b*x + a) + d*\sinh(b*x + a))) + a*\log(2*(d - 1)*\cosh(b*x + a) + 2*(d - 1)*\sinh(b*x + a) + 2*\sqrt{d - 1}) + a*\log(2*(d - 1)*\cosh(b*x + a) + 2*(d - 1)*\sinh(b*x + a) - 2*\sqrt{d - 1}) - (b*x + a)*\log(\sqrt{d - 1}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - (b*x + a)*\log(-\sqrt{d - 1}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - \operatorname{dilog}(\sqrt{d - 1}*(\cosh(b*x + a) + \sinh(b*x + a))) - \operatorname{dilog}(-\sqrt{d - 1}*(\cosh(b*x + a) + \sinh(b*x + a))))/b$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(-d \tanh(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(-d\*tanh(b\*x + a) - d + 1), x)

**maple** [B] time = 0.40, size = 271, normalized size = 3.57

$$\frac{\operatorname{arccoth}(1 - d - d \tanh(bx + a)) \ln(-d \tanh(bx + a) + d)}{2b} + \frac{\operatorname{arccoth}(1 - d - d \tanh(bx + a)) \ln(-d \tanh(bx + a) - d)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-d-d\*tanh(b\*x+a)),x)

[Out]  $-1/2/b*\operatorname{arccoth}(1-d-d*\tanh(b*x+a))*\ln(-d*\tanh(b*x+a)+d)+1/2/b*\operatorname{arccoth}(1-d-d*\tanh(b*x+a))*\ln(-d*\tanh(b*x+a)-d)+1/8/b*\ln(-d*\tanh(b*x+a)-d)^2-1/4/b*\operatorname{dilog}(1-1/2*d*\tanh(b*x+a)-1/2*d)-1/4/b*\ln(-d*\tanh(b*x+a)-d)*\ln(1-1/2*d*\tanh(b*x+a)-1/2*d)+1/4/b*\operatorname{dilog}((-d*\tanh(b*x+a)-d+2)/(-2*d+2))+1/4/b*\ln(-d*\tanh(b*x+a)+d)*\ln((-d*\tanh(b*x+a)-d+2)/(-2*d+2))-1/4/b*\operatorname{dilog}(-1/2*(-d*\tanh(b*x+a)-d)/d)-1/4/b*\ln(-d*\tanh(b*x+a)+d)*\ln(-1/2*(-d*\tanh(b*x+a)-d)/d)$

**maxima** [A] time = 1.10, size = 73, normalized size = 0.96

$$\frac{1}{4}bd\left(\frac{2x^2}{d} - \frac{2bx \log\left(-(d-1)e^{2bx+2a} + 1\right) + \operatorname{Li}_2\left((d-1)e^{2bx+2a}\right)}{b^2d}\right) - x \operatorname{arccoth}(d \tanh(bx + a) + d - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{4}bd\left(\frac{2x^2}{d} - (2bx \log(-(d-1)e^{2bx+2a}) + 1) + \operatorname{dilog}((d-1)e^{2bx+2a})\right) - x \operatorname{arccoth}(d \tanh(bx+a) + d - 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\operatorname{acoth}(d + d \tanh(a + bx) - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-acoth(d + d\*tanh(a + b\*x) - 1),x)

[Out] int(-acoth(d + d\*tanh(a + b\*x) - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(-d \tanh(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-d-d\*tanh(b\*x+a)),x)

[Out] Integral(acoth(-d\*tanh(a + b\*x) - d + 1), x)

$$3.216 \quad \int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx$$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\coth^{-1}(d(-\tanh(a+bx))-d+1)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(1-d-d\*tanh(b\*x+a))/x,x)

**Rubi** [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx = \int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx$$

**Mathematica** [A] time = 3.39, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1-d-d \tanh(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 - d - d\*Tanh[a + b\*x]]/x, x]

**fricas** [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{arccoth}(d \tanh(bx+a)+d-1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(-arccoth(d\*tanh(b\*x + a) + d - 1)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(-d \tanh(bx + a) - d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(-d\*tanh(b\*x + a) - d + 1)/x, x)

**maple** [A] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1 - d - d \tanh(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-d-d\*tanh(b\*x+a))/x,x)

[Out] int(arccoth(1-d-d\*tanh(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\operatorname{arccoth}(d \tanh(bx + a) + d - 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*tanh(b\*x+a))/x,x, algorithm="maxima")

[Out] -integrate(arccoth(d\*tanh(b\*x + a) + d - 1)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{\operatorname{acoth}(d + d \tanh(a + bx) - 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-acoth(d + d\*tanh(a + b\*x) - 1)/x,x)

[Out] int(-acoth(d + d\*tanh(a + b\*x) - 1)/x, x)



sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(-d \tanh(a + bx) - d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-d-d\*tanh(b\*x+a))/x,x)

[Out] Integral(acoth(-d\*tanh(a + b\*x) - d + 1)/x, x)

### 3.217 $\int x^2 \coth^{-1}(c + d \coth(a + bx)) dx$

Optimal. Leaf size=303

$$\frac{\operatorname{Li}_4\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3} - \frac{\operatorname{Li}_4\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3} - \frac{x\operatorname{Li}_3\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2} + \frac{x\operatorname{Li}_3\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2} + \frac{x^2\operatorname{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x^2\operatorname{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

[Out]  $\frac{1}{3}x^3\operatorname{arccoth}(c+d\coth(bx+a)) + \frac{1}{6}x^3\ln(1-(1-c-d)\exp(2bx+2a)/(1-c+d)) - \frac{1}{6}x^3\ln(1-(1+c+d)\exp(2bx+2a)/(1+c-d)) + \frac{1}{4}x^2\operatorname{polylog}(2, (1-c-d)\exp(2bx+2a)/(1-c+d))/b - \frac{1}{4}x^2\operatorname{polylog}(2, (1+c+d)\exp(2bx+2a)/(1+c-d))/b - \frac{1}{4}x\operatorname{polylog}(3, (1-c-d)\exp(2bx+2a)/(1-c+d))/b^2 + \frac{1}{4}x\operatorname{polylog}(3, (1+c+d)\exp(2bx+2a)/(1+c-d))/b^2 + \frac{1}{8}\operatorname{polylog}(4, (1-c-d)\exp(2bx+2a)/(1-c+d))/b^3 - \frac{1}{8}\operatorname{polylog}(4, (1+c+d)\exp(2bx+2a)/(1+c-d))/b^3$

**Rubi [A]** time = 0.47, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6246, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\operatorname{PolyLog}\left(3, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2} + \frac{x\operatorname{PolyLog}\left(3, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2} + \frac{\operatorname{PolyLog}\left(4, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3} - \frac{\operatorname{PolyLog}\left(4, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2\operatorname{ArcCoth}[c + d\operatorname{Coth}[a + bx]], x]$

[Out]  $\frac{x^3\operatorname{ArcCoth}[c + d\operatorname{Coth}[a + bx]]}{3} + \frac{x^3\operatorname{Log}[1 - ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)]}{6} - \frac{x^3\operatorname{Log}[1 - ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)]}{6} + \frac{x^2\operatorname{PolyLog}[2, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)]}{(4*b)} - \frac{x^2\operatorname{PolyLog}[2, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)]}{(4*b)} - \frac{x\operatorname{PolyLog}[3, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)]}{(4*b^2)} + \frac{x\operatorname{PolyLog}[3, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)]}{(4*b^2)} + \frac{\operatorname{PolyLog}[4, ((1 - c - d)E^{(2a + 2bx)})/(1 - c + d)]}{(8*b^3)} - \frac{\operatorname{PolyLog}[4, ((1 + c + d)E^{(2a + 2bx)})/(1 + c - d)]}{(8*b^3)}$

#### Rule 2190

$\operatorname{Int}[(((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)*((c_) + (d_)*(x_))^\wedge(m_))/((a_) + (b_)*((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)), x\_Symbol] \rightarrow \operatorname{Simp}[\frac{(c + dx)^m \operatorname{Log}[1 + (b(F^{g(e+fx)})^n)/a]}{(bfg^n \operatorname{Log}[F])}, x] - \operatorname{Dist}[\frac{(d*m)}{(bfg^n \operatorname{Log}[F])}, \operatorname{Int}[(c + dx)^{m-1} \operatorname{Log}[1 + (b(F^{g(e+fx)})^n)/a]], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6246

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_)^(m
_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(c + d \coth(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(c + d \coth(a + bx)) - \frac{1}{3} (b(1 - c - d)) \int \frac{e^{2a+2bx} x^3}{1 - c + d + (-1 + c + d)e^{2a+2bx}} dx \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.45, size = 353, normalized size = 1.17

$$4b^3 x^3 \log \left( \frac{2(\cosh(a+bx) - \sinh(a+bx))((c-1)\sinh(a+bx) + d \cosh(a+bx))}{c+d-1} \right) - 4b^3 x^3 \log \left( \frac{(c-d+1)(\sinh(2(a+bx)) - \cosh(2(a+bx)))}{c+d+1} + 1 \right) - 6b^2$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*ArcCoth[c + d\*Coth[a + b\*x]],x]

[Out] (x^3\*ArcCoth[c + d\*Coth[a + b\*x]])/3 + (4\*b^3\*x^3\*Log[(2\*(Cosh[a + b\*x] - Sinh[a + b\*x])\*(d\*Cosh[a + b\*x] + (-1 + c)\*Sinh[a + b\*x]))/(-1 + c + d)] - 4\*b^3\*x^3\*Log[1 + ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 6\*b^2\*x^2\*PolyLog[2, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 6\*b^2\*x^2\*PolyLog[2, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 6\*b\*x\*PolyLog[3, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 6\*b\*x\*PolyLog[3, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(1 + c + d)] - 3\*PolyLog[4, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(-1 + c + d)] + 3\*PolyLog[4, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])]/(1 + c + d))]/(24\*b^3)

**fricas [C]** time = 0.56, size = 879, normalized size = 2.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="fricas")`

[Out]  $\frac{1}{6}(b^3x^3 \log((d \cosh(bx+a) + (c+1)\sinh(bx+a))/(d \cosh(bx+a) + (c-1)\sinh(bx+a))) - 3b^2x^2 \operatorname{dilog}(\sqrt{(c+d+1)/(c-d+1)}(\cosh(bx+a) + \sinh(bx+a))) - 3b^2x^2 \operatorname{dilog}(-\sqrt{(c+d+1)/(c-d+1)}(\cosh(bx+a) + \sinh(bx+a))) + 3b^2x^2 \operatorname{dilog}(\sqrt{(c+d-1)/(c-d-1)}(\cosh(bx+a) + \sinh(bx+a))) + 3b^2x^2 \operatorname{dilog}(-\sqrt{(c+d-1)/(c-d-1)}(\cosh(bx+a) + \sinh(bx+a)))) + a^3 \log(2(c+d+1) \cosh(bx+a) + 2(c+d+1) \sinh(bx+a) - 2(c-d+1) \sqrt{(c+d+1)/(c-d+1)}) - a^3 \log(2(c+d-1) \cosh(bx+a) + 2(c+d-1) \sinh(bx+a) - 2(c-d-1) \sqrt{(c+d-1)/(c-d-1)}) + 6bx \operatorname{polylog}(3, \sqrt{(c+d+1)/(c-d+1)}(\cosh(bx+a) + \sinh(bx+a))) + 6bx \operatorname{polylog}(3, -\sqrt{(c+d+1)/(c-d+1)}(\cosh(bx+a) + \sinh(bx+a))) - 6bx \operatorname{polylog}(3, \sqrt{(c+d-1)/(c-d-1)}(\cosh(bx+a) + \sinh(bx+a))) - 6bx \operatorname{polylog}(3, -\sqrt{(c+d-1)/(c-d-1)}(\cosh(bx+a) + \sinh(bx+a))) - (b^3x^3 + a^3) \log(\sqrt{(c+d+1)/(c-d+1)}(\cosh(bx+a) + \sinh(bx+a)) + 1) - (b^3x^3 + a^3) \log(-\sqrt{(c+d+1)/(c-d+1)}(\cosh(bx+a) + \sinh(bx+a)) + 1) + (b^3x^3 + a^3) \log(\sqrt{(c+d-1)/(c-d-1)}(\cosh(bx+a) + \sinh(bx+a)) + 1) + (b^3x^3 + a^3) \log(-\sqrt{(c+d-1)/(c-d-1)}(\cosh(bx+a) + \sinh(bx+a)) + 1) - 6 \operatorname{polylog}(4, \sqrt{(c+d+1)/(c-d+1)}(\cosh(bx+a) + \sinh(bx+a))) - 6 \operatorname{polylog}(4, -\sqrt{(c+d+1)/(c-d+1)}(\cosh(bx+a) + \sinh(bx+a))) + 6 \operatorname{polylog}(4, \sqrt{(c+d-1)/(c-d-1)}(\cosh(bx+a) + \sinh(bx+a))) + 6 \operatorname{polylog}(4, -\sqrt{(c+d-1)/(c-d-1)}(\cosh(bx+a) + \sinh(bx+a))))/b^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(d \coth(bx+a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(d*coth(b*x+a)),x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(d*coth(b*x+a)+c),x)`

**maple** [C] time = 10.36, size = 5222, normalized size = 17.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(c+d*coth(b*x+a)),x)`

[Out] result too large to display

**maxima** [A] time = 0.70, size = 277, normalized size = 0.91

$$\frac{1}{3}x^3 \operatorname{arccoth}(d \operatorname{coth}(bx+a)+c) - \frac{1}{18}bd \left( \frac{4b^3x^3 \log\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1} + 1\right) + 6b^2x^2 \operatorname{Li}_2\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) - 6bx \operatorname{Li}_3\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^4d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")`

[Out]  $\frac{1}{3}x^3 \operatorname{arccoth}(d \operatorname{coth}(bx+a)+c) - \frac{1}{18}bd \left( \frac{4b^3x^3 \log\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1} + 1\right) + 6b^2x^2 \operatorname{dilog}\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) - 6bx \operatorname{polylog}\left(3, \frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right) + 3 \operatorname{polylog}\left(4, \frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)}{b^4d} - \frac{4b^3x^3 \log\left(-\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1} + 1\right) + 6b^2x^2 \operatorname{dilog}\left(\frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right) - 6bx \operatorname{polylog}\left(3, \frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right) + 3 \operatorname{polylog}\left(4, \frac{(c+d-1)e^{(2bx+2a)}}{c-d-1}\right)}{b^4d} \right)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(c + d \operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(c + d*coth(a + b*x)),x)`

[Out] `int(x^2*acoth(c + d*coth(a + b*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(c + d \operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(c+d*coth(b*x+a)),x)`

[Out] `Integral(x**2*acoth(c + d*coth(a + b*x)), x)`

### 3.218 $\int x \coth^{-1}(c + d \coth(a + bx)) dx$

Optimal. Leaf size=229

$$-\frac{\operatorname{Li}_3\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\operatorname{Li}_3\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x\operatorname{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x\operatorname{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{4}x^2 \log\left(1 - \frac{(-c-d+1)}{-c+d+1}\right)$$

[Out]  $1/2*x^2*\operatorname{arccoth}(c+d*\coth(b*x+a))+1/4*x^2*\ln(1-(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))-1/4*x^2*\ln(1-(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))+1/4*x*\operatorname{polylog}(2,(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))/b-1/4*x*\operatorname{polylog}(2,(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))/b-1/8*\operatorname{polylog}(3,(1-c-d)*\exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*\operatorname{polylog}(3,(1+c+d)*\exp(2*b*x+2*a)/(1+c-d))/b^2$

Rubi [A] time = 0.38, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6246, 2190, 2531, 2282, 6589}

$$-\frac{\operatorname{PolyLog}\left(3, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2} + \frac{\operatorname{PolyLog}\left(3, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2} + \frac{x\operatorname{PolyLog}\left(2, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{x\operatorname{PolyLog}\left(2, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{ArcCoth}[c + d*\operatorname{Coth}[a + b*x]], x]$

[Out]  $(x^2*\operatorname{ArcCoth}[c + d*\operatorname{Coth}[a + b*x]])/2 + (x^2*\operatorname{Log}[1 - ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)])/4 - (x^2*\operatorname{Log}[1 - ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)])/4 + (x*\operatorname{PolyLog}[2, ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)])/ (4*b) - (x*\operatorname{PolyLog}[2, ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)])/ (4*b) - \operatorname{PolyLog}[3, ((1 - c - d)*E^{(2*a + 2*b*x)})/(1 - c + d)]/(8*b^2) + \operatorname{PolyLog}[3, ((1 + c + d)*E^{(2*a + 2*b*x)})/(1 + c - d)]/(8*b^2)$

#### Rule 2190

$\operatorname{Int}[(((F_)^{((g_)*(e_)+(f_)*(x_)))})^{(n_)*((c_)+(d_)*(x_))^{(m_)}})/((a_)+(b_)*((F_)^{((g_)*(e_)+(f_)*(x_)))})^{(n_)}), x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^m*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))^n)/a)]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-1)}*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))^n)/a)], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2282

$\operatorname{Int}[u, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& !\operatorname{MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; \operatorname{FreeQ}[$

```
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6246

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*x_]]*(d_.)]*((e_.) + (f_.)*x_)^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*x_)^(p_.)]/((d_.) + (e_.)*x_), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps



$$\begin{aligned}
\int x \coth^{-1}(c + d \coth(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) - \frac{1}{2}(b(1 - c - d)) \int \frac{e^{2a+2bx} x^2}{1 - c + d + (-1 + c + d)e^{2a+2bx}} dx \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 267, normalized size = 1.17

$$2b^2x^2 \log\left(\frac{2(\cosh(a+bx)-\sinh(a+bx))((c-1)\sinh(a+bx)+d\cosh(a+bx))}{c+d-1}\right) - 2b^2x^2 \log\left(\frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1} + 1\right) - 2$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*ArcCoth[c + d\*Coth[a + b\*x]], x]

[Out] (x^2\*ArcCoth[c + d\*Coth[a + b\*x]])/2 + (2\*b^2\*x^2\*Log[(2\*(Cosh[a + b\*x] - Sinh[a + b\*x])\*(d\*Cosh[a + b\*x] + (-1 + c)\*Sinh[a + b\*x]))/(-1 + c + d)] - 2\*b^2\*x^2\*Log[1 + ((1 + c - d)\*(-Cosh[2\*(a + b\*x)] + Sinh[2\*(a + b\*x)])/(1 + c + d)] - 2\*b\*x\*PolyLog[2, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])/(-1 + c + d)] + 2\*b\*x\*PolyLog[2, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])/(1 + c + d)] - PolyLog[3, ((-1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])/(-1 + c + d)] + PolyLog[3, ((1 + c - d)\*(Cosh[2\*(a + b\*x)] - Sinh[2\*(a + b\*x)])/(1 + c + d))]/(8\*b^2)

**fricas [C]** time = 0.62, size = 729, normalized size = 3.18

$$b^2x^2 \log\left(\frac{d \cosh(bx+a)+(c+1) \sinh(bx+a)}{d \cosh(bx+a)+(c-1) \sinh(bx+a)}\right) - 2bx \operatorname{Li}_2\left(\sqrt{\frac{c+d+1}{c-d+1}}(\cosh(bx+a) + \sinh(bx+a))\right) - 2bx \operatorname{Li}_2\left(-\sqrt{\frac{c+d+1}{c-d+1}}(\cosh(bx+a) + \sinh(bx+a))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoath(c+d\*coth(b\*x+a)), x, algorithm="fricas")

```
[Out] 1/4*(b^2*x^2*log((d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a)
+ (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(cos
h(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1))*(
cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d - 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c - d -
1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x + a)
+ 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))
) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(
c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cosh(b*x
+ a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d
- 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a)
- 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*log(sqrt((
c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^2*x^2 - a
^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
+ (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c + d + 1)/(c - d +
1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt((c + d - 1)/(c - d
- 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt((c + d - 1)/(c
- d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(d \coth(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccoth(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arccoth(d*coth(b*x + a) + c), x)
```

**maple** [C] time = 3.78, size = 4918, normalized size = 21.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccoth(c+d*coth(b*x+a)),x)
```

```
[Out] -1/4/b^2*a^2/(c+d-1)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-c+
d+1)-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b
*x+2*a)-1)/(exp(2*b*x+2*a)-1))^3+1/8/b^2/(1+c+d)*polylog(3, (1+c+d)*exp(2*b*
x+2*a)/(1+c-d))-1/4/b^2/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/
4/b^2/(1+c+d)*polylog(2, (1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a+1/2/b^2*a^2/(1+c+
d)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c
```

$$\begin{aligned}
& -d) * (1+c+d))^{(1/2)} + 1/2/b^2*a^2/(1+c+d) * \ln((c*\exp(b*x+a) + \exp(b*x+a)*d + ((1+c-d)*(1+c+d))^{(1/2)} + \exp(b*x+a)) / ((1+c-d)*(1+c+d))^{(1/2)}) + 1/8/b^2*d/(1+c+d) * \text{polylog}(3, (1+c+d)*\exp(2*b*x+2*a)/(1+c-d)) + 1/8/b^2*c/(1+c+d) * \text{polylog}(3, (1+c+d)*\exp(2*b*x+2*a)/(1+c-d)) + 1/2/b^2*a/(1+c+d) * \text{dilog}((-c*\exp(b*x+a) - \exp(b*x+a)*d + ((1+c-d)*(1+c+d))^{(1/2)} - \exp(b*x+a)) / ((1+c-d)*(1+c+d))^{(1/2)}) + 1/2/b^2*a/(1+c+d) * \text{dilog}((c*\exp(b*x+a) + \exp(b*x+a)*d + ((1+c-d)*(1+c+d))^{(1/2)} + \exp(b*x+a)) / ((1+c-d)*(1+c+d))^{(1/2)}) - 1/4/b/(1+c+d) * \text{polylog}(2, (1+c+d)*\exp(2*b*x+2*a)/(1+c-d)) * x - 1/4*d/(1+c+d) * \ln(1 - (1+c+d)*\exp(2*b*x+2*a)/(1+c-d)) * x^2 - 1/4*c/(1+c+d) * \ln(1 - (1+c+d)*\exp(2*b*x+2*a)/(1+c-d)) * x^2 + 1/4*x^2 * \ln((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d + \exp(2*b*x+2*a) - 1) + 1/4/b*c/(c+d-1) * \text{polylog}(2, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x + 1/4/b^2*c/(c+d-1) * \text{polylog}(2, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * a + 1/4/b^2*d/(c+d-1) * \ln(1 - (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * a^2 + 1/4/b*d/(c+d-1) * \text{polylog}(2, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x + 1/4/b^2*d/(c+d-1) * \text{polylog}(2, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * a - 1/2/b/(c+d-1) * \ln(1 - (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x * a + 1/2/b*a/(c+d-1) * \ln((-c*\exp(b*x+a) - \exp(b*x+a)*d + ((c-d-1)*(c+d-1))^{(1/2)} + \exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)}) * x - 1/4/(1+c+d) * \ln(1 - (1+c+d)*\exp(2*b*x+2*a)/(1+c-d)) * x^2 + 1/4*d/(c+d-1) * \ln(1 - (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x^2 - 1/8/b^2*c/(c+d-1) * \text{polylog}(3, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) - 1/8/b^2*d/(c+d-1) * \text{polylog}(3, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) + 1/2/b^2*a/(c+d-1) * \text{dilog}((-c*\exp(b*x+a) - \exp(b*x+a)*d + ((c-d-1)*(c+d-1))^{(1/2)} + \exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)}) + 1/2/b^2*a/(c+d-1) * \text{dilog}((c*\exp(b*x+a) + \exp(b*x+a)*d + ((c-d-1)*(c+d-1))^{(1/2)} - \exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)}) - 1/4/b^2/(c+d-1) * \ln(1 - (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * a^2 - 1/4/b/(c+d-1) * \text{polylog}(2, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x - 1/4/b^2/(c+d-1) * \text{polylog}(2, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * a + 1/2/b^2*a^2/(c+d-1) * \ln((-c*\exp(b*x+a) - \exp(b*x+a)*d + ((c-d-1)*(c+d-1))^{(1/2)} + \exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)}) + 1/4*c/(c+d-1) * \ln(1 - (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x^2 - 1/4/(c+d-1) * \ln(1 - (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x^2 + 1/8/b^2/(c+d-1) * \text{polylog}(3, (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * \exp(2*b*x+2*a)/(c-d-1) - 1/4*x^2 * \ln((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) - 1/4/b^2*a^2/(1+c+d) * \ln(c*\exp(2*b*x+2*a) + d*\exp(2*b*x+2*a) + \exp(2*b*x+2*a) - c+d-1) + 1/8*I*Pi*x^2*csgn(I*((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1))^{-3} - 1/2/b*c*a/(c+d-1) * \ln((c*\exp(b*x+a) + \exp(b*x+a)*d + ((c-d-1)*(c+d-1))^{(1/2)} - \exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)}) * x - 1/2/b*d*a/(c+d-1) * \ln((-c*\exp(b*x+a) - \exp(b*x+a)*d + ((c-d-1)*(c+d-1))^{(1/2)} + \exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)}) * x - 1/2/b*d*a/(c+d-1) * \ln((c*\exp(b*x+a) + \exp(b*x+a)*d + ((c-d-1)*(c+d-1))^{(1/2)} - \exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)}) * x + 1/2/b*c/(c+d-1) * \ln(1 - (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x * a + 1/2/b*d/(c+d-1) * \ln(1 - (c+d-1)*\exp(2*b*x+2*a)/(c-d-1)) * x * a - 1/2/b*c*a/(c+d-1) * \ln((-c*\exp(b*x+a) - \exp(b*x+a)*d + ((c-d-1)*(c+d-1))^{(1/2)} + \exp(b*x+a)) / ((c-d-1)*(c+d-1))^{(1/2)}) * x + 1/2/b^2*a*c/(1+c+d) * \text{dilog}((-c*\exp(b*x+a) - \exp(b*x+a)*d + ((1+c-d)*(1+c+d))^{(1/2)} - \exp(b*x+a)) / ((1+c-d)*(1+c+d))^{(1/2)}) + 1/2/b^2*a*c/(1+c+d) * \text{dilog}((c*\exp(b*x+a) + \exp(b*x+a)*d + ((1+c-d)*(1+c+d))^{(1/2)} + \exp(b*x+a)) / ((1+c-d)*(1+c+d))^{(1/2)}) + 1/2/b^2*a*d/(1+c+d) * \text{dilog}((-c*\exp(b*x+a) - \exp(b*x+a)*d + ((1+c-d)*(1+c+d))^{(1/2)} - \exp(b*x+a)) / ((1+c-d)*(1+c+d))^{(1/2)}) + 1/2/b^2*a*d/(1+c+d)
\end{aligned}$$

$$\begin{aligned}
& ) * \operatorname{dilog}((c \exp(b*x+a) + \exp(b*x+a) * d + ((1+c-d) * (1+c+d))^{1/2} + \exp(b*x+a)) / ((1+c-d) * (1+c+d))^{1/2}) - 1/2/b / (1+c+d) * \ln(1 - (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * x * a \\
& + 1/2/b * a / (1+c+d) * \ln((-c \exp(b*x+a) - \exp(b*x+a) * d + ((1+c-d) * (1+c+d))^{1/2} - \exp(b*x+a)) / ((1+c-d) * (1+c+d))^{1/2}) * x - 1/4/b^2 * d / (1+c+d) * \ln(1 - (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * a^2 \\
& + 1/4/b^2 * c / (c+d-1) * \ln(1 - (c+d-1) * \exp(2*b*x+2*a) / (c+d-1)) * a^2 - 1/8 * I * \pi * x^2 * \operatorname{csgn}(I / (\exp(2*b*x+2*a) - 1)) * \operatorname{csgn}(I * ((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1)))^2 \\
& - 1/4/b^2 * a^2 * d / (1+c+d) * \ln(c \exp(2*b*x+2*a) + d \exp(2*b*x+2*a) + \exp(2*b*x+2*a) - c + d - 1) - 1/4/b^2 * a^2 * c / (1+c+d) * \ln(c \exp(2*b*x+2*a) + d \exp(2*b*x+2*a) + \exp(2*b*x+2*a) - c + d - 1) \\
& - 1/2/b * d / (1+c+d) * \ln(1 - (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * x * a - 1/2/b * c / (1+c+d) * \ln(1 - (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * x * a \\
& + 1/2/b * a / (c+d-1) * \ln((c \exp(b*x+a) + \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} - \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) * x - 1/2/b^2 * c * a^2 / (c+d-1) * \ln((-c \exp(b*x+a) - \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} + \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) \\
& - 1/2/b^2 * c * a^2 / (c+d-1) * \ln((c \exp(b*x+a) + \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} - \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) \\
& - 1/2/b^2 * d * a^2 / (c+d-1) * \ln((-c \exp(b*x+a) - \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} + \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) \\
& - 1/2/b^2 * d * a^2 / (c+d-1) * \ln((c \exp(b*x+a) + \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} - \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) \\
& - 1/2/b^2 * c * a / (c+d-1) * \operatorname{dilog}((-c \exp(b*x+a) - \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} + \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) \\
& - 1/2/b^2 * c * a / (c+d-1) * \operatorname{dilog}((c \exp(b*x+a) + \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} - \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) \\
& - 1/2/b^2 * d * a / (c+d-1) * \operatorname{dilog}((-c \exp(b*x+a) - \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} + \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) \\
& - 1/2/b^2 * d * a / (c+d-1) * \operatorname{dilog}((c \exp(b*x+a) + \exp(b*x+a) * d + ((c-d-1) * (c+d-1))^{1/2} - \exp(b*x+a)) / ((c-d-1) * (c+d-1))^{1/2}) \\
& + 1/8 * I * \pi * x^2 * \operatorname{csgn}(I / (\exp(2*b*x+2*a) - 1)) * \operatorname{csgn}(I * ((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1))) * \operatorname{csgn}(I * ((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1))) \\
& - 1/8 * I * \pi * x^2 * \operatorname{csgn}(I * ((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1))) * \operatorname{csgn}(I * ((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1))) \\
& - 1/8 * I * \pi * x^2 * \operatorname{csgn}(I * ((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1)))^2 \\
& + 1/8 * I * \pi * x^2 * \operatorname{csgn}(I * ((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1))) * \operatorname{csgn}(I * ((\exp(2*b*x+2*a) - 1) * c + (\exp(2*b*x+2*a) + 1) * d - \exp(2*b*x+2*a) + 1) / (\exp(2*b*x+2*a) - 1))) \\
& + 1/2/b * a / (1+c+d) * \ln((c \exp(b*x+a) + \exp(b*x+a) * d + ((1+c-d) * (1+c+d))^{1/2} + \exp(b*x+a)) / ((1+c-d) * (1+c+d))^{1/2}) * x - 1/4/b * d / (1+c+d) * \operatorname{polylog}(2, (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * x - 1/4/b * c / (1+c+d) * \operatorname{polylog}(2, (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * x - 1/4/b^2 * d / (1+c+d) * \operatorname{polylog}(2, (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * a - 1/4/b^2 * c / (1+c+d) * \ln(1 - (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * a^2 - 1/4/b^2 * c / (1+c+d) * \operatorname{polylog}(2, (1+c+d) * \exp(2*b*x+2*a) / (1+c-d)) * a + 1/2/b^2 * a^2 * c / (1+c+d) * \ln((-c \exp(b*x+a) - \exp(b*x+a) * d + ((1+c-d) * (1+c+d))^{1/2} - \exp(b*x+a)) / ((1+c-d) * (1+c+d))^{1/2}) + 1/2/b^2 * a^2 * c / (1+c+d) * \ln((c \exp(b*x+a) + \exp(b*x+a) * d + ((1+c-d) * (1+c+d))^{1/2} + \exp(b*x+a)) / ((1+c-d) * (1+c+d))^{1/2}) + 1/2/b^2 * a^2 * d / (1+c+d) * \ln((-c \exp(b*x+a) - \exp(b*x+a) * d + ((1+c-d) * (1+c+d))^{1/2} + \exp(b*x+a)) / ((1+c-d) * (1+c+d))^{1/2}) + 1/2/b^2 * a^2 * d / (1+c+d) * \ln((c \exp(b*x+a) + \exp(b*x+a) * d + ((1+c-d) * (1+c+d))^{1/2} + \exp(b*x+a)) / ((1+c-d) * (1+c+d))^{1/2}) + \operatorname{ex}
\end{aligned}$$

$$\frac{p(b*x+a)}{((1+c-d)*(1+c+d))^{(1/2)}} + \frac{1/2/b*a*c}{(1+c+d)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)})} + \frac{1/2/b*a*c}{(1+c+d)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)})} + \frac{1/2/b*a*d}{(1+c+d)*\ln((-c*\exp(b*x+a)-\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)})} + \frac{1/2/b*a*d}{(1+c+d)*\ln((c*\exp(b*x+a)+\exp(b*x+a)*d+((1+c-d)*(1+c+d))^{(1/2)})} + \frac{1/8*I*Pi*x^2*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I*((\exp(2*b*x+2*a)-1)*c+(\exp(2*b*x+2*a)+1)*d+\exp(2*b*x+2*a)-1)/(\exp(2*b*x+2*a)-1))^{2+1/4/b^2*d*a^2/(c+d-1)*\ln(c*\exp(2*b*x+2*a)+d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)-c+d+1)+1/4/b^2*c*a^2/(c+d-1)*\ln(c*\exp(2*b*x+2*a)+d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)-c+d+1))}{b^3d}$$

**maxima** [A] time = 0.74, size = 213, normalized size = 0.93

$$-\frac{1}{8}bd \left( \frac{2b^2x^2 \log\left(-\frac{(c+d+1)e^{2bx+2a}}{c-d+1} + 1\right) + 2bx \operatorname{Li}_2\left(\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right) - \operatorname{Li}_3\left(\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right)}{b^3d} - \frac{2b^2x^2 \log\left(-\frac{(c+d-1)e^{2bx+2a}}{c-d-1}\right)}{b^3d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*coth(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/8*b*d*((2*b^2*x^2*\log(-(c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}+1)+2*b*x*dilog((c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}-polylog(3,(c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}))/(b^3*d)-(2*b^2*x^2*\log(-(c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}+1)+2*b*x*dilog((c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}-polylog(3,(c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}))/(b^3*d))+1/2*x^2*arccoth(d*coth(b*x+a)+c)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acoth}(c + d \operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(c + d\*coth(a + b\*x)),x)

[Out] int(x\*acoth(c + d\*coth(a + b\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(c + d \operatorname{coth}(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(c+d\*coth(b\*x+a)),x)

[Out] Integral(x\*acoth(c + d\*coth(a + b\*x)), x)

### 3.219 $\int \coth^{-1}(c + d \coth(a + bx)) dx$

**Optimal.** Leaf size=150

$$\frac{\operatorname{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{\operatorname{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{2}x \log\left(1 - \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right) - \frac{1}{2}x \log\left(1 - \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right) + x \coth^{-1}(c + d \coth(a + bx))$$

[Out] x\*arccoth(c+d\*coth(b\*x+a))+1/2\*x\*ln(1-(1-c-d)\*exp(2\*b\*x+2\*a)/(1-c+d))-1/2\*x\*ln(1-(1+c+d)\*exp(2\*b\*x+2\*a)/(1+c-d))+1/4\*polylog(2,(1-c-d)\*exp(2\*b\*x+2\*a)/(1-c+d))/b-1/4\*polylog(2,(1+c+d)\*exp(2\*b\*x+2\*a)/(1+c-d))/b

**Rubi [A]** time = 0.23, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6238, 2190, 2279, 2391}

$$\frac{\operatorname{PolyLog}\left(2, \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b} - \frac{\operatorname{PolyLog}\left(2, \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b} + \frac{1}{2}x \log\left(1 - \frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right) - \frac{1}{2}x \log\left(1 - \frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right) + x \coth^{-1}(c + d \coth(a + bx))$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[c + d\*Coth[a + b\*x]],x]

[Out] x\*ArcCoth[c + d\*Coth[a + b\*x]] + (x\*Log[1 - ((1 - c - d)\*E^(2\*a + 2\*b\*x))/(1 - c + d)])/2 - (x\*Log[1 - ((1 + c + d)\*E^(2\*a + 2\*b\*x))/(1 + c - d)])/2 + PolyLog[2, ((1 - c - d)\*E^(2\*a + 2\*b\*x))/(1 - c + d)]/(4\*b) - PolyLog[2, ((1 + c + d)\*E^(2\*a + 2\*b\*x))/(1 + c - d)]/(4\*b)

#### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 6238

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Coth[a + b\*x]], x] + (-Dist[b\*(1 - c - d), Int[(x\*E^(2\*a + 2\*b\*x))/(1 - c + d - (1 - c - d)\*E^(2\*a + 2\*b\*x)), x], x] + Dist[b\*(1 + c + d), Int[(x\*E^(2\*a + 2\*b\*x))/(1 + c - d - (1 + c + d)\*E^(2\*a + 2\*b\*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(c + d \coth(a + bx)) dx &= x \coth^{-1}(c + d \coth(a + bx)) - (b(1 - c - d)) \int \frac{e^{2a+2bx} x}{1 - c + d + (-1 + c + d)e^{2a+2bx}} dx \\ &= x \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + d)e^{2a+2bx}}{1 + c - d} \right) \\ &= x \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + d)e^{2a+2bx}}{1 + c - d} \right) \\ &= x \coth^{-1}(c + d \coth(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 + c + d)e^{2a+2bx}}{1 + c - d} \right) \end{aligned}$$

**Mathematica [A]** time = 1.26, size = 131, normalized size = 0.87

$$x \coth^{-1}(d \coth(a+bx)+c) - \frac{-\operatorname{Li}_2\left(\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) + \operatorname{Li}_2\left(\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right) - 2bx \left(\log\left(1 - \frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) - \log\left(1 - \frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right)\right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[c + d\*Coth[a + b\*x]], x]

[Out] x\*ArcCoth[c + d\*Coth[a + b\*x]] - (-2\*b\*x\*(Log[1 - ((-1 + c + d)\*E^(2\*(a + b\*x)))/(-1 + c - d)] - Log[1 - ((1 + c + d)\*E^(2\*(a + b\*x)))/(1 + c - d)]) - PolyLog[2, ((-1 + c + d)\*E^(2\*(a + b\*x)))/(-1 + c - d)] + PolyLog[2, ((1 + c + d)\*E^(2\*(a + b\*x)))/(1 + c - d)]/(4\*b)

**fricas [B]** time = 0.48, size = 539, normalized size = 3.59

$$bx \log \left( \frac{d \cosh(bx+a) + (c+1) \sinh(bx+a)}{d \cosh(bx+a) + (c-1) \sinh(bx+a)} \right) + a \log \left( 2(c+d+1) \cosh(bx+a) + 2(c+d+1) \sinh(bx+a) + 2(c-d+1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(b*x*\log((d*\cosh(b*x + a) + (c + 1)*\sinh(b*x + a))/(d*\cosh(b*x + a) + (c - 1)*\sinh(b*x + a))) + a*\log(2*(c + d + 1)*\cosh(b*x + a) + 2*(c + d + 1)*\sinh(b*x + a) + 2*(c - d + 1)*\sqrt{(c + d + 1)/(c - d + 1)}) + a*\log(2*(c + d + 1)*\cosh(b*x + a) + 2*(c + d + 1)*\sinh(b*x + a) - 2*(c - d + 1)*\sqrt{(c + d + 1)/(c - d + 1)}) - a*\log(2*(c + d - 1)*\cosh(b*x + a) + 2*(c + d - 1)*\sinh(b*x + a) + 2*(c - d - 1)*\sqrt{(c + d - 1)/(c - d - 1)}) - a*\log(2*(c + d - 1)*\cosh(b*x + a) + 2*(c + d - 1)*\sinh(b*x + a) - 2*(c - d - 1)*\sqrt{(c + d - 1)/(c - d - 1)}) - (b*x + a)*\log(\sqrt{(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - (b*x + a)*\log(-\sqrt{(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + (b*x + a)*\log(\sqrt{(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) + (b*x + a)*\log(-\sqrt{(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a)) + 1) - \operatorname{dilog}(\sqrt{(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) - \operatorname{dilog}(-\sqrt{(c + d + 1)/(c - d + 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + \operatorname{dilog}(\sqrt{(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))) + \operatorname{dilog}(-\sqrt{(c + d - 1)/(c - d - 1)}*(\cosh(b*x + a) + \sinh(b*x + a))))/b$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(d \coth(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*coth(b\*x + a) + c), x)

maple [B] time = 0.53, size = 306, normalized size = 2.04

$$-\frac{\operatorname{arccoth}(c + d \coth(bx + a)) \ln(d \coth(bx + a) - d)}{2b} + \frac{\operatorname{arccoth}(c + d \coth(bx + a)) \ln(d \coth(bx + a) + d)}{2b} - \operatorname{dilog}(\dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*coth(b\*x+a)),x)

[Out]  $-\frac{1}{2}*\frac{1}{b}*\operatorname{arccoth}(c+d*\coth(b*x+a))*\ln(d*\coth(b*x+a)-d)+\frac{1}{2}*\frac{1}{b}*\operatorname{arccoth}(c+d*\coth(b*x+a))*\ln(d*\coth(b*x+a)+d)-\frac{1}{4}*\frac{1}{b}*\operatorname{dilog}((d*\coth(b*x+a)+c-1)/(c+d-1))-\frac{1}{4}*\frac{1}{b}*\ln(d*\coth(b*x+a)-d)*\ln((d*\coth(b*x+a)+c-1)/(c+d-1))+\frac{1}{4}*\frac{1}{b}*\operatorname{dilog}((d*\coth(b*x+a)+c+1)/(1+c+d))+\frac{1}{4}*\frac{1}{b}*\ln(d*\coth(b*x+a)-d)*\ln((d*\coth(b*x+a)+c+1)/(1+c+d))+\frac{1}{4}*\frac{1}{b}*\operatorname{dilog}((d*\coth(b*x+a)+c-1)/(c-d-1))+\frac{1}{4}*\frac{1}{b}*\ln(d*\coth(b*x+a)+d)*\ln((d*c$



$\text{coth}(b*x+a)+c-1)/(c-d-1))-1/4/b*\text{dilog}((d*\text{coth}(b*x+a)+c+1)/(1+c-d))-1/4/b*\ln(d*\text{coth}(b*x+a)+d)*\ln((d*\text{coth}(b*x+a)+c+1)/(1+c-d))$

**maxima** [A] time = 0.71, size = 142, normalized size = 0.95

$$-\frac{1}{4}bd \left( \frac{2bx \log\left(-\frac{(c+d+1)e^{2bx+2a}}{c-d+1} + 1\right) + \text{Li}_2\left(\frac{(c+d+1)e^{2bx+2a}}{c-d+1}\right)}{b^2d} - \frac{2bx \log\left(-\frac{(c+d-1)e^{2bx+2a}}{c-d-1} + 1\right) + \text{Li}_2\left(\frac{(c+d-1)e^{2bx+2a}}{c-d-1}\right)}{b^2d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(c+d*coth(b*x+a)),x, algorithm="maxima")`

[Out]  $-1/4*b*d*((2*b*x*\log(-(c+d+1)*e^{(2*b*x+2*a)/(c-d+1)+1}) + \text{dilog}((c+d+1)*e^{(2*b*x+2*a)/(c-d+1)}))/(b^2*d) - (2*b*x*\log(-(c+d-1)*e^{(2*b*x+2*a)/(c-d-1)+1}) + \text{dilog}((c+d-1)*e^{(2*b*x+2*a)/(c-d-1)}))/(b^2*d) + x*\text{arccoth}(d*\text{coth}(b*x+a)+c)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{acoth}(c + d \coth(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(c + d*coth(a + b*x)),x)`

[Out] `int(acoth(c + d*coth(a + b*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{acoth}(c + d \coth(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(c+d*coth(b*x+a)),x)`

[Out] `Integral(acoth(c + d*coth(a + b*x)), x)`

$$3.220 \quad \int \frac{\coth^{-1}(c+d \coth(a+bx))}{x} dx$$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(d \coth(a + bx) + c)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(c+d\*coth(b\*x+a))/x,x)

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(c + d \coth(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[c + d\*Coth[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d\*Coth[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c + d \coth(a + bx))}{x} dx = \int \frac{\coth^{-1}(c + d \coth(a + bx))}{x} dx$$

Mathematica [A] time = 6.23, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(c + d \coth(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[c + d\*Coth[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[c + d\*Coth[a + b\*x]]/x, x]

fricas [A] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcoth}(d \coth(bx + a) + c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*coth(b\*x + a) + c)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \coth(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*coth(b\*x + a) + c)/x, x)

**maple** [A] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(c + d \coth(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*coth(b\*x+a))/x,x)

[Out] int(arccoth(c+d\*coth(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \coth(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*coth(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*coth(b\*x + a) + c)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(c + d \coth(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(c + d\*coth(a + b\*x))/x,x)

[Out] int(acoth(c + d\*coth(a + b\*x))/x, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(c + d \operatorname{coth}(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c+d\*coth(b\*x+a))/x,x)

[Out] Integral(acoth(c + d\*coth(a + b\*x))/x, x)

### 3.221 $\int x^3 \coth^{-1}(1 + d + d \coth(ax + bx)) dx$

**Optimal.** Leaf size=152

$$\frac{3\text{Li}_5((d+1)e^{2a+2bx})}{16b^4} - \frac{3x\text{Li}_4((d+1)e^{2a+2bx})}{8b^3} + \frac{3x^2\text{Li}_3((d+1)e^{2a+2bx})}{8b^2} - \frac{x^3\text{Li}_2((d+1)e^{2a+2bx})}{4b} - \frac{1}{8}x^4 \log(1 - (d +$$

[Out]  $1/20*b*x^5 + 1/4*x^4*\text{arccoth}(1+d+d*\coth(b*x+a)) - 1/8*x^4*\ln(1-(1+d)*\exp(2*b*x+2*a)) - 1/4*x^3*\text{polylog}(2, (1+d)*\exp(2*b*x+2*a))/b + 3/8*x^2*\text{polylog}(3, (1+d)*\exp(2*b*x+2*a))/b^2 - 3/8*x*\text{polylog}(4, (1+d)*\exp(2*b*x+2*a))/b^3 + 3/16*\text{polylog}(5, (1+d)*\exp(2*b*x+2*a))/b^4$

**Rubi [A]** time = 0.31, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}(3, (d+1)e^{2a+2bx})}{8b^2} - \frac{3x\text{PolyLog}(4, (d+1)e^{2a+2bx})}{8b^3} + \frac{3\text{PolyLog}(5, (d+1)e^{2a+2bx})}{16b^4} - \frac{x^3\text{PolyLog}(2, (d+1)e^{2a+2bx})}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{ArcCoth}[1 + d + d*\text{Coth}[a + b*x]], x]$

[Out]  $(b*x^5)/20 + (x^4*\text{ArcCoth}[1 + d + d*\text{Coth}[a + b*x]])/4 - (x^4*\text{Log}[1 - (1 + d)*E^{(2*a + 2*b*x)}])/8 - (x^3*\text{PolyLog}[2, (1 + d)*E^{(2*a + 2*b*x)}])/(4*b) + (3*x^2*\text{PolyLog}[3, (1 + d)*E^{(2*a + 2*b*x)}])/(8*b^2) - (3*x*\text{PolyLog}[4, (1 + d)*E^{(2*a + 2*b*x)}])/(8*b^3) + (3*\text{PolyLog}[5, (1 + d)*E^{(2*a + 2*b*x)}])/(16*b^4)$

#### Rule 2184

$\text{Int}[\frac{(c + d*x)^m}{(a + b*(F^{g*(e + f*x)}))^n}, x\_Symbol] := \text{Simp}[\frac{(c + d*x)^{m+1}}{a*d*(m+1)}, x] - \text{Dist}[\frac{b}{a}, \text{Int}[\frac{(c + d*x)^m*(F^{g*(e + f*x)})^n}{(a + b*(F^{g*(e + f*x)}))^n}, x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

$\text{Int}[\frac{(F^{g*(e + f*x)})^n*(c + d*x)^m}{(a + b*(F^{g*(e + f*x)}))^n}, x\_Symbol] := \text{Simp}[\frac{(c + d*x)^m*\text{Log}[1 + (b*(F^{g*(e + f*x)}))^n/a]}{b*f*g*n*\text{Log}[F]}, x] - \text{Dist}[\frac{d*m}{b*f*g*n*\text{Log}[F]}, \text{Int}[\frac{(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{g*(e + f*x)}))^n/a]}{a}, x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*(
m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(1 + d + d \coth(a + bx)) dx &= \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{4} b \int \frac{x^4}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{4} (b(1 + d)) \int \frac{e^{2a+2bx}}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4} x^4 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{8} x^4 \log(1 - (1 + d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 141, normalized size = 0.93

$$\frac{1}{16} \left( \frac{3 \operatorname{Li}_5\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{6x \operatorname{Li}_4\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x^2 \operatorname{Li}_3\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{4x^3 \operatorname{Li}_2\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 2x^4 \log\left(1 - \frac{e^{-2(a+bx)}}{d+1}\right) + 4x^4 \coth^{-1}\left(\frac{1 + d + d \coth(a + bx)}{1 + (-1 - d)e^{2a+2bx}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[1 + d + d\*Coth[a + b\*x]],x]

[Out] (4\*x^4\*ArcCoth[1 + d + d\*Coth[a + b\*x]] - 2\*x^4\*Log[1 - 1/((1 + d)\*E^(2\*(a + b\*x)))] + (4\*x^3\*PolyLog[2, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b + (6\*x^2\*PolyLog[3, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b^2 + (6\*x\*PolyLog[4, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b^3 + (3\*PolyLog[5, 1/((1 + d)\*E^(2\*(a + b\*x)))]/b^4)/16

**fricas [C]** time = 0.53, size = 423, normalized size = 2.78

$$\frac{2b^5x^5 + 5b^4x^4 \log\left(\frac{d \cosh(bx+a) + (d+2) \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)}\right) - 20b^3x^3 \operatorname{Li}_2\left(\sqrt{d+1} (\cosh(bx+a) + \sinh(bx+a))\right) - 20b^3x^3 \operatorname{Li}_2\left(\frac{1 + d + d \coth(a + bx)}{1 + (-1 - d)e^{2a+2bx}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d\*d\*coth(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{40} \cdot (2b^5x^5 + 5b^4x^4 \log((d \cosh(bx + a) + (d + 2) \sinh(bx + a)) / (d \cosh(bx + a) + d \sinh(bx + a))) - 20b^3x^3 \operatorname{dilog}(\sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a))) - 20b^3x^3 \operatorname{dilog}(-\sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a))) - 5a^4 \log(2(d + 1) \cosh(bx + a) + 2(d + 1) \sinh(bx + a) + 2\sqrt{d + 1}) - 5a^4 \log(2(d + 1) \cosh(bx + a) + 2(d + 1) \sinh(bx + a) - 2\sqrt{d + 1}) + 60b^2x^2 \operatorname{polylog}(3, \sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a))) + 60b^2x^2 \operatorname{polylog}(3, -\sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a))) - 120bx \operatorname{polylog}(4, \sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a))) - 120bx \operatorname{polylog}(4, -\sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a))) - 5(b^4x^4 - a^4) \log(\sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a)) + 1) - 5(b^4x^4 - a^4) \log(-\sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a)) + 1) + 120 \operatorname{polylog}(5, \sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a))) + 120 \operatorname{polylog}(5, -\sqrt{d + 1}(\cosh(bx + a) + \sinh(bx + a)))) / b^4$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(d \coth(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d\*d\*coth(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^3\*arccoth(d\*coth(b\*x + a) + d + 1), x)

maple [C] time = 5.86, size = 1698, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(1+d\*d\*coth(b\*x+a)),x)

[Out]  $-1/16 \cdot I \cdot x^4 \cdot \operatorname{csgn}(I / (\exp(2bx + 2a) - 1) \cdot (d \exp(2bx + 2a) + \exp(2bx + 2a) - 1))^{-3} + 3/16 \cdot b^{-4} \cdot (1 + d) \cdot \operatorname{polylog}(5, (1 + d) \exp(2bx + 2a)) - 1/8 \cdot (1 + d) \cdot \ln(1 - (1 + d) \exp(2bx + 2a)) \cdot x^4 + 1/2 \cdot b^{-4} \cdot a^4 \cdot (1 + d) \cdot \ln(1 - \exp(bx + a) \cdot (1 + d)^{(1/2)}) + 1/2 \cdot b^{-4} \cdot a^4 \cdot (1 + d) \cdot \ln(1 + \exp(bx + a) \cdot (1 + d)^{(1/2)}) - 1/8 \cdot d \cdot (1 + d) \cdot \ln(1 - (1 + d) \exp(2bx + 2a)) \cdot x^4 + 3/16 \cdot b^{-4} \cdot d \cdot (1 + d) \cdot \operatorname{polylog}(5, (1 + d) \exp(2bx + 2a)) + 1/2 \cdot b^{-4} \cdot a^3 \cdot (1 + d) \cdot \operatorname{dilog}(1 - \exp(bx + a) \cdot (1 + d)^{(1/2)}) + 1/2 \cdot b^{-4} \cdot a^3 \cdot (1 + d) \cdot \operatorname{dilog}(1 + \exp(bx + a) \cdot (1 + d)^{(1/2)}) - 1/4 \cdot b \cdot (1 + d) \cdot \operatorname{polylog}(2, (1 + d) \exp(2bx + 2a)) \cdot x^3 - 1/4 \cdot b^{-4} \cdot (1 + d) \cdot \operatorname{polylog}(2, (1 + d) \exp(2bx + 2a)) \cdot a^3 + 3/8 \cdot b^{-2} \cdot (1 + d) \cdot \operatorname{polylog}(3, (1 + d) \exp(2bx + 2a)) \cdot x^2 - 3/8 \cdot b^{-3} \cdot (1 + d) \cdot \operatorname{polylog}(4, (1 + d) \exp(2bx + 2a)) \cdot x - 3/8 \cdot b^{-4} \cdot (1 + d) \cdot \ln(1 - (1 + d) \exp(2bx + 2a)) \cdot a^4 + 1/20 \cdot b \cdot x^5 - 1/8 \cdot b^{-4} \cdot a^4 \cdot (1 + d) \cdot \ln(d \exp(2bx + 2a) + \exp(2bx + 2a) - 1) + 1/16 \cdot I \cdot x^4 \cdot \operatorname{csgn}(I \cdot d / (\exp(2bx + 2a) - 1) \cdot \exp(2bx + 2a))^{-3} + 1/2 \cdot b^{-3} \cdot a^3 \cdot (1 + d) \cdot \ln(1 + \exp(bx + a) \cdot (1 + d)^{(1/2)}) \cdot x + 1/8 \cdot x^4 \cdot \ln(d \exp(2bx + 2a) + \exp(2bx + 2a) - 1)$



$$\begin{aligned} & x^3 \operatorname{arccoth}(1+d+d \coth(bx+a)) + \frac{1}{4} x^4 \operatorname{arccoth}(d \coth(bx+a) + d + 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4x^4 \log(-(d+1)e^{2bx+2a}) + 1) + 4b^3x^3 \operatorname{Li}_2((d+1)e^{2bx+2a})}{d} \right) \\ & - \frac{5(2b^4x^4 \log(-(d+1)e^{2bx+2a}) + 1) + 4b^3x^3 \operatorname{Li}_2((d+1)e^{2bx+2a})}{40d} + \frac{4b^3x^3 \operatorname{Li}_2((d+1)e^{2bx+2a})}{40d} \\ & + \frac{5(2b^4x^4 \log(-(d+1)e^{2bx+2a}) + 1) + 4b^3x^3 \operatorname{Li}_2((d+1)e^{2bx+2a})}{40d} \end{aligned}$$

**maxima** [A] time = 1.09, size = 146, normalized size = 0.96

$$\frac{1}{4} x^4 \operatorname{arccoth}(d \coth(bx+a) + d + 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4x^4 \log(-(d+1)e^{2bx+2a}) + 1) + 4b^3x^3 \operatorname{Li}_2((d+1)e^{2bx+2a})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1+d+d\*coth(b\*x+a)),x, algorithm="maxima")

[Out] 1/4\*x^4\*arccoth(d\*coth(b\*x + a) + d + 1) + 1/40\*(2\*x^5/d - 5\*(2\*b^4\*x^4\*log(-(d + 1)\*e^(2\*b\*x + 2\*a) + 1) + 4\*b^3\*x^3\*dilog((d + 1)\*e^(2\*b\*x + 2\*a)) - 6\*b^2\*x^2\*polylog(3, (d + 1)\*e^(2\*b\*x + 2\*a)) + 6\*b\*x\*polylog(4, (d + 1)\*e^(2\*b\*x + 2\*a)) - 3\*polylog(5, (d + 1)\*e^(2\*b\*x + 2\*a)))/(b^5\*d))\*b\*d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{arccoth}(d + d \coth(a + bx) + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*acoth(d + d*coth(a + b*x) + 1),x)`

[Out] `int(x^3*acoth(d + d*coth(a + b*x) + 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}(d \coth(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(1+d+d*coth(b*x+a)),x)`

[Out] `Integral(x**3*acoth(d*coth(a + b*x) + d + 1), x)`

### 3.222 $\int x^2 \coth^{-1}(1 + d + d \coth(ax + bx)) dx$

**Optimal.** Leaf size=126

$$-\frac{\text{Li}_4((d+1)e^{2a+2bx})}{8b^3} + \frac{x\text{Li}_3((d+1)e^{2a+2bx})}{4b^2} - \frac{x^2\text{Li}_2((d+1)e^{2a+2bx})}{4b} - \frac{1}{6}x^3 \log(1 - (d+1)e^{2a+2bx}) + \frac{1}{3}x^3 \coth^{-1}(d + d \coth(ax + bx))$$

[Out] 1/12\*b\*x^4+1/3\*x^3\*arccoth(1+d+d\*coth(b\*x+a))-1/6\*x^3\*ln(1-(1+d)\*exp(2\*b\*x+2\*a))-1/4\*x^2\*polylog(2,(1+d)\*exp(2\*b\*x+2\*a))/b+1/4\*x\*polylog(3,(1+d)\*exp(2\*b\*x+2\*a))/b^2-1/8\*polylog(4,(1+d)\*exp(2\*b\*x+2\*a))/b^3

**Rubi [A]** time = 0.27, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}(3, (d+1)e^{2a+2bx})}{4b^2} - \frac{\text{PolyLog}(4, (d+1)e^{2a+2bx})}{8b^3} - \frac{x^2\text{PolyLog}(2, (d+1)e^{2a+2bx})}{4b} - \frac{1}{6}x^3 \log(1 - (d+1)e^{2a+2bx}) + \frac{1}{3}x^3 \coth^{-1}(d + d \coth(ax + bx))$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 + d + d\*Coth[a + b\*x]], x]

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 + d + d\*Coth[a + b\*x]])/3 - (x^3\*Log[1 - (1 + d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (x\*PolyLog[3, (1 + d)\*E^(2\*a + 2\*b\*x)])/(4\*b^2) - PolyLog[4, (1 + d)\*E^(2\*a + 2\*b\*x)]/(8\*b^3)

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.), x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 + d + d \coth(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{3} b \int \frac{x^3}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{3} (b(1 + d)) \int \frac{e^{2a+2bx}}{1 + (-1 - d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3} x^3 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 116, normalized size = 0.92

$$\frac{1}{24} \left( \frac{3 \operatorname{Li}_4\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x \operatorname{Li}_3\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{6x^2 \operatorname{Li}_2\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 4x^3 \log\left(1 - \frac{e^{-2(a+bx)}}{d+1}\right) + 8x^3 \coth^{-1}(d \coth(a + bx)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 + d + d\*Coth[a + b\*x]],x]

[Out] (8\*x^3\*ArcCoth[1 + d + d\*Coth[a + b\*x]] - 4\*x^3\*Log[1 - 1/((1 + d)\*E^(2\*(a + b\*x)))] + (6\*x^2\*PolyLog[2, 1/((1 + d)\*E^(2\*(a + b\*x)))])/b + (6\*x\*PolyLog[3, 1/((1 + d)\*E^(2\*(a + b\*x)))])/b^2 + (3\*PolyLog[4, 1/((1 + d)\*E^(2\*(a + b\*x)))])/b^3)/24

**fricas [C]** time = 0.87, size = 359, normalized size = 2.85

$$\frac{b^4 x^4 + 2 b^3 x^3 \log\left(\frac{d \cosh(bx+a) + (d+2) \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)}\right) - 6 b^2 x^2 \operatorname{Li}_2\left(\sqrt{d+1} (\cosh(bx+a) + \sinh(bx+a))\right) - 6 b^2 x^2 \operatorname{Li}_2\left(-\sqrt{d+1} (\cosh(bx+a) + \sinh(bx+a))\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+d+d\*coth(b\*x+a)),x, algorithm="fricas")

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log((d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(d \coth(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d*d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*coth(b*x + a) + d + 1), x)
```

**maple** [C] time = 5.59, size = 1641, normalized size = 13.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+d*d*coth(b*x+a)),x)
```

```
[Out] 1/3/b^3/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/6*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3+1/12*I*x^3*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/12*b*x^4-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d
```

```

*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(
b*x+a)*(1+d)^(1/2))-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^
2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/12*I*x^3*Pi*c
sgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/6/(1+d)*ln(1-(1+d)*exp(2*b*x
+2*a))*x^3-1/8/b^3/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))+1/2/b^2/(1+d)*ln(1
-(1+d)*exp(2*b*x+2*a))*x*a^2+1/6/b^3*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*
x+2*a)-1)+1/3/b^3*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/4/b*d/(1+d)*poly
log(2,(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2
*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x+1/6*x^3*ln(d*exp
(2*b*x+2*a)+exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn
(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/6/
b^3*d*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1/12*I*x^3*Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/2/b^2*d*a^2/(
1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(1+
d)^(1/2))*x+1/2/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2+1/12*I*x^3*Pi*
csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(ex
p(2*b*x+2*a)-1))-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(d*exp(2*b
*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(
2*b*x+2*a)-1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a
)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))

```

**maxima** [A] time = 1.11, size = 123, normalized size = 0.98

$$\frac{1}{3} x^3 \operatorname{arccoth}(d \coth(bx + a) + d + 1) + \frac{1}{36} \left( \frac{3x^4}{d} - \frac{2(4b^3x^3 \log(-(d+1)e^{2bx+2a}) + 1) + 6b^2x^2 \operatorname{Li}_2((d+1)e^{2bx+2a})}{b^4d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccoth(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a)) -
6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b*x
+ 2*a)))/(b^4*d))*b*d
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(d + d \coth(a + bx) + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x^2*acoth(d + d*coth(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(d \coth(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(1+d+d\*coth(b\*x+a)),x)

[Out] Integral(x\*\*2\*acoth(d\*coth(a + b\*x) + d + 1), x)



### 3.223 $\int x \coth^{-1}(1 + d + d \coth(a + bx)) dx$

**Optimal.** Leaf size=100

$$\frac{\text{Li}_3((d+1)e^{2a+2bx})}{8b^2} - \frac{x\text{Li}_2((d+1)e^{2a+2bx})}{4b} - \frac{1}{4}x^2 \log(1 - (d+1)e^{2a+2bx}) + \frac{1}{2}x^2 \coth^{-1}(d \coth(a+bx) + d+1) + \frac{bx^3}{6}$$

[Out]  $1/6*b*x^3 + 1/2*x^2*\text{arccoth}(1+d+d*\coth(b*x+a)) - 1/4*x^2*\ln(1-(1+d)*\exp(2*b*x+2*a)) - 1/4*x*\text{polylog}(2, (1+d)*\exp(2*b*x+2*a))/b + 1/8*\text{polylog}(3, (1+d)*\exp(2*b*x+2*a))/b^2$

**Rubi [A]** time = 0.24, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6242, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}(3, (d+1)e^{2a+2bx})}{8b^2} - \frac{x\text{PolyLog}(2, (d+1)e^{2a+2bx})}{4b} - \frac{1}{4}x^2 \log(1 - (d+1)e^{2a+2bx}) + \frac{1}{2}x^2 \coth^{-1}(d \coth(a +$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*\text{ArcCoth}[1 + d + d*\text{Coth}[a + b*x]], x]$

[Out]  $(b*x^3)/6 + (x^2*\text{ArcCoth}[1 + d + d*\text{Coth}[a + b*x]])/2 - (x^2*\text{Log}[1 - (1 + d)*E^{(2*a + 2*b*x)}])/4 - (x*\text{PolyLog}[2, (1 + d)*E^{(2*a + 2*b*x)}])/(4*b) + \text{PolyLog}[3, (1 + d)*E^{(2*a + 2*b*x)}]/(8*b^2)$

#### Rule 2184

$\text{Int}[\frac{((c_.) + (d_.)*(x_))^{(m_.)}}{((a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)})})}, x\_Symbol] := \text{Simp}[(c + d*x)^{(m + 1)}/(a*d*(m + 1)), x] - \text{Dist}[b/a, \text{Int}[\frac{(c + d*x)^m*(F^{(g*(e + f*x)))^n}}{(a + b*(F^{(g*(e + f*x)))^n})}, x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2190

$\text{Int}[\frac{((F_.)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)})*((c_.) + (d_.)*(x_))^{(m_.)}}{((a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)})})}, x\_Symbol] := \text{Simp}[\frac{(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]}{(b*f*g*n*\text{Log}[F])}, x] - \text{Dist}[\frac{(d*m)}{(b*f*g*n*\text{Log}[F])}, \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2282

$\text{Int}[u, x\_Symbol] := \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w_)*((a_.)*(v_)^{(n_}))^{(m_)} /; \text{FreeQ}[\$

{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
)))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6242

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m  
\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Coth[a + b\*x]])/(f\*  
(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d - c\*E^(2\*a  
+ 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -  
d)^2, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(1 + d + d \coth(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{2}b \int \frac{x^2}{1 + (-1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \coth(a + bx)) + \frac{1}{2}(b(1 + d)) \int \frac{e^{2a+2bx}}{1 + (-1 - d)e^{2a+2bx}} \\
 &= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{4}x^2 \log(1 - (1 + d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{4}x^2 \log(1 - (1 + d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{4}x^2 \log(1 - (1 + d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2}x^2 \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{4}x^2 \log(1 - (1 + d)e^{2a+2bx})
 \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 90, normalized size = 0.90

$$\frac{2b^2x^2 \left( 2 \operatorname{coth}^{-1}(d \operatorname{coth}(a + bx) + d + 1) - \log \left( 1 - \frac{e^{-2(a+bx)}}{d+1} \right) \right) + 2bx \operatorname{Li}_2 \left( \frac{e^{-2(a+bx)}}{d+1} \right) + \operatorname{Li}_3 \left( \frac{e^{-2(a+bx)}}{d+1} \right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 + d + d\*Coth[a + b\*x]],x]

[Out] (2\*b^2\*x^2\*(2\*ArcCoth[1 + d + d\*Coth[a + b\*x]] - Log[1 - 1/((1 + d)\*E^(2\*(a + b\*x))])) + 2\*b\*x\*PolyLog[2, 1/((1 + d)\*E^(2\*(a + b\*x)))] + PolyLog[3, 1/((1 + d)\*E^(2\*(a + b\*x)))])/(8\*b^2)

**fricas [C]** time = 0.64, size = 305, normalized size = 3.05

$$\frac{2b^3x^3 + 3b^2x^2 \log \left( \frac{d \cosh(bx+a) + (d+2) \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)} \right) - 6bx \operatorname{Li}_2 \left( \sqrt{d+1} (\cosh(bx+a) + \sinh(bx+a)) \right) - 6bx \operatorname{Li}_2(-\sqrt{d+1} (\cosh(bx+a) + \sinh(bx+a)))}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+d+d\*coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/12\*(2\*b^3\*x^3 + 3\*b^2\*x^2\*log((d\*cosh(b\*x + a) + (d + 2)\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + d\*sinh(b\*x + a))) - 6\*b\*x\*dilog(sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 6\*b\*x\*dilog(-sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 3\*a^2\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) + 2\*sqrt(d + 1)) - 3\*a^2\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) - 2\*sqrt(d + 1)) - 3\*(b^2\*x^2 - a^2)\*log(sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 3\*(b^2\*x^2 - a^2)\*log(-sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 6\*polylog(3, sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 6\*polylog(3, -sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(d \operatorname{coth}(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+d+d\*coth(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*coth(b\*x + a) + d + 1), x)

**maple [C]** time = 5.09, size = 1560, normalized size = 15.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(1+d+d*coth(b*x+a)),x)`

[Out] 
$$\begin{aligned} & -1/4*d/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))*x^2+1/8/b^2*d/(1+d)*\text{polylog}(3,(1+d) \\ & * \exp(2*b*x+2*a))+1/2/b^2*a^2/(1+d)*\ln(1-\exp(b*x+a)*(1+d)^{(1/2)})+1/2/b^2*a^2 \\ & / (1+d)*\ln(1+\exp(b*x+a)*(1+d)^{(1/2)})+1/2/b^2*a/(1+d)*\text{dilog}(1-\exp(b*x+a)*(1+d) \\ & )^{(1/2)})+1/2/b^2*a/(1+d)*\text{dilog}(1+\exp(b*x+a)*(1+d)^{(1/2)})-1/4/b^2/(1+d)*\ln(1 \\ & -(1+d)*\exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*\text{polylog}(2,(1+d)*\exp(2*b*x+2*a))*x-1/ \\ & 4/b^2/(1+d)*\text{polylog}(2,(1+d)*\exp(2*b*x+2*a))*a-1/8*I*x^2*Pi*csgn(I/(\exp(2*b*x \\ & +2*a)-1))*(d*\exp(2*b*x+2*a)+\exp(2*b*x+2*a)-1))^{-3}-1/2*x^2*\ln(\exp(b*x+a))-1/4 \\ & *x^2*\ln(d)+1/8*I*x^2*Pi*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a))^{-3}+1/8*I \\ & *x^2*Pi*csgn(I*\exp(2*b*x+2*a))^{-3}-1/4/b^2*a^2/(1+d)*\ln(d*\exp(2*b*x+2*a)+\exp( \\ & 2*b*x+2*a)-1)+1/6*b*x^3-1/4/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))*x^2+1/8/b^2/(1 \\ & +d)*\text{polylog}(3,(1+d)*\exp(2*b*x+2*a))-1/2/b/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))* \\ & x*a-1/4/b^2*d/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*\text{polylog}(2, \\ & (1+d)*\exp(2*b*x+2*a))*x-1/4/b^2*d/(1+d)*\text{polylog}(2,(1+d)*\exp(2*b*x+2*a))*a+1 \\ & /2/b*a/(1+d)*\ln(1-\exp(b*x+a)*(1+d)^{(1/2)})*x+1/2/b*a/(1+d)*\ln(1+\exp(b*x+a)*( \\ & 1+d)^{(1/2)})*x+1/2/b^2*d*a^2/(1+d)*\ln(1-\exp(b*x+a)*(1+d)^{(1/2)})+1/2/b^2*d*a^ \\ & 2/(1+d)*\ln(1+\exp(b*x+a)*(1+d)^{(1/2)})+1/2/b^2*d*a/(1+d)*\text{dilog}(1-\exp(b*x+a)*( \\ & 1+d)^{(1/2)})+1/2/b^2*d*a/(1+d)*\text{dilog}(1+\exp(b*x+a)*(1+d)^{(1/2)})-1/8*I*x^2*Pi* \\ & csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))^{-2}+1/8*I*x^ \\ & 2*Pi*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I/(\exp(2*b*x+2*a)-1)*(d*\exp(2*b*x+2*a) \\ & +\exp(2*b*x+2*a)-1))^{-2}+1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1) \\ & )^{-3}+1/4*x^2*\ln(d*\exp(2*b*x+2*a)+\exp(2*b*x+2*a)-1)+1/8*I*x^2*Pi*csgn(I*(d*\exp \\ & (2*b*x+2*a)+\exp(2*b*x+2*a)-1))*csgn(I/(\exp(2*b*x+2*a)-1)*(d*\exp(2*b*x+2*a) \\ & +\exp(2*b*x+2*a)-1))^{-2}-1/8*I*x^2*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1) \\ & )*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a))^{-2}-1/8*I*x^2*Pi*csgn(I/(\exp(2* \\ & b*x+2*a)-1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))^{-2}+1/8*I*x^2*Pi*csgn( \\ & I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))-1/4*I*x^2*Pi*csgn(I*\exp(b*x+a))*csgn \\ & (I*\exp(2*b*x+2*a))^{-2}-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp \\ & (2*b*x+2*a))^{-2}-1/4/b^2*d*a^2/(1+d)*\ln(d*\exp(2*b*x+2*a)+\exp(2*b*x+2*a)-1)+1/ \\ & 2/b*d*a/(1+d)*\ln(1-\exp(b*x+a)*(1+d)^{(1/2)})*x+1/2/b*d*a/(1+d)*\ln(1+\exp(b*x+a) \\ & )*(1+d)^{(1/2))*x-1/2/b*d/(1+d)*\ln(1-(1+d)*\exp(2*b*x+2*a))*x*a+1/8*I*x^2*Pi* \\ & csgn(I*\exp(2*b*x+2*a))*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I*\exp(2*b*x+2*a)/(\exp \\ & (2*b*x+2*a)-1))-1/8*I*x^2*Pi*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I*(d*\exp(2*b* \\ & x+2*a)+\exp(2*b*x+2*a)-1))*csgn(I/(\exp(2*b*x+2*a)-1)*(d*\exp(2*b*x+2*a)+\exp(2 \\ & *b*x+2*a)-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)- \\ & 1))*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a)) \end{aligned}$$

**maxima** [A] time = 1.10, size = 100, normalized size = 1.00

$$\frac{1}{24} \left( \frac{4x^3}{d} - \frac{3 \left( 2b^2x^2 \log \left( -(d+1)e^{(2bx+2a)} + 1 \right) + 2bx \text{Li}_2 \left( (d+1)e^{(2bx+2a)} \right) - \text{Li}_3 \left( (d+1)e^{(2bx+2a)} \right) \right)}{b^3d} \right) bd + \frac{1}{2} x^2 \text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+d+d\*coth(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{24} \left( \frac{4x^3}{d} - 3(2b^2x^2 \log(-(d+1)e^{(2bx+2a)} + 1) + 2bx \operatorname{dilog}((d+1)e^{(2bx+2a)}) - \operatorname{polylog}(3, (d+1)e^{(2bx+2a)})) / (b^3d) \right) b^3d + \frac{1}{2} x^2 \operatorname{arccoth}(d \operatorname{coth}(bx+a) + d + 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(d + d \operatorname{coth}(a + bx) + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(d + d\*coth(a + b\*x) + 1),x)

[Out] int(x\*acoth(d + d\*coth(a + b\*x) + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(d \operatorname{coth}(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1+d+d\*coth(b\*x+a)),x)

[Out] Integral(x\*acoth(d\*coth(a + b\*x) + d + 1), x)

### 3.224 $\int \coth^{-1}(1 + d + d \coth(a + bx)) dx$

**Optimal.** Leaf size=69

$$-\frac{\text{Li}_2\left(\frac{(d+1)e^{2a+2bx}}{4b}\right) - \frac{1}{2}x \log\left(1 - (d+1)e^{2a+2bx}\right) + x \coth^{-1}(d \coth(a + bx) + d + 1) + \frac{bx^2}{2}}$$

[Out]  $1/2*b*x^2+x*\text{arccoth}(1+d+d*\coth(b*x+a))-1/2*x*\ln(1-(1+d)*\exp(2*b*x+2*a))-1/4*\text{polylog}(2,(1+d)*\exp(2*b*x+2*a))/b$

**Rubi [A]** time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6234, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right) - \frac{1}{2}x \log\left(1 - (d+1)e^{2a+2bx}\right) + x \coth^{-1}(d \coth(a + bx) + d + 1) + \frac{bx^2}{2}}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[1 + d + d*Coth[a + b*x]],x]`

[Out]  $(b*x^2)/2 + x*\text{ArcCoth}[1 + d + d*\text{Coth}[a + b*x]] - (x*\text{Log}[1 - (1 + d)*E^{(2*a + 2*b*x)}])/2 - \text{PolyLog}[2, (1 + d)*E^{(2*a + 2*b*x)}]/(4*b)$

#### Rule 2184

`Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(((c + d*x)^m*(F^(g*(e + f*x))))^n)/(a + b*(F^(g*(e + f*x))))^n], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n])/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2279

`Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 6234

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Coth[a + b\*x]], x] + Dist[b, Int[x/(c - d - c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]

Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(1 + d + d \coth(a + bx)) dx &= x \coth^{-1}(1 + d + d \coth(a + bx)) + b \int \frac{x}{1 + (-1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \coth(a + bx)) + (b(1 + d)) \int \frac{e^{2a+2bx} x}{1 + (-1 - d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 + d)e^{2a+2bx}) + \dots \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 + d)e^{2a+2bx}) + \dots \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 + d + d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 + d)e^{2a+2bx}) - \dots
 \end{aligned}$$

**Mathematica [B]** time = 0.87, size = 197, normalized size = 2.86

$$\frac{-2\text{Li}_2(-\sqrt{d+1}e^{a+bx}) - 2\text{Li}_2(\sqrt{d+1}e^{a+bx}) - 2\log(e^{a+bx})\log(1 - \sqrt{d+1}e^{a+bx}) - 2\log(e^{a+bx})\log(\sqrt{d+1}e^{a+bx})}{4b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[1 + d + d\*Coth[a + b\*x]], x]

[Out] x\*ArcCoth[1 + d + d\*Coth[a + b\*x]] + (b^2\*x^2 + Log[E^(a + b\*x)]^2 - 2\*Log[E^(a + b\*x)]\*Log[1 - Sqrt[1 + d]\*E^(a + b\*x)] - 2\*Log[E^(a + b\*x)]\*Log[1 + Sqrt[1 + d]\*E^(a + b\*x)] + 2\*Log[E^(a + b\*x)]\*Log[E^(-a - b\*x)\*(-1 + (1 + d)\*E^(2\*(a + b\*x)))] - 2\*b\*x\*Log[d\*Cosh[a + b\*x] + (2 + d)\*Sinh[a + b\*x]] - 2\*PolyLog[2, -(Sqrt[1 + d]\*E^(a + b\*x))] - 2\*PolyLog[2, Sqrt[1 + d]\*E^(a + b\*x)])/(4\*b)

**fricas** [B] time = 0.49, size = 226, normalized size = 3.28

$$\frac{b^2 x^2 + bx \log\left(\frac{d \cosh(bx+a) + (d+2) \sinh(bx+a)}{d \cosh(bx+a) + d \sinh(bx+a)}\right) + a \log\left(2(d+1) \cosh(bx+a) + 2(d+1) \sinh(bx+a) + 2\sqrt{d+1}\right) + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d\*d\*coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/2\*(b^2\*x^2 + b\*x\*log((d\*cosh(b\*x + a) + (d + 2)\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + d\*sinh(b\*x + a))) + a\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) + 2\*sqrt(d + 1)) + a\*log(2\*(d + 1)\*cosh(b\*x + a) + 2\*(d + 1)\*sinh(b\*x + a) - 2\*sqrt(d + 1)) - (b\*x + a)\*log(sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - (b\*x + a)\*log(-sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - dilog(sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))) - dilog(-sqrt(d + 1)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(d \coth(bx + a) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d\*d\*coth(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*coth(b\*x + a) + d + 1), x)

**maple** [B] time = 0.56, size = 247, normalized size = 3.58

$$\frac{\operatorname{arccoth}(1 + d + d \coth(bx + a)) \ln(d \coth(bx + a) - d)}{2b} + \frac{\operatorname{arccoth}(1 + d + d \coth(bx + a)) \ln(d \coth(bx + a) + d)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+d\*d\*coth(b\*x+a)),x)

[Out] -1/2/b\*arccoth(1+d\*d\*coth(b\*x+a))\*ln(d\*coth(b\*x+a)-d)+1/2/b\*arccoth(1+d\*d\*coth(b\*x+a))\*ln(d\*coth(b\*x+a)+d)-1/4/b\*dilog(1/2\*(d\*coth(b\*x+a)+d)/d)-1/4/b\*ln(d\*coth(b\*x+a)-d)\*ln(1/2\*(d\*coth(b\*x+a)+d)/d)+1/4/b\*dilog((d\*coth(b\*x+a)+d+2)/(2\*d+2))+1/4/b\*ln(d\*coth(b\*x+a)-d)\*ln((d\*coth(b\*x+a)+d+2)/(2\*d+2))+1/8/b\*ln(d\*coth(b\*x+a)+d)^2-1/4/b\*dilog(1+1/2\*d\*coth(b\*x+a)+1/2\*d)-1/4/b\*ln(d\*coth(b\*x+a)+d)\*ln(1+1/2\*d\*coth(b\*x+a)+1/2\*d)

**maxima** [A] time = 1.10, size = 72, normalized size = 1.04

$$\frac{1}{4}bd\left(\frac{2x^2}{d} - \frac{2bx \log\left(-(d+1)e^{2bx+2a} + 1\right) + \operatorname{Li}_2\left((d+1)e^{2bx+2a}\right)}{b^2d}\right) + x \operatorname{arccoth}(d \coth(bx + a) + d + 1)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*coth(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{4}bd\left(\frac{2x^2}{d} - (2bx \log(-(d+1)e^{2bx+2a}) + 1) + \operatorname{dilog}((d+1)e^{2bx+2a})\right) / (b^2d) + x \operatorname{arccoth}(d \operatorname{coth}(bx+a) + d + 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(d + d \operatorname{coth}(a + bx) + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(d + d\*coth(a + b\*x) + 1),x)

[Out] int(acoth(d + d\*coth(a + b\*x) + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(d \operatorname{coth}(a + bx) + d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+d+d\*coth(b\*x+a)),x)

[Out] Integral(acoth(d\*coth(a + b\*x) + d + 1), x)

$$3.225 \quad \int \frac{\coth^{-1}(1+d+d \coth(a+bx))}{x} dx$$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\coth^{-1}(d \coth(a+bx)+d+1)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(1+d+d\*coth(b\*x+a))/x,x)

**Rubi [A]** time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(1+d+d \coth(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 + d + d\*Coth[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 + d + d\*Coth[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1+d+d \coth(a+bx))}{x} dx = \int \frac{\coth^{-1}(1+d+d \coth(a+bx))}{x} dx$$

**Mathematica [A]** time = 3.50, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1+d+d \coth(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 + d + d\*Coth[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 + d + d\*Coth[a + b\*x]]/x, x]

**fricas [A]** time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(d \coth(bx+a)+d+1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*coth(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*coth(b\*x + a) + d + 1)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \coth(bx + a) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*coth(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*coth(b\*x + a) + d + 1)/x, x)

**maple** [A] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1 + d + d \coth(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+d+d\*coth(b\*x+a))/x,x)

[Out] int(arccoth(1+d+d\*coth(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \coth(bx + a) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+d+d\*coth(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*coth(b\*x + a) + d + 1)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acoth}(d + d \coth(a + bx) + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(d + d\*coth(a + b\*x) + 1)/x,x)

[Out] int(acoth(d + d\*coth(a + b\*x) + 1)/x, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(d \coth(a + bx) + d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+d\*d\*coth(b\*x+a))/x,x)

[Out] Integral(acoth(d\*coth(a + b\*x) + d + 1)/x, x)

### 3.226 $\int x^3 \coth^{-1}(1 - d - d \coth(a + bx)) dx$

**Optimal.** Leaf size=165

$$\frac{3\text{Li}_5((1-d)e^{2a+2bx})}{16b^4} - \frac{3x\text{Li}_4((1-d)e^{2a+2bx})}{8b^3} + \frac{3x^2\text{Li}_3((1-d)e^{2a+2bx})}{8b^2} - \frac{x^3\text{Li}_2((1-d)e^{2a+2bx})}{4b} - \frac{1}{8}x^4 \log(1 - (1-d)e^{2a+2bx})$$

[Out]  $1/20*b*x^5 + 1/4*x^4*\text{arccoth}(1-d-d*\coth(b*x+a)) - 1/8*x^4*\ln(1-(1-d)*\exp(2*b*x+2*a)) - 1/4*x^3*\text{polylog}(2, (1-d)*\exp(2*b*x+2*a))/b + 3/8*x^2*\text{polylog}(3, (1-d)*\exp(2*b*x+2*a))/b^2 - 3/8*x*\text{polylog}(4, (1-d)*\exp(2*b*x+2*a))/b^3 + 3/16*\text{polylog}(5, (1-d)*\exp(2*b*x+2*a))/b^4$

**Rubi [A]** time = 0.31, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}(3, (1-d)e^{2a+2bx})}{8b^2} - \frac{3x\text{PolyLog}(4, (1-d)e^{2a+2bx})}{8b^3} + \frac{3\text{PolyLog}(5, (1-d)e^{2a+2bx})}{16b^4} - \frac{x^3\text{PolyLog}(2, (1-d)e^{2a+2bx})}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{ArcCoth}[1 - d - d*\text{Coth}[a + b*x]], x]$

[Out]  $(b*x^5)/20 + (x^4*\text{ArcCoth}[1 - d - d*\text{Coth}[a + b*x]])/4 - (x^4*\text{Log}[1 - (1 - d)*E^{(2*a + 2*b*x)}])/8 - (x^3*\text{PolyLog}[2, (1 - d)*E^{(2*a + 2*b*x)}])/(4*b) + (3*x^2*\text{PolyLog}[3, (1 - d)*E^{(2*a + 2*b*x)}])/(8*b^2) - (3*x*\text{PolyLog}[4, (1 - d)*E^{(2*a + 2*b*x)}])/(8*b^3) + (3*\text{PolyLog}[5, (1 - d)*E^{(2*a + 2*b*x)}])/(16*b^4)$

#### Rule 2184

$\text{Int}[\frac{(c + d*x)^m}{(a + b*x)^n}, x] := \text{Simp}[\frac{(c + d*x)^{m+1}}{a*d*(m+1)}, x] - \text{Dist}[\frac{b}{a}, \text{Int}[\frac{(c + d*x)^m*(F^{g*(e+f*x)})^n}{(a + b*(F^{g*(e+f*x)})^n)}, x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

$\text{Int}[\frac{(F^{g*(e+f*x)})^n*(c + d*x)^m}{(a + b*x)^n}, x] := \text{Simp}[\frac{(c + d*x)^m*\text{Log}[1 + (b*(F^{g*(e+f*x)})^n)/a]}{b*f*g*n*\text{Log}[F]}, x] - \text{Dist}[\frac{d*m}{b*f*g*n*\text{Log}[F]}, \text{Int}[\frac{(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{g*(e+f*x)})^n)/a]}{a}], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a + 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c - d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(1-d-d \coth(a+bx)) dx &= \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{4}b \int \frac{x^4}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{4}(b(1-d)) \int \frac{e^{2a+2bx}}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^5}{20} + \frac{1}{4}x^4 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{8}x^4 \log(1-(1-d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 147, normalized size = 0.89

$$\frac{1}{16} \left( \frac{3\text{Li}_5\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{6x\text{Li}_4\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{4x^3\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 2x^4 \log\left(\frac{e^{-2(a+bx)}}{d-1} + 1\right) + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[1 - d - d\*Coth[a + b\*x]],x]

[Out] (4\*x^4\*ArcCoth[1 - d - d\*Coth[a + b\*x]] - 2\*x^4\*Log[1 + 1/((-1 + d)\*E^(2\*(a + b\*x))]) + (4\*x^3\*PolyLog[2, -(1/((-1 + d)\*E^(2\*(a + b\*x))))])/b + (6\*x^2\*PolyLog[3, -(1/((-1 + d)\*E^(2\*(a + b\*x))))])/b^2 + (6\*x\*PolyLog[4, -(1/((-1 + d)\*E^(2\*(a + b\*x))))])/b^3 + (3\*PolyLog[5, -(1/((-1 + d)\*E^(2\*(a + b\*x))))])/b^4)/16

**fricas [C]** time = 0.57, size = 450, normalized size = 2.73

$$\frac{2b^5x^5 - 5b^4x^4 \log\left(\frac{d \cosh(bx+a)+d \sinh(bx+a)}{d \cosh(bx+a)+(d-2) \sinh(bx+a)}\right) - 20b^3x^3\text{Li}_2\left(\frac{1}{2}\sqrt{-4d+4}(\cosh(bx+a) + \sinh(bx+a))\right) - 20b^2x^2 \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/40\*(2\*b^5\*x^5 - 5\*b^4\*x^4\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + (d - 2)\*sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 20\*b^3\*x^3\*dilog(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*a^4\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + sqrt(-4\*d + 4)) - 5\*a^4\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - sqrt(-4\*d + 4)) + 60\*b^2\*x^2\*polylog(3, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 60\*b^2\*x^2\*polylog(3, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 120\*b\*x\*polylog(4, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 5\*(b^4\*x^4 - a^4)\*log(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 5\*(b^4\*x^4 - a^4)\*log(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 120\*polylog(5, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 120\*polylog(5, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^4

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(-d \coth(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^3\*arccoth(-d\*coth(b\*x + a) - d + 1), x)

maple [C] time = 6.46, size = 1830, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*arccoth(1-d-d\*coth(b\*x+a)),x)

[Out] 1/8\*I\*x^4\*Pi\*csgn(I\*d/(exp(2\*b\*x+2\*a)-1)\*exp(2\*b\*x+2\*a))^2-1/16\*I\*x^4\*Pi\*csgn(I\*d/(exp(2\*b\*x+2\*a)-1)\*exp(2\*b\*x+2\*a))^3-1/8\*I\*x^4\*Pi\*csgn(I/(exp(2\*b\*x+2\*a)-1)\*(d\*exp(2\*b\*x+2\*a)-exp(2\*b\*x+2\*a)+1))^2+1/2/b^4\*d\*a^3/(d-1)\*dilog(1+exp(b\*x+a)\*(1-d)^(1/2))+1/2/b^4\*d\*a^3/(d-1)\*dilog(1-exp(b\*x+a)\*(1-d)^(1/2))+1/2/b^3/(d-1)\*ln(1+(d-1)\*exp(2\*b\*x+2\*a))\*x\*a^3-3/8/b^4\*d/(d-1)\*ln(1+(d-1)\*exp(2\*b\*x+2\*a))\*a^4-1/4/b\*d/(d-1)\*polylog(2,-(d-1)\*exp(2\*b\*x+2\*a))\*x^3-1/4/b^4\*d/(d-1)\*polylog(2,-(d-1)\*exp(2\*b\*x+2\*a))\*a^3+3/8/b^2\*d/(d-1)\*polylog(3,-(d-1)\*exp(2\*b\*x+2\*a))\*x^2-3/8/b^3\*d/(d-1)\*polylog(4,-(d-1)\*exp(2\*b\*x+2\*a))\*x-1/2/b^3\*a^3/(d-1)\*ln(1+exp(b\*x+a)\*(1-d)^(1/2))\*x-1/2/b^3\*a^3/(d-1)\*ln(1-exp(b\*x+a)\*(1-d)^(1/2))\*x+1/8\*x^4\*ln(d\*exp(2\*b\*x+2\*a)-exp(2\*b\*x+2\*a)+1)+1/20\*b\*x^5+1/8/b^4\*a^4/(d-1)\*ln(d\*exp(2\*b\*x+2\*a)-exp(2\*b\*x+2\*a)+1)+1/2/b^4\*d\*a



$$\begin{aligned} &^4/(d-1)*\ln(1+\exp(b*x+a)*(1-d)^{(1/2)})+1/2/b^4*d*a^4/(d-1)*\ln(1-\exp(b*x+a)*(1-d)^{(1/2)})+1/16*I*x^4*Pi*csgn(I/(\exp(2*b*x+2*a)-1)*(d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)+1))^3-1/8*d/(d-1)*\ln(1+(d-1)*\exp(2*b*x+2*a))*x^4+3/8/b^3/(d-1)*\text{polylog}(4,-(d-1)*\exp(2*b*x+2*a))*x+3/8/b^4/(d-1)*\ln(1+(d-1)*\exp(2*b*x+2*a))*a^4+1/4/b/(d-1)*\text{polylog}(2,-(d-1)*\exp(2*b*x+2*a))*x^3+1/4/b^4/(d-1)*\text{polylog}(2,-(d-1)*\exp(2*b*x+2*a))*a^3-3/8/b^2/(d-1)*\text{polylog}(3,-(d-1)*\exp(2*b*x+2*a))*x^2-1/2/b^4*a^4/(d-1)*\ln(1+\exp(b*x+a)*(1-d)^{(1/2)})-1/2/b^4*a^4/(d-1)*\ln(1-\exp(b*x+a)*(1-d)^{(1/2)})-1/2/b^4*a^3/(d-1)*\text{dilog}(1+\exp(b*x+a)*(1-d)^{(1/2)})-1/2/b^4*a^3/(d-1)*\text{dilog}(1-\exp(b*x+a)*(1-d)^{(1/2)})+3/16/b^4*d/(d-1)*\text{polylog}(5,-(d-1)*\exp(2*b*x+2*a))+1/8/(d-1)*\ln(1+(d-1)*\exp(2*b*x+2*a))*x^4-3/16/b^4/(d-1)*\text{polylog}(5,-(d-1)*\exp(2*b*x+2*a))-1/4*x^4*\ln(\exp(b*x+a))-1/8*x^4*\ln(d)-1/16*I*x^4*Pi*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I*(d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)+1))*csgn(I/(\exp(2*b*x+2*a)-1)*(d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)+1))+1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a))^3+1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))^3-1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*\exp(b*x+a))^2*csgn(I*\exp(2*b*x+2*a))-1/8*I*x^4*Pi*csgn(I*\exp(b*x+a))*csgn(I*\exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))*csgn(I*d/(\exp(2*b*x+2*a)-1)*\exp(2*b*x+2*a))+1/16*I*x^4*Pi*csgn(I*\exp(2*b*x+2*a))*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I*\exp(2*b*x+2*a)/(\exp(2*b*x+2*a)-1))+1/2/b^3*d*a^3/(d-1)*\ln(1-\exp(b*x+a)*(1-d)^{(1/2)})*x-1/2/b^3*d/(d-1)*\ln(1+(d-1)*\exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(d-1)*\ln(1+\exp(b*x+a)*(1-d)^{(1/2)})*x+1/16*I*x^4*Pi*csgn(I*(d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)+1))*csgn(I/(\exp(2*b*x+2*a)-1)*(d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I/(\exp(2*b*x+2*a)-1))*csgn(I/(\exp(2*b*x+2*a)-1)*(d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)+1))^2-1/8/b^4*d*a^4/(d-1)*\ln(d*\exp(2*b*x+2*a)-\exp(2*b*x+2*a)+1) \end{aligned}$$

**maxima** [A] time = 1.10, size = 149, normalized size = 0.90

$$-\frac{1}{4}x^4 \operatorname{arccoth}(d \coth(bx+a) + d - 1) + \frac{1}{40} \left( \frac{2x^5}{d} - \frac{5(2b^4x^4 \log((d-1)e^{2bx+2a}) + 1) + 4b^3x^3 \operatorname{Li}_2(-(d-1)e^{2bx+2a})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="maxima")

[Out] -1/4\*x^4\*arccoth(d\*coth(b\*x + a) + d - 1) + 1/40\*(2\*x^5/d - 5\*(2\*b^4\*x^4\*log((d - 1)\*e^(2\*b\*x + 2\*a) + 1) + 4\*b^3\*x^3\*dilog(-(d - 1)\*e^(2\*b\*x + 2\*a)) - 6\*b^2\*x^2\*polylog(3, -(d - 1)\*e^(2\*b\*x + 2\*a)) + 6\*b\*x\*polylog(4, -(d - 1)\*e^(2\*b\*x + 2\*a)) - 3\*polylog(5, -(d - 1)\*e^(2\*b\*x + 2\*a)))/(b^5\*d))\*b\*d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^3 \operatorname{acoth}(d + d \coth(a + bx) - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x^3*acoth(d + d*coth(a + b*x) - 1), x)`

[Out] `int(-x^3*acoth(d + d*coth(a + b*x) - 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{acoth}(-d \coth(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*acoth(1-d-d*coth(b*x+a)), x)`

[Out] `Integral(x**3*acoth(-d*coth(a + b*x) - d + 1), x)`

### 3.227 $\int x^2 \coth^{-1}(1 - d - d \coth(a + bx)) dx$

**Optimal.** Leaf size=137

$$-\frac{\text{Li}_4((1-d)e^{2a+2bx})}{8b^3} + \frac{x\text{Li}_3((1-d)e^{2a+2bx})}{4b^2} - \frac{x^2\text{Li}_2((1-d)e^{2a+2bx})}{4b} - \frac{1}{6}x^3 \log(1 - (1-d)e^{2a+2bx}) + \frac{1}{3}x^3 \coth^{-1}(d - d \coth(a + bx))$$

[Out] 1/12\*b\*x^4+1/3\*x^3\*arccoth(1-d-d\*coth(b\*x+a))-1/6\*x^3\*ln(1-(1-d)\*exp(2\*b\*x+2\*a))-1/4\*x^2\*polylog(2,(1-d)\*exp(2\*b\*x+2\*a))/b+1/4\*x\*polylog(3,(1-d)\*exp(2\*b\*x+2\*a))/b^2-1/8\*polylog(4,(1-d)\*exp(2\*b\*x+2\*a))/b^3

**Rubi [A]** time = 0.27, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6242, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}(3, (1-d)e^{2a+2bx})}{4b^2} - \frac{\text{PolyLog}(4, (1-d)e^{2a+2bx})}{8b^3} - \frac{x^2\text{PolyLog}(2, (1-d)e^{2a+2bx})}{4b} - \frac{1}{6}x^3 \log(1 - (1-d)e^{2a+2bx})$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 - d - d\*Coth[a + b\*x]], x]

[Out] (b\*x^4)/12 + (x^3\*ArcCoth[1 - d - d\*Coth[a + b\*x]])/3 - (x^3\*Log[1 - (1 - d)\*E^(2\*a + 2\*b\*x)])/6 - (x^2\*PolyLog[2, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b) + (x\*PolyLog[3, (1 - d)\*E^(2\*a + 2\*b\*x)])/(4\*b^2) - PolyLog[4, (1 - d)\*E^(2\*a + 2\*b\*x)]/(8\*b^3)

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6242

```
Int[ArcCoth[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.), x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1-d-d \coth(a+bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{3}b \int \frac{x^3}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) + \frac{1}{3}(b(1-d)) \int \frac{e^{2a+2bx}}{1+(-1+d)e^{2a+2bx}} dx \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx}) \\
&= \frac{bx^4}{12} + \frac{1}{3}x^3 \coth^{-1}(1-d-d \coth(a+bx)) - \frac{1}{6}x^3 \log(1-(1-d)e^{2a+2bx})
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 121, normalized size = 0.88

$$\frac{1}{24} \left( \frac{3\text{Li}_4\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{6x^2\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3 \log\left(\frac{e^{-2(a+bx)}}{d-1} + 1\right) + 8x^3 \coth^{-1}(d(-\coth(a+bx))) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 - d - d\*Coth[a + b\*x]],x]

[Out] (8\*x^3\*ArcCoth[1 - d - d\*Coth[a + b\*x]] - 4\*x^3\*Log[1 + 1/((-1 + d)\*E^(2\*(a + b\*x))]) + (6\*x^2\*PolyLog[2, -(1/((-1 + d)\*E^(2\*(a + b\*x))))])/b + (6\*x\*PolyLog[3, -(1/((-1 + d)\*E^(2\*(a + b\*x))))])/b^2 + (3\*PolyLog[4, -(1/((-1 + d)\*E^(2\*(a + b\*x))))])/b^3)/24

**fricas [C]** time = 0.53, size = 381, normalized size = 2.78

$$\frac{b^4x^4 - 2b^3x^3 \log\left(\frac{d \cosh(bx+a)+d \sinh(bx+a)}{d \cosh(bx+a)+(d-2) \sinh(bx+a)}\right) - 6b^2x^2\text{Li}_2\left(\frac{1}{2}\sqrt{-4d+4}(\cosh(bx+a)+\sinh(bx+a))\right) - 6b^2x^2 \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="fricas")

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log((d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b
*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*
x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d +
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d + 4)
*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d + 4)*(cosh
(b*x + a) + sinh(b*x + a))))/b^3
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(-d \coth(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-d-d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(-d*coth(b*x + a) - d + 1), x)
```

**maple** [C] time = 5.46, size = 1771, normalized size = 12.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-d-d*coth(b*x+a)),x)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(
2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*ex
p(2*b*x+2*a))-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/6*
I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))^2+1
/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3-1/12*I*x^3*Pi*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x
+2*a))^2+1/12*b*x^4+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/3*x^3*ln(exp(b
*x+a))-1/6*x^3*ln(d)+1/6/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/8/b^3/(d-1)
*polylog(4,-(d-1)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)
*exp(2*b*x+2*a))^3+1/6*I*x^3*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a)
)^2-1/6/b^3*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/3/b^3/(d-1)*ln
(1+(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*
x^2-1/4/b^3/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polylog
(3,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+
1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/8/b^3*d/(d-1)*polylog(4,-(
```

$(d-1)\exp(2bx+2a)+1/2/b^3a^2/(d-1)\operatorname{dilog}(1+\exp(bx+a)(1-d)^{1/2})+1/2/b^3a^2/(d-1)\operatorname{dilog}(1-\exp(bx+a)(1-d)^{1/2})-1/6*d/(d-1)\ln(1+(d-1)\exp(2bx+2a))*x^3+1/6*x^3*\ln(d*\exp(2bx+2a)-\exp(2bx+2a)+1)-1/2/b^3*d*a^2/(d-1)\operatorname{dilog}(1+\exp(bx+a)(1-d)^{1/2})-1/2/b^3*d*a^2/(d-1)\operatorname{dilog}(1-\exp(bx+a)(1-d)^{1/2})+1/3/b^3*d/(d-1)\ln(1+(d-1)\exp(2bx+2a))*a^3-1/4/b*d/(d-1)*\operatorname{polylog}(2,-(d-1)\exp(2bx+2a))*x^2+1/4/b^3*d/(d-1)*\operatorname{polylog}(2,-(d-1)\exp(2bx+2a))*a^2+1/4/b^2*d/(d-1)*\operatorname{polylog}(3,-(d-1)\exp(2bx+2a))*x-1/2/b^2/(d-1)\ln(1+(d-1)\exp(2bx+2a))*x*a^2+1/2/b^2*a^2/(d-1)\ln(1+\exp(bx+a)(1-d)^{1/2})*x+1/2/b^2*a^2/(d-1)\ln(1-\exp(bx+a)(1-d)^{1/2})*x-1/2/b^3*d*a^3/(d-1)\ln(1+\exp(bx+a)(1-d)^{1/2})-1/2/b^3*d*a^3/(d-1)\ln(1-\exp(bx+a)(1-d)^{1/2})+1/12*I*x^3*Pi*csgn(I/(\exp(2bx+2a)-1))*(d*\exp(2bx+2a)-\exp(2bx+2a)+1))^3+1/12*I*x^3*Pi*csgn(I*(d*\exp(2bx+2a)-\exp(2bx+2a)+1))*csgn(I/(\exp(2bx+2a)-1))*(d*\exp(2bx+2a)-\exp(2bx+2a)+1))^2+1/12*I*x^3*Pi*csgn(I/(\exp(2bx+2a)-1))*csgn(I/(\exp(2bx+2a)-1))*(d*\exp(2bx+2a)-\exp(2bx+2a)+1))^2+1/6/b^3*d*a^3/(d-1)\ln(d*\exp(2bx+2a)-\exp(2bx+2a)+1)-1/12*I*x^3*Pi*csgn(I/(\exp(2bx+2a)-1))*csgn(I*(d*\exp(2bx+2a)-\exp(2bx+2a)+1))*csgn(I/(\exp(2bx+2a)-1))*(d*\exp(2bx+2a)-\exp(2bx+2a)+1))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*\exp(2bx+2a)/(\exp(2bx+2a)-1))*csgn(I*d/(\exp(2bx+2a)-1)*\exp(2bx+2a))-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(\exp(2bx+2a)-1)*\exp(2bx+2a))^2-1/12*I*x^3*Pi*csgn(I*\exp(2bx+2a))*csgn(I*\exp(2bx+2a)/(\exp(2bx+2a)-1))^2-1/12*I*x^3*Pi*csgn(I/(\exp(2bx+2a)-1))*csgn(I*\exp(2bx+2a)/(\exp(2bx+2a)-1))^2+1/2/b^2*d/(d-1)\ln(1+(d-1)\exp(2bx+2a))*x*a^2-1/2/b^2*d*a^2/(d-1)\ln(1+\exp(bx+a)(1-d)^{1/2})*x-1/2/b^2*d*a^2/(d-1)\ln(1-\exp(bx+a)(1-d)^{1/2})*x$

**maxima** [A] time = 1.10, size = 125, normalized size = 0.91

$$-\frac{1}{3}x^3 \operatorname{arccoth}(d \coth(bx+a) + d - 1) + \frac{1}{36} \left( \frac{3x^4}{d} - \frac{2(4b^3x^3 \log((d-1)e^{2bx+2a} + 1) + 6b^2x^2 \operatorname{Li}_2(-(d-1)e^{2bx+2a}))}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="maxima")

[Out] -1/3\*x^3\*arccoth(d\*coth(b\*x + a) + d - 1) + 1/36\*(3\*x^4/d - 2\*(4\*b^3\*x^3\*log((d - 1)\*e^(2\*b\*x + 2\*a) + 1) + 6\*b^2\*x^2\*dilog(-(d - 1)\*e^(2\*b\*x + 2\*a)) - 6\*b\*x\*polylog(3, -(d - 1)\*e^(2\*b\*x + 2\*a)) + 3\*polylog(4, -(d - 1)\*e^(2\*b\*x + 2\*a)))/(b^4\*d))\*b\*d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2 \operatorname{acoth}(d + d \coth(a + bx) - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x^2*acoth(d + d*coth(a + b*x) - 1),x)`

[Out] `int(-x^2*acoth(d + d*coth(a + b*x) - 1), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(-d \coth(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(1-d-d*coth(b*x+a)),x)`

[Out] `Integral(x**2*acoth(-d*coth(a + b*x) - d + 1), x)`



### 3.228 $\int x \coth^{-1}(1 - d - d \coth(a + bx)) dx$

**Optimal.** Leaf size=109

$$\frac{\text{Li}_3((1-d)e^{2a+2bx})}{8b^2} - \frac{x\text{Li}_2((1-d)e^{2a+2bx})}{4b} - \frac{1}{4}x^2 \log(1 - (1-d)e^{2a+2bx}) + \frac{1}{2}x^2 \coth^{-1}(d(-\coth(a+bx)) - d + 1) + \frac{bx}{6}$$

[Out]  $1/6*b*x^3 + 1/2*x^2*\text{arccoth}(1-d-d*\coth(b*x+a)) - 1/4*x^2*\ln(1-(1-d)*\exp(2*b*x+2*a)) - 1/4*x*\text{polylog}(2, (1-d)*\exp(2*b*x+2*a))/b + 1/8*\text{polylog}(3, (1-d)*\exp(2*b*x+2*a))/b^2$

**Rubi [A]** time = 0.24, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {6242, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}(3, (1-d)e^{2a+2bx})}{8b^2} - \frac{x\text{PolyLog}(2, (1-d)e^{2a+2bx})}{4b} - \frac{1}{4}x^2 \log(1 - (1-d)e^{2a+2bx}) + \frac{1}{2}x^2 \coth^{-1}(d(-\coth(a+bx)) - d + 1) + \frac{bx}{6}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[1 - d - d*Coth[a + b*x]], x]`

[Out]  $(b*x^3)/6 + (x^2*\text{ArcCoth}[1 - d - d*\text{Coth}[a + b*x]])/2 - (x^2*\text{Log}[1 - (1 - d)*E^{(2*a + 2*b*x)}])/4 - (x*\text{PolyLog}[2, (1 - d)*E^{(2*a + 2*b*x)}])/(4*b) + \text{PolyLog}[3, (1 - d)*E^{(2*a + 2*b*x)}]/(8*b^2)$

#### Rule 2184

`Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[`

{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*x\_)))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_)^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
)))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6242

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*x\_]]\*(d\_.)\*((e\_.) + (f\_.)\*x\_)^(m  
\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Coth[a + b\*x]])/(f\*  
(m + 1)), x] + Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - d - c\*E^(2\*a  
+ 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -  
d)^2, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*x\_)^(p\_.)]/((d\_.) + (e\_.)\*x\_), x\_S  
ymbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(1 - d - d \coth(a + bx)) dx &= \frac{1}{2} x^2 \coth^{-1}(1 - d - d \coth(a + bx)) + \frac{1}{2} b \int \frac{x^2}{1 + (-1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \coth(a + bx)) + \frac{1}{2} (b(1 - d)) \int \frac{e^{2a+2bx}}{1 + (-1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 - d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 - d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 - d)e^{2a+2bx}) \\
 &= \frac{bx^3}{6} + \frac{1}{2} x^2 \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 - d)e^{2a+2bx})
 \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 94, normalized size = 0.86

$$\frac{2b^2x^2 \left( 2 \coth^{-1}(d(-\coth(a+bx)) - d + 1) - \log\left(\frac{e^{-2(a+bx)}}{d-1} + 1\right) \right) + 2bx \operatorname{Li}_2\left(-\frac{e^{-2(a+bx)}}{d-1}\right) + \operatorname{Li}_3\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 - d - d\*Coth[a + b\*x]], x]

[Out] (2\*b^2\*x^2\*(2\*ArcCoth[1 - d - d\*Coth[a + b\*x]] - Log[1 + 1/((-1 + d)\*E^(2\*(a + b\*x))])) + 2\*b\*x\*PolyLog[2, -(1/((-1 + d)\*E^(2\*(a + b\*x)))] + PolyLog[3, -(1/((-1 + d)\*E^(2\*(a + b\*x)))])/(8\*b^2)

**fricas [C]** time = 0.53, size = 322, normalized size = 2.95

$$\frac{2b^3x^3 - 3b^2x^2 \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{d \cosh(bx+a) + (d-2) \sinh(bx+a)}\right) - 6bx \operatorname{Li}_2\left(\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a))\right) - 6bx \operatorname{Li}_3\left(\frac{1}{2} \sqrt{-4d+4} (\cosh(bx+a) + \sinh(bx+a))\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*coth(b\*x+a)), x, algorithm="fricas")

[Out] 1/12\*(2\*b^3\*x^3 - 3\*b^2\*x^2\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + (d - 2)\*sinh(b\*x + a))) - 6\*b\*x\*dilog(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 6\*b\*x\*dilog(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - 3\*a^2\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + sqrt(-4\*d + 4)) - 3\*a^2\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - sqrt(-4\*d + 4)) - 3\*(b^2\*x^2 - a^2)\*log(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - 3\*(b^2\*x^2 - a^2)\*log(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) + 6\*polylog(3, 1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) + 6\*polylog(3, -1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(-d \coth(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*coth(b\*x+a)), x, algorithm="giac")

[Out] integrate(x\*arccoth(-d\*coth(b\*x + a) - d + 1), x)

**maple [C]** time = 4.88, size = 1688, normalized size = 15.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(1-d-d*coth(b*x+a)),x)`

[Out]  $\frac{1}{8}I^2x^2\text{Picsgn}(I^2d)\text{csgn}(I^2\exp(2bx+2a)/(\exp(2bx+2a)-1))\text{csgn}(I^2d/(\exp(2bx+2a)-1)\exp(2bx+2a))+\frac{1}{8}I^2x^2\text{Picsgn}(I^2\exp(2bx+2a))\text{csgn}(I/(\exp(2bx+2a)-1))\text{csgn}(I^2\exp(2bx+2a)/(\exp(2bx+2a)-1))-1/8I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)-1))\text{csgn}(I^2(\exp(2bx+2a)-\exp(2bx+2a)+1))\text{csgn}(I/(\exp(2bx+2a)-1)(\exp(2bx+2a)-\exp(2bx+2a)+1))-1/4/b^2d/(d-1)\ln(1+(d-1)\exp(2bx+2a))a^2-1/8/b^2/(d-1)\text{polylog}(3,-(d-1)\exp(2bx+2a))+1/4/(d-1)\ln(1+(d-1)\exp(2bx+2a))x^2-1/2x^2\ln(\exp(bx+a))-1/4x^2\ln(d)+1/8I^2x^2\text{Picsgn}(I^2\exp(2bx+2a))^3+1/6b^2x^3-1/4/b^2d/(d-1)\text{polylog}(2,-(d-1)\exp(2bx+2a))x-1/4/b^2d/(d-1)\text{polylog}(2,-(d-1)\exp(2bx+2a))a+1/2/b/(d-1)\ln(1+(d-1)\exp(2bx+2a))xa-1/2/b^2a/(d-1)\ln(1+\exp(bx+a))(1-d)^{1/2})x-1/2/b^2a/(d-1)\ln(1-\exp(bx+a))(1-d)^{1/2})x+1/2/b^2da^2/(d-1)\ln(1+\exp(bx+a))(1-d)^{1/2})+1/2/b^2da^2/(d-1)\ln(1-\exp(bx+a))(1-d)^{1/2})+1/2/b^2da^2/(d-1)\text{dilog}(1+\exp(bx+a))(1-d)^{1/2})+1/2/b^2da^2/(d-1)\text{dilog}(1-\exp(bx+a))(1-d)^{1/2})+1/4I^2x^2\text{Picsgn}(I^2d/(\exp(2bx+2a)-1)\exp(2bx+2a))^2-1/4/b^2da^2/(d-1)\ln(d\exp(2bx+2a)-\exp(2bx+2a)+1)+1/8I^2x^2\text{Picsgn}(I^2\exp(2bx+2a)/(\exp(2bx+2a)-1))^3+1/4/b^2a^2/(d-1)\ln(d\exp(2bx+2a)-\exp(2bx+2a)+1)+1/8I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)-1)(\exp(2bx+2a)-\exp(2bx+2a)+1))^3+1/4/b^2/(d-1)\ln(1+(d-1)\exp(2bx+2a))a^2+1/4/b/(d-1)\text{polylog}(2,-(d-1)\exp(2bx+2a))x+1/4/b^2/(d-1)\text{polylog}(2,-(d-1)\exp(2bx+2a))a-1/2/b^2a^2/(d-1)\ln(1+\exp(bx+a))(1-d)^{1/2})-1/2/b^2a^2/(d-1)\ln(1-\exp(bx+a))(1-d)^{1/2})-1/4d/(d-1)\ln(1+(d-1)\exp(2bx+2a))x^2+1/8/b^2d/(d-1)\text{polylog}(3,-(d-1)\exp(2bx+2a))-1/2/b^2a/(d-1)\text{dilog}(1+\exp(bx+a))(1-d)^{1/2})-1/2/b^2a/(d-1)\text{dilog}(1-\exp(bx+a))(1-d)^{1/2})+1/8I^2x^2\text{Picsgn}(I^2(\exp(2bx+2a)-\exp(2bx+2a)+1))\text{csgn}(I/(\exp(2bx+2a)-1)(\exp(2bx+2a)-\exp(2bx+2a)+1))^2+1/8I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)-1))\text{csgn}(I/(\exp(2bx+2a)-1)(\exp(2bx+2a)-\exp(2bx+2a)+1))^2-1/8I^2x^2\text{Picsgn}(I^2d/(\exp(2bx+2a)-1)\exp(2bx+2a))^3+1/4x^2\ln(d\exp(2bx+2a)-\exp(2bx+2a)+1)-1/8I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)-1))\text{csgn}(I^2\exp(2bx+2a)/(\exp(2bx+2a)-1))^2-1/8I^2x^2\text{Picsgn}(I^2\exp(2bx+2a)/(\exp(2bx+2a)-1))\text{csgn}(I^2d/(\exp(2bx+2a)-1)\exp(2bx+2a))^2-1/4I^2x^2\text{Picsgn}(I/(\exp(2bx+2a)-1)(\exp(2bx+2a)-\exp(2bx+2a)+1))^2-1/8I^2x^2\text{Picsgn}(I^2d)\text{csgn}(I^2d/(\exp(2bx+2a)-1)\exp(2bx+2a))^2-1/8I^2x^2\text{Picsgn}(I^2\exp(2bx+2a))\text{csgn}(I^2\exp(2bx+2a)/(\exp(2bx+2a)-1))^2+1/8I^2x^2\text{Picsgn}(I^2\exp(bx+a))^2\text{csgn}(I^2\exp(2bx+2a))-1/4I^2x^2\text{Picsgn}(I^2\exp(bx+a))\text{csgn}(I^2\exp(2bx+2a))^2+1/2/b^2da/(d-1)\ln(1-\exp(bx+a))(1-d)^{1/2})x-1/2/b^2d/(d-1)\ln(1+(d-1)\exp(2bx+2a))xa+1/2/b^2da/(d-1)\ln(1+\exp(bx+a))(1-d)^{1/2})x$

**maxima** [A] time = 1.12, size = 101, normalized size = 0.93

$$\frac{1}{24} \left( \frac{4x^3}{d} - \frac{3 \left( 2b^2x^2 \log \left( (d-1)e^{2bx+2a} + 1 \right) + 2bx \text{Li}_2 \left( -(d-1)e^{2bx+2a} \right) - \text{Li}_3 \left( -(d-1)e^{2bx+2a} \right) \right)}{b^3d} \right) bd - \frac{1}{2} x^2 \arccoth \left( \frac{1-d-d\coth(bx+a)}{1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{24} \left( \frac{4x^3}{d} - 3(2b^2x^2 \log((d-1)e^{2bx+2a}) + 1) + 2bx \operatorname{dilog}(- (d-1)e^{2bx+2a}) - \operatorname{polylog}(3, -(d-1)e^{2bx+2a}) \right) / (b^3d) - b^2d - \frac{1}{2}x^2 \operatorname{arccoth}(d \operatorname{coth}(bx+a) + d - 1)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x \operatorname{acoth}(d + d \operatorname{coth}(a + bx) - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x\*acoth(d + d\*coth(a + b\*x) - 1),x)

[Out] int(-x\*acoth(d + d\*coth(a + b\*x) - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(-d \operatorname{coth}(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1-d-d\*coth(b\*x+a)),x)

[Out] Integral(x\*acoth(-d\*coth(a + b\*x) - d + 1), x)

### 3.229 $\int \coth^{-1}(1 - d - d \coth(a + bx)) dx$

Optimal. Leaf size=76

$$-\frac{\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1-d)e^{2a+2bx}\right) + x \coth^{-1}(d(-\coth(a+bx)) - d + 1) + \frac{bx^2}{2}$$

[Out]  $1/2*b*x^2+x*\text{arccoth}(1-d-d*\coth(b*x+a))-1/2*x*\ln(1-(1-d)*\exp(2*b*x+2*a))-1/4*\text{polylog}(2,(1-d)*\exp(2*b*x+2*a))/b$

**Rubi [A]** time = 0.14, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6234, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1-d)e^{2a+2bx}\right) + x \coth^{-1}(d(-\coth(a+bx)) - d + 1) + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[1 - d - d*Coth[a + b*x]], x]`

[Out]  $(b*x^2)/2 + x*\text{ArcCoth}[1 - d - d*\text{Coth}[a + b*x]] - (x*\text{Log}[1 - (1 - d)*E^{(2*a + 2*b*x)}])/2 - \text{PolyLog}[2, (1 - d)*E^{(2*a + 2*b*x)}]/(4*b)$

#### Rule 2184

`Int[(((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(((c + d*x)^m*(F^(g*(e + f*x))))^n)/(a + b*(F^(g*(e + f*x))))^n], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n])/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2279

`Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 6234

Int[ArcCoth[(c\_.) + Coth[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Coth[a + b\*x]], x] + Dist[b, Int[x/(c - d - c\*E^(2\*a + 2\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]

Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(1 - d - d \coth(a + bx)) dx &= x \coth^{-1}(1 - d - d \coth(a + bx)) + b \int \frac{x}{1 + (-1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \coth(a + bx)) + (b(1 - d)) \int \frac{e^{2a+2bx} x}{1 + (-1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 - d)e^{2a+2bx}) + \frac{1}{2} \int \frac{e^{2a+2bx} x}{1 + (-1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 - d)e^{2a+2bx}) + \frac{1}{2} \int \frac{e^{2a+2bx} x}{1 + (-1 + d)e^{2a+2bx}} dx \\
 &= \frac{bx^2}{2} + x \coth^{-1}(1 - d - d \coth(a + bx)) - \frac{1}{2} x \log(1 - (1 - d)e^{2a+2bx}) - \frac{1}{2} \int \frac{e^{2a+2bx} x}{1 + (-1 + d)e^{2a+2bx}} dx
 \end{aligned}$$

**Mathematica [B]** time = 0.79, size = 208, normalized size = 2.74

$$\frac{-2\text{Li}_2(-\sqrt{1-d}e^{a+bx}) - 2\text{Li}_2(\sqrt{1-d}e^{a+bx}) - 2\log(e^{a+bx})\log(1 - \sqrt{1-d}e^{a+bx}) - 2\log(e^{a+bx})\log(\sqrt{1-d}e^{a+bx})}{4b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[1 - d - d\*Coth[a + b\*x]], x]

[Out] x\*ArcCoth[1 - d - d\*Coth[a + b\*x]] + (b^2\*x^2 + Log[E^(a + b\*x)]^2 - 2\*Log[E^(a + b\*x)]\*Log[1 - Sqrt[1 - d]\*E^(a + b\*x)] - 2\*Log[E^(a + b\*x)]\*Log[1 + Sqrt[1 - d]\*E^(a + b\*x)] + 2\*Log[E^(a + b\*x)]\*Log[E^(-a - b\*x)\*(1 + (-1 + d)\*E^(2\*(a + b\*x)))] - 2\*b\*x\*Log[d\*Cosh[a + b\*x] + (-2 + d)\*Sinh[a + b\*x]] - 2\*PolyLog[2, -(Sqrt[1 - d]\*E^(a + b\*x))] - 2\*PolyLog[2, Sqrt[1 - d]\*E^(a + b\*x)])/(4\*b)

**fricas** [B] time = 0.84, size = 239, normalized size = 3.14

$$\frac{b^2 x^2 - bx \log\left(\frac{d \cosh(bx+a) + d \sinh(bx+a)}{d \cosh(bx+a) + (d-2) \sinh(bx+a)}\right) + a \log\left(2(d-1) \cosh(bx+a) + 2(d-1) \sinh(bx+a) + \sqrt{-4d+4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="fricas")

[Out] 1/2\*(b^2\*x^2 - b\*x\*log((d\*cosh(b\*x + a) + d\*sinh(b\*x + a))/(d\*cosh(b\*x + a) + (d - 2)\*sinh(b\*x + a))) + a\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) + sqrt(-4\*d + 4)) + a\*log(2\*(d - 1)\*cosh(b\*x + a) + 2\*(d - 1)\*sinh(b\*x + a) - sqrt(-4\*d + 4)) - (b\*x + a)\*log(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - (b\*x + a)\*log(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a)) + 1) - dilog(1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))) - dilog(-1/2\*sqrt(-4\*d + 4)\*(cosh(b\*x + a) + sinh(b\*x + a))))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(-d \coth(bx + a) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(-d\*coth(b\*x + a) - d + 1), x)

**maple** [B] time = 0.52, size = 271, normalized size = 3.57

$$-\frac{\operatorname{arccoth}(1-d-d \coth(bx+a)) \ln(-d \coth(bx+a)+d)}{2b} + \frac{\operatorname{arccoth}(1-d-d \coth(bx+a)) \ln(-d \coth(bx+a)-d)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-d-d\*coth(b\*x+a)),x)

[Out] -1/2/b\*arccoth(1-d-d\*coth(b\*x+a))\*ln(-d\*coth(b\*x+a)+d)+1/2/b\*arccoth(1-d-d\*coth(b\*x+a))\*ln(-d\*coth(b\*x+a)-d)+1/8/b\*ln(-d\*coth(b\*x+a)-d)^2-1/4/b\*dilog(1-1/2\*d\*coth(b\*x+a)-1/2\*d)-1/4/b\*ln(-d\*coth(b\*x+a)-d)\*ln(1-1/2\*d\*coth(b\*x+a)-1/2\*d)-1/4/b\*dilog(-1/2\*(-d\*coth(b\*x+a)-d)/d)-1/4/b\*ln(-d\*coth(b\*x+a)+d)\*ln(-1/2\*(-d\*coth(b\*x+a)-d)/d)+1/4/b\*dilog((-d\*coth(b\*x+a)-d+2)/(-2\*d+2))+1/4/b\*ln(-d\*coth(b\*x+a)+d)\*ln((-d\*coth(b\*x+a)-d+2)/(-2\*d+2))

**maxima** [A] time = 1.11, size = 73, normalized size = 0.96

$$\frac{1}{4}bd \left( \frac{2x^2}{d} - \frac{2bx \log\left((d-1)e^{2bx+2a} + 1\right) + \operatorname{Li}_2\left(- (d-1)e^{2bx+2a}\right)}{b^2d} \right) - x \operatorname{arccoth}(d \coth(bx+a) + d - 1)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{4}bd\left(\frac{2x^2}{d} - (2bx \log((d-1)e^{2bx+2a}) + 1) + \operatorname{dilog}(-(d-1)e^{2bx+2a})\right) / (b^2d) - x \operatorname{arccoth}(d \operatorname{coth}(bx+a) + d - 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\operatorname{acoth}(d + d \operatorname{coth}(a + bx) - 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-acoth(d + d\*coth(a + b\*x) - 1),x)

[Out] int(-acoth(d + d\*coth(a + b\*x) - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(-d \operatorname{coth}(a + bx) - d + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-d-d\*coth(b\*x+a)),x)

[Out] Integral(acoth(-d\*coth(a + b\*x) - d + 1), x)

$$3.230 \quad \int \frac{\coth^{-1}(1-d-d \coth(a+bx))}{x} dx$$

Optimal. Leaf size=22

$$\text{Int}\left(\frac{\coth^{-1}(d(-\coth(a+bx))-d+1)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(1-d-d\*coth(b\*x+a))/x,x)

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(1-d-d \coth(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 - d - d\*Coth[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - d - d\*Coth[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1-d-d \coth(a+bx))}{x} dx = \int \frac{\coth^{-1}(1-d-d \coth(a+bx))}{x} dx$$

**Mathematica [A]** time = 3.63, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1-d-d \coth(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 - d - d\*Coth[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 - d - d\*Coth[a + b\*x]]/x, x]

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{arccoth}(d \coth(bx+a)+d-1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(-arccoth(d\*coth(b\*x + a) + d - 1)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(-d \operatorname{coth}(bx + a) - d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(-d\*coth(b\*x + a) - d + 1)/x, x)

**maple** [A] time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(1 - d - d \operatorname{coth}(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-d-d\*coth(b\*x+a))/x,x)

[Out] int(arccoth(1-d-d\*coth(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\operatorname{arccoth}(d \operatorname{coth}(bx + a) + d - 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-d-d\*coth(b\*x+a))/x,x, algorithm="maxima")

[Out] -integrate(arccoth(d\*coth(b\*x + a) + d - 1)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{\operatorname{acoth}(d + d \operatorname{coth}(a + bx) - 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-acoth(d + d\*coth(a + b\*x) - 1)/x,x)

[Out] int(-acoth(d + d\*coth(a + b\*x) - 1)/x, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(-d \operatorname{coth}(a + bx) - d + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-d-d\*coth(b\*x+a))/x,x)

[Out] Integral(acoth(-d\*coth(a + b\*x) - d + 1)/x, x)

### 3.231 $\int (e + fx)^3 \coth^{-1}(\tan(a + bx)) dx$

**Optimal.** Leaf size=302

$$\frac{3f^3 \operatorname{Li}_5(-ie^{2i(a+bx)})}{16b^4} + \frac{3f^3 \operatorname{Li}_5(ie^{2i(a+bx)})}{16b^4} + \frac{3if^2(e+fx) \operatorname{Li}_4(-ie^{2i(a+bx)})}{8b^3} - \frac{3if^2(e+fx) \operatorname{Li}_4(ie^{2i(a+bx)})}{8b^3} + \frac{3f(e+fx)}{b}$$

[Out]  $\frac{1}{4}*(f*x+e)^4*\operatorname{arccoth}(\tan(b*x+a))/f + \frac{1}{4}*I*(f*x+e)^4*\operatorname{arctan}(\exp(2*I*(b*x+a)))/f - \frac{1}{4}*I*(f*x+e)^3*\operatorname{polylog}(2, -I*\exp(2*I*(b*x+a)))/b + \frac{1}{4}*I*(f*x+e)^3*\operatorname{polylog}(2, I*\exp(2*I*(b*x+a)))/b + \frac{3}{8}*f*(f*x+e)^2*\operatorname{polylog}(3, -I*\exp(2*I*(b*x+a)))/b^2 - \frac{3}{8}*f*(f*x+e)^2*\operatorname{polylog}(3, I*\exp(2*I*(b*x+a)))/b^2 + \frac{3}{8}*I*f^2*(f*x+e)*\operatorname{polylog}(4, -I*\exp(2*I*(b*x+a)))/b^3 - \frac{3}{8}*I*f^2*(f*x+e)*\operatorname{polylog}(4, I*\exp(2*I*(b*x+a)))/b^3 - \frac{3}{16}*f^3*\operatorname{polylog}(5, -I*\exp(2*I*(b*x+a)))/b^4 + \frac{3}{16}*f^3*\operatorname{polylog}(5, I*\exp(2*I*(b*x+a)))/b^4$

**Rubi [A]** time = 0.24, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6252, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\operatorname{PolyLog}(4, -ie^{2i(a+bx)})}{8b^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}(4, ie^{2i(a+bx)})}{8b^3} + \frac{3f(e+fx)^2\operatorname{PolyLog}(3, -ie^{2i(a+bx)})}{8b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^3*\operatorname{ArcCoth}[\operatorname{Tan}[a + b*x]], x]$

[Out]  $((e + f*x)^4*\operatorname{ArcCoth}[\operatorname{Tan}[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*\operatorname{ArcTan}[E^{((2*I)*(a + b*x))}]/f - ((I/4)*(e + f*x)^3*\operatorname{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}])/b + ((I/4)*(e + f*x)^3*\operatorname{PolyLog}[2, I*E^{((2*I)*(a + b*x))}])/b + (3*f*(e + f*x)^2*\operatorname{PolyLog}[3, (-I)*E^{((2*I)*(a + b*x))}])/(8*b^2) - (3*f*(e + f*x)^2*\operatorname{PolyLog}[3, I*E^{((2*I)*(a + b*x))}])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*\operatorname{PolyLog}[4, (-I)*E^{((2*I)*(a + b*x))}])/b^3 - (((3*I)/8)*f^2*(e + f*x)*\operatorname{PolyLog}[4, I*E^{((2*I)*(a + b*x))}])/b^3 - (3*f^3*\operatorname{PolyLog}[5, (-I)*E^{((2*I)*(a + b*x))}])/(16*b^4) + (3*f^3*\operatorname{PolyLog}[5, I*E^{((2*I)*(a + b*x))}])/(16*b^4)$

#### Rule 2282

$\operatorname{Int}[u, x\_Symbol] := \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_))}^{(m\_)} /; \operatorname{FreeQ}[\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n]] \&\& \operatorname{!MatchQ}[u, E^{((c\_)*(a\_)+(b\_)*x)}*(F\_)]/v] /; \operatorname{FreeQ}[\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_)^(m_.), x_Symbol]
:= Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
e, f}, x] && IGtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx)^3 \coth^{-1}(\tan(a + bx)) dx &= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} - \frac{b \int (e + fx)^4 \sec(2a + 2bx) dx}{4f} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} + \frac{1}{2} \int (e + fx)^3 \sec(2a + 2bx) dx \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \sec(2a + 2bx)}{2} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \sec(2a + 2bx)}{2} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \sec(2a + 2bx)}{2} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \sec(2a + 2bx)}{2} \\
&= \frac{(e + fx)^4 \coth^{-1}(\tan(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \sec(2a + 2bx)}{2}
\end{aligned}$$

**Mathematica [B]** time = 0.35, size = 654, normalized size = 2.17

$$\frac{1}{4}x(4e^3 + 6e^2fx + 4ef^2x^2 + f^3x^3) \coth^{-1}(\tan(a+bx)) + \frac{-8b^4e^3x \log(1 - ie^{2i(a+bx)}) + 8b^4e^3x \log(1 + ie^{2i(a+bx)})}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^3\*ArcCoth[Tan[a + b\*x]],x]

[Out] (x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3)\*ArcCoth[Tan[a + b\*x]])/4 + (-8\*b^4\*e^3\*x\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 12\*b^4\*e^2\*f\*x^2\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 8\*b^4\*e\*f^2\*x^3\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 2\*b^4\*f^3\*x^4\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] + 8\*b^4\*e^3\*x\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 12\*b^4\*e^2\*f\*x^2\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 8\*b^4\*e\*f^2\*x^3\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 2\*b^4\*f^3\*x^4\*Log[1 + I\*E^((2\*I)\*(a + b\*x))]) - (4\*I)\*b^3\*(e + f\*x)^3\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + (4\*I)\*b^3\*(e + f\*x)^3\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))] + 6\*b^2\*e^2\*f\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 12\*b^2\*e\*f^2\*x\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 6\*b^2\*f^3\*x^2\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] - 6\*b^2\*e^2\*f\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 12\*b^2\*e\*f^2\*x\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 6\*b^2\*f^3\*x^2\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b\*e\*f^2\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b\*f^3\*x\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))]

$$*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))]/(16*b^4)$$

**fricas** [C] time = 0.77, size = 1808, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*arccoth(tan(b\*x+a)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/32*(3*f^3*polylog(5, (I*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (I*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*\tan(b*x + a)^2 + 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*\tan(b*x + a)^2 - 2*\tan(b*x + a) + I)/(\tan(b*x + a)^2 + 1)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-((I + 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a)^2 + 1) + 1) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-((I + 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-(-(I - 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-(-(I - 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*\tan(b*x + a)^2 + 2*I*\tan(b*x + a) + I - 1)/(\tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*\tan(b*x + a)^2 - 2*I*\tan(b*x + a) + I - 1)/(\tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) - I + 1)/(\tan(b*x + a)^2 + 1)) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*\tan(b*x + a)^2 + 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I - 1)*\tan(b*x + a)^2 - 2*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I - 1)*\tan(b*x + a)^2 + 2*I*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I - 1)*\tan(b*x + a)^2 - 2*I*\tan(b*x + a) + I + 1)/(\tan(b*x + a)^2 + 1)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*log((\tan(b*x + a) + 1)/(\tan(b*x + a) - 1)) - (6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, (I*\tan(b*x + a)^2 + 2*\tan(b*x + a) - I)/(\tan(b*x + a)^2 + 1)) - ($$



```

6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) -
I)/(tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6*I*b*e*f^2)*polylog(4, (-I*tan(
b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6*
I*b*e*f^2)*polylog(4, (-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a
)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (I*tan(b
*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b^2*f^3*x^2 + 2*
b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)
/(tan(b*x + a)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylo
g(3, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + 6*(b^
2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (-I*tan(b*x + a)^2 - 2*ta
n(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^4

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)^3 \operatorname{arccoth}(\tan(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*arccoth(tan(b*x + a)), x)
```

**maple** [C] time = 50.40, size = 7429, normalized size = 24.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arccoth(tan(b*x+a)),x)
```

```
[Out] result too large to display
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16} (f^3 x^4 + 4 e f^2 x^3 + 6 e^2 f x^2 + 4 e^3 x) \log(2 \cos(2 b x + 2 a)^2 + 2 \sin(2 b x + 2 a)^2 + 4 \sin(2 b x + 2 a) + 2) - \frac{1}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
```

+ 4\*b\*e\*f^2\*x^3 + 6\*b\*e^2\*f\*x^2 + 4\*b\*e^3\*x)\*sin(4\*b\*x + 4\*a)\*sin(2\*b\*x + 2\*a) + (b\*f^3\*x^4 + 4\*b\*e\*f^2\*x^3 + 6\*b\*e^2\*f\*x^2 + 4\*b\*e^3\*x)\*cos(2\*b\*x + 2\*a))/(cos(4\*b\*x + 4\*a)^2 + sin(4\*b\*x + 4\*a)^2 + 2\*cos(4\*b\*x + 4\*a) + 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acoth}(\tan(a + bx)) (e + fx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tan(a + b\*x))\*(e + f\*x)^3,x)

[Out] int(acoth(tan(a + b\*x))\*(e + f\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)^3 \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*acoth(tan(b\*x+a)),x)

[Out] Integral((e + f\*x)\*\*3\*acoth(tan(a + b\*x)), x)

### 3.232 $\int (e + fx)^2 \coth^{-1}(\tan(a + bx)) dx$

**Optimal.** Leaf size=234

$$\frac{if^2\text{Li}_4(-ie^{2i(a+bx)})}{8b^3} - \frac{if^2\text{Li}_4(ie^{2i(a+bx)})}{8b^3} + \frac{f(e+fx)\text{Li}_3(-ie^{2i(a+bx)})}{4b^2} - \frac{f(e+fx)\text{Li}_3(ie^{2i(a+bx)})}{4b^2} - \frac{i(e+fx)^2\text{Li}_2(-ie^{2i(a+bx)})}{4b}$$

[Out]  $\frac{1}{3}*(f*x+e)^3*\text{arccoth}(\tan(b*x+a))/f + \frac{1}{3}*I*(f*x+e)^3*\text{arctan}(\exp(2*I*(b*x+a)))/f - \frac{1}{4}*I*(f*x+e)^2*\text{polylog}(2, -I*\exp(2*I*(b*x+a)))/b + \frac{1}{4}*I*(f*x+e)^2*\text{polylog}(2, I*\exp(2*I*(b*x+a)))/b + \frac{1}{4}*f*(f*x+e)*\text{polylog}(3, -I*\exp(2*I*(b*x+a)))/b^2 - \frac{1}{4}*f*(f*x+e)*\text{polylog}(3, I*\exp(2*I*(b*x+a)))/b^2 + \frac{1}{8}*I*f^2*\text{polylog}(4, -I*\exp(2*I*(b*x+a)))/b^3 - \frac{1}{8}*I*f^2*\text{polylog}(4, I*\exp(2*I*(b*x+a)))/b^3$

**Rubi [A]** time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6252, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e+fx)\text{PolyLog}(3, -ie^{2i(a+bx)})}{4b^2} - \frac{f(e+fx)\text{PolyLog}(3, ie^{2i(a+bx)})}{4b^2} + \frac{if^2\text{PolyLog}(4, -ie^{2i(a+bx)})}{8b^3} - \frac{if^2\text{PolyLog}(4, ie^{2i(a+bx)})}{8b^3}$$

Antiderivative was successfully verified.

[In] `Int[(e + f*x)^2*ArcCoth[Tan[a + b*x]], x]`

[Out]  $((e + f*x)^3*\text{ArcCoth}[\text{Tan}[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*\text{ArcTan}[E^{((2*I)*(a + b*x))}]/f - ((I/4)*(e + f*x)^2*\text{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}]/b + ((I/4)*(e + f*x)^2*\text{PolyLog}[2, I*E^{((2*I)*(a + b*x))}]/b + (f*(e + f*x)*\text{PolyLog}[3, (-I)*E^{((2*I)*(a + b*x))}]/(4*b^2) - (f*(e + f*x)*\text{PolyLog}[3, I*E^{((2*I)*(a + b*x))}]/(4*b^2) + ((I/8)*f^2*\text{PolyLog}[4, (-I)*E^{((2*I)*(a + b*x))}]/b^3 - ((I/8)*f^2*\text{PolyLog}[4, I*E^{((2*I)*(a + b*x))}]/b^3$

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

#### Rule 2531

`Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))] * ((f_.) + (g_.) * (x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f`

, g, n}, x] && GtQ[m, 0]

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_.)]]*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(p_.)], x_Symbol]
:> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx)^2 \coth^{-1}(\tan(a + bx)) dx &= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} - \frac{b \int (e + fx)^3 \sec(2a + 2bx) dx}{3f} \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} + \frac{1}{2} \int (e + fx)^2 \sec(2a + 2bx) dx \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \sec(2a + 2bx)}{2} \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \sec(2a + 2bx)}{2} \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \sec(2a + 2bx)}{2} \\
&= \frac{(e + fx)^3 \coth^{-1}(\tan(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \sec(2a + 2bx)}{2}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 409, normalized size = 1.75

$$\frac{1}{3}x(3e^2 + 3efx + f^2x^2) \coth^{-1}(\tan(a+bx)) + \frac{-12b^3e^2x \log(1 - ie^{2i(a+bx)}) + 12b^3e^2x \log(1 + ie^{2i(a+bx)}) - 12b^3efx}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^2\*ArcCoth[Tan[a + b\*x]], x]

[Out] (x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2)\*ArcCoth[Tan[a + b\*x]])/3 + (-12\*b^3\*e^2\*x\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 12\*b^3\*e\*f\*x^2\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 4\*b^3\*f^2\*x^3\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] + 12\*b^3\*e^2\*x\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 12\*b^3\*e\*f\*x^2\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 4\*b^3\*f^2\*x^3\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] - (6\*I)\*b^2\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b^2\*(e + f\*x)^2\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))] + 6\*b\*e\*f\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 6\*b\*f^2\*x\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] - 6\*b\*e\*f\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 6\*b\*f^2\*x\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] + (3\*I)\*f^2\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] - (3\*I)\*f^2\*PolyLog[4, I\*E^((2\*I)\*(a + b\*x))])/(24\*b^3)

**fricas [C]** time = 0.91, size = 1278, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*arccoth(tan(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{48} \cdot (3I^2 f^2 \text{polylog}(4, (I \tan(bx+a))^2 + 2 \tan(bx+a) - I) / (\tan(bx+a)^2 + 1)) + 3I^2 f^2 \text{polylog}(4, (I \tan(bx+a))^2 - 2 \tan(bx+a) - I) / (\tan(bx+a)^2 + 1) - 3I^2 f^2 \text{polylog}(4, (-I \tan(bx+a))^2 + 2 \tan(bx+a) + I) / (\tan(bx+a)^2 + 1) - 3I^2 f^2 \text{polylog}(4, (-I \tan(bx+a))^2 - 2 \tan(bx+a) + I) / (\tan(bx+a)^2 + 1) + (6I^2 b^2 f^2 x^2 + 12I^2 b^2 e f x + 6I^2 b^2 e^2) \text{dilog}(-((I+1) \tan(bx+a))^2 + 2 \tan(bx+a) - I + 1) / (\tan(bx+a)^2 + 1) + 1) + (6I^2 b^2 f^2 x^2 + 12I^2 b^2 e f x + 6I^2 b^2 e^2) \text{dilog}(-((I+1) \tan(bx+a))^2 - 2 \tan(bx+a) - I + 1) / (\tan(bx+a)^2 + 1) + 1) + (-6I^2 b^2 f^2 x^2 - 12I^2 b^2 e f x - 6I^2 b^2 e^2) \text{dilog}(-(-(I-1) \tan(bx+a))^2 + 2 \tan(bx+a) + I + 1) / (\tan(bx+a)^2 + 1) + 1) + (-6I^2 b^2 f^2 x^2 - 12I^2 b^2 e f x - 6I^2 b^2 e^2) \text{dilog}(-(-(I-1) \tan(bx+a))^2 - 2 \tan(bx+a) + I + 1) / (\tan(bx+a)^2 + 1) + 1) - 4(b^3 f^2 x^3 + 3b^3 e f x^2 + 3b^3 e^2 x + 3a b^2 e^2 - 3a^2 b e f + a^3 f^2) \log(((I+1) \tan(bx+a))^2 + 2 \tan(bx+a) - I + 1) / (\tan(bx+a)^2 + 1)) + 4(3a b^2 e^2 - 3a^2 b e f + a^3 f^2) \log(((I+1) \tan(bx+a))^2 + 2 \tan(bx+a) - I + 1) / (\tan(bx+a)^2 + 1)) - 4(3a b^2 e^2 - 3a^2 b e f + a^3 f^2) \log(((I+1) \tan(bx+a))^2 - 2 \tan(bx+a) + I - 1) / (\tan(bx+a)^2 + 1)) + 4(b^3 f^2 x^3 + 3b^3 e f x^2 + 3b^3 e^2 x + 3a b^2 e^2 - 3a^2 b e f + a^3 f^2) \log(((I+1) \tan(bx+a))^2 - 2 \tan(bx+a) - I + 1) / (\tan(bx+a)^2 + 1)) - 4(b^3 f^2 x^3 + 3b^3 e f x^2 + 3b^3 e^2 x + 3a b^2 e^2 - 3a^2 b e f + a^3 f^2) \log((- (I-1) \tan(bx+a))^2 + 2 \tan(bx+a) + I + 1) / (\tan(bx+a)^2 + 1)) + 4(b^3 f^2 x^3 + 3b^3 e f x^2 + 3b^3 e^2 x + 3a b^2 e^2 - 3a^2 b e f + a^3 f^2) \log((- (I-1) \tan(bx+a))^2 - 2 \tan(bx+a) + I + 1) / (\tan(bx+a)^2 + 1)) + 4(3a b^2 e^2 - 3a^2 b e f + a^3 f^2) \log(((I-1) \tan(bx+a))^2 + 2 \tan(bx+a) + I + 1) / (\tan(bx+a)^2 + 1)) - 4(3a b^2 e^2 - 3a^2 b e f + a^3 f^2) \log(((I-1) \tan(bx+a))^2 - 2 \tan(bx+a) + I + 1) / (\tan(bx+a)^2 + 1)) + 8(b^3 f^2 x^3 + 3b^3 e f x^2 + 3b^3 e^2 x) \log((\tan(bx+a) + 1) / (\tan(bx+a) - 1)) + 6(b f^2 x + b e f) \text{polylog}(3, (I \tan(bx+a))^2 + 2 \tan(bx+a) - I) / (\tan(bx+a)^2 + 1)) - 6(b f^2 x + b e f) \text{polylog}(3, (I \tan(bx+a))^2 - 2 \tan(bx+a) - I) / (\tan(bx+a)^2 + 1)) + 6(b f^2 x + b e f) \text{polylog}(3, (-I \tan(bx+a))^2 + 2 \tan(bx+a) + I) / (\tan(bx+a)^2 + 1)) - 6(b f^2 x + b e f) \text{polylog}(3, (-I \tan(bx+a))^2 - 2 \tan(bx+a) + I) / (\tan(bx+a)^2 + 1)) / b^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)^2 \operatorname{arccoth}(\tan(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*arccoth(tan(b\*x+a)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*arccoth(tan(b\*x + a)), x)

**maple** [C] time = 36.63, size = 5543, normalized size = 23.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^2*arccoth(tan(b*x+a)),x)`

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12} (f^2x^3 + 3efx^2 + 3e^2x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{12} (f^2x^3 + 3efx^2 + 3e^2x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 - 4 \sin(2bx + 2a) + 2) - \int \frac{(f^2x^3 + 3efx^2 + 3e^2x) \cos(4bx + 4a) \cos(2bx + 2a) + (f^2x^3 + 3efx^2 + 3e^2x) \sin(4bx + 4a) \sin(2bx + 2a) + (f^2x^3 + 3efx^2 + 3e^2x) \cos(2bx + 2a)}{\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 \cos(4bx + 4a) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*arccoth(tan(b*x+a)),x, algorithm="maxima")`

[Out] `1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acoth}(\tan(a + bx)) (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tan(a + b*x))*(e + f*x)^2,x)`

[Out] `int(acoth(tan(a + b*x))*(e + f*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)^2 \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**2*acoth(tan(b*x+a)),x)`

[Out] `Integral((e + f*x)**2*acoth(tan(a + b*x)), x)`

### 3.233 $\int (e + fx) \coth^{-1}(\tan(a + bx)) dx$

**Optimal.** Leaf size=162

$$\frac{f \operatorname{Li}_3(-ie^{2i(a+bx)})}{8b^2} - \frac{f \operatorname{Li}_3(ie^{2i(a+bx)})}{8b^2} - \frac{i(e+fx) \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} + \frac{i(e+fx) \operatorname{Li}_2(ie^{2i(a+bx)})}{4b} + \frac{i(e+fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f}$$

[Out]  $1/2*(f*x+e)^2*\operatorname{arccoth}(\tan(b*x+a))/f+1/2*I*(f*x+e)^2*\operatorname{arctan}(\exp(2*I*(b*x+a)))/f-1/4*I*(f*x+e)*\operatorname{polylog}(2,-I*\exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*\operatorname{polylog}(2,I*\exp(2*I*(b*x+a)))/b+1/8*f*\operatorname{polylog}(3,-I*\exp(2*I*(b*x+a)))/b^2-1/8*f*\operatorname{polylog}(3,I*\exp(2*I*(b*x+a)))/b^2$

**Rubi [A]** time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6252, 4181, 2531, 2282, 6589}

$$\frac{f \operatorname{PolyLog}(3, -ie^{2i(a+bx)})}{8b^2} - \frac{f \operatorname{PolyLog}(3, ie^{2i(a+bx)})}{8b^2} - \frac{i(e+fx) \operatorname{PolyLog}(2, -ie^{2i(a+bx)})}{4b} + \frac{i(e+fx) \operatorname{PolyLog}(2, ie^{2i(a+bx)})}{4b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)*\operatorname{ArcCoth}[\operatorname{Tan}[a + b*x]], x]$

[Out]  $((e + f*x)^2*\operatorname{ArcCoth}[\operatorname{Tan}[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*\operatorname{ArcTan}[E^{((2*I)*(a + b*x))}])/f - ((I/4)*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}])/b + ((I/4)*(e + f*x)*\operatorname{PolyLog}[2, I*E^{((2*I)*(a + b*x))}])/b + (f*\operatorname{PolyLog}[3, (-I)*E^{((2*I)*(a + b*x))}])/(8*b^2) - (f*\operatorname{PolyLog}[3, I*E^{((2*I)*(a + b*x))}])/(8*b^2)$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] \rightarrow \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$   $\operatorname{FunctionOfExponentialQ}[u, x] \ \&\& \ !\operatorname{MatchQ}[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /;$   $\operatorname{FreeQ}[\{a, m, n\}, x] \ \&\& \ \operatorname{IntegerQ}[m*n] \ \&\& \ !\operatorname{MatchQ}[u, E^{((c_.)*((a_.) + (b_.)*x))}*(F_)[v_] /;$   $\operatorname{FreeQ}[\{a, b, c\}, x] \ \&\& \ \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_)))})^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x\_Symbol] \rightarrow -\operatorname{Simp}[(f + g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n)]/(b*c*n*\operatorname{Log}[F]), x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f + g*x)^(m-1)*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n)], x], x] /;$   $\operatorname{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \operatorname{GtQ}[m, 0]$



Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6252

```
Int[ArcCoth[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Tan[a + b*x]]/(f*(m + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (e + fx) \coth^{-1}(\tan(a + bx)) dx &= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} - \frac{b \int (e + fx)^2 \sec(2a + 2bx) dx}{2f} \\ &= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} + \frac{1}{2} \int (e + fx) \\ &= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \text{Li}_2}{4} \\ &= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \text{Li}_2}{4} \\ &= \frac{(e + fx)^2 \coth^{-1}(\tan(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \text{Li}_2}{4} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 263, normalized size = 1.62

$$-be \left( \frac{i \text{Li}_2(-ie^{i(2a+2bx)})}{4b^2} - \frac{i \text{Li}_2(ie^{i(2a+2bx)})}{4b^2} - \frac{ix \tan^{-1}(e^{2ia+2ibx})}{b} \right) + \frac{f(4ib^2x^2 \tan^{-1}(\cos(2(a + bx)) + i \sin(2(a + bx)))}{4b^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(e + f\*x)\*ArcCoth[Tan[a + b\*x]],x]

[Out] e\*x\*ArcCoth[Tan[a + b\*x]] + (f\*x^2\*ArcCoth[Tan[a + b\*x]])/2 - b\*e\*(((-I)\*x\*ArcTan[E^((2\*I)\*a + (2\*I)\*b\*x)]/b + ((I/4)\*PolyLog[2, (-I)\*E^(I\*(2\*a + 2\*b\*x))])/b^2 - ((I/4)\*PolyLog[2, I\*E^(I\*(2\*a + 2\*b\*x))])/b^2) + (f\*((4\*I)\*b^2\*x^2\*ArcTan[Cos[2\*(a + b\*x)] + I\*Sin[2\*(a + b\*x)]] + (2\*I)\*b\*x\*PolyLog[2, I\*Cos[2\*(a + b\*x)] - Sin[2\*(a + b\*x)]] - (2\*I)\*b\*x\*PolyLog[2, (-I)\*Cos[2\*(a + b\*x)] + Sin[2\*(a + b\*x)]] - PolyLog[3, I\*Cos[2\*(a + b\*x)] - Sin[2\*(a + b\*x)]] + PolyLog[3, (-I)\*Cos[2\*(a + b\*x)] + Sin[2\*(a + b\*x)]]))/(8\*b^2)

fricas [C] time = 0.66, size = 830, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*arccoth(tan(b\*x+a)),x, algorithm="fricas")

[Out] 1/16\*((2\*I\*b\*f\*x + 2\*I\*b\*e)\*dilog(-((I + 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1) + 1) + (2\*I\*b\*f\*x + 2\*I\*b\*e)\*dilog(-((I + 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1) + 1) + (-2\*I\*b\*f\*x - 2\*I\*b\*e)\*dilog(-(-(I - 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1) + 1) + (-2\*I\*b\*f\*x - 2\*I\*b\*e)\*dilog(-(-(I - 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1) + 1) - 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1)) + 2\*(2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*I\*tan(b\*x + a) + I - 1)/(tan(b\*x + a)^2 + 1)) - 2\*(2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 - 2\*I\*tan(b\*x + a) + I - 1)/(tan(b\*x + a)^2 + 1)) + 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log(((I + 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1)) - 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log((- (I - 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) + 2\*(b^2\*f\*x^2 + 2\*b^2\*e\*x + 2\*a\*b\*e - a^2\*f)\*log((- (I - 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) + 2\*(2\*a\*b\*e - a^2\*f)\*log(((I - 1)\*tan(b\*x + a)^2 + 2\*I\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) - 2\*(2\*a\*b\*e - a^2\*f)\*log(((I - 1)\*tan(b\*x + a)^2 - 2\*I\*tan(b\*x + a) + I + 1)/(tan(b\*x + a)^2 + 1)) + 4\*(b^2\*f\*x^2 + 2\*b^2\*e\*x)\*log((tan(b\*x + a) + 1)/(tan(b\*x + a) - 1)) + f\*polylog(3, (I\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I)/(tan(b\*x + a)^2 + 1)) - f\*polylog(3, (I\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) - I)/(tan(b\*x + a)^2 + 1)) + f\*polylog(3, (-I\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) + I)/(tan(b\*x + a)^2 + 1)) - f\*polylog(3, (-I\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) + I)/(tan(b\*x + a)^2 + 1)))/b^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e) \operatorname{arccoth}(\tan(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arccoth(tan(b*x + a)), x)
```

**maple** [C] time = 4.74, size = 2543, normalized size = 15.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arccoth(tan(b*x+a)),x)
```

```
[Out] 1/4*I*Pi*x*e+1/8*I*Pi*f*x^2+1/4/b^2*f*(I*b*x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))
)+1/4/b^2*f*(I*b*x+I*a)*polylog(2,I*exp(2*I*(b*x+a)))-1/4*ln(exp(2*I*(b*x+a))
)-I)*x^2*f-1/2*ln(exp(2*I*(b*x+a))-I)*x*e+1/8*f*polylog(3,-I*exp(2*I*(b*x+a))
)/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^2-1/2*I/b*e*dilog(1+exp(I*(b
*x+a)))*(-1)^(3/4))-1/2*I/b*e*dilog(1-exp(I*(b*x+a)))*(-1)^(3/4))+1/4*I*Pi*x*
e*csgn(((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/4*I*Pi*x*e*csgn
(((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/4*I*Pi*x*e*csgn(I/(ex
p(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)
/(exp(2*I*(b*x+a))+1))+1/2*I/b*e*dilog(((1-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/
2))+1/2*I/b*e*dilog(((1-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/2/b*a*e*ln(-e
xp(2*I*(b*x+a))+I)+1/2/b*a*e*ln(exp(2*I*(b*x+a))+I)-1/4/b^2*a^2*f*ln(exp(2*
I*(b*x+a))+I)-1/4/b^2*f*(I*b*x+I*a)^2*ln(1+I*exp(2*I*(b*x+a)))-1/4/b^2*f*(I
*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))+(1/4*f*x^2+1/2*e*x)*ln(exp(2*I*(b
*x+a))+I)+1/4/b^2*a^2*f*ln(-exp(2*I*(b*x+a))+I)+1/4*I*Pi*x*e*csgn(I*(exp(2*I
*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn(((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I
*(b*x+a))+1))-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1)
))*csgn(((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(I
*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)
/(exp(2*I*(b*x+a))+1))*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*
I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/4*
I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*
(b*x+a))+1))^2-1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))
+1))^2*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x
^2-1/8*I*Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2+1
/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/4*I*Pi*x*
e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3-1/8*I*Pi*f*csgn(I*(ex
p(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b
*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b
*x+a))+1))^2*x^2-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1+exp(I*
(b*x+a)))*(-1)^(3/4))+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1-exp(I*(b*x+a)))*(-1)^(3/
4))+1/2*I/b*e*(I*b*x+I*a)*ln(((1-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/8*I*
```

```

Pi*f*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/2*I/b^2*
f*a*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)
-exp(I*(b*x+a)))/(-I)^(1/2))+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I
*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*x^
2+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I))*cs
gn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/4*I*Pi*x*e*csgn(I*(exp(2*
I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi
*f*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a)
))+1))^2*x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*c
sgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*csg
n(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a)
+I)/(exp(2*I*(b*x+a))+1))+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(
b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*x^2+1/2*I
/b^2*f*a*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x
+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(
b*x+a))+1))*x^2-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2)
)+1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/
2*I/b^2*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^
2*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/4*I*Pi*x*e*c
sgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3+1/4*I*Pi*x*e*csgn((1
+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/2*I/b^2*f*a*dilog(((-I)^(
1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b*e*(I*b*x+I*a)*ln(1+exp(I*(b*x+a)
))*(-1)^(3/4))-1/2*I/b*e*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/4*I*Pi
*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x
+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a)
))+I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(
I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/8*I*Pi*f*csgn(I*(exp(2
*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2

```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} (fx^2 + 2ex) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{8} (fx^2 + 2ex) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)*co
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(\tan(a + bx)) (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(tan(a + b*x))*(e + f*x), x)`

[Out] `int(acoth(tan(a + b*x))*(e + f*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx) \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*acoth(tan(b*x+a)), x)`

[Out] `Integral((e + f*x)*acoth(tan(a + b*x)), x)`

### 3.234 $\int \coth^{-1}(\tan(a + bx)) dx$

Optimal. Leaf size=79

$$-\frac{i\text{Li}_2(-ie^{2i(a+bx)})}{4b} + \frac{i\text{Li}_2(ie^{2i(a+bx)})}{4b} + ix \tan^{-1}(e^{2i(a+bx)}) + x \coth^{-1}(\tan(a + bx))$$

[Out]  $x \cdot \text{arccoth}(\tan(b \cdot x + a)) + I \cdot x \cdot \text{arctan}(\exp(2 \cdot I \cdot (b \cdot x + a))) - 1/4 \cdot I \cdot \text{polylog}(2, -I \cdot \exp(2 \cdot I \cdot (b \cdot x + a))) / b + 1/4 \cdot I \cdot \text{polylog}(2, I \cdot \exp(2 \cdot I \cdot (b \cdot x + a))) / b$

**Rubi [A]** time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6248, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}(2, -ie^{2i(a+bx)})}{4b} + \frac{i\text{PolyLog}(2, ie^{2i(a+bx)})}{4b} + ix \tan^{-1}(e^{2i(a+bx)}) + x \coth^{-1}(\tan(a + bx))$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[Tan[a + b\*x]], x]

[Out]  $x \cdot \text{ArcCoth}[\text{Tan}[a + b \cdot x]] + I \cdot x \cdot \text{ArcTan}[E^{((2 \cdot I) \cdot (a + b \cdot x))}] - ((I/4) \cdot \text{PolyLog}[2, (-I) \cdot E^{((2 \cdot I) \cdot (a + b \cdot x))}]) / b + ((I/4) \cdot \text{PolyLog}[2, I \cdot E^{((2 \cdot I) \cdot (a + b \cdot x))}]) / b$

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_))], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4181

Int[csc[(e\_) + Pi\*(k\_) + (f\_)\*(x\_)]\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))]/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

#### Rule 6248

Int[ArcCoth[Tan[(a\_.) + (b\_.)\*(x\_)]], x\_Symbol] := Simp[x\*ArcCoth[Tan[a + b\*x]], x] - Dist[b, Int[x\*Sec[2\*a + 2\*b\*x], x], x] /; FreeQ[{a, b}, x]

### Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(\tan(a + bx)) dx &= x \coth^{-1}(\tan(a + bx)) - b \int x \sec(2a + 2bx) dx \\
 &= x \coth^{-1}(\tan(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) + \frac{1}{2} \int \log(1 - ie^{i(2a+2bx)}) dx - \frac{1}{2} \int \log(1 + ie^{i(2a+2bx)}) dx \\
 &= x \coth^{-1}(\tan(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) - \frac{i \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, e^{i(2a+2bx)}\right)}{4b} + \frac{i \operatorname{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{i(2a+2bx)}\right)}{4b} \\
 &= x \coth^{-1}(\tan(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) - \frac{i \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} + \frac{i \operatorname{Li}_2(ie^{2i(a+bx)})}{4b}
 \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 78, normalized size = 0.99

$$\frac{-i \operatorname{Li}_2(-ie^{2i(a+bx)}) + i \operatorname{Li}_2(ie^{2i(a+bx)}) + 4bx \left( \coth^{-1}(\tan(a + bx)) + i \tan^{-1}(e^{2i(a+bx)}) \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[Tan[a + b\*x]], x]

[Out] (4\*b\*x\*(ArcCoth[Tan[a + b\*x]] + I\*ArcTan[E^((2\*I)\*(a + b\*x))]) - I\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + I\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))])/(4\*b)

**fricas [B]** time = 1.01, size = 498, normalized size = 6.30

$$\frac{4bx \log\left(\frac{\tan(bx+a)+1}{\tan(bx+a)-1}\right) - 2(bx+a) \log\left(\frac{(i+1)\tan(bx+a)^2 + 2\tan(bx+a) - i + 1}{\tan(bx+a)^2 + 1}\right) + 2a \log\left(\frac{(i+1)\tan(bx+a)^2 + 2i\tan(bx+a) + i - 1}{\tan(bx+a)^2 + 1}\right) - 2a \log\left(\frac{(i-1)\tan(bx+a)^2 + 2i\tan(bx+a) - i - 1}{\tan(bx+a)^2 + 1}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tan(b\*x+a)), x, algorithm="fricas")

[Out] 1/8\*(4\*b\*x\*log((tan(b\*x + a) + 1)/(tan(b\*x + a) - 1)) - 2\*(b\*x + a)\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1)) + 2\*a\*log(((I + 1)\*tan(b\*x + a)^2 + 2\*I\*tan(b\*x + a) + I - 1)/(tan(b\*x + a)^2 + 1)) - 2\*a\*log(((I + 1)\*tan(b\*x + a)^2 - 2\*I\*tan(b\*x + a) + I - 1)/(tan(b\*x + a)^2 + 1)) + 2\*(b\*x + a)\*log(((I + 1)\*tan(b\*x + a)^2 - 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1)) - 2\*(b\*x + a)\*log((-I - 1)\*tan(b\*x + a)^2 + 2\*tan(b\*x + a) - I + 1)/(tan(b\*x + a)^2 + 1))

```

an(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-I - 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I -
1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\tan(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arccoth(tan(b*x + a)), x)
```

**maple** [B] time = 0.55, size = 180, normalized size = 2.28

$$\frac{\arctan(\tan(bx + a)) \operatorname{arccoth}(\tan(bx + a))}{b} + \frac{\arctan(\tan(bx + a)) \ln\left(1 + \frac{i(1+i\tan(bx+a))^2}{1+\tan^2(bx+a)}\right)}{2b} - \frac{\arctan(\tan(bx + a))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccoth(tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arccoth(tan(b*x+a))+1/2/b*arctan(tan(b*x+a))*ln(1+I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/2/b*arctan(tan(b*x+a))*ln(1-I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/4*I/b*dilog(1+I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))+1/4*I/b*dilog(1-I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))
```

**maxima** [B] time = 0.46, size = 182, normalized size = 2.30

$$\frac{4(bx + a) \operatorname{arccoth}(\tan(bx + a)) + \left(\arctan\left(\frac{1}{2} \tan(bx + a) + \frac{1}{2}, \frac{1}{2} \tan(bx + a) + \frac{1}{2}\right) - \arctan\left(\frac{1}{2} \tan(bx + a) - \frac{1}{2}, \frac{1}{2} \tan(bx + a) - \frac{1}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arccoth(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2, 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a) - 1/2))
```



+ 1/2))\*log(tan(b\*x + a)^2 + 1) - (b\*x + a)\*log(1/2\*tan(b\*x + a)^2 + tan(b\*x + a) + 1/2) + (b\*x + a)\*log(1/2\*tan(b\*x + a)^2 - tan(b\*x + a) + 1/2) - I\*dilog((1/2\*I + 1/2)\*tan(b\*x + a) - 1/2\*I + 1/2) + I\*dilog(-(1/2\*I - 1/2)\*tan(b\*x + a) + 1/2\*I + 1/2) + I\*dilog((1/2\*I - 1/2)\*tan(b\*x + a) + 1/2\*I + 1/2) - I\*dilog(-(1/2\*I + 1/2)\*tan(b\*x + a) - 1/2\*I + 1/2))/b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tan(a + b\*x)),x)

[Out] int(acoth(tan(a + b\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(\tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tan(b\*x+a)),x)

[Out] Integral(acoth(tan(a + b\*x)), x)

$$3.235 \quad \int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(\tan(a+bx))}{e+fx}, x\right)$$

[Out] CannotIntegrate(arccoth(tan(b\*x+a))/(f\*x+e), x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

[Out] Defer[Int][ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

Rubi steps

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx = \int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

Mathematica [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(\tan(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

[Out] Integrate[ArcCoth[Tan[a + b\*x]]/(e + f\*x), x]

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(\tan(bx+a))}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tan(b\*x+a))/(f\*x+e),x, algorithm="fricas")

[Out] integral(arccoth(tan(b\*x + a))/(f\*x + e), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tan(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tan(b\*x+a))/(f\*x+e),x, algorithm="giac")

[Out] integrate(arccoth(tan(b\*x + a))/(f\*x + e), x)

**maple** [A] time = 4.15, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tan(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(tan(b\*x+a))/(f\*x+e),x)

[Out] int(arccoth(tan(b\*x+a))/(f\*x+e),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\tan(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(tan(b\*x+a))/(f\*x+e),x, algorithm="maxima")

[Out] integrate(arccoth(tan(b\*x + a))/(f\*x + e), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(\tan(a + bx))}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(tan(a + b\*x))/(e + f\*x),x)

[Out] int(acoth(tan(a + b\*x))/(e + f\*x), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(\tan(a + bx))}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(tan(b\*x+a))/(f\*x+e), x)

[Out] Integral(acoth(tan(a + b\*x))/(e + f\*x), x)

### 3.236 $\int x^2 \coth^{-1}(c + d \tan(a + bx)) dx$

**Optimal.** Leaf size=395

$$\frac{i\text{Li}_4\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3} - \frac{i\text{Li}_4\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^3} + \frac{x\text{Li}_3\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2} - \frac{x\text{Li}_3\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2} - \frac{ix^2\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{ix^2\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}$$

[Out]  $\frac{1}{3}x^3 \operatorname{arccoth}(c+d \tan(bx+a)) + \frac{1}{6}x^3 \ln(1+(1-c+I*d) \exp(2I*a+2I*b*x)) / (1-c-I*d) - \frac{1}{6}x^3 \ln(1+(1+c-I*d) \exp(2I*a+2I*b*x)) / (1+c+I*d) - \frac{1}{4}I*x^2 \operatorname{polylog}(2, -(1-c+I*d) \exp(2I*a+2I*b*x)) / (1-c-I*d) / b + \frac{1}{4}I*x^2 \operatorname{polylog}(2, -(1+c-I*d) \exp(2I*a+2I*b*x)) / (1+c+I*d) / b + \frac{1}{4}x \operatorname{polylog}(3, -(1-c+I*d) \exp(2I*a+2I*b*x)) / (1-c-I*d) / b^2 - \frac{1}{4}x \operatorname{polylog}(3, -(1+c-I*d) \exp(2I*a+2I*b*x)) / (1+c+I*d) / b^2 + \frac{1}{8}I \operatorname{polylog}(4, -(1-c+I*d) \exp(2I*a+2I*b*x)) / (1-c-I*d) / b^3 - \frac{1}{8}I \operatorname{polylog}(4, -(1+c-I*d) \exp(2I*a+2I*b*x)) / (1+c+I*d) / b^3$

**Rubi [A]** time = 0.50, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6268, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2} - \frac{x\text{PolyLog}\left(3, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2} + \frac{i\text{PolyLog}\left(4, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3} - \frac{i\text{PolyLog}\left(4, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2 \operatorname{ArcCoth}[c + d \operatorname{Tan}[a + b*x]], x]$

[Out]  $\frac{x^3 \operatorname{ArcCoth}[c + d \operatorname{Tan}[a + b*x]]}{3} + \frac{x^3 \operatorname{Log}[1 + ((1 - c + I*d)E^{(2I)*a + (2I)*b*x}) / (1 - c - I*d)]}{6} - \frac{x^3 \operatorname{Log}[1 + ((1 + c - I*d)E^{(2I)*a + (2I)*b*x}) / (1 + c + I*d)]}{6} - \frac{(I/4)x^2 \operatorname{PolyLog}[2, -(((1 - c + I*d)E^{(2I)*a + (2I)*b*x}) / (1 - c - I*d))]}{b} + \frac{(I/4)x^2 \operatorname{PolyLog}[2, -(((1 + c - I*d)E^{(2I)*a + (2I)*b*x}) / (1 + c + I*d))]}{b} + \frac{x \operatorname{PolyLog}[3, -(((1 - c + I*d)E^{(2I)*a + (2I)*b*x}) / (1 - c - I*d))]}{(4*b^2)} - \frac{x \operatorname{PolyLog}[3, -(((1 + c - I*d)E^{(2I)*a + (2I)*b*x}) / (1 + c + I*d))]}{(4*b^2)} + \frac{(I/8) \operatorname{PolyLog}[4, -(((1 - c + I*d)E^{(2I)*a + (2I)*b*x}) / (1 - c - I*d))]}{b^3} - \frac{(I/8) \operatorname{PolyLog}[4, -(((1 + c - I*d)E^{(2I)*a + (2I)*b*x}) / (1 + c + I*d))]}{b^3}$

**Rule 2190**

$\text{Int}[((F_)^((g_)*(e_) + (f_)*(x_)))^((n_)*((c_) + (d_)*(x_))^(m_)) / ((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^((n_))), x\_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^m \operatorname{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a]}{(b*f*g*n \operatorname{Log}[F])}, x] - \text{Dist}[\frac{(d*m)}{(b*f*g*n \operatorname{Log}[F])}, \text{Int}[(c + d*x)^(m-1) \operatorname{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGTQ}[m, 0]$

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6268

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
+ 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*e^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_
.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(c + d \tan(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{3}(b(i(1 - c) - d)) \int \frac{e^{2ia+2ibx} x^3}{1 - c - id + (1 - c + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{3}x^3 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{6}x^3 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.34, size = 346, normalized size = 0.88

$$\frac{1}{3}x^3 \coth^{-1}(d \tan(a+bx)+c) + \frac{4b^3 x^3 \log\left(1 + \frac{(c-id-1)e^{2i(a+bx)}}{c+id-1}\right) - 4b^3 x^3 \log\left(1 + \frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right) - 6ib^2 x^2 \operatorname{Li}_2\left(\frac{(-c+id+1)e^{2i(a+bx)}}{c+id+1}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[c + d\*Tan[a + b\*x]], x]

[Out] (x^3\*ArcCoth[c + d\*Tan[a + b\*x]])/3 + (4\*b^3\*x^3\*Log[1 + ((-1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] - 4\*b^3\*x^3\*Log[1 + ((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d)] - (6\*I)\*b^2\*x^2\*PolyLog[2, ((1 - c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] + (6\*I)\*b^2\*x^2\*PolyLog[2, -(((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d))] + 6\*b\*x\*PolyLog[3, ((1 - c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] - 6\*b\*x\*PolyLog[3, -(((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d))] + (3\*I)\*PolyLog[4, ((1 - c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] - (3\*I)\*PolyLog[4, -(((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d))]/(24\*b^3)

**fricas [C]** time = 0.96, size = 2160, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(c+d\*tan(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{48} \cdot (8b^3x^3 \log((d \tan(bx + a) + c + 1)/(d \tan(bx + a) + c - 1)) + 6Ib^2x^2 \operatorname{dilog}(-((2I(c + 1)d + 2d^2) \tan(bx + a)^2 + 2c^2 - 2I(c + 1)d + (2Ic^2 + 4(c + 1)d - 2Id^2 + 4Ic + 2I) \tan(bx + a) + 4c + 2)/((c^2 + d^2 + 2c + 1) \tan(bx + a)^2 + c^2 + d^2 + 2c + 1) + 1) - 6Ib^2x^2 \operatorname{dilog}(-((-2I(c + 1)d + 2d^2) \tan(bx + a)^2 + 2c^2 + 2I(c + 1)d + (-2Ic^2 + 4(c + 1)d + 2Id^2 - 4Ic - 2I) \tan(bx + a) + 4c + 2)/((c^2 + d^2 + 2c + 1) \tan(bx + a)^2 + c^2 + d^2 + 2c + 1) + 1) + 6Ib^2x^2 \operatorname{dilog}((2(I(c - 1)d - d^2) \tan(bx + a)^2 - 2c^2 - 2I(c - 1)d - (-2Ic^2 + 4(c - 1)d + 2Id^2 + 4Ic - 2I) \tan(bx + a) + 4c - 2)/((c^2 + d^2 - 2c + 1) \tan(bx + a)^2 + c^2 + d^2 - 2c + 1) + 1) - 6Ib^2x^2 \operatorname{dilog}((2(-I(c - 1)d - d^2) \tan(bx + a)^2 - 2c^2 + 2I(c - 1)d - (2Ic^2 + 4(c - 1)d - 2Id^2 - 4Ic + 2I) \tan(bx + a) + 4c - 2)/((c^2 + d^2 - 2c + 1) \tan(bx + a)^2 + c^2 + d^2 - 2c + 1) + 1) + 4a^3 \log(((I(c + 1)d + d^2) \tan(bx + a)^2 - c^2 + I(c + 1)d + (Ic^2 + Id^2 + 2Ic + I) \tan(bx + a) - 2c - 1)/(\tan(bx + a)^2 + 1)) + 4a^3 \log(((I(c + 1)d - d^2) \tan(bx + a)^2 + c^2 + I(c + 1)d + (Ic^2 + Id^2 + 2Ic + I) \tan(bx + a) + 2c + 1)/(\tan(bx + a)^2 + 1)) - 4a^3 \log(((I(c - 1)d + d^2) \tan(bx + a)^2 - c^2 + I(c - 1)d + (Ic^2 + Id^2 - 2Ic + I) \tan(bx + a) + 2c - 1)/(\tan(bx + a)^2 + 1)) - 4a^3 \log(((I(c - 1)d - d^2) \tan(bx + a)^2 + c^2 + I(c - 1)d + (Ic^2 + Id^2 - 2Ic + I) \tan(bx + a) - 2c + 1)/(\tan(bx + a)^2 + 1)) - 6bxx \operatorname{polylog}(3, ((c^2 + 2I(c + 1)d - d^2 + 2c + 1) \tan(bx + a)^2 - c^2 - 2I(c + 1)d + d^2 + (2Ic^2 - 4(c + 1)d - 2Id^2 + 4Ic + 2I) \tan(bx + a) - 2c - 1)/((c^2 + d^2 + 2c + 1) \tan(bx + a)^2 + c^2 + d^2 + 2c + 1)) - 6bxx \operatorname{polylog}(3, ((c^2 - 2I(c + 1)d - d^2 + 2c + 1) \tan(bx + a)^2 - c^2 + 2I(c + 1)d + d^2 + (-2Ic^2 - 4(c + 1)d + 2Id^2 - 4Ic - 2I) \tan(bx + a) - 2c - 1)/((c^2 + d^2 + 2c + 1) \tan(bx + a)^2 + c^2 + d^2 + 2c + 1)) + 6bxx \operatorname{polylog}(3, ((c^2 + 2I(c - 1)d - d^2 - 2c + 1) \tan(bx + a)^2 - c^2 - 2I(c - 1)d + d^2 + (2Ic^2 - 4(c - 1)d - 2Id^2 - 4Ic + 2I) \tan(bx + a) + 2c - 1)/((c^2 + d^2 - 2c + 1) \tan(bx + a)^2 + c^2 + d^2 - 2c + 1)) + 6bxx \operatorname{polylog}(3, ((c^2 - 2I(c - 1)d - d^2 - 2c + 1) \tan(bx + a)^2 - c^2 + 2I(c - 1)d + d^2 + (-2Ic^2 - 4(c - 1)d + 2Id^2 + 4Ic - 2I) \tan(bx + a) + 2c - 1)/((c^2 + d^2 - 2c + 1) \tan(bx + a)^2 + c^2 + d^2 - 2c + 1)) - 4(b^3x^3 + a^3) \log(((2I(c + 1)d + 2d^2) \tan(bx + a)^2 + 2c^2 - 2I(c + 1)d + (2Ic^2 + 4(c + 1)d - 2Id^2 + 4Ic + 2I) \tan(bx + a) + 4c + 2)/((c^2 + d^2 + 2c + 1) \tan(bx + a)^2 + c^2 + d^2 + 2c + 1)) - 4(b^3x^3 + a^3) \log((( -2I(c + 1)d + 2d^2) \tan(bx + a)^2 + 2c^2 + 2I(c + 1)d + (-2Ic^2 + 4(c + 1)d + 2Id^2 - 4Ic - 2I) \tan(bx + a) + 4c + 2)/((c^2 + d^2 + 2c + 1) \tan(bx + a)^2 + c^2 + d^2 + 2c + 1)) + 4(b^3x^3 + a^3) \log(- (2(I(c - 1)d - d^2) \tan(bx + a)^2 - 2c^2 - 2I(c - 1)d - (-2Ic^2 + 4(c - 1)d + 2Id^2 + 4Ic - 2I) \tan(bx + a) + 4c - 2)/((c^2 + d^2 - 2c + 1) \tan(bx + a)^2 + c^2 + d^2 - 2c + 1)) + 4(b^3x^3 + a^3) \log(- (2(-I(c - 1)d - d^2) \tan(bx$



+ a)^2 - 2\*c^2 + 2\*I\*(c - 1)\*d - (2\*I\*c^2 + 4\*(c - 1)\*d - 2\*I\*d^2 - 4\*I\*c + 2\*I)\*tan(b\*x + a) + 4\*c - 2)/((c^2 + d^2 - 2\*c + 1)\*tan(b\*x + a)^2 + c^2 + d^2 - 2\*c + 1)) + 3\*I\*polylog(4, ((c^2 + 2\*I\*(c + 1)\*d - d^2 + 2\*c + 1)\*tan(b\*x + a)^2 - c^2 - 2\*I\*(c + 1)\*d + d^2 + (2\*I\*c^2 - 4\*(c + 1)\*d - 2\*I\*d^2 + 4\*I\*c + 2\*I)\*tan(b\*x + a) - 2\*c - 1)/((c^2 + d^2 + 2\*c + 1)\*tan(b\*x + a)^2 + c^2 + d^2 + 2\*c + 1)) - 3\*I\*polylog(4, ((c^2 - 2\*I\*(c + 1)\*d - d^2 + 2\*c + 1)\*tan(b\*x + a)^2 - c^2 + 2\*I\*(c + 1)\*d + d^2 + (-2\*I\*c^2 - 4\*(c + 1)\*d + 2\*I\*d^2 - 4\*I\*c - 2\*I)\*tan(b\*x + a) - 2\*c - 1)/((c^2 + d^2 + 2\*c + 1)\*tan(b\*x + a)^2 + c^2 + d^2 + 2\*c + 1)) - 3\*I\*polylog(4, ((c^2 + 2\*I\*(c - 1)\*d - d^2 - 2\*c + 1)\*tan(b\*x + a)^2 - c^2 - 2\*I\*(c - 1)\*d + d^2 + (2\*I\*c^2 - 4\*(c - 1)\*d - 2\*I\*d^2 - 4\*I\*c + 2\*I)\*tan(b\*x + a) + 2\*c - 1)/((c^2 + d^2 - 2\*c + 1)\*tan(b\*x + a)^2 + c^2 + d^2 - 2\*c + 1)) + 3\*I\*polylog(4, ((c^2 - 2\*I\*(c - 1)\*d - d^2 - 2\*c + 1)\*tan(b\*x + a)^2 - c^2 + 2\*I\*(c - 1)\*d + d^2 + (-2\*I\*c^2 - 4\*(c - 1)\*d + 2\*I\*d^2 + 4\*I\*c - 2\*I)\*tan(b\*x + a) + 2\*c - 1)/((c^2 + d^2 - 2\*c + 1)\*tan(b\*x + a)^2 + c^2 + d^2 - 2\*c + 1)))/b^3

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(d \tan(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(c+d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(d\*tan(b\*x + a) + c), x)

**maple** [C] time = 34.68, size = 6892, normalized size = 17.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(c+d\*tan(b\*x+a)),x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12} x^3 \log\left(\left(c^2 + d^2 + 2c + 1\right) \cos(2bx + 2a)^2 + 4(c + 1)d \sin(2bx + 2a) + \left(c^2 + d^2 + 2c + 1\right) \sin(2bx + 2a)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(c+d\*tan(b\*x+a)),x, algorithm="maxima")

[Out] 1/12\*x^3\*log((c^2 + d^2 + 2\*c + 1)\*cos(2\*b\*x + 2\*a)^2 + 4\*(c + 1)\*d\*sin(2\*b\*x + 2\*a) + (c^2 + d^2 + 2\*c + 1)\*sin(2\*b\*x + 2\*a)^2 + c^2 + d^2 + 2\*(c^2 -

$$\begin{aligned}
& d^2 + 2c + 1) \cos(2bx + 2a) + 2c + 1) - 1/12x^3 \log((c^2 + d^2 - 2c \\
& + 1) \cos(2bx + 2a)^2 + 4(c - 1)d \sin(2bx + 2a) + (c^2 + d^2 - 2c \\
& + 1) \sin(2bx + 2a)^2 + c^2 + d^2 + 2(c^2 - d^2 - 2c + 1) \cos(2bx + 2 \\
& a) - 2c + 1) - 4b \int (-1/3(2(c^2 + d^2 - 1)x^3 \cos(2bx + 2 \\
& a)^2 + 2c d x^3 \sin(2bx + 2a) + 2(c^2 + d^2 - 1)x^3 \sin(2bx + 2a)^ \\
& 2 + (c^2 - d^2 - 1)x^3 \cos(2bx + 2a) - (2c d x^3 \sin(2bx + 2a) - (c \\
& ^2 - d^2 - 1)x^3 \cos(2bx + 2a)) \cos(4bx + 4a) + (2c d x^3 \cos(2bx \\
& + 2a) + (c^2 - d^2 - 1)x^3 \sin(2bx + 2a)) \sin(4bx + 4a)) / (c^4 + d^ \\
& 4 + 2(c^2 + 1)d^2 + (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1) \cos(4bx + \\
& 4a)^2 + 4(c^4 + d^4 + 2(c^2 - 1)d^2 - 2c^2 + 1) \cos(2bx + 2a)^2 + \\
& (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1) \sin(4bx + 4a)^2 + 4(c^4 + d^4 \\
& + 2(c^2 - 1)d^2 - 2c^2 + 1) \sin(2bx + 2a)^2 - 2c^2 + 2(c^4 + d^4 - \\
& 2(3c^2 - 1)d^2 - 2c^2 + 2(c^4 - d^4 - 2c^2 + 1) \cos(2bx + 2a) - 4 \\
& (c d^3 + (c^3 - c)d) \sin(2bx + 2a) + 1) \cos(4bx + 4a) + 4(c^4 - d^ \\
& 4 - 2c^2 + 1) \cos(2bx + 2a) - 4(2c d^3 - 2(c^3 - c)d - 2(c d^3 + ( \\
& c^3 - c)d) \cos(2bx + 2a) - (c^4 - d^4 - 2c^2 + 1) \sin(2bx + 2a)) \sin \\
& (4bx + 4a) + 8(c d^3 + (c^3 - c)d) \sin(2bx + 2a) + 1), x)
\end{aligned}$$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(c + d \tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(c + d*tan(a + b*x)),x)`

[Out] `int(x^2*acoth(c + d*tan(a + b*x)), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(c + d \tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(c+d*tan(b*x+a)),x)`

[Out] `Integral(x**2*acoth(c + d*tan(a + b*x)), x)`

### 3.237 $\int x \coth^{-1}(c + d \tan(a + bx)) dx$

**Optimal.** Leaf size=295

$$\frac{\operatorname{Li}_3\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2} - \frac{\operatorname{Li}_3\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2} - \frac{ix\operatorname{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{ix\operatorname{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b} + \frac{1}{4}x^2 \log\left(1 + \dots\right)$$

[Out]  $1/2*x^2*\operatorname{arccoth}(c+d*\tan(b*x+a))+1/4*x^2*\ln(1+(1-c+I*d)*\exp(2*I*a+2*I*b*x)/(1-c-I*d))-1/4*x^2*\ln(1+(1+c-I*d)*\exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x*\operatorname{polylog}(2,-(1-c+I*d)*\exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x*\operatorname{polylog}(2,-(1+c-I*d)*\exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/8*\operatorname{polylog}(3,-(1-c+I*d)*\exp(2*I*a+2*I*b*x)/(1-c-I*d))/b^2-1/8*\operatorname{polylog}(3,-(1+c-I*d)*\exp(2*I*a+2*I*b*x)/(1+c+I*d))/b^2$

**Rubi [A]** time = 0.40, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6268, 2190, 2531, 2282, 6589}

$$\frac{\operatorname{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2} - \frac{\operatorname{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2} - \frac{ix\operatorname{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{ArcCoth}[c + d*\operatorname{Tan}[a + b*x]], x]$

[Out]  $(x^2*\operatorname{ArcCoth}[c + d*\operatorname{Tan}[a + b*x]])/2 + (x^2*\operatorname{Log}[1 + ((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)/(1 - c - I*d)})]/4 - (x^2*\operatorname{Log}[1 + ((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)/(1 + c + I*d)})]/4 - ((I/4)*x*\operatorname{PolyLog}[2, -(((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)/(1 - c - I*d)})]/b + ((I/4)*x*\operatorname{PolyLog}[2, -(((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)/(1 + c + I*d)})]/b + \operatorname{PolyLog}[3, -(((1 - c + I*d)*E^{((2*I)*a + (2*I)*b*x)/(1 - c - I*d)})]/(8*b^2) - \operatorname{PolyLog}[3, -(((1 + c - I*d)*E^{((2*I)*a + (2*I)*b*x)/(1 + c + I*d)})]/(8*b^2)$

#### Rule 2190

$\operatorname{Int}[(((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_))*((c_) + (d_)*(x_))^\wedge(m_)]/((a_) + (b_)*((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^\wedge m * \operatorname{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n)/a]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^\wedge(m - 1)*\operatorname{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n)/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2282

$\operatorname{Int}[u, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialFunction}[u, x] \neq 0$

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6268

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
+ 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
)*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(c + d \tan(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{2}(b(i(1 - c) - d)) \int \frac{e^{2ia+2ibx}x^2}{1 - c - id + (1 - c + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c - id}\right) \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{4}x^2 \log\left(1 + \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c - id}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 257, normalized size = 0.87

$$\frac{1}{2}x^2 \coth^{-1}(d \tan(a+bx)+c) + \frac{2b^2x^2 \log\left(1 + \frac{(c-id-1)e^{2i(a+bx)}}{c+id-1}\right) - 2b^2x^2 \log\left(1 + \frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right) - 2ibx \operatorname{Li}_2\left(\frac{(-c+id+1)e^{2i(a+bx)}}{c+id-1}\right)}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[c + d\*Tan[a + b\*x]], x]

[Out] (x^2\*ArcCoth[c + d\*Tan[a + b\*x]])/2 + (2\*b^2\*x^2\*Log[1 + ((-1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] - 2\*b^2\*x^2\*Log[1 + ((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d)] - (2\*I)\*b\*x\*PolyLog[2, ((1 - c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] + (2\*I)\*b\*x\*PolyLog[2, -(((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d))] + PolyLog[3, ((1 - c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c + I\*d)] - PolyLog[3, -(((1 + c - I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c + I\*d))]/(8\*b^2)

**fricas [C]** time = 0.83, size = 1688, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tan(b\*x+a)), x, algorithm="fricas")

[Out] 1/16\*(4\*b^2\*x^2\*log((d\*tan(b\*x + a) + c + 1)/(d\*tan(b\*x + a) + c - 1)) + 2\*I\*b\*x\*dilog(-((2\*I\*(c + 1)\*d + 2\*d^2)\*tan(b\*x + a)^2 + 2\*c^2 - 2\*I\*(c + 1)\*

$$\begin{aligned}
& d + (2Ic^2 + 4(c + 1)d - 2Id^2 + 4Ic + 2I)\tan(bx + a) + 4c + 2 \\
& /((c^2 + d^2 + 2c + 1)\tan(bx + a)^2 + c^2 + d^2 + 2c + 1) + 1 - 2Ibx \\
& \times \operatorname{dilog}\left(-\frac{(-2I(c + 1)d + 2d^2)\tan(bx + a)^2 + 2c^2 + 2I(c + 1)d + (-2Ic^2 + 4(c + 1)d + 2Id^2 - 4Ic - 2I)\tan(bx + a) + 4c + 2}{(c^2 + d^2 + 2c + 1)\tan(bx + a)^2 + c^2 + d^2 + 2c + 1} + 1\right) + 2Ibx \\
& \times \operatorname{dilog}\left(\frac{(2(I(c - 1)d - d^2)\tan(bx + a)^2 - 2c^2 - 2I(c - 1)d - (-2Ic^2 + 4(c - 1)d + 2Id^2 + 4Ic - 2I)\tan(bx + a) + 4c - 2)}{(c^2 + d^2 - 2c + 1)\tan(bx + a)^2 + c^2 + d^2 - 2c + 1} + 1\right) - 2Ibx \\
& \times \operatorname{dilog}\left(\frac{(2(-I(c - 1)d - d^2)\tan(bx + a)^2 - 2c^2 + 2I(c - 1)d - (2Ic^2 + 4(c - 1)d - 2Id^2 - 4Ic + 2I)\tan(bx + a) + 4c - 2)}{(c^2 + d^2 - 2c + 1)\tan(bx + a)^2 + c^2 + d^2 - 2c + 1} + 1\right) - 2a^2 \log\left(\frac{(I(c + 1)d + d^2)\tan(bx + a)^2 - c^2 + I(c + 1)d + (Ic^2 + Id^2 + 2Ic + I)\tan(bx + a) - 2c - 1}{(\tan(bx + a)^2 + 1)}\right) - 2a^2 \log\left(\frac{(I(c + 1)d - d^2)\tan(bx + a)^2 + c^2 + I(c + 1)d + (Ic^2 + Id^2 + 2Ic + I)\tan(bx + a) + 2c + 1}{(\tan(bx + a)^2 + 1)}\right) + 2a^2 \log\left(\frac{(I(c - 1)d + d^2)\tan(bx + a)^2 - c^2 + I(c - 1)d + (Ic^2 + Id^2 - 2Ic + I)\tan(bx + a) + 2c - 1}{(\tan(bx + a)^2 + 1)}\right) + 2a^2 \log\left(\frac{(I(c - 1)d - d^2)\tan(bx + a)^2 + c^2 + I(c - 1)d + (Ic^2 + Id^2 - 2Ic + I)\tan(bx + a) - 2c + 1}{(\tan(bx + a)^2 + 1)}\right) - 2(b^2x^2 - a^2) \log\left(\frac{(2I(c + 1)d + 2d^2)\tan(bx + a)^2 + 2c^2 - 2I(c + 1)d + (2Ic^2 + 4(c + 1)d - 2Id^2 + 4Ic + 2I)\tan(bx + a) + 4c + 2}{(c^2 + d^2 + 2c + 1)\tan(bx + a)^2 + c^2 + d^2 + 2c + 1}\right) - 2(b^2x^2 - a^2) \log\left(\frac{(-2I(c + 1)d + 2d^2)\tan(bx + a)^2 + 2c^2 + 2I(c + 1)d + (-2Ic^2 + 4(c + 1)d + 2Id^2 - 4Ic - 2I)\tan(bx + a) + 4c + 2}{(c^2 + d^2 + 2c + 1)\tan(bx + a)^2 + c^2 + d^2 + 2c + 1}\right) + 2(b^2x^2 - a^2) \log\left(\frac{-(2(I(c - 1)d - d^2)\tan(bx + a)^2 - 2c^2 - 2I(c - 1)d - (-2Ic^2 + 4(c - 1)d + 2Id^2 + 4Ic - 2I)\tan(bx + a) + 4c - 2)}{(c^2 + d^2 - 2c + 1)\tan(bx + a)^2 + c^2 + d^2 - 2c + 1}\right) + 2(b^2x^2 - a^2) \log\left(\frac{-(2(-I(c - 1)d - d^2)\tan(bx + a)^2 - 2c^2 + 2I(c - 1)d - (2Ic^2 + 4(c - 1)d - 2Id^2 - 4Ic + 2I)\tan(bx + a) + 4c - 2)}{(c^2 + d^2 - 2c + 1)\tan(bx + a)^2 + c^2 + d^2 - 2c + 1}\right) - \operatorname{polylog}(3, ((c^2 + 2I(c + 1)d - d^2 + 2c + 1)\tan(bx + a)^2 - c^2 - 2I(c + 1)d + d^2 + (2Ic^2 - 4(c + 1)d - 2Id^2 + 4Ic + 2I)\tan(bx + a) - 2c - 1)/((c^2 + d^2 + 2c + 1)\tan(bx + a)^2 + c^2 + d^2 + 2c + 1)) - \operatorname{polylog}(3, ((c^2 - 2I(c + 1)d - d^2 + 2c + 1)\tan(bx + a)^2 - c^2 + 2I(c + 1)d + d^2 + (-2Ic^2 - 4(c + 1)d + 2Id^2 - 4Ic - 2I)\tan(bx + a) - 2c - 1)/((c^2 + d^2 + 2c + 1)\tan(bx + a)^2 + c^2 + d^2 + 2c + 1)) + \operatorname{polylog}(3, ((c^2 + 2I(c - 1)d - d^2 - 2c + 1)\tan(bx + a)^2 - c^2 - 2I(c - 1)d + d^2 + (2Ic^2 - 4(c - 1)d - 2Id^2 - 4Ic + 2I)\tan(bx + a) + 2c - 1)/((c^2 + d^2 - 2c + 1)\tan(bx + a)^2 + c^2 + d^2 - 2c + 1)) + \operatorname{polylog}(3, ((c^2 - 2I(c - 1)d - d^2 - 2c + 1)\tan(bx + a)^2 - c^2 + 2I(c - 1)d + d^2 + (-2Ic^2 - 4(c - 1)d + 2Id^2 + 4Ic - 2I)\tan(bx + a) + 2c - 1)/((c^2 + d^2 - 2c + 1)\tan(bx + a)^2 + c^2 + d^2 - 2c + 1))) / b^2
\end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(d \tan(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*tan(b\*x + a) + c), x)

**maple** [C] time = 4.68, size = 6518, normalized size = 22.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(c+d\*tan(b\*x+a)),x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2bd \int \frac{1}{c^4 + d^4 + 2(c^2 + 1)d^2 + (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1) \cos(4bx + 4a)^2 + 4(c^4 + d^4 + 2(c^2 - 1)d^2 - 2c^2 + 1) \sin(4bx + 4a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*tan(b\*x+a)),x, algorithm="maxima")

[Out]  $-2*b*d*\integrate(-(2*(c^2 + d^2 - 1)*x^2*\cos(2*b*x + 2*a)^2 + 2*c*d*x^2*\sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*\sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1)*x^2*\cos(2*b*x + 2*a) - (2*c*d*x^2*\sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2*\cos(2*b*x + 2*a))*\cos(4*b*x + 4*a) + (2*c*d*x^2*\cos(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*\sin(2*b*x + 2*a))*\sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*\cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*\sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*\cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*\sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1))*\cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*\sin(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2 + 1)*\cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)*d))*\cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*\sin(2*b*x + 2*a))*\sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*\sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2*c + 1)*\cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*\sin(2*b*x + 2*a) + (c^2 + d^2 + 2*c + 1)*\sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c + 1)*\cos(2*b*x +$

$2*a) + 2*c + 1) - 1/8*x^2*\log((c^2 + d^2 - 2*c + 1)*\cos(2*b*x + 2*a)^2 + 4$   
 $*(c - 1)*d*\sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*\sin(2*b*x + 2*a)^2 + c^$   
 $2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) - 2*c + 1)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acoth}(c + d \tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(c + d*tan(a + b*x)),x)`

[Out] `int(x*acoth(c + d*tan(a + b*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(c + d \tan(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(c+d*tan(b*x+a)),x)`

[Out] `Integral(x*acoth(c + d*tan(a + b*x)), x)`



### 3.238 $\int \coth^{-1}(c + d \tan(a + bx)) dx$

**Optimal.** Leaf size=194

$$-\frac{i\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{i\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b} + \frac{1}{2}x \log\left(1 + \frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right) - \frac{1}{2}x \log\left(1 + \frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)$$

[Out]  $x \cdot \text{arccoth}(c + d \cdot \tan(b \cdot x + a)) + 1/2 \cdot x \cdot \ln(1 + ((1 - c + I \cdot d) \cdot \exp(2 \cdot I \cdot a + 2 \cdot I \cdot b \cdot x)) / (1 - c - I \cdot d)) - 1/2 \cdot x \cdot \ln(1 + ((1 + c - I \cdot d) \cdot \exp(2 \cdot I \cdot a + 2 \cdot I \cdot b \cdot x)) / (1 + c + I \cdot d)) - 1/4 \cdot I \cdot \text{polylog}(2, -(1 - c + I \cdot d) \cdot \exp(2 \cdot I \cdot a + 2 \cdot I \cdot b \cdot x) / (1 - c - I \cdot d)) / b + 1/4 \cdot I \cdot \text{polylog}(2, -(1 + c - I \cdot d) \cdot \exp(2 \cdot I \cdot a + 2 \cdot I \cdot b \cdot x) / (1 + c + I \cdot d)) / b$

**Rubi [A]** time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6260, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2, -\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b} + \frac{i\text{PolyLog}\left(2, -\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b} + \frac{1}{2}x \log\left(1 + \frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right) - \frac{1}{2}x \log\left(1 + \frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcCoth}[c + d \cdot \tan[a + b \cdot x]], x]$

[Out]  $x \cdot \text{ArcCoth}[c + d \cdot \tan[a + b \cdot x]] + (x \cdot \text{Log}[1 + ((1 - c + I \cdot d) \cdot E^{((2 \cdot I) \cdot a + (2 \cdot I) \cdot b \cdot x)) / (1 - c - I \cdot d)}]) / 2 - (x \cdot \text{Log}[1 + ((1 + c - I \cdot d) \cdot E^{((2 \cdot I) \cdot a + (2 \cdot I) \cdot b \cdot x)) / (1 + c + I \cdot d)}]) / 2 - ((I/4) \cdot \text{PolyLog}[2, -(((1 - c + I \cdot d) \cdot E^{((2 \cdot I) \cdot a + (2 \cdot I) \cdot b \cdot x)) / (1 - c - I \cdot d)})]) / b + ((I/4) \cdot \text{PolyLog}[2, -(((1 + c - I \cdot d) \cdot E^{((2 \cdot I) \cdot a + (2 \cdot I) \cdot b \cdot x)) / (1 + c + I \cdot d)})]) / b$

#### Rule 2190

$\text{Int}[(((F_)^\wedge((g_) \cdot ((e_) + (f_) \cdot (x_)))^\wedge(n_) \cdot ((c_) + (d_) \cdot (x_))^\wedge(m_)) / ((a_) + (b_) \cdot ((F_)^\wedge((g_) \cdot ((e_) + (f_) \cdot (x_)))^\wedge(n_))), x\_Symbol] \rightarrow \text{Simp} [((c + d \cdot x)^\wedge m \cdot \text{Log}[1 + (b \cdot (F)^\wedge(g \cdot (e + f \cdot x)))^\wedge n] / a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^\wedge(m - 1) \cdot \text{Log}[1 + (b \cdot (F)^\wedge(g \cdot (e + f \cdot x)))^\wedge n] / a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

#### Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_) \cdot ((F_)^\wedge((e_) \cdot ((c_) + (d_) \cdot (x_)))^\wedge(n_)]], x\_Symbol] \rightarrow \text{Dist}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F)^\wedge(e \cdot (c + d \cdot x))^\wedge n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

#### Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 6260

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] := Simp[x*ArcCoth[c + d*Tan[a + b*x]], x] + (-Dist[I*b*(1 + c - I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[I*b*(1 - c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(c + d \tan(a + bx)) dx &= x \coth^{-1}(c + d \tan(a + bx)) + (b(i(1 - c) - d)) \int \frac{e^{2ia+2ibx} x}{1 - c - id + (1 - c + id)e^{2ia+2ibx}} dx \\ &= x \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) \\ &= x \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) \\ &= x \coth^{-1}(c + d \tan(a + bx)) + \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) - \frac{1}{2} x \log \left( 1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id} \right) \end{aligned}$$

**Mathematica [B]** time = 13.41, size = 4654, normalized size = 23.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c + d*Tan[a + b*x]], x]
```

```
[Out] x*ArcCoth[c + d*Tan[a + b*x]] + (d*(-(a*Log[-(Sec[(a + b*x)/2]^2*((-1 + c)*Cos[a + b*x] + d*Sin[a + b*x]))]) + a*Log[Sec[(a + b*x)/2]^2*(Cos[a + b*x] + c*Cos[a + b*x] + d*Sin[a + b*x])) + (a + b*x)*Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(1 + I*Tan[(a + b*x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*c + c^2 + d^2]))*Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] - I*Log[-(((1 + c)*(1 + Tan[(a + b*x)/2]))/(1 - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Log[(-d + Sqrt[1
```

$$\begin{aligned}
& - 2*c + c^2 + d^2)/(-1 + c) + \text{Tan}[(a + b*x)/2]] + (a + b*x)*\text{Log}[(d + \text{Sqrt}[ \\
& 1 - 2*c + c^2 + d^2])/(1 - c) + \text{Tan}[(a + b*x)/2]] + I*\text{Log}[((-1 + c)*(-I + \text{T} \\
& \text{an}[(a + b*x)/2]))/(I - I*c + d + \text{Sqrt}[1 - 2*c + c^2 + d^2])] * \text{Log}[(d + \text{Sqrt}[ \\
& 1 - 2*c + c^2 + d^2])/(1 - c) + \text{Tan}[(a + b*x)/2]] - I*\text{Log}[((-1 + c)*(I + \text{T} \\
& \text{an}[(a + b*x)/2]))/(-I + I*c + d + \text{Sqrt}[1 - 2*c + c^2 + d^2])] * \text{Log}[(d + \text{Sqrt}[ \\
& 1 - 2*c + c^2 + d^2])/(1 - c) + \text{Tan}[(a + b*x)/2]] - (a + b*x)*\text{Log}[-((d + \text{S} \\
& \text{qrt}[1 + 2*c + c^2 + d^2])/(1 + c)) + \text{Tan}[(a + b*x)/2]] - I*\text{Log}[((1 + c)*(-I \\
& + \text{Tan}[(a + b*x)/2]))/(-I - I*c + d + \text{Sqrt}[1 + 2*c + c^2 + d^2])] * \text{Log}[-((d + \\
& \text{Sqrt}[1 + 2*c + c^2 + d^2])/(1 + c)) + \text{Tan}[(a + b*x)/2]] + I*\text{Log}[((1 + c)*( \\
& I + \text{Tan}[(a + b*x)/2]))/(I + I*c + d + \text{Sqrt}[1 + 2*c + c^2 + d^2])] * \text{Log}[-((d \\
& + \text{Sqrt}[1 + 2*c + c^2 + d^2])/(1 + c)) + \text{Tan}[(a + b*x)/2]] - (a + b*x)*\text{Log}[( \\
& -d + \text{Sqrt}[1 + 2*c + c^2 + d^2] + (1 + c)*\text{Tan}[(a + b*x)/2])/(1 + c)] + I*\text{Log} \\
& [((1 + c)*(1 - I*\text{Tan}[(a + b*x)/2]))/(1 + c - I*d + I*\text{Sqrt}[1 + 2*c + c^2 + d \\
& ^2])] * \text{Log}[-(d + \text{Sqrt}[1 + 2*c + c^2 + d^2] + (1 + c)*\text{Tan}[(a + b*x)/2])/(1 + \\
& c)] - I*\text{Log}[((1 + c)*(1 + I*\text{Tan}[(a + b*x)/2]))/(1 + c + I*d - I*\text{Sqrt}[1 + 2* \\
& c + c^2 + d^2])] * \text{Log}[-(d + \text{Sqrt}[1 + 2*c + c^2 + d^2] + (1 + c)*\text{Tan}[(a + b*x \\
& )/2])/(1 + c)] + I*\text{PolyLog}[2, (d + \text{Sqrt}[1 - 2*c + c^2 + d^2] - (-1 + c)*\text{T} \\
& \text{an}[(a + b*x)/2])/(I - I*c + d + \text{Sqrt}[1 - 2*c + c^2 + d^2])] - I*\text{PolyLog}[2, (d \\
& + \text{Sqrt}[1 - 2*c + c^2 + d^2] - (-1 + c)*\text{Tan}[(a + b*x)/2])/(-I + I*c + d + \text{S} \\
& \text{qrt}[1 - 2*c + c^2 + d^2])] - I*\text{PolyLog}[2, (-d + \text{Sqrt}[1 - 2*c + c^2 + d^2] + \\
& (-1 + c)*\text{Tan}[(a + b*x)/2])/(I - I*c - d + \text{Sqrt}[1 - 2*c + c^2 + d^2])] + I* \\
& \text{PolyLog}[2, (-d + \text{Sqrt}[1 - 2*c + c^2 + d^2] + (-1 + c)*\text{Tan}[(a + b*x)/2])/(-I \\
& + I*c - d + \text{Sqrt}[1 - 2*c + c^2 + d^2])] - I*\text{PolyLog}[2, (d + \text{Sqrt}[1 + 2*c + \\
& c^2 + d^2] - (1 + c)*\text{Tan}[(a + b*x)/2])/(-I - I*c + d + \text{Sqrt}[1 + 2*c + c^2 \\
& + d^2])] + I*\text{PolyLog}[2, (d + \text{Sqrt}[1 + 2*c + c^2 + d^2] - (1 + c)*\text{Tan}[(a + b \\
& *x)/2])/(I + I*c + d + \text{Sqrt}[1 + 2*c + c^2 + d^2])] + I*\text{PolyLog}[2, (-d + \text{S} \\
& \text{qrt}[1 + 2*c + c^2 + d^2] + (1 + c)*\text{Tan}[(a + b*x)/2])/(-I - I*c - d + \text{Sqrt}[1 + \\
& 2*c + c^2 + d^2])] - I*\text{PolyLog}[2, (-d + \text{Sqrt}[1 + 2*c + c^2 + d^2] + (1 + c \\
& )*\text{Tan}[(a + b*x)/2])/(I + I*c - d + \text{Sqrt}[1 + 2*c + c^2 + d^2])]*((-2*a)/(b* \\
& (-1 + c^2 + d^2 - \text{Cos}[2*(a + b*x)] + c^2*\text{Cos}[2*(a + b*x)] - d^2*\text{Cos}[2*(a + \\
& b*x)] + 2*c*d*\text{Sin}[2*(a + b*x)])) + (2*(a + b*x))/(b*(-1 + c^2 + d^2 - \text{Cos}[2 \\
& *(a + b*x)] + c^2*\text{Cos}[2*(a + b*x)] - d^2*\text{Cos}[2*(a + b*x)] + 2*c*d*\text{Sin}[2*(a \\
& + b*x)])))]/(Log[-(d + \text{Sqrt}[1 - 2*c + c^2 + d^2])/(-1 + c) + \text{Tan}[(a + b*x)/ \\
& 2]] + Log[(d + \text{Sqrt}[1 - 2*c + c^2 + d^2])/(1 - c) + \text{Tan}[(a + b*x)/2]] - Log \\
& [-((d + \text{Sqrt}[1 + 2*c + c^2 + d^2])/(1 + c)) + \text{Tan}[(a + b*x)/2]] - Log[-(d + \\
& \text{Sqrt}[1 + 2*c + c^2 + d^2] + (1 + c)*\text{Tan}[(a + b*x)/2])/(1 + c)] + (Log[-(d \\
& + \text{Sqrt}[1 + 2*c + c^2 + d^2] + (1 + c)*\text{Tan}[(a + b*x)/2])/(1 + c)]*Sec[(a + b \\
& *x)/2]^2)/(2*(1 - I*\text{Tan}[(a + b*x)/2])) - (Log[-(d + \text{Sqrt}[1 - 2*c + c^2 + d^ \\
& 2])/(-1 + c) + \text{Tan}[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(2*(1 + I*\text{Tan}[(a + b*x \\
& )/2])) + (Log[-(d + \text{Sqrt}[1 + 2*c + c^2 + d^2] + (1 + c)*\text{Tan}[(a + b*x)/2])/( \\
& 1 + c)]*Sec[(a + b*x)/2]^2)/(2*(1 + I*\text{Tan}[(a + b*x)/2])) + ((I/2)*Log[(d + \\
& \text{Sqrt}[1 - 2*c + c^2 + d^2])/(1 - c) + \text{Tan}[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/ \\
& (-I + \text{Tan}[(a + b*x)/2]) - ((I/2)*Log[-(d + \text{Sqrt}[1 + 2*c + c^2 + d^2])/(1 + \\
& c)) + \text{Tan}[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + \text{Tan}[(a + b*x)/2]) - ((I/ \\
& 2)*Log[-(d + \text{Sqrt}[1 - 2*c + c^2 + d^2])/(-1 + c) + \text{Tan}[(a + b*x)/2]]*Sec[(a
\end{aligned}$$

$$\begin{aligned}
& + b*x)/2]^2)/(I + \tan[(a + b*x)/2]) - ((I/2)*\log[(d + \sqrt{1 - 2*c + c^2 + d^2})]/(1 - c) + \tan[(a + b*x)/2])* \sec[(a + b*x)/2]^2)/(I + \tan[(a + b*x)/2]) \\
& + ((I/2)*\log[-((d + \sqrt{1 + 2*c + c^2 + d^2})/(1 + c)) + \tan[(a + b*x)/2])* \sec[(a + b*x)/2]^2)/(I + \tan[(a + b*x)/2]) + ((a + b*x)* \sec[(a + b*x)/2]^2)/(2*((-d + \sqrt{1 - 2*c + c^2 + d^2})/(-1 + c) + \tan[(a + b*x)/2])) + \\
& ((I/2)*\log[((-1 + c)*(1 + I*\tan[(a + b*x)/2]))/(-1 + c + I*d - I*\sqrt{1 - 2*c + c^2 + d^2})])* \sec[(a + b*x)/2]^2)/((-d + \sqrt{1 - 2*c + c^2 + d^2})/(-1 + c) + \tan[(a + b*x)/2]) - ((I/2)*\log[-(((-1 + c)*(I + \tan[(a + b*x)/2]))/(I - I*c - d + \sqrt{1 - 2*c + c^2 + d^2}))])* \sec[(a + b*x)/2]^2)/((-d + \sqrt{1 - 2*c + c^2 + d^2})/(-1 + c) + \tan[(a + b*x)/2]) + ((a + b*x)* \sec[(a + b*x)/2]^2)/(2*((d + \sqrt{1 - 2*c + c^2 + d^2})/(1 - c) + \tan[(a + b*x)/2])) + \\
& ((I/2)*\log[((-1 + c)*(-I + \tan[(a + b*x)/2]))/(I - I*c + d + \sqrt{1 - 2*c + c^2 + d^2})])* \sec[(a + b*x)/2]^2)/((d + \sqrt{1 - 2*c + c^2 + d^2})/(1 - c) + \tan[(a + b*x)/2]) - ((I/2)*\log[((-1 + c)*(I + \tan[(a + b*x)/2]))/(-I + I*c + d + \sqrt{1 - 2*c + c^2 + d^2})])* \sec[(a + b*x)/2]^2)/((d + \sqrt{1 - 2*c + c^2 + d^2})/(1 - c) + \tan[(a + b*x)/2]) - ((a + b*x)* \sec[(a + b*x)/2]^2)/(2*(-((d + \sqrt{1 + 2*c + c^2 + d^2})/(1 + c)) + \tan[(a + b*x)/2])) - ((I/2)*\log[((1 + c)*(-I + \tan[(a + b*x)/2]))/(-I - I*c + d + \sqrt{1 + 2*c + c^2 + d^2})])* \sec[(a + b*x)/2]^2)/(-((d + \sqrt{1 + 2*c + c^2 + d^2})/(1 + c)) + \tan[(a + b*x)/2]) + ((I/2)*\log[((1 + c)*(I + \tan[(a + b*x)/2]))/(I + I*c + d + \sqrt{1 + 2*c + c^2 + d^2})])* \sec[(a + b*x)/2]^2)/(-((d + \sqrt{1 + 2*c + c^2 + d^2})/(1 + c)) + \tan[(a + b*x)/2]) + ((I/2)*(-1 + c)*\log[1 - (d + \sqrt{1 - 2*c + c^2 + d^2}) - (-1 + c)*\tan[(a + b*x)/2]])/(I - I*c + d + \sqrt{1 - 2*c + c^2 + d^2}) + \sec[(a + b*x)/2]^2)/(d + \sqrt{1 - 2*c + c^2 + d^2}) - (-1 + c)*\tan[(a + b*x)/2]) - ((I/2)*(-1 + c)*\log[1 - (d + \sqrt{1 - 2*c + c^2 + d^2}) - (-1 + c)*\tan[(a + b*x)/2]])/(-I + I*c + d + \sqrt{1 - 2*c + c^2 + d^2}) + \sec[(a + b*x)/2]^2)/(d + \sqrt{1 - 2*c + c^2 + d^2}) - (-1 + c)*\tan[(a + b*x)/2]) + ((I/2)*(-1 + c)*\log[1 - (-d + \sqrt{1 - 2*c + c^2 + d^2}) + (-1 + c)*\tan[(a + b*x)/2]])/(I - I*c - d + \sqrt{1 - 2*c + c^2 + d^2}) + \sec[(a + b*x)/2]^2)/(-d + \sqrt{1 - 2*c + c^2 + d^2}) + (-1 + c)*\tan[(a + b*x)/2]) - ((I/2)*(-1 + c)*\log[1 - (-d + \sqrt{1 - 2*c + c^2 + d^2}) + (-1 + c)*\tan[(a + b*x)/2]])/(-I + I*c - d + \sqrt{1 - 2*c + c^2 + d^2}) + \sec[(a + b*x)/2]^2)/(-d + \sqrt{1 - 2*c + c^2 + d^2}) + (-1 + c)*\tan[(a + b*x)/2]) - ((I/2)*(1 + c)*\log[1 - (d + \sqrt{1 + 2*c + c^2 + d^2}) - (1 + c)*\tan[(a + b*x)/2]])/(-I - I*c + d + \sqrt{1 + 2*c + c^2 + d^2}) + \sec[(a + b*x)/2]^2)/(d + \sqrt{1 + 2*c + c^2 + d^2}) - (1 + c)*\tan[(a + b*x)/2]) + ((I/2)*(1 + c)*\log[1 - (d + \sqrt{1 + 2*c + c^2 + d^2}) - (1 + c)*\tan[(a + b*x)/2]])/(I + I*c + d + \sqrt{1 + 2*c + c^2 + d^2}) + \sec[(a + b*x)/2]^2)/(d + \sqrt{1 + 2*c + c^2 + d^2}) - (1 + c)*\tan[(a + b*x)/2]) - ((1 + c)*(a + b*x)* \sec[(a + b*x)/2]^2)/(2*(-d + \sqrt{1 + 2*c + c^2 + d^2}) + (1 + c)*\tan[(a + b*x)/2])) + ((I/2)*(1 + c)*\log[((1 + c)*(1 - I*\tan[(a + b*x)/2]))/(1 + c - I*d + I*\sqrt{1 + 2*c + c^2 + d^2})])* \sec[(a + b*x)/2]^2)/(-d + \sqrt{1 + 2*c + c^2 + d^2}) + (1 + c)*\tan[(a + b*x)/2]) - ((I/2)*(1 + c)*\log[((1 + c)*(1 + I*\tan[(a + b*x)/2]))/(1 + c + I*d - I*\sqrt{1 + 2*c + c^2 + d^2})])* \sec[(a + b*x)/2]^2)/(-d + \sqrt{1 + 2*c + c^2 + d^2}) + (1 + c)*\tan[(a + b*x)/2]) - ((I/2)*(1 + c)*\log[1 - (-d + \sqrt{1 + 2*c + c^2 + d^2}) + (1 + c)*\tan[(a + b*x)/2]])/(I - I*c + d + \sqrt{1 + 2*c + c^2 + d^2}) + \sec[(a + b*x)/2]^2)/(d + \sqrt{1 + 2*c + c^2 + d^2}) - (1 + c)*\tan[(a + b*x)/2])
\end{aligned}$$



$$\frac{(2*(I*(c-1)*d-d^2)*\tan(b*x+a)^2-2*c^2-2*I*(c-1)*d-(-2*I*c^2+4*(c-1)*d+2*I*d^2+4*I*c-2*I)*\tan(b*x+a)+4*c-2)/((c^2+d^2-2*c+1)*\tan(b*x+a)^2+c^2+d^2-2*c+1)+1-I*d\log((2*(-I*(c-1)*d-d^2)*\tan(b*x+a)^2-2*c^2+2*I*(c-1)*d-(2*I*c^2+4*(c-1)*d-2*I*d^2-4*I*c+2*I)*\tan(b*x+a)+4*c-2)/((c^2+d^2-2*c+1)*\tan(b*x+a)^2+c^2+d^2-2*c+1)+1))/b}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(d \tan(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*tan(b\*x + a) + c), x)

**maple** [B] time = 0.33, size = 612, normalized size = 3.15

$$\frac{\arctan(\tan(bx+a)) \operatorname{arccoth}(c+d \tan(bx+a))}{b} - \frac{\arctan\left(\frac{c+d \tan(bx+a)}{d} - \frac{c}{d}\right) \ln\left(d\left(\frac{c+d \tan(bx+a)}{d} - \frac{c}{d}\right) + c + 1\right)}{2b} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*tan(b\*x+a)),x)

[Out]  $\frac{1}{b} \arctan(\tan(bx+a)) \operatorname{arccoth}(c+d \tan(bx+a)) - \frac{1}{2b} \arctan\left(\frac{c+d \tan(bx+a)}{d-c/d}\right) \ln\left(d \frac{c+d \tan(bx+a)}{d-c/d} + c + 1\right) + \frac{1}{2b} \arctan\left(\frac{c+d \tan(bx+a)}{d-c/d}\right) \ln\left(d \frac{c+d \tan(bx+a)}{d-c/d} + c - 1\right) + \frac{1}{4b} \ln\left(d \frac{c+d \tan(bx+a)}{d-c/d} + c - 1\right) \ln\left(\frac{I*d-d*((c+d \tan(bx+a))/d-c/d)}{I*d+c-1}\right) - \frac{1}{4b} \ln\left(d \frac{c+d \tan(bx+a)}{d-c/d} + c - 1\right) \ln\left(\frac{I*d+d*((c+d \tan(bx+a))/d-c/d)}{1-c+I*d}\right) + \frac{1}{4b} \operatorname{dilog}\left(\frac{I*d-d*((c+d \tan(bx+a))/d-c/d)}{I*d+c-1}\right) - \frac{1}{4b} \operatorname{dilog}\left(\frac{I*d+d*((c+d \tan(bx+a))/d-c/d)}{1-c+I*d}\right) - \frac{1}{4b} \ln\left(d \frac{c+d \tan(bx+a)}{d-c/d} + c + 1\right) \ln\left(\frac{I*d-d*((c+d \tan(bx+a))/d-c/d)}{1+c+I*d}\right) + \frac{1}{4b} \ln\left(d \frac{c+d \tan(bx+a)}{d-c/d} + c + 1\right) \ln\left(\frac{I*d+d*((c+d \tan(bx+a))/d-c/d)}{I*d-c-1}\right) - \frac{1}{4b} \operatorname{dilog}\left(\frac{I*d-d*((c+d \tan(bx+a))/d-c/d)}{1+c+I*d}\right) + \frac{1}{4b} \operatorname{dilog}\left(\frac{I*d+d*((c+d \tan(bx+a))/d-c/d)}{I*d-c-1}\right)$

**maxima** [B] time = 0.51, size = 372, normalized size = 1.92

$$\frac{4(bx+a) \operatorname{arccoth}(d \tan(bx+a) + c) + \left(\arctan\left(\frac{d^2 \tan(bx+a) + (c+1)d}{c^2 + d^2 + 2c + 1}\right), \frac{(c+1)d \tan(bx+a) + c^2 + 2c + 1}{c^2 + d^2 + 2c + 1}\right) - \arctan\left(\frac{d^2 \tan(bx+a) + (c+1)d}{c^2 + d^2 - 2c + 1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tan(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{4} * (4 * (b * x + a) * \operatorname{arccoth}(d * \tan(b * x + a) + c) + (\arctan2((d^2 * \tan(b * x + a) + (c + 1) * d) / (c^2 + d^2 + 2 * c + 1), ((c + 1) * d * \tan(b * x + a) + c^2 + 2 * c + 1) / (c^2 + d^2 + 2 * c + 1)) - \arctan2((d^2 * \tan(b * x + a) + (c - 1) * d) / (c^2 + d^2 - 2 * c + 1), ((c - 1) * d * \tan(b * x + a) + c^2 - 2 * c + 1) / (c^2 + d^2 - 2 * c + 1))) * \log(\tan(b * x + a)^2 + 1) - (b * x + a) * \log((d^2 * \tan(b * x + a)^2 + 2 * (c + 1) * d * \tan(b * x + a) + c^2 + 2 * c + 1) / (c^2 + d^2 + 2 * c + 1)) + (b * x + a) * \log((d^2 * \tan(b * x + a)^2 + 2 * (c - 1) * d * \tan(b * x + a) + c^2 - 2 * c + 1) / (c^2 + d^2 - 2 * c + 1)) - I * \operatorname{dilog}(-(I * d * \tan(b * x + a) - d) / (I * c + d + I)) + I * \operatorname{dilog}(-(I * d * \tan(b * x + a) - d) / (I * c + d - I)) - I * \operatorname{dilog}((I * d * \tan(b * x + a) + d) / (-I * c + d + I)) + I * \operatorname{dilog}((I * d * \tan(b * x + a) + d) / (-I * c + d - I))) / b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(c + d \tan(a + b x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(c + d\*tan(a + b\*x)),x)

[Out] int(acoth(c + d\*tan(a + b\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(c + d \tan(a + b x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c+d\*tan(b\*x+a)),x)

[Out] Integral(acoth(c + d\*tan(a + b\*x)), x)

$$3.239 \quad \int \frac{\coth^{-1}(c+d \tan(a+bx))}{x} dx$$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(d \tan(a + bx) + c)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(c+d\*tan(b\*x+a))/x,x)

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(c + d \tan(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[c + d\*Tan[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d\*Tan[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c + d \tan(a + bx))}{x} dx = \int \frac{\coth^{-1}(c + d \tan(a + bx))}{x} dx$$

Mathematica [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(c + d \tan(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[c + d\*Tan[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[c + d\*Tan[a + b\*x]]/x, x]

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcoth}(d \tan(bx + a) + c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(arccoth(c+d\*tan(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*tan(b\*x + a) + c)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \tan (b x+a)+c)}{x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tan(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*tan(b\*x + a) + c)/x, x)

**maple** [A] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(c+d \tan (b x+a))}{x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*tan(b\*x+a))/x,x)

[Out] int(arccoth(c+d\*tan(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \tan (b x+a)+c)}{x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*tan(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*tan(b\*x + a) + c)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(c+d \tan (a+b x))}{x} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(c + d\*tan(a + b\*x))/x,x)

[Out] int(acoth(c + d\*tan(a + b\*x))/x, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(c + d \tan(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c+d\*tan(b\*x+a))/x,x)

[Out] Integral(acoth(c + d\*tan(a + b\*x))/x, x)

### 3.240 $\int x^2 \coth^{-1}(1 - id + d \tan(a + bx)) dx$

**Optimal.** Leaf size=170

$$\frac{i\text{Li}_4\left(-\left(1-id\right)e^{2ia+2ibx}\right)}{8b^3} - \frac{x\text{Li}_3\left(-\left(1-id\right)e^{2ia+2ibx}\right)}{4b^2} + \frac{ix^2\text{Li}_2\left(-\left(1-id\right)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3 \log\left(1 + \left(1-id\right)e^{2ia+2ibx}\right)$$

[Out] 1/12\*I\*b\*x^4+1/3\*x^3\*arccoth(1-I\*d+d\*tan(b\*x+a))-1/6\*x^3\*ln(1+(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))+1/4\*I\*x^2\*polylog(2,-(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b-1/4\*x\*polylog(3,-(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^2-1/8\*I\*polylog(4,-(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^3

**Rubi [A]** time = 0.30, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {6264, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-\left(1-id\right)e^{2ia+2ibx}\right)}{4b^2} - \frac{i\text{PolyLog}\left(4,-\left(1-id\right)e^{2ia+2ibx}\right)}{8b^3} + \frac{ix^2\text{PolyLog}\left(2,-\left(1-id\right)e^{2ia+2ibx}\right)}{4b} - \frac{1}{6}x^3 \log\left(1 + \left(1-id\right)e^{2ia+2ibx}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]],x]

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/3 - (x^3\*Log[1 + (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/6 + ((I/4)\*x^2\*PolyLog[2, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - (x\*PolyLog[3, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/((4\*b^2) - ((I/8)\*PolyLog[4, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b^3

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6264

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c + I*d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 - id + d \tan(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) + \frac{1}{3} (ib) \int \frac{x^3}{1 + (1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{3} (b(i + d)) \int \frac{e^{2ia}}{1 + (1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 - id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 155, normalized size = 0.91

$$\frac{1}{3} x^3 \coth^{-1}(d \tan(a+bx) - id + 1) - \frac{4b^3 x^3 \log\left(1 + \frac{ie^{-2i(a+bx)}}{d+i}\right) + 6ib^2 x^2 \operatorname{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{d+i}\right) + 6bx \operatorname{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{d+i}\right) - 3i \operatorname{Li}_4\left(-\frac{ie^{-2i(a+bx)}}{d+i}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]], x]

[Out] (x^3\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/3 - (4\*b^3\*x^3\*Log[1 + I/((I + d)\*E^((2\*I)\*(a + b\*x)))] + (6\*I)\*b^2\*x^2\*PolyLog[2, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))] + 6\*b\*x\*PolyLog[3, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))] - (3\*I)\*PolyLog[4, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))])/(24\*b^3)

**fricas [C]** time = 0.51, size = 348, normalized size = 2.05

$$\frac{ib^4 x^4 + 2b^3 x^3 \log\left(\frac{((d+i)e^{2ibx+2ia}+i)e^{-2ibx-2ia}}{d}\right) + 6ib^2 x^2 \operatorname{Li}_2\left(\frac{1}{2} \sqrt{4id-4} e^{(ibx+ia)}\right) + 6ib^2 x^2 \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{4id-4} e^{(ibx+ia)}\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d+d\*tan(b\*x+a)), x, algorithm="fricas")

```
[Out] 1/12*(I*b^4*x^4 + 2*b^3*x^3*log(((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) + 6*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log(((2*d + 2*I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) + 2*a^3*log(((2*d + 2*I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 12*b*x*polylog(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*polylog(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arcoth}(d \tan(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*tan(b*x + a) - I*d + 1), x)
```

**maple** [C] time = 5.88, size = 2339, normalized size = 13.76

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/6*x^3*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/3*x^3*ln(exp(I*(b*x+a)))+1/12*I*b*x^4+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2+1/3/b^3*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/2*I/b^3*a^3/(I+d)*ln(1-I*exp(I*(b*x+a)))*(-I*(I+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a)))*(-I*(I+d))^(1/2))-1/8*I/b^3*d/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))+1))^3-1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3-1/6*x^3*ln(d)+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(I
```

```

*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+
a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+exp(
2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/8/b^3/(I+d)*polylog(4,I*(I+d)*e
xp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a
)))^3+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/2/b^3
*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dil
og(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,I*(I+d)*exp(2
*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/6*
d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/6*I/(I+d)*ln(1-I*(I+d)*exp(2*I
*(b*x+a)))*x^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a))
)*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))+1/2/b^2*d/(I+d)*ln(1-I*(I+d
)*exp(2*I*(b*x+a)))*x*a^2-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+
d))^(1/2))*x-1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+
1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/3*I/b^3/(
I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,I*(I+d)*e
xp(2*I*(b*x+a)))*x-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/
2))*x-1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/4*I/b
*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/4*I/b^3*d/(I+d)*polylog(
2,I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(
b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))/(exp(2
*I*(b*x+a))+1)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/12*I*x^
3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))+1)*csgn(I*d/(exp(
2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1)
)*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+
a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))-1/12*I*x^3*Pi*csgn(I*d/(exp
(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x
+a)))^2-1/2/b^3*a^3*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3
*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2*I/b^2/(I+d)*ln(1-I
*(I+d)*exp(2*I*(b*x+a)))*x*a^2+1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b*x+a
)))*(-I*(I+d))^(1/2))+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+
d))^(1/2))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-
1/4/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x-1/12*I*x^3*Pi*csgn(I*
(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp
(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Piscsg
n(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))/
(exp(2*I*(b*x+a))+1))

```

**maxima [B]** time = 0.35, size = 341, normalized size = 2.01

$$\frac{12((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2)\operatorname{arccoth}(d\tan(bx+a)-id+1)}{b^2} - \frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2+(8i(bx+a)^3-18i(bx+a)^2a+18i(bx+a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="maxima")

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arccoth(d*tan(b*x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x + a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((I*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b^2)/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(d \tan(a + bx) + 1 - d i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(d*tan(a + b*x) - d*I + 1),x)
```

```
[Out] int(x^2*acoth(d*tan(a + b*x) - d*I + 1), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(d \tan(a + bx) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(x**2*acoth(d*tan(a + b*x) - I*d + 1), x)
```



### 3.241 $\int x \coth^{-1}(1 - id + d \tan(a + bx)) dx$

**Optimal.** Leaf size=133

$$-\frac{\text{Li}_3\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^2} + \frac{ix\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b} - \frac{1}{4}x^2 \log\left(1 + (1-id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d \tan(a+bx))$$

[Out]  $1/6*I*b*x^3 + 1/2*x^2*\text{arccoth}(1-I*d+d*\tan(b*x+a)) - 1/4*x^2*\ln(1+(1-I*d)*\exp(2*I*a+2*I*b*x)) + 1/4*I*x*\text{polylog}(2, -(1-I*d)*\exp(2*I*a+2*I*b*x))/b - 1/8*\text{polylog}(3, -(1-I*d)*\exp(2*I*a+2*I*b*x))/b^2$

**Rubi [A]** time = 0.25, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6264, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, -(1-id)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\text{PolyLog}\left(2, -(1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 + (1-id)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}(d \tan(a+bx))$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[1 - I*d + d*Tan[a + b*x]], x]`

[Out]  $(I/6)*b*x^3 + (x^2*\text{ArcCoth}[1 - I*d + d*\text{Tan}[a + b*x]])/2 - (x^2*\text{Log}[1 + (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)}])/4 + ((I/4)*x*\text{PolyLog}[2, -((1 - I*d)*E^{((2*I)*a + (2*I)*b*x)})])/b - \text{PolyLog}[3, -((1 - I*d)*E^{((2*I)*a + (2*I)*b*x)})]/(8*b^2)$

#### Rule 2184

`Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi`

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6264

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
+ 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1 - id + d \tan(a + bx)) dx &= \frac{1}{2} x^2 \coth^{-1}(1 - id + d \tan(a + bx)) + \frac{1}{2} (ib) \int \frac{x^2}{1 + (1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{2} (b(i + d)) \int \frac{e^{2ia+2ibx}}{1 + (1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 - id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2 \operatorname{coth}^{-1}(d \tan(ax+bx) - id + 1) - \frac{2b^2x^2 \log\left(1 + \frac{ie^{-2i(ax+bx)}}{d+i}\right) + 2ibx \operatorname{Li}_2\left(-\frac{ie^{-2i(ax+bx)}}{d+i}\right) + \operatorname{Li}_3\left(-\frac{ie^{-2i(ax+bx)}}{d+i}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]], x]

[Out] (x^2\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]])/2 - (2\*b^2\*x^2\*Log[1 + I/((I + d)\*E^((2\*I)\*(a + b\*x)))] + (2\*I)\*b\*x\*PolyLog[2, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))] + PolyLog[3, (-I)/((I + d)\*E^((2\*I)\*(a + b\*x)))])/(8\*b^2)

**fricas [C]** time = 0.58, size = 296, normalized size = 2.23

$$\frac{2i b^3 x^3 + 3 b^2 x^2 \log\left(\frac{((d+i)e^{2i bx+2i a}+i)e^{-2i bx-2i a}}{d}\right) + 2i a^3 + 6i b x \operatorname{Li}_2\left(\frac{1}{2} \sqrt{4i d - 4} e^{(i b x + i a)}\right) + 6i b x \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{4i d - 4}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d+d\*tan(b\*x+a)), x, algorithm="fricas")

[Out] 1/12\*(2\*I\*b^3\*x^3 + 3\*b^2\*x^2\*log(((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) + I)\*e^(-2\*I\*b\*x - 2\*I\*a)/d) + 2\*I\*a^3 + 6\*I\*b\*x\*dilog(1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) + 6\*I\*b\*x\*dilog(-1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) - 3\*a^2\*log(((2\*d + 2\*I)\*e^(I\*b\*x + I\*a) + I\*sqrt(4\*I\*d - 4))/(2\*d + 2\*I)) - 3\*a^2\*log(((2\*d + 2\*I)\*e^(I\*b\*x + I\*a) - I\*sqrt(4\*I\*d - 4))/(2\*d + 2\*I)) - 3\*(b^2\*x^2 - a^2)\*log(1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) - 3\*(b^2\*x^2 - a^2)\*log(-1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) - 6\*polylog(3, 1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) - 6\*polylog(3, -1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a)))/b^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(d \tan(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d+d\*tan(b\*x+a)), x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*tan(b\*x + a) - I\*d + 1), x)

**maple [C]** time = 5.06, size = 2249, normalized size = 16.91

Expression too large to display



$x+a)) * \operatorname{csgn}(I \exp(2I(b*x+a)) / (\exp(2I(b*x+a))+1)) - 1/8 I x^2 \operatorname{Pi} \operatorname{csgn}(I(I \exp(2I(b*x+a)) + \exp(2I(b*x+a)) * d + I) / (\exp(2I(b*x+a))+1)) * \operatorname{csgn}((I \exp(2I(b*x+a)) + \exp(2I(b*x+a)) * d + I) / (\exp(2I(b*x+a))+1)) - 1/4 I / b^2 a^2 / (I+d) * \ln(I \exp(2I(b*x+a)) + \exp(2I(b*x+a)) * d + I) + 1/8 I x^2 \operatorname{Pi} \operatorname{csgn}(I(I \exp(2I(b*x+a)) + \exp(2I(b*x+a)) * d + I)) * \operatorname{csgn}(I(I \exp(2I(b*x+a)) + \exp(2I(b*x+a)) * d + I) / (\exp(2I(b*x+a))+1))^{2+1/8 I x^2 \operatorname{Pi} \operatorname{csgn}(I(I \exp(2I(b*x+a)) + \exp(2I(b*x+a)) * d + I) / (\exp(2I(b*x+a))+1)) * \operatorname{csgn}((I \exp(2I(b*x+a)) + \exp(2I(b*x+a)) * d + I) / (\exp(2I(b*x+a))+1))^{2+1/8 I x^2 \operatorname{Pi} \operatorname{csgn}(I / (\exp(2I(b*x+a))+1)) * \operatorname{csgn}(I(I \exp(2I(b*x+a)) + \exp(2I(b*x+a)) * d + I) / (\exp(2I(b*x+a))+1))^{2}}$

**maxima** [B] time = 0.34, size = 247, normalized size = 1.86

$$\frac{12((bx+a)^2 - 2(bx+a)a) \operatorname{arccoth}(d \tan(bx+a) - id + 1)}{b} - \frac{-4i(bx+a)^3 + 12i(bx+a)^2 a - 6i b x \operatorname{Li}_2((id-1)e^{2i bx + 2ia}) + (6i(bx+a)^2 - 12i(bx+a)a) \operatorname{arctan}(-d \cos(2bx + 2a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")`

[Out]  $1/24 * (12 * ((b*x + a)^2 - 2 * (b*x + a) * a) * \operatorname{arccoth}(d * \tan(b*x + a) - I * d + 1) / b - (-4 * I * (b*x + a)^3 + 12 * I * (b*x + a)^2 * a - 6 * I * b * x * \operatorname{dilog}((I * d - 1) * e^{(2 * I * b * x + 2 * I * a)}) + (6 * I * (b*x + a)^2 - 12 * I * (b*x + a) * a) * \operatorname{arctan}2(-d * \cos(2 * b * x + 2 * a) + \sin(2 * b * x + 2 * a), d * \sin(2 * b * x + 2 * a) + \cos(2 * b * x + 2 * a) + 1) + 3 * ((b * x + a)^2 - 2 * (b * x + a) * a) * \log((d^2 + 1) * \cos(2 * b * x + 2 * a)^2 + (d^2 + 1) * \sin(2 * b * x + 2 * a)^2 + 2 * d * \sin(2 * b * x + 2 * a) + 2 * \cos(2 * b * x + 2 * a) + 1) + 3 * \operatorname{polylog}(3, (I * d - 1) * e^{(2 * I * b * x + 2 * I * a)})) / b) / b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(d \tan(a + bx) + 1 - d i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*acoth(d*tan(a + b*x) - d*i + 1),x)`

[Out] `int(x*acoth(d*tan(a + b*x) - d*i + 1), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(d \tan(a + bx) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*acoth(1-I*d+d*tan(b*x+a)),x)`

[Out] `Integral(x*acoth(d*tan(a + b*x) - I*d + 1), x)`

### 3.242 $\int \coth^{-1}(1 - id + d \tan(a + bx)) dx$

**Optimal.** Leaf size=93

$$\frac{i\text{Li}_2\left(-\left(1-id\right)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 + \left(1-id\right)e^{2ia+2ibx}\right) + x \coth^{-1}\left(d \tan(a+bx) - id + 1\right) + \frac{1}{2}ibx^2$$

[Out] 1/2\*I\*b\*x^2+x\*arccoth(1-I\*d+d\*tan(b\*x+a))-1/2\*x\*ln(1+(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))+1/4\*I\*polylog(2,-(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b

**Rubi [A]** time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6256, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2, -\left(1-id\right)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 + \left(1-id\right)e^{2ia+2ibx}\right) + x \coth^{-1}\left(d \tan(a+bx) - id + 1\right) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]], x]

[Out] (I/2)\*b\*x^2 + x\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]] - (x\*Log[1 + (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/2 + ((I/4)\*PolyLog[2, -((1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_.))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 6256

Int[ArcCoth[(c\_.) + (d\_.)\*Tan[(a\_.) + (b\_.)\*(x\_)]], x\_Symbol] := Simp[x\*ArcCoth[c + d\*Tan[a + b\*x]], x] + Dist[I\*b, Int[x/(c + I\*d + c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I\*d)^2, 1]

Rubi steps

$$\begin{aligned} \int \coth^{-1}(1 - id + d \tan(a + bx)) dx &= x \coth^{-1}(1 - id + d \tan(a + bx)) + (ib) \int \frac{x}{1 + (1 - id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id + d \tan(a + bx)) - (b(i + d)) \int \frac{e^{2ia+2ibx} x}{1 + (1 - id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 - id)e^{2ia+2ibx}) \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 - id)e^{2ia+2ibx}) \\ &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id + d \tan(a + bx)) - \frac{1}{2} x \log(1 + (1 - id)e^{2ia+2ibx}) \end{aligned}$$

**Mathematica [B]** time = 3.85, size = 766, normalized size = 8.24

$$\frac{x \sec^2(a + bx)(\cos(bx) + i \sin(bx))(\sin(bx) + i \cos(bx))(d \sin(a + bx) + (2 - id) \cos(a + bx)) \left( \text{Li}_2 \left( -\frac{1}{2}(\cos(a) + i \sin(a)) \frac{\sec^2(bx)(d \sin(a + bx) + (2 - id) \cos(a + bx))}{\tan(bx) - i} \right) \right)}{(\tan(a + bx) - i)(d \tan(a + bx) - id + 2)(id \sin(a + bx) + (d + 2i) \cos(a + bx)) \left( \frac{\sec^2(bx) \log \left( \frac{\sec(bx)(d \sin(a + bx) + (2 - id) \cos(a + bx))}{2 \cos(bx) - i} \right)}{\tan(bx) - i} \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]], x]

[Out] x\*ArcCoth[1 - I\*d + d\*Tan[a + b\*x]] + (x\*((2\*I)\*b\*x\*Log[2\*Cos[b\*x]\*(Cos[b\*x] - I\*Sin[b\*x])] - Log[(Sec[b\*x]\*(Cos[a] - I\*Sin[a])\*((2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x]))/(2\*(I + d))]\*Log[1 - I\*Tan[b\*x]] + Log[(Sec[b\*x]\*((2 - I\*d)\*Cos[a + b\*x] + d\*Sin[a + b\*x]))/(2\*Cos[a] - (2\*I)\*Sin[a])]\*Log[1 + I\*Tan[b\*x]] - PolyLog[2, -Cos[2\*b\*x] + I\*Sin[2\*b\*x]] - PolyLog[2, (Sec[b\*x]\*(d\*Cos[a] + I\*(2\*I + d)\*Sin[a])\*(Cos[a + b\*x] - I\*Sin[a + b\*x]))/(2\*(I + d

))] + PolyLog[2, -1/2\*((Cos[a] + I\*Sin[a])\*(d\*Cos[a] + I\*(2\*I + d)\*Sin[a])\*(-I + Tan[b\*x]))]\*Sec[a + b\*x]^2\*(Cos[b\*x] + I\*Sin[b\*x])\*(I\*Cos[b\*x] + Sin[b\*x])\*((2 - I\*d)\*Cos[a + b\*x] + d\*Sin[a + b\*x])/(((2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x])\*((I\*Log[1 + I\*Tan[b\*x]]\*Sec[b\*x]\*(d\*Cos[a] + I\*(2\*I + d)\*Sin[a]))/((2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a])) + (Log[1 - I\*Tan[b\*x]]\*Sec[b\*x]\*((-I)\*d\*Cos[a] + (2\*I + d)\*Sin[a]))/((2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a] + 2\*b\*x\*(1 - I\*Tan[b\*x])) + (Log[(Sec[b\*x]\*((2 - I\*d)\*Cos[a + b\*x] + d\*Sin[a + b\*x]))/(2\*Cos[a] - (2\*I)\*Sin[a]))\*Sec[b\*x]^2)/(-I + Tan[b\*x]) - (Log[1 + ((Cos[a] + I\*Sin[a])\*(d\*Cos[a] + I\*(2\*I + d)\*Sin[a])\*(-I + Tan[b\*x]))/2]\*Sec[b\*x]^2)/(-I + Tan[b\*x]) + Log[(Sec[b\*x]\*(Cos[a] - I\*Sin[a])\*((2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x]))/(2\*(I + d))]\*(-I + Tan[b\*x]) - (Log[(Sec[b\*x]\*(Cos[a] - I\*Sin[a])\*((2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x]))/(2\*(I + d))]\*Sec[b\*x]^2)/(I + Tan[b\*x])\*(-I + Tan[a + b\*x])\*(2 - I\*d + d\*Tan[a + b\*x]))

**fricas [B]** time = 0.63, size = 221, normalized size = 2.38

$$i b^2 x^2 + b x \log\left(\frac{((d+i)e^{2i b x+2i a}+i)e^{(-2i b x-2i a)}}{d}\right) - i a^2 - (b x + a) \log\left(\frac{1}{2} \sqrt{4i d - 4} e^{(i b x+i a)} + 1\right) - (b x + a) \log\left(-\frac{1}{2} \sqrt{4i d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="fricas")

[Out] 1/2\*(I\*b^2\*x^2 + b\*x\*log(((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) + I)\*e^(-2\*I\*b\*x - 2\*I\*a)/d) - I\*a^2 - (b\*x + a)\*log(1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) - (b\*x + a)\*log(-1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) + a\*log(((2\*d + 2\*I)\*e^(I\*b\*x + I\*a) + I\*sqrt(4\*I\*d - 4))/(2\*d + 2\*I)) + a\*log(((2\*d + 2\*I)\*e^(I\*b\*x + I\*a) - I\*sqrt(4\*I\*d - 4))/(2\*d + 2\*I)) + I\*dilog(1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) + I\*dilog(-1/2\*sqrt(4\*I\*d - 4)\*e^(I\*b\*x + I\*a)))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(d \tan(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*tan(b\*x + a) - I\*d + 1), x)

**maple [B]** time = 0.52, size = 292, normalized size = 3.14

$$\frac{\operatorname{arccoth}(1 - id + d \tan(bx + a)) \ln(id + d \tan(bx + a))}{2b} - \frac{\operatorname{arccoth}(1 - id + d \tan(bx + a)) \ln(-id + d \tan(bx + a))}{2b}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arccoth(1-I*d+d*tan(b*x+a)),x)`

[Out]  $\frac{1}{2}I/b \operatorname{arccoth}(1-I*d+d*\tan(b*x+a)) * \ln(I*d+d*\tan(b*x+a)) - \frac{1}{2}I/b \operatorname{arccoth}(1-I*d+d*\tan(b*x+a)) * \ln(-I*d+d*\tan(b*x+a)) - \frac{1}{8}I/b \ln(-I*d+d*\tan(b*x+a))^2 + \frac{1}{4}I/b \operatorname{dilog}(1-1/2*I*d+1/2*d*\tan(b*x+a)) + \frac{1}{4}I/b \ln(-I*d+d*\tan(b*x+a)) * \ln(1-1/2*I*d+1/2*d*\tan(b*x+a)) + \frac{1}{4}I/b \operatorname{dilog}(1/2*I*(-I*d+d*\tan(b*x+a))/d) + \frac{1}{4}I/b * \ln(I*d+d*\tan(b*x+a)) * \ln(1/2*I*(-I*d+d*\tan(b*x+a))/d) - \frac{1}{4}I/b \operatorname{dilog}((2-I*d+d*\tan(b*x+a))/(-2*I*d+2)) - \frac{1}{4}I/b \ln(I*d+d*\tan(b*x+a)) * \ln((2-I*d+d*\tan(b*x+a))/(-2*I*d+2))$

**maxima** [B] time = 0.43, size = 265, normalized size = 2.85

$$4(bx+a)d \left( \frac{\log(d \tan(bx+a) - id+2)}{d} - \frac{\log(\tan(bx+a) - i)}{d} \right) - d \left( \frac{2i \left( \log(d \tan(bx+a) - id+2) \log\left(-\frac{id \tan(bx+a) + d + 2i}{2d+2i} + 1\right) + \operatorname{Li}_2\left(\frac{id \tan(bx+a) + d + 2i}{2d+2i}\right) \right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")`

[Out]  $-\frac{1}{8}*(4*(b*x+a)*d*(\log(d*\tan(b*x+a)-I*d+2)/d - \log(\tan(b*x+a)-I)/d) - d*(2*I*(\log(d*\tan(b*x+a)-I*d+2)*\log(-(I*d*\tan(b*x+a)+d+2*I)/(2*d+2*I)+1) + \operatorname{dilog}((I*d*\tan(b*x+a)+d+2*I)/(2*d+2*I)))/d - (2*I*\log(d*\tan(b*x+a)-I*d+2)*\log(\tan(b*x+a)-I) - I*\log(\tan(b*x+a)-I)^2)/d + 2*I*(\log(1/2*d*\tan(b*x+a)-1/2*I*d+1)*\log(\tan(b*x+a)-I) + \operatorname{dilog}(-1/2*d*\tan(b*x+a)+1/2*I*d))/d - 2*I*(\log(\tan(b*x+a)-I)*\log(-1/2*I*\tan(b*x+a)+1/2) + \operatorname{dilog}(1/2*I*\tan(b*x+a)+1/2))/d) - 8*(b*x+a)*\operatorname{arccoth}(d*\tan(b*x+a)-I*d+1))/b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(d \tan(a + bx) - d1i + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(d*tan(a + b*x) - d*1i + 1),x)`

[Out] `int(acoth(d*tan(a + b*x) - d*1i + 1), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(d \tan(a + bx) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(acoth(d*tan(a + b*x) - I*d + 1), x)
```

$$3.243 \quad \int \frac{\coth^{-1}(1-id+d \tan(a+bx))}{x} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\coth^{-1}(d \tan(a+bx) - id + 1)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x)

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(1-id+d \tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1-id+d \tan(a+bx))}{x} dx = \int \frac{\coth^{-1}(1-id+d \tan(a+bx))}{x} dx$$

**Mathematica [A]** time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1-id+d \tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 - I\*d + d\*Tan[a + b\*x]]/x, x]

**fricas [A]** time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{((d+i)e^{2ibx+2ia}+i)e^{-2ibx-2ia}}{d}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(1/2\*log(((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) + I)\*e^(-2\*I\*b\*x - 2\*I\*a)/d)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}\left(\frac{d \tan(bx + a) - id + 1}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*tan(b\*x + a) - I\*d + 1)/x, x)

**maple** [A] time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}\left(\frac{1 - id + d \tan(bx + a)}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x)

[Out] int(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-ibx + \frac{1}{4}(-i\pi - 4ia - 2\log(d))\log(x) - \frac{1}{2}i \int \frac{\arctan\left(\frac{-d \cos(2bx + 2a) + \sin(2bx + 2a)}{-d \sin(2bx + 2a) - \cos(2bx + 2a) - 1}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d+d\*tan(b\*x+a))/x,x, algorithm="maxima")

[Out] -I\*b\*x + 1/4\*(-I\*pi - 4\*I\*a - 2\*log(d))\*log(x) - 1/2\*I\*integrate(arctan2(-d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) - cos(2\*b\*x + 2\*a) - 1)/x, x) + 1/4\*integrate(log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 + 2\*d\*sin(2\*b\*x + 2\*a) + 2\*cos(2\*b\*x + 2\*a) + 1)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}\left(\frac{d \tan(a + bx) + 1 - d1i}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(d*tan(a + b*x) - d*I + 1)/x,x)`

[Out] `int(acoth(d*tan(a + b*x) - d*I + 1)/x, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(d \tan(a + bx) - id + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(1-I*d+d*tan(b*x+a))/x,x)`

[Out] `Integral(acoth(d*tan(a + b*x) - I*d + 1)/x, x)`

### 3.244 $\int x^2 \coth^{-1}(1 + id - d \tan(a + bx)) dx$

**Optimal.** Leaf size=171

$$-\frac{i\text{Li}_4\left(-\left(id+1\right)e^{2ia+2ibx}\right)}{8b^3}-\frac{x\text{Li}_3\left(-\left(id+1\right)e^{2ia+2ibx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-\left(id+1\right)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1+\left(1+id\right)e^{2ia+2ibx}\right)$$

[Out] 1/12\*I\*b\*x^4+1/3\*x^3\*arccoth(1+I\*d-d\*tan(b\*x+a))-1/6\*x^3\*ln(1+(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))+1/4\*I\*x^2\*polylog(2,-(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b-1/4\*x\*polylog(3,-(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^2-1/8\*I\*polylog(4,-(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^3

**Rubi [A]** time = 0.30, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6264, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,-\left(1+id\right)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-\left(1+id\right)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-\left(1+id\right)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1+\left(1+id\right)e^{2ia+2ibx}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]],x]

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/3 - (x^3\*Log[1 + (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/6 + ((I/4)\*x^2\*PolyLog[2, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - (x\*PolyLog[3, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/((4\*b^2) - ((I/8)\*PolyLog[4, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b^3

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6264

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_)^(m_
.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
+ 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 + id - d \tan(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) + \frac{1}{3} (ib) \int \frac{x^3}{1 + (1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{3} (b(i - d)) \int \frac{e^{2ia+2ibx}}{1 + (1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{6} x^3 \log(1 + (1 + id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 156, normalized size = 0.91

$$\frac{1}{3} x^3 \coth^{-1}(d(-\tan(a+bx))+id+1) - \frac{4b^3 x^3 \log\left(1 - \frac{ie^{-2i(a+bx)}}{d-i}\right) + 6ib^2 x^2 \operatorname{Li}_2\left(\frac{ie^{-2i(a+bx)}}{d-i}\right) + 6bx \operatorname{Li}_3\left(\frac{ie^{-2i(a+bx)}}{d-i}\right) - 3i \operatorname{Li}_4\left(\frac{ie^{-2i(a+bx)}}{d-i}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]], x]

[Out] (x^3\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/3 - (4\*b^3\*x^3\*Log[1 - I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + (6\*I)\*b^2\*x^2\*PolyLog[2, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + 6\*b\*x\*PolyLog[3, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] - (3\*I)\*PolyLog[4, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))])/(24\*b^3)

**fricas [C]** time = 0.67, size = 348, normalized size = 2.04

$$\frac{ib^4 x^4 - 2b^3 x^3 \log\left(\frac{de^{(2ibx+2ia)}}{(d-i)e^{(2ibx+2ia)-i}}\right) + 6ib^2 x^2 \operatorname{Li}_2\left(\frac{1}{2} \sqrt{-4id-4} e^{(ibx+ia)}\right) + 6ib^2 x^2 \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{-4id-4} e^{(ibx+ia)}\right) - i}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+I\*d-d\*tan(b\*x+a)), x, algorithm="fricas")

[Out] 1/12\*(I\*b^4\*x^4 - 2\*b^3\*x^3\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d - I)\*e^(2\*I\*b\*x + 2\*I\*a) - I)) + 6\*I\*b^2\*x^2\*dilog(1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) + 6



\*I\*b^2\*x^2\*dilog(-1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) - I\*a^4 + 2\*a^3\*log  
 (((2\*d - 2\*I)\*e^(I\*b\*x + I\*a) + I\*sqrt(-4\*I\*d - 4))/(2\*d - 2\*I)) + 2\*a^3\*log  
 (((2\*d - 2\*I)\*e^(I\*b\*x + I\*a) - I\*sqrt(-4\*I\*d - 4))/(2\*d - 2\*I)) - 12\*b\*x\*  
 polylog(3, 1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) - 12\*b\*x\*polylog(3, -1/2\*  
 sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) - 2\*(b^3\*x^3 + a^3)\*log(1/2\*sqrt(-4\*I\*d -  
 4)\*e^(I\*b\*x + I\*a) + 1) - 2\*(b^3\*x^3 + a^3)\*log(-1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*  
 b\*x + I\*a) + 1) - 12\*I\*polylog(4, 1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) - 1  
 2\*I\*polylog(4, -1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)))/b^3

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(-d \tan(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(-d\*tan(b\*x + a) + I\*d + 1), x)

maple [C] time = 5.93, size = 2449, normalized size = 14.32

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1+I\*d-d\*tan(b\*x+a)),x)

[Out] -1/3\*x^3\*ln(exp(I\*(b\*x+a)))+1/12\*I\*b\*x^4-1/12\*I\*x^3\*Pi\*csgn(I/(exp(2\*I\*(b\*x+a))+1))\*csgn(I\*exp(2\*I\*(b\*x+a))/(exp(2\*I\*(b\*x+a))+1))^2-1/12\*I\*x^3\*Pi\*csgn(I\*exp(2\*I\*(b\*x+a)))\*csgn(I\*exp(2\*I\*(b\*x+a))/(exp(2\*I\*(b\*x+a))+1))^2+1/12\*I\*x^3\*Pi\*csgn(I\*exp(2\*I\*(b\*x+a)))^3+1/12\*I\*x^3\*Pi\*csgn(I\*exp(2\*I\*(b\*x+a)))/(exp(2\*I\*(b\*x+a))+1)^3-1/6\*x^3\*ln(d)+1/12\*I\*x^3\*Pi\*csgn(I\*exp(I\*(b\*x+a)))^2\*csgn(I\*exp(2\*I\*(b\*x+a)))-1/6\*I\*x^3\*Pi\*csgn(I\*exp(I\*(b\*x+a)))\*csgn(I\*exp(2\*I\*(b\*x+a)))^2+1/3\*I/b^3/(I-d)\*ln(1-I\*(I-d)\*exp(2\*I\*(b\*x+a)))\*a^3-1/12\*I\*x^3\*Pi\*csgn((exp(2\*I\*(b\*x+a))\*d-I\*exp(2\*I\*(b\*x+a))-I)/(exp(2\*I\*(b\*x+a))+1))^2+1/6\*I/b^3\*a^3/(I-d)\*ln(I\*exp(2\*I\*(b\*x+a))-exp(2\*I\*(b\*x+a))\*d+I)+1/12\*I\*x^3\*Pi\*csgn(I\*d/(exp(2\*I\*(b\*x+a))+1)\*exp(2\*I\*(b\*x+a)))^3+1/12\*I\*x^3\*Pi\*csgn(I\*d/(exp(2\*I\*(b\*x+a))+1)\*exp(2\*I\*(b\*x+a)))\*csgn(d/(exp(2\*I\*(b\*x+a))+1)\*exp(2\*I\*(b\*x+a)))^2-1/2\*I/b^3\*a^3/(I-d)\*ln(1+I\*exp(I\*(b\*x+a))\*(-I\*(I-d))^(1/2))-1/2\*I/b^3\*a^3/(I-d)\*ln(1-I\*exp(I\*(b\*x+a))\*(-I\*(I-d))^(1/2))-1/12\*I\*x^3\*Pi\*csgn(d/(exp(2\*I\*(b\*x+a))+1)\*exp(2\*I\*(b\*x+a)))^3-1/12\*I\*x^3\*Pi\*csgn(I\*(exp(2\*I\*(b\*x+a))\*d-I\*exp(2\*I\*(b\*x+a))-I)/(exp(2\*I\*(b\*x+a))+1))^3-1/6\*I/(I-d)\*ln(1-I\*(I-d)\*exp(2\*I\*(b\*x+a)))\*x^3-1/2/b^3\*a^2/(I-d)\*dilog(1+I\*exp(I\*(b\*x+a))\*(-I\*(I-d))^(1/2))-1/2/b^3\*a^2/(I-d)\*dilog(1-I\*exp(I\*(b\*x+a))\*(-I\*(I-d))^(1/2))-1/4/b/(I-d)\*polylog(2,I\*(I-d)\*exp(2\*I\*(b\*x+a)))\*x^2+1/4/b^3/(I-d)\*polylog(2,I\*(I-d)\*exp(2\*I\*(b\*x+a)))\*a^2+1/6\*d/(I-d)\*ln(1-I\*(I-d)\*exp(2\*I\*(b\*x+a)))\*x^3

$$\begin{aligned}
& +1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/2/b^3*a^3* \\
& d/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/12*I*x^3*Pi*csgn(I*d)*csg \\
& n(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I \\
& *(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+ \\
& a)))^2-1/4*I/b^2/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))*x+1/8/b^3/(I-d)* \\
& polylog(4,I*(I-d)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*I* \\
& (b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a \\
& )))+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)+1/12*I*x^3*Pi*csgn( \\
& (exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/12*I*x^ \\
& 3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a \\
& ))+1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2* \\
& I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b \\
& *x+a))+1))^2+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I) \\
& /(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp( \\
& 2*I*(b*x+a))+1))^2+1/2/b^2*a^2*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/ \\
& 2))*x-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2 \\
& *I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b \\
& x+a))+1))+1/8*I/b^3*d/(I-d)*polylog(4,I*(I-d)*exp(2*I*(b*x+a)))+1/12*I*x^3* \\
& Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a \\
& ))-I)/(exp(2*I*(b*x+a))+1))^2+1/4/b^2*d/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b \\
& x+a)))*x+1/2/b^3*a^3*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/3/b^ \\
& 3*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^3-1/6/b^3*a^3*d/(I-d)*ln(I*exp(2 \\
& *I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1) \\
& )*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))-1/ \\
& 2/b^2*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x*a^2-1/2*I/b^2*a^2/(I-d)*ln(1 \\
& +I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x \\
& +a))*(-I*(I-d))^(1/2))*x-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I* \\
& (exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp \\
& (2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/2*I/b^3*a^2*d/(I-d)*dilog(1+I*exp( \\
& I*(b*x+a))*(-I*(I-d))^(1/2))-1/4*I/b*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x \\
& +a)))*x^2+1/2*I/b^2/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x*a^2-1/2*I/b^3*a^ \\
& 2*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/4*I/b^3*d/(I-d)*poly \\
& log(2,I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/2*I/b^2*a^2/(I-d)*ln(1-I*exp(I*(b*x+a \\
& ))*(-I*(I-d))^(1/2))*x
\end{aligned}$$

**maxima [B]** time = 0.35, size = 340, normalized size = 1.99

$$\frac{12((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2)\operatorname{arccoth}(d\tan(bx+a)-id-1)}{b^2} + \frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2+(8i(bx+a)^3-18i(bx+a)^2a+18i(bx+a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="maxima")

[Out] -1/36\*(12\*((b\*x + a)^3 - 3\*(b\*x + a)^2\*a + 3\*(b\*x + a)\*a^2)\*arccoth(d\*tan(b\*x + a) - I\*d - 1)/b^2 + (-3\*I\*(b\*x + a)^4 + 12\*I\*(b\*x + a)^3\*a - 18\*I\*(b\*x

$+ a)^2 a^2 + (8I(bx + a)^3 - 18I(bx + a)^2 a + 18I(bx + a)a^2) a$   
 $\operatorname{rctan2}(d \cos(2bx + 2a) + \sin(2bx + 2a), -d \sin(2bx + 2a) + \cos(2bx$   
 $*x + 2a) + 1) + (-12I(bx + a)^2 + 18I(bx + a)a - 9Ia^2) \operatorname{dilog}((-I$   
 $*d - 1)e^{(2Ibx + 2Ia)}) + (4(bx + a)^3 - 9(bx + a)^2 a + 9(bx +$   
 $a)a^2) \log((d^2 + 1) \cos(2bx + 2a)^2 + (d^2 + 1) \sin(2bx + 2a)^2 - 2$   
 $*d \sin(2bx + 2a) + 2 \cos(2bx + 2a) + 1) + 3(4bx + a) \operatorname{polylog}(3, (-$   
 $I*d - 1)e^{(2Ibx + 2Ia)}) + 6I \operatorname{polylog}(4, (-I*d - 1)e^{(2Ibx + 2I*$   
 $a)))/b^2)/b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(1 - d \tan(a + bx) + d1i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(d*1i - d*tan(a + b*x) + 1), x)`

[Out] `int(x^2*acoth(d*1i - d*tan(a + b*x) + 1), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(-d \tan(a + bx) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(1+I*d-d*tan(b*x+a)), x)`

[Out] `Integral(x**2*acoth(-d*tan(a + b*x) + I*d + 1), x)`

### 3.245 $\int x \coth^{-1}(1 + id - d \tan(a + bx)) dx$

Optimal. Leaf size=134

$$-\frac{\operatorname{Li}_3\left(-\left(id+1\right)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\operatorname{Li}_2\left(-\left(id+1\right)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 + \left(id+1\right)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}\left(d - \tan(a+bx)\right)$$

[Out] 1/6\*I\*b\*x^3+1/2\*x^2\*arccoth(1+I\*d-d\*tan(b\*x+a))-1/4\*x^2\*ln(1+(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))+1/4\*I\*x\*polylog(2,-(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b-1/8\*polylog(3,-(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^2

**Rubi [A]** time = 0.25, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {6264, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\operatorname{PolyLog}\left(3, -\left(1 + id\right)e^{2ia+2ibx}\right)}{8b^2} + \frac{ix\operatorname{PolyLog}\left(2, -\left(1 + id\right)e^{2ia+2ibx}\right)}{4b} - \frac{1}{4}x^2 \log\left(1 + \left(id+1\right)e^{2ia+2ibx}\right) + \frac{1}{2}x^2 \coth^{-1}\left(d - \tan(a+bx)\right)$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]],x]

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/2 - (x^2\*Log[1 + (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))])/b - PolyLog[3, -((1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))]/(8\*b^2)

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6264

```
Int[ArcCoth[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.)
, x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Tan[a + b*x]])/(f*(m
+ 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(1 + id - d \tan(a + bx)) dx &= \frac{1}{2} x^2 \coth^{-1}(1 + id - d \tan(a + bx)) + \frac{1}{2} (ib) \int \frac{x^2}{1 + (1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{2} (b(i - d)) \int \frac{e^{2ia+2ibx}}{1 + (1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{4} x^2 \log(1 + (1 + id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 120, normalized size = 0.90

$$\frac{1}{2}x^2 \coth^{-1}(d(-\tan(a+bx))+id+1) - \frac{2b^2x^2 \log\left(1 - \frac{ie^{-2i(a+bx)}}{d-i}\right) + 2ibx \operatorname{Li}_2\left(\frac{ie^{-2i(a+bx)}}{d-i}\right) + \operatorname{Li}_3\left(\frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]], x]

[Out] (x^2\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]])/2 - (2\*b^2\*x^2\*Log[1 - I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + (2\*I)\*b\*x\*PolyLog[2, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + PolyLog[3, I/((-I + d)\*E^((2\*I)\*(a + b\*x)))])/(8\*b^2)

**fricas [C]** time = 0.74, size = 296, normalized size = 2.21

$$\frac{2i b^3 x^3 - 3 b^2 x^2 \log\left(\frac{d e^{2i b x + 2i a}}{(d-i)e^{2i b x + 2i a} - i}\right) + 2i a^3 + 6i b x \operatorname{Li}_2\left(\frac{1}{2} \sqrt{-4i d - 4} e^{i b x + i a}\right) + 6i b x \operatorname{Li}_2\left(-\frac{1}{2} \sqrt{-4i d - 4} e^{i b x + i a}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d-d\*tan(b\*x+a)), x, algorithm="fricas")

[Out] 1/12\*(2\*I\*b^3\*x^3 - 3\*b^2\*x^2\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d - I)\*e^(2\*I\*b\*x + 2\*I\*a) - I)) + 2\*I\*a^3 + 6\*I\*b\*x\*dilog(1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) + 6\*I\*b\*x\*dilog(-1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) - 3\*a^2\*log(((2\*d - 2\*I)\*e^(I\*b\*x + I\*a) + I\*sqrt(-4\*I\*d - 4))/(2\*d - 2\*I)) - 3\*a^2\*log(((2\*d - 2\*I)\*e^(I\*b\*x + I\*a) - I\*sqrt(-4\*I\*d - 4))/(2\*d - 2\*I)) - 3\*(b^2\*x^2 - a^2)\*log(1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) - 3\*(b^2\*x^2 - a^2)\*log(-1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) - 6\*polylog(3, 1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) - 6\*polylog(3, -1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)))/b^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(-d \tan(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d-d\*tan(b\*x+a)), x, algorithm="giac")

[Out] integrate(x\*arccoth(-d\*tan(b\*x + a) + I\*d + 1), x)

**maple [C]** time = 4.91, size = 2351, normalized size = 17.54

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x \cdot \text{arccoth}(1 + I \cdot d - d \cdot \tan(b \cdot x + a)), x)$

[Out]  $\frac{1}{6} I b x^3 + \frac{1}{8} I x^2 \pi \text{csgn}(I \exp(2 I (b x + a)))^3 - \frac{1}{4} x^2 \ln(d) - \frac{1}{8} I x^2 \pi \text{csgn}(I \exp(2 I (b x + a)) / (\exp(2 I (b x + a)) + 1)) \text{csgn}(I d / (\exp(2 I (b x + a)) + 1)) \exp(2 I (b x + a))^{2 - 1/4} I / b^2 / (I - d) \ln(1 - I (I - d) \exp(2 I (b x + a))) a^2 + \frac{1}{8} I x^2 \pi \text{csgn}(I d / (\exp(2 I (b x + a)) + 1)) \exp(2 I (b x + a)) \text{csgn}(d / (\exp(2 I (b x + a)) + 1)) \exp(2 I (b x + a)) - \frac{1}{8} I x^2 \pi \text{csgn}(I (\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1))^{3 - 1/8} I x^2 \pi \text{csgn}((\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1))^{2 + 1/4} d / (I - d) \ln(1 - I (I - d) \exp(2 I (b x + a))) x^2 - \frac{1}{8} I x^2 \pi \text{csgn}(I \exp(2 I (b x + a))) \text{csgn}(I \exp(2 I (b x + a)) / (\exp(2 I (b x + a)) + 1))^{2 - 1/8} I x^2 \pi \text{csgn}(I d) \text{csgn}(I d / (\exp(2 I (b x + a)) + 1)) \exp(2 I (b x + a))^{2 - 1/2} I / b^2 a^2 d / (I - d) \ln(1 + I \exp(I (b x + a))) * (-I (I - d))^{(1/2)} - \frac{1}{2} x^2 \ln(\exp(I (b x + a))) + \frac{1}{2} I / b^2 a d / (I - d) \text{dilog}(1 + I \exp(I (b x + a))) * (-I (I - d))^{(1/2)} - \frac{1}{4} I / b d / (I - d) \text{polylog}(2, I (I - d) \exp(2 I (b x + a))) * x + \frac{1}{8} I x^2 \pi \text{csgn}((\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1))^{3 + 1/2} I / b d / (I - d) \ln(1 - I (I - d) \exp(2 I (b x + a))) * x a + \frac{1}{2} I / b^2 a / (I - d) \text{dilog}(1 + I \exp(I (b x + a))) * (-I (I - d))^{(1/2)} + \frac{1}{8} I x^2 \pi \text{csgn}(I d / (\exp(2 I (b x + a)) + 1)) \exp(2 I (b x + a))^{3 + 1/2} I / b^2 a / (I - d) \text{dilog}(1 - I \exp(I (b x + a))) * (-I (I - d))^{(1/2)} - \frac{1}{4} I / b / (I - d) \text{polylog}(2, I (I - d) \exp(2 I (b x + a))) * x - \frac{1}{8} I / b^2 / (I - d) \text{polylog}(3, I (I - d) \exp(2 I (b x + a))) - \frac{1}{4} I / (I - d) \ln(1 - I (I - d) \exp(2 I (b x + a))) * x^2 - \frac{1}{8} I x^2 \pi \text{csgn}(I / (\exp(2 I (b x + a)) + 1)) \text{csgn}(I (\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I)) \text{csgn}(I (\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1)) + \frac{1}{4} x^2 \ln(\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) + \frac{1}{8} I x^2 \pi \text{csgn}(I \exp(2 I (b x + a)) / (\exp(2 I (b x + a)) + 1))^{3 + 1/8} I / b^2 d / (I - d) \text{polylog}(3, I (I - d) \exp(2 I (b x + a))) - \frac{1}{4} I / b^2 / (I - d) \text{polylog}(2, I (I - d) \exp(2 I (b x + a))) a - \frac{1}{8} I x^2 \pi \text{csgn}(I d / (\exp(2 I (b x + a)) + 1)) \exp(2 I (b x + a)) \text{csgn}(d / (\exp(2 I (b x + a)) + 1)) \exp(2 I (b x + a))^{2 + 1/8} I x^2 \pi \text{csgn}(I (\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1)) \text{csgn}((\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1))^{2 - 1/8} I x^2 \pi \text{csgn}(I (\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1)) \text{csgn}((\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1)) - \frac{1}{2} I / b a d / (I - d) \ln(1 + I \exp(I (b x + a))) * (-I (I - d))^{(1/2)} * x - \frac{1}{2} I / b a d / (I - d) \ln(1 - I \exp(I (b x + a))) * (-I (I - d))^{(1/2)} * x + \frac{1}{8} I x^2 \pi \text{csgn}(I \exp(I (b x + a)))^{2} \text{csgn}(I \exp(2 I (b x + a))) - \frac{1}{4} I x^2 \pi \text{csgn}(I \exp(I (b x + a))) \text{csgn}(I \exp(2 I (b x + a)))^{2 + 1/8} I x^2 \pi \text{csgn}(I / (\exp(2 I (b x + a)) + 1)) \text{csgn}(I (\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1))^{2 - 1/8} I x^2 \pi \text{csgn}(I / (\exp(2 I (b x + a)) + 1)) \text{csgn}(I \exp(2 I (b x + a)) / (\exp(2 I (b x + a)) + 1))^{2 + 1/8} I x^2 \pi \text{csgn}(I (\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1)) \text{csgn}(I (\exp(2 I (b x + a)) d - I \exp(2 I (b x + a)) - I) / (\exp(2 I (b x + a)) + 1))^{2 + 1/8} I x^2 \pi \text{csgn}(d / (\exp(2 I (b x + a)) + 1)) \exp(2 I (b x + a))^{2 - 1/2} I / b^2 a^2 d / (I - d) \ln(1 - I \exp(I (b x + a))) * (-I (I - d))^{(1/2)} + \frac{1}{4} I / b^2 d / (I - d) \ln(1 - I (I - d) \exp(2 I (b x + a))) a^2 + \frac{1}{4} I / b^2 a^2 d / (I - d) \ln(I \exp(2 I (b x + a)) - \exp(2 I (b x + a)) d + I) + \frac{1}{2} I / b^2 a^2 / (I - d) \ln(1 + I \exp(I (b x + a))) * (-I (I - d))^{(1/2)} + \frac{1}{2} I / b^2 a^2 / (I - d) \ln($

$1 - I \cdot \exp(I \cdot (b \cdot x + a)) \cdot (-I \cdot (I - d))^{(1/2)} - 1/4 \cdot I / b^2 \cdot a^2 / (I - d) \cdot \ln(I \cdot \exp(2 \cdot I \cdot (b \cdot x + a)) - \exp(2 \cdot I \cdot (b \cdot x + a)) \cdot d + I) - 1/8 \cdot I \cdot x^2 \cdot \text{Pcsgn}(d / (\exp(2 \cdot I \cdot (b \cdot x + a)) + 1) \cdot \exp(2 \cdot I \cdot (b \cdot x + a)))^3 + 1/2 \cdot I / b \cdot a / (I - d) \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a)) \cdot (-I \cdot (I - d))^{(1/2)}) \cdot x + 1/2 \cdot I / b \cdot a / (I - d) \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a)) \cdot (-I \cdot (I - d))^{(1/2)}) \cdot x + 1/2 \cdot I / b^2 \cdot a \cdot d / (I - d) \cdot \text{dilog}(1 - I \cdot \exp(I \cdot (b \cdot x + a)) \cdot (-I \cdot (I - d))^{(1/2)}) - 1/4 \cdot I / b^2 \cdot d / (I - d) \cdot \text{polylog}(2, I \cdot (I - d) \cdot \exp(2 \cdot I \cdot (b \cdot x + a))) \cdot a - 1/2 \cdot I / b / (I - d) \cdot \ln(1 - I \cdot (I - d) \cdot \exp(2 \cdot I \cdot (b \cdot x + a))) \cdot x \cdot a + 1/8 \cdot I \cdot x^2 \cdot \text{Pcsgn}(I \cdot d) \cdot \text{csgn}(I \cdot \exp(2 \cdot I \cdot (b \cdot x + a)) / (\exp(2 \cdot I \cdot (b \cdot x + a)) + 1)) \cdot \text{csgn}(I \cdot d / (\exp(2 \cdot I \cdot (b \cdot x + a)) + 1) \cdot \exp(2 \cdot I \cdot (b \cdot x + a))) + 1/8 \cdot I \cdot x^2 \cdot \text{Pcsgn}(I / (\exp(2 \cdot I \cdot (b \cdot x + a)) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot I \cdot (b \cdot x + a))) \cdot \text{csgn}(I \cdot \exp(2 \cdot I \cdot (b \cdot x + a)) / (\exp(2 \cdot I \cdot (b \cdot x + a)) + 1))$

**maxima** [B] time = 0.34, size = 246, normalized size = 1.84

$$\frac{12((bx+a)^2 - 2(bx+a)a) \operatorname{arccoth}(d \tan(bx+a) - id - 1)}{b} + \frac{-4i(bx+a)^3 + 12i(bx+a)^2 a - 6i b x \operatorname{Li}_2((-id-1)e^{2i(bx+2ia)}) + (6i(bx+a)^2 - 12i(bx+a)a) \operatorname{arctan}(d \tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/24 \cdot (12 \cdot ((b \cdot x + a)^2 - 2 \cdot (b \cdot x + a) \cdot a) \cdot \operatorname{arccoth}(d \cdot \tan(b \cdot x + a) - I \cdot d - 1) / b + (-4 \cdot I \cdot (b \cdot x + a)^3 + 12 \cdot I \cdot (b \cdot x + a)^2 \cdot a - 6 \cdot I \cdot b \cdot x \cdot \operatorname{dilog}((-I \cdot d - 1) \cdot e^{(2 \cdot I \cdot b \cdot x + 2 \cdot I \cdot a)}) + (6 \cdot I \cdot (b \cdot x + a)^2 - 12 \cdot I \cdot (b \cdot x + a) \cdot a) \cdot \operatorname{arctan}2(d \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + \sin(2 \cdot b \cdot x + 2 \cdot a), -d \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) + \cos(2 \cdot b \cdot x + 2 \cdot a) + 1) + 3 \cdot ((b \cdot x + a)^2 - 2 \cdot (b \cdot x + a) \cdot a) \cdot \log((d^2 + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a)^2 + (d^2 + 1) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a)^2 - 2 \cdot d \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) + 2 \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + 1) + 3 \cdot \operatorname{polylog}(3, (-I \cdot d - 1) \cdot e^{(2 \cdot I \cdot b \cdot x + 2 \cdot I \cdot a)})) / b) / b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(1 - d \tan(a + b x) + d i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(d\*i - d\*tan(a + b\*x) + 1),x)

[Out] int(x\*acoth(d\*i - d\*tan(a + b\*x) + 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(-d \tan(a + b x) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1+I\*d-d\*tan(b\*x+a)),x)

[Out] Integral(x\*acoth(-d\*tan(a + b\*x) + I\*d + 1), x)



### 3.246 $\int \coth^{-1}(1 + id - d \tan(a + bx)) dx$

Optimal. Leaf size=94

$$\frac{i\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b} - \frac{1}{2}x \log\left(1 + (1+id)e^{2ia+2ibx}\right) + x \coth^{-1}(d(-\tan(a+bx))+id+1) + \frac{1}{2}ibx^2$$

[Out]  $1/2*I*b*x^2 + x*\text{arccoth}(1+I*d-d*\tan(b*x+a)) - 1/2*x*\ln(1+(1+I*d)*\exp(2*I*a+2*I*b*x)) + 1/4*I*\text{polylog}(2, -(1+I*d)*\exp(2*I*a+2*I*b*x))/b$

**Rubi [A]** time = 0.16, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6256, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2, -(1+id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 + (1+id)e^{2ia+2ibx}\right) + x \coth^{-1}(d(-\tan(a+bx))+id+1) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[1 + I*d - d*Tan[a + b*x]], x]`

[Out]  $(I/2)*b*x^2 + x*\text{ArcCoth}[1 + I*d - d*\text{Tan}[a + b*x]] - (x*\text{Log}[1 + (1 + I*d)*E^{((2*I)*a + (2*I)*b*x)}])/2 + ((I/4)*\text{PolyLog}[2, -((1 + I*d)*E^{((2*I)*a + (2*I)*b*x)})])/b$

#### Rule 2184

`Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2279

`Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2391

$\text{Int}[\text{Log}[(c\_.)*((d\_.) + (e\_.)*(x\_.)^{(n\_.)})]/(x\_.), x\_Symbol] \text{ :> } -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] \text{ /; FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

Rule 6256

$\text{Int}[\text{ArcCoth}[(c\_.) + (d\_.)*\text{Tan}[(a\_.) + (b\_.)*(x\_.)]], x\_Symbol] \text{ :> } \text{Simp}[x*\text{ArcCoth}[c + d*\text{Tan}[a + b*x]], x] + \text{Dist}[I*b, \text{Int}[x/(c + I*d + c*E^{(2*I*a + 2*I*b*x)})], x], x] \text{ /; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[(c + I*d)^2, 1]$

Rubi steps

$$\begin{aligned} \int \coth^{-1}(1 + id - d \tan(a + bx)) dx &= x \coth^{-1}(1 + id - d \tan(a + bx)) + (ib) \int \frac{x}{1 + (1 + id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id - d \tan(a + bx)) - (b(i - d)) \int \frac{e^{2ia+2ibx} x}{1 + (1 + id)e^{2ia+2ibx}} dx \\ &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{2}x \log(1 + (1 + id)e^{2ia+2ibx}) + \\ &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{2}x \log(1 + (1 + id)e^{2ia+2ibx}) - \\ &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id - d \tan(a + bx)) - \frac{1}{2}x \log(1 + (1 + id)e^{2ia+2ibx}) + \end{aligned}$$

**Mathematica [B]** time = 3.24, size = 723, normalized size = 7.69

$$x \coth^{-1}(d(-\tan(a+bx))+id+1) - \frac{x \sec(a+bx)(\cos(bx) + i \sin(bx))(\sin(bx) + i \cos(bx)) \left( -\text{Li}_2 \left( \frac{1}{2}(\cos(a) + i \sin(a)) \right) \right)}{(\tan(a+bx) - i)(id \sin(a+bx) + (d - 2i) \cos(a+bx))} \left( -\frac{\sec^2(bx) \log \left( \frac{\sec(bx) - d \sin(bx)}{2 \cos(bx)} \right)}{\tan(bx)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]], x]

[Out] x\*ArcCoth[1 + I\*d - d\*Tan[a + b\*x]] - (x\*((-2\*I)\*b\*x\*Log[2\*Cos[b\*x]\*(Cos[b\*x] - I\*Sin[b\*x])] + Log[(Sec[b\*x]\*(Cos[a] - I\*Sin[a])\*((-2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x])]/(2\*(-I + d))]\*Log[1 - I\*Tan[b\*x]] - Log[(Sec[b\*x]\*((2 + I\*d)\*Cos[a + b\*x] - d\*Sin[a + b\*x])]/(2\*Cos[a] - (2\*I)\*Sin[a])]\*Log[1 + I\*Tan[b\*x]] + PolyLog[2, -Cos[2\*b\*x] + I\*Sin[2\*b\*x]] + PolyLog[2, (Sec[b\*x]\*(d\*Cos[a] + (2 + I\*d)\*Sin[a])\*(Cos[a + b\*x] - I\*Sin[a + b\*x]))]/(2\*(-I +

d))] - PolyLog[2, ((Cos[a] + I\*Sin[a])\*(d\*Cos[a] + (2 + I\*d)\*Sin[a]))\*(-I + Tan[b\*x]))/2])\*Sec[a + b\*x]\*(Cos[b\*x] + I\*Sin[b\*x])\*(I\*Cos[b\*x] + Sin[b\*x])/((( -2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x]))\*(I\*Log[1 - I\*Tan[b\*x]]\*Sec[b\*x]\*(d\*Cos[a] + (2 + I\*d)\*Sin[a]))/((-2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x]) + (Log[1 + I\*Tan[b\*x]]\*Sec[b\*x]\*((-I)\*d\*Cos[a] + (-2\*I + d)\*Sin[a]))/((-2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x]) - (Log[(Sec[b\*x]\*((2 + I\*d)\*Cos[a + b\*x] - d\*Sin[a + b\*x]))/(2\*Cos[a] - (2\*I)\*Sin[a])]\*Sec[b\*x]^2)/(-I + Tan[b\*x]) + (Log[1 - ((Cos[a] + I\*Sin[a])\*(d\*Cos[a] + (2 + I\*d)\*Sin[a]))\*(-I + Tan[b\*x]))/2])\*Sec[b\*x]^2)/(-I + Tan[b\*x]) - Log[1 - (Sec[b\*x]\*(d\*Cos[a] + (2 + I\*d)\*Sin[a])\*(Cos[a + b\*x] - I\*Sin[a + b\*x]))/(2\*(-I + d))]\*(-I + Tan[b\*x]) + (Log[(Sec[b\*x]\*(Cos[a] - I\*Sin[a])\*((-2\*I + d)\*Cos[a + b\*x] + I\*d\*Sin[a + b\*x]))/(2\*(-I + d))]\*Sec[b\*x]^2)/(I + Tan[b\*x]) + (2\*I)\*b\*x\*(I + Tan[b\*x]))\*(-I + Tan[a + b\*x]))

**fricas [B]** time = 0.48, size = 222, normalized size = 2.36

$$i b^2 x^2 - b x \log\left(\frac{d e^{2i b x + 2i a}}{(d-i)e^{2i b x + 2i a} - i}\right) - i a^2 - (b x + a) \log\left(\frac{1}{2} \sqrt{-4i d - 4} e^{i b x + i a} + 1\right) - (b x + a) \log\left(-\frac{1}{2} \sqrt{-4i d - 4} e^{i b x + i a} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="fricas")

[Out] 1/2\*(I\*b^2\*x^2 - b\*x\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d - I)\*e^(2\*I\*b\*x + 2\*I\*a) - I)) - I\*a^2 - (b\*x + a)\*log(1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) - (b\*x + a)\*log(-1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a) + 1) + a\*log(((2\*d - 2\*I)\*e^(I\*b\*x + I\*a) + I\*sqrt(-4\*I\*d - 4))/(2\*d - 2\*I)) + a\*log(((2\*d - 2\*I)\*e^(I\*b\*x + I\*a) - I\*sqrt(-4\*I\*d - 4))/(2\*d - 2\*I)) + I\*dilog(1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)) + I\*dilog(-1/2\*sqrt(-4\*I\*d - 4)\*e^(I\*b\*x + I\*a)))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(-d \tan(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(-d\*tan(b\*x + a) + I\*d + 1), x)

**maple [B]** time = 0.51, size = 297, normalized size = 3.16

$$\frac{i \operatorname{arccoth}(1 + id - d \tan(bx + a)) \ln(id + d \tan(bx + a))}{2b} - \frac{i \operatorname{arccoth}(1 + id - d \tan(bx + a)) \ln(id - d \tan(bx + a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+I\*d-d\*tan(b\*x+a)),x)

[Out]  $\frac{1}{2} \frac{I}{b} \operatorname{arccoth}(1+I*d-d*\tan(b*x+a)) * \ln(I*d+d*\tan(b*x+a)) - \frac{1}{2} \frac{I}{b} \operatorname{arccoth}(1+I*d-d*\tan(b*x+a)) * \ln(I*d-d*\tan(b*x+a)) - \frac{1}{8} \frac{I}{b} \ln(I*d-d*\tan(b*x+a))^2 + \frac{1}{4} \frac{I}{b} \operatorname{dilog}(1+\frac{1}{2} \frac{I*d-1/2*d*\tan(b*x+a)}{d}) + \frac{1}{4} \frac{I}{b} \ln(I*d-d*\tan(b*x+a)) * \ln(1+\frac{1}{2} \frac{I*d-1/2*d*\tan(b*x+a)}{d}) - \frac{1}{4} \frac{I}{b} \operatorname{dilog}(\frac{-2-I*d+d*\tan(b*x+a)}{-2*I*d-2}) - \frac{1}{4} \frac{I}{b} \ln(I*d+d*\tan(b*x+a)) * \ln(\frac{-2-I*d+d*\tan(b*x+a)}{-2*I*d-2}) + \frac{1}{4} \frac{I}{b} \operatorname{dilog}(\frac{1/2*I*(-I*d+d*\tan(b*x+a))/d}{d}) + \frac{1}{4} \frac{I}{b} \ln(I*d+d*\tan(b*x+a)) * \ln(\frac{1/2*I*(-I*d+d*\tan(b*x+a))/d}{d})$

**maxima** [B] time = 0.42, size = 263, normalized size = 2.80

$$4(bx+a)d \left( \frac{\log(d \tan(bx+a)-id-2)}{d} - \frac{\log(\tan(bx+a)-i)}{d} \right) + d \left( -\frac{2i \left( \log(d \tan(bx+a)-id-2) \log\left(-\frac{id \tan(bx+a)+d-2i}{2d-2i}+1\right) \right) + \operatorname{Li}_2\left(\frac{id \tan(bx+a)+d-2i}{2d-2i}\right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a)),x, algorithm="maxima")

[Out]  $-\frac{1}{8} * (4 * (b*x + a) * d * (\log(d*\tan(b*x + a) - I*d - 2)/d - \log(\tan(b*x + a) - I)/d) + d * (-2*I * (\log(d*\tan(b*x + a) - I*d - 2) * \log(-\frac{I*d*\tan(b*x + a) + d - 2*I}{2*d - 2*I} + 1) + \operatorname{dilog}(\frac{I*d*\tan(b*x + a) + d - 2*I}{2*d - 2*I}))) / d + (2*I * \log(d*\tan(b*x + a) - I*d - 2) * \log(\tan(b*x + a) - I) - I * \log(\tan(b*x + a) - I)^2) / d - 2*I * (\log(-1/2*d*\tan(b*x + a) + 1/2*I*d + 1) * \log(\tan(b*x + a) - I) + \operatorname{dilog}(1/2*d*\tan(b*x + a) - 1/2*I*d)) / d + 2*I * (\log(\tan(b*x + a) - I) * \log(-1/2*I*\tan(b*x + a) + 1/2) + \operatorname{dilog}(1/2*I*\tan(b*x + a) + 1/2)) / d + 8 * (b*x + a) * \operatorname{arccoth}(d*\tan(b*x + a) - I*d - 1)) / b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(1 - d \tan(a + bx) + d i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(d\*i - d\*tan(a + b\*x) + 1),x)

[Out] int(acoth(d\*i - d\*tan(a + b\*x) + 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(-d \tan(a + bx) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1+I\*d-d\*tan(b\*x+a)),x)

[Out] Integral(acoth(-d\*tan(a + b\*x) + I\*d + 1), x)

$$3.247 \quad \int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\coth^{-1}(d(-\tan(a+bx))+id+1)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x)

**Rubi** [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx = \int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$$

**Mathematica** [A] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1+id-d \tan(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 + I\*d - d\*Tan[a + b\*x]]/x, x]

**fricas** [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(\frac{de^{2ibx+2ia}}{(d-i)e^{2ibx+2ia}-i}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(-1/2\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d - I)\*e^(2\*I\*b\*x + 2\*I\*a) - I))/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}\left(\frac{-d \tan(bx + a) + id + 1}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(-d\*tan(b\*x + a) + I\*d + 1)/x, x)

**maple** [A] time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}\left(\frac{1 + id - d \tan(bx + a)}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x)

[Out] int(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-ibx + \frac{1}{4}(-i\pi - 4ia - 2\log(d))\log(x) + \frac{1}{2}i \int \frac{\arctan(d \cos(2bx + 2a) + \sin(2bx + 2a), -d \sin(2bx + 2a) + \cos(2bx + 2a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d-d\*tan(b\*x+a))/x,x, algorithm="maxima")

[Out] -I\*b\*x + 1/4\*(-I\*pi - 4\*I\*a - 2\*log(d))\*log(x) + 1/2\*I\*integrate(arctan2(d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) + cos(2\*b\*x + 2\*a) + 1)/x, x) + 1/4\*integrate(log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 - 2\*d\*sin(2\*b\*x + 2\*a) + 2\*cos(2\*b\*x + 2\*a) + 1)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}\left(\frac{1 - d \tan(a + bx) + d i}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(d*1i - d*tan(a + b*x) + 1)/x,x)`

[Out] `int(acoth(d*1i - d*tan(a + b*x) + 1)/x, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(-d \tan(a + bx) + id + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(1+I*d-d*tan(b*x+a))/x,x)`

[Out] `Integral(acoth(-d*tan(a + b*x) + I*d + 1)/x, x)`

### 3.248 $\int (e + fx)^3 \coth^{-1}(\cot(a + bx)) dx$

Optimal. Leaf size=302

$$-\frac{3f^3 \operatorname{Li}_5(-ie^{2i(a+bx)})}{16b^4} + \frac{3f^3 \operatorname{Li}_5(ie^{2i(a+bx)})}{16b^4} + \frac{3if^2(e+fx) \operatorname{Li}_4(-ie^{2i(a+bx)})}{8b^3} - \frac{3if^2(e+fx) \operatorname{Li}_4(ie^{2i(a+bx)})}{8b^3} + \frac{3f(e+fx)^2}{8b^2}$$

[Out]  $\frac{1}{4}*(f*x+e)^4*\operatorname{arccoth}(\cot(b*x+a))/f + \frac{1}{4}*I*(f*x+e)^4*\operatorname{arctan}(\exp(2*I*(b*x+a)))/f - \frac{1}{4}*I*(f*x+e)^3*\operatorname{polylog}(2, -I*\exp(2*I*(b*x+a)))/b + \frac{1}{4}*I*(f*x+e)^3*\operatorname{polylog}(2, I*\exp(2*I*(b*x+a)))/b + \frac{3}{8}*f*(f*x+e)^2*\operatorname{polylog}(3, -I*\exp(2*I*(b*x+a)))/b^2 - \frac{3}{8}*f*(f*x+e)^2*\operatorname{polylog}(3, I*\exp(2*I*(b*x+a)))/b^2 + \frac{3}{8}*I*f^2*(f*x+e)*\operatorname{polylog}(4, -I*\exp(2*I*(b*x+a)))/b^3 - \frac{3}{8}*I*f^2*(f*x+e)*\operatorname{polylog}(4, I*\exp(2*I*(b*x+a)))/b^3 - \frac{3}{16}*f^3*\operatorname{polylog}(5, -I*\exp(2*I*(b*x+a)))/b^4 + \frac{3}{16}*f^3*\operatorname{polylog}(5, I*\exp(2*I*(b*x+a)))/b^4$

**Rubi [A]** time = 0.24, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6254, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\operatorname{PolyLog}(4, -ie^{2i(a+bx)})}{8b^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}(4, ie^{2i(a+bx)})}{8b^3} + \frac{3f(e+fx)^2\operatorname{PolyLog}(3, -ie^{2i(a+bx)})}{8b^2} - \frac{3f(e+fx)^2\operatorname{PolyLog}(3, ie^{2i(a+bx)})}{8b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^3*\operatorname{ArcCoth}[\operatorname{Cot}[a + b*x]], x]$

[Out]  $((e + f*x)^4*\operatorname{ArcCoth}[\operatorname{Cot}[a + b*x]])/(4*f) + ((I/4)*(e + f*x)^4*\operatorname{ArcTan}[E^{((2*I)*(a + b*x))}]/f - ((I/4)*(e + f*x)^3*\operatorname{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}])/b + ((I/4)*(e + f*x)^3*\operatorname{PolyLog}[2, I*E^{((2*I)*(a + b*x))}])/b + (3*f*(e + f*x)^2*\operatorname{PolyLog}[3, (-I)*E^{((2*I)*(a + b*x))}])/(8*b^2) - (3*f*(e + f*x)^2*\operatorname{PolyLog}[3, I*E^{((2*I)*(a + b*x))}])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*\operatorname{PolyLog}[4, (-I)*E^{((2*I)*(a + b*x))}])/b^3 - (((3*I)/8)*f^2*(e + f*x)*\operatorname{PolyLog}[4, I*E^{((2*I)*(a + b*x))}])/b^3 - (3*f^3*\operatorname{PolyLog}[5, (-I)*E^{((2*I)*(a + b*x))}])/(16*b^4) + (3*f^3*\operatorname{PolyLog}[5, I*E^{((2*I)*(a + b*x))}])/(16*b^4)$

#### Rule 2282

$\operatorname{Int}[u, x\_Symbol] := \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; \operatorname{FreeQ}[\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n] \&\& \operatorname{!MatchQ}[u, E^{((c_.)*((a_.) + (b_.)*x))}*(F_)[v_]] /; \operatorname{FreeQ}[\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531



```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x)
)], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 6254

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:= Simp[((e + f*x)^(m + 1)*ArcCoth[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
e, f}, x] && IGtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int (e + fx)^3 \coth^{-1}(\cot(a + bx)) dx &= \frac{(e + fx)^4 \coth^{-1}(\cot(a + bx))}{4f} - \frac{b \int (e + fx)^4 \sec(2a + 2bx) dx}{4f} \\
&= \frac{(e + fx)^4 \coth^{-1}(\cot(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} + \frac{1}{2} \int (e + fx)^3 \\
&= \frac{(e + fx)^4 \coth^{-1}(\cot(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\cot(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\cot(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\cot(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\cot(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2}{4b} \\
&= \frac{(e + fx)^4 \coth^{-1}(\cot(a + bx))}{4f} + \frac{i(e + fx)^4 \tan^{-1}(e^{2i(a+bx)})}{4f} - \frac{i(e + fx)^3 \text{Li}_2}{4b}
\end{aligned}$$

**Mathematica [B]** time = 0.31, size = 654, normalized size = 2.17

$$\frac{1}{4}x(4e^3 + 6e^2fx + 4ef^2x^2 + f^3x^3) \coth^{-1}(\cot(a+bx)) + \frac{-8b^4e^3x \log(1 - ie^{2i(a+bx)}) + 8b^4e^3x \log(1 + ie^{2i(a+bx)})}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^3\*ArcCoth[Cot[a + b\*x]],x]

[Out] (x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3)\*ArcCoth[Cot[a + b\*x]])/4 + (-8\*b^4\*e^3\*x\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 12\*b^4\*e^2\*f\*x^2\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 8\*b^4\*e\*f^2\*x^3\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 2\*b^4\*f^3\*x^4\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] + 8\*b^4\*e^3\*x\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 12\*b^4\*e^2\*f\*x^2\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 8\*b^4\*e\*f^2\*x^3\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 2\*b^4\*f^3\*x^4\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] - (4\*I)\*b^3\*(e + f\*x)^3\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + (4\*I)\*b^3\*(e + f\*x)^3\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))] + 6\*b^2\*e^2\*f\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 12\*b^2\*e\*f^2\*x\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 6\*b^2\*f^3\*x^2\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] - 6\*b^2\*e^2\*f\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 12\*b^2\*e\*f^2\*x\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 6\*b^2\*f^3\*x^2\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b\*e\*f^2\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b\*f^3\*x\*PolyLog[4, (-I)\*E^((2\*I)

$$*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))]/(16*b^4)$$

**fricas** [C] time = 0.80, size = 1566, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*arccoth(cot(b\*x+a)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/32*(3*f^3*polylog(5, I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - 3*f^3*polylog(5, I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*log((\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a) + 1)/(\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a) + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a) + 1) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a) + 1) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a) + 1) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-\cos(2*b*x + 2*a) + I*\sin(2*b*x + 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-\cos(2*b*x + 2*a) - I*\sin(2*b*x + 2*a) + I) - (-6*I*b*f^3*x - 6*I*b*e*f^2)*polylog(4, I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - (-6*I*b*f^3*x - 6*I*b*e*f^2)*polylog(4, I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - (6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, -I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - (6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, -I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) + 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, -I*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, -I*\cos(2*b*x + 2*a) - \sin(2*b*x + 2*a)) \end{aligned}$$

$*f^2*x + b^2*e^2*f)*polylog(3, -I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))/b^4$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)^3 \operatorname{arccoth}(\cot(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*arccoth(cot(b\*x+a)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*arccoth(cot(b\*x + a)), x)

**maple** [C] time = 49.15, size = 7429, normalized size = 24.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*arccoth(cot(b\*x+a)),x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16} (f^3x^4 + 4ef^2x^3 + 6e^2fx^2 + 4e^3x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{16} ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*arccoth(cot(b\*x+a)),x, algorithm="maxima")

[Out]  $1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*\log(2*\cos(2*b*x + 2*a)^2 + 2*\sin(2*b*x + 2*a)^2 + 4*\sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*\log(2*\cos(2*b*x + 2*a)^2 + 2*\sin(2*b*x + 2*a)^2 - 4*\sin(2*b*x + 2*a) + 2) - \operatorname{integrate}(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*\cos(4*b*x + 4*a)*\cos(2*b*x + 2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*\sin(4*b*x + 4*a)*\sin(2*b*x + 2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*\cos(2*b*x + 2*a))/(\cos(4*b*x + 4*a)^2 + \sin(4*b*x + 4*a)^2 + 2*\cos(4*b*x + 4*a) + 1), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acoth}(\cot(a + bx)) (e + fx)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(cot(a + b*x))*(e + f*x)^3,x)`

[Out] `int(acoth(cot(a + b*x))*(e + f*x)^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)^3 \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**3*acoth(cot(b*x+a)),x)`

[Out] `Integral((e + f*x)**3*acoth(cot(a + b*x)), x)`

### 3.249 $\int (e + fx)^2 \coth^{-1}(\cot(a + bx)) dx$

**Optimal.** Leaf size=234

$$\frac{if^2\text{Li}_4(-ie^{2i(a+bx)})}{8b^3} - \frac{if^2\text{Li}_4(ie^{2i(a+bx)})}{8b^3} + \frac{f(e+fx)\text{Li}_3(-ie^{2i(a+bx)})}{4b^2} - \frac{f(e+fx)\text{Li}_3(ie^{2i(a+bx)})}{4b^2} - \frac{i(e+fx)^2\text{Li}_2(-ie^{2i(a+bx)})}{4b}$$

[Out]  $1/3*(f*x+e)^3*\text{arccoth}(\cot(b*x+a))/f+1/3*I*(f*x+e)^3*\text{arctan}(\exp(2*I*(b*x+a)))/f-1/4*I*(f*x+e)^2*\text{polylog}(2,-I*\exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*\text{polylog}(2,I*\exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*\text{polylog}(3,-I*\exp(2*I*(b*x+a)))/b^2-1/4*f*(f*x+e)*\text{polylog}(3,I*\exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*\text{polylog}(4,-I*\exp(2*I*(b*x+a)))/b^3-1/8*I*f^2*\text{polylog}(4,I*\exp(2*I*(b*x+a)))/b^3$

**Rubi [A]** time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6254, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e+fx)\text{PolyLog}(3,-ie^{2i(a+bx)})}{4b^2} - \frac{f(e+fx)\text{PolyLog}(3,ie^{2i(a+bx)})}{4b^2} + \frac{if^2\text{PolyLog}(4,-ie^{2i(a+bx)})}{8b^3} - \frac{if^2\text{PolyLog}(4,ie^{2i(a+bx)})}{8b^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e + f*x)^2*\text{ArcCoth}[\text{Cot}[a + b*x]], x]$

[Out]  $((e + f*x)^3*\text{ArcCoth}[\text{Cot}[a + b*x]])/(3*f) + ((I/3)*(e + f*x)^3*\text{ArcTan}[E^{((2*I)*(a + b*x))}]/f - ((I/4)*(e + f*x)^2*\text{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}]/b + ((I/4)*(e + f*x)^2*\text{PolyLog}[2, I*E^{((2*I)*(a + b*x))}]/b + (f*(e + f*x)*\text{PolyLog}[3, (-I)*E^{((2*I)*(a + b*x))}]/(4*b^2) - (f*(e + f*x)*\text{PolyLog}[3, I*E^{((2*I)*(a + b*x))}]/(4*b^2) + ((I/8)*f^2*\text{PolyLog}[4, (-I)*E^{((2*I)*(a + b*x))}]/b^3 - ((I/8)*f^2*\text{PolyLog}[4, I*E^{((2*I)*(a + b*x))}]/b^3$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] \text{ :> With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_)})^{(m\_)} /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n]] \ \&\& \ !\text{MatchQ}[u, E^{(c\_)*((a\_)+(b\_)*x)}*(F\_)[v\_]] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\text{Int}[\text{Log}[1 + (e\_)*((F\_)^{(c\_)*((a\_)+(b\_)*(x\_)))^{(n\_)}]*(f\_)+(g\_)*(x\_)^{(m\_)}], x\_Symbol] \text{ :> -Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n)]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n)], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f$

, g, n}, x] && GtQ[m, 0]

### Rule 4181

Int[csc[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(I\*k\*Pi)\*E^(I\*(e + f\*x))])/f, x] + (-Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 - E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Log[1 + E^(I\*k\*Pi)\*E^(I\*(e + f\*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 6254

Int[ArcCoth[Cot[(a\_.) + (b\_.)\*(x\_)]]\*((e\_.) + (f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((e + f\*x)^(m + 1)\*ArcCoth[Cot[a + b\*x]])/(f\*(m + 1)), x] - Dist[b/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)\*Sec[2\*a + 2\*b\*x], x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int (e + fx)^2 \coth^{-1}(\cot(a + bx)) dx &= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} - \frac{b \int (e + fx)^3 \sec(2a + 2bx) dx}{3f} \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} + \frac{1}{2} \int (e + fx)^2 dx \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2}{4b} \\
&= \frac{(e + fx)^3 \coth^{-1}(\cot(a + bx))}{3f} + \frac{i(e + fx)^3 \tan^{-1}(e^{2i(a+bx)})}{3f} - \frac{i(e + fx)^2 \text{Li}_2}{4b}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 409, normalized size = 1.75

$$\frac{1}{3}x(3e^2 + 3efx + f^2x^2) \coth^{-1}(\cot(a+bx)) + \frac{-12b^3e^2x \log(1 - ie^{2i(a+bx)}) + 12b^3e^2x \log(1 + ie^{2i(a+bx)}) - 12b^3efx^2}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^2\*ArcCoth[Cot[a + b\*x]], x]

[Out] (x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2)\*ArcCoth[Cot[a + b\*x]])/3 + (-12\*b^3\*e^2\*x\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 12\*b^3\*e\*f\*x^2\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] - 4\*b^3\*f^2\*x^3\*Log[1 - I\*E^((2\*I)\*(a + b\*x))] + 12\*b^3\*e^2\*x\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 12\*b^3\*e\*f\*x^2\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] + 4\*b^3\*f^2\*x^3\*Log[1 + I\*E^((2\*I)\*(a + b\*x))] - (6\*I)\*b^2\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^((2\*I)\*(a + b\*x))] + (6\*I)\*b^2\*(e + f\*x)^2\*PolyLog[2, I\*E^((2\*I)\*(a + b\*x))] + 6\*b\*e\*f\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] + 6\*b\*f^2\*x\*PolyLog[3, (-I)\*E^((2\*I)\*(a + b\*x))] - 6\*b\*e\*f\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] - 6\*b\*f^2\*x\*PolyLog[3, I\*E^((2\*I)\*(a + b\*x))] + (3\*I)\*f^2\*PolyLog[4, (-I)\*E^((2\*I)\*(a + b\*x))] - (3\*I)\*f^2\*PolyLog[4, I\*E^((2\*I)\*(a + b\*x))])/(24\*b^3)

**fricas [C]** time = 0.72, size = 1080, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((f\*x+e)^2\*arccoth(cot(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{48}(-3I^2f^2 \text{polylog}(4, I\cos(2bx+2a) + \sin(2bx+2a)) - 3I^2f^2 \text{polylog}(4, I\cos(2bx+2a) - \sin(2bx+2a)) + 3I^2f^2 \text{polylog}(4, -I\cos(2bx+2a) + \sin(2bx+2a)) + 3I^2f^2 \text{polylog}(4, -I\cos(2bx+2a) - \sin(2bx+2a)) + (6I^2b^2f^2x^2 + 12I^2b^2efx + 6I^2b^2e^2) \text{dilog}(I\cos(2bx+2a) + \sin(2bx+2a)) + (6I^2b^2f^2x^2 + 12I^2b^2efx + 6I^2b^2e^2) \text{dilog}(I\cos(2bx+2a) - \sin(2bx+2a)) + (-6I^2b^2f^2x^2 - 12I^2b^2efx - 6I^2b^2e^2) \text{dilog}(-I\cos(2bx+2a) + \sin(2bx+2a)) + (-6I^2b^2f^2x^2 - 12I^2b^2efx - 6I^2b^2e^2) \text{dilog}(-I\cos(2bx+2a) - \sin(2bx+2a)) + 8(b^3f^2x^3 + 3b^3efx^2 + 3b^3e^2x) \log\left(\frac{\cos(2bx+2a) + \sin(2bx+2a) + 1}{\cos(2bx+2a) - \sin(2bx+2a) + 1}\right) + 4(3ab^2e^2 - 3a^2b^2ef + a^3f^2) \log(\cos(2bx+2a) + I\sin(2bx+2a) + I) - 4(3ab^2e^2 - 3a^2b^2ef + a^3f^2) \log(\cos(2bx+2a) - I\sin(2bx+2a) + I) - 4(b^3f^2x^3 + 3b^3efx^2 + 3b^3e^2x + 3ab^2e^2 - 3a^2b^2ef + a^3f^2) \log(I\cos(2bx+2a) + \sin(2bx+2a) + 1) + 4(b^3f^2x^3 + 3b^3efx^2 + 3b^3e^2x + 3ab^2e^2 - 3a^2b^2ef + a^3f^2) \log(I\cos(2bx+2a) - \sin(2bx+2a) + 1) - 4(b^3f^2x^3 + 3b^3efx^2 + 3b^3e^2x + 3ab^2e^2 - 3a^2b^2ef + a^3f^2) \log(-I\cos(2bx+2a) + \sin(2bx+2a) + 1) + 4(b^3f^2x^3 + 3b^3efx^2 + 3b^3e^2x + 3ab^2e^2 - 3a^2b^2ef + a^3f^2) \log(-I\cos(2bx+2a) - \sin(2bx+2a) + 1) + 4(3ab^2e^2 - 3a^2b^2ef + a^3f^2) \log(-\cos(2bx+2a) + I\sin(2bx+2a) + I) - 4(3ab^2e^2 - 3a^2b^2ef + a^3f^2) \log(-\cos(2bx+2a) - I\sin(2bx+2a) + I) + 6(bf^2x + b^2ef) \text{polylog}(3, I\cos(2bx+2a) + \sin(2bx+2a)) - 6(bf^2x + b^2ef) \text{polylog}(3, I\cos(2bx+2a) - \sin(2bx+2a)) + 6(bf^2x + b^2ef) \text{polylog}(3, -I\cos(2bx+2a) + \sin(2bx+2a)) - 6(bf^2x + b^2ef) \text{polylog}(3, -I\cos(2bx+2a) - \sin(2bx+2a)))/b^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e)^2 \operatorname{arccoth}(\cot(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*arccoth(cot(b\*x+a)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*arccoth(cot(b\*x + a)), x)

**maple** [C] time = 40.20, size = 5543, normalized size = 23.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^2*arccoth(cot(b*x+a)),x)`

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12} (f^2x^3 + 3efx^2 + 3e^2x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{12} (f^2x^3 + 3efx^2 + 3e^2x) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 - 4 \sin(2bx + 2a) + 2) - \int \frac{(f^2x^3 + 3efx^2 + 3e^2x) \cos(4bx + 4a) \cos(2bx + 2a) + (bf^2x^3 + 3b*efx^2 + 3b*e^2x) \sin(4bx + 4a) \sin(2bx + 2a) + (bf^2x^3 + 3b*efx^2 + 3b*e^2x) \cos(2bx + 2a)}{\cos(4bx + 4a)^2 + \sin(4bx + 4a)^2 + 2 \cos(4bx + 4a) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*arccoth(cot(b*x+a)),x, algorithm="maxima")`

[Out] `1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acoth}(\cot(a + bx)) (e + fx)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(cot(a + b*x))*(e + f*x)^2,x)`

[Out] `int(acoth(cot(a + b*x))*(e + f*x)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e + fx)^2 \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**2*acoth(cot(b*x+a)),x)`

[Out] `Integral((e + f*x)**2*acoth(cot(a + b*x)), x)`

### 3.250 $\int (e + fx) \coth^{-1}(\cot(a + bx)) dx$

**Optimal.** Leaf size=162

$$\frac{f\text{Li}_3(-ie^{2i(a+bx)})}{8b^2} - \frac{f\text{Li}_3(ie^{2i(a+bx)})}{8b^2} - \frac{i(e+fx)\text{Li}_2(-ie^{2i(a+bx)})}{4b} + \frac{i(e+fx)\text{Li}_2(ie^{2i(a+bx)})}{4b} + \frac{i(e+fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f}$$

[Out]  $1/2*(f*x+e)^2*\text{arccoth}(\cot(b*x+a))/f+1/2*I*(f*x+e)^2*\text{arctan}(\exp(2*I*(b*x+a)))/f-1/4*I*(f*x+e)*\text{polylog}(2,-I*\exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*\text{polylog}(2,I*\exp(2*I*(b*x+a)))/b+1/8*f*\text{polylog}(3,-I*\exp(2*I*(b*x+a)))/b^2-1/8*f*\text{polylog}(3,I*\exp(2*I*(b*x+a)))/b^2$

**Rubi [A]** time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6254, 4181, 2531, 2282, 6589}

$$\frac{f\text{PolyLog}(3,-ie^{2i(a+bx)})}{8b^2} - \frac{f\text{PolyLog}(3,ie^{2i(a+bx)})}{8b^2} - \frac{i(e+fx)\text{PolyLog}(2,-ie^{2i(a+bx)})}{4b} + \frac{i(e+fx)\text{PolyLog}(2,ie^{2i(a+bx)})}{4b}$$

Antiderivative was successfully verified.

[In] `Int[(e + f*x)*ArcCoth[Cot[a + b*x]],x]`

[Out]  $((e + f*x)^2*\text{ArcCoth}[\text{Cot}[a + b*x]])/(2*f) + ((I/2)*(e + f*x)^2*\text{ArcTan}[E^{((2*I)*(a + b*x))}]/f - ((I/4)*(e + f*x)*\text{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}])/b + ((I/4)*(e + f*x)*\text{PolyLog}[2, I*E^{((2*I)*(a + b*x))}])/b + (f*\text{PolyLog}[3, (-I)*E^{((2*I)*(a + b*x))}]/(8*b^2) - (f*\text{PolyLog}[3, I*E^{((2*I)*(a + b*x))}]/(8*b^2))$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x, x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6254

```
Int[ArcCoth[Cot[(a_.) + (b_.)*(x_.)]]*((e_.) + (f_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcCoth[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (e + fx) \coth^{-1}(\cot(a + bx)) dx &= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} - \frac{b \int (e + fx)^2 \sec(2a + 2bx) dx}{2f} \\ &= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} + \frac{1}{2} \int (e + fx) \sec(2a + 2bx) dx \\ &= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \operatorname{Li}_2(-e^{2i(a+bx)})}{4b} \\ &= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \operatorname{Li}_2(-e^{2i(a+bx)})}{4b} \\ &= \frac{(e + fx)^2 \coth^{-1}(\cot(a + bx))}{2f} + \frac{i(e + fx)^2 \tan^{-1}(e^{2i(a+bx)})}{2f} - \frac{i(e + fx) \operatorname{Li}_2(-e^{2i(a+bx)})}{4b} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 263, normalized size = 1.62

$$-be \left( \frac{i \operatorname{Li}_2(-ie^{i(2a+2bx)})}{4b^2} - \frac{i \operatorname{Li}_2(ie^{i(2a+2bx)})}{4b^2} - \frac{ix \tan^{-1}(e^{2ia+2ibx})}{b} \right) + \frac{f(4ib^2x^2 \tan^{-1}(\cos(2(a + bx))) + i \sin(2(a + bx)))}{4b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*ArcCoth[Cot[a + b*x]],x]
```

```
[Out] e*x*ArcCoth[Cot[a + b*x]] + (f*x^2*ArcCoth[Cot[a + b*x]])/2 - b*e*((( -I)*x*
ArcTan[E^((2*I)*a + (2*I)*b*x)))/b + ((I/4)*PolyLog[2, (-I)*E^(I*(2*a + 2*b
*x))])/b^2 - ((I/4)*PolyLog[2, I*E^(I*(2*a + 2*b*x))])/b^2 + (f*((4*I)*b^2
*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I
*cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a
+ b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*
x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

```
fricas [C] time = 0.56, size = 676, normalized size = 4.17
```

$$(2ibfx + 2ibe) \operatorname{Li}_2(i \cos(2bx + 2a) + \sin(2bx + 2a)) + (2ibfx + 2ibe) \operatorname{Li}_2(i \cos(2bx + 2a) - \sin(2bx + 2a))$$


---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*I*b*f*x + 2*I*b*e)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) +
(2*I*b*f*x + 2*I*b*e)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (-2*I*
b*f*x - 2*I*b*e)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (-2*I*b*f*
x - 2*I*b*e)*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 4*(b^2*f*x^2 +
2*b^2*e*x)*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
- sin(2*b*x + 2*a) + 1)) + 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a) + I) - 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) - I*sin(2*b*
x + 2*a) + I) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(I*cos(2*b*x
+ 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*
f)*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 + 2*b^2*e*
x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b
^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x
+ 2*a) + 1) + 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*
a) + I) - 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) +
I) + f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - f*polylog(3, I*c
os(2*b*x + 2*a) - sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) + si
n(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)))/b^2
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (fx + e) \operatorname{arccoth}(\cot(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arccoth(cot(b*x+a)),x, algorithm="giac")
```

[Out] integrate((f\*x + e)\*arccoth(cot(b\*x + a)), x)

maple [C] time = 4.35, size = 2543, normalized size = 15.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*arccoth(cot(b\*x+a)),x)

[Out] 
$$-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{3+1/4}/b^{2*f} * (I*b*x+I*a)^{2*ln(1-I*exp(2*I*(b*x+a)))+1/4}/b^{2*f} * (I*b*x+I*a)*polylog(2, I*exp(2*I*(b*x+a))) - 1/4*ln(exp(2*I*(b*x+a))-I)*x^{2*f} - 1/2*ln(exp(2*I*(b*x+a))-I)*x*e + 1/8*f*polylog(3, -I*exp(2*I*(b*x+a)))/b^{2-1/8*f} * polylog(3, I*exp(2*I*(b*x+a)))/b^{2-1/2*I/b*e*dilog(1+exp(I*(b*x+a))*(-1)^{(3/4)})} - 1/2*I/b*e*dilog(1-exp(I*(b*x+a))*(-1)^{(3/4)}) + 1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^{3+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{2*x^2+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))} * csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{2*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{2+1/2*I/b*e*dilog((-1)^{(1/2)}-exp(I*(b*x+a)))/(-1)^{(1/2)})} + 1/2*I/b*e*dilog((-1)^{(1/2)}+exp(I*(b*x+a)))/(-1)^{(1/2)}) - 1/2/b*a*e*ln(-exp(2*I*(b*x+a))+I) + 1/2/b*a*e*ln(exp(2*I*(b*x+a))+I) - 1/4/b^{2*a^2*f} * ln(exp(2*I*(b*x+a))+I) - 1/4/b^{2*f} * (I*b*x+I*a)^{2*ln(1+I*exp(2*I*(b*x+a)))-1/4}/b^{2*f} * (I*b*x+I*a)*polylog(2, -I*exp(2*I*(b*x+a))) - 1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{3*x^2+(1/4*f*x^2+1/2*e*x)*ln(exp(2*I*(b*x+a))+I)+1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{2*x^2+1/4/b^{2*a^2*f} * ln(-exp(2*I*(b*x+a))+I) - 1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{2*x^2-1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^{2+1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^{2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1)) - 1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1)) - 1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*x^{2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1)) + 1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*x^{2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1)) + 1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))$$

$$(2*I*(b*x+a))-1)) * x^{2-1/4} * I * \pi * x * e * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I)) * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{2-1/8} * I * \pi * f * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I)) * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{2*x^{2-1/4} * I * \pi * x * e * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}((1+I) * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{2-1/8} * I * \pi * f * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}((1+I) * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{2*x^{2+1/2} * I / b^{2*f*a} * (I*b*x+I*a) * \ln(1+\exp(I*(b*x+a)) * (-1)^{(3/4)}) + 1/2 * I / b^{2*f*a} * (I*b*x+I*a) * \ln(1-\exp(I*(b*x+a)) * (-1)^{(3/4)}) + 1/2 * I / b * e * (I*b*x+I*a) * \ln((( -1)^{(1/2)} + \exp(I*(b*x+a))) / (-1)^{(1/2)}) + 1/2 * I / b^{2*f*a} * \operatorname{dilog}(1-\exp(I*(b*x+a)) * (-1)^{(3/4)}) + 1/2 * I / b * e * (I*b*x+I*a) * \ln((( -1)^{(1/2)} - \exp(I*(b*x+a))) / (-1)^{(1/2)}) + 1/4 * I * \pi * x * e * \operatorname{csgn}((1-I) * (\exp(2*I*(b*x+a))+I) / (\exp(2*I*(b*x+a))-1))^{3+1/8} * I * \pi * f * \operatorname{csgn}((1-I) * (\exp(2*I*(b*x+a))+I) / (\exp(2*I*(b*x+a))-1))^{3*x^{2+1/8} * I * \pi * f * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{3*x^{2+1/2} * I / b^{2*f*a} * \operatorname{dilog}(1+\exp(I*(b*x+a)) * (-1)^{(3/4)}) - 1/2 * I / b^{2*f*a} * \operatorname{dilog}((( -1)^{(1/2)} + \exp(I*(b*x+a))) / (-1)^{(1/2)}) - 1/4 * I * \pi * x * e * \operatorname{csgn}((1+I) * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{3-1/8} * I * \pi * f * \operatorname{csgn}((1+I) * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{3*x^{2+1/8} * I * \pi * f * \operatorname{csgn}((1+I) * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{2*x^{2-1/2} * I / b^{2*f*a} * (I*b*x+I*a) * \ln((( -1)^{(1/2)} - \exp(I*(b*x+a))) / (-1)^{(1/2)}) - 1/2 * I / b^{2*f*a} * (I*b*x+I*a) * \ln((( -1)^{(1/2)} + \exp(I*(b*x+a))) / (-1)^{(1/2)}) - 1/8 * I * \pi * f * x^{2-1/4} * I * \pi * x * e - 1/2 * I / b^{2*f*a} * \operatorname{dilog}((( -1)^{(1/2)} - \exp(I*(b*x+a))) / (-1)^{(1/2)}) - 1/2 * I / b * e * (I*b*x+I*a) * \ln(1+\exp(I*(b*x+a)) * (-1)^{(3/4)}) - 1/2 * I / b * e * (I*b*x+I*a) * \ln(1-\exp(I*(b*x+a)) * (-1)^{(3/4)}) - 1/8 * I * \pi * f * \operatorname{csgn}(I / (\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{2*x^{2-1/4} * I * \pi * x * e * \operatorname{csgn}(I / (\exp(2*I*(b*x+a))-1)) * \operatorname{csgn}(I * (\exp(2*I*(b*x+a))-I) / (\exp(2*I*(b*x+a))-1))^{2}}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} (fx^2 + 2ex) \log(2 \cos(2bx + 2a)^2 + 2 \sin(2bx + 2a)^2 + 4 \sin(2bx + 2a) + 2) - \frac{1}{8} (fx^2 + 2ex) \log(2 \cos$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*arccoth(cot(b\*x+a)),x, algorithm="maxima")

[Out] 1/8\*(f\*x^2 + 2\*e\*x)\*log(2\*cos(2\*b\*x + 2\*a)^2 + 2\*sin(2\*b\*x + 2\*a)^2 + 4\*sin(2\*b\*x + 2\*a) + 2) - 1/8\*(f\*x^2 + 2\*e\*x)\*log(2\*cos(2\*b\*x + 2\*a)^2 + 2\*sin(2\*b\*x + 2\*a)^2 - 4\*sin(2\*b\*x + 2\*a) + 2) - integrate(((b\*f\*x^2 + 2\*b\*e\*x)\*cos(4\*b\*x + 4\*a)\*cos(2\*b\*x + 2\*a) + (b\*f\*x^2 + 2\*b\*e\*x)\*sin(4\*b\*x + 4\*a)\*sin(2\*b\*x + 2\*a) + (b\*f\*x^2 + 2\*b\*e\*x)\*cos(2\*b\*x + 2\*a))/(cos(4\*b\*x + 4\*a)^2 + sin(4\*b\*x + 4\*a)^2 + 2\*cos(4\*b\*x + 4\*a) + 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(\cot(a + bx)) (e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(cot(a + b*x))*(e + f*x),x)
```

```
[Out] int(acoth(cot(a + b*x))*(e + f*x), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (e + fx) \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*acoth(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*acoth(cot(a + b*x)), x)
```



### 3.251 $\int \coth^{-1}(\cot(a + bx)) dx$

Optimal. Leaf size=79

$$-\frac{i\text{Li}_2(-ie^{2i(a+bx)})}{4b} + \frac{i\text{Li}_2(ie^{2i(a+bx)})}{4b} + ix \tan^{-1}(e^{2i(a+bx)}) + x \coth^{-1}(\cot(a + bx))$$

[Out]  $x*\text{arccoth}(\cot(b*x+a))+I*x*\text{arctan}(\exp(2*I*(b*x+a)))-1/4*I*\text{polylog}(2,-I*\exp(2*I*(b*x+a)))/b+1/4*I*\text{polylog}(2,I*\exp(2*I*(b*x+a)))/b$

**Rubi [A]** time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6250, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}(2,-ie^{2i(a+bx)})}{4b} + \frac{i\text{PolyLog}(2,ie^{2i(a+bx)})}{4b} + ix \tan^{-1}(e^{2i(a+bx)}) + x \coth^{-1}(\cot(a + bx))$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[Cot[a + b*x]],x]`

[Out]  $x*\text{ArcCoth}[\text{Cot}[a + b*x]] + I*x*\text{ArcTan}[E^{((2*I)*(a + b*x))}] - ((I/4)*\text{PolyLog}[2, (-I)*E^{((2*I)*(a + b*x))}])/b + ((I/4)*\text{PolyLog}[2, I*E^{((2*I)*(a + b*x))}])/b$

#### Rule 2279

`Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]`  
`> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /;` `FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

#### Rule 2391

`Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] > -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /;` `FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

#### Rule 4181

`Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]`  
`> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f, x] + (-Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))], x], x) /;` `FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

#### Rule 6250

`Int[ArcCoth[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] := Simp[x*ArcCoth[Cot[a + b*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]`

### Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(\cot(a + bx)) dx &= x \coth^{-1}(\cot(a + bx)) - b \int x \sec(2a + 2bx) dx \\
 &= x \coth^{-1}(\cot(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) + \frac{1}{2} \int \log(1 - ie^{i(2a+2bx)}) dx - \frac{1}{2} \int \log(1 + ie^{i(2a+2bx)}) dx \\
 &= x \coth^{-1}(\cot(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) - \frac{i \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, e^{i(2a+2bx)}\right)}{4b} + \frac{i \operatorname{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{i(2a+2bx)}\right)}{4b} \\
 &= x \coth^{-1}(\cot(a + bx)) + ix \tan^{-1}(e^{2i(a+bx)}) - \frac{i \operatorname{Li}_2(-ie^{2i(a+bx)})}{4b} + \frac{i \operatorname{Li}_2(ie^{2i(a+bx)})}{4b}
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 78, normalized size = 0.99

$$\frac{-i \operatorname{Li}_2(-ie^{2i(a+bx)}) + i \operatorname{Li}_2(ie^{2i(a+bx)}) + 4bx \left( \coth^{-1}(\cot(a + bx)) + i \tan^{-1}(e^{2i(a+bx)}) \right)}{4b}$$

Antiderivative was successfully verified.

[In] `Integrate[ArcCoth[Cot[a + b*x]], x]`

[Out] `(4*b*x*(ArcCoth[Cot[a + b*x]] + I*ArcTan[E^((2*I)*(a + b*x))]) - I*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + I*PolyLog[2, I*E^((2*I)*(a + b*x))])/(4*b)`

**fricas [B]** time = 0.69, size = 388, normalized size = 4.91

$$\frac{4bx \log\left(\frac{\cos(2bx+2a)+\sin(2bx+2a)+1}{\cos(2bx+2a)-\sin(2bx+2a)+1}\right) + 2a \log(\cos(2bx+2a) + i \sin(2bx+2a) + i) - 2a \log(\cos(2bx+2a) - i \sin(2bx+2a) + i)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(cot(b*x+a)),x, algorithm="fricas")`

[Out] `1/8*(4*b*x*log((cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)`

) + 2\*a\*log(-cos(2\*b\*x + 2\*a) + I\*sin(2\*b\*x + 2\*a) + I) - 2\*a\*log(-cos(2\*b\*x + 2\*a) - I\*sin(2\*b\*x + 2\*a) + I) + I\*dilog(I\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a)) + I\*dilog(I\*cos(2\*b\*x + 2\*a) - sin(2\*b\*x + 2\*a)) - I\*dilog(-I\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a)) - I\*dilog(-I\*cos(2\*b\*x + 2\*a) - sin(2\*b\*x + 2\*a)))/b

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\cot(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(cot(b\*x + a)), x)

**maple** [B] time = 0.55, size = 267, normalized size = 3.38

$$\frac{\operatorname{arccoth}(\cot(bx + a)) \pi}{2b} + \frac{\operatorname{arccoth}(\cot(bx + a)) \operatorname{arccot}(\cot(bx + a))}{b} - \frac{\ln\left(1 + \frac{i(1+i \cot(bx+a))^2}{\cot^2(bx+a)+1}\right) \pi}{4b} + \frac{\ln\left(1 + \frac{i(1+i \cot(bx+a))^2}{\cot^2(bx+a)+1}\right)}{\cot^2(bx+a)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(cot(b\*x+a)),x)

[Out] -1/2/b\*arccoth(cot(b\*x+a))\*Pi+1/b\*arccoth(cot(b\*x+a))\*arccot(cot(b\*x+a))-1/4/b\*ln(1+I\*(1+I\*cot(b\*x+a))^2/(cot(b\*x+a)^2+1))\*Pi+1/2/b\*ln(1+I\*(1+I\*cot(b\*x+a))^2/(cot(b\*x+a)^2+1))\*arccot(cot(b\*x+a))+1/4/b\*ln(1-I\*(1+I\*cot(b\*x+a))^2/(cot(b\*x+a)^2+1))\*Pi-1/2/b\*ln(1-I\*(1+I\*cot(b\*x+a))^2/(cot(b\*x+a)^2+1))\*arccot(cot(b\*x+a))+1/4\*I/b\*dilog(1+I\*(1+I\*cot(b\*x+a))^2/(cot(b\*x+a)^2+1))-1/4\*I/b\*dilog(1-I\*(1+I\*cot(b\*x+a))^2/(cot(b\*x+a)^2+1))

**maxima** [B] time = 0.47, size = 184, normalized size = 2.33

$$4(bx + a) \operatorname{arccoth}\left(\frac{1}{\tan(bx+a)}\right) + \left(\arctan\left(\frac{1}{2} \tan(bx + a) + \frac{1}{2}, \frac{1}{2} \tan(bx + a) + \frac{1}{2}\right) - \arctan\left(\frac{1}{2} \tan(bx + a) - \frac{1}{2}, -\frac{1}{2} \tan(bx + a) - \frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a)),x, algorithm="maxima")

[Out] 1/4\*(4\*(b\*x + a)\*arccoth(1/tan(b\*x + a)) + (arctan2(1/2\*tan(b\*x + a) + 1/2, 1/2\*tan(b\*x + a) + 1/2) - arctan2(1/2\*tan(b\*x + a) - 1/2, -1/2\*tan(b\*x + a) + 1/2))\*log(tan(b\*x + a)^2 + 1) - (b\*x + a)\*log(1/2\*tan(b\*x + a)^2 + tan(b\*x + a) + 1/2) + (b\*x + a)\*log(1/2\*tan(b\*x + a)^2 - tan(b\*x + a) + 1/2) -

$I \cdot \operatorname{dilog}\left(\left(\frac{1}{2}I + \frac{1}{2}\right) \tan(bx + a) - \frac{1}{2}I + \frac{1}{2}\right) + I \cdot \operatorname{dilog}\left(-\left(\frac{1}{2}I - \frac{1}{2}\right) \tan(bx + a) + \frac{1}{2}I + \frac{1}{2}\right) + I \cdot \operatorname{dilog}\left(\left(\frac{1}{2}I - \frac{1}{2}\right) \tan(bx + a) + \frac{1}{2}I + \frac{1}{2}\right) - I \cdot \operatorname{dilog}\left(-\left(\frac{1}{2}I + \frac{1}{2}\right) \tan(bx + a) - \frac{1}{2}I + \frac{1}{2}\right) / b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(cot(a + b*x)), x)`

[Out] `int(acoth(cot(a + b*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(\cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(cot(b*x+a)), x)`

[Out] `Integral(acoth(cot(a + b*x)), x)`

$$3.252 \quad \int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(\cot(a+bx))}{e+fx}, x\right)$$

[Out] CannotIntegrate(arccoth(cot(b\*x+a))/(f\*x+e), x)

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]

[Out] Defer[Int][ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]

Rubi steps

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx = \int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

**Mathematica** [A] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(\cot(a+bx))}{e+fx} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]

[Out] Integrate[ArcCoth[Cot[a + b\*x]]/(e + f\*x), x]

**fricas** [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arccoth}(\cot(bx+a))}{fx+e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a))/(f\*x+e),x, algorithm="fricas")

[Out] integral(arccoth(cot(b\*x + a))/(f\*x + e), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\cot(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a))/(f\*x+e),x, algorithm="giac")

[Out] integrate(arccoth(cot(b\*x + a))/(f\*x + e), x)

**maple** [A] time = 4.19, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\cot(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(cot(b\*x+a))/(f\*x+e),x)

[Out] int(arccoth(cot(b\*x+a))/(f\*x+e),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(\cot(bx + a))}{fx + e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(cot(b\*x+a))/(f\*x+e),x, algorithm="maxima")

[Out] integrate(arccoth(cot(b\*x + a))/(f\*x + e), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(\cot(a + bx))}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(cot(a + b\*x))/(e + f\*x),x)

[Out] int(acoth(cot(a + b\*x))/(e + f\*x), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(\cot(a + bx))}{e + fx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(cot(b\*x+a))/(f\*x+e), x)

[Out] Integral(acoth(cot(a + b\*x))/(e + f\*x), x)

### 3.253 $\int x^2 \coth^{-1}(c + d \cot(a + bx)) dx$

**Optimal.** Leaf size=391

$$\frac{i\text{Li}_4\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3} - \frac{i\text{Li}_4\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^3} + \frac{x\text{Li}_3\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2} - \frac{x\text{Li}_3\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2} - \frac{ix^2\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}$$

[Out]  $\frac{1}{3}x^3 \operatorname{arccoth}(c+d \cot(bx+a)) + \frac{1}{6}x^3 \ln(1 - (1-c-I*d) \exp(2I*a+2I*b*x) / (1-c+I*d)) - \frac{1}{6}x^3 \ln(1 - (1+c+I*d) \exp(2I*a+2I*b*x) / (1+c-I*d)) - \frac{1}{4}I*x^2 \operatorname{polylog}(2, (1-c-I*d) \exp(2I*a+2I*b*x) / (1-c+I*d)) / b + \frac{1}{4}I*x^2 \operatorname{polylog}(2, (1+c+I*d) \exp(2I*a+2I*b*x) / (1+c-I*d)) / b + \frac{1}{4}x \operatorname{polylog}(3, (1-c-I*d) \exp(2I*a+2I*b*x) / (1-c+I*d)) / b^2 - \frac{1}{4}x \operatorname{polylog}(3, (1+c+I*d) \exp(2I*a+2I*b*x) / (1+c-I*d)) / b^2 + \frac{1}{8}I \operatorname{polylog}(4, (1-c-I*d) \exp(2I*a+2I*b*x) / (1-c+I*d)) / b^3 - \frac{1}{8}I \operatorname{polylog}(4, (1+c+I*d) \exp(2I*a+2I*b*x) / (1+c-I*d)) / b^3$

**Rubi [A]** time = 0.50, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6270, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2} - \frac{x\text{PolyLog}\left(3, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2} + \frac{i\text{PolyLog}\left(4, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3} - \frac{i\text{PolyLog}\left(4, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2 \operatorname{ArcCoth}[c + d \cot[a + bx]], x]$

[Out]  $\frac{x^3 \operatorname{ArcCoth}[c + d \cot[a + bx]]}{3} + \frac{x^3 \operatorname{Log}[1 - ((1 - c - I*d) \exp((2I)*a + (2I)*b*x)) / (1 - c + I*d)]}{6} - \frac{x^3 \operatorname{Log}[1 - ((1 + c + I*d) \exp((2I)*a + (2I)*b*x)) / (1 + c - I*d)]}{6} - \frac{(I/4) x^2 \operatorname{PolyLog}[2, ((1 - c - I*d) \exp((2I)*a + (2I)*b*x)) / (1 - c + I*d)]}{b} + \frac{(I/4) x^2 \operatorname{PolyLog}[2, ((1 + c + I*d) \exp((2I)*a + (2I)*b*x)) / (1 + c - I*d)]}{b} + \frac{x \operatorname{PolyLog}[3, ((1 - c - I*d) \exp((2I)*a + (2I)*b*x)) / (1 - c + I*d)]}{4b^2} - \frac{x \operatorname{PolyLog}[3, ((1 + c + I*d) \exp((2I)*a + (2I)*b*x)) / (1 + c - I*d)]}{4b^2} + \frac{(I/8) \operatorname{PolyLog}[4, ((1 - c - I*d) \exp((2I)*a + (2I)*b*x)) / (1 - c + I*d)]}{b^3} - \frac{(I/8) \operatorname{PolyLog}[4, ((1 + c + I*d) \exp((2I)*a + (2I)*b*x)) / (1 + c - I*d)]}{b^3}$

**Rule 2190**

$\text{Int}[(((F_)^\wedge((g_) * ((e_) + (f_) * (x_))))^\wedge(n_) * ((c_) + (d_) * (x_))^\wedge(m_)) / ((a_) + (b_) * ((F_)^\wedge((g_) * ((e_) + (f_) * (x_))))^\wedge(n_)), x\_Symbol] := \text{Simp} [((c + d*x)^\wedge m * \text{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n] / a)] / (b*f*g*n * \text{Log}[F]), x] - \text{Dist} [(d*m) / (b*f*g*n * \text{Log}[F]), \text{Int} [(c + d*x)^\wedge (m - 1) * \text{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n] / a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$



Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6270

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
+ 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_
.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(c + d \cot(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{3} (b(i(1 + c) - d)) \int \frac{e^{2ia+2ibx} x^3}{1 + c - id + (-1 - c - i)} \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{6} x^3 \log \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{6} x^3 \log \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{6} x^3 \log \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{6} x^3 \log \\
&= \frac{1}{3} x^3 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{6} x^3 \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{6} x^3 \log
\end{aligned}$$

**Mathematica [A]** time = 0.33, size = 339, normalized size = 0.87

$$\frac{1}{3} x^3 \coth^{-1}(d \cot(a+bx)+c) + \frac{4b^3 x^3 \log \left( 1 - \frac{(c+id-1)e^{2i(a+bx)}}{c-id-1} \right) - 4b^3 x^3 \log \left( 1 - \frac{(c+id+1)e^{2i(a+bx)}}{c-id+1} \right) - 6ib^2 x^2 \text{Li}_2 \left( \frac{(c+id-1)e^{2i(a+bx)}}{c-id-1} \right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[c + d\*Cot[a + b\*x]],x]

[Out] (x^3\*ArcCoth[c + d\*Cot[a + b\*x]])/3 + (4\*b^3\*x^3\*Log[1 - ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - 4\*b^3\*x^3\*Log[1 - ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] - (6\*I)\*b^2\*x^2\*PolyLog[2, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] + (6\*I)\*b^2\*x^2\*PolyLog[2, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] + 6\*b\*x\*PolyLog[3, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - 6\*b\*x\*PolyLog[3, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] + (3\*I)\*PolyLog[4, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - (3\*I)\*PolyLog[4, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)]/(24\*b^3)

**fricas [C]** time = 0.83, size = 1798, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{48} \cdot (8b^3x^3 \log((d \cos(2bx + 2a) + (c + 1) \sin(2bx + 2a) + d) / (d \cos(2bx + 2a) + (c - 1) \sin(2bx + 2a) + d)) + 6Ib^2x^2 \operatorname{dilog}(-(c^2 + d^2 - (c^2 + 2I(c + 1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c + 1)d + Id^2 - 2Ic - I) \sin(2bx + 2a) + 2c + 1) / (c^2 + d^2 + 2c + 1) + 1) - 6Ib^2x^2 \operatorname{dilog}(-(c^2 + d^2 - (c^2 - 2I(c + 1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c + 1)d - Id^2 + 2Ic + I) \sin(2bx + 2a) + 2c + 1) / (c^2 + d^2 + 2c + 1) + 1) - 6Ib^2x^2 \operatorname{dilog}(-(c^2 + d^2 - (c^2 + 2I(c - 1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c - 1)d + Id^2 + 2Ic - I) \sin(2bx + 2a) - 2c + 1) / (c^2 + d^2 - 2c + 1) + 1) + 6Ib^2x^2 \operatorname{dilog}(-(c^2 + d^2 - (c^2 - 2I(c - 1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c - 1)d - Id^2 - 2Ic + I) \sin(2bx + 2a) - 2c + 1) / (c^2 + d^2 - 2c + 1) + 1) + 4a^3 \log(1/2c^2 + I(c + 1)d - 1/2d^2 - 1/2(c^2 + d^2 + 2c + 1) \cos(2bx + 2a) + 1/2(Ic^2 + Id^2 + 2Ic + I) \sin(2bx + 2a) + c + 1/2) - 4a^3 \log(1/2c^2 + I(c - 1)d - 1/2d^2 - 1/2(c^2 + d^2 - 2c + 1) \cos(2bx + 2a) + 1/2(Ic^2 + Id^2 - 2Ic + I) \sin(2bx + 2a) - c + 1/2) + 4a^3 \log(-1/2c^2 + I(c + 1)d + 1/2d^2 + 1/2(c^2 + d^2 + 2c + 1) \cos(2bx + 2a) + 1/2(Ic^2 + Id^2 + 2Ic + I) \sin(2bx + 2a) - c - 1/2) - 4a^3 \log(-1/2c^2 + I(c - 1)d + 1/2d^2 + 1/2(c^2 + d^2 - 2c + 1) \cos(2bx + 2a) + 1/2(Ic^2 + Id^2 - 2Ic + I) \sin(2bx + 2a) + c - 1/2) - 6bxx \operatorname{polylog}(3, ((c^2 + 2I(c + 1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (Ic^2 - 2(c + 1)d - Id^2 + 2Ic + I) \sin(2bx + 2a)) / (c^2 + d^2 + 2c + 1)) - 6bxx \operatorname{polylog}(3, ((c^2 - 2I(c + 1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (-Ic^2 - 2(c + 1)d + Id^2 - 2Ic - I) \sin(2bx + 2a)) / (c^2 + d^2 + 2c + 1)) + 6bxx \operatorname{polylog}(3, ((c^2 + 2I(c - 1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (Ic^2 - 2(c - 1)d - Id^2 - 2Ic + I) \sin(2bx + 2a)) / (c^2 + d^2 - 2c + 1)) + 6bxx \operatorname{polylog}(3, ((c^2 - 2I(c - 1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (-Ic^2 - 2(c - 1)d + Id^2 + 2Ic - I) \sin(2bx + 2a)) / (c^2 + d^2 - 2c + 1)) - 4(b^3x^3 + a^3) \log((c^2 + d^2 - (c^2 + 2I(c + 1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c + 1)d + Id^2 - 2Ic - I) \sin(2bx + 2a) + 2c + 1) / (c^2 + d^2 + 2c + 1)) - 4(b^3x^3 + a^3) \log((c^2 + d^2 - (c^2 - 2I(c + 1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c + 1)d - Id^2 + 2Ic + I) \sin(2bx + 2a) + 2c + 1) / (c^2 + d^2 + 2c + 1)) + 4(b^3x^3 + a^3) \log((c^2 + d^2 - (c^2 + 2I(c - 1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c - 1)d + Id^2 + 2Ic - I) \sin(2bx + 2a) - 2c + 1) / (c^2 + d^2 - 2c + 1)) + 4(b^3x^3 + a^3) \log((c^2 + d^2 - (c^2 - 2I(c - 1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c - 1)d - Id^2 - 2Ic + I) \sin(2bx + 2a) - 2c + 1) / (c^2 + d^2 - 2c + 1)) - 3I \operatorname{polylog}(4, ((c^2 + 2I(c + 1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (Ic^2 - 2(c + 1)d - Id^2 + 2Ic + I) \sin(2bx + 2a)) / (c^2 + d^2 + 2c + 1)) + 3I \operatorname{polylog}(4, ((c^2 - 2I(c + 1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (-Ic^2 - 2(c + 1)d + Id^2 - 2Ic - I) \sin(2bx + 2a)) / (c^2 + d^2 + 2c + 1)) + 3I \operatorname{polylog}(4, ((c^2 + 2I(c$

$$- 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) - 3*I*\text{polylog}(4, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*\sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^3$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(d \cot(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(d\*cot(b\*x + a) + c), x)

**maple** [C] time = 39.34, size = 6698, normalized size = 17.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(c+d\*cot(b\*x+a)),x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12} x^3 \log\left(\left(c^2 + d^2 + 2c + 1\right) \cos(2bx + 2a)^2 + 4(c + 1)d \sin(2bx + 2a) + \left(c^2 + d^2 + 2c + 1\right) \sin(2bx + 2a)^2 + \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{12}x^3 \log\left(\left(c^2 + d^2 + 2c + 1\right) \cos(2bx + 2a)^2 + 4(c + 1)d \sin(2bx + 2a) + \left(c^2 + d^2 + 2c + 1\right) \sin(2bx + 2a)^2 + \right) - \frac{1}{12}x^3 \log\left(\left(c^2 + d^2 - 2c + 1\right) \cos(2bx + 2a)^2 + 4(c - 1)d \sin(2bx + 2a) + \left(c^2 + d^2 - 2c + 1\right) \sin(2bx + 2a)^2 + \right) - 2(c^2 + d^2 - 2c + 1) \cos(2bx + 2a) - 2(c^2 - d^2 - 2c + 1) \sin(2bx + 2a) - 4b \int \frac{1}{3} \left(2(c^2 + d^2 - 1)x^3 \cos(2bx + 2a)^2 + 2c d x^3 \sin(2bx + 2a) + 2(c^2 + d^2 - 1)x^3 \sin(2bx + 2a)^2 - (c^2 - d^2 - 1)x^3 \cos(2bx + 2a) - (2c d x^3 \sin(2bx + 2a) + (c^2 - d^2 - 1)x^3 \cos(2bx + 2a)) \cos(4bx + 4a) + (2c d x^3 \cos(2bx + 2a) - (c^2 - d^2 - 1)x^3 \sin(2bx + 2a)) \sin(4bx + 4a)\right) / (c^4 + d^4 + 2(c^2 + 1)d^2 + (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1) \cos(4bx + 4a)^2 + 4(c^4 + d^4 + 2(c^2 - 1)d^2 - 2c^2 + 1) \cos(2bx + 2a)^2 + \right)$

$c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*\sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*\sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*\cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d)*\sin(2*b*x + 2*a) + 1)*\cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2 + 1)*\cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*d)*\cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*\sin(2*b*x + 2*a))*\sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*\sin(2*b*x + 2*a) + 1), x$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(c + d \cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(c + d*cot(a + b*x)),x)`

[Out] `int(x^2*acoth(c + d*cot(a + b*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(c + d \cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(c+d*cot(b*x+a)),x)`

[Out] `Integral(x**2*acoth(c + d*cot(a + b*x)), x)`

### 3.254 $\int x \coth^{-1}(c + d \cot(a + bx)) dx$

**Optimal.** Leaf size=293

$$\frac{\operatorname{Li}_3\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2} - \frac{\operatorname{Li}_3\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2} - \frac{ix\operatorname{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b} + \frac{ix\operatorname{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b} + \frac{1}{4}x^2 \log\left(1 - \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)$$

[Out]  $1/2*x^2*\operatorname{arccoth}(c+d*\cot(b*x+a))+1/4*x^2*\ln(1-(1-c-I*d)*\exp(2*I*a+2*I*b*x)/(1-c+I*d))-1/4*x^2*\ln(1-(1+c+I*d)*\exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x*\operatorname{polylog}(2,(1-c-I*d)*\exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x*\operatorname{polylog}(2,(1+c+I*d)*\exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/8*\operatorname{polylog}(3,(1-c-I*d)*\exp(2*I*a+2*I*b*x)/(1-c+I*d))/b^2-1/8*\operatorname{polylog}(3,(1+c+I*d)*\exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^2$

**Rubi [A]** time = 0.40, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6270, 2190, 2531, 2282, 6589}

$$\frac{\operatorname{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2} - \frac{\operatorname{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2} - \frac{ix\operatorname{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[c + d*Cot[a + b*x]],x]`

[Out]  $(x^2*\operatorname{ArcCoth}[c + d*\cot[a + b*x]])/2 + (x^2*\log[1 - ((1 - c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c + I*d)])/4 - (x^2*\log[1 - ((1 + c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c - I*d)])/4 - ((I/4)*x*\operatorname{PolyLog}[2, ((1 - c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c + I*d)])/b + ((I/4)*x*\operatorname{PolyLog}[2, ((1 + c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c - I*d)])/b + \operatorname{PolyLog}[3, ((1 - c - I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 - c + I*d)]/(8*b^2) - \operatorname{PolyLog}[3, ((1 + c + I*d)*E^{((2*I)*a + (2*I)*b*x)})/(1 + c - I*d)]/(8*b^2)$

#### Rule 2190

`Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi`

```
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6270

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
+ 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
)*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(c + d \cot(a + bx)) dx &= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{2}(b(i(1 + c) - d)) \int \frac{e^{2ia+2ibx}x^2}{1 + c - id + (-1 - c - id)} \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \log \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \log \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \log \\
&= \frac{1}{2}x^2 \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{4}x^2 \log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \log
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 253, normalized size = 0.86

$$\frac{1}{2}x^2 \coth^{-1}(d \cot(a+bx)+c) + \frac{2b^2x^2 \log\left(1 - \frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right) - 2b^2x^2 \log\left(1 - \frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right) - 2ibx \operatorname{Li}_2\left(\frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[c + d\*Cot[a + b\*x]],x]

[Out] (x^2\*ArcCoth[c + d\*Cot[a + b\*x]])/2 + (2\*b^2\*x^2\*Log[1 - ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - 2\*b^2\*x^2\*Log[1 - ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] - (2\*I)\*b\*x\*PolyLog[2, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] + (2\*I)\*b\*x\*PolyLog[2, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)] + PolyLog[3, ((-1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(-1 + c - I\*d)] - PolyLog[3, ((1 + c + I\*d)\*E^((2\*I)\*(a + b\*x)))/(1 + c - I\*d)])/(8\*b^2)

**fricas [C]** time = 0.89, size = 1462, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="fricas")

[Out] 1/16\*(4\*b^2\*x^2\*log((d\*cos(2\*b\*x + 2\*a) + (c + 1)\*sin(2\*b\*x + 2\*a) + d)/(d\*cos(2\*b\*x + 2\*a) + (c - 1)\*sin(2\*b\*x + 2\*a) + d)) + 2\*I\*b\*x\*dilog(-(c^2 + d



$$\begin{aligned}
& \int x \operatorname{arccoth}(d \cot(bx + a) + c) dx \\
&= \frac{1}{b^2} \left( \frac{1}{2} (c^2 + 2I(c+1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c+1)d + Id^2 - 2Ic - I) \sin(2bx + 2a) + 2c + 1 \right) / (c^2 + d^2 + 2c + 1) \\
&+ \frac{1}{2} \operatorname{dilog}\left(-\frac{c^2 + d^2 - (c^2 - 2I(c+1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c+1)d - Id^2 + 2Ic + I) \sin(2bx + 2a) + 2c + 1}{c^2 + d^2 + 2c + 1} + 1\right) \\
&- \frac{1}{2} \operatorname{dilog}\left(-\frac{c^2 + d^2 - (c^2 - 2I(c-1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c-1)d + Id^2 + 2Ic - I) \sin(2bx + 2a) - 2c + 1}{c^2 + d^2 - 2c + 1} + 1\right) \\
&+ \frac{1}{2} \operatorname{dilog}\left(-\frac{c^2 + d^2 - (c^2 - 2I(c-1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c-1)d - Id^2 - 2Ic + I) \sin(2bx + 2a) - 2c + 1}{c^2 + d^2 - 2c + 1} + 1\right) \\
&- \frac{1}{2} a^2 \log\left(\frac{1}{2} c^2 + I(c+1)d - \frac{1}{2} d^2 - \frac{1}{2} (c^2 + d^2 + 2c + 1) \cos(2bx + 2a) + \frac{1}{2} (Ic^2 + Id^2 + 2Ic + I) \sin(2bx + 2a) + c + \frac{1}{2}\right) \\
&+ \frac{1}{2} a^2 \log\left(\frac{1}{2} c^2 + I(c-1)d - \frac{1}{2} d^2 - \frac{1}{2} (c^2 + d^2 - 2c + 1) \cos(2bx + 2a) + \frac{1}{2} (Ic^2 + Id^2 - 2Ic + I) \sin(2bx + 2a) - c + \frac{1}{2}\right) \\
&- \frac{1}{2} a^2 \log\left(-\frac{1}{2} c^2 + I(c+1)d + \frac{1}{2} d^2 + \frac{1}{2} (c^2 + d^2 + 2c + 1) \cos(2bx + 2a) + \frac{1}{2} (Ic^2 + Id^2 + 2Ic + I) \sin(2bx + 2a) - c - \frac{1}{2}\right) \\
&+ \frac{1}{2} a^2 \log\left(-\frac{1}{2} c^2 + I(c-1)d + \frac{1}{2} d^2 + \frac{1}{2} (c^2 + d^2 - 2c + 1) \cos(2bx + 2a) + \frac{1}{2} (Ic^2 + Id^2 - 2Ic + I) \sin(2bx + 2a) + c - \frac{1}{2}\right) \\
&- 2(b^2 x^2 - a^2) \log\left(\frac{c^2 + d^2 - (c^2 + 2I(c+1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c+1)d + Id^2 - 2Ic - I) \sin(2bx + 2a) + 2c + 1}{c^2 + d^2 + 2c + 1}\right) \\
&- 2(b^2 x^2 - a^2) \log\left(\frac{c^2 + d^2 - (c^2 - 2I(c+1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c+1)d - Id^2 + 2Ic + I) \sin(2bx + 2a) + 2c + 1}{c^2 + d^2 + 2c + 1}\right) \\
&+ 2(b^2 x^2 - a^2) \log\left(\frac{c^2 + d^2 - (c^2 + 2I(c-1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (-Ic^2 + 2(c-1)d + Id^2 + 2Ic - I) \sin(2bx + 2a) - 2c + 1}{c^2 + d^2 - 2c + 1}\right) \\
&+ 2(b^2 x^2 - a^2) \log\left(\frac{c^2 + d^2 - (c^2 - 2I(c-1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (Ic^2 + 2(c-1)d - Id^2 - 2Ic + I) \sin(2bx + 2a) - 2c + 1}{c^2 + d^2 - 2c + 1}\right) \\
&- \operatorname{polylog}\left(3, \frac{(c^2 + 2I(c+1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (Ic^2 - 2(c+1)d - Id^2 + 2Ic + I) \sin(2bx + 2a)}{c^2 + d^2 + 2c + 1}\right) \\
&- \operatorname{polylog}\left(3, \frac{(c^2 - 2I(c+1)d - d^2 + 2c + 1) \cos(2bx + 2a) + (-Ic^2 - 2(c+1)d + Id^2 - 2Ic - I) \sin(2bx + 2a)}{c^2 + d^2 + 2c + 1}\right) \\
&+ \operatorname{polylog}\left(3, \frac{(c^2 + 2I(c-1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (Ic^2 - 2(c-1)d - Id^2 - 2Ic + I) \sin(2bx + 2a)}{c^2 + d^2 - 2c + 1}\right) \\
&+ \operatorname{polylog}\left(3, \frac{(c^2 - 2I(c-1)d - d^2 - 2c + 1) \cos(2bx + 2a) + (-Ic^2 - 2(c-1)d + Id^2 + 2Ic - I) \sin(2bx + 2a)}{c^2 + d^2 - 2c + 1}\right) / b^2
\end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(d \cot(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*cot(b\*x + a) + c), x)

**maple** [C] time = 4.92, size = 6348, normalized size = 21.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(c+d\*cot(b\*x+a)),x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2bd \int \frac{1}{c^4 + d^4 + 2(c^2 + 1)d^2 + (c^4 + d^4 + 2(c^2 + 1)d^2 - 2c^2 + 1) \cos(4bx + 4a)^2 + 4(c^4 + d^4 + 2(c^2 - 1)d^2 - 2c^2 + 1) \sin(4bx + 4a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(c+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] -2\*b\*d\*integrate((2\*(c^2 + d^2 - 1)\*x^2\*cos(2\*b\*x + 2\*a)^2 + 2\*c\*d\*x^2\*sin(2\*b\*x + 2\*a) + 2\*(c^2 + d^2 - 1)\*x^2\*sin(2\*b\*x + 2\*a)^2 - (c^2 - d^2 - 1)\*x^2\*cos(2\*b\*x + 2\*a) - (2\*c\*d\*x^2\*sin(2\*b\*x + 2\*a) + (c^2 - d^2 - 1)\*x^2\*cos(2\*b\*x + 2\*a))\*cos(4\*b\*x + 4\*a) + (2\*c\*d\*x^2\*cos(2\*b\*x + 2\*a) - (c^2 - d^2 - 1)\*x^2\*sin(2\*b\*x + 2\*a))\*sin(4\*b\*x + 4\*a))/(c^4 + d^4 + 2\*(c^2 + 1)\*d^2 + (c^4 + d^4 + 2\*(c^2 + 1)\*d^2 - 2\*c^2 + 1)\*cos(4\*b\*x + 4\*a)^2 + 4\*(c^4 + d^4 + 2\*(c^2 - 1)\*d^2 - 2\*c^2 + 1)\*sin(4\*b\*x + 4\*a)^2 + 4\*(c^4 + d^4 + 2\*(c^2 - 1)\*d^2 - 2\*c^2 + 1)\*sin(2\*b\*x + 2\*a)^2 - 2\*c^2 + 2\*(c^4 + d^4 - 2\*(3\*c^2 - 1)\*d^2 - 2\*c^2 - 2\*(c^4 - d^4 - 2\*c^2 + 1)\*cos(2\*b\*x + 2\*a) - 4\*(c\*d^3 + (c^3 - c)\*d)\*sin(2\*b\*x + 2\*a) + 1)\*cos(4\*b\*x + 4\*a) - 4\*(c^4 - d^4 - 2\*c^2 + 1)\*cos(2\*b\*x + 2\*a) + 4\*(2\*c\*d^3 - 2\*(c^3 - c)\*d + 2\*(c\*d^3 + (c^3 - c)\*d)\*cos(2\*b\*x + 2\*a) - (c^4 - d^4 - 2\*c^2 + 1)\*sin(2\*b\*x + 2\*a))\*sin(4\*b\*x + 4\*a) + 8\*(c\*d^3 + (c^3 - c)\*d)\*sin(2\*b\*x + 2\*a) + 1), x) + 1/8\*x^2\*log((c^2 + d^2 + 2\*c + 1)\*cos(2\*b\*x + 2\*a)^2 + 4\*(c + 1)\*d\*sin(2\*b\*x + 2\*a) + (c^2 + d^2 + 2\*c + 1)\*sin(2\*b\*x + 2\*a)^2 + c^2 + d^2 - 2\*(c^2 - d^2 + 2\*c + 1)\*cos(2\*b\*x + 2\*a) + 2\*c + 1) - 1/8\*x^2\*log((c^2 + d^2 - 2\*c + 1)\*cos(2\*b\*x + 2\*a)^2 + 4\*(c - 1)\*d\*sin(2\*b\*x + 2\*a) + (c^2 + d^2 - 2\*c + 1)\*sin(2\*b\*x + 2\*a)^2 + c^2 + d^2 - 2\*(c^2 - d^2 - 2\*c + 1)\*cos(2\*b\*x + 2\*a) - 2\*c + 1)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acoth}(c + d \cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(c + d*cot(a + b*x)),x)
```

```
[Out] int(x*acoth(c + d*cot(a + b*x)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x \operatorname{acoth}(c + d \cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(c+d*cot(b*x+a)),x)
```

```
[Out] Integral(x*acoth(c + d*cot(a + b*x)), x)
```

### 3.255 $\int \coth^{-1}(c + d \cot(a + bx)) dx$

**Optimal.** Leaf size=194

$$-\frac{i\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b} + \frac{i\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b} + \frac{1}{2}x \log\left(1 - \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right) - \frac{1}{2}x \log\left(1 - \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)$$

[Out] x\*arccoth(c+d\*cot(b\*x+a))+1/2\*x\*ln(1-(1-c-I\*d)\*exp(2\*I\*a+2\*I\*b\*x)/(1-c+I\*d))-1/2\*x\*ln(1-(1+c+I\*d)\*exp(2\*I\*a+2\*I\*b\*x)/(1+c-I\*d))-1/4\*I\*polylog(2,(1-c-I\*d)\*exp(2\*I\*a+2\*I\*b\*x)/(1-c+I\*d))/b+1/4\*I\*polylog(2,(1+c+I\*d)\*exp(2\*I\*a+2\*I\*b\*x)/(1+c-I\*d))/b

**Rubi [A]** time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6262, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2, \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b} + \frac{i\text{PolyLog}\left(2, \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b} + \frac{1}{2}x \log\left(1 - \frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right) - \frac{1}{2}x \log\left(1 - \frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[c + d\*Cot[a + b\*x]], x]

[Out] x\*ArcCoth[c + d\*Cot[a + b\*x]] + (x\*Log[1 - ((1 - c - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))/(1 - c + I\*d)]/2 - (x\*Log[1 - ((1 + c + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))/(1 + c - I\*d)]/2 - ((I/4)\*PolyLog[2, ((1 - c - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))/(1 - c + I\*d)])/b + ((I/4)\*PolyLog[2, ((1 + c + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x))/(1 + c - I\*d)])/b

#### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_))), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_))], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 6262

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] := Simp[x*ArcCoth[c + d*Cot[a + b*x]], x] + (-Dist[I*b*(1 - c - I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[I*b*(1 + c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2, 1]
```

### Rubi steps

$$\begin{aligned} \int \coth^{-1}(c + d \cot(a + bx)) dx &= x \coth^{-1}(c + d \cot(a + bx)) + (b(i(1 + c) - d)) \int \frac{e^{2ia+2ibx} x}{1 + c - id + (-1 - c - id)e^{2ia}} \\ &= x \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) \\ &= x \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) \\ &= x \coth^{-1}(c + d \cot(a + bx)) + \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) - \frac{1}{2} x \log \left( 1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id} \right) \end{aligned}$$

**Mathematica [B]** time = 13.10, size = 4463, normalized size = 23.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCoth[c + d*Cot[a + b*x]], x]
```

```
[Out] x*ArcCoth[c + d*Cot[a + b*x]] - (d*(a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] + (-1 + c)*Sin[a + b*x]))] - a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] + Sin[a + b*x] + c*Sin[a + b*x]))] - (a + b*x)*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(-I + Tan[(a + b*x)/2]))]/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2]))*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(I + Tan[(a + b*x)/2]))]/(-1 + c + I*d + Sqrt[1 - 2*c + c^2 + d^2]))*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 +
```

$$\begin{aligned}
& d^2)/d) + \tan[(a + b*x)/2] + (a + b*x)*\log[-((1 + c + \sqrt{1 + 2*c + c^2} \\
& + d^2))/d) + \tan[(a + b*x)/2] + I*\log[(d*(-I + \tan[(a + b*x)/2]))/(1 + c \\
& - I*d + \sqrt{1 + 2*c + c^2 + d^2}))*\log[-((1 + c + \sqrt{1 + 2*c + c^2 + d^2} \\
& )/d) + \tan[(a + b*x)/2] - I*\log[(d*(I + \tan[(a + b*x)/2]))/(1 + c + I*d + \\
& \sqrt{1 + 2*c + c^2 + d^2}))*\log[-((1 + c + \sqrt{1 + 2*c + c^2 + d^2}))/d) + \\
& \tan[(a + b*x)/2] - (a + b*x)*\log[(1 - c + \sqrt{1 - 2*c + c^2 + d^2} + d*T \\
& \tan[(a + b*x)/2])/d] - I*\log[-((d*(-I + \tan[(a + b*x)/2]))/(1 - c + I*d + S \\
& \sqrt{1 - 2*c + c^2 + d^2}))*\log[(1 - c + \sqrt{1 - 2*c + c^2 + d^2} + d*\tan[( \\
& a + b*x)/2])/d] + I*\log[-((d*(I + \tan[(a + b*x)/2]))/(1 - c - I*d + \sqrt{1 \\
& - 2*c + c^2 + d^2}))*\log[(1 - c + \sqrt{1 - 2*c + c^2 + d^2} + d*\tan[(a + b \\
& *x)/2])/d] + (a + b*x)*\log[(-1 - c + \sqrt{1 + 2*c + c^2 + d^2} + d*\tan[(a + \\
& b*x)/2])/d] + I*\log[-((d*(-I + \tan[(a + b*x)/2]))/(-1 - c + I*d + \sqrt{1 + \\
& 2*c + c^2 + d^2}))*\log[(-1 - c + \sqrt{1 + 2*c + c^2 + d^2} + d*\tan[(a + b \\
& *x)/2])/d] - I*\log[-((d*(I + \tan[(a + b*x)/2]))/(-1 - c - I*d + \sqrt{1 + 2* \\
& c + c^2 + d^2}))*\log[(-1 - c + \sqrt{1 + 2*c + c^2 + d^2} + d*\tan[(a + b*x) \\
& /2])/d] - I*\text{PolyLog}[2, (-1 + c + \sqrt{1 - 2*c + c^2 + d^2} - d*\tan[(a + b*x) \\
& )/2])/(-1 + c - I*d + \sqrt{1 - 2*c + c^2 + d^2}]] + I*\text{PolyLog}[2, (-1 + c + \\
& \sqrt{1 - 2*c + c^2 + d^2} - d*\tan[(a + b*x)/2])/(-1 + c + I*d + \sqrt{1 - 2* \\
& c + c^2 + d^2}]] - I*\text{PolyLog}[2, (1 + c - \sqrt{1 + 2*c + c^2 + d^2} - d*\tan[ \\
& (a + b*x)/2])/((1 + c + I*d - \sqrt{1 + 2*c + c^2 + d^2}))] + I*\text{PolyLog}[2, (1 \\
& + c + \sqrt{1 + 2*c + c^2 + d^2} - d*\tan[(a + b*x)/2])/((1 + c - I*d + \sqrt{1 \\
& + 2*c + c^2 + d^2}))] - I*\text{PolyLog}[2, (1 + c + \sqrt{1 + 2*c + c^2 + d^2} - d \\
& *\tan[(a + b*x)/2])/((1 + c + I*d + \sqrt{1 + 2*c + c^2 + d^2}))] + I*\text{PolyLog}[2 \\
& , (1 - c + \sqrt{1 - 2*c + c^2 + d^2} + d*\tan[(a + b*x)/2])/((1 - c - I*d + S \\
& \sqrt{1 - 2*c + c^2 + d^2}))] - I*\text{PolyLog}[2, (1 - c + \sqrt{1 - 2*c + c^2 + d^2} \\
& ] + d*\tan[(a + b*x)/2])/((1 - c + I*d + \sqrt{1 - 2*c + c^2 + d^2}))] + I*\text{Poly} \\
& \text{Log}[2, (-1 - c + \sqrt{1 + 2*c + c^2 + d^2} + d*\tan[(a + b*x)/2])/(-1 - c + \\
& I*d + \sqrt{1 + 2*c + c^2 + d^2}))*((2*a)/(b*(1 - c^2 - d^2 - \cos[2*(a + b*x) \\
& ] + c^2*\cos[2*(a + b*x)] - d^2*\cos[2*(a + b*x)] - 2*c*d*\sin[2*(a + b*x) \\
& ]) - (2*(a + b*x))/(b*(1 - c^2 - d^2 - \cos[2*(a + b*x)] + c^2*\cos[2*(a + b*x) \\
& ]) - d^2*\cos[2*(a + b*x)] - 2*c*d*\sin[2*(a + b*x)])))/(-\log[-((-1 + c + S \\
& \sqrt{1 - 2*c + c^2 + d^2}))/d) + \tan[(a + b*x)/2] + \log[-((1 + c + \sqrt{1 + 2 \\
& *c + c^2 + d^2}))/d) + \tan[(a + b*x)/2] - \log[(1 - c + \sqrt{1 - 2*c + c^2 + \\
& d^2} + d*\tan[(a + b*x)/2])/d] + \log[(-1 - c + \sqrt{1 + 2*c + c^2 + d^2} + \\
& d*\tan[(a + b*x)/2])/d] - ((I/2)*\log[-((-1 + c + \sqrt{1 - 2*c + c^2 + d^2}))/ \\
& d) + \tan[(a + b*x)/2]]*\sec[(a + b*x)/2]^2)/(-I + \tan[(a + b*x)/2]) + ((I/2) \\
& *\log[-((1 + c + \sqrt{1 + 2*c + c^2 + d^2}))/d) + \tan[(a + b*x)/2]]*\sec[(a + \\
& b*x)/2]^2)/(-I + \tan[(a + b*x)/2]) - ((I/2)*\log[(1 - c + \sqrt{1 - 2*c + c^2 \\
& + d^2} + d*\tan[(a + b*x)/2])/d]*\sec[(a + b*x)/2]^2)/(-I + \tan[(a + b*x)/2] \\
& ) + ((I/2)*\log[(-1 - c + \sqrt{1 + 2*c + c^2 + d^2} + d*\tan[(a + b*x)/2])/d] \\
& *\sec[(a + b*x)/2]^2)/(-I + \tan[(a + b*x)/2]) + ((I/2)*\log[-((-1 + c + \sqrt{1 \\
& - 2*c + c^2 + d^2}))/d) + \tan[(a + b*x)/2]]*\sec[(a + b*x)/2]^2)/(I + \tan[( \\
& a + b*x)/2]) - ((I/2)*\log[-((1 + c + \sqrt{1 + 2*c + c^2 + d^2}))/d) + \tan[(a \\
& + b*x)/2]]*\sec[(a + b*x)/2]^2)/(I + \tan[(a + b*x)/2]) + ((I/2)*\log[(1 - c \\
& + \sqrt{1 - 2*c + c^2 + d^2} + d*\tan[(a + b*x)/2])/d]*\sec[(a + b*x)/2]^2)/(I
\end{aligned}$$



$$\begin{aligned} & 2]^2)/(-1 - c + \text{Sqrt}[1 + 2*c + c^2 + d^2] + d*\text{Tan}[(a + b*x)/2]) - (a*\text{Cos}[(a \\ & + b*x)/2]^2*(-(\text{Sec}[(a + b*x)/2]^2*(-1 + c)*\text{Cos}[a + b*x] - d*\text{Sin}[a + b*x]) \\ & ) - \text{Sec}[(a + b*x)/2]^2*(d*\text{Cos}[a + b*x] + (-1 + c)*\text{Sin}[a + b*x])* \text{Tan}[(a + b* \\ & x)/2]))/(d*\text{Cos}[a + b*x] + (-1 + c)*\text{Sin}[a + b*x]) + (a*\text{Cos}[(a + b*x)/2]^2*(- \\ & (\text{Sec}[(a + b*x)/2]^2*(\text{Cos}[a + b*x] + c*\text{Cos}[a + b*x] - d*\text{Sin}[a + b*x])) - \text{Sec} \\ & [(a + b*x)/2]^2*(d*\text{Cos}[a + b*x] + \text{Sin}[a + b*x] + c*\text{Sin}[a + b*x])* \text{Tan}[(a + b \\ & *x)/2]))/(d*\text{Cos}[a + b*x] + \text{Sin}[a + b*x] + c*\text{Sin}[a + b*x])) \end{aligned}$$

**fricas [B]** time = 0.75, size = 1098, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{8}*(4*b*x*\log((d*\cos(2*b*x + 2*a) + (c + 1)*\sin(2*b*x + 2*a) + d)/(d*\cos(2*b*x + 2*a) + (c - 1)*\sin(2*b*x + 2*a) + d)) + 2*a*\log(1/2*c^2 + I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a) + c + 1/2) - 2*a*\log(1/2*c^2 + I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a) - c + 1/2) + 2*a*\log(-1/2*c^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a) - c - 1/2) - 2*a*\log(-1/2*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*\log((c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*\sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*\log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*\log((c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*\sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 2*(b*x + a)*\log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + I*\text{dilog}(-(c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*\sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - I*\text{dilog}(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*\sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - I*\text{dilog}(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*\sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + I*\text{dilog}(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*\cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*\sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1))/b$



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(d \cot(bx + a) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*cot(b\*x + a) + c), x)

**maple** [B] time = 0.58, size = 629, normalized size = 3.24

$$-\frac{\operatorname{arccoth}(c + d \cot(bx + a)) \pi}{2b} + \frac{\operatorname{arccoth}(c + d \cot(bx + a)) \operatorname{arccot}(\cot(bx + a))}{b} + \frac{\operatorname{arctan}\left(\frac{c+d \cot(bx+a)}{d} - \frac{c}{d}\right) \ln}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*cot(b\*x+a)),x)

[Out] 
$$-1/2/b*\operatorname{arccoth}(c+d*\cot(b*x+a))*\pi+1/b*\operatorname{arccoth}(c+d*\cot(b*x+a))*\operatorname{arccot}(\cot(b*x+a))+1/2/b*\operatorname{arctan}((c+d*\cot(b*x+a))/d-c/d)*\ln(d*((c+d*\cot(b*x+a))/d-c/d)+c+1)-1/2/b*\operatorname{arctan}((c+d*\cot(b*x+a))/d-c/d)*\ln(d*((c+d*\cot(b*x+a))/d-c/d)+c-1)-1/4*I/b*\ln(d*((c+d*\cot(b*x+a))/d-c/d)+c-1)*\ln((I*d-d*((c+d*\cot(b*x+a))/d-c/d))/(I*d+c-1))+1/4*I/b*\ln(d*((c+d*\cot(b*x+a))/d-c/d)+c-1)*\ln((I*d+d*((c+d*\cot(b*x+a))/d-c/d))/(1-c+I*d))-1/4*I/b*\operatorname{dilog}((I*d-d*((c+d*\cot(b*x+a))/d-c/d))/(I*d+c-1))+1/4*I/b*\operatorname{dilog}((I*d+d*((c+d*\cot(b*x+a))/d-c/d))/(1-c+I*d))+1/4*I/b*\ln(d*((c+d*\cot(b*x+a))/d-c/d)+c+1)*\ln((I*d-d*((c+d*\cot(b*x+a))/d-c/d))/(1+c+I*d))-1/4*I/b*\ln(d*((c+d*\cot(b*x+a))/d-c/d)+c+1)*\ln((I*d+d*((c+d*\cot(b*x+a))/d-c/d))/(I*d-c-1))+1/4*I/b*\operatorname{dilog}((I*d-d*((c+d*\cot(b*x+a))/d-c/d))/(1+c+I*d))-1/4*I/b*\operatorname{dilog}((I*d+d*((c+d*\cot(b*x+a))/d-c/d))/(I*d-c-1))$$

**maxima** [B] time = 0.54, size = 392, normalized size = 2.02

$$4(bx + a) \operatorname{arccoth}\left(c + \frac{d}{\tan(bx+a)}\right) + \left(\operatorname{arctan}\left(\frac{(c+1)d+(c^2+2c+1)\tan(bx+a)}{c^2+d^2+2c+1}, \frac{(c+1)d \tan(bx+a)+d^2}{c^2+d^2+2c+1}\right) - \operatorname{arctan}\left(\frac{(c-1)d+(c^2-2c+1)}{c^2+d^2-2c+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] 
$$1/4*(4*(bx + a)*\operatorname{arccoth}(c + d/\tan(b*x + a)) + (\operatorname{arctan}2(((c + 1)*d + (c^2 + 2*c + 1)*\tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*\tan(b*x + a) + d^2)/(c^2 + d^2 + 2*c + 1)) - \operatorname{arctan}2(((c - 1)*d + (c^2 - 2*c + 1)*\tan(b*x + a) + d^2)/(c^2 + d^2 - 2*c + 1))$$

$a)/((c^2 + d^2 - 2*c + 1), ((c - 1)*d*\tan(b*x + a) + d^2)/(c^2 + d^2 - 2*c + 1)))*\log(\tan(b*x + a)^2 + 1) - (b*x + a)*\log((2*(c + 1)*d*\tan(b*x + a) + (c^2 + 2*c + 1)*\tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*\log((2*(c - 1)*d*\tan(b*x + a) + (c^2 - 2*c + 1)*\tan(b*x + a)^2 + d^2)/(c^2 + d^2 - 2*c + 1)) + I*\operatorname{dilog}(-((c + 1)*\tan(b*x + a) - I*c - I)/(I*c + d + I)) - I*\operatorname{dilog}(-((c - 1)*\tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*\operatorname{dilog}(-((c - 1)*\tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*\operatorname{dilog}(-((c + 1)*\tan(b*x + a) + I*c + I)/(-I*c + d - I)))/b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}(c + d \cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(c + d*cot(a + b*x)),x)`

[Out] `int(acoth(c + d*cot(a + b*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(c + d \cot(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(c+d*cot(b*x+a)),x)`

[Out] `Integral(acoth(c + d*cot(a + b*x)), x)`

$$3.256 \quad \int \frac{\coth^{-1}(c+d \cot(a+bx))}{x} dx$$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\coth^{-1}(d \cot(a + bx) + c)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(c+d\*cot(b\*x+a))/x,x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(c + d \cot(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[c + d\*Cot[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[c + d\*Cot[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(c + d \cot(a + bx))}{x} dx = \int \frac{\coth^{-1}(c + d \cot(a + bx))}{x} dx$$

Mathematica [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(c + d \cot(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[c + d\*Cot[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[c + d\*Cot[a + b\*x]]/x, x]

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcoth}(d \cot(bx + a) + c)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(arccoth(d\*cot(b\*x + a) + c)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \cot(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*cot(b\*x + a) + c)/x, x)

**maple** [A] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(c + d \cot(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(c+d\*cot(b\*x+a))/x,x)

[Out] int(arccoth(c+d\*cot(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}(d \cot(bx + a) + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(c+d\*cot(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(arccoth(d\*cot(b\*x + a) + c)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{acoth}(c + d \cot(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(c + d\*cot(a + b\*x))/x,x)

[Out] int(acoth(c + d\*cot(a + b\*x))/x, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(c + d \cot(a + bx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(c+d\*cot(b\*x+a))/x,x)

[Out] Integral(acoth(c + d\*cot(a + b\*x))/x, x)

### 3.257 $\int x^2 \coth^{-1}(1 + id + d \cot(a + bx)) dx$

Optimal. Leaf size=168

$$-\frac{i\text{Li}_4((id+1)e^{2ia+2ibx})}{8b^3} - \frac{x\text{Li}_3((id+1)e^{2ia+2ibx})}{4b^2} + \frac{ix^2\text{Li}_2((id+1)e^{2ia+2ibx})}{4b} - \frac{1}{6}x^3 \log(1 - (1+id)e^{2ia+2ibx}) + \frac{1}{3}x^3 \coth^{-1}(1 + id + d \cot(a + bx))$$

[Out] 1/12\*I\*b\*x^4+1/3\*x^3\*arccoth(1+I\*d+d\*cot(b\*x+a))-1/6\*x^3\*ln(1-(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))+1/4\*I\*x^2\*polylog(2,(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b-1/4\*x\*polylog(3,(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^2-1/8\*I\*polylog(4,(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^3

**Rubi [A]** time = 0.31, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {6266, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}(3,(1+id)e^{2ia+2ibx})}{4b^2} - \frac{i\text{PolyLog}(4,(1+id)e^{2ia+2ibx})}{8b^3} + \frac{ix^2\text{PolyLog}(2,(1+id)e^{2ia+2ibx})}{4b} - \frac{1}{6}x^3 \log(1 - (1+id)e^{2ia+2ibx}) + \frac{1}{3}x^3 \coth^{-1}(1 + id + d \cot(a + bx))$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]],x]

[Out] (I/12)\*b\*x^4 + (x^3\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/3 - (x^3\*Log[1 - (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/6 + ((I/4)\*x^2\*PolyLog[2, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b - (x\*PolyLog[3, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b^2 - ((I/8)\*PolyLog[4, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)])/b^3

#### Rule 2184

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6266

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
+ 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 + id + d \cot(a + bx)) dx &= \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) + \frac{1}{3} (ib) \int \frac{x^3}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) + \frac{1}{3} (b(i - d)) \int \frac{e^{2ia+2ibx}}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx}) \\
&= \frac{1}{12} ibx^4 + \frac{1}{3} x^3 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{6} x^3 \log(1 - (1 + id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 155, normalized size = 0.92

$$\frac{1}{3} x^3 \coth^{-1}(d \cot(a+bx)+id+1) - \frac{4b^3 x^3 \log\left(1 + \frac{ie^{-2i(a+bx)}}{d-i}\right) + 6ib^2 x^2 \text{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right) + 6bx \text{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right) - 3i \text{Li}_4\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]], x]

[Out] (x^3\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/3 - (4\*b^3\*x^3\*Log[1 + I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + (6\*I)\*b^2\*x^2\*PolyLog[2, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + 6\*b\*x\*PolyLog[3, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))] - (3\*I)\*PolyLog[4, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))])/(24\*b^3)

**fricas [C]** time = 0.67, size = 179, normalized size = 1.07

$$\frac{2i b^4 x^4 + 4 b^3 x^3 \log\left(\frac{((d-i)e^{2i bx+2i a}+i)e^{-2i bx-2i a}}{d}\right) + 6i b^2 x^2 \text{Li}_2\left(-(-i d - 1)e^{(2i bx+2i a)}\right) - 2i a^4 + 4 a^3 \log\left(\frac{(d-i)e^{(2i bx+2i a)}}{d-i}\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+I\*d+d\*cot(b\*x+a)), x, algorithm="fricas")



```
[Out] 1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I
*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I
*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*polylog
(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e^(2*
I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(d \cot(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccoth(d*cot(b*x + a) + I*d + 1), x)
```

**maple** [C] time = 6.23, size = 2449, normalized size = 14.58

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I-d)*polylog
(2,-I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/2/b^3*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a)
)*(I*(I-d))^(1/2))-1/2/b^3*a^2/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/
2))+1/6*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/3*x^3*ln(exp(I*(b*x+a)
))+1/12*I*b*x^4-1/4*I/b*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/
4*I/b^3*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2/(I-d)*ln
(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a^2-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I
*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I*exp(2*I*(
b*x+a)))^3-1/6*x^3*ln(d)+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(
2*I*(b*x+a)))-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^
2-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/8/b^3/(I-
d)*polylog(4,-I*(I-d)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+
a))-1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2
*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/6*I/(I-d)*ln(1+I*(I-d)*exp(2*I*(b
*x+a)))*x^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1))^3+1/
12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi
*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/1
2*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*
I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b
*x+a))-1))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-
1))*csgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*d)*c
sgn(I*exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*
```

```

exp(2*I*(b*x+a))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1
))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/2*I/b^2*a^2/(I-d)*ln
(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*
x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^3*a^2*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*
(I-d))^(1/2))-1/2*I/b^3*a^2*d/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2
))-1/3/b^3*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^3+1/4/b^2*d/(I-d)*polyl
og(3,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-
1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/2/b^
3*a^3*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2/b^3*a^3*d/(I-d)*ln
(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/6/b^3*a^3*d/(I-d)*ln(I*exp(2*I*(b*x+
a))-exp(2*I*(b*x+a))*d-I)-1/2*I/b^3*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d
))^(1/2))+1/6*I/b^3*a^3/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1
/8*I/b^3*d/(I-d)*polylog(4,-I*(I-d)*exp(2*I*(b*x+a)))-1/4*I/b^2/(I-d)*polyl
og(3,-I*(I-d)*exp(2*I*(b*x+a)))*x+1/3*I/b^3/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x
+a)))*a^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csg
n(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b
*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a)
)*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/2*I/b^3*a^3/(I-d)*ln(1-I
*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b^2*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+
a)))*x*a^2+1/2/b^2*a^2*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x+1/2
/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x+1/6*x^3*ln(exp(2*
I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d
-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp
(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a)
))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I
*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/12*I*x
^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1)
)^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp
(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*e
xp(2*I*(b*x+a)))^2

```

**maxima** [B] time = 0.38, size = 342, normalized size = 2.04

$$\frac{12((bx+a)^3 - 3(bx+a)^2a + 3(bx+a)a^2) \operatorname{arccoth}(d \cot(bx+a) + id + 1)}{b^2} - \frac{-3i(bx+a)^4 + 12i(bx+a)^3a - 18i(bx+a)^2a^2 + (-8i(bx+a)^3 + 18i(bx+a)^2a - 18i(bx+a)a^2)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] 1/36\*(12\*((b\*x + a)^3 - 3\*(b\*x + a)^2\*a + 3\*(b\*x + a)\*a^2)\*arccoth(d\*cot(b\*x + a) + I\*d + 1)/b^2 - (-3\*I\*(b\*x + a)^4 + 12\*I\*(b\*x + a)^3\*a - 18\*I\*(b\*x + a)^2\*a^2 + (-8\*I\*(b\*x + a)^3 + 18\*I\*(b\*x + a)^2\*a - 18\*I\*(b\*x + a)\*a^2)\*arctan2(d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), d\*sin(2\*b\*x + 2\*a) - cos(2\*b\*

```
x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((I*d
+ 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
+ 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(d \cot(a + bx) + 1 + d1i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(d*1i + d*cot(a + b*x) + 1), x)
```

```
[Out] int(x^2*acoth(d*1i + d*cot(a + b*x) + 1), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(d \cot(a + bx) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1+I*d+d*cot(b*x+a)), x)
```

```
[Out] Integral(x**2*acoth(d*cot(a + b*x) + I*d + 1), x)
```

### 3.258 $\int x \coth^{-1}(1 + id + d \cot(a + bx)) dx$

**Optimal.** Leaf size=132

$$-\frac{\operatorname{Li}_3((id+1)e^{2ia+2ibx})}{8b^2} + \frac{ix\operatorname{Li}_2((id+1)e^{2ia+2ibx})}{4b} - \frac{1}{4}x^2 \log(1 - (1+id)e^{2ia+2ibx}) + \frac{1}{2}x^2 \coth^{-1}(d \cot(a+bx)+id+1) +$$

[Out] 1/6\*I\*b\*x^3+1/2\*x^2\*arccoth(1+I\*d+d\*cot(b\*x+a))-1/4\*x^2\*ln(1-(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))+1/4\*I\*x\*polylog(2,(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b-1/8\*polylog(3,(1+I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^2

**Rubi [A]** time = 0.26, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6266, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\operatorname{PolyLog}(3,(1+id)e^{2ia+2ibx})}{8b^2} + \frac{ix\operatorname{PolyLog}(2,(1+id)e^{2ia+2ibx})}{4b} - \frac{1}{4}x^2 \log(1 - (1+id)e^{2ia+2ibx}) + \frac{1}{2}x^2 \coth^{-1}(d \cot(a+bx)+id+1) +$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]],x]

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/2 - (x^2\*Log[1 - (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/b - PolyLog[3, (1 + I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/(8\*b^2))

#### Rule 2184

Int[(((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x))))^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
)))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6266

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m\_  
.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Cot[a + b\*x]])/(f\*(m  
+ 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - I\*d - c\*E^(  
2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&  
EqQ[(c - I\*d)^2, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(1 + id + d \cot(a + bx)) dx &= \frac{1}{2} x^2 \coth^{-1}(1 + id + d \cot(a + bx)) + \frac{1}{2} (ib) \int \frac{x^2}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id + d \cot(a + bx)) + \frac{1}{2} (b(i - d)) \int \frac{e^{2ia+2ibx}}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 + id)e^{2ia+2ibx}) \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 + id)e^{2ia+2ibx}) \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 + id)e^{2ia+2ibx}) \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 + id)e^{2ia+2ibx})
 \end{aligned}$$

**Mathematica** [A] time = 0.12, size = 119, normalized size = 0.90

$$\frac{1}{2}x^2 \operatorname{coth}^{-1}(d \cot(ax+bx)+id+1) - \frac{2b^2x^2 \log\left(1 + \frac{ie^{-2i(a+bx)}}{d-i}\right) + 2ibx \operatorname{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right) + \operatorname{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]], x]

[Out] (x^2\*ArcCoth[1 + I\*d + d\*Cot[a + b\*x]])/2 - (2\*b^2\*x^2\*Log[1 + I/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + (2\*I)\*b\*x\*PolyLog[2, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))] + PolyLog[3, (-I)/((-I + d)\*E^((2\*I)\*(a + b\*x)))])/(8\*b^2)

**fricas** [C] time = 0.73, size = 156, normalized size = 1.18

$$\frac{4i b^3 x^3 + 6 b^2 x^2 \log\left(\frac{((d-i)e^{2i bx+2i a}+i)e^{(-2i bx-2i a)}}{d}\right) + 4i a^3 + 6i bx \operatorname{Li}_2\left(-(-i d - 1)e^{(2i bx+2i a)}\right) - 6 a^2 \log\left(\frac{(d-i)e^{(2i bx+2i a)}+i}{d-i}\right)}{24 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d+d\*cot(b\*x+a)), x, algorithm="fricas")

[Out] 1/24\*(4\*I\*b^3\*x^3 + 6\*b^2\*x^2\*log(((d - I)\*e^(2\*I\*b\*x + 2\*I\*a) + I)\*e^(-2\*I\*b\*x - 2\*I\*a)/d) + 4\*I\*a^3 + 6\*I\*b\*x\*dilog(-(-I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a)) - 6\*a^2\*log(((d - I)\*e^(2\*I\*b\*x + 2\*I\*a) + I)/(d - I)) - 6\*(b^2\*x^2 - a^2)\*log((-I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a) + 1) - 3\*polylog(3, (I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)))/b^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(d \cot(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d+d\*cot(b\*x+a)), x, algorithm="giac")

[Out] integrate(x\*arccoth(d\*cot(b\*x + a) + I\*d + 1), x)

**maple** [C] time = 5.27, size = 2351, normalized size = 17.81

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1+I\*d+d\*cot(b\*x+a)), x)

```
[Out] 1/2*I/b^2*a*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2*I/b^2*a*d
/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/4*I/b*d/(I-d)*polylog(2,
-I*(I-d)*exp(2*I*(b*x+a)))*x-1/4*I/b^2*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(
b*x+a)))*a-1/2*I/b/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a+1/8*I*x^2*Pi*cs
gn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/8*I*
x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))
^2+1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a)
))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*d
)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1
))*exp(2*I*(b*x+a))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/4/b^2*d/(I-d)
*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/4*x^2*ln(d)-1/2/b^2*a^2*d/(I-d)*ln(1-
I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))
*(I*(I-d))^(1/2))-1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/4/b^
2/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a+1/4*d/(I-d)*ln(1+I*(I-d)*exp
(2*I*(b*x+a)))*x^2+1/2/b^2*a/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2)
)-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b
*x+a))-1))^3-1/8*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))-1/4*I/(I-
d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a)
))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x
+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))-1/8*I*x^2*Pi*csgn(d/(exp
(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)
*exp(2*I*(b*x+a)))^2-1/2*x^2*ln(exp(I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*d/(exp
(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x
+a)))+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/8*I
*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/4*x^2
*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(
b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+
a))+I)/(exp(2*I*(b*x+a))-1))^2+1/2/b^2*a/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*
(I-d))^(1/2))+1/8/b^2*d/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))+1/8*I*x^
2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/4/b^2*a^2*d/(I-d)*ln
(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+
a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I/(e
xp(2*I*(b*x+a))-1)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x
^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+
a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b
*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*c
sgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))-1))^2-1/2/b*a*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(
1/2))*x-1/2/b*a*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x+1/2/b*d/(I
-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^
2*csgn(I*exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2
*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp
(2*I*(b*x+a))-1))^2-1/4*I/b^2*a^2/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+
a))*d-I)-1/4*I/b^2/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2*a^2/(
```

$$\begin{aligned} & (I-d) \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a)) \cdot (I \cdot (I-d))^{1/2}) + 1/2 \cdot I/b^2 \cdot a^2 / (I-d) \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a)) \cdot (I \cdot (I-d))^{1/2}) \\ & - 1/8 \cdot I \cdot x^2 \cdot \pi \cdot \operatorname{csgn}(I \cdot (\exp(2 \cdot I \cdot (b \cdot x + a)) \cdot d - I \cdot \exp(2 \cdot I \cdot (b \cdot x + a)) + I) / (\exp(2 \cdot I \cdot (b \cdot x + a)) - 1)) \\ & \cdot \operatorname{csgn}((\exp(2 \cdot I \cdot (b \cdot x + a)) \cdot d - I \cdot \exp(2 \cdot I \cdot (b \cdot x + a)) + I) / (\exp(2 \cdot I \cdot (b \cdot x + a)) - 1)) \\ & + 1/2 \cdot I/b \cdot a / (I-d) \cdot \ln(1 + I \cdot \exp(I \cdot (b \cdot x + a)) \cdot (I \cdot (I-d))^{1/2}) \\ & \cdot x + 1/2 \cdot I/b \cdot a / (I-d) \cdot \ln(1 - I \cdot \exp(I \cdot (b \cdot x + a)) \cdot (I \cdot (I-d))^{1/2}) \cdot x \end{aligned}$$

**maxima** [B] time = 0.37, size = 248, normalized size = 1.88

$$\frac{12((bx+a)^2 - 2(bx+a)a) \operatorname{arccoth}(d \cot(bx+a) + id + 1)}{b} - \frac{-4i(bx+a)^3 + 12i(bx+a)^2 a - 6i b x \operatorname{Li}_2((id+1)e^{2i bx + 2i a}) + (-6i(bx+a)^2 + 12i(bx+a)a) \operatorname{arctan}(d \cos(2bx + 2a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="maxima")

[Out]  $\frac{1}{24} \cdot (12 \cdot ((b \cdot x + a)^2 - 2 \cdot (b \cdot x + a) \cdot a) \cdot \operatorname{arccoth}(d \cdot \cot(b \cdot x + a) + I \cdot d + 1) / b - (-4 \cdot I \cdot (b \cdot x + a)^3 + 12 \cdot I \cdot (b \cdot x + a)^2 \cdot a - 6 \cdot I \cdot b \cdot x \cdot \operatorname{dilog}((I \cdot d + 1) \cdot e^{(2 \cdot I \cdot b \cdot x + 2 \cdot I \cdot a)}) + (-6 \cdot I \cdot (b \cdot x + a)^2 + 12 \cdot I \cdot (b \cdot x + a) \cdot a) \cdot \operatorname{arctan}^2(d \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + \sin(2 \cdot b \cdot x + 2 \cdot a), d \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) - \cos(2 \cdot b \cdot x + 2 \cdot a) + 1) + 3 \cdot ((b \cdot x + a)^2 - 2 \cdot (b \cdot x + a) \cdot a) \cdot \log((d^2 + 1) \cdot \cos(2 \cdot b \cdot x + 2 \cdot a)^2 + (d^2 + 1) \cdot \sin(2 \cdot b \cdot x + 2 \cdot a)^2 + 2 \cdot d \cdot \sin(2 \cdot b \cdot x + 2 \cdot a) - 2 \cdot \cos(2 \cdot b \cdot x + 2 \cdot a) + 1) + 3 \cdot \operatorname{polylog}(3, (I \cdot d + 1) \cdot e^{(2 \cdot I \cdot b \cdot x + 2 \cdot I \cdot a)}) / b) / b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(d \cot(a + b x) + 1 + d 1i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(d\*1i + d\*cot(a + b\*x) + 1),x)

[Out] int(x\*acoth(d\*1i + d\*cot(a + b\*x) + 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(d \cot(a + b x) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1+I\*d+d\*cot(b\*x+a)),x)

[Out] Integral(x\*acoth(d\*cot(a + b\*x) + I\*d + 1), x)



### 3.259 $\int \coth^{-1}(1 + id + d \cot(a + bx)) dx$

Optimal. Leaf size=93

$$\frac{i\text{Li}_2\left(\frac{(id+1)e^{2ia+2ibx}}{4b}\right) - \frac{1}{2}x \log\left(1 - (1+id)e^{2ia+2ibx}\right) + x \coth^{-1}(d \cot(a+bx) + id + 1) + \frac{1}{2}ibx^2}{4b}$$

[Out]  $\frac{1}{2}I*b*x^2 + x*\text{arccoth}(1+I*d+d*\cot(b*x+a)) - \frac{1}{2}*x*\ln(1-(1+I*d)*\exp(2*I*a+2*I*b*x)) + \frac{1}{4}*I*\text{polylog}(2, (1+I*d)*\exp(2*I*a+2*I*b*x))/b$

**Rubi** [A] time = 0.16, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6258, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2, \frac{(1+id)e^{2ia+2ibx}}{4b}\right) - \frac{1}{2}x \log\left(1 - (1+id)e^{2ia+2ibx}\right) + x \coth^{-1}(d \cot(a+bx) + id + 1) + \frac{1}{2}ibx^2}{4b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]], x]

[Out]  $(I/2)*b*x^2 + x*\text{ArcCoth}[1 + I*d + d*\text{Cot}[a + b*x]] - (x*\text{Log}[1 - (1 + I*d)*E^{((2*I)*a + (2*I)*b*x)}])/2 + ((I/4)*\text{PolyLog}[2, (1 + I*d)*E^{((2*I)*a + (2*I)*b*x)}])/b$

#### Rule 2184

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[((c + d\*x)^m\*(F^(g\*(e + f\*x)))^n)/(a + b\*(F^(g\*(e + f\*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(g\_.)\*((c\_.) + (d\_.)\*(x\_)))]^(n\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

`Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

Rule 6258

`Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)])*(d_.)], x_Symbol] :> Simp[x*ArcCoth[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]`

Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(1 + id + d \cot(a + bx)) dx &= x \coth^{-1}(1 + id + d \cot(a + bx)) + (ib) \int \frac{x}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
 &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id + d \cot(a + bx)) + (b(i - d)) \int \frac{e^{2ia+2ibx} x}{1 + (-1 - id)e^{2ia+2ibx}} dx \\
 &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 + id)e^{2ia+2ibx}) + \\
 &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 + id)e^{2ia+2ibx}) - \\
 &= \frac{1}{2}ibx^2 + x \coth^{-1}(1 + id + d \cot(a + bx)) - \frac{1}{2}x \log(1 - (1 + id)e^{2ia+2ibx}) +
 \end{aligned}$$

**Mathematica [B]** time = 3.79, size = 709, normalized size = 7.62

$$\frac{x \csc^2(a + bx)(\cos(bx) - i \sin(bx))(\cos(bx) + i \sin(bx)) \left( i \cot(a + bx) + i \right) (d \cot(a + bx) + id + 2) \left( \frac{(d-2i) \cos(a+bx)(\log(1-i \tan(bx))-\log(1+i \tan(bx)))}{d \cos(a+bx)+(2+id) \sin(a+bx)} + \frac{d \sin(a+bx)(\log(1-i \tan(bx))-\log(1+i \tan(bx)))}{(d-2i) \sin(a+bx)-id \cos(a+bx)} \right)}{1}$$

Warning: Unable to verify antiderivative.

`[In] Integrate[ArcCoth[1 + I*d + d*Cot[a + b*x]], x]`

`[Out] x*ArcCoth[1 + I*d + d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])]/(2*(-I + d))] * Log[1 - I*Tan[b*x]] - I*Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])/2] * Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - I*PolyLog[2, (Sec[b*x]*((2 + I*d)*Cos[a] - d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))])`

$$\frac{\begin{aligned} & *x)))/2] + I*\text{PolyLog}[2, ((\text{Cos}[a] - I*\text{Sin}[a])*((-2 - I*d)*\text{Cos}[a] + d*\text{Sin}[a]) \\ & *(I + \text{Tan}[b*x]))/(2*(-I + d)))]*(\text{Cos}[b*x] - I*\text{Sin}[b*x])*(\text{Cos}[b*x] + I*\text{Sin}[b \\ & *x]))/((I + \text{Cot}[a + b*x])*(2 + I*d + d*\text{Cot}[a + b*x]))*((2*I)*b*x + \text{Log}[1 + ( \\ & \text{Sec}[b*x]*((-2 - I*d)*\text{Cos}[a] + d*\text{Sin}[a])*(\text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]))/2] \\ & - \text{Log}[(\text{Sec}[b*x]*((-I)*\text{Cos}[a] + \text{Sin}[a])*(d*\text{Cos}[a + b*x] + (2 + I*d)*\text{Sin}[a + \\ & b*x]))/2] + ((-2*I + d)*\text{Cos}[a + b*x]*(\text{Log}[1 - I*\text{Tan}[b*x]] - \text{Log}[1 + I*\text{Tan}[ \\ & b*x]])))/(d*\text{Cos}[a + b*x] + (2 + I*d)*\text{Sin}[a + b*x]) + (d*(\text{Log}[1 - I*\text{Tan}[b*x]] \\ & - \text{Log}[1 + I*\text{Tan}[b*x]])*\text{Sin}[a + b*x])/((-I)*d*\text{Cos}[a + b*x] + (-2*I + d)*\text{Sin} \\ & [a + b*x]) + 2*b*x*\text{Tan}[b*x] - I*\text{Log}[1 + (\text{Sec}[b*x]*((-2 - I*d)*\text{Cos}[a] + d*\text{Si} \\ & n[a])*(\text{Cos}[a + b*x] + I*\text{Sin}[a + b*x]))/2]*\text{Tan}[b*x] + I*\text{Log}[(\text{Sec}[b*x]*((-I)* \\ & \text{Cos}[a] + \text{Sin}[a])*(d*\text{Cos}[a + b*x] + (2 + I*d)*\text{Sin}[a + b*x]))/2]*\text{Tan}[b*x] - I \\ & *\text{Log}[1 - I*\text{Tan}[b*x]]*\text{Tan}[b*x] + I*\text{Log}[1 + I*\text{Tan}[b*x]]*\text{Tan}[b*x]) \end{aligned}}$$

**fricas [A]** time = 0.52, size = 121, normalized size = 1.30

$$\frac{2i b^2 x^2 + 2 b x \log\left(\frac{(d-i)e^{2i b x+2i a}+i}{d}e^{(-2i b x-2i a)}\right) - 2i a^2 - 2(bx+a) \log((-i d-1)e^{2i b x+2i a}+1) + 2 a \log\left(\frac{(d-i)e^{2i b x+2i a}+i}{d-i}\right)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="fricas")

[Out]  $\frac{1}{4}*(2*I*b^2*x^2 + 2*b*x*\log(((d - I)*e^{(2*I*b*x + 2*I*a)} + I)*e^{(-2*I*b*x - 2*I*a)}/d) - 2*I*a^2 - 2*(b*x + a)*\log((-I*d - 1)*e^{(2*I*b*x + 2*I*a)} + 1) + 2*a*\log(((d - I)*e^{(2*I*b*x + 2*I*a)} + I)/(d - I)) + I*\text{dilog}(-(-I*d - 1)*e^{(2*I*b*x + 2*I*a)}))/b$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \text{arccoth}(d \cot(bx + a) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d+d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(d\*cot(b\*x + a) + I\*d + 1), x)

**maple [B]** time = 0.76, size = 299, normalized size = 3.22

$$\frac{i \text{arccoth}(1 + id + d \cot(bx + a)) \ln(id + d \cot(bx + a))}{2b} + \frac{i \text{arccoth}(1 + id + d \cot(bx + a)) \ln(id - d \cot(bx + a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+I\*d+d\*cot(b\*x+a)),x)

```
[Out] -1/2*I/b*arccoth(1+I*d+d*cot(b*x+a))*ln(I*d+d*cot(b*x+a))+1/2*I/b*arccoth(1
+I*d+d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))-1/8*I/b*ln(I*d+d*cot(b*x+a))^2+1/4*
I/b*dilog(1+1/2*I*d+1/2*d*cot(b*x+a))+1/4*I/b*ln(I*d+d*cot(b*x+a))*ln(1+1/2
*I*d+1/2*d*cot(b*x+a))-1/4*I/b*dilog((-2-I*d-d*cot(b*x+a))/(-2*I*d-2))-1/4*
I/b*ln(I*d-d*cot(b*x+a))*ln((-2-I*d-d*cot(b*x+a))/(-2*I*d-2))+1/4*I/b*dilog
(1/2*I*(-I*d-d*cot(b*x+a))/d)+1/4*I/b*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d
*cot(b*x+a))/d)
```

**maxima** [B] time = 0.45, size = 286, normalized size = 3.08

$$4(bx+a)d\left(\frac{\log((id+2)\tan(bx+a)+d)}{d} - \frac{\log(i\tan(bx+a)+1)}{d}\right) + d\left(-\frac{2i\left(\log((id+2)\tan(bx+a)+d)\log\left(\frac{(d-2i)\tan(bx+a)-id}{2id+2}+1\right)+\text{Li}_2\left(-\frac{(d-2i)\tan(bx+a)-id}{2id+2}\right)\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccoth(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
+ 1)/d) + d*(-2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*x
+ a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*I*d
+ 2)))/d - 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*x +
a) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d + (2*I*log((I
*d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) +
1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dil
og(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arccoth(I*d + d/tan(b*x + a)
+ 1))/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \text{acoth}(d \cot(a + bx) + 1 + d1i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(d*1i + d*cot(a + b*x) + 1),x)
```

```
[Out] int(acoth(d*1i + d*cot(a + b*x) + 1), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{acoth}(d \cot(a + bx) + id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Integral(acoth(d*cot(a + b*x) + I*d + 1), x)
```

$$3.260 \quad \int \frac{\coth^{-1}(1+id+d \cot(a+bx))}{x} dx$$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\coth^{-1}(d \cot(a+bx) + id + 1)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(1+I\*d+d\*cot(b\*x+a))/x,x)

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(1 + id + d \cot(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1 + id + d \cot(a + bx))}{x} dx = \int \frac{\coth^{-1}(1 + id + d \cot(a + bx))}{x} dx$$

**Mathematica [A]** time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1 + id + d \cot(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 + I\*d + d\*Cot[a + b\*x]]/x, x]

**fricas [A]** time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(\frac{((d-i)e^{2ibx+2ia}+i)e^{-2ibx-2ia}}{d}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d+d\*cot(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(1/2\*log(((d - I)\*e^(2\*I\*b\*x + 2\*I\*a) + I)\*e^(-2\*I\*b\*x - 2\*I\*a)/d)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}\left(\frac{d \cot(bx + a) + id + 1}{x}\right) dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d+d\*cot(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(d\*cot(b\*x + a) + I\*d + 1)/x, x)

**maple** [A] time = 2.06, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}\left(1 + id + \frac{d \cot(bx + a)}{x}\right) dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1+I\*d+d\*cot(b\*x+a))/x,x)

[Out] int(arccoth(1+I\*d+d\*cot(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-ibx + \frac{1}{4}(-i\pi - 4ia - 2\log(d))\log(x) + \frac{1}{2}i \int \frac{\arctan\left(\frac{d \cos(2bx + 2a) + \sin(2bx + 2a)}{x}, -d \sin(2bx + 2a) + \cos(2bx + 2a)\right) dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1+I\*d+d\*cot(b\*x+a))/x,x, algorithm="maxima")

[Out] -I\*b\*x + 1/4\*(-I\*pi - 4\*I\*a - 2\*log(d))\*log(x) + 1/2\*I\*integrate(arctan2(d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) + cos(2\*b\*x + 2\*a) - 1)/x, x) + 1/4\*integrate(log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 + 2\*d\*sin(2\*b\*x + 2\*a) - 2\*cos(2\*b\*x + 2\*a) + 1)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{acoth}\left(\frac{d \cot(a + bx) + 1 + d i}{x}\right) dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(d*1i + d*cot(a + b*x) + 1)/x,x)`

[Out] `int(acoth(d*1i + d*cot(a + b*x) + 1)/x, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(d \cot(a + bx) + id + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(1+I*d+d*cot(b*x+a))/x,x)`

[Out] `Integral(acoth(d*cot(a + b*x) + I*d + 1)/x, x)`

### 3.261 $\int x^2 \coth^{-1}(1 - id - d \cot(a + bx)) dx$

**Optimal.** Leaf size=169

$$-\frac{i\text{Li}_4((1-id)e^{2ia+2ibx})}{8b^3} - \frac{x\text{Li}_3((1-id)e^{2ia+2ibx})}{4b^2} + \frac{ix^2\text{Li}_2((1-id)e^{2ia+2ibx})}{4b} - \frac{1}{6}x^3 \log(1 - (1-id)e^{2ia+2ibx}) + \frac{1}{3}x^3 \cot$$

[Out]  $1/12*I*b*x^4 + 1/3*x^3*\text{arccoth}(1-I*d-d*\cot(b*x+a)) - 1/6*x^3*\ln(1-(1-I*d)*\exp(2*I*a+2*I*b*x)) + 1/4*I*x^2*\text{polylog}(2, (1-I*d)*\exp(2*I*a+2*I*b*x))/b - 1/4*x*\text{polylog}(3, (1-I*d)*\exp(2*I*a+2*I*b*x))/b^2 - 1/8*I*\text{polylog}(4, (1-I*d)*\exp(2*I*a+2*I*b*x))/b^3$

**Rubi [A]** time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6266, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}(3, (1-id)e^{2ia+2ibx})}{4b^2} - \frac{i\text{PolyLog}(4, (1-id)e^{2ia+2ibx})}{8b^3} + \frac{ix^2\text{PolyLog}(2, (1-id)e^{2ia+2ibx})}{4b} - \frac{1}{6}x^3 \log(1 - (1-id)e^{2ia+2ibx})$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{ArcCoth}[1 - I*d - d*\text{Cot}[a + b*x]], x]$

[Out]  $(I/12)*b*x^4 + (x^3*\text{ArcCoth}[1 - I*d - d*\text{Cot}[a + b*x]])/3 - (x^3*\text{Log}[1 - (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)}])/6 + ((I/4)*x^2*\text{PolyLog}[2, (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)}])/b - (x*\text{PolyLog}[3, (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)}])/(4*b^2) - ((I/8)*\text{PolyLog}[4, (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)}])/b^3$

#### Rule 2184

$\text{Int}[\frac{(c + d*x)^m}{(a + b*(F^g(e + f*x)))^n}, x\_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^{m+1}}{a*d*(m+1)}, x] - \text{Dist}[\frac{b}{a}, \text{Int}[\frac{(c + d*x)^m*(F^g(e + f*x))^n}{(a + b*(F^g(e + f*x)))^n}, x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

$\text{Int}[\frac{(F^g(e + f*x))^n*(c + d*x)^m}{(a + b*(F^g(e + f*x)))^n}, x\_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^m*\text{Log}[1 + (b*(F^g(e + f*x)))^n/a]}{b*f*g*n*\text{Log}[F]}, x] - \text{Dist}[\frac{d*m}{b*f*g*n*\text{Log}[F]}, \text{Int}[\frac{(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^g(e + f*x)))^n/a]}{a}, x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282



```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6266

```
Int[ArcCoth[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] := Simp[((e + f*x)^(m + 1)*ArcCoth[c + d*Cot[a + b*x]])/(f*(m
+ 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_
.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(1 - id - d \cot(a + bx)) dx &= \frac{1}{3}x^3 \coth^{-1}(1 - id - d \cot(a + bx)) + \frac{1}{3}(ib) \int \frac{x^3}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3 \coth^{-1}(1 - id - d \cot(a + bx)) + \frac{1}{3}(b(i + d)) \int \frac{e^{2ia+2ibx}}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6}x^3 \log(1 - (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6}x^3 \log(1 - (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6}x^3 \log(1 - (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6}x^3 \log(1 - (1 - id)e^{2ia+2ibx}) \\
&= \frac{1}{12}ibx^4 + \frac{1}{3}x^3 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{6}x^3 \log(1 - (1 - id)e^{2ia+2ibx})
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3 \coth^{-1}(d(-\cot(a+bx))-id+1) - \frac{4b^3x^3 \log\left(1 + \frac{e^{-2i(a+bx)}}{-1+id}\right) + 6ib^2x^2 \operatorname{Li}_2\left(\frac{ie^{-2i(a+bx)}}{d+i}\right) + 6bx \operatorname{Li}_3\left(\frac{ie^{-2i(a+bx)}}{d+i}\right) - 3i \operatorname{Li}_4\left(\frac{ie^{-2i(a+bx)}}{d+i}\right)}{24b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]], x]

[Out] (x^3\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/3 - (4\*b^3\*x^3\*Log[1 + 1/((-1 + I\*d)\*E^((2\*I)\*(a + b\*x)))] + (6\*I)\*b^2\*x^2\*PolyLog[2, I/((I + d)\*E^((2\*I)\*(a + b\*x)))] + 6\*b\*x\*PolyLog[3, I/((I + d)\*E^((2\*I)\*(a + b\*x)))] - (3\*I)\*PolyLog[4, I/((I + d)\*E^((2\*I)\*(a + b\*x)))])/(24\*b^3)

**fricas [C]** time = 0.68, size = 179, normalized size = 1.06

$$\frac{2ib^4x^4 - 4b^3x^3 \log\left(\frac{de^{2ibx+2ia}}{(d+i)e^{2ibx+2ia}-i}\right) + 6ib^2x^2 \operatorname{Li}_2(-id-1)e^{2ibx+2ia} - 2ia^4 + 4a^3 \log\left(\frac{(d+i)e^{2ibx+2ia}-i}{d+i}\right) - 6bx \operatorname{Li}_3\left(\frac{(d+i)e^{2ibx+2ia}-i}{d+i}\right) - 3i \operatorname{Li}_4\left(\frac{(d+i)e^{2ibx+2ia}-i}{d+i}\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d-d\*cot(b\*x+a)), x, algorithm="fricas")

[Out] 1/24\*(2\*I\*b^4\*x^4 - 4\*b^3\*x^3\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) - I)) + 6\*I\*b^2\*x^2\*dilog(-(I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a)) - 2\*I\*a

$$\frac{d^4 + 4a^3 \log\left(\frac{(d+I)e^{(2Ibx+2Ia)} - I}{d+I}\right) - 6bx \operatorname{polylog}(3, (-I d + 1)e^{(2Ibx+2Ia)}) - 4(b^3 x^3 + a^3) \log\left(\frac{I d - 1}{I d + 1} e^{(2Ibx+2Ia)} + 1\right) - 3I \operatorname{polylog}(4, (-I d + 1)e^{(2Ibx+2Ia)})}{b^3}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(-d \cot(bx+a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(-d\*cot(b\*x + a) - I\*d + 1), x)

**maple** [C] time = 5.97, size = 2339, normalized size = 13.84

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(1-I\*d-d\*cot(b\*x+a)),x)

[Out]  $\frac{1}{2} \frac{d}{b^2} \frac{1}{(I+d)} \ln(1+I(I+d)\exp(2I(bx+a))) x a^2 - \frac{1}{2} \frac{d}{b^2} \frac{a^2}{(I+d)} \ln(1+I\exp(I(bx+a))(I(I+d))^{1/2}) x - \frac{1}{2} \frac{d}{b^2} \frac{a^2}{(I+d)} \ln(1-I\exp(I(bx+a))(I(I+d))^{1/2}) x - \frac{1}{12} I x^3 \operatorname{Pisgn}(I/(\exp(2I(bx+a))-1)) \operatorname{csgn}(I(I\exp(2I(bx+a))+\exp(2I(bx+a))d-I)) \operatorname{csgn}(I(I\exp(2I(bx+a))+\exp(2I(bx+a))d-I)/(\exp(2I(bx+a))-1)) + \frac{1}{12} I x^3 \operatorname{Pisgn}((I\exp(2I(bx+a))+\exp(2I(bx+a))d-I)/(\exp(2I(bx+a))-1))^{2+1/4} \frac{1}{b^3} \frac{1}{(I+d)} \operatorname{polylog}(2, -I(I+d)\exp(2I(bx+a))) a^2 - \frac{1}{2} \frac{d}{b^3} \frac{a^2}{(I+d)} \operatorname{dilog}(1+I\exp(I(bx+a))(I(I+d))^{1/2}) - \frac{1}{2} \frac{d}{b^3} \frac{a^2}{(I+d)} \operatorname{dilog}(1-I\exp(I(bx+a))(I(I+d))^{1/2}) - \frac{1}{6} \frac{d}{(I+d)} \ln(1+I(I+d)\exp(2I(bx+a))) x^3 + \frac{1}{6} \frac{d}{b^3} \frac{a^3}{(I+d)} \ln(I\exp(2I(bx+a))+\exp(2I(bx+a))d-I) - \frac{1}{12} I x^3 \operatorname{Pisgn}(d/(\exp(2I(bx+a))-1)) \exp(2I(bx+a))^{2-1/3} x^3 \ln(\exp(I(bx+a))) + \frac{1}{12} I b x^4 - \frac{1}{12} I x^3 \operatorname{Pisgn} \operatorname{csgn}((I\exp(2I(bx+a))+\exp(2I(bx+a))d-I)/(\exp(2I(bx+a))-1))^{3-1/6} \frac{I}{(I+d)} \ln(1+I(I+d)\exp(2I(bx+a))) x^3 + \frac{1}{12} I x^3 \operatorname{Pisgn}(I\exp(2I(bx+a)))^{3-1/6} x^3 \ln(d) + \frac{1}{12} I x^3 \operatorname{Pisgn}(I\exp(I(bx+a)))^2 \operatorname{csgn}(I\exp(2I(bx+a))) - \frac{1}{2} \frac{I}{b^3} \frac{a^3}{(I+d)} \ln(1+I\exp(I(bx+a))(I(I+d))^{1/2}) - \frac{1}{6} I x^3 \operatorname{Pisgn}(I\exp(I(bx+a))) \operatorname{csgn}(I\exp(2I(bx+a)))^{2-1/12} I x^3 \operatorname{Pisgn} \operatorname{csgn}(I(I\exp(2I(bx+a))+\exp(2I(bx+a))d-I)/(\exp(2I(bx+a))-1)) \operatorname{csgn}((I\exp(2I(bx+a))+\exp(2I(bx+a))d-I)/(\exp(2I(bx+a))-1)) + \frac{1}{12} I x^3 \operatorname{Pisgn} \operatorname{csgn}(I d/(\exp(2I(bx+a))-1) \exp(2I(bx+a)))^{3+1/12} I x^3 \operatorname{Pisgn}(d/(\exp(2I(bx+a))-1) \exp(2I(bx+a)))^{3+1/8} \frac{1}{b^3} \frac{1}{(I+d)} \operatorname{polylog}(4, -I(I+d)\exp(2I(bx+a))) - \frac{1}{12} I x^3 \operatorname{Pisgn}(I d) \operatorname{csgn}(I d/(\exp(2I(bx+a))-1) \exp(2I(bx+a)))^{2+1/6} x^3 \ln(I\exp(2I(bx+a))+\exp(2I(bx+a))d-I) + \frac{1}{12} I x^3 \operatorname{Pisgn} \operatorname{csgn}(I\exp(2I(bx+a)))/(\exp(2I(bx+a))-1))^{3-1/12} I x^3 \operatorname{Pisgn}(I(I\exp(2I(bx+a))+\exp(2I(bx+a))d-I)/(\exp(2I(bx+a))-1))^{3-1/4} \frac{I}{b^2}$

$$\begin{aligned} &/(I+d)*\text{polylog}(3,-I*(I+d)*\exp(2*I*(b*x+a)))*x+1/6*I/b^3*a^3/(I+d)*\ln(I*\exp( \\ &2*I*(b*x+a))+\exp(2*I*(b*x+a))*d-I)+1/12*I*x^3*Pi*csgn(I*\exp(2*I*(b*x+a)))*c \\ &sgn(I/(\exp(2*I*(b*x+a))-1))*csgn(I*\exp(2*I*(b*x+a))/(\exp(2*I*(b*x+a))-1))+1 \\ &/12*I*x^3*Pi*csgn(I*d)*csgn(I*\exp(2*I*(b*x+a))/(\exp(2*I*(b*x+a))-1))*csgn(I \\ &*d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))-1/2*I/b^2*a^2/(I+d)*\ln(1+I*\exp(I* \\ &(b*x+a))*(I*(I+d))^(1/2))*x-1/2*I/b^2*a^2/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(I*(I \\ &+d))^(1/2))*x+1/2*I/b^2/(I+d)*\ln(1+I*(I+d)*\exp(2*I*(b*x+a)))*x*a^2+1/4*I/b* \\ &d/(I+d)*\text{polylog}(2,-I*(I+d)*\exp(2*I*(b*x+a)))*x^2-1/4*I/b^3*d/(I+d)*\text{polylog}( \\ &2,-I*(I+d)*\exp(2*I*(b*x+a)))*a^2+1/2*I/b^3*a^2*d/(I+d)*\text{dilog}(1+I*\exp(I*(b*x \\ &+a))*(I*(I+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(I*(I+d))^( \\ &1/2))-1/8*I/b^3*d/(I+d)*\text{polylog}(4,-I*(I+d)*\exp(2*I*(b*x+a)))+1/2*I/b^3*a^2* \\ &d/(I+d)*\text{dilog}(1-I*\exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*d/(e \\ &xp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))*csgn(d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b \\ &*x+a)))^2-1/2/b^3*a^3*d/(I+d)*\ln(1+I*\exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2/b^ \\ &3*a^3*d/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/3/b^3*d/(I+d)*\ln(1+I \\ &*(I+d)*\exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I*d/(\exp(2*I*(b*x+a))-1)*ex \\ &p(2*I*(b*x+a)))*csgn(d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))-1/12*I*x^3*Pi \\ &*csgn(I*\exp(2*I*(b*x+a)))*csgn(I*\exp(2*I*(b*x+a))/(\exp(2*I*(b*x+a))-1))^2-1 \\ &/12*I*x^3*Pi*csgn(I/(\exp(2*I*(b*x+a))-1))*csgn(I*\exp(2*I*(b*x+a))/(\exp(2*I* \\ &(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*\exp(2*I*(b*x+a))/(\exp(2*I*(b*x+a))-1))* \\ &csgn(I*d/(\exp(2*I*(b*x+a))-1)*\exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I*(Ie \\ &xp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d-I)/(\exp(2*I*(b*x+a))-1))*csgn((I*\exp(2*I \\ &*(b*x+a))+\exp(2*I*(b*x+a))*d-I)/(\exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*Pi*csgn( \\ &I*(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d-I))*csgn(I*(I*\exp(2*I*(b*x+a))+\exp \\ &(2*I*(b*x+a))*d-I)/(\exp(2*I*(b*x+a))-1))^2-1/4/b^2*d/(I+d)*\text{polylog}(3,-I*(I+ \\ &d)*\exp(2*I*(b*x+a)))*x+1/3*I/b^3/(I+d)*\ln(1+I*(I+d)*\exp(2*I*(b*x+a)))*a^3+1 \\ &/12*I*x^3*Pi*csgn(I/(\exp(2*I*(b*x+a))-1))*csgn(I*(I*\exp(2*I*(b*x+a))+\exp(2* \\ &I*(b*x+a))*d-I)/(\exp(2*I*(b*x+a))-1))^2-1/4/b/(I+d)*\text{polylog}(2,-I*(I+d)*\exp( \\ &2*I*(b*x+a)))*x^2 \end{aligned}$$

**maxima** [B] time = 0.41, size = 343, normalized size = 2.03

$$\frac{12((bx+a)^3-3(bx+a)^2a+3(bx+a)a^2)\operatorname{arccoth}(d \cot(bx+a)+id-1)}{b^2} + \frac{-3i(bx+a)^4+12i(bx+a)^3a-18i(bx+a)^2a^2+(-8i(bx+a)^3+18i(bx+a)^2a-18i(bx+a))\operatorname{arctan}^2(-d \cos(2bx+2a)+\sin(2bx+2a),-d \sin(2bx+2a)-\cos(2bx+2a)+1)+(-12I*(bx+a)^2+18I*(bx+a)*a-9I*a^2)*\text{dilog}((-I*d+1)*e^{(2I*bx+2I*a)})+(4*(bx+a)^3-9*(bx+a)^2a+9*(bx+a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] -1/36\*(12\*((b\*x + a)^3 - 3\*(b\*x + a)^2\*a + 3\*(b\*x + a)\*a^2)\*arccoth(d\*cot(b\*x + a) + I\*d - 1)/b^2 + (-3\*I\*(b\*x + a)^4 + 12\*I\*(b\*x + a)^3\*a - 18\*I\*(b\*x + a)^2\*a^2 + (-8\*I\*(b\*x + a)^3 + 18\*I\*(b\*x + a)^2\*a - 18\*I\*(b\*x + a)\*a^2)\*arctan^2(-d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) - cos(2\*b\*x + 2\*a) + 1) + (-12\*I\*(b\*x + a)^2 + 18\*I\*(b\*x + a)\*a - 9\*I\*a^2)\*dilog((-I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)) + (4\*(b\*x + a)^3 - 9\*(b\*x + a)^2\*a + 9\*(b\*x + a))

```
+ a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 -
  2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3,
  (-I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*
  I*a)))/b^2)/b
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2 \operatorname{acoth}(d \cot(a + bx) - 1 + d1i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*acoth(d*1i + d*cot(a + b*x) - 1), x)
```

```
[Out] int(-x^2*acoth(d*1i + d*cot(a + b*x) - 1), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(-d \cot(a + bx) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(1-I*d-d*cot(b*x+a)), x)
```

```
[Out] Integral(x**2*acoth(-d*cot(a + b*x) - I*d + 1), x)
```

### 3.262 $\int x \coth^{-1}(1 - id - d \cot(a + bx)) dx$

**Optimal.** Leaf size=133

$$-\frac{\text{Li}_3((1-id)e^{2ia+2ibx})}{8b^2} + \frac{ix\text{Li}_2((1-id)e^{2ia+2ibx})}{4b} - \frac{1}{4}x^2 \log(1 - (1-id)e^{2ia+2ibx}) + \frac{1}{2}x^2 \coth^{-1}(d(-\cot(a+bx))-id+1)$$

[Out] 1/6\*I\*b\*x^3+1/2\*x^2\*arccoth(1-I\*d-d\*cot(b\*x+a))-1/4\*x^2\*ln(1-(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))+1/4\*I\*x\*polylog(2,(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b-1/8\*polylog(3,(1-I\*d)\*exp(2\*I\*a+2\*I\*b\*x))/b^2

**Rubi [A]** time = 0.25, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {6266, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}(3,(1-id)e^{2ia+2ibx})}{8b^2} + \frac{ix\text{PolyLog}(2,(1-id)e^{2ia+2ibx})}{4b} - \frac{1}{4}x^2 \log(1 - (1-id)e^{2ia+2ibx}) + \frac{1}{2}x^2 \coth^{-1}(d(-\cot(a+bx))-id+1)$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]],x]

[Out] (I/6)\*b\*x^3 + (x^2\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/2 - (x^2\*Log[1 - (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/4 + ((I/4)\*x\*PolyLog[2, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/b - PolyLog[3, (1 - I\*d)\*E^((2\*I)\*a + (2\*I)\*b\*x)]/(8\*b^2))

#### Rule 2184

Int[(((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] :> Simp[(c + d\*x)^(m + 1)/(a\*d\*(m + 1)), x] - Dist[b/a, Int[(((c + d\*x)^m\*(F^(g\*(e + f\*x))))^n)/(a + b\*(F^(g\*(e + f\*x))))^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[

{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
)))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 6266

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_)]\*(d\_.)]\*((e\_.) + (f\_.)\*(x\_))^(m\_  
.), x\_Symbol] := Simp[((e + f\*x)^(m + 1)\*ArcCoth[c + d\*Cot[a + b\*x]])/(f\*(m  
+ 1)), x] + Dist[(I\*b)/(f\*(m + 1)), Int[(e + f\*x)^(m + 1)/(c - I\*d - c\*E^(  
2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&  
EqQ[(c - I\*d)^2, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(1 - id - d \cot(a + bx)) dx &= \frac{1}{2} x^2 \coth^{-1}(1 - id - d \cot(a + bx)) + \frac{1}{2} (ib) \int \frac{x^2}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id - d \cot(a + bx)) + \frac{1}{2} (b(i + d)) \int \frac{e^{2ia+2ibx}}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 - id)e^{2ia+2ibx}) \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 - id)e^{2ia+2ibx}) \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 - id)e^{2ia+2ibx}) \\
 &= \frac{1}{6} ibx^3 + \frac{1}{2} x^2 \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{4} x^2 \log(1 - (1 - id)e^{2ia+2ibx})
 \end{aligned}$$

**Mathematica** [A] time = 0.11, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2 \coth^{-1}(d(-\cot(a+bx))-id+1) - \frac{2b^2x^2 \log\left(1 + \frac{e^{-2i(a+bx)}}{-1+id}\right) + 2ibx \operatorname{Li}_2\left(\frac{ie^{-2i(a+bx)}}{d+i}\right) + \operatorname{Li}_3\left(\frac{ie^{-2i(a+bx)}}{d+i}\right)}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]], x]

[Out] (x^2\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]])/2 - (2\*b^2\*x^2\*Log[1 + 1/((-1 + I\*d)\*E^((2\*I)\*(a + b\*x)))] + (2\*I)\*b\*x\*PolyLog[2, I/((I + d)\*E^((2\*I)\*(a + b\*x)))]) + PolyLog[3, I/((I + d)\*E^((2\*I)\*(a + b\*x)))])/(8\*b^2)

**fricas** [C] time = 0.63, size = 156, normalized size = 1.17

$$\frac{4i b^3 x^3 - 6 b^2 x^2 \log\left(\frac{de^{(2i bx+2i a)}}{(d+i)e^{(2i bx+2i a)-i}}\right) + 4i a^3 + 6i bx \operatorname{Li}_2\left(-i(d-1)e^{(2i bx+2i a)}\right) - 6 a^2 \log\left(\frac{(d+i)e^{(2i bx+2i a)-i}}{d+i}\right) - 6(b^2 x^2 - a^2)}{24 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d-d\*cot(b\*x+a)), x, algorithm="fricas")

[Out] 1/24\*(4\*I\*b^3\*x^3 - 6\*b^2\*x^2\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) - I)) + 4\*I\*a^3 + 6\*I\*b\*x\*dilog(-(I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a)) - 6\*a^2\*log(((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) - I)/(d + I)) - 6\*(b^2\*x^2 - a^2)\*log((I\*d - 1)\*e^(2\*I\*b\*x + 2\*I\*a) + 1) - 3\*polylog(3, (-I\*d + 1)\*e^(2\*I\*b\*x + 2\*I\*a)))/b^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(-d \cot(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d-d\*cot(b\*x+a)), x, algorithm="giac")

[Out] integrate(x\*arccoth(-d\*cot(b\*x + a) - I\*d + 1), x)

**maple** [C] time = 5.51, size = 2249, normalized size = 16.91

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(1-I\*d-d\*cot(b\*x+a)), x)



```

[Out] -1/4/b^2/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a-1/4*d/(I+d)*ln(1+I*(I
+d)*exp(2*I*(b*x+a)))*x^2-1/8/b^2*d/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a
))) +1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b
*x+a))-1))^2+1/4*I/b^2*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*a+1/2*I
/b*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2*I/b*a/(I+d)*ln(1-I*
exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/4*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x
+a)))*a^2+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/6*I*
b*x^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*cs
gn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp
(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I
*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/8*I/b^2/(I+d)*polylog(
3,-I*(I+d)*exp(2*I*(b*x+a)))-1/4*I/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^2
-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3
-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/2*x^2*ln(ex
p(I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*
csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*d/(exp(2*
I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp
(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*
(b*x+a))*(I*(I+d))^(1/2))+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d)
)^(1/2))-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/8*I*x^2*Pi*cs
gn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn
(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3-1/8*I*
x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1
))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/8
*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/8*I*x^2*Pi*csgn
(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+ex
p(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*
x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I/
(exp(2*I*(b*x+a))-1)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I
*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*
x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*
(b*x+a))-1)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/4*x^2*ln(I
*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))
^2*csgn(I*exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(
2*I*(b*x+a)))^2-1/2*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2
))-1/2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b/(I
+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a)
))*(I*(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/2/b
*a*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/8*I*x^2*Pi*csgn(I/(ex
p(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I
*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/2/b^2*a^
2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/8*I*x^2*Pi*csgn(I*(I*exp
(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(
b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I/(
exp(2*I*(b*x+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(

```

$$2*I*(b*x+a)-1))^2+1/2*I/b^2*a^2/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4*I/b^2*a^2/(I+d)*\ln(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d-I)+1/2/b^2*a^2*d/(I+d)*\ln(1-I*\exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b^2*a^2*d/(I+d)*\ln(I*\exp(2*I*(b*x+a))+\exp(2*I*(b*x+a))*d-I)-1/4/b^2*d/(I+d)*\ln(1+I*(I+d)*\exp(2*I*(b*x+a)))*a^2+1/4*I/b*d/(I+d)*\text{polylog}(2,-I*(I+d)*\exp(2*I*(b*x+a)))*x$$

**maxima** [B] time = 0.38, size = 249, normalized size = 1.87

$$\frac{12((bx+a)^2-2(bx+a)a)\operatorname{arccoth}(d\cot(bx+a)+id-1)}{b} + \frac{-4i(bx+a)^3+12i(bx+a)^2a-6ibx\operatorname{Li}_2((-id+1)e^{2ibx+2ia})+(-6i(bx+a)^2+12i(bx+a)a)\operatorname{arctan}(-$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="maxima")

[Out] 
$$\frac{-1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*\operatorname{arccoth}(d*\cot(b*x + a) + I*d - 1)/b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*\operatorname{dilog}((-I*d + 1)*e^{(2*I*b*x + 2*I*a)}) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*\operatorname{arctan}2(-d*\cos(2*b*x + 2*a) + \sin(2*b*x + 2*a), -d*\sin(2*b*x + 2*a) - \cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*\log((d^2 + 1)*\cos(2*b*x + 2*a)^2 + (d^2 + 1)*\sin(2*b*x + 2*a)^2 - 2*d*\sin(2*b*x + 2*a) - 2*\cos(2*b*x + 2*a) + 1) + 3*\operatorname{polylog}(3, (-I*d + 1)*e^{(2*I*b*x + 2*I*a)}))/b}{b}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x \operatorname{acoth}(d \cot(a + bx) - 1 + d1i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x\*acoth(d\*1i + d\*cot(a + b\*x) - 1),x)

[Out] int(-x\*acoth(d\*1i + d\*cot(a + b\*x) - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(-d \cot(a + bx) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(1-I\*d-d\*cot(b\*x+a)),x)

[Out] Integral(x\*acoth(-d\*cot(a + b\*x) - I\*d + 1), x)

### 3.263 $\int \coth^{-1}(1 - id - d \cot(a + bx)) dx$

Optimal. Leaf size=94

$$\frac{i\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1-id)e^{2ia+2ibx}\right) + x \coth^{-1}(d(-\cot(a+bx)) - id + 1) + \frac{1}{2}ibx^2$$

[Out]  $1/2*I*b*x^2 + x*\text{arccoth}(1-I*d-d*\cot(b*x+a)) - 1/2*x*\ln(1-(1-I*d)*\exp(2*I*a+2*I*b*x)) + 1/4*I*\text{polylog}(2, (1-I*d)*\exp(2*I*a+2*I*b*x))/b$

**Rubi [A]** time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6258, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2, (1-id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x \log\left(1 - (1-id)e^{2ia+2ibx}\right) + x \coth^{-1}(d(-\cot(a+bx)) - id + 1) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[1 - I*d - d*Cot[a + b*x]], x]`

[Out]  $(I/2)*b*x^2 + x*\text{ArcCoth}[1 - I*d - d*\text{Cot}[a + b*x]] - (x*\text{Log}[1 - (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)}])/2 + ((I/4)*\text{PolyLog}[2, (1 - I*d)*E^{((2*I)*a + (2*I)*b*x)}])/b$

#### Rule 2184

`Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[(((c + d*x)^m*(F^(g*(e + f*x))))^n)/(a + b*(F^(g*(e + f*x))))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2190

`Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2279

`Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 6258

Int[ArcCoth[(c\_.) + Cot[(a\_.) + (b\_.)\*(x\_)]]\*(d\_.)], x\_Symbol] :> Simp[x\*ArcCoth[c + d\*Cot[a + b\*x]], x] + Dist[I\*b, Int[x/(c - I\*d - c\*E^(2\*I\*a + 2\*I\*b\*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I\*d)^2, 1]

Rubi steps

$$\begin{aligned}
 \int \coth^{-1}(1 - id - d \cot(a + bx)) dx &= x \coth^{-1}(1 - id - d \cot(a + bx)) + (ib) \int \frac{x}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
 &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id - d \cot(a + bx)) + (b(i + d)) \int \frac{e^{2ia+2ibx} x}{1 + (-1 + id)e^{2ia+2ibx}} dx \\
 &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{2} x \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{i}{2} x \log(1 + (1 - id)e^{2ia+2ibx}) \\
 &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{2} x \log(1 - (1 - id)e^{2ia+2ibx}) \\
 &= \frac{1}{2} ibx^2 + x \coth^{-1}(1 - id - d \cot(a + bx)) - \frac{1}{2} x \log(1 - (1 - id)e^{2ia+2ibx}) + \frac{i}{2} x \log(1 + (1 - id)e^{2ia+2ibx})
 \end{aligned}$$

**Mathematica [B]** time = 2.94, size = 605, normalized size = 6.44

$$\frac{x \csc^2(a + bx)(\cos(bx) - i \sin(bx))(\cos(bx) + i \sin(bx)) \left( i \operatorname{Li}_2 \left( \frac{(\cos(a) - i \sin(a))((2 - id) \cos(a) + d \sin(a))(\tan(bx) + i)}{2(d+i)} \right) - i \operatorname{Li}_2 \left( \frac{1}{2} \right) \right)}{(\cot(a + bx) + i)(d \cot(a + bx) + id - 2) \left( \frac{\sec^2(bx) \log \left( \frac{i \sec(bx)(d)}{1+i} \right)}{1+i} \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]], x]

[Out] x\*ArcCoth[1 - I\*d - d\*Cot[a + b\*x]] + (x\*Csc[a + b\*x]^2\*(2\*b\*x\*Log[2\*Cos[b\*x]\*(Cos[b\*x] - I\*Sin[b\*x])] + I\*Log[(Sec[b\*x]\*(Cos[a] - I\*Sin[a])\*(d\*Cos[a + b\*x] + I\*(2\*I + d)\*Sin[a + b\*x])]/(2\*(I + d))]\*Log[1 - I\*Tan[b\*x]] - I\*Log[(I\*Sec[b\*x]\*(d\*Cos[a + b\*x] + I\*(2\*I + d)\*Sin[a + b\*x])]/(2\*Cos[a] - (2\*I)\*Sin[a])]\*Log[1 + I\*Tan[b\*x]] + I\*PolyLog[2, -Cos[2\*b\*x] + I\*Sin[2\*b\*x]] - I\*PolyLog[2, (Sec[b\*x]\*((2 - I\*d)\*Cos[a] + d\*Sin[a]))\*(Cos[a + b\*x] + I\*Sin

$[a + b*x]))/2 + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((2 - I*d)*Cos[a] + d*Sin[a])*(I + Tan[b*x]))/(2*(I + d))]*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(-2 + I*d + d*Cot[a + b*x])*(-((Log[1 - I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x])) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x])) + (Log[(I*Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a]))*Sec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*(I + Tan[b*x]) + I*Log[1 - (Sec[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*(I + Tan[b*x]))]$

**fricas [A]** time = 0.71, size = 121, normalized size = 1.29

$$\frac{2i b^2 x^2 - 2 b x \log\left(\frac{d e^{(2i b x + 2i a)}}{(d+i)e^{(2i b x + 2i a)} - i}\right) - 2i a^2 - 2(bx + a) \log\left((id - 1)e^{(2i b x + 2i a)} + 1\right) + 2a \log\left(\frac{(d+i)e^{(2i b x + 2i a)} - i}{d+i}\right) + i I}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="fricas")

[Out]  $1/4*(2*I*b^2*x^2 - 2*b*x*\log(d*e^{(2*I*b*x + 2*I*a)}/((d + I)*e^{(2*I*b*x + 2*I*a)} - I)) - 2*I*a^2 - 2*(b*x + a)*\log((I*d - 1)*e^{(2*I*b*x + 2*I*a)} + 1) + 2*a*\log(((d + I)*e^{(2*I*b*x + 2*I*a)} - I)/(d + I)) + I*\operatorname{dilog}(-(I*d - 1)*e^{(2*I*b*x + 2*I*a)}))/b$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(-d \cot(bx + a) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="giac")

[Out] integrate(arccoth(-d\*cot(b\*x + a) - I\*d + 1), x)

**maple [B]** time = 0.70, size = 304, normalized size = 3.23

$$\frac{i \operatorname{arccoth}(1 - id - d \cot(bx + a)) \ln(id - d \cot(bx + a))}{2b} - \frac{i \operatorname{arccoth}(1 - id - d \cot(bx + a)) \ln(-id - d \cot(bx + a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-I\*d-d\*cot(b\*x+a)),x)

[Out]  $1/2*I/b*\operatorname{arccoth}(1-I*d-d*\cot(b*x+a))*\ln(I*d-d*\cot(b*x+a))-1/2*I/b*\operatorname{arccoth}(1-I*d-d*\cot(b*x+a))*\ln(-I*d-d*\cot(b*x+a))-1/8*I/b*\ln(-I*d-d*\cot(b*x+a))^2+1/4$

$*I/b*\operatorname{dilog}(1-1/2*I*d-1/2*d*\cot(b*x+a))+1/4*I/b*\ln(-I*d-d*\cot(b*x+a))*\ln(1-1/2*I*d-1/2*d*\cot(b*x+a))-1/4*I/b*\operatorname{dilog}((2-I*d-d*\cot(b*x+a))/(-2*I*d+2))-1/4*I/b*\ln(I*d-d*\cot(b*x+a))*\ln((2-I*d-d*\cot(b*x+a))/(-2*I*d+2))+1/4*I/b*\operatorname{dilog}(1/2*I*(-I*d-d*\cot(b*x+a))/d)+1/4*I/b*\ln(I*d-d*\cot(b*x+a))*\ln(1/2*I*(-I*d-d*\cot(b*x+a))/d)$

**maxima** [B] time = 0.45, size = 288, normalized size = 3.06

$$4(bx+a)d\left(\frac{\log((id-2)\tan(bx+a)+d)}{d} - \frac{\log(i\tan(bx+a)+1)}{d}\right) - d\left(\frac{2i\left(\log((id-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-id}{2id-2}+1\right)\right)+\operatorname{Li}_2\left(-\frac{(d+2i)\tan(bx+a)-id}{2id-2}\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/8*(4*(b*x+a)*d*(\log((I*d-2)*\tan(b*x+a)+d)/d - \log(I*\tan(b*x+a)+1)/d) - d*(2*I*(\log((I*d-2)*\tan(b*x+a)+d)*\log(((d+2*I)*\tan(b*x+a)-I*d)/(2*I*d-2)+1) + \operatorname{dilog}(-((d+2*I)*\tan(b*x+a)-I*d)/(2*I*d-2)))/d + 2*I*(\log(-1/2*(d+2*I)*\tan(b*x+a)+1/2*I*d)*\log(I*\tan(b*x+a)+1) + \operatorname{dilog}(1/2*(d+2*I)*\tan(b*x+a)-1/2*I*d+1))/d - (2*I*\log((I*d-2)*\tan(b*x+a)+d)*\log(I*\tan(b*x+a)+1) - I*\log(I*\tan(b*x+a)+1)^2)/d - 2*I*(\log(I*\tan(b*x+a)+1)*\log(-1/2*I*\tan(b*x+a)+1/2) + \operatorname{dilog}(1/2*I*\tan(b*x+a)+1/2))/d + 8*(b*x+a)*\operatorname{arccoth}(I*d+d/\tan(b*x+a)-1))/b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\operatorname{acoth}(d \cot(a + bx) - 1 + d i) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-acoth(d\*i + d\*cot(a + b\*x) - 1),x)

[Out] int(-acoth(d\*i + d\*cot(a + b\*x) - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(-d \cot(a + bx) - id + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acoth(1-I\*d-d\*cot(b\*x+a)),x)

[Out] Integral(acoth(-d\*cot(a + b\*x) - I\*d + 1), x)

$$3.264 \quad \int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\coth^{-1}(d(-\cot(a+bx))-id+1)}{x}, x\right)$$

[Out] CannotIntegrate(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x)

**Rubi** [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x,x]

[Out] Defer[Int][ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x, x]

Rubi steps

$$\int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx = \int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$$

**Mathematica** [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\coth^{-1}(1-id-d \cot(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x,x]

[Out] Integrate[ArcCoth[1 - I\*d - d\*Cot[a + b\*x]]/x, x]

**fricas** [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(\frac{de^{2ibx+2ia}}{(d+i)e^{2ibx+2ia}-i}\right)}{2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(-1/2\*log(d\*e^(2\*I\*b\*x + 2\*I\*a)/((d + I)\*e^(2\*I\*b\*x + 2\*I\*a) - I))/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}\left(\frac{-d \cot(bx + a) - id + 1}{x}\right) dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(arccoth(-d\*cot(b\*x + a) - I\*d + 1)/x, x)

**maple** [A] time = 2.07, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccoth}\left(\frac{1 - id - d \cot(bx + a)}{x}\right) dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x)

[Out] int(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-ibx + \frac{1}{4}(-i\pi - 4ia - 2\log(d))\log(x) - \frac{1}{2}i \int \frac{\arctan\left(\frac{-d \cos(2bx + 2a) + \sin(2bx + 2a)}{x}\right), -d \sin(2bx + 2a) - \cos(2bx + 2a) + 1}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(1-I\*d-d\*cot(b\*x+a))/x,x, algorithm="maxima")

[Out] -I\*b\*x + 1/4\*(-I\*pi - 4\*I\*a - 2\*log(d))\*log(x) - 1/2\*I\*integrate(arctan2(-d\*cos(2\*b\*x + 2\*a) + sin(2\*b\*x + 2\*a), -d\*sin(2\*b\*x + 2\*a) - cos(2\*b\*x + 2\*a) + 1)/x, x) + 1/4\*integrate(log((d^2 + 1)\*cos(2\*b\*x + 2\*a)^2 + (d^2 + 1)\*sin(2\*b\*x + 2\*a)^2 - 2\*d\*sin(2\*b\*x + 2\*a) - 2\*cos(2\*b\*x + 2\*a) + 1)/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{\operatorname{acoth}\left(\frac{d \cot(a + bx) - 1 + d1i}{x}\right) dx}{x}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(-acoth(d*I + d*cot(a + b*x) - 1)/x,x)`

[Out] `int(-acoth(d*I + d*cot(a + b*x) - 1)/x, x)`

**sympy [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acoth}(-d \cot(a + bx) - id + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(1-I*d-d*cot(b*x+a))/x,x)`

[Out] `Integral(acoth(-d*cot(a + b*x) - I*d + 1)/x, x)`

$$3.265 \quad \int \frac{(a+b \coth^{-1}(cx^n))(d+e \log(fx^m))}{x} dx$$

**Optimal.** Leaf size=160

$$ad \log(x) + \frac{ae \log^2(fx^m)}{2m} + \frac{bd \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} - \frac{bd \operatorname{Li}_2\left(\frac{x^{-n}}{c}\right)}{2n} + \frac{be \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right) \log(fx^m)}{2n} - \frac{be \operatorname{Li}_2\left(\frac{x^{-n}}{c}\right) \log(fx^m)}{2n} + \frac{bem \operatorname{Li}_3}{2n}$$

[Out] a\*d\*ln(x)+1/2\*a\*e\*ln(f\*x^m)^2/m+1/2\*b\*d\*polylog(2,-1/c/(x^n))/n+1/2\*b\*e\*ln(f\*x^m)\*polylog(2,-1/c/(x^n))/n-1/2\*b\*d\*polylog(2,1/c/(x^n))/n-1/2\*b\*e\*ln(f\*x^m)\*polylog(2,1/c/(x^n))/n+1/2\*b\*e\*m\*polylog(3,-1/c/(x^n))/n^2-1/2\*b\*e\*m\*polylog(3,1/c/(x^n))/n^2

**Rubi [A]** time = 0.57, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2301, 6742, 6096, 5913, 6072, 6070, 2374, 6589}

$$\frac{bd \operatorname{PolyLog}\left(2, -\frac{x^{-n}}{c}\right)}{2n} - \frac{bd \operatorname{PolyLog}\left(2, \frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{PolyLog}\left(2, -\frac{x^{-n}}{c}\right)}{2n} - \frac{be \log(fx^m) \operatorname{PolyLog}\left(2, \frac{x^{-n}}{c}\right)}{2n} + \dots$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x^n])\*(d + e\*Log[f\*x^m]))/x, x]

[Out] a\*d\*Log[x] + (a\*e\*Log[f\*x^m]^2)/(2\*m) + (b\*d\*PolyLog[2, -(1/(c\*x^n))])/(2\*n) + (b\*e\*Log[f\*x^m]\*PolyLog[2, -(1/(c\*x^n))])/(2\*n) - (b\*d\*PolyLog[2, 1/(c\*x^n)])/(2\*n) - (b\*e\*Log[f\*x^m]\*PolyLog[2, 1/(c\*x^n)])/(2\*n) + (b\*e\*m\*PolyLog[3, -(1/(c\*x^n))])/(2\*n^2) - (b\*e\*m\*PolyLog[3, 1/(c\*x^n)])/(2\*n^2)

**Rule 2301**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

**Rule 2374**

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p-1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

**Rule 5913**

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/2, x]) /; FreeQ[{a, b, c}, x]
```

### Rule 6070

```
Int[(ArcCoth[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)]/(x_), x_Symbol] := Dist[1/2, Int[(Log[d*x^m]*Log[1 + 1/(c*x^n)])/x, x], x] - Dist[1/2, Int[(Log[d*x^m]*Log[1 - 1/(c*x^n)])/x, x], x] /; FreeQ[{c, d, m, n}, x]
```

### Rule 6072

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcCoth[(c_.)*(x_)^(n_.)]*(b_.) + (a_.)))/(x_), x_Symbol] := Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*ArcCoth[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

### Rule 6096

```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*ArcCoth[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

### Rule 6589

```
Int[PolyLog[n, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx^n))(d + e \log(fx^m))}{x} dx &= \int \left( \frac{d(a + b \coth^{-1}(cx^n))}{x} + \frac{e(a + b \coth^{-1}(cx^n)) \log(fx^m)}{x} \right) dx \\
&= d \int \frac{a + b \coth^{-1}(cx^n)}{x} dx + e \int \frac{(a + b \coth^{-1}(cx^n)) \log(fx^m)}{x} dx \\
&= (ae) \int \frac{\log(fx^m)}{x} dx + (be) \int \frac{\coth^{-1}(cx^n) \log(fx^m)}{x} dx + \frac{d \operatorname{Sub}}{x} \\
&= ad \log(x) + \frac{ae \log^2(fx^m)}{2m} + \frac{bd \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} - \frac{bd \operatorname{Li}_2\left(\frac{x^{-n}}{c}\right)}{2n} - \frac{1}{2}(be) \\
&= ad \log(x) + \frac{ae \log^2(fx^m)}{2m} + \frac{bd \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} \\
&= ad \log(x) + \frac{ae \log^2(fx^m)}{2m} + \frac{bd \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n} + \frac{be \log(fx^m) \operatorname{Li}_2\left(-\frac{x^{-n}}{c}\right)}{2n}
\end{aligned}$$

**Mathematica [C]** time = 0.30, size = 131, normalized size = 0.82

$$\frac{bcx^n (d + e \log(fx^m)) {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; c^2 x^{2n}\right)}{n} - \frac{bcemx^n {}_4F_3\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; c^2 x^{2n}\right)}{n^2} - \frac{1}{2} \log(x) (em \log(x) - 2(d + e \log(fx^m)))$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*ArcCoth[c\*x^n])\*(d + e\*Log[f\*x^m]))/x,x]

[Out] -((b\*c\*e\*m\*x^n\*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2\*x^(2\*n)])/n^2) + (b\*c\*x^n\*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2\*x^(2\*n)]\*(d + e\*Log[f\*x^m]))/n - ((a + b\*ArcCoth[c\*x^n] - b\*ArcTanh[c\*x^n])\*Log[x]\*(e\*m\*Log[x] - 2\*(d + e\*Log[f\*x^m]))) / 2

**fricas [C]** time = 0.60, size = 326, normalized size = 2.04

$$2 aem n^2 \log(x)^2 - 2 bempolylog\left(3, c \cosh(n \log(x)) + c \sinh(n \log(x))\right) + 2 bempolylog\left(3, -c \cosh(n \log(x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x^n))\*(d+e\*log(f\*x^m))/x,x, algorithm="fricas")

```
[Out] 1/4*(2*a*e*m*n^2*log(x)^2 - 2*b*e*m*polylog(3, c*cosh(n*log(x)) + c*sinh(n*log(x))) + 2*b*e*m*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log(x))) + 2*(b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(c*cosh(n*log(x)) + c*sinh(n*log(x))) - 2*(b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(-c*cosh(n*log(x)) - c*sinh(n*log(x))) - (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1) + (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(-c*cosh(n*log(x)) - c*sinh(n*log(x)) + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x) + (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(x))*log((c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)))/n^2
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx^n) + a)(e \log(fx^m) + d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

**maple** [C] time = 0.54, size = 920, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccoth(c*x^n))*(d+e*ln(f*x^m))/x,x)
```

```
[Out] 1/4*I/n*Pi*ln(c*x^n-1)*ln(c*x^n)*b*e*csgn(I*f*x^m)^3-1/4*I/n*Pi*dilog(c*x^n)*b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I/n*Pi*dilog(c*x^n)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/n*ln(x^n)*a*d+1/2*e*a/m*ln(x^m)^2-1/2/n*dilog(c*x^n)*b*d-1/2/n*dilog(c*x^n+1)*b*d+1/4*I/n*Pi*ln(c*x^n-1)*ln(c*x^n)*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I/n*dilog(c*x^n+1)*Pi*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I/n*dilog(c*x^n+1)*Pi*b*e*csgn(I*f*x^m)^3-1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*f*x^m)^3-1/2/n*ln(c*x^n-1)*ln(c*x^n)*b*d+1/4*e*b*ln(c*x^n-1)*ln(x)^2*m-1/2*e*b*ln(c*x^n-1)*ln(x^m)*ln(x)-1/4*e*b*m*ln(x)^2*ln(1-c*x^n)-1/4*I/n*Pi*ln(c*x^n-1)*ln(c*x^n)*b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/4*I/n*Pi*dilog(c*x^n)*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4*I/n*dilog(c*x^n+1)*Pi*b*e*csgn(I*f)*csgn(I*f*x^m)^2+1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*f)*csgn(I*f*x^m)^2-1/4*I/n*dilog(c*x^n+1)*Pi*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*b*e*m*polylog(3,-c*x^n)/n^2-1/2*b*e*m*polylog(3,c*x^n)/n^2-1/4*I/n*Pi*ln(c*x^n-1)*ln(c*x^n)*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2-1/2*e*b/n*m*ln(x)*polylog(2,-c*x^n)+1/2*e*b/n*dilog(c*x^n+1)*m*ln(x)+1/2*e*
```

$b/n*m*\ln(x)*\text{polylog}(2, c*x^n) - 1/2*e*b/n*\ln(1-c*x^n)*\ln(c*x^n)*\ln(x^m) + 1/2*e*b/n*\text{dilog}(c*x^n)*m*\ln(x) + 1/2*e*b/n*\ln(1-c*x^n)*\ln(c*x^n)*m*\ln(x) + 1/4*I/n*\text{Pi}*\text{dilog}(c*x^n)*b*e*\text{csgn}(I*f*x^m)^3 - 1/2/n*\ln(f)*\ln(c*x^n-1)*\ln(c*x^n)*b*e - 1/2*e*b/n*\text{dilog}(c*x^n+1)*\ln(x^m) + 1/2*e*b*\ln(x^m)*\ln(1-c*x^n)*\ln(x) - 1/2*e*b/n*\text{dilog}(c*x^n)*\ln(x^m) + 1/n*\ln(x^n)*\ln(f)*a*e - 1/2/n*\text{dilog}(c*x^n+1)*\ln(f)*b*e - 1/2/n*\ln(f)*\text{dilog}(c*x^n)*b*e$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae \log(fx^m)^2}{2m} + ad \log(x) - \frac{1}{4} (bem \log(x)^2 - 2be \log(x) \log(x^m) - 2(e \log(f) + d)b \log(x)) \log(cx^n + 1) + \frac{1}{4} (bem$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x^n))\*(d+e\*log(f\*x^m))/x,x, algorithm="maxima")

[Out]  $1/2*a*e*\log(f*x^m)^2/m + a*d*\log(x) - 1/4*(b*e*m*\log(x)^2 - 2*b*e*\log(x)*\log(x^m) - 2*(e*\log(f) + d)*b*\log(x))*\log(c*x^n + 1) + 1/4*(b*e*m*\log(x)^2 - 2*b*e*\log(x)*\log(x^m) - 2*(e*\log(f) + d)*b*\log(x))*\log(c*x^n - 1) + \text{integrate}(1/2*(2*b*c*e*n*x^n*\log(x)*\log(x^m) - (b*c*e*m*n*\log(x)^2 - 2*(e*n*\log(f) + d*n)*b*c*\log(x))*x^n)/(c^2*x*x^(2*n) - x), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acoth}(cx^n)) (d + e \ln(fx^m))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*acoth(c\*x^n))\*(d + e\*log(f\*x^m)))/x,x)

[Out] int(((a + b\*acoth(c\*x^n))\*(d + e\*log(f\*x^m)))/x, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x\*\*n))\*(d+e\*ln(f\*x\*\*m))/x,x)

[Out] Timed out

### 3.266 $\int x^5 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$

**Optimal.** Leaf size=297

$$\frac{ex^2(a + b \coth^{-1}(cx))}{6c^4} + \frac{1}{6}x^6(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d) - \frac{ex^4(a + b \coth^{-1}(cx))}{12c^2} - \frac{e \log(1 - c^2x^2)}{6c^4}$$

[Out]  $1/36*b*(6*d-11*e)*x/c^5-23/45*b*e*x/c^5+1/108*b*(6*d-5*e)*x^3/c^3-8/135*b*e*x^3/c^3+1/90*b*(3*d-e)*x^5/c-1/75*b*e*x^5/c-1/6*e*x^2*(a+b*\operatorname{arccoth}(c*x))/c^4-1/12*e*x^4*(a+b*\operatorname{arccoth}(c*x))/c^2-1/18*e*x^6*(a+b*\operatorname{arccoth}(c*x))-1/36*b*(6*d-11*e)*\operatorname{arctanh}(c*x)/c^6+23/45*b*e*\operatorname{arctanh}(c*x)/c^6+1/6*b*e*x*\ln(-c^2*x^2+1)/c^5+1/18*b*e*x^3*\ln(-c^2*x^2+1)/c^3+1/30*b*e*x^5*\ln(-c^2*x^2+1)/c-1/6*e*(a+b*\operatorname{arccoth}(c*x))*\ln(-c^2*x^2+1)/c^6+1/6*x^6*(a+b*\operatorname{arccoth}(c*x))*(d+e*\ln(-c^2*x^2+1))$

**Rubi [A]** time = 0.39, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$ , Rules used = {2454, 2395, 43, 6084, 321, 207, 302, 2528, 2448, 206, 2455}

$$\frac{1}{6}x^6(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d) - \frac{ex^4(a + b \coth^{-1}(cx))}{12c^2} - \frac{ex^2(a + b \coth^{-1}(cx))}{6c^4} - \frac{e \log(1 - c^2x^2)}{6c^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5*(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]), x]$

[Out]  $(b*(3*d - e)*x)/(18*c^5) - (137*b*e*x)/(180*c^5) + (b*(3*d - e)*x^3)/(54*c^3) - (47*b*e*x^3)/(540*c^3) + (b*(3*d - e)*x^5)/(90*c) - (b*e*x^5)/(75*c) - (e*x^2*(a + b*\operatorname{ArcCoth}[c*x]))/(6*c^4) - (e*x^4*(a + b*\operatorname{ArcCoth}[c*x]))/(12*c^2) - (e*x^6*(a + b*\operatorname{ArcCoth}[c*x]))/18 - (b*(3*d - e)*\operatorname{ArcTanh}[c*x])/(18*c^6) + (137*b*e*\operatorname{ArcTanh}[c*x])/(180*c^6) + (b*e*x*\operatorname{Log}[1 - c^2*x^2])/(6*c^5) + (b*e*x^3*\operatorname{Log}[1 - c^2*x^2])/(18*c^3) + (b*e*x^5*\operatorname{Log}[1 - c^2*x^2])/(30*c) - (e*(a + b*\operatorname{ArcCoth}[c*x])*\operatorname{Log}[1 - c^2*x^2])/(6*c^6) + (x^6*(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]))/6$

#### Rule 43

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0]) \ || \ \text{GtQ}[m + n + 2, 0])$

#### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/Rt[a, 2]\*Rt[-b, 2], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 207

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[Rt[b, 2]\*x]/Rt[-a, 2])/Rt[-a, 2]\*Rt[b, 2], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 2395

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]\*(b\_))\*((f\_) + (g\_)\*(x\_)^(q\_)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 2448

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(p\_)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

### Rule 2454

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(p\_)])\*(b\_)^(q\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&



!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 6084

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.)\*(x\_)^(m\_.), x\_Symbol] :> With[{u = IntHide[x^m\*(d + e\*Log[f + g\*x^2]), x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[ExpandIntegrand[u/(1 - c^2\*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2, 0]

#### Rubi steps

$$\begin{aligned}
\int x^5 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx &= -\frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} - \frac{ex^4 (a + b \coth^{-1}(cx))}{12c^2} - \frac{1}{18}ex^6 (a \\
&= -\frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} - \frac{ex^4 (a + b \coth^{-1}(cx))}{12c^2} - \frac{1}{18}ex^6 (a \\
&= \frac{bex}{6c^5} - \frac{ex^2 (a + b \coth^{-1}(cx))}{6c^4} - \frac{ex^4 (a + b \coth^{-1}(cx))}{12c^2} - \frac{1}{18}ex^6 (a \\
&= \frac{b(3d - e)x}{18c^5} - \frac{bex}{4c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{bex^3}{36c^3} + \frac{b(3d - e)x^5}{90c} - \frac{ex^2 (a \\
&= \frac{b(3d - e)x}{18c^5} - \frac{bex}{4c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{bex^3}{36c^3} + \frac{b(3d - e)x^5}{90c} - \frac{ex^2 (a \\
&= \frac{b(3d - e)x}{18c^5} - \frac{7bex}{12c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{bex^3}{36c^3} + \frac{b(3d - e)x^5}{90c} - \frac{ex^2 (a \\
&= \frac{b(3d - e)x}{18c^5} - \frac{137bex}{180c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{47bex^3}{540c^3} + \frac{b(3d - e)x^5}{90c} \\
&= \frac{b(3d - e)x}{18c^5} - \frac{137bex}{180c^5} + \frac{b(3d - e)x^3}{54c^3} - \frac{47bex^3}{540c^3} + \frac{b(3d - e)x^5}{90c}
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 236, normalized size = 0.79

$$\frac{20e \log(1 - c^2x^2) (15ac^6x^6 + 15b(c^6x^6 - 1) \coth^{-1}(cx) + bcx(3c^4x^4 + 5c^2x^2 + 15)) + 15 \log(1 - cx)(-20ae + 10)}{1800c^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (30\*b\*c\*(10\*d - 49\*e)\*x - 300\*a\*c^2\*e\*x^2 + 10\*b\*c^3\*(10\*d - 19\*e)\*x^3 - 150\*a\*c^4\*e\*x^4 + 4\*b\*c^5\*(15\*d - 11\*e)\*x^5 + 100\*a\*c^6\*(3\*d - e)\*x^6 - 50\*b\*c^2\*x^2\*(-6\*c^4\*d\*x^4 + e\*(6 + 3\*c^2\*x^2 + 2\*c^4\*x^4))\*ArcCoth[c\*x] + 15\*(10\*b\*d - 20\*a\*e - 49\*b\*e)\*Log[1 - c\*x] - 15\*(10\*b\*d + 20\*a\*e - 49\*b\*e)\*Log[1 + c\*x] + 20\*e\*(15\*a\*c^6\*x^6 + b\*c\*x\*(15 + 5\*c^2\*x^2 + 3\*c^4\*x^4) + 15\*b\*(-1 + c^6\*x^6))\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(1800\*c^6)

**fricas** [A] time = 0.67, size = 246, normalized size = 0.83

$$150 ac^4 ex^4 - 100 (3 ac^6 d - ac^6 e) x^6 + 300 ac^2 ex^2 - 4 (15 bc^5 d - 11 bc^5 e) x^5 - 10 (10 bc^3 d - 19 bc^3 e) x^3 - 30 (10 b$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="fricas")

[Out] 
$$-1/1800*(150*a*c^4*e*x^4 - 100*(3*a*c^6*d - a*c^6*e)*x^6 + 300*a*c^2*e*x^2 - 4*(15*b*c^5*d - 11*b*c^5*e)*x^5 - 10*(10*b*c^3*d - 19*b*c^3*e)*x^3 - 30*(10*b*c*d - 49*b*c*e)*x - 20*(15*a*c^6*e*x^6 + 3*b*c^5*e*x^5 + 5*b*c^3*e*x^3 + 15*b*c*e*x - 15*a*e)*\log(-c^2*x^2 + 1) + 5*(15*b*c^4*e*x^4 - 10*(3*b*c^6*d - b*c^6*e)*x^6 + 30*b*c^2*e*x^2 + 30*b*d - 147*b*e - 30*(b*c^6*e*x^6 - b*e)*\log(-c^2*x^2 + 1))*\log((c*x + 1)/(c*x - 1))/c^6$$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [C] time = 16.95, size = 4034, normalized size = 13.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1)),x)

[Out] 
$$-49/60*b*e*x/c^5-19/180*b*e*x^3/c^3-11/450*b*e*x^5/c-23/90/c^6*b*d+1/12*I*b*\arccoth(c*x)*\text{Pi}*c\text{sgn}(I*((c*x+1)/(c*x-1)-1)^2)^3*x^6*e+1/12*I*b*\arccoth(c*x)*\text{Pi}*c\text{sgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^6*e+1/60*I/c*b*\text{Pi}*c\text{sgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^5*e+1/36*I/c^3*b*\text{Pi}*c\text{sgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^3*e+1/12*I/c^5*b*\text{Pi}*c\text{sgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x*e+1/60*I/c*b*\text{Pi}*c\text{sgn}(I*((c*x+1)/(c*x-1)-1)^2)^3*x^5*e+1/36*I/c^3*b*\text{Pi}*c\text{sgn}(I*((c*x+1)/(c*x-1)-1)^2)^3*x^3*e+1/12*I/c^5*b*\text{Pi}*c\text{sgn}(I*((c*x+1)/(c*x-1)-1)^2)^3*x*e-1/12*I*b*\arccoth(c*x)*\text{Pi}*c\text{sgn}(I/((c*x-1)/(c*x+1))^(1/2))^2*c\text{sgn}(I*(c*x+1)/(c*x-1))*x^6*e+1/12*I*b*\arcc$$



```

e/c^6*ln(c^2*x^2-1)+1/18/c^3*b*x^3*d+1/6/c^5*b*x*d+1/30/c*b*x^5*d+1/6*b*ar
c coth(c*x)*x^6*d-1/18*b*arccoth(c*x)*x^6*e+239/180/c^6*b*arccoth(c*x)*e-1/6
/c^6*b*arccoth(c*x)*d-23/90*I/c^6*b*e*Pi-1/6*I*b*arccoth(c*x)*Pi*csgn(I*(c*
x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^6*e-1/12*I*b*arccoth(c*x)*Pi*csgn(I
*(c*x+1)/(c*x-1))^3*x^6*e-1/60*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^5*e-1/1
2*I/c^5*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x*e-1/30*I/c*b*Pi*csgn(I*(c*x+1)/(c*
x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^5*e-1/18*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1)/
((c*x+1)/(c*x-1)-1)^2)^2*x^3*e-1/6*I/c^5*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+
1)/(c*x-1)-1)^2)^2*x*e-23/180*I/c^6*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*cs
gn(I*((c*x+1)/(c*x-1)-1)^2)-1/12*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/
(c*x-1)-1)^2)^3-1/12*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x
+1)/(c*x-1)-1)^2)^3+1/6*I/c^6*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((
c*x+1)/(c*x-1)-1)^2)^2+23/180*I/c^6*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^
2*csgn(I*(c*x+1)/(c*x-1))-23/180*I/c^6*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(
I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/12*I/c^6*b*arccoth(c*x)*Pi*e*c
sgn(I*(c*x+1)/(c*x-1))^3+71/75*e/c^6*b-1/18*a*e*x^6+1/6*x^6*a*d+23/90*I/c^6
*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2-23/90*I
/c^6*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2-1/90/
c^6*b*e*(15*arccoth(c*x)*x^5*c^5+15*arccoth(c*x)*x^4*c^4+3*c^4*x^4+15*arcco
th(c*x)*x^3*c^3+3*c^3*x^3+15*arccoth(c*x)*x^2*c^2+8*c^2*x^2+15*arccoth(c*x)
*x*c+8*c*x+15*arccoth(c*x)+23)*(c*x-1)*ln((c*x-1)/(c*x+1))-1/6*I/c^6*b*arcc
oth(c*x)*Pi*e+23/180*I/c^6*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3+23/90*I/c^6*b*e
*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-23/180*I/c^6*b*e*Pi*csg
n(I*((c*x+1)/(c*x-1)-1)^2)^3-23/180*I/c^6*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c
*x+1)/(c*x-1)-1)^2)^3+1/30*I/c*b*Pi*x^5*e+1/18*I/c^3*b*Pi*x^3*e+1/6*I/c^5*b
*Pi*x*e+1/6*I*b*arccoth(c*x)*Pi*x^6*e-23/45/c^6*b*e*ln(2)+1/3/c^6*b*arccoth
(c*x)*e*ln((c*x+1)/(c*x-1)-1)-1/3/c^6*b*arccoth(c*x)*ln(2)*e-1/15/c*b*ln((c
*x+1)/(c*x-1)-1)*x^5*e-1/9/c^3*b*ln((c*x+1)/(c*x-1)-1)*x^3*e-1/3/c^5*b*ln((
c*x+1)/(c*x-1)-1)*x*e+1/15/c*b*ln(2)*x^5*e+1/9/c^3*b*ln(2)*x^3*e+1/3/c^5*b*
ln(2)*x*e-1/12/c^2*b*arccoth(c*x)*x^4*e-1/6/c^4*b*arccoth(c*x)*x^2*e-1/3*b*
arccoth(c*x)*ln((c*x+1)/(c*x-1)-1)*x^6*e+1/3*b*arccoth(c*x)*ln(2)*x^6*e-1/3
6*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^3*e-23/180*I/c^6*b*e*Pi*csgn(I/((c
*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2

```

**maxima [C]** time = 0.35, size = 328, normalized size = 1.10

$$\frac{1}{6} adx^6 + \frac{1}{36} \left( 6x^6 \log(-c^2x^2 + 1) - c^2 \left( \frac{2c^4x^6 + 3c^2x^4 + 6x^2}{c^6} + \frac{6 \log(c^2x^2 - 1)}{c^8} \right) \right) be \operatorname{arccoth}(cx) + \frac{1}{180} \left( 30x^6 \operatorname{arccoth}(cx) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(x^5*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")

```

```

[Out] 1/6*a*d*x^6 + 1/36*(6*x^6*log(-c^2*x^2 + 1) - c^2*((2*c^4*x^6 + 3*c^2*x^4 +
6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*b*e*arccoth(c*x) + 1/180*(30*x^6*arc

```

$\coth(cx) + c(2(3c^4x^5 + 5c^2x^3 + 15x)/c^6 - 15\log(cx + 1)/c^7 + 15\log(cx - 1)/c^7)*bd + 1/36(6x^6\log(-c^2x^2 + 1) - c^2((2c^4x^6 + 3c^2x^4 + 6x^2)/c^6 + 6\log(c^2x^2 - 1)/c^8))*ae + 1/1800((60I\pi ic^5 - 44c^5)*x^5 + (100I\pi ic^3 - 190c^3)*x^3 + (300I\pi ic - 1470c)*x - 5(30I\pi - 12c^5x^5 - 20c^3x^3 - 60cx - 147)*\log(cx + 1) - 5(-30I\pi - 12c^5x^5 - 20c^3x^3 - 60cx + 147)*\log(cx - 1))*b*e/c^6$

**mupad [B]** time = 2.29, size = 510, normalized size = 1.72

$$\ln(1 - c^2x^2) \left( \frac{aex^6}{6} + \frac{bex}{6c^5} + \frac{bex^5}{30c} + \frac{bex^3}{18c^3} \right) - \ln\left(\frac{1}{cx} + 1\right) \left( \ln(1 - c^2x^2) \left( \frac{be}{12c^6} - \frac{bex^6}{12} \right) - \frac{bdx^6}{12} + \frac{bex^6}{36} + \frac{b}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b*acoth(cx))*(d + e*log(1 - c^2*x^2)),x)`

[Out]  $\log(1 - c^2x^2)*((aex^6)/6 + (bex)/(6c^5) + (bex^5)/(30c) + (bex^3)/(18c^3)) - \log(1/(cx) + 1)*(\log(1 - c^2x^2)*((be)/(12c^6) - (bex^6)/12) - (bdx^6)/12 + (bex^5)/36 + (bex^4)/(24c^2) + (bex^2)/(12c^4)) + \log(1 - 1/(cx))*(((bdx^7)/6 - (bc^2dx^9)/6)/(2*(x + cx^2)*(cx - 1)) + ((bex^7)/36 + (bex^5)/(12c^2) - (bex^3)/(6c^4) + (bc^2ex^9)/18)/(2*(x + cx^2)*(cx - 1)) + (\log(1 - c^2x^2)*((bex^7)/6 - (bc^2ex^9)/6))/(2*(x + cx^2)*(cx - 1)) - (be*log(1 - c^2x^2)*(x - c^2x^3))/(12c^6*(x + cx^2)*(cx - 1)) + x^4*((a(3d - e))/(12c^2) - (ad)/(4c^2)) + x^3*((b(15d - 11e))/(270c^3) - (7be)/(108c^3)) + x*((b(15d - 11e))/(90c^3) - (7be)/(36c^3))/c^2 - (be)/(2c^5)) + (ax^6*(3d - e))/18 + (x^2*((a(3d - e))/(3c^2) - (ad)/c^2))/(2c^2) - (\log(cx - 1)*(20ae - 10bd + 49be))/(120c^6) - (\log(cx + 1)*(20ae + 10bd - 49be))/(120c^6) + (bx^5*(15d - 11e))/(450c)$

**sympy [A]** time = 23.40, size = 362, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{adx^6}{6} + \frac{aex^6 \log(-c^2x^2+1)}{6} - \frac{aex^6}{18} - \frac{aex^4}{12c^2} - \frac{aex^2}{6c^4} - \frac{ae \log(-c^2x^2+1)}{6c^6} + \frac{bdx^6 \operatorname{acoth}(cx)}{6} + \frac{bex^6 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{6} - \frac{bex^6 \operatorname{acoth}(cx)}{18} \\ \frac{dx^6 \left( a + \frac{i\pi b}{2} \right)}{6} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*acoth(cx))*(d+e*ln(-c**2*x**2+1)),x)`

[Out] `Piecewise((ad*x**6/6 + a*e*x**6*log(-c**2*x**2 + 1)/6 - a*e*x**6/18 - a*e*x**4/(12*c**2) - a*e*x**2/(6*c**4) - a*e*log(-c**2*x**2 + 1)/(6*c**6) + b*d*x**6*acoth(cx)/6 + b*e*x**6*log(-c**2*x**2 + 1)*acoth(cx)/6 - b*e*x**6*a`

```

coth(c*x)/18 + b*d*x**5/(30*c) + b*e*x**5*log(-c**2*x**2 + 1)/(30*c) - 11*b
*e*x**5/(450*c) - b*e*x**4*acoth(c*x)/(12*c**2) + b*d*x**3/(18*c**3) + b*e*
x**3*log(-c**2*x**2 + 1)/(18*c**3) - 19*b*e*x**3/(180*c**3) - b*e*x**2*acot
h(c*x)/(6*c**4) + b*d*x/(6*c**5) + b*e*x*log(-c**2*x**2 + 1)/(6*c**5) - 49*
b*e*x/(60*c**5) - b*d*acoth(c*x)/(6*c**6) - b*e*log(-c**2*x**2 + 1)*acoth(c
*x)/(6*c**6) + 49*b*e*acoth(c*x)/(60*c**6), Ne(c, 0)), (d*x**6*(a + I*pi*b/
2)/6, True))

```

### 3.267 $\int x^3 \left( a + b \coth^{-1}(cx) \right) \left( d + e \log \left( 1 - c^2 x^2 \right) \right) dx$

**Optimal.** Leaf size=225

$$\frac{1}{4}x^4 \left( a + b \coth^{-1}(cx) \right) \left( e \log \left( 1 - c^2 x^2 \right) + d \right) - \frac{ex^2 \left( a + b \coth^{-1}(cx) \right)}{4c^2} - \frac{e \log \left( 1 - c^2 x^2 \right) \left( a + b \coth^{-1}(cx) \right)}{4c^4} - \frac{1}{8}ex^4 \left( a + b \coth^{-1}(cx) \right)$$

[Out]  $\frac{1}{8}bx^2(2d-3e)x/c^3 - \frac{2}{3}b^2ex/c^3 + \frac{1}{24}b^2(2d-e)x^3/c - \frac{1}{18}b^2ex^3/c - \frac{1}{4}e^2x^2(a+b\operatorname{arccoth}(cx))/c^2 - \frac{1}{8}e^2x^4(a+b\operatorname{arccoth}(cx)) - \frac{1}{8}b(2d-3e)\operatorname{arctanh}(cx)/c^4 + \frac{2}{3}b^2e\operatorname{arctanh}(cx)/c^4 + \frac{1}{4}b^2ex\ln(-c^2x^2+1)/c^3 + \frac{1}{12}b^2ex^3\ln(-c^2x^2+1)/c - \frac{1}{4}e(a+b\operatorname{arccoth}(cx))\ln(-c^2x^2+1)/c^4 + \frac{1}{4}x^4(a+b\operatorname{arccoth}(cx))(d+e\ln(-c^2x^2+1))$

**Rubi [A]** time = 0.26, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$ , Rules used = {2454, 2395, 43, 6084, 459, 321, 206, 2471, 2448, 2455, 302}

$$\frac{1}{4}x^4 \left( a + b \coth^{-1}(cx) \right) \left( e \log \left( 1 - c^2 x^2 \right) + d \right) - \frac{ex^2 \left( a + b \coth^{-1}(cx) \right)}{4c^2} - \frac{e \log \left( 1 - c^2 x^2 \right) \left( a + b \coth^{-1}(cx) \right)}{4c^4} - \frac{1}{8}ex^4 \left( a + b \coth^{-1}(cx) \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3(a + b\text{ArcCoth}[c*x])*(d + e\text{Log}[1 - c^2*x^2]), x]$

[Out]  $(b(2d - 3e)x)/(8c^3) - (2b^2ex)/(3c^3) + (b(2d - e)x^3)/(24c) - (b^2ex^3)/(18c) - (e^2x^2(a + b\text{ArcCoth}[c*x]))/(4c^2) - (e^2x^4(a + b\text{ArcCoth}[c*x]))/8 - (b(2d - 3e)\text{ArcTanh}[c*x])/(8c^4) + (2b^2e\text{ArcTanh}[c*x])/(3c^4) + (b^2ex\text{Log}[1 - c^2x^2])/(4c^3) + (b^2ex^3\text{Log}[1 - c^2x^2])/(12c) - (e(a + b\text{ArcCoth}[c*x])\text{Log}[1 - c^2x^2])/(4c^4) + (x^4(a + b\text{ArcCoth}[c*x])*(d + e\text{Log}[1 - c^2x^2]))/4$

#### Rule 43

$\text{Int}[(a_.) + (b_.)(x_)^{(m_.)}*((c_.) + (d_.)(x_)^{(n_.)}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

#### Rule 206

$\text{Int}[(a_.) + (b_.)(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$



Rule 302

$\text{Int}[(x_)^m / ((a_) + (b_.) * (x_)^n), x\_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, 2*n - 1]$

Rule 321

$\text{Int}[(c_.) * (x_)^m * ((a_) + (b_.) * (x_)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(c^{n-1} * (c*x)^{m-n+1} * (a + b*x^n)^{p+1}) / (b*(m + n*p + 1)), x] - \text{Dist}[(a*c^n * (m - n + 1)) / (b*(m + n*p + 1)), \text{Int}[(c*x)^{m-n} * (a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 459

$\text{Int}[(c_.) * (x_)^m * ((a_) + (b_.) * (x_)^n)^p * ((c_) + (d_.) * (x_)^n), x\_Symbol] \rightarrow \text{Simp}[(d * (e*x)^{m+1} * (a + b*x^n)^{p+1}) / (b*e*(m + n*(p + 1) + 1)), x] - \text{Dist}[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1)) / (b*(m + n*(p + 1) + 1)), \text{Int}[(e*x)^m * (a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m + n*(p + 1) + 1, 0]$

Rule 2395

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_) + (e_.) * (x_)^n)] * (b_.) * ((f_.) + (g_.) * (x_)^q), x\_Symbol] \rightarrow \text{Simp}[(f + g*x)^{q+1} * (a + b*\text{Log}[c*(d + e*x)^n]) / (g*(q + 1)), x] - \text{Dist}[(b*e^n) / (g*(q + 1)), \text{Int}[(f + g*x)^{q+1} / (d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, q\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[q, -1]$

Rule 2448

$\text{Int}[\text{Log}[(c_.) * ((d_) + (e_.) * (x_)^n)^p], x\_Symbol] \rightarrow \text{Simp}[x * \text{Log}[c*(d + e*x^n)^p], x] - \text{Dist}[e*n*p, \text{Int}[x^n / (d + e*x^n), x], x] /; \text{FreeQ}[\{c, d, e, n, p\}, x]$

Rule 2454

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_) + (e_.) * (x_)^n)^p] * (b_.)^q * (x_)^m, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) * (a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n, p, q\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \ \&\& \ (\text{GtQ}[(m + 1)/n, 0] \ || \ \text{IGtQ}[q, 0]) \ \&\& \ !(\text{EqQ}[q, 1] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 0])$

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(m_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])/(f*(m + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

### Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] := With[{t = ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && IntegerQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s, 0] && LtQ[r, 0]))
```

### Rule 6084

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(e_.)*(x_)^(m_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx &= -\frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e(a + b \coth^{-1}(cx))}{8c} \\
&= -\frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e(a + b \coth^{-1}(cx))}{8c} \\
&= \frac{b(2d - e)x^3}{24c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 (a + b \coth^{-1}(cx)) - \frac{e(a + b \coth^{-1}(cx))}{8c} \\
&= \frac{b(2d - 3e)x}{8c^3} + \frac{b(2d - e)x^3}{24c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 \\
&= \frac{b(2d - 3e)x}{8c^3} + \frac{b(2d - e)x^3}{24c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} - \frac{1}{8}ex^4 \\
&= \frac{b(2d - 3e)x}{8c^3} - \frac{bex}{2c^3} + \frac{b(2d - e)x^3}{24c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} \\
&= \frac{b(2d - 3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d - e)x^3}{24c} - \frac{bex^3}{18c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2} \\
&= \frac{b(2d - 3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d - e)x^3}{24c} - \frac{bex^3}{18c} - \frac{ex^2 (a + b \coth^{-1}(cx))}{4c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 192, normalized size = 0.85

$$\frac{12e \log(1 - c^2x^2) (3ac^4x^4 + 3b(c^4x^4 - 1) \coth^{-1}(cx) + bcx(c^2x^2 + 3)) + 3 \log(1 - cx)(-12ae + 6bd - 25be) - 3}{1}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (6\*b\*c\*(6\*d - 25\*e)\*x - 36\*a\*c^2\*e\*x^2 + 2\*b\*c^3\*(6\*d - 7\*e)\*x^3 + 18\*a\*c^4\*(2\*d - e)\*x^4 - 18\*b\*c^2\*x^2\*(-2\*c^2\*d\*x^2 + e\*(2 + c^2\*x^2))\*ArcCoth[c\*x] + 3\*(6\*b\*d - 12\*a\*e - 25\*b\*e)\*Log[1 - c\*x] - 3\*(6\*b\*d + 12\*a\*e - 25\*b\*e)\*Log[1 + c\*x] + 12\*e\*(3\*a\*c^4\*x^4 + b\*c\*x\*(3 + c^2\*x^2) + 3\*b\*(-1 + c^4\*x^4))\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(144\*c^4)

**fricas [A]** time = 0.52, size = 195, normalized size = 0.87

$$\frac{36ac^2ex^2 - 18(2ac^4d - ac^4e)x^4 - 2(6bc^3d - 7bc^3e)x^3 - 6(6bcd - 25bce)x - 12(3ac^4ex^4 + bc^3ex^3 + 3bcex - 3e)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] -1/144*(36*a*c^2*e*x^2 - 18*(2*a*c^4*d - a*c^4*e)*x^4 - 2*(6*b*c^3*d - 7*b*c^3*e)*x^3 - 6*(6*b*c*d - 25*b*c*e)*x - 12*(3*a*c^4*e*x^4 + b*c^3*e*x^3 + 3*b*c*e*x - 3*a*e)*log(-c^2*x^2 + 1) + 3*(6*b*c^2*e*x^2 - 3*(2*b*c^4*d - b*c^4*e)*x^4 + 6*b*d - 25*b*e - 6*(b*c^4*e*x^4 - b*e)*log(-c^2*x^2 + 1))*log((c*x + 1)/(c*x - 1))/c^4
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value
```

**maple** [C] time = 12.63, size = 3320, normalized size = 14.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -25/24*b*e*x/c^3-7/72*b*e*x^3/c+1/4*b*d*x/c^3+1/12*b*d*x^3/c+1/4*I*b*arccot
h(c*x)*csgn(I*(c*x+1)/(c*x-1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*Pi*x^4*e-1
/8*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x-1)/(c*x+1))^(1/2))
^2*Pi*x^4*e+1/8*I/c^3*b*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)
)/((c*x+1)/(c*x-1)-1)^2)^2*Pi*x*e-2/3/c^4*b*e*ln(2)-1/24*I/c*b*csgn(I/((c*x
+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x
+1)/(c*x-1))*Pi*x^3*e-1/8*I/c^3*b*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x
+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1))*Pi*x*e+1/8*I/c^4
*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+1)/(c*x-1)-1)^2)*
csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)-1/8*I*b*arccoth(c*x)*csgn(I/((
c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I*
(c*x+1)/(c*x-1))*Pi*x^4*e+1/8*I/c^3*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-
1)-1)^2)^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x*e+1/8*I/c^3*b*csgn(I*((c*x+1)/(c*x-
1)-1))^2*csgn(I*((c*x+1)/(c*x-1)-1)^2)*Pi*x*e-1/4*I/c^3*b*csgn(I*((c*x+1)/(
c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*Pi*x*e+1/4*I/c^3*b*csgn(I*(c*x+1)
)/(c*x-1))^2*csgn(I/((c*x-1)/(c*x+1))^(1/2))*Pi*x*e-1/8*I/c^3*b*csgn(I*(c*x
```

$$\begin{aligned}
& +1)/(c*x-1)) * \operatorname{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})^2 * \pi * x * e^{-1/8} * I/c^4 * b * \operatorname{arccoth}(c \\
& *x) * \pi * e * \operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x \\
& -1)-1)^2)^2 - 1/8 * I/c^4 * b * \operatorname{arccoth}(c*x) * \pi * e * \operatorname{csgn}(I*(c*x+1)/(c*x-1)) * \operatorname{csgn}(I*(c \\
& *x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 - 1/8 * I/c^4 * b * \operatorname{arccoth}(c*x) * e * \pi * \operatorname{csgn}(I \\
& *((c*x+1)/(c*x-1)-1))^2 * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2) + 1/4 * I/c^4 * b * \operatorname{arccoth}(c \\
& *x) * e * \pi * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)) * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2 - 1/4 * I/ \\
& c^4 * b * \operatorname{arccoth}(c*x) * \pi * e * \operatorname{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)}) * \operatorname{csgn}(I*(c*x+1)/(c*x \\
& -1))^2 + 1/8 * I/c^4 * b * \operatorname{arccoth}(c*x) * \pi * e * \operatorname{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})^2 * \operatorname{csgn} \\
& (I*(c*x+1)/(c*x-1)) + 1/6 * I/c^4 * b * e * \pi * \operatorname{csgn}(I*(c*x+1)/(c*x-1)) * \operatorname{csgn}(I/((c*x+1) \\
& )/(c*x-1)-1)^2) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2) + 1/8 * I * b * \operatorname{arcco} \\
& \operatorname{th}(c*x) * \operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x- \\
& 1)-1)^2)^2 * \pi * x^4 * e + 1/8 * I * b * \operatorname{arccoth}(c*x) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c \\
& *x-1)-1)^2)^2 * \operatorname{csgn}(I*(c*x+1)/(c*x-1)) * \pi * x^4 * e + 1/8 * I * b * \operatorname{arccoth}(c*x) * \operatorname{csgn}(I* \\
& ((c*x+1)/(c*x-1)-1))^2 * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2) * \pi * x^4 * e - 1/4 * I * b * \operatorname{arcco} \\
& \operatorname{th}(c*x) * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)) * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2 * \pi * x^4 * e \\
& + 1/24 * I/c * b * \operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/ \\
& (c*x-1)-1)^2)^2 * \pi * x^3 * e + 1/24 * I/c * b * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1) \\
& -1)^2)^2 * \operatorname{csgn}(I*(c*x+1)/(c*x-1)) * \pi * x^3 * e + 1/24 * I/c * b * \operatorname{csgn}(I*((c*x+1)/(c*x-1) \\
& )-1))^2 * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2) * \pi * x^3 * e - 1/12 * I/c * b * \operatorname{csgn}(I*((c*x+1)/(c \\
& *x-1)-1)) * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2 * \pi * x^3 * e + 1/12 * I/c * b * \operatorname{csgn}(I*(c*x+ \\
& 1)/(c*x-1))^2 * \operatorname{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)}) * \pi * x^3 * e - 1/24 * I/c * b * \operatorname{csgn}(I*(c \\
& *x+1)/(c*x-1)) * \operatorname{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})^2 * \pi * x^3 * e + 41/36 * e/c^4 * b - 1/3 \\
& /c^4 * b * d + 1/6 * I/c^4 * b * e * \pi * \operatorname{csgn}(I/((c*x-1)/(c*x+1))^{(1/2)})^2 * \operatorname{csgn}(I*(c*x+1)/ \\
& (c*x-1)) + 1/12 * I/c * b * \pi * x^3 * e - 1/6 * I/c^4 * b * e * \pi * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+ \\
& 1)/(c*x-1)-1)^2)^3 - 1/6 * I/c^4 * b * e * \pi * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^3 + 1/6 * I/c \\
& ^4 * b * e * \pi * \operatorname{csgn}(I*(c*x+1)/(c*x-1))^3 + 1/3 * I/c^4 * b * e * \pi * \operatorname{csgn}(I*(c*x+1)/(c*x-1) \\
& /((c*x+1)/(c*x-1)-1)^2)^2 + 1/4 * I/c^3 * b * \pi * x * e - 1/4 * I/c^4 * b * \pi * e * \operatorname{arccoth}(c*x) + \\
& 1/4 * I * b * \operatorname{arccoth}(c*x) * \pi * x^4 * e - 1/12/c^4 * b * e * (3 * \operatorname{arccoth}(c*x) * x^3 * c^3 + 3 * \operatorname{arccot} \\
& h(c*x) * x^2 * c^2 + c^2 * x^2 + 3 * \operatorname{arccoth}(c*x) * x * c + c * x + 3 * \operatorname{arccoth}(c*x) + 4) * (c*x-1) * \ln( \\
& (c*x-1)/(c*x+1)) + 1/8 * I * b * \operatorname{arccoth}(c*x) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x- \\
& 1)-1)^2)^3 * \pi * x^4 * e + 1/8 * I * b * \operatorname{arccoth}(c*x) * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^3 * \pi \\
& * x^4 * e - 1/8 * I * b * \operatorname{arccoth}(c*x) * \operatorname{csgn}(I*(c*x+1)/(c*x-1))^3 * \pi * x^4 * e - 1/4 * I * b * \operatorname{arcc} \\
& \operatorname{oth}(c*x) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 * \pi * x^4 * e + 1/24 * I/c * \\
& b * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3 * \pi * x^3 * e + 1/24 * I/c * b * \operatorname{csgn}(I \\
& *((c*x+1)/(c*x-1)-1)^2)^3 * \pi * x^3 * e - 1/24 * I/c * b * \operatorname{csgn}(I*(c*x+1)/(c*x-1))^3 * \pi \\
& * x^3 * e - 1/12 * I/c * b * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 * \pi * x^3 * e + \\
& 1/8 * I/c^3 * b * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3 * \pi * x * e + 1/8 * I/c^ \\
& 3 * b * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^3 * \pi * x * e - 1/8 * I/c^3 * b * \operatorname{csgn}(I*(c*x+1)/(c*x- \\
& 1))^3 * \pi * x * e - 1/8 * I/c^4 * b * \operatorname{arccoth}(c*x) * e * \pi * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/ \\
& (c*x-1)-1)^2)^3 - 1/8 * I/c^4 * b * \operatorname{arccoth}(c*x) * e * \pi * \operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2) \\
& ^3 + 1/8 * I/c^4 * b * \operatorname{arccoth}(c*x) * e * \pi * \operatorname{csgn}(I*(c*x+1)/(c*x-1))^3 - 1/4 * I/c^3 * b * \operatorname{csgn} \\
& (I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 * \pi * x * e + 1/4 * I/c^4 * b * \operatorname{arccoth}(c*x) \\
& * e * \pi * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 - 1/6 * I/c^4 * b * e * \pi * \operatorname{csgn} \\
& (I/((c*x+1)/(c*x-1)-1)^2) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 - 1 \\
& /6 * I/c^4 * b * e * \pi * \operatorname{csgn}(I*(c*x+1)/(c*x-1)) * \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*
\end{aligned}$$

$(x-1)^{-1})^2)^{-2-1/6} I/c^4 b e \pi \operatorname{csgn}(I((c*x+1)/(c*x-1)-1))^2 \operatorname{csgn}(I((c*x+1)/(c*x-1)-1)^2) + 1/3 I/c^4 b e \pi \operatorname{csgn}(I((c*x+1)/(c*x-1)-1)) \operatorname{csgn}(I((c*x+1)/(c*x-1)-1)^2)^{-2-1/3} I/c^4 b e \pi \operatorname{csgn}(I((c*x+1)/(c*x-1)-1)^{1/2}) \operatorname{csgn}(I((c*x+1)/(c*x-1)-1)^2)^{-1/3} I/c^4 b e \pi - 1/2 b \operatorname{arccoth}(c*x) \ln((c*x+1)/(c*x-1)-1) * x^4 e^{1/2} b \operatorname{arccoth}(c*x) \ln(2) * x^4 e^{-1/6} / c * b \ln((c*x+1)/(c*x-1)-1) * x^3 e^{1/6} / c * b \ln(2) * x^3 e^{-1/4} / c^2 b \operatorname{arccoth}(c*x) * x^2 e^{-1/2} / c^3 b \ln((c*x+1)/(c*x-1)-1) * x e^{1/2} / c^3 b \ln(2) * x e^{1/2} / c^4 b \operatorname{arccoth}(c*x) e \ln((c*x+1)/(c*x-1)-1) - 1/2 / c^4 b \operatorname{arccoth}(c*x) \ln(2) * e^{1/4} x^4 a d - 1/8 a e x^4 - 1/4 a e / c^2 x^2 + 1/4 a e x^4 \ln(-c^2 x^2 + 1) - 1/4 a e / c^4 \ln(c^2 x^2 - 1) - 1/4 / c^4 b \operatorname{arccoth}(c*x) * d + 4/24 / c^4 b \operatorname{arccoth}(c*x) * e + 1/4 b \operatorname{arccoth}(c*x) * x^4 d - 1/8 b \operatorname{arccoth}(c*x) * x^4 e$

**maxima** [C] time = 0.35, size = 269, normalized size = 1.20

$$\frac{1}{4} a d x^4 + \frac{1}{8} \left( 2 x^4 \log(-c^2 x^2 + 1) - c^2 \left( \frac{c^2 x^4 + 2 x^2}{c^4} + \frac{2 \log(c^2 x^2 - 1)}{c^6} \right) \right) b e \operatorname{arccoth}(c x) + \frac{1}{24} \left( 6 x^4 \operatorname{arccoth}(c x) + c \left( \frac{2}{c^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

[Out] 1/4\*a\*d\*x^4 + 1/8\*(2\*x^4\*log(-c^2\*x^2 + 1) - c^2\*((c^2\*x^4 + 2\*x^2)/c^4 + 2\*log(c^2\*x^2 - 1)/c^6))\*b\*e\*arccoth(c\*x) + 1/24\*(6\*x^4\*arccoth(c\*x) + c\*(2\*(c^2\*x^3 + 3\*x)/c^4 - 3\*log(c\*x + 1)/c^5 + 3\*log(c\*x - 1)/c^5))\*b\*d + 1/8\*(2\*x^4\*log(-c^2\*x^2 + 1) - c^2\*((c^2\*x^4 + 2\*x^2)/c^4 + 2\*log(c^2\*x^2 - 1)/c^6))\*a\*e + 1/144\*((12\*I\*pi\*c^3 - 14\*c^3)\*x^3 + (36\*I\*pi\*c - 150\*c)\*x - 3\*(6\*I\*pi - 4\*c^3\*x^3 - 12\*c\*x - 25)\*log(c\*x + 1) - 3\*(-6\*I\*pi - 4\*c^3\*x^3 - 12\*c\*x + 25)\*log(c\*x - 1))\*b\*e/c^4

**mupad** [B] time = 2.28, size = 414, normalized size = 1.84

$$\ln\left(1 - \frac{1}{c x}\right) \left( \frac{\frac{b e x^5}{8} - \frac{b e x^3}{4 c^2} + \frac{b c^2 e x^7}{8}}{2(c x^2 + x)(c x - 1)} + \frac{\frac{b d x^5}{4} - \frac{b c^2 d x^7}{4}}{2(c x^2 + x)(c x - 1)} + \frac{\ln(1 - c^2 x^2) \left( \frac{b e x^5}{4} - \frac{b c^2 e x^7}{4} \right)}{2(c x^2 + x)(c x - 1)} - \frac{b e \ln(1 - c^2 x^2)}{8 c^4 (c x^2 + x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)),x)

[Out] log(1 - 1/(c\*x))\*(((b\*e\*x^5)/8 - (b\*e\*x^3)/(4\*c^2) + (b\*c^2\*e\*x^7)/8)/(2\*(x + c\*x^2)\*(c\*x - 1)) + ((b\*d\*x^5)/4 - (b\*c^2\*d\*x^7)/4)/(2\*(x + c\*x^2)\*(c\*x - 1)) + (log(1 - c^2\*x^2)\*((b\*e\*x^5)/4 - (b\*c^2\*e\*x^7)/4))/(2\*(x + c\*x^2)\*(c\*x - 1)) - (b\*e\*log(1 - c^2\*x^2)\*(x - c^2\*x^3))/(8\*c^4\*(x + c\*x^2)\*(c\*x - 1))) + x\*((b\*(6\*d - 7\*e))/(24\*c^3) - (3\*b\*e)/(4\*c^3)) + log(1 - c^2\*x^2)\*((a\*e\*x^4)/4 + (b\*e\*x)/(4\*c^3) + (b\*e\*x^3)/(12\*c)) - log(1/(c\*x) + 1)\*(log(1

$$-c^2x^2*((b*e)/(8*c^4) - (b*e*x^4)/8) - (b*d*x^4)/8 + (b*e*x^4)/16 + (b*e*x^2)/(8*c^2) + x^2*((a*(2*d - e))/(4*c^2) - (a*d)/(2*c^2)) + (a*x^4*(2*d - e))/8 - (\log(cx - 1)*(12*a*e - 6*b*d + 25*b*e))/(48*c^4) - (\log(cx + 1)*(12*a*e + 6*b*d - 25*b*e))/(48*c^4) + (b*x^3*(6*d - 7*e))/(72*c)$$

**sympy** [A] time = 9.88, size = 286, normalized size = 1.27

$$\left\{ \begin{array}{l} \frac{adx^4}{4} + \frac{aex^4 \log(-c^2x^2+1)}{4} - \frac{aex^4}{8} - \frac{aex^2}{4c^2} - \frac{ae \log(-c^2x^2+1)}{4c^4} + \frac{bdx^4 \operatorname{acoth}(cx)}{4} + \frac{bex^4 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{4} - \frac{bex^4 \operatorname{acoth}(cx)}{8} + \frac{bdx^4}{12} \\ \frac{dx^4 \left(a + \frac{i\pi b}{2}\right)}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1)),x)

[Out] Piecewise((a\*d\*x\*\*4/4 + a\*e\*x\*\*4\*log(-c\*\*2\*x\*\*2 + 1)/4 - a\*e\*x\*\*4/8 - a\*e\*x\*\*2/(4\*c\*\*2) - a\*e\*log(-c\*\*2\*x\*\*2 + 1)/(4\*c\*\*4) + b\*d\*x\*\*4\*acoth(c\*x)/4 + b\*e\*x\*\*4\*log(-c\*\*2\*x\*\*2 + 1)\*acoth(c\*x)/4 - b\*e\*x\*\*4\*acoth(c\*x)/8 + b\*d\*x\*\*3/(12\*c) + b\*e\*x\*\*3\*log(-c\*\*2\*x\*\*2 + 1)/(12\*c) - 7\*b\*e\*x\*\*3/(72\*c) - b\*e\*x\*\*2\*acoth(c\*x)/(4\*c\*\*2) + b\*d\*x/(4\*c\*\*3) + b\*e\*x\*log(-c\*\*2\*x\*\*2 + 1)/(4\*c\*\*3) - 25\*b\*e\*x/(24\*c\*\*3) - b\*d\*acoth(c\*x)/(4\*c\*\*4) - b\*e\*log(-c\*\*2\*x\*\*2 + 1)\*acoth(c\*x)/(4\*c\*\*4) + 25\*b\*e\*acoth(c\*x)/(24\*c\*\*4), Ne(c, 0)), (d\*x\*\*4\*(a + I\*pi\*b/2)/4, True))

### 3.268 $\int x \left( a + b \coth^{-1}(cx) \right) \left( d + e \log \left( 1 - c^2 x^2 \right) \right) dx$

**Optimal.** Leaf size=140

$$-\frac{e(1-c^2x^2)\log(1-c^2x^2)(a+b\coth^{-1}(cx))}{2c^2} + \frac{1}{2}dx^2(a+b\coth^{-1}(cx)) - \frac{1}{2}ex^2(a+b\coth^{-1}(cx)) - \frac{b(d-e)\tanh^{-1}(cx)}{2c^2}$$

[Out]  $\frac{1}{2}b(d-e)x/c - bex/c + \frac{1}{2}d*x^2*(a+b*\operatorname{arccoth}(c*x)) - \frac{1}{2}e*x^2*(a+b*\operatorname{arccoth}(c*x)) - \frac{1}{2}b(d-e)*\operatorname{arctanh}(c*x)/c^2 + b*e*\operatorname{arctanh}(c*x)/c^2 + \frac{1}{2}b*e*x*\ln(-c^2*x^2+1)/c - \frac{1}{2}e*(-c^2*x^2+1)*(a+b*\operatorname{arccoth}(c*x))*\ln(-c^2*x^2+1)/c^2$

**Rubi [A]** time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$ , Rules used = {2454, 2389, 2295, 6084, 321, 207, 2448, 206}

$$-\frac{e(1-c^2x^2)\log(1-c^2x^2)(a+b\coth^{-1}(cx))}{2c^2} + \frac{1}{2}dx^2(a+b\coth^{-1}(cx)) - \frac{1}{2}ex^2(a+b\coth^{-1}(cx)) - \frac{b(d-e)\tanh^{-1}(cx)}{2c^2}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]),x]`

[Out]  $(b*(d - e)*x)/(2*c) - (b*e*x)/c + (d*x^2*(a + b*\operatorname{ArcCoth}[c*x]))/2 - (e*x^2*(a + b*\operatorname{ArcCoth}[c*x]))/2 - (b*(d - e)*\operatorname{ArcTanh}[c*x])/(2*c^2) + (b*e*\operatorname{ArcTanh}[c*x])/c^2 + (b*e*x*\operatorname{Log}[1 - c^2*x^2])/(2*c) - (e*(1 - c^2*x^2)*(a + b*\operatorname{ArcCoth}[c*x])*\operatorname{Log}[1 - c^2*x^2])/(2*c^2)$

#### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

#### Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

#### Rule 321

`Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p, 0]`



+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

### Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

### Rule 6084

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.)\*(x\_)^(m\_.), x\_Symbol] := With[{u = IntHide[x^m\*(d + e\*Log[f + g\*x^2]), x]}, Dist[a + b\*ArcCoth[c\*x], u, x] - Dist[b\*c, Int[ExpandIntegrand[u/(1 - c^2\*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)/2, 0]

### Rubi steps

$$\begin{aligned}
\int x(a + b \operatorname{coth}^{-1}(cx))(d + e \log(1 - c^2x^2)) dx &= \frac{1}{2}dx^2(a + b \operatorname{coth}^{-1}(cx)) - \frac{1}{2}ex^2(a + b \operatorname{coth}^{-1}(cx)) - \frac{e(1 - c^2x^2)}{2c} \\
&= \frac{1}{2}dx^2(a + b \operatorname{coth}^{-1}(cx)) - \frac{1}{2}ex^2(a + b \operatorname{coth}^{-1}(cx)) - \frac{e(1 - c^2x^2)}{2c} \\
&= \frac{b(d - e)x}{2c} + \frac{1}{2}dx^2(a + b \operatorname{coth}^{-1}(cx)) - \frac{1}{2}ex^2(a + b \operatorname{coth}^{-1}(cx)) \\
&= \frac{b(d - e)x}{2c} - \frac{bex}{c} + \frac{1}{2}dx^2(a + b \operatorname{coth}^{-1}(cx)) - \frac{1}{2}ex^2(a + b \operatorname{coth}^{-1}(cx)) \\
&= \frac{b(d - e)x}{2c} - \frac{bex}{c} + \frac{1}{2}dx^2(a + b \operatorname{coth}^{-1}(cx)) - \frac{1}{2}ex^2(a + b \operatorname{coth}^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 129, normalized size = 0.92

$$\frac{2e \log(1 - c^2x^2)(cx(acx + b) + b(c^2x^2 - 1) \operatorname{coth}^{-1}(cx)) + \log(1 - cx)(b(d - 3e) - 2ae) - \log(cx + 1)(2ae + b(d - 3e))}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (2\*b\*c\*(d - 3\*e)\*x + 2\*a\*c^2\*(d - e)\*x^2 + 2\*b\*c^2\*(d - e)\*x^2\*ArcCoth[c\*x] + (b\*(d - 3\*e) - 2\*a\*e)\*Log[1 - c\*x] - (b\*(d - 3\*e) + 2\*a\*e)\*Log[1 + c\*x] + 2\*e\*(c\*x\*(b + a\*c\*x) + b\*(-1 + c^2\*x^2)\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/(4\*c^2)

**fricas [A]** time = 0.58, size = 138, normalized size = 0.99

$$\frac{2(ac^2d - ac^2e)x^2 + 2(bcd - 3bce)x + 2(ac^2ex^2 + bcex - ae) \log(-c^2x^2 + 1) + ((bc^2d - bc^2e)x^2 - bd + 3be + (bc^2d - bc^2e)) \log((cx + 1)/(cx - 1))}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="fricas")

[Out] 1/4\*(2\*(a\*c^2\*d - a\*c^2\*e)\*x^2 + 2\*(b\*c\*d - 3\*b\*c\*e)\*x + 2\*(a\*c^2\*e\*x^2 + b\*c\*e\*x - a\*e)\*log(-c^2\*x^2 + 1) + ((b\*c^2\*d - b\*c^2\*e)\*x^2 - b\*d + 3\*b\*e + (b\*c^2\*e\*x^2 - b\*e)\*log(-c^2\*x^2 + 1))\*log((c\*x + 1)/(c\*x - 1)))/c^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] integrate((b*arccoth(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)*x, x)
```

**maple** [C] time = 3.36, size = 2616, normalized size = 18.69

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccoth(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/c^2*b*e*ln(2)-1/2/c^2*b*d+1/2*a*x^2*d-1/2*a*e*x^2+1/2*a*e/c^2+3/2*e/c^2*
b-1/2/c^2*b*arccoth(c*x)*d+5/2/c^2*b*arccoth(c*x)*e+1/2*a*e*ln(-c^2*x^2+1)*
x^2-1/2*a*e/c^2*ln(-c^2*x^2+1)+1/2*b*arccoth(c*x)*x^2*d-1/2*b*arccoth(c*x)*
x^2*e+1/2*b*d*x/c-3/2*b*e*x/c-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)
/(c*x-1)-1)^2)^3-1/2*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*
Pi*x*e+1/2*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)
)-1)^2)^2+1/4*I/c^2*b*Pi*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)
/(c*x-1))-1/2*I/c^2*b*Pi*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c
*x-1))^2-1/4*I/c^2*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/(
(c*x+1)/(c*x-1)-1)^2)^2-1/4*I/c^2*b*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn
(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*I/c^2*b*e*Pi*csgn(I*((c*x+1)
)/(c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2-1/4*I/c^2*b*e*Pi*csgn(I*((c*x
+1)/(c*x-1)-1))^2*csgn(I*((c*x+1)/(c*x-1)-1)^2)+1/4*I/c^2*b*arccoth(c*x)*e*
Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*
x-1)/((c*x+1)/(c*x-1)-1)^2)-1/4*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1))*cs
gn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I/((c*x+1)/(c*x-1)-1)^2)*P
i*x^2*e-1/4*I/c*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(
c*x-1)-1)^2)*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x*e-1/c*b*ln((c*x+1)/(c*x-1)-
1)*x*e+1/c*b*ln(2)*x*e+1/c^2*b*arccoth(c*x)*e*ln((c*x+1)/(c*x-1)-1)-1/c^2*b
*arccoth(c*x)*e*ln(2)-b*arccoth(c*x)*ln((c*x+1)/(c*x-1)-1)*x^2*e+b*arccoth(
c*x)*ln(2)*x^2*e+1/4*I/c^2*b*Pi*e*csgn(I*(c*x+1)/(c*x-1))^3-1/4*I/c^2*b*e*P
i*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/4*I/c^2*b*e*Pi*csgn(I*(
(c*x+1)/(c*x-1)-1)^2)^3+1/2*I/c^2*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c
*x-1)-1)^2)^2+1/2*I/c*b*Pi*x*e-1/2*I/c^2*b*Pi*e*arccoth(c*x)+1/2*I*b*arccot
h(c*x)*Pi*x^2*e-1/2/c^2*b*e*(arccoth(c*x)*x+c+arccoth(c*x)+1)*(c*x-1)*ln((c
*x-1)/(c*x+1))+1/2*I/c*b*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*
x-1))^2*Pi*x*e+1/4*I/c*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c
*x+1)/(c*x-1)-1)^2)^2*Pi*x*e+1/4*I/c*b*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x
-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x*e-1/2*I/c*b*csgn(I*((c*x+1)/
(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*x*e+1/4*I/c*b*csgn(I*((c*x+1)
)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2*Pi*x*e+1/4*I/c^2*b*arccoth(c*
x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))-1/2*I/c^2
```

```

*b*arccoth(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1)
)^2-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c
*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I/((c*x+1
)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*I/c^2*b
*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2
)^2-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1))^2*csgn(I*((c*
x+1)/(c*x-1)-1)^2)+1/4*I/c^2*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+1)
/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)+1/2*I*b*arccot
h(c*x)*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*Pi*x^2*e+1
/4*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)
/(c*x-1)-1)^2)^2*Pi*x^2*e-1/4*I*b*arccoth(c*x)*csgn(I/((c*x-1)/(c*x+1))^(1/
2))^2*csgn(I*(c*x+1)/(c*x-1))*Pi*x^2*e+1/4*I*b*arccoth(c*x)*csgn(I*(c*x+1)/
(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x^2*e-1/2
*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1
))*Pi*x^2*e+1/4*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x
+1)/(c*x-1)-1))^2*Pi*x^2*e-1/4*I/c*b*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn
(I*(c*x+1)/(c*x-1))*Pi*x*e-1/2*I/c^2*b*Pi*e-1/4*I*b*arccoth(c*x)*csgn(I*(c*
x+1)/(c*x-1))^3*Pi*x^2*e+1/4*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1)/((c*x+
1)/(c*x-1)-1)^2)^3*Pi*x^2*e+1/4*I*b*arccoth(c*x)*csgn(I*((c*x+1)/(c*x-1)-1)
^2)^3*Pi*x^2*e-1/2*I*b*arccoth(c*x)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)
-1)^2)^2*Pi*x^2*e-1/4*I/c*b*csgn(I*(c*x+1)/(c*x-1))^3*Pi*x*e+1/4*I/c*b*csgn
(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*Pi*x*e+1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-
1))^3-1/4*I/c^2*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)
-1)^2)^3

```

**maxima** [A] time = 0.34, size = 171, normalized size = 1.22

$$\frac{1}{2} adx^2 + \frac{1}{4} \left( 2x^2 \operatorname{arccoth}(cx) + c \left( \frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3} \right) \right) bd - \frac{(c^2x^2 - (c^2x^2 - 1) \log(-c^2x^2 + 1) - 1) be}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

```

[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arccoth(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*b
*e*arccoth(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*a
*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*b*e
/c^2

```

**mupad** [B] time = 2.05, size = 329, normalized size = 2.35

$$\ln\left(1 - \frac{1}{cx}\right) \left( \frac{\frac{bdx^3}{2} - \frac{bc^2dx^5}{2}}{2(cx^2+x)(cx-1)} - \frac{\frac{bex^3}{2} - \frac{bc^2ex^5}{2}}{2(cx^2+x)(cx-1)} + \frac{\ln(1-c^2x^2) \left( \frac{bex^3}{2} - \frac{bc^2ex^5}{2} \right)}{2(cx^2+x)(cx-1)} - \frac{be \ln(1-c^2x^2)}{4c^2(cx^2+x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)`

[Out]  $\log(1 - 1/(c*x)) * (((b*d*x^3)/2 - (b*c^2*d*x^5)/2)/(2*(x + c*x^2)*(c*x - 1)) - ((b*e*x^3)/2 - (b*c^2*e*x^5)/2)/(2*(x + c*x^2)*(c*x - 1)) + (\log(1 - c^2*x^2)*((b*e*x^3)/2 - (b*c^2*e*x^5)/2))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*\log(1 - c^2*x^2)*(x - c^2*x^3))/(4*c^2*(x + c*x^2)*(c*x - 1))) + \log(1 - c^2*x^2)*((a*e*x^2)/2 + (b*e*x)/(2*c)) - \log(1/(c*x) + 1)*(\log(1 - c^2*x^2)*((b*e)/(4*c^2) - (b*e*x^2)/4) - (b*d*x^2)/4 + (b*e*x^2)/4) + (a*x^2*(d - e))/2 - (\log(c*x + 1)*(2*a*e + b*d - 3*b*e))/(4*c^2) - (\log(c*x - 1)*(2*a*e - b*d + 3*b*e))/(4*c^2) + (b*x*(d - 3*e))/(2*c)$

**sympy** [A] time = 3.93, size = 209, normalized size = 1.49

$$\left\{ \begin{array}{l} \frac{adx^2}{2} + \frac{aex^2 \log(-c^2x^2+1)}{2} - \frac{aex^2}{2} - \frac{ae \log(-c^2x^2+1)}{2c^2} + \frac{bdx^2 \operatorname{acoth}(cx)}{2} + \frac{bex^2 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{2} - \frac{bex^2 \operatorname{acoth}(cx)}{2} + \frac{bdx}{2c} + \frac{bex}{2} \\ \frac{dx^2 \left( a + \frac{i\pi b}{2} \right)}{2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)`

[Out] `Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*log(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*acoth(c*x)/2 + b*e*x**2*log(-c**2*x**2 + 1)*acoth(c*x)/2 - b*e*x**2*acoth(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-c**2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*acoth(c*x)/(2*c**2) - b*e*log(-c**2*x**2 + 1)*acoth(c*x)/(2*c**2) + 3*b*e*acoth(c*x)/(2*c**2), Ne(c, 0)), (d*x**2*(a + I*pi*b/2)/2, True))`

$$3.269 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x} dx$$

**Optimal.** Leaf size=381

$$-\frac{1}{2}ae\text{Li}_2(c^2x^2)+ad \log(x)+\frac{1}{2}be\text{Li}_2\left(-\frac{1}{cx}\right) \log(-c^2x^2)-\frac{1}{2}be\text{Li}_2\left(-\frac{1}{cx}\right) \left(\log(-c^2x^2) - \log(1-c^2x^2) + \log\left(1-\frac{1}{cx}\right)\right)$$

[Out]  $-1/2*b*e*\ln(1+1/c/x)^2*\ln(-1/c/x)+1/2*b*e*\ln(1-1/c/x)^2*\ln(1/c/x)+a*d*\ln(x)$   
 $-b*e*\ln((c+1/x)/c)*\text{polylog}(2,(c+1/x)/c)+b*e*\ln(1-1/c/x)*\text{polylog}(2,1-1/c/x)+$   
 $1/2*b*d*\text{polylog}(2,-1/c/x)+1/2*b*e*\ln(-c^2*x^2)*\text{polylog}(2,-1/c/x)-1/2*b*e*(1$   
 $n(1-1/c/x)+\ln(1+1/c/x)+\ln(-c^2*x^2)-\ln(-c^2*x^2+1))*\text{polylog}(2,-1/c/x)-1/2*b$   
 $*d*\text{polylog}(2,1/c/x)-1/2*b*e*\ln(-c^2*x^2)*\text{polylog}(2,1/c/x)+1/2*b*e*(\ln(1-1/c$   
 $/x)+\ln(1+1/c/x)+\ln(-c^2*x^2)-\ln(-c^2*x^2+1))*\text{polylog}(2,1/c/x)-1/2*a*e*\text{polyl}$   
 $\text{og}(2,c^2*x^2)+b*e*\text{polylog}(3,(c+1/x)/c)-b*e*\text{polylog}(3,1-1/c/x)+b*e*\text{polylog}(3$   
 $, -1/c/x)-b*e*\text{polylog}(3,1/c/x)$

**Rubi [A]** time = 0.44, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$ , Rules used = {6080, 5913, 6078, 2391, 6076, 2454, 2396, 2433, 2374, 6589, 6070}

$$-\frac{1}{2}ae\text{PolyLog}\left(2,c^2x^2\right)+\frac{1}{2}be \log(-c^2x^2) \text{PolyLog}\left(2,-\frac{1}{cx}\right)-\frac{1}{2}be \left(\log(-c^2x^2) - \log(1-c^2x^2) + \log\left(1-\frac{1}{cx}\right)\right) +$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x,x]

[Out]  $-(b*e*\text{Log}[1 + 1/(c*x)]^2*\text{Log}[-(1/(c*x))])/2 + (b*e*\text{Log}[1 - 1/(c*x)]^2*\text{Log}[1$   
 $/ (c*x)])/2 + a*d*\text{Log}[x] - b*e*\text{Log}[(c + x^(-1))/c]*\text{PolyLog}[2, (c + x^(-1))/c$   
 $] + b*e*\text{Log}[1 - 1/(c*x)]*\text{PolyLog}[2, 1 - 1/(c*x)] + (b*d*\text{PolyLog}[2, -(1/(c*x$   
 $))])/2 + (b*e*\text{Log}[-(c^2*x^2)]*\text{PolyLog}[2, -(1/(c*x))])/2 - (b*e*(\text{Log}[1 - 1/(c$   
 $*x)] + \text{Log}[1 + 1/(c*x)] + \text{Log}[-(c^2*x^2)] - \text{Log}[1 - c^2*x^2])* \text{PolyLog}[2, -$   
 $(1/(c*x))])/2 - (b*d*\text{PolyLog}[2, 1/(c*x)])/2 - (b*e*\text{Log}[-(c^2*x^2)]*\text{PolyLog}[$   
 $2, 1/(c*x)])/2 + (b*e*(\text{Log}[1 - 1/(c*x)] + \text{Log}[1 + 1/(c*x)] + \text{Log}[-(c^2*x^2)$   
 $] - \text{Log}[1 - c^2*x^2])* \text{PolyLog}[2, 1/(c*x)])/2 - (a*e*\text{PolyLog}[2, c^2*x^2])/2$   
 $+ b*e*\text{PolyLog}[3, (c + x^(-1))/c] - b*e*\text{PolyLog}[3, 1 - 1/(c*x)] + b*e*\text{PolyLo}$   
 $g[3, -(1/(c*x))] - b*e*\text{PolyLog}[3, 1/(c*x)]$

**Rule 2374**

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_.)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x

$\wedge n))^p)/m, x] + \text{Dist}[(b*n*p)/m, \text{Int}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^{(p-1)})/x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[d*e, 1]$

### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

### Rule 2396

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})]*(b_.))^{(p_.)}/((f_.) + (g_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])^p)/g, x] - \text{Dist}[(b*e*n*p)/g, \text{Int}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])^{(p-1)})/(d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{IGtQ}[p, 1]$

### Rule 2433

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})]*(b_.))^{(p_.)*((f_.) + \text{Log}[(h_.)*((i_.) + (j_.)*(x_)^{(m_.)})*(g_.))*((k_.) + (l_.)*(x_)^{(r_.)})], x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(k*x)/d]^r*(a + b*\text{Log}[c*x^n])^p*(f + g*\text{Log}[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r\}, x] \&\& \text{EqQ}[e*k - d*l, 0]$

### Rule 2454

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)*(x_)^{(m_.)}], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, e, m, n, p, q\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& (\text{GtQ}[(m+1)/n, 0] \|\| \text{IGtQ}[q, 0]) \&\& !(\text{EqQ}[q, 1] \&\& \text{ILtQ}[n, 0] \&\& \text{IGtQ}[m, 0])$

### Rule 5913

$\text{Int}[(a_.) + \text{ArcCoth}[(c_.)*(x_)]*(b_.))/(x_), x\_Symbol] \rightarrow \text{Simp}[a*\text{Log}[x], x] + (\text{Simp}[(b*\text{PolyLog}[2, -(c*x)^{-1}]]/2, x] - \text{Simp}[(b*\text{PolyLog}[2, 1/(c*x)]]/2, x)) /; \text{FreeQ}[\{a, b, c\}, x]$

### Rule 6070

$\text{Int}[(\text{ArcCoth}[(c_.)*(x_)^{(n_.)}]*\text{Log}[(d_.)*(x_)^{(m_.)}])/(x_), x\_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[(\text{Log}[d*x^m]*\text{Log}[1 + 1/(c*x^n)])]/x, x], x] - \text{Dist}[1/2, \text{Int}[(\text{Log}[d*x^m]*\text{Log}[1 - 1/(c*x^n)])]/x, x], x] /; \text{FreeQ}[\{c, d, m, n\}, x]$

Rule 6076

```
Int[(ArcCoth[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] := Dist
[Log[f + g*x^2] - Log[-(c^2*x^2)] - Log[1 - 1/(c*x)] - Log[1 + 1/(c*x)], In
t[ArcCoth[c*x]/x, x], x] + (-Dist[1/2, Int[Log[1 - 1/(c*x)]^2/x, x], x] + D
ist[1/2, Int[Log[1 + 1/(c*x)]^2/x, x], x] + Int[(Log[-(c^2*x^2)]*ArcCoth[c*
x])/x, x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

Rule 6078

```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcCoth[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] := Dist[a, Int[Log[f + g*x^2]/x, x], x] + Dist[b, Int[(Log[f + g*x^
2]*ArcCoth[c*x])/x, x], x] /; FreeQ[{a, b, c, f, g}, x]
```

Rule 6080

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
(d_)))/(x_), x_Symbol] := Dist[d, Int[(a + b*ArcCoth[c*x])/x, x], x] + Dis
t[e, Int[(Log[f + g*x^2]*(a + b*ArcCoth[c*x]))/x, x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps



$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} dx &= d \int \frac{a + b \coth^{-1}(cx)}{x} dx + e \int \frac{(a + b \coth^{-1}(cx)) \log(1 - c^2x^2)}{x} dx \\
&= ad \log(x) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right) + (ae) \int \frac{\log(1 - c^2x^2)}{x} dx \\
&= ad \log(x) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}bd\text{Li}_2\left(\frac{1}{cx}\right) - \frac{1}{2}ae\text{Li}_2(c^2x^2) - \frac{1}{2}ae \log^2(1 - c^2x^2) \\
&= ad \log(x) + \frac{1}{2}bd\text{Li}_2\left(-\frac{1}{cx}\right) - \frac{1}{2}be \left( \log\left(1 - \frac{1}{cx}\right) + \log\left(1 + \frac{1}{cx}\right) \right) \\
&= -\frac{1}{2}be \log^2\left(1 + \frac{1}{cx}\right) \log\left(-\frac{1}{cx}\right) + \frac{1}{2}be \log^2\left(1 - \frac{1}{cx}\right) \log\left(\frac{1}{cx}\right) + \\
&= -\frac{1}{2}be \log^2\left(1 + \frac{1}{cx}\right) \log\left(-\frac{1}{cx}\right) + \frac{1}{2}be \log^2\left(1 - \frac{1}{cx}\right) \log\left(\frac{1}{cx}\right) + \\
&= -\frac{1}{2}be \log^2\left(1 + \frac{1}{cx}\right) \log\left(-\frac{1}{cx}\right) + \frac{1}{2}be \log^2\left(1 - \frac{1}{cx}\right) \log\left(\frac{1}{cx}\right) + \\
&= -\frac{1}{2}be \log^2\left(1 + \frac{1}{cx}\right) \log\left(-\frac{1}{cx}\right) + \frac{1}{2}be \log^2\left(1 - \frac{1}{cx}\right) \log\left(\frac{1}{cx}\right) +
\end{aligned}$$

**Mathematica** [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x,x]

[Out] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x, x]

**fricas** [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x, x)

**maple** [C] time = 4.56, size = 864, normalized size = 2.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x,x)

[Out] b\*e\*polylog(3,-c\*x+1)-b\*e\*polylog(3,c\*x+1)-(-1/2\*I\*Pi\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^2-1/4\*I\*Pi\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))+1/4\*I\*Pi\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/4\*I\*Pi\*b\*e\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/4\*I\*Pi\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^3+1/2\*I\*e\*Pi\*b+a\*e+1/2\*b\*d)\*dilog(c\*x+1)-1/2\*I\*ln(c\*x-1)\*Pi\*ln(c\*x)\*b\*e+ln(c\*x)\*ln(c\*x-1)\*a\*e-1/2\*ln(c\*x)\*ln(c\*x-1)\*b\*d-1/2\*ln(c\*x)\*ln(c\*x-1)^2\*b\*e-polylog(2,-c\*x+1)\*ln(c\*x-1)\*b\*e+1/2\*b\*e\*ln(-c\*x)\*ln(c\*x+1)^2+b\*e\*ln(c\*x+1)\*polylog(2,c\*x+1)-1/4\*I\*ln(c\*x)\*ln(c\*x-1)\*Pi\*b\*e\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/4\*I\*dilog(c\*x)\*Pi\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))-1/2\*I\*ln(c\*x)\*Pi\*a\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))-1/4\*I\*ln(c\*x)\*ln(c\*x-1)\*Pi\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+dilog(c\*x)\*a\*e-1/2\*dilog(c\*x)\*b\*d+ln(c\*x)\*a\*d-1/4\*I\*dilog(c\*x)\*Pi\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^3+1/4\*I\*ln(c\*x)\*ln(c\*x-1)\*Pi\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))+I\*ln(c\*x)\*Pi\*a\*e-1/4\*I\*ln(c\*x)\*ln(c\*x-1)\*Pi\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^3+1/2\*I\*ln(c\*x)\*ln(c\*x-1)\*Pi\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^2-1/2\*I\*Pi\*dilog(c\*x)\*b\*e-1/4\*I\*dilog(c\*x)\*Pi\*b\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2-1/4\*I\*dilog(c\*x)\*Pi\*b\*e\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/2\*I\*ln(c\*x)\*Pi\*a\*e\*csgn(I\*(c\*x-1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/2\*I\*ln(c\*x)\*Pi\*a\*e\*csgn(I\*(c\*x+1))\*csgn(I\*(c\*x-1)\*(c\*x+1))^2+1/2\*I\*ln(c\*x)\*Pi\*a\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^3+1/2\*I\*dilog(c\*x)\*Pi\*b\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^2-I\*ln(c\*x)\*Pi\*a\*e\*csgn(I\*(c\*x-1)\*(c\*x+1))^2

**maxima** [C] time = 0.54, size = 167, normalized size = 0.44

$$i \pi a e \log(x) - \frac{1}{2} \left( \log(cx-1)^2 \log(cx) + 2 \operatorname{Li}_2(-cx+1) \log(cx-1) - 2 \operatorname{Li}_3(-cx+1) \right) b e + \frac{1}{2} \left( \log(cx+1)^2 \log(-cx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x,x, algorithm="maxima")

[Out] I\*pi\*a\*e\*log(x) - 1/2\*(log(c\*x - 1)^2\*log(c\*x) + 2\*dilog(-c\*x + 1)\*log(c\*x - 1) - 2\*polylog(3, -c\*x + 1))\*b\*e + 1/2\*(log(c\*x + 1)^2\*log(-c\*x) + 2\*dilog(c\*x + 1)\*log(c\*x + 1) - 2\*polylog(3, c\*x + 1))\*b\*e + a\*d\*log(x) - 1/2\*(I\*pi\*b\*e + b\*d - 2\*a\*e)\*(log(c\*x - 1)\*log(c\*x) + dilog(-c\*x + 1)) - 1/2\*(-I\*pi\*b\*e - b\*d - 2\*a\*e)\*(log(c\*x + 1)\*log(-c\*x) + dilog(c\*x + 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \ln(1 - c^2 x^2))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)))/x,x)

[Out] int(((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)))/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2 x^2 + 1))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x,x)

[Out] Integral((a + b\*acoth(c\*x))\*(d + e\*log(-c\*\*2\*x\*\*2 + 1))/x, x)

$$3.270 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^3} dx$$

**Optimal.** Leaf size=247

$$-\frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{2x^2} + \frac{1}{2}c^2e(a+b) \log(1-cx) + \frac{1}{2}c^2e(a-b) \log(cx+1) - ac^2e \log(x) - \frac{bc(e \log(1-c^2x^2)+d)}{2x^2}$$

[Out]  $-1/2*b*c^2*e*\operatorname{arccoth}(c*x)^2 - 1/2*b*c^2*e*\operatorname{arctanh}(c*x)^2 - a*c^2*e*\ln(x) + b*c^2*e*\operatorname{arctanh}(c*x)*\ln(2/(-c*x+1)) + 1/2*(a+b)*c^2*e*\ln(-c*x+1) + 1/2*(a-b)*c^2*e*\ln(c*x+1) - 1/2*b*c*(d+e*\ln(-c^2*x^2+1))/x - 1/2*(a+b*\operatorname{arccoth}(c*x))*(d+e*\ln(-c^2*x^2+1))/x^2 + 1/2*b*c^2*\operatorname{arctanh}(c*x)*(d+e*\ln(-c^2*x^2+1)) - b*c^2*e*\operatorname{arccoth}(c*x)*\ln(2-2/(c*x+1)) + 1/2*b*c^2*e*\operatorname{polylog}(2, 1-2/(-c*x+1)) + 1/2*b*c^2*e*\operatorname{polylog}(2, -1+2/(c*x+1))$

**Rubi [A]** time = 0.49, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.482$ , Rules used = {5917, 325, 206, 6086, 6725, 801, 5989, 5933, 2447, 5984, 5918, 2402, 2315}

$$\frac{1}{2}bc^2e \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) + \frac{1}{2}bc^2e \operatorname{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{2x^2} + \frac{1}{2}c^2e(a$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2])/x^3, x]$

[Out]  $-(b*c^2*e*\operatorname{ArcCoth}[c*x]^2)/2 - (b*c^2*e*\operatorname{ArcTanh}[c*x]^2)/2 - a*c^2*e*\operatorname{Log}[x] + b*c^2*e*\operatorname{ArcTanh}[c*x]*\operatorname{Log}[2/(1-c*x)] + ((a+b)*c^2*e*\operatorname{Log}[1-c*x])/2 + ((a-b)*c^2*e*\operatorname{Log}[1+c*x])/2 - (b*c*(d+e*\operatorname{Log}[1-c^2*x^2]))/(2*x) - ((a+b*\operatorname{ArcCoth}[c*x])*(d+e*\operatorname{Log}[1-c^2*x^2]))/(2*x^2) + (b*c^2*\operatorname{ArcTanh}[c*x]*(d+e*\operatorname{Log}[1-c^2*x^2]))/2 - b*c^2*e*\operatorname{ArcCoth}[c*x]*\operatorname{Log}[2-2/(1+c*x)] + (b*c^2*e*\operatorname{PolyLog}[2, 1-2/(1-c*x)])/2 + (b*c^2*e*\operatorname{PolyLog}[2, -1+2/(1+c*x)])/2$

### Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt}Q[a, 0] \ || \operatorname{Lt}Q[b, 0])$

### Rule 325

$\operatorname{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c*(m+1)), x] - \operatorname{Dist}[(b*(m+n*(p+1))$

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
  b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
  x]
```

### Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
  x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
  x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
  c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
  t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
  c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
  /D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
  PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
  x][[2]], Expon[Pq, x]]
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
  *p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*
  x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
  tegerQ[m]) && NeQ[m, -1]
```

### Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
  := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
  p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
  ), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
  ]
```

Rule 5933

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.))), x
_Symbol] := Simp[((a + b*ArcCoth[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcCoth[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/
(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_.))/((d_.) + (e_.)*(x_.)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5989

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_.)^2)),
x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcCoth[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_.)^2]*
(e_.)*(x_.)^(m_.), x_Symbol] := With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{coth}^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^3} dx &= -\frac{bc(d + e \log(1 - c^2x^2))}{2x} - \frac{(a + b \operatorname{coth}^{-1}(cx))(d + e \log(1 - c^2x^2))}{2x^2} \\
&= -\frac{bc(d + e \log(1 - c^2x^2))}{2x} - \frac{(a + b \operatorname{coth}^{-1}(cx))(d + e \log(1 - c^2x^2))}{2x^2} \\
&= -\frac{1}{2}bc^2e \tanh^{-1}(cx)^2 - \frac{bc(d + e \log(1 - c^2x^2))}{2x} - \frac{(a + b \operatorname{coth}^{-1}(cx))(d + e \log(1 - c^2x^2))}{2x^2} \\
&= -\frac{1}{2}bc^2e \tanh^{-1}(cx)^2 + bc^2e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{2x} \\
&= -\frac{1}{2}bc^2e \operatorname{coth}^{-1}(cx)^2 - \frac{1}{2}bc^2e \tanh^{-1}(cx)^2 + bc^2e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) \\
&= -\frac{1}{2}bc^2e \operatorname{coth}^{-1}(cx)^2 - \frac{1}{2}bc^2e \tanh^{-1}(cx)^2 - ac^2e \log(x) + bc^2e \log(1 - cx) \\
&= -\frac{1}{2}bc^2e \operatorname{coth}^{-1}(cx)^2 - \frac{1}{2}bc^2e \tanh^{-1}(cx)^2 - ac^2e \log(x) + bc^2e \log(1 - cx)
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 161, normalized size = 0.65

$$\frac{1}{2} \left( -\frac{e \log(1 - c^2x^2)(a + (b - bc^2x^2) \operatorname{coth}^{-1}(cx) + bcx)}{x^2} + c^2e(a + b) \log(1 - cx) + c^2e(a - b) \log(cx + 1) - 2ac^2e \log(x) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^3,x]

[Out] (-((a\*d)/x^2) - 2\*a\*c^2\*e\*Log[x] + (a + b)\*c^2\*e\*Log[1 - c\*x] + (a - b)\*c^2\*e\*Log[1 + c\*x] - (b\*d\*(2\*ArcCoth[c\*x] + c\*x\*(2 + c\*x\*Log[1 - c\*x] - c\*x\*Log[1 + c\*x])))/(2\*x^2) - (e\*(a + b\*c\*x + (b - b\*c^2\*x^2)\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2])/x^2 - b\*c^2\*e\*(PolyLog[2, -(1/(c\*x))] - PolyLog[2, 1/(c\*x)]))/2

**fricas [F]** time = 0.87, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{bd \operatorname{arcoth}(cx) + ad + (be \operatorname{arcoth}(cx) + ae) \log(-c^2x^2 + 1)}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^3,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^3,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^3, x)

**maple** [F] time = 11.44, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(-c^2x^2 + 1))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^3,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} \left( \left( c \log(cx + 1) - c \log(cx - 1) - \frac{2}{x} \right) c - \frac{2 \operatorname{arccoth}(cx)}{x^2} \right) bd + \frac{1}{2} \left( c^2 (\log(c^2x^2 - 1) - \log(x^2)) - \frac{\log(-c^2x^2 + 1)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^3,x, algorithm="maxima")

[Out] 1/4\*((c\*log(c\*x + 1) - c\*log(c\*x - 1) - 2/x)\*c - 2\*arccoth(c\*x)/x^2)\*b\*d + 1/2\*(c^2\*(log(c^2\*x^2 - 1) - log(x^2)) - log(-c^2\*x^2 + 1)/x^2)\*a\*e - 1/4\*b\*e\*(log(c\*x + 1)^2/x^2 - 2\*integrate(-((c\*x + 1)\*log(c\*x - 1)^2 - (I\*pi + (I\*pi\*c + c)\*x)\*log(c\*x + 1) - (-I\*pi - I\*pi\*c\*x)\*log(c\*x - 1))/(c\*x^4 + x^3), x)) - 1/2\*a\*d/x^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(cx))(d + e \ln(1 - c^2x^2))}{x^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^3, x)`

[Out] `int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^3, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2 x^2 + 1))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**3, x)`

[Out] `Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)`

$$3.271 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^5} dx$$

**Optimal.** Leaf size=339

$$\frac{1}{12}c^4e(3a+4b) \log(1-cx) + \frac{1}{12}c^4e(3a-4b) \log(cx+1) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2) + d)}{4x^4} - \frac{1}{2}ac^4e \log(x) + \frac{ac^2e}{4x^2}$$

[Out]  $\frac{1}{4}ac^2e/x^2 + \frac{5}{12}b^3c^3e/x + \frac{1}{4}b^2c^2e \operatorname{arccoth}(cx)/x^2 - \frac{1}{4}b^4c^4e \operatorname{arccoth}(cx)^2 - \frac{1}{4}b^4c^4e \operatorname{arctanh}(cx) - \frac{1}{4}b^4c^4e \operatorname{arctanh}(cx)^2 - \frac{1}{2}a^4c^4e \ln(x) + \frac{1}{2}b^4c^4e \operatorname{arctanh}(cx) \ln(2/(-cx+1)) + \frac{1}{12}(3a+4b)c^4e \ln(-cx+1) + \frac{1}{12}(3a-4b)c^4e \ln(cx+1) - \frac{1}{12}b^3c^3e(d+e \ln(-c^2x^2+1))/x^3 - \frac{1}{4}b^4c^4e(d+e \ln(-c^2x^2+1))/x - \frac{1}{4}(a+b \operatorname{arccoth}(cx))(d+e \ln(-c^2x^2+1))/x^4 + \frac{1}{4}b^4c^4e \operatorname{arctanh}(cx)(d+e \ln(-c^2x^2+1)) - \frac{1}{2}b^4c^4e \operatorname{arccoth}(cx) \ln(2-2/(cx+1)) + \frac{1}{4}b^4c^4e \operatorname{polylog}(2, 1-2/(-cx+1)) + \frac{1}{4}b^4c^4e \operatorname{polylog}(2, -1+2/(cx+1))$

**Rubi [A]** time = 0.72, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.518$ , Rules used = {5917, 325, 206, 6086, 6725, 1802, 5983, 5989, 5933, 2447, 5984, 5918, 2402, 2315}

$$\frac{1}{4}bc^4e \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) + \frac{1}{4}bc^4e \operatorname{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2) + d)}{4x^4} + \frac{1}{12}c^4e(3a+4b) \log(1-cx) + \frac{1}{12}c^4e(3a-4b) \log(cx+1)$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^5, x]

[Out]  $(a*c^2*e)/(4*x^2) + (5*b*c^3*e)/(12*x) + (b*c^2*e*ArcCoth[c*x])/(4*x^2) - (b*c^4*e*ArcCoth[c*x]^2)/4 - (b*c^4*e*ArcTanh[c*x])/4 - (b*c^4*e*ArcTanh[c*x]^2)/4 - (a*c^4*e*Log[x])/2 + (b*c^4*e*ArcTanh[c*x]*Log[2/(1-c*x)])/2 + ((3*a+4*b)*c^4*e*Log[1-c*x])/12 + ((3*a-4*b)*c^4*e*Log[1+c*x])/12 - (b*c*(d+e*Log[1-c^2*x^2]))/(12*x^3) - (b*c^3*(d+e*Log[1-c^2*x^2]))/(4*x) - ((a+b*ArcCoth[c*x])*(d+e*Log[1-c^2*x^2]))/(4*x^4) + (b*c^4*ArcTanh[c*x]*(d+e*Log[1-c^2*x^2]))/4 - (b*c^4*e*ArcCoth[c*x]*Log[2-2/(1+c*x)])/2 + (b*c^4*e*PolyLog[2, 1-2/(1-c*x)])/4 + (b*c^4*e*PolyLog[2, -1+2/(1+c*x)])/4$

**Rule 206**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_)*((d_.)*(x_))^(m_), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2))
```

, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5933

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcCoth[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c^p)/d, Int[((a + b\*ArcCoth[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/ (1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5983

Int((((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_)^(m\_)))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 5984

Int((((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*(x\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcTanh[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5989

Int((((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2))), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcCoth[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 6086

Int((((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.)\*(x\_)^(m\_.)), x\_Symbol] := With[{u = IntHide[x^m\*(a + b\*ArcCoth[c\*x]), x]}, Dist[d + e\*Log[f + g\*x^2], u, x] - Dist[2\*e\*g, Int[ExpandIntegrand[(x\*u)/(f + g\*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m] && NeQ[m, -1]

### Rule 6725

Int[(u\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ

[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^5} dx &= -\frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{4x^4} \\
&= -\frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{4x^4} \\
&= -\frac{1}{4}bc^4e \tanh^{-1}(cx)^2 - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{4x^4} \\
&= -\frac{1}{4}bc^4e \tanh^{-1}(cx)^2 + \frac{1}{2}bc^4e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{4x^4} \\
&= -\frac{1}{4}bc^4e \tanh^{-1}(cx)^2 + \frac{1}{2}bc^4e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{4x^4} \\
&= \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} + \frac{bc^2e \coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e \coth^{-1}(cx)^2 - \frac{1}{4}bc^4e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{4x^4} \\
&= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e \coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e \coth^{-1}(cx)^2 - \frac{1}{4}bc^4e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{4x^4} \\
&= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e \coth^{-1}(cx)}{4x^2} - \frac{1}{4}bc^4e \coth^{-1}(cx)^2 - \frac{1}{4}bc^4e \tanh^{-1}(cx) \log\left(\frac{2}{1 - cx}\right) - \frac{bc(d + e \log(1 - c^2x^2))}{12x^3} - \frac{bc^3(d + e \log(1 - c^2x^2))}{4x} - \frac{(a + b \coth^{-1}(cx))}{4x^4}
\end{aligned}$$

**Mathematica** [A] time = 0.15, size = 307, normalized size = 0.91

$$\frac{1}{12} \log(1-cx) (3ac^4e + 4bc^4e) + \frac{1}{12} \log(cx+1) (3ac^4e - 4bc^4e) + \frac{e \log(1 - c^2x^2) (-3a + 3bc^4x^4 \coth^{-1}(cx) - 3bc^3x^4)}{12x^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^5,x]

```

[Out] -1/4*(a*d)/x^4 + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2 +
((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-1/2*ArcCoth[c*x]/(c^2*x^2) + (-1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2))/2 + b*c^4*d*(-1/4*ArcCoth[c*x]/(c^4*x^4) + (-1/3*1/(c^3*x^3) - 1/(c*x) - Log[1 - c*x]/2 +

```

$\text{Log}[1 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*\text{Log}[1 + c*x])/12 + (e*(-3*a - b*c*x - 3*b*c^3*x^3 - 3*b*\text{ArcCoth}[c*x] + 3*b*c^4*x^4*\text{ArcCoth}[c*x])*\text{Log}[1 - c^2*x^2])/(12*x^4) - (b*c^4*e*(\text{PolyLog}[2, -(1/(c*x))] - \text{PolyLog}[2, 1/(c*x)])))/4$

**fricas** [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^5,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^5, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^5,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^5, x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(-c^2x^2 + 1))}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^5,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^5,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{24} \left( \left( 3c^3 \log(cx + 1) - 3c^3 \log(cx - 1) - \frac{2(3c^2x^2 + 1)}{x^3} \right) c - \frac{6 \operatorname{arccoth}(cx)}{x^4} \right) bd + \frac{1}{4} \left( \left( c^2 \log(c^2x^2 - 1) - c^2 \log(x^2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^5,x, algorithm="maxima")

[Out]  $\frac{1}{24}((3c^3 \log(cx + 1) - 3c^3 \log(cx - 1) - 2(3c^2 x^2 + 1)/x^3)c - 6 \operatorname{arccoth}(cx)/x^4)bd + \frac{1}{4}((c^2 \log(c^2 x^2 - 1) - c^2 \log(x^2) + 1/x^2)c^2 - \log(-c^2 x^2 + 1)/x^4)ae - \frac{1}{8}b e (\log(cx + 1)^2/x^4 - 4 \operatorname{integrate}(-1/2(2(cx + 1)\log(cx - 1)^2 - (2I\pi + (2I\pi c + c)x)\log(cx + 1) - (-2I\pi - 2I\pi cx)\log(cx - 1))/(c^2 x^6 + x^5), x)) - 1/4 a d/x^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \ln(1 - c^2 x^2))}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)))/x^5,x)

[Out] int(((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)))/x^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2 x^2 + 1))}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x\*\*5,x)

[Out] Integral((a + b\*acoth(c\*x))\*(d + e\*log(-c\*\*2\*x\*\*2 + 1))/x\*\*5, x)

$$3.272 \quad \int x^4 \left( a + b \coth^{-1}(cx) \right) \left( d + e \log \left( 1 - c^2 x^2 \right) \right) dx$$

**Optimal.** Leaf size=315

$$-\frac{e(4a+3b)\log(1-cx)}{20c^5} + \frac{e(4a-3b)\log(cx+1)}{20c^5} + \frac{1}{5}x^5 \left( a + b \coth^{-1}(cx) \right) \left( e \log \left( 1 - c^2 x^2 \right) + d \right) - \frac{2aex}{5c^4} - \frac{2aex^3}{15c^2} - \frac{2}{25}ae$$

[Out]  $-2/5*a*e*x/c^4 - 77/300*b*e*x^2/c^3 - 2/15*a*e*x^3/c^2 - 9/200*b*e*x^4/c - 2/25*a*e*x^5 - 2/5*b*e*x*\operatorname{arccoth}(c*x)/c^4 - 2/15*b*e*x^3*\operatorname{arccoth}(c*x)/c^2 - 2/25*b*e*x^5*\operatorname{arccoth}(c*x) + 1/5*b*e*\operatorname{arccoth}(c*x)^2/c^5 - 1/20*(4*a+3*b)*e*\ln(-c*x+1)/c^5 + 1/20*(4*a-3*b)*e*\ln(c*x+1)/c^5 - 23/75*b*e*\ln(-c^2*x^2+1)/c^5 - 1/20*b*e*\ln(-c^2*x^2+1)^2/c^5 + 1/10*b*x^2*(d+e*\ln(-c^2*x^2+1))/c^3 + 1/20*b*x^4*(d+e*\ln(-c^2*x^2+1))/c + 1/5*x^5*(a+b*\operatorname{arccoth}(c*x))*(d+e*\ln(-c^2*x^2+1)) + 1/10*b*\ln(-c^2*x^2+1)*(d+e*\ln(-c^2*x^2+1))/c^5$

**Rubi [A]** time = 0.75, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {5917, 266, 43, 6086, 6725, 1802, 633, 31, 5981, 5911, 260, 5949, 2475, 2390, 2301}

$$\frac{1}{5}x^5 \left( a + b \coth^{-1}(cx) \right) \left( e \log \left( 1 - c^2 x^2 \right) + d \right) - \frac{e(4a+3b)\log(1-cx)}{20c^5} + \frac{e(4a-3b)\log(cx+1)}{20c^5} - \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} - \frac{2}{25}ae$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^4*(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]), x]$

[Out]  $(-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*\operatorname{ArcCoth}[c*x])/(5*c^4) - (2*b*e*x^3*\operatorname{ArcCoth}[c*x])/(15*c^2) - (2*b*e*x^5*\operatorname{ArcCoth}[c*x])/25 + (b*e*\operatorname{ArcCoth}[c*x]^2)/(5*c^5) - ((4*a + 3*b)*e*\operatorname{Log}[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*\operatorname{Log}[1 + c*x])/(20*c^5) - (23*b*e*\operatorname{Log}[1 - c^2*x^2])/(75*c^5) - (b*e*\operatorname{Log}[1 - c^2*x^2]^2)/(20*c^5) + (b*x^2*(d + e*\operatorname{Log}[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*\operatorname{Log}[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]))/5 + (b*\operatorname{Log}[1 - c^2*x^2]*(d + e*\operatorname{Log}[1 - c^2*x^2]))/(10*c^5)$

**Rule 31**

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

**Rule 43**

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\},$



$x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& ( !\text{IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \parallel \text{LtQ}[9*m + 5*(n + 1), 0] \parallel \text{GtQ}[m + n + 2, 0])$

### Rule 260

$\text{Int}[(x_)^{(m_.)} / ((a_) + (b_.)*(x_)^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \&\& \text{EqQ}[m, n - 1]$

### Rule 266

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 633

$\text{Int}[(d_) + (e_.)*(x_)] / ((a_) + (c_.)*(x_)^2), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NiceSqrtQ}[-(a*c)]$

### Rule 1802

$\text{Int}[(Pq_)*((c_.)*(x_))^{(m_.)} * ((a_) + (b_.)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m * Pq*(a + b*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, m\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[p, -2]$

### Rule 2301

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}] * (b_.)] / (x_), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2 / (2*b*n), x] /; \text{FreeQ}[\{a, b, c, n\}, x]$

### Rule 2390

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_.)}) * (b_.)]^{(p_.)} * ((f_) + (g_.)*(x_)^{(q_.)}), x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q * (a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p, q\}, x] \&\& \text{EqQ}[e*f - d*g, 0]$

### Rule 2475

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_.)})^{(p_.)}] * (b_.)]^{(q_.)} * (x_)^{(m_.)} * ((f_) + (g_.)*(x_)^{(s_.)})^{(r_.)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(f + g*x^{(s/n)})^r * (a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, n, p, q, r, s\}, x] \&\& \text{IntegerQ}$

[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 5917

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 5949

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcCoth[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5981

Int((((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((f\_.)\*(x\_))^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 6086

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_)]\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.)\*(x\_)^(m\_.), x\_Symbol] := With[{u = IntHide[x^m\*(a + b\*ArcCoth[c\*x]), x]}, Dist[d + e\*Log[f + g\*x^2], u, x] - Dist[2\*e\*g, Int[ExpandIntegrand[(x\*u)/(f + g\*x^2), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m] && NeQ[m, -1]

### Rule 6725

Int[(u\_)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := With[{v = RationalFunctionExpand[u/(a + b\*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int x^4 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) dx &= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5}x^5 \\
&= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5}x^5 \\
&= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5}x^5 \\
&= \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5}x^5 \\
&= -\frac{be \log^2(1 - c^2 x^2)}{20c^5} + \frac{bx^2 (d + e \log(1 - c^2 x^2))}{10c^3} + \frac{bx^4 (d + e \log(1 - c^2 x^2))}{20c} + \frac{1}{5}x^5 \\
&= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25}aex^5 - \frac{2}{25}bex^5 \coth^{-1}(cx) \\
&= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25}aex^5 - \frac{2bex^3 \coth^{-1}(cx)}{15c^2} \\
&= -\frac{2aex}{5c^4} - \frac{3bex^2}{20c^3} - \frac{2aex^3}{15c^2} - \frac{bex^4}{40c} - \frac{2}{25}aex^5 - \frac{2bex \coth^{-1}(cx)}{5c^4} \\
&= -\frac{2aex}{5c^4} - \frac{19bex^2}{100c^3} - \frac{2aex^3}{15c^2} - \frac{9bex^4}{200c} - \frac{2}{25}aex^5 - \frac{2bex \coth^{-1}(cx)}{5c^4} \\
&= -\frac{2aex}{5c^4} - \frac{77bex^2}{300c^3} - \frac{2aex^3}{15c^2} - \frac{9bex^4}{200c} - \frac{2}{25}aex^5 - \frac{2bex \coth^{-1}(cx)}{5c^4}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 236, normalized size = 0.75

$$30c^2ex^2 \log(1 - c^2x^2) (4ac^3x^3 + 4bc^3x^3 \coth^{-1}(cx) + b(c^2x^2 + 2)) + 2 \log(1 - cx)(-60ae + 30bd - 137be) + 21$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (-240\*a\*c\*e\*x + 2\*b\*c^2\*(30\*d - 77\*e))\*x^2 - 80\*a\*c^3\*e\*x^3 + 3\*b\*c^4\*(10\*d - 9\*e)\*x^4 + 24\*a\*c^5\*(5\*d - 2\*e)\*x^5 - 8\*b\*c\*x\*(-15\*c^4\*d\*x^4 + 2\*e\*(15 +

$5*c^2*x^2 + 3*c^4*x^4) * \text{ArcCoth}[c*x] + 120*b*e * \text{ArcCoth}[c*x]^2 + 2*(30*b*d - 60*a*e - 137*b*e) * \text{Log}[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e) * \text{Log}[1 + c*x] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3 * \text{ArcCoth}[c*x]) * \text{Log}[1 - c^2*x^2] + 30*b*e * \text{Log}[1 - c^2*x^2]^2 / (600*c^5)$

**fricas [A]** time = 0.61, size = 249, normalized size = 0.79

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$$80 ac^3 ex^3 - 24 (5 ac^5 d - 2 ac^5 e) x^5 - 3 (10 bc^4 d - 9 bc^4 e) x^4 + 240 acex - 30 be \log(-c^2 x^2 + 1)^2 - 30 be \log\left(\frac{cx+1}{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="fricas")

[Out]  $-1/600*(80*a*c^3*e*x^3 - 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d - 9*b*c^4*e)*x^4 + 240*a*c*e*x - 30*b*e*\log(-c^2*x^2 + 1)^2 - 30*b*e*\log((c*x + 1)/(c*x - 1))^2 - 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 - 2*(60*a*c^5*e*x^5 + 15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 + 30*b*d - 137*b*e)*\log(-c^2*x^2 + 1) - 4*(15*b*c^5*e*x^5*\log(-c^2*x^2 + 1) - 10*b*c^3*e*x^3 + 3*(5*b*c^5*d - 2*b*c^5*e)*x^5 - 30*b*c*e*x + 30*a*e)*\log((c*x + 1)/(c*x - 1)))/c^5$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2 x^2 + 1) + d)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)\*x^4, x)

**maple [C]** time = 4.75, size = 4194, normalized size = 13.31

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1)),x)

[Out]  $-2/5*b*\operatorname{arccoth}(c*x)*\ln((c*x+1)/(c*x-1)-1)*x^5*e+2/5*b*\operatorname{arccoth}(c*x)*\ln(2)*x^5*e-1/10/c*b*\ln((c*x+1)/(c*x-1)-1)*x^4*e-1/5/c^3*b*\ln((c*x+1)/(c*x-1)-1)*x^2*e-1/20/c^5*b*e*(4*\operatorname{arccoth}(c*x)*x^5*c^5+c^4*x^4+2*c^2*x^2+4*\operatorname{arccoth}(c*x)-4*\ln((c*x+1)/(c*x-1)-1)-3)*\ln((c*x-1)/(c*x+1))-2/25*a*e*x^5-2/5*b*e*x*\operatorname{arccoth}(c*x)/c^4-2/15*b*e*x^3*\operatorname{arccoth}(c*x)/c^2-2/25*b*e*x^5*\operatorname{arccoth}(c*x)-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2/15*a*e*x^3/c^2-9/200*b*e*x^4/c+181/600*e/c^5*b+1$

$$\begin{aligned}
& /10/c^3*b*x^2*d+1/20/c*b*x^4*d-1/5/c^5*b*d*\ln((c*x+1)/(c*x-1)-1)+1/5/c^5*b* \\
& \ln((c*x+1)/(c*x-1)-1)^2*e+137/150/c^5*b*e*\ln((c*x+1)/(c*x-1)-1)+1/5/c^5*b*a \\
& \operatorname{rccoth}(c*x)*d-46/75/c^5*b*\operatorname{arccoth}(c*x)*e+1/5*b*\operatorname{arccoth}(c*x)*x^5*d+1/5*a*e*x \\
& ^5*\ln(-c^2*x^2+1)+1/5*a*e/c^5*\ln(c*x+1)-1/5*a*e/c^5*\ln(c*x-1)+3/40*I/c^5*b* \\
& \operatorname{Pi}*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*(c*x+1)/( \\
& c*x-1)/((c*x+1)/(c*x-1)-1)^2)+1/10*I/c^5*b*\operatorname{arccoth}(c*x)*\operatorname{Pi}*e*\operatorname{csgn}(I/((c*x+1) \\
& )/(c*x-1)-1)^2)*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/5*I/c^5*b \\
& *\operatorname{arccoth}(c*x)*\operatorname{Pi}*e*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^ \\
& 2-1/40*I/c*b*\operatorname{Pi}*e*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))^2*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*x \\
& ^4*e-1/20*I/c^3*b*\operatorname{Pi}*e*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))^2*\operatorname{csgn}(I*(c*x+1)/(c*x- \\
& 1))*x^2*e+1/40*I/c*b*\operatorname{Pi}*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*((c*x+1)/(c*x- \\
& 1)-1))^2*x^4*e+1/20*I/c^3*b*\operatorname{Pi}*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*((c*x+1) \\
& )/(c*x-1)-1))^2*x^2*e-1/10*I/c^5*b*\operatorname{Pi}*e*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I*(c*x+1) \\
& )/(c*x-1))*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/40*I/c*b*\operatorname{Pi}*e* \\
& \operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2) \\
& *x^4*e+1/20*I/c^3*b*\operatorname{Pi}*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\operatorname{csgn} \\
& (I/((c*x+1)/(c*x-1)-1)^2)*x^2*e+1/10*I/c^5*b*\operatorname{Pi}*e*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csg} \\
& n(I/((c*x-1)/(c*x+1))^(1/2))^2*\operatorname{csgn}(I*(c*x+1)/(c*x-1))+1/10*I/c^5*b*\operatorname{arccoth} \\
& (c*x)*\operatorname{Pi}*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)- \\
& 1)^2)^2-1/5*I/c^5*b*\operatorname{arccoth}(c*x)*\operatorname{Pi}*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))*\operatorname{csgn}(I*(( \\
& c*x+1)/(c*x-1)-1)^2)^2+1/10*I/c^5*b*\operatorname{arccoth}(c*x)*\operatorname{Pi}*e*\operatorname{csgn}(I*((c*x+1)/(c*x- \\
& 1)-1))^2*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)-1/10*I/c^5*b*\operatorname{arccoth}(c*x)*\operatorname{Pi}*e*\operatorname{csgn}( \\
& I/((c*x-1)/(c*x+1))^(1/2))^2*\operatorname{csgn}(I*(c*x+1)/(c*x-1))+1/5*I/c^5*b*\operatorname{Pi}*e*\ln((c*x \\
& +1)/(c*x-1)-1)*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^ \\
& 2-1/10*I/c^5*b*\operatorname{Pi}*e*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csg} \\
& n(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/20*I/c*b*\operatorname{Pi}*e*\operatorname{csgn}(I/((c*x-1)/ \\
& (c*x+1))^(1/2))*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^2*x^4*e+1/5*x^5*a*d-1/20*I/c^3*b*\operatorname{Pi} \\
& *e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn} \\
& (I/((c*x+1)/(c*x-1)-1)^2)*x^2*e+1/10*I/c^5*b*\operatorname{Pi}*e*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csg} \\
& n(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/( \\
& (c*x+1)/(c*x-1)-1)^2)-1/10*I*b*\operatorname{arccoth}(c*x)*\operatorname{Pi}*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn} \\
& (I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*x^5 \\
& *e-1/10*I/c^5*b*\operatorname{arccoth}(c*x)*e*\operatorname{Pi}*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I/((c*x+1)/( \\
& c*x-1)-1)^2)*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)-1/40*I/c*b*\operatorname{Pi}*e* \\
& \operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I/ \\
& ((c*x+1)/(c*x-1)-1)^2)*x^4*e-3/10/c^5*b*e*\ln(2)-3/20/c^5*b*d+1/10*I/c^3*b*\operatorname{P} \\
& i*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^2*x^2*e+1/40*I/c* \\
& b*\operatorname{Pi}*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^ \\
& 2*x^4*e+1/20*I/c^3*b*\operatorname{Pi}*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c* \\
& x+1)/(c*x-1)-1)^2)^2*x^2*e-1/20*I/c*b*\operatorname{Pi}*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*\operatorname{cs} \\
& \operatorname{gn}(I*((c*x+1)/(c*x-1)-1))*x^4*e-1/10*I/c^3*b*\operatorname{Pi}*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^ \\
& 2)^2*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))*x^2*e-1/10*I/c^5*b*\operatorname{Pi}*e*\ln((c*x+1)/(c*x-1)-1) \\
& )*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))^2*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)-1/5*I/c^5*b \\
& *\operatorname{Pi}*e*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))*\operatorname{csgn}(I*(c*x+1)/ \\
& (c*x-1))^2+1/10*I*b*\operatorname{arccoth}(c*x)*\operatorname{Pi}*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)
\end{aligned}$$

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-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*x^5*e+1/5*I*b*arccoth(c*x)*Pi*csgn(I
/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*x^5*e+1/10*I*b*arccoth(
c*x)*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)*csgn(I*((c*x+1)/(c*x-1)-1))^2*x^5*e-1
/10*I*b*arccoth(c*x)*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c
*x-1))*x^5*e+1/10*I*b*arccoth(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1
)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^5*e-1/5*I*b*arccoth(c*x)*Pi*csgn(I*((c
*x+1)/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*x^5*e+1/10/c*b*ln(2)*x^4*
e+1/5/c^3*b*ln(2)*x^2*e-2/5/c^5*b*ln((c*x+1)/(c*x-1)-1)*ln(2)*e+2/5/c^5*b*a
rccoth(c*x)*ln(2)*e-3/20*I/c^5*b*Pi*e-1/5*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*
e+1/20*I/c*b*Pi*x^4*e+1/10*I/c^3*b*Pi*x^2*e+3/40*I/c^5*b*e*Pi*csgn(I*(c*x+1
)/(c*x-1))^3-3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2
)^3+3/20*I/c^5*b*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/5*I
/c^5*b*Pi*e*arccoth(c*x)-3/40*I/c^5*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3+
1/5*I*b*arccoth(c*x)*Pi*x^5*e-3/40*I/c^5*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1))
^2*csgn(I*((c*x+1)/(c*x-1)-1)^2)+1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*((c*
x+1)/(c*x-1)-1)^2)^3-3/20*I/c^5*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn
(I*(c*x+1)/(c*x-1))^2+1/5*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1
)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*c
sgn(I*((c*x+1)/(c*x-1)-1)^2)^3-1/10*I/c^5*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn
(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3+1/40*I/c*b*Pi*csgn(I*(c*x+1)/(c
*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1)
/((c*x+1)/(c*x-1)-1)^2)^3*x^2*e-1/20*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+
1)/(c*x-1)-1)^2)^2*x^4*e-1/10*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c
*x-1)-1)^2)^2*x^2*e+1/40*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x^4*e+1/2
0*I/c^3*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x^2*e-1/40*I/c*b*Pi*csgn(I*(c*
x+1)/(c*x-1))^3*x^4*e-1/20*I/c^3*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^2*e+1/10*
I*b*arccoth(c*x)*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x^5*e-1/10*I*b*arccoth(
c*x)*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^5*e+1/10*I*b*arccoth(c*x)*Pi*csgn(I*(c*
x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^5*e-1/5*I*b*arccoth(c*x)*Pi*csgn(I*
(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^5*e+1/10*I/c^5*b*Pi*ln((c*x+1)/(
c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1))^3-3/40*I/c^5*b*e*Pi*csgn(I*(c*x+1)/(c*x
-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+3/20*I/c^5*b*Pi*e*csgn
(I*((c*x+1)/(c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2-1/5*I/c^5*b*arccoth
(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-3/40*I/c^5*b*Pi*
e*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^
2)^2-1/10*I/c^5*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1))^3+1/10*I/c^5*b*
arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3+3/40*I/c^
5*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))

```

**maxima** [C] time = 0.34, size = 314, normalized size = 1.00

$$\frac{1}{5} adx^5 + \frac{1}{75} \left( 15x^5 \log(-c^2x^2 + 1) - c^2 \left( \frac{2(3c^4x^5 + 5c^2x^3 + 15x)}{c^6} - \frac{15 \log(cx + 1)}{c^7} + \frac{15 \log(cx - 1)}{c^7} \right) \right) be \operatorname{arccoth}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

[Out]  $\frac{1}{5}a*d*x^5 + \frac{1}{75}(15*x^5*\log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*\log(c*x + 1)/c^7 + 15*\log(c*x - 1)/c^7))*b*e*arccoth(c*x) + \frac{1}{20}(4*x^5*arccoth(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*\log(c^2*x^2 - 1)/c^6))*b*d + \frac{1}{75}(15*x^5*\log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*\log(c*x + 1)/c^7 + 15*\log(c*x - 1)/c^7))*a*e + \frac{1}{600}((30*I*pi*c^4 - 27*c^4)*x^4 + (60*I*pi*c^2 - 154*c^2)*x^2 + (60*I*pi + 30*c^4*x^4 + 60*c^2*x^2 + 120*\log(c*x - 1) - 274)*\log(c*x + 1) - 2*(-30*I*pi - 15*c^4*x^4 - 30*c^2*x^2 + 137)*\log(c*x - 1))*b*e/c^5$

**mupad [B]** time = 2.34, size = 497, normalized size = 1.58

$$\ln\left(\frac{1}{cx} + 1\right) \left( \frac{bdx^5}{10} - \frac{\frac{2bec^5x^5}{5} + \frac{2bec^3x^3}{3} + 2becx}{10c^5} + \frac{bex^5 \ln(1 - c^2x^2)}{10} \right) + \ln\left(1 - \frac{1}{cx}\right) \left( \frac{\frac{bdx^6}{5} - \frac{bc^2dx^8}{5}}{2(cx^2 + x)(cx - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)),x)

[Out]  $\log(1/(c*x) + 1)*((b*d*x^5)/10 - (2*b*c*e*x + (2*b*c^3*e*x^3)/3 + (2*b*c^5*e*x^5)/5)/(10*c^5) + (b*e*x^5*\log(1 - c^2*x^2))/10) + \log(1 - 1/(c*x))*((b*d*x^6)/5 - (b*c^2*d*x^8)/5)/(2*(x + c*x^2)*(c*x - 1)) + ((4*b*e*x^6)/75 + (4*b*e*x^4)/(15*c^2) - (2*b*e*x^2)/(5*c^4) + (2*b*c^2*e*x^8)/25)/(2*(x + c*x^2)*(c*x - 1)) + (\log(1 - c^2*x^2)*((b*e*x^6)/5 - (b*c^2*e*x^8)/5))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*\log(1/(c*x) + 1))/(10*c^5) + x^3*((a*(5*d - 2*e))/(15*c^2) - (a*d)/(3*c^2)) + x^2*((b*(10*d - 9*e))/(100*c^3) - (b*e)/(6*c^3)) + (x*((a*(5*d - 2*e))/(5*c^2) - (a*d)/c^2))/c^2 + (a*x^5*(5*d - 2*e))/25 + c^2*\log(1 - c^2*x^2)*((a*e*x^5)/(5*c^2) + (b*e*x^4)/(20*c^3) + (b*e*x^2)/(10*c^5)) - (\log(c*x - 1)*(60*a*e - 30*b*d + 137*b*e))/(300*c^5) + (\log(c*x + 1)*(60*a*e + 30*b*d - 137*b*e))/(300*c^5) + (b*e*\log(1/(c*x) + 1)^2)/(20*c^5) + (b*e*\log(1 - 1/(c*x))^2)/(20*c^5) + (b*e*\log(1 - c^2*x^2)^2)/(20*c^5) + (b*x^4*(10*d - 9*e))/(200*c)$

**sympy [A]** time = 15.30, size = 345, normalized size = 1.10

$$\left\{ \begin{array}{l} \frac{adx^5}{5} + \frac{aex^5 \log(-c^2x^2+1)}{5} - \frac{2aex^5}{25} - \frac{2aex^3}{15c^2} - \frac{2aex}{5c^4} + \frac{2ae \operatorname{acoth}(cx)}{5c^5} + \frac{bdx^5 \operatorname{acoth}(cx)}{5} + \frac{bex^5 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{5} - \frac{2bex^5 \operatorname{acoth}(cx)}{25} \\ \frac{dx^5 \left(a + \frac{ib}{2}\right)}{5} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2*
a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*acoth(c*x)/(5*c**5) + b*d*x**
5*acoth(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*acoth(c*x)/5 - 2*b*e*x**5*aco
th(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*b*e*
x**4/(200*c) - 2*b*e*x**3*acoth(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x
**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*acoth(
c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x**2 + 1)
**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*acoth(c*x)**2/
(5*c**5), Ne(c, 0)), (d*x**5*(a + I*pi*b/2)/5, True))
```



$$3.273 \quad \int x^2 \left( a + b \coth^{-1}(cx) \right) \left( d + e \log(1 - c^2 x^2) \right) dx$$

**Optimal.** Leaf size=247

$$\frac{e(2a+b)\log(1-cx)}{6c^3} + \frac{e(2a-b)\log(cx+1)}{6c^3} + \frac{1}{3}x^3 \left( a + b \coth^{-1}(cx) \right) \left( e \log(1 - c^2 x^2) + d \right) - \frac{2aex}{3c^2} - \frac{2}{9}aex^3 + \frac{bex^2}{9}$$

[Out]  $-2/3*a*e*x/c^2-5/18*b*e*x^2/c-2/9*a*e*x^3-2/3*b*e*x*\operatorname{arccoth}(c*x)/c^2-2/9*b*e*x^3*\operatorname{arccoth}(c*x)+1/3*b*e*\operatorname{arccoth}(c*x)^2/c^3-1/6*(2*a+b)*e*\ln(-c*x+1)/c^3+1/6*(2*a-b)*e*\ln(c*x+1)/c^3-4/9*b*e*\ln(-c^2*x^2+1)/c^3-1/12*b*e*\ln(-c^2*x^2+1)^2/c^3+1/6*b*x^2*(d+e*\ln(-c^2*x^2+1))/c+1/3*x^3*(a+b*\operatorname{arccoth}(c*x))*(d+e*\ln(-c^2*x^2+1))+1/6*b*\ln(-c^2*x^2+1)*(d+e*\ln(-c^2*x^2+1))/c^3$

**Rubi [A]** time = 0.62, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {5917, 266, 43, 6086, 6725, 801, 633, 31, 5981, 5911, 260, 5949, 2475, 2390, 2301}

$$\frac{1}{3}x^3 \left( a + b \coth^{-1}(cx) \right) \left( e \log(1 - c^2 x^2) + d \right) - \frac{e(2a+b)\log(1-cx)}{6c^3} + \frac{e(2a-b)\log(cx+1)}{6c^3} - \frac{2aex}{3c^2} - \frac{2}{9}aex^3 + \frac{bex^2}{9}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]), x]$

[Out]  $(-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*\operatorname{ArcCoth}[c*x])/(3*c^2) - (2*b*e*x^3*\operatorname{ArcCoth}[c*x])/9 + (b*e*\operatorname{ArcCoth}[c*x]^2)/(3*c^3) - ((2*a + b)*e*\operatorname{Log}[1 - c*x])/(6*c^3) + ((2*a - b)*e*\operatorname{Log}[1 + c*x])/(6*c^3) - (4*b*e*\operatorname{Log}[1 - c^2*x^2])/(9*c^3) - (b*e*\operatorname{Log}[1 - c^2*x^2]^2)/(12*c^3) + (b*x^2*(d + e*\operatorname{Log}[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]))/3 + (b*\operatorname{Log}[1 - c^2*x^2]*(d + e*\operatorname{Log}[1 - c^2*x^2]))/(6*c^3)$

### Rule 31

$\operatorname{Int}[(a_) + (b_)*(x_)^(-1), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

### Rule 43

$\operatorname{Int}[(a_.) + (b_.)*(x_)^(m_.)*((c_.) + (d_.)*(x_)^(n_.), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{IGtQ}[m, 0] \ \&\& (!\operatorname{IntegerQ}[n] \ || (\operatorname{EqQ}[c, 0] \ \&\& \operatorname{LeQ}[7*m + 4*n + 4, 0]) \ || \operatorname{LtQ}[9*m + 5*(n + 1), 0]) \ || \operatorname{GtQ}[m + n + 2, 0])$

### Rule 260

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

### Rule 266

$\text{Int}[(x_)^{(m_.)*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 633

$\text{Int}[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(a*c), 2]\}, \text{Dist}[e/2 + (c*d)/(2*q), \text{Int}[1/(-q + c*x), x], x] + \text{Dist}[e/2 - (c*d)/(2*q), \text{Int}[1/(q + c*x), x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NiceSqrtQ}[-(a*c)]$

### Rule 801

$\text{Int}[(d_) + (e_.)*(x_)]^{(m_.)*((f_.) + (g_.)*(x_))}/((a_) + (c_.)*(x_)^2), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IntegerQ}[m]$

### Rule 2301

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]/(x_), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /; \text{FreeQ}\{a, b, c, n\}, x\}$

### Rule 2390

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_.)})*(b_.)]^{(p_.)}*((f_.) + (g_.)*(x_)^{(q_.)}), x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x\} \ \&\& \ \text{EqQ}[e*f - d*g, 0]$

### Rule 2475

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_.)})^{(p_.)}]*(b_.)]^{(q_.)}*(x_)^{(m_.)*((f_.) + (g_.)*(x_)^{(s_.)})^{(r_.)}], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(f + g*x^{(s/n)})^r*(a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p, q, r, s\}, x\} \ \&\& \ \text{IntegerQ}[r] \ \&\& \ \text{IntegerQ}[s/n] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \ \&\& \ (\text{GtQ}[(m + 1)/n, 0] \ || \ \text{IGtQ}[q, 0])$

### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

### Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(e_.)*(x_)^(m_.)), x_Symbol] := With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]), x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[m] && NeQ[m, -1]
```

### Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx &= \frac{bx^2 (d + e \log(1 - c^2x^2))}{6c} + \frac{1}{3}x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= \frac{bx^2 (d + e \log(1 - c^2x^2))}{6c} + \frac{1}{3}x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= \frac{bx^2 (d + e \log(1 - c^2x^2))}{6c} + \frac{1}{3}x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= \frac{bx^2 (d + e \log(1 - c^2x^2))}{6c} + \frac{1}{3}x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= -\frac{be \log^2(1 - c^2x^2)}{12c^3} + \frac{bx^2 (d + e \log(1 - c^2x^2))}{6c} + \frac{1}{3}x^3 (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2}{9}bex^3 \coth^{-1}(cx) - \frac{be \log^2(1 - c^2x^2)}{12c^3} \\
&= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2bex \coth^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3 \coth^{-1}(cx) \\
&= -\frac{2aex}{3c^2} - \frac{bex^2}{6c} - \frac{2}{9}aex^3 - \frac{2bex \coth^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3 \coth^{-1}(cx) \\
&= -\frac{2aex}{3c^2} - \frac{5bex^2}{18c} - \frac{2}{9}aex^3 - \frac{2bex \coth^{-1}(cx)}{3c^2} - \frac{2}{9}bex^3 \coth^{-1}(cx)
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 183, normalized size = 0.74

$$\frac{6c^2ex^2 \log(1 - c^2x^2) (2acx + 2bcx \coth^{-1}(cx) + b) + 2 \log(1 - cx)(-6ae + 3bd - 11be) + 2 \log(cx + 1)(6ae + 3bd)}{(36c^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]),x]

[Out] (-24\*a\*c\*e\*x + 2\*b\*c^2\*(3\*d - 5\*e)\*x^2 + 4\*a\*c^3\*(3\*d - 2\*e)\*x^3 + 4\*b\*c\*x\*(3\*c^2\*d\*x^2 - 2\*e\*(3 + c^2\*x^2))\*ArcCoth[c\*x] + 12\*b\*e\*ArcCoth[c\*x]^2 + 2\*(3\*b\*d - 6\*a\*e - 11\*b\*e)\*Log[1 - c\*x] + 2\*(3\*b\*d + 6\*a\*e - 11\*b\*e)\*Log[1 + c\*x] + 6\*c^2\*e\*x^2\*(b + 2\*a\*c\*x + 2\*b\*c\*x\*ArcCoth[c\*x])\*Log[1 - c^2\*x^2] + 3\*b\*e\*Log[1 - c^2\*x^2]^2)/(36\*c^3)

**fricas** [A] time = 0.48, size = 198, normalized size = 0.80

$$24acex - 4(3ac^3d - 2ac^3e)x^3 - 3be \log(-c^2x^2 + 1)^2 - 3be \log\left(\frac{cx+1}{cx-1}\right)^2 - 2(3bc^2d - 5bc^2e)x^2 - 2(6ac^3ex^3 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="fricas")

[Out] 
$$-1/36*(24*a*c*e*x - 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 3*b*e*\log(-c^2*x^2 + 1)^2 - 3*b*e*\log((c*x + 1)/(c*x - 1))^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 - 2*(6*a*c^3*e*x^3 + 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*\log(-c^2*x^2 + 1) - 2*(3*b*c^3*e*x^3*\log(-c^2*x^2 + 1) - 6*b*c*e*x + (3*b*c^3*d - 2*b*c^3*e)*x^3 + 6*a*e)*\log((c*x + 1)/(c*x - 1)))/c^3$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)\*x^2, x)

**maple** [C] time = 3.72, size = 3514, normalized size = 14.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1)),x)

[Out] 
$$-2/3*b*e*x*\operatorname{arccoth}(c*x)/c^2 - 1/6/c^3*b*e*(2*\operatorname{arccoth}(c*x)*x^3*c^3 + c^2*x^2 + 2*a*\operatorname{arccoth}(c*x) - 2*\ln((c*x+1)/(c*x-1)) - 1)*\ln((c*x-1)/(c*x+1)) - 1/3/c^3*b*e*\ln(2) - 2/9*a*e*x^3 - 2/9*b*e*x^3*\operatorname{arccoth}(c*x) - 1/6/c^3*b*d - 2/3*a*e*x/c^2 - 5/18*b*e*x^2/c + 1/6*b*d*x^2/c + 5/18*e/c^3*b + 1/3*a*e*x^3*\ln(-c^2*x^2+1) + 1/3*a*e/c^3*\ln(c*x+1) - 1/3*a*e/c^3*\ln(c*x-1) - 1/6*I/c^3*b*e*Pi + 1/3*x^3*a*d + 1/6*I/c*b*Pi*x^2*e + 1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))^3 - 1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3 - 1/12*I/c^3*b*Pi*e*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3 + 1/6*I/c^3*b*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2 + 1/3*I/c^3*b*Pi*\operatorname{arccoth}(c*x)*e - 1/3*I/c^3*b*Pi*\ln((c*x+1)/(c*x-1)-1)*e + 1/3*I*b*\operatorname{arccoth}(c*x)*Pi*x^3*e - 1/6*I/c^3*b*Pi*\ln((c*x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1)/(c*x-1)-1))^2*csgn(I*((c*x+1)/(c*x-1)-1)^2) + 1/3*I/c^3*b*csgn(I*((c*x+1)$$

$$\begin{aligned}
& )/(c*x-1)-1)^2)^2*csgn(I*((c*x+1)/(c*x-1)-1))*Pi*\ln((c*x+1)/(c*x-1)-1)*e+1/ \\
& 12*I/c^3*b*Pi*e*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn( \\
& I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)+1/3/c^3*b*\ln((c*x+1)/(c*x-1)-1)^2* \\
& e+1/3/c^3*b*\operatorname{arccoth}(c*x)*d-8/9/c^3*b*\operatorname{arccoth}(c*x)*e-1/3/c^3*b*d*\ln((c*x+1)/ \\
& (c*x-1)-1)+11/9/c^3*b*e*\ln((c*x+1)/(c*x-1)-1)+1/3*b*\operatorname{arccoth}(c*x)*x^3*d-1/6* \\
& I*b*\operatorname{arccoth}(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1) \\
& /((c*x-1)-1)^2)*csgn(I/((c*x+1)/(c*x-1)-1)^2)*x^3*e-1/12*I/c*b*Pi*csgn(I*(c* \\
& x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*csgn(I/((c*x+1) \\
& /((c*x-1)-1)^2)*x^2*e-1/6*I/c^3*b*\operatorname{arccoth}(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1))* \\
& csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2) \\
& +1/6*I/c^3*b*Pi*\ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1))*csgn(I/((c* \\
& x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)-1/12*I/c*b \\
& *Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))*x^2*e+1/6*I/c \\
& *b*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2*x^2*e-1/6*I \\
& *b*\operatorname{arccoth}(c*x)*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1) \\
& )*x^3*e+1/3*I*b*\operatorname{arccoth}(c*x)*Pi*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I*(c*x \\
& +1)/(c*x-1))^2*x^3*e+1/6*I*b*\operatorname{arccoth}(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1))*csgn(I \\
& *(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^3*e+1/6*I*b*\operatorname{arccoth}(c*x)*Pi*csg \\
& n(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)* \\
& x^3*e+1/6*I*b*\operatorname{arccoth}(c*x)*Pi*csgn(I*((c*x+1)/(c*x-1)-1))^2*csgn(I*((c*x+1) \\
& /((c*x-1)-1)^2)*x^3*e-1/3*I*b*\operatorname{arccoth}(c*x)*Pi*csgn(I*((c*x+1)/(c*x-1)-1))*csg \\
& n(I*((c*x+1)/(c*x-1)-1)^2)^2*x^3*e+1/12*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1))*c \\
& sgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^2*e+1/12*I/c*b*Pi*csgn(I*( \\
& c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*x^2*e \\
& +1/12*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1))^2*csgn(I*((c*x+1)/(c*x-1)-1)^2)* \\
& x^2*e-1/6*I/c*b*Pi*csgn(I*((c*x+1)/(c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2 \\
& )^2*x^2*e-1/6*I/c^3*b*Pi*\operatorname{arccoth}(c*x)*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*c \\
& sgn(I*(c*x+1)/(c*x-1))+1/3*I/c^3*b*\operatorname{arccoth}(c*x)*e*Pi*csgn(I/((c*x-1)/(c*x+1) \\
& ))^(1/2))*csgn(I*(c*x+1)/(c*x-1))^2+1/6*I/c^3*b*\operatorname{arccoth}(c*x)*e*Pi*csgn(I*(c \\
& *x+1)/(c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/6*I/c^3*b* \\
& \operatorname{arccoth}(c*x)*e*Pi*csgn(I/((c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c* \\
& x+1)/(c*x-1)-1)^2)^2+1/6*I/c^3*b*\operatorname{arccoth}(c*x)*Pi*e*csgn(I*((c*x+1)/(c*x-1)- \\
& 1))^2*csgn(I*((c*x+1)/(c*x-1)-1)^2)-1/3*I/c^3*b*\operatorname{arccoth}(c*x)*e*Pi*csgn(I*(( \\
& c*x+1)/(c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2+1/6*I/c^3*b*Pi*\ln((c*x+1 \\
& )/(c*x-1)-1)*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))^2*csgn(I*(c*x+1)/(c*x-1))-1/ \\
& 3*I/c^3*b*Pi*\ln((c*x+1)/(c*x-1)-1)*e*csgn(I/((c*x-1)/(c*x+1))^(1/2))*csgn(I \\
& *(c*x+1)/(c*x-1))^2-1/6*I/c^3*b*Pi*\ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/( \\
& c*x-1))*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/6*I/c^3*b*csgn(I* \\
& (c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*csgn(I/((c*x+1)/(c*x-1)-1)^2)*Pi*\ln \\
& ((c*x+1)/(c*x-1)-1)*e-1/3/c*b*\ln((c*x+1)/(c*x-1)-1)*x^2*e+2/3/c^3*b*\operatorname{arccot} \\
& h(c*x)*e*\ln(2)-2/3/c^3*b*\ln((c*x+1)/(c*x-1)-1)*\ln(2)*e+2/3*b*\operatorname{arccoth}(c*x)*\ln \\
& (2)*x^3*e-2/3*b*\operatorname{arccoth}(c*x)*\ln((c*x+1)/(c*x-1)-1)*x^3*e+1/3/c*b*\ln(2)*x^2 \\
& *e+1/6*I*b*\operatorname{arccoth}(c*x)*Pi*csgn(I*((c*x+1)/(c*x-1)-1)^2)^3*x^3*e-1/3*I*b*\operatorname{ar} \\
& ccoth(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*x^3*e-1/12*I/ \\
& c*b*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^2*e+1/12*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)
\end{aligned}$$

```

/((c*x+1)/(c*x-1)-1)^2)^3*x^2*e+1/6*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I*((c*x+
1)/(c*x-1)-1)^2)^3+1/6*I/c^3*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c
*x-1))^3-1/6*I/c^3*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(c*x-1)/((c*
x+1)/(c*x-1)-1)^2)^3-1/6*I/c^3*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*((c*x+1)
/(c*x-1)-1)^2)^3-1/3*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x
+1)/(c*x-1)-1)^2)^2+1/3*I/c^3*b*Pi*ln((c*x+1)/(c*x-1)-1)*e*csgn(I*(c*x+1)/(
c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/12*I/c^3*b*csgn(I*(c*x+1)/(c*x-1))*csgn(I
/((c*x-1)/(c*x+1))^(1/2))^2*e*Pi-1/6*I/c^3*b*e*Pi*csgn(I/((c*x-1)/(c*x+1))^(
1/2))*csgn(I*(c*x+1)/(c*x-1))^2-1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)/(c*x-1))*
csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/12*I/c^3*b*Pi*e*csgn(I/((
c*x+1)/(c*x-1)-1)^2)*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2-1/12*I
/c^3*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1))^2*csgn(I*((c*x+1)/(c*x-1)-1)^2)+1/6
*I/c^3*b*e*Pi*csgn(I*((c*x+1)/(c*x-1)-1))*csgn(I*((c*x+1)/(c*x-1)-1)^2)^2-1
/6*I*b*arccoth(c*x)*Pi*csgn(I*(c*x+1)/(c*x-1))^3*x^3*e+1/6*I*b*arccoth(c*x)
*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*x^3*e+1/12*I/c*b*Pi*csg
n(I*((c*x+1)/(c*x-1)-1)^2)^3*x^2*e-1/6*I/c*b*Pi*csgn(I*(c*x+1)/(c*x-1)/((c*
x+1)/(c*x-1)-1)^2)^2*x^2*e-1/6*I/c^3*b*arccoth(c*x)*e*Pi*csgn(I*(c*x+1)/(c*
x-1))^3+1/6*I/c^3*b*arccoth(c*x)*Pi*e*csgn(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-
1)-1)^2)^3

```

**maxima** [C] time = 0.33, size = 252, normalized size = 1.02

$$\frac{1}{3} adx^3 + \frac{1}{9} \left( 3x^3 \log(-c^2x^2 + 1) - c^2 \left( \frac{2(c^2x^3 + 3x)}{c^4} - \frac{3 \log(cx + 1)}{c^5} + \frac{3 \log(cx - 1)}{c^5} \right) \right) be \operatorname{arccoth}(cx) + \frac{1}{6} \left( 2x^3 a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")

```

```

[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*e*arccoth(c*x) + 1/6*(2*x^3*arcc
oth(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2*x^
2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c
^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*log(c*x
- 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*e/c^3

```

**mupad** [B] time = 2.33, size = 414, normalized size = 1.68

$$\ln\left(\frac{1}{cx} + 1\right) \left( \frac{bdx^3}{6} - \frac{2bec^3x^3 + 2becx}{6c^3} + \frac{bex^3 \ln(1 - c^2x^2)}{6} \right) + x \left( \frac{a(3d - 2e)}{3c^2} - \frac{ad}{c^2} \right) + \ln\left(1 - \frac{1}{cx}\right) \left( \frac{4bex^4}{9} - \frac{1}{2(c^2x^2 + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)),x)`

[Out]  $\log(1/(c*x) + 1)*((b*d*x^3)/6 - (2*b*c*e*x + (2*b*c^3*e*x^3)/3)/(6*c^3) + (b*e*x^3*\log(1 - c^2*x^2))/6) + x*((a*(3*d - 2*e))/(3*c^2) - (a*d)/c^2) + \log(1 - 1/(c*x))*(((4*b*e*x^4)/9 - (2*b*e*x^2)/(3*c^2) + (2*b*c^2*e*x^6)/9)/(2*(x + c*x^2)*(c*x - 1)) + ((b*d*x^4)/3 - (b*c^2*d*x^6)/3)/(2*(x + c*x^2)*(c*x - 1)) + (\log(1 - c^2*x^2)*((b*e*x^4)/3 - (b*c^2*e*x^6)/3))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*\log(1/(c*x) + 1))/(6*c^3) + (a*x^3*(3*d - 2*e))/9 + c^2*\log(1 - c^2*x^2)*((a*e*x^3)/(3*c^2) + (b*e*x^2)/(6*c^3)) - (\log(c*x - 1)*(6*a*e - 3*b*d + 11*b*e))/(18*c^3) + (\log(c*x + 1)*(6*a*e + 3*b*d - 11*b*e))/(18*c^3) + (b*e*\log(1/(c*x) + 1)^2)/(12*c^3) + (b*e*\log(1 - 1/(c*x))^2)/(12*c^3) + (b*e*\log(1 - c^2*x^2)^2)/(12*c^3) + (b*x^2*(3*d - 5*e))/(18*c)$

**sympy [A]** time = 6.78, size = 265, normalized size = 1.07

$$\left\{ \begin{array}{l} \frac{adx^3}{3} + \frac{aex^3 \log(-c^2x^2+1)}{3} - \frac{2aex^3}{9} - \frac{2aex}{3c^2} + \frac{2ae \operatorname{acoth}(cx)}{3c^3} + \frac{bdx^3 \operatorname{acoth}(cx)}{3} + \frac{bex^3 \log(-c^2x^2+1) \operatorname{acoth}(cx)}{3} - \frac{2bex^3 \operatorname{acoth}(cx)}{9} + \frac{bdx^2}{6c} \\ \frac{dx^3 \left(a + \frac{i\pi b}{2}\right)}{3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)`

[Out] `Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*a*e*x/(3*c**2) + 2*a*e*acoth(c*x)/(3*c**3) + b*d*x**3*acoth(c*x)/3 + b*e*x**3*log(-c**2*x**2 + 1)*acoth(c*x)/3 - 2*b*e*x**3*acoth(c*x)/9 + b*d*x**2/(6*c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*acoth(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2 + 1)**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*acoth(c*x)**2/(3*c**3), Ne(c, 0)), (d*x**3*(a + I*pi*b/2)/3, True))`



### 3.274 $\int (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx$

**Optimal.** Leaf size=104

$$x(a + b \coth^{-1}(cx)) (e \log(1 - c^2x^2) + d) + \frac{e(a + b \coth^{-1}(cx))^2}{bc} - 2aex + \frac{b(e \log(1 - c^2x^2) + d)^2}{4ce} - \frac{be \log(1 - c^2x^2)}{c}$$

[Out]  $-2*a*e*x - 2*b*e*x*\operatorname{arccoth}(c*x) + e*(a + b*\operatorname{arccoth}(c*x))^2/b/c - b*e*\ln(-c^2*x^2 + 1)/c + x*(a + b*\operatorname{arccoth}(c*x))*(d + e*\ln(-c^2*x^2 + 1)) + 1/4*b*(d + e*\ln(-c^2*x^2 + 1))^2/c$   
/e

**Rubi [A]** time = 0.20, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6074, 2475, 2390, 2301, 5981, 5911, 260, 5949}

$$x(a + b \coth^{-1}(cx)) (e \log(1 - c^2x^2) + d) + \frac{e(a + b \coth^{-1}(cx))^2}{bc} - 2aex + \frac{b(e \log(1 - c^2x^2) + d)^2}{4ce} - \frac{be \log(1 - c^2x^2)}{c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]), x]$

[Out]  $-2*a*e*x - 2*b*e*x*\operatorname{ArcCoth}[c*x] + (e*(a + b*\operatorname{ArcCoth}[c*x])^2)/(b*c) - (b*e*\operatorname{Log}[1 - c^2*x^2])/c + x*(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]) + (b*(d + e*\operatorname{Log}[1 - c^2*x^2])^2)/(4*c*e)$

#### Rule 260

$\operatorname{Int}[(x_)^m / ((a_) + (b_)*(x_)^n), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]] / (b*n), x] /;$  FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 2301

$\operatorname{Int}[(a_) + \operatorname{Log}[(c_)*(x_)^n] * (b_)] / (x_), x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*\operatorname{Log}[c*x^n])^2 / (2*b*n), x] /;$  FreeQ[{a, b, c, n}, x]

#### Rule 2390

$\operatorname{Int}[(a_) + \operatorname{Log}[(c_)*((d_) + (e_)*(x_)^n)] * (b_)]^p * ((f_) + (g_)*(x_)^q), x\_Symbol] \rightarrow \operatorname{Dist}[1/e, \operatorname{Subst}[\operatorname{Int}[(f*x)/d]^q * (a + b*\operatorname{Log}[c*x^n])^p, x], x, d + e*x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.)^(q_.)*(x_)^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

### Rule 5911

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)^(p_.), x_Symbol] := Simp[x*(a + b*ArcCoth[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5981

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)^(p_.))*((f_.)*(x_)^(m_))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

### Rule 6074

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(e_.)), x_Symbol] := Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]), x] + (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[2*e*g, Int[(x^2*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g}, x]
```

### Rubi steps

$$\begin{aligned}
\int (a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) dx &= x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) - (bc) \int \frac{x(d + e \log(1 - c^2x^2))}{1 - c^2x^2} dx \\
&= x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) - \frac{1}{2}(bc) \text{Subst} \left( \int \frac{d + e \log(1 - c^2x^2)}{1 - c^2x^2} dx, cx \right) \\
&= -2aex + \frac{e(a + b \coth^{-1}(cx))^2}{bc} + x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= -2aex - 2bex \coth^{-1}(cx) + \frac{e(a + b \coth^{-1}(cx))^2}{bc} + x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2x^2)) \\
&= -2aex - 2bex \coth^{-1}(cx) + \frac{e(a + b \coth^{-1}(cx))^2}{bc} - \frac{be \log(1 - c^2x^2)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 144, normalized size = 1.38

$$aex \log(1 - c^2x^2) + \frac{2ae \tanh^{-1}(cx)}{c} + adx - 2aex + \frac{bd \log(1 - c^2x^2)}{2c} + \frac{be \log^2(1 - c^2x^2)}{4c} - \frac{be \log(1 - c^2x^2)}{c} + bex \log(1 - c^2x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]), x]

[Out] a\*d\*x - 2\*a\*e\*x + b\*d\*x\*ArcCoth[c\*x] - 2\*b\*e\*x\*ArcCoth[c\*x] + (b\*e\*ArcCoth[c\*x]^2)/c + (2\*a\*e\*ArcTanh[c\*x])/c + (b\*d\*Log[1 - c^2\*x^2])/(2\*c) - (b\*e\*Log[1 - c^2\*x^2])/c + a\*e\*x\*Log[1 - c^2\*x^2] + b\*e\*x\*ArcCoth[c\*x]\*Log[1 - c^2\*x^2] + (b\*e\*Log[1 - c^2\*x^2]^2)/(4\*c)

**fricas [A]** time = 0.57, size = 130, normalized size = 1.25

$$\frac{be \log(-c^2x^2 + 1)^2 + be \log\left(\frac{cx+1}{cx-1}\right)^2 + 4(acd - 2ace)x + 2(2acex + bd - 2be) \log(-c^2x^2 + 1) + 2(bcex \log(-c^2x^2 + 1) + 2aex + (b*c*d - 2*b*c*e)*x) \log\left(\frac{cx+1}{cx-1}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)), x, algorithm="fricas")

[Out] 1/4\*(b\*e\*log(-c^2\*x^2 + 1)^2 + b\*e\*log((c\*x + 1)/(c\*x - 1))^2 + 4\*(a\*c\*d - 2\*a\*c\*e)\*x + 2\*(2\*a\*c\*e\*x + b\*d - 2\*b\*e)\*log(-c^2\*x^2 + 1) + 2\*(b\*c\*e\*x\*log(-c^2\*x^2 + 1) + 2\*a\*e + (b\*c\*d - 2\*b\*c\*e)\*x)\*log((c\*x + 1)/(c\*x - 1)))/c

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d), x)

**maple** [C] time = 1.80, size = 2210, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1)),x)

[Out]  $2/c*b*\operatorname{arccoth}(c*x)*e*\ln(2)-2/c*b*\ln((c*x+1)/(c*x-1)-1)*\ln(2)*e+2*b*\operatorname{arccoth}(c*x)*\ln(2)*x*e-2*b*e*x*\operatorname{arccoth}(c*x)+a*d*x-2*a*x*e+I/c*b*e*\operatorname{arccoth}(c*x)*\pi-1/2*I/c*b*\operatorname{arccoth}(c*x)*e*\pi*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^3-1/2*I/c*b*\pi*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3-1/2*I/c*b*\pi*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^3+1/c*b*\ln((c*x+1)/(c*x-1)-1)^2*e+1/c*b*\operatorname{arccoth}(c*x)*d-2/c*b*\operatorname{arccoth}(c*x)*e-1/c*b*d*\ln((c*x+1)/(c*x-1)-1)+2/c*b*e*\ln((c*x+1)/(c*x-1)-1)+b*\operatorname{arccoth}(c*x)*x*d+a*e*x*\ln(-c^2*x^2+1)+a*e/c*\ln(c*x+1)-a*e/c*\ln(c*x-1)-2*b*\operatorname{arccoth}(c*x)*\ln((c*x+1)/(c*x-1)-1)*x*e+I/c*b*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^2*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))*e*\operatorname{arccoth}(c*x)*\pi+I/c*b*\pi*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2+I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^2*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))*\pi*x*e+1/2*I/c*b*\operatorname{arccoth}(c*x)*e*\pi*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*I/c*b*\operatorname{arccoth}(c*x)*e*\pi*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))^2*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2+1/2*I/c*b*\operatorname{arccoth}(c*x)*e*\pi*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))^2*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)-1/2*I/c*b*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))^2*e*\operatorname{arccoth}(c*x)*\pi-1/2*I/c*b*\pi*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2-1/2*I/c*b*\pi*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)^2*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))^2*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)-I/c*b*\pi*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^2+1/2*I/c*b*\pi*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn}(I/((c*x-1)/(c*x+1))^(1/2))^2*\operatorname{csgn}(I*(c*x+1)/(c*x-1))+1/2*I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\pi*x*e-I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^2*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1))*\pi*x*e-\ln((c*x$

$$\begin{aligned}
& -1/(c*x+1))*(\operatorname{arccoth}(c*x)*x*c+\operatorname{arccoth}(c*x)-\ln((c*x+1)/(c*x-1)-1))*b*e/c-1/ \\
& 2*I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*(c* \\
& x+1)/(c*x-1))*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*\operatorname{Pi}*x*e^{-1/2*I/c*b*\operatorname{arccoth}(c*x)*e} \\
& *Pi*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*(c*x+1)/(c \\
& *x-1)/((c*x+1)/(c*x-1)-1)^2)+1/2*I/c*b*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x \\
& -1)-1)^2)*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*e*\ln((c*x+1 \\
& )/(c*x-1)-1)*Pi+I*b*\operatorname{arccoth}(c*x)*Pi*x*e^{-I/c*b*e*\ln((c*x+1)/(c*x-1)-1)*Pi+1/} \\
& 2*I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1) \\
& )^2*Pi*x*e^{-1/2*I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(c*x+1)/(c*x-1))*\operatorname{csgn}(I/((c*x-1)/(c* \\
& x+1))^{(1/2)}})^2*Pi*x*e^{1/2*I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/ \\
& (c*x-1)-1)^2}^2*\operatorname{csgn}(I/((c*x+1)/(c*x-1)-1)^2)*Pi*x*e^{1/2*I/c*b*Pi*\ln((c*x+1 \\
& )/(c*x-1)-1)*e*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^3-I/c*b*\operatorname{arccoth}(c*x)*e*Pi*\operatorname{csgn}(I*(c* \\
& x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+I/c*b*Pi*\ln((c*x+1)/(c*x-1)-1)*e*\operatorname{csgn} \\
& (I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2+1/2*I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(c* \\
& x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^3*Pi*x*e^{1/2*I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(( \\
& c*x+1)/(c*x-1)-1)^2)^3*Pi*x*e^{-1/2*I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(c*x+1)/(c*x-1))^} \\
& 3*Pi*x*e^{-I*b*\operatorname{arccoth}(c*x)*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1)^2)^2*P} \\
& i*x*e^{1/2*I/c*b*\operatorname{arccoth}(c*x)*e*Pi*\operatorname{csgn}(I*(c*x+1)/(c*x-1)/((c*x+1)/(c*x-1)-1} \\
& )^2)^3+1/2*I/c*b*\operatorname{arccoth}(c*x)*e*Pi*\operatorname{csgn}(I*((c*x+1)/(c*x-1)-1)^2)^3
\end{aligned}$$

**maxima** [C] time = 0.33, size = 178, normalized size = 1.71

$$-\left(c^2\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right) - x \log(-c^2x^2+1)\right) b e \operatorname{arccoth}(cx) - \left(c^2\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right) - x \log(-c^2x^2+1)\right) b e \operatorname{arccoth}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1)),x, algorithm="maxima")

[Out]  $-(c^2*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3) - x*\log(-c^2*x^2 + 1)) * b * e * \operatorname{arccoth}(c*x) - (c^2*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3) - x*\log(-c^2*x^2 + 1)) * a * e + a*d*x + 1/2*(2*c*x*\operatorname{arccoth}(c*x) + \log(-c^2*x^2 + 1)) * b*d/c + 1/2*((I*pi + 2*\log(c*x - 1) - 2)*\log(c*x + 1) + (I*pi - 2)*\log(c*x - 1)) * b*e/c$

**mupad** [B] time = 2.09, size = 315, normalized size = 3.03

$$\ln\left(\frac{1}{cx} + 1\right) \left(\frac{bdx}{2} - bex + \frac{bex \ln(1 - c^2x^2)}{2}\right) + \ln\left(1 - \frac{1}{cx}\right) \left(\frac{bdx^2 - bc^2dx^4}{2(cx^2 + x)(cx - 1)} - \frac{2bex^2 - 2bc^2ex^4}{2(cx^2 + x)(cx - 1)} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)),x)

```
[Out] log(1/(c*x) + 1)*((b*d*x)/2 - b*e*x + (b*e*x*log(1 - c^2*x^2))/2) + log(1 -
1/(c*x))*((b*d*x^2 - b*c^2*d*x^4)/(2*(x + c*x^2)*(c*x - 1)) - (2*b*e*x^2 -
2*b*c^2*e*x^4)/(2*(x + c*x^2)*(c*x - 1)) + (log(1 - c^2*x^2)*(b*e*x^2 - b*
c^2*e*x^4))/(2*(x + c*x^2)*(c*x - 1)) - (b*e*log(1/(c*x) + 1))/(2*c)) + a*x
*(d - 2*e) + (log(c*x + 1)*(2*a*e + b*d - 2*b*e))/(2*c) - (log(c*x - 1)*(2*
a*e - b*d + 2*b*e))/(2*c) + (b*e*log(1/(c*x) + 1)^2)/(4*c) + (b*e*log(1 - 1
/(c*x))^2)/(4*c) + (b*e*log(1 - c^2*x^2)^2)/(4*c) + a*e*x*log(1 - c^2*x^2)
```

**sympy [A]** time = 2.30, size = 155, normalized size = 1.49

$$\left\{ \begin{array}{l} adx + aex \log(-c^2x^2 + 1) - 2aex + \frac{2ae \operatorname{acoth}(cx)}{c} + bdx \operatorname{acoth}(cx) + bex \log(-c^2x^2 + 1) \operatorname{acoth}(cx) - 2bex \operatorname{acoth}(cx) \\ dx \left( a + \frac{i\pi b}{2} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*acoth(c*x)/c
+ b*d*x*acoth(c*x) + b*e*x*log(-c**2*x**2 + 1)*acoth(c*x) - 2*b*e*x*acoth(
c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c) - b
*e*log(-c**2*x**2 + 1)/c + b*e*acoth(c*x)**2/c, Ne(c, 0)), (d*x*(a + I*pi*b
/2), True))
```

$$3.275 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^2} dx$$

Optimal. Leaf size=105

$$\frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{x} - \frac{ce(a+b \coth^{-1}(cx))^2}{b} + \frac{1}{2}bc \log\left(1 - \frac{1}{1-c^2x^2}\right)(e \log(1-c^2x^2)+d)$$

[Out]  $-c*e*(a+b*\operatorname{arccoth}(c*x))^2/b - (a+b*\operatorname{arccoth}(c*x))*(d+e*\ln(-c^2*x^2+1))/x + 1/2*b*c*(d+e*\ln(-c^2*x^2+1))*\ln(1-1/(-c^2*x^2+1)) - 1/2*b*c*e*\operatorname{polylog}(2, 1/(-c^2*x^2+1))$

**Rubi [A]** time = 0.27, antiderivative size = 94, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$ , Rules used = {6082, 2475, 2411, 2344, 2301, 2316, 2315, 5949}

$$-\frac{1}{2}bce \operatorname{PolyLog}(2, c^2x^2) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{x} - \frac{ce(a+b \coth^{-1}(cx))^2}{b} - \frac{bc(e \log(1-c^2x^2))}{4e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2])/x^2, x]$

[Out]  $-((c*e*(a + b*\operatorname{ArcCoth}[c*x])^2)/b) + b*c*d*\operatorname{Log}[x] - ((a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]))/x - (b*c*(d + e*\operatorname{Log}[1 - c^2*x^2])^2)/(4*e) - (b*c*e*\operatorname{PolyLog}[2, c^2*x^2])/2$

#### Rule 2301

$\operatorname{Int}[(a + b*\operatorname{Log}[(c_*)*(x_)^(n_)]*(b_*))/(x_), x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*\operatorname{Log}[c*x^n])^2/(2*b*n), x] /; \operatorname{FreeQ}\{a, b, c, n\}, x]$

#### Rule 2315

$\operatorname{Int}[\operatorname{Log}[(c_*)*(x_)]/((d_) + (e_)*(x_)), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, 1 - c*x]/e, x] /; \operatorname{FreeQ}\{c, d, e\}, x \ \&\& \operatorname{EqQ}[e + c*d, 0]$

#### Rule 2316

$\operatorname{Int}[(a + b*\operatorname{Log}[(c_*)*(x_)]*(b_*))/((d_) + (e_)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*\operatorname{Log}[-(c*d)/e])* \operatorname{Log}[d + e*x]/e, x] + \operatorname{Dist}[b, \operatorname{Int}[\operatorname{Log}[-(e*x)/d]/(d + e*x), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x \ \&\& \operatorname{GtQ}[-(c*d)/e, 0]$

#### Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
  x_Symbol] := Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_
_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[(g*x)/e]^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

### Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_)^(s_.))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
|| IGtQ[q, 0])
```

### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6082

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^2} dx &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} + (bc) \int \frac{d + e \log(1 - c^2x^2)}{x(1 - c^2x^2)} dx \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} + bcd \log(x) - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x} \\
&= -\frac{ce(a + b \coth^{-1}(cx))^2}{b} + bcd \log(x) - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x}
\end{aligned}$$

**Mathematica [B]** time = 0.21, size = 332, normalized size = 3.16

$$4ae \log(1 - c^2x^2) + 8acex \tanh^{-1}(cx) + 4ad + 2bcdx \log(1 - c^2x^2) - 4bcex \log(x) \log(1 - c^2x^2) + 2bcex \log(1 - c^2x^2)$$

Antiderivative was successfully verified.

[In] Integrate(((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^2,x)

[Out] -1/4\*(4\*a\*d + 4\*b\*d\*ArcCoth[c\*x] + 4\*b\*c\*e\*x\*ArcCoth[c\*x]^2 + 8\*a\*c\*e\*x\*ArcTanh[c\*x] - 4\*b\*c\*d\*x\*Log[x] - b\*c\*e\*x\*Log[-c^(-1) + x]^2 - b\*c\*e\*x\*Log[c^(-1) + x]^2 - 2\*b\*c\*e\*x\*Log[c^(-1) + x]\*Log[(1 - c\*x)/2] + 4\*b\*c\*e\*x\*Log[x]\*Log[1 - c\*x] - 2\*b\*c\*e\*x\*Log[-c^(-1) + x]\*Log[(1 + c\*x)/2] + 4\*b\*c\*e\*x\*Log[x]\*Log[1 + c\*x] + 4\*a\*e\*Log[1 - c^2\*x^2] + 2\*b\*c\*d\*x\*Log[1 - c^2\*x^2] + 4\*b\*c\*e\*ArcCoth[c\*x]\*Log[1 - c^2\*x^2] - 4\*b\*c\*e\*x\*Log[x]\*Log[1 - c^2\*x^2] + 2\*b\*c\*e\*x\*Log[-c^(-1) + x]\*Log[1 - c^2\*x^2] + 2\*b\*c\*e\*x\*Log[c^(-1) + x]\*Log[1 - c^2\*x^2] + 4\*b\*c\*e\*x\*PolyLog[2, -(c\*x)] + 4\*b\*c\*e\*x\*PolyLog[2, c\*x] - 2\*b\*c\*e\*x\*PolyLog[2, 1/2 - (c\*x)/2] - 2\*b\*c\*e\*x\*PolyLog[2, (1 + c\*x)/2])/x

**fricas [F]** time = 0.57, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^2,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^2,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^2, x)

**maple** [F] time = 5.65, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(-c^2x^2 + 1))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^2,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( c(\log(c^2x^2 - 1) - \log(x^2)) + \frac{2 \operatorname{arccoth}(cx)}{x} \right) bd - \left( c^2 \left( \frac{\log(cx + 1)}{c} - \frac{\log(cx - 1)}{c} \right) + \frac{\log(-c^2x^2 + 1)}{x} \right) ae - \frac{1}{2} b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^2,x, algorithm="maxima")

[Out] -1/2\*(c\*(log(c^2\*x^2 - 1) - log(x^2)) + 2\*arccoth(c\*x)/x)\*b\*d - (c^2\*(log(c\*x + 1)/c - log(c\*x - 1)/c) + log(-c^2\*x^2 + 1)/x)\*a\*e - 1/2\*b\*e\*(log(c\*x + 1)^2/x - integrate(-((c\*x + 1)\*log(c\*x - 1)^2 - (I\*pi + (I\*pi\*c + 2\*c)\*x)\*log(c\*x + 1) - (-I\*pi - I\*pi\*c\*x)\*log(c\*x - 1))/(c\*x^3 + x^2), x)) - a\*d/x

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acoth}(cx))(d + e \ln(1 - c^2x^2))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^2, x)`

[Out] `int(((a + b*acoth(c*x))*(d + e*log(1 - c^2*x^2)))/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2 x^2 + 1))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acoth(c*x))*(d+e*ln(-c**2*x**2+1))/x**2, x)`

[Out] `Integral((a + b*acoth(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)`

$$3.276 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^4} dx$$

**Optimal.** Leaf size=197

$$\frac{c^3e(a+b \coth^{-1}(cx))^2}{3b} - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{3x^3} + \frac{2c^2e(a+b \coth^{-1}(cx))}{3x} - bc^3e \log(x) - \frac{bc(1-c^2x^2)}{3x^2}$$

[Out]  $2/3*c^2*e*(a+b*arccoth(c*x))/x-1/3*c^3*e*(a+b*arccoth(c*x))^2/b-b*c^3*e*\ln(x)+1/3*b*c^3*e*\ln(-c^2*x^2+1)-1/6*b*c*(-c^2*x^2+1)*(d+e*\ln(-c^2*x^2+1))/x^2-1/3*(a+b*arccoth(c*x))*(d+e*\ln(-c^2*x^2+1))/x^3+1/6*b*c^3*(d+e*\ln(-c^2*x^2+1))*\ln(1/(-c^2*x^2+1))-1/6*b*c^3*e*polylog(2,1/(-c^2*x^2+1))$

**Rubi [A]** time = 0.46, antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$ , Rules used = {6082, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 5983, 5917, 266, 36, 29, 5949}

$$-\frac{1}{6}bc^3e \text{PolyLog}(2, c^2x^2) - \frac{(a+b \coth^{-1}(cx))(e \log(1-c^2x^2)+d)}{3x^3} - \frac{c^3e(a+b \coth^{-1}(cx))^2}{3b} + \frac{2c^2e(a+b \coth^{-1}(cx))}{3x}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^4, x]

[Out]  $(2*c^2*e*(a + b*ArcCoth[c*x]))/(3*x) - (c^3*e*(a + b*ArcCoth[c*x])^2)/(3*b) + (b*c^3*d*Log[x])/3 - b*c^3*e*Log[x] + (b*c^3*e*Log[1 - c^2*x^2])/3 - (b*c*(1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(6*x^2) - ((a + b*ArcCoth[c*x])*(d + e*Log[1 - c^2*x^2]))/(3*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2])^2)/(12*e) - (b*c^3*e*PolyLog[2, c^2*x^2])/6$

**Rule 29**

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 36**

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x],

$x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

### Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 2301

$\text{Int}[((a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)) / (x_), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2 / (2*b*n), x] /; \text{FreeQ}\{a, b, c, n\}, x]$

### Rule 2314

$\text{Int}[((a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))*((d_) + (e_.)*(x_)^{(r_.)})^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[(x*(d + e*x^r)^{(q + 1)}*(a + b*\text{Log}[c*x^n]))/d, x] - \text{Dist}[(b*n)/d, \text{Int}[(d + e*x^r)^{(q + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, n, q, r\}, x] \&\& \text{EqQ}[r*(q + 1) + 1, 0]$

### Rule 2315

$\text{Int}[\text{Log}[(c_.)*(x_)] / ((d_) + (e_.)*(x_)), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

### Rule 2316

$\text{Int}[((a_.) + \text{Log}[(c_.)*(x_)]*(b_.)) / ((d_) + (e_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[-((c*d)/e)])*\text{Log}[d + e*x]/e, x] + \text{Dist}[b, \text{Int}[\text{Log}[-((e*x)/d)] / (d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{GtQ}[-((c*d)/e), 0]$

### Rule 2344

$\text{Int}[((a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))^{(p_.)} / ((x_)*((d_) + (e_.)*(x_))), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/x, x], x] - \text{Dist}[e/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p / (d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{IGtQ}[p, 0]$

### Rule 2347

$\text{Int}[(((a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))^{(p_.)}*((d_) + (e_.)*(x_))^{(q_.)}) / (x_), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(d + e*x)^{(q + 1)}*(a + b*\text{Log}[c*x^n])^p/x, x], x] - \text{Dist}[e/d, \text{Int}[(d + e*x)^q*(a + b*\text{Log}[c*x^n])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{LtQ}[q, -1] \&\& \text{IntegerQ}[2*q]$

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rule 5917

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCoth[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5983

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x], x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 6082

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(e_.)*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
```

g}, x] && ILtQ[m/2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^4} dx &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{3x^3} + \frac{1}{3}(bc) \int \frac{d + e \log(1 - c^2x^2)}{x^3} dx \\
 &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{3x^3} + \frac{1}{6}(bc) \operatorname{Subst} \left( \int \frac{d + e \log(1 - c^2x^2)}{x^3} dx, x, \frac{1}{c} \right) \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{3x^3} \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{3x^3} \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} - \frac{bc(1 - c^2x^2)(d + e \log(1 - c^2x^2))}{3x^3} \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} + \frac{1}{3}bc^3d \log(x) \\
 &= \frac{2c^2e(a + b \coth^{-1}(cx))}{3x} - \frac{c^3e(a + b \coth^{-1}(cx))^2}{3b} + \frac{1}{3}bc^3d \log(x)
 \end{aligned}$$

**Mathematica [B]** time = 0.37, size = 457, normalized size = 2.32

$$\frac{1}{6} \left( -4ac^3e \tanh^{-1}(cx) - \frac{2ae \log(1 - c^2x^2)}{x^3} + \frac{4ac^2e}{x} - \frac{2ad}{x^3} + 2bc^3d \log(x) - 2bc^3e \operatorname{Li}_2(-cx) - 2bc^3e \operatorname{Li}_2(cx) + bc^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^4, x]

[Out] ((-2\*a\*d)/x^3 - (b\*c\*d)/x^2 + (4\*a\*c^2\*e)/x - (2\*b\*d\*ArcCoth[c\*x])/x^3 + (4\*b\*c^2\*e\*ArcCoth[c\*x])/x - 2\*b\*c^3\*e\*ArcCoth[c\*x]^2 - 4\*a\*c^3\*e\*ArcTanh[c\*x])

$$] - 4*b*c^3*e*Log[1/Sqrt[1 - 1/(c^2*x^2)]] + 2*b*c^3*d*Log[x] - 2*b*c^3*e*Log[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 + (b*c^3*e*Log[c^(-1) + x]^2)/2 + b*c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 - c*x] + b*c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] - 2*b*c^3*e*Log[x]*Log[1 + c*x] - b*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 - c^2*x^2])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcCoth[c*x]*Log[1 - c^2*x^2])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) + x]*Log[1 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*e*PolyLog[2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2 - (c*x)/2] + b*c^3*e*PolyLog[2, (1 + c*x)/2])/6$$

**fricas** [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^4,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^4, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^4,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^4, x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(-c^2x^2 + 1))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^4,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^4,x)



**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6} \left( \left( c^2 \log(c^2 x^2 - 1) - c^2 \log(x^2) + \frac{1}{x^2} \right) c + \frac{2 \operatorname{arccoth}(cx)}{x^3} \right) b d - \frac{1}{3} \left( \left( c \log(cx + 1) - c \log(cx - 1) - \frac{2}{x} \right) c^2 + \frac{\log(-c^2 x^2 + 1)}{x^3} \right) a e - \frac{1}{6} b e \left( \frac{\log(c^2 x^2 - 1)}{x^3} - 3 \operatorname{integrate} \left( \frac{-1/3 (3(c^2 x^2 - 1) \log(c^2 x^2 - 1) - (3I\pi + (3I\pi c + 2c)x) \log(c^2 x^2 - 1) - (-3I\pi - 3I\pi c)x) \log(c^2 x^2 - 1)}{c^2 x^5 + x^4}, x \right) - 1/3 a d / x^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^4,x, algorithm="maxima")

[Out] -1/6\*((c^2\*log(c^2\*x^2 - 1) - c^2\*log(x^2) + 1/x^2)\*c + 2\*arccoth(c\*x)/x^3)\*b\*d - 1/3\*((c\*log(c\*x + 1) - c\*log(c\*x - 1) - 2/x)\*c^2 + log(-c^2\*x^2 + 1)/x^3)\*a\*e - 1/6\*b\*e\*(log(c\*x + 1)^2/x^3 - 3\*integrate(-1/3\*(3\*(c\*x + 1)\*log(c\*x - 1)^2 - (3\*I\*pi + (3\*I\*pi\*c + 2\*c)\*x)\*log(c\*x + 1) - (-3\*I\*pi - 3\*I\*pi\*c\*x)\*log(c\*x - 1))/(c\*x^5 + x^4), x)) - 1/3\*a\*d/x^3

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \ln(1 - c^2 x^2))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)))/x^4,x)

[Out] int(((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)))/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2 x^2 + 1))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x\*\*4,x)

[Out] Integral((a + b\*acoth(c\*x))\*(d + e\*log(-c\*\*2\*x\*\*2 + 1))/x\*\*4, x)

$$3.277 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(1-c^2x^2))}{x^6} dx$$

**Optimal.** Leaf size=256

$$\frac{c^5 e (a + b \coth^{-1}(cx))^2}{5b} + \frac{2c^4 e (a + b \coth^{-1}(cx))}{5x} - \frac{(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d)}{5x^5} + \frac{2c^2 e (a + b \coth^{-1}(cx))}{15x^3}$$

[Out]  $7/60*b*c^3*e/x^2+2/15*c^2*e*(a+b*\operatorname{arccoth}(c*x))/x^3+2/5*c^4*e*(a+b*\operatorname{arccoth}(c*x))/x-1/5*c^5*e*(a+b*\operatorname{arccoth}(c*x))^2/b-5/6*b*c^5*e*\ln(x)+19/60*b*c^5*e*\ln(-c^2*x^2+1)-1/20*b*c*(d+e*\ln(-c^2*x^2+1))/x^4-1/10*b*c^3*(-c^2*x^2+1)*(d+e*\ln(-c^2*x^2+1))/x^2-1/5*(a+b*\operatorname{arccoth}(c*x))*(d+e*\ln(-c^2*x^2+1))/x^5+1/10*b*c^5*(d+e*\ln(-c^2*x^2+1))*\ln(1-1/(-c^2*x^2+1))-1/10*b*c^5*e*\operatorname{polylog}(2,1/(-c^2*x^2+1))$

**Rubi [A]** time = 0.67, antiderivative size = 250, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 18, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6082, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 5983, 5917, 266, 36, 29, 5949}

$$-\frac{1}{10}bc^5e\operatorname{PolyLog}(2, c^2x^2) - \frac{(a + b \coth^{-1}(cx))(e \log(1 - c^2x^2) + d)}{5x^5} + \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} - \frac{c^5e(a + b \coth^{-1}(cx))^2}{5b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2])/x^6, x]$

[Out]  $(7*b*c^3*e)/(60*x^2) + (2*c^2*e*(a + b*\operatorname{ArcCoth}[c*x]))/(15*x^3) + (2*c^4*e*(a + b*\operatorname{ArcCoth}[c*x]))/(5*x) - (c^5*e*(a + b*\operatorname{ArcCoth}[c*x])^2)/(5*b) + (b*c^5*d*\operatorname{Log}[x])/5 - (5*b*c^5*e*\operatorname{Log}[x])/6 + (19*b*c^5*e*\operatorname{Log}[1 - c^2*x^2])/60 - (b*c*(d + e*\operatorname{Log}[1 - c^2*x^2]))/(20*x^4) - (b*c^3*(1 - c^2*x^2)*(d + e*\operatorname{Log}[1 - c^2*x^2]))/(10*x^2) - ((a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[1 - c^2*x^2]))/(5*x^5) - (b*c^5*(d + e*\operatorname{Log}[1 - c^2*x^2])^2)/(20*e) - (b*c^5*e*\operatorname{PolyLog}[2, c^2*x^2])/10$

**Rule 29**

$\operatorname{Int}[(x_)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[x], x]$

**Rule 31**

$\operatorname{Int}[(a_) + (b_)*(x_)^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}[\{a, b\}, x]$

**Rule 36**

```
Int[1/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 44

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] := Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

#### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

#### Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

#### Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_.))^(q_.),
x_Symbol] := Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
```

1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, -1] && (EqQ[p, 1] || (IntegersQ[2\*p, 2\*q] && !IGtQ[q, 0]) || (EqQ[p, 2] && NeQ[q, 1]))

#### Rule 2344

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] :> Dist[1/d, Int[(a + b\*Log[c\*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b\*Log[c\*x^n])^p/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

#### Rule 2347

Int((((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_))/(x\_), x\_Symbol] :> Dist[1/d, Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e\*x)^q\*(a + b\*Log[c\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2\*q]

#### Rule 2411

Int(((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2475

Int(((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(f + g\*x^(s/n))^r\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])

#### Rule 5917

Int(((a\_.) + ArcCoth[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 5949

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5983

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_)^(m_))/((d_) + (e_.)*(x_)^2), x_Symbol]
:> Dist[1/d, Int[(f*x)^m*(a + b*ArcCoth[c*x])^p, x], x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcCoth[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

### Rule 6082

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2])*(e_.)*(x_)^(m_.), x_Symbol]
:> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a + b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g}, x] && ILtQ[m/2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^6} dx &= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{5x^5} + \frac{1}{5}(bc) \int \frac{d + e \log(1 - c^2x^2)}{x^5} dx \\
&= -\frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{5x^5} + \frac{1}{10}(bc) \text{Subst} \left( \int \frac{d + e \log(1 - c^2x^2)}{x^5} dx, x, \frac{1}{c} \right) \\
&= \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} - \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{5x^5} \\
&= \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5x^5} \\
&= \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5x^5} \\
&= \frac{bc^3e}{15x^2} + \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5x^5} \\
&= \frac{7bc^3e}{60x^2} + \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5x^5} \\
&= \frac{7bc^3e}{60x^2} + \frac{2c^2e(a + b \coth^{-1}(cx))}{15x^3} + \frac{2c^4e(a + b \coth^{-1}(cx))}{5x} - \frac{c^5e(a + b \coth^{-1}(cx))}{5x^5}
\end{aligned}$$

**Mathematica** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(a + b \coth^{-1}(cx))(d + e \log(1 - c^2x^2))}{x^6} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^6, x]

[Out] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[1 - c^2\*x^2]))/x^6, x]

**fricas** [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(-c^2x^2 + 1)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^6,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(-c^2\*x^2 + 1))/x^6, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(-c^2x^2 + 1) + d)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^6,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(-c^2\*x^2 + 1) + d)/x^6, x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(-c^2x^2 + 1))}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^6,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(-c^2\*x^2+1))/x^6,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{20} \left( \left( 2c^4 \log(c^2x^2 - 1) - 2c^4 \log(x^2) + \frac{2c^2x^2 + 1}{x^4} \right) c + \frac{4 \operatorname{arccoth}(cx)}{x^5} \right) bd - \frac{1}{15} \left( \left( 3c^3 \log(cx + 1) - 3c^3 \log(cx - 1) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(-c^2\*x^2+1))/x^6,x, algorithm="maxima")

[Out] -1/20\*((2\*c^4\*log(c^2\*x^2 - 1) - 2\*c^4\*log(x^2) + (2\*c^2\*x^2 + 1)/x^4)\*c + 4\*arccoth(c\*x)/x^5)\*b\*d - 1/15\*((3\*c^3\*log(c\*x + 1) - 3\*c^3\*log(c\*x - 1) -

$2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*\log(-c^2*x^2 + 1)/x^5)*a*e - 1/10*b*e*(\log(c*x + 1)^2/x^5 - 5*\integrate(-1/5*(5*(c*x + 1)*\log(c*x - 1)^2 - (5*I*pi + 5*I*pi*c + 2*c)*x)*\log(c*x + 1) - (-5*I*pi - 5*I*pi*c*x)*\log(c*x - 1))/(c*x^7 + x^6), x)) - 1/5*a*d/x^5$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \ln(1 - c^2 x^2))}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)))/x^6,x)

[Out] int(((a + b\*acoth(c\*x))\*(d + e\*log(1 - c^2\*x^2)))/x^6, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \log(-c^2 x^2 + 1))}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(-c\*\*2\*x\*\*2+1))/x\*\*6,x)

[Out] Integral((a + b\*acoth(c\*x))\*(d + e\*log(-c\*\*2\*x\*\*2 + 1))/x\*\*6, x)



### 3.278 $\int x \left( a + b \coth^{-1}(cx) \right) \left( d + e \log \left( f + gx^2 \right) \right) dx$

**Optimal.** Leaf size=512

$$\frac{1}{2} dx^2 \left( a + b \coth^{-1}(cx) \right) + \frac{e \left( f + gx^2 \right) \log \left( f + gx^2 \right) \left( a + b \coth^{-1}(cx) \right)}{2g} - \frac{1}{2} ex^2 \left( a + b \coth^{-1}(cx) \right) - \frac{b(d-e) \tanh^{-1}\left(\frac{cx}{g}\right)}{2c^2}$$

[Out]  $\frac{1}{2} b (d - e) x / c - b e x / c + \frac{1}{2} d x^2 (a + b \operatorname{arccoth}(c x)) - \frac{1}{2} e x^2 (a + b \operatorname{arccoth}(c x)) - \frac{1}{2} b (d - e) \operatorname{arctanh}(c x) / c^2 - b e (c^2 f + g) \operatorname{arctanh}(c x) \ln(2 / (c x + 1)) / c^2 / g + \frac{1}{2} b e x \ln(g x^2 + f) / c + \frac{1}{2} e (g x^2 + f) (a + b \operatorname{arccoth}(c x)) \ln(g x^2 + f) / g - \frac{1}{2} b e (c^2 f + g) \operatorname{arctanh}(c x) \ln(g x^2 + f) / c^2 / g + \frac{1}{2} b e (c^2 f + g) \operatorname{arctanh}(c x) \ln(2 c ((-f)^{1/2} - x g^{1/2})) / (c x + 1) / (c (-f)^{1/2} - g^{1/2}) / c^2 / g + \frac{1}{2} b e (c^2 f + g) \operatorname{arctanh}(c x) \ln(2 c ((-f)^{1/2} + x g^{1/2})) / (c x + 1) / (c (-f)^{1/2} + g^{1/2}) / c^2 / g + \frac{1}{2} b e (c^2 f + g) \operatorname{polylog}(2, 1 - 2 / (c x + 1)) / c^2 / g - \frac{1}{4} b e (c^2 f + g) \operatorname{polylog}(2, 1 - 2 c ((-f)^{1/2} - x g^{1/2})) / (c x + 1) / (c (-f)^{1/2} - g^{1/2}) / c^2 / g - \frac{1}{4} b e (c^2 f + g) \operatorname{polylog}(2, 1 - 2 c ((-f)^{1/2} + x g^{1/2})) / (c x + 1) / (c (-f)^{1/2} + g^{1/2}) / c^2 / g + b e \operatorname{arctan}(x g^{1/2} / f^{1/2}) * f^{1/2} / c / g^{1/2}$

**Rubi [A]** time = 0.77, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 17, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$ , Rules used = {2454, 2389, 2295, 6084, 321, 207, 517, 2528, 2448, 205, 2470, 12, 5992, 5920, 2402, 2315, 2447}

$$\frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2}{cx+1}\right)}{2c^2g} - \frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f} - \sqrt{g}x)}{(cx+1)(c\sqrt{-f} - \sqrt{g})}\right)}{4c^2g} + \frac{be(c^2f + g) \operatorname{PolyLog}\left(2, 1 - \frac{2c(\sqrt{-f} + \sqrt{g}x)}{(cx+1)(c\sqrt{-f} + \sqrt{g})}\right)}{4c^2g}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x(a + b \operatorname{ArcCoth}[c x]) * (d + e \operatorname{Log}[f + g x^2]), x]$

[Out]  $(b(d - e)x) / (2c) - (bex) / c + (dx^2(a + b \operatorname{ArcCoth}[c x])) / 2 - (ex^2(a + b \operatorname{ArcCoth}[c x])) / 2 + (be \operatorname{Sqrt}[f] \operatorname{ArcTan}[(\operatorname{Sqrt}[g] x) / \operatorname{Sqrt}[f]]) / (c \operatorname{Sqrt}[g]) - (b(d - e) \operatorname{ArcTanh}[c x]) / (2c^2) - (be(c^2f + g) \operatorname{ArcTanh}[c x] \operatorname{Log}[2 / (1 + c x)]) / (c^2g) + (be(c^2f + g) \operatorname{ArcTanh}[c x] \operatorname{Log}[(2c(\operatorname{Sqrt}[-f] - \operatorname{Sqrt}[g] x)) / ((c \operatorname{Sqrt}[-f] - \operatorname{Sqrt}[g]) * (1 + c x))]) / (2c^2g) + (be(c^2f + g) \operatorname{ArcTanh}[c x] \operatorname{Log}[(2c(\operatorname{Sqrt}[-f] + \operatorname{Sqrt}[g] x)) / ((c \operatorname{Sqrt}[-f] + \operatorname{Sqrt}[g]) * (1 + c x))]) / (2c^2g) + (bex \operatorname{Log}[f + g x^2]) / (2c) + (e(f + g x^2)(a + b \operatorname{ArcCoth}[c x]) \operatorname{Log}[f + g x^2]) / (2g) - (be(c^2f + g) \operatorname{ArcTanh}[c x] \operatorname{Log}[f + g x^2]) / (2c^2g) + (be(c^2f + g) \operatorname{PolyLog}[2, 1 - 2 / (1 + c x)]) / (2c^2g) - (be(c^2f + g) \operatorname{PolyLog}[2, 1 - (2c(\operatorname{Sqrt}[-f] - \operatorname{Sqrt}[g] x)) / ((c \operatorname{Sqrt}[-f] - \operatorname{Sqrt}[g]) * (1 + c x))]) / (4c^2g) - (be(c^2f + g) \operatorname{PolyLog}[2, 1 - (2c(\operatorname{Sqrt}[-f] + \operatorname{Sqrt}[g] x)) / ((c \operatorname{Sqrt}[-f] + \operatorname{Sqrt}[g]) * (1 + c x))]) / (4c^2g)$

Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 205

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] * \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 207

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 321

$\text{Int}[(c_*)(x_)^{(m_*)} * ((a_*) + (b_*)(x_)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)} * (c*x)^{(m-n+1)} * (a + b*x^n)^{(p+1)}) / (b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(m-n+1)}) / (b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)} * (a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p+1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 517

$\text{Int}[(u_*) * ((c_*) + (d_*)(x_)^{(n_*)})^{(q_*)} * ((a1_*) + (b1_*)(x_)^{(non2_*)})^{(p_*)} * ((a2_*) + (b2_*)(x_)^{(non2_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Int}[u*(a1*a2 + b1*b2*x^n)^p * (c + d*x^n)^q, x] /; \text{FreeQ}[\{a1, b1, a2, b2, c, d, n, p, q\}, x] \ \&\& \ \text{EqQ}[non2, n/2] \ \&\& \ \text{EqQ}[a2*b1 + a1*b2, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[a1, 0] \ \&\& \ \text{GtQ}[a2, 0]))$

Rule 2295

$\text{Int}[\text{Log}[(c_*)(x_)^{(n_*)}], x\_Symbol] \rightarrow \text{Simp}[x * \text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /; \text{FreeQ}[\{c, n\}, x]$

Rule 2315

$\text{Int}[\text{Log}[(c_*)(x_)] / ((d_*) + (e_*)(x_)), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}[\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2447

```
Int[Log[u]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
+ e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

### Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x^n)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

### Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 5992

```
Int((((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6084

```
Int(((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2])*
(e_.)*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcCoth[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
- c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rubi steps



**Mathematica** [A] time = 4.43, size = 677, normalized size = 1.32

$$2ac^2d gx^2 + 2ac^2egx^2 \log(f + gx^2) + 2ac^2ef \log(f + gx^2) - 2ac^2egx^2 + 2bc^2d gx^2 \coth^{-1}(cx) + bc^2ef \operatorname{Li}_2\left(\frac{e^{2\coth^{-1}(cx)}}{fc^2+2}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]),x]

[Out] (2\*b\*c\*d\*g\*x - 6\*b\*c\*e\*g\*x + 2\*a\*c^2\*d\*g\*x^2 - 2\*a\*c^2\*e\*g\*x^2 - 2\*b\*d\*g\*ArcCoth[c\*x] + 2\*b\*e\*g\*ArcCoth[c\*x] + 2\*b\*c^2\*d\*g\*x^2\*ArcCoth[c\*x] - 2\*b\*c^2\*e\*g\*x^2\*ArcCoth[c\*x] - 4\*b\*c^2\*e\*f\*ArcCoth[c\*x]^2 - 4\*b\*e\*g\*ArcCoth[c\*x]^2 + 4\*b\*c\*e\*Sqrt[f]\*Sqrt[g]\*ArcTan[(Sqrt[g]\*x)/Sqrt[f]] - 4\*b\*c^2\*e\*f\*ArcCoth[c\*x]\*Log[1 - E^(-2\*ArcCoth[c\*x])] - 4\*b\*e\*g\*ArcCoth[c\*x]\*Log[1 - E^(-2\*ArcCoth[c\*x])] + 2\*b\*c^2\*e\*f\*ArcCoth[c\*x]\*Log[1 + (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(-c^2\*f - 2\*c\*Sqrt[-f]\*Sqrt[g] + g)] + 2\*b\*e\*g\*ArcCoth[c\*x]\*Log[1 + (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(-c^2\*f - 2\*c\*Sqrt[-f]\*Sqrt[g] + g)] + 2\*b\*c^2\*e\*f\*ArcCoth[c\*x]\*Log[1 + (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(-c^2\*f + 2\*c\*Sqrt[-f]\*Sqrt[g] + g)] + 2\*b\*e\*g\*ArcCoth[c\*x]\*Log[1 + (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(-c^2\*f + 2\*c\*Sqrt[-f]\*Sqrt[g] + g)] + 2\*a\*c^2\*e\*f\*Log[f + g\*x^2] + 2\*b\*c\*e\*g\*x\*Log[f + g\*x^2] + 2\*a\*c^2\*e\*g\*x^2\*Log[f + g\*x^2] - 2\*b\*e\*g\*ArcCoth[c\*x]\*Log[f + g\*x^2] + 2\*b\*c^2\*e\*g\*x^2\*ArcCoth[c\*x]\*Log[f + g\*x^2] + 2\*b\*e\*(c^2\*f + g)\*PolyLog[2, E^(-2\*ArcCoth[c\*x])] + b\*e\*(c^2\*f + g)\*PolyLog[2, (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(c^2\*f - 2\*c\*Sqrt[-f]\*Sqrt[g] - g)] + b\*c^2\*e\*f\*PolyLog[2, (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(c^2\*f + 2\*c\*Sqrt[-f]\*Sqrt[g] - g)] + b\*e\*g\*PolyLog[2, (E^(2\*ArcCoth[c\*x])\*(c^2\*f + g))/(c^2\*f + 2\*c\*Sqrt[-f]\*Sqrt[g] - g)]/(4\*c^2\*g)

**fricas** [F] time = 0.60, size = 0, normalized size = 0.00

$$\operatorname{integral}(b dx \operatorname{arccoth}(cx) + a dx + (bex \operatorname{arccoth}(cx) + aex) \log(gx^2 + f), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="fricas")

[Out] integral(b\*d\*x\*arccoth(c\*x) + a\*d\*x + (b\*e\*x\*arccoth(c\*x) + a\*e\*x)\*log(g\*x^2 + f), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(g\*x^2 + f) + d)\*x, x)

**maple** [C] time = 3.14, size = 8491, normalized size = 16.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f)),x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} adx^2 + \frac{1}{4} \left( 2x^2 \operatorname{arccoth}(cx) + c \left( \frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3} \right) \right) bd - \frac{1}{4} \left( 2c^2g \int \frac{x^3 \log(cx+1)}{c^2gx^2 + c^2f} dx - 2c^2g \int \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="maxima")

[Out]  $\frac{1}{2} a d x^2 + \frac{1}{4} (2 x^2 \operatorname{arccoth}(c x) + c (2 x / c^2 - \log(c x + 1) / c^3 + \log(c x - 1) / c^3)) b d - \frac{1}{4} (2 c^2 g \int \frac{x^3 \log(c x + 1)}{c^2 g x^2 + c^2 f}, x) - 2 c^2 g \int \frac{x^3 \log(c x - 1)}{c^2 g x^2 + c^2 f}, x) - 2 c g (-I f (\log(I g x / \sqrt{f g}) + 1) - \log(-I g x / \sqrt{f g}) + 1) / (\sqrt{f g}) c^2 g - 2 x / (c^2 g) - 2 g \int \frac{x \log(c x + 1)}{c^2 g x^2 + c^2 f}, x) + 2 g \int \frac{x \log(c x - 1)}{c^2 g x^2 + c^2 f}, x) - (2 c x + (c^2 x^2 - 1) \log(c x + 1) - (c^2 x^2 - 1) \log(c x - 1)) \log(g x^2 + f) / c^2) b e - \frac{1}{2} (g x^2 - (g x^2 + f) \log(g x^2 + f) + f) a e / g$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x (a + b \operatorname{acoth}(c x)) (d + e \ln(g x^2 + f)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*acoth(c\*x))\*(d + e\*log(f + g\*x^2)),x)

[Out] int(x\*(a + b\*acoth(c\*x))\*(d + e\*log(f + g\*x^2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```



### 3.279 $\int (a + b \coth^{-1}(cx)) (d + e \log(f + gx^2)) dx$

**Optimal.** Leaf size=546

$$x(a + b \coth^{-1}(cx))(d + e \log(f + gx^2)) + \frac{2ae\sqrt{f} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}} - 2aex + \frac{b \log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d + e \log(f + gx^2))}{2c} + \dots$$

[Out]  $-2*a*e*x - 2*b*e*x*\operatorname{arccoth}(c*x) - b*e*\ln(-c^2*x^2+1)/c + x*(a+b*\operatorname{arccoth}(c*x))*(d+e*\ln(g*x^2+f)) + 1/2*b*\ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*\ln(g*x^2+f))/c + 1/2*b*e*\operatorname{polylog}(2, c^2*(g*x^2+f)/(c^2*f+g))/c + 2*a*e*\operatorname{arctan}(x*g^(1/2)/f^(1/2))*f^(1/2)/g^(1/2) - b*e*\operatorname{arctan}(x*g^(1/2)/f^(1/2))*\ln(1-1/c/x)*f^(1/2)/g^(1/2) + b*e*\operatorname{arctan}(x*g^(1/2)/f^(1/2))*\ln(1+1/c/x)*f^(1/2)/g^(1/2) + b*e*\operatorname{arctan}(x*g^(1/2)/f^(1/2))*\ln(-2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)-g^(1/2)))/(f^(1/2)-I*x*g^(1/2))*f^(1/2)/g^(1/2) - b*e*\operatorname{arctan}(x*g^(1/2)/f^(1/2))*\ln(2*(c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)+g^(1/2)))/(f^(1/2)-I*x*g^(1/2))*f^(1/2)/g^(1/2) - 1/2*I*b*e*\operatorname{polylog}(2, 1+2*(-c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)-g^(1/2)))/(f^(1/2)-I*x*g^(1/2))*f^(1/2)/g^(1/2) + 1/2*I*b*e*\operatorname{polylog}(2, 1-2*(c*x+1)*f^(1/2)*g^(1/2)/(I*c*f^(1/2)+g^(1/2)))/(f^(1/2)-I*x*g^(1/2))*f^(1/2)/g^(1/2)$

**Rubi [A]** time = 1.39, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 20, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.952$ , Rules used = {6074, 2475, 2394, 2393, 2391, 5981, 5911, 260, 5975, 205, 5973, 2470, 12, 6688, 4876, 4848, 4856, 2402, 2315, 2447}

$$\frac{be \operatorname{PolyLog}\left(2, \frac{c^2(f+gx^2)}{c^2f+g}\right)}{2c} - \frac{ibe\sqrt{f} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{g}} + \frac{ibe\sqrt{f} \operatorname{PolyLog}\left(2, 1 - \frac{2\sqrt{f}\sqrt{g}(cx+1)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}+i\sqrt{g}x)}\right)}{2\sqrt{g}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[f + g*x^2]), x]$

[Out]  $-2*a*e*x - 2*b*e*x*\operatorname{ArcCoth}[c*x] + (2*a*e*\operatorname{Sqrt}[f]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]])/\operatorname{Sqrt}[g] - (b*e*\operatorname{Sqrt}[f]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]]*\operatorname{Log}[1 - 1/(c*x)])/\operatorname{Sqrt}[g] + (b*e*\operatorname{Sqrt}[f]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]]*\operatorname{Log}[1 + 1/(c*x)])/\operatorname{Sqrt}[g] + (b*e*\operatorname{Sqrt}[f]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]]*\operatorname{Log}[(-2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[g]*(1 - c*x))/(I*c*\operatorname{Sqrt}[f] - \operatorname{Sqrt}[g])*(\operatorname{Sqrt}[f] - I*\operatorname{Sqrt}[g]*x)])/\operatorname{Sqrt}[g] - (b*e*\operatorname{Sqrt}[f]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]]*\operatorname{Log}[(2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[g]*(1 + c*x))/(I*c*\operatorname{Sqrt}[f] + \operatorname{Sqrt}[g])*(\operatorname{Sqrt}[f] - I*\operatorname{Sqrt}[g]*x)])/\operatorname{Sqrt}[g] - (b*e*\operatorname{Log}[1 - c^2*x^2])/c + x*(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[f + g*x^2]) + (b*\operatorname{Log}[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*\operatorname{Log}[f + g*x^2]))/(2*c) + (b*e*\operatorname{PolyLog}[2, (c^2*(f + g*x^2))/(c^2*f + g)]/(2*c) - ((I/2)*b*e*\operatorname{Sqrt}[f]*\operatorname{PolyLog}[2, 1 + (2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[g]*(1 - c*x))/(I*c*\operatorname{Sqrt}[f] - \operatorname{Sqrt}[g])*(\operatorname{Sqrt}[f] - I*\operatorname{Sqrt}[g]*x)])/\operatorname{Sqrt}[g]$

+ ((I/2)\*b\*e\*Sqrt[f]\*PolyLog[2, 1 - (2\*Sqrt[f]\*Sqrt[g]\*(1 + c\*x))/((I\*c\*Sqrt[f] + Sqrt[g])\*(Sqrt[f] - I\*Sqrt[g]\*x)))]/Sqrt[g]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 2315

Int[Log[(c\_)\*(x\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))])\*(b\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))])\*(b\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2402

Int[Log[(c\_)/((d\_) + (e\_)\*(x\_))]/((f\_) + (g\_)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{

c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

#### Rule 2447

Int[Log[u\_]\*(Pq\_)^(m\_), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

#### Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

#### Rule 2475

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(s\_))^(r\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(f + g\*x^(s/n))^r\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])

#### Rule 4848

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (Dist[(I\*b)/2, Int[Log[1 - I\*c\*x]/x, x], x] - Dist[(I\*b)/2, Int[Log[1 + I\*c\*x]/x, x], x) /; FreeQ[{a, b, c}, x]

#### Rule 4856

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTan[c\*x])\*Log[2/(1 - I\*c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 - I\*c\*x)]/(1 + c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))]/(1 + c^2\*x^2), x], x] + Simp[((a + b\*ArcTan[c\*x])\*Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 + e^2, 0]

#### Rule 4876

Int[((a\_.) + ArcTan[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((f\_.)\*(x\_)^(m\_.))\*((d\_) + (e\_.)\*(x\_)^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcTan[c\*x])^p, (f\*x)^m\*(d + e\*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &

& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])

### Rule 5911

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 5973

Int[ArcCoth[(c\_.)\*(x\_.)]/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[Log[1 + 1/(c\*x)]/(d + e\*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c\*x)]/(d + e\*x^2), x], x] /; FreeQ[{c, d, e}, x]

### Rule 5975

Int[(ArcCoth[(c\_.)\*(x\_.)]\*(b\_.) + (a\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[a, Int[1/(d + e\*x^2), x], x] + Dist[b, Int[ArcCoth[c\*x]/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]

### Rule 5981

Int[(((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_)^(m\_)))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcCoth[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 6074

Int[((a\_.) + ArcCoth[(c\_.)\*(x\_.)]\*(b\_.))\*((d\_.) + Log[(f\_.) + (g\_.)\*(x\_)^2]\*(e\_.)), x\_Symbol] := Simp[x\*(d + e\*Log[f + g\*x^2])\*(a + b\*ArcCoth[c\*x]), x] + (-Dist[b\*c, Int[(x\*(d + e\*Log[f + g\*x^2]))/(1 - c^2\*x^2), x], x] - Dist[2\*e\*g, Int[(x^2\*(a + b\*ArcCoth[c\*x]))/(f + g\*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g}, x]

### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

### Rubi steps



**Mathematica [B]** time = 3.22, size = 1287, normalized size = 2.36

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]),x]

[Out] a\*d\*x - 2\*a\*e\*x + b\*d\*x\*ArcCoth[c\*x] + (2\*a\*e\*Sqrt[f]\*ArcTan[(Sqrt[g]\*x)/Sqrt[f]])/Sqrt[g] + (b\*d\*Log[1 - c^2\*x^2])/(2\*c) + a\*e\*x\*Log[f + g\*x^2] + b\*e\*(x\*ArcCoth[c\*x] + Log[1 - c^2\*x^2]/(2\*c))\*Log[f + g\*x^2] + (b\*e\*(-4\*c\*x\*ArcCoth[c\*x] + 4\*Log[1/(c\*Sqrt[1 - 1/(c^2\*x^2)])\*x]) + (Sqrt[c^2\*f\*g]\*((-2\*I)\*ArcCos[(c^2\*f - g)/(c^2\*f + g)]\*ArcTan[Sqrt[c^2\*f\*g]/(c\*g\*x)] + 4\*ArcCoth[c\*x]\*ArcTan[(c\*g\*x)/Sqrt[c^2\*f\*g]] - (ArcCos[(c^2\*f - g)/(c^2\*f + g)] + 2\*ArcTan[Sqrt[c^2\*f\*g]/(c\*g\*x)])\*Log[((2\*I)\*g\*(I\*c^2\*f + Sqrt[c^2\*f\*g])\*(-1 + 1/(c\*x)))/((c^2\*f + g)\*(g + (I\*Sqrt[c^2\*f\*g])/(c\*x)))]) - (ArcCos[(c^2\*f - g)/(c^2\*f + g)] - 2\*ArcTan[Sqrt[c^2\*f\*g]/(c\*g\*x)])\*Log[(2\*g\*(c^2\*f + I\*Sqrt[c^2\*f\*g])\*(1 + 1/(c\*x)))/((c^2\*f + g)\*(g + (I\*Sqrt[c^2\*f\*g])/(c\*x)))]) + (ArcCos[(c^2\*f - g)/(c^2\*f + g)] + 2\*(ArcTan[Sqrt[c^2\*f\*g]/(c\*g\*x)] + ArcTan[(c\*g\*x)/Sqrt[c^2\*f\*g]]))\*Log[(Sqrt[2]\*Sqrt[c^2\*f\*g])/(E^ArcCoth[c\*x]\*Sqrt[c^2\*f + g]\*Sqrt[-(c^2\*f) + g + (c^2\*f + g)\*Cosh[2\*ArcCoth[c\*x]])]) + (ArcCos[(c^2\*f - g)/(c^2\*f + g)] - 2\*(ArcTan[Sqrt[c^2\*f\*g]/(c\*g\*x)] + ArcTan[(c\*g\*x)/Sqrt[c^2\*f\*g]]))\*Log[(Sqrt[2]\*E^ArcCoth[c\*x]\*Sqrt[c^2\*f\*g])/(Sqrt[c^2\*f + g]\*Sqrt[-(c^2\*f) + g + (c^2\*f + g)\*Cosh[2\*ArcCoth[c\*x]])]) + I\*(-PolyLog[2, ((-(c^2\*f) + g + (2\*I)\*Sqrt[c^2\*f\*g])\*(g - (I\*Sqrt[c^2\*f\*g])/(c\*x)))/((c^2\*f + g)\*(g + (I\*Sqrt[c^2\*f\*g])/(c\*x)))]) + PolyLog[2, ((c^2\*f - g + (2\*I)\*Sqrt[c^2\*f\*g])\*(I\*g + Sqrt[c^2\*f\*g]/(c\*x)))/((c^2\*f + g)\*((-I)\*g + Sqrt[c^2\*f\*g]/(c\*x)))])/g)/(2\*c) - (b\*e\*g\*((-Log[-c^(-1) + x] - Log[c^(-1) + x] + Log[1 - c^2\*x^2])\*Log[f + g\*x^2])/(2\*g) + (Log[-c^(-1) + x]\*Log[1 - (Sqrt[g]\*(-c^(-1) + x))/((-I)\*Sqrt[f] - Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]\*(-c^(-1) + x))/((-I)\*Sqrt[f] - Sqrt[g]/c)])/(2\*g) + (Log[-c^(-1) + x]\*Log[1 - (Sqrt[g]\*(-c^(-1) + x))/(I\*Sqrt[f] - Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]\*(-c^(-1) + x))/(I\*Sqrt[f] - Sqrt[g]/c)])/(2\*g) + (Log[c^(-1) + x]\*Log[1 - (Sqrt[g]\*(c^(-1) + x))/((-I)\*Sqrt[f] + Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]\*(c^(-1) + x))/((-I)\*Sqrt[f] + Sqrt[g]/c)])/(2\*g) + (Log[c^(-1) + x]\*Log[1 - (Sqrt[g]\*(c^(-1) + x))/(I\*Sqrt[f] + Sqrt[g]/c)] + PolyLog[2, (Sqrt[g]\*(c^(-1) + x))/(I\*Sqrt[f] + Sqrt[g]/c)])/(2\*g)))/c

**fricas [F]** time = 0.64, size = 0, normalized size = 0.00

integral((bd arccoth(cx) + ad + (be arccoth(cx) + ae) log(gx^2 + f)), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="fricas")

[Out] integral(b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(g\*x^2 + f), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(g\*x^2 + f) + d), x)

maple [F] time = 2.96, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccoth}(cx)) (d + e \ln(gx^2 + f)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f)),x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left( 2g \left( \frac{f \arctan\left(\frac{gx}{\sqrt{fg}}\right)}{\sqrt{fg}g} - \frac{x}{g} \right) + x \log(gx^2 + f) \right) ae + adx + \frac{1}{2} be \left( \frac{((cx + 1) \log(cx + 1) - (cx - 1) \log(cx - 1)) \log(gx^2 + f)}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f)),x, algorithm="maxima")

[Out] (2\*g\*(f\*arctan(g\*x/sqrt(f\*g))/(sqrt(f\*g)\*g) - x/g) + x\*log(g\*x^2 + f))\*a\*e + a\*d\*x + 1/2\*b\*e\*((c\*x + 1)\*log(c\*x + 1) - (c\*x - 1)\*log(c\*x - 1))\*log(g\*x^2 + f)/c - integrate(2\*((c\*g\*x^2 + g\*x)\*log(c\*x + 1) - (c\*g\*x^2 - g\*x)\*log(c\*x - 1))/(c\*g\*x^2 + c\*f), x) + 1/2\*(2\*c\*x\*arccoth(c\*x) + log(-c^2\*x^2 + 1))\*b\*d/c

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{acoth}(cx)) (d + e \ln(gx^2 + f)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acoth(c*x))*(d + e*log(f + g*x^2)),x)
```

```
[Out] int((a + b*acoth(c*x))*(d + e*log(f + g*x^2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```



$$3.280 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x} dx$$

**Optimal.** Leaf size=101

$$be \operatorname{Int} \left( \frac{\coth^{-1}(cx) \log(f+gx^2)}{x}, x \right) + ad \log(x) + \frac{1}{2} ae \operatorname{Li}_2 \left( \frac{gx^2}{f} + 1 \right) + \frac{1}{2} ae \log \left( -\frac{gx^2}{f} \right) \log(f+gx^2) + \frac{1}{2} bd \operatorname{Li}_2 \left( -\frac{1}{cx} \right)$$

[Out] b\*e\*CannotIntegrate(arccoth(c\*x)\*ln(g\*x^2+f)/x,x)+a\*d\*ln(x)+1/2\*a\*e\*ln(-g\*x^2/f)\*ln(g\*x^2+f)+1/2\*b\*d\*polylog(2,-1/c/x)-1/2\*b\*d\*polylog(2,1/c/x)+1/2\*a\*e\*polylog(2,1+g\*x^2/f)

**Rubi [A]** time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x,x]

[Out] a\*d\*Log[x] + (a\*e\*Log[-((g\*x^2)/f)]\*Log[f + g\*x^2])/2 + (b\*d\*PolyLog[2, -(1/(c\*x))])/2 - (b\*d\*PolyLog[2, 1/(c\*x)])/2 + (a\*e\*PolyLog[2, 1 + (g\*x^2)/f])/2 + b\*e\*Defer[Int] [(ArcCoth[c\*x]\*Log[f + g\*x^2])/x, x]

Rubi steps

$$\begin{aligned} \int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x} dx &= d \int \frac{a+b \coth^{-1}(cx)}{x} dx + e \int \frac{(a+b \coth^{-1}(cx)) \log(f+gx^2)}{x} dx \\ &= ad \log(x) + \frac{1}{2} bd \operatorname{Li}_2 \left( -\frac{1}{cx} \right) - \frac{1}{2} bd \operatorname{Li}_2 \left( \frac{1}{cx} \right) + (ae) \int \frac{\log(f+gx^2)}{x} dx \\ &= ad \log(x) + \frac{1}{2} bd \operatorname{Li}_2 \left( -\frac{1}{cx} \right) - \frac{1}{2} bd \operatorname{Li}_2 \left( \frac{1}{cx} \right) + \frac{1}{2} (ae) \operatorname{Subst} \left( \int \frac{\log(u)}{u} du, f+gx^2 \right) \\ &= ad \log(x) + \frac{1}{2} ae \log \left( -\frac{gx^2}{f} \right) \log(f+gx^2) + \frac{1}{2} bd \operatorname{Li}_2 \left( -\frac{1}{cx} \right) - \frac{1}{2} bd \operatorname{Li}_2 \left( \frac{1}{cx} \right) \\ &= ad \log(x) + \frac{1}{2} ae \log \left( -\frac{gx^2}{f} \right) \log(f+gx^2) + \frac{1}{2} bd \operatorname{Li}_2 \left( -\frac{1}{cx} \right) - \frac{1}{2} bd \operatorname{Li}_2 \left( \frac{1}{cx} \right) \end{aligned}$$

**Mathematica** [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{coth}^{-1}(cx))(d + e \log(f + gx^2))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x,x]

[Out] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x, x]

**fricas** [A] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(gx^2 + f)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x,x, algorithm="fricas")

[Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(g\*x^2 + f))/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x,x, algorithm="giac")

[Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(g\*x^2 + f) + d)/x, x)

**maple** [A] time = 1.49, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(gx^2 + f))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f))/x,x)

[Out] int((a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$ad \log(x) + \int \frac{be \left( \log\left(\frac{1}{cx} + 1\right) - \log\left(-\frac{1}{cx} + 1\right) \right) \log(gx^2 + f)}{2x} + \frac{bd \left( \log\left(\frac{1}{cx} + 1\right) - \log\left(-\frac{1}{cx} + 1\right) \right)}{2x} + \frac{ae \log(gx^2 + f)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + integrate(1/2*b*e*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))*log(g
*x^2 + f)/x + 1/2*b*d*(log(1/(c*x) + 1) - log(-1/(c*x) + 1))/x + a*e*log(g*
x^2 + f)/x, x)
```

**mupad** [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \ln(gx^2 + f))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

```
[Out] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

```
[Out] Timed out
```

$$3.281 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x^2} dx$$

**Optimal.** Leaf size=560

$$-\frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x} + \frac{2ae\sqrt{g} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{1}{2}bc \log\left(\frac{g(1-c^2x^2)}{c^2f+g}\right)(d+e \log(f+gx^2)) - \frac{1}{2}bce \log\left(\frac{c^2(f+gx^2)}{c^2f+g}\right) + \frac{1}{2}bce \log\left(\frac{gx^2}{f} + 1\right) - \frac{ibe\sqrt{g} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}} + \frac{ibe\sqrt{g} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}+i\sqrt{g}x)}\right)}{2\sqrt{f}}$$

[Out]  $-(a+b*\operatorname{arccoth}(c*x))*(d+e*\ln(g*x^2+f))/x+1/2*b*c*\ln(-g*x^2/f)*(d+e*\ln(g*x^2+f))-1/2*b*c*\ln(g*(-c^2*x^2+1)/(c^2*f+g))*(d+e*\ln(g*x^2+f))-1/2*b*c*e*\operatorname{polylog}(2, c^2*(g*x^2+f)/(c^2*f+g))+1/2*b*c*e*\operatorname{polylog}(2, 1+g*x^2/f)+2*a*e*\operatorname{arctan}(x*g^{(1/2)}/f^{(1/2)})*g^{(1/2)}/f^{(1/2)}-b*e*\operatorname{arctan}(x*g^{(1/2)}/f^{(1/2)})*\ln(1-1/c/x)*g^{(1/2)}/f^{(1/2)}+b*e*\operatorname{arctan}(x*g^{(1/2)}/f^{(1/2)})*\ln(1+1/c/x)*g^{(1/2)}/f^{(1/2)}+b*e*\operatorname{arctan}(x*g^{(1/2)}/f^{(1/2)})*\ln(-2*(-c*x+1)*f^{(1/2)*g^{(1/2)}}/(I*c*f^{(1/2)}-g^{(1/2)})/(f^{(1/2)}-I*x*g^{(1/2)}))*g^{(1/2)}/f^{(1/2)}-b*e*\operatorname{arctan}(x*g^{(1/2)}/f^{(1/2)})*\ln(2*(c*x+1)*f^{(1/2)*g^{(1/2)}}/(I*c*f^{(1/2)}+g^{(1/2)})/(f^{(1/2)}-I*x*g^{(1/2)}))*g^{(1/2)}/f^{(1/2)}-1/2*I*b*e*\operatorname{polylog}(2, 1+2*(-c*x+1)*f^{(1/2)*g^{(1/2)}}/(I*c*f^{(1/2)}-g^{(1/2)})/(f^{(1/2)}-I*x*g^{(1/2)}))*g^{(1/2)}/f^{(1/2)}+1/2*I*b*e*\operatorname{polylog}(2, 1-2*(c*x+1)*f^{(1/2)*g^{(1/2)}}/(I*c*f^{(1/2)}+g^{(1/2)})/(f^{(1/2)}-I*x*g^{(1/2)}))*g^{(1/2)}/f^{(1/2)}$

**Rubi [A]** time = 1.26, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 22, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$ , Rules used = {6082, 2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 5975, 205, 5973, 2470, 12, 260, 6688, 4876, 4848, 4856, 2402, 2447}

$$-\frac{1}{2}bce \operatorname{PolyLog}\left(2, \frac{c^2(f+gx^2)}{c^2f+g}\right) + \frac{1}{2}bce \operatorname{PolyLog}\left(2, \frac{gx^2}{f} + 1\right) - \frac{ibe\sqrt{g} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}} + \frac{ibe\sqrt{g} \operatorname{PolyLog}\left(2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}+i\sqrt{g}x)}\right)}{2\sqrt{f}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[f + g*x^2])/x^2, x]$

[Out]  $(2*a*e*\operatorname{Sqrt}[g]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]])/\operatorname{Sqrt}[f] - (b*e*\operatorname{Sqrt}[g]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]]*\operatorname{Log}[1 - 1/(c*x)])/\operatorname{Sqrt}[f] + (b*e*\operatorname{Sqrt}[g]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]]*\operatorname{Log}[1 + 1/(c*x)])/\operatorname{Sqrt}[f] + (b*e*\operatorname{Sqrt}[g]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]]*\operatorname{Log}[(-2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[g]*(1 - c*x))/((I*c*\operatorname{Sqrt}[f] - \operatorname{Sqrt}[g])*(\operatorname{Sqrt}[f] - I*\operatorname{Sqrt}[g]*x))])/ \operatorname{Sqrt}[f] - (b*e*\operatorname{Sqrt}[g]*\operatorname{ArcTan}[(\operatorname{Sqrt}[g]*x)/\operatorname{Sqrt}[f]]*\operatorname{Log}[(2*\operatorname{Sqrt}[f]*\operatorname{Sqrt}[g]*(1 + c*x))/((I*c*\operatorname{Sqrt}[f] + \operatorname{Sqrt}[g])*(\operatorname{Sqrt}[f] - I*\operatorname{Sqrt}[g]*x))])/ \operatorname{Sqrt}[f] - ((a + b*\operatorname{ArcCoth}[c*x])*(d + e*\operatorname{Log}[f + g*x^2])/x + (b*c*\operatorname{Log}[-((g*x^2)/f)]*(d + e*\operatorname{Log}[f + g*x^2])/2 - (b*c*\operatorname{Log}[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*\operatorname{Log}[f + g*x^2])/2 - (b*c*e*\operatorname{PolyLog}[2, (c^2*(f + g*x^2))/(c^2*f + g)]*(d + e*\operatorname{Log}[f + g*x^2])/2 - (b*c*e*\operatorname{PolyLog}[2, (gx^2)/f + 1])*(d + e*\operatorname{Log}[f + g*x^2])/2 - (ibe*\sqrt{g}*\operatorname{PolyLog}[2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(-\sqrt{g}+ic\sqrt{f})(\sqrt{f}-i\sqrt{g}x)}])/2\sqrt{f} + (ibe*\sqrt{g}*\operatorname{PolyLog}[2, 1 + \frac{2\sqrt{f}\sqrt{g}(1-cx)}{(\sqrt{g}+ic\sqrt{f})(\sqrt{f}+i\sqrt{g}x)}])/2\sqrt{f}))/x$

$$2*f + g))/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2 - ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(1 - c*x))/((I*c*Sqrt[f] - Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f] + ((I/2)*b*e*Sqrt[g]*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(1 + c*x))/((I*c*Sqrt[f] + Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/Sqrt[f]$$
Rule 12

$$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] \text{ /; FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_) \text{ /; FreeQ}[b, x]$$
Rule 29

$$\text{Int}[(x_)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$$
Rule 31

$$\text{Int}[(a_*) + (b_*)(x_)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ /; FreeQ}[\{a, b\}, x]$$
Rule 36

$$\text{Int}[1/((a_*) + (b_*)(x_))*((c_*) + (d_*)(x_)), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] \text{ /; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$$
Rule 205

$$\text{Int}[(a_*) + (b_*)(x_)^2)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \text{ /; FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$$
Rule 260

$$\text{Int}[(x_)^{(m_*)}/((a_*) + (b_*)(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] \text{ /; FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$$
Rule 2315

$$\text{Int}[\text{Log}[(c_*)(x_)]/((d_*) + (e_*)(x_)), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] \text{ /; FreeQ}[\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$$
Rule 2391

$$\text{Int}[\text{Log}[(c_*)((d_*) + (e_*)(x_)^{(n_)}))]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] \text{ /; FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$$

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))])*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))])*(b_.)^(p_.)*((h_.)*(x_)
)^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))])^(p_.)*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] := With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p], x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))])^(p_.)*(b_.)^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Sim
```

```

plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
|| IGtQ[q, 0])

```

### Rule 4848

```

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]

```

### Rule 4856

```

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] := -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
)))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]

```

### Rule 4876

```

Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])

```

### Rule 5973

```

Int[ArcCoth[(c_.)*(x_)])/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[1/2, Int[
Log[1 + 1/(c*x)]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - 1/(c*x)]/(d +
e*x^2), x], x] /; FreeQ[{c, d, e}, x]

```

### Rule 5975

```

Int[(ArcCoth[(c_.)*(x_)])*(b_.) + (a_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] :=
Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcCoth[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]

```

### Rule 6082

```

Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)*(x_)^(m_.), x_Symbol] := Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
b*ArcCoth[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +

```

```
2)*(a + b*ArcCoth[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g}, x] && ILtQ[m/2, 0]
```

### Rule 6688

```
Int[u_, x_Symbol] :=> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]
```

### Rubi steps





**Mathematica [B]** time = 3.59, size = 1236, normalized size = 2.21

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x^2,x]

[Out] 
$$-\frac{(a*d)}{x} - \frac{(b*d*ArcCoth[c*x])}{x} + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2])/2 + a*e*((2*sqrt{g}*ArcTan[(sqrt{g}*x)/sqrt{f}])/sqrt{f} - Log[f + g*x^2]/x) + (b*e*(-(((2*ArcCoth[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[f + g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*sqrt{g}*x)/sqrt{f}] + Log[1 + (I*sqrt{g}*x)/sqrt{f}]) + PolyLog[2, ((-I)*sqrt{g}*x)/sqrt{f}] + PolyLog[2, (I*sqrt{g}*x)/sqrt{f}]) + c*(Log[-c^{(-1)} + x]*Log[(c*(sqrt{f} - I*sqrt{g}*x))/(c*sqrt{f} - I*sqrt{g}]) + Log[-c^{(-1)} + x]*Log[(c*(sqrt{f} - I*sqrt{g}*x))/(c*sqrt{f} + I*sqrt{g}]) + Log[-c^{(-1)} + x]*Log[(c*(sqrt{f} + I*sqrt{g}*x))/(c*sqrt{f} + I*sqrt{g}]) - (Log[-c^{(-1)} + x] + Log[c^{(-1)} + x] - Log[1 - c^2*x^2])*Log[f + g*x^2] + Log[c^{(-1)} + x]*Log[1 - (sqrt{g}*(1 + c*x))/(I*c*sqrt{f} + sqrt{g}]) + PolyLog[2, (c*sqrt{g}*(c^{(-1)} + x))/(I*c*sqrt{f} + sqrt{g}]) + PolyLog[2, (I*sqrt{g}*(-1 + c*x))/(c*sqrt{f} - I*sqrt{g}]) + PolyLog[2, ((-I)*sqrt{g}*(-1 + c*x))/(c*sqrt{f} + I*sqrt{g}]) + PolyLog[2, (I*sqrt{g}*(1 + c*x))/(c*sqrt{f} + I*sqrt{g}]) - (c*g*((2*I)*ArcCos[(c^2*f - g)/(c^2*f + g)]*ArcTan[(c*f)/(sqrt{c^2*f*g}*x)] - 4*ArcCoth[c*x]*ArcTan[(c*g*x)/sqrt{c^2*f*g}] + (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*ArcTan[(c*f)/(sqrt{c^2*f*g}*x)])*Log[(2*g*(c^2*f - I*sqrt{c^2*f*g})*(-1 + c*x))/((c^2*f + g)*(I*sqrt{c^2*f*g} + c*g*x))] + (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*ArcTan[(c*f)/(sqrt{c^2*f*g}*x)])*Log[(2*g*(c^2*f + I*sqrt{c^2*f*g})*(1 + c*x))/((c^2*f + g)*(I*sqrt{c^2*f*g} + c*g*x))] - (ArcCos[(c^2*f - g)/(c^2*f + g)] + 2*(ArcTan[(c*f)/(sqrt{c^2*f*g}*x)] + ArcTan[(c*g*x)/sqrt{c^2*f*g}]))*Log[(sqrt{2}*sqrt{c^2*f*g})/(E^ArcCoth[c*x]*sqrt{c^2*f + g}*sqrt{-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]])]) - (ArcCos[(c^2*f - g)/(c^2*f + g)] - 2*(ArcTan[(c*f)/(sqrt{c^2*f*g}*x)] + ArcTan[(c*g*x)/sqrt{c^2*f*g}]))*Log[(sqrt{2}*E^ArcCoth[c*x]*sqrt{c^2*f*g})/(sqrt{c^2*f + g}*sqrt{-(c^2*f) + g + (c^2*f + g)*Cosh[2*ArcCoth[c*x]])]) + I*(PolyLog[2, ((c^2*f - g - (2*I)*sqrt{c^2*f*g})*(sqrt{c^2*f*g} + I*c*g*x))/((c^2*f + g)*(sqrt{c^2*f*g} - I*c*g*x))] - PolyLog[2, ((c^2*f - g + (2*I)*sqrt{c^2*f*g})*(sqrt{c^2*f*g} + I*c*g*x))/((c^2*f + g)*(sqrt{c^2*f*g} - I*c*g*x))])))/sqrt{c^2*f*g})/2$$

**fricas [F]** time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(gx^2 + f)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x^2,x, algorithm="fricas")

[Out]  $\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)}{x^2} dx$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")`

[Out]  $\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)}{x^2} dx$

**maple** [F] time = 2.20, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(gx^2 + f))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccoth(c*x))*(d+e*ln(g*x^2+f))/x^2,x)`

[Out]  $\int \frac{(a + b \operatorname{arccoth}(cx))(d + e \ln(gx^2 + f))}{x^2} dx$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( c(\log(c^2x^2 - 1) - \log(x^2)) + \frac{2 \operatorname{arccoth}(cx)}{x} \right) bd + \left( \frac{2g \arctan\left(\frac{gx}{\sqrt{fg}}\right)}{\sqrt{fg}} - \frac{\log(gx^2 + f)}{x} \right) ae + \frac{1}{2} be \int \frac{\log\left(\frac{1}{cx}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccoth(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")`

[Out]  $-1/2*(c*(\log(c^2*x^2 - 1) - \log(x^2)) + 2*\operatorname{arccoth}(c*x)/x)*b*d + (2*g*\arctan(g*x/\sqrt{f*g})/\sqrt{f*g} - \log(g*x^2 + f)/x)*a*e + 1/2*b*e*\int(\log(1/(c*x) + 1) - \log(-1/(c*x) + 1))*\log(g*x^2 + f)/x^2, x) - a*d/x$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(cx))(d + e \ln(gx^2 + f))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^2,x)`

```
[Out] int(((a + b*acoth(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acoth(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

```
[Out] Timed out
```

$$3.282 \quad \int \frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{x^3} dx$$

Optimal. Leaf size=712

$$\frac{(a+b \coth^{-1}(cx))(d+e \log(f+gx^2))}{2x^2} - \frac{aeg \log(f+gx^2)}{2f} + \frac{aeg \log(x)}{f} + \frac{1}{2}bc^2 \tanh^{-1}(cx)(d+e \log(f+gx^2))$$

[Out] a\*e\*g\*ln(x)/f+b\*e\*g\*arccoth(c\*x)\*ln(2/(c\*x+1))/f+b\*c^2\*e\*arctanh(c\*x)\*ln(2/(c\*x+1))-1/2\*a\*e\*g\*ln(g\*x^2+f)/f-1/2\*b\*c\*(d+e\*ln(g\*x^2+f))/x-1/2\*(a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f))/x^2+1/2\*b\*c^2\*arctanh(c\*x)\*(d+e\*ln(g\*x^2+f))-1/2\*b\*e\*g\*arccoth(c\*x)\*ln(2\*c\*((-f)^(1/2)-x\*g^(1/2))/(c\*x+1)/(c\*(-f)^(1/2)-g^(1/2)))/f-1/2\*b\*c^2\*e\*arctanh(c\*x)\*ln(2\*c\*((-f)^(1/2)-x\*g^(1/2))/(c\*x+1)/(c\*(-f)^(1/2)-g^(1/2)))-1/2\*b\*e\*g\*arccoth(c\*x)\*ln(2\*c\*((-f)^(1/2)+x\*g^(1/2))/(c\*x+1)/(c\*(-f)^(1/2)+g^(1/2)))/f-1/2\*b\*c^2\*e\*arctanh(c\*x)\*ln(2\*c\*((-f)^(1/2)+x\*g^(1/2))/(c\*x+1)/(c\*(-f)^(1/2)+g^(1/2)))+1/2\*b\*e\*g\*polylog(2,-1/c/x)/f-1/2\*b\*e\*g\*polylog(2,1/c/x)/f-1/2\*b\*c^2\*e\*polylog(2,1-2/(c\*x+1))-1/2\*b\*e\*g\*polylog(2,1-2/(c\*x+1))/f+1/4\*b\*c^2\*e\*polylog(2,1-2\*c\*((-f)^(1/2)-x\*g^(1/2))/(c\*x+1)/(c\*(-f)^(1/2)-g^(1/2)))/f+1/4\*b\*c^2\*e\*polylog(2,1-2\*c\*((-f)^(1/2)-x\*g^(1/2))/(c\*x+1)/(c\*(-f)^(1/2)-g^(1/2)))/f+1/4\*b\*c^2\*e\*polylog(2,1-2\*c\*((-f)^(1/2)+x\*g^(1/2))/(c\*x+1)/(c\*(-f)^(1/2)+g^(1/2)))/f+1/4\*b\*e\*g\*polylog(2,1-2\*c\*((-f)^(1/2)+x\*g^(1/2))/(c\*x+1)/(c\*(-f)^(1/2)+g^(1/2)))/f+b\*c\*e\*arctan(x\*g^(1/2)/f^(1/2))\*g^(1/2)/f^(1/2)

**Rubi [A]** time = 1.11, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 17, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$ , Rules used = {5917, 325, 206, 6086, 6725, 801, 635, 205, 260, 5993, 5913, 5921, 2402, 2315, 2447, 5992, 5920}

$$\frac{1}{4}bc^2e \text{PolyLog} \left( 2, 1 - \frac{2c(\sqrt{-f} - \sqrt{g}x)}{(cx+1)(c\sqrt{-f} - \sqrt{g})} \right) + \frac{1}{4}bc^2e \text{PolyLog} \left( 2, 1 - \frac{2c(\sqrt{-f} + \sqrt{g}x)}{(cx+1)(c\sqrt{-f} + \sqrt{g})} \right) - \frac{1}{2}bc^2e \text{PolyLog}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x^3,x]

[Out] (b\*c\*e\*sqrt[g]\*ArcTan[(sqrt[g]\*x)/sqrt[f]])/sqrt[f] + (a\*e\*g\*Log[x])/f + (b\*e\*g\*ArcCoth[c\*x]\*Log[2/(1 + c\*x)])/f + b\*c^2\*e\*ArcTanh[c\*x]\*Log[2/(1 + c\*x)] - (b\*e\*g\*ArcCoth[c\*x]\*Log[(2\*c\*(sqrt[-f] - sqrt[g]\*x))/((c\*sqrt[-f] - sqrt[g])\*(1 + c\*x))])/(2\*f) - (b\*c^2\*e\*ArcTanh[c\*x]\*Log[(2\*c\*(sqrt[-f] - sqrt[g]\*x))/((c\*sqrt[-f] - sqrt[g])\*(1 + c\*x))])/2 - (b\*e\*g\*ArcCoth[c\*x]\*Log[(2\*c\*(sqrt[-f] + sqrt[g]\*x))/((c\*sqrt[-f] + sqrt[g])\*(1 + c\*x))])/(2\*f) - (b\*

$$c^2 e \operatorname{ArcTanh}[c x] \operatorname{Log}[(2 c (\sqrt{-f} + \sqrt{g} x)) / ((c \sqrt{-f} + \sqrt{g}) (1 + c x))] / 2 - (a e g \operatorname{Log}[f + g x^2]) / (2 f) - (b c (d + e \operatorname{Log}[f + g x^2])) / (2 x) - ((a + b \operatorname{ArcCoth}[c x]) (d + e \operatorname{Log}[f + g x^2])) / (2 x^2) + (b c^2 \operatorname{ArcTanh}[c x] (d + e \operatorname{Log}[f + g x^2])) / 2 + (b e g \operatorname{PolyLog}[2, -(1 / (c x))]) / (2 f) - (b e g \operatorname{PolyLog}[2, 1 / (c x)]) / (2 f) - (b c^2 e \operatorname{PolyLog}[2, 1 - 2 / (1 + c x)]) / 2 - (b e g \operatorname{PolyLog}[2, 1 - 2 / (1 + c x)]) / (2 f) + (b c^2 e \operatorname{PolyLog}[2, 1 - (2 c (\sqrt{-f} - \sqrt{g} x)) / ((c \sqrt{-f} - \sqrt{g}) (1 + c x))]) / 4 + (b e g \operatorname{PolyLog}[2, 1 - (2 c (\sqrt{-f} - \sqrt{g} x)) / ((c \sqrt{-f} - \sqrt{g}) (1 + c x))]) / (4 f) + (b c^2 e \operatorname{PolyLog}[2, 1 - (2 c (\sqrt{-f} + \sqrt{g} x)) / ((c \sqrt{-f} + \sqrt{g}) (1 + c x))]) / 4 + (b e g \operatorname{PolyLog}[2, 1 - (2 c (\sqrt{-f} + \sqrt{g} x)) / ((c \sqrt{-f} + \sqrt{g}) (1 + c x))]) / (4 f)$$

### Rule 205

$$\operatorname{Int}[(a + (b \cdot) (x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2] \operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{PosQ}[a/b]$$

### Rule 206

$$\operatorname{Int}[(a + (b \cdot) (x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1 \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2] x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ \|\ \operatorname{LtQ}[b, 0])$$

### Rule 260

$$\operatorname{Int}[(x)^{(m \cdot)} / ((a + (b \cdot) (x)^n)), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b x^n, x]] / (b n), x] /; \operatorname{FreeQ}\{a, b, m, n\}, x \ \&\& \operatorname{EqQ}[m, n - 1]$$

### Rule 325

$$\operatorname{Int}[(c \cdot) (x)^{(m \cdot)} ((a + (b \cdot) (x)^n)^{(p \cdot)}), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(c x)^{(m+1)} (a + b x^n)^{(p+1)} / (a c (m+1)), x] - \operatorname{Dist}[(b (m+n(p+1)+1)) / (a c^n (m+1)), \operatorname{Int}[(c x)^{(m+n)} (a + b x^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{LtQ}[m, -1] \ \&\& \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$$

### Rule 635

$$\operatorname{Int}[(d + (e \cdot) (x)) / ((a + (c \cdot) (x)^2), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[d, \operatorname{Int}[1 / (a + c x^2), x], x] + \operatorname{Dist}[e, \operatorname{Int}[x / (a + c x^2), x], x] /; \operatorname{FreeQ}\{a, c, d, e\}, x \ \&\& \operatorname{!NiceSqrtQ}[-(a c)]$$

### Rule 801

$$\operatorname{Int}[(d + (e \cdot) (x))^{(m \cdot)} ((f + (g \cdot) (x)) / ((a + (c \cdot) (x)^2)), x_{\text{Symbol}}] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e x)^m (f + g x) / (a + c x^2), x],$$

$x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x\} \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{IntegerQ}[m]$

### Rule 2315

$\text{Int}[\text{Log}[(c\_)*(x\_)]/((d\_)+(e\_)*(x\_)), x\_Symbol] := -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x\} \&\& \text{EqQ}[e + c*d, 0]$

### Rule 2402

$\text{Int}[\text{Log}[(c\_)/((d\_)+(e\_)*(x_))]/((f\_)+(g\_)*(x_)^2), x\_Symbol] := -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x\} \&\& \text{EqQ}[c, 2*d] \&\& \text{EqQ}[e^2*f + d^2*g, 0]$

### Rule 2447

$\text{Int}[\text{Log}[u_]*(Pq_)^{(m_)}, x\_Symbol] := \text{With}[\{C = \text{FullSimplify}[(Pq^m*(1 - u))/D[u, x]]\}, \text{Simp}[C*\text{PolyLog}[2, 1 - u], x] /; \text{FreeQ}[C, x] /; \text{IntegerQ}[m] \&\& \text{PolyQ}[Pq, x] \&\& \text{RationalFunctionQ}[u, x] \&\& \text{LeQ}[\text{RationalFunctionExponents}[u, x][[2]], \text{Expon}[Pq, x]]]$

### Rule 5913

$\text{Int}[(a\_ + \text{ArcCoth}[(c\_)*(x_)]*(b\_))/(x_), x\_Symbol] := \text{Simp}[a*\text{Log}[x], x] + (\text{Simp}[(b*\text{PolyLog}[2, -(c*x)^{-1}])/2, x] - \text{Simp}[(b*\text{PolyLog}[2, 1/(c*x)])/2, x]) /; \text{FreeQ}\{a, b, c\}, x\}$

### Rule 5917

$\text{Int}[(a\_ + \text{ArcCoth}[(c\_)*(x_)]*(b\_))^{(p\_)*((d\_)*(x_))^{(m_)}, x\_Symbol] := \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*\text{ArcCoth}[c*x])^{(p-1)}/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] \|\ \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

### Rule 5920

$\text{Int}[(a\_ + \text{ArcTanh}[(c\_)*(x_)]*(b\_))/((d\_)+(e\_)*(x_)), x\_Symbol] := -\text{Simp}[(a + b*\text{ArcTanh}[c*x])*Log[2/(1 + c*x)]/e, x] + (\text{Dist}[(b*c)/e, \text{Int}[\text{Log}[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - \text{Dist}[(b*c)/e, \text{Int}[\text{Log}[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + \text{Simp}[(a + b*\text{ArcTanh}[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x]) /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[c^2*d^2 - e^2, 0]$

### Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -
Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

### Rule 5993

```
Int[(((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcCoth[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

### Rule 6086

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)*(x_)^(m_.), x_Symbol] := With[{u = IntHide[x^m*(a + b*ArcCoth[c*x]),
x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

### Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{x^3} dx &= -\frac{bc(d + e \log(f + gx^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{2x^2} \\
&= -\frac{bc(d + e \log(f + gx^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{2x^2} \\
&= -\frac{bc(d + e \log(f + gx^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{2x^2} \\
&= -\frac{bc(d + e \log(f + gx^2))}{2x} - \frac{(a + b \coth^{-1}(cx))(d + e \log(f + gx^2))}{2x^2} \\
&= bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) - \frac{1}{2} bc^2 e \tanh^{-1}(cx) \log\left(\frac{2c(\sqrt{f+gx^2})}{c\sqrt{-f}}\right) \\
&= \frac{aeg \log(x)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) - \frac{1}{2} bc^2 e \tanh^{-1}(cx) \log\left(\frac{2c(\sqrt{f+gx^2})}{c\sqrt{-f}}\right) \\
&= \frac{aeg \log(x)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) - \frac{1}{2} bc^2 e \tanh^{-1}(cx) \log\left(\frac{2c(\sqrt{f+gx^2})}{c\sqrt{-f}}\right) \\
&= \frac{bce\sqrt{g} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg \log(x)}{f} + bc^2 e \tanh^{-1}(cx) \log\left(\frac{2}{1+cx}\right) \\
&= \frac{bce\sqrt{g} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg \log(x)}{f} + \frac{beg \coth^{-1}(cx) \log\left(\frac{2}{1+cx}\right)}{f} \\
&= \frac{bce\sqrt{g} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg \log(x)}{f} + \frac{beg \coth^{-1}(cx) \log\left(\frac{2}{1+cx}\right)}{f} \\
&= \frac{bce\sqrt{g} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} + \frac{aeg \log(x)}{f} + \frac{beg \coth^{-1}(cx) \log\left(\frac{2}{1+cx}\right)}{f}
\end{aligned}$$

**Mathematica [C]** time = 5.90, size = 1318, normalized size = 1.85

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((a + b\*ArcCoth[c\*x])\*(d + e\*Log[f + g\*x^2]))/x^3,x]

[Out] 
$$-1/4*(2*a*d*f - 4*b*c*e*\text{Sqrt}[f]*\text{Sqrt}[g]*x^2*\text{ArcTan}[(\text{Sqrt}[g]*x)/\text{Sqrt}[f]] - 4*a*e*g*x^2*\text{Log}[x] + 2*a*e*g*x^2*\text{Log}[f + g*x^2] + 2*e*f*(a + b*c*x + (b - b*c^2*x^2)*\text{ArcCoth}[c*x])*\text{Log}[f + g*x^2] + b*c^2*e*f*x^2*(-4*\text{ArcCoth}[c*x]^2 - 4*\text{ArcCoth}[c*x]*\text{Log}[1 - E^{(-2*\text{ArcCoth}[c*x])}] + 2*\text{ArcCoth}[c*x]*\text{Log}[1 + (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(-c^2*f) - 2*c*\text{Sqrt}[-f]*\text{Sqrt}[g] + g)] + 2*\text{ArcCoth}[c*x]*\text{Log}[1 + (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(-c^2*f) + 2*c*\text{Sqrt}[-f]*\text{Sqrt}[g] + g)] + 2*\text{PolyLog}[2, E^{(-2*\text{ArcCoth}[c*x])}] + \text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(c^2*f - 2*c*\text{Sqrt}[-f]*\text{Sqrt}[g] - g)] + \text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(c^2*f + 2*c*\text{Sqrt}[-f]*\text{Sqrt}[g] - g)]) - b*d*(-2*(c*f*x + g*x^2*\text{ArcCoth}[c*x]^2 + \text{ArcCoth}[c*x]*(f - c^2*f*x^2 + 2*g*x^2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[c*x])}])) - g*x^2*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[c*x])}]) + g*x^2*(2*\text{ArcCoth}[c*x]*(-\text{ArcCoth}[c*x] + \text{Log}[1 + (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(-c^2*f) - 2*c*\text{Sqrt}[-f]*\text{Sqrt}[g] + g)] + \text{Log}[1 + (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(-c^2*f) + 2*c*\text{Sqrt}[-f]*\text{Sqrt}[g] + g)]) + \text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(c^2*f - 2*c*\text{Sqrt}[-f]*\text{Sqrt}[g] - g)] + \text{PolyLog}[2, (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))/(c^2*f + 2*c*\text{Sqrt}[-f]*\text{Sqrt}[g] - g)])) + b*d*g*x^2*(2*\text{ArcCoth}[c*x]^2 - (4*I)*\text{ArcSin}[\text{Sqrt}[g/(c^2*f + g)]]*\text{ArcTanh}[(c*f)/(\text{Sqrt}[-(c^2*f*g)]*x)] - 2*\text{ArcCoth}[c*x]*(\text{ArcCoth}[c*x] + 2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[c*x])}])) + 2*(\text{ArcCoth}[c*x] - I*\text{ArcSin}[\text{Sqrt}[g/(c^2*f + g)]])*\text{Log}[(c^2*(-1 + E^{(2*\text{ArcCoth}[c*x])})*f + g + E^{(2*\text{ArcCoth}[c*x])}*g - 2*\text{Sqrt}[-(c^2*f*g)])/ (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] + 2*(\text{ArcCoth}[c*x] + I*\text{ArcSin}[\text{Sqrt}[g/(c^2*f + g)]])*\text{Log}[(c^2*(-1 + E^{(2*\text{ArcCoth}[c*x])})*f + g + E^{(2*\text{ArcCoth}[c*x])}*g + 2*\text{Sqrt}[-(c^2*f*g)])/ (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] + 2*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[c*x])}] - \text{PolyLog}[2, (c^2*f - g + 2*\text{Sqrt}[-(c^2*f*g)])/ (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] - \text{PolyLog}[2, -((-(c^2*f) + g + 2*\text{Sqrt}[-(c^2*f*g)])/ (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g)))] + b*e*g*x^2*(2*\text{ArcCoth}[c*x]^2 - (4*I)*\text{ArcSin}[\text{Sqrt}[g/(c^2*f + g)]]*\text{ArcTanh}[(c*f)/(\text{Sqrt}[-(c^2*f*g)]*x)] - 2*\text{ArcCoth}[c*x]*(\text{ArcCoth}[c*x] + 2*\text{Log}[1 + E^{(-2*\text{ArcCoth}[c*x])}])) + 2*(\text{ArcCoth}[c*x] - I*\text{ArcSin}[\text{Sqrt}[g/(c^2*f + g)]])*\text{Log}[(c^2*(-1 + E^{(2*\text{ArcCoth}[c*x])})*f + g + E^{(2*\text{ArcCoth}[c*x])}*g - 2*\text{Sqrt}[-(c^2*f*g)])/ (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] + 2*(\text{ArcCoth}[c*x] + I*\text{ArcSin}[\text{Sqrt}[g/(c^2*f + g)]])*\text{Log}[(c^2*(-1 + E^{(2*\text{ArcCoth}[c*x])})*f + g + E^{(2*\text{ArcCoth}[c*x])}*g + 2*\text{Sqrt}[-(c^2*f*g)])/ (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] + 2*\text{PolyLog}[2, -E^{(-2*\text{ArcCoth}[c*x])}] - \text{PolyLog}[2, (c^2*f - g + 2*\text{Sqrt}[-(c^2*f*g)])/ (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g))] - \text{PolyLog}[2, -((-(c^2*f) + g + 2*\text{Sqrt}[-(c^2*f*g)])/ (E^{(2*\text{ArcCoth}[c*x])}*(c^2*f + g)))])))/(f*x^2)$$

**fricas** [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bd \operatorname{arccoth}(cx) + ad + (be \operatorname{arccoth}(cx) + ae) \log(gx^2 + f)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x^3,x, algorithm="fricas")  
 [Out] integral((b\*d\*arccoth(c\*x) + a\*d + (b\*e\*arccoth(c\*x) + a\*e)\*log(g\*x^2 + f))  
 /x^3, x)  
**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arccoth}(cx) + a)(e \log(gx^2 + f) + d)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x^3,x, algorithm="giac")  
 [Out] integrate((b\*arccoth(c\*x) + a)\*(e\*log(g\*x^2 + f) + d)/x^3, x)  
**maple** [A] time = 4.68, size = 936, normalized size = 1.31

$$\frac{db c^2 \ln(cx + 1)}{4} - \frac{db \ln(cx + 1)}{4x^2} + \frac{aeg \ln(x)}{f} - \frac{aeg \ln(gx^2 + f)}{2f} - \frac{da}{2x^2} - \frac{bcd}{2x} + \left( -\frac{be \ln(cx + 1)}{4x^2} + \frac{e(b c^2 \ln(cx + 1) x^2)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccoth(c\*x))\*(d+e\*ln(g\*x^2+f))/x^3,x)  
 [Out] 1/4\*d\*b\*c^2\*ln(c\*x+1)-1/4\*d\*b\*ln(c\*x+1)/x^2+a\*e\*g\*ln(x)/f-1/2\*a\*e\*g\*ln(g\*x^2+f)/f-1/2/x^2\*d\*a-1/2\*b\*c\*d/x+(-1/4\*b\*e/x^2\*ln(c\*x+1)+1/4\*e\*(b\*c^2\*ln(c\*x+1)\*x^2-b\*c^2\*ln(c\*x-1)\*x^2-2\*x\*b\*c+b\*ln(c\*x-1)-2\*a)/x^2)\*ln(g\*x^2+f)-1/2\*g\*b\*e\*dilog(c\*x+1)/f-1/4\*b\*e\*dilog((c\*(-f\*g)^(1/2)-(c\*x+1)\*g+g)/(c\*(-f\*g)^(1/2)+g))\*c^2-1/4\*b\*e\*dilog((c\*(-f\*g)^(1/2)+(c\*x+1)\*g-g)/(c\*(-f\*g)^(1/2)-g))\*c^2-1/4\*b\*e\*ln(c\*x+1)\*ln((c\*(-f\*g)^(1/2)-(c\*x+1)\*g+g)/(c\*(-f\*g)^(1/2)+g))\*c^2-1/4\*b\*e\*ln(c\*x+1)\*ln((c\*(-f\*g)^(1/2)+(c\*x+1)\*g-g)/(c\*(-f\*g)^(1/2)-g))\*c^2-1/4\*g\*b\*e/f\*dilog((c\*(-f\*g)^(1/2)-(c\*x+1)\*g+g)/(c\*(-f\*g)^(1/2)+g))-1/4\*g\*b\*e/f\*dilog((c\*(-f\*g)^(1/2)+(c\*x+1)\*g-g)/(c\*(-f\*g)^(1/2)-g))-1/4\*g\*b\*e/f\*ln(c\*x+1)\*ln((c\*(-f\*g)^(1/2)-(c\*x+1)\*g+g)/(c\*(-f\*g)^(1/2)+g))-1/4\*g\*b\*e/f\*ln(c\*x+1)\*ln((c\*(-f\*g)^(1/2)+(c\*x+1)\*g-g)/(c\*(-f\*g)^(1/2)-g))+g\*e\*b\*c/(f\*g)^(1/2)\*arctan(x\*g/(f\*g)^(1/2))-1/4\*d\*b\*c^2\*ln(c\*x-1)+1/4\*d\*b\*ln(c\*x-1)/x^2+1/4\*b\*e\*ln(c\*x-1)\*ln((c\*(-f\*g)^(1/2)-(c\*x-1)\*g-g)/(c\*(-f\*g)^(1/2)-g))\*c^2+1/4\*b\*e\*ln(c\*x-1)\*ln((c\*(-f\*g)^(1/2)+(c\*x-1)\*g+g)/(c\*(-f\*g)^(1/2)+g))\*c^2+1/4\*g\*b\*e/f\*dilog((c\*(-f\*g)^(1/2)-(c\*x-1)\*g-g)/(c\*(-f\*g)^(1/2)-g))+1/4\*b\*e\*dilog((c\*(-f\*g)^(1/2)-(c\*x-1)\*g-g)/(c\*(-f\*g)^(1/2)-g))\*c^2+1/4\*b\*e\*dilog((c\*(-f\*g)^(1/2)+(c\*x-1)\*g+g)/(c\*(-f\*g)^(1/2)+g))\*c^2+1/4\*g\*b\*e/f\*ln(c\*x-1)\*ln((c\*(-f\*g)^(1/2)-(c\*x-1)\*g-g)/(c\*(-f\*g)^(1/2)-g))+1/4\*g\*b\*e/f\*ln(c\*x-1)\*ln((c\*(-f\*g)^(1/2)+(c\*x-1)\*g+g)/(c\*(-f\*g)^(1/2)+g))-1/2\*g\*b\*e/f\*ln(c\*x-1)\*ln(c\*x)+1/4\*g\*b\*e/f\*dilog((c\*(-f\*g)^(1/2)+(c\*x-1)\*g+g)/(c\*(-f\*g)^(1/2)+g))-1/2\*g\*b\*e/f\*dilog(c\*x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} \left( \left( c \log(cx+1) - c \log(cx-1) - \frac{2}{x} \right) c - \frac{2 \operatorname{arccoth}(cx)}{x^2} \right) b d - \frac{1}{2} \left( g \left( \frac{\log(gx^2+f)}{f} - \frac{\log(x^2)}{f} \right) + \frac{\log(gx^2+f)}{x^2} \right) a e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccoth(c\*x))\*(d+e\*log(g\*x^2+f))/x^3,x, algorithm="maxima")

[Out] 1/4\*((c\*log(c\*x + 1) - c\*log(c\*x - 1) - 2/x)\*c - 2\*arccoth(c\*x)/x^2)\*b\*d - 1/2\*(g\*(log(g\*x^2 + f)/f - log(x^2)/f) + log(g\*x^2 + f)/x^2)\*a\*e - 1/4\*(2\*c^2\*g\*integrate(x^2\*log(c\*x + 1)/(g\*x^3 + f\*x), x) - 2\*c^2\*g\*integrate(x^2\*log(c\*x - 1)/(g\*x^3 + f\*x), x) + 2\*I\*c\*g\*(log(I\*g\*x/sqrt(f\*g) + 1) - log(-I\*g\*x/sqrt(f\*g) + 1))/sqrt(f\*g) - 2\*g\*integrate(log(c\*x + 1)/(g\*x^3 + f\*x), x) + 2\*g\*integrate(log(c\*x - 1)/(g\*x^3 + f\*x), x) + (2\*c\*x - (c^2\*x^2 - 1)\*log(c\*x + 1) + (c^2\*x^2 - 1)\*log(c\*x - 1))\*log(g\*x^2 + f)/x^2)\*b\*e - 1/2\*a\*d/x^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acoth}(cx)) (d + e \ln(gx^2 + f))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*acoth(c\*x))\*(d + e\*log(f + g\*x^2)))/x^3,x)

[Out] int(((a + b\*acoth(c\*x))\*(d + e\*log(f + g\*x^2)))/x^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acoth(c\*x))\*(d+e\*ln(g\*x\*\*2+f))/x\*\*3,x)

[Out] Timed out

### 3.283 $\int \coth^{-1}(e^x) dx$

Optimal. Leaf size=25

$$\frac{\text{Li}_2(-e^{-x})}{2} - \frac{\text{Li}_2(e^{-x})}{2}$$

[Out] 1/2\*polylog(2,-1/exp(x))-1/2\*polylog(2,exp(-x))

**Rubi [A]** time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {2282, 5913}

$$\frac{1}{2}\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}\text{PolyLog}(2, e^{-x})$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[E^x], x]

[Out] PolyLog[2, -E^(-x)]/2 - PolyLog[2, E^(-x)]/2

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])]/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])]/2, x) /; FreeQ[{a, b, c}, x]
```

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(e^x) dx &= \text{Subst} \left( \int \frac{\coth^{-1}(x)}{x} dx, x, e^x \right) \\ &= \frac{\text{Li}_2(-e^{-x})}{2} - \frac{\text{Li}_2(e^{-x})}{2} \end{aligned}$$

**Mathematica** [B] time = 0.04, size = 51, normalized size = 2.04

$$-\frac{\operatorname{Li}_2(-e^x)}{2} + \frac{\operatorname{Li}_2(e^x)}{2} + \frac{1}{2}x \log(1 - e^x) - \frac{1}{2}x \log(e^x + 1) + x \operatorname{coth}^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[E^x], x]

[Out] x\*ArcCoth[E^x] + (x\*Log[1 - E^x])/2 - (x\*Log[1 + E^x])/2 - PolyLog[2, -E^x]/2 + PolyLog[2, E^x]/2

**fricas** [B] time = 0.58, size = 64, normalized size = 2.56

$$\frac{1}{2}x \log\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{2}x \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{2}x \log(-\cosh(x) - \sinh(x) + 1) + \frac{1}{2}\operatorname{Li}_2(\cosh(x) + \sinh(x) + 1) - \frac{1}{2}\operatorname{Li}_2(-\cosh(x) - \sinh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(exp(x)), x, algorithm="fricas")

[Out] 1/2\*x\*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2\*x\*log(cosh(x) + sinh(x) + 1) + 1/2\*x\*log(-cosh(x) - sinh(x) + 1) + 1/2\*dilog(cosh(x) + sinh(x)) - 1/2\*dilog(-cosh(x) - sinh(x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(exp(x)), x, algorithm="giac")

[Out] integrate(arccoth(e^x), x)

**maple** [A] time = 0.10, size = 31, normalized size = 1.24

$$\ln(e^x) \operatorname{arccoth}(e^x) - \frac{\operatorname{dilog}(e^x)}{2} - \frac{\operatorname{dilog}(e^x + 1)}{2} - \frac{\ln(e^x) \ln(e^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(exp(x)), x)

[Out] ln(exp(x))\*arccoth(exp(x))-1/2\*dilog(exp(x))-1/2\*dilog(exp(x)+1)-1/2\*ln(exp(x))\*ln(exp(x)+1)

**maxima** [B] time = 0.32, size = 58, normalized size = 2.32

$$-\frac{1}{2}x(\log(e^x + 1) - \log(e^x - 1)) + x \operatorname{arccoth}(e^x) + \frac{1}{2} \log(-e^x) \log(e^x + 1) - \frac{1}{2}x \log(e^x - 1) + \frac{1}{2} \operatorname{Li}_2(e^x + 1) - \frac{1}{2} \operatorname{Li}_2(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(exp(x)), x, algorithm="maxima")`

[Out] `-1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arccoth(e^x) + 1/2*log(-e^x)*log(e^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(exp(x)), x)`

[Out] `int(acoth(exp(x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(exp(x)), x)`

[Out] `Integral(acoth(exp(x)), x)`

### 3.284 $\int x \coth^{-1}(e^x) dx$

Optimal. Leaf size=51

$$\frac{1}{2}x\text{Li}_2(-e^{-x}) - \frac{1}{2}x\text{Li}_2(e^{-x}) + \frac{\text{Li}_3(-e^{-x})}{2} - \frac{\text{Li}_3(e^{-x})}{2}$$

[Out] 1/2\*x\*polylog(2,-1/exp(x))-1/2\*x\*polylog(2,exp(-x))+1/2\*polylog(3,-1/exp(x))-1/2\*polylog(3,exp(-x))

**Rubi [A]** time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6214, 2531, 2282, 6589}

$$\frac{1}{2}x\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}x\text{PolyLog}(2, e^{-x}) + \frac{1}{2}\text{PolyLog}(3, -e^{-x}) - \frac{1}{2}\text{PolyLog}(3, e^{-x})$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[E^x], x]

[Out] (x\*PolyLog[2, -E^(-x)])/2 - (x\*PolyLog[2, E^(-x)])/2 + PolyLog[3, -E^(-x)]/2 - PolyLog[3, E^(-x)]/2

#### Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :=> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

#### Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :=> Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m, 0]
```



Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(e^x) dx &= -\left(\frac{1}{2} \int x \log(1 - e^{-x}) dx\right) + \frac{1}{2} \int x \log(1 + e^{-x}) dx \\
 &= \frac{1}{2} x \text{Li}_2(-e^{-x}) - \frac{1}{2} x \text{Li}_2(e^{-x}) - \frac{1}{2} \int \text{Li}_2(-e^{-x}) dx + \frac{1}{2} \int \text{Li}_2(e^{-x}) dx \\
 &= \frac{1}{2} x \text{Li}_2(-e^{-x}) - \frac{1}{2} x \text{Li}_2(e^{-x}) + \frac{1}{2} \text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x} dx, x, e^{-x}\right) - \frac{1}{2} \text{Subst}\left(\int \frac{\text{Li}_2(x)}{x} dx, x, e^{-x}\right) \\
 &= \frac{1}{2} x \text{Li}_2(-e^{-x}) - \frac{1}{2} x \text{Li}_2(e^{-x}) + \frac{\text{Li}_3(-e^{-x})}{2} - \frac{\text{Li}_3(e^{-x})}{2}
 \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 71, normalized size = 1.39

$$\frac{1}{4} \left( -2x \text{Li}_2(-e^x) + 2x \text{Li}_2(e^x) + 2\text{Li}_3(-e^x) - 2\text{Li}_3(e^x) + x^2 \log(1 - e^x) - x^2 \log(e^x + 1) + 2x^2 \coth^{-1}(e^x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[E^x], x]

[Out] (2\*x^2\*ArcCoth[E^x] + x^2\*Log[1 - E^x] - x^2\*Log[1 + E^x] - 2\*x\*PolyLog[2, -E^x] + 2\*x\*PolyLog[2, E^x] + 2\*PolyLog[3, -E^x] - 2\*PolyLog[3, E^x])/4

**fricas** [C] time = 0.58, size = 94, normalized size = 1.84

$$\frac{1}{4} x^2 \log\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{4} x^2 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{4} x^2 \log(-\cosh(x) - \sinh(x) + 1) + \frac{1}{2} x \text{Li}_2\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{2} x \text{Li}_2(\cosh(x) + \sinh(x) + 1) + \frac{1}{2} x \text{Li}_2(-\cosh(x) - \sinh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(exp(x)), x, algorithm="fricas")

[Out] 1/4\*x^2\*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4\*x^2\*log(cosh(x) + sinh(x) + 1) + 1/4\*x^2\*log(-cosh(x) - sinh(x) + 1) + 1/2\*x\*dilog(cosh(x) + sinh(x)) - 1/2\*x\*dilog(-cosh(x) - sinh(x)) - 1/2\*polylog(3, cosh(x) + sinh(x)) + 1/2\*polylog(3, -cosh(x) - sinh(x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(exp(x)),x, algorithm="giac")

[Out] integrate(x\*arccoth(e^x), x)

**maple** [A] time = 0.06, size = 62, normalized size = 1.22

$$\frac{x^2 \operatorname{arccoth}(e^x)}{2} - \frac{x^2 \ln(e^x + 1)}{4} - \frac{x \operatorname{polylog}(2, -e^x)}{2} + \frac{\operatorname{polylog}(3, -e^x)}{2} + \frac{x^2 \ln(1 - e^x)}{4} + \frac{x \operatorname{polylog}(2, e^x)}{2} - \frac{\operatorname{polylog}(3, e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(exp(x)),x)

[Out] 1/2\*x^2\*arccoth(exp(x))-1/4\*x^2\*ln(exp(x)+1)-1/2\*x\*polylog(2,-exp(x))+1/2\*polylog(3,-exp(x))+1/4\*x^2\*ln(1-exp(x))+1/2\*x\*polylog(2,exp(x))-1/2\*polylog(3,exp(x))

**maxima** [A] time = 0.32, size = 59, normalized size = 1.16

$$\frac{1}{2} x^2 \operatorname{arccoth}(e^x) - \frac{1}{4} x^2 \log(e^x + 1) + \frac{1}{4} x^2 \log(-e^x + 1) - \frac{1}{2} x \operatorname{Li}_2(-e^x) + \frac{1}{2} x \operatorname{Li}_2(e^x) + \frac{1}{2} \operatorname{Li}_3(-e^x) - \frac{1}{2} \operatorname{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(exp(x)),x, algorithm="maxima")

[Out] 1/2\*x^2\*arccoth(e^x) - 1/4\*x^2\*log(e^x + 1) + 1/4\*x^2\*log(-e^x + 1) - 1/2\*x\*dilog(-e^x) + 1/2\*x\*dilog(e^x) + 1/2\*polylog(3, -e^x) - 1/2\*polylog(3, e^x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(exp(x)),x)

[Out] int(x\*acoth(exp(x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(exp(x)),x)
```

```
[Out] Integral(x*acoth(exp(x)), x)
```

### 3.285 $\int x^2 \coth^{-1}(e^x) dx$

**Optimal.** Leaf size=70

$$\frac{1}{2}x^2\text{Li}_2(-e^{-x}) - \frac{1}{2}x^2\text{Li}_2(e^{-x}) + x\text{Li}_3(-e^{-x}) - x\text{Li}_3(e^{-x}) + \text{Li}_4(-e^{-x}) - \text{Li}_4(e^{-x})$$

[Out]  $\frac{1}{2}x^2\text{polylog}(2, -1/\exp(x)) - \frac{1}{2}x^2\text{polylog}(2, \exp(-x)) + x\text{polylog}(3, -1/\exp(x)) - x\text{polylog}(3, \exp(-x)) + \text{polylog}(4, -1/\exp(x)) - \text{polylog}(4, \exp(-x))$

**Rubi [A]** time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {6214, 2531, 6609, 2282, 6589}

$$\frac{1}{2}x^2\text{PolyLog}(2, -e^{-x}) - \frac{1}{2}x^2\text{PolyLog}(2, e^{-x}) + x\text{PolyLog}(3, -e^{-x}) - x\text{PolyLog}(3, e^{-x}) + \text{PolyLog}(4, -e^{-x}) - \text{PolyLog}(4, e^{-x})$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[E^x], x]

[Out]  $(x^2*\text{PolyLog}[2, -E^{(-x)}])/2 - (x^2*\text{PolyLog}[2, E^{(-x)}])/2 + x*\text{PolyLog}[3, -E^{(-x)}] - x*\text{PolyLog}[3, E^{(-x)}] + \text{PolyLog}[4, -E^{(-x)}] - \text{PolyLog}[4, E^{(-x)}]$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

#### Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] := Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m, 0]
```

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int x^2 \coth^{-1}(e^x) dx &= -\left(\frac{1}{2} \int x^2 \log(1 - e^{-x}) dx\right) + \frac{1}{2} \int x^2 \log(1 + e^{-x}) dx \\
 &= \frac{1}{2} x^2 \text{Li}_2(-e^{-x}) - \frac{1}{2} x^2 \text{Li}_2(e^{-x}) - \int x \text{Li}_2(-e^{-x}) dx + \int x \text{Li}_2(e^{-x}) dx \\
 &= \frac{1}{2} x^2 \text{Li}_2(-e^{-x}) - \frac{1}{2} x^2 \text{Li}_2(e^{-x}) + x \text{Li}_3(-e^{-x}) - x \text{Li}_3(e^{-x}) - \int \text{Li}_3(-e^{-x}) dx + \int \text{Li}_3(e^{-x}) dx \\
 &= \frac{1}{2} x^2 \text{Li}_2(-e^{-x}) - \frac{1}{2} x^2 \text{Li}_2(e^{-x}) + x \text{Li}_3(-e^{-x}) - x \text{Li}_3(e^{-x}) + \text{Subst}\left(\int \frac{\text{Li}_3(-x)}{x} dx, x, e^{-x}\right) \\
 &= \frac{1}{2} x^2 \text{Li}_2(-e^{-x}) - \frac{1}{2} x^2 \text{Li}_2(e^{-x}) + x \text{Li}_3(-e^{-x}) - x \text{Li}_3(e^{-x}) + \text{Li}_4(-e^{-x}) - \text{Li}_4(e^{-x})
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 93, normalized size = 1.33

$$\frac{1}{6} \left( -3x^2 \text{Li}_2(-e^x) + 3x^2 \text{Li}_2(e^x) + 6x \text{Li}_3(-e^x) - 6x \text{Li}_3(e^x) - 6\text{Li}_4(-e^x) + 6\text{Li}_4(e^x) + x^3 \log(1 - e^x) - x^3 \log(e^x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[E^x], x]

[Out] (2\*x^3\*ArcCoth[E^x] + x^3\*Log[1 - E^x] - x^3\*Log[1 + E^x] - 3\*x^2\*PolyLog[2, -E^x] + 3\*x^2\*PolyLog[2, E^x] + 6\*x\*PolyLog[3, -E^x] - 6\*x\*PolyLog[3, E^x] - 6\*PolyLog[4, -E^x] + 6\*PolyLog[4, E^x])/6

**fricas [C]** time = 0.69, size = 119, normalized size = 1.70

$$\frac{1}{6} x^3 \log\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right) - \frac{1}{6} x^3 \log(\cosh(x) + \sinh(x) + 1) + \frac{1}{6} x^3 \log(-\cosh(x) - \sinh(x) + 1) + \frac{1}{2} x^2 \text{Li}_2\left(\frac{\cosh(x) + \sinh(x) + 1}{\cosh(x) + \sinh(x) - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(x)),x, algorithm="fricas")

[Out] 1/6\*x^3\*log((cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6\*x^3\*log(cosh(x) + sinh(x) + 1) + 1/6\*x^3\*log(-cosh(x) - sinh(x) + 1) + 1/2\*x^2\*dilog(cosh(x) + sinh(x)) - 1/2\*x^2\*dilog(-cosh(x) - sinh(x)) - x\*polylog(3, cosh(x) + sinh(x)) + x\*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x) + sinh(x)) - polylog(4, -cosh(x) - sinh(x))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(x)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(e^x), x)

**maple** [A] time = 0.06, size = 79, normalized size = 1.13

$$\frac{x^3 \operatorname{arccoth}(e^x)}{3} - \frac{x^3 \ln(e^x + 1)}{6} - \frac{x^2 \operatorname{polylog}(2, -e^x)}{2} + x \operatorname{polylog}(3, -e^x) - \operatorname{polylog}(4, -e^x) + \frac{x^3 \ln(1 - e^x)}{6} + \frac{x^2 \operatorname{polylog}(2, -e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(exp(x)),x)

[Out] 1/3\*x^3\*arccoth(exp(x))-1/6\*x^3\*ln(exp(x)+1)-1/2\*x^2\*polylog(2,-exp(x))+x\*polylog(3,-exp(x))-polylog(4,-exp(x))+1/6\*x^3\*ln(1-exp(x))+1/2\*x^2\*polylog(2,exp(x))-x\*polylog(3,exp(x))+polylog(4,exp(x))

**maxima** [A] time = 0.32, size = 76, normalized size = 1.09

$$\frac{1}{3} x^3 \operatorname{arccoth}(e^x) - \frac{1}{6} x^3 \log(e^x + 1) + \frac{1}{6} x^3 \log(-e^x + 1) - \frac{1}{2} x^2 \operatorname{Li}_2(-e^x) + \frac{1}{2} x^2 \operatorname{Li}_2(e^x) + x \operatorname{Li}_3(-e^x) - x \operatorname{Li}_3(e^x) - \operatorname{Li}_4(-e^x) + \operatorname{Li}_4(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(x)),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(e^x) - 1/6\*x^3\*log(e^x + 1) + 1/6\*x^3\*log(-e^x + 1) - 1/2\*x^2\*dilog(-e^x) + 1/2\*x^2\*dilog(e^x) + x\*polylog(3, -e^x) - x\*polylog(3, e^x) - polylog(4, -e^x) + polylog(4, e^x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acoth(exp(x)),x)
```

```
[Out] int(x^2*acoth(exp(x)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x^2 \operatorname{acoth}(e^x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acoth(exp(x)),x)
```

```
[Out] Integral(x**2*acoth(exp(x)), x)
```

### 3.286 $\int \coth^{-1}(e^{a+bx}) dx$

Optimal. Leaf size=41

$$\frac{\operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{\operatorname{Li}_2(e^{-a-bx})}{2b}$$

[Out] 1/2\*polylog(2,-exp(-b\*x-a))/b-1/2\*polylog(2,exp(-b\*x-a))/b

**Rubi [A]** time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2282, 5913}

$$\frac{\operatorname{PolyLog}(2, -e^{-a-bx})}{2b} - \frac{\operatorname{PolyLog}(2, e^{-a-bx})}{2b}$$

Antiderivative was successfully verified.

[In] Int[ArcCoth[E^(a + b\*x)],x]

[Out] PolyLog[2, -E^(-a - b\*x)]/(2\*b) - PolyLog[2, E^(-a - b\*x)]/(2\*b)

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 5913

```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x] + (Simp[(b*PolyLog[2, -(c*x)^(-1)])/2, x] - Simp[(b*PolyLog[2, 1/(c*x)])/2, x]) /; FreeQ[{a, b, c}, x]
```

#### Rubi steps

$$\begin{aligned} \int \coth^{-1}(e^{a+bx}) dx &= \frac{\operatorname{Subst}\left(\int \frac{\coth^{-1}(x)}{x} dx, x, e^{a+bx}\right)}{b} \\ &= \frac{\operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{\operatorname{Li}_2(e^{-a-bx})}{2b} \end{aligned}$$



**Mathematica [A]** time = 0.08, size = 68, normalized size = 1.66

$$\frac{-\operatorname{Li}_2(-e^{a+bx}) + \operatorname{Li}_2(e^{a+bx}) + bx(\log(1 - e^{a+bx}) - \log(e^{a+bx} + 1)) + 2 \operatorname{coth}^{-1}(e^{a+bx})}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[E^(a + b\*x)], x]

[Out] (b\*x\*(2\*ArcCoth[E^(a + b\*x)] + Log[1 - E^(a + b\*x)] - Log[1 + E^(a + b\*x)]) - PolyLog[2, -E^(a + b\*x)] + PolyLog[2, E^(a + b\*x)])/(2\*b)

**fricas [B]** time = 0.63, size = 137, normalized size = 3.34

$$\frac{bx \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - bx \log(\cosh(bx+a) + \sinh(bx+a) + 1) - a \log(\cosh(bx+a) + \sinh(bx+a))}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(exp(b\*x+a)), x, algorithm="fricas")

[Out] 1/2\*(b\*x\*log((cosh(b\*x + a) + sinh(b\*x + a) + 1)/(cosh(b\*x + a) + sinh(b\*x + a) - 1)) - b\*x\*log(cosh(b\*x + a) + sinh(b\*x + a) + 1) - a\*log(cosh(b\*x + a) + sinh(b\*x + a) - 1) + (b\*x + a)\*log(-cosh(b\*x + a) - sinh(b\*x + a) + 1) + dilog(cosh(b\*x + a) + sinh(b\*x + a)) - dilog(-cosh(b\*x + a) - sinh(b\*x + a)))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(e^{(bx+a)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(exp(b\*x+a)), x, algorithm="giac")

[Out] integrate(arccoth(e^(b\*x + a)), x)

**maple [A]** time = 0.07, size = 67, normalized size = 1.63

$$\frac{\ln(e^{bx+a}) \operatorname{arccoth}(e^{bx+a})}{b} - \frac{\operatorname{dilog}(e^{bx+a})}{2b} - \frac{\operatorname{dilog}(e^{bx+a} + 1)}{2b} - \frac{\ln(e^{bx+a}) \ln(e^{bx+a} + 1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(exp(b\*x+a)), x)

[Out]  $\frac{1}{b} \ln(\exp(bx+a)) \operatorname{arccoth}(\exp(bx+a)) - \frac{1}{2b} \operatorname{dilog}(\exp(bx+a)) - \frac{1}{2b} \operatorname{dilog}(\exp(bx+a)+1) - \frac{1}{2b} \ln(\exp(bx+a)) \ln(\exp(bx+a)+1)$

**maxima** [B] time = 0.33, size = 107, normalized size = 2.61

$$\frac{(bx+a) \operatorname{arccoth}(e^{(bx+a)})}{b} - \frac{(bx+a)(\log(e^{(bx+a)}+1) - \log(e^{(bx+a)}-1)) - \log(-e^{(bx+a)}) \log(e^{(bx+a)}+1) + (bx+a) \log(e^{(bx+a)}-1) - \operatorname{dilog}(e^{(bx+a)}+1) + \operatorname{dilog}(-e^{(bx+a)}+1))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arccoth(exp(b*x+a)),x, algorithm="maxima")`

[Out]  $(b*x + a) \operatorname{arccoth}(e^{(b*x + a)})/b - 1/2 * ((b*x + a) * (\log(e^{(b*x + a)} + 1) - \log(e^{(b*x + a)} - 1)) - \log(-e^{(b*x + a)}) * \log(e^{(b*x + a)} + 1) + (b*x + a) * \log(e^{(b*x + a)} - 1) - \operatorname{dilog}(e^{(b*x + a)} + 1) + \operatorname{dilog}(-e^{(b*x + a)} + 1))/b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{acoth}(e^{a+bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(acoth(exp(a + b*x)),x)`

[Out] `int(acoth(exp(a + b*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acoth}(e^{a+bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(acoth(exp(b*x+a)),x)`

[Out] `Integral(acoth(exp(a + b*x)), x)`

### 3.287 $\int x \coth^{-1} (e^{a+bx}) dx$

Optimal. Leaf size=83

$$\frac{\operatorname{Li}_3(-e^{-a-bx})}{2b^2} - \frac{\operatorname{Li}_3(e^{-a-bx})}{2b^2} + \frac{x\operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{x\operatorname{Li}_2(e^{-a-bx})}{2b}$$

[Out] 1/2\*x\*polylog(2,-exp(-b\*x-a))/b-1/2\*x\*polylog(2,exp(-b\*x-a))/b+1/2\*polylog(3,-exp(-b\*x-a))/b^2-1/2\*polylog(3,exp(-b\*x-a))/b^2

**Rubi [A]** time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {6214, 2531, 2282, 6589}

$$\frac{\operatorname{PolyLog}(3, -e^{-a-bx})}{2b^2} - \frac{\operatorname{PolyLog}(3, e^{-a-bx})}{2b^2} + \frac{x\operatorname{PolyLog}(2, -e^{-a-bx})}{2b} - \frac{x\operatorname{PolyLog}(2, e^{-a-bx})}{2b}$$

Antiderivative was successfully verified.

[In] Int[x\*ArcCoth[E^(a + b\*x)],x]

[Out] (x\*PolyLog[2, -E^(-a - b\*x)])/(2\*b) - (x\*PolyLog[2, E^(-a - b\*x)])/(2\*b) + PolyLog[3, -E^(-a - b\*x)]/(2\*b^2) - PolyLog[3, E^(-a - b\*x)]/(2\*b^2)

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

#### Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] := Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
```

IGtQ[m, 0]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\begin{aligned}
 \int x \coth^{-1}(e^{a+bx}) dx &= -\left(\frac{1}{2} \int x \log(1 - e^{-a-bx}) dx\right) + \frac{1}{2} \int x \log(1 + e^{-a-bx}) dx \\
 &= \frac{x\text{Li}_2(-e^{-a-bx})}{2b} - \frac{x\text{Li}_2(e^{-a-bx})}{2b} - \frac{\int \text{Li}_2(-e^{-a-bx}) dx}{2b} + \frac{\int \text{Li}_2(e^{-a-bx}) dx}{2b} \\
 &= \frac{x\text{Li}_2(-e^{-a-bx})}{2b} - \frac{x\text{Li}_2(e^{-a-bx})}{2b} + \frac{\text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x} dx, x, e^{-a-bx}\right)}{2b^2} - \frac{\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x} dx, x, e^{-a-bx}\right)}{2b^2} \\
 &= \frac{x\text{Li}_2(-e^{-a-bx})}{2b} - \frac{x\text{Li}_2(e^{-a-bx})}{2b} + \frac{\text{Li}_3(-e^{-a-bx})}{2b^2} - \frac{\text{Li}_3(e^{-a-bx})}{2b^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 113, normalized size = 1.36

$$\frac{b^2 x^2 \log(1 - e^{a+bx}) - b^2 x^2 \log(e^{a+bx} + 1) + 2b^2 x^2 \coth^{-1}(e^{a+bx}) - 2bx\text{Li}_2(-e^{a+bx}) + 2bx\text{Li}_2(e^{a+bx}) + 2\text{Li}_3(-e^{a+bx})}{4b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[E^(a + b\*x)], x]

[Out] (2\*b^2\*x^2\*ArcCoth[E^(a + b\*x)] + b^2\*x^2\*Log[1 - E^(a + b\*x)] - b^2\*x^2\*Log[1 + E^(a + b\*x)] - 2\*b\*x\*PolyLog[2, -E^(a + b\*x)] + 2\*b\*x\*PolyLog[2, E^(a + b\*x)] + 2\*PolyLog[3, -E^(a + b\*x)] - 2\*PolyLog[3, E^(a + b\*x)])/(4\*b^2)

**fricas [C]** time = 0.64, size = 198, normalized size = 2.39

$$\frac{b^2 x^2 \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^2 x^2 \log(\cosh(bx+a) + \sinh(bx+a) + 1) + 2bx\text{Li}_2(\cosh(bx+a) + \sinh(bx+a) + 1) - 2bx\text{Li}_2(\cosh(bx+a) + \sinh(bx+a) - 1)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(exp(b\*x+a)), x, algorithm="fricas")

[Out]  $\frac{1}{4}*(b^2*x^2*\log((\cosh(b*x + a) + \sinh(b*x + a) + 1)/(\cosh(b*x + a) + \sinh(b*x + a) - 1)) - b^2*x^2*\log(\cosh(b*x + a) + \sinh(b*x + a) + 1) + 2*b*x*\operatorname{dilog}(\cosh(b*x + a) + \sinh(b*x + a)) - 2*b*x*\operatorname{dilog}(-\cosh(b*x + a) - \sinh(b*x + a)) + a^2*\log(\cosh(b*x + a) + \sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*\log(-\cosh(b*x + a) - \sinh(b*x + a) + 1) - 2*\operatorname{polylog}(3, \cosh(b*x + a) + \sinh(b*x + a)) + 2*\operatorname{polylog}(3, -\cosh(b*x + a) - \sinh(b*x + a)))/b^2$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(e^{(bx+a)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(exp(b*x+a)),x, algorithm="giac")`

[Out] `integrate(x*arccoth(e^(b*x + a)), x)`

**maple** [B] time = 0.08, size = 153, normalized size = 1.84

$$\frac{x^2 \operatorname{arccoth}(e^{bx+a})}{2} - \frac{\ln(e^{bx+a} + 1)x^2}{4} + \frac{\ln(e^{bx+a} + 1)a^2}{4b^2} - \frac{x \operatorname{polylog}(2, -e^{bx+a})}{2b} + \frac{\operatorname{polylog}(3, -e^{bx+a})}{2b^2} + \frac{\ln(1 - e^{bx+a})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arccoth(exp(b*x+a)),x)`

[Out]  $\frac{1}{2}*x^2*\operatorname{arccoth}(\exp(b*x+a)) - \frac{1}{4}*b*\ln(\exp(b*x+a)+1)*x^2 + \frac{1}{4}/b^2*\ln(\exp(b*x+a)+1)*a^2 - \frac{1}{2}*x*\operatorname{polylog}(2, -\exp(b*x+a))/b + \frac{1}{2}*x*\operatorname{polylog}(3, -\exp(b*x+a))/b^2 + \frac{1}{4}*b*\ln(1-\exp(b*x+a))*x^2 - \frac{1}{4}/b^2*\ln(1-\exp(b*x+a))*a^2 + \frac{1}{2}*x*\operatorname{polylog}(2, \exp(b*x+a))/b - \frac{1}{2}*x*\operatorname{polylog}(3, \exp(b*x+a))/b^2 - \frac{1}{2}/b^2*a^2*\operatorname{arctanh}(\exp(b*x+a))$

**maxima** [A] time = 0.34, size = 108, normalized size = 1.30

$$\frac{1}{2} x^2 \operatorname{arccoth}(e^{(bx+a)}) - \frac{1}{4} b \left( \frac{b^2 x^2 \log(e^{(bx+a)} + 1) + 2 b x \operatorname{Li}_2(-e^{(bx+a)}) - 2 \operatorname{Li}_3(-e^{(bx+a)})}{b^3} - \frac{b^2 x^2 \log(-e^{(bx+a)} + 1)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arccoth(exp(b*x+a)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}*x^2*\operatorname{arccoth}(e^{(b*x + a)}) - \frac{1}{4}*b*((b^2*x^2*\log(e^{(b*x + a)} + 1) + 2*b*x*\operatorname{dilog}(-e^{(b*x + a)}) - 2*\operatorname{polylog}(3, -e^{(b*x + a)}))/b^3 - (b^2*x^2*\log(-e^{(b*x + a)} + 1) + 2*b*x*\operatorname{dilog}(e^{(b*x + a)}) - 2*\operatorname{polylog}(3, e^{(b*x + a)}))/b^3)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{acoth}(e^{a+bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acoth(exp(a + b*x)),x)
```

```
[Out] int(x*acoth(exp(a + b*x)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int x \operatorname{acoth}(e^a e^{bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acoth(exp(b*x+a)),x)
```

```
[Out] Integral(x*acoth(exp(a)*exp(b*x)), x)
```

### 3.288 $\int x^2 \coth^{-1}(e^{a+bx}) dx$

**Optimal.** Leaf size=119

$$\frac{\text{Li}_4(-e^{-a-bx})}{b^3} - \frac{\text{Li}_4(e^{-a-bx})}{b^3} + \frac{x\text{Li}_3(-e^{-a-bx})}{b^2} - \frac{x\text{Li}_3(e^{-a-bx})}{b^2} + \frac{x^2\text{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2\text{Li}_2(e^{-a-bx})}{2b}$$

[Out] 1/2\*x^2\*polylog(2,-exp(-b\*x-a))/b-1/2\*x^2\*polylog(2,exp(-b\*x-a))/b+x\*polylog(3,-exp(-b\*x-a))/b^2-x\*polylog(3,exp(-b\*x-a))/b^2+polylog(4,-exp(-b\*x-a))/b^3-polylog(4,exp(-b\*x-a))/b^3

**Rubi [A]** time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6214, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}(3, -e^{-a-bx})}{b^2} - \frac{x\text{PolyLog}(3, e^{-a-bx})}{b^2} + \frac{\text{PolyLog}(4, -e^{-a-bx})}{b^3} - \frac{\text{PolyLog}(4, e^{-a-bx})}{b^3} + \frac{x^2\text{PolyLog}(2, -e^{-a-bx})}{2b} - \frac{x^2\text{PolyLog}(2, e^{-a-bx})}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^2\*ArcCoth[E^(a + b\*x)], x]

[Out] (x^2\*PolyLog[2, -E^(-a - b\*x)])/(2\*b) - (x^2\*PolyLog[2, E^(-a - b\*x)])/(2\*b) + (x\*PolyLog[3, -E^(-a - b\*x)]/b^2 - (x\*PolyLog[3, E^(-a - b\*x)]/b^2 + PolyLog[4, -E^(-a - b\*x)]/b^3 - PolyLog[4, E^(-a - b\*x)]/b^3

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/ (b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m-1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 6214

Int[ArcCoth[(a\_.) + (b\_.)\*(f\_)^((c\_.) + (d\_.)\*(x\_))]\*(x\_)^(m\_.), x\_Symbol] := Dist[1/2, Int[x^m\*Log[1 + 1/(a + b\*f^(c + d\*x))], x], x] - Dist[1/2, Int

```
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IGtQ[m, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
)*(x_)))]^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 \coth^{-1}(e^{a+bx}) dx &= -\left(\frac{1}{2} \int x^2 \log(1 - e^{-a-bx}) dx\right) + \frac{1}{2} \int x^2 \log(1 + e^{-a-bx}) dx \\
&= \frac{x^2 \operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2 \operatorname{Li}_2(e^{-a-bx})}{2b} - \frac{\int x \operatorname{Li}_2(-e^{-a-bx}) dx}{b} + \frac{\int x \operatorname{Li}_2(e^{-a-bx}) dx}{b} \\
&= \frac{x^2 \operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2 \operatorname{Li}_2(e^{-a-bx})}{2b} + \frac{x \operatorname{Li}_3(-e^{-a-bx})}{b^2} - \frac{x \operatorname{Li}_3(e^{-a-bx})}{b^2} - \frac{\int \operatorname{Li}_3(-e^{-a-bx}) dx}{b^2} \\
&= \frac{x^2 \operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2 \operatorname{Li}_2(e^{-a-bx})}{2b} + \frac{x \operatorname{Li}_3(-e^{-a-bx})}{b^2} - \frac{x \operatorname{Li}_3(e^{-a-bx})}{b^2} + \frac{\operatorname{Subst}\left(\int \frac{\operatorname{Li}_3(-x)}{x} dx\right)}{b^3} \\
&= \frac{x^2 \operatorname{Li}_2(-e^{-a-bx})}{2b} - \frac{x^2 \operatorname{Li}_2(e^{-a-bx})}{2b} + \frac{x \operatorname{Li}_3(-e^{-a-bx})}{b^2} - \frac{x \operatorname{Li}_3(e^{-a-bx})}{b^2} + \frac{\operatorname{Li}_4(-e^{-a-bx})}{b^3} - \frac{\operatorname{Li}_4(e^{-a-bx})}{b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 149, normalized size = 1.25

$$\frac{b^3 x^3 \log(1 - e^{a+bx}) - b^3 x^3 \log(e^{a+bx} + 1) + 2b^3 x^3 \coth^{-1}(e^{a+bx}) - 3b^2 x^2 \operatorname{Li}_2(-e^{a+bx}) + 3b^2 x^2 \operatorname{Li}_2(e^{a+bx}) + 6bx \operatorname{Li}_3(-e^{a+bx}) - 6bx \operatorname{Li}_3(e^{a+bx})}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCoth[E^(a + b*x)], x]
```



[Out]  $(2*b^3*x^3*ArcCoth[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*Log[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyLog[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(a + b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3)$

**fricas** [C] time = 0.57, size = 247, normalized size = 2.08

$$b^3 x^3 \log\left(\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^3 x^3 \log(\cosh(bx+a) + \sinh(bx+a) + 1) + 3 b^2 x^2 \text{Li}_2(\cosh(bx+a) + \sinh(bx+a) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="fricas")`

[Out]  $1/6*(b^3*x^3*\log((\cosh(b*x + a) + \sinh(b*x + a) + 1)/(\cosh(b*x + a) + \sinh(b*x + a) - 1)) - b^3*x^3*\log(\cosh(b*x + a) + \sinh(b*x + a) + 1) + 3*b^2*x^2*\text{dilog}(\cosh(b*x + a) + \sinh(b*x + a)) - 3*b^2*x^2*\text{dilog}(-\cosh(b*x + a) - \sinh(b*x + a)) - a^3*\log(\cosh(b*x + a) + \sinh(b*x + a) - 1) - 6*b*x*\text{polylog}(3, \cosh(b*x + a) + \sinh(b*x + a)) + 6*b*x*\text{polylog}(3, -\cosh(b*x + a) - \sinh(b*x + a)) + (b^3*x^3 + a^3)*\log(-\cosh(b*x + a) - \sinh(b*x + a) + 1) + 6*\text{polylog}(4, \cosh(b*x + a) + \sinh(b*x + a)) - 6*\text{polylog}(4, -\cosh(b*x + a) - \sinh(b*x + a)))/b^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(e^{(bx+a)}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arccoth(exp(b*x+a)),x, algorithm="giac")`

[Out] `integrate(x^2*arccoth(e^(b*x + a)), x)`

**maple** [A] time = 0.07, size = 185, normalized size = 1.55

$$\frac{x^3 \operatorname{arccoth}(e^{bx+a})}{3} - \frac{x^2 \operatorname{polylog}(2, -e^{bx+a})}{2b} + \frac{x \operatorname{polylog}(3, -e^{bx+a})}{b^2} + \frac{\ln(1 - e^{bx+a}) x^3}{6} + \frac{x^2 \operatorname{polylog}(2, e^{bx+a})}{2b} + \frac{a^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arccoth(exp(b*x+a)),x)`

[Out]  $1/3*x^3*\operatorname{arccoth}(\exp(b*x+a)) - 1/2*x^2*\text{polylog}(2, -\exp(b*x+a))/b + x*\text{polylog}(3, -\exp(b*x+a))/b^2 + 1/6*\ln(1 - \exp(b*x+a))*x^3 + 1/2*x^2*\text{polylog}(2, \exp(b*x+a))/b + 1/3/b^3*a^3*\operatorname{arctanh}(\exp(b*x+a)) + \text{polylog}(4, \exp(b*x+a))/b^3 - \text{polylog}(4, -\exp(b*x+a))$

))/b^3-x\*polylog(3,exp(b\*x+a))/b^2-1/6\*ln(exp(b\*x+a)+1)\*x^3-1/6/b^3\*ln(exp(b\*x+a)+1)\*a^3+1/6/b^3\*ln(1-exp(b\*x+a))\*a^3

**maxima** [A] time = 0.35, size = 142, normalized size = 1.19

$$\frac{1}{3}x^3 \operatorname{arccoth}(e^{(bx+a)}) - \frac{1}{6}b \left( \frac{b^3x^3 \log(e^{(bx+a)} + 1) + 3b^2x^2 \operatorname{Li}_2(-e^{(bx+a)}) - 6bx \operatorname{Li}_3(-e^{(bx+a)}) + 6 \operatorname{Li}_4(-e^{(bx+a)})}{b^4} - \frac{b^3x^3}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(exp(b\*x+a)),x, algorithm="maxima")

[Out] 1/3\*x^3\*arccoth(e^(b\*x + a)) - 1/6\*b\*((b^3\*x^3\*log(e^(b\*x + a) + 1) + 3\*b^2\*x^2\*dilog(-e^(b\*x + a)) - 6\*b\*x\*polylog(3, -e^(b\*x + a)) + 6\*polylog(4, -e^(b\*x + a)))/b^4 - (b^3\*x^3\*log(-e^(b\*x + a) + 1) + 3\*b^2\*x^2\*dilog(e^(b\*x + a)) - 6\*b\*x\*polylog(3, e^(b\*x + a)) + 6\*polylog(4, e^(b\*x + a)))/b^4)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acoth}(e^{a+bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*acoth(exp(a + b\*x)),x)

[Out] int(x^2\*acoth(exp(a + b\*x)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{acoth}(e^a e^{bx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*acoth(exp(b\*x+a)),x)

[Out] Integral(x\*\*2\*acoth(exp(a)\*exp(b\*x)), x)

### 3.289 $\int \coth^{-1} \left( a + b f^{c+dx} \right) dx$

**Optimal.** Leaf size=168

$$\frac{\operatorname{Li}_2\left(1 - \frac{2}{b f^{c+dx} + a + 1}\right)}{2d \log(f)} - \frac{\operatorname{Li}_2\left(1 - \frac{2b f^{c+dx}}{(1-a)(b f^{c+dx} + a + 1)}\right)}{2d \log(f)} - \frac{\log\left(\frac{2}{a + b f^{c+dx} + 1}\right) \coth^{-1}\left(a + b f^{c+dx}\right)}{d \log(f)} + \frac{\log\left(\frac{2b f^{c+dx}}{(1-a)(a + b f^{c+dx} + 1)}\right) \coth^{-1}\left(a + b f^{c+dx}\right)}{d \log(f)}$$

[Out]  $-\operatorname{arccoth}(a + b f^{c+dx}) \ln\left(\frac{2}{1 + a + b f^{c+dx}}\right) / d \ln(f) + \operatorname{arccoth}(a + b f^{c+dx}) \ln\left(\frac{2b f^{c+dx}}{(1-a)(a + b f^{c+dx} + 1)}\right) / d \ln(f) + \frac{1}{2} \operatorname{polylog}\left(2, 1 - \frac{2}{1 + a + b f^{c+dx}}\right) / d \ln(f) - \frac{1}{2} \operatorname{polylog}\left(2, 1 - \frac{2b f^{c+dx}}{(1-a)(a + b f^{c+dx} + 1)}\right) / d \ln(f)$

**Rubi [A]** time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {2282, 6112, 5921, 2402, 2315, 2447}

$$\frac{\operatorname{PolyLog}\left(2, 1 - \frac{2}{a + b f^{c+dx} + 1}\right)}{2d \log(f)} - \frac{\operatorname{PolyLog}\left(2, 1 - \frac{2b f^{c+dx}}{(1-a)(a + b f^{c+dx} + 1)}\right)}{2d \log(f)} - \frac{\log\left(\frac{2}{a + b f^{c+dx} + 1}\right) \coth^{-1}\left(a + b f^{c+dx}\right)}{d \log(f)} + \frac{\log\left(\frac{2b f^{c+dx}}{(1-a)(a + b f^{c+dx} + 1)}\right) \coth^{-1}\left(a + b f^{c+dx}\right)}{d \log(f)}$$

Antiderivative was successfully verified.

[In] `Int[ArcCoth[a + b*f^(c + d*x)], x]`

[Out]  $-\left(\operatorname{ArcCoth}[a + b f^{c+dx}] \operatorname{Log}\left[\frac{2}{1 + a + b f^{c+dx}}\right]\right) / (d \operatorname{Log}[f]) + \left(\operatorname{ArcCoth}[a + b f^{c+dx}] \operatorname{Log}\left[\frac{2b f^{c+dx}}{(1-a)(a + b f^{c+dx} + 1)}\right]\right) / (d \operatorname{Log}[f]) + \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 + a + b f^{c+dx}}\right] / (2d \operatorname{Log}[f]) - \operatorname{PolyLog}\left[2, 1 - \frac{2b f^{c+dx}}{(1-a)(a + b f^{c+dx} + 1)}\right] / (2d \operatorname{Log}[f])$

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

#### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

### Rule 5921

```
Int[((a_.) + ArcCoth[(c_.)*(x_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcCoth[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + e*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcCoth[c*x])*Log[(2*c*(d + e*x))/(c*d + e*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 6112

```
Int[((a_.) + ArcCoth[(c_) + (d_.)*(x_)])*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] := Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*ArcCoth[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
```

### Rubi steps

$$\begin{aligned}
\int \coth^{-1}(a + bf^{c+dx}) dx &= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(a+bx)}{x} dx, x, f^{c+dx}\right)}{d \log(f)} \\
&= \frac{\text{Subst}\left(\int \frac{\coth^{-1}(x)}{-\frac{a}{b} + \frac{x}{b}} dx, x, a + bf^{c+dx}\right)}{bd \log(f)} \\
&= -\frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d \log(f)} + \frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+bf^{c+dx})}\right)}{d \log(f)} \\
&= -\frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d \log(f)} + \frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+bf^{c+dx})}\right)}{d \log(f)} \\
&= -\frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d \log(f)} + \frac{\coth^{-1}(a + bf^{c+dx}) \log\left(\frac{2bf^{c+dx}}{(1-a)(1+a+bf^{c+dx})}\right)}{d \log(f)}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 108, normalized size = 0.64

$$\frac{\text{Li}_2\left(-\frac{bf^{c+dx}}{a-1}\right) - \text{Li}_2\left(-\frac{bf^{c+dx}}{a+1}\right) + dx \log(f) \left(\log\left(\frac{a+bf^{c+dx}-1}{a-1}\right) - \log\left(\frac{a+bf^{c+dx}+1}{a+1}\right) + 2 \coth^{-1}(a + bf^{c+dx})\right)}{2d \log(f)}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCoth[a + b\*f^(c + d\*x)], x]

[Out] (d\*x\*Log[f]\*(2\*ArcCoth[a + b\*f^(c + d\*x)] + Log[(-1 + a + b\*f^(c + d\*x))/(1 + a)] - Log[(1 + a + b\*f^(c + d\*x))/(1 + a)]) + PolyLog[2, -((b\*f^(c + d\*x))/(-1 + a))] - PolyLog[2, -((b\*f^(c + d\*x))/(1 + a))]/(2\*d\*Log[f])

**fricas [A]** time = 0.58, size = 283, normalized size = 1.68

$$\frac{dx \log(f) \log\left(\frac{b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)) + a + 1}{b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)) + a - 1}\right) + c \log(b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)))}{2d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a+b\*f^(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2} \frac{(d*x*\log(f)*\log((b*\cosh((d*x + c)*\log(f)) + b*\sinh((d*x + c)*\log(f)) + a + 1)/(b*\cosh((d*x + c)*\log(f)) + b*\sinh((d*x + c)*\log(f)) + a - 1)) + c*\log(b*\cosh((d*x + c)*\log(f)) + b*\sinh((d*x + c)*\log(f)) + a + 1)*\log(f) - c*\log(b*\cosh((d*x + c)*\log(f)) + b*\sinh((d*x + c)*\log(f)) + a - 1)*\log(f) - (d*x + c)*\log(f)*\log((b*\cosh((d*x + c)*\log(f)) + b*\sinh((d*x + c)*\log(f)) + a + 1)/(a + 1)) + (d*x + c)*\log(f)*\log((b*\cosh((d*x + c)*\log(f)) + b*\sinh((d*x + c)*\log(f)) + a - 1)/(a - 1)) - \operatorname{dilog}(-(b*\cosh((d*x + c)*\log(f)) + b*\sinh((d*x + c)*\log(f)) + a + 1)/(a + 1) + 1) + \operatorname{dilog}(-(b*\cosh((d*x + c)*\log(f)) + b*\sinh((d*x + c)*\log(f)) + a - 1)/(a - 1) + 1))/(d*\log(f))$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(b f^{dx+c} + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a+b\*f^(d\*x+c)),x, algorithm="giac")

[Out] integrate(arccoth(b\*f^(d\*x + c) + a), x)

**maple** [A] time = 0.15, size = 164, normalized size = 0.98

$$\frac{\ln(b f^{dx+c}) \operatorname{arccoth}(a + b f^{dx+c})}{d \ln(f)} + \frac{\operatorname{dilog}\left(\frac{b f^{dx+c} + a - 1}{a - 1}\right)}{2d \ln(f)} + \frac{\ln(b f^{dx+c}) \ln\left(\frac{b f^{dx+c} + a - 1}{a - 1}\right)}{2d \ln(f)} - \frac{\operatorname{dilog}\left(\frac{1 + a + b f^{dx+c}}{1 + a}\right)}{2d \ln(f)} - \frac{\ln(b f^{dx+c}) \ln\left(\frac{1 + a + b f^{dx+c}}{1 + a}\right)}{2d \ln(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccoth(a+b\*f^(d\*x+c)),x)

[Out]  $\frac{1}{d} \frac{\ln(b*f^{d*x+c})*\operatorname{arccoth}(a+b*f^{d*x+c}) + 1/2/d/\ln(f)*\operatorname{dilog}((b*f^{d*x+c}+a-1)/(a-1)) + 1/2/d/\ln(f)*\ln(b*f^{d*x+c})*\ln((b*f^{d*x+c}+a-1)/(a-1)) - 1/2/d/\ln(f)*\operatorname{dilog}((1+a+b*f^{d*x+c})/(1+a)) - 1/2/d/\ln(f)*\ln(b*f^{d*x+c})*\ln((1+a+b*f^{d*x+c})/(1+a))}{d}$

**maxima** [A] time = 0.33, size = 202, normalized size = 1.20

$$\frac{(dx + c) \operatorname{arccoth}(b f^{dx+c} + a)}{d} - \frac{(dx + c) b \left( \frac{\log(b f^{dx+c} + a + 1)}{b} - \frac{\log(b f^{dx+c} + a - 1)}{b} \right) \log(f) - b \left( \frac{\log(b f^{dx+c} + a + 1) \log\left(-\frac{b f^{dx+c} + a + 1}{a + 1}\right) + \log(b f^{dx+c} + a - 1) \log\left(-\frac{b f^{dx+c} + a - 1}{a - 1}\right)}{b}}{2d \log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccoth(a+b\*f^(d\*x+c)),x, algorithm="maxima")

```
[Out] (d*x + c)*arccoth(b*f^(d*x + c) + a)/d - 1/2*((d*x + c)*b*(log(b*f^(d*x + c)
) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f) - b*((log(b*f^(d*x + c)
+ a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog((b*f^(d*x + c)
+ a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*f^(d*x + c) + a
- 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))/b))/(d*log(f))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acoth}\left(a + b f^{c+dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acoth(a + b*f^(c + d*x)),x)
```

```
[Out] int(acoth(a + b*f^(c + d*x)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acoth(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

### 3.290 $\int x \coth^{-1} \left( a + b f^{c+dx} \right) dx$

**Optimal.** Leaf size=216

$$-\frac{\text{Li}_3\left(\frac{b f^{c+dx}}{1-a}\right)}{2d^2 \log^2(f)} + \frac{\text{Li}_3\left(-\frac{b f^{c+dx}}{a+1}\right)}{2d^2 \log^2(f)} + \frac{x \text{Li}_2\left(\frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)} - \frac{x \text{Li}_2\left(-\frac{b f^{c+dx}}{a+1}\right)}{2d \log(f)} + \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1-a}\right) - \frac{1}{4} x^2 \log\left(\frac{b f^{c+dx}}{a+1} + 1\right) - \frac{1}{4} x^2 \log\left(\frac{b f^{c+dx}}{1-a}\right)$$

[Out]  $\frac{1}{4} x^2 \ln\left(1 - \frac{b f^{c+dx}}{1-a}\right) - \frac{1}{4} x^2 \ln\left(1 + \frac{b f^{c+dx}}{1+a}\right) - \frac{1}{4} x^2 \ln\left(1 - \frac{b f^{c+dx}}{1-a}\right) + \frac{1}{4} x^2 \ln\left(1 + \frac{b f^{c+dx}}{1+a}\right) + \frac{1}{2} x \text{polylog}\left(2, \frac{b f^{c+dx}}{1-a}\right) / d \ln(f) - \frac{1}{2} x \text{polylog}\left(2, -\frac{b f^{c+dx}}{1+a}\right) / d \ln(f) - \frac{1}{2} \text{polylog}\left(3, \frac{b f^{c+dx}}{1-a}\right) / d^2 \ln(f)^2 + \frac{1}{2} \text{polylog}\left(3, -\frac{b f^{c+dx}}{1+a}\right) / d^2 \ln(f)^2$

**Rubi [A]** time = 2.71, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6214, 2551, 12, 6742, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3, \frac{b f^{c+dx}}{1-a}\right)}{2d^2 \log^2(f)} + \frac{\text{PolyLog}\left(3, -\frac{b f^{c+dx}}{a+1}\right)}{2d^2 \log^2(f)} + \frac{x \text{PolyLog}\left(2, \frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)} - \frac{x \text{PolyLog}\left(2, -\frac{b f^{c+dx}}{a+1}\right)}{2d \log(f)} + \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1-a}\right)$$

Antiderivative was successfully verified.

[In] `Int[x*ArcCoth[a + b*f^(c + d*x)],x]`

[Out]  $(x^2 \text{Log}[1 - (b f^{c+dx}) / (1-a)]) / 4 - (x^2 \text{Log}[1 + (b f^{c+dx}) / (1+a)]) / 4 - (x^2 \text{Log}[1 - (a + b f^{c+dx})^{-1}]) / 4 + (x^2 \text{Log}[1 + (a + b f^{c+dx})^{-1}]) / 4 + (x \text{PolyLog}[2, (b f^{c+dx}) / (1-a)]) / (2 d \text{Log}[f]) - (x \text{PolyLog}[2, -((b f^{c+dx}) / (1+a))]) / (2 d \text{Log}[f]) - \text{PolyLog}[3, (b f^{c+dx}) / (1-a)] / (2 d^2 \text{Log}[f]^2) + \text{PolyLog}[3, -((b f^{c+dx}) / (1+a))] / (2 d^2 \text{Log}[f]^2)$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_)) / ((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n] / a)) / (b*f*g*n*Log[F]), x] - Dist[(d*m) / (b*f*g*n*Log[F]), Int[(c + d*x)^(m-1) * Log[1 + (b*(F^(g*(e + f*x)))^n] / a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`



Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_] * ((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[((a + b*x)^(m + 1)
)*Log[u]]/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 6214

```
Int[ArcCoth[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:= Dist[1/2, Int[x^m*Log[1 + 1/(a + b*f^(c + d*x))], x], x] - Dist[1/2, Int
[x^m*Log[1 - 1/(a + b*f^(c + d*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] &&
IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x \coth^{-1}(a + b f^{c+dx}) dx &= -\left(\frac{1}{2} \int x \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) dx\right) + \frac{1}{2} \int x \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) dx \\
&= -\frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} \int \frac{b d f^{c+dx} x^2 \log(f)}{(-1 + a + b f^{c+dx})} dx \\
&= -\frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} (b d \log(f)) \int \frac{x^2}{(-1 + a + b f^{c+dx})} dx \\
&= -\frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} (b d \log(f)) \int \left(\frac{f^{c+dx}}{-a - b f^{c+dx}}\right) dx \\
&= -\frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} (b d \log(f)) \int \frac{f^{c+dx}}{-1 + a + b f^{c+dx}} dx \\
&= \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{4} x^2 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{4} x^2 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{4} x^2 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) \\
&= \frac{1}{4} x^2 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{4} x^2 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{4} x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{4} x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 177, normalized size = 0.82

$$\frac{d^2 x^2 \log^2(f) \log\left(\frac{b f^{c+dx}}{a-1} + 1\right) - d^2 x^2 \log^2(f) \log\left(\frac{b f^{c+dx}}{a+1} + 1\right) + 2 d^2 x^2 \log^2(f) \coth^{-1}(a + b f^{c+dx}) - 2 \text{Li}_3\left(-\frac{b f^{c+dx}}{a-1}\right)}{4 d^2 \log^2(f)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*ArcCoth[a + b\*f^(c + d\*x)],x]

[Out] (2\*d^2\*x^2\*ArcCoth[a + b\*f^(c + d\*x)]\*Log[f]^2 + d^2\*x^2\*Log[f]^2\*Log[1 + (b\*f^(c + d\*x))/(-1 + a)] - d^2\*x^2\*Log[f]^2\*Log[1 + (b\*f^(c + d\*x))/(1 + a)] + 2\*d\*x\*Log[f]\*PolyLog[2, -((b\*f^(c + d\*x))/(-1 + a))] - 2\*d\*x\*Log[f]\*PolyLog[2, -((b\*f^(c + d\*x))/(1 + a))] - 2\*PolyLog[3, -((b\*f^(c + d\*x))/(-1 + a))] + 2\*PolyLog[3, -((b\*f^(c + d\*x))/(1 + a))])/(4\*d^2\*Log[f]^2)

**fricas** [C] time = 0.66, size = 395, normalized size = 1.83

$$d^2 x^2 \log(f)^2 \log\left(\frac{b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)) + a + 1}{b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)) + a - 1}\right) - c^2 \log(b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="fricas")

[Out] 1/4\*(d^2\*x^2\*log(f)^2\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)) - c^2\*log(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)\*log(f)^2 + c^2\*log(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)\*log(f)^2 - 2\*d\*x\*dilog(-(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(a + 1) + 1)\*log(f) + 2\*d\*x\*dilog(-(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)/(a - 1) + 1)\*log(f) - (d^2\*x^2 - c^2)\*log(f)^2\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(a + 1)) + (d^2\*x^2 - c^2)\*log(f)^2\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)/(a - 1)) + 2\*polylog(3, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a + 1)) - 2\*polylog(3, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a - 1)))/(d^2\*log(f)^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{arccoth}(b f^{dx+c} + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="giac")

[Out] integrate(x\*arccoth(b\*f^(d\*x + c) + a), x)

**maple** [B] time = 0.18, size = 590, normalized size = 2.73

$$\frac{x^2 \ln(b f^{dx+c} + a - 1)}{4} + \frac{x^2 \ln(1 + a + b f^{dx+c})}{4} - \frac{\ln\left(1 - \frac{b f^{dx} f^c}{-1-a}\right) x^2}{4} - \frac{\ln\left(1 - \frac{b f^{dx} f^c}{-1-a}\right) x c}{2d} - \frac{\ln\left(1 - \frac{b f^{dx} f^c}{-1-a}\right) c^2}{4d^2} - \text{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*arccoth(a+b\*f^(d\*x+c)),x)

[Out] -1/4\*x^2\*ln(b\*f^(d\*x+c)+a-1)+1/4\*x^2\*ln(1+a+b\*f^(d\*x+c))-1/4\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*x^2-1/2/d\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*x\*c-1/4/d^2\*ln(1-b\*f^(d\*x)\*f^c/(-1-a))\*c^2-1/2/ln(f)/d\*polylog(2,b\*f^(d\*x)\*f^c/(-1-a))\*x-1/2/ln(f)/d

$$\begin{aligned} & \int \frac{d^2 x^2 \log\left(\frac{b f^{dx} f^c}{a+1} + 1\right) \log(f)^2 + 2 dx \operatorname{Li}_2\left(-\frac{b f^{dx} f^c}{a+1}\right) \log(f) - 2 \operatorname{Li}_3\left(-\frac{b f^{dx} f^c}{a+1}\right)}{b d^3 \log(f)^3} - \frac{d^2 x^2 \log\left(\frac{b f^{dx} f^c}{a-1} + 1\right) \log(f)^2 + 2 dx \operatorname{Li}_2\left(-\frac{b f^{dx} f^c}{a-1}\right) \log(f) - 2 \operatorname{Li}_3\left(-\frac{b f^{dx} f^c}{a-1}\right)}{b d^3 \log(f)^3} \\ & - \frac{1}{4} b d \left( \frac{d^2 x^2 \log\left(\frac{b f^{dx} f^c}{a+1} + 1\right) \log(f)^2 + 2 dx \operatorname{Li}_2\left(-\frac{b f^{dx} f^c}{a+1}\right) \log(f) - 2 \operatorname{Li}_3\left(-\frac{b f^{dx} f^c}{a+1}\right)}{b d^3 \log(f)^3} - \frac{d^2 x^2 \log\left(\frac{b f^{dx} f^c}{a-1} + 1\right) \log(f)^2 + 2 dx \operatorname{Li}_2\left(-\frac{b f^{dx} f^c}{a-1}\right) \log(f) - 2 \operatorname{Li}_3\left(-\frac{b f^{dx} f^c}{a-1}\right)}{b d^3 \log(f)^3} \right) \end{aligned}$$

**maxima** [A] time = 0.38, size = 194, normalized size = 0.90

$$-\frac{1}{4} b d \left( \frac{d^2 x^2 \log\left(\frac{b f^{dx} f^c}{a+1} + 1\right) \log(f)^2 + 2 dx \operatorname{Li}_2\left(-\frac{b f^{dx} f^c}{a+1}\right) \log(f) - 2 \operatorname{Li}_3\left(-\frac{b f^{dx} f^c}{a+1}\right)}{b d^3 \log(f)^3} - \frac{d^2 x^2 \log\left(\frac{b f^{dx} f^c}{a-1} + 1\right) \log(f)^2 + 2 dx \operatorname{Li}_2\left(-\frac{b f^{dx} f^c}{a-1}\right) \log(f) - 2 \operatorname{Li}_3\left(-\frac{b f^{dx} f^c}{a-1}\right)}{b d^3 \log(f)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/4*b*d*((d^2*x^2*\log(b*f^(d*x)*f^c/(a+1)+1)*\log(f)^2+2*d*x*dilog(-b*f^(d*x)*f^c/(a+1))*\log(f)-2*\operatorname{polylog}(3,-b*f^(d*x)*f^c/(a+1)))/(b*d^3*\log(f)^3)-(d^2*x^2*\log(b*f^(d*x)*f^c/(a-1)+1)*\log(f)^2+2*d*x*dilog(-b*f^(d*x)*f^c/(a-1))*\log(f)-2*\operatorname{polylog}(3,-b*f^(d*x)*f^c/(a-1)))/(b*d^3*\log(f)^3))*\log(f)+1/2*x^2*\operatorname{arccoth}(b*f^(d*x+c)+a)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{acoth}(a + b f^{c+dx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*acoth(a + b\*f^(c + d\*x)),x)

[Out] int(x\*acoth(a + b\*f^(c + d\*x)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*acoth(a+b\*f\*\*(d\*x+c)),x)

[Out] Timed out

### 3.291 $\int x^2 \coth^{-1} \left( a + b f^{c+dx} \right) dx$

**Optimal.** Leaf size=269

$$\frac{\operatorname{Li}_4\left(\frac{b f^{c+dx}}{1-a}\right)}{d^3 \log^3(f)} - \frac{\operatorname{Li}_4\left(-\frac{b f^{c+dx}}{a+1}\right)}{d^3 \log^3(f)} - \frac{x \operatorname{Li}_3\left(\frac{b f^{c+dx}}{1-a}\right)}{d^2 \log^2(f)} + \frac{x \operatorname{Li}_3\left(-\frac{b f^{c+dx}}{a+1}\right)}{d^2 \log^2(f)} + \frac{x^2 \operatorname{Li}_2\left(\frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)} - \frac{x^2 \operatorname{Li}_2\left(-\frac{b f^{c+dx}}{a+1}\right)}{2d \log(f)} + \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1-a}\right)$$

[Out]  $\frac{1}{6} x^3 \ln\left(\frac{1 - b f^{c+dx}}{1-a}\right) - \frac{1}{6} x^3 \ln\left(\frac{1 + b f^{c+dx}}{1+a}\right) - \frac{1}{6} x^3 \ln\left(\frac{1 - 1/(a + b f^{c+dx})}{1 + 1/(a + b f^{c+dx})}\right) + \frac{1}{2} x^2 \operatorname{polylog}\left(2, \frac{b f^{c+dx}}{1-a}\right) / d \ln(f) - \frac{1}{2} x^2 \operatorname{polylog}\left(2, -\frac{b f^{c+dx}}{1+a}\right) / d \ln(f) - x \operatorname{polylog}\left(3, \frac{b f^{c+dx}}{1-a}\right) / d^2 \ln(f)^2 + x \operatorname{polylog}\left(3, -\frac{b f^{c+dx}}{1+a}\right) / d^2 \ln(f)^2 + \operatorname{polylog}\left(4, \frac{b f^{c+dx}}{1-a}\right) / d^3 \ln(f)^3 - \operatorname{polylog}\left(4, -\frac{b f^{c+dx}}{1+a}\right) / d^3 \ln(f)^3$

**Rubi [A]** time = 2.59, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {6214, 2551, 12, 6742, 2190, 2531, 6609, 2282, 6589}

$$\frac{x \operatorname{PolyLog}\left(3, \frac{b f^{c+dx}}{1-a}\right)}{d^2 \log^2(f)} + \frac{x \operatorname{PolyLog}\left(3, -\frac{b f^{c+dx}}{a+1}\right)}{d^2 \log^2(f)} + \frac{\operatorname{PolyLog}\left(4, \frac{b f^{c+dx}}{1-a}\right)}{d^3 \log^3(f)} - \frac{\operatorname{PolyLog}\left(4, -\frac{b f^{c+dx}}{a+1}\right)}{d^3 \log^3(f)} + \frac{x^2 \operatorname{PolyLog}\left(2, \frac{b f^{c+dx}}{1-a}\right)}{2d \log(f)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 \operatorname{ArcCoth}[a + b f^{c+dx}], x]$

[Out]  $\frac{x^3 \operatorname{Log}[1 - (b f^{c+dx}) / (1 - a)]}{6} - \frac{x^3 \operatorname{Log}[1 + (b f^{c+dx}) / (1 + a)]}{6} - \frac{x^3 \operatorname{Log}[1 - (a + b f^{c+dx})^{-1}]}{6} + \frac{x^3 \operatorname{Log}[1 + (a + b f^{c+dx})^{-1}]}{6} + \frac{x^2 \operatorname{PolyLog}[2, (b f^{c+dx}) / (1 - a)]}{2d \operatorname{Log}[f]} - \frac{x^2 \operatorname{PolyLog}[2, -(b f^{c+dx}) / (1 + a)]}{2d \operatorname{Log}[f]} - \frac{x \operatorname{PolyLog}[3, (b f^{c+dx}) / (1 - a)]}{d^2 \operatorname{Log}[f]^2} + \frac{x \operatorname{PolyLog}[3, -(b f^{c+dx}) / (1 + a)]}{d^2 \operatorname{Log}[f]^2} + \frac{\operatorname{PolyLog}[4, (b f^{c+dx}) / (1 - a)]}{d^3 \operatorname{Log}[f]^3} - \frac{\operatorname{PolyLog}[4, -(b f^{c+dx}) / (1 + a)]}{d^3 \operatorname{Log}[f]^3}$

#### Rule 12

$\operatorname{Int}[(a_*) (u_*), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_\*) (v\_\*) /; FreeQ[b, x]]

#### Rule 2190

$\operatorname{Int}[((F_*)^{((g_*) * ((e_*) + (f_*) (x_*)))})^{(n_*) * ((c_*) + (d_*) (x_*))^{(m_*)}} / ((a_*) + (b_*) * ((F_*)^{((g_*) * ((e_*) + (f_*) (x_*)))})^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Simp}[(c + dx)^m \operatorname{Log}[1 + (b (F^{(g(e + fx))))^n] / a] / (b f g^n \operatorname{Log}[F]), x] - \operatorname{Dist}[(d m) / (b f g^n \operatorname{Log}[F]), \operatorname{Int}[(c + dx)^{m-1} \operatorname{Log}[1 + (b (F^{(g(e + fx))))^n]$

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :=> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2551

Int[Log[u\_]\*((a\_.) + (b\_.)\*(x\_))^(m\_.), x\_Symbol] :=> Simp[((a + b\*x)^(m + 1)\*Log[u])/(b\*(m + 1)), x] - Dist[1/(b\*(m + 1)), Int[SimplifyIntegrand[((a + b\*x)^(m + 1)\*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunctionFreeQ[u, x] && NeQ[m, -1]

### Rule 6214

Int[ArcCoth[(a\_.) + (b\_.)\*(f\_)^((c\_.) + (d\_.)\*(x\_))]\*(x\_)^(m\_.), x\_Symbol] :=> Dist[1/2, Int[x^m\*Log[1 + 1/(a + b\*f^(c + d\*x))], x], x] - Dist[1/2, Int[x^m\*Log[1 - 1/(a + b\*f^(c + d\*x))], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :=> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] :=> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c,

d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]

### Rubi steps

$$\begin{aligned}
 \int x^2 \coth^{-1}(a + b f^{c+dx}) dx &= -\left(\frac{1}{2} \int x^2 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) dx\right) + \frac{1}{2} \int x^2 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) dx \\
 &= -\frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} \int \frac{b d f^{c+dx} x^3}{(-1 + a + b f^{c+dx})} \\
 &= -\frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} (b d \log(f)) \int \frac{f^c}{(-1 + a + b f^{c+dx})} \\
 &= -\frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} (b d \log(f)) \int \left(\frac{f^c}{-a - b f^{c+dx}}\right) \\
 &= -\frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \log\left(1 + \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} (b d \log(f)) \int \frac{f^c}{-1 + a + b f^{c+dx}} \\
 &= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \\
 &= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \\
 &= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \\
 &= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3 \\
 &= \frac{1}{6} x^3 \log\left(1 - \frac{b f^{c+dx}}{1 - a}\right) - \frac{1}{6} x^3 \log\left(1 + \frac{b f^{c+dx}}{1 + a}\right) - \frac{1}{6} x^3 \log\left(1 - \frac{1}{a + b f^{c+dx}}\right) + \frac{1}{6} x^3
 \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 235, normalized size = 0.87

$$d^3x^3 \log^3(f) \log\left(\frac{bf^{c+dx}}{a-1} + 1\right) - d^3x^3 \log^3(f) \log\left(\frac{bf^{c+dx}}{a+1} + 1\right) + 2d^3x^3 \log^3(f) \coth^{-1}\left(a + bf^{c+dx}\right) + 3d^2x^2 \log^2(f)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*ArcCoth[a + b\*f^(c + d\*x)],x]

[Out] (2\*d^3\*x^3\*ArcCoth[a + b\*f^(c + d\*x)]\*Log[f]^3 + d^3\*x^3\*Log[f]^3\*Log[1 + (b\*f^(c + d\*x))/(-1 + a)] - d^3\*x^3\*Log[f]^3\*Log[1 + (b\*f^(c + d\*x))/(1 + a)] + 3\*d^2\*x^2\*Log[f]^2\*PolyLog[2, -((b\*f^(c + d\*x))/(-1 + a))] - 3\*d^2\*x^2\*Log[f]^2\*PolyLog[2, -((b\*f^(c + d\*x))/(1 + a))] - 6\*d\*x\*Log[f]\*PolyLog[3, -((b\*f^(c + d\*x))/(-1 + a))] + 6\*d\*x\*Log[f]\*PolyLog[3, -((b\*f^(c + d\*x))/(1 + a))] + 6\*PolyLog[4, -((b\*f^(c + d\*x))/(-1 + a))] - 6\*PolyLog[4, -((b\*f^(c + d\*x))/(1 + a))])/(6\*d^3\*Log[f]^3)

**fricas [C]** time = 0.70, size = 479, normalized size = 1.78

$$d^3x^3 \log(f)^3 \log\left(\frac{b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)) + a + 1}{b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)) + a - 1}\right) - 3d^2x^2 \text{Li}_2\left(-\frac{b \cosh((dx+c) \log(f)) + b \sinh((dx+c) \log(f)) + a + 1}{a + 1}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="fricas")

[Out] 1/6\*(d^3\*x^3\*log(f)^3\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)) - 3\*d^2\*x^2\*dilog(-(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(a + 1) + 1)\*log(f)^2 + 3\*d^2\*x^2\*dilog(-(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)/(a - 1) + 1)\*log(f)^2 + c^3\*log(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)\*log(f)^3 - c^3\*log(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)\*log(f)^3 - (d^3\*x^3 + c^3)\*log(f)^3\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a + 1)/(a + 1)) + (d^3\*x^3 + c^3)\*log(f)^3\*log((b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)) + a - 1)/(a - 1)) + 6\*d\*x\*log(f)\*polylog(3, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a + 1)) - 6\*d\*x\*log(f)\*polylog(3, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a - 1)) - 6\*polylog(4, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a + 1)) + 6\*polylog(4, -(b\*cosh((d\*x + c)\*log(f)) + b\*sinh((d\*x + c)\*log(f)))/(a - 1)))/(d^3\*log(f)^3)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{arccoth}(bf^{dx+c} + a) dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="giac")

[Out] integrate(x^2\*arccoth(b\*f^(d\*x + c) + a), x)

**maple [B]** time = 0.18, size = 666, normalized size = 2.48

$$\frac{x^3 \ln(b f^{dx+c} + a - 1)}{6} + \frac{x^3 \ln(1 + a + b f^{dx+c})}{6} - \frac{\ln\left(1 - \frac{b f^{dx} f^c}{-1-a}\right) x^3}{6} + \frac{\ln\left(1 - \frac{b f^{dx} f^c}{-1-a}\right) x c^2}{2d^2} + \frac{\ln\left(1 - \frac{b f^{dx} f^c}{-1-a}\right) c^3}{3d^3} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*arccoth(a+b\*f^(d\*x+c)),x)

[Out] 
$$-1/6*x^3*\ln(b*f^(d*x+c)+a-1)+1/6*x^3*\ln(1+a+b*f^(d*x+c))-1/6*\ln(1-b*f^(d*x)*f^c/(-1-a))*x^3+1/2/d^2*\ln(1-b*f^(d*x)*f^c/(-1-a))*x*c^2+1/3/d^3*\ln(1-b*f^(d*x)*f^c/(-1-a))*c^3-1/2/\ln(f)/d*\text{polylog}(2,b*f^(d*x)*f^c/(-1-a))*x^2+1/2/\ln(f)/d^3*\text{polylog}(2,b*f^(d*x)*f^c/(-1-a))*c^2+1/\ln(f)^2/d^2*\text{polylog}(3,b*f^(d*x)*f^c/(-1-a))*x-1/\ln(f)^3/d^3*\text{polylog}(4,b*f^(d*x)*f^c/(-1-a))+1/6/d^3*c^3*\ln(1+a+f^c*f^(d*x)*b)-1/2/\ln(f)/d^3*c^2*dilog((1+a+f^c*f^(d*x)*b)/(1+a))-1/2/d^2*c^2*\ln((1+a+f^c*f^(d*x)*b)/(1+a))*x-1/2/d^3*c^3*\ln((1+a+f^c*f^(d*x)*b)/(1+a))+1/6*\ln(1-b*f^(d*x)*f^c/(1-a))*x^3-1/2/d^2*\ln(1-b*f^(d*x)*f^c/(1-a))*x*c^2-1/3/d^3*\ln(1-b*f^(d*x)*f^c/(1-a))*c^3+1/2/\ln(f)/d*\text{polylog}(2,b*f^(d*x)*f^c/(1-a))*x^2-1/2/\ln(f)/d^3*\text{polylog}(2,b*f^(d*x)*f^c/(1-a))*c^2-1/\ln(f)^2/d^2*\text{polylog}(3,b*f^(d*x)*f^c/(1-a))*x+1/\ln(f)^3/d^3*\text{polylog}(4,b*f^(d*x)*f^c/(1-a))-1/6/d^3*c^3*\ln(f^c*f^(d*x)*b+a-1)+1/2/\ln(f)/d^3*c^2*dilog((f^c*f^(d*x)*b+a-1)/(a-1))+1/2/d^2*c^2*\ln((f^c*f^(d*x)*b+a-1)/(a-1))*x+1/2/d^3*c^3*\ln((f^c*f^(d*x)*b+a-1)/(a-1))$$

**maxima [A]** time = 0.37, size = 254, normalized size = 0.94

$$\frac{1}{3} x^3 \operatorname{arccoth}(b f^{dx+c} + a) - \frac{1}{6} b d \left( \frac{d^3 x^3 \log\left(\frac{b f^{dx} f^c}{a+1} + 1\right) \log(f)^3 + 3 d^2 x^2 \operatorname{Li}_2\left(-\frac{b f^{dx} f^c}{a+1}\right) \log(f)^2 - 6 d x \log(f) \operatorname{Li}_3\left(-\frac{b f^{dx} f^c}{a+1}\right)}{b d^4 \log(f)^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*arccoth(a+b\*f^(d\*x+c)),x, algorithm="maxima")

[Out] 
$$1/3*x^3*\operatorname{arccoth}(b*f^(d*x + c) + a) - 1/6*b*d*((d^3*x^3*\log(b*f^(d*x)*f^c/(a + 1) + 1)*\log(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a + 1))*\log(f)^2 - 6*d*x*\log(f)*\text{polylog}(3, -b*f^(d*x)*f^c/(a + 1)) + 6*\text{polylog}(4, -b*f^(d*x)*f^c/(a + 1)))/(b*d^4*\log(f)^4) - (d^3*x^3*\log(b*f^(d*x)*f^c/(a - 1) + 1)*\log(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a - 1))*\log(f)^2 - 6*d*x*\log(f)*\text{polylog}(3, -b*f^(d*x)*f^c/(a - 1)) + 6*\text{polylog}(4, -b*f^(d*x)*f^c/(a - 1)))/(b*d^4*\log(f)^4)$$

$\log(3, -b*f^{(d*x)}*f^c/(a - 1)) + 6*\text{polylog}(4, -b*f^{(d*x)}*f^c/(a - 1)))/(b*d^4*\log(f)^4))*\log(f)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{acoth}(a + b f^{c+dx}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*acoth(a + b*f^(c + d*x)),x)`

[Out] `int(x^2*acoth(a + b*f^(c + d*x)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*acoth(a+b*f**(d*x+c)),x)`

[Out] Timed out

$$3.292 \quad \int \frac{1}{(a-ax^2)(b-2b \coth^{-1}(x))} dx$$

Optimal. Leaf size=17

$$-\frac{\log(1-2 \coth^{-1}(x))}{2ab}$$

[Out]  $-1/2*\ln(1-2*\operatorname{arccoth}(x))/a/b$

**Rubi [A]** time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$ , Rules used = {5947}

$$-\frac{\log(1-2 \coth^{-1}(x))}{2ab}$$

Antiderivative was successfully verified.

[In] `Int[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])), x]`

[Out] `-Log[1 - 2*ArcCoth[x]]/(2*a*b)`

Rule 5947

`Int[1/(((a_.) + ArcCoth[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_)^2)), x_Symbol] :> Simp[Log[RemoveContent[a + b*ArcCoth[c*x], x]]/(b*c*d), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]`

Rubi steps

$$\int \frac{1}{(a-ax^2)(b-2b \coth^{-1}(x))} dx = -\frac{\log(1-2 \coth^{-1}(x))}{2ab}$$

**Mathematica [A]** time = 0.06, size = 17, normalized size = 1.00

$$-\frac{\log(2 \coth^{-1}(x) - 1)}{2ab}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((a - a*x^2)*(b - 2*b*ArcCoth[x])), x]`

[Out] `-1/2*Log[-1 + 2*ArcCoth[x]]/(a*b)`

**fricas** [A] time = 0.73, size = 21, normalized size = 1.24

$$\frac{\log\left(\log\left(\frac{x+1}{x-1}\right) - 1\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x^2+a)/(b-2\*b\*arccoth(x)),x, algorithm="fricas")

[Out] -1/2\*log(log((x + 1)/(x - 1)) - 1)/(a\*b)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^2 - a)(2b \operatorname{arccoth}(x) - b)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x^2+a)/(b-2\*b\*arccoth(x)),x, algorithm="giac")

[Out] integrate(1/((a\*x^2 - a)\*(2\*b\*arccoth(x) - b)), x)

**maple** [A] time = 0.07, size = 19, normalized size = 1.12

$$\frac{\ln(2b \operatorname{arccoth}(x) - b)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-a\*x^2+a)/(b-2\*b\*arccoth(x)),x)

[Out] -1/2/a\*ln(2\*b\*arccoth(x)-b)/b

**maxima** [A] time = 0.33, size = 21, normalized size = 1.24

$$\frac{\log\left(\log(x + 1) - \log(x - 1) - 1\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a\*x^2+a)/(b-2\*b\*arccoth(x)),x, algorithm="maxima")

[Out] -1/2\*log(log(x + 1) - log(x - 1) - 1)/(a\*b)

**mupad** [B] time = 1.40, size = 15, normalized size = 0.88

$$\frac{\ln(2a \operatorname{coth}(x) - 1)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a - a*x^2)*(b - 2*b*acoth(x))),x)`

[Out] `-log(2*acoth(x) - 1)/(2*a*b)`

sympy [A] time = 0.57, size = 14, normalized size = 0.82

$$-\frac{\log\left(\operatorname{acoth}(x) - \frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-a*x**2+a)/(b-2*b*acoth(x)),x)`

[Out] `-log(acoth(x) - 1/2)/(2*a*b)`

### 3.293 $\int x^3 \coth^{-1}(a + bx^4) dx$

**Optimal.** Leaf size=44

$$\frac{\log\left(1 - (a + bx^4)^2\right)}{8b} + \frac{(a + bx^4) \coth^{-1}(a + bx^4)}{4b}$$

[Out] 1/4\*(b\*x^4+a)\*arccoth(b\*x^4+a)/b+1/8\*ln(1-(b\*x^4+a)^2)/b

**Rubi [A]** time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6715, 6104, 5911, 260}

$$\frac{\log\left(1 - (a + bx^4)^2\right)}{8b} + \frac{(a + bx^4) \coth^{-1}(a + bx^4)}{4b}$$

Antiderivative was successfully verified.

[In] Int[x^3\*ArcCoth[a + b\*x^4],x]

[Out] ((a + b\*x^4)\*ArcCoth[a + b\*x^4])/(4\*b) + Log[1 - (a + b\*x^4)^2]/(8\*b)

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 5911

Int[((a\_) + ArcCoth[(c\_)\*(x\_)])\*(b\_)^(p\_), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 6104

Int[((a\_) + ArcCoth[(c\_) + (d\_)\*(x\_)])\*(b\_)^(p\_), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned}
\int x^3 \coth^{-1}(a + bx^4) dx &= \frac{1}{4} \text{Subst} \left( \int \coth^{-1}(a + bx) dx, x, x^4 \right) \\
&= \frac{\text{Subst} \left( \int \coth^{-1}(x) dx, x, a + bx^4 \right)}{4b} \\
&= \frac{(a + bx^4) \coth^{-1}(a + bx^4)}{4b} - \frac{\text{Subst} \left( \int \frac{x}{1-x^2} dx, x, a + bx^4 \right)}{4b} \\
&= \frac{(a + bx^4) \coth^{-1}(a + bx^4)}{4b} + \frac{\log \left( 1 - (a + bx^4)^2 \right)}{8b}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 0.89

$$\frac{\log \left( 1 - (a + bx^4)^2 \right) + 2(a + bx^4) \coth^{-1}(a + bx^4)}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*ArcCoth[a + b\*x^4],x]

[Out] (2\*(a + b\*x^4)\*ArcCoth[a + b\*x^4] + Log[1 - (a + b\*x^4)^2])/(8\*b)

**fricas [A]** time = 0.61, size = 58, normalized size = 1.32

$$\frac{bx^4 \log \left( \frac{bx^4+a+1}{bx^4+a-1} \right) + (a+1) \log(bx^4+a+1) - (a-1) \log(bx^4+a-1)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x^4+a),x, algorithm="fricas")

[Out] 1/8\*(b\*x^4\*log((b\*x^4 + a + 1)/(b\*x^4 + a - 1)) + (a + 1)\*log(b\*x^4 + a + 1) - (a - 1)\*log(b\*x^4 + a - 1))/b

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{arccoth}(bx^4 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*arccoth(b\*x^4+a),x, algorithm="giac")

[Out] integrate( $x^3 \operatorname{arccoth}(bx^4 + a)$ , x)

**maple** [A] time = 0.07, size = 46, normalized size = 1.05

$$\frac{\operatorname{arccoth}(bx^4 + a)x^4}{4} + \frac{\operatorname{arccoth}(bx^4 + a)a}{4b} + \frac{\ln\left((bx^4 + a)^2 - 1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int( $x^3 \operatorname{arccoth}(bx^4 + a)$ , x)

[Out]  $1/4 \operatorname{arccoth}(bx^4 + a)x^4 + 1/4/b \operatorname{arccoth}(bx^4 + a)a + 1/8/b \ln((bx^4 + a)^2 - 1)$

**maxima** [A] time = 0.32, size = 37, normalized size = 0.84

$$\frac{2(bx^4 + a) \operatorname{arccoth}(bx^4 + a) + \log\left(-(bx^4 + a)^2 + 1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^3 \operatorname{arccoth}(bx^4 + a)$ , x, algorithm="maxima")

[Out]  $1/8 * (2 * (bx^4 + a) * \operatorname{arccoth}(bx^4 + a) + \log(-(bx^4 + a)^2 + 1)) / b$

**mupad** [B] time = 1.56, size = 107, normalized size = 2.43

$$\frac{x^4 \ln\left(\frac{bx^4 + a + 1}{bx^4 + a}\right)}{8} - \frac{x^4 \ln\left(\frac{bx^4 + a - 1}{bx^4 + a}\right)}{8} + \frac{\ln(bx^4 + a - 1)}{8b} + \frac{\ln(bx^4 + a + 1)}{8b} - \frac{a \ln(bx^4 + a - 1)}{8b} + \frac{a \ln(bx^4 + a + 1)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int( $x^3 \operatorname{acoth}(a + bx^4)$ , x)

[Out]  $(x^4 * \log((a + bx^4 + 1)/(a + bx^4)))/8 - (x^4 * \log((a + bx^4 - 1)/(a + bx^4)))/8 + \log(a + bx^4 - 1)/(8*b) + \log(a + bx^4 + 1)/(8*b) - (a * \log(a + bx^4 - 1))/(8*b) + (a * \log(a + bx^4 + 1))/(8*b)$

**sympy** [A] time = 4.22, size = 60, normalized size = 1.36

$$\begin{cases} \frac{a \operatorname{acoth}(a + bx^4)}{4b} + \frac{x^4 \operatorname{acoth}(a + bx^4)}{4} + \frac{\log(a + bx^4 + 1)}{4b} - \frac{\operatorname{acoth}(a + bx^4)}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{acoth}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x**3*acoth(b*x**4+a),x)
```

```
[Out] Piecewise((a*acoth(a + b*x**4)/(4*b) + x**4*acoth(a + b*x**4)/4 + log(a + b  
*x**4 + 1)/(4*b) - acoth(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*acoth(a)/4, Tr  
ue))
```

### 3.294 $\int x^{-1+n} \coth^{-1}(a + bx^n) dx$

Optimal. Leaf size=47

$$\frac{\log(1 - (a + bx^n)^2)}{2bn} + \frac{(a + bx^n) \coth^{-1}(a + bx^n)}{bn}$$

[Out] (a+b\*x^n)\*arccoth(a+b\*x^n)/b/n+1/2\*ln(1-(a+b\*x^n)^2)/b/n

**Rubi [A]** time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6715, 6104, 5911, 260}

$$\frac{\log(1 - (a + bx^n)^2)}{2bn} + \frac{(a + bx^n) \coth^{-1}(a + bx^n)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)\*ArcCoth[a + b\*x^n], x]

[Out] ((a + b\*x^n)\*ArcCoth[a + b\*x^n])/(b\*n) + Log[1 - (a + b\*x^n)^2]/(2\*b\*n)

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 5911

Int[((a\_) + ArcCoth[(c\_)\*(x\_)])\*(b\_)^(p\_), x\_Symbol] := Simp[x\*(a + b\*ArcCoth[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcCoth[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 6104

Int[((a\_) + ArcCoth[(c\_) + (d\_)\*(x\_)])\*(b\_)^(p\_), x\_Symbol] := Dist[1/d, Subst[Int[(a + b\*ArcCoth[x])^p, x], x, c + d\*x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0]

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOf[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned}
\int x^{-1+n} \coth^{-1}(a + bx^n) dx &= \frac{\text{Subst}\left(\int \coth^{-1}(a + bx) dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \coth^{-1}(x) dx, x, a + bx^n\right)}{bn} \\
&= \frac{(a + bx^n) \coth^{-1}(a + bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{1-x^2} dx, x, a + bx^n\right)}{bn} \\
&= \frac{(a + bx^n) \coth^{-1}(a + bx^n)}{bn} + \frac{\log(1 - (a + bx^n)^2)}{2bn}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 42, normalized size = 0.89

$$\frac{\log(1 - (a + bx^n)^2) + 2(a + bx^n) \coth^{-1}(a + bx^n)}{2bn}$$

Antiderivative was successfully verified.

[In] Integrate[x<sup>(-1 + n)</sup>\*ArcCoth[a + b\*x<sup>n</sup>], x]

[Out] (2\*(a + b\*x<sup>n</sup>)\*ArcCoth[a + b\*x<sup>n</sup>] + Log[1 - (a + b\*x<sup>n</sup>)<sup>2</sup>])/(2\*b\*n)

**fricas [B]** time = 0.64, size = 108, normalized size = 2.30

$$\frac{(a + 1) \log(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a + 1) - (a - 1) \log(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a - 1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*arccoth(a+b\*x<sup>n</sup>), x, algorithm="fricas")

[Out] 1/2\*((a + 1)\*log(b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a + 1) - (a - 1)\*log(b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a - 1) + (b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)))\*log((b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a + 1)/(b\*cosh(n\*log(x)) + b\*sinh(n\*log(x)) + a - 1)))/(b\*n)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{n-1} \operatorname{arccoth}(bx^n + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*arccoth(a+b\*x<sup>n</sup>),x, algorithm="giac")

[Out] integrate(x<sup>(n - 1)</sup>\*arccoth(b\*x<sup>n</sup> + a), x)

**maple [B]** time = 0.13, size = 118, normalized size = 2.51

$$\frac{x^n \ln(1 + a + b x^n)}{2n} - \frac{x^n \ln(-1 + a + b x^n)}{2n} - \frac{\ln\left(x^n + \frac{a-1}{b}\right) a}{2nb} + \frac{\ln\left(x^n + \frac{1+a}{b}\right) a}{2nb} + \frac{\ln\left(x^n + \frac{a-1}{b}\right)}{2nb} + \frac{\ln\left(x^n + \frac{1+a}{b}\right)}{2nb}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(-1+n)</sup>\*arccoth(a+b\*x<sup>n</sup>),x)

[Out] 1/2/n\*x<sup>n</sup>\*ln(1+a+b\*x<sup>n</sup>)-1/2/n\*x<sup>n</sup>\*ln(-1+a+b\*x<sup>n</sup>)-1/2/n/b\*ln(x<sup>n</sup>+(a-1)/b)\*a+1/2/n/b\*ln(x<sup>n</sup>+(1+a)/b)\*a+1/2/n/b\*ln(x<sup>n</sup>+(a-1)/b)+1/2/n/b\*ln(x<sup>n</sup>+(1+a)/b)

**maxima [A]** time = 0.30, size = 40, normalized size = 0.85

$$\frac{2(bx^n + a) \operatorname{arccoth}(bx^n + a) + \log(-(bx^n + a)^2 + 1)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*arccoth(a+b\*x<sup>n</sup>),x, algorithm="maxima")

[Out] 1/2\*(2\*(b\*x<sup>n</sup> + a)\*arccoth(b\*x<sup>n</sup> + a) + log(-(b\*x<sup>n</sup> + a)<sup>2</sup> + 1))/(b\*n)

**mupad [B]** time = 2.51, size = 58, normalized size = 1.23

$$\frac{\frac{\ln(a^2 + b^2 x^{2n} + 2abx^n - 1)}{2}}{bn} + a \operatorname{acoth}(a + b x^n) + \frac{x^n \operatorname{acoth}(a + b x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(n - 1)</sup>\*acoth(a + b\*x<sup>n</sup>),x)

[Out] (log(a<sup>2</sup> + b<sup>2</sup>\*x<sup>(2\*n)</sup> + 2\*a\*b\*x<sup>n</sup> - 1)/2 + a\*acoth(a + b\*x<sup>n</sup>))/(b\*n) + (x<sup>n</sup>\*acoth(a + b\*x<sup>n</sup>))/n

**sympy [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>\*\*(-1+n)</sup>\*acoth(a+b\*x<sup>\*\*n</sup>),x)

[Out] Exception raised: HeuristicGCDFailed

### 3.295 $\int e^{c(a+bx)} \coth^{-1}(\sinh(ac + bcx)) dx$

**Optimal.** Leaf size=107

$$\frac{(1 - \sqrt{2}) \log(-e^{2c(a+bx)} + 3 - 2\sqrt{2})}{2bc} + \frac{(1 + \sqrt{2}) \log(-e^{2c(a+bx)} + 3 + 2\sqrt{2})}{2bc} + \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc}$$

[Out] exp(b\*c\*x+a\*c)\*arccoth(sinh(c\*(b\*x+a)))/b/c+1/2\*ln(3-exp(2\*c\*(b\*x+a))-2\*2^(1/2))\*(1-2^(1/2))/b/c+1/2\*ln(3-exp(2\*c\*(b\*x+a))+2\*2^(1/2))\*(1+2^(1/2))/b/c

**Rubi [A]** time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {2194, 6276, 2282, 12, 1247, 632, 31}

$$\frac{(1 - \sqrt{2}) \log(-e^{2c(a+bx)} + 3 - 2\sqrt{2})}{2bc} + \frac{(1 + \sqrt{2}) \log(-e^{2c(a+bx)} + 3 + 2\sqrt{2})}{2bc} + \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In] Int[E^(c\*(a + b\*x))\*ArcCoth[Sinh[a\*c + b\*c\*x]],x]

[Out] (E^(a\*c + b\*c\*x)\*ArcCoth[Sinh[c\*(a + b\*x)]])/(b\*c) + ((1 - Sqrt[2])\*Log[3 - 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c) + ((1 + Sqrt[2])\*Log[3 + 2\*Sqrt[2] - E^(2\*c\*(a + b\*x))])/(2\*b\*c)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 632

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(c\*d - e\*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c\*x), x], x] - Dist[(c\*d - e\*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && NiceSqrtQ[b^2 - 4\*a\*c]

#### Rule 1247

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(
p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

### Rule 2194

```
Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6276

```
Int[((a_) + ArcCoth[u_]*(b_))*(v_), x_Symbol] := With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] &&
InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_) + (d_)*x)^(m_) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

### Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \coth^{-1}(\sinh(ac + bcx)) dx &= \frac{\text{Subst}\left(\int e^x \coth^{-1}(\sinh(x)) dx, x, ac + bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x \cosh(x)}{1-\sinh^2(x)} dx, x, ac + bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x(-1-x^2)}{1-6x^2+x^4} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc} - \frac{2 \text{Subst}\left(\int \frac{x(-1-x^2)}{1-6x^2+x^4} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1-x}{1-6x+x^2} dx, x, e^{2ac+2bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc} + \frac{(1 - \sqrt{2}) \text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x} dx, x, e^{2ac+2bcx}\right)}{2bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\sinh(c(a + bx)))}{bc} + \frac{(1 - \sqrt{2}) \log(3 - 2\sqrt{2} - e^{2ac+2bcx})}{2bc} +
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 153, normalized size = 1.43

$$\frac{\log\left(-2e^{c(a+bx)} - e^{2c(a+bx)} + 1\right) + \log\left(2e^{c(a+bx)} - e^{2c(a+bx)} + 1\right) - 2\sqrt{2} \tanh^{-1}\left(\frac{e^{c(a+bx)}-1}{\sqrt{2}}\right) + 2\sqrt{2} \tanh^{-1}\left(\frac{e^{c(a+bx)}+1}{\sqrt{2}}\right)}{2bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Sinh[a\*c + b\*c\*x]], x]

[Out] (-2\*E^(c\*(a + b\*x))\*ArcCoth[1/(2\*E^(c\*(a + b\*x)))] - E^(c\*(a + b\*x))/2] - 2\*  
Sqrt[2]\*ArcTanh[(-1 + E^(c\*(a + b\*x)))/Sqrt[2]] + 2\*Sqrt[2]\*ArcTanh[(1 + E^(  
c\*(a + b\*x)))/Sqrt[2]] + Log[1 - 2\*E^(c\*(a + b\*x)) - E^(2\*c\*(a + b\*x))] +  
Log[1 + 2\*E^(c\*(a + b\*x)) - E^(2\*c\*(a + b\*x))]/(2\*b\*c)

**fricas [B]** time = 0.59, size = 233, normalized size = 2.18

$$\frac{(\cosh(bcx + ac) + \sinh(bcx + ac)) \log\left(\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right) + \sqrt{2} \log\left(\frac{3(2\sqrt{2}+3) \cosh(bcx+ac)^2 - 4(3\sqrt{2}+4) \cosh(bcx+ac) \sinh(bcx+ac) + 3}{\cosh(bcx+ac)^2 + \sinh(bcx+ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(sinh(b\*c\*x+a\*c)),x, algorithm="fricas")

[Out]  $\frac{1}{2} * ((\cosh(b*c*x + a*c) + \sinh(b*c*x + a*c)) * \log((\sinh(b*c*x + a*c) + 1) / (\sinh(b*c*x + a*c) - 1)) + \sqrt{2} * \log((3 * (2 * \sqrt{2}) + 3) * \cosh(b*c*x + a*c)^2 - 4 * (3 * \sqrt{2}) + 4) * \cosh(b*c*x + a*c) * \sinh(b*c*x + a*c) + 3 * (2 * \sqrt{2}) + 3) * \sinh(b*c*x + a*c)^2 - 2 * \sqrt{2} - 3) / (\cosh(b*c*x + a*c)^2 + \sinh(b*c*x + a*c)^2 - 3)) + \log(2 * (\cosh(b*c*x + a*c)^2 + \sinh(b*c*x + a*c)^2 - 3) / (\cosh(b*c*x + a*c)^2 - 2 * \cosh(b*c*x + a*c) * \sinh(b*c*x + a*c) + \sinh(b*c*x + a*c)^2)) / (b*c)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\sinh(bc x + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(sinh(b\*c\*x+a\*c)),x, algorithm="giac")

[Out] integrate(arccoth(sinh(b\*c\*x + a\*c))\*e^((b\*x + a)\*c), x)

**maple** [C] time = 0.51, size = 794, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(b\*x+a))\*arccoth(sinh(b\*c\*x+a\*c)),x)

[Out]  $\frac{1}{2} / b / c * \exp(c * (b * x + a)) * \ln(\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) - 1) - 1 / 4 * I / b / c * \text{Pi} * \text{csgn}(I * \exp(-c * (b * x + a))) * (-\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) + 1)^3 * \exp(c * (b * x + a)) - 1 / 4 * I / b / c * \text{Pi} * \text{csgn}(I * \exp(-c * (b * x + a))) * \text{csgn}(I * \exp(-c * (b * x + a))) * (-\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) + 1)^2 * \exp(c * (b * x + a)) + 1 / 4 * I / b / c * \text{Pi} * \text{csgn}(I * (-\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) + 1)) * \text{csgn}(I * \exp(-c * (b * x + a))) * (-\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) + 1)^2 * \exp(c * (b * x + a)) + 1 / 4 * I / b / c * \text{Pi} * \text{csgn}(I * (-\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) + 1)) * \text{csgn}(I * \exp(-c * (b * x + a))) * \text{csgn}(I * \exp(-c * (b * x + a))) * (-\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) + 1)) * \exp(c * (b * x + a)) + 1 / 4 * I / b / c * \text{Pi} * \text{csgn}(I * (\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) - 1)) * \text{csgn}(I * \exp(-c * (b * x + a))) * (\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) - 1)^2 * \exp(c * (b * x + a)) - 1 / 4 * I / b / c * \text{Pi} * \text{csgn}(I * (\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) - 1)) * \text{csgn}(I * \exp(-c * (b * x + a))) * \text{csgn}(I * \exp(-c * (b * x + a))) * (\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) - 1)) * \exp(c * (b * x + a)) - 1 / 4 * I / b / c * \text{Pi} * \text{csgn}(I * \exp(-c * (b * x + a))) * (\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) - 1))^3 * \exp(c * (b * x + a)) + 1 / 4 * I / b / c * \text{Pi} * \text{csgn}(I * \exp(-c * (b * x + a))) * \text{csgn}(I * \exp(-c * (b * x + a))) * (\exp(2 * c * (b * x + a)) + 2 * \exp(c * (b * x + a)) - 1))^2 * \exp(c * (b * x + a)) - 1 / 2 / b / c * \exp(c * (b * x + a)) * \ln(\exp(2 * c * (b * x + a)) - 2 * \exp(c * (b * x + a)) - 1) - 1 / 2 / b / c * \ln(\exp(2 * c * (b * x + a)) - (2^{(1/2)} - 1)^2) * 2^{(1/2)} + 1 / 2 / b / c * \ln(\exp(2 * c * (b * x + a)) - (1 + 2^{(1/2)})^2) * 2^{(1/2)} - 2 * a / b + 1 / 2 / b / c * \ln(\exp(2 * c * (b * x + a)) - (2^{(1/2)} - 1)^2) + 1 / 2 / b / c * \ln(\exp(2 * c * (b * x + a)) - (1 + 2^{(1/2)})^2)$



**maxima** [B] time = 0.41, size = 184, normalized size = 1.72

$$\frac{\operatorname{arccoth}(\sinh(bc x + ac)) e^{(bx+a)c}}{bc} + \frac{\sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(bcx+ac)+1}}{\sqrt{2}+e^{(bcx+ac)-1}}\right)}{2bc} - \frac{\sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(bcx+ac)-1}}{\sqrt{2}+e^{(bcx+ac)+1}}\right)}{2bc} + \frac{\log\left(e^{2bcx+2ac} + 2e^{bcx}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(sinh(b\*c\*x+a\*c)),x, algorithm="maxima")

[Out] arccoth(sinh(b\*c\*x + a\*c))\*e^((b\*x + a)\*c)/(b\*c) + 1/2\*sqrt(2)\*log(-(sqrt(2) - e^(b\*c\*x + a\*c) + 1)/(sqrt(2) + e^(b\*c\*x + a\*c) - 1))/(b\*c) - 1/2\*sqrt(2)\*log(-(sqrt(2) - e^(b\*c\*x + a\*c) - 1)/(sqrt(2) + e^(b\*c\*x + a\*c) + 1))/(b\*c) + 1/2\*log(e^(2\*b\*c\*x + 2\*a\*c) + 2\*e^(b\*c\*x + a\*c) - 1)/(b\*c) + 1/2\*log(e^(2\*b\*c\*x + 2\*a\*c) - 2\*e^(b\*c\*x + a\*c) - 1)/(b\*c)

**mupad** [B] time = 1.67, size = 187, normalized size = 1.75

$$\frac{\ln\left(6\sqrt{2}e^{2c(a+bx)} - 2\sqrt{2} - 8e^{2c(a+bx)}\right)(\sqrt{2} + 1)}{2bc} - \frac{e^{ac+bcx} \ln\left(1 - \frac{1}{\frac{e^{bcx}e^{ac}}{2} - \frac{e^{-bcx}e^{-ac}}{2}}\right)}{2bc} - \frac{\ln\left(2\sqrt{2} - 8e^{2c(a+bx)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(a + b\*x))\*acoth(sinh(a\*c + b\*c\*x)),x)

[Out] (log(6\*2^(1/2)\*exp(2\*c\*(a + b\*x)) - 2\*2^(1/2) - 8\*exp(2\*c\*(a + b\*x)))\*(2^(1/2) + 1))/(2\*b\*c) - (exp(a\*c + b\*c\*x)\*log(1 - 1/((exp(b\*c\*x)\*exp(a\*c))/2 - (exp(-b\*c\*x)\*exp(-a\*c))/2)))/(2\*b\*c) - (log(2\*2^(1/2) - 8\*exp(2\*c\*(a + b\*x)) - 6\*2^(1/2)\*exp(2\*c\*(a + b\*x)))\*(2^(1/2) - 1))/(2\*b\*c) + (log(1/((exp(b\*c\*x)\*exp(a\*c))/2 - (exp(-b\*c\*x)\*exp(-a\*c))/2) + 1)\*exp(a\*c + b\*c\*x))/(2\*b\*c)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*acoth(sinh(b\*c\*x+a\*c)),x)

[Out] Timed out

### 3.296 $\int e^{c(a+bx)} \coth^{-1}(\cosh(ac + bcx)) dx$

Optimal. Leaf size=49

$$\frac{\log(1 - e^{2c(a+bx)})}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a + bx)))}{bc}$$

[Out]  $\exp(b*c*x+a*c)*\operatorname{arccoth}(\cosh(c*(b*x+a)))/b/c + \ln(1-\exp(2*c*(b*x+a)))/b/c$

**Rubi [A]** time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2194, 6276, 2282, 12, 260}

$$\frac{\log(1 - e^{2c(a+bx)})}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(c*(a + b*x))*\text{ArcCoth}[\text{Cosh}[a*c + b*c*x]]}, x]$

[Out]  $(E^{(a*c + b*c*x)*\text{ArcCoth}[\text{Cosh}[c*(a + b*x)]])/(b*c) + \text{Log}[1 - E^{(2*c*(a + b*x))}]/(b*c)$

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 260

$\text{Int}[(x_)^{(m_)} / ((a_) + (b_)*(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

#### Rule 2194

$\text{Int}[(F_)^{((c_)*((a_) + (b_)*(x_)))^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n / (b*c*n*\text{Log}[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

#### Rule 2282

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c_)*((a_) + (b_)*x))* (F_)}[v_] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$

Rule 6276

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] := With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] &&
InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \coth^{-1}(\cosh(ac+bcx)) dx &= \frac{\text{Subst}\left(\int e^x \coth^{-1}(\cosh(x)) dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int e^x \text{csch}(x) dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\text{Subst}\left(\int \frac{2x}{-1+x^2} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{2 \text{Subst}\left(\int \frac{x}{-1+x^2} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\cosh(c(a+bx)))}{bc} + \frac{\log(1 - e^{2c(a+bx)})}{bc}
\end{aligned}$$

**Mathematica** [A] time = 0.09, size = 60, normalized size = 1.22

$$\frac{\log(1 - e^{2c(a+bx)}) + e^{c(a+bx)} \coth^{-1}\left(\frac{1}{2}e^{-c(a+bx)}(e^{2c(a+bx)} + 1)\right)}{bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Cosh[a\*c + b\*c\*x]], x]

[Out] (E^(c\*(a + b\*x))\*ArcCoth[(1 + E^(2\*c\*(a + b\*x)))/(2\*E^(c\*(a + b\*x)))] + Log[1 - E^(2\*c\*(a + b\*x))]/(b\*c)

**fricas** [A] time = 0.50, size = 92, normalized size = 1.88

$$\frac{(\cosh(bcx + ac) + \sinh(bcx + ac)) \log\left(\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right) + 2 \log\left(\frac{2 \sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(cosh(b\*c\*x+a\*c)),x, algorithm="fricas")

[Out] 1/2\*((cosh(b\*c\*x + a\*c) + sinh(b\*c\*x + a\*c))\*log((cosh(b\*c\*x + a\*c) + 1)/(cosh(b\*c\*x + a\*c) - 1)) + 2\*log(2\*sinh(b\*c\*x + a\*c)/(cosh(b\*c\*x + a\*c) - sinh(b\*c\*x + a\*c))))/(b\*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\cosh(bc x + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(cosh(b\*c\*x+a\*c)),x, algorithm="giac")

[Out] integrate(arccoth(cosh(b\*c\*x + a\*c))\*e^((b\*x + a)\*c), x)

maple [C] time = 0.43, size = 824, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(b\*x+a))\*arccoth(cosh(b\*c\*x+a\*c)),x)

[Out] 1/b/c\*exp(c\*(b\*x+a))\*ln(exp(c\*(b\*x+a))+1)+1/4\*I/b/c\*Pi\*csgn(I\*exp(-c\*(b\*x+a)))\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)\*csgn(I\*exp(-c\*(b\*x+a))\*(exp(c\*(b\*x+a))-1)^2)\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)^3\*exp(c\*(b\*x+a))+1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1))^2\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)\*exp(c\*(b\*x+a))-1/2\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1))\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)^2\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)\*csgn(I\*exp(-c\*(b\*x+a))\*(exp(c\*(b\*x+a))-1)^2)^2\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*exp(-c\*(b\*x+a)))\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)\*csgn(I\*exp(-c\*(b\*x+a))\*(exp(c\*(b\*x+a))+1)^2)\*exp(c\*(b\*x+a))+1/4\*I/b/c\*Pi\*csgn(I\*exp(-c\*(b\*x+a)))\*csgn(I\*exp(-c\*(b\*x+a))\*(exp(c\*(b\*x+a))+1)^2)^2\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)\*csgn(I\*exp(-c\*(b\*x+a))\*(exp(c\*(b\*x+a))+1)^2)^3\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*exp(-c\*(b\*x+a)))\*csgn(I\*exp(-c\*(b\*x+a))\*(exp(c\*(b\*x+a))-1)^2)^2\*exp(c\*(b\*x+a))+1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)^3\*exp(c\*(b\*x+a))+1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)\*csgn(I\*exp(-c\*(b\*x+a))\*(exp(c\*(b\*x+a))+1)^2)^2\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*exp(-c\*(b\*x+a))\*(exp(c\*(b\*x+a))+1)^2)^3\*exp(c\*(b\*x+a))-1/b/c\*exp(c\*(b\*x+a))\*ln(exp(c\*(b\*x+a))-1)-2\*a/b+1/b/c\*ln(exp(2\*c\*(b\*x+a))-1)

maxima [A] time = 0.33, size = 64, normalized size = 1.31

$$\frac{\operatorname{arccoth}(\cosh(bc x + ac)) e^{(bx+a)c}}{bc} + \frac{\log(e^{(bcx+ac)} + 1)}{bc} + \frac{\log(e^{(bcx+ac)} - 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="maxima")`

[Out]  $\operatorname{arccoth}(\cosh(b*c*x + a*c))*e^{((b*x + a)*c)/(b*c)} + \log(e^{(b*c*x + a*c)} + 1)/(b*c) + \log(e^{(b*c*x + a*c)} - 1)/(b*c)$

**mupad [B]** time = 1.83, size = 119, normalized size = 2.43

$$\frac{\ln(e^{2bcx}e^{2ac}-1)}{bc} - \frac{e^{ac+bcx} \ln\left(1 - \frac{1}{\frac{e^{bcx}e^{ac}}{2} + \frac{e^{-bcx}e^{-ac}}{2}}\right)}{2bc} + \frac{\ln\left(\frac{1}{\frac{e^{bcx}e^{ac}}{2} + \frac{e^{-bcx}e^{-ac}}{2}} + 1\right) e^{ac+bcx}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(c*(a + b*x))*acoth(cosh(a*c + b*c*x)),x)`

[Out]  $\log(\exp(2*b*c*x)*\exp(2*a*c) - 1)/(b*c) - (\exp(a*c + b*c*x)*\log(1 - 1/((\exp(b*c*x)*\exp(a*c))/2 + (\exp(-b*c*x)*\exp(-a*c))/2)))/(2*b*c) + (\log(1/((\exp(b*c*x)*\exp(a*c))/2 + (\exp(-b*c*x)*\exp(-a*c))/2) + 1)*\exp(a*c + b*c*x))/(2*b*c)$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(c*(b*x+a))*acoth(cosh(b*c*x+a*c)),x)`

[Out] Timed out

$$3.297 \quad \int e^{c(a+bx)} \coth^{-1}(\tanh(ac + bcx)) dx$$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a + bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

[Out]  $-\exp(b*c*x+a*c)/b/c+\exp(b*c*x+a*c)*\operatorname{arccoth}(\tanh(c*(b*x+a)))/b/c$

**Rubi [A]** time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2194, 6276}

$$\frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a + bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[E^{(c*(a + b*x))*\text{ArcCoth}[\text{Tanh}[a*c + b*c*x]]}, x]$

[Out]  $-(E^{(a*c + b*c*x)/(b*c)}) + (E^{(a*c + b*c*x)*\text{ArcCoth}[\text{Tanh}[c*(a + b*x)]])/(b*c)$

Rule 2194

$\text{Int}[(F_)^{((c_.)*((a_.) + (b_.)*(x_)))}^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\text{Log}[F]), x] /;$   $\text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 6276

$\text{Int}[(a_.) + \text{ArcCoth}[u_]*(b_.)]*(v_.), x\_Symbol] \rightarrow \text{With}\{w = \text{IntHide}[v, x]\}$   
 $, \text{Dist}[a + b*\text{ArcCoth}[u], w, x] - \text{Dist}[b, \text{Int}[\text{SimplifyIntegrand}[(w*D[u, x])/(1 - u^2), x], x], x] /;$   
 $\text{InverseFunctionFreeQ}[w, x] /;$   $\text{FreeQ}\{a, b\}, x] \&\&$   
 $\text{InverseFunctionFreeQ}[u, x] \&\& !\text{MatchQ}[v, ((c_.) + (d_.)*x)^{(m_.)} /;$   $\text{FreeQ}\{c, d, m\}, x] \&\& \text{FalseQ}[\text{FunctionOfLinear}[v*(a + b*\text{ArcCoth}[u]), x]]$

Rubi steps

$$\begin{aligned} \int e^{c(a+bx)} \coth^{-1}(\tanh(ac + bcx)) dx &= \frac{\text{Subst}\left(\int e^x \coth^{-1}(\tanh(x)) dx, x, ac + bcx\right)}{bc} \\ &= \frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a + bx)))}{bc} - \frac{\text{Subst}\left(\int e^x dx, x, ac + bcx\right)}{bc} \\ &= -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\tanh(c(a + bx)))}{bc} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)} \left( \coth^{-1} \left( \frac{e^{2c(a+bx)} - 1}{e^{2c(a+bx)} + 1} \right) - 1 \right)}{bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Tanh[a\*c + b\*c\*x]], x]

[Out] (E^(c\*(a + b\*x))\*(-1 + ArcCoth[(-1 + E^(2\*c\*(a + b\*x))]/(1 + E^(2\*c\*(a + b\*x)))))/(b\*c)

**fricas [A]** time = 0.58, size = 25, normalized size = 0.56

$$\frac{(bcx + ac - 1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(tanh(b\*c\*x+a\*c)), x, algorithm="fricas")

[Out] (b\*c\*x + a\*c - 1)\*e^(b\*c\*x + a\*c)/(b\*c)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\tanh(bc x + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(tanh(b\*c\*x+a\*c)), x, algorithm="giac")

[Out] integrate(arccoth(tanh(b\*c\*x + a\*c))\*e^((b\*x + a)\*c), x)

**maple [C]** time = 0.38, size = 349, normalized size = 7.76

$$\frac{e^{c(bx+a)} \ln(e^{c(bx+a)})}{bc} - \frac{i \left( -2\pi \operatorname{csgn} \left( \frac{i}{e^{2c(bx+a)} + 1} \right)^2 + \pi \operatorname{csgn} \left( i e^{c(bx+a)} \right)^2 \operatorname{csgn} \left( i e^{2c(bx+a)} \right) - 2\pi \operatorname{csgn} \left( i e^{c(bx+a)} \right) \operatorname{csgn} \left( i \right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(b\*x+a))\*arccoth(tanh(b\*c\*x+a\*c)), x)

[Out] 1/b/c\*exp(c\*(b\*x+a))\*ln(exp(c\*(b\*x+a)))-1/4\*I\*(-2\*Pi\*csgn(I/(exp(2\*c\*(b\*x+a))+1))^2+Pi\*csgn(I\*exp(c\*(b\*x+a)))^2\*csgn(I\*exp(2\*c\*(b\*x+a)))-2\*Pi\*csgn(I\*exp(c\*(b\*x+a)))\*csgn(I\*exp(2\*c\*(b\*x+a)))^2+Pi\*csgn(I\*exp(2\*c\*(b\*x+a)))^3+Pi\*

$\text{csgn}(I \cdot \exp(2 \cdot c \cdot (b \cdot x + a))) \cdot \text{csgn}(I / (\exp(2 \cdot c \cdot (b \cdot x + a)) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot c \cdot (b \cdot x + a))) / (\exp(2 \cdot c \cdot (b \cdot x + a)) + 1) - \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot c \cdot (b \cdot x + a))) \cdot \text{csgn}(I \cdot \exp(2 \cdot c \cdot (b \cdot x + a))) / (\exp(2 \cdot c \cdot (b \cdot x + a)) + 1)^2 + 2 \cdot \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot c \cdot (b \cdot x + a)) + 1))^3 - \text{Pi} \cdot \text{csgn}(I / (\exp(2 \cdot c \cdot (b \cdot x + a)) + 1)) \cdot \text{csgn}(I \cdot \exp(2 \cdot c \cdot (b \cdot x + a))) / (\exp(2 \cdot c \cdot (b \cdot x + a)) + 1)^2 + \text{Pi} \cdot \text{csgn}(I \cdot \exp(2 \cdot c \cdot (b \cdot x + a))) / (\exp(2 \cdot c \cdot (b \cdot x + a)) + 1))^3 - 4 \cdot I \cdot 2 \cdot \text{Pi}) / b / c \cdot \exp(c \cdot (b \cdot x + a))$

**maxima [A]** time = 0.33, size = 43, normalized size = 0.96

$$\frac{\text{arccoth}(\tanh(bc x + ac)) e^{(bx+a)c}}{bc} - \frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(tanh(b\*c\*x+a\*c)),x, algorithm="maxima")

[Out] arccoth(tanh(b\*c\*x + a\*c))\*e^((b\*x + a)\*c)/(b\*c) - e^(b\*c\*x + a\*c)/(b\*c)

**mupad [B]** time = 0.10, size = 28, normalized size = 0.62

$$\frac{e^{ac+bcx} (\text{acoth}(\tanh(ac + bcx)) - 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(a + b\*x))\*acoth(tanh(a\*c + b\*c\*x)),x)

[Out] (exp(a\*c + b\*c\*x)\*(acoth(tanh(a\*c + b\*c\*x)) - 1))/(b\*c)

**sympy [A]** time = 3.12, size = 66, normalized size = 1.47

$$\begin{cases} \frac{i\pi x}{2} & \text{for } b = 0 \wedge c = 0 \\ x e^{ac} \text{acoth}(\tanh(ac)) & \text{for } b = 0 \\ \frac{i\pi x}{2} & \text{for } c = 0 \\ \frac{e^{ac} e^{bcx} \text{acoth}(\tanh(ac+bcx))}{bc} - \frac{e^{ac} e^{bcx}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*acoth(tanh(b\*c\*x+a\*c)),x)

[Out] Piecewise((I\*pi\*x/2, Eq(b, 0) & Eq(c, 0)), (x\*exp(a\*c)\*acoth(tanh(a\*c)), Eq(b, 0)), (I\*pi\*x/2, Eq(c, 0)), (exp(a\*c)\*exp(b\*c\*x)\*acoth(tanh(a\*c + b\*c\*x))/(b\*c) - exp(a\*c)\*exp(b\*c\*x)/(b\*c), True))



### 3.298 $\int e^{c(a+bx)} \coth^{-1}(\coth(ac + bcx)) dx$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx} \coth^{-1}(\coth(c(a + bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

[Out]  $-\exp(b*c*x+a*c)/b/c+\exp(b*c*x+a*c)*\operatorname{arccoth}(\coth(c*(b*x+a)))/b/c$

**Rubi [A]** time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2194, 6276}

$$\frac{e^{ac+bcx} \coth^{-1}(\coth(c(a + bx)))}{bc} - \frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{c*(a + b*x)}*ArcCoth[Coth[a*c + b*c*x]], x]$

[Out]  $-(E^{a*c + b*c*x}/(b*c)) + (E^{a*c + b*c*x}*ArcCoth[Coth[c*(a + b*x)]])/(b*c)$

#### Rule 2194

$\operatorname{Int}[(F_{-})^{((c_{-})*(a_{-}) + (b_{-})*(x_{-}))}^{(n_{-})}, x\_Symbol] := \operatorname{Simp}[(F^{c*(a + b*x)})^n/(b*c*n*\operatorname{Log}[F]), x] /; \operatorname{FreeQ}[\{F, a, b, c, n\}, x]$

#### Rule 6276

$\operatorname{Int}[(a_{-}) + \operatorname{ArcCoth}[u_{-}*(b_{-})]*(v_{-}), x\_Symbol] := \operatorname{With}[\{w = \operatorname{IntHide}[v, x]\}, \operatorname{Dist}[a + b*\operatorname{ArcCoth}[u], w, x] - \operatorname{Dist}[b, \operatorname{Int}[\operatorname{SimplifyIntegrand}[(w*D[u, x])/(1 - u^2), x], x], x] /; \operatorname{InverseFunctionFreeQ}[w, x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{InverseFunctionFreeQ}[u, x] \&\& !\operatorname{MatchQ}[v, ((c_{-}) + (d_{-})*x)^{(m_{-})} /; \operatorname{FreeQ}[\{c, d, m\}, x]] \&\& \operatorname{FalseQ}[\operatorname{FunctionOfLinear}[v*(a + b*\operatorname{ArcCoth}[u]), x]]]$

#### Rubi steps

$$\begin{aligned} \int e^{c(a+bx)} \coth^{-1}(\coth(ac + bcx)) dx &= \frac{\operatorname{Subst}\left(\int e^x \coth^{-1}(\coth(x)) dx, x, ac + bcx\right)}{bc} \\ &= \frac{e^{ac+bcx} \coth^{-1}(\coth(c(a + bx)))}{bc} - \frac{\operatorname{Subst}\left(\int e^x dx, x, ac + bcx\right)}{bc} \\ &= -\frac{e^{ac+bcx}}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\coth(c(a + bx)))}{bc} \end{aligned}$$

**Mathematica** [A] time = 0.09, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)} \left( \coth^{-1} \left( \frac{e^{2c(a+bx)} + 1}{e^{2c(a+bx)} - 1} \right) - 1 \right)}{bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Coth[a\*c + b\*c\*x]], x]

[Out] (E^(c\*(a + b\*x))\*(-1 + ArcCoth[(1 + E^(2\*c\*(a + b\*x)))/(-1 + E^(2\*c\*(a + b\*x))]))/(b\*c)

**fricas** [A] time = 0.53, size = 25, normalized size = 0.56

$$\frac{(bcx + ac - 1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(coth(b\*c\*x+a\*c)), x, algorithm="fricas")

[Out] (b\*c\*x + a\*c - 1)\*e^(b\*c\*x + a\*c)/(b\*c)

**giac** [A] time = 0.14, size = 35, normalized size = 0.78

$$\frac{(b^2c^2x + abc^2 - bc)e^{(bcx+ac)}}{b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(coth(b\*c\*x+a\*c)), x, algorithm="giac")

[Out] (b^2\*c^2\*x + a\*b\*c^2 - b\*c)\*e^(b\*c\*x + a\*c)/(b^2\*c^2)

**maple** [A] time = 0.29, size = 68, normalized size = 1.51

$$\frac{(xbc + ac)e^{xbc+ac} - e^{xbc+ac} + e^{xbc+ac} (\operatorname{arccoth}(\coth(xbc + ac)) - xbc - ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(b\*x+a))\*arccoth(coth(b\*c\*x+a\*c)), x)

[Out] 1/b/c\*((b\*c\*x+a\*c)\*exp(b\*c\*x+a\*c)-exp(b\*c\*x+a\*c)+exp(b\*c\*x+a\*c)\*(arccoth(coth(b\*c\*x+a\*c))-x\*b\*c-a\*c))

**maxima** [A] time = 0.31, size = 42, normalized size = 0.93

$$\frac{ae^{(bcx+ac)}}{b} + \frac{(bcxe^{(ac)} - e^{(ac)})e^{(bcx)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(coth(b\*c\*x+a\*c)),x, algorithm="maxima")

[Out] a\*e^(b\*c\*x + a\*c)/b + (b\*c\*x\*e^(a\*c) - e^(a\*c))\*e^(b\*c\*x)/(b\*c)

**mupad** [B] time = 1.21, size = 28, normalized size = 0.62

$$\frac{e^{ac+bcx} (\operatorname{acoth}(\operatorname{coth}(ac + bcx)) - 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(a + b\*x))\*acoth(coth(a\*c + b\*c\*x)),x)

[Out] (exp(a\*c + b\*c\*x)\*(acoth(coth(a\*c + b\*c\*x)) - 1))/(b\*c)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int e^{bcx} \operatorname{acoth}(\operatorname{coth}(ac + bcx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*acoth(coth(b\*c\*x+a\*c)),x)

[Out] exp(a\*c)\*Integral(exp(b\*c\*x)\*acoth(coth(a\*c + b\*c\*x)), x)

### 3.299 $\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac + bcx)) dx$

Optimal. Leaf size=49

$$\frac{\log(1 - e^{2c(a+bx)})}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a + bx)))}{bc}$$

[Out]  $\exp(b*c*x+a*c)*\operatorname{arccoth}(\operatorname{sech}(c*(b*x+a)))/b/c + \ln(1 - \exp(2*c*(b*x+a)))/b/c$

**Rubi [A]** time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {2194, 6276, 2282, 12, 260}

$$\frac{\log(1 - e^{2c(a+bx)})}{bc} + \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(c*(a + b*x))*\operatorname{ArcCoth}[\operatorname{Sech}[a*c + b*c*x]]], x]$

[Out]  $(E^{(a*c + b*c*x)*\operatorname{ArcCoth}[\operatorname{Sech}[c*(a + b*x)]])/(b*c) + \operatorname{Log}[1 - E^{(2*c*(a + b*x))}]/(b*c)$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 260

$\operatorname{Int}[(x_)^{(m_*)}/((a_) + (b_*)*(x_)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \operatorname{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \operatorname{EqQ}[m, n - 1]$

#### Rule 2194

$\operatorname{Int}[(F_)^{((c_*)*((a_*) + (b_*)*(x_)))^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(c*(a + b*x))})^n/(b*c*n*\operatorname{Log}[F]), x] /; \operatorname{FreeQ}[\{F, a, b, c, n\}, x]$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] \rightarrow \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \operatorname{FunctionOfExponentialQ}[u, x] \ \&\& \ !\operatorname{MatchQ}[u, (w_*)*((a_*)*(v_)^{(n_*)})^{(m_*)} /; \operatorname{FreeQ}[\{a, m, n\}, x] \ \&\& \ \operatorname{IntegerQ}[m*n]] \ \&\& \ !\operatorname{MatchQ}[u, E^{((c_*)*((a_*) + (b_*)*x))* (F_)}[v_] /; \operatorname{FreeQ}[\{a, b, c\}, x] \ \&\& \ \operatorname{InverseFunctionQ}[F[x]]]$

Rule 6276

```
Int[((a_.) + ArcCoth[u_]*(b_.))*(v_), x_Symbol] := With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] &&
InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \coth^{-1}(\operatorname{sech}(ac+bcx)) dx &= \frac{\operatorname{Subst}\left(\int e^x \coth^{-1}(\operatorname{sech}(x)) dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int e^x \operatorname{csch}(x) dx, x, ac+bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{2x}{-1+x^2} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{2 \operatorname{Subst}\left(\int \frac{x}{-1+x^2} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\log(1 - e^{2c(a+bx)})}{bc}
\end{aligned}$$

**Mathematica** [A] time = 0.09, size = 59, normalized size = 1.20

$$\frac{\log(1 - e^{2c(a+bx)}) + e^{c(a+bx)} \coth^{-1}\left(\frac{2e^{c(a+bx)}}{e^{2c(a+bx)}+1}\right)}{bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Sech[a\*c + b\*c\*x]], x]

[Out] (E^(c\*(a + b\*x))\*ArcCoth[(2\*E^(c\*(a + b\*x)))/(1 + E^(2\*c\*(a + b\*x)))] + Log[1 - E^(2\*c\*(a + b\*x))]/(b\*c)

**fricas** [A] time = 0.67, size = 93, normalized size = 1.90

$$\frac{(\cosh(bcx + ac) + \sinh(bcx + ac)) \log\left(\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right) + 2 \log\left(\frac{2 \sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(sech(b\*c\*x+a\*c)),x, algorithm="fricas")

[Out] 1/2\*((cosh(b\*c\*x + a\*c) + sinh(b\*c\*x + a\*c))\*log(-(cosh(b\*c\*x + a\*c) + 1)/(cosh(b\*c\*x + a\*c) - 1)) + 2\*log(2\*sinh(b\*c\*x + a\*c)/(cosh(b\*c\*x + a\*c) - sinh(b\*c\*x + a\*c))))/(b\*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\operatorname{sech}(bcx + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(sech(b\*c\*x+a\*c)),x, algorithm="giac")

[Out] integrate(arccoth(sech(b\*c\*x + a\*c))\*e^((b\*x + a)\*c), x)

maple [C] time = 0.48, size = 939, normalized size = 19.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(c\*(b\*x+a))\*arccoth(sech(b\*c\*x+a\*c)),x)

[Out] 1/2\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1)^2/(exp(2\*c\*(b\*x+a))+1))^2\*exp(c\*(b\*x+a))+1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)^3\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I/(exp(2\*c\*(b\*x+a))+1))\*csgn(I\*(exp(c\*(b\*x+a))-1)^2/(exp(2\*c\*(b\*x+a))+1))^2\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1))^2\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)\*exp(c\*(b\*x+a))+1/2\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1))\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)^2\*exp(c\*(b\*x+a))-1/2\*I/b/c\*exp(c\*(b\*x+a))\*Pi+1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)\*csgn(I\*(exp(c\*(b\*x+a))+1)^2/(exp(2\*c\*(b\*x+a))+1))^2\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1)^2/(exp(2\*c\*(b\*x+a))+1))^3\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)\*csgn(I/(exp(2\*c\*(b\*x+a))+1))\*csgn(I\*(exp(c\*(b\*x+a))+1)^2/(exp(2\*c\*(b\*x+a))+1))\*exp(c\*(b\*x+a))+1/4\*I/b/c\*Pi\*csgn(I/(exp(2\*c\*(b\*x+a))+1))\*csgn(I\*(exp(c\*(b\*x+a))+1)^2/(exp(2\*c\*(b\*x+a))+1))^2\*exp(c\*(b\*x+a))+1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)\*csgn(I/(exp(2\*c\*(b\*x+a))+1))\*csgn(I\*(exp(c\*(b\*x+a))-1)^2/(exp(2\*c\*(b\*x+a))+1))\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1)^2)^3\*exp(c\*(b\*x+a))-1/2\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))-1))\*csgn(I\*(exp(c\*(b\*x+a))-1)^2)^2\*exp(c\*(b\*x+a))-1/4\*I/b/c\*Pi\*csgn(I\*(exp(c\*(b\*x+a))+1)^2/(exp(2\*c\*(b\*x+a))+1))^3\*exp(c\*(b\*x+a))-1/b/c\*exp(c\*(b\*x+a))\*ln(exp(c\*(b\*x+a))-1)+1/b/c\*exp(c\*(b\*x+a))\*ln(exp(c\*(b\*x+a))+1)-2\*a/b+1/b/c\*ln(exp(2\*c\*(b\*x+a))-1)

**maxima [A]** time = 0.33, size = 64, normalized size = 1.31

$$\frac{\operatorname{arccoth}(\operatorname{sech}(bcx + ac)) e^{(bx+ac)}}{bc} + \frac{\log(e^{(bcx+ac)} + 1)}{bc} + \frac{\log(e^{(bcx+ac)} - 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(sech(b\*c\*x+a\*c)),x, algorithm="maxima")

[Out] arccoth(sech(b\*c\*x + a\*c))\*e^((b\*x + a)\*c)/(b\*c) + log(e^(b\*c\*x + a\*c) + 1)/(b\*c) + log(e^(b\*c\*x + a\*c) - 1)/(b\*c)

**mupad [B]** time = 1.44, size = 111, normalized size = 2.27

$$\frac{\ln(e^{2bcx} e^{2ac} - 1)}{bc} - \frac{e^{bcx} e^{ac} \ln\left(1 - \frac{e^{-bcx} e^{-ac}}{2} - \frac{e^{bcx} e^{ac}}{2}\right)}{2bc} + \frac{e^{bcx} e^{ac} \ln\left(\frac{e^{bcx} e^{ac}}{2} + \frac{e^{-bcx} e^{-ac}}{2} + 1\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(1/cosh(a\*c + b\*c\*x))\*exp(c\*(a + b\*x)),x)

[Out] log(exp(2\*b\*c\*x)\*exp(2\*a\*c) - 1)/(b\*c) - (exp(b\*c\*x)\*exp(a\*c)\*log(1 - (exp(-b\*c\*x)\*exp(-a\*c))/2 - (exp(b\*c\*x)\*exp(a\*c))/2))/(2\*b\*c) + (exp(b\*c\*x)\*exp(a\*c)\*log((exp(b\*c\*x)\*exp(a\*c))/2 + (exp(-b\*c\*x)\*exp(-a\*c))/2 + 1))/(2\*b\*c)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*acoth(sech(b\*c\*x+a\*c)),x)

[Out] Timed out

### 3.300 $\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac + bcx)) dx$

**Optimal.** Leaf size=107

$$\frac{(1 - \sqrt{2}) \log(-e^{2c(a+bx)} + 3 - 2\sqrt{2})}{2bc} + \frac{(1 + \sqrt{2}) \log(-e^{2c(a+bx)} + 3 + 2\sqrt{2})}{2bc} + \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc}$$

[Out]  $\exp(b*c*x+a*c)*\operatorname{arccoth}(\operatorname{csch}(c*(b*x+a)))/b/c+1/2*\ln(3-\exp(2*c*(b*x+a))-2*2^{(1/2)}*(1-2^{(1/2)}))/b/c+1/2*\ln(3-\exp(2*c*(b*x+a))+2*2^{(1/2)}*(1+2^{(1/2)}))/b/c$

**Rubi [A]** time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {2194, 6276, 2282, 12, 1247, 632, 31}

$$\frac{(1 - \sqrt{2}) \log(-e^{2c(a+bx)} + 3 - 2\sqrt{2})}{2bc} + \frac{(1 + \sqrt{2}) \log(-e^{2c(a+bx)} + 3 + 2\sqrt{2})}{2bc} + \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[E^{(c*(a + b*x))*\operatorname{ArcCoth}[\operatorname{Csch}[a*c + b*c*x]]], x]$

[Out]  $(E^{(a*c + b*c*x)*\operatorname{ArcCoth}[\operatorname{Csch}[c*(a + b*x)]])/(b*c) + ((1 - \operatorname{Sqrt}[2])*Log[3 - 2*\operatorname{Sqrt}[2] - E^{(2*c*(a + b*x))}])/(2*b*c) + ((1 + \operatorname{Sqrt}[2])*Log[3 + 2*\operatorname{Sqrt}[2] - E^{(2*c*(a + b*x))}])/(2*b*c)$

#### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 31

$\operatorname{Int}[((a_) + (b_.)*(x_))^{(-1)}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

#### Rule 632

$\operatorname{Int}[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[(c*d - e*(b/2 - q/2))/q, \operatorname{Int}[1/(b/2 - q/2 + c*x), x], x] - \operatorname{Dist}[(c*d - e*(b/2 + q/2))/q, \operatorname{Int}[1/(b/2 + q/2 + c*x), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \operatorname{NeQ}[2*c*d - b*e, 0] \ \&\& \ \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \operatorname{NiceSqrtQ}[b^2 - 4*a*c]$

#### Rule 1247



```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(
p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

#### Rule 2194

```
Int[((F_)^((c_)*((a_) + (b_)*(x_))))^(n_), x_Symbol] := Simp[(F^(c*(a +
b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 6276

```
Int[((a_) + ArcCoth[u_]*(b_))*(v_), x_Symbol] := With[{w = IntHide[v, x]}
, Dist[a + b*ArcCoth[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x] /; FreeQ[{a, b}, x] &&
InverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_) + (d_)*x)^(m_)] /; FreeQ
[{c, d, m}, x] && FalseQ[FunctionOfLinear[v*(a + b*ArcCoth[u]), x]]
```

#### Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac + bcx)) dx &= \frac{\operatorname{Subst}\left(\int e^x \coth^{-1}(\operatorname{csch}(x)) dx, x, ac + bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{e^x \coth(x) \operatorname{csch}(x)}{1 - \operatorname{csch}^2(x)} dx, x, ac + bcx\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{2x(1+x^2)}{1-6x^2+x^4} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc} + \frac{2 \operatorname{Subst}\left(\int \frac{x(1+x^2)}{1-6x^2+x^4} dx, x, e^{ac+bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{1+x}{1-6x+x^2} dx, x, e^{2ac+2bcx}\right)}{bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc} + \frac{(1 - \sqrt{2}) \operatorname{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x} dx, x, e^{2ac+2bcx}\right)}{2bc} \\
&= \frac{e^{ac+bcx} \coth^{-1}(\operatorname{csch}(c(a + bx)))}{bc} + \frac{(1 - \sqrt{2}) \log(3 - 2\sqrt{2} - e^{2ac+2bcx})}{2bc} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.17, size = 150, normalized size = 1.40

$$\frac{\log\left(-2e^{c(a+bx)} - e^{2c(a+bx)} + 1\right) + \log\left(2e^{c(a+bx)} - e^{2c(a+bx)} + 1\right) - 2\sqrt{2} \tanh^{-1}\left(\frac{e^{c(a+bx)} - 1}{\sqrt{2}}\right) + 2\sqrt{2} \tanh^{-1}\left(\frac{e^{c(a+bx)} + 1}{\sqrt{2}}\right)}{2bc}$$

Warning: Unable to verify antiderivative.

[In] Integrate[E^(c\*(a + b\*x))\*ArcCoth[Csch[a\*c + b\*c\*x]], x]

[Out] (2\*E^(c\*(a + b\*x))\*ArcCoth[(2\*E^(c\*(a + b\*x)))/(-1 + E^(2\*c\*(a + b\*x)))] - 2\*sqrt[2]\*ArcTanh[(-1 + E^(c\*(a + b\*x)))/sqrt[2]] + 2\*sqrt[2]\*ArcTanh[(1 + E^(c\*(a + b\*x)))/sqrt[2]] + Log[1 - 2\*E^(c\*(a + b\*x)) - E^(2\*c\*(a + b\*x))] + Log[1 + 2\*E^(c\*(a + b\*x)) - E^(2\*c\*(a + b\*x))]/(2\*b\*c)

**fricas [B]** time = 0.46, size = 234, normalized size = 2.19

$$\frac{(\cosh(bcx + ac) + \sinh(bcx + ac)) \log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right) + \sqrt{2} \log\left(\frac{3(2\sqrt{2}+3) \cosh(bcx+ac)^2 - 4(3\sqrt{2}+4) \cosh(bcx+ac) \sinh(bcx+ac) + 3}{\cosh(bcx+ac)^2 + \sinh(bcx+ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="fricas")
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/(sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arccoth}(\operatorname{csch}(bcx + ac)) e^{(bx+a)c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="giac")
[Out] integrate(arccoth(csch(b*c*x + a*c))*e^((b*x + a)*c), x)
```

**maple** [C] time = 0.65, size = 920, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x)
[Out] 1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*Pi-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^3*exp(c*(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))-1)+1/2/b/c*ln(exp(2*c*(
```

$b*x+a))-(1+2^{(1/2)})^2)*2^{(1/2)}-1/2/b/c*\ln(\exp(2*c*(b*x+a))-(2^{(1/2)}-1)^2)*2^{(1/2)}-2*a/b+1/2/b/c*\ln(\exp(2*c*(b*x+a))-(1+2^{(1/2)})^2)+1/2/b/c*\ln(\exp(2*c*(b*x+a))-(2^{(1/2)}-1)^2)$

**maxima [B]** time = 0.42, size = 184, normalized size = 1.72

$$\frac{\operatorname{arcoth}(\operatorname{csch}(bcx + ac)) e^{((bx+a)c)}}{bc} + \frac{\sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(bcx+ac)+1}}{\sqrt{2}+e^{(bcx+ac)-1}}\right)}{2bc} - \frac{\sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(bcx+ac)-1}}{\sqrt{2}+e^{(bcx+ac)+1}}\right)}{2bc} + \frac{\log\left(e^{(2bcx+2ac)} + 2e^{(bcx+a)c}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*arccoth(csch(b\*c\*x+a\*c)), x, algorithm="maxima")

[Out] arccoth(csch(b\*c\*x + a\*c))\*e^((b\*x + a)\*c)/(b\*c) + 1/2\*sqrt(2)\*log(-(sqrt(2) - e^(b\*c\*x + a\*c) + 1)/(sqrt(2) + e^(b\*c\*x + a\*c) - 1))/(b\*c) - 1/2\*sqrt(2)\*log(-(sqrt(2) - e^(b\*c\*x + a\*c) - 1)/(sqrt(2) + e^(b\*c\*x + a\*c) + 1))/(b\*c) + 1/2\*log(e^(2\*b\*c\*x + 2\*a\*c) + 2\*e^(b\*c\*x + a\*c) - 1)/(b\*c) + 1/2\*log(e^(2\*b\*c\*x + 2\*a\*c) - 2\*e^(b\*c\*x + a\*c) - 1)/(b\*c)

**mupad [B]** time = 1.60, size = 179, normalized size = 1.67

$$\frac{e^{ac+bcx} \ln\left(\frac{e^{bcx} e^{ac}}{2} - \frac{e^{-bcx} e^{-ac}}{2} + 1\right)}{2bc} - \frac{e^{ac+bcx} \ln\left(\frac{e^{-bcx} e^{-ac}}{2} - \frac{e^{bcx} e^{ac}}{2} + 1\right)}{2bc} + \frac{\ln\left(6\sqrt{2} e^{2c(a+bx)} - 2\sqrt{2} - 8e^{2c(a+bx)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acoth(1/sinh(a\*c + b\*c\*x))\*exp(c\*(a + b\*x)), x)

[Out] (exp(a\*c + b\*c\*x)\*log((exp(b\*c\*x)\*exp(a\*c))/2 - (exp(-b\*c\*x)\*exp(-a\*c))/2 + 1))/(2\*b\*c) - (exp(a\*c + b\*c\*x)\*log((exp(-b\*c\*x)\*exp(-a\*c))/2 - (exp(b\*c\*x)\*exp(a\*c))/2 + 1))/(2\*b\*c) + (log(6\*2^(1/2)\*exp(2\*c\*(a + b\*x)) - 2\*2^(1/2) - 8\*exp(2\*c\*(a + b\*x)))\*(2^(1/2) + 1))/(2\*b\*c) - (log(2\*2^(1/2) - 8\*exp(2\*c\*(a + b\*x)) - 6\*2^(1/2)\*exp(2\*c\*(a + b\*x)))\*(2^(1/2) - 1))/(2\*b\*c)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$e^{ac} \int e^{bcx} \operatorname{acoth}(\operatorname{csch}(ac + bcx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(c\*(b\*x+a))\*acoth(csch(b\*c\*x+a\*c)), x)

[Out] exp(a\*c)\*Integral(exp(b\*c\*x)\*acoth(csch(a\*c + b\*c\*x)), x)

# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],

```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```



```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```



```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```