Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.7-Inverse-hyperbolic-tangent-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [361]. This is test number [197].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (361)	% 0.00 (0)
Mathematica	% 100.00 (361)	% 0.00 (0)
Maple	% 94.74 (342)	% 5.26 (19)
Maxima	% 73.96 (267)	% 26.04 (94)
Fricas	% 93.63 (338)	% 6.37 (23)
Sympy	% 21.61 (78)	% 78.39 (283)
Giac	% 71.19 (257)	% 28.81 (104)
Mupad	% 66.20 (239)	% 33.80 (122)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

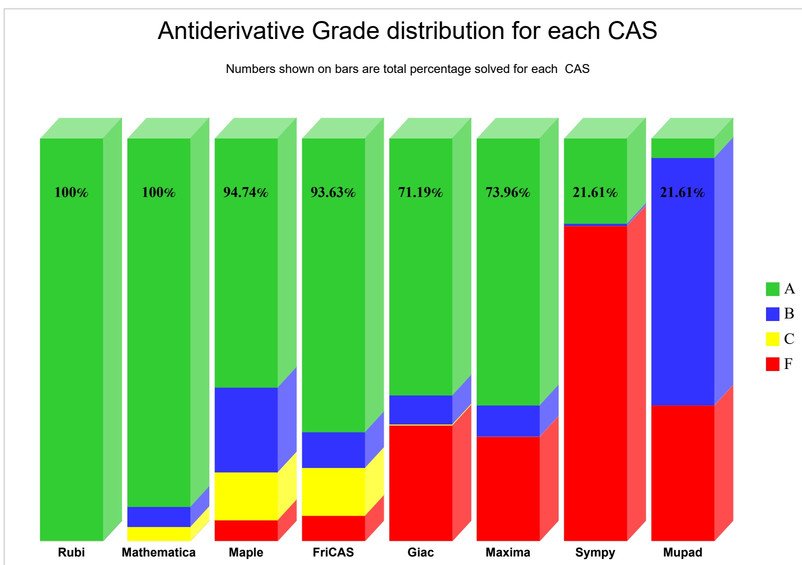
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

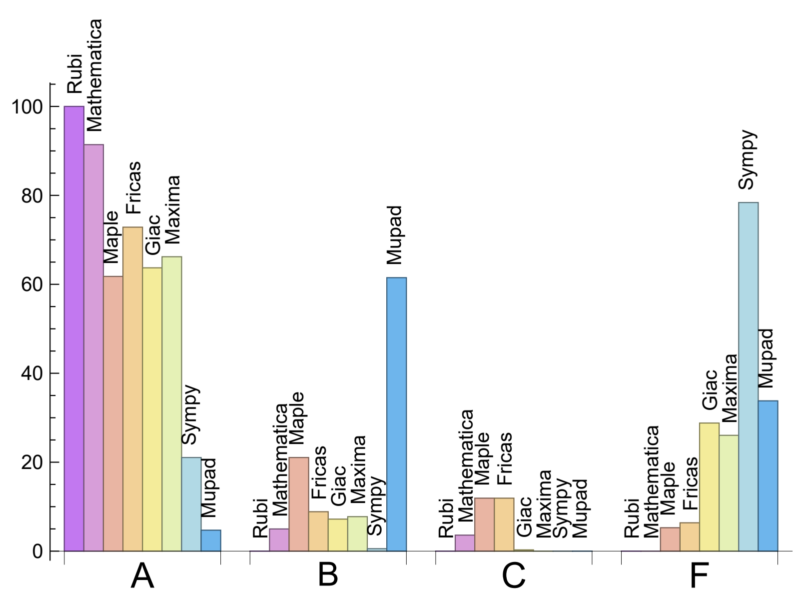
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	91.41	4.99	3.60	0.00
Maple	61.77	21.05	11.91	5.26
Maxima	66.20	7.76	0.00	26.04
Fricas	72.85	8.86	11.91	6.37
Sympy	21.05	0.55	0.00	78.39
Giac	63.71	7.20	0.28	28.81
Mupad	4.71	61.50	0.00	33.80

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	19	36.84 %	0.00 %	63.16 %
Maxima	94	98.94 %	1.06 %	0.00 %
Fricas	23	100.00 %	0.00 %	0.00 %
Sympy	283	77.03 %	18.37 %	4.59 %
Giac	104	78.85 %	0.00 %	21.15 %
Mupad	122	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

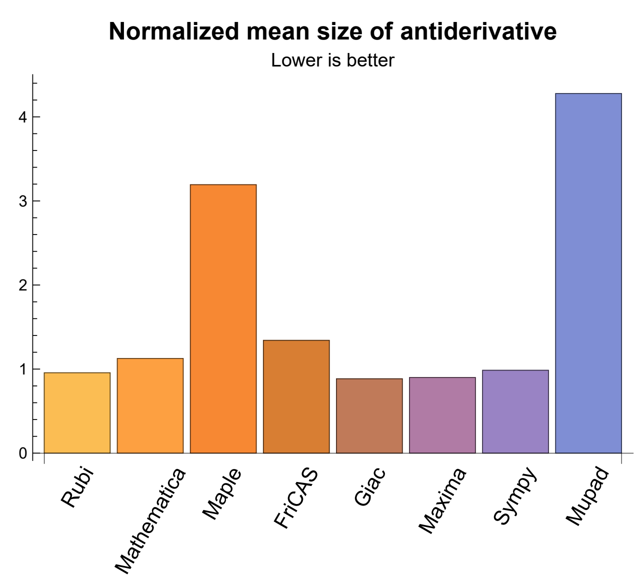
1.3 Performance

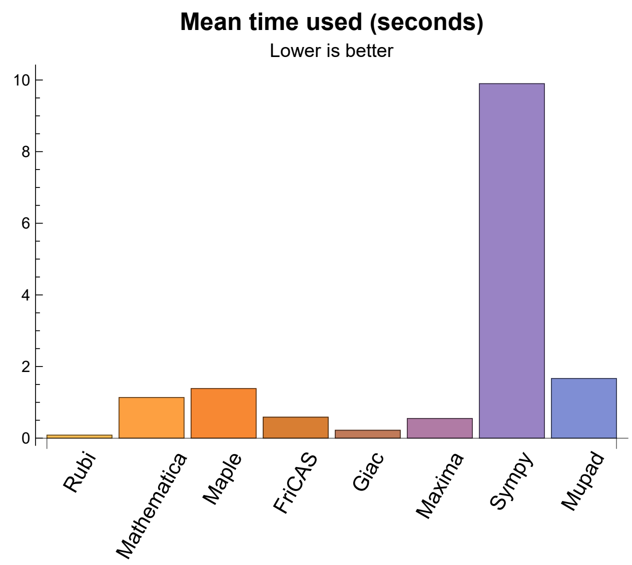
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	95.79	0.96	80.00	1.00
Mathematica	1.14	119.43	1.13	74.00	0.88
Maple	1.38	477.21	3.19	105.00	1.11
Maxima	0.55	71.30	0.90	53.00	0.83
Fricas	0.59	160.70	1.34	67.00	0.97
Sympy	9.90	42.77	0.99	41.00	0.99
Giac	0.22	59.20	0.88	46.00	0.57
Mupad	1.67	365.98	4.28	211.00	3.17

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{30, 34, 35, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {4, 33, 291, 296, 298, 299, 305, 310, 314, 319, 323, 327, 331, 336, 340, 344, 355, 356, 357, 358, 359, 360}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

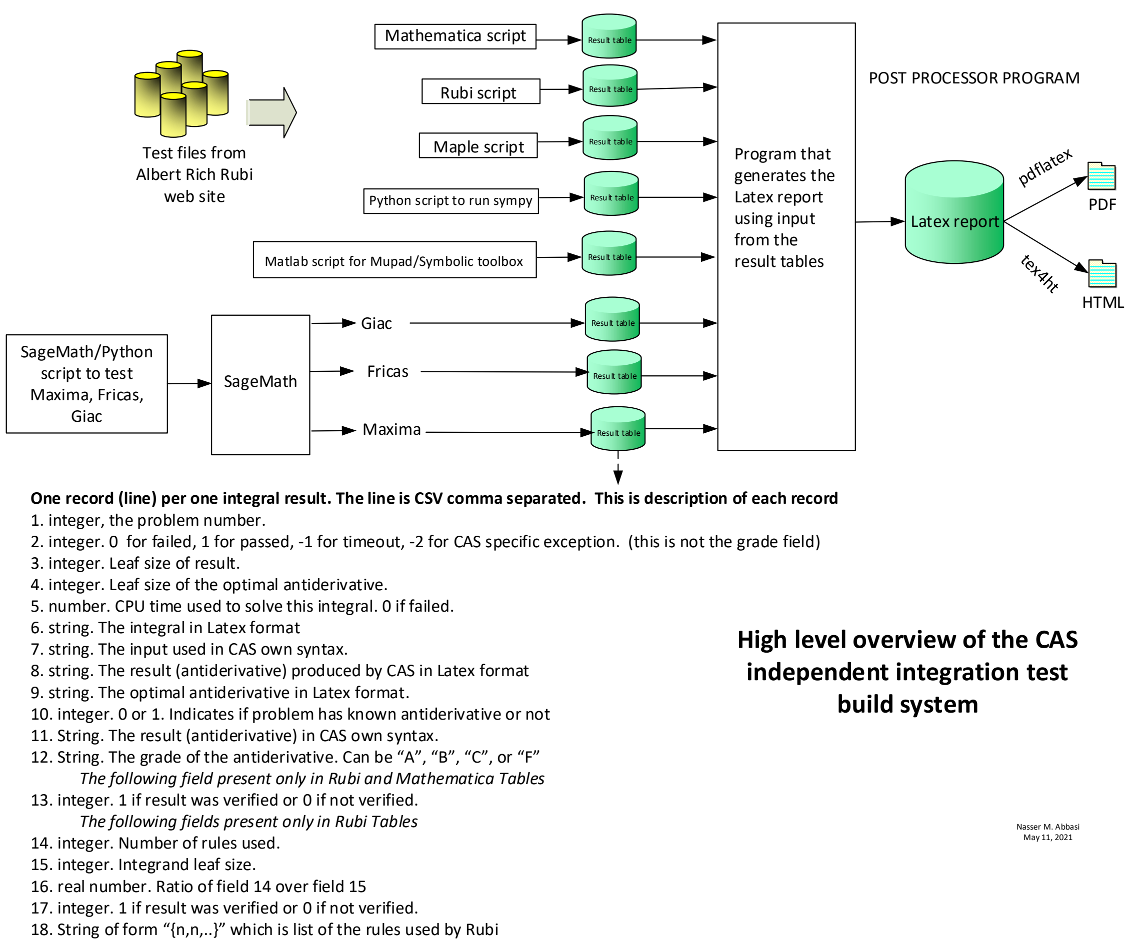
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 320, 321, 322, 324, 325, 326, 328, 330, 331, 332, 333, 334, 335, 337, 338, 339, 341, 342, 343, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360 }

B grade: { 47, 57, 71, 78, 84, 291, 296, 305, 310, 312, 319, 323, 327, 329, 336, 340, 344, 346 }

C grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 361 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 13, 14, 15, 28, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 88, 89, 90, 91, 92, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 202, 203, 204, 205, 206, 209, 210, 211, 212, 213, 214, 217, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 259, 261, 262, 263, 264, 270, 275, 276, 277, 278, 279, 280, 281, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345, 346, 347, 348, 352, 357 }

B grade: { 9, 10, 11, 12, 29, 31, 32, 71, 78, 84, 86, 87, 94, 95, 96, 103, 104, 105, 134, 149, 150, 151, 158, 159, 191, 192, 193, 194, 199, 200, 201, 207, 208, 215, 216, 218, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 249, 250, 257, 258, 260, 266, 267, 268, 269, 286, 291, 296, 300, 305, 310, 315, 319, 323, 327, 332, 336, 340, 344, 349, 350, 351, 353, 354 }

C grade: { 274, 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 298, 299, 302, 303, 304, 307, 308, 309, 312, 313, 314, 317, 318, 321, 322, 325, 326, 329, 330, 331, 334, 335, 338, 339, 342, 343, 355, 356, 358, 359, 360, 361 }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 85, 93, 102, 265, 271, 272, 273 }

2.1.4 Maxima

A grade: { 5, 6, 7, 8, 9, 10, 11, 28, 29, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 158, 159, 160, 161, 162, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 245, 246, 247, 248, 254, 255, 256, 262, 263, 264, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 316, 320, 324, 328, 333, 337, 341, 345, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359 }

B grade: { 12, 48, 58, 71, 72, 78, 84, 315, 319, 321, 322, 323, 325, 326, 327, 332, 336, 338, 339, 340, 342, 343, 344, 346, 347, 349, 355, 360 }

C grade: { }

F grade: { 1, 2, 3, 4, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 54, 85, 93, 102, 116, 117, 118, 119, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 145, 146, 147, 148, 154, 155, 156, 157, 163, 164, 165, 166, 215, 216, 217, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 243, 244, 249, 250, 251, 252, 253, 257, 258, 259, 260, 261, 265, 271, 272, 273, 312, 313, 314, 317, 318, 329, 330, 331, 334, 335, 361 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 28, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 269, 270, 274, 275, 276, 277, 278, 279, 280, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 340, 341, 344, 345, 352, 356, 357, 358, 359 }

B grade: { 13, 14, 15, 29, 44, 54, 58, 65, 72, 84, 133, 162, 266, 267, 268, 281, 286, 291, 296, 300, 305, 310, 315, 319, 323, 327, 332, 336, 346, 349, 355, 360 }

C grade: { 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 298, 299, 302, 303, 304, 307, 308, 309, 312, 313, 314, 317, 318, 321, 322, 325, 326, 329, 330, 331, 334, 335, 338, 339, 342, 343, 347, 348, 350, 351, 353, 354, 361 }

F grade: { 4, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 85, 93, 102, 265, 271, 272, 273 }

2.1.6 Sympy

A grade: { 1, 2, 3, 9, 10, 11, 12, 28, 30, 34, 35, 37, 38, 39, 41, 42, 43, 45, 46, 47, 48, 51, 52, 53, 55, 56, 57, 58, 62, 64, 66, 67, 68, 69, 70, 71, 72, 77, 79, 80, 81, 82, 83, 84, 115, 123, 124, 169, 172, 173, 174, 181, 182, 189, 190, 270, 275, 276, 277, 279, 280, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345, 357 }

B grade: { 63, 78 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 32, 33, 36, 40, 44, 49, 50, 54, 59, 60, 61, 65, 73, 74, 75, 76, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 180, 183, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 278, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361 }

2.1.7 Giac

A grade: { 5, 6, 7, 8, 24, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 269, 270, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345, 355, 357, 358, 360 }

B grade: { 28, 29, 58, 72, 84, 113, 114, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 219, 266, 275, 276, 277, 279, 280, 356, 359 }

C grade: { 278 }

F grade: { 1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 31, 32, 33, 44, 54, 65, 85, 93, 102, 225, 226, 227, 233, 234, 235, 236, 265, 271, 272, 273, 274, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361 }

2.1.8 Mupad

A grade: { 30, 34, 35, 287, 292, 297, 301, 306, 311, 316, 320, 324, 328, 333, 337, 341, 345 }

B grade: { 12, 28, 29, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 219, 220, 221, 222, 228, 229, 230, 231, 237, 238, 239, 240, 245, 246, 247, 248, 253, 254, 255, 256, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 355, 356, 357, 358, 359, 360 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 85, 93, 102, 215, 216, 217, 218, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 241, 242, 243, 244, 249, 250, 251, 252, 257, 258, 259, 265, 271, 272, 273, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 298, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 321, 322, 323, 325, 326, 327, 329, 330, 331, 332, 334, 335, 336, 338, 339, 340, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	99	172	0	86	121	0	-1
normalized size	1	1.00	0.78	1.35	0.00	0.68	0.95	0.00	-0.01
time (sec)	N/A	0.052	0.080	0.037	0.000	0.600	5.032	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	88	134	0	75	95	0	-1
normalized size	1	1.00	0.87	1.33	0.00	0.74	0.94	0.00	-0.01
time (sec)	N/A	0.039	0.051	0.036	0.000	0.693	1.715	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	76	97	0	59	66	0	-1
normalized size	1	1.00	1.01	1.29	0.00	0.79	0.88	0.00	-0.01
time (sec)	N/A	0.023	0.033	0.031	0.000	0.606	0.673	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	167	209	0	0	0	0	-1
normalized size	1	1.00	0.70	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	2.478	0.306	0.000	0.573	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	60	51	54	0	71	-1
normalized size	1	1.00	0.94	1.13	0.96	1.02	0.00	1.34	-0.02
time (sec)	N/A	0.019	0.039	0.031	0.373	0.603	0.000	0.283	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	62	61	67	0	107	-1
normalized size	1	1.00	0.80	0.78	0.77	0.85	0.00	1.35	-0.01
time (sec)	N/A	0.026	0.046	0.032	0.346	0.701	0.000	0.388	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	74	110	102	78	0	134	-1
normalized size	1	1.00	0.70	1.05	0.97	0.74	0.00	1.28	-0.01
time (sec)	N/A	0.037	0.054	0.033	0.342	0.571	0.000	0.447	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	85	158	125	89	0	161	-1
normalized size	1	1.00	0.65	1.21	0.95	0.68	0.00	1.23	-0.01
time (sec)	N/A	0.049	0.063	0.036	0.345	1.130	0.000	1.388	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	79	224	155	88	116	0	-1
normalized size	1	1.00	0.69	1.96	1.36	0.77	1.02	0.00	-0.01
time (sec)	N/A	0.064	0.061	0.041	0.336	0.566	8.511	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	176	127	77	90	0	-1
normalized size	1	1.00	0.75	1.93	1.40	0.85	0.99	0.00	-0.01
time (sec)	N/A	0.049	0.064	0.034	0.335	0.401	2.938	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	128	99	65	65	0	-1
normalized size	1	1.00	0.82	1.88	1.46	0.96	0.96	0.00	-0.01
time (sec)	N/A	0.035	0.053	0.033	0.335	0.490	1.031	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	76	65	51	36	0	32
normalized size	1	1.00	1.00	1.90	1.62	1.28	0.90	0.00	0.80
time (sec)	N/A	0.009	0.014	0.031	0.342	0.765	0.628	0.000	1.057

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	61	53	0	273	0	0	-1
normalized size	1	1.00	1.11	0.96	0.00	4.96	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.049	0.033	0.000	0.678	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	92	90	0	340	0	0	-1
normalized size	1	1.00	1.08	1.06	0.00	4.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.110	0.034	0.000	0.642	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	107	130	0	383	0	0	-1
normalized size	1	1.00	0.96	1.17	0.00	3.45	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.164	0.033	0.000	1.502	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	161	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.691	180.000	0.000	0.629	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	147	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.503	180.000	0.000	0.495	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	135	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.307	180.000	0.000	0.809	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.134	180.000	0.000	0.479	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	142	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.243	180.000	0.000	0.589	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	154	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.340	180.000	0.000	0.582	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	163	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.102	0.473	180.000	0.000	0.515	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	124	0	0	0	0	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.149	180.000	0.000	0.579	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	109	0	0	0	0	1	-1
normalized size	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.157	0.104	180.000	0.000	0.850	0.000	0.318	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	85	0	0	0	0	0	-1
normalized size	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.112	180.000	0.000	0.623	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	118	0	0	0	0	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.162	0.117	180.000	0.000	0.602	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	131	0	0	0	0	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.110	180.000	0.000	0.577	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	48	37	59	60	223	90
normalized size	1	1.00	0.89	1.09	0.84	1.34	1.36	5.07	2.05
time (sec)	N/A	0.049	0.019	0.026	0.323	0.505	4.180	0.205	0.316

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	121	40	109	0	124	56
normalized size	1	1.00	0.89	2.57	0.85	2.32	0.00	2.64	1.19
time (sec)	N/A	0.049	0.039	0.067	0.320	0.566	0.000	0.213	1.466

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.099	0.885	0.000	0.653	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	482	1449	0	0	0	0	-1
normalized size	1	1.00	1.18	3.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.506	0.190	1.421	0.000	0.508	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	324	676	0	0	0	0	-1
normalized size	1	1.00	1.21	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.109	1.007	0.000	0.550	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	43	118	0	0	0	0	-1
normalized size	1	1.00	0.48	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.295	0.621	0.000	0.480	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.099	0.931	0.000	0.464	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.838	0.921	0.000	0.593	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	38	62	0	43	96
normalized size	1	1.00	0.92	1.11	1.03	1.68	0.00	1.16	2.59
time (sec)	N/A	0.025	0.065	0.151	0.329	0.481	0.000	0.322	1.493

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	19	13	19
normalized size	1	1.00	0.87	0.87	0.83	0.57	0.83	0.57	0.83
time (sec)	N/A	0.008	0.018	0.135	0.381	0.367	0.389	0.224	1.005

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	19	13	19
normalized size	1	1.00	0.87	0.87	0.83	0.57	0.83	0.57	0.83
time (sec)	N/A	0.007	0.016	0.132	0.378	0.616	0.228	0.141	0.976

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	16	10	19	10	16
normalized size	1	1.00	1.12	0.94	1.00	0.62	1.19	0.62	1.00
time (sec)	N/A	0.003	0.010	0.028	0.389	0.491	0.162	0.925	0.046

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	34	8	0	9	58
normalized size	1	1.00	0.90	1.00	1.62	0.38	0.00	0.43	2.76
time (sec)	N/A	0.036	0.015	0.148	0.328	0.659	0.000	0.187	1.101

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	18	17	13	14	12	17
normalized size	1	1.00	1.06	1.06	1.00	0.76	0.82	0.71	1.00
time (sec)	N/A	0.008	0.021	0.138	0.384	0.449	0.238	0.164	0.091

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	19	11	19	11	16
normalized size	1	1.00	0.78	0.87	0.83	0.48	0.83	0.48	0.70
time (sec)	N/A	0.009	0.015	0.138	0.387	0.466	0.513	0.150	0.951

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	20	13	19
normalized size	1	1.00	0.87	0.87	0.83	0.57	0.87	0.57	0.83
time (sec)	N/A	0.008	0.016	0.142	0.384	0.515	0.785	0.151	0.066

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	98	73	161	0	0	203
normalized size	1	1.00	0.87	1.38	1.03	2.27	0.00	0.00	2.86
time (sec)	N/A	0.032	0.162	0.151	0.401	0.678	0.000	0.000	1.126

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	38	36	24	78	24	36
normalized size	1	1.00	0.88	0.90	0.86	0.57	1.86	0.57	0.86
time (sec)	N/A	0.023	0.034	0.142	0.459	0.506	2.445	0.352	0.998

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	38	36	24	60	24	36
normalized size	1	1.00	0.88	0.90	0.86	0.57	1.43	0.57	0.86
time (sec)	N/A	0.022	0.050	0.140	0.459	0.490	1.231	0.256	0.975

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	74	38	36	24	41	24	36
normalized size	1	1.00	2.18	1.12	1.06	0.71	1.21	0.71	1.06
time (sec)	N/A	0.021	0.078	0.140	0.454	0.496	0.631	0.163	0.938

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	33	20	20	20	33
normalized size	1	1.00	1.00	0.94	2.06	1.25	1.25	1.25	2.06
time (sec)	N/A	0.004	0.007	0.027	0.454	0.643	0.296	0.163	0.073

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	55	20	20	0	21	183
normalized size	1	1.00	1.08	1.12	0.41	0.41	0.00	0.43	3.73
time (sec)	N/A	0.034	0.068	0.169	0.709	0.575	0.000	0.172	0.288

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	42	54	24	0	21	198
normalized size	1	1.00	0.95	1.08	1.38	0.62	0.00	0.54	5.08
time (sec)	N/A	0.024	0.051	0.150	0.400	0.507	0.000	0.168	0.187

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	42	35	34	26	32	22	34
normalized size	1	1.00	1.17	0.97	0.94	0.72	0.89	0.61	0.94
time (sec)	N/A	0.021	0.036	0.148	0.463	0.419	0.522	0.180	0.932

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	34	38	36	22	37	22	32
normalized size	1	1.00	1.10	1.23	1.16	0.71	1.19	0.71	1.03
time (sec)	N/A	0.013	0.041	0.143	0.466	0.457	0.801	0.813	0.944

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	64	37	38	36	24	39	24	36
normalized size	1	1.52	0.88	0.90	0.86	0.57	0.93	0.57	0.86
time (sec)	N/A	0.030	0.030	0.143	0.481	0.481	1.265	0.396	0.955

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	177	0	300	0	0	332
normalized size	1	1.00	0.88	1.61	0.00	2.73	0.00	0.00	3.02
time (sec)	N/A	0.058	0.243	0.145	0.000	0.397	0.000	0.000	1.233

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	56	54	35	80	35	53
normalized size	1	1.00	0.89	0.92	0.89	0.57	1.31	0.57	0.87
time (sec)	N/A	0.044	0.029	0.144	0.516	1.016	4.095	0.214	1.048

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	56	54	35	60	35	53
normalized size	1	1.00	1.02	1.06	1.02	0.66	1.13	0.66	1.00
time (sec)	N/A	0.029	0.033	0.150	0.519	0.562	2.400	0.160	0.141

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	99	56	54	34	41	34	53
normalized size	1	1.00	2.91	1.65	1.59	1.00	1.21	1.00	1.56
time (sec)	N/A	0.014	0.082	0.149	0.520	0.501	1.197	0.165	0.977

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	51	31	20	31	47
normalized size	1	1.00	1.00	0.94	3.19	1.94	1.25	1.94	2.94
time (sec)	N/A	0.004	0.006	0.030	0.521	0.731	0.609	0.174	0.098

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	104	92	31	31	0	32	306
normalized size	1	1.00	1.35	1.19	0.40	0.40	0.00	0.42	3.97
time (sec)	N/A	0.081	0.098	0.173	0.708	0.592	0.000	0.174	0.142

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	76	65	36	0	33	415
normalized size	1	1.00	0.91	1.12	0.96	0.53	0.00	0.49	6.10
time (sec)	N/A	0.041	0.052	0.175	0.781	0.741	0.000	0.191	1.071

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	66	59	72	37	0	31	365
normalized size	1	1.00	1.10	0.98	1.20	0.62	0.00	0.52	6.08
time (sec)	N/A	0.040	0.049	0.148	0.472	0.456	0.000	0.200	0.205

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	52	52	37	51	35	51
normalized size	1	1.00	1.09	0.95	0.95	0.67	0.93	0.64	0.93
time (sec)	N/A	0.037	0.025	0.210	0.525	0.452	0.840	0.158	1.004

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	50	56	53	33	56	33	48
normalized size	1	1.00	1.61	1.81	1.71	1.06	1.81	1.06	1.55
time (sec)	N/A	0.013	0.023	0.142	0.525	0.924	1.306	0.225	0.969

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	56	54	35	60	35	53
normalized size	1	1.00	0.84	0.88	0.84	0.55	0.94	0.55	0.83
time (sec)	N/A	0.032	0.035	0.146	0.529	0.373	2.067	0.164	0.991

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	137	278	145	483	0	0	479
normalized size	1	1.00	0.89	1.81	0.94	3.14	0.00	0.00	3.11
time (sec)	N/A	0.099	0.414	0.152	0.542	0.637	0.000	0.000	1.341

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	134	46	242
normalized size	1	1.00	0.89	0.92	0.90	0.58	1.68	0.58	3.02
time (sec)	N/A	0.063	0.068	0.151	0.586	0.454	29.343	0.466	1.032

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	117	46	242
normalized size	1	1.00	0.89	0.92	0.90	0.58	1.46	0.58	3.02
time (sec)	N/A	0.056	0.034	0.147	0.575	0.518	18.905	0.195	0.151

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	100	46	77
normalized size	1	1.00	0.89	0.92	0.90	0.58	1.25	0.58	0.96
time (sec)	N/A	0.055	0.035	0.147	0.572	0.521	11.720	0.224	1.055

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	74	72	45	78	45	70
normalized size	1	1.00	0.99	1.03	1.00	0.62	1.08	0.62	0.97
time (sec)	N/A	0.047	0.027	0.146	0.580	0.624	6.722	0.199	1.020

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	71	74	72	45	60	45	70
normalized size	1	1.00	1.34	1.40	1.36	0.85	1.13	0.85	1.32
time (sec)	N/A	0.030	0.059	0.151	0.575	0.464	4.006	0.186	0.153

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	125	74	72	46	41	46	70
normalized size	1	1.00	3.68	2.18	2.12	1.35	1.21	1.35	2.06
time (sec)	N/A	0.014	0.097	0.155	0.577	0.703	2.335	0.166	1.001

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	69	42	20	42	67
normalized size	1	1.00	1.00	0.94	4.31	2.62	1.25	2.62	4.19
time (sec)	N/A	0.005	0.006	0.030	0.575	0.443	1.149	0.161	0.979

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	175	127	42	42	0	43	423
normalized size	1	1.00	1.67	1.21	0.40	0.40	0.00	0.41	4.03
time (sec)	N/A	0.065	0.171	0.170	0.716	0.541	0.000	0.212	0.135

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	112	77	47	0	44	553
normalized size	1	1.00	0.89	1.18	0.81	0.49	0.00	0.46	5.82
time (sec)	N/A	0.064	0.091	0.179	0.785	0.507	0.000	0.328	0.144

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	93	83	47	0	43	672
normalized size	1	1.00	0.93	1.07	0.95	0.54	0.00	0.49	7.72
time (sec)	N/A	0.062	0.038	0.182	0.855	0.745	0.000	0.229	1.537

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	82	76	91	48	0	42	571
normalized size	1	1.00	1.06	0.99	1.18	0.62	0.00	0.55	7.42
time (sec)	N/A	0.058	0.054	0.157	0.544	0.663	0.000	0.211	1.336

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	69	72	48	70	46	68
normalized size	1	1.00	1.05	0.93	0.97	0.65	0.95	0.62	0.92
time (sec)	N/A	0.054	0.033	0.150	0.597	0.610	1.329	0.179	1.135

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	66	74	70	44	75	44	64
normalized size	1	1.00	2.13	2.39	2.26	1.42	2.42	1.42	2.06
time (sec)	N/A	0.013	0.056	0.150	0.600	0.494	2.101	0.173	1.196

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	71	74	72	46	78	46	70
normalized size	1	1.00	1.11	1.16	1.12	0.72	1.22	0.72	1.09
time (sec)	N/A	0.031	0.036	0.154	0.601	0.404	3.287	0.157	1.039

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	71	74	72	46	80	46	70
normalized size	1	1.00	0.72	0.76	0.73	0.47	0.82	0.47	0.71
time (sec)	N/A	0.053	0.039	0.151	0.598	0.558	5.406	0.212	1.005

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	132	71	74	72	46	76	46	70
normalized size	1	1.65	0.89	0.92	0.90	0.58	0.95	0.58	0.88
time (sec)	N/A	0.075	0.037	0.152	0.600	0.465	8.248	0.153	1.072

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	76	46	70
normalized size	1	1.00	0.89	0.92	0.90	0.58	0.95	0.58	0.88
time (sec)	N/A	0.055	0.066	0.153	0.605	0.549	12.999	0.186	1.105

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	72	46	78	46	70
normalized size	1	1.00	0.89	0.92	0.90	0.58	0.98	0.58	0.88
time (sec)	N/A	0.056	0.036	0.149	0.606	0.498	19.489	0.256	1.039

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	177	110	110	68	41	68	104
normalized size	1	1.00	5.21	3.24	3.24	2.00	1.21	2.00	3.06
time (sec)	N/A	0.014	0.135	0.166	0.720	0.469	6.536	0.190	1.100

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.092	1.040	0.000	0.797	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	79	202	42	41	0	43	354
normalized size	1	1.00	0.98	2.49	0.52	0.51	0.00	0.53	4.37
time (sec)	N/A	0.058	0.047	0.145	0.514	0.547	0.000	0.232	0.129

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	111	29	29	0	30	234
normalized size	1	1.00	0.98	1.98	0.52	0.52	0.00	0.54	4.18
time (sec)	N/A	0.035	0.045	0.139	0.517	0.383	0.000	0.176	0.273

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	49	18	17	0	19	108
normalized size	1	1.00	1.00	1.58	0.58	0.55	0.00	0.61	3.48
time (sec)	N/A	0.015	0.026	0.145	0.530	0.434	0.000	0.172	0.154

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	10	0	11	12
normalized size	1	1.00	1.00	1.08	1.08	0.83	0.00	0.92	1.00
time (sec)	N/A	0.004	0.052	0.028	0.432	0.488	0.000	0.404	1.060

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	43	18	16	0	20	107
normalized size	1	1.00	0.66	0.98	0.41	0.36	0.00	0.45	2.43
time (sec)	N/A	0.027	0.029	0.142	0.513	0.457	0.000	0.210	2.870

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	64	28	26	0	30	210
normalized size	1	1.00	0.69	0.98	0.43	0.40	0.00	0.46	3.23
time (sec)	N/A	0.044	0.034	0.144	0.521	0.379	0.000	0.270	2.832

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	87	40	41	0	45	286
normalized size	1	1.00	0.72	0.95	0.43	0.45	0.00	0.49	3.11
time (sec)	N/A	0.068	0.036	0.147	0.517	0.502	0.000	0.143	2.893

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.550	2.777	0.000	0.582	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	106	350	70	73	0	62	669
normalized size	1	1.00	1.08	3.57	0.71	0.74	0.00	0.63	6.83
time (sec)	N/A	0.081	0.095	0.154	0.751	0.446	0.000	0.157	0.177

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	83	223	59	62	0	48	490
normalized size	1	1.00	1.11	2.97	0.79	0.83	0.00	0.64	6.53
time (sec)	N/A	0.053	0.056	0.151	0.746	0.453	0.000	0.167	1.033

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	127	44	47	0	34	302
normalized size	1	1.00	1.12	2.54	0.88	0.94	0.00	0.68	6.04
time (sec)	N/A	0.034	0.074	0.147	0.760	0.569	0.000	0.177	1.055

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	56	26	28	0	24	28
normalized size	1	1.00	0.96	2.00	0.93	1.00	0.00	0.86	1.00
time (sec)	N/A	0.014	0.057	0.142	0.739	0.431	0.000	0.156	0.078

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	12	13	0	12	14
normalized size	1	1.00	1.00	1.07	0.86	0.93	0.00	0.86	1.00
time (sec)	N/A	0.005	0.007	0.028	0.421	0.445	0.000	0.203	0.081

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	67	28	39	0	31	359
normalized size	1	1.00	0.76	0.96	0.40	0.56	0.00	0.44	5.13
time (sec)	N/A	0.046	0.078	0.151	0.750	0.626	0.000	0.445	3.390

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	70	91	45	63	0	45	432
normalized size	1	1.00	0.69	0.89	0.44	0.62	0.00	0.44	4.24
time (sec)	N/A	0.072	0.069	0.151	0.737	0.542	0.000	0.159	3.444

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	92	116	64	86	0	64	660
normalized size	1	1.00	0.64	0.81	0.45	0.60	0.00	0.45	4.62
time (sec)	N/A	0.096	0.053	0.148	1.023	0.455	0.000	0.188	3.716

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.516	2.987	0.000	0.491	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	114	371	81	95	0	61	867
normalized size	1	1.00	1.24	4.03	0.88	1.03	0.00	0.66	9.42
time (sec)	N/A	0.073	0.046	0.154	2.382	0.923	0.000	0.204	1.336

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	86	239	69	83	0	44	620
normalized size	1	1.00	1.21	3.37	0.97	1.17	0.00	0.62	8.73
time (sec)	N/A	0.048	0.047	0.149	2.411	1.563	0.000	0.184	1.473

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	136	48	61	0	37	46
normalized size	1	1.00	1.04	2.89	1.02	1.30	0.00	0.79	0.98
time (sec)	N/A	0.029	0.040	0.145	3.221	1.071	0.000	0.482	1.028

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	43	32	32	0	18	25
normalized size	1	1.00	0.79	1.26	0.94	0.94	0.00	0.53	0.74
time (sec)	N/A	0.014	0.050	0.161	3.187	0.511	0.000	0.166	0.082

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	24	0	12	14
normalized size	1	1.00	1.00	0.94	0.75	1.50	0.00	0.75	0.88
time (sec)	N/A	0.004	0.006	0.027	0.557	0.400	0.000	0.166	0.059

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	74	92	51	80	0	43	645
normalized size	1	1.00	0.76	0.95	0.53	0.82	0.00	0.44	6.65
time (sec)	N/A	0.066	0.099	0.149	4.318	0.500	0.000	0.188	3.661

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	93	117	69	109	0	60	804
normalized size	1	1.00	0.71	0.89	0.53	0.83	0.00	0.46	6.14
time (sec)	N/A	0.093	0.062	0.151	3.127	0.519	0.000	0.168	3.665

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	107	145	86	130	0	73	909
normalized size	1	1.00	0.63	0.85	0.51	0.76	0.00	0.43	5.35
time (sec)	N/A	0.121	0.050	0.156	3.144	0.459	0.000	0.179	2.728

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	64	64	0	150	811
normalized size	1	1.00	0.82	1.52	0.63	0.63	0.00	1.49	8.03
time (sec)	N/A	0.065	0.034	0.172	1.839	0.521	0.000	0.620	1.052

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	53	53	0	125	648
normalized size	1	1.00	0.82	1.55	0.66	0.66	0.00	1.56	8.10
time (sec)	N/A	0.049	0.035	0.150	0.523	0.458	0.000	0.186	0.990

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	42	42	0	102	485
normalized size	1	1.00	0.83	1.17	0.71	0.71	0.00	1.73	8.22
time (sec)	N/A	0.030	0.035	0.148	0.542	0.621	0.000	0.145	0.992

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	30	30	0	75	151
normalized size	1	1.00	0.84	1.11	0.79	0.79	0.00	1.97	3.97
time (sec)	N/A	0.014	0.062	0.151	0.525	0.510	0.000	0.177	1.038

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	12	26	18	95
normalized size	1	1.00	1.00	0.83	0.67	0.67	1.44	1.00	5.28
time (sec)	N/A	0.005	0.007	0.029	0.492	0.530	0.576	0.163	1.121

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	61	54	0	73	0	40	308
normalized size	1	1.00	0.97	0.86	0.00	1.16	0.00	0.63	4.89
time (sec)	N/A	0.060	0.083	0.152	0.000	0.495	0.000	0.186	2.364

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	63	0	93	0	51	341
normalized size	1	1.00	0.98	0.95	0.00	1.41	0.00	0.77	5.17
time (sec)	N/A	0.032	0.048	0.161	0.000	0.546	0.000	0.161	6.975

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	89	92	0	119	0	75	741
normalized size	1	1.00	0.71	0.74	0.00	0.95	0.00	0.60	5.93
time (sec)	N/A	0.072	0.102	0.161	0.000	0.623	0.000	0.172	5.739

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	115	185	0	145	0	93	964
normalized size	1	1.00	0.64	1.03	0.00	0.81	0.00	0.52	5.39
time (sec)	N/A	0.121	0.104	0.177	0.000	0.510	0.000	0.923	5.513

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	64	75	0	241	1813
normalized size	1	1.00	0.82	1.52	0.63	0.74	0.00	2.39	17.95
time (sec)	N/A	0.067	0.039	0.138	0.528	0.477	0.000	0.190	1.186

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	53	64	0	205	1483
normalized size	1	1.00	0.82	1.55	0.66	0.80	0.00	2.56	18.54
time (sec)	N/A	0.046	0.037	0.137	0.528	0.484	0.000	0.198	1.124

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	42	53	0	168	1153
normalized size	1	1.00	0.83	1.17	0.71	0.90	0.00	2.85	19.54
time (sec)	N/A	0.029	0.037	0.144	0.529	0.420	0.000	0.161	1.119

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	31	41	49	131	823
normalized size	1	1.00	0.84	1.11	0.82	1.08	1.29	3.45	21.66
time (sec)	N/A	0.014	0.076	0.142	0.538	0.527	26.898	0.549	1.081

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	28	26	84	97
normalized size	1	1.00	1.00	0.83	0.67	1.56	1.44	4.67	5.39
time (sec)	N/A	0.005	0.006	0.028	0.490	0.524	11.531	0.196	1.162

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	80	131	0	88	0	57	501
normalized size	1	1.00	0.88	1.44	0.00	0.97	0.00	0.63	5.51
time (sec)	N/A	0.050	0.089	0.143	0.000	0.642	0.000	0.217	5.999

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	79	85	0	102	0	69	459
normalized size	1	1.00	0.98	1.05	0.00	1.26	0.00	0.85	5.67
time (sec)	N/A	0.047	0.049	0.152	0.000	0.675	0.000	0.404	2.217

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	91	0	124	0	73	609
normalized size	1	1.00	0.96	0.99	0.00	1.35	0.00	0.79	6.62
time (sec)	N/A	0.049	0.066	0.148	0.000	0.567	0.000	0.189	6.032

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	117	116	0	145	0	93	1019
normalized size	1	1.00	0.80	0.79	0.00	0.99	0.00	0.64	6.98
time (sec)	N/A	0.094	0.098	0.151	0.000	0.456	0.000	0.211	5.419

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	154	64	86	0	344	2681
normalized size	1	1.00	0.82	1.52	0.63	0.85	0.00	3.41	26.54
time (sec)	N/A	0.065	0.040	0.144	0.531	0.522	0.000	0.165	1.280

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	124	53	75	0	296	2235
normalized size	1	1.00	0.82	1.55	0.66	0.94	0.00	3.70	27.94
time (sec)	N/A	0.047	0.038	0.136	0.544	0.455	0.000	0.135	1.152

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	42	64	0	248	1789
normalized size	1	1.00	0.83	1.17	0.71	1.08	0.00	4.20	30.32
time (sec)	N/A	0.029	0.038	0.145	0.539	0.533	0.000	1.142	1.139

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	42	31	52	0	197	773
normalized size	1	1.00	0.84	1.11	0.82	1.37	0.00	5.18	20.34
time (sec)	N/A	0.014	0.080	0.140	0.521	0.600	0.000	0.177	1.105

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	39	0	136	337
normalized size	1	1.00	1.00	0.83	0.67	2.17	0.00	7.56	18.72
time (sec)	N/A	0.005	0.007	0.027	0.497	0.798	0.000	0.151	1.120

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	99	222	0	114	0	73	789
normalized size	1	1.00	0.82	1.83	0.00	0.94	0.00	0.60	6.52
time (sec)	N/A	0.070	0.096	0.144	0.000	0.720	0.000	0.192	4.816

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	106	193	0	126	0	89	616
normalized size	1	1.00	0.96	1.75	0.00	1.15	0.00	0.81	5.60
time (sec)	N/A	0.069	0.068	0.155	0.000	0.620	0.000	0.193	4.967

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	142	0	133	0	92	614
normalized size	1	1.00	0.98	1.29	0.00	1.21	0.00	0.84	5.58
time (sec)	N/A	0.067	0.047	0.155	0.000	0.963	0.000	0.187	2.066

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	144	0	146	0	88	669
normalized size	1	1.00	0.95	1.27	0.00	1.29	0.00	0.78	5.92
time (sec)	N/A	0.068	0.083	0.148	0.000	0.656	0.000	0.195	5.675

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	134	169	0	167	0	108	1069
normalized size	1	1.00	0.80	1.01	0.00	1.00	0.00	0.65	6.40
time (sec)	N/A	0.117	0.123	0.153	0.000	0.619	0.000	0.385	5.848

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	150	262	0	189	0	123	1292
normalized size	1	1.00	0.68	1.19	0.00	0.86	0.00	0.56	5.85
time (sec)	N/A	0.174	0.133	0.182	0.000	0.843	0.000	0.307	6.506

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	83	153	64	53	0	61	496
normalized size	1	1.00	0.84	1.55	0.65	0.54	0.00	0.62	5.01
time (sec)	N/A	0.065	0.047	0.166	0.530	0.655	0.000	0.159	1.090

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	123	53	42	0	49	385
normalized size	1	1.00	0.87	1.62	0.70	0.55	0.00	0.64	5.07
time (sec)	N/A	0.048	0.041	0.175	0.523	0.469	0.000	0.135	1.066

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	68	42	31	0	37	211
normalized size	1	1.00	0.86	1.19	0.74	0.54	0.00	0.65	3.70
time (sec)	N/A	0.031	0.043	0.171	0.532	0.473	0.000	0.165	1.145

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	32	56	30	19	0	23	105
normalized size	1	1.00	0.89	1.56	0.83	0.53	0.00	0.64	2.92
time (sec)	N/A	0.015	0.063	0.166	0.545	0.495	0.000	0.386	1.213

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	12	0	12	52
normalized size	1	1.00	1.00	0.94	0.75	0.75	0.00	0.75	3.25
time (sec)	N/A	0.004	0.007	0.029	0.530	0.445	0.000	0.190	1.176

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	42	0	56	0	21	285
normalized size	1	1.00	0.96	0.86	0.00	1.14	0.00	0.43	5.82
time (sec)	N/A	0.016	0.060	0.173	0.000	0.533	0.000	0.165	7.189

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	78	95	0	93	0	47	570
normalized size	1	1.00	0.83	1.01	0.00	0.99	0.00	0.50	6.06
time (sec)	N/A	0.053	0.083	0.172	0.000	0.581	0.000	0.238	7.018

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	98	148	0	123	0	69	802
normalized size	1	1.00	0.62	0.94	0.00	0.78	0.00	0.44	5.08
time (sec)	N/A	0.096	0.097	0.174	0.000	0.579	0.000	0.176	6.009

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	117	200	0	145	0	84	1086
normalized size	1	1.00	0.55	0.94	0.00	0.68	0.00	0.40	5.12
time (sec)	N/A	0.151	0.131	0.181	0.000	0.564	0.000	0.210	5.749

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	319	64	63	0	77	1057
normalized size	1	1.00	0.87	3.36	0.67	0.66	0.00	0.81	11.13
time (sec)	N/A	0.067	0.053	0.150	0.545	0.516	0.000	0.186	1.450

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	201	52	51	0	61	660
normalized size	1	1.00	0.89	2.72	0.70	0.69	0.00	0.82	8.92
time (sec)	N/A	0.049	0.047	0.149	0.526	0.518	0.000	0.185	1.258

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	106	41	40	0	46	259
normalized size	1	1.00	0.89	1.93	0.75	0.73	0.00	0.84	4.71
time (sec)	N/A	0.030	0.063	0.178	0.531	0.737	0.000	0.154	1.293

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	29	40	30	29	0	29	152
normalized size	1	1.00	0.85	1.18	0.88	0.85	0.00	0.85	4.47
time (sec)	N/A	0.015	0.082	0.143	0.528	0.460	0.000	0.190	1.334

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	12	20	0	12	97
normalized size	1	1.00	1.00	0.94	0.75	1.25	0.00	0.75	6.06
time (sec)	N/A	0.005	0.015	0.030	0.498	0.417	0.000	0.179	1.310

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	75	68	0	110	0	37	614
normalized size	1	1.00	0.96	0.87	0.00	1.41	0.00	0.47	7.87
time (sec)	N/A	0.036	0.119	0.153	0.000	0.629	0.000	0.271	6.144

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	91	105	0	151	0	64	807
normalized size	1	1.00	0.73	0.85	0.00	1.22	0.00	0.52	6.51
time (sec)	N/A	0.074	0.104	0.156	0.000	0.544	0.000	0.222	5.856

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	115	131	0	189	0	80	1028
normalized size	1	1.00	0.60	0.69	0.00	0.99	0.00	0.42	5.38
time (sec)	N/A	0.128	0.129	0.155	0.000	0.626	0.000	0.179	6.072

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	133	186	0	211	0	95	1258
normalized size	1	1.00	0.54	0.76	0.00	0.86	0.00	0.39	5.13
time (sec)	N/A	0.183	0.147	0.159	0.000	0.577	0.000	0.209	5.092

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	83	295	64	74	0	75	817
normalized size	1	1.00	0.84	2.98	0.65	0.75	0.00	0.76	8.25
time (sec)	N/A	0.066	0.056	0.153	0.520	0.487	0.000	0.246	1.423

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	65	186	52	62	0	59	533
normalized size	1	1.00	0.86	2.45	0.68	0.82	0.00	0.78	7.01
time (sec)	N/A	0.049	0.041	0.154	0.535	0.600	0.000	0.239	1.296

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	91	42	52	0	39	259
normalized size	1	1.00	0.81	1.54	0.71	0.88	0.00	0.66	4.39
time (sec)	N/A	0.030	0.052	0.151	0.541	0.417	0.000	0.318	1.262

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	42	31	41	0	20	152
normalized size	1	1.00	0.82	1.11	0.82	1.08	0.00	0.53	4.00
time (sec)	N/A	0.014	0.063	0.150	0.527	0.585	0.000	0.230	1.382

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	31	0	12	103
normalized size	1	1.00	1.00	0.83	0.67	1.72	0.00	0.67	5.72
time (sec)	N/A	0.005	0.008	0.027	0.498	0.521	0.000	0.232	1.367

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	91	93	0	177	0	45	886
normalized size	1	1.00	0.84	0.86	0.00	1.64	0.00	0.42	8.20
time (sec)	N/A	0.054	0.167	0.146	0.000	0.509	0.000	0.241	6.263

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	113	130	0	221	0	65	1230
normalized size	1	1.00	0.73	0.84	0.00	1.43	0.00	0.42	7.94
time (sec)	N/A	0.100	0.148	0.159	0.000	0.596	0.000	0.217	7.338

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	133	157	0	255	0	93	1514
normalized size	1	1.00	0.59	0.70	0.00	1.14	0.00	0.42	6.76
time (sec)	N/A	0.152	0.127	0.152	0.000	0.569	0.000	0.176	8.471

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	150	211	0	277	0	115	2359
normalized size	1	1.00	0.54	0.76	0.00	1.00	0.00	0.41	8.49
time (sec)	N/A	0.222	0.221	0.157	0.000	0.509	0.000	0.223	6.417

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	0	13	57
normalized size	1	1.00	0.85	0.74	0.70	0.67	0.00	0.48	2.11
time (sec)	N/A	0.010	0.051	0.242	0.330	0.469	0.000	0.185	1.242

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	0	13	57
normalized size	1	1.00	0.85	0.74	0.70	0.67	0.00	0.48	2.11
time (sec)	N/A	0.009	0.030	0.239	0.335	0.523	0.000	0.253	1.087

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	18	26	13	57
normalized size	1	1.00	0.85	0.74	0.70	0.67	0.96	0.48	2.11
time (sec)	N/A	0.010	0.030	0.237	0.329	0.450	14.462	0.147	1.100

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	16	0	13	57
normalized size	1	1.00	0.85	0.74	0.70	0.59	0.00	0.48	2.11
time (sec)	N/A	0.008	0.029	0.243	0.333	0.571	0.000	1.504	1.092

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	19	12	0	13	56
normalized size	1	1.00	0.92	0.80	0.76	0.48	0.00	0.52	2.24
time (sec)	N/A	0.008	0.020	0.238	0.328	0.576	0.000	0.211	1.127

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	12	22	13	56
normalized size	1	1.00	0.87	0.87	0.83	0.52	0.96	0.57	2.43
time (sec)	N/A	0.008	0.027	0.238	0.332	0.402	1.055	0.193	1.116

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	21	20	19	11	27	11	52
normalized size	1	1.00	0.78	0.74	0.70	0.41	1.00	0.41	1.93
time (sec)	N/A	0.009	0.021	0.237	0.334	0.570	10.541	0.167	1.241

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	19	13	27	13	57
normalized size	1	1.00	0.85	0.74	0.70	0.48	1.00	0.48	2.11
time (sec)	N/A	0.009	0.021	0.249	0.334	0.485	155.355	0.221	1.282

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	29	0	24	122
normalized size	1	1.00	0.83	0.79	0.75	0.60	0.00	0.50	2.54
time (sec)	N/A	0.023	0.057	0.281	0.347	0.613	0.000	0.178	1.154

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	29	0	24	122
normalized size	1	1.00	0.83	0.79	0.75	0.60	0.00	0.50	2.54
time (sec)	N/A	0.022	0.049	0.253	0.340	0.414	0.000	0.218	1.131

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	29	0	24	122
normalized size	1	1.00	0.83	0.79	0.75	0.60	0.00	0.50	2.54
time (sec)	N/A	0.023	0.044	0.246	0.347	0.471	0.000	0.151	1.141

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	27	0	24	122
normalized size	1	1.00	0.83	0.79	0.75	0.56	0.00	0.50	2.54
time (sec)	N/A	0.022	0.055	0.252	0.340	0.535	0.000	0.742	1.119

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	47	36	24	0	24	122
normalized size	1	1.00	0.87	1.02	0.78	0.52	0.00	0.52	2.65
time (sec)	N/A	0.021	0.032	0.263	0.338	0.489	0.000	0.165	1.154

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	40	37	36	23	0	24	122
normalized size	1	1.00	0.91	0.84	0.82	0.52	0.00	0.55	2.77
time (sec)	N/A	0.022	0.049	0.246	0.344	0.477	0.000	0.206	1.162

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	24	48	23	122
normalized size	1	1.00	0.83	0.79	0.75	0.50	1.00	0.48	2.54
time (sec)	N/A	0.023	0.053	0.248	0.343	0.591	10.495	0.171	1.132

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	38	36	24	49	24	122
normalized size	1	1.00	0.83	0.79	0.75	0.50	1.02	0.50	2.54
time (sec)	N/A	0.023	0.048	0.247	0.353	0.532	154.701	0.246	1.134

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	40	0	35	182
normalized size	1	1.00	0.83	0.81	0.80	0.58	0.00	0.51	2.64
time (sec)	N/A	0.039	0.031	0.293	0.360	0.428	0.000	0.418	1.165

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	40	0	35	182
normalized size	1	1.00	0.83	0.81	0.80	0.58	0.00	0.51	2.64
time (sec)	N/A	0.038	0.054	0.250	0.355	0.495	0.000	0.204	1.177

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	40	0	35	182
normalized size	1	1.00	0.83	0.81	0.80	0.58	0.00	0.51	2.64
time (sec)	N/A	0.037	0.033	0.253	0.353	0.620	0.000	0.184	1.170

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	38	0	35	182
normalized size	1	1.00	0.83	0.81	0.80	0.55	0.00	0.51	2.64
time (sec)	N/A	0.037	0.029	0.259	0.357	0.678	0.000	0.157	1.157

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	69	55	35	0	35	182
normalized size	1	1.00	0.88	1.06	0.85	0.54	0.00	0.54	2.80
time (sec)	N/A	0.036	0.031	0.250	0.357	0.527	0.000	0.294	1.186

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	64	55	34	0	35	182
normalized size	1	1.00	0.90	1.02	0.87	0.54	0.00	0.56	2.89
time (sec)	N/A	0.036	0.034	0.254	0.355	0.630	0.000	0.417	1.214

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	55	55	34	66	34	182
normalized size	1	1.00	0.85	0.85	0.85	0.52	1.02	0.52	2.80
time (sec)	N/A	0.038	0.032	0.255	0.354	0.522	10.581	0.163	1.216

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	57	56	55	35	70	34	182
normalized size	1	1.00	0.83	0.81	0.80	0.51	1.01	0.49	2.64
time (sec)	N/A	0.038	0.034	0.260	0.362	0.540	155.326	0.170	1.185

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	129	481	65	153	0	70	475
normalized size	1	1.00	0.90	3.36	0.45	1.07	0.00	0.49	3.32
time (sec)	N/A	0.127	0.069	0.267	0.429	0.501	0.000	0.142	1.721

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	330	54	132	0	59	415
normalized size	1	1.00	0.93	2.84	0.47	1.14	0.00	0.51	3.58
time (sec)	N/A	0.079	0.129	0.266	0.418	0.484	0.000	0.145	1.388

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	207	42	103	0	45	354
normalized size	1	1.00	0.97	2.33	0.47	1.16	0.00	0.51	3.98
time (sec)	N/A	0.056	0.094	0.267	0.417	0.388	0.000	0.183	1.852

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	112	31	85	0	31	296
normalized size	1	1.00	0.97	1.75	0.48	1.33	0.00	0.48	4.62
time (sec)	N/A	0.033	0.046	0.289	0.415	0.518	0.000	0.141	2.228

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	41	18	68	0	18	347
normalized size	1	1.00	0.96	0.77	0.34	1.28	0.00	0.34	6.55
time (sec)	N/A	0.018	0.030	0.261	0.420	0.622	0.000	0.157	4.005

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	73	76	31	93	0	31	464
normalized size	1	1.00	0.96	1.00	0.41	1.22	0.00	0.41	6.11
time (sec)	N/A	0.036	0.066	0.260	0.423	0.583	0.000	0.159	1.968

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	89	98	41	118	0	41	642
normalized size	1	1.00	0.88	0.97	0.41	1.17	0.00	0.41	6.36
time (sec)	N/A	0.057	0.196	0.265	0.435	0.540	0.000	0.188	1.832

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	107	120	52	144	0	52	822
normalized size	1	1.00	0.84	0.94	0.41	1.12	0.00	0.41	6.42
time (sec)	N/A	0.084	0.161	0.265	0.420	0.565	0.000	0.167	1.634

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	144	452	75	188	0	76	523
normalized size	1	1.00	1.07	3.35	0.56	1.39	0.00	0.56	3.87
time (sec)	N/A	0.103	0.205	0.270	0.424	0.405	0.000	0.161	1.612

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	119	294	64	161	0	65	463
normalized size	1	1.00	1.10	2.72	0.59	1.49	0.00	0.60	4.29
time (sec)	N/A	0.076	0.156	0.266	0.424	0.516	0.000	0.191	1.532

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	81	160	50	134	0	46	403
normalized size	1	1.00	0.98	1.93	0.60	1.61	0.00	0.55	4.86
time (sec)	N/A	0.054	0.087	0.265	0.420	0.531	0.000	0.521	1.771

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	61	37	115	0	36	344
normalized size	1	1.00	0.96	0.84	0.51	1.58	0.00	0.49	4.71
time (sec)	N/A	0.034	0.061	0.264	0.420	0.432	0.000	0.176	1.769

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	80	82	35	116	0	35	516
normalized size	1	1.00	0.82	0.85	0.36	1.20	0.00	0.36	5.32
time (sec)	N/A	0.054	0.064	0.269	0.419	0.393	0.000	0.250	2.182

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	105	51	147	0	49	705
normalized size	1	1.00	0.87	0.88	0.42	1.22	0.00	0.41	5.88
time (sec)	N/A	0.080	0.120	0.275	0.418	0.457	0.000	0.163	1.830

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	120	128	64	184	0	58	871
normalized size	1	1.00	0.83	0.88	0.44	1.27	0.00	0.40	6.01
time (sec)	N/A	0.106	0.221	0.326	0.422	0.574	0.000	0.179	2.163

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	139	151	75	210	0	70	1051
normalized size	1	1.00	0.81	0.88	0.44	1.22	0.00	0.41	6.11
time (sec)	N/A	0.137	0.275	0.274	0.425	0.606	0.000	0.509	2.138

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	147	418	86	227	0	77	571
normalized size	1	1.00	1.09	3.10	0.64	1.68	0.00	0.57	4.23
time (sec)	N/A	0.097	0.113	0.269	0.445	0.571	0.000	0.163	1.977

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	104	249	73	200	0	59	511
normalized size	1	1.00	0.95	2.26	0.66	1.82	0.00	0.54	4.65
time (sec)	N/A	0.071	0.103	0.264	0.442	0.531	0.000	0.207	1.691

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	85	61	185	0	47	667
normalized size	1	1.00	0.98	0.87	0.62	1.89	0.00	0.48	6.81
time (sec)	N/A	0.053	0.081	0.269	0.429	0.497	0.000	0.171	1.867

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	107	98	64	186	0	52	580
normalized size	1	1.00	0.86	0.78	0.51	1.49	0.00	0.42	4.64
time (sec)	N/A	0.071	0.151	0.268	0.430	0.516	0.000	0.208	1.842

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	118	112	60	186	0	47	741
normalized size	1	1.00	0.78	0.74	0.39	1.22	0.00	0.31	4.88
time (sec)	N/A	0.097	0.089	0.267	0.431	0.506	0.000	0.175	1.898

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	141	181	73	214	0	59	1077
normalized size	1	1.00	0.80	1.03	0.41	1.22	0.00	0.34	6.12
time (sec)	N/A	0.128	0.178	0.278	0.426	0.469	0.000	0.296	2.168

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	156	207	86	250	0	71	1362
normalized size	1	1.00	0.78	1.03	0.43	1.24	0.00	0.35	6.78
time (sec)	N/A	0.159	0.278	0.286	0.433	0.540	0.000	0.431	2.495

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	174	229	97	276	0	80	2151
normalized size	1	1.00	0.76	1.00	0.43	1.21	0.00	0.35	9.43
time (sec)	N/A	0.202	0.355	0.326	0.428	0.647	0.000	0.205	1.947

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	104	304	0	141	0	60	-1
normalized size	1	1.00	0.73	2.14	0.00	0.99	0.00	0.42	-0.01
time (sec)	N/A	0.078	0.093	0.288	0.000	0.674	0.000	0.174	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	84	174	0	114	0	48	-1
normalized size	1	1.00	0.81	1.67	0.00	1.10	0.00	0.46	-0.01
time (sec)	N/A	0.051	0.069	0.266	0.000	0.552	0.000	0.180	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	62	75	0	93	0	36	-1
normalized size	1	1.00	1.02	1.23	0.00	1.52	0.00	0.59	-0.02
time (sec)	N/A	0.029	0.040	0.272	0.000	0.489	0.000	0.464	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	52	149	40	89	0	57	-1
normalized size	1	1.00	1.06	3.04	0.82	1.82	0.00	1.16	-0.02
time (sec)	N/A	0.029	0.042	0.263	0.462	0.557	0.000	0.228	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	15	0	59	210
normalized size	1	1.00	0.97	0.83	0.43	0.43	0.00	1.69	6.00
time (sec)	N/A	0.014	0.036	0.268	0.422	0.530	0.000	0.199	1.714

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	34	0	112	174
normalized size	1	1.00	0.67	0.82	0.47	0.47	0.00	1.56	2.42
time (sec)	N/A	0.033	0.040	0.274	0.436	0.565	0.000	0.188	1.413

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	45	45	0	138	234
normalized size	1	1.00	0.60	0.95	0.41	0.41	0.00	1.25	2.13
time (sec)	N/A	0.054	0.064	0.280	0.430	0.601	0.000	0.202	1.594

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	56	56	0	166	294
normalized size	1	1.00	0.55	1.02	0.38	0.38	0.00	1.12	1.99
time (sec)	N/A	0.078	0.070	0.302	0.431	0.746	0.000	0.196	1.519

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	471	0	163	0	147	-1
normalized size	1	1.00	0.69	2.66	0.00	0.92	0.00	0.83	-0.01
time (sec)	N/A	0.104	0.096	0.242	0.000	0.515	0.000	0.721	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	105	304	0	140	0	122	-1
normalized size	1	1.00	0.76	2.19	0.00	1.01	0.00	0.88	-0.01
time (sec)	N/A	0.071	0.076	0.240	0.000	0.485	0.000	0.324	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	165	0	119	0	0	-1
normalized size	1	1.00	0.82	1.63	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.063	0.244	0.000	0.563	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	280	0	109	0	0	-1
normalized size	1	1.00	0.95	3.46	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.059	0.241	0.000	0.495	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	74	315	0	109	0	0	-1
normalized size	1	1.00	1.06	4.50	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.049	0.240	0.000	0.578	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	31	0	33	332
normalized size	1	1.00	0.97	0.83	0.43	0.89	0.00	0.94	9.49
time (sec)	N/A	0.014	0.044	0.256	0.440	0.441	0.000	0.149	1.510

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	45	0	59	228
normalized size	1	1.00	0.67	0.82	0.47	0.62	0.00	0.82	3.17
time (sec)	N/A	0.033	0.049	0.251	0.430	0.498	0.000	0.162	1.541

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	45	56	0	78	288
normalized size	1	1.00	0.60	0.95	0.41	0.51	0.00	0.71	2.62
time (sec)	N/A	0.052	0.054	0.270	0.436	0.528	0.000	0.150	1.635

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	56	67	0	97	348
normalized size	1	1.00	0.55	1.02	0.38	0.45	0.00	0.66	2.35
time (sec)	N/A	0.076	0.077	0.389	0.441	0.509	0.000	0.166	1.741

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	121	471	0	162	0	204	-1
normalized size	1	1.00	0.70	2.71	0.00	0.93	0.00	1.17	-0.01
time (sec)	N/A	0.098	0.086	0.242	0.000	0.405	0.000	0.170	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	101	286	0	141	0	0	-1
normalized size	1	1.00	0.74	2.10	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.063	0.244	0.000	0.562	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	101	460	0	137	0	0	-1
normalized size	1	1.00	0.83	3.80	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.077	0.247	0.000	0.478	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	97	501	0	138	0	0	-1
normalized size	1	1.00	0.92	4.73	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.075	0.248	0.000	0.483	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	95	532	0	137	0	0	-1
normalized size	1	1.00	1.02	5.72	0.00	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.057	0.250	0.000	0.899	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	29	15	42	0	33	396
normalized size	1	1.00	0.97	0.83	0.43	1.20	0.00	0.94	11.31
time (sec)	N/A	0.013	0.049	0.251	0.419	0.418	0.000	0.181	1.654

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	59	34	56	0	59	293
normalized size	1	1.00	0.67	0.82	0.47	0.78	0.00	0.82	4.07
time (sec)	N/A	0.033	0.056	0.264	0.430	0.453	0.000	0.190	1.625

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	45	67	0	78	353
normalized size	1	1.00	0.60	0.95	0.41	0.61	0.00	0.71	3.21
time (sec)	N/A	0.057	0.056	0.304	0.422	0.565	0.000	0.196	1.657

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	56	78	0	97	413
normalized size	1	1.00	0.55	1.02	0.38	0.53	0.00	0.66	2.79
time (sec)	N/A	0.079	0.088	0.388	0.420	0.490	0.000	0.197	1.744

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	105	304	0	140	0	64	-1
normalized size	1	1.00	0.72	2.10	0.00	0.97	0.00	0.44	-0.01
time (sec)	N/A	0.081	0.102	0.298	0.000	0.637	0.000	0.143	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	88	174	0	119	0	52	-1
normalized size	1	1.00	0.82	1.63	0.00	1.11	0.00	0.49	-0.01
time (sec)	N/A	0.052	0.079	0.302	0.000	0.587	0.000	0.161	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	66	80	0	91	0	38	-1
normalized size	1	1.00	1.05	1.27	0.00	1.44	0.00	0.60	-0.02
time (sec)	N/A	0.028	0.062	0.293	0.000	0.465	0.000	0.155	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	24	0	57	0	23	-1
normalized size	1	1.00	1.10	0.80	0.00	1.90	0.00	0.77	-0.03
time (sec)	N/A	0.012	0.033	0.332	0.000	0.767	0.000	0.147	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	15	15	0	30	101
normalized size	1	1.00	0.97	0.88	0.45	0.45	0.00	0.91	3.06
time (sec)	N/A	0.013	0.043	0.303	0.431	0.492	0.000	0.152	1.620

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	59	33	23	0	55	218
normalized size	1	1.00	0.64	0.82	0.46	0.32	0.00	0.76	3.03
time (sec)	N/A	0.032	0.046	0.300	0.429	0.562	0.000	0.139	1.573

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	105	45	34	0	77	227
normalized size	1	1.00	0.60	0.95	0.41	0.31	0.00	0.70	2.06
time (sec)	N/A	0.056	0.057	0.301	0.439	0.469	0.000	0.156	1.527

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	82	151	55	45	0	103	287
normalized size	1	1.00	0.55	1.02	0.37	0.30	0.00	0.70	1.94
time (sec)	N/A	0.083	0.061	0.296	0.434	0.712	0.000	0.147	1.636

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	122	428	0	196	0	75	-1
normalized size	1	1.00	0.73	2.58	0.00	1.18	0.00	0.45	-0.01
time (sec)	N/A	0.107	0.114	0.250	0.000	0.509	0.000	0.140	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	261	0	175	0	63	-1
normalized size	1	1.00	0.81	2.04	0.00	1.37	0.00	0.49	-0.01
time (sec)	N/A	0.076	0.099	0.247	0.000	0.567	0.000	0.160	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	130	0	145	0	48	-1
normalized size	1	1.00	0.94	1.51	0.00	1.69	0.00	0.56	-0.01
time (sec)	N/A	0.049	0.086	0.255	0.000	0.576	0.000	0.138	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	55	42	0	119	0	39	-1
normalized size	1	1.00	1.06	0.81	0.00	2.29	0.00	0.75	-0.02
time (sec)	N/A	0.026	0.054	0.249	0.000	0.474	0.000	0.142	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	0	22	0	15	163
normalized size	1	1.00	0.97	0.88	0.00	0.67	0.00	0.45	4.94
time (sec)	N/A	0.013	0.033	0.250	0.000	0.506	0.000	0.144	1.736

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	43	59	32	34	0	50	281
normalized size	1	1.00	0.63	0.87	0.47	0.50	0.00	0.74	4.13
time (sec)	N/A	0.033	0.052	0.246	0.427	0.427	0.000	0.163	1.490

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	64	105	45	49	0	107	286
normalized size	1	1.00	0.58	0.95	0.41	0.45	0.00	0.97	2.60
time (sec)	N/A	0.054	0.060	0.248	0.425	0.575	0.000	0.173	1.694

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	80	151	54	58	0	161	346
normalized size	1	1.00	0.54	1.02	0.36	0.39	0.00	1.09	2.34
time (sec)	N/A	0.080	0.070	0.245	0.435	0.583	0.000	0.198	1.774

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	121	348	0	241	0	75	-1
normalized size	1	1.00	0.79	2.27	0.00	1.58	0.00	0.49	-0.01
time (sec)	N/A	0.096	0.109	0.252	0.000	0.526	0.000	0.149	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	101	180	0	214	0	61	-1
normalized size	1	1.00	0.91	1.62	0.00	1.93	0.00	0.55	-0.01
time (sec)	N/A	0.066	0.094	0.247	0.000	0.804	0.000	0.147	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	78	59	0	186	0	49	-1
normalized size	1	1.00	1.04	0.79	0.00	2.48	0.00	0.65	-0.01
time (sec)	N/A	0.042	0.070	0.249	0.000	0.515	0.000	0.166	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	92	0	33	0	15	229
normalized size	1	1.00	0.97	2.63	0.00	0.94	0.00	0.43	6.54
time (sec)	N/A	0.012	0.048	0.248	0.000	0.550	0.000	0.135	1.704

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	47	58	0	43	0	25	346
normalized size	1	1.00	0.66	0.82	0.00	0.61	0.00	0.35	4.87
time (sec)	N/A	0.032	0.042	0.241	0.000	0.458	0.000	0.130	1.659

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	104	45	58	0	62	348
normalized size	1	1.00	0.62	0.98	0.42	0.55	0.00	0.58	3.28
time (sec)	N/A	0.055	0.060	0.244	0.437	0.382	0.000	0.160	1.687

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	79	150	56	71	0	119	406
normalized size	1	1.00	0.54	1.03	0.38	0.49	0.00	0.82	2.78
time (sec)	N/A	0.079	0.071	0.273	0.435	0.397	0.000	0.192	1.765

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	100	196	67	82	0	173	466
normalized size	1	1.00	0.54	1.05	0.36	0.44	0.00	0.93	2.51
time (sec)	N/A	0.104	0.079	0.256	0.438	0.735	0.000	0.240	1.861

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.140	1.582	0.000	0.466	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	146	654	139	374	0	332	546
normalized size	1	1.00	0.88	3.96	0.84	2.27	0.00	2.01	3.31
time (sec)	N/A	0.134	0.112	0.139	0.532	0.403	0.000	0.141	1.671

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	492	101	255	0	226	418
normalized size	1	1.00	0.88	4.07	0.83	2.11	0.00	1.87	3.45
time (sec)	N/A	0.075	0.097	0.110	0.535	0.508	0.000	0.140	1.422

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	315	68	168	0	140	304
normalized size	1	1.00	0.87	3.84	0.83	2.05	0.00	1.71	3.71
time (sec)	N/A	0.046	0.072	0.105	0.540	0.667	0.000	0.138	1.291

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	175	42	91	0	76	205
normalized size	1	1.00	0.85	3.65	0.88	1.90	0.00	1.58	4.27
time (sec)	N/A	0.019	0.046	0.104	0.540	0.559	0.000	0.141	1.231

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	21	39	51	28	121
normalized size	1	1.00	1.00	1.05	1.05	1.95	2.55	1.40	6.05
time (sec)	N/A	0.007	0.016	0.030	0.497	0.506	0.689	0.137	1.182

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.092	0.390	0.000	0.687	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.052	0.879	0.000	1.755	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	67	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.058	0.951	0.000	0.488	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	676	38	33	0	0	96
normalized size	1	1.00	0.92	18.27	1.03	0.89	0.00	0.00	2.59
time (sec)	N/A	0.018	0.070	0.289	0.329	0.454	0.000	0.000	1.596

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	49	71	19
normalized size	1	1.00	0.87	0.87	0.83	0.57	2.13	3.09	0.83
time (sec)	N/A	0.020	0.046	0.235	0.387	0.688	11.101	0.158	1.105

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	13	49	71	19
normalized size	1	1.00	0.87	0.87	0.83	0.57	2.13	3.09	0.83
time (sec)	N/A	0.007	0.016	0.135	0.386	0.602	5.555	0.153	0.061

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	16	10	46	69	16
normalized size	1	1.00	1.12	0.94	1.00	0.62	2.88	4.31	1.00
time (sec)	N/A	0.003	0.008	0.026	0.384	0.493	2.825	0.140	0.020

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	21	34	8	0	15	59
normalized size	1	1.00	0.90	1.00	1.62	0.38	0.00	0.71	2.81
time (sec)	N/A	0.030	0.017	0.145	0.330	0.706	0.000	0.144	1.204

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	18	17	13	42	70	17
normalized size	1	1.00	1.06	1.06	1.00	0.76	2.47	4.12	1.00
time (sec)	N/A	0.008	0.020	0.136	0.382	0.615	5.777	0.157	0.076

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	20	19	11	49	71	16
normalized size	1	1.00	0.78	0.87	0.83	0.48	2.13	3.09	0.70
time (sec)	N/A	0.009	0.016	0.135	0.390	0.473	10.939	0.161	1.066

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	47	21	33	58	0	0	-1
normalized size	1	1.00	1.74	0.78	1.22	2.15	0.00	0.00	-0.04
time (sec)	N/A	0.032	0.024	0.310	0.372	0.609	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	81	479	56	88	0	0	-1
normalized size	1	1.00	1.59	9.39	1.10	1.73	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.016	0.501	0.379	0.405	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	109	501	78	118	0	0	-1
normalized size	1	1.00	1.42	6.51	1.01	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.036	0.442	0.376	0.833	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	345	5366	281	900	0	0	-1
normalized size	1	1.00	1.12	17.48	0.92	2.93	0.00	0.00	-0.00
time (sec)	N/A	0.469	12.695	11.993	0.718	0.474	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	259	5062	215	746	0	0	-1
normalized size	1	1.00	1.12	21.91	0.93	3.23	0.00	0.00	-0.00
time (sec)	N/A	0.371	10.378	3.945	0.716	0.581	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	142	552	0	0	-1
normalized size	1	1.00	0.87	2.04	0.95	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.222	6.808	0.300	0.695	0.937	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.141	14.685	0.625	0.000	0.533	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	144	1769	149	451	0	0	-1
normalized size	1	1.00	0.93	11.41	0.96	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.296	4.671	5.084	1.127	0.586	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	1710	125	382	0	0	-1
normalized size	1	1.00	0.92	13.36	0.98	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.254	5.285	4.592	1.107	0.840	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	1627	101	323	0	0	-1
normalized size	1	1.00	0.90	16.11	1.00	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.224	5.273	4.181	1.102	0.640	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	201	247	72	239	0	0	-1
normalized size	1	1.00	2.91	3.58	1.04	3.46	0.00	0.00	-0.01
time (sec)	N/A	0.137	4.762	0.339	1.111	0.576	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.079	4.759	0.952	0.000	0.541	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	144	1773	146	424	0	0	-1
normalized size	1	1.00	0.86	10.55	0.87	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.297	4.741	4.996	1.133	0.632	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	119	1716	123	360	0	0	-1
normalized size	1	1.00	0.86	12.35	0.88	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.258	5.544	4.961	1.123	0.734	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	93	1635	100	306	0	0	-1
normalized size	1	1.00	0.85	14.86	0.91	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.224	5.372	4.490	1.109	0.559	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	200	280	73	228	0	0	-1
normalized size	1	1.00	2.63	3.68	0.96	3.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	4.696	0.341	1.110	0.497	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.067	5.887	0.945	0.000	0.450	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	353	5294	277	880	0	0	-1
normalized size	1	1.00	1.17	17.47	0.91	2.90	0.00	0.00	-0.00
time (sec)	N/A	0.457	11.274	11.710	0.703	0.917	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	267	4990	213	730	0	0	-1
normalized size	1	1.00	1.17	21.79	0.93	3.19	0.00	0.00	-0.00
time (sec)	N/A	0.372	9.541	3.602	0.718	0.778	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	131	306	142	540	0	0	-1
normalized size	1	1.00	0.87	2.04	0.95	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.227	5.417	0.348	0.723	0.469	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.141	14.621	0.571	0.000	0.734	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	141	1741	146	424	0	0	-1
normalized size	1	1.00	0.93	11.45	0.96	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.304	4.651	5.187	1.101	1.528	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	116	1684	123	360	0	0	-1
normalized size	1	1.00	0.92	13.37	0.98	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.259	5.421	4.672	1.124	0.995	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	1603	100	306	0	0	-1
normalized size	1	1.00	0.90	16.03	1.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.230	5.225	4.347	1.093	0.583	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	197	247	72	227	0	0	-1
normalized size	1	1.00	2.86	3.58	1.04	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.142	3.884	0.432	1.096	0.486	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.078	4.681	0.985	0.000	0.790	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	147	1801	149	451	0	0	-1
normalized size	1	1.00	0.89	10.92	0.90	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.299	4.669	4.852	1.120	0.529	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	121	1742	125	382	0	0	-1
normalized size	1	1.00	0.88	12.72	0.91	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.264	5.354	4.869	1.102	0.531	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	1659	101	323	0	0	-1
normalized size	1	1.00	0.86	15.22	0.93	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.231	5.378	4.375	1.104	0.634	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	208	280	73	240	0	0	-1
normalized size	1	1.00	2.74	3.68	0.96	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.137	4.397	0.450	1.119	0.540	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.089	4.753	0.954	0.000	0.388	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	1809	0	0	-1
normalized size	1	1.00	2.17	24.60	0.00	5.99	0.00	0.00	-0.00
time (sec)	N/A	0.232	1.317	43.609	0.000	1.237	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	1279	0	0	-1
normalized size	1	1.00	1.75	23.69	0.00	5.47	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.710	31.862	0.000	0.613	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2543	0	831	0	0	-1
normalized size	1	1.00	1.62	15.70	0.00	5.13	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.309	4.038	0.000	0.771	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	74	180	182	499	0	0	-1
normalized size	1	1.00	0.94	2.28	2.30	6.32	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.027	0.499	0.482	0.694	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.040	5.066	1.796	0.000	0.634	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	346	6967	0	2161	0	0	-1
normalized size	1	1.00	0.88	17.64	0.00	5.47	0.00	0.00	-0.00
time (sec)	N/A	0.499	0.935	44.582	0.000	0.754	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	257	6593	0	1689	0	0	-1
normalized size	1	1.00	0.87	22.35	0.00	5.73	0.00	0.00	-0.00
time (sec)	N/A	0.403	0.602	4.948	0.000	1.064	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	4654	612	372	1189	0	0	-1
normalized size	1	1.00	23.99	3.15	1.92	6.13	0.00	0.00	-0.01
time (sec)	N/A	0.239	32.978	0.262	0.523	0.655	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.150	4.802	1.006	0.000	0.501	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	155	2346	341	349	0	0	-1
normalized size	1	1.00	0.91	13.80	2.01	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.442	6.098	0.372	0.625	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2256	247	297	0	0	-1
normalized size	1	1.00	0.89	16.96	1.86	2.23	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.315	4.411	0.357	0.505	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	766	292	263	222	0	0	-1
normalized size	1	1.00	8.24	3.14	2.83	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.151	14.367	0.477	0.434	1.010	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.096	1.076	1.595	0.000	0.519	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	156	2456	340	349	0	0	-1
normalized size	1	1.00	0.91	14.36	1.99	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.411	5.688	0.366	0.582	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	120	2358	246	297	0	0	-1
normalized size	1	1.00	0.90	17.60	1.84	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.315	4.473	0.357	0.471	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	723	328	265	223	0	0	-1
normalized size	1	1.00	7.69	3.49	2.82	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.148	15.535	0.502	0.431	0.708	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.091	1.061	1.648	0.000	1.409	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	654	7429	0	1567	0	0	-1
normalized size	1	1.00	2.17	24.60	0.00	5.19	0.00	0.00	-0.00
time (sec)	N/A	0.233	0.304	41.706	0.000	0.674	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	409	5543	0	1081	0	0	-1
normalized size	1	1.00	1.75	23.69	0.00	4.62	0.00	0.00	-0.00
time (sec)	N/A	0.170	0.182	32.115	0.000	0.974	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	263	2544	0	677	0	0	-1
normalized size	1	1.00	1.62	15.70	0.00	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.128	3.711	0.000	0.737	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	74	160	184	389	0	0	-1
normalized size	1	1.00	0.94	2.03	2.33	4.92	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.037	0.421	0.468	0.609	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.042	5.067	3.621	0.000	0.436	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	339	6775	0	1799	0	0	-1
normalized size	1	1.00	0.87	17.33	0.00	4.60	0.00	0.00	-0.00
time (sec)	N/A	0.486	0.972	51.088	0.000	1.023	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	253	6425	0	1463	0	0	-1
normalized size	1	1.00	0.86	21.93	0.00	4.99	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.567	4.972	0.000	0.636	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	4463	629	392	1099	0	0	-1
normalized size	1	1.00	23.01	3.24	2.02	5.66	0.00	0.00	-0.01
time (sec)	N/A	0.241	31.708	0.471	0.531	0.996	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.142	5.258	1.112	0.000	0.552	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	155	2456	342	180	0	0	-1
normalized size	1	1.00	0.92	14.62	2.04	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.296	0.457	5.641	0.369	0.457	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	119	2358	248	157	0	0	-1
normalized size	1	1.00	0.90	17.86	1.88	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.316	4.456	0.354	1.122	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	709	299	288	122	0	0	-1
normalized size	1	1.00	7.62	3.22	3.10	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.149	39.254	0.608	0.493	0.449	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.086	1.066	1.837	0.000	0.550	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	155	2346	343	180	0	0	-1
normalized size	1	1.00	0.92	13.88	2.03	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.295	0.435	5.562	0.380	0.604	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	119	2256	249	157	0	0	-1
normalized size	1	1.00	0.89	16.96	1.87	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.307	4.336	0.356	0.649	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	605	335	286	122	0	0	-1
normalized size	1	1.00	6.44	3.56	3.04	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.149	30.010	0.553	0.429	0.717	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.096	1.093	1.531	0.000	0.418	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	51	31	58	65	0	0	-1
normalized size	1	1.00	2.43	1.48	2.76	3.10	0.00	0.00	-0.05
time (sec)	N/A	0.012	0.034	0.048	0.315	0.842	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	62	59	95	0	0	-1
normalized size	1	1.00	1.65	1.44	1.37	2.21	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.027	0.042	0.313	0.544	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	93	79	76	120	0	0	-1
normalized size	1	1.00	1.60	1.36	1.31	2.07	0.00	0.00	-0.02
time (sec)	N/A	0.067	0.027	0.041	0.312	0.530	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	68	67	107	138	0	0	-1
normalized size	1	1.00	1.94	1.91	3.06	3.94	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.080	0.050	0.309	0.668	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	113	153	108	199	0	0	-1
normalized size	1	1.00	1.59	2.15	1.52	2.80	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.051	0.049	0.332	0.896	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	149	185	142	248	0	0	-1
normalized size	1	1.00	1.48	1.83	1.41	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.047	0.052	0.326	0.605	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	108	164	202	284	0	0	-1
normalized size	1	1.00	0.64	0.98	1.20	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.078	0.076	0.325	0.488	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	177	596	194	396	0	0	-1
normalized size	1	1.00	0.84	2.82	0.92	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.120	0.134	0.361	0.684	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	235	672	254	480	0	0	-1
normalized size	1	1.00	0.89	2.55	0.96	1.82	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.085	0.131	0.366	0.528	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	153	868	184	234	0	157	179
normalized size	1	1.00	1.43	8.11	1.72	2.19	0.00	1.47	1.67
time (sec)	N/A	0.171	0.174	0.686	0.421	0.476	0.000	0.431	1.594

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	60	887	64	93	0	147	111
normalized size	1	1.00	1.22	18.10	1.31	1.90	0.00	3.00	2.27
time (sec)	N/A	0.075	0.088	0.551	0.328	0.455	0.000	0.206	1.517

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	68	43	46	56	35	28
normalized size	1	1.00	1.02	1.51	0.96	1.02	1.24	0.78	0.62
time (sec)	N/A	0.054	0.088	0.174	0.328	0.473	3.306	0.142	0.157

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	351	43	25	0	40	28
normalized size	1	1.00	1.02	7.80	0.96	0.56	0.00	0.89	0.62
time (sec)	N/A	0.054	0.090	0.308	0.328	0.597	0.000	0.130	0.072

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	59	872	64	92	0	98	119
normalized size	1	1.00	1.20	17.80	1.31	1.88	0.00	2.00	2.43
time (sec)	N/A	0.068	0.084	0.379	0.328	0.478	0.000	0.317	1.711

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	150	842	184	233	0	167	187
normalized size	1	1.00	1.40	7.87	1.72	2.18	0.00	1.56	1.75
time (sec)	N/A	0.170	0.171	0.559	0.417	1.473	0.000	0.427	1.620

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	114	668	0	327	0	0	-1
normalized size	1	1.00	0.84	4.91	0.00	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.538	0.286	0.403	0.000	0.552	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [281] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.00	23	0.174
2	A	5	4	1.00	23	0.174
3	A	4	4	1.00	21	0.190
4	A	8	8	1.00	23	0.348
5	A	2	2	1.00	23	0.087
6	A	3	3	1.00	23	0.130
7	A	4	3	1.00	23	0.130
8	A	5	3	1.00	23	0.130
9	A	4	3	1.00	23	0.130
10	A	4	3	1.00	23	0.130
11	A	4	3	1.00	23	0.130
12	A	2	2	1.00	19	0.105
13	A	4	4	1.00	23	0.174

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	5	5	1.00	23	0.217
15	A	6	5	1.00	23	0.217
16	A	6	4	1.00	25	0.160
17	A	5	4	1.00	25	0.160
18	A	4	4	1.00	25	0.160
19	A	3	3	1.00	25	0.120
20	A	4	4	1.00	25	0.160
21	A	5	4	1.00	25	0.160
22	A	6	4	1.00	25	0.160
23	A	7	6	1.00	25	0.240
24	A	6	6	1.00	25	0.240
25	A	5	5	1.00	25	0.200
26	A	6	6	1.00	25	0.240
27	A	7	6	1.00	25	0.240
28	A	4	4	1.00	12	0.333
29	A	4	4	1.00	14	0.286
30	A	0	0	0.00	0	0.000
31	A	9	7	1.00	40	0.175
32	A	7	6	1.00	40	0.150
33	A	2	3	1.00	38	0.079
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	2	2	1.00	11	0.182
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	9	0.222
39	A	2	2	1.00	7	0.286
40	A	2	2	1.00	11	0.182
41	A	2	2	1.00	11	0.182
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	11	0.182
44	A	3	2	1.00	13	0.154
45	A	3	2	1.00	13	0.154
46	A	3	2	1.00	13	0.154
47	A	3	3	1.00	11	0.273
48	A	2	2	1.00	9	0.222
49	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	3	3	1.00	13	0.231
51	A	3	2	1.00	13	0.154
52	A	1	1	1.00	13	0.077
53	A	2	2	1.52	13	0.154
54	A	4	2	1.00	13	0.154
55	A	4	2	1.00	13	0.154
56	A	4	3	1.00	13	0.231
57	A	3	3	1.00	11	0.273
58	A	2	2	1.00	9	0.222
59	A	4	3	1.00	13	0.231
60	A	4	4	1.00	13	0.308
61	A	4	3	1.00	13	0.231
62	A	4	2	1.00	13	0.154
63	A	1	1	1.00	13	0.077
64	A	2	2	1.00	13	0.154
65	A	5	2	1.00	13	0.154
66	A	5	2	1.00	13	0.154
67	A	5	2	1.00	13	0.154
68	A	5	2	1.00	13	0.154
69	A	5	3	1.00	13	0.231
70	A	4	3	1.00	13	0.231
71	A	3	3	1.00	11	0.273
72	A	2	2	1.00	9	0.222
73	A	5	3	1.00	13	0.231
74	A	5	4	1.00	13	0.308
75	A	5	4	1.00	13	0.308
76	A	5	3	1.00	13	0.231
77	A	5	2	1.00	13	0.154
78	A	1	1	1.00	13	0.077
79	A	2	2	1.00	13	0.154
80	A	3	2	1.00	13	0.154
81	A	4	2	1.65	13	0.154
82	A	5	2	1.00	13	0.154
83	A	5	2	1.00	13	0.154
84	A	3	3	1.00	11	0.273
85	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	4	1.00	13	0.308
87	A	4	4	1.00	13	0.308
88	A	3	3	1.00	11	0.273
89	A	2	2	1.00	9	0.222
90	A	4	3	1.00	13	0.231
91	A	5	4	1.00	13	0.308
92	A	6	4	1.00	13	0.308
93	A	2	2	1.00	13	0.154
94	A	6	5	1.00	13	0.385
95	A	5	5	1.00	13	0.385
96	A	4	4	1.00	13	0.308
97	A	3	3	1.00	11	0.273
98	A	2	2	1.00	9	0.222
99	A	5	4	1.00	13	0.308
100	A	6	5	1.00	13	0.385
101	A	7	5	1.00	13	0.385
102	A	3	2	1.00	13	0.154
103	A	6	5	1.00	13	0.385
104	A	5	4	1.00	13	0.308
105	A	4	3	1.00	13	0.231
106	A	3	3	1.00	11	0.273
107	A	2	2	1.00	9	0.222
108	A	6	4	1.00	13	0.308
109	A	7	5	1.00	13	0.385
110	A	8	5	1.00	13	0.385
111	A	6	3	1.00	15	0.200
112	A	5	3	1.00	15	0.200
113	A	4	3	1.00	15	0.200
114	A	3	3	1.00	13	0.231
115	A	2	2	1.00	11	0.182
116	A	2	2	1.00	15	0.133
117	A	2	2	1.00	15	0.133
118	A	4	3	1.00	15	0.200
119	A	6	3	1.00	15	0.200
120	A	6	3	1.00	15	0.200
121	A	5	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
122	A	4	3	1.00	15	0.200
123	A	3	3	1.00	13	0.231
124	A	2	2	1.00	11	0.182
125	A	3	2	1.00	15	0.133
126	A	3	3	1.00	15	0.200
127	A	3	2	1.00	15	0.133
128	A	5	3	1.00	15	0.200
129	A	6	3	1.00	15	0.200
130	A	5	3	1.00	15	0.200
131	A	4	3	1.00	15	0.200
132	A	3	3	1.00	13	0.231
133	A	2	2	1.00	11	0.182
134	A	4	2	1.00	15	0.133
135	A	4	3	1.00	15	0.200
136	A	4	3	1.00	15	0.200
137	A	4	2	1.00	15	0.133
138	A	6	3	1.00	15	0.200
139	A	8	3	1.00	15	0.200
140	A	6	3	1.00	15	0.200
141	A	5	3	1.00	15	0.200
142	A	4	3	1.00	15	0.200
143	A	3	3	1.00	13	0.231
144	A	2	2	1.00	11	0.182
145	A	1	1	1.00	15	0.067
146	A	3	3	1.00	15	0.200
147	A	5	3	1.00	15	0.200
148	A	7	3	1.00	15	0.200
149	A	6	3	1.00	15	0.200
150	A	5	3	1.00	15	0.200
151	A	4	3	1.00	15	0.200
152	A	3	3	1.00	13	0.231
153	A	2	2	1.00	11	0.182
154	A	2	2	1.00	15	0.133
155	A	4	3	1.00	15	0.200
156	A	6	3	1.00	15	0.200
157	A	8	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	6	3	1.00	15	0.200
159	A	5	3	1.00	15	0.200
160	A	4	3	1.00	15	0.200
161	A	3	3	1.00	13	0.231
162	A	2	2	1.00	11	0.182
163	A	3	2	1.00	15	0.133
164	A	5	3	1.00	15	0.200
165	A	7	3	1.00	15	0.200
166	A	9	3	1.00	15	0.200
167	A	2	2	1.00	13	0.154
168	A	2	2	1.00	13	0.154
169	A	2	2	1.00	13	0.154
170	A	2	2	1.00	13	0.154
171	A	2	2	1.00	13	0.154
172	A	2	2	1.00	13	0.154
173	A	2	2	1.00	13	0.154
174	A	2	2	1.00	13	0.154
175	A	3	2	1.00	15	0.133
176	A	3	2	1.00	15	0.133
177	A	3	2	1.00	15	0.133
178	A	3	2	1.00	15	0.133
179	A	3	2	1.00	15	0.133
180	A	3	2	1.00	15	0.133
181	A	3	2	1.00	15	0.133
182	A	3	2	1.00	15	0.133
183	A	4	2	1.00	15	0.133
184	A	4	2	1.00	15	0.133
185	A	4	2	1.00	15	0.133
186	A	4	2	1.00	15	0.133
187	A	4	2	1.00	15	0.133
188	A	4	2	1.00	15	0.133
189	A	4	2	1.00	15	0.133
190	A	4	2	1.00	15	0.133
191	A	5	2	1.00	15	0.133
192	A	4	2	1.00	15	0.133
193	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
194	A	2	2	1.00	15	0.133
195	A	1	1	1.00	15	0.067
196	A	2	2	1.00	15	0.133
197	A	3	2	1.00	15	0.133
198	A	4	2	1.00	15	0.133
199	A	5	3	1.00	15	0.200
200	A	4	3	1.00	15	0.200
201	A	3	3	1.00	15	0.200
202	A	2	2	1.00	15	0.133
203	A	3	3	1.00	15	0.200
204	A	4	3	1.00	15	0.200
205	A	5	3	1.00	15	0.200
206	A	6	3	1.00	15	0.200
207	A	5	3	1.00	15	0.200
208	A	4	3	1.00	15	0.200
209	A	3	2	1.00	15	0.133
210	A	4	3	1.00	15	0.200
211	A	5	3	1.00	15	0.200
212	A	6	3	1.00	15	0.200
213	A	7	3	1.00	15	0.200
214	A	8	3	1.00	15	0.200
215	A	4	2	1.00	17	0.118
216	A	3	2	1.00	17	0.118
217	A	2	2	1.00	17	0.118
218	A	2	2	1.00	17	0.118
219	A	1	1	1.00	17	0.059
220	A	2	2	1.00	17	0.118
221	A	3	2	1.00	17	0.118
222	A	4	2	1.00	17	0.118
223	A	5	2	1.00	17	0.118
224	A	4	2	1.00	17	0.118
225	A	3	2	1.00	17	0.118
226	A	3	3	1.00	17	0.176
227	A	3	2	1.00	17	0.118
228	A	1	1	1.00	17	0.059
229	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
230	A	3	2	1.00	17	0.118
231	A	4	2	1.00	17	0.118
232	A	5	2	1.00	17	0.118
233	A	4	2	1.00	17	0.118
234	A	4	3	1.00	17	0.176
235	A	4	3	1.00	17	0.176
236	A	4	2	1.00	17	0.118
237	A	1	1	1.00	17	0.059
238	A	2	2	1.00	17	0.118
239	A	3	2	1.00	17	0.118
240	A	4	2	1.00	17	0.118
241	A	4	2	1.00	17	0.118
242	A	3	2	1.00	17	0.118
243	A	2	2	1.00	17	0.118
244	A	1	1	1.00	17	0.059
245	A	1	1	1.00	17	0.059
246	A	2	2	1.00	17	0.118
247	A	3	2	1.00	17	0.118
248	A	4	2	1.00	17	0.118
249	A	5	3	1.00	17	0.176
250	A	4	3	1.00	17	0.176
251	A	3	3	1.00	17	0.176
252	A	2	2	1.00	17	0.118
253	A	1	1	1.00	17	0.059
254	A	2	2	1.00	17	0.118
255	A	3	2	1.00	17	0.118
256	A	4	2	1.00	17	0.118
257	A	5	3	1.00	17	0.176
258	A	4	3	1.00	17	0.176
259	A	3	2	1.00	17	0.118
260	A	1	1	1.00	17	0.059
261	A	2	2	1.00	17	0.118
262	A	3	2	1.00	17	0.118
263	A	4	2	1.00	17	0.118
264	A	5	2	1.00	17	0.118
265	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
266	A	6	3	1.00	13	0.231
267	A	5	3	1.00	13	0.231
268	A	4	3	1.00	13	0.231
269	A	3	3	1.00	11	0.273
270	A	2	2	1.00	9	0.222
271	A	1	1	1.00	13	0.077
272	A	2	2	1.00	13	0.154
273	A	3	2	1.00	13	0.154
274	A	2	2	1.00	11	0.182
275	A	2	2	1.00	11	0.182
276	A	2	2	1.00	9	0.222
277	A	2	2	1.00	7	0.286
278	A	2	2	1.00	11	0.182
279	A	2	2	1.00	11	0.182
280	A	2	2	1.00	11	0.182
281	A	6	4	1.00	3	1.333
282	A	8	5	1.00	5	1.000
283	A	10	6	1.00	7	0.857
284	A	11	6	1.00	15	0.400
285	A	9	5	1.00	13	0.385
286	A	7	4	1.00	11	0.364
287	A	0	0	0.00	0	0.000
288	A	8	7	1.00	16	0.438
289	A	7	7	1.00	16	0.438
290	A	6	6	1.00	14	0.429
291	A	5	5	1.00	12	0.417
292	A	0	0	0.00	0	0.000
293	A	8	7	1.00	19	0.368
294	A	7	7	1.00	19	0.368
295	A	6	6	1.00	17	0.353
296	A	5	5	1.00	15	0.333
297	A	0	0	0.00	0	0.000
298	A	11	6	1.00	15	0.400
299	A	9	5	1.00	13	0.385
300	A	7	4	1.00	11	0.364
301	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	A	8	7	1.00	16	0.438
303	A	7	7	1.00	16	0.438
304	A	6	6	1.00	14	0.429
305	A	5	5	1.00	12	0.417
306	A	0	0	0.00	0	0.000
307	A	8	7	1.00	19	0.368
308	A	7	7	1.00	19	0.368
309	A	6	6	1.00	17	0.353
310	A	5	5	1.00	15	0.333
311	A	0	0	0.00	0	0.000
312	A	12	6	1.00	15	0.400
313	A	10	6	1.00	15	0.400
314	A	8	5	1.00	13	0.385
315	A	6	4	1.00	7	0.571
316	A	0	0	0.00	0	0.000
317	A	11	6	1.00	15	0.400
318	A	9	5	1.00	13	0.385
319	A	7	4	1.00	11	0.364
320	A	0	0	0.00	0	0.000
321	A	7	7	1.00	20	0.350
322	A	6	6	1.00	18	0.333
323	A	5	5	1.00	16	0.312
324	A	0	0	0.00	0	0.000
325	A	7	7	1.00	21	0.333
326	A	6	6	1.00	19	0.316
327	A	5	5	1.00	17	0.294
328	A	0	0	0.00	0	0.000
329	A	12	6	1.00	15	0.400
330	A	10	6	1.00	15	0.400
331	A	8	5	1.00	13	0.385
332	A	6	4	1.00	7	0.571
333	A	0	0	0.00	0	0.000
334	A	11	6	1.00	15	0.400
335	A	9	5	1.00	13	0.385
336	A	7	4	1.00	11	0.364
337	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	A	7	7	1.00	20	0.350
339	A	6	6	1.00	18	0.333
340	A	5	5	1.00	16	0.312
341	A	0	0	0.00	0	0.000
342	A	7	7	1.00	21	0.333
343	A	6	6	1.00	19	0.316
344	A	5	5	1.00	17	0.294
345	A	0	0	0.00	0	0.000
346	A	2	2	1.00	4	0.500
347	A	7	4	1.00	6	0.667
348	A	9	5	1.00	8	0.625
349	A	2	2	1.00	8	0.250
350	A	7	4	1.00	10	0.400
351	A	9	5	1.00	12	0.417
352	A	6	6	1.00	12	0.500
353	A	9	5	1.00	14	0.357
354	A	11	6	1.00	16	0.375
355	A	8	7	1.00	20	0.350
356	A	5	5	1.00	20	0.250
357	A	3	2	1.00	20	0.100
358	A	3	2	1.00	20	0.100
359	A	5	5	1.00	20	0.250
360	A	8	7	1.00	20	0.350
361	A	11	8	1.00	24	0.333

Chapter 3

Listing of integrals

3.1 $\int x^5 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=127

$$\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3}-\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}$$

```
[Out] 5/96*d^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^3+1/6*x^6*arctanh(x*e^(1/2)/(
e*x^2+d)^(1/2))-5/96*d^2*x*(e*x^2+d)^(1/2)/e^(5/2)+5/144*d*x^3*(e*x^2+d)^(1
/2)/e^(3/2)-1/36*x^5*(e*x^2+d)^(1/2)/e^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6221, 321, 217, 206}

$$-\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-5*d^2*x*Sqrt[d + e*x^2])/(96*e^(5/2)) + (5*d*x^3*Sqrt[d + e*x^2])/(144*e^
(3/2)) - (x^5*Sqrt[d + e*x^2])/(36*Sqrt[e]) + (5*d^3*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(96*e^3) + (x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\int\frac{x^6}{\sqrt{d+ex^2}}dx\\
&= -\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{(5d)\int\frac{x^4}{\sqrt{d+ex^2}}dx}{36\sqrt{e}}\\
&= \frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(5d^2\right)\int\frac{x^2}{\sqrt{d+ex^2}}dx}{48e^{3/2}}\\
&= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{(5\ldots}{\ldots}\\
&= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{1}{6}x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{(5\ldots}{\ldots}\\
&= -\frac{5d^2x\sqrt{d+ex^2}}{96e^{5/2}}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{3/2}}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3}+\frac{1}{6}x^6\text{ta}\ldots\end{aligned}$$

Mathematica [A] time = 0.08, size = 99, normalized size = 0.78

$$\frac{15d^3\log\left(\sqrt{d+ex^2}+\sqrt{e}x\right)+\sqrt{e}x\sqrt{d+ex^2}\left(-15d^2+10dex^2-8e^2x^4\right)+48e^3x^6\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{288e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 48*e^3*x^6*
ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + 15*d^3*Log[Sqrt[e]*x + Sqrt[d + e*x^
2]])/(288*e^3)
```

fricas [A] time = 0.60, size = 86, normalized size = 0.68

$$-\frac{2\left(8e^2x^5-10dex^3+15d^2x\right)\sqrt{ex^2+d}\sqrt{e}-3\left(16e^3x^6+5d^3\right)\log\left(\frac{2ex^2+2\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{576e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/576*(2*(8*e^2*x^5 - 10*d*e*x^3 + 15*d^2*x)*sqrt(e*x^2 + d)*sqrt(e) - 3*(
16*e^3*x^6 + 5*d^3)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/e^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.04, size = 172, normalized size = 1.35

$$\frac{x^6 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{6}+\frac{\sqrt{e}\, x^7\sqrt{e x^2+d}}{48d}-\frac{7x^5\sqrt{e x^2+d}}{288\sqrt{e}}+\frac{35d\, x^3\sqrt{e x^2+d}}{1152e^{\frac{3}{2}}}-\frac{5d^2x\sqrt{e x^2+d}}{128e^{\frac{5}{2}}}+\frac{5d^3\ln\left(x\sqrt{e}+\sqrt{e}\right.}{96e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/6*x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/48*e^(1/2)/d*x^7*(e*x^2+d)^(1/
2)-7/288*x^5*(e*x^2+d)^(1/2)/e^(1/2)+35/1152*d*x^3*(e*x^2+d)^(1/2)/e^(3/2)-
5/128*d^2*x*(e*x^2+d)^(1/2)/e^(5/2)+5/96/e^3*d^3*ln(x*e^(1/2)+(e*x^2+d)^(1/
2))-1/48/e^(1/2)/d*x^5*(e*x^2+d)^(3/2)+5/288/e^(3/2)*x^3*(e*x^2+d)^(3/2)-5/
384/e^(5/2)*d*x*(e*x^2+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}x^6\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{12}x^6\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{2}d\sqrt{e}\int -\frac{\sqrt{ex^2+d}\,x^6}{3\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*x^6*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/12*x^6*log(-sqrt(e)*x + sqrt(
e*x^2 + d)) - 1/2*d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)*x^6/(e^2*x^4 + d
*e*x^2 - (e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^5*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 5.03, size = 121, normalized size = 0.95

$$\begin{cases} \frac{5d^3\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^3}-\frac{5d^2x\sqrt{d+ex^2}}{96e^{\frac{5}{2}}}+\frac{5dx^3\sqrt{d+ex^2}}{144e^{\frac{3}{2}}}+\frac{x^6\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6}-\frac{x^5\sqrt{d+ex^2}}{36\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((5*d**3*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(96*e**3) - 5*d**2*x*sq
rt(d + e*x**2)/(96*e**(5/2)) + 5*d*x**3*sqrt(d + e*x**2)/(144*e**(3/2)) + x
**6*atanh(sqrt(e)*x/sqrt(d + e*x**2))/6 - x**5*sqrt(d + e*x**2)/(36*sqrt(e)
), Ne(e, 0)), (0, True))
```

3.2 $\int x^3 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=101

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}+\frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}$$

```
[Out] -3/32*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^2+1/4*x^4*arctanh(x*e^(1/2)/
(e*x^2+d)^(1/2))+3/32*d*x*(e*x^2+d)^(1/2)/e^(3/2)-1/16*x^3*(e*x^2+d)^(1/2)/
e^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6221, 321, 217, 206}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{3dx\sqrt{d+ex^2}}{32e^{3/2}}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}}+\frac{1}{4}x^4\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (3*d*x*Sqrt[d + e*x^2])/(32*e^(3/2)) - (x^3*Sqrt[d + e*x^2])/(16*Sqrt[e]) -
 (3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(32*e^2) + (x^4*ArcTanh[(Sqrt
[e]*x)/Sqrt[d + e*x^2]])/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{e}\int \frac{x^4}{\sqrt{d+ex^2}} dx$$
$$= -\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} + \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(3d)\int \frac{x^2}{\sqrt{d+ex^2}} dx}{16\sqrt{e}}$$
$$= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} + \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{(3d^2)\int \frac{1}{\sqrt{d+ex^2}} dx}{32e^{3/2}}$$
$$= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} + \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{(3d^2)\operatorname{Subst}\left(\int \frac{1}{1-ex}\right.}{32e^{3/}}$$
$$= \frac{3dx\sqrt{d+ex^2}}{32e^{3/2}} - \frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} - \frac{3d^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2} + \frac{1}{4}x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right.$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.87

$$\frac{-3d^2 \log\left(\sqrt{d+ex^2} + \sqrt{e}x\right) + 8e^2x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \sqrt{e}x\left(3d-2ex^2\right)\sqrt{d+ex^2}}{32e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + 8*e^2*x^4*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]] - 3*d^2*Log[Sqrt[e]*x + Sqrt[d + e*x^2]])/(32*e^2)
```

fricas [A] time = 0.69, size = 75, normalized size = 0.74

$$-\frac{2\left(2ex^3 - 3dx\right)\sqrt{ex^2+d}\sqrt{e} - \left(8e^2x^4 - 3d^2\right)\log\left(\frac{2ex^2+2\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{64e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/64*(2*(2*e*x^3 - 3*d*x)*sqrt(e*x^2 + d)*sqrt(e) - (8*e^2*x^4 - 3*d^2)*lo
g((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/e^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.04, size = 134, normalized size = 1.33

$$\frac{x^4 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{4} + \frac{\sqrt{e}\,x^5\sqrt{ex^2+d}}{24d} - \frac{5x^3\sqrt{ex^2+d}}{96\sqrt{e}} + \frac{dx\sqrt{ex^2+d}}{16e^{\frac{3}{2}}} - \frac{3d^2 \ln\left(x\sqrt{e} + \sqrt{ex^2+d}\right)}{32e^2} - \frac{x^3\left(ex^2+d\right.}{24\sqrt{e}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/4*x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/24*e^(1/2)/d*x^5*(e*x^2+d)^(1/
2)-5/96*x^3*(e*x^2+d)^(1/2)/e^(1/2)+1/16*d*x*(e*x^2+d)^(1/2)/e^(3/2)-3/32/e
^2*d^2*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/24/e^(1/2)/d*x^3*(e*x^2+d)^(3/2)+1/3
2/e^(3/2)*x*(e*x^2+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,x^4\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\frac{1}{8}\,x^4\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\frac{1}{2}\,d\sqrt{e}\int-\frac{\sqrt{ex^2+d}\,x^4}{2\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/8*x^4*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/8*x^4*log(-sqrt(e)*x + sqrt(e*
x^2 + d)) - 1/2*d*sqrt(e)*integrate(-1/2*sqrt(e*x^2 + d)*x^4/(e^2*x^4 + d*e
*x^2 - (e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^3*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 1.72, size = 95, normalized size = 0.94

$$\begin{cases} -\frac{3d^2\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^2}+\frac{3dx\sqrt{d+ex^2}}{32e^{\frac{3}{2}}}+\frac{x^4\,\mathrm{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4}-\frac{x^3\sqrt{d+ex^2}}{16\sqrt{e}} & \text{for}\ e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-3*d**2*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(32*e**2) + 3*d*x*sqrt
(d + e*x**2)/(32*e**(3/2)) + x**4*atanh(sqrt(e)*x/sqrt(d + e*x**2))/4 - x**
3*sqrt(d + e*x**2)/(16*sqrt(e)), Ne(e, 0)), (0, True))
```

3.3 $\int x \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=75

$$-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e}$$

```
[Out] 1/4*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e+1/2*x^2*arctanh(x*e^(1/2)/(e*x^2
+d)^(1/2))-1/4*x*(e*x^2+d)^(1/2)/e^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6221, 321, 217, 206}

$$-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}}+\frac{1}{2}x^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(x*Sqrt[d + e*x^2])/(4*Sqrt[e]) + (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/(4*e) + (x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{2}\sqrt{e}\int \frac{x^2}{\sqrt{d+ex^2}} dx$$
$$= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} + \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{d\int \frac{1}{\sqrt{d+ex^2}}dx}{4\sqrt{e}}$$
$$= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} + \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{d\,\text{Subst}\left(\int \frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{4\sqrt{e}}$$
$$= -\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e} + \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Mathematica [A] time = 0.03, size = 76, normalized size = 1.01

$$-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} + \frac{d\log\left(\sqrt{d+ex^2}+\sqrt{e}x\right)}{4e} + \frac{1}{2}x^2 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -1/4*(x*Sqrt[d + e*x^2])/Sqrt[e] + (x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]
])/2 + (d*Log[Sqrt[e]*x + Sqrt[d + e*x^2]])/(4*e)
```

fricas [A] time = 0.61, size = 59, normalized size = 0.79

$$-\frac{2\sqrt{ex^2+d}\sqrt{e}x - \left(2ex^2+d\right)\log\left(\frac{2ex^2+2\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(e*x^2 + d)*sqrt(e)*x - (2*e*x^2 + d)*log((2*e*x^2 + 2*sqrt(e*x
^2 + d)*sqrt(e)*x + d)/d))/e
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to transpose Error: Bad Argument Valu
e
```

maple [A] time = 0.03, size = 97, normalized size = 1.29

$$\frac{x^2\,\text{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{2} + \frac{\sqrt{e}\,x^3\sqrt{e\,x^2+d}}{8d} - \frac{x\sqrt{e\,x^2+d}}{8\sqrt{e}} + \frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{4e} - \frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{8\sqrt{e}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/2*x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/8*e^(1/2)/d*x^3*(e*x^2+d)^(1/2
)-1/8*x*(e*x^2+d)^(1/2)/e^(1/2)+1/4/e*d*ln(x*e^(1/2)+(e*x^2+d)^(1/2))-1/8/e
^(1/2)/d*x*(e*x^2+d)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,x^2\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\frac{1}{4}\,x^2\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\frac{1}{2}\,d\sqrt{e}\int-\frac{\sqrt{ex^2+d}\,x^2}{e^2x^4+dex^2-\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/4*x^2*log(-sqrt(e)*x + sqrt(e*
x^2 + d)) - 1/2*d*sqrt(e)*integrate(-sqrt(e*x^2 + d)*x^2/(e^2*x^4 + d*e*x^2
 - (e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.67, size = 66, normalized size = 0.88

$$\begin{cases} \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4e}+\frac{x^2\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2}-\frac{x\sqrt{d+ex^2}}{4\sqrt{e}} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((d*atanh(sqrt(e)*x/sqrt(d + e*x**2))/(4*e) + x**2*atanh(sqrt(e)*x
/sqrt(d + e*x**2))/2 - x*sqrt(d + e*x**2)/(4*sqrt(e)), Ne(e, 0)), (0, True)
)
```

3.4 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

Optimal. Leaf size=238

$$\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\,\mathrm{Li}_2\left(e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{2\sqrt{d+ex^2}}-\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*ln(x)-1/2*arcsinh(x*e^(1/2)/d^(1/2))^2*d
^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2+d)^(1/2)+arcsinh(x*e^(1/2)/d^(1/2))*ln(1-(x
*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2+d)^
(1/2)-arcsinh(x*e^(1/2)/d^(1/2))*ln(x)*d^(1/2)*(1+e*x^2/d)^(1/2)/(e*x^2+d)^
(1/2)+1/2*polylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(1+e*x
^2/d)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6219, 2327, 2325, 5659, 3716, 2190, 2279, 2391}

$$\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\,\mathrm{PolyLog}\left(2,e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{2\sqrt{d+ex^2}}-\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}}+\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] -(Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2)/(2*Sqrt[d + e
*x^2]) + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[1 -
E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/Sqrt[d + e*x^2] - (Sqrt[d]*Sqrt[1 + (e
*x^2)/d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[x])/Sqrt[d + e*x^2] + ArcTanh[(Sq
rt[e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*PolyLog[2,
E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])])/(2*Sqrt[d + e*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2325

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[e, 2], x]
 - Dist[(b*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]*x)/Sqrt[d]]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2327

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Dist[Sqrt[1 + (e*x^2)/d]/Sqrt[d + e*x^2], Int[(a + b*Log[c*x^n])/Sqr
t[1 + (e*x^2)/d], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6219

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Sim
p[ArcTanh[(c*x)/Sqrt[a + b*x^2]]*Log[x], x] - Dist[c, Int[Log[x]/Sqrt[a + b
*x^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b, c^2]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) - \sqrt{e}\int \frac{\log(x)}{\sqrt{d+ex^2}}\,dx$$

$$= \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) - \frac{\left(\sqrt{e}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{\log(x)}{\sqrt{1+\frac{ex^2}{d}}}\,dx}{\sqrt{d+ex^2}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\right)\int -\ldots}{\sqrt{d+ex\ldots}}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\left(\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\right)\text{Sub}\ldots}{\ldots}$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} - \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{d+ex^2}} + \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\ldots\right.$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}} - \ldots$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}} - \ldots$$

$$= -\frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{d+ex^2}} + \frac{\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\sinh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{d+ex^2}} - \ldots$$

Mathematica [A] time = 2.48, size = 167, normalized size = 0.70

$$\frac{\sqrt{e}\sqrt{\frac{ex^2}{d}+1}\left(-\text{Li}_2\left(e^{-2\sinh^{-1}\left(\sqrt{\frac{e}{d}}x\right)}\right) - 2\log(x)\log\left(\sqrt{\frac{ex^2}{d}+1}+x\sqrt{\frac{e}{d}}\right) + \sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)^2 + 2\sinh^{-1}\left(x\sqrt{\frac{e}{d}}\right)\log\left(\ldots\right.\right.}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x,x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[e]*Sqrt[1 + (e*x^2)/d]*
(ArcSinh[Sqrt[e/d]*x]^2 + 2*ArcSinh[Sqrt[e/d]*x]*Log[1 - E^(-2*ArcSinh[Sqrt
[e/d]*x])] - 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[1 + (e*x^2)/d]] - PolyLog[2, E
^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")
```

[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")

[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x, x)

maple [A] time = 0.31, size = 209, normalized size = 0.88

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)^2}{2}+\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)\ln\left(1+\frac{\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}+1}{\sqrt{-\frac{x^2e}{e\,x^2+d}+1}}\right)+\operatorname{polylog}\left(2,-\frac{\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}+1}{\sqrt{-\frac{x^2e}{e\,x^2+d}+1}}\right)+\operatorname{arctanh}\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x)

[Out] -1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))^2+arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)
)*ln(1+(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+polylog(2,
-(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))+arctanh(x*e^(1/2
)/(e*x^2+d)^(1/2))*ln(1-(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^
(1/2))+polylog(2,(x*e^(1/2)/(e*x^2+d)^(1/2)+1)/(-x^2*e/(e*x^2+d)+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")

[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x,x)

[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x,x)

[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x, x)

3.5 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3} dx$

Optimal. Leaf size=53

$$-\frac{\sqrt{e}\sqrt{d+ex^2}}{2dx}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

```
[Out] -1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*e^(1/2)*(e*x^2+d)^(1/2)/d/x
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6221, 264}

$$-\frac{\sqrt{e}\sqrt{d+ex^2}}{2dx}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(2*d*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(
2*x^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^3} dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}+\frac{1}{2}\sqrt{e}\int \frac{1}{x^2\sqrt{d+ex^2}} dx\\ &= -\frac{\sqrt{e}\sqrt{d+ex^2}}{2dx}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.94

$$-\frac{\sqrt{e}x\sqrt{d+ex^2}+d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

```
[Out] -1/2*(Sqrt[e]*x*Sqrt[d + e*x^2] + d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(
d*x^2)
```

fricas [A] time = 0.60, size = 54, normalized size = 1.02

$$-\frac{2\sqrt{ex^2+d}\sqrt{e}x + d\log\left(\frac{2ex^2+2\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)}{4dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(e*x^2 + d)*sqrt(e)*x + d*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt
(e)*x + d)/d))/(d*x^2)
```

giac [A] time = 0.28, size = 71, normalized size = 1.34

$$\frac{e}{\left(xe^{\frac{1}{2}} - \sqrt{x^2e+d}\right)^2 - d} - \frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] e/((x*e^(1/2) - sqrt(x^2*e + d))^2 - d) - 1/4*log(-(x*e^(1/2)/sqrt(x^2*e +
d) + 1)/(x*e^(1/2)/sqrt(x^2*e + d) - 1))/x^2
```

maple [A] time = 0.03, size = 60, normalized size = 1.13

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{ex^2+d}}\right)}{2x^2} - \frac{\sqrt{e}\left(ex^2+d\right)^{\frac{3}{2}}}{2d^2x} + \frac{e^{\frac{3}{2}}x\sqrt{ex^2+d}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x)
```

```
[Out] -1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*e^(1/2)/d^2/x*(e*x^2+d)^(3/
2)+1/2*e^(3/2)/d^2*x*(e*x^2+d)^(1/2)
```

maxima [A] time = 0.37, size = 51, normalized size = 0.96

$$-\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{2x^2} - \frac{e^{\frac{3}{2}}x^2 + d\sqrt{e}}{2\sqrt{ex^2+d}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^2 - 1/2*(e^(3/2)*x^2 + d*sqrt(e))
/(sqrt(e*x^2 + d)*d*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^3,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**3, x)
```

3.6 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5} dx$

Optimal. Leaf size=79

$$\frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4} - \frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3}$$

```
[Out] -1/4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4+1/6*e^(3/2)*(e*x^2+d)^(1/2)/d^2
/x-1/12*e^(1/2)*(e*x^2+d)^(1/2)/d/x^3
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6221, 271, 264}

$$\frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(12*d*x^3) + (e^(3/2)*Sqrt[d + e*x^2])/(6*d^2*x)
 - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5} dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4} + \frac{1}{4}\sqrt{e} \int \frac{1}{x^4\sqrt{d+ex^2}} dx \\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4} - \frac{e^{3/2}\int \frac{1}{x^2\sqrt{d+ex^2}} dx}{6d} \\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{12dx^3} + \frac{e^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.80

$$\frac{\sqrt{e}x\sqrt{d+ex^2}\left(2ex^2-d\right)-3d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/(12*d^2*x^4)
```

fricas [A] time = 0.70, size = 67, normalized size = 0.85

$$-\frac{3\,d^2\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\,\sqrt{e}\,x+d}{d}\right)-2\left(2\,ex^3-dx\right)\sqrt{ex^2+d}\,\sqrt{e}}{24\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*d^2*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(2*e*x^
3 - d*x)*sqrt(e*x^2 + d)*sqrt(e))/(d^2*x^4)
```

giac [A] time = 0.39, size = 107, normalized size = 1.35

$$\frac{\left(3\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2de-d^2e\right)e}{3\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-d\right)^3d}-\frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(x*e^(1/2) - sqrt(x^2*e + d))^2*d*e - d^2*e)*e/(((x*e^(1/2) - sqrt(x
^2*e + d))^2 - d)^3*d) - 1/8*log(-(x*e^(1/2)/sqrt(x^2*e + d) + 1)/(x*e^(1/2
)/sqrt(x^2*e + d) - 1))/x^4
```

maple [A] time = 0.03, size = 62, normalized size = 0.78

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{4x^4}+\frac{e^{\frac{3}{2}}\sqrt{e\,x^2+d}}{4d^2x}-\frac{\sqrt{e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{12d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x)
```

```
[Out] -1/4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*e^(3/2)*(e*x^2+d)^(1/2)/d^2
/x-1/12*e^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

maxima [A] time = 0.35, size = 61, normalized size = 0.77

$$\frac{\sqrt{ex^2+d}\,e^{\frac{3}{2}}}{4\,d^2x}-\frac{\left(ex^2+d\right)^{\frac{3}{2}}\sqrt{e}}{12\,d^2x^3}-\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(e*x^2 + d)*e^(3/2)/(d^2*x) - 1/12*(e*x^2 + d)^(3/2)*sqrt(e)/(d^2*x
^3) - 1/4*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^5,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**5, x)
```

3.7 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

Optimal. Leaf size=105

$$-\frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}-\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5}$$

```
[Out] -1/6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6+2/45*e^(3/2)*(e*x^2+d)^(1/2)/d^
2/x^3-4/45*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x-1/30*e^(1/2)*(e*x^2+d)^(1/2)/d/x^5
```

Rubi [A] time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6221, 271, 264}

$$-\frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x}+\frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3}-\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(30*d*x^5) + (2*e^(3/2)*Sqrt[d + e*x^2])/(45*d^2
*x^3) - (4*e^(5/2)*Sqrt[d + e*x^2])/(45*d^3*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]]/(6*x^6)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{1}{6}\sqrt{e}\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6} - \frac{\left(2e^{3/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{15d}$$
$$= -\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5} + \frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6} + \frac{\left(4e^{5/2}\right)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{45d^2}$$
$$= -\frac{\sqrt{e}\sqrt{d+ex^2}}{30dx^5} + \frac{2e^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{4e^{5/2}\sqrt{d+ex^2}}{45d^3x} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 0.70

$$\frac{\sqrt{e}x\sqrt{d+ex^2}\left(-3d^2+4dex^2-8e^2x^4\right)-15d^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

fricas [A] time = 0.57, size = 78, normalized size = 0.74

$$-\frac{15\,d^3\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\,\sqrt{e}\,x+d}{d}\right)+2\left(8\,e^2x^5-4\,dex^3+3\,d^2x\right)\sqrt{ex^2+d}\,\sqrt{e}}{180\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

```
[Out] -1/180*(15*d^3*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) + 2*(8*e^
2*x^5 - 4*d*e*x^3 + 3*d^2*x)*sqrt(e*x^2 + d)*sqrt(e))/(d^3*x^6)
```

giac [A] time = 0.45, size = 134, normalized size = 1.28

$$\frac{8\left(10\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^4d^2e^2-5\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2d^3e^2+d^4e^2\right)e}{45\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-d\right)^5d^2}-\frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

```
[Out] 8/45*(10*(x*e^(1/2) - sqrt(x^2*e + d))^4*d^2*e^2 - 5*(x*e^(1/2) - sqrt(x^2*
e + d))^2*d^3*e^2 + d^4*e^2)*e/(((x*e^(1/2) - sqrt(x^2*e + d))^2 - d)^5*d^2
) - 1/12*log(-(x*e^(1/2)/sqrt(x^2*e + d) + 1)/(x*e^(1/2)/sqrt(x^2*e + d) -
1))/x^6
```

maple [A] time = 0.03, size = 110, normalized size = 1.05

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{6x^6}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{6d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x)
```

```
[Out] -1/6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6-1/6*e^(3/2)/d*(-1/3/d/x^3*(e*x^
2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))+1/6*e^(1/2)/d*(-1/5/d/x^5*(e*x^2+d)
^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2))
```

maxima [A] time = 0.34, size = 102, normalized size = 0.97

$$-\frac{\left(2\,e^2x^4+dex^2-d^2\right)e^{\frac{3}{2}}}{18\,\sqrt{ex^2+d}\,d^3x^3}-\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{6\,x^6}+\frac{\left(2\,e^2x^4-dex^2-3\,d^2\right)\sqrt{ex^2+d}\,\sqrt{e}}{90\,d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

```
[Out] -1/18*(2*e^2*x^4 + d*e*x^2 - d^2)*e^(3/2)/(sqrt(e*x^2 + d)*d^3*x^3) - 1/6*a
rctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^6 + 1/90*(2*e^2*x^4 - d*e*x^2 - 3*d^2)*
sqrt(e*x^2 + d)*sqrt(e)/(d^3*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^7,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**7, x)
```

3.8 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9} dx$

Optimal. Leaf size=131

$$\frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8} - \frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7}$$

```
[Out] -1/8*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^8+3/140*e^(3/2)*(e*x^2+d)^(1/2)/d
^2/x^5-1/35*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x^3+2/35*e^(7/2)*(e*x^2+d)^(1/2)/d^
4/x-1/56*e^(1/2)*(e*x^2+d)^(1/2)/d/x^7
```

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6221, 271, 264}

$$\frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(56*d*x^7) + (3*e^(3/2)*Sqrt[d + e*x^2])/(140*d^
2*x^5) - (e^(5/2)*Sqrt[d + e*x^2])/(35*d^3*x^3) + (2*e^(7/2)*Sqrt[d + e*x^2
])/(35*d^4*x) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{1}{8}\sqrt{e}\int \frac{1}{x^8\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8} - \frac{\left(3e^{3/2}\right)\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{28d}$$
$$= -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8} + \frac{\left(3e^{5/2}\right)\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{35d^2}$$
$$= -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8} - \frac{\left(2e^{7/2}\right)\int \frac{1}{x^2\sqrt{d+ex^2}}}{35d^3}$$
$$= -\frac{\sqrt{e}\sqrt{d+ex^2}}{56dx^7} + \frac{3e^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{e^{5/2}\sqrt{d+ex^2}}{35d^3x^3} + \frac{2e^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 0.65

$$\frac{\sqrt{e}x\sqrt{d+ex^2}\left(-5d^3+6d^2ex^2-8de^2x^4+16e^3x^6\right)-35d^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x^6
) - 35*d^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

fricas [A] time = 1.13, size = 89, normalized size = 0.68

$$-\frac{35\,d^4\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\,\sqrt{e}\,x+d}{d}\right)-2\left(16\,e^3x^7-8\,de^2x^5+6\,d^2ex^3-5\,d^3x\right)\sqrt{ex^2+d}\,\sqrt{e}}{560\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
```

```
[Out] -1/560*(35*d^4*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(16*e
^3*x^7 - 8*d*e^2*x^5 + 6*d^2*e*x^3 - 5*d^3*x)*sqrt(e*x^2 + d)*sqrt(e))/(d^4
*x^8)
```

giac [A] time = 1.39, size = 161, normalized size = 1.23

$$-\frac{\log\left(-\frac{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}+1}{\frac{xe^{\frac{1}{2}}}{\sqrt{x^2e+d}}-1}\right)}{16\,x^8}+\frac{4\left(35\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^6d^3e^3-21\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^4d^4e^3+7\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2d^5e^3-d^6e^3\right)}{35\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-d\right)^7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

```
[Out] -1/16*log(-(x*e^(1/2)/sqrt(x^2*e + d) + 1)/(x*e^(1/2)/sqrt(x^2*e + d) - 1))
/x^8 + 4/35*(35*(x*e^(1/2) - sqrt(x^2*e + d))^6*d^3*e^3 - 21*(x*e^(1/2) - s
```

```
qrt(x^2*e + d))^4*d^4*e^3 + 7*(x*e^(1/2) - sqrt(x^2*e + d))^2*d^5*e^3 - d^6
*e^3)*e/(((x*e^(1/2) - sqrt(x^2*e + d))^2 - d)^7*d^3)
```

maple [A] time = 0.04, size = 158, normalized size = 1.21

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{8x^8}-\frac{e^{\frac{3}{2}}\left(-\frac{\sqrt{e\,x^2+d}}{5d\,x^5}-\frac{4e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{5d}\right)}{8d}+\frac{\sqrt{e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{7d\,x^7}-\frac{4e\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}\right)}{7d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x)
```

```
[Out] -1/8*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^8-1/8*e^(3/2)/d*(-1/5/d/x^5*(e*x^
2+d)^(1/2)-4/5*e/d*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))
)+1/8*e^(1/2)/d*(-1/7/d/x^7*(e*x^2+d)^(3/2)-4/7*e/d*(-1/5/d/x^5*(e*x^2+d)^(
3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2)))
```

maxima [A] time = 0.34, size = 125, normalized size = 0.95

$$\frac{\left(8\,e^3x^6+4\,de^2x^4-d^2ex^2+3\,d^3\right)e^{\frac{3}{2}}}{120\,\sqrt{ex^2+d}\,d^4x^5}-\frac{\operatorname{artanh}\left(\frac{\sqrt{e}\,x}{\sqrt{ex^2+d}}\right)}{8\,x^8}-\frac{\left(8\,e^3x^6-4\,de^2x^4+3\,d^2ex^2+15\,d^3\right)\sqrt{ex^2+d}\,\sqrt{e}}{840\,d^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

```
[Out] 1/120*(8*e^3*x^6 + 4*d*e^2*x^4 - d^2*e*x^2 + 3*d^3)*e^(3/2)/(sqrt(e*x^2 + d
)*d^4*x^5) - 1/8*arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^8 - 1/840*(8*e^3*x^6
- 4*d*e^2*x^4 + 3*d^2*e*x^2 + 15*d^3)*sqrt(e*x^2 + d)*sqrt(e)/(d^4*x^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^9,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**9, x)
```

3.9 $\int x^6 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=114

$$\frac{d^3\sqrt{d+ex^2}}{7e^{7/2}}-\frac{d^2\left(d+ex^2\right)^{3/2}}{7e^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49e^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35e^{7/2}}+\frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/7*d^2*(e*x^2+d)^(3/2)/e^(7/2)+3/35*d*(e*x^2+d)^(5/2)/e^(7/2)-1/49*(e*x^2
+d)^(7/2)/e^(7/2)+1/7*x^7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/7*d^3*(e*x^2
+d)^(1/2)/e^(7/2)
```

Rubi [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6221, 266, 43}

$$-\frac{d^2\left(d+ex^2\right)^{3/2}}{7e^{7/2}}+\frac{d^3\sqrt{d+ex^2}}{7e^{7/2}}-\frac{\left(d+ex^2\right)^{7/2}}{49e^{7/2}}+\frac{3d\left(d+ex^2\right)^{5/2}}{35e^{7/2}}+\frac{1}{7}x^7\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d^3*Sqrt[d + e*x^2])/(7*e^(7/2)) - (d^2*(d + e*x^2)^(3/2))/(7*e^(7/2)) + (
3*d*(d + e*x^2)^(5/2))/(35*e^(7/2)) - (d + e*x^2)^(7/2)/(49*e^(7/2)) + (x^7
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^6 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{7}x^7 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\sqrt{e}\int \frac{x^7}{\sqrt{d+ex^2}} dx$$
$$= \frac{1}{7}x^7 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{e}\,\text{Subst}\left(\int \frac{x^3}{\sqrt{d+ex}} dx, x, x^2\right)$$
$$= \frac{1}{7}x^7 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{e}\,\text{Subst}\left(\int \left(-\frac{d^3}{e^3\sqrt{d+ex}} + \frac{3d^2\sqrt{d+ex}}{e^3} - \frac{3d}{}\right.\right.$$
$$= \frac{d^3\sqrt{d+ex^2}}{7e^{7/2}} - \frac{d^2\left(d+ex^2\right)^{3/2}}{7e^{7/2}} + \frac{3d\left(d+ex^2\right)^{5/2}}{35e^{7/2}} - \frac{\left(d+ex^2\right)^{7/2}}{49e^{7/2}} + \frac{1}{7}x^7 \tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.69

$$\frac{\sqrt{d+ex^2}\left(16d^3 - 8d^2ex^2 + 6de^2x^4 - 5e^3x^6\right)}{245e^{7/2}} + \frac{1}{7}x^7 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*e^(
7/2)) + (x^7*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7
```

fricas [A] time = 0.57, size = 88, normalized size = 0.77

$$\frac{35\,e^4x^7\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\,\sqrt{e}\,x+d}{d}\right) - 2\left(5\,e^3x^6 - 6\,de^2x^4 + 8\,d^2ex^2 - 16\,d^3\right)\sqrt{ex^2+d}\,\sqrt{e}}{490\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/490*(35*e^4*x^7*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(5
*e^3*x^6 - 6*d*e^2*x^4 + 8*d^2*e*x^2 - 16*d^3)*sqrt(e*x^2 + d)*sqrt(e))/e^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
x^7/7*ln((1+(sqrt(exp(1)*x^2+d))^-1*exp(1/2)*x)/(1-(sqrt(exp(1)*x^2+d))^-1*
exp(1/2)*x))-2*d*exp(1/2)*((2401/5*sqrt(d+x^2*exp(1))*(d+x^2*exp(1))^2*exp(
1)^16-9604/5*sqrt(d+x^2*exp(1))*(d+x^2*exp(1))^2*exp(1)^15*exp(1/2)^2+14406
/5*sqrt(d+x^2*exp(1))*(d+x^2*exp(1))^2*exp(1)^14*exp(1/2)^4-9604/5*sqrt(d+x
^2*exp(1))*(d+x^2*exp(1))^2*exp(1)^13*exp(1/2)^6+2401/5*sqrt(d+x^2*exp(1))*
(d+x^2*exp(1))^2*exp(1)^12*exp(1/2)^8-2401*sqrt(d+x^2*exp(1))*(d+x^2*exp(1)
)*d*exp(1)^16+26411/3*sqrt(d+x^2*exp(1))*(d+x^2*exp(1))*d*exp(1)^15*exp(1/2
)^2-12005*sqrt(d+x^2*exp(1))*(d+x^2*exp(1))*d*exp(1)^14*exp(1/2)^4+7203*sqr
t(d+x^2*exp(1))*(d+x^2*exp(1))*d*exp(1)^13*exp(1/2)^6-4802/3*sqrt(d+x^2*exp
(1))*(d+x^2*exp(1))*d*exp(1)^12*exp(1/2)^8+7203*sqrt(d+x^2*exp(1))*d^2*exp(
1)^16-21609*sqrt(d+x^2*exp(1))*d^2*exp(1)^15*exp(1/2)^2+24010*sqrt(d+x^2*ex
p(1))*d^2*exp(1)^14*exp(1/2)^4-12005*sqrt(d+x^2*exp(1))*d^2*exp(1)^13*exp(1
/2)^6+2401*sqrt(d+x^2*exp(1))*d^2*exp(1)^12*exp(1/2)^8)/(16807*exp(1)^20-84
```

```
035*exp(1)^19*exp(1/2)^2+168070*exp(1)^18*exp(1/2)^4-168070*exp(1)^17*exp(1
/2)^6+84035*exp(1)^16*exp(1/2)^8-16807*exp(1)^15*exp(1/2)^10)-2*d^3/2/(7*ex
p(1)^3-21*exp(1)^2*exp(1/2)^2+21*exp(1)*exp(1/2)^4-7*exp(1/2)^6)/sqrt(-d*ex
p(1/2)^2+d*exp(1))/exp(1/2)*atan((sqrt(d+x^2*exp(1))*exp(1)-sqrt(d+x^2*exp(
1))*exp(1/2)^2)/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2))))
```

maple [B] time = 0.04, size = 224, normalized size = 1.96

$$\frac{x^7 \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e x^2+d}}\right)}{7}+\frac{e^{\frac{3}{2}}\left(\frac{x^8\sqrt{e x^2+d}}{9e}-\frac{8d\left(\frac{x^6\sqrt{e x^2+d}}{7e}-\frac{6d\left(\frac{x^4\sqrt{e x^2+d}}{5e}-\frac{4d\left(\frac{x^2\sqrt{e x^2+d}}{3e}-\frac{2d\sqrt{e x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{9e}\right)}{7d}-\frac{\sqrt{e}\left(\frac{x^6\left(e x^2+d\right)^{\frac{3}{2}}}{9e}-\frac{2d\left(\frac{x^4\left(e x^2+d\right.}{7e}\right.}{}\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/7*x^7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/7*e^(3/2)/d*(1/9*x^8/e*(e*x^2+
d)^(1/2)-8/9*d/e*(1/7*x^6/e*(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1
/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))))-1/7*e^
(1/2)/d*(1/9*x^6*(e*x^2+d)^(3/2)/e-2/3*d/e*(1/7*x^4*(e*x^2+d)^(3/2)/e-4/7*d
/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))))
```

maxima [A] time = 0.34, size = 155, normalized size = 1.36

$$\frac{1}{7}x^7 \operatorname{artanh}\left(\frac{\sqrt{e}\,x}{\sqrt{ex^2+d}}\right)-\frac{35\left(ex^2+d\right)^{\frac{9}{2}}-135\left(ex^2+d\right)^{\frac{7}{2}}d+189\left(ex^2+d\right)^{\frac{5}{2}}d^2-105\left(ex^2+d\right)^{\frac{3}{2}}d^3}{2205\,de^{\frac{7}{2}}}+\frac{35\left(ex^2+d\right)^{\frac{9}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/7*x^7*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/2205*(35*(e*x^2 + d)^(9/2) -
 135*(e*x^2 + d)^(7/2)*d + 189*(e*x^2 + d)^(5/2)*d^2 - 105*(e*x^2 + d)^(3/2
)*d^3)/(d*e^(7/2)) + 1/2205*(35*(e*x^2 + d)^(9/2) - 180*(e*x^2 + d)^(7/2)*d
 + 378*(e*x^2 + d)^(5/2)*d^2 - 420*(e*x^2 + d)^(3/2)*d^3 + 315*sqrt(e*x^2 +
 d)*d^4)/(d*e^(7/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^6*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 8.51, size = 116, normalized size = 1.02

$$\begin{cases} \frac{16d^3\sqrt{d+ex^2}}{245e^{\frac{7}{2}}} - \frac{8d^2x^2\sqrt{d+ex^2}}{245e^{\frac{5}{2}}} + \frac{6dx^4\sqrt{d+ex^2}}{245e^{\frac{3}{2}}} + \frac{x^7\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^6\sqrt{d+ex^2}}{49\sqrt{e}} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((16*d**3*sqrt(d + e*x**2)/(245*e**(7/2)) - 8*d**2*x**2*sqrt(d + e
*x**2)/(245*e**(5/2)) + 6*d*x**4*sqrt(d + e*x**2)/(245*e**(3/2)) + x**7*ata
nh(sqrt(e)*x/sqrt(d + e*x**2))/7 - x**6*sqrt(d + e*x**2)/(49*sqrt(e)), Ne(e
, 0)), (0, True))
```

3.10 $\int x^4 \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=91

$$-\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}}-\frac{\left(d+ex^2\right)^{5/2}}{25e^{5/2}}+\frac{2d\left(d+ex^2\right)^{3/2}}{15e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] 2/15*d*(e*x^2+d)^(3/2)/e^(5/2)-1/25*(e*x^2+d)^(5/2)/e^(5/2)+1/5*x^5*arctanh
(x*e^(1/2)/(e*x^2+d)^(1/2))-1/5*d^2*(e*x^2+d)^(1/2)/e^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6221, 266, 43}

$$-\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}}-\frac{\left(d+ex^2\right)^{5/2}}{25e^{5/2}}+\frac{2d\left(d+ex^2\right)^{3/2}}{15e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(d^2*Sqrt[d + e*x^2])/(5*e^(5/2)) + (2*d*(d + e*x^2)^(3/2))/(15*e^(5/2)) -
 (d + e*x^2)^(5/2)/(25*e^(5/2)) + (x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^4\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\sqrt{e}\int\frac{x^5}{\sqrt{d+ex^2}}dx\\
&= \frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{e}\,\text{Subst}\left(\int\frac{x^2}{\sqrt{d+ex}}dx,x,x^2\right)\\
&= \frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{e}\,\text{Subst}\left(\int\left(\frac{d^2}{e^2\sqrt{d+ex}}-\frac{2d\sqrt{d+ex}}{e^2}+\frac{(d+ex)}{e^2}\right.\right.\\
&= -\frac{d^2\sqrt{d+ex^2}}{5e^{5/2}}+\frac{2d\left(d+ex^2\right)^{3/2}}{15e^{5/2}}-\frac{\left(d+ex^2\right)^{5/2}}{25e^{5/2}}+\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 0.75

$$\frac{1}{5}x^5\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}\left(8d^2-4dex^2+3e^2x^4\right)}{75e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -1/75*(Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/e^(5/2) + (x^5*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/5
```

fricas [A] time = 0.40, size = 77, normalized size = 0.85

$$\frac{15\,e^3x^5\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\,\sqrt{e}\,x+d}{d}\right)-2\left(3\,e^2x^4-4\,dex^2+8\,d^2\right)\sqrt{ex^2+d}\,\sqrt{e}}{150\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/150*(15*e^3*x^5*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*(3
*e^2*x^4 - 4*d*e*x^2 + 8*d^2)*sqrt(e*x^2 + d)*sqrt(e))/e^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
x^5/5*ln((1+(sqrt(exp(1)*x^2+d))^-1*exp(1/2)*x)/(1-(sqrt(exp(1)*x^2+d))^-1*
exp(1/2)*x))-2*d*exp(1/2)*((25/3*sqrt(d+x^2*exp(1))*(d+x^2*exp(1))*exp(1)^6
-50/3*sqrt(d+x^2*exp(1))*(d+x^2*exp(1))*exp(1)^5*exp(1/2)^2+25/3*sqrt(d+x^2
*exp(1))*(d+x^2*exp(1))*exp(1)^4*exp(1/2)^4-50*sqrt(d+x^2*exp(1))*d*exp(1)^
6+75*sqrt(d+x^2*exp(1))*d*exp(1)^5*exp(1/2)^2-25*sqrt(d+x^2*exp(1))*d*exp(1
)^4*exp(1/2)^4)/(125*exp(1)^9-375*exp(1)^8*exp(1/2)^2+375*exp(1)^7*exp(1/2)
^4-125*exp(1)^6*exp(1/2)^6)+2*d^2/2/(5*exp(1)^2-10*exp(1)*exp(1/2)^2+5*exp(
1/2)^4)/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2)*atan((sqrt(d+x^2*exp(1))*exp(
1)-sqrt(d+x^2*exp(1))*exp(1/2)^2)/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2))))
```

maple [B] time = 0.03, size = 176, normalized size = 1.93

$$\frac{x^5\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{5}+\frac{e^{\frac{3}{2}}\left(\frac{x^6\sqrt{e\,x^2+d}}{7e}-\frac{6d\left(\frac{x^4\sqrt{e\,x^2+d}}{5e}-\frac{4d\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{5d}-\frac{\sqrt{e}\left(\frac{x^4\left(e\,x^2+d\right)^{\frac{3}{2}}}{7e}-\frac{4d\left(\frac{x^2\left(e\,x^2+d\right)^{\frac{3}{2}}}{5e}-\frac{2d\left(e\,x^2+\right.}{15e^2}\right.}{7e}\right.}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/5*x^5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/5*e^(3/2)/d*(1/7*x^6/e*(e*x^2+
d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1
/2)-2/3*d/e^2*(e*x^2+d)^(1/2))))-1/5*e^(1/2)/d*(1/7*x^4*(e*x^2+d)^(3/2)/e-4
/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2)))
```

maxima [A] time = 0.33, size = 127, normalized size = 1.40

$$\frac{1}{5}x^5\,\text{artanh}\left(\frac{\sqrt{e}\,x}{\sqrt{ex^2+d}}\right)-\frac{15\left(ex^2+d\right)^{\frac{7}{2}}-42\left(ex^2+d\right)^{\frac{5}{2}}d+35\left(ex^2+d\right)^{\frac{3}{2}}d^2}{525\,de^{\frac{5}{2}}}+\frac{5\left(ex^2+d\right)^{\frac{7}{2}}-21\left(ex^2+d\right)^{\frac{5}{2}}d+35}{175\,de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/525*(15*(e*x^2 + d)^(7/2) -
42*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2)/(d*e^(5/2)) + 1/175*(5*(
e*x^2 + d)^(7/2) - 21*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2 - 35*s
qrt(e*x^2 + d)*d^3)/(d*e^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\text{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^4*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 2.94, size = 90, normalized size = 0.99

$$\begin{cases} -\frac{8d^2\sqrt{d+ex^2}}{75e^{\frac{5}{2}}}+\frac{4dx^2\sqrt{d+ex^2}}{75e^{\frac{3}{2}}}+\frac{x^5\,\text{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{5}-\frac{x^4\sqrt{d+ex^2}}{25\sqrt{e}} & \text{for } e\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((-8*d**2*sqrt(d + e*x**2)/(75*e**(5/2)) + 4*d*x**2*sqrt(d + e*x**
2)/(75*e**(3/2)) + x**5*atanh(sqrt(e)*x/sqrt(d + e*x**2))/5 - x**4*sqrt(d +
 e*x**2)/(25*sqrt(e)), Ne(e, 0)), (0, True))
```

3.11 $\int x^2 \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=68

$$-\frac{\left(d+ex^2\right)^{3/2}}{9e^{3/2}}+\frac{d\sqrt{d+ex^2}}{3e^{3/2}}+\frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

```
[Out] -1/9*(e*x^2+d)^(3/2)/e^(3/2)+1/3*x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/3
*d*(e*x^2+d)^(1/2)/e^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6221, 266, 43}

$$-\frac{\left(d+ex^2\right)^{3/2}}{9e^{3/2}}+\frac{d\sqrt{d+ex^2}}{3e^{3/2}}+\frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (d*Sqrt[d + e*x^2])/(3*e^(3/2)) - (d + e*x^2)^(3/2)/(9*e^(3/2)) + (x^3*ArcT
anh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx &= \frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{3}\sqrt{e}\int\frac{x^3}{\sqrt{d+ex^2}}\,dx\\
&= \frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\,\text{Subst}\left(\int\frac{x}{\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{e}\,\text{Subst}\left(\int\left(-\frac{d}{e\sqrt{d+ex}}+\frac{\sqrt{d+ex}}{e}\right)dx,x,x^2\right)\\
&= \frac{d\sqrt{d+ex^2}}{3e^{3/2}}-\frac{\left(d+ex^2\right)^{3/2}}{9e^{3/2}}+\frac{1}{3}x^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.82

$$\frac{1}{9}\left(\frac{(2d-ex^2)\sqrt{d+ex^2}}{e^{3/2}}+3x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (((2*d - e*x^2)*Sqrt[d + e*x^2])/e^(3/2) + 3*x^3*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/9
```

fricas [A] time = 0.49, size = 65, normalized size = 0.96

$$\frac{3e^2x^3\log\left(\frac{2ex^2+2\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-2\sqrt{ex^2+d}\left(ex^2-2d\right)\sqrt{e}}{18e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*(3*e^2*x^3*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*sqrt
(e*x^2 + d)*(e*x^2 - 2*d)*sqrt(e))/e^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
x^3/3*ln((1+(sqrt(exp(1)*x^2+d))^-1*exp(1/2)*x)/(1-(sqrt(exp(1)*x^2+d))^-1*
exp(1/2)*x))-2*d*exp(1/2)/exp(1)*(sqrt(d+x^2*exp(1))/(3*exp(1)-3*exp(1/2)^2
)-2*d*exp(1)/2/(3*exp(1)-3*exp(1/2)^2)/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2
)*atan((sqrt(d+x^2*exp(1))*exp(1)-sqrt(d+x^2*exp(1))*exp(1/2)^2)/sqrt(-d*ex
p(1/2)^2+d*exp(1))/exp(1/2))))
```

maple [B] time = 0.03, size = 128, normalized size = 1.88

$$\frac{x^3\,\mathrm{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{3}+\frac{e^{\frac{3}{2}}\left(\frac{x^4\sqrt{e\,x^2+d}}{5e}-\frac{4d\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{5e}\right)}{3d}-\frac{\sqrt{e}\left(\frac{x^2\left(e\,x^2+d\right)^{\frac{3}{2}}}{5e}-\frac{2d\left(e\,x^2+d\right)^{\frac{3}{2}}}{15e^2}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] 1/3*x^3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+1/3*e^(3/2)/d*(1/5*x^4/e*(e*x^2+
d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))-1/3
*e^(1/2)/d*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))
```

maxima [A] time = 0.33, size = 99, normalized size = 1.46

$$\frac{1}{3}x^3\,\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)-\frac{3\left(ex^2+d\right)^{\frac{5}{2}}-5\left(ex^2+d\right)^{\frac{3}{2}}d}{45de^{\frac{3}{2}}}+\frac{3\left(ex^2+d\right)^{\frac{5}{2}}-10\left(ex^2+d\right)^{\frac{3}{2}}d+15\sqrt{ex^2+d}\,d^2}{45de^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/45*(3*(e*x^2 + d)^(5/2) - 5*
(e*x^2 + d)^(3/2)*d)/(d*e^(3/2)) + 1/45*(3*(e*x^2 + d)^(5/2) - 10*(e*x^2 +
d)^(3/2)*d + 15*sqrt(e*x^2 + d)*d^2)/(d*e^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^2*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 1.03, size = 65, normalized size = 0.96

$$\begin{cases} \frac{2d\sqrt{d+ex^2}}{9e^{\frac{3}{2}}} + \frac{x^3 \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3} - \frac{x^2\sqrt{d+ex^2}}{9\sqrt{e}} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((2*d*sqrt(d + e*x**2)/(9*e**(3/2)) + x**3*atanh(sqrt(e)*x/sqrt(d
+ e*x**2))/3 - x**2*sqrt(d + e*x**2)/(9*sqrt(e)), Ne(e, 0)), (0, True))
```

3.12 $\int \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=40

$$x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}}$$

```
[Out] x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-(e*x^2+d)^(1/2)/e^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = $\{6217, 261\}$

$$x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6217

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*A
rcTanh[(c*x)/Sqrt[a + b*x^2]], x] - Dist[c, Int[x/Sqrt[a + b*x^2], x], x] /
; FreeQ[{a, b, c}, x] && EqQ[b, c^2]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx &= x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\sqrt{e}\int \frac{x}{\sqrt{d+ex^2}}\,dx\\ &= -\frac{\sqrt{d+ex^2}}{\sqrt{e}}+x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$x\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] -(Sqrt[d + e*x^2]/Sqrt[e]) + x*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
```

fricas [A] time = 0.77, size = 51, normalized size = 1.28

$$\frac{ex\log\left(\frac{2\,ex^2+2\,\sqrt{ex^2+d}\,\sqrt{e}\,x+d}{d}\right)-2\,\sqrt{ex^2+d}\,\sqrt{e}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(e*x*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - 2*sqrt(e*x^2
+ d)*sqrt(e))/e
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
x*ln((1+(sqrt(exp(1)*x^2+d))^-1*exp(1/2)*x)/(1-(sqrt(exp(1)*x^2+d))^-1*exp(
1/2)*x))-2*d*exp(1/2)/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2)*atan((sqrt(d+x^
2*exp(1))*exp(1)-sqrt(d+x^2*exp(1))*exp(1/2)^2)/sqrt(-d*exp(1/2)^2+d*exp(1)
)/exp(1/2)))
```

maple [B] time = 0.03, size = 76, normalized size = 1.90

$$x \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)+\frac{e^{\frac{3}{2}}\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{3\sqrt{e}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] x*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+e^(3/2)/d*(1/3*x^2/e*(e*x^2+d)^(1/2)-2
/3*d/e^2*(e*x^2+d)^(1/2))-1/3/e^(1/2)/d*(e*x^2+d)^(3/2)
```

maxima [B] time = 0.34, size = 65, normalized size = 1.62

$$x \operatorname{artanh}\left(\frac{\sqrt{e}\,x}{\sqrt{ex^2+d}}\right)-\frac{\left(ex^2+d\right)^{\frac{3}{2}}}{3\,d\sqrt{e}}+\frac{\left(ex^2+d\right)^{\frac{3}{2}}-3\,\sqrt{ex^2+d}\,d}{3\,d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)) - 1/3*(e*x^2 + d)^(3/2)/(d*sqrt(e)) +
1/3*((e*x^2 + d)^(3/2) - 3*sqrt(e*x^2 + d)*d)/(d*sqrt(e))
```

mupad [B] time = 1.06, size = 32, normalized size = 0.80

$$x \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)-\frac{\sqrt{e\,x^2+d}}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] x*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)) - (d + e*x^2)^(1/2)/e^(1/2)
```

sympy [A] time = 0.63, size = 36, normalized size = 0.90

$$\begin{cases} x \operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{\sqrt{e}} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Piecewise((x*atanh(sqrt(e)*x/sqrt(d + e*x**2)) - sqrt(d + e*x**2)/sqrt(e),
Ne(e, 0)), (0, True))
```

3.13 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2} dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

```
[Out] -arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x-arctanh((e*x^2+d)^(1/2)/d^(1/2))*e^(1
/2)/d^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6221, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[e]*ArcTanh[Sqrt[d + e*x^2
]/Sqrt[d]])/Sqrt[d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x} + \sqrt{e}\int \frac{1}{x\sqrt{d+ex^2}}\,dx \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x} + \frac{1}{2}\sqrt{e}\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx, x, x^2\right) \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx, x, \sqrt{d+ex^2}\right)}{\sqrt{e}} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.11

$$
\frac{\sqrt{e}\left(\log(x)-\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)\right)}{\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x) + (Sqrt[e]*(Log[x] - Log[d + Sqrt
[d]*Sqrt[d + e*x^2]]))/Sqrt[d]
```

fricas [B] time = 0.68, size = 273, normalized size = 4.96

$$
\left[\frac{x\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2-2\sqrt{ex^2+d}\,d\sqrt{e}\sqrt{\frac{e}{d}}+2de}{x^2}\right)+(x-1)\log\left(\frac{2ex^2+2\sqrt{ex^2+d}\sqrt{e}x+d}{d}\right)-x\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right)+x\log\left(\frac{ex-\sqrt{ex^2+d}}{x}\right.}{2x}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(e/d) + 2
*d*e)/x^2) + (x - 1)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d) - x
*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + x*log((e*x - sqrt(e*x^2 + d)*sqrt
(e))/x))/x, 1/2*(2*x*sqrt(-e/d)*arctan(sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(-e/d)
/(e^2*x^2 + d*e)) + (x - 1)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)
/d) - x*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + x*log((e*x - sqrt(e*x^2 +
d)*sqrt(e))/x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-1/x*ln((1+(sqrt(exp(1)*x^2+d))^-1*exp(1/2)*x)/(1-(sqrt(exp(1)*x^2+d))^-1*e
xp(1/2)*x))-2*d*exp(1/2)*(-2/d/2/sqrt(-d)*atan(sqrt(d+x^2*exp(1))/sqrt(-d))
```

```
+(2*exp(1)-2*exp(1/2)^2)/d/2/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2)*atan((sq
rt(d+x^2*exp(1))*exp(1)-sqrt(d+x^2*exp(1))*exp(1/2)^2)/sqrt(-d*exp(1/2)^2+d
*exp(1))/exp(1/2))))
```

maple [A] time = 0.03, size = 53, normalized size = 0.96

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x}-\frac{\sqrt{e}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x)
```

```
[Out] -arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x-e^(1/2)/d^(1/2)*ln((2*d+2*d^(1/2)*(e*
x^2+d)^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d\sqrt{e}\int -\frac{\sqrt{ex^2+d}}{e^2x^5+dex^3-\left(ex^3+dx\right)\left(ex^2+d\right)}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] d*sqrt(e)*integrate(-sqrt(e*x^2 + d)/(e^2*x^5 + d*e*x^3 - (e*x^3 + d*x)*(e*
x^2 + d)), x) - 1/2*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(e)*x + sq
rt(e*x^2 + d)))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^2,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**2, x)
```

3.14 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4} dx$

Optimal. Leaf size=85

$$\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}} - \frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

```
[Out] -1/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3+1/6*e^(3/2)*arctanh((e*x^2+d)^(
1/2)/d^(1/2))/d^(3/2)-1/6*e^(1/2)*(e*x^2+d)^(1/2)/d/x^2
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6221, 266, 51, 63, 208}

$$\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}} - \frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(6*d*x^2) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
/(3*x^3) + (e^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(6*d^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{3}\sqrt{e}\int \frac{1}{x^3\sqrt{d+ex^2}}\,dx\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{1}{6}\sqrt{e}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{e^{3/2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}\,dx,x,x^2\right)}{12d}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}-\frac{\sqrt{e}\,\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}\,dx,x,\sqrt{d+ex^2}\right)}{6d}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{6dx^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^3}+\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 92, normalized size = 1.08

$$
-\frac{\frac{\sqrt{e}x\left(\sqrt{d}\sqrt{d+ex^2}-ex^2\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)+ex^2\log(x)\right)}{d^{3/2}}+2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

```
[Out] -1/6*(2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + (Sqrt[e]*x*(Sqrt[d]*Sqrt[d +
 e*x^2] + e*x^2*Log[x] - e*x^2*Log[d + Sqrt[d]*Sqrt[d + e*x^2]]))/d^(3/2))/
x^3
```

fricas [B] time = 0.64, size = 340, normalized size = 4.00

$$
\left[\frac{ex^3\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2+2\sqrt{ex^2+d}\,d\sqrt{e}\sqrt{\frac{e}{d}}+2de}{x^2}\right)-2dx^3\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right)+2dx^3\log\left(\frac{ex-\sqrt{ex^2+d}\sqrt{e}}{x}\right)-2\sqrt{ex^2+d}\sqrt{e}}{12dx^3}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(e*x^3*sqrt(e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(e/d
) + 2*d*e)/x^2) - 2*d*x^3*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + 2*d*x^3*
log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) - 2*sqrt(e*x^2 + d)*sqrt(e)*x + 2*(d
*x^3 - d)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/(d*x^3), -1/6
*(e*x^3*sqrt(-e/d)*arctan(sqrt(e*x^2 + d)*d*sqrt(e)*sqrt(-e/d)/(e^2*x^2 + d
*e)) + d*x^3*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) - d*x^3*log((e*x - sqrt
(e*x^2 + d)*sqrt(e))/x) + sqrt(e*x^2 + d)*sqrt(e)*x - (d*x^3 - d)*log((2*e*
x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x + d)/d))/(d*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-1/3/x^3*ln((1+(sqrt(exp(1)*x^2+d))^-1*exp(1/2)*x)/(1-(sqrt(exp(1)*x^2+d))^
-1*exp(1/2)*x))-2*d*exp(1/2)*(-(2*exp(1)^2-4*exp(1)*exp(1/2)^2+2*exp(1/2)^4
)/3/d^2/2/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2)*atan((sqrt(d+x^2*exp(1))*ex
p(1)-sqrt(d+x^2*exp(1))*exp(1/2)^2)/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2))+
(3*exp(1)-2*exp(1/2)^2)/3/d^2/2/sqrt(-d)*atan(sqrt(d+x^2*exp(1))/sqrt(-d))+
sqrt(d+x^2*exp(1))*exp(1)/6/d^2/(d+x^2*exp(1)-d)))
```

maple [A] time = 0.03, size = 90, normalized size = 1.06

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{3x^3}+\frac{e^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{6d^{\frac{3}{2}}}-\frac{\sqrt{e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{6d^2x^2}+\frac{e^{\frac{3}{2}}\sqrt{e\,x^2+d}}{6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x)
```

```
[Out] -1/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^3+1/6*e^(3/2)/d^(3/2)*ln((2*d+2*d
^(1/2)*(e*x^2+d)^(1/2))/x)-1/6*e^(1/2)/d^2/x^2*(e*x^2+d)^(3/2)+1/6*e^(3/2)/
d^2*(e*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d\sqrt{e}\int-\frac{\sqrt{ex^2+d}}{3\left(e^2x^7+dex^5-\left(ex^5+dx^3\right)\left(ex^2+d\right)\right)}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

```
[Out] d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)/(e^2*x^7 + d*e*x^5 - (e*x^5 + d*x^
3)*(e*x^2 + d)), x) - 1/6*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(e)*
x + sqrt(e*x^2 + d)))/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^4,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**4, x)
```

3.15 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$

Optimal. Leaf size=111

$$-\frac{3e^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}$$

```
[Out] -1/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5-3/40*e^(5/2)*arctanh((e*x^2+d)^
(1/2)/d^(1/2))/d^(5/2)+3/40*e^(3/2)*(e*x^2+d)^(1/2)/d^2/x^2-1/20*e^(1/2)*(e
*x^2+d)^(1/2)/d/x^4
```

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6221, 266, 51, 63, 208}

$$\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{3e^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}-\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] -(Sqrt[e]*Sqrt[d + e*x^2])/(20*d*x^4) + (3*e^(3/2)*Sqrt[d + e*x^2])/(40*d^2
*x^2) - ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(5*x^5) - (3*e^(5/2)*ArcTanh[S
qrt[d + e*x^2]/Sqrt[d]])/(40*d^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^6}dx &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{5}\sqrt{e}\int \frac{1}{x^5\sqrt{d+ex^2}}dx\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{1}{10}\sqrt{e}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{d+ex}}dx,x,x^2\right)\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{\left(3e^{3/2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{d+ex}}dx,x,x^2\right)}{40d}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3e^{5/2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{d+ex}}dx,x,x^2\right)}{80d^2}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}+\frac{\left(3e^{3/2}\right)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x^2}{e}}dx,x,\sqrt{d}\right)}{40d^2}\\
&= -\frac{\sqrt{e}\sqrt{d+ex^2}}{20dx^4}+\frac{3e^{3/2}\sqrt{d+ex^2}}{40d^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^5}-\frac{3e^{5/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 107, normalized size = 0.96

$$\frac{\frac{\sqrt{e}x\left(-3e^2x^4\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)+\sqrt{d}\sqrt{d+ex^2}\left(3ex^2-2d\right)+3e^2x^4\log(x)\right)}{d^{5/2}}-8\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{40x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

```
[Out] (-8*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] + (Sqrt[e]*x*(Sqrt[d]*Sqrt[d + e*x
^2]*(-2*d + 3*e*x^2) + 3*e^2*x^4*Log[x] - 3*e^2*x^4*Log[d + Sqrt[d]*Sqrt[d
+ e*x^2]]))/d^(5/2))/(40*x^5)
```

fricas [B] time = 1.50, size = 383, normalized size = 3.45

$$\left[\frac{3e^2x^5\sqrt{\frac{e}{d}}\log\left(-\frac{e^2x^2-2\sqrt{ex^2+d}\,d\sqrt{e}\sqrt{\frac{e}{d}}+2de}{x^2}\right)-8d^2x^5\log\left(\frac{ex+\sqrt{ex^2+d}\sqrt{e}}{x}\right)+8d^2x^5\log\left(\frac{ex-\sqrt{ex^2+d}\sqrt{e}}{x}\right)+2\left(3ex^3-2d\right.}{80d^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

```
[Out] [1/80*(3*e^2*x^5*sqrt(e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(e)*sqrt
(e/d) + 2*d*e)/x^2) - 8*d^2*x^5*log((e*x + sqrt(e*x^2 + d)*sqrt(e))/x) + 8*
d^2*x^5*log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) + 2*(3*e*x^3 - 2*d*x)*sqrt(e
*x^2 + d)*sqrt(e) + 8*(d^2*x^5 - d^2)*log((2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt
(e)*x + d)/d))/(d^2*x^5), 1/40*(3*e^2*x^5*sqrt(-e/d)*arctan(sqrt(e*x^2 + d)
```

```
*d*sqrt(e)*sqrt(-e/d)/(e^2*x^2 + d*e)) - 4*d^2*x^5*log((e*x + sqrt(e*x^2 +
d)*sqrt(e))/x) + 4*d^2*x^5*log((e*x - sqrt(e*x^2 + d)*sqrt(e))/x) + (3*e*x^
3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(e) + 4*(d^2*x^5 - d^2)*log((2*e*x^2 + 2*sqr
t(e*x^2 + d)*sqrt(e)*x + d)/d))/(d^2*x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-1/5/x^5*ln((1+(sqrt(exp(1)*x^2+d))^-1*exp(1/2)*x)/(1-(sqrt(exp(1)*x^2+d))^
-1*exp(1/2)*x))-2*d*exp(1/2)*((2*exp(1)^3-6*exp(1)^2*exp(1/2)^2+6*exp(1)*ex
p(1/2)^4-2*exp(1/2)^6)/5/d^3/2/sqrt(-d*exp(1/2)^2+d*exp(1))/exp(1/2)*atan((
sqrt(d+x^2*exp(1))*exp(1)-sqrt(d+x^2*exp(1))*exp(1/2)^2)/sqrt(-d*exp(1/2)^2
+d*exp(1))/exp(1/2))+(-15*exp(1)^2+20*exp(1)*exp(1/2)^2-8*exp(1/2)^4)/20/d^
3/2/sqrt(-d)*atan(sqrt(d+x^2*exp(1))/sqrt(-d))+(-7*sqrt(d+x^2*exp(1))*(d+x^
2*exp(1))*exp(1)^2+4*sqrt(d+x^2*exp(1))*(d+x^2*exp(1))*exp(1)*exp(1/2)^2+9*
sqrt(d+x^2*exp(1))*d*exp(1)^2-4*sqrt(d+x^2*exp(1))*d*exp(1)*exp(1/2)^2)/40/
d^3/(d+x^2*exp(1)-d)^2))
```

maple [A] time = 0.03, size = 130, normalized size = 1.17

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{5x^5}+\frac{e^{\frac{3}{2}}\sqrt{e\,x^2+d}}{10d^2x^2}-\frac{3e^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)}{40d^{\frac{5}{2}}}-\frac{\sqrt{e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{20d^2x^4}+\frac{e^{\frac{3}{2}}\left(e\,x^2+d\right)^{\frac{3}{2}}}{40d^3x^2}-\frac{e^{\frac{5}{2}}\sqrt{e\,x^2+d}}{40d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x)
```

```
[Out] -1/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^5+1/10*e^(3/2)*(e*x^2+d)^(1/2)/d^
2/x^2-3/40*e^(5/2)/d^(5/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-1/20*e^(1/
2)/d^2/x^4*(e*x^2+d)^(3/2)+1/40*e^(3/2)/d^3/x^2*(e*x^2+d)^(3/2)-1/40*e^(5/2
)/d^3*(e*x^2+d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d\sqrt{e}\int -\frac{\sqrt{ex^2+d}}{5\left(e^2x^9+dex^7-\left(ex^7+dx^5\right)\left(ex^2+d\right)\right)}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
```

```
[Out] d*sqrt(e)*integrate(-1/5*sqrt(e*x^2 + d)/(e^2*x^9 + d*e*x^7 - (e*x^7 + d*x^
5)*(e*x^2 + d)), x) - 1/10*(log(sqrt(e)*x + sqrt(e*x^2 + d)) - log(-sqrt(e)
*x + sqrt(e*x^2 + d)))/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^6,x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**6, x)
```

3.16 $\int x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=196

$$\frac{30d^{11/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{847e^{11/4}\sqrt{d+ex^2}}-\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847e^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}$$

```
[Out] 2/11*x^(11/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+36/847*d*x^(5/2)*(e*x^2+d)
^(1/2)/e^(3/2)-4/121*x^(9/2)*(e*x^2+d)^(1/2)/e^(1/2)-60/847*d^2*x^(1/2)*(e*
x^2+d)^(1/2)/e^(5/2)+30/847*d^(11/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))
)^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^
(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2
)+x*e^(1/2))^2)^(1/2)/e^(11/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6221, 321, 329, 220}

$$-\frac{60d^2\sqrt{x}\,\sqrt{d+ex^2}}{847e^{5/2}}+\frac{30d^{11/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{847e^{11/4}\sqrt{d+ex^2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}}-\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-60*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(847*e^(5/2)) + (36*d*x^(5/2)*Sqrt[d + e*
x^2])/(847*e^(3/2)) - (4*x^(9/2)*Sqrt[d + e*x^2])/(121*Sqrt[e]) + (2*x^(11/
2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/11 + (30*d^(11/4)*(Sqrt[d] + Sqrt[
e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)
*Sqrt[x])/d^(1/4)], 1/2])/(847*e^(11/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
```

```
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{11}\left(2\sqrt{e}\right)\int\frac{x^{11/2}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}} + \frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{(18d)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}\,dx}{121\sqrt{e}}$$

$$= \frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}} - \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}} + \frac{2}{11}x^{11/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(90d^2\right)\int \ldots}{847e^{3}\ldots}$$

$$= -\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}} - \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}} + \frac{2}{11}x^{11/2}\tanh^{-1}\Big(\ldots$$

$$= -\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}} - \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}} + \frac{2}{11}x^{11/2}\tanh^{-1}\Big(\ldots$$

$$= -\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847e^{5/2}} + \frac{36dx^{5/2}\sqrt{d+ex^2}}{847e^{3/2}} - \frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{e}} + \frac{2}{11}x^{11/2}\tanh^{-1}\Big(\ldots$$

Mathematica [C] time = 0.69, size = 161, normalized size = 0.82

$$\frac{2}{847}\sqrt{x}\left(77x^5\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{2\sqrt{d+ex^2}\left(15d^2-9dex^2+7e^2x^4\right)}{e^{5/2}}\right) + \frac{60d^{5/2}x\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\ldots\right.\right.}{847e^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((-2*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/e^(5/2) +
 77*x^5*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/847 + (60*d^(5/2)*Sqrt[(I*Sq
rt[d])/Sqrt[e]]*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/
Sqrt[e]]/Sqrt[x]], -1])/(847*e^2*Sqrt[d + e*x^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{\frac{9}{2}}\,\text{artanh}\left(\frac{\sqrt{e}\,x}{\sqrt{ex^2+d}}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(x^(9/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2
=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)
exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2
)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)Unable to tran
spose Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{\frac{9}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{11}\,x^{\frac{11}{2}}\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\frac{1}{11}\,x^{\frac{11}{2}}\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)-2\,d\sqrt{e}\int -\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{9}{2}\log(x)\right)}}{11\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/11*x^(11/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/11*x^(11/2)*log(-sqrt(e)
*x + sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/11*x*e^(1/2*log(e*x^2 + d)
 + 9/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{9/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\;x}{\sqrt{e\,x^2+d}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(9/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Timed out
```

3.17 $\int x^{5/2} \tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=168

$$-\frac{10 d^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{147 e^{7/4}\sqrt{d+ex^2}}+\frac{20 d\sqrt{x}\,\sqrt{d+ex^2}}{147 e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\,\cdots}\right.$$

```
[Out] 2/7*x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/49*x^(5/2)*(e*x^2+d)^(1/2)
/e^(1/2)+20/147*d*x^(1/2)*(e*x^2+d)^(1/2)/e^(3/2)-10/147*d^(7/4)*(cos(2*arc
tan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)
))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x
*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6221, 321, 329, 220}

$$-\frac{10 d^{7/4}\left(\sqrt{d}+\sqrt{e}\,x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\,\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{147 e^{7/4}\sqrt{d+ex^2}}+\frac{20 d\sqrt{x}\,\sqrt{d+ex^2}}{147 e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\,\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (20*d*Sqrt[x]*Sqrt[d + e*x^2])/(147*e^(3/2)) - (4*x^(5/2)*Sqrt[d + e*x^2])/
(49*Sqrt[e]) + (2*x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/7 - (10*d^(
7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellipt
icF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(147*e^(7/4)*Sqrt[d + e*x^2]
)
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{7}\left(2\sqrt{e}\right)\int\frac{x^{7/2}}{\sqrt{d+ex^2}}dx$$
$$= -\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{(10d)\int\frac{x^{3/2}}{\sqrt{d+ex^2}}dx}{49\sqrt{e}}$$
$$= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(10d^2\right)\int \ldots}{147\ldots}$$
$$= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\left(20d^2\right)\mathrm{Sub}\ldots}{\ldots}$$
$$= \frac{20d\sqrt{x}\sqrt{d+ex^2}}{147e^{3/2}}-\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{e}}+\frac{2}{7}x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{10d^{7/4}\left(\sqrt{d}\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.50, size = 147, normalized size = 0.88

$$\frac{2}{147}\sqrt{x}\left(\frac{2\left(5d-3ex^2\right)\sqrt{d+ex^2}}{e^{3/2}}+21x^3\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)+\frac{20\sqrt{d}\,x\left(\frac{i\sqrt{d}}{\sqrt{e}}\right)^{5/2}\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\right|\ldots}{147\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/e^(3/2) + 21*x^3*ArcTanh[(S
qrt[e]*x)/Sqrt[d + e*x^2]]))/147 + (20*Sqrt[d]*((I*Sqrt[d])/Sqrt[e])^(5/2)*
Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]
], -1])/(147*Sqrt[d + e*x^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^{\frac{5}{2}}\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(x^(5/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2
```

```
=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)
exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2
)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)Unable to tran
spose Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
[Out] int(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{7}\,x^{\frac{7}{2}}\log\left(\sqrt{e}\,x+\sqrt{ex^{2}+d}\right)-\frac{1}{7}\,x^{\frac{7}{2}}\log\left(-\sqrt{e}\,x+\sqrt{ex^{2}+d}\right)-2\,d\sqrt{e}\int -\frac{xe^{\left(\frac{1}{2}\,\log\left(ex^{2}+d\right)+\frac{5}{2}\,\log\left(x\right)\right)}}{7\left(e^{2}x^{4}+dex^{2}-\left(ex^{2}+d\right)^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
[Out] 1/7*x^(7/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/7*x^(7/2)*log(-sqrt(e)*x +
 sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/7*x*e^(1/2*log(e*x^2 + d) + 5/
2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(5/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \operatorname{atanh}{\left(\frac{\sqrt{e} x}{\sqrt{d + e x^{2}}} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Integral(x**(5/2)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

3.18 $\int \sqrt{x} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=142

$$\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{9e^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

```
[Out] 2/3*x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/9*x^(1/2)*(e*x^2+d)^(1/2)/
e^(1/2)+2/9*d^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*
arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(
1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(
1/2)/e^(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6221, 321, 329, 220}

$$\frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{9e^{3/4}\sqrt{d+ex^2}}-\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}}+\frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[e]) + (2*x^(3/2)*ArcTanh[(Sqrt[e]*x)/S
qrt[d + e*x^2]])/3 + (2*d^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqr
t[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(
9*e^(3/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{3}\left(2\sqrt{e}\right)\int \frac{x^{3/2}}{\sqrt{d+ex^2}}dx$$
$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}} + \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(2d)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}dx}{9\sqrt{e}}$$
$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}} + \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{(4d)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}dx, x, \sqrt{x}\right)}{9\sqrt{e}}$$
$$= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{e}} + \frac{2}{3}x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + \frac{2d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,F}{9e^{3/4}\sqrt{d+ex}}$$

Mathematica [C] time = 0.31, size = 135, normalized size = 0.95

$$\frac{2}{9}\sqrt{x}\left(3x\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) - \frac{2\sqrt{d+ex^2}}{\sqrt{e}}\right) + \frac{4\sqrt{d}x\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{9\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*Sqrt[x]*((-2*Sqrt[d + e*x^2])/Sqrt[e] + 3*x*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]]))/9 + (4*Sqrt[d]*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[1 + d/(e*x^2)]*x*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(9*Sqrt[d + e*x
^2])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{x}\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2
=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)
exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2
```

```
)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)Unable to tran
spose Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\ \operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\,d\sqrt{e}\int-\frac{xe^{\left(\frac{1}{2}\,\log\left(ex^2+d\right)+\frac{1}{2}\,\log(x)\right)}}{3\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx+\frac{1}{3}\,x^{\frac{3}{2}}\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\frac{1}{3}\,x^{\frac{3}{2}}\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*d*sqrt(e)*integrate(-1/3*x*e^(1/2*log(e*x^2 + d) + 1/2*log(x))/(e^2*x^4
+ d*e*x^2 - (e*x^2 + d)^2), x) + 1/3*x^(3/2)*log(sqrt(e)*x + sqrt(e*x^2 + d
)) - 1/3*x^(3/2)*log(-sqrt(e)*x + sqrt(e*x^2 + d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\ \operatorname{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{e\,x^2+d}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

```
[Out] int(x^(1/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\ \operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
```

```
[Out] Integral(sqrt(x)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

3.19 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{d+ex^2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

```
[Out] -2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2)+2*e^(1/4)*(cos(2*arctan(e^(1/
4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellipt
icF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))
*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(1/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {6221, 329, 220}

$$\frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{d+ex^2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*e^(1/4)*(Sqrt[d] + S
qrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(
1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(2\sqrt{e}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \left(4\sqrt{e}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)\middle|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.13, size = 111, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{e}x\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[e]*Sqrt[1 +
 d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/
(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,d\sqrt{e}\int-\frac{\sqrt{ex^2+d}\,x}{\left(e^2x^4+dex^2\right)x^{\frac{3}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{3}{2}\,\log\left(x\right)\right)}}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{\sqrt{x}}+\frac{\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*d*sqrt(e)*integrate(-sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(3/2) - (e*
x^2 + d)*e^(log(e*x^2 + d) + 3/2*log(x))), x) - log(sqrt(e)*x + sqrt(e*x^2
+ d))/sqrt(x) + log(-sqrt(e)*x + sqrt(e*x^2 + d))/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2),x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{atanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(3/2), x)
```

3.20 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

```
[Out] -2/5*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2)-4/15*e^(1/2)*(e*x^2+d)^(1/2
)/d/x^(3/2)-2/15*e^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2
)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))
^2)^(1/2)/d^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6221, 325, 329, 220}

$$-\frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(15*d*x^(3/2)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d
 + e*x^2]])/(5*x^(5/2)) - (2*e^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)
/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/
2])/(15*d^(5/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} + \frac{1}{5}\left(2\sqrt{e}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx$$
$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{\left(2e^{3/2}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{15d}$$
$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{\left(4e^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{15d}$$
$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{2e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)\right.}{15d^{5/4}\sqrt{d+ex^2}}$$

Mathematica [C] time = 0.24, size = 142, normalized size = 0.98

$$-\frac{2\left(2\sqrt{e}x\sqrt{d+ex^2}+3d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}} - \frac{4e^2x\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{15d^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

```
[Out] (-2*(2*Sqrt[e]*x*Sqrt[d + e*x^2] + 3*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
))/(15*d*x^(5/2)) - (4*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^2*Sqrt[1 + d/(e*x^2)]*x*
EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(15*d^(3/2)*Sq
rt[d + e*x^2])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}\,x}{5\left(\left(e^2x^4+dex^2\right)x^{\frac{7}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{7}{2}\,\log\left(x\right)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{5\,x^{\frac{5}{2}}}+\frac{\log\left(-\sqrt{e}\,x+\sqrt{ex\cdots}\right.}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2*d*sqrt(e)*integrate(-1/5*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(7/2) -
 (e*x^2 + d)*e^(log(e*x^2 + d) + 7/2*log(x))), x) - 1/5*log(sqrt(e)*x + sqr
t(e*x^2 + d))/x^(5/2) + 1/5*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2),x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(7/2), x)
```

3.21 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

Optimal. Leaf size=173

$$\frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}+\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

```
[Out] -2/9*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2)+20/189*e^(3/2)*(e*x^2+d)^(1
/2)/d^2/x^(3/2)-4/63*e^(1/2)*(e*x^2+d)^(1/2)/d/x^(7/2)+10/189*e^(9/4)*(cos(
2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^
(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1
/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(9/4)/(e*x^2+d)^(1
/2)
```

Rubi [A] time = 0.09, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6221, 325, 329, 220}

$$\frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}}+\frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(63*d*x^(7/2)) + (20*e^(3/2)*Sqrt[d + e*x^2])/
(189*d^2*x^(3/2)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)) +
(10*e^(9/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]
*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(189*d^(9/4)*Sqrt[d +
 e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
```

```
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{1}{9}\left(2\sqrt{e}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} - \frac{\left(10e^{3/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{63d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}} + \frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(10e^{5/2}\right)\int \frac{1}{\sqrt{x}\sqrt{d+ex^2}}\,dx}{189d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}} + \frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{\left(20e^{5/2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\right.}{189d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}} + \frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{10e^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{\cdot}{(\sqrt{d}}}}{189d^{9/}}$$

Mathematica [C] time = 0.34, size = 154, normalized size = 0.89

$$\frac{4\sqrt{e}x\sqrt{d+ex^2}\left(5ex^2-3d\right)-42d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}}+\frac{20e^3x\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{189d^{5/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

```
[Out] (4*Sqrt[e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTanh[(Sqrt[e]*x)/
Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (20*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^3*Sqr
t[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]],
-1])/(189*d^(5/2)*Sqrt[d + e*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="fricas"
)
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^{2}+d}\,x}{9\left(\left(e^{2}x^{4}+dex^{2}\right)x^{\frac{11}{2}}-\left(ex^{2}+d\right)e^{\left(\log\left(ex^{2}+d\right)+\frac{11}{2}\,\log\left(x\right)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^{2}+d}\right)}{9\,x^{\frac{9}{2}}}+\frac{\log\left(-\sqrt{e}\,x+\sqrt{ex^{2}\cdots}\right)}{9\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxima"
)
```

```
[Out] 2*d*sqrt(e)*integrate(-1/9*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(11/2)
- (e*x^2 + d)*e^(log(e*x^2 + d) + 11/2*log(x))), x) - 1/9*log(sqrt(e)*x + s
qrt(e*x^2 + d))/x^(9/2) + 1/9*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(9/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2),x)
```

```
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

```
[Out] Timed out
```

3.22 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}} \, dx$

Optimal. Leaf size=201

$$-\frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}+\frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}}-$$

```
[Out] -2/13*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(13/2)+36/1001*e^(3/2)*(e*x^2+d)
^(1/2)/d^2/x^(7/2)-60/1001*e^(5/2)*(e*x^2+d)^(1/2)/d^3/x^(3/2)-4/143*e^(1/2
)*(e*x^2+d)^(1/2)/d/x^(11/2)-30/1001*e^(13/4)*(cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*
arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d
)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(13/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6221, 325, 329, 220}

$$-\frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}}+\frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}}-\frac{30e^{13/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}-\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(143*d*x^(11/2)) + (36*e^(3/2)*Sqrt[d + e*x^2]
)/(1001*d^2*x^(7/2)) - (60*e^(5/2)*Sqrt[d + e*x^2])/(1001*d^3*x^(3/2)) - (2
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) - (30*e^(13/4)*(Sqrt[d
] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan
[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(1001*d^(13/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
```

```
x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{1}{13}\left(2\sqrt{e}\right)\int \frac{1}{x^{13/2}\sqrt{d+ex^2}}\,dx$$
$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{\left(18e^{3/2}\right)\int \frac{1}{x^{9/2}\sqrt{d+ex^2}}\,dx}{143d}$$
$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} + \frac{\left(90e^{5/2}\right)\int \frac{1}{x^{5/2}\sqrt{d+ex^2}}\,dx}{1001d^2}$$
$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{\left(30e^{7/2}\right)\int}{1(}$$
$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{\left(60e^{7/2}\right)\mathrm{S}}{}$$
$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{30e^{13/4}\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 0.47, size = 163, normalized size = 0.81

$$\frac{2\left(-\frac{30e^4x^{15/2}\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{\frac{d}{ex^2}+1}\,F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right)\middle|-1\right)}{d^{7/2}\sqrt{d+ex^2}} - \frac{2\sqrt{e}x\sqrt{d+ex^2}\left(7d^2-9dex^2+15e^2x^4\right)}{d^3} - 77\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\right)}{1001x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

```
[Out] (2*((-2*Sqrt[e]*x*Sqrt[d + e*x^2]*(7*d^2 - 9*d*e*x^2 + 15*e^2*x^4))/d^3 - 7
7*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - (30*Sqrt[(I*Sqrt[d])/Sqrt[e]]*e^4*
Sqrt[1 + d/(e*x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/
Sqrt[x]], -1])/(d^(7/2)*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="fricas"
)
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac")
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}\,x}{13\left(\left(e^2x^4+dex^2\right)x^{\frac{15}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{15}{2}\log(x)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{13\,x^{\frac{13}{2}}}+\frac{\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{13\,x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxima"
)
[Out] 2*d*sqrt(e)*integrate(-1/13*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(15/2)
 - (e*x^2 + d)*e^(log(e*x^2 + d) + 15/2*log(x))), x) - 1/13*log(sqrt(e)*x +
 sqrt(e*x^2 + d))/x^(13/2) + 1/13*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(13/2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2),x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
[Out] Timed out
```

3.23 $\int x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)dx$

Optimal. Leaf size=297

$$-\frac{14d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{9/4}\sqrt{d+ex^2}}+\frac{28d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{9/4}\sqrt{d+ex^2}}$$

```
[Out] 2/9*x^(9/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))+28/405*d*x^(3/2)*(e*x^2+d)^(
1/2)/e^(3/2)-4/81*x^(7/2)*(e*x^2+d)^(1/2)/e^(1/2)-28/135*d^2*x^(1/2)*(e*x^2
+d)^(1/2)/e^2/(d^(1/2)+x*e^(1/2))+28/135*d^(9/4)*(cos(2*arctan(e^(1/4)*x^(1
/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin
(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^
2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4)/(e*x^2+d)^(1/2)-14/135*d^(9/4)*(c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d
^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(9/4)/(e*x^2+d)
^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6221, 321, 329, 305, 220, 1196}

$$-\frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}x\right)}-\frac{14d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{9/4}\sqrt{d+ex^2}}+\frac{28d^{9/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\ldots}{135e^{9/4}\sqrt{d+e\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (28*d*x^(3/2)*Sqrt[d + e*x^2])/(405*e^(3/2)) - (4*x^(7/2)*Sqrt[d + e*x^2])/
(81*Sqrt[e]) - (28*d^2*Sqrt[x]*Sqrt[d + e*x^2])/(135*e^2*(Sqrt[d] + Sqrt[e]
*x)) + (2*x^(9/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/9 + (28*d^(9/4)*(Sq
rt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*Ar
cTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(9/4)*Sqrt[d + e*x^2]) - (14*
d^(9/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ell
ipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(135*e^(9/4)*Sqrt[d + e*x
^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^{7/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{1}{9}\left(2\sqrt{e}\right)\int\frac{x^{9/2}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) + \frac{(14d)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}\,dx}{81\sqrt{e}}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(14d^2\right)\int \cdots}{135e \cdots}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(28d^2\right)\text{Sul}\cdots}{\cdots}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right) - \frac{\left(28d^{5/2}\right)\text{S}\cdots}{\cdots}$$

$$= \frac{28dx^{3/2}\sqrt{d+ex^2}}{405e^{3/2}} - \frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{e}} - \frac{28d^2\sqrt{x}\sqrt{d+ex^2}}{135e^2\left(\sqrt{d}+\sqrt{e}\,x\right)} + \frac{2}{9}x^{9/2}\tanh^{-1}\left(\frac{\cdots}{\sqrt{\cdots}}\right.$$

Mathematica [C] time = 0.15, size = 124, normalized size = 0.42

$$\frac{2x^{3/2}\left(-14d^2\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)+14d^2+45e^{3/2}x^3\sqrt{d+ex^2}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+4dex^2-10e^2x^4\right)}{405e^{3/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(14*d^2 + 4*d*e*x^2 - 10*e^2*x^4 + 45*e^(3/2)*x^3*Sqrt[d + e*x^2
]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[1 + (e*x^2)/d]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^(3/2)*Sqrt[d + e*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^{\frac{7}{2}}\,\mathrm{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(x^(7/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2
=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)
exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2
)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)exp(1/2)^2=exp(1)Unable to tran
spose Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{\frac{7}{2}}\,\mathrm{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

```
[Out] int(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9}x^{\frac{9}{2}}\log\left(\sqrt{e}x+\sqrt{ex^2+d}\right)-\frac{1}{9}x^{\frac{9}{2}}\log\left(-\sqrt{e}x+\sqrt{ex^2+d}\right)-2d\sqrt{e}\int-\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{7}{2}\log(x)\right)}}{9\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/9*x^(9/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/9*x^(9/2)*log(-sqrt(e)*x +
 sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/9*x*e^(1/2*log(e*x^2 + d) + 7/
2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{7/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(7/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Timed out
```

3.24 $\int x^{3/2} \tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) dx$

Optimal. Leaf size=269

$$\frac{6d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}+$$

```
[Out] 2/5*x^(5/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4/25*x^(3/2)*(e*x^2+d)^(1/2)
/e^(1/2)+12/25*d*x^(1/2)*(e*x^2+d)^(1/2)/e/(d^(1/2)+x*e^(1/2))-12/25*d^(5/4
)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2)
)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(5/4)/(e*x^
2+d)^(1/2)+6/25*d^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/co
s(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^
2)^(1/2)/e^(5/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6221, 321, 329, 305, 220, 1196}

$$\frac{6d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}-\frac{12d^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{5/4}\sqrt{d+ex^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (-4*x^(3/2)*Sqrt[d + e*x^2])/(25*Sqrt[e]) + (12*d*Sqrt[x]*Sqrt[d + e*x^2])/
(25*e*(Sqrt[d] + Sqrt[e]*x)) + (2*x^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/5 - (12*d^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt
[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25*e^(5/4)*
Sqrt[d + e*x^2]) + (6*d^(5/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[
d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(25
*e^(5/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx = \frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\left(2\sqrt{e}\right)\int\frac{x^{5/2}}{\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{(6d)\int\frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{25\sqrt{e}}$$

$$= -\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{(12d)\,\mathrm{Subst}\left(\int\frac{x^2}{\sqrt{d+ex^4}}\,dx,x,\right.}{25\sqrt{e}}$$

$$= -\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)+\frac{\left(12d^{3/2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{d+ex^4}}\,dx,\right.}{25e}$$

$$= -\frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{e}}+\frac{12d\sqrt{x}\,\sqrt{d+ex^2}}{25e\left(\sqrt{d}+\sqrt{e}\,x\right)}+\frac{2}{5}x^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)-\frac{12d^{5/4}\left(\right.}{}$$

Mathematica [C] time = 0.10, size = 109, normalized size = 0.41

$$\frac{2x^{3/2}\left(2d\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)-2\left(d+ex^2\right)+5\sqrt{e}\,x\sqrt{d+ex^2}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\right)}{25\sqrt{e}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]],x]
```

```
[Out] (2*x^(3/2)*(-2*(d + e*x^2) + 5*Sqrt[e]*x*Sqrt[d + e*x^2]*ArcTanh[(Sqrt[e]*x
)/Sqrt[d + e*x^2]] + 2*d*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/
4, -((e*x^2)/d)]))/(25*Sqrt[e]*Sqrt[d + e*x^2])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(x^{\frac{3}{2}}\operatorname{artanh}\left(\frac{\sqrt{e}\,x}{\sqrt{ex^2+d}}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
[Out] integral(x^(3/2)*arctanh(sqrt(e)*x/sqrt(e*x^2 + d)), x)
```

giac [A] time = 0.32, size = 1, normalized size = 0.00

$$+\infty$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
[Out] +Infinity
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
[Out] int(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}\,x^{\frac{5}{2}}\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)-\frac{1}{5}\,x^{\frac{5}{2}}\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)-2\,d\sqrt{e}\int-\frac{xe^{\left(\frac{1}{2}\log\left(ex^2+d\right)+\frac{3}{2}\log(x)\right)}}{5\left(e^2x^4+dex^2-\left(ex^2+d\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
[Out] 1/5*x^(5/2)*log(sqrt(e)*x + sqrt(e*x^2 + d)) - 1/5*x^(5/2)*log(-sqrt(e)*x +
 sqrt(e*x^2 + d)) - 2*d*sqrt(e)*integrate(-1/5*x*e^(1/2*log(e*x^2 + d) + 3/
2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)),x)
[Out] int(x^(3/2)*atanh((e^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(x*e**(1/2)/(e*x**2+d)**(1/2)),x)
[Out] Integral(x**(3/2)*atanh(sqrt(e)*x/sqrt(d + e*x**2)), x)
```

3.25 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=232

$$-\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}x}+2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\left(\sqrt{d}\right.}{}$$

```
[Out] 2*x^(1/2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))-4*x^(1/2)*(e*x^2+d)^(1/2)/(d^(
1/2)+x*e^(1/2))+4*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/
cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2)
)^2)^(1/2)/e^(1/4)/(e*x^2+d)^(1/2)-2*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/
d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*a
rctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)
/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(1/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6221, 329, 305, 220, 1196}

$$-\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}x}+2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{2\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}+\frac{4\sqrt[4]{d}\left(\sqrt{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] (-4*Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x) + 2*Sqrt[x]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]] + (4*d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2
)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1
/2])/(e^(1/4)*Sqrt[d + e*x^2]) - (2*d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/
4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\left(2\sqrt{e}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx\\
&= 2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\left(4\sqrt{e}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\left(4\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx,x,\sqrt{x}\right)+\left(4\sqrt{d}\right)\text{Subst}\Bigg(\int\\
&= -\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}x}+2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+\frac{4\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,E\Big(2\tan}{\sqrt[4]{e}\sqrt{d+ex^2}}
\end{aligned}$$

Mathematica [C] time = 0.11, size = 85, normalized size = 0.37

$$2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{4\sqrt{e}x^{3/2}\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[e]*x^(3/2)*Sqrt[1
+ (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d + e*
x^2])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac")
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^{2}+d}}\right)}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\,d\sqrt{e}\int \frac{\sqrt{ex^{2}+d}\,x}{\left(ex^{2}+d\right)e^{\left(\log\left(ex^{2}+d\right)+\frac{1}{2}\,\log\left(x\right)\right)}-\left(e^{2}x^{4}+dex^{2}\right)\sqrt{x}}\,dx+\sqrt{x}\,\log\left(\sqrt{e}\,x+\sqrt{ex^{2}+d}\right)-\sqrt{x}\,\log\left(-\sqrt{e}\,x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] -2*d*sqrt(e)*integrate(sqrt(e*x^2 + d)*x/((e*x^2 + d)*e^(log(e*x^2 + d) + 1
/2*log(x)) - (e^2*x^4 + d*e*x^2)*sqrt(x)), x) + sqrt(x)*log(sqrt(e)*x + sqr
t(e*x^2 + d)) - sqrt(x)*log(-sqrt(e)*x + sqrt(e*x^2 + d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^{2}+d}}\right)}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2),x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^{2}}}\right)}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/sqrt(x), x)
```

3.26 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=272

$$\frac{2e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}+\frac{4e}{3d}\cdots$$

```
[Out] -2/3*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(3/2)-4/3*e^(1/2)*(e*x^2+d)^(1/2)
/d/x^(1/2)+4/3*e*x^(1/2)*(e*x^2+d)^(1/2)/d/(d^(1/2)+x*e^(1/2))-4/3*e^(3/4)*
(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/
2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*
(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(3/4)/(e*x^2+
d)^(1/2)+2/3*e^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2
*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^
(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^
(1/2)/d^(3/4)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6221, 325, 329, 305, 220, 1196}

$$\frac{2e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}-\frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}+\frac{4e}{3d}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) + (4*e*Sqrt[x]*Sqrt[d + e*x^2])/
(3*d*(Sqrt[d] + Sqrt[e]*x)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(3*x
^(3/2)) - (4*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt
[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d^(3/4)*S
qrt[d + e*x^2]) + (2*e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d
] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d
^(3/4)*Sqrt[d + e*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{1}{3}\left(2\sqrt{e}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{\left(2e^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{3d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{\left(4e^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{3d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} + \frac{(4e)\,\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{d}} - \frac{(4e)\,\text{Subs}}{}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} + \frac{4e\sqrt{x}\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{4e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d}{(\sqrt{d}}}}{3d^{3/4}}$$

Mathematica [C] time = 0.12, size = 118, normalized size = 0.43

$$\frac{4e^{3/2}x^{3/2}\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{9d\sqrt{d+ex^2}} - \frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(3*d*Sqrt[x]) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/(3*x^(3/2)) + (4*e^(3/2)*x^(3/2)*Sqrt[1 + (e*x^2)/d]*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(9*d*Sqrt[d + e*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="fricas")
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac")
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2d\sqrt{e}\int -\frac{\sqrt{ex^2+d}\,x}{3\left(\left(e^2x^4+dex^2\right)x^{\frac{5}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{5}{2}\log(x)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{3\,x^{\frac{3}{2}}}+\frac{\log\left(-\sqrt{e}\,x+\sqrt{ex^2+\right.}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxima")
[Out] 2*d*sqrt(e)*integrate(-1/3*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(5/2) -
 (e*x^2 + d)*e^(log(e*x^2 + d) + 5/2*log(x))), x) - 1/3*log(sqrt(e)*x + sqr
t(e*x^2 + d))/x^(3/2) + 1/3*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2),x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

```
[Out] Integral(atanh(sqrt(e)*x/sqrt(d + e*x**2))/x**(5/2), x)
```

3.27 $\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

Optimal. Leaf size=302

$$-\frac{6e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+$$

```
[Out] -2/7*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(7/2)-4/35*e^(1/2)*(e*x^2+d)^(1/2
)/d/x^(5/2)+12/35*e^(3/2)*(e*x^2+d)^(1/2)/d^2/x^(1/2)-12/35*e^2*x^(1/2)*(e*
x^2+d)^(1/2)/d^2/(d^(1/2)+x*e^(1/2))+12/35*e^(7/4)*(cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(s
in(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d^(1/2)+x*e^(1/2))*((e*
x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)^(1/2)-6/35*e^(7/4)*(c
os(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)
/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(d
^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)
^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {6221, 325, 329, 305, 220, 1196}

$$-\frac{12e^2\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}}-\frac{6e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}+\frac{12e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (-4*Sqrt[e]*Sqrt[d + e*x^2])/(35*d*x^(5/2)) + (12*e^(3/2)*Sqrt[d + e*x^2])/
(35*d^2*Sqrt[x]) - (12*e^2*Sqrt[x]*Sqrt[d + e*x^2])/(35*d^2*(Sqrt[d] + Sqrt
[e]*x)) - (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(7*x^(7/2)) + (12*e^(7/4
)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE
[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e*x^2]) -
(6*e^(7/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*
EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(35*d^(7/4)*Sqrt[d + e
*x^2])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 6221

```
Int[ArcTanh[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_
Symbol] :> Simp[((d*x)^(m + 1)*ArcTanh[(c*x)/Sqrt[a + b*x^2]])/(d*(m + 1)),
 x] - Dist[c/(d*(m + 1)), Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; Free
Q[{a, b, c, d, m}, x] && EqQ[b, c^2] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = -\frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{1}{7}\left(2\sqrt{e}\right)\int \frac{1}{x^{7/2}\sqrt{d+ex^2}}\,dx$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6e^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{d+ex^2}}\,dx}{35d}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(6e^{5/2}\right)\int \frac{\sqrt{x}}{\sqrt{d+ex^2}}\,dx}{35d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12e^{5/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{d+ex^4}}\right.}{35d^2}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} - \frac{\left(12e^{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{d+ex^4}}\,d\right.}{35d^{3/2}}$$

$$= -\frac{4\sqrt{e}\sqrt{d+ex^2}}{35dx^{5/2}} + \frac{12e^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} - \frac{12e^2\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} + \frac{12e^{7}}{}$$

Mathematica [C] time = 0.11, size = 131, normalized size = 0.43

$$\frac{4\sqrt{e}x\left(-d^2+2dex^2+3e^2x^4\right)-10d^2\sqrt{d+ex^2}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-4e^{5/2}x^5\sqrt{\frac{ex^2}{d}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ex^2}{d}\right)}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

```
[Out] (4*Sqrt[e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]] - 4*e^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(\frac{\sqrt{e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(sqrt(e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

```
[Out] int(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,d\sqrt{e}\int -\frac{\sqrt{ex^2+d}\,x}{7\left(\left(e^2x^4+dex^2\right)x^{\frac{9}{2}}-\left(ex^2+d\right)e^{\left(\log\left(ex^2+d\right)+\frac{9}{2}\log(x)\right)}\right)}\,dx-\frac{\log\left(\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{7\,x^{\frac{7}{2}}}+\frac{\log\left(-\sqrt{e}\,x+\sqrt{ex^2+d}\right)}{7\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxima")
```

```
[Out] 2*d*sqrt(e)*integrate(-1/7*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^(9/2) -
 (e*x^2 + d)*e^(log(e*x^2 + d) + 9/2*log(x))), x) - 1/7*log(sqrt(e)*x + sqr
t(e*x^2 + d))/x^(7/2) + 1/7*log(-sqrt(e)*x + sqrt(e*x^2 + d))/x^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\text{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2),x)
[Out] int(atanh((e^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x*e**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)
[Out] Timed out
```

3.28 $\int x^3 \tanh^{-1}\left(a+bx^4\right) dx$

Optimal. Leaf size=44

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}+\frac{\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arctanh(b*x^4+a)/b+1/8*ln(1-(b*x^4+a)^2)/b
```

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 6103, 5910, 260}

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}+\frac{\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[a + b*x^4],x]
```

```
[Out] ((a + b*x^4)*ArcTanh[a + b*x^4])/(4*b) + Log[1 - (a + b*x^4)^2]/(8*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \tanh^{-1}\left(a+bx^4\right) dx &= \frac{1}{4}\text{Subst}\left(\int \tanh^{-1}(a+bx)\,dx,x,x^4\right)\\
&= \frac{\text{Subst}\left(\int \tanh^{-1}(x)\,dx,x,a+bx^4\right)}{4b}\\
&= \frac{\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx,x,a+bx^4\right)}{4b}\\
&= \frac{\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{4b}+\frac{\log\left(1-\left(a+bx^4\right)^2\right)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.89

$$\frac{\log\left(1-\left(a+bx^4\right)^2\right)+2\left(a+bx^4\right)\tanh^{-1}\left(a+bx^4\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[a + b*x^4],x]
```

```
[Out] (2*(a + b*x^4)*ArcTanh[a + b*x^4] + Log[1 - (a + b*x^4)^2])/(8*b)
```

fricas [A] time = 0.51, size = 59, normalized size = 1.34

$$\frac{bx^4\log\left(-\frac{bx^4+a+1}{bx^4+a-1}\right)+(a+1)\log\left(bx^4+a+1\right)-(a-1)\log\left(bx^4+a-1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/8*(b*x^4*log(-(b*x^4 + a + 1)/(b*x^4 + a - 1)) + (a + 1)*log(b*x^4 + a +
1) - (a - 1)*log(b*x^4 + a - 1))/b
```

giac [B] time = 0.20, size = 223, normalized size = 5.07

$$\frac{1}{8}\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{\left|-bx^4-a-1\right|}{\left|bx^4+a-1\right|}\right)}{b^2}-\frac{\log\left(\left|-\frac{bx^4+a+1}{bx^4+a-1}+1\right|\right)}{b^2}+\frac{\log\left(-\frac{a-\frac{\left(\frac{\left(bx^4+a+1\right)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{\left(bx^4+a+1\right)b}{bx^4+a-1}-b}+1}{a-\frac{\left(\frac{\left(bx^4+a+1\right)(a-1)}{bx^4+a-1}-a-1\right)b}{\frac{\left(bx^4+a+1\right)b}{bx^4+a-1}-b}-1}\right)}{b^2\left(\frac{bx^4+a+1}{bx^4+a-1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*((a + 1)*b - (a - 1)*b)*(log(abs(-b*x^4 - a - 1)/abs(b*x^4 + a - 1))/b^
2 - log(abs(-(b*x^4 + a + 1)/(b*x^4 + a - 1) + 1))/b^2 + log(-(a - ((b*x^4
+ a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*x^4 + a + 1)*b/(b*x^4 + a -
 1) - b) + 1)/(a - ((b*x^4 + a + 1)*(a - 1)/(b*x^4 + a - 1) - a - 1)*b/((b*
x^4 + a + 1)*b/(b*x^4 + a - 1) - b) - 1))/(b^2*((b*x^4 + a + 1)/(b*x^4 + a
- 1) - 1)))
```

maple [A] time = 0.03, size = 48, normalized size = 1.09

$$\frac{\operatorname{arctanh}\left(b\,x^4+a\right)x^4}{4}+\frac{\operatorname{arctanh}\left(b\,x^4+a\right)a}{4b}+\frac{\ln\left(1-\left(b\,x^4+a\right)^2\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(b*x^4+a),x)
```

```
[Out] 1/4*arctanh(b*x^4+a)*x^4+1/4/b*arctanh(b*x^4+a)*a+1/8*ln(1-(b*x^4+a)^2)/b
```

maxima [A] time = 0.32, size = 37, normalized size = 0.84

$$\frac{2\left(bx^4+a\right)\operatorname{artanh}\left(bx^4+a\right)+\log\left(-\left(bx^4+a\right)^2+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(b*x^4 + a)*arctanh(b*x^4 + a) + log(-(b*x^4 + a)^2 + 1))/b
```

mupad [B] time = 0.32, size = 90, normalized size = 2.05

$$\frac{\ln\left(b\,x^4+a-1\right)}{8\,b}-\frac{x^4\,\ln\left(-b\,x^4-a+1\right)}{8}+\frac{\ln\left(b\,x^4+a+1\right)}{8\,b}+\frac{x^4\,\ln\left(b\,x^4+a+1\right)}{8}-\frac{a\,\ln\left(b\,x^4+a-1\right)}{8\,b}+\frac{a\,\ln\left(b\,x^4+\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(a + b*x^4),x)
```

```
[Out] log(a + b*x^4 - 1)/(8*b) - (x^4*log(1 - b*x^4 - a))/8 + log(a + b*x^4 + 1)/
(8*b) + (x^4*log(a + b*x^4 + 1))/8 - (a*log(a + b*x^4 - 1))/(8*b) + (a*log(
a + b*x^4 + 1))/(8*b)
```

sympy [A] time = 4.18, size = 60, normalized size = 1.36

$$\begin{cases} \frac{a \operatorname{atanh}\left(a+bx^4\right)}{4b}+\frac{x^4 \operatorname{atanh}\left(a+bx^4\right)}{4}+\frac{\log\left(a+bx^4+1\right)}{4b}-\frac{\operatorname{atanh}\left(a+bx^4\right)}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{atanh}(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(b*x**4+a),x)
```

```
[Out] Piecewise((a*atanh(a + b*x**4)/(4*b) + x**4*atanh(a + b*x**4)/4 + log(a + b
*x**4 + 1)/(4*b) - atanh(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*atanh(a)/4, Tr
ue))
```

3.29 $\int x^{-1+n} \tanh^{-1}(a+bx^n)\,dx$

Optimal. Leaf size=47

$$\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}+\frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}$$

```
[Out] (a+b*x^n)*arctanh(a+b*x^n)/b/n+1/2*ln(1-(a+b*x^n)^2)/b/n
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 6103, 5910, 260}

$$\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}+\frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*ArcTanh[a + b*x^n],x]
```

```
[Out] ((a + b*x^n)*ArcTanh[a + b*x^n])/(b*n) + Log[1 - (a + b*x^n)^2]/(2*b*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 6103

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d}
, x] && IGtQ[p, 0]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+n}\tanh^{-1}(a+bx^n)\,dx &= \frac{\text{Subst}\left(\int \tanh^{-1}(a+bx)\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \tanh^{-1}(x)\,dx,x,a+bx^n\right)}{bn}\\
&= \frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}-\frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx,x,a+bx^n\right)}{bn}\\
&= \frac{(a+bx^n)\tanh^{-1}(a+bx^n)}{bn}+\frac{\log\left(1-(a+bx^n)^2\right)}{2bn}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.89

$$\frac{\log\left(1-(a+bx^n)^2\right)+2\left(a+bx^n\right)\tanh^{-1}\left(a+bx^n\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcTanh[a + b*x^n],x]
```

```
[Out] (2*(a + b*x^n)*ArcTanh[a + b*x^n] + Log[1 - (a + b*x^n)^2])/(2*b*n)
```

fricas [B] time = 0.57, size = 109, normalized size = 2.32

$$\frac{(a+1)\log\left(b\cosh\left(n\log(x)\right)+b\sinh\left(n\log(x)\right)+a+1\right)-(a-1)\log\left(b\cosh\left(n\log(x)\right)+b\sinh\left(n\log(x)\right)+a\right.}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*((a + 1)*log(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1) - (a - 1)*log
(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a - 1) + (b*cosh(n*log(x)) + b*sinh(
n*log(x)))*log(-(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a + 1)/(b*cosh(n*log
(x)) + b*sinh(n*log(x)) + a - 1)))/(b*n)
```

giac [B] time = 0.21, size = 124, normalized size = 2.64

$$\frac{\left((a+1)b-(a-1)b\right)\left(\frac{\log\left(\frac{|-bx^n-a-1|}{|bx^n+a-1|}\right)}{b^2}-\frac{\log\left(\left|-\frac{bx^n+a+1}{bx^n+a-1}+1\right|\right)}{b^2}+\frac{\log\left(-\frac{bx^n+a+1}{bx^n+a-1}\right)}{b^2\left(\frac{bx^n+a+1}{bx^n+a-1}-1\right)}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*((a + 1)*b - (a - 1)*b)*(log(abs(-b*x^n - a - 1)/abs(b*x^n + a - 1))/b^
2 - log(abs(-(b*x^n + a + 1)/(b*x^n + a - 1) + 1))/b^2 + log(-(b*x^n + a +
1)/(b*x^n + a - 1))/(b^2*((b*x^n + a + 1)/(b*x^n + a - 1) - 1)))/n
```

maple [B] time = 0.07, size = 121, normalized size = 2.57

$$\frac{x^n\ln\left(1+a+b\,x^n\right)}{2n}-\frac{x^n\ln\left(1-a-b\,x^n\right)}{2n}-\frac{\ln\left(x^n+\frac{a-1}{b}\right)a}{2bn}+\frac{\ln\left(x^n+\frac{1+a}{b}\right)a}{2bn}+\frac{\ln\left(x^n+\frac{a-1}{b}\right)}{2bn}+\frac{\ln\left(x^n+\frac{1+a}{b}\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arctanh(a+b*x^n),x)
```

```
[Out] 1/2/n*x^n*ln(1+a+b*x^n)-1/2/n*x^n*ln(1-a-b*x^n)-1/2/b/n*ln(x^n+(a-1)/b)*a+1
/2/b/n*ln(x^n+(1+a)/b)*a+1/2/b/n*ln(x^n+(a-1)/b)+1/2/b/n*ln(x^n+(1+a)/b)
```

maxima [A] time = 0.32, size = 40, normalized size = 0.85

$$\frac{2\left(bx^n+a\right)\operatorname{artanh}\left(bx^n+a\right)+\log\left(-(bx^n+a)^2+1\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arctanh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*x^n + a)*arctanh(b*x^n + a) + log(-(b*x^n + a)^2 + 1))/(b*n)
```

mupad [B] time = 1.47, size = 56, normalized size = 1.19

$$\frac{x^n\,\mathrm{atanh}\left(a+b\,x^n\right)}{n}-\frac{\ln\left(a+b\,x^n-1\right)\,\left(a-1\right)}{2\,b\,n}+\frac{\ln\left(a+b\,x^n+1\right)\,\left(a+1\right)}{2\,b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*atanh(a + b*x^n),x)
```

```
[Out] (x^n*atanh(a + b*x^n))/n - (log(a + b*x^n - 1)*(a - 1))/(2*b*n) + (log(a +
b*x^n + 1)*(a + 1))/(2*b*n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*atanh(a+b*x**n),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\mathrm{Int}\left(\frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Defer[Int][(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
[Out] integral(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(b\,\mathrm{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^{n}}{c^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] -Integral((a + b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x
)
```

3.31 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=409

$$-\frac{3b^2\text{Li}_3\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b^2\text{Li}_3\left(\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right.}{}$$

```
[Out] 2*arctanh(-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arctanh((-c*x+1)^(1/2
)/(c*x+1)^(1/2)))^3/c+3/2*b*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*p
olylog(2,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b*(a+b*arctanh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/
c-3/2*b^2*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,1-2/(1-(-c*
x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/2*b^2*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1
/2)))*polylog(3,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*b^3*polylog(4,
1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/4*b^3*polylog(4,-1+2/(1-(-c*x+1)^
(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A] time = 0.51, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6681, 5914, 6052, 5948, 6058, 6062, 6610}

$$-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b^2\text{PolyLog}\left(3,\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c + (3*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b*(a
 + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2, -1 + 2/(1 - Sqrt[1
- c*x]/Sqrt[1 + c*x])])/(2*c) - (3*b^2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1
+ c*x]])*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^
2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -1 + 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/(2*c) + (3*b^3*PolyLog[4, 1 - 2/(1 - Sqrt[1 - c*x
]/Sqrt[1 + c*x])])/(4*c) - (3*b^3*PolyLog[4, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt
[1 + c*x])])/(4*c)
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
```

```
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(6b)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(1-\frac{2}{1-x}\right)\cdots}{1-\cdots}\right.}{c\cdots}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)\cdots}{1-x\cdots}\right.}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{\cdots}}{2c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{\cdots}}{2c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^{\cdots}}{2c}$$

Mathematica [A] time = 0.19, size = 482, normalized size = 1.18

$$-\frac{-6b^2\text{Li}_3\left(-\frac{\sqrt{1-cx}+\sqrt{cx+1}}{\sqrt{1-cx}-\sqrt{cx+1}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+6b^2\text{Li}_3\left(\frac{\sqrt{1-cx}+\sqrt{cx+1}}{\sqrt{1-cx}-\sqrt{cx+1}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+6b\text{Li}_2\left(-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] -1/4*(8*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 - S
qrt[1 - c*x]/Sqrt[1 + c*x])] + 6*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])^2*PolyLog[2, -((Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1
 + c*x]))] - 6*b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2,
(Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])] - 6*b^2*(a
 + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -((Sqrt[1 - c*x] + Sq
rt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]))] + 6*b^2*(a + b*ArcTanh[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, (Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1
 - c*x] - Sqrt[1 + c*x])] + 3*b^3*PolyLog[4, -((Sqrt[1 - c*x] + Sqrt[1 + c*
x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]))] - 3*b^3*PolyLog[4, (Sqrt[1 - c*x] + S
qrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])])/c
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b^3\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3+3\,ab^2\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2+3\,a^2b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a^3}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^3*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctanh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^3}{c^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

maple [B] time = 1.42, size = 1449, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)+6*a*b^2/c*polylog(3,-((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+6*a*b^2/c*polylog(3,((-c*x+
1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-3/2*a*b^2/c*polylog(
3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-3*a^2*b/c*dilo
g(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2*(-(-c*x+1)/(c*x+1)+1))+3/4*a^2*b/c*d
ilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^4*(-(-c*x+1)/(c*x+1)+1)^2)-3/2*b^3/
c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^
(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2
))^3*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-3*b
^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,((-c*x+1)^(1/2)/(c*x
+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+6*b^3/c*arctanh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))*polylog(3,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+
1)^(1/2))-b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x+1)^(1/2
)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-3*b^3/c*arctanh((-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)
/(c*x+1)+1)^(1/2))+6*b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,
-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+b^3/c*arctan
h((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(
-(-c*x+1)/(c*x+1)+1))+3/2*b^3/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*pol
ylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-3*a*b^2/c
*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)
+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(
1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/
2))-6*b^3/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+
1)^(1/2))+3/4*b^3/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1
)/(c*x+1)+1))-6*b^3/c*polylog(4,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)
/(c*x+1)+1)^(1/2))-3*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(
(-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-6*a*b^2/c*arct
anh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1
)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+3*a*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/
2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))+3*a*b^
2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1
)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^3\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)-\frac{\left(b^3\log(cx+1)-b^3\log(-cx+1)\right)\log\left(\sqrt{cx+1}-\sqrt{-cx+1}\right)^3}{16c}-\int\frac{4\left(\sqrt{cx+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/16*(b^3*log(c*x + 1) - b^3*lo
g(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))^3/c - integrate(1/32*(4*(s
qrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^
3 + 24*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x + 1) + sqr
t(-c*x + 1))^2 + 3*(4*(sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x
 + 1) + sqrt(-c*x + 1)) + (8*a*b^2 - (b^3*c*x - b^3)*log(c*x + 1) + (b^3*c*
x - b^3)*log(-c*x + 1))*sqrt(c*x + 1) - (8*a*b^2 - (b^3*c*x + b^3)*log(c*x
+ 1) + (b^3*c*x + b^3)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(c*x + 1) - s
qrt(-c*x + 1))^2 + 48*(sqrt(c*x + 1)*a^2*b - sqrt(-c*x + 1)*a^2*b)*log(sqrt
(c*x + 1) + sqrt(-c*x + 1)) - 12*(4*sqrt(c*x + 1)*a^2*b - 4*sqrt(-c*x + 1)*
a^2*b + (sqrt(c*x + 1)*b^3 - sqrt(-c*x + 1)*b^3)*log(sqrt(c*x + 1) + sqrt(-
c*x + 1))^2 + 4*(sqrt(c*x + 1)*a*b^2 - sqrt(-c*x + 1)*a*b^2)*log(sqrt(c*x +
 1) + sqrt(-c*x + 1)))*log(sqrt(c*x + 1) - sqrt(-c*x + 1)))/((c^2*x^2 - 1)*
sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int-\frac{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^3}{c^2x^2-1}\,dx-\int\frac{b^3\,\mathrm{atanh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{3ab^2\,\mathrm{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{3a^2b\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*atanh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*atanh(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*atanh(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.32 $\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=268

$$\frac{b\text{Li}_2\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b\text{Li}_2\left(\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right.}{}$$

```
[Out] 2*arctanh(-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))*(a+b*arctanh((-c*x+1)^(1/2
)/(c*x+1)^(1/2)))^2/c+b*(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog
(2,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-b*(a+b*arctanh((-c*x+1)^(1/2)/(c
*x+1)^(1/2)))*polylog(2,-1+2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-1/2*b^2*po
lylog(3,1-2/(1-(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+1/2*b^2*polylog(3,-1+2/(1-(
-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

Rubi [A] time = 0.31, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6681, 5914, 6052, 5948, 6058, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b\text{PolyLog}\left(2,\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}-1\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{b^2\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (-2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 - Sqrt[
1 - c*x]/Sqrt[1 + c*x])])/c + (b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]
])*PolyLog[2, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/c - (b*(a + b*ArcTa
nh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[
1 + c*x])])/c - (b^2*PolyLog[3, 1 - 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(
2*c) + (b^2*PolyLog[3, -1 + 2/(1 - Sqrt[1 - c*x]/Sqrt[1 + c*x])])/(2*c)
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(4b)\,\text{Subst}\left(\int \frac{\tanh^{-1}\left(1-\frac{2}{1-x}\right)}{1-}\right.}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\tanh^{-1}(x)\right)}{1-x^2}\right.}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{L}}{c}$$

$$= -\frac{2\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\text{L}}{c}$$

Mathematica [A] time = 0.11, size = 324, normalized size = 1.21

$$-\frac{b\text{Li}_2\left(-\frac{\sqrt{1-cx}+\sqrt{cx+1}}{\sqrt{1-cx}-\sqrt{cx+1}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-b\text{Li}_2\left(\frac{\sqrt{1-cx}+\sqrt{cx+1}}{\sqrt{1-cx}-\sqrt{cx+1}}\right)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)+2\tanh^{-1}\left(1-\frac{}{1-}\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] -((2*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 - Sqrt
[1 - c*x]/Sqrt[1 + c*x])] + b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*
PolyLog[2, -((Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]
))] - b*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, (Sqrt[1 - c
```

```
*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])] - (b^2*PolyLog[3, -((
Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x]))])/2 + (b^2*
PolyLog[3, (Sqrt[1 - c*x] + Sqrt[1 + c*x])/(Sqrt[1 - c*x] - Sqrt[1 + c*x])]
)/2)/c)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b^2 \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2\,ab \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^2}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b^2*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctanh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(b \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a\right)^2}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

maple [B] time = 1.01, size = 676, normalized size = 2.52

$$\frac{a^2\ln(cx+1)}{2c} - \frac{a^2\ln(cx-1)}{2c} - \frac{b^2 \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1 + \frac{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1}{\sqrt{-\frac{-cx+1}{cx+1}+1}}\right)}{c} - \frac{2b^2 \operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) \operatorname{polylog}\left(2, -\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)-b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1
)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2
))-2*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)
/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+2*b^2/c*polylog(3,-((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))-b^2/c*arctanh((-c*x+1)
^(1/2)/(c*x+1)^(1/2))^2*ln(1-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c
*x+1)+1)^(1/2))-2*b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,((-
c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+2*b^2/c*polylog(
3,((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)/(-(-c*x+1)/(c*x+1)+1)^(1/2))+b^2/c*arcta
nh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/
(-(-c*x+1)/(c*x+1)+1))+b^2/c*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(
2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-1/2*b^2/c*poly
log(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2/(-(-c*x+1)/(c*x+1)+1))-2*a*b/c*di
log(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^2*(-(-c*x+1)/(c*x+1)+1))+1/2*a*b/c*d
ilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^4*(-(-c*x+1)/(c*x+1)+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(b^2\log(cx+1)-b^2\log(-cx+1)\right)\log\left(\sqrt{cx+1}-\sqrt{-cx+1}\right)^2}{8c}+\int-\frac{2\left(\sqrt{cx\cdot\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/8*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))^2/c + integrate(-1/8*(2*(sq
rt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sqrt(-c*x + 1))^2
 + 8*(sqrt(c*x + 1)*a*b - sqrt(-c*x + 1)*a*b)*log(sqrt(c*x + 1) + sqrt(-c*x
 + 1)) - (4*(sqrt(c*x + 1)*b^2 - sqrt(-c*x + 1)*b^2)*log(sqrt(c*x + 1) + sq
rt(-c*x + 1)) + (8*a*b - (b^2*c*x - b^2)*log(c*x + 1) + (b^2*c*x - b^2)*log
(-c*x + 1))*sqrt(c*x + 1) - (8*a*b - (b^2*c*x + b^2)*log(c*x + 1) + (b^2*c*
x + b^2)*log(-c*x + 1))*sqrt(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1))
)/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2}{c^2x^2-1}\,dx-\int\frac{b^2\,\mathrm{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{2ab\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*atanh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*atanh(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.33 $\int \frac{a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}+\frac{b\text{Li}_2\left(-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{b\text{Li}_2\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}$$

```
[Out] -a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c+1/2*b*polylog(2,-(-c*x+1)^(1/2)/(c*x+
1)^(1/2))/c-1/2*b*polylog(2,(-c*x+1)^(1/2)/(c*x+1)^(1/2))/c
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {206, 6681, 5912}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{2c}-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] -((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/c) + (b*PolyLog[2, -(Sqrt[1 - c*x]/S
qrt[1 + c*x])])/(2*c) - (b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{a+b\tanh^{-1}(x)}{x}\,dx,x,\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\ &= -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}+\frac{b\text{Li}_2\left(-\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}-\frac{b\text{Li}_2\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.30, size = 43, normalized size = 0.48

$$\frac{a\tanh^{-1}(cx)}{c}+\frac{b\left(\text{Li}_2\left(-e^{-\tanh^{-1}(cx)}\right)-\text{Li}_2\left(e^{-\tanh^{-1}(cx)}\right)\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a*ArcTanh[c*x])/c + (b*(PolyLog[2, -E^(-ArcTanh[c*x])] - PolyLog[2, E^(-Ar
cTanh[c*x])]))/(2*c)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] integrate(-(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

maple [A] time = 0.62, size = 118, normalized size = 1.33

$$\frac{a\ln(cx+1)}{2c}-\frac{a\ln(cx-1)}{2c}-\frac{b\,\text{dilog}\left(\frac{-\frac{-cx+1}{cx+1}+1}{\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1\right)^2}\right)}{c}+\frac{b\,\text{dilog}\left(\frac{\left(-\frac{-cx+1}{cx+1}+1\right)^2}{\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+1\right)^4}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)-b/c*dilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/
2)+1)^2*(-(-c*x+1)/(c*x+1)+1))+1/4*b/c*dilog(1/((-c*x+1)^(1/2)/(c*x+1)^(1/2
)+1)^4*(-(-c*x+1)/(c*x+1)+1)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}b\left(\frac{\left(\log(cx+1)-\log(-cx+1)\right)\log\left(\sqrt{cx+1}+\sqrt{-cx+1}\right)-\left(\log(cx+1)-\log(-cx+1)\right)\log\left(\sqrt{cx+1}-\sqrt{-}\right.}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] 1/4*b*(((log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) + sqrt(-c*x + 1))
- (log(c*x + 1) - log(-c*x + 1))*log(sqrt(c*x + 1) - sqrt(-c*x + 1)))/c - 2
*integrate(-1/2*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)
*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 2*integrate(1/2*sqrt(c
```

```
*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c*x + 1) - (c^2*
x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a + b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2 - 1}\,dx - \int \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*atanh(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

3.34 $\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{ac^2x^2+(bc^2x^2-b)\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctanh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

maple [A] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)\left(a+b\arctan\text{h}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(c^2x^2-1\right)\left(b\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\text{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ac^2x^2-a+bc^2x^2\,\text{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\,\text{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] -Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atanh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\tanh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{a^2c^2x^2+(b^2c^2x^2-b^2)\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2-a^2+2\,(abc^2x^2-ab)\,\text{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctanh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctanh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(c^2x^2-1\right)\left(b\operatorname{artanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] integrate(-1/((c^2*x^2 - 1)*(b*arctanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

maple [A] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)\left(a+b\operatorname{arctanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,cx}{\sqrt{cx+1}\,\sqrt{-cx+1}\,b^2c\log\left(\sqrt{cx+1}+\sqrt{-cx+1}\right)-\sqrt{cx+1}\,\sqrt{-cx+1}\,b^2c\log\left(\sqrt{cx+1}-\sqrt{-cx+1}\right)+2\,\sqrt{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arctanh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] 4*c*x/(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) + sqrt(-c*x + 1
)) - sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*c*log(sqrt(c*x + 1) - sqrt(-c*x + 1))
 + 2*sqrt(c*x + 1)*sqrt(-c*x + 1)*a*b*c) - integrate(-4/((b^2*c^2*x^2 - b^2
)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) + sqrt(-c*x + 1)) - (b^2*c
^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(sqrt(c*x + 1) - sqrt(-c*x +
1)) + 2*(a*b*c^2*x^2 - a*b)*sqrt(c*x + 1)*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\left(a+b\,\mathrm{atanh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)}^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*atanh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2c^2x^2 - a^2 + 2abc^2x^2 \operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - 2ab \operatorname{atanh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + b^2c^2x^2 \operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) - b^2 \operatorname{atanh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*atanh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] -Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atanh(sqrt(-c*x + 1)/s
qrt(c*x + 1)) - 2*a*b*atanh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*
atanh(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atanh(sqrt(-c*x + 1)/sqrt(c*x
 + 1))**2), x)
```

3.36 $\int x^m \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arctanh(tanh(b*x+a))/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]])/(1 +
m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m\tanh^{-1}(\tanh(a+bx))\,dx&=\frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\&=-\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.92

$$x^m\left(\frac{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{m+1}+\frac{bx^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(1 + m))
```

fricas [A] time = 0.48, size = 62, normalized size = 1.68

$$\frac{\left((bm+b)x^2+(am+2\,a)x\right)\cosh\left(m\log(x)\right)+\left((bm+b)x^2+(am+2\,a)x\right)\sinh\left(m\log(x)\right)}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] (((b*m + b)*x^2 + (a*m + 2*a)*x)*cosh(m*log(x)) + ((b*m + b)*x^2 + (a*m + 2
*a)*x)*sinh(m*log(x)))/(m^2 + 3*m + 2)
```

giac [A] time = 0.32, size = 43, normalized size = 1.16

$$\frac{bmx^2x^m+amxx^m+bx^2x^m+2\,axx^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

maple [A] time = 0.15, size = 41, normalized size = 1.11

$$\frac{b\,x^2\mathrm{e}^{m\ln(x)}}{2+m}+\frac{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x\,\mathrm{e}^{m\ln(x)}}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a)),x)
```

```
[Out] b/(2+m)*x^2*exp(m*ln(x))+(arctanh(tanh(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

maxima [A] time = 0.33, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)}+\frac{x^{m+1}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arctanh(tanh(b*x + a))/(m + 1)
```

mupad [B] time = 1.49, size = 96, normalized size = 2.59

$$\frac{2\,b\,x^m\,x^2\,\left(m+1\right)}{2\,m^2+6\,m+4}-\frac{x\,x^m\,\left(m+2\right)\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x)),x)
```

```
[Out] (2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x))/(6*m + 2*m^2 + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b\log\left(x\right)-\frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{x} & \text{for}\: m=-2 \\ \int\frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{x}\,dx & \text{for}\: m=-1 \\ -\frac{bx^2x^m}{m^2+3m+2}+\frac{mxx^m\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{m^2+3m+2}+\frac{2xx^m\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - atanh(tanh(a + b*x))/x, Eq(m, -2)), (Integral(atanh(t
anh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m*
atanh(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*atanh(tanh(a + b*x))/(m**2
 + 3*m + 2), True))
```

3.37 $\int x^2 \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))-\frac{bx^4}{12}$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -(b*x^4)/12 + (x^3*ArcTanh[Tanh[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))-\frac{1}{3}b\int x^3\,dx\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -1/12*(x^3*(b*x - 4*ArcTanh[Tanh[a + b*x]]))
```

fricas [A] time = 0.37, size = 13, normalized size = 0.57

$$\frac{1}{4}bx^4+\frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

giac [A] time = 0.22, size = 13, normalized size = 0.57

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

maple [A] time = 0.14, size = 20, normalized size = 0.87

$$-\frac{b\,x^4}{12}+\frac{x^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a)),x)
[Out] -1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a))
```

maxima [A] time = 0.38, size = 19, normalized size = 0.83

$$-\frac{1}{12}\,bx^4+\frac{1}{3}\,x^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -1/12*b*x^4 + 1/3*x^3*arctanh(tanh(b*x + a))
```

mupad [B] time = 1.00, size = 19, normalized size = 0.83

$$\frac{x^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{3}-\frac{b\,x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x)),x)
[Out] (x^3*atanh(tanh(a + b*x)))/3 - (b*x^4)/12
```

sympy [A] time = 0.39, size = 19, normalized size = 0.83

$$-\frac{bx^4}{12}+\frac{x^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a)),x)
[Out] -b*x**4/12 + x**3*atanh(tanh(a + b*x))/3
```

3.38 $\int x \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

[Out] -1/6*b*x^3+1/2*x^2*arctanh(tanh(b*x+a))

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6239, 30}

$$\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

[In] Int[x*ArcTanh[Tanh[a + b*x]],x]

[Out] -(b*x^3)/6 + (x^2*ArcTanh[Tanh[a + b*x]])/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))-\frac{1}{2}b\int x^2\,dx\\ &= -\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcTanh[Tanh[a + b*x]],x]

[Out] -1/6*(x^2*(b*x - 3*ArcTanh[Tanh[a + b*x]]))

fricas [A] time = 0.62, size = 13, normalized size = 0.57

$$\frac{1}{3}bx^3+\frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="fricas")

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

giac [A] time = 0.14, size = 13, normalized size = 0.57

$$\frac{1}{3}\,bx^{3}+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

maple [A] time = 0.13, size = 20, normalized size = 0.87

$$-\frac{b\,x^{3}}{6}+\frac{x^{2}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a)),x)
[Out] -1/6*b*x^3+1/2*x^2*arctanh(tanh(b*x+a))
```

maxima [A] time = 0.38, size = 19, normalized size = 0.83

$$-\frac{1}{6}\,bx^{3}+\frac{1}{2}\,x^{2}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] -1/6*b*x^3 + 1/2*x^2*arctanh(tanh(b*x + a))
```

mupad [B] time = 0.98, size = 19, normalized size = 0.83

$$\frac{x^{2}\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{2}-\frac{b\,x^{3}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x)),x)
[Out] (x^2*atanh(tanh(a + b*x)))/2 - (b*x^3)/6
```

sympy [A] time = 0.23, size = 19, normalized size = 0.83

$$-\frac{bx^{3}}{6}+\frac{x^{2}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a)),x)
[Out] -b*x**3/6 + x**2*atanh(tanh(a + b*x))/2
```

3.39 $\int \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}$$

```
[Out] 1/2*arctanh(tanh(b*x+a))^2/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]],x]
[Out] ArcTanh[Tanh[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.12

$$x\tanh^{-1}(\tanh(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]],x]
[Out] -1/2*(b*x^2) + x*ArcTanh[Tanh[a + b*x]]
```

fricas [A] time = 0.49, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*x
```

giac [A] time = 0.92, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

maple [A] time = 0.03, size = 15, normalized size = 0.94

$$\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/2*arctanh(tanh(b*x+a))^2/b
```

maxima [A] time = 0.39, size = 16, normalized size = 1.00

$$-\frac{1}{2}bx^2 + x\operatorname{artanh}(\tanh(bx+a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + x*arctanh(tanh(b*x + a))
```

mupad [B] time = 0.05, size = 16, normalized size = 1.00

$$x\operatorname{atanh}(\tanh(a+b\,x)) - \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x)),x)
```

```
[Out] x*atanh(tanh(a + b*x)) - (b*x^2)/2
```

sympy [A] time = 0.16, size = 19, normalized size = 1.19

$$\begin{cases} \frac{\operatorname{atanh}^2(\tanh(a+bx))}{2b} & \text{for } b \neq 0 \\ x\operatorname{atanh}(\tanh(a)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a)),x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**2/(2*b), Ne(b, 0)), (x*atanh(tanh(a)), Tru
e))
```

3.40 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x} dx$

Optimal. Leaf size=21

$$bx - \log(x)\left(bx - \tanh^{-1}(\tanh(a + bx))\right)$$

```
[Out] b*x-(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2158, 29}

$$bx - \log(x)\left(bx - \tanh^{-1}(\tanh(a + bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x,x]
[Out] b*x - (b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x} dx &= bx - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{x} dx\\ &= bx - \left(bx - \tanh^{-1}(\tanh(a+bx))\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 0.90

$$\log(x)\left(\tanh^{-1}(\tanh(a + bx)) - bx\right) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x,x]
[Out] b*x + (-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[x]
```

fricas [A] time = 0.66, size = 8, normalized size = 0.38

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="fricas")
[Out] b*x + a*log(x)
```

giac [A] time = 0.19, size = 9, normalized size = 0.43

$$bx + a\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x + a*log(abs(x))
```

maple [A] time = 0.15, size = 21, normalized size = 1.00

$$\ln(x)\operatorname{arctanh}(\tanh(bx+a)) - \ln(x)xb + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x,x)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))-ln(x)*x*b+b*x
```

maxima [A] time = 0.33, size = 34, normalized size = 1.62

$$-b\left(x+\frac{a}{b}\right)\log(x) + b\left(x+\frac{a\log(x)}{b}\right) + \operatorname{artanh}(\tanh(bx+a))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arctanh(tanh(b*x + a))*log(x)
```

mupad [B] time = 1.10, size = 58, normalized size = 2.76

$$b\,x - \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x)}{2} + \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln(x)}{2} - b\,x\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x,x)
```

```
[Out] b*x - (log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 + (log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - b*x*log(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))/x, x)
```

3.41 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2}\,dx$

Optimal. Leaf size=17

$$b\log(x)-\frac{\tanh^{-1}(\tanh(a+bx))}{x}$$

```
[Out] -arctanh(tanh(b*x+a))/x+b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 29}

$$b\log(x)-\frac{\tanh^{-1}(\tanh(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]/x) + b*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))}{x}+b\int \frac{1}{x}\,dx\\ &= -\frac{\tanh^{-1}(\tanh(a+bx))}{x}+b\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.06

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{x}+b\log(x)+b$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^2,x]
```

```
[Out] b - ArcTanh[Tanh[a + b*x]]/x + b*Log[x]
```

fricas [A] time = 0.45, size = 13, normalized size = 0.76

$$\frac{bx\log(x)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

giac [A] time = 0.16, size = 12, normalized size = 0.71

$$b \log(|x|) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - a/x
```

maple [A] time = 0.14, size = 18, normalized size = 1.06

$$-\frac{\operatorname{arctanh}(\tanh(bx+a))}{x} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^2,x)
```

```
[Out] -arctanh(tanh(b*x+a))/x+b*ln(x)
```

maxima [A] time = 0.38, size = 17, normalized size = 1.00

$$b\log(x) - \frac{\operatorname{artanh}(\tanh(bx+a))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - arctanh(tanh(b*x + a))/x
```

mupad [B] time = 0.09, size = 17, normalized size = 1.00

$$b\,\ln(x) - \frac{\operatorname{atanh}(\tanh(a+b\,x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^2,x)
```

```
[Out] b*log(x) - atanh(tanh(a + b*x))/x
```

sympy [A] time = 0.24, size = 14, normalized size = 0.82

$$b\log(x) - \frac{\operatorname{atanh}(\tanh(a+bx))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**2,x)
```

```
[Out] b*log(x) - atanh(tanh(a + b*x))/x
```

3.42 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^3} dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

```
[Out] -1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -b/(2*x) - ArcTanh[Tanh[a + b*x]]/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^3}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2}\,dx\\ &= -\frac{b}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{\tanh^{-1}(\tanh(a+bx))+bx}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^3,x]
```

```
[Out] -1/2*(b*x + ArcTanh[Tanh[a + b*x]])/x^2
```

fricas [A] time = 0.47, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

giac [A] time = 0.15, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

maple [A] time = 0.14, size = 20, normalized size = 0.87

$$-\frac{b}{2x}-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^3,x)
```

```
[Out] -1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2
```

maxima [A] time = 0.39, size = 19, normalized size = 0.83

$$-\frac{b}{2\,x}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b/x - 1/2*arctanh(tanh(b*x + a))/x^2
```

mupad [B] time = 0.95, size = 16, normalized size = 0.70

$$-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)+b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^3,x)
```

```
[Out] -(atanh(tanh(a + b*x)) + b*x)/(2*x^2)
```

sympy [A] time = 0.51, size = 19, normalized size = 0.83

$$-\frac{b}{2x}-\frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**3,x)
```

```
[Out] -b/(2*x) - atanh(tanh(a + b*x))/(2*x**2)
```

3.43 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}-\frac{b}{6x^2}$$

```
[Out] -1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}-\frac{b}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^4,x]
```

```
[Out] -b/(6*x^2) - ArcTanh[Tanh[a + b*x]]/(3*x^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^4}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}+\frac{1}{3}b\int \frac{1}{x^3}\,dx\\ &= -\frac{b}{6x^2}-\frac{\tanh^{-1}(\tanh(a+bx))}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$-\frac{2\tanh^{-1}(\tanh(a+bx))+bx}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^4,x]
```

```
[Out] -1/6*(b*x + 2*ArcTanh[Tanh[a + b*x]])/x^3
```

fricas [A] time = 0.52, size = 13, normalized size = 0.57

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="fricas")
[Out] -1/6*(3*b*x + 2*a)/x^3
```

giac [A] time = 0.15, size = 13, normalized size = 0.57

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="giac")
[Out] -1/6*(3*b*x + 2*a)/x^3
```

maple [A] time = 0.14, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^4,x)
[Out] -1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3
```

maxima [A] time = 0.38, size = 19, normalized size = 0.83

$$-\frac{b}{6\,x^2}-\frac{\operatorname{artanh}(\tanh(bx+a))}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^4,x, algorithm="maxima")
[Out] -1/6*b/x^2 - 1/3*arctanh(tanh(b*x + a))/x^3
```

mupad [B] time = 0.07, size = 19, normalized size = 0.83

$$-\frac{\operatorname{atanh}(\tanh(a+b\,x))}{3\,x^3}-\frac{b}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^4,x)
[Out] - atanh(tanh(a + b*x))/(3*x^3) - b/(6*x^2)
```

sympy [A] time = 0.78, size = 20, normalized size = 0.87

$$-\frac{b}{6x^2}-\frac{\operatorname{atanh}(\tanh(a+bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**4,x)
[Out] -b/(6*x**2) - atanh(tanh(a + b*x))/(3*x**3)
```

3.44 $\int x^m \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=71

$$-\frac{2bx^{m+2}\tanh^{-1}(\tanh(a+bx))}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^2}{m+1}+\frac{2b^2x^{m+3}}{m^3+6m^2+11m+6}$$

```
[Out] 2*b^2*x^(3+m)/(m^3+6*m^2+11*m+6)-2*b*x^(2+m)*arctanh(tanh(b*x+a))/(m^2+3*m+
2)+x^(1+m)*arctanh(tanh(b*x+a))^2/(1+m)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{2bx^{m+2}\tanh^{-1}(\tanh(a+bx))}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^2}{m+1}+\frac{2b^2x^{m+3}}{m^3+6m^2+11m+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*b^2*x^(3 + m))/(6 + 11*m + 6*m^2 + m^3) - (2*b*x^(2 + m)*ArcTanh[Tanh[a
+ b*x]])/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^2)/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^m \tanh^{-1}(\tanh(a+bx))^2 \, dx &= \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^2}{1+m}-\frac{(2b)\int x^{1+m}\tanh^{-1}(\tanh(a+bx))\,dx}{1+m}\\
&= -\frac{2bx^{2+m}\tanh^{-1}(\tanh(a+bx))}{2+3m+m^2}+\frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^2}{1+m}+\frac{\left(2b^2\right)\int x^{2+m}
\end{aligned}$$

$$= \frac{2b^2x^{3+m}}{6+11m+6m^2+m^3}-\frac{2bx^{2+m}\tanh^{-1}(\tanh(a+bx))}{2+3m+m^2}+\frac{x^{1+m}\tanh^{-1}(\tanh(a}{1+m}$$

Mathematica [A] time = 0.16, size = 62, normalized size = 0.87

$$\frac{x^{m+1}\left(\left(m^2+5m+6\right)\tanh^{-1}(\tanh(a+bx))^2-2b(m+3)x\tanh^{-1}(\tanh(a+bx))+2b^2x^2\right)}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^(1 + m)*(2*b^2*x^2 - 2*b*(3 + m)*x*ArcTanh[Tanh[a + b*x]] + (6 + 5*m + m
^2)*ArcTanh[Tanh[a + b*x]]^2))/((1 + m)*(2 + m)*(3 + m))
```

fricas [B] time = 0.68, size = 161, normalized size = 2.27

$$\frac{\left(\left(b^2m^2+3\,b^2m+2\,b^2\right)x^3+2\left(abm^2+4\,abm+3\,ab\right)x^2+\left(a^2m^2+5\,a^2m+6\,a^2\right)x\right)\cosh\left(m\log(x)\right)+\left(\left(b^2m^2\right.\right.}{m^3+6\,m^2+11\,m+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] (((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^
2*m^2 + 5*a^2*m + 6*a^2)*x)*cosh(m*log(x)) + ((b^2*m^2 + 3*b^2*m + 2*b^2)*x
^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2*m^2 + 5*a^2*m + 6*a^2)*x)*sin
h(m*log(x)))/(m^3 + 6*m^2 + 11*m + 6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^2, x)
```

maple [A] time = 0.15, size = 98, normalized size = 1.38

$$\frac{b^2x^3\mathrm{e}^{m\ln(x)}}{3+m}+\frac{\left(a^2+2a\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)+\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)^2\right)x\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] b^2/(3+m)*x^3*exp(m*ln(x))+(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(t
anh(b*x+a))-b*x-a)^2)/(1+m)*x*exp(m*ln(x))+2*b*(arctanh(tanh(b*x+a))-b*x)/(
2+m)*x^2*exp(m*ln(x))
```

maxima [A] time = 0.40, size = 73, normalized size = 1.03

$$\frac{2\,b^2x^3x^m}{(m+3)(m+2)(m+1)}-\frac{2\,bx^2x^m\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{(m+2)(m+1)}+\frac{x^{m+1}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 2*b^2*x^3*x^m/((m + 3)*(m + 2)*(m + 1)) - 2*b*x^2*x^m*arctanh(tanh(b*x + a)
)/((m + 2)*(m + 1)) + x^(m + 1)*arctanh(tanh(b*x + a))^2/(m + 1)
```

mupad [B] time = 1.13, size = 203, normalized size = 2.86

$$\frac{4\,b^2\,x^m\,x^3\,\left(m^2+3\,m+2\right)}{4\,m^3+24\,m^2+44\,m+24}+\frac{x\,x^m\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2\,\left(m^2+5\,m+6\right)}{4\,m^3+24\,m^2+44\,m+24}-\frac{4\,b\,x^m\,x^2\,\left(\ln\left(\frac{}{{\mathrm{e}}^{2\,a}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (4*b^2*x^m*x^3*(3*m + m^2 + 2))/(44*m + 24*m^2 + 4*m^3 + 24) + (x*x^m*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^2*(5*m + m^2 + 6))/(44*m + 24*m^2 + 4*m^3 + 24) - (4*b*
x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(4*m + m^2 + 3))/(44*m + 24*m^2 + 4*m^3 +
24)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b^2\log(x) - \frac{b\,\text{atanh}(\tanh(a+bx))}{x} - \frac{\text{atanh}^2(\tanh(a+bx))}{2x^2} \\ \int \frac{\text{atanh}^2(\tanh(a+bx))}{x^2}\,dx \\ \int \frac{\text{atanh}^2(\tanh(a+bx))}{x}\,dx \\ \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} - \frac{2bmx^2x^m\,\text{atanh}(\tanh(a+bx))}{m^3+6m^2+11m+6} - \frac{6bx^2x^m\,\text{atanh}(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{m^2xx^m\,\text{atanh}^2(\tanh(a+bx))}{m^3+6m^2+11m+6} + \frac{5mxx^m\,\text{atanh}^2(\tanh(a+bx}{m^3+6m^2+11m+6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((b**2*log(x) - b*atanh(tanh(a + b*x))/x - atanh(tanh(a + b*x))**2
/(2*x**2), Eq(m, -3)), (Integral(atanh(tanh(a + b*x))**2/x**2, x), Eq(m, -2
)), (Integral(atanh(tanh(a + b*x))**2/x, x), Eq(m, -1)), (2*b**2*x**3*x**m/
(m**3 + 6*m**2 + 11*m + 6) - 2*b*m*x**2*x**m*atanh(tanh(a + b*x))/(m**3 + 6
*m**2 + 11*m + 6) - 6*b*x**2*x**m*atanh(tanh(a + b*x))/(m**3 + 6*m**2 + 11*
m + 6) + m**2*x*x**m*atanh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 5
*m*x*x**m*atanh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6) + 6*x*x**m*ata
nh(tanh(a + b*x))**2/(m**3 + 6*m**2 + 11*m + 6), True))
```

3.45 $\int x^3 \tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=42

$$-\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2+\frac{b^2x^6}{60}$$

```
[Out] 1/60*b^2*x^6-1/10*b*x^5*arctanh(tanh(b*x+a))+1/4*x^4*arctanh(tanh(b*x+a))^2
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2+\frac{b^2x^6}{60}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^6)/60 - (b*x^5*ArcTanh[Tanh[a + b*x]])/10 + (x^4*ArcTanh[Tanh[a + b*
x]]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^3\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{2}b\int x^4\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{10}b^2\int x^5\,dx\\ &= \frac{b^2x^6}{60}-\frac{1}{10}bx^5\tanh^{-1}(\tanh(a+bx))+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.88

$$\frac{1}{60}x^4\left(-6bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2+b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^4*(b^2*x^2 - 6*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a + b*x]]^2)
)/60
```

fricas [A] time = 0.51, size = 24, normalized size = 0.57

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

giac [A] time = 0.35, size = 24, normalized size = 0.57

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

maple [A] time = 0.14, size = 38, normalized size = 0.90

$$\frac{x^4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{4}-\frac{b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a))}{5}-\frac{x^6b}{30}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5*x^5*arctanh(tanh(b*x+a))-1/30*x^6
*b)
```

maxima [A] time = 0.46, size = 36, normalized size = 0.86

$$\frac{1}{60}\,b^2x^6-\frac{1}{10}\,bx^5\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{1}{4}\,x^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/60*b^2*x^6 - 1/10*b*x^5*arctanh(tanh(b*x + a)) + 1/4*x^4*arctanh(tanh(b*x
 + a))^2
```

mupad [B] time = 1.00, size = 36, normalized size = 0.86

$$\frac{b^2\,x^6}{60}-\frac{b\,x^5\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}{10}+\frac{x^4\,{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^4*atanh(tanh(a + b*x))^2)/4 + (b^2*x^6)/60 - (b*x^5*atanh(tanh(a + b*x))
)/10
```

sympy [A] time = 2.45, size = 78, normalized size = 1.86

$$\begin{cases}\frac{x^3\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3b}-\frac{x^2\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{4b^2}+\frac{x\operatorname{atanh}^5\left(\tanh\left(a+bx\right)\right)}{10b^3}-\frac{\operatorname{atanh}^6\left(\tanh\left(a+bx\right)\right)}{60b^4} & \text{for}\ b\neq 0\\ \frac{x^4\operatorname{atanh}^2\left(\tanh\left(a\right)\right)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((x**3*atanh(tanh(a + b*x))**3/(3*b) - x**2*atanh(tanh(a + b*x))**
4/(4*b**2) + x*atanh(tanh(a + b*x))**5/(10*b**3) - atanh(tanh(a + b*x))**6/
(60*b**4), Ne(b, 0)), (x**4*atanh(tanh(a))**2/4, True))
```

3.46 $\int x^2 \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=42

$$-\frac{1}{6}bx^4\tanh^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))^2+\frac{b^2x^5}{30}$$

```
[Out] 1/30*b^2*x^5-1/6*b*x^4*arctanh(tanh(b*x+a))+1/3*x^3*arctanh(tanh(b*x+a))^2
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{1}{6}bx^4\tanh^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))^2+\frac{b^2x^5}{30}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b^2*x^5)/30 - (b*x^4*ArcTanh[Tanh[a + b*x]])/6 + (x^3*ArcTanh[Tanh[a + b*x
]]^2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{3}(2b)\int x^3\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{1}{6}bx^4\tanh^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{6}b^2\int x^4\,dx\\ &= \frac{b^2x^5}{30}-\frac{1}{6}bx^4\tanh^{-1}(\tanh(a+bx))+\frac{1}{3}x^3\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.05, size = 37, normalized size = 0.88

$$\frac{1}{30}x^3\left(-5bx\tanh^{-1}(\tanh(a+bx))+10\tanh^{-1}(\tanh(a+bx))^2+b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^3*(b^2*x^2 - 5*b*x*ArcTanh[Tanh[a + b*x]] + 10*ArcTanh[Tanh[a + b*x]]^2)
)/30
```

fricas [A] time = 0.49, size = 24, normalized size = 0.57

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

giac [A] time = 0.26, size = 24, normalized size = 0.57

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

maple [A] time = 0.14, size = 38, normalized size = 0.90

$$\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))^2}{3} - \frac{2b\left(\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))}{4} - \frac{b x^5}{20}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^2,x)
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^2-2/3*b*(1/4*x^4*arctanh(tanh(b*x+a))-1/20*b*x
^5)
```

maxima [A] time = 0.46, size = 36, normalized size = 0.86

$$\frac{1}{30}b^2x^5 - \frac{1}{6}bx^4 \operatorname{artanh}(\tanh(bx+a)) + \frac{1}{3}x^3 \operatorname{artanh}(\tanh(bx+a))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] 1/30*b^2*x^5 - 1/6*b*x^4*arctanh(tanh(b*x + a)) + 1/3*x^3*arctanh(tanh(b*x
+ a))^2
```

mupad [B] time = 0.97, size = 36, normalized size = 0.86

$$\frac{b^2 x^5}{30} - \frac{b x^4 \operatorname{atanh}(\tanh(a+b x))}{6} + \frac{x^3 \operatorname{atanh}(\tanh(a+b x))^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^2,x)
[Out] (x^3*atanh(tanh(a + b*x))^2)/3 + (b^2*x^5)/30 - (b*x^4*atanh(tanh(a + b*x))
)/6
```

sympy [A] time = 1.23, size = 60, normalized size = 1.43

$$\begin{cases} \frac{x^2 \operatorname{atanh}^3(\tanh(a+bx))}{3b} - \frac{x \operatorname{atanh}^4(\tanh(a+bx))}{6b^2} + \frac{\operatorname{atanh}^5(\tanh(a+bx))}{30b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{atanh}^2(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((x**2*atanh(tanh(a + b*x))**3/(3*b) - x*atanh(tanh(a + b*x))**4/(
6*b**2) + atanh(tanh(a + b*x))**5/(30*b**3), Ne(b, 0)), (x**3*atanh(tanh(a)
)**2/3, True))
```

3.47 $\int x \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=34

$$\frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}$$

```
[Out] 1/3*x*arctanh(tanh(b*x+a))^3/b-1/12*arctanh(tanh(b*x+a))^4/b^2
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^3)/(3*b) - ArcTanh[Tanh[a + b*x]]^4/(12*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\int \tanh^{-1}(\tanh(a+bx))^3\,dx}{3b}\\
&= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\text{Subst}\left(\int x^3\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}\\
&= \frac{x\tanh^{-1}(\tanh(a+bx))^3}{3b}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{12b^2}
\end{aligned}$$

Mathematica [B] time = 0.08, size = 74, normalized size = 2.18

$$\frac{(a+bx)\left(4\left(2a^2+abx-b^2x^2\right)\tanh^{-1}(\tanh(a+bx))-\left((3a-bx)(a+bx)^2\right)-6(a-bx)\tanh^{-1}(\tanh(a+bx))^2\right.}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] ((a + b*x)*(-((3*a - b*x)*(a + b*x)^2) + 4*(2*a^2 + a*b*x - b^2*x^2)*ArcTan
h[Tanh[a + b*x]] - 6*(a - b*x)*ArcTanh[Tanh[a + b*x]]^2))/(12*b^2)
```

fricas [A] time = 0.50, size = 24, normalized size = 0.71

$$\frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.71

$$\frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

maple [A] time = 0.14, size = 38, normalized size = 1.12

$$\frac{x^2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{2} - b\left(-\frac{b\,x^4}{12} + \frac{x^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^2-b*(-1/12*b*x^4+1/3*x^3*arctanh(tanh(b*x+a)))
```

maxima [A] time = 0.45, size = 36, normalized size = 1.06

$$\frac{1}{12}\,b^2x^4 - \frac{1}{3}\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{1}{2}\,x^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^4 - 1/3*b*x^3*arctanh(tanh(b*x + a)) + 1/2*x^2*arctanh(tanh(b*x
+ a))^2
```

mupad [B] time = 0.94, size = 36, normalized size = 1.06

$$\frac{b^2\,x^4}{12} - \frac{b\,x^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{3} + \frac{x^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^2*atanh(tanh(a + b*x))^2)/2 + (b^2*x^4)/12 - (b*x^3*atanh(tanh(a + b*x))
)/3
```

sympy [A] time = 0.63, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{3b} - \frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{12b^2} & \text{for}\: b \neq 0 \\ \frac{x^2\operatorname{atanh}^{2}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**3/(3*b) - atanh(tanh(a + b*x))**4/(12*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**2/2, True))
```

3.48 $\int \tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

```
[Out] 1/3*arctanh(tanh(b*x+a))^3/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*b)
```

fricas [A] time = 0.64, size = 20, normalized size = 1.25

$$\frac{1}{3}b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

giac [**A**] time = 0.16, size = 20, normalized size = 1.25

$$\frac{1}{3}\,b^{2}x^{3} + abx^{2} + a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="giac")
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

maple [**A**] time = 0.03, size = 15, normalized size = 0.94

$$\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2,x)
[Out] 1/3*arctanh(tanh(b*x+a))^3/b
```

maxima [**B**] time = 0.45, size = 33, normalized size = 2.06

$$\frac{1}{3}\,b^{2}x^{3} - bx^{2}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right) + x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 - b*x^2*arctanh(tanh(b*x + a)) + x*arctanh(tanh(b*x + a))^2
```

mupad [**B**] time = 0.07, size = 33, normalized size = 2.06

$$\frac{b^{2}\,x^{3}}{3} - b\,x^{2}\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right) + x\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2,x)
[Out] x*atanh(tanh(a + b*x))^2 + (b^2*x^3)/3 - b*x^2*atanh(tanh(a + b*x))
```

sympy [**A**] time = 0.30, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{3b} & \text{for}\ b \neq 0 \\ x\operatorname{atanh}^{2}\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2,x)
[Out] Piecewise((atanh(tanh(a + b*x))**3/(3*b), Ne(b, 0)), (x*atanh(tanh(a))**2,
True))
```

3.49 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x} dx$

Optimal. Leaf size=49

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2+\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2$$

```
[Out] -b*x*(b*x-arctanh(tanh(b*x+a)))+1/2*arctanh(tanh(b*x+a))^2+(b*x-arctanh(tan
h(b*x+a)))^2*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2159, 2158, 29}

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2+\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x,x]
```

```
[Out] -(b*x*(b*x - ArcTanh[Tanh[a + b*x]])) + ArcTanh[Tanh[a + b*x]]^2/2 + (b*x -
 ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x} dx = \frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2-\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a+bx)}{x}$$
$$= -bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2-\left(\left(bx-\tanh^{-1}(\tan\right.\right.$$
$$= -bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2+\left(bx-\tanh^{-1}(\tan\right.$$

Mathematica [A] time = 0.07, size = 53, normalized size = 1.08

$$\frac{1}{2}(a+bx)^2-(a+bx)\left(-2\tanh^{-1}(\tanh(a+bx))+a+2bx\right)+\log(bx)\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x,x]
```

```
[Out] (a + b*x)^2/2 - (a + b*x)*(a + 2*b*x - 2*ArcTanh[Tanh[a + b*x]]) + (-(b*x)
+ ArcTanh[Tanh[a + b*x]])^2*Log[b*x]
```

fricas [A] time = 0.58, size = 20, normalized size = 0.41

$$\frac{1}{2}b^2x^2 + 2abx + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="fricas")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

giac [A] time = 0.17, size = 21, normalized size = 0.43

$$\frac{1}{2}b^2x^2 + 2abx + a^2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

maple [A] time = 0.17, size = 55, normalized size = 1.12

$$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2+b^2x^2\ln(x)-\frac{3b^2x^2}{2}-2b\operatorname{arctanh}(\tanh(bx+a))\ln(x)x+2b\operatorname{arctanh}(\tanh(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x,x)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^2+b^2*x^2*ln(x)-3/2*b^2*x^2-2*b*arctanh(tanh(b*x
+a))*ln(x)*x+2*b*arctanh(tanh(b*x+a))*x
```

maxima [A] time = 0.71, size = 20, normalized size = 0.41

$$\frac{1}{2}b^2x^2 + 2abx + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

mupad [B] time = 0.29, size = 183, normalized size = 3.73

$$\ln(x)\left(\frac{\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}{4}-a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x,x)
```

```
[Out] log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/4 - a*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x) + a^2) + (b^2*x^2)/2 - b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x, x)
```

3.50 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x}-2b\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+2b^2x$$

```
[Out] 2*b^2*x-arctanh(tanh(b*x+a))^2/x-2*b*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2158, 29}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x}-2b\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] 2*b^2*x - ArcTanh[Tanh[a + b*x]]^2/x - 2*b*(b*x - ArcTanh[Tanh[a + b*x]])*L
og[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^2}{x}+(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x}\,dx\\
&= 2b^2x-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x}-\left(2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{x}\,dx\\
&= 2b^2x-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x}-2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 37, normalized size = 0.95

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{x}+2b(\log(x)+1)\tanh^{-1}(\tanh(a+bx))-2b^2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^2/x) - 2*b^2*x*Log[x] + 2*b*ArcTanh[Tanh[a + b*x]]
*(1 + Log[x])
```

fricas [A] time = 0.51, size = 24, normalized size = 0.62

$$\frac{b^2x^2 + 2\,abx\log(x) - a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

giac [A] time = 0.17, size = 21, normalized size = 0.54

$$b^2x + 2\,ab\log\left(|x|\right) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="giac")
```

```
[Out] b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

maple [A] time = 0.15, size = 42, normalized size = 1.08

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{x} - 2\ln(x)x\,b^2 + 2\ln(x)\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)b + 2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^2,x)
```

```
[Out] -arctanh(tanh(b*x+a))^2/x-2*ln(x)*x*b^2+2*ln(x)*arctanh(tanh(b*x+a))*b+2*b^
2*x
```

maxima [A] time = 0.40, size = 54, normalized size = 1.38

$$2\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\log(x) - 2\left(b\left(x+\frac{a}{b}\right)\log(x) - b\left(x+\frac{a\log(x)}{b}\right)\right)b - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*arctanh(tanh(b*x + a))*log(x) - 2*(b*(x + a/b)*log(x) - b*(x + a*log(x)
/b))*b - arctanh(tanh(b*x + a))^2/x
```

mupad [B] time = 0.19, size = 198, normalized size = 5.08

$$b\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4\,x} - b\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4\,x} + b\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\ln(x) - 2\,b^2\,x\,\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^2,x)
```

```
[Out] b*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))^2/(4*x) - b*log(1/(exp(2*a)*exp(2*b*x) + 1)) - log(
```

```
1/(exp(2*a)*exp(2*b*x) + 1))^2/(4*x) + b*log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))*log(x) - 2*b^2*x*log(x) - b*log(1/(exp(2*a)*exp(2*b*x) +
 1))*log(x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)))/(2*x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x**2, x)
```

3.51 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3} dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\tanh^{-1}(\tanh(a+bx))}{x}+b^2\log(x)$$

```
[Out] -b*arctanh(tanh(b*x+a))/x-1/2*arctanh(tanh(b*x+a))^2/x^2+b^2*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 29}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\tanh^{-1}(\tanh(a+bx))}{x}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -((b*ArcTanh[Tanh[a + b*x]])/x) - ArcTanh[Tanh[a + b*x]]^2/(2*x^2) + b^2*Lo
g[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2}+b\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^2}\,dx\\
&= -\frac{b\tanh^{-1}(\tanh(a+bx))}{x}-\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2}+b^2\int \frac{1}{x}\,dx\\
&= -\frac{b\tanh^{-1}(\tanh(a+bx))}{x}-\frac{\tanh^{-1}(\tanh(a+bx))^2}{2x^2}+b^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.17

$$-\frac{2bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2-b^2x^2(2\log(x)+3)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^3,x]
```

```
[Out] -1/2*(2*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2 - b^2*x^2*(3
+ 2*Log[x]))/x^2
```

fricas [**A**] time = 0.42, size = 26, normalized size = 0.72

$$\frac{2\,b^2x^2\log(x) - 4\,abx - a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

giac [**A**] time = 0.18, size = 22, normalized size = 0.61

$$b^2\log\left(|x|\right) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="giac")
[Out] b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

maple [**A**] time = 0.15, size = 35, normalized size = 0.97

$$-\frac{b\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}{x} - \frac{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{2x^2} + b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^3,x)
[Out] -b*arctanh(tanh(b*x+a))/x-1/2*arctanh(tanh(b*x+a))^2/x^2+b^2*ln(x)
```

maxima [**A**] time = 0.46, size = 34, normalized size = 0.94

$$b^2\log(x) - \frac{b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x} - \frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^3,x, algorithm="maxima")
[Out] b^2*log(x) - b*arctanh(tanh(b*x + a))/x - 1/2*arctanh(tanh(b*x + a))^2/x^2
```

mupad [**B**] time = 0.93, size = 34, normalized size = 0.94

$$b^2\,\ln(x) - \frac{\frac{\mathrm{atanh}(\tanh(a+b\,x))^2}{2} + b\,x\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^3,x)
[Out] b^2*log(x) - (atanh(tanh(a + b*x))^2/2 + b*x*atanh(tanh(a + b*x)))/x^2
```

sympy [**A**] time = 0.52, size = 32, normalized size = 0.89

$$b^2\log\left(x\right) - \frac{b\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{x} - \frac{\mathrm{atanh}^2\left(\tanh\left(a+bx\right)\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**3,x)
[Out] b**2*log(x) - b*atanh(tanh(a + b*x))/x - atanh(tanh(a + b*x))**2/(2*x**2)
```

3.52 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4} \, dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/3*arctanh(tanh(b*x+a))^3/x^3/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^3/(3*x^3*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4} \, dx = \frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.10

$$-\frac{bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2+b^2x^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^4,x]
```

```
[Out] -1/3*(b^2*x^2 + b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2)/x^3
```

fricas [A] time = 0.46, size = 22, normalized size = 0.71

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

giac [A] time = 0.81, size = 22, normalized size = 0.71

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

maple [A] time = 0.14, size = 38, normalized size = 1.23

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{2}}{3x^{3}}+\frac{2b\left(-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^4,x)
```

```
[Out] -1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^
2)
```

maxima [A] time = 0.47, size = 36, normalized size = 1.16

$$-\frac{b^{2}}{3\,x}-\frac{b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^{2}}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2/x - 1/3*b*arctanh(tanh(b*x + a))/x^2 - 1/3*arctanh(tanh(b*x + a))^
2/x^3
```

mupad [B] time = 0.94, size = 32, normalized size = 1.03

$$-\frac{b^{2}\,x^{2}+b\,x\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)+\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{2}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^4,x)
```

```
[Out] -(atanh(tanh(a + b*x))^2 + b^2*x^2 + b*x*atanh(tanh(a + b*x)))/(3*x^3)
```

sympy [A] time = 0.80, size = 37, normalized size = 1.19

$$-\frac{b^{2}}{3x}-\frac{b\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3x^{2}}-\frac{\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**4,x)
```

```
[Out] -b**2/(3*x) - b*atanh(tanh(a + b*x))/(3*x**2) - atanh(tanh(a + b*x))**2/(3*
x**3)
```

3.53 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^5}\,dx$

Optimal. Leaf size=42

$$-\frac{\tanh^{-1}(\tanh(a+bx))^2}{4x^4}-\frac{b\tanh^{-1}(\tanh(a+bx))}{6x^3}-\frac{b^2}{12x^2}$$

```
[Out] -1/12*b^2/x^2-1/6*b*arctanh(tanh(b*x+a))/x^3-1/4*arctanh(tanh(b*x+a))^2/x^4
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^3)/(12*x^3*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^3/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^5}\,dx &= \frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^4}\,dx}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\tanh^{-1}(\tanh(a+bx))^3}{12x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^3}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.88

$$-\frac{2bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2+b^2x^2}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^5,x]
```

```
[Out] -1/12*(b^2*x^2 + 2*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)
/x^4
```

fricas [A] time = 0.48, size = 24, normalized size = 0.57

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

giac [A] time = 0.40, size = 24, normalized size = 0.57

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

maple [A] time = 0.14, size = 38, normalized size = 0.90

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{4x^4}+\frac{b\left(-\frac{b}{6x^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^5,x)
```

```
[Out] -1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/
x^3)
```

maxima [A] time = 0.48, size = 36, normalized size = 0.86

$$-\frac{b^2}{12\,x^2}-\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{6\,x^3}-\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*b^2/x^2 - 1/6*b*arctanh(tanh(b*x + a))/x^3 - 1/4*arctanh(tanh(b*x + a
))^2/x^4
```

mupad [B] time = 0.95, size = 36, normalized size = 0.86

$$-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{4\,x^4}-\frac{b^2}{12\,x^2}-\frac{b\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^5,x)
```

```
[Out] - atanh(tanh(a + b*x))^2/(4*x^4) - b^2/(12*x^2) - (b*atanh(tanh(a + b*x)))/
(6*x^3)
```

sympy [A] time = 1.27, size = 39, normalized size = 0.93

$$-\frac{b^2}{12x^2}-\frac{b\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{6x^3}-\frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**5,x)
```

```
[Out] -b**2/(12*x**2) - b*atanh(tanh(a + b*x))/(6*x**3) - atanh(tanh(a + b*x))**2
/(4*x**4)
```

3.54 $\int x^m \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=110

$$\frac{6b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))}{m^3+6m^2+11m+6}-\frac{3bx^{m+2}\tanh^{-1}(\tanh(a+bx))^2}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^3}{m+1}-\frac{6b}{(m+1)\left(m^3+\right.}$$

```
[Out] -6*b^3*x^(4+m)/(1+m)/(m^3+9*m^2+26*m+24)+6*b^2*x^(3+m)*arctanh(tanh(b*x+a))
/(m^3+6*m^2+11*m+6)-3*b*x^(2+m)*arctanh(tanh(b*x+a))^2/(m^2+3*m+2)+x^(1+m)*
arctanh(tanh(b*x+a))^3/(1+m)
```

Rubi [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2168, 30}

$$\frac{6b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))}{m^3+6m^2+11m+6}-\frac{3bx^{m+2}\tanh^{-1}(\tanh(a+bx))^2}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-1}(\tanh(a+bx))^3}{m+1}-\frac{6b}{(m+1)\left(m^3+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-6*b^3*x^(4 + m))/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (6*b^2*x^(3 + m)*A
rcTanh[Tanh[a + b*x]])/(6 + 11*m + 6*m^2 + m^3) - (3*b*x^(2 + m)*ArcTanh[Ta
nh[a + b*x]]^2)/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^3)/(1 +
 m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m \tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^3}{1+m}-\frac{(3b)\int x^{1+m}\tanh^{-1}(\tanh(a+bx))^2\,dx}{1+m}\\ &= -\frac{3bx^{2+m}\tanh^{-1}(\tanh(a+bx))^2}{2+3m+m^2}+\frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^3}{1+m}+\frac{\left(6b^2\right)\int}{}\\ &= \frac{6b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3}-\frac{3bx^{2+m}\tanh^{-1}(\tanh(a+bx))^2}{2+3m+m^2}+\frac{x^{1+m}\,\text{ta}}{}\\ &= -\frac{6b^3x^{4+m}}{(4+m)\left(6+11m+6m^2+m^3\right)}+\frac{6b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))}{6+11m+6m^2+m^3}-\frac{3bx^{2+}}{}\end{aligned}$$

Mathematica [A] time = 0.24, size = 97, normalized size = 0.88

$$\frac{x^{m+1}\left(6b^2(m+4)x^2\tanh^{-1}(\tanh(a+bx))-3b\left(m^2+7m+12\right)x\tanh^{-1}(\tanh(a+bx))^2+\left(m^3+9m^2+26m\right.\right.}{(m+1)(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*(-6*b^3*x^3 + 6*b^2*(4 + m)*x^2*ArcTanh[Tanh[a + b*x]] - 3*b*(12
 + 7*m + m^2)*x*ArcTanh[Tanh[a + b*x]]^2 + (24 + 26*m + 9*m^2 + m^3)*ArcTan
h[Tanh[a + b*x]]^3))/((1 + m)*(2 + m)*(3 + m)*(4 + m))
```

fricas [B] time = 0.40, size = 300, normalized size = 2.73

$$\frac{\left(\left(b^3m^3 + 6\,b^3m^2 + 11\,b^3m + 6\,b^3\right)x^4 + 3\left(ab^2m^3 + 7\,ab^2m^2 + 14\,ab^2m + 8\,ab^2\right)x^3 + 3\left(a^2bm^3 + 8\,a^2bm^2 + 19\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] (((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12
*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*cosh(m*log(x)) +
 ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
 + 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12
*a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*sinh(m*log(x)))/
(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{artanh}\left(\tanh\left(bx + a\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^3, x)
```

maple [A] time = 0.14, size = 177, normalized size = 1.61

$$\frac{b^3x^4e^{m\ln(x)}}{4+m} + \frac{\left(a^3 + 3a^2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx - a\right) + 3a\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx - a\right)^2 + \left(\operatorname{arctanh}\left(\mathrm{t}\right.\right.\right.}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] b^3/(4+m)*x^4*exp(m*ln(x))+(a^3+3*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3*a*(arc
tanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x-a)^3)/(1+m)*x*exp(m*ln
(x))+3*b*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)
^2)/(2+m)*x^2*exp(m*ln(x))+3*b^2*(arctanh(tanh(b*x+a))-b*x)/(3+m)*x^3*exp(m
*ln(x))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 1.23, size = 332, normalized size = 3.02

$$\frac{8\,b^3\,x^m\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192} - \frac{x\,x^m\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) + 2\,b\,x\right)^3\,\left(m^3+9\,m^2+26\,m+24\right)}{8\,m^4+80\,m^3+280\,m^2+400\,m+192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (8*b^3*x^m*x^4*(11*m + 6*m^2 + m^3 + 6))/(400*m + 280*m^2 + 80*m^3 + 8*m^4
+ 192) - (x*x^m*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3*(26*m + 9*m^2 + m^3 + 24))/(400*
m + 280*m^2 + 80*m^3 + 8*m^4 + 192) - (12*b^2*x^m*x^3*(log(2/(exp(2*a)*exp(
2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x)*(14*m + 7*m^2 + m^3 + 8))/(400*m + 280*m^2 + 80*m^3 + 8*m^4 + 192) + (6
*b*x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2*(19*m + 8*m^2 + m^3 + 12))/(400*m + 2
80*m^2 + 80*m^3 + 8*m^4 + 192)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b^3\log(x) - \frac{b^2\operatorname{atanh}(\tanh(a+bx))}{x} - \frac{b\operatorname{atanh}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{atanh}^3(\tanh(a+bx))}{3x^3} \\ \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x^3}\,dx \\ \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x^2}\,dx \\ \int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x}\,dx \\ -\frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^2mx^3x^m\operatorname{atanh}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} + \frac{24b^2x^3x^m\operatorname{atanh}(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{3bm^2x^2x^m\operatorname{atanh}^2(\tanh(a+bx))}{m^4+10m^3+35m^2+50m+24} - \frac{21bm}{m^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((b**3*log(x) - b**2*atanh(tanh(a + b*x))/x - b*atanh(tanh(a + b*x
))**2/(2*x**2) - atanh(tanh(a + b*x))**3/(3*x**3), Eq(m, -4)), (Integral(at
anh(tanh(a + b*x))**3/x**3, x), Eq(m, -3)), (Integral(atanh(tanh(a + b*x))*
*3/x**2, x), Eq(m, -2)), (Integral(atanh(tanh(a + b*x))**3/x, x), Eq(m, -1)
), (-6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**2*m*x**
3*x**m*atanh(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*b**
2*x**3*x**m*atanh(tanh(a + b*x))/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) - 3
*b*m**2*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2 + 50*m
+ 24) - 21*b*m*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*m**2
+ 50*m + 24) - 36*b*x**2*x**m*atanh(tanh(a + b*x))**2/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + m**3*x*x**m*atanh(tanh(a + b*x))**3/(m**4 + 10*m**3 + 3
5*m**2 + 50*m + 24) + 9*m**2*x*x**m*atanh(tanh(a + b*x))**3/(m**4 + 10*m**3
 + 35*m**2 + 50*m + 24) + 26*m*x*x**m*atanh(tanh(a + b*x))**3/(m**4 + 10*m*
*3 + 35*m**2 + 50*m + 24) + 24*x*x**m*atanh(tanh(a + b*x))**3/(m**4 + 10*m*
*3 + 35*m**2 + 50*m + 24), True))
```

3.55 $\int x^3 \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=61

$$\frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3-\frac{1}{140}b^3x^7$$

```
[Out] -1/140*b^3*x^7+1/20*b^2*x^6*arctanh(tanh(b*x+a))-3/20*b*x^5*arctanh(tanh(b*
x+a))^2+1/4*x^4*arctanh(tanh(b*x+a))^3
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2168, 30}

$$\frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3-\frac{1}{140}b^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -(b^3*x^7)/140 + (b^2*x^6*ArcTanh[Tanh[a + b*x]])/20 - (3*b*x^5*ArcTanh[Tan
h[a + b*x]]^2)/20 + (x^4*ArcTanh[Tanh[a + b*x]]^3)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^3\tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3-\frac{1}{4}(3b)\int x^4\tanh^{-1}(\tanh(a+bx))^2\,dx\\
&= -\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{10}\left(3b^2\right)\int x^5\\
&= \frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}x^4\tanh^{-1}(\mathrm{ta}\\
&= -\frac{1}{140}b^3x^7+\frac{1}{20}b^2x^6\tanh^{-1}(\tanh(a+bx))-\frac{3}{20}bx^5\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.89

$$-\frac{1}{140}x^4\left(-7b^2x^2\tanh^{-1}(\tanh(a+bx))+21bx\tanh^{-1}(\tanh(a+bx))^2-35\tanh^{-1}(\tanh(a+bx))^3+b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/140*(x^4*(b^3*x^3 - 7*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 21*b*x*ArcTanh[Ta
nh[a + b*x]]^2 - 35*ArcTanh[Tanh[a + b*x]]^3))
```

fricas [A] time = 1.02, size = 35, normalized size = 0.57

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

giac [A] time = 0.21, size = 35, normalized size = 0.57

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

maple [A] time = 0.14, size = 56, normalized size = 0.92

$$\frac{x^4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{4}-\frac{3b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a))^2}{5}-\frac{2b\left(\frac{x^6\operatorname{arctanh}(\tanh(bx+a))}{6}-\frac{x^7b}{42}\right)}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^3,x)
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^3-3/4*b*(1/5*x^5*arctanh(tanh(b*x+a))^2-2/5*b*
(1/6*x^6*arctanh(tanh(b*x+a))-1/42*x^7*b))
```

maxima [A] time = 0.52, size = 54, normalized size = 0.89

$$-\frac{3}{20}\,bx^5\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\frac{1}{4}\,x^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{140}\,\left(b^2x^7-7\,bx^6\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
[Out] -3/20*b*x^5*arctanh(tanh(b*x + a))^2 + 1/4*x^4*arctanh(tanh(b*x + a))^3 - 1
/140*(b^2*x^7 - 7*b*x^6*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 1.05, size = 53, normalized size = 0.87

$$-\frac{b^3\,x^7}{140}+\frac{b^2\,x^6\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}{20}-\frac{3\,b\,x^5\,{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}^2}{20}+\frac{x^4\,{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^3,x)
[Out] (x^4*atanh(tanh(a + b*x))^3)/4 - (b^3*x^7)/140 - (3*b*x^5*atanh(tanh(a + b*
x))^2)/20 + (b^2*x^6*atanh(tanh(a + b*x)))/20
```

sympy [A] time = 4.09, size = 80, normalized size = 1.31

$$\begin{cases} \frac{x^3 \operatorname{atanh}^4(\tanh(a+bx))}{4b} - \frac{3x^2 \operatorname{atanh}^5(\tanh(a+bx))}{20b^2} + \frac{x \operatorname{atanh}^6(\tanh(a+bx))}{20b^3} - \frac{\operatorname{atanh}^7(\tanh(a+bx))}{140b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{atanh}^3(\tanh(a))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((x**3*atanh(tanh(a + b*x))**4/(4*b) - 3*x**2*atanh(tanh(a + b*x))
**5/(20*b**2) + x*atanh(tanh(a + b*x))**6/(20*b**3) - atanh(tanh(a + b*x))*
*7/(140*b**4), Ne(b, 0)), (x**4*atanh(tanh(a))**3/4, True))
```

3.56 $\int x^2 \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}(\tanh(a+bx))^6}{60b^3}-\frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

```
[Out] 1/4*x^2*arctanh(tanh(b*x+a))^4/b-1/10*x*arctanh(tanh(b*x+a))^5/b^2+1/60*arc
tanh(tanh(b*x+a))^6/b^3
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{2168, 2157, 30\}$

$$\frac{\tanh^{-1}(\tanh(a+bx))^6}{60b^3}-\frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - (x*ArcTanh[Tanh[a + b*x]]^5)/(10*b^2
) + ArcTanh[Tanh[a + b*x]]^6/(60*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b}-\frac{\int x\tanh^{-1}(\tanh(a+bx))^4\,dx}{2b}\\ &= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b}-\frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{\int \tanh^{-1}(\tanh(a+\\ &= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b}-\frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{\text{Subst}\left(\int x^5\,dx,x,t\right.}{1}\\ &= \frac{x^2\tanh^{-1}(\tanh(a+bx))^4}{4b}-\frac{x\tanh^{-1}(\tanh(a+bx))^5}{10b^2}+\frac{\tanh^{-1}(\tanh(a+b}{60b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.02

$$-\frac{1}{60}x^3\left(-6b^2x^2\tanh^{-1}(\tanh(a+bx))+15bx\tanh^{-1}(\tanh(a+bx))^2-20\tanh^{-1}(\tanh(a+bx))^3+b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/60*(x^3*(b^3*x^3 - 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 15*b*x*ArcTanh[Tan
h[a + b*x]]^2 - 20*ArcTanh[Tanh[a + b*x]]^3))
```

fricas [A] time = 0.56, size = 35, normalized size = 0.66

$$\frac{1}{6}b^3x^6 + \frac{3}{5}ab^2x^5 + \frac{3}{4}a^2bx^4 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

giac [A] time = 0.16, size = 35, normalized size = 0.66

$$\frac{1}{6}b^3x^6 + \frac{3}{5}ab^2x^5 + \frac{3}{4}a^2bx^4 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

maple [A] time = 0.15, size = 56, normalized size = 1.06

$$\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))^3}{3} - b\left(\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))^2}{4} - \frac{b\left(\frac{x^5 \operatorname{arctanh}(\tanh(bx+a))}{5} - \frac{x^6 b}{30}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^3-b*(1/4*x^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5
*x^5*arctanh(tanh(b*x+a))-1/30*x^6*b))
```

maxima [A] time = 0.52, size = 54, normalized size = 1.02

$$-\frac{1}{4}bx^4 \operatorname{artanh}(\tanh(bx+a))^2 + \frac{1}{3}x^3 \operatorname{artanh}(\tanh(bx+a))^3 - \frac{1}{60}\left(b^2x^6 - 6\,bx^5 \operatorname{artanh}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*x^4*arctanh(tanh(b*x + a))^2 + 1/3*x^3*arctanh(tanh(b*x + a))^3 - 1/
60*(b^2*x^6 - 6*b*x^5*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 0.14, size = 53, normalized size = 1.00

$$-\frac{b^3\,x^6}{60} + \frac{b^2\,x^5\,\operatorname{atanh}(\tanh(a+b\,x))}{10} - \frac{b\,x^4\,\operatorname{atanh}(\tanh(a+b\,x))^2}{4} + \frac{x^3\,\operatorname{atanh}(\tanh(a+b\,x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (x^3*atanh(tanh(a + b*x))^3)/3 - (b^3*x^6)/60 - (b*x^4*atanh(tanh(a + b*x))
^2)/4 + (b^2*x^5*atanh(tanh(a + b*x)))/10
```

sympy [A] time = 2.40, size = 60, normalized size = 1.13

$$\begin{cases} \frac{x^2\operatorname{atanh}^4(\tanh(a+bx))}{4b} - \frac{x\operatorname{atanh}^5(\tanh(a+bx))}{10b^2} + \frac{\operatorname{atanh}^6(\tanh(a+bx))}{60b^3} & \text{for } b \neq 0 \\ \frac{x^3\operatorname{atanh}^3(\tanh(a))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((x**2*atanh(tanh(a + b*x))**4/(4*b) - x*atanh(tanh(a + b*x))**5/(
10*b**2) + atanh(tanh(a + b*x))**6/(60*b**3), Ne(b, 0)), (x**3*atanh(tanh(a
))**3/3, True))
```

3.57 $\int x \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=34

$$\frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}$$

```
[Out] 1/4*x*arctanh(tanh(b*x+a))^4/b-1/20*arctanh(tanh(b*x+a))^5/b^2
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^4)/(4*b) - ArcTanh[Tanh[a + b*x]]^5/(20*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x \tanh^{-1}(\tanh(a+bx))^3 \, dx &= \frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\int \tanh^{-1}(\tanh(a+bx))^4 \, dx}{4b}\\ &= \frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\text{Subst}\left(\int x^4 \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{4b^2}\\ &= \frac{x \tanh^{-1}(\tanh(a+bx))^4}{4b} - \frac{\tanh^{-1}(\tanh(a+bx))^5}{20b^2}\end{aligned}$$

Mathematica [B] time = 0.08, size = 99, normalized size = 2.91

$$\frac{(a+bx)\left(10\left(2a^2+abx-b^2x^2\right)\tanh^{-1}(\tanh(a+bx))^2+(4a-bx)(a+bx)^3-5(3a-bx)(a+bx)^2\tanh^{-1}(\tanh(a\right.}{20b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ((a + b*x)*((4*a - b*x)*(a + b*x)^3 - 5*(3*a - b*x)*(a + b*x)^2*ArcTanh[Tan
h[a + b*x]] + 10*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*x]]^2 - 10*(a
 - b*x)*ArcTanh[Tanh[a + b*x]]^3))/(20*b^2)
```

fricas [A] time = 0.50, size = 34, normalized size = 1.00

$$\frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

giac [A] time = 0.17, size = 34, normalized size = 1.00

$$\frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

maple [A] time = 0.15, size = 56, normalized size = 1.65

$$\frac{x^2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{2} - \frac{3b\left(\frac{x^3\operatorname{arctanh}(\tanh(bx+a))^2}{3} - \frac{2b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))}{4} - \frac{b\,x^5}{20}\right)}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^3-3/2*b*(1/3*x^3*arctanh(tanh(b*x+a))^2-2/3*b*
(1/4*x^4*arctanh(tanh(b*x+a))-1/20*b*x^5))
```

maxima [A] time = 0.52, size = 54, normalized size = 1.59

$$-\frac{1}{2}\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\frac{1}{2}\,x^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{1}{20}\,\left(b^2x^5-5\,bx^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*x^3*arctanh(tanh(b*x + a))^2 + 1/2*x^2*arctanh(tanh(b*x + a))^3 - 1/
20*(b^2*x^5 - 5*b*x^4*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 0.98, size = 53, normalized size = 1.56

$$-\frac{b^3\,x^5}{20}+\frac{b^2\,x^4\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}{4}-\frac{b\,x^3\,{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}^2}{2}+\frac{x^2\,{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (x^2*atanh(tanh(a + b*x))^3)/2 - (b^3*x^5)/20 - (b*x^3*atanh(tanh(a + b*x))
^2)/2 + (b^2*x^4*atanh(tanh(a + b*x)))/4
```

sympy [A] time = 1.20, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x \operatorname{atanh}^4(\tanh(a+bx))}{4b} - \frac{\operatorname{atanh}^5(\tanh(a+bx))}{20b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{atanh}^3(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**4/(4*b) - atanh(tanh(a + b*x))**5/(20*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**3/2, True))
```

3.58 $\int \tanh^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

```
[Out] 1/4*arctanh(tanh(b*x+a))^4/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*b)
```

fricas [B] time = 0.73, size = 31, normalized size = 1.94

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

giac [B] time = 0.17, size = 31, normalized size = 1.94

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)a^2 + \frac{1}{4}\left(bx^2 + 2\,ax\right)^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*a^2 + 1/4*(b*x^2 + 2*a*x)^2*b
```

maple [A] time = 0.03, size = 15, normalized size = 0.94

$$\frac{\operatorname{arctanh}\left(\tanh\left(bx + a\right)\right)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/4*arctanh(tanh(b*x+a))^4/b
```

maxima [B] time = 0.52, size = 51, normalized size = 3.19

$$-\frac{3}{2}\,bx^2\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)^2 + x\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)^3 - \frac{1}{4}\left(b^2x^4 - 4\,bx^3\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*x^2*arctanh(tanh(b*x + a))^2 + x*arctanh(tanh(b*x + a))^3 - 1/4*(b^2
*x^4 - 4*b*x^3*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 0.10, size = 47, normalized size = 2.94

$$\frac{x\,\left(2\operatorname{atanh}\left(\tanh\left(a + b\,x\right)\right) - b\,x\right)\,\left(b^2\,x^2 - 2\,b\,x\operatorname{atanh}\left(\tanh\left(a + b\,x\right)\right) + 2\operatorname{atanh}\left(\tanh\left(a + b\,x\right)\right)^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3,x)
```

```
[Out] (x*(2*atanh(tanh(a + b*x)) - b*x)*(2*atanh(tanh(a + b*x))^2 + b^2*x^2 - 2*b
*x*atanh(tanh(a + b*x))))/4
```

sympy [A] time = 0.61, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{4b} & \text{for } b \neq 0 \\ x\operatorname{atanh}^{3}\left(\tanh\left(a\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3,x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**4/(4*b), Ne(b, 0)), (x*atanh(tanh(a))**3,
True))
```

3.59 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} \, dx$

Optimal. Leaf size=77

$$bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{3}\tanh^{-1}(\tanh(a+bx$$

```
[Out] b*x*(b*x-arctanh(tanh(b*x+a)))^2-1/2*(b*x-arctanh(tanh(b*x+a)))*arctanh(tan
h(b*x+a))^2+1/3*arctanh(tanh(b*x+a))^3-(b*x-arctanh(tanh(b*x+a)))^3*ln(x)
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2159, 2158, 29}

$$bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{1}{3}\tanh^{-1}(\tanh(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x,x]
```

```
[Out] b*x*(b*x - ArcTanh[Tanh[a + b*x]])^2 - ((b*x - ArcTanh[Tanh[a + b*x]])*ArcT
anh[Tanh[a + b*x]]^2)/2 + ArcTanh[Tanh[a + b*x]]^3/3 - (b*x - ArcTanh[Tanh[
a + b*x]])^3*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} \, dx &= \frac{1}{3}\tanh^{-1}(\tanh(a+bx))^3-\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a+\\ &= -\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{3}\tanh^{-1}(\tanh(a+\\ &= bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan\\ &= bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan\end{aligned}$$

Mathematica [A] time = 0.10, size = 104, normalized size = 1.35

$$(a+bx)\left(a^2-3a\left(-\tanh^{-1}(\tanh(a+bx))+a+bx\right)+3\left(-\tanh^{-1}(\tanh(a+bx))+a+bx\right)^2\right)+\frac{1}{3}(a+bx)^3-\frac{1}{2}(a+$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x,x]
```

```
[Out] (a + b*x)^3/3 + (a + b*x)*(a^2 - 3*a*(a + b*x - ArcTanh[Tanh[a + b*x]]) + 3
*(a + b*x - ArcTanh[Tanh[a + b*x]])^2) - ((a + b*x)^2*(2*a + 3*b*x - 3*ArcT
anh[Tanh[a + b*x]]))/2 + (-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[b*x]
```

fricas [A] time = 0.59, size = 31, normalized size = 0.40

$$\frac{1}{3}b^3x^3 + \frac{3}{2}ab^2x^2 + 3a^2bx + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

giac [A] time = 0.17, size = 32, normalized size = 0.42

$$\frac{1}{3}b^3x^3 + \frac{3}{2}ab^2x^2 + 3a^2bx + a^3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

maple [A] time = 0.17, size = 92, normalized size = 1.19

$$\ln(x)\operatorname{arctanh}(\tanh(bx+a))^3+3\operatorname{arctanh}(\tanh(bx+a))\ln(x)x^2b^2+3b\operatorname{arctanh}(\tanh(bx+a))^2x+\frac{11x^3b^3}{6}-b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x,x)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^3+3*arctanh(tanh(b*x+a))*ln(x)*x^2*b^2+3*b*arcta
nh(tanh(b*x+a))^2*x+11/6*x^3*b^3-b^3*x^3*ln(x)-3*b*arctanh(tanh(b*x+a))^2*l
n(x)*x-9/2*arctanh(tanh(b*x+a))*x^2*b^2
```

maxima [A] time = 0.71, size = 31, normalized size = 0.40

$$\frac{1}{3}b^3x^3 + \frac{3}{2}ab^2x^2 + 3a^2bx + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

mupad [B] time = 0.14, size = 306, normalized size = 3.97

$$\frac{b^3x^3}{3}-\ln(x)\left(\frac{\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}{8}-a^3-\frac{3a\left(2a-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)+2\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x,x)
```

```
[Out] (b^3*x^3)/3 - log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/8 - a^3 - (3*a*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^2)/4 + (3*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/2)
- (3*b^2*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/4 + (3*b*x*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
2)/4
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x, x)
```

3.60 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2} dx$

Optimal. Leaf size=68

$$-3b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{x}+\frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2+3b\log(x)\left(bx-\tanh^{-1}(t\right.$$

```
[Out] -3*b^2*x*(b*x-arctanh(tanh(b*x+a)))+3/2*b*arctanh(tanh(b*x+a))^2-arctanh(ta
nh(b*x+a))^3/x+3*b*(b*x-arctanh(tanh(b*x+a)))^2*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2159, 2158, 29}

$$-3b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{x}+\frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2+3b\log(x)\left(bx-\tanh^{-1}(t\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -3*b^2*x*(b*x - ArcTanh[Tanh[a + b*x]]) + (3*b*ArcTanh[Tanh[a + b*x]]^2)/2
- ArcTanh[Tanh[a + b*x]]^3/x + 3*b*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^3}{x} + (3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x}\,dx$$
$$= \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} - \left(3b\left(bx - \tanh^{-1}(\tanh(\right.\right.$$
$$= -3b^2x\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tan}{}$$
$$= -3b^2x\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + \frac{3}{2}b\tanh^{-1}(\tanh(a+bx))^2 - \frac{\tanh^{-1}(\tan}{}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.91

$$-6b^2x\log(x)\tanh^{-1}(\tanh(a+bx)) - \frac{\tanh^{-1}(\tanh(a+bx))^3}{x} + 3b(\log(x)+1)\tanh^{-1}(\tanh(a+bx))^2 + \frac{3}{2}b^3x^2(2\log($$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^3/x) - 6*b^2*x*ArcTanh[Tanh[a + b*x]]*Log[x] + 3*b
*ArcTanh[Tanh[a + b*x]]^2*(1 + Log[x]) + (3*b^3*x^2*(-1 + 2*Log[x]))/2
```

fricas [A] time = 0.74, size = 36, normalized size = 0.53

$$\frac{b^3x^3 + 6\,ab^2x^2 + 6\,a^2bx\log(x) - 2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

giac [A] time = 0.19, size = 33, normalized size = 0.49

$$\frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log(|x|) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

maple [A] time = 0.18, size = 76, normalized size = 1.12

$$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{x} + 3\ln(x)\operatorname{arctanh}(\tanh(bx+a))^2 b + 3b^3x^2\ln(x) - \frac{9x^2b^3}{2} - 6b^2\operatorname{arctanh}(\tanh(bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^2,x)
```

```
[Out] -arctanh(tanh(b*x+a))^3/x+3*ln(x)*arctanh(tanh(b*x+a))^2*b+3*b^3*x^2*ln(x)-
9/2*x^2*b^3-6*b^2*arctanh(tanh(b*x+a))*ln(x)*x+6*b^2*arctanh(tanh(b*x+a))*x
```

maxima [A] time = 0.78, size = 65, normalized size = 0.96

$$3\,b\operatorname{artanh}(\tanh(bx+a))^2\log(x) + \frac{3}{2}\left(b^2x^2 + 4\,abx + 2\,a^2\log(x) - 2\,\operatorname{artanh}(\tanh(bx+a))^2\log(x)\right)b - \frac{\operatorname{artanl}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^2,x, algorithm="maxima")
```

```
[Out] 3*b*arctanh(tanh(b*x + a))^2*log(x) + 3/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x)
 - 2*arctanh(tanh(b*x + a))^2*log(x))*b - arctanh(tanh(b*x + a))^3/x
```

mupad [B] time = 1.07, size = 415, normalized size = 6.10

$$\frac{3\,b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4}-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{8\,x}+\frac{3\,b\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{4}+\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{8\,x}-\frac{3\,b^3\,x^2}{2}+\frac{3\,b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2\,\ln(x)}{4}+\frac{3\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^2,x)
```

```
[Out] (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/4 - log((exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))^3/(8*x) + (3*b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))^2)/4 + log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(8*x) - (3*b^3*x^
2)/2 + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log(x))/4 + (3*log(1/(exp(2*
a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)
/(8*x) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)))/(8*x) + (3*b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))^2*log(x))/4 - (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/2 + 3*b^3*x^2*log(x) - (3*b
*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))*log(x))/2 + 3*b^2*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x) - 3
*b^2*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{3}{\left(\tanh{\left(a + bx \right)} \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**2, x)
```

3.61 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3} dx$

Optimal. Leaf size=60

$$-3b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}-\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}+3b^3x$$

```
[Out] 3*b^3*x-3/2*b*arctanh(tanh(b*x+a))^2/x-1/2*arctanh(tanh(b*x+a))^3/x^2-3*b^2
*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2158, 29}

$$-3b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}-\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}+3b^3x$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] 3*b^3*x - (3*b*ArcTanh[Tanh[a + b*x]]^2)/(2*x) - ArcTanh[Tanh[a + b*x]]^3/(
2*x^2) - 3*b^2*(b*x - ArcTanh[Tanh[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3} dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}+\frac{1}{2}(3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^2} dx\\
&= -\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}+\left(3b^2\right)\int \frac{\tanh^{-1}(\tanh}{x}\\
&= 3b^3x-\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}-\left(3b^2\left(bx-\tanh\right.\right.\\
&= 3b^3x-\frac{3b\tanh^{-1}(\tanh(a+bx))^2}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{2x^2}-3b^2\left(bx-\tanh^{-}\right.
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 1.10

$$3b^2\log(x)\left(\tanh^{-1}(\tanh(a+bx))-bx\right)-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}{2x^2}-\frac{3b\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{x}+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^3,x]
```

```
[Out] b^3*x - (3*b*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)/x - (-(b*x) + ArcTanh[Tan
h[a + b*x]])^3/(2*x^2) + 3*b^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[x]
```

fricas [A] time = 0.46, size = 37, normalized size = 0.62

$$\frac{2\,b^3x^3+6\,ab^2x^2\log(x)-6\,a^2bx-a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

giac [A] time = 0.20, size = 31, normalized size = 0.52

$$b^3x+3\,ab^2\log\left(|x|\right)-\frac{6\,a^2bx+a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="giac")
```

```
[Out] b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

maple [A] time = 0.15, size = 59, normalized size = 0.98

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{2x^2}-\frac{3b\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{2x}-3\ln(x)x\,b^3+3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\ln(x)b^2+3b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^3,x)
```

```
[Out] -1/2*arctanh(tanh(b*x+a))^3/x^2-3/2*b*arctanh(tanh(b*x+a))^2/x-3*ln(x)*x*b^
3+3*arctanh(tanh(b*x+a))*ln(x)*b^2+3*b^3*x
```

maxima [A] time = 0.47, size = 72, normalized size = 1.20

$$3\left(b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\log(x)-\left(b\left(x+\frac{a}{b}\right)\log(x)-b\left(x+\frac{a\log(x)}{b}\right)\right)b\right)b-\frac{3\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{2\,x}-\frac{\operatorname{artan}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3*(b*arctanh(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(x)/
b))*b)*b - 3/2*b*arctanh(tanh(b*x + a))^2/x - 1/2*arctanh(tanh(b*x + a))^3/
x^2
```

mupad [B] time = 0.20, size = 365, normalized size = 6.08

$$\frac{9\,b^2\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4}-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{16\,x^2}-\frac{9\,b^2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{4}-\frac{3\,b^3\,x}{2}+\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{16\,x^2}-\frac{3\,b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2}{8\,x}-\frac{3\,b^2\,\ln\left(\frac{}{e}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^3,x)
```

```
[Out] (9*b^2*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)))/4 - log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))^3/(16*x^2) - (9*b^2*log(1/(exp(2*a)*exp(2*b
*x) + 1)))/4 - (3*b^3*x)/2 + log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(16*x^2) -
(3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/(8*x) - (3*b^2*log(1/(exp(2*a)*exp
(2*b*x) + 1))*log(x))/2 + (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(16*x^2) - (3*log(1/(exp(2*a)*ex
p(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(16*
x^2) - (3*b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(8*x) +
 (3*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x))/2 - 3*
b^3*x*log(x) + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)))/(4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**3, x)
```

3.62 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4}\,dx$

Optimal. Leaf size=55

$$-\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3}-\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2}+b^3\log(x)$$

```
[Out] -b^2*arctanh(tanh(b*x+a))/x-1/2*b*arctanh(tanh(b*x+a))^2/x^2-1/3*arctanh(ta
nh(b*x+a))^3/x^3+b^3*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2168, 29}

$$-\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] -((b^2*ArcTanh[Tanh[a + b*x]])/x) - (b*ArcTanh[Tanh[a + b*x]]^2)/(2*x^2) -
ArcTanh[Tanh[a + b*x]]^3/(3*x^3) + b^3*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3}+b\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^3}\,dx\\ &= -\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3}+b^2\int \frac{\tanh^{-1}(\tanh(a+bx}{x^2}\\ &= -\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3}\\ &= -\frac{b^2\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.09

$$\frac{-6b^2x^2\tanh^{-1}(\tanh(a+bx))-3bx\tanh^{-1}(\tanh(a+bx))^2-2\tanh^{-1}(\tanh(a+bx))^3+b^3x^3(6\log(x)+11)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^4,x]
```

```
[Out] (-6*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 3*b*x*ArcTanh[Tanh[a + b*x]]^2 - 2*Arc
Tanh[Tanh[a + b*x]]^3 + b^3*x^3*(11 + 6*Log[x]))/(6*x^3)
```

fricas [A] time = 0.45, size = 37, normalized size = 0.67

$$\frac{6\,b^3x^3\log(x) - 18\,ab^2x^2 - 9\,a^2bx - 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

giac [A] time = 0.16, size = 35, normalized size = 0.64

$$b^3\log(|x|) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

maple [A] time = 0.21, size = 52, normalized size = 0.95

$$-\frac{b^2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}{x} - \frac{b\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{2x^2} - \frac{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{3x^3} + b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^4,x)
```

```
[Out] -b^2*arctanh(tanh(b*x+a))/x-1/2*b*arctanh(tanh(b*x+a))^2/x^2-1/3*arctanh(ta
nh(b*x+a))^3/x^3+b^3*ln(x)
```

maxima [A] time = 0.52, size = 52, normalized size = 0.95

$$\left(b^2\log(x) - \frac{b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x}\right)b - \frac{b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{2\,x^2} - \frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*log(x) - b*arctanh(tanh(b*x + a))/x)*b - 1/2*b*arctanh(tanh(b*x + a))^
2/x^2 - 1/3*arctanh(tanh(b*x + a))^3/x^3
```

mupad [B] time = 1.00, size = 51, normalized size = 0.93

$$b^3\,\ln(x) - \frac{b^2\,x^2\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right) + \frac{b\,x\,\mathrm{atanh}(\tanh(a+b\,x))^2}{2} + \frac{\mathrm{atanh}(\tanh(a+b\,x))^3}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^4,x)
```

```
[Out] b^3*log(x) - (atanh(tanh(a + b*x))^3/3 + (b*x*atanh(tanh(a + b*x))^2)/2 + b
^2*x^2*atanh(tanh(a + b*x)))/x^3
```

<u>**sympy**</u> **[A]** time = 0.84, size = 51, normalized size = 0.93

$$b^3 \log(x) - \frac{b^2 \operatorname{atanh}(\tanh(a+bx))}{x} - \frac{b \operatorname{atanh}^2(\tanh(a+bx))}{2x^2} - \frac{\operatorname{atanh}^3(\tanh(a+bx))}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**4,x)
```

```
[Out] b**3*log(x) - b**2*atanh(tanh(a + b*x))/x - b*atanh(tanh(a + b*x))**2/(2*x*
*2) - atanh(tanh(a + b*x))**3/(3*x**3)
```

3.63 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/4*arctanh(tanh(b*x+a))^4/x^4/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^4/(4*x^4*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5}\,dx=\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.61

$$-\frac{b^2x^2\tanh^{-1}(\tanh(a+bx))+bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3+b^3x^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^5,x]
```

```
[Out] -1/4*(b^3*x^3 + b^2*x^2*ArcTanh[Tanh[a + b*x]] + b*x*ArcTanh[Tanh[a + b*x]]
^2 + ArcTanh[Tanh[a + b*x]]^3)/x^4
```

fricas [A] time = 0.92, size = 33, normalized size = 1.06

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

giac [A] time = 0.22, size = 33, normalized size = 1.06

$$-\frac{4\, b^3 x^3 + 6\, a b^2 x^2 + 4\, a^2 b x + a^3}{4\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

maple [A] time = 0.14, size = 56, normalized size = 1.81

$$-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{4x^4} + \frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{3x^3} + \frac{2b\left(-\frac{b}{2x} - \frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^2}\right)}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^5,x)
```

```
[Out] -1/4*arctanh(tanh(b*x+a))^3/x^4+3/4*b*(-1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*
b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))/x^2))
```

maxima [A] time = 0.53, size = 53, normalized size = 1.71

$$-\frac{1}{4}\, b\left(\frac{b^2}{x} + \frac{b \operatorname{artanh}(\tanh(bx+a))}{x^2}\right) - \frac{b \operatorname{artanh}(\tanh(bx+a))^2}{4\, x^3} - \frac{\operatorname{artanh}(\tanh(bx+a))^3}{4\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b*(b^2/x + b*arctanh(tanh(b*x + a))/x^2) - 1/4*b*arctanh(tanh(b*x + a)
)^2/x^3 - 1/4*arctanh(tanh(b*x + a))^3/x^4
```

mupad [B] time = 0.97, size = 48, normalized size = 1.55

$$-\frac{b^3\, x^3 + b^2\, x^2 \operatorname{atanh}(\tanh(a + b\, x)) + b\, x \operatorname{atanh}(\tanh(a + b\, x))^2 + \operatorname{atanh}(\tanh(a + b\, x))^3}{4\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^5,x)
```

```
[Out] -(atanh(tanh(a + b*x))^3 + b^3*x^3 + b*x*atanh(tanh(a + b*x))^2 + b^2*x^2*a
tanh(tanh(a + b*x)))/(4*x^4)
```

sympy [B] time = 1.31, size = 56, normalized size = 1.81

$$-\frac{b^3}{4x} - \frac{b^2 \operatorname{atanh}(\tanh(a+bx))}{4x^2} - \frac{b \operatorname{atanh}^2(\tanh(a+bx))}{4x^3} - \frac{\operatorname{atanh}^3(\tanh(a+bx))}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**5,x)
```

```
[Out] -b**3/(4*x) - b**2*atanh(tanh(a + b*x))/(4*x**2) - b*atanh(tanh(a + b*x))**
2/(4*x**3) - atanh(tanh(a + b*x))**3/(4*x**4)
```

3.64 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^6}\,dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 1/20*b*arctanh(tanh(b*x+a))^4/x^4/(b*x-arctanh(tanh(b*x+a)))^2+1/5*arctanh(
tanh(b*x+a))^4/x^5/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^4)/(20*x^4*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^4/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^6}\,dx &= \frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^5}\,dx}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\tanh^{-1}(\tanh(a+bx))^4}{20x^4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^4}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.84

$$-\frac{2b^2x^2\tanh^{-1}(\tanh(a+bx))+3bx\tanh^{-1}(\tanh(a+bx))^2+4\tanh^{-1}(\tanh(a+bx))^3+b^3x^3}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^6,x]
```

```
[Out] -1/20*(b^3*x^3 + 2*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 3*b*x*ArcTanh[Tanh[a +
b*x]]^2 + 4*ArcTanh[Tanh[a + b*x]]^3)/x^5
```

fricas [A] time = 0.37, size = 35, normalized size = 0.55

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

giac [A] time = 0.16, size = 35, normalized size = 0.55

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

maple [A] time = 0.15, size = 56, normalized size = 0.88

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{5x^5} + \frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{4x^4} + \frac{b\left(-\frac{b}{6x^2} - \frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^3}\right)}{2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^6,x)
```

```
[Out] -1/5*arctanh(tanh(b*x+a))^3/x^5+3/5*b*(-1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*
b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a))/x^3))
```

maxima [A] time = 0.53, size = 54, normalized size = 0.84

$$-\frac{1}{20}\,b\left(\frac{b^2}{x^2} + \frac{2\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^3}\right) - \frac{3\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{20\,x^4} - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*b*(b^2/x^2 + 2*b*arctanh(tanh(b*x + a))/x^3) - 3/20*b*arctanh(tanh(b*
x + a))^2/x^4 - 1/5*arctanh(tanh(b*x + a))^3/x^5
```

mupad [B] time = 0.99, size = 53, normalized size = 0.83

$$-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^3}{5\,x^5} - \frac{b^3}{20\,x^2} - \frac{b^2\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{10\,x^3} - \frac{3\,b\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^6,x)
```

```
[Out] - atanh(tanh(a + b*x))^3/(5*x^5) - b^3/(20*x^2) - (b^2*atanh(tanh(a + b*x))
)/(10*x^3) - (3*b*atanh(tanh(a + b*x))^2)/(20*x^4)
```

<u>**sympy**</u> **[A]** time = 2.07, size = 60, normalized size = 0.94

$$-\frac{b^3}{20x^2} - \frac{b^2 \operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{10x^3} - \frac{3b \operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{20x^4} - \frac{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**6,x)
```

```
[Out] -b**3/(20*x**2) - b**2*atanh(tanh(a + b*x))/(10*x**3) - 3*b*atanh(tanh(a +
b*x))**2/(20*x**4) - atanh(tanh(a + b*x))**3/(5*x**5)
```

3.65 $\int x^m \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=154

$$-\frac{24b^3x^{m+4}\tanh^{-1}(\tanh(a+bx))}{(m+1)\left(m^3+9m^2+26m+24\right)}+\frac{12b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))^2}{m^3+6m^2+11m+6}-\frac{4bx^{m+2}\tanh^{-1}(\tanh(a+bx))^3}{m^2+3m+2}+\frac{x^{m+1}\tanh}{}$$

```
[Out] 24*b^4*x^(5+m)/(1+m)/(2+m)/(3+m)/(m^2+9*m+20)-24*b^3*x^(4+m)*arctanh(tanh(b
*x+a))/(1+m)/(m^3+9*m^2+26*m+24)+12*b^2*x^(3+m)*arctanh(tanh(b*x+a))^2/(m^3
+6*m^2+11*m+6)-4*b*x^(2+m)*arctanh(tanh(b*x+a))^3/(m^2+3*m+2)+x^(1+m)*arcta
nh(tanh(b*x+a))^4/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2168, 30}

$$\frac{12b^2x^{m+3}\tanh^{-1}(\tanh(a+bx))^2}{m^3+6m^2+11m+6}-\frac{24b^3x^{m+4}\tanh^{-1}(\tanh(a+bx))}{(m+1)\left(m^3+9m^2+26m+24\right)}-\frac{4bx^{m+2}\tanh^{-1}(\tanh(a+bx))^3}{m^2+3m+2}+\frac{x^{m+1}\tanh^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (24*b^4*x^(5 + m))/((1 + m)*(2 + m)*(3 + m)*(20 + 9*m + m^2)) - (24*b^3*x^(
4 + m)*ArcTanh[Tanh[a + b*x]])/((1 + m)*(24 + 26*m + 9*m^2 + m^3)) + (12*b^
2*x^(3 + m)*ArcTanh[Tanh[a + b*x]]^2)/(6 + 11*m + 6*m^2 + m^3) - (4*b*x^(2
+ m)*ArcTanh[Tanh[a + b*x]]^3)/(2 + 3*m + m^2) + (x^(1 + m)*ArcTanh[Tanh[a
+ b*x]]^4)/(1 + m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^m \tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^4}{1+m}-\frac{(4b)\int x^{1+m}\tanh^{-1}(\tanh(a+bx))^3\,dx}{1+m}\\
&= -\frac{4bx^{2+m}\tanh^{-1}(\tanh(a+bx))^3}{2+3m+m^2}+\frac{x^{1+m}\tanh^{-1}(\tanh(a+bx))^4}{1+m}+\frac{\left(12b^2\right)\int x^2}{}\\
&= \frac{12b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))^2}{6+11m+6m^2+m^3}-\frac{4bx^{2+m}\tanh^{-1}(\tanh(a+bx))^3}{2+3m+m^2}+\frac{x^{1+m}\tan}{}\\
&= -\frac{24b^3x^{4+m}\tanh^{-1}(\tanh(a+bx))}{(4+m)\left(6+11m+6m^2+m^3\right)}+\frac{12b^2x^{3+m}\tanh^{-1}(\tanh(a+bx))^2}{6+11m+6m^2+m^3}-\frac{4bx^{2+}}{}\\
&= \frac{24b^4x^{5+m}}{(4+m)(5+m)\left(6+11m+6m^2+m^3\right)}-\frac{24b^3x^{4+m}\tanh^{-1}(\tanh(a+bx))}{(4+m)\left(6+11m+6m^2+m^3\right)}+\frac{1}{}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 137, normalized size = 0.89

$$\frac{x^{m+1}\left(-24b^3(m+5)x^3\tanh^{-1}(\tanh(a+bx))+12b^2\left(m^2+9m+20\right)x^2\tanh^{-1}(\tanh(a+bx))^2-4b\left(m^3+12m\right.\right.}{(m+1)(m+2)(m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^(1 + m)*(24*b^4*x^4 - 24*b^3*(5 + m)*x^3*ArcTanh[Tanh[a + b*x]] + 12*b^2
*(20 + 9*m + m^2)*x^2*ArcTanh[Tanh[a + b*x]]^2 - 4*b*(60 + 47*m + 12*m^2 +
m^3)*x*ArcTanh[Tanh[a + b*x]]^3 + (120 + 154*m + 71*m^2 + 14*m^3 + m^4)*Arc
Tanh[Tanh[a + b*x]]^4))/((1 + m)*(2 + m)*(3 + m)*(4 + m)*(5 + m))
```

fricas [B] time = 0.64, size = 483, normalized size = 3.14

$$\frac{\left(\left(b^4m^4+10\,b^4m^3+35\,b^4m^2+50\,b^4m+24\,b^4\right)x^5+4\left(ab^3m^4+11\,ab^3m^3+41\,ab^3m^2+61\,ab^3m+30\,ab^3\right)x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] (((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b^3*m^
4 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2*b^2*m
^4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^3 + 4*(
a^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x^2 + (a^
4*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*cosh(m*log(x)) +
((b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)*x^5 + 4*(a*b^3*m^4
 + 11*a*b^3*m^3 + 41*a*b^3*m^2 + 61*a*b^3*m + 30*a*b^3)*x^4 + 6*(a^2*b^2*m^
4 + 12*a^2*b^2*m^3 + 49*a^2*b^2*m^2 + 78*a^2*b^2*m + 40*a^2*b^2)*x^3 + 4*(a
^3*b*m^4 + 13*a^3*b*m^3 + 59*a^3*b*m^2 + 107*a^3*b*m + 60*a^3*b)*x^2 + (a^4
*m^4 + 14*a^4*m^3 + 71*a^4*m^2 + 154*a^4*m + 120*a^4)*x)*sinh(m*log(x)))/(m
^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(tanh(b*x + a))^4, x)
```

maple [A] time = 0.15, size = 278, normalized size = 1.81

$$\frac{b^4x^5e^{m\ln(x)}}{5+m}+\frac{\left(a^4+4a^3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)+6a^2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)^2+4a\left(\operatorname{arct}\right.\right.}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] b^4/(5+m)*x^5*exp(m*ln(x))+(a^4+4*a^3*(arctanh(tanh(b*x+a))-b*x-a)+6*a^2*(a
rctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(b*x+a))-b*x-a)^3+(arctanh(ta
nh(b*x+a))-b*x-a)^4)/(1+m)*x*exp(m*ln(x))+4*b*(a^3+3*a^2*(arctanh(tanh(b*x+
a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))-b*x-a)^
3)/(2+m)*x^2*exp(m*ln(x))+6*b^2*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arct
anh(tanh(b*x+a))-b*x-a)^2)/(3+m)*x^3*exp(m*ln(x))+4*b^3*(arctanh(tanh(b*x+a
))-b*x)/(4+m)*x^4*exp(m*ln(x))
```

maxima [A] time = 0.54, size = 145, normalized size = 0.94

$$-\frac{4\,bx^2x^m\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{(m+2)(m+1)}+\frac{x^{m+1}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^4}{m+1}+\frac{12\left(\frac{bx^3x^m\,\mathrm{artanh}(\tanh(bx+a))^2}{(m+3)(m+2)}+\frac{2\left(\frac{b^2x^5x^m}{(m+5)(m+4)(m+3)}-\frac{bx^4}{}\right.}{m-}\right.}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -4*b*x^2*x^m*arctanh(tanh(b*x + a))^3/((m + 2)*(m + 1)) + x^(m + 1)*arctanh
(tanh(b*x + a))^4/(m + 1) + 12*(b*x^3*x^m*arctanh(tanh(b*x + a))^2/((m + 3)
*(m + 2)) + 2*(b^2*x^5*x^m/((m + 5)*(m + 4)*(m + 3)) - b*x^4*x^m*arctanh(ta
nh(b*x + a))/((m + 4)*(m + 3)))*b/(m + 2))*b/(m + 1)
```

mupad [B] time = 1.34, size = 479, normalized size = 3.11

$$\frac{x\,x^m\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^4\left(m^4+14\,m^3+71\,m^2+154\,m+120\right)}{16\,m^5+240\,m^4+1360\,m^3+3600\,m^2+4384\,m+1920}+\frac{16\,b^4\,x^m\,x^5\,\left(m^4+10\,n\right.}{16\,m^5+240\,m^4+1360\,m^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x*x^m*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4*(154*m + 71*m^2 + 14*m^3 + m^4 + 120))/(4
384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 1920) + (16*b^4*x^m*x^5*(5
0*m + 35*m^2 + 10*m^3 + m^4 + 24))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4
+ 16*m^5 + 1920) + (24*b^2*x^m*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2*(78*m + 49*m^
2 + 12*m^3 + m^4 + 40))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 +
1920) - (32*b^3*x^m*x^4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(61*m + 41*m^2 + 11*m^3 +
m^4 + 30))/(4384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 1920) - (8*b*
x^m*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3*(107*m + 59*m^2 + 13*m^3 + m^4 + 60))/(4
384*m + 3600*m^2 + 1360*m^3 + 240*m^4 + 16*m^5 + 1920)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((b**4*log(x) - b**3*atanh(tanh(a + b*x))/x - b**2*atanh(tanh(a +
b*x))**2/(2*x**2) - b*atanh(tanh(a + b*x))**3/(3*x**3) - atanh(tanh(a + b*x
))**4/(4*x**4), Eq(m, -5)), (Integral(atanh(tanh(a + b*x))**4/x**4, x), Eq(
m, -4)), (Integral(atanh(tanh(a + b*x))**4/x**3, x), Eq(m, -3)), (Integral(
atanh(tanh(a + b*x))**4/x**2, x), Eq(m, -2)), (Integral(atanh(tanh(a + b*x)
)**4/x, x), Eq(m, -1)), (24*b**4*x**5*x**m/(m**5 + 15*m**4 + 85*m**3 + 225*
m**2 + 274*m + 120) - 24*b**3*m*x**4*x**m*atanh(tanh(a + b*x))/(m**5 + 15*m
**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 120*b**3*x**4*x**m*atanh(tanh(a +
 b*x))/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 12*b**2*m**2*x
**3*x**m*atanh(tanh(a + b*x))**2/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274
*m + 120) + 108*b**2*m*x**3*x**m*atanh(tanh(a + b*x))**2/(m**5 + 15*m**4 +
85*m**3 + 225*m**2 + 274*m + 120) + 240*b**2*x**3*x**m*atanh(tanh(a + b*x))
**2/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 4*b*m**3*x**2*x**
```

```
m*atanh(tanh(a + b*x))**3/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 12
0) - 48*b*m**2*x**2*x**m*atanh(tanh(a + b*x))**3/(m**5 + 15*m**4 + 85*m**3
+ 225*m**2 + 274*m + 120) - 188*b*m*x**2*x**m*atanh(tanh(a + b*x))**3/(m**5
 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) - 240*b*x**2*x**m*atanh(tanh
(a + b*x))**3/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + m**4*x*
x**m*atanh(tanh(a + b*x))**4/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m +
 120) + 14*m**3*x*x**m*atanh(tanh(a + b*x))**4/(m**5 + 15*m**4 + 85*m**3 +
225*m**2 + 274*m + 120) + 71*m**2*x*x**m*atanh(tanh(a + b*x))**4/(m**5 + 15
*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 154*m*x*x**m*atanh(tanh(a + b*x
))**4/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120) + 120*x*x**m*atan
h(tanh(a + b*x))**4/(m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120), Tr
ue))
```

3.66 $\int x^6 \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=80

$$-\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx))+\frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{7}x^7\tanh^{-1}(\tanh($$

```
[Out] 1/2310*b^4*x^11-1/210*b^3*x^10*arctanh(tanh(b*x+a))+1/42*b^2*x^9*arctanh(ta
nh(b*x+a))^2-1/14*b*x^8*arctanh(tanh(b*x+a))^3+1/7*x^7*arctanh(tanh(b*x+a))
^4
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2168, 30}

$$-\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx))+\frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{7}x^7\tanh^{-1}(\tanh($$

Antiderivative was successfully verified.

```
[In] Int[x^6*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (b^4*x^11)/2310 - (b^3*x^10*ArcTanh[Tanh[a + b*x]])/210 + (b^2*x^9*ArcTanh[
Tanh[a + b*x]]^2)/42 - (b*x^8*ArcTanh[Tanh[a + b*x]]^3)/14 + (x^7*ArcTanh[T
anh[a + b*x]]^4)/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^6\tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{1}{7}x^7\tanh^{-1}(\tanh(a+bx))^4-\frac{1}{7}(4b)\int x^7\tanh^{-1}(\tanh(a+bx))^3\,dx\\ &= -\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{7}x^7\tanh^{-1}(\tanh(a+bx))^4+\frac{1}{14}\left(3b^2\right)\int x^8\\ &= \frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}bx^8\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{7}x^7\tanh^{-1}(t\\ &= -\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx))+\frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}bx^8\tan\\ &= \frac{b^4x^{11}}{2310}-\frac{1}{210}b^3x^{10}\tanh^{-1}(\tanh(a+bx))+\frac{1}{42}b^2x^9\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{14}\end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.89

$$\frac{x^7\left(-11b^3x^3\tanh^{-1}(\tanh(a+bx))+55b^2x^2\tanh^{-1}(\tanh(a+bx))^2-165bx\tanh^{-1}(\tanh(a+bx))^3+330\tanh^{-1}\right.}{2310}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^7*(b^4*x^4 - 11*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 55*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 165*b*x*ArcTanh[Tanh[a + b*x]]^3 + 330*ArcTanh[Tanh[a + b*x]
]^4))/2310
```

fricas [A] time = 0.45, size = 46, normalized size = 0.58

$$\frac{1}{11}\,b^4x^{11}+\frac{2}{5}\,ab^3x^{10}+\frac{2}{3}\,a^2b^2x^9+\frac{1}{2}\,a^3bx^8+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/11*b^4*x^11 + 2/5*a*b^3*x^10 + 2/3*a^2*b^2*x^9 + 1/2*a^3*b*x^8 + 1/7*a^4*
x^7
```

giac [A] time = 0.47, size = 46, normalized size = 0.58

$$\frac{1}{11}\,b^4x^{11}+\frac{2}{5}\,ab^3x^{10}+\frac{2}{3}\,a^2b^2x^9+\frac{1}{2}\,a^3bx^8+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11 + 2/5*a*b^3*x^10 + 2/3*a^2*b^2*x^9 + 1/2*a^3*b*x^8 + 1/7*a^4*
x^7
```

maple [A] time = 0.15, size = 74, normalized size = 0.92

$$\frac{x^7\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{7}-\frac{4b\left(\frac{x^8\operatorname{arctanh}(\tanh(bx+a))^3}{8}-\frac{3b\left(\frac{x^9\operatorname{arctanh}(\tanh(bx+a))^2}{9}-\frac{2b\left(\frac{x^{10}\operatorname{arctanh}(\tanh(bx+a))}{10}-\frac{x^{11}b}{110}\right)}{9}\right)}{8}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/7*x^7*arctanh(tanh(b*x+a))^4-4/7*b*(1/8*x^8*arctanh(tanh(b*x+a))^3-3/8*b*
(1/9*x^9*arctanh(tanh(b*x+a))^2-2/9*b*(1/10*x^10*arctanh(tanh(b*x+a))-1/110
*x^11*b)))
```

maxima [A] time = 0.59, size = 72, normalized size = 0.90

$$-\frac{1}{14}\,bx^8\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{7}\,x^7\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{2310}\,\left(55\,bx^9\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/14*b*x^8*arctanh(tanh(b*x + a))^3 + 1/7*x^7*arctanh(tanh(b*x + a))^4 + 1
/2310*(55*b*x^9*arctanh(tanh(b*x + a))^2 + (b^2*x^11 - 11*b*x^10*arctanh(ta
nh(b*x + a)))*b)*b
```

mupad [B] time = 1.03, size = 242, normalized size = 3.02

$$\frac{x^7\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}{112}+\frac{b^4\,x^{11}}{11}-\frac{b\,x^8\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{16}-\frac{b^3\,x^{10}\left(\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^7*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/112 + (b^4*x^11)/11 - (b*x^8*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^3)/16 - (b^3*x^10*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/5 + (b^2*x^9*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^2)/6
```

sympy [A] time = 29.34, size = 134, normalized size = 1.68

$$\begin{cases} \frac{x^6 \operatorname{atanh}^5(\tanh(a+bx))}{5b} - \frac{x^5 \operatorname{atanh}^6(\tanh(a+bx))}{5b^2} + \frac{x^4 \operatorname{atanh}^7(\tanh(a+bx))}{7b^3} - \frac{x^3 \operatorname{atanh}^8(\tanh(a+bx))}{14b^4} + \frac{x^2 \operatorname{atanh}^9(\tanh(a+bx))}{42b^5} - \frac{x \operatorname{atanh} \\ \frac{x^7 \operatorname{atanh}^4(\tanh(a))}{7} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((x**6*atanh(tanh(a + b*x))**5/(5*b) - x**5*atanh(tanh(a + b*x))**
6/(5*b**2) + x**4*atanh(tanh(a + b*x))**7/(7*b**3) - x**3*atanh(tanh(a + b*
x))**8/(14*b**4) + x**2*atanh(tanh(a + b*x))**9/(42*b**5) - x*atanh(tanh(a
+ b*x))**10/(210*b**6) + atanh(tanh(a + b*x))**11/(2310*b**7), Ne(b, 0)), (
x**7*atanh(tanh(a))**4/7, True))
```

3.67 $\int x^5 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=80

$$-\frac{1}{126}b^3x^9\tanh^{-1}(\tanh(a+bx))+\frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{21}bx^7\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{6}x^6\tanh^{-1}(\tan$$

```
[Out] 1/1260*b^4*x^10-1/126*b^3*x^9*arctanh(tanh(b*x+a))+1/28*b^2*x^8*arctanh(tan
h(b*x+a))^2-2/21*b*x^7*arctanh(tanh(b*x+a))^3+1/6*x^6*arctanh(tanh(b*x+a))^
4
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2168, 30}

$$-\frac{1}{126}b^3x^9\tanh^{-1}(\tanh(a+bx))+\frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{21}bx^7\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{6}x^6\tanh^{-1}(\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^5*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (b^4*x^10)/1260 - (b^3*x^9*ArcTanh[Tanh[a + b*x]])/126 + (b^2*x^8*ArcTanh[T
anh[a + b*x]]^2)/28 - (2*b*x^7*ArcTanh[Tanh[a + b*x]]^3)/21 + (x^6*ArcTanh[
Tanh[a + b*x]]^4)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^5\tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{1}{6}x^6\tanh^{-1}(\tanh(a+bx))^4-\frac{1}{3}(2b)\int x^6\tanh^{-1}(\tanh(a+bx))^3\,dx\\
&= -\frac{2}{21}bx^7\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{6}x^6\tanh^{-1}(\tanh(a+bx))^4+\frac{1}{7}\left(2b^2\right)\int x\\
&= \frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{21}bx^7\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{6}x^6\tanh\\
&= -\frac{1}{126}b^3x^9\tanh^{-1}(\tanh(a+bx))+\frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{21}bx^7\tanh\\
&= \frac{b^4x^{10}}{1260}-\frac{1}{126}b^3x^9\tanh^{-1}(\tanh(a+bx))+\frac{1}{28}b^2x^8\tanh^{-1}(\tanh(a+bx))^2-
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.89

$$\frac{x^6\left(-10b^3x^3\tanh^{-1}(\tanh(a+bx))+45b^2x^2\tanh^{-1}(\tanh(a+bx))^2-120bx\tanh^{-1}(\tanh(a+bx))^3+210\tanh\right.}{1260}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^6*(b^4*x^4 - 10*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 45*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 - 120*b*x*ArcTanh[Tanh[a + b*x]]^3 + 210*ArcTanh[Tanh[a + b*x]
]^4))/1260
```

fricas [A] time = 0.52, size = 46, normalized size = 0.58

$$\frac{1}{10}\,b^4x^{10}+\frac{4}{9}\,ab^3x^9+\frac{3}{4}\,a^2b^2x^8+\frac{4}{7}\,a^3bx^7+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/10*b^4*x^10 + 4/9*a*b^3*x^9 + 3/4*a^2*b^2*x^8 + 4/7*a^3*b*x^7 + 1/6*a^4*x
^6
```

giac [A] time = 0.20, size = 46, normalized size = 0.58

$$\frac{1}{10}\,b^4x^{10}+\frac{4}{9}\,ab^3x^9+\frac{3}{4}\,a^2b^2x^8+\frac{4}{7}\,a^3bx^7+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/10*b^4*x^10 + 4/9*a*b^3*x^9 + 3/4*a^2*b^2*x^8 + 4/7*a^3*b*x^7 + 1/6*a^4*x
^6
```

maple [A] time = 0.15, size = 74, normalized size = 0.92

$$\frac{x^6\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{6}-\frac{2b\left(\frac{x^7\operatorname{arctanh}(\tanh(bx+a))^3}{7}-\frac{3b\left(\frac{x^8\operatorname{arctanh}(\tanh(bx+a))^2}{8}-\frac{b\left(\frac{x^9\operatorname{arctanh}(\tanh(bx+a))}{9}-\frac{x^{10}b}{90}\right)}{4}\right)}{7}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/6*x^6*arctanh(tanh(b*x+a))^4-2/3*b*(1/7*x^7*arctanh(tanh(b*x+a))^3-3/7*b*
(1/8*x^8*arctanh(tanh(b*x+a))^2-1/4*b*(1/9*x^9*arctanh(tanh(b*x+a))-1/90*x^
10*b)))
```

maxima [A] time = 0.58, size = 72, normalized size = 0.90

$$-\frac{2}{21}\,bx^7\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{6}\,x^6\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{1260}\left(45\,bx^8\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^{10}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/21*b*x^7*arctanh(tanh(b*x + a))^3 + 1/6*x^6*arctanh(tanh(b*x + a))^4 + 1
/1260*(45*b*x^8*arctanh(tanh(b*x + a))^2 + (b^2*x^10 - 10*b*x^9*arctanh(tan
h(b*x + a)))*b)*b
```

mupad [B] time = 0.15, size = 242, normalized size = 3.02

$$\frac{x^6\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}{96}+\frac{b^4\,x^{10}}{10}-\frac{b\,x^7\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{14}-\frac{2\,b^3\,x^9\,\left(\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^6*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/96 + (b^4*x^10)/10 - (b*x^7*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^3)/14 - (2*b^3*x^9*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/9 + (3*b^2*x^8*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^2)/16
```

sympy [A] time = 18.91, size = 117, normalized size = 1.46

$$\begin{cases} \frac{x^5 \operatorname{atanh}^5(\tanh(a+bx))}{5b} - \frac{x^4 \operatorname{atanh}^6(\tanh(a+bx))}{6b^2} + \frac{2x^3 \operatorname{atanh}^7(\tanh(a+bx))}{21b^3} - \frac{x^2 \operatorname{atanh}^8(\tanh(a+bx))}{28b^4} + \frac{x \operatorname{atanh}^9(\tanh(a+bx))}{126b^5} - \operatorname{ata} \\ \frac{x^6 \operatorname{atanh}^4(\tanh(a))}{6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((x**5*atanh(tanh(a + b*x))**5/(5*b) - x**4*atanh(tanh(a + b*x))**
6/(6*b**2) + 2*x**3*atanh(tanh(a + b*x))**7/(21*b**3) - x**2*atanh(tanh(a +
 b*x))**8/(28*b**4) + x*atanh(tanh(a + b*x))**9/(126*b**5) - atanh(tanh(a +
 b*x))**10/(1260*b**6), Ne(b, 0)), (x**6*atanh(tanh(a))**4/6, True))
```

3.68 $\int x^4 \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=80

$$-\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx))+\frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}x^5\tanh^{-1}(\tanh(a+$$

```
[Out] 1/630*b^4*x^9-1/70*b^3*x^8*arctanh(tanh(b*x+a))+2/35*b^2*x^7*arctanh(tanh(b
*x+a))^2-2/15*b*x^6*arctanh(tanh(b*x+a))^3+1/5*x^5*arctanh(tanh(b*x+a))^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2168, 30}

$$-\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx))+\frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}x^5\tanh^{-1}(\tanh(a+$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (b^4*x^9)/630 - (b^3*x^8*ArcTanh[Tanh[a + b*x]])/70 + (2*b^2*x^7*ArcTanh[Ta
nh[a + b*x]]^2)/35 - (2*b*x^6*ArcTanh[Tanh[a + b*x]]^3)/15 + (x^5*ArcTanh[T
anh[a + b*x]]^4)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^4\tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{1}{5}x^5\tanh^{-1}(\tanh(a+bx))^4-\frac{1}{5}(4b)\int x^5\tanh^{-1}(\tanh(a+bx))^3\,dx\\
&= -\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}x^5\tanh^{-1}(\tanh(a+bx))^4+\frac{1}{5}\left(2b^2\right)\int x^6\,\mathrm{t}\\
&= \frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx^6\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}x^5\tanh^{-1}(\mathrm{t}\\
&= -\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx))+\frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx^6\,\mathrm{tanh}\\
&= \frac{b^4x^9}{630}-\frac{1}{70}b^3x^8\tanh^{-1}(\tanh(a+bx))+\frac{2}{35}b^2x^7\tanh^{-1}(\tanh(a+bx))^2-\frac{2}{15}bx
\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.89

$$\frac{1}{630}x^5\left(-9b^3x^3\tanh^{-1}(\tanh(a+bx))+36b^2x^2\tanh^{-1}(\tanh(a+bx))^2-84bx\tanh^{-1}(\tanh(a+bx))^3+126\,\mathrm{tanh}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^5*(b^4*x^4 - 9*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 36*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 84*b*x*ArcTanh[Tanh[a + b*x]]^3 + 126*ArcTanh[Tanh[a + b*x]]^
4))/630
```

fricas [A] time = 0.52, size = 46, normalized size = 0.58

$$\frac{1}{9}\,b^4x^9+\frac{1}{2}\,ab^3x^8+\frac{6}{7}\,a^2b^2x^7+\frac{2}{3}\,a^3bx^6+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/9*b^4*x^9 + 1/2*a*b^3*x^8 + 6/7*a^2*b^2*x^7 + 2/3*a^3*b*x^6 + 1/5*a^4*x^5
```

giac [A] time = 0.22, size = 46, normalized size = 0.58

$$\frac{1}{9}\,b^4x^9+\frac{1}{2}\,ab^3x^8+\frac{6}{7}\,a^2b^2x^7+\frac{2}{3}\,a^3bx^6+\frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/9*b^4*x^9 + 1/2*a*b^3*x^8 + 6/7*a^2*b^2*x^7 + 2/3*a^3*b*x^6 + 1/5*a^4*x^5
```

maple [A] time = 0.15, size = 74, normalized size = 0.92

$$\frac{x^5\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{5}-\frac{4b\left(\frac{x^6\operatorname{arctanh}(\tanh(bx+a))^3}{6}-\frac{b\left(\frac{x^7\operatorname{arctanh}(\tanh(bx+a))^2}{7}-\frac{2b\left(\frac{x^8\operatorname{arctanh}(\tanh(bx+a))}{8}-\frac{x^9b}{72}\right)}{7}\right)}{2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/5*x^5*arctanh(tanh(b*x+a))^4-4/5*b*(1/6*x^6*arctanh(tanh(b*x+a))^3-1/2*b*
(1/7*x^7*arctanh(tanh(b*x+a))^2-2/7*b*(1/8*x^8*arctanh(tanh(b*x+a))-1/72*x^
9*b)))
```

maxima [A] time = 0.57, size = 72, normalized size = 0.90

$$-\frac{2}{15}\,bx^6\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{5}\,x^5\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{630}\,\left(36\,bx^7\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^9-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/15*b*x^6*arctanh(tanh(b*x + a))^3 + 1/5*x^5*arctanh(tanh(b*x + a))^4 + 1
/630*(36*b*x^7*arctanh(tanh(b*x + a))^2 + (b^2*x^9 - 9*b*x^8*arctanh(tanh(b
*x + a)))*b)*b
```

mupad [B] time = 1.06, size = 77, normalized size = 0.96

$$\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^5\,\left(126\,b^4\,x^4-84\,b^3\,x^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)+36\,b^2\,x^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2-9\,b\,x\,\mathrm{a}\right.}{630\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (atanh(tanh(a + b*x))^5*(atanh(tanh(a + b*x))^4 + 126*b^4*x^4 + 36*b^2*x^2*
atanh(tanh(a + b*x))^2 - 9*b*x*atanh(tanh(a + b*x))^3 - 84*b^3*x^3*atanh(ta
nh(a + b*x))))/(630*b^5)
```

sympy [A] time = 11.72, size = 100, normalized size = 1.25

$$\begin{cases} \frac{x^4 \operatorname{atanh}^5(\tanh(a+bx))}{5b} - \frac{2x^3 \operatorname{atanh}^6(\tanh(a+bx))}{15b^2} + \frac{2x^2 \operatorname{atanh}^7(\tanh(a+bx))}{35b^3} - \frac{x \operatorname{atanh}^8(\tanh(a+bx))}{70b^4} + \frac{\operatorname{atanh}^9(\tanh(a+bx))}{630b^5} & \text{for } b \neq \\ \frac{x^5 \operatorname{atanh}^4(\tanh(a))}{5} & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((x**4*atanh(tanh(a + b*x))**5/(5*b) - 2*x**3*atanh(tanh(a + b*x))
**6/(15*b**2) + 2*x**2*atanh(tanh(a + b*x))**7/(35*b**3) - x*atanh(tanh(a +
 b*x))**8/(70*b**4) + atanh(tanh(a + b*x))**9/(630*b**5), Ne(b, 0)), (x**5*
atanh(tanh(a))**4/5, True))
```

3.69 $\int x^3 \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=72

$$-\frac{\tanh^{-1}(\tanh(a+bx))^8}{280b^4}+\frac{x\tanh^{-1}(\tanh(a+bx))^7}{35b^3}-\frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2}+\frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

```
[Out] 1/5*x^3*arctanh(tanh(b*x+a))^5/b-1/10*x^2*arctanh(tanh(b*x+a))^6/b^2+1/35*x
*arctanh(tanh(b*x+a))^7/b^3-1/280*arctanh(tanh(b*x+a))^8/b^4
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{2168, 2157, 30\}$

$$-\frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{280b^4}+\frac{x\tanh^{-1}(\tanh(a+bx))^7}{35b^3}+\frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^3*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (x^2*ArcTanh[Tanh[a + b*x]]^6)/(10*b
^2) + (x*ArcTanh[Tanh[a + b*x]]^7)/(35*b^3) - ArcTanh[Tanh[a + b*x]]^8/(280
*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^3\tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{3\int x^2\tanh^{-1}(\tanh(a+bx))^5\,dx}{5b}\\
&= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2}+\frac{\int x\tanh^{-1}(\tanh(}{5b^2}\\
&= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2}+\frac{x\tanh^{-1}(\tanh(a+}{35b^3}\\
&= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2}+\frac{x\tanh^{-1}(\tanh(a+}{35b^3}\\
&= \frac{x^3\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{x^2\tanh^{-1}(\tanh(a+bx))^6}{10b^2}+\frac{x\tanh^{-1}(\tanh(a+}{35b^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.99

$$\frac{1}{280}x^4\left(-8b^3x^3\tanh^{-1}(\tanh(a+bx))+28b^2x^2\tanh^{-1}(\tanh(a+bx))^2-56bx\tanh^{-1}(\tanh(a+bx))^3+70\tanh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^4*(b^4*x^4 - 8*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 28*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 56*b*x*ArcTanh[Tanh[a + b*x]]^3 + 70*ArcTanh[Tanh[a + b*x]]^4
))/280
```

fricas [A] time = 0.62, size = 45, normalized size = 0.62

$$\frac{1}{8}b^4x^8+\frac{4}{7}ab^3x^7+a^2b^2x^6+\frac{4}{5}a^3bx^5+\frac{1}{4}a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*x^8 + 4/7*a*b^3*x^7 + a^2*b^2*x^6 + 4/5*a^3*b*x^5 + 1/4*a^4*x^4
```

giac [A] time = 0.20, size = 45, normalized size = 0.62

$$\frac{1}{8}b^4x^8+\frac{4}{7}ab^3x^7+a^2b^2x^6+\frac{4}{5}a^3bx^5+\frac{1}{4}a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8 + 4/7*a*b^3*x^7 + a^2*b^2*x^6 + 4/5*a^3*b*x^5 + 1/4*a^4*x^4
```

maple [A] time = 0.15, size = 74, normalized size = 1.03

$$\frac{x^4\operatorname{arctanh}(\tanh(bx+a))^4}{4}-b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a))^3}{5}-\frac{3b\left(\frac{x^6\operatorname{arctanh}(\tanh(bx+a))^2}{6}-\frac{b\left(\frac{x^7\operatorname{arctanh}(\tanh(bx+a))}{7}-\frac{x^8b}{56}\right)}{3}\right)}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/4*x^4*arctanh(tanh(b*x+a))^4-b*(1/5*x^5*arctanh(tanh(b*x+a))^3-3/5*b*(1/6
*x^6*arctanh(tanh(b*x+a))^2-1/3*b*(1/7*x^7*arctanh(tanh(b*x+a))-1/56*x^8*b)
))
```

maxima [A] time = 0.58, size = 72, normalized size = 1.00

$$-\frac{1}{5}bx^5\operatorname{artanh}(\tanh(bx+a))^3+\frac{1}{4}x^4\operatorname{artanh}(\tanh(bx+a))^4+\frac{1}{280}\left(28\,bx^6\operatorname{artanh}(\tanh(bx+a))^2+\left(b^2x^8-8\,b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/5*b*x^5*arctanh(tanh(b*x + a))^3 + 1/4*x^4*arctanh(tanh(b*x + a))^4 + 1/
280*(28*b*x^6*arctanh(tanh(b*x + a))^2 + (b^2*x^8 - 8*b*x^7*arctanh(tanh(b*
x + a)))*b)*b
```

mupad [B] time = 1.02, size = 70, normalized size = 0.97

$$\frac{b^4 x^8}{280}-\frac{b^3 x^7 \operatorname{atanh}(\tanh(a+b x))}{35}+\frac{b^2 x^6 \operatorname{atanh}(\tanh(a+b x))^2}{10}-\frac{b x^5 \operatorname{atanh}(\tanh(a+b x))^3}{5}+\frac{x^4 \operatorname{atanh}(\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^4*atanh(tanh(a + b*x))^4)/4 + (b^4*x^8)/280 + (b^2*x^6*atanh(tanh(a + b*
x))^2)/10 - (b*x^5*atanh(tanh(a + b*x))^3)/5 - (b^3*x^7*atanh(tanh(a + b*x)
))/35
```

sympy [A] time = 6.72, size = 78, normalized size = 1.08

$$\begin{cases} \frac{x^3 \operatorname{atanh}^5(\tanh(a+bx))}{5b} - \frac{x^2 \operatorname{atanh}^6(\tanh(a+bx))}{10b^2} + \frac{x \operatorname{atanh}^7(\tanh(a+bx))}{35b^3} - \frac{\operatorname{atanh}^8(\tanh(a+bx))}{280b^4} & \text{for } b \neq 0 \\ \frac{x^4 \operatorname{atanh}^4(\tanh(a))}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((x**3*atanh(tanh(a + b*x))**5/(5*b) - x**2*atanh(tanh(a + b*x))**
6/(10*b**2) + x*atanh(tanh(a + b*x))**7/(35*b**3) - atanh(tanh(a + b*x))**8
/(280*b**4), Ne(b, 0)), (x**4*atanh(tanh(a))**4/4, True))
```

3.70 $\int x^2 \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}(\tanh(a+bx))^7}{105b^3}-\frac{x\tanh^{-1}(\tanh(a+bx))^6}{15b^2}+\frac{x^2\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

```
[Out] 1/5*x^2*arctanh(tanh(b*x+a))^5/b-1/15*x*arctanh(tanh(b*x+a))^6/b^2+1/105*ar
ctanh(tanh(b*x+a))^7/b^3
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {2168, 2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^7}{105b^3}-\frac{x\tanh^{-1}(\tanh(a+bx))^6}{15b^2}+\frac{x^2\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - (x*ArcTanh[Tanh[a + b*x]]^6)/(15*b^2
) + ArcTanh[Tanh[a + b*x]]^7/(105*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{x^2\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{2\int x\tanh^{-1}(\tanh(a+bx))^5\,dx}{5b}\\ &= \frac{x^2\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{x\tanh^{-1}(\tanh(a+bx))^6}{15b^2}+\frac{\int \tanh^{-1}(\tanh(a+bx\\ &= \frac{x^2\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{x\tanh^{-1}(\tanh(a+bx))^6}{15b^2}+\frac{\text{Subst}\left(\int x^6\,dx,x,\tanh\right.}{15b^3}\\ &= \frac{x^2\tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{x\tanh^{-1}(\tanh(a+bx))^6}{15b^2}+\frac{\tanh^{-1}(\tanh(a+bx))^7}{105b^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 1.34

$$\frac{1}{105}x^3\left(-7b^3x^3\tanh^{-1}(\tanh(a+bx))+21b^2x^2\tanh^{-1}(\tanh(a+bx))^2-35bx\tanh^{-1}(\tanh(a+bx))^3+35\tanh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x^3*(b^4*x^4 - 7*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 21*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 - 35*b*x*ArcTanh[Tanh[a + b*x]]^3 + 35*ArcTanh[Tanh[a + b*x]]^4
))/105
```

fricas [A] time = 0.46, size = 45, normalized size = 0.85

$$\frac{1}{7}\,b^4x^7+\frac{2}{3}\,ab^3x^6+\frac{6}{5}\,a^2b^2x^5+a^3bx^4+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

giac [A] time = 0.19, size = 45, normalized size = 0.85

$$\frac{1}{7}\,b^4x^7+\frac{2}{3}\,ab^3x^6+\frac{6}{5}\,a^2b^2x^5+a^3bx^4+\frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/7*b^4*x^7 + 2/3*a*b^3*x^6 + 6/5*a^2*b^2*x^5 + a^3*b*x^4 + 1/3*a^4*x^3
```

maple [A] time = 0.15, size = 74, normalized size = 1.40

$$\frac{x^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{3}-\frac{4b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))^3}{4}-\frac{3b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a))^2}{5}-\frac{2b\left(\frac{x^6\operatorname{arctanh}(\tanh(bx+a))}{6}-\frac{x^7b}{42}\right)}{5}\right)}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/3*x^3*arctanh(tanh(b*x+a))^4-4/3*b*(1/4*x^4*arctanh(tanh(b*x+a))^3-3/4*b*
(1/5*x^5*arctanh(tanh(b*x+a))^2-2/5*b*(1/6*x^6*arctanh(tanh(b*x+a))-1/42*x^
7*b)))
```

maxima [A] time = 0.58, size = 72, normalized size = 1.36

$$-\frac{1}{3}\,bx^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3+\frac{1}{3}\,x^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{105}\left(21\,bx^5\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^7-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -1/3*b*x^4*arctanh(tanh(b*x + a))^3 + 1/3*x^3*arctanh(tanh(b*x + a))^4 + 1/
105*(21*b*x^5*arctanh(tanh(b*x + a))^2 + (b^2*x^7 - 7*b*x^6*arctanh(tanh(b*
x + a)))*b)*b
```

mupad [B] time = 0.15, size = 70, normalized size = 1.32

$$\frac{b^4\,x^7}{105}-\frac{b^3\,x^6\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{15}+\frac{b^2\,x^5\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{5}-\frac{b\,x^4\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^3}{3}+\frac{x^3\operatorname{atanh}\left(\tan\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^3*atanh(tanh(a + b*x))^4)/3 + (b^4*x^7)/105 + (b^2*x^5*atanh(tanh(a + b*
x))^2)/5 - (b*x^4*atanh(tanh(a + b*x))^3)/3 - (b^3*x^6*atanh(tanh(a + b*x))
)/15
```

sympy [A] time = 4.01, size = 60, normalized size = 1.13

$$\begin{cases} \frac{x^2 \operatorname{atanh}^5 (\tanh (a+bx))}{5b} - \frac{x \operatorname{atanh}^6 (\tanh (a+bx))}{15b^2} + \frac{\operatorname{atanh}^7 (\tanh (a+bx))}{105b^3} & \text{for } b \neq 0 \\ \frac{x^3 \operatorname{atanh}^4 (\tanh (a))}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((x**2*atanh(tanh(a + b*x))**5/(5*b) - x*atanh(tanh(a + b*x))**6/(
15*b**2) + atanh(tanh(a + b*x))**7/(105*b**3), Ne(b, 0)), (x**3*atanh(tanh(
a))**4/3, True))
```

3.71 $\int x \tanh^{-1}(\tanh(a+bx))^4 \, dx$

Optimal. Leaf size=34

$$\frac{x \tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}$$

```
[Out] 1/5*x*arctanh(tanh(b*x+a))^5/b-1/30*arctanh(tanh(b*x+a))^6/b^2
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x \tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^5)/(5*b) - ArcTanh[Tanh[a + b*x]]^6/(30*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x \tanh^{-1}(\tanh(a+bx))^4 \, dx &= \frac{x \tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\int \tanh^{-1}(\tanh(a+bx))^5 \, dx}{5b}\\
&= \frac{x \tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\text{Subst}\left(\int x^5 \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{5b^2}\\
&= \frac{x \tanh^{-1}(\tanh(a+bx))^5}{5b}-\frac{\tanh^{-1}(\tanh(a+bx))^6}{30b^2}
\end{aligned}$$

Mathematica [B] time = 0.10, size = 125, normalized size = 3.68

$$-\frac{(a+bx)\left(-20\left(2a^2+abx-b^2x^2\right)\tanh^{-1}(\tanh(a+bx))^3+(5a-bx)(a+bx)^4-6(4a-bx)(a+bx)^3\tanh^{-1}(\text{t}}{30b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] -1/30*((a + b*x)*((5*a - b*x)*(a + b*x)^4 - 6*(4*a - b*x)*(a + b*x)^3*ArcTa
nh[Tanh[a + b*x]] + 15*(3*a - b*x)*(a + b*x)^2*ArcTanh[Tanh[a + b*x]]^2 - 2
0*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a + b*x]]^3 + 15*(a - b*x)*ArcTanh
[Tanh[a + b*x]]^4))/b^2
```

fricas [A] time = 0.70, size = 46, normalized size = 1.35

$$\frac{1}{6}b^4x^6 + \frac{4}{5}ab^3x^5 + \frac{3}{2}a^2b^2x^4 + \frac{4}{3}a^3bx^3 + \frac{1}{2}a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^2
```

giac [A] time = 0.17, size = 46, normalized size = 1.35

$$\frac{1}{6}b^4x^6 + \frac{4}{5}ab^3x^5 + \frac{3}{2}a^2b^2x^4 + \frac{4}{3}a^3bx^3 + \frac{1}{2}a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/6*b^4*x^6 + 4/5*a*b^3*x^5 + 3/2*a^2*b^2*x^4 + 4/3*a^3*b*x^3 + 1/2*a^4*x^2
```

maple [B] time = 0.16, size = 74, normalized size = 2.18

$$\frac{x^2 \operatorname{arctanh}(\tanh(bx+a))^4}{2} - 2b\left(\frac{x^3 \operatorname{arctanh}(\tanh(bx+a))^3}{3} - b\left(\frac{x^4 \operatorname{arctanh}(\tanh(bx+a))^2}{4} - \frac{b\left(\frac{x^5 \operatorname{arctanh}(\tanh}{5}\right.}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^4-2*b*(1/3*x^3*arctanh(tanh(b*x+a))^3-b*(1/4*x
^4*arctanh(tanh(b*x+a))^2-1/2*b*(1/5*x^5*arctanh(tanh(b*x+a))-1/30*x^6*b)))
```

maxima [B] time = 0.58, size = 72, normalized size = 2.12

$$-\frac{2}{3}bx^3 \operatorname{artanh}(\tanh(bx+a))^3 + \frac{1}{2}x^2 \operatorname{artanh}(\tanh(bx+a))^4 + \frac{1}{30}\left(15bx^4 \operatorname{artanh}(\tanh(bx+a))^2 + \left(b^2x^6 - 6bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2/3*b*x^3*arctanh(tanh(b*x + a))^3 + 1/2*x^2*arctanh(tanh(b*x + a))^4 + 1/
30*(15*b*x^4*arctanh(tanh(b*x + a))^2 + (b^2*x^6 - 6*b*x^5*arctanh(tanh(b*x
 + a)))*b)*b
```

mupad [B] time = 1.00, size = 70, normalized size = 2.06

$$\frac{b^4\,x^6}{30} - \frac{b^3\,x^5\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}{5} + \frac{b^2\,x^4\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{2} - \frac{2\,b\,x^3\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^3}{3} + \frac{x^2\,\mathrm{atanh}\left(\mathrm{tanh}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^4,x)
```

```
[Out] (x^2*atanh(tanh(a + b*x))^4)/2 + (b^4*x^6)/30 + (b^2*x^4*atanh(tanh(a + b*x
))^2)/2 - (2*b*x^3*atanh(tanh(a + b*x))^3)/3 - (b^3*x^5*atanh(tanh(a + b*x)
))/5
```

sympy [A] time = 2.33, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x \operatorname{atanh}^{5}\left(\tanh\left(a+bx\right)\right)}{5b} - \frac{\operatorname{atanh}^{6}\left(\tanh\left(a+bx\right)\right)}{30b^{2}} & \text{for } b \neq 0 \\ \frac{x^{2} \operatorname{atanh}^{4}\left(\tanh\left(a\right)\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**5/(5*b) - atanh(tanh(a + b*x))**6/(30*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**4/2, True))
```

3.72 $\int \tanh^{-1}(\tanh(a+bx))^4\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

```
[Out] 1/5*arctanh(tanh(b*x+a))^5/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^4\,dx &= \frac{\text{Subst}\left(\int x^4\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*b)
```

fricas [B] time = 0.44, size = 42, normalized size = 2.62

$$\frac{1}{5}\,b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="fricas")
```

```
[Out] 1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

giac [B] time = 0.16, size = 42, normalized size = 2.62

$$\frac{1}{5}\,b^4x^5 + ab^3x^4 + 2\,a^2b^2x^3 + 2\,a^3bx^2 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + a*b^3*x^4 + 2*a^2*b^2*x^3 + 2*a^3*b*x^2 + a^4*x
```

maple [A] time = 0.03, size = 15, normalized size = 0.94

$$\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4,x)
```

```
[Out] 1/5*arctanh(tanh(b*x+a))^5/b
```

maxima [B] time = 0.58, size = 69, normalized size = 4.31

$$-2\,bx^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3+x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4+\frac{1}{5}\left(10\,bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\left(b^2x^5-5\,bx^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4,x, algorithm="maxima")
```

```
[Out] -2*b*x^2*arctanh(tanh(b*x + a))^3 + x*arctanh(tanh(b*x + a))^4 + 1/5*(10*b*
x^3*arctanh(tanh(b*x + a))^2 + (b^2*x^5 - 5*b*x^4*arctanh(tanh(b*x + a)))*b
)*b
```

mupad [B] time = 0.98, size = 67, normalized size = 4.19

$$\frac{b^4\,x^5}{5}-b^3\,x^4\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)+2\,b^2\,x^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2-2\,b\,x^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^3+x\operatorname{atanh}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4,x)
```

```
[Out] x*atanh(tanh(a + b*x))^4 + (b^4*x^5)/5 + 2*b^2*x^3*atanh(tanh(a + b*x))^2 -
 2*b*x^2*atanh(tanh(a + b*x))^3 - b^3*x^4*atanh(tanh(a + b*x))
```

sympy [A] time = 1.15, size = 20, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atanh}^{5}{\left(\tanh{\left(a+bx \right)} \right)}}{5b} & \text{for}\: b \neq 0 \\ x \operatorname{atanh}^{4}{\left(\tanh{\left(a \right)} \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4,x)
```

```
[Out] Piecewise((atanh(tanh(a + b*x))**5/(5*b), Ne(b, 0)), (x*atanh(tanh(a))**4,
True))
```

3.73 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x}\,dx$

Optimal. Leaf size=105

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{3}\tanh^{-1}(\tanh(a+bx)$$

```
[Out] -b*x*(b*x-arctanh(tanh(b*x+a)))^3+1/2*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(
tanh(b*x+a))^2-1/3*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^3+1/4*ar
ctanh(tanh(b*x+a))^4+(b*x-arctanh(tanh(b*x+a)))^4*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2159, 2158, 29}

$$-bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\tanh^{-1}(\tanh(a+bx))^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{1}{3}\tanh^{-1}(\tanh(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x,x]
```

```
[Out] -(b*x*(b*x - ArcTanh[Tanh[a + b*x]])^3) + ((b*x - ArcTanh[Tanh[a + b*x]])^2
*ArcTanh[Tanh[a + b*x]]^2)/2 - ((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh
[a + b*x]]^3)/3 + ArcTanh[Tanh[a + b*x]]^4/4 + (b*x - ArcTanh[Tanh[a + b*x]
])^4*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x}\,dx &= \frac{1}{4}\tanh^{-1}(\tanh(a+bx))^4-\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a+bx)}{x}\\
&= -\frac{1}{3}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{4}\tanh^{-1}(\tanh(a+bx\\
&= \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{3}\left(bx-\tanh^{-1}(\tanh(a\right.\\
&= -bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tan\\
&= -bx\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3+\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tan
\end{aligned}$$

Mathematica [A] time = 0.17, size = 175, normalized size = 1.67

$$\frac{1}{2}(a+bx)^2\left(a^2-4a\left(-\tanh^{-1}(\tanh(a+bx))+a+bx\right)+6\left(-\tanh^{-1}(\tanh(a+bx))+a+bx\right)^2\right)+(a+bx)\left(a^3-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x,x]
```

```
[Out] (a + b*x)^4/4 + ((a + b*x)^2*(a^2 - 4*a*(a + b*x - ArcTanh[Tanh[a + b*x]])
+ 6*(a + b*x - ArcTanh[Tanh[a + b*x]])^2))/2 + (a + b*x)*(a^3 - 4*a^2*(a +
b*x - ArcTanh[Tanh[a + b*x]]) + 6*a*(a + b*x - ArcTanh[Tanh[a + b*x]])^2 -
4*(a + b*x - ArcTanh[Tanh[a + b*x]])^3) - ((a + b*x)^3*(3*a + 4*b*x - 4*Arc
Tanh[Tanh[a + b*x]]))/3 + (-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*x]
```

fricas [A] time = 0.54, size = 42, normalized size = 0.40

$$\frac{1}{4}\,b^4x^4+\frac{4}{3}\,ab^3x^3+3\,a^2b^2x^2+4\,a^3bx+a^4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

giac [A] time = 0.21, size = 43, normalized size = 0.41

$$\frac{1}{4}\,b^4x^4+\frac{4}{3}\,ab^3x^3+3\,a^2b^2x^2+4\,a^3bx+a^4\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(abs(x))
```

maple [A] time = 0.17, size = 127, normalized size = 1.21

$$\ln(x)\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^4-4\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\ln(x)x^3b^3+6\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^2\ln(x)x^2b^2+4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x,x)
```

```
[Out] ln(x)*arctanh(tanh(b*x+a))^4-4*arctanh(tanh(b*x+a))*ln(x)*x^3*b^3+6*arctanh
(tanh(b*x+a))^2*ln(x)*x^2*b^2+4*b*arctanh(tanh(b*x+a))^3*x+b^4*x^4*ln(x)-25
/12*b^4*x^4-4*b*arctanh(tanh(b*x+a))^3*ln(x)*x-9*arctanh(tanh(b*x+a))^2*x^2
*b^2+22/3*arctanh(tanh(b*x+a))*x^3*b^3
```

maxima [A] time = 0.72, size = 42, normalized size = 0.40

$$\frac{1}{4}\,b^4x^4+\frac{4}{3}\,ab^3x^3+3\,a^2b^2x^2+4\,a^3bx+a^4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + a^4*log(x)
```

mupad [B] time = 0.14, size = 423, normalized size = 4.03

$$\ln(x)\left(\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}{16}+\frac{3\,a^2\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x,x)
```

```
[Out] log(x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4/16 + (3*a^2*(2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^2)/2 + a^4 - (a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/2 - 2*a^3*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)) + (b^4*x^4)/4 - (2*b^3*x^3*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x))/3 + (3*b^2*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/4 - (b*x*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^3)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^4(\tanh(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x, x)
```

3.74 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^2}\,dx$

Optimal. Leaf size=95

$$4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(\tanh(a+bx))^4}{x}+\frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3-2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

```
[Out] 4*b^2*x*(b*x-arctanh(tanh(b*x+a)))^2-2*b*(b*x-arctanh(tanh(b*x+a)))*arctanh
(tanh(b*x+a))^2+4/3*b*arctanh(tanh(b*x+a))^3-arctanh(tanh(b*x+a))^4/x-4*b*(
b*x-arctanh(tanh(b*x+a)))^3*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2159, 2158, 29}

$$4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(\tanh(a+bx))^4}{x}+\frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3-2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^2,x]
```

```
[Out] 4*b^2*x*(b*x - ArcTanh[Tanh[a + b*x]])^2 - 2*b*(b*x - ArcTanh[Tanh[a + b*x]
])*ArcTanh[Tanh[a + b*x]]^2 + (4*b*ArcTanh[Tanh[a + b*x]]^3)/3 - ArcTanh[Ta
nh[a + b*x]]^4/x - 4*b*(b*x - ArcTanh[Tanh[a + b*x]])^3*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{x} + (4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x}\,dx$$
$$= \frac{4}{3}b\tanh^{-1}(\tanh(a+bx))^3 - \frac{\tanh^{-1}(\tanh(a+bx))^4}{x} - \left(4b\left(bx-\tanh^{-1}(\tanh(a\right.\right.$$
$$= -2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2 + \frac{4}{3}b\tanh^{-1}(\tanh(a+$$
$$= 4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 - 2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan$$
$$= 4b^2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 - 2b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan$$

Mathematica [A] time = 0.09, size = 85, normalized size = 0.89

$$6b^3x^2(2\log(x)-1)\tanh^{-1}(\tanh(a+bx))-12b^2x\log(x)\tanh^{-1}(\tanh(a+bx))^2-\frac{\tanh^{-1}(\tanh(a+bx))^4}{x}+4b(\log(x)+$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^4/x) + (2*b^4*x^3*(5 - 6*Log[x]))/3 - 12*b^2*x*Arc
Tanh[Tanh[a + b*x]]^2*Log[x] + 4*b*ArcTanh[Tanh[a + b*x]]^3*(1 + Log[x]) +
6*b^3*x^2*ArcTanh[Tanh[a + b*x]]*(-1 + 2*Log[x])
```

fricas [A] time = 0.51, size = 47, normalized size = 0.49

$$\frac{b^4x^4 + 6\,ab^3x^3 + 18\,a^2b^2x^2 + 12\,a^3bx\log(x) - 3\,a^4}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 + 6*a*b^3*x^3 + 18*a^2*b^2*x^2 + 12*a^3*b*x*log(x) - 3*a^4)/x
```

giac [A] time = 0.33, size = 44, normalized size = 0.46

$$\frac{1}{3}\,b^4x^3 + 2\,ab^3x^2 + 6\,a^2b^2x + 4\,a^3b\log(|x|) - \frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="giac")
```

```
[Out] 1/3*b^4*x^3 + 2*a*b^3*x^2 + 6*a^2*b^2*x + 4*a^3*b*log(abs(x)) - a^4/x
```

maple [A] time = 0.18, size = 112, normalized size = 1.18

$$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{x}+4\ln(x)\operatorname{arctanh}(\tanh(bx+a))^3b+12\operatorname{arctanh}(\tanh(bx+a))\ln(x)x^2b^3+12b^2\operatorname{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^2,x)
```

```
[Out] -arctanh(tanh(b*x+a))^4/x+4*ln(x)*arctanh(tanh(b*x+a))^3*b+12*arctanh(tanh(
b*x+a))*ln(x)*x^2*b^3+12*b^2*arctanh(tanh(b*x+a))^2*x+22/3*x^3*b^4-4*b^4*x^
3*ln(x)-12*b^2*arctanh(tanh(b*x+a))^2*ln(x)*x-18*arctanh(tanh(b*x+a))*x^2*b
^3
```

maxima [A] time = 0.78, size = 77, normalized size = 0.81

$$4\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}\log(x) - \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{4}}{x} + \frac{2}{3}\left(2\,b^{3}x^{3} + 9\,ab^{2}x^{2} + 18\,a^{2}bx + 6\,a^{3}\log(x) - 6\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^2,x, algorithm="maxima")
```

```
[Out] 4*b*arctanh(tanh(b*x + a))^3*log(x) - arctanh(tanh(b*x + a))^4/x + 2/3*(2*b
^3*x^3 + 9*a*b^2*x^2 + 18*a^2*b*x + 6*a^3*log(x) - 6*arctanh(tanh(b*x + a))
^3*log(x))*b
```

mupad [B] time = 0.14, size = 553, normalized size = 5.82

$$\ln(x)\left(4\,a^{3}\,b - \frac{b\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^{3}}{2} + 3\,a\,b\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^2,x)
```

```
[Out] log(x)*(4*a^3*b - (b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/2 + 3*a*b*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^2 - 6*a^2*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((2*a -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 16*a^
4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x))/(16*x) + (b^4*x^3)/3 + (3*b^2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - b^3*
x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**2, x)
```

3.75 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^3}\,dx$

Optimal. Leaf size=87

$$-6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+3b^2\tanh^{-1}(\tanh(a+bx))^2+6b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}}{}$$

```
[Out] -6*b^3*x*(b*x-arctanh(tanh(b*x+a)))+3*b^2*arctanh(tanh(b*x+a))^2-2*b*arctan
h(tanh(b*x+a))^3/x-1/2*arctanh(tanh(b*x+a))^4/x^2+6*b^2*(b*x-arctanh(tanh(b
*x+a)))^2*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2159, 2158, 29}

$$3b^2\tanh^{-1}(\tanh(a+bx))^2-6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+6b^2\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{\tanh^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^3,x]
```

```
[Out] -6*b^3*x*(b*x - ArcTanh[Tanh[a + b*x]]) + 3*b^2*ArcTanh[Tanh[a + b*x]]^2 -
(2*b*ArcTanh[Tanh[a + b*x]]^3)/x - ArcTanh[Tanh[a + b*x]]^4/(2*x^2) + 6*b^2
*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} + (2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^2}\,dx$$
$$= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x} - \frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2} + \left(6b^2\right)\int \frac{\tanh^{-1}(\tanh}{x}$$
$$= 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-1}(\tanh(a+bx))^3}{x} - \frac{\tanh^{-1}(\tanh(a+bx}{2x^2}$$
$$= -6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-1}}{}$$
$$= -6b^3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + 3b^2\tanh^{-1}(\tanh(a+bx))^2 - \frac{2b\tanh^{-1}}{}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 0.93

$$-6b^3x(2\log(x)+1)\tanh^{-1}(\tanh(a+bx))+3b^2(2\log(x)+3)\tanh^{-1}(\tanh(a+bx))^2-\frac{\tanh^{-1}(\tanh(a+bx))^4}{2x^2}-\frac{2b\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^3,x]
[Out] (-2*b*ArcTanh[Tanh[a + b*x]]^3)/x - ArcTanh[Tanh[a + b*x]]^4/(2*x^2) + 6*b^
4*x^2*Log[x] - 6*b^3*x*ArcTanh[Tanh[a + b*x]]*(1 + 2*Log[x]) + 3*b^2*ArcTan
h[Tanh[a + b*x]]^2*(3 + 2*Log[x])
```

fricas [A] time = 0.75, size = 47, normalized size = 0.54

$$\frac{b^4x^4 + 8\,ab^3x^3 + 12\,a^2b^2x^2\log(x) - 8\,a^3bx - a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="fricas")
[Out] 1/2*(b^4*x^4 + 8*a*b^3*x^3 + 12*a^2*b^2*x^2*log(x) - 8*a^3*b*x - a^4)/x^2
```

giac [A] time = 0.23, size = 43, normalized size = 0.49

$$\frac{1}{2}\,b^4x^2 + 4\,ab^3x + 6\,a^2b^2\log\left(|x|\right) - \frac{8\,a^3bx + a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="giac")
[Out] 1/2*b^4*x^2 + 4*a*b^3*x + 6*a^2*b^2*log(abs(x)) - 1/2*(8*a^3*b*x + a^4)/x^2
```

maple [A] time = 0.18, size = 93, normalized size = 1.07

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{2x^2}-\frac{2b\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{x}+6b^2\ln(x)\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2+6b^4x^2\ln(x)-9x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^3,x)
[Out] -1/2*arctanh(tanh(b*x+a))^4/x^2-2*b*arctanh(tanh(b*x+a))^3/x+6*b^2*ln(x)*ar
ctanh(tanh(b*x+a))^2+6*b^4*x^2*ln(x)-9*x^2*b^4-12*b^3*arctanh(tanh(b*x+a))*
ln(x)*x+12*b^3*arctanh(tanh(b*x+a))*x
```

maxima [A] time = 0.85, size = 83, normalized size = 0.95

$$-\frac{2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{x}+3\left(2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{2}\log(x)+\left(b^{2}x^{2}+4\,abx+2\,a^{2}\log(x)-2\,\mathrm{artanh}\left(\tan\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^3,x, algorithm="maxima")
```

```
[Out] -2*b*arctanh(tanh(b*x + a))^3/x + 3*(2*b*arctanh(tanh(b*x + a))^2*log(x) +
(b^2*x^2 + 4*a*b*x + 2*a^2*log(x) - 2*arctanh(tanh(b*x + a))^2*log(x))*b)*b
 - 1/2*arctanh(tanh(b*x + a))^4/x^2
```

mupad [B] time = 1.54, size = 672, normalized size = 7.72

$$\frac{9\,b^{2}\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{2}}{4}-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{4}}{32\,x^{2}}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{4}}{32\,x^{2}}+\frac{9\,b^{2}\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{2}}{4}-3\,b^{3}\,x\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\frac{b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,\cdots}}\right.}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^3,x)
```

```
[Out] (9*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/4 - log((exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))^4/(32*x^2) - log(1/(exp(2*a)*exp(2*b*x) + 1))^4/(3
2*x^2) + (9*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/4 -
 3*b^3*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + (b*log(1/(e
xp(2*a)*exp(2*b*x) + 1))^3)/(4*x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(8*x^2) + (log(1/(exp(2*
a)*exp(2*b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))
/(8*x^2) - (b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(4*x)
 + (3*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log(x))/2 - (3*log(1/(exp(2*a)
*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)
/(16*x^2) + (3*b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*l
og(x))/2 + 6*b^4*x^2*log(x) - (9*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/2 + 3*b^3*x*log(1/(exp(2*a
)*exp(2*b*x) + 1)) + (3*b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(4*x) - (3*b*log(1/(exp(2*a)*exp(2*
b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(4*x) -
3*b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))*log(x) + 6*b^3*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log(x) -
 6*b^3*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**3, x)
```

3.76 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^4} dx$

Optimal. Leaf size=77

$$-4b^3 \log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3}-\frac{2b\tanh^{-1}(\tan}{3x^{}}$$

```
[Out] 4*b^4*x-2*b^2*arctanh(tanh(b*x+a))^2/x-2/3*b*arctanh(tanh(b*x+a))^3/x^2-1/3
*arctanh(tanh(b*x+a))^4/x^3-4*b^3*(b*x-arctanh(tanh(b*x+a)))*ln(x)
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2168, 2158, 29}

$$-\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-4b^3\log(x)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}-\frac{\tanh^{-1}(\tan}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^4,x]
```

```
[Out] 4*b^4*x - (2*b^2*ArcTanh[Tanh[a + b*x]]^2)/x - (2*b*ArcTanh[Tanh[a + b*x]]^
3)/(3*x^2) - ArcTanh[Tanh[a + b*x]]^4/(3*x^3) - 4*b^3*(b*x - ArcTanh[Tanh[a
 + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^4}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3}+\frac{1}{3}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^3}\,dx\\
&= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{3x^3}+\left(2b^2\right)\int \frac{\tanh^{-1}(\tanh}{x^2}\\
&= -\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}-\frac{\tanh^{-1}(\tanh(a+}{3x^3}\\
&= 4b^4x-\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}-\frac{\tanh^{-1}(\tan}{3x}\\
&= 4b^4x-\frac{2b^2\tanh^{-1}(\tanh(a+bx))^2}{x}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{3x^2}-\frac{\tanh^{-1}(\tan}{3x}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 1.06

$$-\frac{-2b^3x^3(6\log(x)+11)\tanh^{-1}(\tanh(a+bx))+6b^2x^2\tanh^{-1}(\tanh(a+bx))^2+2bx\tanh^{-1}(\tanh(a+bx))^3+\tan}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^4,x]
```

```
[Out] -1/3*(6*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 + 2*b*x*ArcTanh[Tanh[a + b*x]]^3 +
 ArcTanh[Tanh[a + b*x]]^4 + 2*b^4*x^4*(5 + 6*Log[x]) - 2*b^3*x^3*ArcTanh[Ta
nh[a + b*x]]*(11 + 6*Log[x]))/x^3
```

fricas [A] time = 0.66, size = 48, normalized size = 0.62

$$\frac{3\,b^4x^4+12\,ab^3x^3\log(x)-18\,a^2b^2x^2-6\,a^3bx-a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x^4 + 12*a*b^3*x^3*log(x) - 18*a^2*b^2*x^2 - 6*a^3*b*x - a^4)/x^
3
```

giac [A] time = 0.21, size = 42, normalized size = 0.55

$$b^4x+4\,ab^3\log(|x|)-\frac{18\,a^2b^2x^2+6\,a^3bx+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="giac")
```

```
[Out] b^4*x + 4*a*b^3*log(abs(x)) - 1/3*(18*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/x^3
```

maple [A] time = 0.16, size = 76, normalized size = 0.99

$$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{3x^3}-\frac{2b\operatorname{arctanh}(\tanh(bx+a))^3}{3x^2}-\frac{2b^2\operatorname{arctanh}(\tanh(bx+a))^2}{x}-4\ln(x)x\,b^4+4\operatorname{arctanh}(\mathrm{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^4,x)
```

```
[Out] -1/3*arctanh(tanh(b*x+a))^4/x^3-2/3*b*arctanh(tanh(b*x+a))^3/x^2-2*b^2*arct
anh(tanh(b*x+a))^2/x-4*ln(x)*x*b^4+4*arctanh(tanh(b*x+a))*ln(x)*b^3+4*b^4*x
```

maxima [A] time = 0.54, size = 91, normalized size = 1.18

$$2\left(2\left(b\operatorname{artanh}(\tanh(bx+a))\log(x)-\left(b\left(x+\frac{a}{b}\right)\log(x)-b\left(x+\frac{a\log(x)}{b}\right)\right)b\right)b-\frac{b\operatorname{artanh}(\tanh(bx+a))^2}{x}\right)b-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^4,x, algorithm="maxima")
```

```
[Out] 2*(2*(b*arctanh(tanh(b*x + a))*log(x) - (b*(x + a/b)*log(x) - b*(x + a*log(
x)/b))*b)*b - b*arctanh(tanh(b*x + a))^2/x)*b - 2/3*b*arctanh(tanh(b*x + a)
)^3/x^2 - 1/3*arctanh(tanh(b*x + a))^4/x^3
```

mupad [B] time = 1.34, size = 571, normalized size = 7.42

$$\frac{11\,b^3\,\ln\left(\frac{e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{3}-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^4}{48\,x^3}-\frac{11\,b^3\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{3}-\frac{10\,b^4\,x}{3}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^4}{48\,x^3}+\frac{b\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^3}{12\,x^2}-2\,b^3\,\ln\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^4,x)
```

```
[Out] (11*b^3*log(exp(2*b*x)/(exp(2*a)*exp(2*b*x) + 1)))/3 - log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))^4/(48*x^3) - (11*b^3*log(1/(exp(2*a)*exp(2
*b*x) + 1)))/3 - (10*b^4*x)/3 - log(1/(exp(2*a)*exp(2*b*x) + 1))^4/(48*x^3)
 + (b*log(1/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x^2) - 2*b^3*log(1/(exp(2*a)*
exp(2*b*x) + 1))*log(x) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x^3) + (log(1/(exp(2*a)*exp(2*
b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(12*x^3)
 - (b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3)/(12*x^2) + 2*
b^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(x) - 4*b^4*x*l
og(x) - (b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))^2)/(2*x) - (log(1/(exp(2*a)*e
xp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(
8*x^3) - (b^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(2*x)
 + (b^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)))/x + (b*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(4*x^2) - (b*log(1/(exp(2*a)*exp(2
*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**4,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**4/x**4, x)
```

3.77 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^5} dx$

Optimal. Leaf size=74

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3}+b^4\log($$

```
[Out] -b^3*arctanh(tanh(b*x+a))/x-1/2*b^2*arctanh(tanh(b*x+a))^2/x^2-1/3*b*arctan
h(tanh(b*x+a))^3/x^3-1/4*arctanh(tanh(b*x+a))^4/x^4+b^4*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 29}

$$-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4}+b^4\log($$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^5,x]
```

```
[Out] -((b^3*ArcTanh[Tanh[a + b*x]])/x) - (b^2*ArcTanh[Tanh[a + b*x]]^2)/(2*x^2)
- (b*ArcTanh[Tanh[a + b*x]]^3)/(3*x^3) - ArcTanh[Tanh[a + b*x]]^4/(4*x^4) +
 b^4*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^5} dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4}+b\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^4} dx \\
&= -\frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4}+b^2\int \frac{\tanh^{-1}(\tanh(a+bx}{x^3} \\
&= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{3x^3}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{4x^4} \\
&= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\tanh^{-1}(\tanh(a+bx)}{3x^3} \\
&= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{x}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{2x^2}-\frac{b\tanh^{-1}(\tanh(a+bx)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.05

$$-\frac{12b^3x^3\tanh^{-1}(\tanh(a+bx))+6b^2x^2\tanh^{-1}(\tanh(a+bx))^2+4bx\tanh^{-1}(\tanh(a+bx))^3+3\tanh^{-1}(\tanh(a+}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^5,x]
```

```
[Out] -1/12*(12*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 6*b^2*x^2*ArcTanh[Tanh[a + b*x]]
^2 + 4*b*x*ArcTanh[Tanh[a + b*x]]^3 + 3*ArcTanh[Tanh[a + b*x]]^4 - b^4*x^4*
(25 + 12*Log[x]))/x^4
```

fricas [A] time = 0.61, size = 48, normalized size = 0.65

$$\frac{12\,b^4x^4\log(x) - 48\,ab^3x^3 - 36\,a^2b^2x^2 - 16\,a^3bx - 3\,a^4}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^4*x^4*log(x) - 48*a*b^3*x^3 - 36*a^2*b^2*x^2 - 16*a^3*b*x - 3*a^
4)/x^4
```

giac [A] time = 0.18, size = 46, normalized size = 0.62

$$b^4\log\left(|x|\right) - \frac{48\,ab^3x^3 + 36\,a^2b^2x^2 + 16\,a^3bx + 3\,a^4}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="giac")
```

```
[Out] b^4*log(abs(x)) - 1/12*(48*a*b^3*x^3 + 36*a^2*b^2*x^2 + 16*a^3*b*x + 3*a^4)
/x^4
```

maple [A] time = 0.15, size = 69, normalized size = 0.93

$$-\frac{b^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{x}-\frac{b^2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{2x^2}-\frac{b\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{3x^3}-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right.\right.}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^5,x)
```

```
[Out] -b^3*arctanh(tanh(b*x+a))/x-1/2*b^2*arctanh(tanh(b*x+a))^2/x^2-1/3*b*arctan
h(tanh(b*x+a))^3/x^3-1/4*arctanh(tanh(b*x+a))^4/x^4+b^4*ln(x)
```

maxima [A] time = 0.60, size = 72, normalized size = 0.97

$$\frac{1}{2}\left(2\left(b^2\log(x)-\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x}\right)b-\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^2}\right)b-\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{3\,x^3}-\operatorname{artan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b^2*log(x) - b*arctanh(tanh(b*x + a))/x)*b - b*arctanh(tanh(b*x + a
))^2/x^2)*b - 1/3*b*arctanh(tanh(b*x + a))^3/x^3 - 1/4*arctanh(tanh(b*x + a
))^4/x^4
```

mupad [B] time = 1.14, size = 68, normalized size = 0.92

$$b^4\,\ln(x)-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^4}{4\,x^4}-\frac{b^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{2\,x^2}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{x}-\frac{b\operatorname{atanh}\left(\tanh\left(a+b\right.\right.}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^5,x)
```

```
[Out] b^4*log(x) - atanh(tanh(a + b*x))^4/(4*x^4) - (b^2*atanh(tanh(a + b*x))^2)/
(2*x^2) - (b^3*atanh(tanh(a + b*x)))/x - (b*atanh(tanh(a + b*x))^3)/(3*x^3)
```

sympy [A] time = 1.33, size = 70, normalized size = 0.95

$$b^4 \log(x) - \frac{b^3 \operatorname{atanh}(\tanh(a + bx))}{x} - \frac{b^2 \operatorname{atanh}^2(\tanh(a + bx))}{2x^2} - \frac{b \operatorname{atanh}^3(\tanh(a + bx))}{3x^3} - \frac{\operatorname{atanh}^4(\tanh(a + bx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**5,x)
```

```
[Out] b**4*log(x) - b**3*atanh(tanh(a + b*x))/x - b**2*atanh(tanh(a + b*x))**2/(2
*x**2) - b*atanh(tanh(a + b*x))**3/(3*x**3) - atanh(tanh(a + b*x))**4/(4*x*
*4)
```

3.78 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6} \, dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 1/5*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^6,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^5/(5*x^5*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6} \, dx = \frac{\tanh^{-1}(\tanh(a+bx))^5}{5x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [B] time = 0.06, size = 66, normalized size = 2.13

$$-\frac{b^3x^3\tanh^{-1}(\tanh(a+bx))+b^2x^2\tanh^{-1}(\tanh(a+bx))^2+bx\tanh^{-1}(\tanh(a+bx))^3+\tanh^{-1}(\tanh(a+bx}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^6,x]
```

```
[Out] -1/5*(b^4*x^4 + b^3*x^3*ArcTanh[Tanh[a + b*x]] + b^2*x^2*ArcTanh[Tanh[a + b
*x]]^2 + b*x*ArcTanh[Tanh[a + b*x]]^3 + ArcTanh[Tanh[a + b*x]]^4)/x^5
```

fricas [A] time = 0.49, size = 44, normalized size = 1.42

$$-\frac{5\,b^4x^4+10\,ab^3x^3+10\,a^2b^2x^2+5\,a^3bx+a^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="fricas")
```

```
[Out] -1/5*(5*b^4*x^4 + 10*a*b^3*x^3 + 10*a^2*b^2*x^2 + 5*a^3*b*x + a^4)/x^5
```

giac [A] time = 0.17, size = 44, normalized size = 1.42

$$-\frac{5\,b^{4}x^{4}+10\,ab^{3}x^{3}+10\,a^{2}b^{2}x^{2}+5\,a^{3}bx+a^{4}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="giac")
```

```
[Out] -1/5*(5*b^4*x^4 + 10*a*b^3*x^3 + 10*a^2*b^2*x^2 + 5*a^3*b*x + a^4)/x^5
```

maple [B] time = 0.15, size = 74, normalized size = 2.39

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{4}}{5x^{5}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{3}}{4x^{4}}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{3x^{3}}+\frac{2b\left(-\frac{b}{2x}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2x^{2}}\right)}{3}\right)}{4}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^6,x)
```

```
[Out] -1/5*arctanh(tanh(b*x+a))^4/x^5+4/5*b*(-1/4*arctanh(tanh(b*x+a))^3/x^4+3/4*
b*(-1/3*arctanh(tanh(b*x+a))^2/x^3+2/3*b*(-1/2*b/x-1/2*arctanh(tanh(b*x+a))
/x^2)))
```

maxima [B] time = 0.60, size = 70, normalized size = 2.26

$$-\frac{1}{5}\left(b\left(\frac{b^{2}}{x}+\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^{2}}\right)+\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{x^{3}}\right)b-\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{5\,x^{4}}-\frac{\operatorname{artanh}\left(\tanh\left(\right.\right.}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(b*(b^2/x + b*arctanh(tanh(b*x + a))/x^2) + b*arctanh(tanh(b*x + a))^2
/x^3)*b - 1/5*b*arctanh(tanh(b*x + a))^3/x^4 - 1/5*arctanh(tanh(b*x + a))^4
/x^5
```

mupad [B] time = 1.20, size = 64, normalized size = 2.06

$$-\frac{b^{4}\,x^{4}+b^{3}\,x^{3}\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)+b^{2}\,x^{2}\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{2}+b\,x\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3}+\operatorname{atanh}\left(\tanh\right.}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^6,x)
```

```
[Out] -(atanh(tanh(a + b*x))^4 + b^4*x^4 + b^2*x^2*atanh(tanh(a + b*x))^2 + b*x*a
tanh(tanh(a + b*x))^3 + b^3*x^3*atanh(tanh(a + b*x)))/(5*x^5)
```

sympy [B] time = 2.10, size = 75, normalized size = 2.42

$$-\frac{b^{4}}{5x}-\frac{b^{3}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{5x^{2}}-\frac{b^{2}\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}{5x^{3}}-\frac{b\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}{5x^{4}}-\frac{\operatorname{atanh}^{4}\left(\tanh\left(a+bx\right)\right)}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**6,x)
```

```
[Out] -b**4/(5*x) - b**3*atanh(tanh(a + b*x))/(5*x**2) - b**2*atanh(tanh(a + b*x)
)**2/(5*x**3) - b*atanh(tanh(a + b*x))**3/(5*x**4) - atanh(tanh(a + b*x))**
4/(5*x**5)
```

3.79 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7} dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 1/30*b*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))^2+1/6*arctanh(
tanh(b*x+a))^5/x^6/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^7,x]
```

```
[Out] (b*ArcTanh[Tanh[a + b*x]]^5)/(30*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^2) + Ar
cTanh[Tanh[a + b*x]]^5/(6*x^6*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7}\,dx &= \frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^6}\,dx}{6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\tanh^{-1}(\tanh(a+bx))^5}{30x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{6x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.11

$$-\frac{2b^3x^3\tanh^{-1}(\tanh(a+bx))+3b^2x^2\tanh^{-1}(\tanh(a+bx))^2+4bx\tanh^{-1}(\tanh(a+bx))^3+5\tanh^{-1}(\tanh(a}{30x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^7,x]
```

```
[Out] -1/30*(b^4*x^4 + 2*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 3*b^2*x^2*ArcTanh[Tanh[
a + b*x]]^2 + 4*b*x*ArcTanh[Tanh[a + b*x]]^3 + 5*ArcTanh[Tanh[a + b*x]]^4)/
x^6
```

fricas [A] time = 0.40, size = 46, normalized size = 0.72

$$-\frac{15\,b^4x^4+40\,ab^3x^3+45\,a^2b^2x^2+24\,a^3bx+5\,a^4}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*x^4 + 40*a*b^3*x^3 + 45*a^2*b^2*x^2 + 24*a^3*b*x + 5*a^4)/x^6
```

giac [A] time = 0.16, size = 46, normalized size = 0.72

$$-\frac{15\,b^4x^4+40\,ab^3x^3+45\,a^2b^2x^2+24\,a^3bx+5\,a^4}{30\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*x^4 + 40*a*b^3*x^3 + 45*a^2*b^2*x^2 + 24*a^3*b*x + 5*a^4)/x^6
```

maple [A] time = 0.15, size = 74, normalized size = 1.16

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{6x^6}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{5x^5}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{4x^4}+\frac{b\left(-\frac{b}{6x^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^3}\right)}{2}\right)}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^7,x)
```

```
[Out] -1/6*arctanh(tanh(b*x+a))^4/x^6+2/3*b*(-1/5*arctanh(tanh(b*x+a))^3/x^5+3/5*
b*(-1/4*arctanh(tanh(b*x+a))^2/x^4+1/2*b*(-1/6*b/x^2-1/3*arctanh(tanh(b*x+a
))/x^3)))
```

maxima [A] time = 0.60, size = 72, normalized size = 1.12

$$-\frac{1}{30}\left(b\left(\frac{b^2}{x^2}+\frac{2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{x^3}\right)+\frac{3\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^4}\right)b-\frac{2\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{15\,x^5}-\frac{\mathrm{artanh}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^7,x, algorithm="maxima")
```

```
[Out] -1/30*(b*(b^2/x^2 + 2*b*arctanh(tanh(b*x + a))/x^3) + 3*b*arctanh(tanh(b*x
+ a))^2/x^4)*b - 2/15*b*arctanh(tanh(b*x + a))^3/x^5 - 1/6*arctanh(tanh(b*x
 + a))^4/x^6
```

mupad [B] time = 1.04, size = 70, normalized size = 1.09

$$-\frac{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^4}{6\,x^6}-\frac{b^4}{30\,x^2}-\frac{b^2\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{10\,x^4}-\frac{b^3\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}{15\,x^3}-\frac{2\,b\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right.}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^7,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(6*x^6) - b^4/(30*x^2) - (b^2*atanh(tanh(a + b*x))
^2)/(10*x^4) - (b^3*atanh(tanh(a + b*x)))/(15*x^3) - (2*b*atanh(tanh(a + b*
x))^3)/(15*x^5)
```

sympy [A] time = 3.29, size = 78, normalized size = 1.22

$$-\frac{b^4}{30x^2}-\frac{b^3\operatorname{atanh}(\tanh(a+bx))}{15x^3}-\frac{b^2\operatorname{atanh}^2(\tanh(a+bx))}{10x^4}-\frac{2b\operatorname{atanh}^3(\tanh(a+bx))}{15x^5}-\frac{\operatorname{atanh}^4(\tanh(a+bx))}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**7,x)
```

```
[Out] -b**4/(30*x**2) - b**3*atanh(tanh(a + b*x))/(15*x**3) - b**2*atanh(tanh(a +
 b*x))**2/(10*x**4) - 2*b*atanh(tanh(a + b*x))**3/(15*x**5) - atanh(tanh(a
+ b*x))**4/(6*x**6)
```

3.80 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8}\,dx$

Optimal. Leaf size=98

$$\frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{105x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^5}{21x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 1/105*b^2*arctanh(tanh(b*x+a))^5/x^5/(b*x-arctanh(tanh(b*x+a)))^3+1/21*b*ar
ctanh(tanh(b*x+a))^5/x^6/(b*x-arctanh(tanh(b*x+a)))^2+1/7*arctanh(tanh(b*x+
a))^5/x^7/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{105x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\tanh^{-1}(\tanh(a+bx))^5}{21x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^8,x]
```

```
[Out] (b^2*ArcTanh[Tanh[a + b*x]]^5)/(105*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^3) +
 (b*ArcTanh[Tanh[a + b*x]]^5)/(21*x^6*(b*x - ArcTanh[Tanh[a + b*x]])^2) + A
rcTanh[Tanh[a + b*x]]^5/(7*x^7*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8}\,dx &= \frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^7}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{b\tanh^{-1}(\tanh(a+bx))^5}{21x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{7x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b^2}{21\left(bx\right.}\\ &= \frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{105x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{b\tanh^{-1}(\tanh(a+bx))^5}{21x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.72

$$-\frac{3b^3x^3\tanh^{-1}(\tanh(a+bx))+6b^2x^2\tanh^{-1}(\tanh(a+bx))^2+10bx\tanh^{-1}(\tanh(a+bx))^3+15\tanh^{-1}(\tanh(a}{105x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^8,x]
```

```
[Out] -1/105*(b^4*x^4 + 3*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 6*b^2*x^2*ArcTanh[Tanh
[a + b*x]]^2 + 10*b*x*ArcTanh[Tanh[a + b*x]]^3 + 15*ArcTanh[Tanh[a + b*x]]^
4)/x^7
```

fricas [A] time = 0.56, size = 46, normalized size = 0.47

$$-\frac{35\,b^4x^4 + 105\,ab^3x^3 + 126\,a^2b^2x^2 + 70\,a^3bx + 15\,a^4}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*x^4 + 105*a*b^3*x^3 + 126*a^2*b^2*x^2 + 70*a^3*b*x + 15*a^4)
/x^7
```

giac [A] time = 0.21, size = 46, normalized size = 0.47

$$-\frac{35\,b^4x^4 + 105\,ab^3x^3 + 126\,a^2b^2x^2 + 70\,a^3bx + 15\,a^4}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*x^4 + 105*a*b^3*x^3 + 126*a^2*b^2*x^2 + 70*a^3*b*x + 15*a^4)
/x^7
```

maple [A] time = 0.15, size = 74, normalized size = 0.76

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^4}{7x^7}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{6x^6}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{5x^5}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{4x^4}-\frac{b}{12x^3}\right)}{5}\right)}{2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^8,x)
```

```
[Out] -1/7*arctanh(tanh(b*x+a))^4/x^7+4/7*b*(-1/6*arctanh(tanh(b*x+a))^3/x^6+1/2*
b*(-1/5*arctanh(tanh(b*x+a))^2/x^5+2/5*b*(-1/4*arctanh(tanh(b*x+a))/x^4-1/1
2*b/x^3)))
```

maxima [A] time = 0.60, size = 72, normalized size = 0.73

$$-\frac{1}{105}\left(b\left(\frac{b^2}{x^3}+\frac{3\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^4}\right)+\frac{6\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{x^5}\right)b-\frac{2\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{21\,x^6}-\operatorname{arta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(b*(b^2/x^3 + 3*b*arctanh(tanh(b*x + a))/x^4) + 6*b*arctanh(tanh(b*x
 + a))^2/x^5)*b - 2/21*b*arctanh(tanh(b*x + a))^3/x^6 - 1/7*arctanh(tanh(b*
x + a))^4/x^7
```

mupad [B] time = 1.01, size = 70, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^4}{7\,x^7}-\frac{b^4}{105\,x^3}-\frac{2\,b^2\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{35\,x^5}-\frac{b^3\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{35\,x^4}-\frac{2\,b\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^3}{21\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^8,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(7*x^7) - b^4/(105*x^3) - (2*b^2*atanh(tanh(a + b*
x))^2)/(35*x^5) - (b^3*atanh(tanh(a + b*x)))/(35*x^4) - (2*b*atanh(tanh(a +
 b*x))^3)/(21*x^6)
```

sympy [A] time = 5.41, size = 80, normalized size = 0.82

$$-\frac{b^4}{105x^3}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{35x^4}-\frac{2b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{35x^5}-\frac{2b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{21x^6}-\frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**8,x)
```

```
[Out] -b**4/(105*x**3) - b**3*atanh(tanh(a + b*x))/(35*x**4) - 2*b**2*atanh(tanh(
a + b*x))**2/(35*x**5) - 2*b*atanh(tanh(a + b*x))**3/(21*x**6) - atanh(tanh
(a + b*x))**4/(7*x**7)
```

3.81 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^9}\,dx$

Optimal. Leaf size=80

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{70x^5}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{28x^6}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{8x^8}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{14x^7}-\frac{b^4}{280x^4}$$

```
[Out] -1/280*b^4/x^4-1/70*b^3*arctanh(tanh(b*x+a))/x^5-1/28*b^2*arctanh(tanh(b*x+
a))^2/x^6-1/14*b*arctanh(tanh(b*x+a))^3/x^7-1/8*arctanh(tanh(b*x+a))^4/x^8
```

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.65, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2171, 2167}

$$\frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{b^3\tanh^{-1}(\tanh(a+bx))^5}{280x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^9,x]
```

```
[Out] (b^3*ArcTanh[Tanh[a + b*x]]^5)/(280*x^5*(b*x - ArcTanh[Tanh[a + b*x]])^4) +
 (b^2*ArcTanh[Tanh[a + b*x]]^5)/(56*x^6*(b*x - ArcTanh[Tanh[a + b*x]])^3) +
 (3*b*ArcTanh[Tanh[a + b*x]]^5)/(56*x^7*(b*x - ArcTanh[Tanh[a + b*x]])^2) +
 ArcTanh[Tanh[a + b*x]]^5/(8*x^8*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^9}\,dx &= \frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(3b)\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^8}\,dx}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\
&= \frac{3b\tanh^{-1}(\tanh(a+bx))^5}{56x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{\tanh^{-1}(\tanh(a+bx))^5}{8x^8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\ldots\\
&= \frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{3b\tanh^{-1}(\tanh(a+bx))^5}{56x^7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\ldots\\
&= \frac{b^3\tanh^{-1}(\tanh(a+bx))^5}{280x^5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{b^2\tanh^{-1}(\tanh(a+bx))^5}{56x^6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\ldots
\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.89

$$-\frac{4b^3x^3\tanh^{-1}(\tanh(a+bx))+10b^2x^2\tanh^{-1}(\tanh(a+bx))^2+20bx\tanh^{-1}(\tanh(a+bx))^3+35\tanh^{-1}(\tanh(}{280x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^9,x]
```

```
[Out] -1/280*(b^4*x^4 + 4*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 10*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 + 20*b*x*ArcTanh[Tanh[a + b*x]]^3 + 35*ArcTanh[Tanh[a + b*x]]
^4)/x^8
```

fricas [A] time = 0.47, size = 46, normalized size = 0.58

$$-\frac{70\,b^4x^4+224\,ab^3x^3+280\,a^2b^2x^2+160\,a^3bx+35\,a^4}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="fricas")
```

```
[Out] -1/280*(70*b^4*x^4 + 224*a*b^3*x^3 + 280*a^2*b^2*x^2 + 160*a^3*b*x + 35*a^4
)/x^8
```

giac [A] time = 0.15, size = 46, normalized size = 0.58

$$-\frac{70\,b^4x^4+224\,ab^3x^3+280\,a^2b^2x^2+160\,a^3bx+35\,a^4}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="giac")
```

```
[Out] -1/280*(70*b^4*x^4 + 224*a*b^3*x^3 + 280*a^2*b^2*x^2 + 160*a^3*b*x + 35*a^4
)/x^8
```

maple [A] time = 0.15, size = 74, normalized size = 0.92

$$-\frac{\operatorname{arctanh}(\tanh(bx+a))^4}{8x^8}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^3}{7x^7}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^2}{6x^6}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{5x^5}-\frac{b}{20x^4}\right)}{3}\right)}{7}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^9,x)
```

```
[Out] -1/8*arctanh(tanh(b*x+a))^4/x^8+1/2*b*(-1/7/x^7*arctanh(tanh(b*x+a))^3+3/7*
b*(-1/6/x^6*arctanh(tanh(b*x+a))^2+1/3*b*(-1/5/x^5*arctanh(tanh(b*x+a))-1/2
0/x^4*b)))
```

maxima [A] time = 0.60, size = 72, normalized size = 0.90

$$-\frac{1}{280}\left(b\left(b\left(\frac{b^2}{x^4}+\frac{4\,b\,\operatorname{artanh}(\tanh(bx+a))}{x^5}\right)+\frac{10\,b\,\operatorname{artanh}(\tanh(bx+a))^2}{x^6}\right)b-\frac{b\,\operatorname{artanh}(\tanh(bx+a))^3}{14\,x^7}-\frac{\operatorname{artanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^9,x, algorithm="maxima")
```

```
[Out] -1/280*(b*(b^2/x^4 + 4*b*arctanh(tanh(b*x + a))/x^5) + 10*b*arctanh(tanh(b*
x + a))^2/x^6)*b - 1/14*b*arctanh(tanh(b*x + a))^3/x^7 - 1/8*arctanh(tanh(b
*x + a))^4/x^8
```

mupad [B] time = 1.07, size = 70, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^4}{8\,x^8}-\frac{b^4}{280\,x^4}-\frac{b^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{28\,x^6}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{70\,x^5}-\frac{b\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^3}{14\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^9,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(8*x^8) - b^4/(280*x^4) - (b^2*atanh(tanh(a + b*x)
)^2)/(28*x^6) - (b^3*atanh(tanh(a + b*x)))/(70*x^5) - (b*atanh(tanh(a + b*x
))^3)/(14*x^7)
```

sympy [A] time = 8.25, size = 76, normalized size = 0.95

$$-\frac{b^4}{280x^4}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{70x^5}-\frac{b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{28x^6}-\frac{b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{14x^7}-\frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**9,x)
```

```
[Out] -b**4/(280*x**4) - b**3*atanh(tanh(a + b*x))/(70*x**5) - b**2*atanh(tanh(a
+ b*x))**2/(28*x**6) - b*atanh(tanh(a + b*x))**3/(14*x**7) - atanh(tanh(a +
 b*x))**4/(8*x**8)
```

3.82 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{10}}\,dx$

Optimal. Leaf size=80

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8}-\frac{b^4}{630x^5}$$

```
[Out] -1/630*b^4/x^5-1/126*b^3*arctanh(tanh(b*x+a))/x^6-1/42*b^2*arctanh(tanh(b*x
+a))^2/x^7-1/18*b*arctanh(tanh(b*x+a))^3/x^8-1/9*arctanh(tanh(b*x+a))^4/x^9
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9}-\frac{b^4}{630x^5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^10,x]
```

```
[Out] -b^4/(630*x^5) - (b^3*ArcTanh[Tanh[a + b*x]])/(126*x^6) - (b^2*ArcTanh[Tanh
[a + b*x]]^2)/(42*x^7) - (b*ArcTanh[Tanh[a + b*x]]^3)/(18*x^8) - ArcTanh[Ta
nh[a + b*x]]^4/(9*x^9)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{10}}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9}+\frac{1}{9}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^9}\,dx\\ &= -\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9}+\frac{1}{6}b^2\int \frac{\tanh^{-1}(\tanh(a+b\\ &= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{b\tanh^{-1}(\tanh(a+bx))^3}{18x^8}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{9x^9}\\ &= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{b\tanh^{-1}(\tanh(a+bx))}{18x^8}\\ &= -\frac{b^4}{630x^5}-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{126x^6}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{42x^7}-\frac{b\tanh^{-1}(\tanh}{18x^8}\end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.89

$$-\frac{5b^3x^3\tanh^{-1}(\tanh(a+bx))+15b^2x^2\tanh^{-1}(\tanh(a+bx))^2+35bx\tanh^{-1}(\tanh(a+bx))^3+70\tanh^{-1}(\tanh(}{630x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^10,x]
```

```
[Out] -1/630*(b^4*x^4 + 5*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 15*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 + 35*b*x*ArcTanh[Tanh[a + b*x]]^3 + 70*ArcTanh[Tanh[a + b*x]]
^4)/x^9
```

fricas [A] time = 0.55, size = 46, normalized size = 0.58

$$-\frac{126\, b^{4} x^{4} + 420\, a b^{3} x^{3} + 540\, a^{2} b^{2} x^{2} + 315\, a^{3} b x + 70\, a^{4}}{630\, x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

giac [A] time = 0.19, size = 46, normalized size = 0.58

$$-\frac{126\, b^{4} x^{4} + 420\, a b^{3} x^{3} + 540\, a^{2} b^{2} x^{2} + 315\, a^{3} b x + 70\, a^{4}}{630\, x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

maple [A] time = 0.15, size = 74, normalized size = 0.92

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{4}}{9x^{9}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{3}}{8x^{8}}+\frac{3b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{7x^{7}}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{6x^{6}}-\frac{b}{30x^{5}}\right)}{7}\right)}{8}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^10,x)
```

```
[Out] -1/9*arctanh(tanh(b*x+a))^4/x^9+4/9*b*(-1/8/x^8*arctanh(tanh(b*x+a))^3+3/8*
b*(-1/7/x^7*arctanh(tanh(b*x+a))^2+2/7*b*(-1/6/x^6*arctanh(tanh(b*x+a))-1/3
0/x^5*b)))
```

maxima [A] time = 0.61, size = 72, normalized size = 0.90

$$-\frac{1}{630}\left(b\left(\frac{b^{2}}{x^{5}}+\frac{5\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^{6}}\right)+\frac{15\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{x^{7}}\right)b-\frac{b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{18\,x^{8}}-\operatorname{artan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^10,x, algorithm="maxima")
```

```
[Out] -1/630*(b*(b^2/x^5 + 5*b*arctanh(tanh(b*x + a))/x^6) + 15*b*arctanh(tanh(b*
x + a))^2/x^7)*b - 1/18*b*arctanh(tanh(b*x + a))^3/x^8 - 1/9*arctanh(tanh(b
*x + a))^4/x^9
```

mupad [B] time = 1.10, size = 70, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^4}{9\,x^9}-\frac{b^4}{630\,x^5}-\frac{b^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{42\,x^7}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{126\,x^6}-\frac{b\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^3}{18\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^10,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(9*x^9) - b^4/(630*x^5) - (b^2*atanh(tanh(a + b*x)
)^2)/(42*x^7) - (b^3*atanh(tanh(a + b*x)))/(126*x^6) - (b*atanh(tanh(a + b*
x))^3)/(18*x^8)
```

sympy [A] time = 13.00, size = 76, normalized size = 0.95

$$-\frac{b^4}{630x^5}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{126x^6}-\frac{b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{42x^7}-\frac{b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{18x^8}-\frac{\operatorname{atanh}^4\left(\tanh\left(a+bx\right)\right)}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**10,x)
```

```
[Out] -b**4/(630*x**5) - b**3*atanh(tanh(a + b*x))/(126*x**6) - b**2*atanh(tanh(a
 + b*x))**2/(42*x**7) - b*atanh(tanh(a + b*x))**3/(18*x**8) - atanh(tanh(a
+ b*x))**4/(9*x**9)
```

3.83 $\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{11}}\,dx$

Optimal. Leaf size=80

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9}-\frac{12}{}$$

```
[Out] -1/1260*b^4/x^6-1/210*b^3*arctanh(tanh(b*x+a))/x^7-1/60*b^2*arctanh(tanh(b*
x+a))^2/x^8-2/45*b*arctanh(tanh(b*x+a))^3/x^9-1/10*arctanh(tanh(b*x+a))^4/x
^10
```

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}}-\frac{12}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^4/x^11,x]
```

```
[Out] -b^4/(1260*x^6) - (b^3*ArcTanh[Tanh[a + b*x]])/(210*x^7) - (b^2*ArcTanh[Tan
h[a + b*x]]^2)/(60*x^8) - (2*b*ArcTanh[Tanh[a + b*x]]^3)/(45*x^9) - ArcTanh
[Tanh[a + b*x]]^4/(10*x^10)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^4}{x^{11}}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}}+\frac{1}{5}(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{10}}\,dx\\
&= -\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9}-\frac{\tanh^{-1}(\tanh(a+bx))^4}{10x^{10}}+\frac{1}{15}\left(2b^2\right)\int \frac{\tanh^{-1}(\mathrm{ta}}{}\\
&= -\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{2b\tanh^{-1}(\tanh(a+bx))^3}{45x^9}-\frac{\tanh^{-1}(\tanh(a+b}{10x^{10}}\\
&= -\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{2b\tanh^{-1}(\tanh(a+}{45x^9}\\
&= -\frac{b^4}{1260x^6}-\frac{b^3\tanh^{-1}(\tanh(a+bx))}{210x^7}-\frac{b^2\tanh^{-1}(\tanh(a+bx))^2}{60x^8}-\frac{2b\tanh^{-1}}{}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.89

$$-\frac{6b^3x^3\tanh^{-1}(\tanh(a+bx))+21b^2x^2\tanh^{-1}(\tanh(a+bx))^2+56bx\tanh^{-1}(\tanh(a+bx))^3+126\tanh^{-1}(\mathrm{ta}}{1260x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^4/x^11,x]
```

```
[Out] -1/1260*(b^4*x^4 + 6*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 21*b^2*x^2*ArcTanh[Ta
nh[a + b*x]]^2 + 56*b*x*ArcTanh[Tanh[a + b*x]]^3 + 126*ArcTanh[Tanh[a + b*x
]]^4)/x^10
```

fricas [A] time = 0.50, size = 46, normalized size = 0.58

$$-\frac{210\,b^{4}x^{4}+720\,ab^{3}x^{3}+945\,a^{2}b^{2}x^{2}+560\,a^{3}bx+126\,a^{4}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(210*b^4*x^4 + 720*a*b^3*x^3 + 945*a^2*b^2*x^2 + 560*a^3*b*x + 126*
a^4)/x^10
```

giac [A] time = 0.26, size = 46, normalized size = 0.58

$$-\frac{210\,b^{4}x^{4}+720\,ab^{3}x^{3}+945\,a^{2}b^{2}x^{2}+560\,a^{3}bx+126\,a^{4}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="giac")
```

```
[Out] -1/1260*(210*b^4*x^4 + 720*a*b^3*x^3 + 945*a^2*b^2*x^2 + 560*a^3*b*x + 126*
a^4)/x^10
```

maple [A] time = 0.15, size = 74, normalized size = 0.92

$$-\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{4}}{10x^{10}}+\frac{2b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{3}}{9x^{9}}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{8x^{8}}+\frac{b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{7x^{7}}-\frac{b}{42x^{6}}\right)}{4}\right)}{3}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^4/x^11,x)
```

```
[Out] -1/10*arctanh(tanh(b*x+a))^4/x^10+2/5*b*(-1/9/x^9*arctanh(tanh(b*x+a))^3+1/
3*b*(-1/8/x^8*arctanh(tanh(b*x+a))^2+1/4*b*(-1/7/x^7*arctanh(tanh(b*x+a))-1
/42/x^6*b)))
```

maxima [A] time = 0.61, size = 72, normalized size = 0.90

$$-\frac{1}{1260}\left(b\left(\frac{b^{2}}{x^{6}}+\frac{6\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{x^{7}}\right)+\frac{21\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{x^{8}}\right)b-\frac{2\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{45\,x^{9}}-\mathrm{artan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^4/x^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(b*(b^2/x^6 + 6*b*arctanh(tanh(b*x + a))/x^7) + 21*b*arctanh(tanh(b
*x + a))^2/x^8)*b - 2/45*b*arctanh(tanh(b*x + a))^3/x^9 - 1/10*arctanh(tanh
(b*x + a))^4/x^10
```

mupad [B] time = 1.04, size = 70, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^4}{10\,x^{10}}-\frac{b^4}{1260\,x^6}-\frac{b^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}{60\,x^8}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{210\,x^7}-\frac{2\,b\operatorname{atanh}\left(\tanh\left(a+\right.\right.}{45\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^4/x^11,x)
```

```
[Out] - atanh(tanh(a + b*x))^4/(10*x^10) - b^4/(1260*x^6) - (b^2*atanh(tanh(a + b
*x))^2)/(60*x^8) - (b^3*atanh(tanh(a + b*x)))/(210*x^7) - (2*b*atanh(tanh(a
 + b*x))^3)/(45*x^9)
```

sympy [A] time = 19.49, size = 78, normalized size = 0.98

$$-\frac{b^4}{1260x^6}-\frac{b^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{210x^7}-\frac{b^2\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{60x^8}-\frac{2b\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{45x^9}-\frac{\operatorname{atanh}^4\left(\tanh\left(a+\right.\right.}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**4/x**11,x)
```

```
[Out] -b**4/(1260*x**6) - b**3*atanh(tanh(a + b*x))/(210*x**7) - b**2*atanh(tanh(
a + b*x))**2/(60*x**8) - 2*b*atanh(tanh(a + b*x))**3/(45*x**9) - atanh(tanh
(a + b*x))**4/(10*x**10)
```

3.84 $\int x \tanh^{-1}(\tanh(a+bx))^6\,dx$

Optimal. Leaf size=34

$$\frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}$$

```
[Out] 1/7*x*arctanh(tanh(b*x+a))^7/b-1/56*arctanh(tanh(b*x+a))^8/b^2
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^6,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^7)/(7*b) - ArcTanh[Tanh[a + b*x]]^8/(56*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^6\,dx &= \frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\int \tanh^{-1}(\tanh(a+bx))^7\,dx}{7b}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\text{Subst}\left(\int x^7\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{7b^2}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^7}{7b}-\frac{\tanh^{-1}(\tanh(a+bx))^8}{56b^2}\end{aligned}$$

Mathematica [B] time = 0.14, size = 177, normalized size = 5.21

$$-\frac{(a+bx)\left(-56\left(2a^2+abx-b^2x^2\right)\tanh^{-1}(\tanh(a+bx))^5+(7a-bx)(a+bx)^6-8(6a-bx)(a+bx)^5\tanh^{-1}(\tanh\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^6,x]
```

```
[Out] -1/56*((a + b*x)*((7*a - b*x)*(a + b*x)^6 - 8*(6*a - b*x)*(a + b*x)^5*ArcTa
nh[Tanh[a + b*x]] + 28*(5*a - b*x)*(a + b*x)^4*ArcTanh[Tanh[a + b*x]]^2 - 5
6*(4*a - b*x)*(a + b*x)^3*ArcTanh[Tanh[a + b*x]]^3 + 70*(3*a - b*x)*(a + b*
x)^2*ArcTanh[Tanh[a + b*x]]^4 - 56*(2*a^2 + a*b*x - b^2*x^2)*ArcTanh[Tanh[a
 + b*x]]^5 + 28*(a - b*x)*ArcTanh[Tanh[a + b*x]]^6))/b^2
```

fricas [B] time = 0.47, size = 68, normalized size = 2.00

$$\frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="fricas")
```

```
[Out] 1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b^
2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

giac [B] time = 0.19, size = 68, normalized size = 2.00

$$\frac{1}{8}\,b^6x^8 + \frac{6}{7}\,ab^5x^7 + \frac{5}{2}\,a^2b^4x^6 + 4\,a^3b^3x^5 + \frac{15}{4}\,a^4b^2x^4 + 2\,a^5bx^3 + \frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8 + 6/7*a*b^5*x^7 + 5/2*a^2*b^4*x^6 + 4*a^3*b^3*x^5 + 15/4*a^4*b^
2*x^4 + 2*a^5*b*x^3 + 1/2*a^6*x^2
```

maple [B] time = 0.17, size = 110, normalized size = 3.24

$$\frac{x^2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^6}{2} - 3b\left(\frac{x^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^5}{3} - \frac{5b\left(\frac{x^4\operatorname{arctanh}(\tanh(bx+a))^4}{4} - b\left(\frac{x^5\operatorname{arctanh}(\tanh(bx+a}{5}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^6,x)
```

```
[Out] 1/2*x^2*arctanh(tanh(b*x+a))^6-3*b*(1/3*x^3*arctanh(tanh(b*x+a))^5-5/3*b*(1
/4*x^4*arctanh(tanh(b*x+a))^4-b*(1/5*x^5*arctanh(tanh(b*x+a))^3-3/5*b*(1/6*
x^6*arctanh(tanh(b*x+a))^2-1/3*b*(1/7*x^7*arctanh(tanh(b*x+a))-1/56*x^8*b))
)))
```

maxima [B] time = 0.72, size = 110, normalized size = 3.24

$$-bx^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^5 + \frac{1}{2}\,x^2\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^6 + \frac{1}{56}\left(70\,bx^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^4 - \left(56\,bx^5\operatorname{arta}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^6,x, algorithm="maxima")
```

```
[Out] -b*x^3*arctanh(tanh(b*x + a))^5 + 1/2*x^2*arctanh(tanh(b*x + a))^6 + 1/56*(
70*b*x^4*arctanh(tanh(b*x + a))^4 - (56*b*x^5*arctanh(tanh(b*x + a))^3 - (2
8*b*x^6*arctanh(tanh(b*x + a))^2 + (b^2*x^8 - 8*b*x^7*arctanh(tanh(b*x + a)
))*b)*b)*b)*b
```

mupad [B] time = 1.10, size = 104, normalized size = 3.06

$$\frac{b^6 x^8}{56} - \frac{b^5 x^7 \operatorname{atanh}(\tanh(a + b x))}{7} + \frac{b^4 x^6 \operatorname{atanh}(\tanh(a + b x))^2}{2} - b^3 x^5 \operatorname{atanh}(\tanh(a + b x))^3 + \frac{5 b^2 x^4 \operatorname{atanh}(\mathrm{ta}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^6,x)
```

```
[Out] (x^2*atanh(tanh(a + b*x))^6)/2 + (b^6*x^8)/56 + (5*b^2*x^4*atanh(tanh(a + b
*x))^4)/4 - b^3*x^5*atanh(tanh(a + b*x))^3 + (b^4*x^6*atanh(tanh(a + b*x))^
2)/2 - b*x^3*atanh(tanh(a + b*x))^5 - (b^5*x^7*atanh(tanh(a + b*x)))/7
```

sympy [A] time = 6.54, size = 41, normalized size = 1.21

$$\begin{cases} \frac{x \operatorname{atanh}^{7}(\tanh(a+bx))}{7b} - \frac{\operatorname{atanh}^{8}(\tanh(a+bx))}{56b^2} & \text{for } b \neq 0 \\ \frac{x^2 \operatorname{atanh}^{6}(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**6,x)
```

```
[Out] Piecewise((x*atanh(tanh(a + b*x))**7/(7*b) - atanh(tanh(a + b*x))**8/(56*b*
*2), Ne(b, 0)), (x**2*atanh(tanh(a))**6/2, True))
```

3.85 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=53

$$-\frac{x^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] -x^(1+m)*hypergeom([1, 1+m],[2+m],b*x/(b*x-arctanh(tanh(b*x+a))))/(1+m)/(b*
x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2164}

$$-\frac{x^{m+1} {}_2F_1\left(1,m+1;m+2;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(m+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] -((x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (b*x)/(b*x - ArcTanh[Tanh[a
 + b*x]])])/((1 + m)*(b*x - ArcTanh[Tanh[a + b*x]])))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))} \, dx = -\frac{x^{1+m} {}_2F_1\left(1,1+m;2+m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(1+m)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.96

$$\frac{x^{m+1} {}_2F_1\left(1,m+1;m+2;-\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\text{artanh}\left(\tanh\left(bx+a\right)\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(x^m/arctanh(tanh(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a)), x)
```

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arctanh}\left(\tanh\left(bx + a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a)),x)
```

```
[Out] int(x^m/arctanh(tanh(b*x+a)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/atanh(tanh(a + b*x)),x)
```

```
[Out] int(x^m/atanh(tanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**m/atanh(tanh(a + b*x)), x)
```

3.86 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=81

$$\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}+\frac{x^2\left(bx-\tanh^{-1}\right.}{2b}$$

```
[Out] 1/3*x^3/b+1/2*x^2*(b*x-arctanh(tanh(b*x+a)))/b^2+x*(b*x-arctanh(tanh(b*x+a)
))^2/b^3+(b*x-arctanh(tanh(b*x+a)))^3*ln(arctanh(tanh(b*x+a)))/b^4
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2159, 2158, 2157, 29}

$$\frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \log\left(\tanh^{-1}\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) + (x^2*(b*x - ArcTanh[Tanh[a + b*x]]))/(2*b^2) + (x*(b*x - ArcTan
h[Tanh[a + b*x]])^2)/b^3 + ((b*x - ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh[Ta
nh[a + b*x]]])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{x^3}{3b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \int \frac{}{\tanh^{-1}(\text{t}}}{b^2}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{(-bx+}{}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3} - \frac{(-bx+}{}$$

$$= \frac{x^3}{3b} + \frac{x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{2b^2} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3} + \frac{(bx-}{}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.98

$$-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4} + \frac{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{b^3} - \frac{x^2\left(\tanh^{-1}(\tanh(\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^3/(3*b) - (x^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(2*b^2) + (x*(-(b*x) +
ArcTanh[Tanh[a + b*x]])^2)/b^3 - ((-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[A
rcTanh[Tanh[a + b*x]]])/b^4
```

fricas [A] time = 0.55, size = 41, normalized size = 0.51

$$\frac{2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

giac [A] time = 0.23, size = 43, normalized size = 0.53

$$-\frac{a^3\log\left(|bx+a|\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

maple [B] time = 0.14, size = 202, normalized size = 2.49

$$\frac{x^3}{3b} - \frac{a\,x^2}{2b^2} - \frac{x^2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{2b^2} + \frac{a^2x}{b^3} + \frac{2xa\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{b^3} + \frac{x\left(\operatorname{arctanh}\left(\tanh\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/3*x^3/b-1/2/b^2*a*x^2-1/2/b^2*x^2*(arctanh(tanh(b*x+a))-b*x-a)+1/b^3*a^2*
x+2/b^3*x*a*(arctanh(tanh(b*x+a))-b*x-a)+1/b^3*x*(arctanh(tanh(b*x+a))-b*x-
a)^2-1/b^4*ln(arctanh(tanh(b*x+a)))*a^3-3/b^4*ln(arctanh(tanh(b*x+a)))*a^2*
(arctanh(tanh(b*x+a))-b*x-a)-3/b^4*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh
(b*x+a))-b*x-a)^2-1/b^4*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-
a)^3
```

maxima [A] time = 0.51, size = 42, normalized size = 0.52

$$-\frac{a^3 \log\left(bx+a\right)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

mupad [B] time = 0.13, size = 354, normalized size = 4.37

$$\frac{x^3}{3\,b}+\frac{x^2\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{4\,b^2}+\frac{x\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4\,b^3}+\frac{\ln\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x)),x)
```

```
[Out] x^3/(3*b) + (x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(4*b^2) + (x*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2)/(4*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*((2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 -
 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)))/(8*b^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x)), x)
```

3.87 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=56

$$\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x^2}{2b}$$

```
[Out] 1/2*x^2/b+x*(b*x-arctanh(tanh(b*x+a)))/b^2+(b*x-arctanh(tanh(b*x+a)))^2*ln(
arctanh(tanh(b*x+a)))/b^3
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2159, 2158, 2157, 29}

$$\frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) + (x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^2 + ((b*x - ArcTanh[Tanh[a
 + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{x^2}{2b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \int \frac{}{\tanh^{-}}}{b^2}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \text{Subst}\left(\right.}{b^3}$$
$$= \frac{x^2}{2b} + \frac{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2} + \frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \log\left(\tanh\right.}{b^3}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.98

$$\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x^2/(2*b) - (x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^2 + ((-(b*x) + ArcTanh[
Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

fricas [A] time = 0.38, size = 29, normalized size = 0.52

$$\frac{b^2x^2 - 2\,abx + 2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

giac [A] time = 0.18, size = 30, normalized size = 0.54

$$\frac{a^2\log\left(|bx+a|\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

maple [B] time = 0.14, size = 111, normalized size = 1.98

$$\frac{x^2}{2b} - \frac{ax}{b^2} - \frac{x\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{b^2} + \frac{\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)a^2}{b^3} + \frac{2\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/2*x^2/b-1/b^2*a*x-1/b^2*x*(arctanh(tanh(b*x+a))-b*x-a)+1/b^3*ln(arctanh(t
anh(b*x+a)))*a^2+2/b^3*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-b*x
-a)+1/b^3*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

maxima [A] time = 0.52, size = 29, normalized size = 0.52

$$\frac{a^2\log(bx+a)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

mupad [B] time = 0.27, size = 234, normalized size = 4.18

$$\frac{x^2}{2\,b}+\frac{x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b^2}+\frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\left(\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x)),x)
```

```
[Out] x^2/(2*b) + (x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^2) + (log(log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*((2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*a^2
))/(4*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**2/atanh(tanh(a + b*x)), x)
```

3.88 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=31

$$\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}$$

```
[Out] x/b+(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(tanh(b*x+a)))/b^2
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2158, 2157, 29}

$$\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x/b + ((b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\tanh^{-1}(\tanh(a+bx))} dx &= \frac{x}{b}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\tanh^{-1}(\tanh(a+bx))} dx}{b}\\
&= \frac{x}{b}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\operatorname{Subst}\left(\int \frac{1}{x} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2}\\
&= \frac{x}{b}+\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 1.00

$$\frac{x}{b}-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] x/b - ((-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

fricas [A] time = 0.43, size = 17, normalized size = 0.55

$$\frac{bx - a\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

giac [A] time = 0.17, size = 19, normalized size = 0.61

$$\frac{x}{b} - \frac{a\log(|bx + a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

maple [A] time = 0.14, size = 49, normalized size = 1.58

$$\frac{x}{b} - \frac{\ln(\operatorname{arctanh}(\tanh(bx + a)))\,a}{b^2} - \frac{\ln(\operatorname{arctanh}(\tanh(bx + a)))\,(\operatorname{arctanh}(\tanh(bx + a)) - bx - a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a)),x)
```

```
[Out] x/b-1/b^2*ln(arctanh(tanh(b*x+a)))*a-1/b^2*ln(arctanh(tanh(b*x+a)))*(arctan
h(tanh(b*x+a))-b*x-a)
```

maxima [A] time = 0.53, size = 18, normalized size = 0.58

$$\frac{x}{b} - \frac{a\log(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

mupad [B] time = 0.15, size = 108, normalized size = 3.48

$$\frac{x}{b} + \frac{\ln\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x)),x)
```

```
[Out] x/b + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(
exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{atanh}(\tanh(a + bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x/atanh(tanh(a + b*x)), x)
```

3.89 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=12

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

```
[Out] ln(arctanh(tanh(b*x+a)))/b
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcTanh[Tanh[a + b*x]]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\tanh^{-1}(\tanh(a+bx))} dx &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b} \\ &= \frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.05, size = 12, normalized size = 1.00

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-1),x]
```

```
[Out] Log[ArcTanh[Tanh[a + b*x]]]/b
```

fricas [A] time = 0.49, size = 10, normalized size = 0.83

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

giac [**A**] time = 0.40, size = 11, normalized size = 0.92

$$\frac{\log\left(|bx+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] log(abs(b*x + a))/b
```

maple [**A**] time = 0.03, size = 13, normalized size = 1.08

$$\frac{\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a)),x)
[Out] ln(arctanh(tanh(b*x+a)))/b
```

maxima [**A**] time = 0.43, size = 13, normalized size = 1.08

$$\frac{\log\left(-bx-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a)),x, algorithm="maxima")
[Out] log(-b*x - a)/b
```

mupad [**B**] time = 1.06, size = 12, normalized size = 1.00

$$\frac{\ln\left(\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x)),x)
[Out] log(atanh(tanh(a + b*x)))/b
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a)),x)
[Out] Exception raised: TypeError
```

3.90 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=44

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -ln(x)/(b*x-arctanh(tanh(b*x+a)))+ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(tan
h(b*x+a)))
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2160, 2157, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] -(Log[x]/(b*x - ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/(b*x
 - ArcTanh[Tanh[a + b*x]])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))}\,dx &= -\frac{\int \frac{1}{x}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}+\frac{b\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}\\
&= -\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))}+\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}\\
&= -\frac{\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))}+\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{bx-\tanh^{-1}(\tanh(a+bx))}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.66

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)-\log(x)}{bx-\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-Log[x] + Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])
```

fricas [A] time = 0.46, size = 16, normalized size = 0.36

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

giac [A] time = 0.21, size = 20, normalized size = 0.45

$$-\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

maple [A] time = 0.14, size = 43, normalized size = 0.98

$$\frac{\ln(x)}{\operatorname{arctanh}(\tanh(bx+a))-bx}-\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{\operatorname{arctanh}(\tanh(bx+a))-bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a)),x)
```

```
[Out] 1/(arctanh(tanh(b*x+a))-b*x)*ln(x)-1/(arctanh(tanh(b*x+a))-b*x)*ln(arctanh(
tanh(b*x+a)))
```

maxima [A] time = 0.51, size = 18, normalized size = 0.41

$$-\frac{\log(bx+a)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

mupad [B] time = 2.87, size = 107, normalized size = 2.43

$$-\frac{4\,\mathrm{atanh}\left(\frac{4\,b\,x}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}-1\right)}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))),x)
```

```
[Out] -(4*atanh((4*b*x)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 1))/(log(1/(exp(2*a)*exp(2*b*x)
+ 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a)),x)
[Out] Integral(1/(x*atanh(tanh(a + b*x))), x)
```

3.91 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))} dx$

Optimal. Leaf size=65

$$\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{b\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 1/x/(b*x-arctanh(tanh(b*x+a)))-b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^2+b*ln(ar
ctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^2
```

Rubi **[A]** time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{b\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] 1/(x*(b*x - ArcTanh[Tanh[a + b*x]])) - (b*Log[x])/(b*x - ArcTanh[Tanh[a + b
*x]])^2 + (b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{b\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x}\,dx}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b^2\int \cdots}{\left(bx-\tan\cdots\right.}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b\,\mathrm{Subst}\left(\cdots\right.}{\left(bx\cdots\right.}$$

$$= \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{b\log\left(\tan\cdots\right.}{\left(bx-\tan\cdots\right.}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.69

$$\frac{bx\left(\log\left(\tanh^{-1}(\tanh(a+bx))\right)-\log(x)+1\right)-\tanh^{-1}(\tanh(a+bx))}{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]),x]
[Out] (-ArcTanh[Tanh[a + b*x]] + b*x*(1 - Log[x] + Log[ArcTanh[Tanh[a + b*x]]]))/
(x*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.38, size = 26, normalized size = 0.40

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

giac [A] time = 0.27, size = 30, normalized size = 0.46

$$\frac{b\log(|bx+a|)}{a^2} - \frac{b\log(|x|)}{a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

maple [A] time = 0.14, size = 64, normalized size = 0.98

$$-\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\,x} - \frac{b\ln(x)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2} + \frac{b\ln(\operatorname{arctanh}(\tanh(bx+a)))}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a)),x)
[Out] -1/(arctanh(tanh(b*x+a))-b*x)/x-1/(arctanh(tanh(b*x+a))-b*x)^2*b*ln(x)+1/(a
rctanh(tanh(b*x+a))-b*x)^2*b*ln(arctanh(tanh(b*x+a)))
```

maxima [A] time = 0.52, size = 28, normalized size = 0.43

$$\frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

mupad [B] time = 2.83, size = 210, normalized size = 3.23

$$\frac{2\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x+b\,x\,\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\mathrm{i}+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\right)\,8\mathrm{i}}{x\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))),x)
```

```
[Out] (2*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 4*b*x + b*x*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(
exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x))*8i)/(x*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\operatorname{atanh}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))), x)
```

3.92 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=92

$$-\frac{b^2 \log(x)}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{b^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{1}{2x^2\left(bx - \tanh^{-1}(\tanh(a+bx))\right)} + \frac{}{x\left(bx - \tanh\right.}$$

```
[Out] b/x/(b*x-arctanh(tanh(b*x+a)))^2+1/2/x^2/(b*x-arctanh(tanh(b*x+a)))-b^2*ln(
x)/(b*x-arctanh(tanh(b*x+a)))^3+b^2*ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(t
anh(b*x+a)))^3
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2163, 2160, 2157, 29}

$$-\frac{b^2 \log(x)}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{b^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{1}{2x^2\left(bx - \tanh^{-1}(\tanh(a+bx))\right)} + \frac{}{x\left(bx - \tanh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] b/(x*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 1/(2*x^2*(b*x - ArcTanh[Tanh[a + b
*x]])) - (b^2*Log[x])/(b*x - ArcTanh[Tanh[a + b*x]])^3 + (b^2*Log[ArcTanh[T
anh[a + b*x]]])/(b*x - ArcTanh[Tanh[a + b*x]])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{\left(bx-\text{ta}\right.}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{}{\left(bx-\text{ta}\right.}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{\left(bx-\text{ta}\right.}$$

$$= \frac{b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{\left(bx-\text{ta}\right.}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.72

$$\frac{b^2x^2\left(2\log\left(\tanh^{-1}(\tanh(a+bx))\right)-2\log(x)+3\right)-4bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-4*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2 + b^2*x^2*(3 - 2*
Log[x] + 2*Log[ArcTanh[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x
]])^3)
```

fricas [A] time = 0.50, size = 41, normalized size = 0.45

$$-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

giac [A] time = 0.14, size = 45, normalized size = 0.49

$$-\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

maple [A] time = 0.15, size = 87, normalized size = 0.95

$$-\frac{1}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^2}+\frac{b^2\ln(x)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3}+\frac{b}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2x}-\frac{b^2\ln}{(\text{arct}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a)),x)
```

```
[Out] -1/2/(arctanh(tanh(b*x+a))-b*x)/x^2+1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*ln(x
)+1/(arctanh(tanh(b*x+a))-b*x)^2*b/x-1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*ln(
arctanh(tanh(b*x+a)))
```

maxima [A] time = 0.52, size = 40, normalized size = 0.43

$$-\frac{b^2\log\left(bx+a\right)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

mupad [B] time = 2.89, size = 286, normalized size = 3.11

$$\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\left(2\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)+8\,b\,x\right)+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^2+12\,b^2\,x^2+8\,b\,x\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)+}{x^2\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))),x)
```

```
[Out] (log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))*(2*log(1/(exp(2*a)*exp(2*b*x) + 1)) + 8*b*x) + log((exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 12*b^2*x^2 + 8*b*x*log(1/(exp(
2*a)*exp(2*b*x) + 1)) + b^2*x^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(
exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x))*16i)/(x^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))), x)
```

3.93 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=65

$$-\frac{x^m \, _2F_1\left(1,m;m+1;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -x^m/b/arctanh(tanh(b*x+a))-x^m*hypergeom([1, m],[1+m],b*x/(b*x-arctanh(tan
h(b*x+a))))/b/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$-\frac{x^m \, _2F_1\left(1,m;m+1;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x^m/(b*ArcTanh[Tanh[a + b*x]])) - (x^m*Hypergeometric2F1[1, m, 1 + m, (b*
x)/(b*x - ArcTanh[Tanh[a + b*x]])])/(b*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^2} \, dx &= -\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}+\frac{m\int \frac{x^{-1+m}}{\tanh^{-1}(\tanh(a+bx))} \, dx}{b}\\ &= -\frac{x^m}{b\tanh^{-1}(\tanh(a+bx))}-\frac{x^m \, _2F_1\left(1,m;1+m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.55, size = 51, normalized size = 0.78

$$\frac{x^{m+1} \, _2F_1\left(2,m+1;m+2;-\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/arctanh(tanh(b*x + a))^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^2, x)
```

maple [F] time = 2.78, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] int(x^m/arctanh(tanh(b*x+a))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/atanh(tanh(a + b*x))^2,x)
```

```
[Out] int(x^m/atanh(tanh(a + b*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**m/atanh(tanh(a + b*x))**2, x)
```

3.94 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=98

$$\frac{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^5}+\frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{2x^2\left(bx-\tan\right.}{}$$

```
[Out] 4/3*x^3/b^2+2*x^2*(b*x-arctanh(tanh(b*x+a)))/b^3+4*x*(b*x-arctanh(tanh(b*x+
a)))^2/b^4-x^4/b/arctanh(tanh(b*x+a))+4*(b*x-arctanh(tanh(b*x+a)))^3*ln(arc
tanh(tanh(b*x+a)))/b^5
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\tan\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (4*x^3)/(3*b^2) + (2*x^2*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 + (4*x*(b*x -
ArcTanh[Tanh[a + b*x]])^2)/b^4 - x^4/(b*ArcTanh[Tanh[a + b*x]]) + (4*(b*x -
 ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} + \frac{4\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{4x^3}{3b^2} - \frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(4\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{x^2}{\tanh^{-1}(\tanh(}}{b^2}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^4}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(4\left(-bx+\text{t}\right.\right.}{}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\,\text{t}}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\,\text{t}}$$

$$= \frac{4x^3}{3b^2} + \frac{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} + \frac{4x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4} - \frac{}{b\,\text{t}}$$

Mathematica [A] time = 0.10, size = 106, normalized size = 1.08

$$-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}{b^5\tanh^{-1}(\tanh(a+bx))} - \frac{4\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^5} + \frac{3x\left(\tanh^{-1}(\tanh(\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] x^3/(3*b^2) - (x^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (3*x*(-(b*x) +
ArcTanh[Tanh[a + b*x]])^2)/b^4 - (-(b*x) + ArcTanh[Tanh[a + b*x]])^4/(b^5*A
rcTanh[Tanh[a + b*x]]) - (4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3*Log[ArcTanh
[Tanh[a + b*x]]])/b^5
```

fricas [A] time = 0.45, size = 73, normalized size = 0.74

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\left(a^3bx + a^4\right)\log\left(bx+a\right)}{3\left(b^6x + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

giac [A] time = 0.16, size = 62, normalized size = 0.63

$$-\frac{4\,a^3\log\left(|bx+a|\right)}{b^5} - \frac{a^4}{(bx+a)b^5} + \frac{b^4x^3 - 3\,ab^3x^2 + 9\,a^2b^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

maple [B] time = 0.15, size = 350, normalized size = 3.57

$$\frac{x^3}{3b^2}-\frac{a\,x^2}{b^3}-\frac{x^2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{b^3}+\frac{3a^2x}{b^4}+\frac{6a\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)x}{b^4}+\frac{3\,(\operatorname{arctanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/3*x^3/b^2-1/b^3*a*x^2-1/b^3*x^2*(arctanh(tanh(b*x+a))-b*x-a)+3/b^4*a^2*x+
6/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x+3/b^4*(arctanh(tanh(b*x+a))-b*x-a)^2
*x-4/b^5*ln(arctanh(tanh(b*x+a)))*a^3-12/b^5*ln(arctanh(tanh(b*x+a)))*a^2*(
arctanh(tanh(b*x+a))-b*x-a)-12/b^5*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh
(b*x+a))-b*x-a)^2-4/b^5*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-
a)^3-1/b^5/arctanh(tanh(b*x+a))*a^4-4/b^5/arctanh(tanh(b*x+a))*a^3*(arctanh
(tanh(b*x+a))-b*x-a)-6/b^5/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b
*x-a)^2-4/b^5/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^3-1/b^5/a
rctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^4
```

maxima [A] time = 0.75, size = 70, normalized size = 0.71

$$\frac{b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4}{3\left(b^6x+ab^5\right)}-\frac{4\,a^3\log\left(bx+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4)/(b^6*x + a*
b^5) - 4*a^3*log(b*x + a)/b^5
```

mupad [B] time = 0.18, size = 669, normalized size = 6.83

$$\frac{x^3}{3\,b^2}-\frac{\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4+24\,a^2\left(2\,a-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2+}{2\,b\left(8\,a\,b^4+8\,b^5\,x-4\,b^4\left(2\,a-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^2,x)
```

```
[Out] x^3/(3*b^2) - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*
a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x))/(2*b*(8*a*b^4 + 8*b^5*x - 4*b^4*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x))) + (x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^3) + (3*x*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^2)/(4*b^4) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*((2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - lo
```

```
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)))/(2*b^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**2, x)
```

3.95 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=75

$$\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^3}{b\tanh^{-1}(\tanh$$

```
[Out] 3/2*x^2/b^2+3*x*(b*x-arctanh(tanh(b*x+a)))/b^3-x^3/b/arctanh(tanh(b*x+a))+3
*(b*x-arctanh(tanh(b*x+a)))^2*ln(arctanh(tanh(b*x+a)))/b^4
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2168, 2159, 2158, 2157, 29}

$$\frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}+\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{b\tanh^{-1}(\tanh$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*x^2)/(2*b^2) + (3*x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 - x^3/(b*ArcTanh
[Tanh[a + b*x]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a +
 b*x]]])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{3x^2}{2b^2} - \frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{x}{\tanh^{-1}(\tanh(}}{b^2}$$

$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(3\left(-bx+\text{ta}\right.\right.}{}$$

$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(3\left(-bx+\text{ta}\right.\right.}{}$$

$$= \frac{3x^2}{2b^2} + \frac{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^3}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\text{tanh}\right.}{}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 1.11

$$\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}{b^4\tanh^{-1}(\tanh(a+bx))} + \frac{3\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{2x\left(\tanh^{-1}(\tanh(a\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] x^2/(2*b^2) - (2*x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (-(b*x) + ArcTa
nh[Tanh[a + b*x]])^3/(b^4*ArcTanh[Tanh[a + b*x]]) + (3*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^4
```

fricas [A] time = 0.45, size = 62, normalized size = 0.83

$$\frac{b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3 + 6\left(a^2bx + a^3\right)\log\left(bx+a\right)}{2\left(b^5x + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

giac [A] time = 0.17, size = 48, normalized size = 0.64

$$\frac{3\,a^2\log\left(|bx+a|\right)}{b^4} + \frac{a^3}{(bx+a)b^4} + \frac{b^2x^2 - 4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

maple [B] time = 0.15, size = 223, normalized size = 2.97

$$\frac{x^2}{2b^2} - \frac{2ax}{b^3} - \frac{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)x}{b^3} + \frac{3\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)a^2}{b^4} + \frac{6\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 1/2*x^2/b^2-2*a*x/b^3-2/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x+3/b^4*ln(arctanh
(tanh(b*x+a)))*a^2+6/b^4*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-b
*x-a)+3/b^4*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2+1/b^4/a
rctanh(tanh(b*x+a))*a^3+3/b^4/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a)
)-b*x-a)+3/b^4/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+1/b^4/
arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

maxima [A] time = 0.75, size = 59, normalized size = 0.79

$$\frac{b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3}{2\left(b^5x + ab^4\right)} + \frac{3\,a^2\log\left(bx + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3)/(b^5*x + a*b^4) + 3*a^2*log
(b*x + a)/b^4
```

mupad [B] time = 1.03, size = 490, normalized size = 6.53

$$\frac{x^2}{2\,b^2}+\frac{\ln\left(\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)\right)\,\left(3\,{\left(2\,a-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2-12\,a\,\left(2\,a-\ln\left(\right.\right.\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^2,x)
```

```
[Out] x^2/(2*b^2) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
 log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 12
*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 12*a^2))/(4*b^4) - ((2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))/(4*b*(2*a*b^3 + 2*b^4*x - b^3*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x))) + (x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{atanh}^2\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**2, x)
```

3.96 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=50

$$\frac{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))}+\frac{2x}{b^2}$$

```
[Out] 2*x/b^2-x^2/b/arctanh(tanh(b*x+a))+2*(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(
tanh(b*x+a)))/b^3
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2158, 2157, 29}

$$\frac{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))}+\frac{2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x)/b^2 - x^2/(b*ArcTanh[Tanh[a + b*x]]) + (2*(b*x - ArcTanh[Tanh[a + b*x
]])*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^2}{b\tanh^{-1}(\tanh(a+bx))} + \frac{2\int \frac{x}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x}{b^2} - \frac{x^2}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{\tanh^{-1}(\tan}}{b^2}$$

$$= \frac{2x}{b^2} - \frac{x^2}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(2\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\mathrm{Subst}\left(\int \frac{1}{x}\right.}{b^3}$$

$$= \frac{2x}{b^2} - \frac{x^2}{b\tanh^{-1}(\tanh(a+bx))} + \frac{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\mathrm{ta}\right.}{b^3}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 1.12

$$\frac{-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{\tanh^{-1}(\tanh(a+bx))} + 2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right) + bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (b*x - (-(b*x) + ArcTanh[Tanh[a + b*x]])^2/ArcTanh[Tanh[a + b*x]] + 2*(b*x
- ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]])/b^3
```

fricas [A] time = 0.57, size = 47, normalized size = 0.94

$$\frac{b^2x^2 + abx - a^2 - 2\left(abx + a^2\right)\log(bx+a)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

giac [A] time = 0.18, size = 34, normalized size = 0.68

$$\frac{x}{b^2} - \frac{2a\log(|bx+a|)}{b^3} - \frac{a^2}{(bx+a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

maple [B] time = 0.15, size = 127, normalized size = 2.54

$$\frac{x}{b^2} - \frac{a^2}{b^3\operatorname{arctanh}(\tanh(bx+a))} - \frac{2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{b^3\operatorname{arctanh}(\tanh(bx+a))} - \frac{(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2}{b^3\operatorname{arctanh}(\tanh(bx+a))} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] x/b^2-1/b^3/arctanh(tanh(b*x+a))*a^2-2/b^3/arctanh(tanh(b*x+a))*a*(arctanh(
tanh(b*x+a))-b*x-a)-1/b^3/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)
```

```
^2-2/b^3*ln(arctanh(tanh(b*x+a)))*a-2/b^3*ln(arctanh(tanh(b*x+a)))*(arctanh
(tanh(b*x+a))-b*x-a)
```

maxima [A] time = 0.76, size = 44, normalized size = 0.88

$$\frac{b^2x^2 + abx - a^2}{b^4x + ab^3} - \frac{2\,a\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] (b^2*x^2 + a*b*x - a^2)/(b^4*x + a*b^3) - 2*a*log(b*x + a)/b^3
```

mupad [B] time = 1.06, size = 302, normalized size = 6.04

$$\frac{x}{b^2} - \frac{\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2 - 4\,a\,\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right) + 4\,a^2}{2\,b\,\left(2\,a\,b^2 + 2\,b^3\,x - b^2\,\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)\right)} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^2,x)
```

```
[Out] x/b^2 - ((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x) + 4*a^2)/(2*b*(2*a*b^2 + 2*b^3*x - b^2*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))
 + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp
(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{atanh}^2\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**2/atanh(tanh(a + b*x))**2, x)
```

3.97 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=28

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -x/b/arctanh(tanh(b*x+a))+ln(arctanh(tanh(b*x+a)))/b^2
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 29}

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{x}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(x/(b*ArcTanh[Tanh[a + b*x]])) + Log[ArcTanh[Tanh[a + b*x]]]/b^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2} dx &= -\frac{x}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))} dx}{b}\\
&= -\frac{x}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2}\\
&= -\frac{x}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 0.96

$$\frac{-\frac{bx}{\tanh^{-1}(\tanh(a+bx))}+\log\left(\tanh^{-1}(\tanh(a+bx))\right)+1}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (1 - (b*x)/ArcTanh[Tanh[a + b*x]] + Log[ArcTanh[Tanh[a + b*x]]])/b^2
```

fricas [A] time = 0.43, size = 28, normalized size = 1.00

$$\frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

giac [A] time = 0.16, size = 24, normalized size = 0.86

$$\frac{\log(|bx+a|)}{b^2}+\frac{a}{(bx+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

maple [A] time = 0.14, size = 56, normalized size = 2.00

$$\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{b^2}+\frac{a}{b^2\operatorname{arctanh}(\tanh(bx+a))}+\frac{\operatorname{arctanh}(\tanh(bx+a))-bx-a}{b^2\operatorname{arctanh}(\tanh(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] ln(arctanh(tanh(b*x+a)))/b^2+1/b^2/arctanh(tanh(b*x+a))*a+1/b^2/arctanh(tan
h(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)
```

maxima [A] time = 0.74, size = 26, normalized size = 0.93

$$\frac{a}{b^3x+ab^2}+\frac{\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

mupad [B] time = 0.08, size = 28, normalized size = 1.00

$$\frac{\ln(\operatorname{atanh}(\tanh(a+b\,x)))}{b^2}-\frac{x}{b\operatorname{atanh}(\tanh(a+b\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x))^2,x)
```

```
[Out] log(atanh(tanh(a + b*x)))/b^2 - x/(b*atanh(tanh(a + b*x)))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Exception raised: TypeError
```

3.98 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2} dx$

Optimal. Leaf size=14

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -1/b/arctanh(tanh(b*x+a))
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-2),x]
```

```
[Out] -(1/(b*ArcTanh[Tanh[a + b*x]]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{1}{b\tanh^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-2),x]
```

```
[Out] -(1/(b*ArcTanh[Tanh[a + b*x]]))
```

fricas [A] time = 0.44, size = 13, normalized size = 0.93

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

giac [A] time = 0.20, size = 12, normalized size = 0.86

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
[Out] -1/((b*x + a)*b)
```

maple [A] time = 0.03, size = 15, normalized size = 1.07

$$-\frac{1}{b\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^2,x)
[Out] -1/b/arctanh(tanh(b*x+a))
```

maxima [A] time = 0.42, size = 12, normalized size = 0.86

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] -1/((b*x + a)*b)
```

mupad [B] time = 0.08, size = 14, normalized size = 1.00

$$-\frac{1}{b\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^2,x)
[Out] -1/(b*atanh(tanh(a + b*x)))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**2,x)
[Out] Exception raised: TypeError
```

3.99 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=70

$$-\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{\log\left(\tanh^{-1}(\tanh(a+\right.}{\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

```
[Out] -1/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))+ln(x)/(b*x-arctanh(tanh(
b*x+a)))^2-ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^2
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2163, 2160, 2157, 29}

$$-\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{\log\left(\tanh^{-1}(\tanh(a+\right.}{\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] -(1/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]])) + Log[x]/(b*x
- ArcTanh[Tanh[a + b*x]])^2 - Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Ta
nh[a + b*x]])^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))}}{-bx+\tanh^{-1}(\tanh(a+}$$

$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} - \frac{}{\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

$$= -\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{\log(x)}{\left(bx-\tanh^{-1}(\tanh(a+\right.}$$

Mathematica [A] time = 0.08, size = 53, normalized size = 0.76

$$\frac{\tanh^{-1}(\tanh(a+bx))\left(-\log\left(\tanh^{-1}(\tanh(a+bx))\right)+\log(bx)+1\right)-bx}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^2),x]
[Out] (-(b*x) + ArcTanh[Tanh[a + b*x]]*(1 + Log[b*x] - Log[ArcTanh[Tanh[a + b*x]]
]))/(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.63, size = 39, normalized size = 0.56

$$-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

giac [A] time = 0.44, size = 31, normalized size = 0.44

$$-\frac{\log(|bx+a|)}{a^2}+\frac{\log(|x|)}{a^2}+\frac{1}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

maple [A] time = 0.15, size = 67, normalized size = 0.96

$$\frac{\ln(x)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}-\frac{\ln(\operatorname{arctanh}(\tanh(bx+a)))}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2}+\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^2,x)
[Out] 1/(arctanh(tanh(b*x+a))-b*x)^2*ln(x)-1/(arctanh(tanh(b*x+a))-b*x)^2*ln(arct
anh(tanh(b*x+a)))+1/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))
```

maxima [A] time = 0.75, size = 28, normalized size = 0.40

$$\frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

mupad [B] time = 3.39, size = 359, normalized size = 5.13

$$\frac{8\,b\,x-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\left(-4+\operatorname{atan}\left(\frac{-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\right)8\mathrm{i}\right)+\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\left(-4+\operatorname{atan}\left(\frac{-\ln\left(\cdots\right.}{\ln\cdots}\right.\right.}{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^2),x)
```

```
[Out] (8*b*x - log(1/(exp(2*a)*exp(2*b*x) + 1))*(atan((log((exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2
i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))*8i - 4) + log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))*(atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*
1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b
*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*
8i - 4))/((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**2), x)
```

3.100 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=102

$$-\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{}{(b}$$

```
[Out] -2*b/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))+1/x/(b*x-arctanh(tan
h(b*x+a)))/arctanh(tanh(b*x+a))+2*b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^3-2*b*
ln(arctanh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^3
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$-\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))}+\frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))}+\frac{}{(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-2*b)/((b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) + 1/(x*(b*
x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (2*b*Log[x])/(b*x - A
rcTanh[Tanh[a + b*x]])^3 - (2*b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh
[Tanh[a + b*x]])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} - \frac{(2b)\int \frac{1}{x\tanh^{-1}(\tanh(}{-bx+\tanh^{-1}(\tanh}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\tan\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\tan\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\tan\right.}$$

$$= -\frac{2b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{x\left(bx-\tanh^{-1}(\tan\right.}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 0.69

$$\frac{\tanh^{-1}(\tanh(a+bx))^2 + 2bx\tanh^{-1}(\tanh(a+bx))\left(\log(x)-\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right) - b^2x^2}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-(b^2*x^2) + ArcTanh[Tanh[a + b*x]]^2 + 2*b*x*ArcTanh[Tanh[a + b*x]]*(Log[
x] - Log[ArcTanh[Tanh[a + b*x]]]))/(x*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcT
anh[Tanh[a + b*x]])
```

fricas [A] time = 0.54, size = 63, normalized size = 0.62

$$-\frac{2\,abx + a^2 - 2\left(b^2x^2 + abx\right)\log\left(bx+a\right) + 2\left(b^2x^2 + abx\right)\log(x)}{a^3bx^2 + a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

giac [A] time = 0.16, size = 45, normalized size = 0.44

$$\frac{2\,b\log\left(|bx+a|\right)}{a^3} - \frac{2\,b\log\left(|x|\right)}{a^3} - \frac{2\,bx+a}{\left(bx^2+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

maple [A] time = 0.15, size = 91, normalized size = 0.89

$$-\frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2 x} - \frac{2b\ln(x)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3} - \frac{b}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2\operatorname{arctanh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^2/x-2/(arctanh(tanh(b*x+a))-b*x)^3*b*ln(x)-1/
(arctanh(tanh(b*x+a))-b*x)^2*b/arctanh(tanh(b*x+a))+2/(arctanh(tanh(b*x+a))
-b*x)^3*b*ln(arctanh(tanh(b*x+a)))
```

maxima [A] time = 0.74, size = 45, normalized size = 0.44

$$-\frac{2\,bx + a}{a^2bx^2 + a^3x} + \frac{2\,b\log\left(bx + a\right)}{a^3} - \frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

mupad [B] time = 3.44, size = 432, normalized size = 4.24

$$-\frac{4\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2 - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\left(8\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right) + b\,x\,\mathrm{atan}\left(\frac{-\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)1\mathrm{i}+\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)1\mathrm{i}+b\,x\,2\mathrm{i}}{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\right)32\mathrm{i}\right) + 4\ln}{x\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\right)\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^2),x)
```

```
[Out] -(4*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))*(8*log(1/(exp(2*a)*exp(2*b*x) + 1)) + b*x*atan((log((exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x
) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*32i) + 4*log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 16*b^2*x^2 + b*x*log(1/(exp(2*a)*exp(2*
b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i -
log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) +
 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*32i)/
(x*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\,\mathrm{atanh}^2\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**2), x)
```

3.101 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=143

$$-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}+\frac{3b^2 \log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{3b^2 \log\left(\tanh^{-1}(\tanh(\right.}{\left(bx-\tanh^{-1}(\tanh(a\right.}$$

```
[Out] -3*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))+3/2*b/x/(b*x-arcta
nh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))+1/2/x^2/(b*x-arctanh(tanh(b*x+a)))/
arctanh(tanh(b*x+a))+3*b^2*ln(x)/(b*x-arctanh(tanh(b*x+a)))^4-3*b^2*ln(arct
anh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^4
```

Rubi [A] time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2171, 2163, 2160, 2157, 29}

$$-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}+\frac{3b^2 \log(x)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{3b^2 \log\left(\tanh^{-1}(\tanh(\right.}{\left(bx-\tanh^{-1}(\tanh(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-3*b^2)/((b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]) + (3*b)/
(2*x*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) + 1/(2*x^2*(b
*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]) + (3*b^2*Log[x])/(b*x
- ArcTanh[Tanh[a + b*x]])^4 - (3*b^2*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - Ar
cTanh[Tanh[a + b*x]])^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^2}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))} + \frac{(3b)\int \frac{1}{x^2\tanh^{-1}(\tanh}}{2\left(bx-\tanh^{-1}(\tanh\right.}$$

$$= \frac{3b}{2x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x^2\left(bx-\tanh^{-1}(\tanh\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-1}(\tanh\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-1}(\tanh\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-1}(\tanh\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))} + \frac{}{2x\left(bx-\tanh^{-1}(\tanh\right.}$$

Mathematica [A] time = 0.05, size = 92, normalized size = 0.64

$$-\frac{-3b^2x^2\tanh^{-1}(\tanh(a+bx))\left(-2\log\left(\tanh^{-1}(\tanh(a+bx))\right)+2\log(x)-1\right)-6bx\tanh^{-1}(\tanh(a+bx))^2+\tan}{2x^2\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] -1/2*(2*b^3*x^3 - 6*b*x*ArcTanh[Tanh[a + b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3
 - 3*b^2*x^2*ArcTanh[Tanh[a + b*x]]*(-1 + 2*Log[x] - 2*Log[ArcTanh[Tanh[a +
 b*x]]]))/(x^2*ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.45, size = 86, normalized size = 0.60

$$\frac{6\,ab^2x^2+3\,a^2bx-a^3-6\left(b^3x^3+ab^2x^2\right)\log\left(bx+a\right)+6\left(b^3x^3+ab^2x^2\right)\log(x)}{2\left(a^4bx^3+a^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

giac [A] time = 0.19, size = 64, normalized size = 0.45

$$-\frac{3\,b^2\log\left(|bx+a|\right)}{a^4}+\frac{3\,b^2\log\left(|x|\right)}{a^4}+\frac{6\,ab^2x^2+3\,a^2bx-a^3}{2\,(bx+a)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

maple [A] time = 0.15, size = 116, normalized size = 0.81

$$-\frac{1}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}x^{2}}+\frac{3b^{2}\ln(x)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}}+\frac{2b}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}x}-\frac{3}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -1/2/(arctanh(tanh(b*x+a))-b*x)^2/x^2+3/(arctanh(tanh(b*x+a))-b*x)^4*b^2*ln
(x)+2/(arctanh(tanh(b*x+a))-b*x)^3*b/x-3/(arctanh(tanh(b*x+a))-b*x)^4*b^2*l
n(arctanh(tanh(b*x+a)))+1/(arctanh(tanh(b*x+a))-b*x)^3*b^2/arctanh(tanh(b*x
+a))
```

maxima [A] time = 1.02, size = 64, normalized size = 0.45

$$\frac{6\,b^{2}x^{2}+3\,abx-a^{2}}{2\left(a^{3}bx^{3}+a^{4}x^{2}\right)}-\frac{3\,b^{2}\log\left(bx+a\right)}{a^{4}}+\frac{3\,b^{2}\log(x)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

mupad [B] time = 3.72, size = 660, normalized size = 4.62

$$-\frac{6\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{2}-6\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{3}-2\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{3}-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^2),x)
```

```
[Out] -(6*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))^2 - 6*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*log(1/(exp(2*a)*exp(2*b*x) + 1))^3 -
2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3 - 32*b^3*x^3 + 24*
b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 24*b^2*x^2*log
(1/(exp(2*a)*exp(2*b*x) + 1)) - 24*b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 24*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 + b^2*x^2*
log(1/(exp(2*a)*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/
(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x))*96i - b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - lo
g(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1
)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*96i - 4
8*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)))/(x^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{atanh}^2(\tanh(a+bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**2), x)
```

3.102 $\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=94

$$-\frac{mx^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{mx^{m-1}}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/2*x^m/b/arctanh(tanh(b*x+a))^2-1/2*m*x^(-1+m)/b^2/arctanh(tanh(b*x+a))-1
/2*m*x^(-1+m)*hypergeom([1, -1+m],[m],b*x/(b*x-arctanh(tanh(b*x+a))))/b^2/(
b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$-\frac{mx^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2b^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{mx^{m-1}}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -x^m/(2*b*ArcTanh[Tanh[a + b*x]]^2) - (m*x^(-1 + m))/(2*b^2*ArcTanh[Tanh[a
+ b*x]]) - (m*x^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, (b*x)/(b*x - ArcTa
nh[Tanh[a + b*x]])])/(2*b^2*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{\tanh^{-1}(\tanh(a+bx))^3} dx &= -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{m\int \frac{x^{-1+m}}{\tanh^{-1}(\tanh(a+bx))^2} dx}{2b}\\
&= -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{mx^{-1+m}}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{((1-m)m)\int \frac{}{\tanh^{-1}}\\
&= -\frac{x^m}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{mx^{-1+m}}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{mx^{-1+m}\,{}_2F_1\left(1,-1\right.}{2b^2\left(bx-\tanh\right.}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 51, normalized size = 0.54

$$\frac{x^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}\right)}{(m+1)\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/(-(b*x) + ArcTanh[Tan
h[a + b*x]]))])/((1 + m)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{x^m}{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/arctanh(tanh(b*x + a))^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^3, x)
```

maple [F] time = 2.99, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] int(x^m/arctanh(tanh(b*x+a))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/arctanh(tanh(b*x + a))^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/atanh(tanh(a + b*x))^3,x)
```

```
[Out] int(x^m/atanh(tanh(a + b*x))^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**m/atanh(tanh(a + b*x))**3, x)
```

3.103 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=92

$$\frac{6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^5}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{2x^3}{b^2\tanh^{-1}(\tanh(a$$

```
[Out] 3*x^2/b^3+6*x*(b*x-arctanh(tanh(b*x+a)))/b^4-1/2*x^4/b/arctanh(tanh(b*x+a))
^2-2*x^3/b^2/arctanh(tanh(b*x+a))+6*(b*x-arctanh(tanh(b*x+a)))^2*ln(arctanh
(tanh(b*x+a)))/b^5
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2168, 2159, 2158, 2157, 29}

$$-\frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4}+\frac{6\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\log\left(\tanh^{-1}(\tanh(a\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x^2)/b^3 + (6*x*(b*x - ArcTanh[Tanh[a + b*x]]))/b^4 - x^4/(2*b*ArcTanh[T
anh[a + b*x]]^2) - (2*x^3)/(b^2*ArcTanh[Tanh[a + b*x]]) + (6*(b*x - ArcTanh
[Tanh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{2\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{b}$$

$$= -\frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))} + \frac{6\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{3x^2}{b^3} - \frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{2x^3}{b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(6\left(-bx+\tanh\right.\right.$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{}{b^2\tanh}$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{}{b^2\tanh}$$

$$= \frac{3x^2}{b^3} + \frac{6x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^4} - \frac{x^4}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{}{b^2\tanh}$$

Mathematica [A] time = 0.05, size = 114, normalized size = 1.24

$$-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}{2b^5\tanh^{-1}(\tanh(a+bx))^2} + \frac{4\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}{b^5\tanh^{-1}(\tanh(a+bx))} + \frac{6\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\tanh^{-1}\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] x^2/(2*b^3) - (3*x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^4 + (4*(-(b*x) + Ar
cTanh[Tanh[a + b*x]])^3)/(b^5*ArcTanh[Tanh[a + b*x]]) - (-(b*x) + ArcTanh[T
anh[a + b*x]])^4/(2*b^5*ArcTanh[Tanh[a + b*x]]^2) + (6*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2*Log[ArcTanh[Tanh[a + b*x]]])/b^5
```

fricas [A] time = 0.92, size = 95, normalized size = 1.03

$$\frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4 + 12\left(a^2b^2x^2 + 2\,a^3bx + a^4\right)\log\left(bx+a\right)}{2\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b
^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

giac [A] time = 0.20, size = 61, normalized size = 0.66

$$\frac{6\,a^2\log\left(|bx+a|\right)}{b^5} + \frac{b^3x^2 - 6\,ab^2x}{2\,b^6} + \frac{8\,a^3bx + 7\,a^4}{2\,(bx+a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b*
x + 7*a^4)/((b*x + a)^2*b^5)
```

maple [B] time = 0.15, size = 371, normalized size = 4.03

$$\frac{x^2}{2b^3}-\frac{3ax}{b^4}-\frac{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)x}{b^4}+\frac{6\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)a^2}{b^5}+\frac{12\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/2*x^2/b^3-3/b^4*a*x-3/b^4*(arctanh(tanh(b*x+a))-b*x-a)*x+6/b^5*ln(arctanh
(tanh(b*x+a)))*a^2+12/b^5*ln(arctanh(tanh(b*x+a)))*a*(arctanh(tanh(b*x+a))-
b*x-a)+6/b^5*ln(arctanh(tanh(b*x+a)))*(arctanh(tanh(b*x+a))-b*x-a)^2-1/2/b^
5/arctanh(tanh(b*x+a))^2*a^4-2/b^5/arctanh(tanh(b*x+a))^2*a^3*(arctanh(tanh
(b*x+a))-b*x-a)-3/b^5/arctanh(tanh(b*x+a))^2*a^2*(arctanh(tanh(b*x+a))-b*x-
a)^2-2/b^5/arctanh(tanh(b*x+a))^2*a*(arctanh(tanh(b*x+a))-b*x-a)^3-1/2/b^5/
arctanh(tanh(b*x+a))^2*(arctanh(tanh(b*x+a))-b*x-a)^4+4/b^5/arctanh(tanh(b*
x+a))*a^3+12/b^5/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+12/b
^5/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+4/b^5/arctanh(tanh
(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

maxima [A] time = 2.38, size = 81, normalized size = 0.88

$$\frac{b^4x^4-4\,ab^3x^3-11\,a^2b^2x^2+2\,a^3bx+7\,a^4}{2\left(b^7x^2+2\,ab^6x+a^2b^5\right)}+\frac{6\,a^2\log\left(bx+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4)/(b^7*x^2 +
 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a)/b^5
```

mupad [B] time = 1.34, size = 867, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^3,x)
```

```
[Out] ((7*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)))/(4*b) - x*(4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3
- 24*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 48*a^2*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)))/(2*b^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + x*(16*a*b^5 - 8*b^5*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)) + 8*a^2*b^4 + 8*b^6*x^2 - 8*a*b^4*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)) + x^2/(2*b^3) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 12*a^2))/(2*b^5) + (
```

```
3*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b^4)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{atanh}^3(\tanh(a + bx))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**3, x)
```

3.104 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=71

$$\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] 3*x/b^3-1/2*x^3/b/arctanh(tanh(b*x+a))^2-3/2*x^2/b^2/arctanh(tanh(b*x+a))+3
*(b*x-arctanh(tanh(b*x+a)))*ln(arctanh(tanh(b*x+a)))/b^4
```

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2168, 2158, 2157, 29}

$$-\frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))}+\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^4}-\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (3*x)/b^3 - x^3/(2*b*ArcTanh[Tanh[a + b*x]]^2) - (3*x^2)/(2*b^2*ArcTanh[Tan
h[a + b*x]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])*Log[ArcTanh[Tanh[a + b*x]]
])/b^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a
*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{3\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{2b}$$
$$= -\frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{x}{\tanh^{-1}(\tanh(a+bx)}}{b^2}$$
$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh\right.\right.}{}$$
$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh\right.\right.}{}$$
$$= \frac{3x}{b^3} - \frac{x^3}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3x^2}{2b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\left(bx-\tanh^{-1}\right.}{}$$

Mathematica [A] time = 0.05, size = 86, normalized size = 1.21

$$-\frac{3b^2x^2\tanh^{-1}(\tanh(a+bx)) - bx\tanh^{-1}(\tanh(a+bx))^2\left(6\log\left(\tanh^{-1}(\tanh(a+bx))\right)+11\right) + \tanh^{-1}(\tanh}{2b^4\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*(b^3*x^3 + 3*b^2*x^2*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^3
*(5 + 6*Log[ArcTanh[Tanh[a + b*x]]]) - b*x*ArcTanh[Tanh[a + b*x]]^2*(11 + 6
*Log[ArcTanh[Tanh[a + b*x]]]))/(b^4*ArcTanh[Tanh[a + b*x]]^2)
```

fricas [A] time = 1.56, size = 83, normalized size = 1.17

$$\frac{2\,b^3x^3 + 4\,ab^2x^2 - 4\,a^2bx - 5\,a^3 - 6\left(ab^2x^2 + 2\,a^2bx + a^3\right)\log\left(bx+a\right)}{2\left(b^6x^2 + 2\,ab^5x + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

giac [A] time = 0.18, size = 44, normalized size = 0.62

$$\frac{x}{b^3} - \frac{3\,a\log\left(|bx+a|\right)}{b^4} - \frac{6\,a^2bx + 5\,a^3}{2\,(bx+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

maple [B] time = 0.15, size = 239, normalized size = 3.37

$$\frac{x}{b^3} + \frac{a^3}{2b^4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2} + \frac{3a^2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx - a\right)}{2b^4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2} + \frac{3a\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx -\right.}{2b^4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] x/b^3+1/2/b^4/arctanh(tanh(b*x+a))^2*a^3+3/2/b^4/arctanh(tanh(b*x+a))^2*a^2
*(arctanh(tanh(b*x+a))-b*x-a)+3/2/b^4/arctanh(tanh(b*x+a))^2*a*(arctanh(tan
h(b*x+a))-b*x-a)^2+1/2/b^4/arctanh(tanh(b*x+a))^2*(arctanh(tanh(b*x+a))-b*x
-a)^3-3/b^4/arctanh(tanh(b*x+a))*a^2-6/b^4/arctanh(tanh(b*x+a))*a*(arctanh(
tanh(b*x+a))-b*x-a)-3/b^4/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)
^2-3/b^4*ln(arctanh(tanh(b*x+a)))*a-3/b^4*ln(arctanh(tanh(b*x+a)))*(arctanh
(tanh(b*x+a))-b*x-a)
```

maxima [A] time = 2.41, size = 69, normalized size = 0.97

$$\frac{2\,b^3x^3 + 4\,ab^2x^2 - 4\,a^2bx - 5\,a^3}{2\left(b^6x^2 + 2\,ab^5x + a^2b^4\right)} - \frac{3\,a\log\left(bx + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^
2*b^4) - 3*a*log(b*x + a)/b^4
```

mupad [B] time = 1.47, size = 620, normalized size = 8.73

$$\frac{x}{b^3} - \frac{x\left(3\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2 - 12\,a\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right) + 1}{b^3\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + \ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2 + x\left(8\,a\,b^4 - 4\,b^4\left(2\,a - \ln\left(\frac{2\,e^{2\,a}\,e^{2}}{e^{2\,a}\,e^{2\,b\,x}}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^3,x)
```

```
[Out] x/b^3 - (x*(3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 12*a*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x) + 12*a^2) - (5*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(4
*b))/(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + l
og(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + x*(8*a*b^4 - 4*b^4*(2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)) + 4*a^2*b^3 + 4*b^5*x^2 - 4*a*b^3*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)) + (log(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
 log(2/(exp(2*a)*exp(2*b*x) + 1)))*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x))/(2*b^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{atanh}^3\left(\tanh\left(a + bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**3, x)
```

3.105 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=47

$$\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x}{b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/2*x^2/b/arctanh(tanh(b*x+a))^2-x/b^2/arctanh(tanh(b*x+a))+ln(arctanh(tan
h(b*x+a)))/b^3
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 29}

$$-\frac{x}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{\log\left(\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -x^2/(2*b*ArcTanh[Tanh[a + b*x]]^2) - x/(b^2*ArcTanh[Tanh[a + b*x]]) + Log[
ArcTanh[Tanh[a + b*x]]]/b^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^3}\,dx &= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{b}\\
&= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{x}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))}\,d\ldots}{b^2}\\
&= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{x}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \text{ta}\ldots\right.}{\ldots}\\
&= -\frac{x^2}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{x}{b^2\tanh^{-1}(\tanh(a+bx))}+\frac{\log\left(\tanh^{-1}(\tanh(a\ldots\right.}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 1.04

$$\frac{-\frac{b^2x^2}{\tanh^{-1}(\tanh(a+bx))^2}-\frac{2bx}{\tanh^{-1}(\tanh(a+bx))}+2\log\left(\tanh^{-1}(\tanh(a+bx))\right)+3}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^3,x]
[Out] (3 - (b^2*x^2)/ArcTanh[Tanh[a + b*x]]^2 - (2*b*x)/ArcTanh[Tanh[a + b*x]] +
2*Log[ArcTanh[Tanh[a + b*x]]])/(2*b^3)
```

fricas [A] time = 1.07, size = 61, normalized size = 1.30

$$\frac{4\,abx+3\,a^2+2\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

giac [A] time = 0.48, size = 37, normalized size = 0.79

$$\frac{\log\left(|bx+a|\right)}{b^3}+\frac{4\,ax+\frac{3\,a^2}{b}}{2\left(bx+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

maple [B] time = 0.14, size = 136, normalized size = 2.89

$$-\frac{a^2}{2b^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}-\frac{a\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{b^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}-\frac{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)^2}{2b^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}+\frac{}{b^3\operatorname{ar}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^3,x)
[Out] -1/2/b^3/arctanh(tanh(b*x+a))^2*a^2-1/b^3/arctanh(tanh(b*x+a))^2*a*(arctanh
(tanh(b*x+a))-b*x-a)-1/2/b^3/arctanh(tanh(b*x+a))^2*(arctanh(tanh(b*x+a))-b
*x-a)^2+2/b^3/arctanh(tanh(b*x+a))*a+2/b^3/arctanh(tanh(b*x+a))*(arctanh(ta
nh(b*x+a))-b*x-a)+ln(arctanh(tanh(b*x+a)))/b^3
```

maxima [A] time = 3.22, size = 48, normalized size = 1.02

$$\frac{4\,abx+3\,a^2}{2\left(b^5x^2+2\,ab^4x+a^2b^3\right)}+\frac{\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

<u>**mupad**</u> **[B]** time = 1.03, size = 46, normalized size = 0.98

$$\frac{\ln\left(\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)\right)}{b^3}-\frac{\frac{b^2\,x^2}{2}+b\,x\,\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}{b^3\,{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^3,x)
```

```
[Out] log(atanh(tanh(a + b*x)))/b^3 - ((b^2*x^2)/2 + b*x*atanh(tanh(a + b*x)))/(b
^3*atanh(tanh(a + b*x))^2)
```

<u>**sympy**</u> **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: TypeError
```

3.106 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/2*x/b/arctanh(tanh(b*x+a))^2-1/2/b^2/arctanh(tanh(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$-\frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -x/(2*b*ArcTanh[Tanh[a + b*x]]^2) - 1/(2*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^3}\,dx &= -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{2b}\\ &= -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}+\frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{2b^2}\\ &= -\frac{x}{2b\tanh^{-1}(\tanh(a+bx))^2}-\frac{1}{2b^2\tanh^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 0.79

$$-\frac{\tanh^{-1}(\tanh(a+bx))+bx}{2b^2\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^3,x]
[Out] -1/2*(b*x + ArcTanh[Tanh[a + b*x]])/(b^2*ArcTanh[Tanh[a + b*x]]^2)
```

fricas [A] time = 0.51, size = 32, normalized size = 0.94

$$-\frac{2\,bx+a}{2\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

giac [A] time = 0.17, size = 18, normalized size = 0.53

$$-\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

maple [A] time = 0.16, size = 43, normalized size = 1.26

$$-\frac{bx-\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{2b^2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}-\frac{1}{b^2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^3,x)
[Out] -1/2*(b*x-arctanh(tanh(b*x+a)))/b^2/arctanh(tanh(b*x+a))^2-1/b^2/arctanh(ta
nh(b*x+a))
```

maxima [A] time = 3.19, size = 32, normalized size = 0.94

$$-\frac{2\,bx+a}{2\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

mupad [B] time = 0.08, size = 25, normalized size = 0.74

$$-\frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)+b\,x}{2\,b^2\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x))^3,x)
[Out] -(atanh(tanh(a + b*x)) + b*x)/(2*b^2*atanh(tanh(a + b*x))^2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: TypeError
```

3.107 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/2/b/arctanh(tanh(b*x+a))^2
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/(2*b*ArcTanh[Tanh[a + b*x]]^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{1}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-3),x]
```

```
[Out] -1/2*1/(b*ArcTanh[Tanh[a + b*x]]^2)
```

fricas [A] time = 0.40, size = 24, normalized size = 1.50

$$-\frac{1}{2\left(b^3x^2 + 2\,ab^2x + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

giac [**A**] time = 0.17, size = 12, normalized size = 0.75

$$-\frac{1}{2\,(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

maple [**A**] time = 0.03, size = 15, normalized size = 0.94

$$-\frac{1}{2b\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/2/b/arctanh(tanh(b*x+a))^2
```

maxima [**A**] time = 0.56, size = 12, normalized size = 0.75

$$-\frac{1}{2\,(bx+a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

mupad [**B**] time = 0.06, size = 14, normalized size = 0.88

$$-\frac{1}{2\,b\,\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^3,x)
```

```
[Out] -1/(2*b*atanh(tanh(a + b*x))^2)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Exception raised: TypeError
```

3.108 $\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=97

$$\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2}-$$

```
[Out] -1/2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^2+1/(b*x-arctanh(tanh(
b*x+a)))^2/arctanh(tanh(b*x+a))-ln(x)/(b*x-arctanh(tanh(b*x+a)))^3+ln(arcta
nh(tanh(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^3
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2163, 2160, 2157, 29}

$$\frac{1}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))}-\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] -1/(2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + 1/((b*x -
ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]) - Log[x]/(b*x - ArcTanh[T
anh[a + b*x]])^3 + Log[ArcTanh[Tanh[a + b*x]]]/(b*x - ArcTanh[Tanh[a + b*x]
])^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} - \frac{\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))}}{bx-\tanh^{-1}(\tanh(a}$$

$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\tanh^{-1}(\tanh(a\right.}$$

$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\tanh^{-1}(\tanh(a\right.}$$

$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\tanh^{-1}(\tanh(a\right.}$$

$$= -\frac{1}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{\left(bx-\tanh^{-1}(\tanh(a\right.}$$

Mathematica [A] time = 0.10, size = 74, normalized size = 0.76

$$\frac{-4bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2\left(-2\log\left(\tanh^{-1}(\tanh(a+bx))\right)+2\log(bx)+3\right)+b^2x^2}{2\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (b^2*x^2 - 4*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2*(3 + 2*L
og[b*x] - 2*Log[ArcTanh[Tanh[a + b*x]]]))/(2*ArcTanh[Tanh[a + b*x]]^2*(-(b*
x) + ArcTanh[Tanh[a + b*x]])^3)
```

fricas [A] time = 0.50, size = 80, normalized size = 0.82

$$\frac{2\,abx+3\,a^2-2\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)+2\left(b^2x^2+2\,abx+a^2\right)\log(x)}{2\left(a^3b^2x^2+2\,a^4bx+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

giac [A] time = 0.19, size = 43, normalized size = 0.44

$$-\frac{\log\left(|bx+a|\right)}{a^3}+\frac{\log\left(|x|\right)}{a^3}+\frac{2\,abx+3\,a^2}{2\left(bx+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

maple [A] time = 0.15, size = 92, normalized size = 0.95

$$\frac{\ln(x)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3}-\frac{\ln\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3}+\frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2\operatorname{arctanh}\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 1/(arctanh(tanh(b*x+a))-b*x)^3*ln(x)-1/(arctanh(tanh(b*x+a))-b*x)^3*ln(arct
anh(tanh(b*x+a)))+1/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b*x+a))+1/2/(
arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^2
```

maxima [A] time = 4.32, size = 51, normalized size = 0.53

$$\frac{2\,bx+3\,a}{2\left(a^2b^2x^2+2\,a^3bx+a^4\right)}-\frac{\log\left(bx+a\right)}{a^3}+\frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

mupad [B] time = 3.66, size = 645, normalized size = 6.65

$$-\frac{12\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^2-24\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+12\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^2+16\,b^2\,x^2+b\,x\,\left(32\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^3),x)
```

```
[Out] -(12*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - 24*log(1/(exp(2*a)*exp(2*b*x) + 1
))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(l
og(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))*16i - log(1/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b
*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*16i + 12*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 16*b^2*x^2 + log(1/(exp(2*a)*exp(2*b*
x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*atan((log((ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*
x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*32i + b*x*(32*log(1/(exp(2*a)
*exp(2*b*x) + 1)) - 32*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
))/((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)))^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**3), x)
```

3.109 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=131

$$\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}-\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^2}+\frac{}{x}$$

```
[Out] -3/2*b/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^2+1/x/(b*x-arctanh
(tanh(b*x+a)))/arctanh(tanh(b*x+a))^2+3*b/(b*x-arctanh(tanh(b*x+a)))^3/arct
anh(tanh(b*x+a))-3*b*ln(x)/(b*x-arctanh(tanh(b*x+a)))^4+3*b*ln(arctanh(tanh
(b*x+a)))/(b*x-arctanh(tanh(b*x+a)))^4
```

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))}-\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^2}+\frac{}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*b)/(2*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^2) + 1/(x
*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (3*b)/((b*x - A
rcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]) - (3*b*Log[x])/(b*x - ArcT
anh[Tanh[a + b*x]])^4 + (3*b*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTanh[Ta
nh[a + b*x]])^4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^3} \, dx = \frac{1}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} - \frac{(3b)\int \frac{1}{x\tanh^{-1}(\tanh}}{-bx+\tanh^{-1}(\tan}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

$$= -\frac{3b}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\right.}$$

Mathematica [A] time = 0.06, size = 93, normalized size = 0.71

$$-\frac{-6b^2x^2\tanh^{-1}(\tanh(a+bx)) + 2\tanh^{-1}(\tanh(a+bx))^3 + 3bx\tanh^{-1}(\tanh(a+bx))^2\left(-2\log\left(\tanh^{-1}(\tanh\right.\right.}{2x\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] -1/2*(b^3*x^3 - 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 2*ArcTanh[Tanh[a + b*x]]
^3 + 3*b*x*ArcTanh[Tanh[a + b*x]]^2*(1 + 2*Log[x] - 2*Log[ArcTanh[Tanh[a +
b*x]]]))/(x*ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.52, size = 109, normalized size = 0.83

$$-\frac{6\,ab^2x^2 + 9\,a^2bx + 2\,a^3 - 6\left(b^3x^3 + 2\,ab^2x^2 + a^2bx\right)\log\left(bx+a\right) + 6\left(b^3x^3 + 2\,ab^2x^2 + a^2bx\right)\log(x)}{2\left(a^4b^2x^3 + 2\,a^5bx^2 + a^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

giac [A] time = 0.17, size = 60, normalized size = 0.46

$$\frac{3\,b\log\left(|bx+a|\right)}{a^4} - \frac{3\,b\log\left(|x|\right)}{a^4} - \frac{6\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{2\left(bx+a\right)^2a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

maple [A] time = 0.15, size = 117, normalized size = 0.89

$$-\frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}x}-\frac{3b\ln\left(x\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}}-\frac{b}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\operatorname{arctanh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/(arctanh(tanh(b*x+a))-b*x)^3/x-3/(arctanh(tanh(b*x+a))-b*x)^4*b*ln(x)-1/
2/(arctanh(tanh(b*x+a))-b*x)^2*b/arctanh(tanh(b*x+a))^2+3/(arctanh(tanh(b*x
+a))-b*x)^4*b*ln(arctanh(tanh(b*x+a)))-2/(arctanh(tanh(b*x+a))-b*x)^3*b/arc
tanh(tanh(b*x+a))
```

maxima [A] time = 3.13, size = 69, normalized size = 0.53

$$-\frac{6\,b^{2}x^{2}+9\,abx+2\,a^{2}}{2\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x\right)}+\frac{3\,b\log\left(bx+a\right)}{a^{4}}-\frac{3\,b\log\left(x\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

mupad [B] time = 3.67, size = 804, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^3),x)
```

```
[Out] -(24*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - 24*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 8*log(1/(exp(2*a)*exp(2*b*x) + 1))^3
+ 8*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3 + 32*b^3*x^3 + 2
4*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 96*b^2*x^2*l
og(1/(exp(2*a)*exp(2*b*x) + 1)) - 96*b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 24*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))^2 - b*x*lo
g((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*
1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*96i - b*x*log(1/(exp(2*a)*exp(2*b*x) +
 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log(
1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1))
 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*96i - 48*
b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(
exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x))*192i)/(x*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))^2*(log(1/(exp(2*a)*exp(2*b*x) + 1))
 - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{2}\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**3), x)
```

3.110 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=170

$$\frac{6b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4 \tanh^{-1}(\tanh(a+bx))}-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))^2}-\ldots (bx$$

```
[Out] -3*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^2+2*b/x/(b*x-arcta
nh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^2+1/2/x^2/(b*x-arctanh(tanh(b*x+a))
)/arctanh(tanh(b*x+a))^2+6*b^2/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*
x+a))-6*b^2*ln(x)/(b*x-arctanh(tanh(b*x+a)))^5+6*b^2*ln(arctanh(tanh(b*x+a)
))/(b*x-arctanh(tanh(b*x+a)))^5
```

Rubi [A] time = 0.12, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2171, 2163, 2160, 2157, 29}

$$\frac{6b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4 \tanh^{-1}(\tanh(a+bx))}-\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \tanh^{-1}(\tanh(a+bx))^2}-\ldots (bx$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*b^2)/((b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^2) + (2*b
)/(x*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^2) + 1/(2*x^2*
(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^2) + (6*b^2)/((b*x -
ArcTanh[Tanh[a + b*x]])^4*ArcTanh[Tanh[a + b*x]]) - (6*b^2*Log[x])/(b*x - A
rcTanh[Tanh[a + b*x]])^5 + (6*b^2*Log[ArcTanh[Tanh[a + b*x]]])/(b*x - ArcTa
nh[Tanh[a + b*x]])^5
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
```

```
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^3}\,dx = \frac{1}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^2} + \frac{(2b)\int \frac{1}{x^2 \tanh^{-1}(\text{t}}}{bx-\tanh^{-1}(\text{ta}}$$

$$= \frac{2b}{x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{2x^2\left(bx-\tanh^{-1}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\text{ta}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\text{ta}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\text{ta}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\text{ta}\right.}$$

$$= -\frac{3b^2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{x\left(bx-\tanh^{-1}(\text{ta}\right.}$$

Mathematica [A] time = 0.05, size = 107, normalized size = 0.63

$$\frac{8b^3x^3\tanh^{-1}(\tanh(a+bx)) - 12b^2x^2\tanh^{-1}(\tanh(a+bx))^2\left(\log(x)-\log\left(\tanh^{-1}(\tanh(a+bx))\right)\right) - 8bx\tan}{2x^2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-(b^4*x^4) + 8*b^3*x^3*ArcTanh[Tanh[a + b*x]] - 8*b*x*ArcTanh[Tanh[a + b*x
]]^3 + ArcTanh[Tanh[a + b*x]]^4 - 12*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2*(Log[
x] - Log[ArcTanh[Tanh[a + b*x]]]))/(2*x^2*(b*x - ArcTanh[Tanh[a + b*x]])^5*
ArcTanh[Tanh[a + b*x]]^2)
```

fricas [A] time = 0.46, size = 130, normalized size = 0.76

$$\frac{12\,ab^3x^3 + 18\,a^2b^2x^2 + 4\,a^3bx - a^4 - 12\left(b^4x^4 + 2\,ab^3x^3 + a^2b^2x^2\right)\log(bx+a) + 12\left(b^4x^4 + 2\,ab^3x^3 + a^2b^2x^2\right)}{2\left(a^5b^2x^4 + 2\,a^6bx^3 + a^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

giac [A] time = 0.18, size = 73, normalized size = 0.43

$$-\frac{6\,b^2\log\left(|bx+a|\right)}{a^5}+\frac{6\,b^2\log\left(|x|\right)}{a^5}+\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,\left(bx^2+ax\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

maple [A] time = 0.16, size = 145, normalized size = 0.85

$$-\frac{1}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}x^{2}}+\frac{6b^{2}\ln(x)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{5}}+\frac{3b}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}x}-\frac{6b^{2}\,\mathrm{l}}{(\mathrm{arc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -1/2/(arctanh(tanh(b*x+a))-b*x)^3/x^2+6/(arctanh(tanh(b*x+a))-b*x)^5*b^2*ln
(x)+3/(arctanh(tanh(b*x+a))-b*x)^4*b/x-6/(arctanh(tanh(b*x+a))-b*x)^5*b^2*l
n(arctanh(tanh(b*x+a)))+3/(arctanh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x
+a))+1/2/(arctanh(tanh(b*x+a))-b*x)^3*b^2/arctanh(tanh(b*x+a))^2
```

maxima [A] time = 3.14, size = 86, normalized size = 0.51

$$\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}-\frac{6\,b^2\log\left(bx+a\right)}{a^5}+\frac{6\,b^2\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

mupad [B] time = 2.73, size = 909, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^3),x)
```

```
[Out] (4*log(1/(exp(2*a)*exp(2*b*x) + 1))^4 - 16*log(1/(exp(2*a)*exp(2*b*x) + 1))
^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - 16*log(1/(exp(2*a
)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3 +
 4*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^4 + 24*log(1/(exp(2
*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
^2 - 64*b^4*x^4 - 64*b*x*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))^3 - 256*b^3*x^3*log(1/(exp(2*a)*exp(2*b*x) + 1)) + 256*b^3*x^3*log((exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 64*b*x*log(1/(exp(2*a)*exp(2*
b*x) + 1))^3 + 192*b*x*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - 192*b*x*log(1/(exp(2*a)*exp(2*b*x) +
1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + b^2*x^2*log(1/
(exp(2*a)*exp(2*b*x) + 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1))*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp
(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))*384i + b^2*x^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
```

```
+ 1))^2*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*1i - log
(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2*b*x) + 1)
) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*384i - b
^2*x^2*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))*atan((log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)*1i - log(1/(exp(2*a)*exp(2*b*x) + 1))*1i + b*x*2i)/(log(1/(exp(2*a)*exp(2
*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
)*768i)/(x^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)))^2*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{atanh}^3 (\tanh (a + bx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**3), x)
```

3.111 $\int x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=101

$$\frac{256\tanh^{-1}(\tanh(a+bx))^{11/2}}{3465b^5}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^4}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{16x^3\tanh^{-1}(\tanh(\ldots}{15b^2}$$

```
[Out] 2/3*x^4*arctanh(tanh(b*x+a))^(3/2)/b-16/15*x^3*arctanh(tanh(b*x+a))^(5/2)/b
^2+32/35*x^2*arctanh(tanh(b*x+a))^(7/2)/b^3-128/315*x*arctanh(tanh(b*x+a))^
(9/2)/b^4+256/3465*arctanh(tanh(b*x+a))^(11/2)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{2168, 2157, 30\}$

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{11/2}}{3465b^5}-\frac{128x\tanh^{-1}(\tanh(\ldots}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(5/2))/(15*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^3) - (128*x*
ArcTanh[Tanh[a + b*x]]^(9/2))/(315*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(11/2
))/(3465*b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{8\int x^3\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{3b}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{16\int x^2\tan}{}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2\tanh}{}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2\tanh}{}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2\tanh}{}$$
$$= \frac{2x^4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2} + \frac{32x^2\tanh}{}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-1848b^3x^3\tanh^{-1}(\tanh(a+bx)) + 1584b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 704bx\tanh\right.}{3465b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(1155*b^4*x^4 - 1848*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 1584*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 704*b*x*ArcTanh[Tanh[a +
b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(3465*b^5)
```

fricas [A] time = 0.52, size = 64, normalized size = 0.63

$$\frac{2\left(315\,b^5x^5 + 35\,ab^4x^4 - 40\,a^2b^3x^3 + 48\,a^3b^2x^2 - 64\,a^4bx + 128\,a^5\right)\sqrt{bx+a}}{3465\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*a
^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

giac [A] time = 0.62, size = 150, normalized size = 1.49

$$\frac{\sqrt{2}\left(\frac{11\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a)^{\frac{5}{2}}a^2 - 420\,(bx+a)^{\frac{3}{2}}a^3 + 315\,\sqrt{bx+a}\,a^4\right)a}{b^4} + \frac{5\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}} - 385\,(bx+a)^{\frac{9}{2}}a + 990\,(bx+a)^{\frac{7}{2}}a^2 -\right.}{b}\right.}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3465*sqrt(2)*(11*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 37
8*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/
b^4 + 5*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a
)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqr
t(b*x + a)*a^5)/b^4)/b
```

maple [A] time = 0.17, size = 154, normalized size = 1.52

$$\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2\left(2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2}+(-2\operatorname{arctanh}(\tanh(bx+a))+\right.}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^5*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-4*arctanh(tanh(b*x+a))+4*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arctanh
(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(7/2)+2/5*(b*x-arctanh(tanh(b*
x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(5/2)+1/3*(b*
x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(3/2))
```

maxima [A] time = 1.84, size = 64, normalized size = 0.63

$$\frac{2\left(315\,b^{5}x^{5}+35\,ab^{4}x^{4}-40\,a^{2}b^{3}x^{3}+48\,a^{3}b^{2}x^{2}-64\,a^{4}bx+128\,a^{5}\right)\sqrt{bx+a}}{3465\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48*a^3*b^2*x^2 - 64*a
^4*b*x + 128*a^5)*sqrt(b*x + a)/b^5
```

mupad [B] time = 1.05, size = 811, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 - (x^4*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/11 + (2*b*x)/11))/(9*b) - (128*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 + b*x)^4*(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 + (2*b*x)/11))/(315*b^5)
 - (8*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*(log(2/(exp(
2*a)*exp(2*b*x) + 1))/11 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))/11 + (2*b*x)/11))/(63*b^2) - (64*x*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))/2 + b*x)^3*(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 + (2*b*x)/11))/(315*b^4) - (16
*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2*(log(2/(exp(2*
a)*exp(2*b*x) + 1))/11 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/11 + (2*b*x)/11))/(105*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(atanh(tanh(a + b*x))), x)
```

3.112 $\int x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=80

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2}+\frac{2x^3\tanh^{-1}(\tanh(a+$$

```
[Out] 2/3*x^3*arctanh(tanh(b*x+a))^(3/2)/b-4/5*x^2*arctanh(tanh(b*x+a))^(5/2)/b^2
+16/35*x*arctanh(tanh(b*x+a))^(7/2)/b^3-32/315*arctanh(tanh(b*x+a))^(9/2)/b
^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^3}+\frac{2x^3\tanh^{-1}(\tanh(a+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*x^2*ArcTanh[Tanh[a + b*x]]^
(5/2))/(5*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^3) - (32*ArcTanh
[Tanh[a + b*x]]^(9/2))/(315*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{2\int x^2\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{b}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{8\int x\tanh^{-}}{}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x\tanh^{-1}(}{}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x\tanh^{-1}(}{}$$
$$= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^2} + \frac{16x\tanh^{-1}(}{}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-126b^2x^2\tanh^{-1}(\tanh(a+bx)) + 72bx\tanh^{-1}(\tanh(a+bx))^2 - 16\tanh^{-1}(\tanh(\right.}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(105*b^3*x^3 - 126*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 72*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(3
15*b^4)
```

fricas [A] time = 0.46, size = 53, normalized size = 0.66

$$\frac{2\left(35\,b^4x^4 + 5\,ab^3x^3 - 6\,a^2b^2x^2 + 8\,a^3bx - 16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

giac [A] time = 0.19, size = 125, normalized size = 1.56

$$\frac{\sqrt{2}\left(\frac{9\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}\,a^3\right)a}{b^3} + \frac{\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a)^{\frac{5}{2}}a^2 - 420\,(bx+a)^{\frac{3}{2}}a^3 + 315\,\sqrt{b}\right.}{b^3}\right.}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(9*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*
x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^3 + sqrt(2)*(35*(b*x + a)^(9/2
) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a
^3 + 315*sqrt(b*x + a)*a^4)/b^3)/b
```

maple [A] time = 0.15, size = 124, normalized size = 1.55

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9} + \frac{2(-3\operatorname{arctanh}(\tanh(bx+a))+3bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7} + \frac{2\big((bx-\operatorname{arctanh}(\tanh(bx+a)))(-2\operatorname{arctanh}(\tanh(bx+a))+2}{}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^4*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-3*arctanh(tanh(b*x+a))+3*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tanh(
b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(5/2)+1/3
*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(3/2))
```

maxima [A] time = 0.52, size = 53, normalized size = 0.66

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

mupad [B] time = 0.99, size = 648, normalized size = 8.10

$$\frac{2\,x^4\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{9}-\frac{x^3\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}+\frac{2\,b\,x}{9}\right)}{7\,b}-\frac{16\,\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/9 - (x^3*(log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/9 + (2*b*x)/9))/(7*b) - (16*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))/2 + b*x)^3*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 + (2*b*x)/9))/(35*b^4) - (6*x^2*
(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*(log(2/(exp(2*a)*exp(
2*b*x) + 1))/9 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 +
 (2*b*x)/9))/(35*b^2) - (8*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
 + b*x)^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/9 + (2*b*x)/9))/(35*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(atanh(tanh(a + b*x))), x)
```

3.113 $\int x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=59

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{7/2}}{105b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

```
[Out] 2/3*x^2*arctanh(tanh(b*x+a))^(3/2)/b-8/15*x*arctanh(tanh(b*x+a))^(5/2)/b^2+
16/105*arctanh(tanh(b*x+a))^(7/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{7/2}}{105b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(5
/2))/(15*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(7/2))/(105*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\int x\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{3b}\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{8\int\tanh^{-1}(\text{t}\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{8\,\text{Subst}\left(\int x\right.\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}+\frac{16\tanh^{-1}(\text{ta}}{10}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-28bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2+35b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(35*b^2*x^2 - 28*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(105*b^3)
```

fricas [A] time = 0.62, size = 42, normalized size = 0.71

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

giac [B] time = 0.15, size = 102, normalized size = 1.73

$$\frac{\sqrt{2}\left(\frac{7\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a}{b^2}+\frac{3\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*sqrt(2)*(7*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqr
t(b*x + a)*a^2)*a/b^2 + 3*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a
 + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

maple [A] time = 0.15, size = 69, normalized size = 1.17

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))^2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^3*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(5/2)+1/3*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x
+a))^(3/2))
```

maxima [A] time = 0.54, size = 42, normalized size = 0.71

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

mupad [B] time = 0.99, size = 485, normalized size = 8.22

$$\frac{2\,x^3\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{7}-\frac{x^2\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}+\frac{2\,b\,x}{7}\right)}{5\,b}-\frac{8\,\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/7 - (x^2*(log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/7 + (2*b*x)/7))/(5*b) - (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/
(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 + b*x)^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/7 + (2*b*x)/7))/(15*b^3) - (4*x*(lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*ex
p(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)*(log(2/(exp(2*a)*exp(2*b
*x) + 1))/7 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/7 + (2
*b*x)/7))/(15*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(atanh(tanh(a + b*x))), x)
```

3.114 $\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}$$

```
[Out] 2/3*x*arctanh(tanh(b*x+a))^(3/2)/b-4/15*arctanh(tanh(b*x+a))^(5/2)/b^2
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b) - (4*ArcTanh[Tanh[a + b*x]]^(5/2))
/(15*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx&=\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\int\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx}{3b}\\&=\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{2\,\text{Subst}\left(\int x^{3/2}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}\\&=\frac{2x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.84

$$\frac{2\left(5bx-2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
[Out] (2*(5*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*b^2
)
```

fricas [A] time = 0.51, size = 30, normalized size = 0.79

$$\frac{2\left(3\,b^{2}x^{2}+abx-2\,a^{2}\right)\sqrt{bx+a}}{15\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

giac [B] time = 0.18, size = 75, normalized size = 1.97

$$\frac{\sqrt{2}\left(\frac{5\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a}{b}+\frac{\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
[Out] 1/15*sqrt(2)*(5*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + sqrt(2)
*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

maple [A] time = 0.15, size = 42, normalized size = 1.11

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(1/2),x)
[Out] 2/b^2*(1/5*arctanh(tanh(b*x+a))^(5/2)+1/3*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(3/2))
```

maxima [A] time = 0.52, size = 30, normalized size = 0.79

$$\frac{2\left(3\,b^{2}x^{2}+abx-2\,a^{2}\right)\sqrt{bx+a}}{15\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

mupad [B] time = 1.04, size = 151, normalized size = 3.97

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+5\,b\,x\right)}{15\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 5*b*x))/(15*
b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x*sqrt(atanh(tanh(a + b*x))), x)
```

3.115 $\int \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2157, 30}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]],x]
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]],x]
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b)
```

fricas [A] time = 0.53, size = 12, normalized size = 0.67

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

[Out] 2/3*(b*x + a)^(3/2)/b

giac [A] time = 0.16, size = 18, normalized size = 1.00

$$\frac{\sqrt{2}\,(2\,bx + 2\,a)^{\frac{3}{2}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(2*b*x + 2*a)^(3/2)/b
```

maple [A] time = 0.03, size = 15, normalized size = 0.83

$$\frac{2\,\mathrm{arctanh}\,(\tanh\,(bx + a))^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)/b
```

maxima [A] time = 0.49, size = 12, normalized size = 0.67

$$\frac{2\,(bx + a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

mupad [B] time = 1.12, size = 95, normalized size = 5.28

$$-\frac{\left(\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] -((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b)
```

sympy [A] time = 0.58, size = 26, normalized size = 1.44

$$\begin{cases} \frac{2\,\mathrm{atanh}^{\frac{3}{2}}\,(\tanh\,(a+bx))}{3b} & \text{for } b \neq 0 \\ x\sqrt{\mathrm{atanh}\,(\tanh\,(a))} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((2*atanh(tanh(a + b*x))**(3/2)/(3*b), Ne(b, 0)), (x*sqrt(atanh(ta
nh(a))), True))
```

3.116 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\,dx$

Optimal. Leaf size=63

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}-2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\,\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)$$

```
[Out] -2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x
-arctanh(tanh(b*x+a)))^(1/2)+2*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2161}

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}-2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\,\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x,x]
```

```
[Out] -2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*
Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 2*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\,dx = 2\sqrt{\tanh^{-1}(\tanh(a+bx))}-\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh}$$

$$= -2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}+2\sqrt{}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 0.97

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}-2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x,x]
```

```
[Out] 2*Sqrt[ArcTanh[Tanh[a + b*x]]] - 2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqr
t[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

fricas [A] time = 0.49, size = 73, normalized size = 1.16

$$\left[\sqrt{a}\log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + 2\sqrt{bx+a}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + 2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

giac [A] time = 0.19, size = 40, normalized size = 0.63

$$\sqrt{2}\left(\frac{\sqrt{2}\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{2}\sqrt{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2)*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(2)*sqrt(b
*x + a))
```

maple [A] time = 0.15, size = 54, normalized size = 0.86

$$2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)} - 2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx}\;\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x,x)
```

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)-2*(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arc
tanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x, x)
```

mupad [B] time = 2.36, size = 308, normalized size = 4.89

$$2\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}} + \ln\left(-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) + 2\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}\sqrt{\frac{\ln(\ldots}{\ldots}}}{x\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}} - b\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x,x)
```

```
[Out] 2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2) + log(-(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log
((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*(log((exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2)*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*
a)*exp(2*b*x) + 1))/2 - b*x)^(1/2) + b*x)/(x*(log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2 - b*x)^(1/2)
))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2 - b*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x, x)
```

3.117 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx$

Optimal. Leaf size=66

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

```
[Out] b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(1/2)-arctanh(tanh(b*x+a))^(1/2)/x
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 2161}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^2,x]
```

```
[Out] (b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] - Sqrt[ArcTanh[Tanh[a + b*x]]]/x
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx &= -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}+\frac{1}{2}b\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx\\ &= \frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 0.98

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}-\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^2,x]
```

```
[Out] -(Sqrt[ArcTanh[Tanh[a + b*x]]]/x) - (b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]
/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]
```

fricas [A] time = 0.55, size = 93, normalized size = 1.41

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2ax},\ \frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

giac [A] time = 0.16, size = 51, normalized size = 0.77

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}\sqrt{bx+a}\,b}{x}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(2)*
sqrt(b*x + a)*b/x)/b
```

maple [A] time = 0.16, size = 63, normalized size = 0.95

$$2b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2bx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^2,x)
```

```
[Out] 2*b*(-1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-1/2/(arctanh(tanh(b*x+a))-b*x)^(1/
2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{artanh}(\tanh(bx+a))}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^2, x)
```

mupad [B] time = 6.97, size = 341, normalized size = 5.17

$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{x}+\frac{\sqrt{2}\,b\,\ln\left(\frac{\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\left(\sqrt{2}\,b\,x-\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b \cdots\right)\right)}{x}\right)}{2\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2} \cdots}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^2,x)
```

```
[Out] (2^(1/2)*b*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1
/2)*b*x)*1i)/x)*1i)/(2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) - (log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/
2)^(1/2)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**2, x)
```

3.118 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3}\,dx$

Optimal. Leaf size=125

$$\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{\sqrt{\tanh^{-1}(\tanh(a}{2x^2}$$

```
[Out] 1/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(3/2)-1/4*b/x/arctanh(tanh(b*x+a))^(1/2)+1/4*b^
2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)-1/2*arctanh(tanh(b*
x+a))^(1/2)/x^2
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{\sqrt{\tanh^{-1}(\tanh(a}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^3,x]
```

```
[Out] (b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - b/(4*x*Sqrt[ArcTanh[Tanh[a +
b*x]]]) + b^2/(4*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]
]) - Sqrt[ArcTanh[Tanh[a + b*x]]]/(2*x^2)
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3}\,dx = -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2} + \frac{1}{4}b\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$
$$= -\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2x^2} - \frac{1}{8}b^2\int \frac{1}{x\tanh^{-1}(\tan}$$
$$= -\frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(}}$$
$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{b}{4x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{4\left(bx-\tanh\right.}$$

Mathematica [A] time = 0.10, size = 89, normalized size = 0.71

$$\frac{1}{4}\left(\frac{b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}} + \frac{\left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}-2\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^3,x]
```

```
[Out] (((-2 + (b*x)/(b*x - ArcTanh[Tanh[a + b*x]]))*Sqrt[ArcTanh[Tanh[a + b*x]]])
/x^2 + (b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh
[a + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2))/4
```

fricas [A] time = 0.62, size = 119, normalized size = 0.95

$$\left[\frac{\sqrt{a}\,b^2x^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(abx+2a^2\right)\sqrt{bx+a}}{8a^2x^2}, -\frac{\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(abx+2a^2\right)\sqrt{bx+a}}{4a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

giac [A] time = 0.17, size = 75, normalized size = 0.60

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a} + \frac{\sqrt{2}\left((bx+a)^{\frac{3}{2}}b^3+\sqrt{bx+a}\,ab^3\right)}{ab^2x^2}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(sqrt(2)*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqr
t(2)*((b*x + a)^(3/2)*b^3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

maple [A] time = 0.16, size = 92, normalized size = 0.74

$$2b^{2}\left(\frac{-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8}}{b^{2}x^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{8\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^3,x)
```

```
[Out] 2*b^2*((-1/8/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(3/2)-1/8*arct
anh(tanh(b*x+a))^(1/2))/b^2/x^2+1/8/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctan
h(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^3, x)
```

mupad [B] time = 5.74, size = 741, normalized size = 5.93

$$\frac{b\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{2\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+\right.}{x^{2}\,\left(2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^3,x)
```

```
[Out] (b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x
) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(x^2*(2*log(2/(exp(2*a)*exp(2*b*
x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x
)) + (2^(1/2)*b^2*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*
b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*4i)/(x*(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
```

```
+ 2*b*x)^(1/2)))*1i)/(4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**3, x)
```

3.119 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^4}\, dx$

Optimal. Leaf size=179

$$\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx$$

```
[Out] 1/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/24*b^2/x/arctanh(tanh(b*x+a))^(3/2)-1/2
4*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/12*b/x^2/arct
anh(tanh(b*x+a))^(1/2)+1/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*
x+a))^(1/2)-1/3*arctanh(tanh(b*x+a))^(1/2)/x^3
```

Rubi [A] time = 0.12, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^4,x]
```

```
[Out] (b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b^2/(24*x*ArcTanh[Tanh[a + b*
x]]^(3/2)) - b^3/(24*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^
(3/2)) - b/(12*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + b^3/(8*(b*x - ArcTanh[Ta
nh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) - Sqrt[ArcTanh[Tanh[a + b*x]]
]/(3*x^3)
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^4}\,dx = -\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^3} + \frac{1}{6}b\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$$

$$= -\frac{b}{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^3} - \frac{1}{24}b^2\int \frac{1}{x^2\tanh^{-1}}$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b}{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{\sqrt{\tanh^{-1}(\tanh}}{3x^3}$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$

$$= \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$

$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b^2}{24x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{}{24\left(bx-\tan\right.}$$

Mathematica [A] time = 0.10, size = 115, normalized size = 0.64

$$\frac{1}{24}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(14bx\tanh^{-1}(\tanh(a+bx))-8\tanh^{-1}(\tanh(a+bx))^2-3b^2x^2\right)}{x^3\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{3b^3\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{\tan}}\right.}{\left(\tanh^{-1}(\tanh(\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^4,x]
```

```
[Out] ((-3*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (Sqrt[ArcTanh[Tanh[a +
 b*x]]]*(-3*b^2*x^2 + 14*b*x*ArcTanh[Tanh[a + b*x]] - 8*ArcTanh[Tanh[a + b*
x]]^2))/(x^3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2))/24
```

fricas [A] time = 0.51, size = 145, normalized size = 0.81

$$\left[\frac{3\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3ab^2x^2-2a^2bx-8a^3\right)\sqrt{bx+a}}{48a^3x^3}, \frac{3\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3ab^2\right.}{24a^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-a)
*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x + a))/(a^3*x^3)]
```

giac [A] time = 0.92, size = 93, normalized size = 0.52

$$\frac{\sqrt{2}\left(\frac{3\,\sqrt{2}\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}b^4-8\,(bx+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx+a}\,a^2b^4\right)}{a^2b^3x^3}\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*(3*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) +
 sqrt(2)*(3*(b*x + a)^(5/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)
*a^2*b^4)/(a^2*b^3*x^3))/b
```

maple [A] time = 0.18, size = 185, normalized size = 1.03

$$2b^3\left(\frac{\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16a^2+32a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+16(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2}-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{6(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{16}}{b^3x^3}-\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^4,x)
```

```
[Out] 2*b^3*((1/16/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*
x-a)^2)*arctanh(tanh(b*x+a))^(5/2)-1/6/(arctanh(tanh(b*x+a))-b*x)*arctanh(t
anh(b*x+a))^(3/2)-1/16*arctanh(tanh(b*x+a))^(1/2))/b^3/x^3-1/16/(a^2+2*a*(a
rctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*
x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b
*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{artanh}(\tanh(bx+a))}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/x^4, x)
```

mupad [B] time = 5.51, size = 964, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^4,x)
```

```
[Out] (b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (b^2*
(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2))/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/2)*b^3
*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
```

```
)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^4*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x))*4i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*1i)/(4*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^(5/2)) - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x))/(x^3*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**4, x)
```

3.120 $\int x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{256\tanh^{-1}(\tanh(a+bx))^{13/2}}{15015b^5}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{11/2}}{1155b^4}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3}-\frac{16x^3\tanh^{-1}(\ta$$

```
[Out] 2/5*x^4*arctanh(tanh(b*x+a))^(5/2)/b-16/35*x^3*arctanh(tanh(b*x+a))^(7/2)/b
^2+32/105*x^2*arctanh(tanh(b*x+a))^(9/2)/b^3-128/1155*x*arctanh(tanh(b*x+a)
)^(11/2)/b^4+256/15015*arctanh(tanh(b*x+a))^(13/2)/b^5
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2157, 30}

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{13/2}}{15015b^5}-\frac{128x\tanh^{-1}(\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(7/2))/(35*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(9/2))/(105*b^3) - (128*x
*ArcTanh[Tanh[a + b*x]]^(11/2))/(1155*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(1
3/2))/(15015*b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{8\int x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{48\int x^2 \tanh$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tanh^{-1}(}{1}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tanh^{-1}(}{1}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tanh^{-1}(}{1}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2} + \frac{32x^2 \tanh^{-1}(}{1}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-3432b^3x^3\tanh^{-1}(\tanh(a+bx)) + 2288b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 832bx\tanh^{-1}(}{15015b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(3003*b^4*x^4 - 3432*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 2288*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 832*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(15015*b^5)
```

fricas [A] time = 0.48, size = 75, normalized size = 0.74

$$\frac{2\left(1155\,b^6x^6 + 1470\,ab^5x^5 + 35\,a^2b^4x^4 - 40\,a^3b^3x^3 + 48\,a^4b^2x^2 - 64\,a^5bx + 128\,a^6\right)\sqrt{bx+a}}{15015\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

giac [B] time = 0.19, size = 241, normalized size = 2.39

$$\sqrt{2}\left(\frac{143\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a)^{\frac{5}{2}}a^2 - 420\,(bx+a)^{\frac{3}{2}}a^3 + 315\,\sqrt{bx+a}\,a^4\right)a^2}{b^4} + \frac{130\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}} - 385\,(bx+a)^{\frac{9}{2}}a + 990\,(bx+a)^{\frac{7}{2}}a^2}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(143*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a +
378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*
a^2/b^4 + 130*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b
*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 6
93*sqrt(b*x + a)*a^5)*a/b^4 + 15*sqrt(2)*(231*(b*x + a)^(13/2) - 1638*(b*x
+ a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*
```

```
(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^
4)/b
```

maple [A] time = 0.14, size = 154, normalized size = 1.52

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13}+\frac{2(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2\left(2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2}+(-2\operatorname{arctanh}(\tanh(bx+\right.}{9}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^5*(1/13*arctanh(tanh(b*x+a))^(13/2)+1/11*(-4*arctanh(tanh(b*x+a))+4*b*x
)*arctanh(tanh(b*x+a))^(11/2)+1/9*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arcta
nh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(9/2)+2/7*(b*x-arctanh(tanh(
b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(7/2)+1/5*(
b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(5/2))
```

maxima [A] time = 0.53, size = 64, normalized size = 0.63

$$\frac{2\left(1155\,b^{5}x^{5}+315\,ab^{4}x^{4}-280\,a^{2}b^{3}x^{3}+240\,a^{3}b^{2}x^{2}-192\,a^{4}bx+128\,a^{5}\right)(bx+a)^{\frac{3}{2}}}{15015\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*b^5*x^5 + 315*a*b^4*x^4 - 280*a^2*b^3*x^3 + 240*a^3*b^2*x^2 -
 192*a^4*b*x + 128*a^5)*(b*x + a)^(3/2)/b^5
```

mupad [B] time = 1.19, size = 1813, normalized size = 17.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (2*b*x^6*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/13 + (x^5*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
(24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(11*b
) + (x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (10*((2
4*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(
exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1))/2 + b*x))/(11*b)))/(9*b) + (128*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^2/2 + (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*
a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1))/2 + b*x)^4)/(315*b^5) + (8*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
```

```
xp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^2/2 + (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(
2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
 b*x))/(63*b^2) + (64*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^2/2 + (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^3)/(3
15*b^4) + (16*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2
+ (10*((24*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13 - 2*b*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(105*b^3
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**4*atanh(tanh(a + b*x))**(3/2), x)
```

3.121 $\int x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=80

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{11/2}}{1155b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3}-\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{2x^3\tanh^{-1}(\tanh}{5b}$$

```
[Out] 2/5*x^3*arctanh(tanh(b*x+a))^(5/2)/b-12/35*x^2*arctanh(tanh(b*x+a))^(7/2)/b
^2+16/105*x*arctanh(tanh(b*x+a))^(9/2)/b^3-32/1155*arctanh(tanh(b*x+a))^(11
/2)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$-\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{11/2}}{1155b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{9/2}}{105b^3}+\frac{2x^3\tanh^{-1}(\tanh}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (12*x^2*ArcTanh[Tanh[a + b*x]]
^(7/2))/(35*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(9/2))/(105*b^3) - (32*ArcT
anh[Tanh[a + b*x]]^(11/2))/(1155*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^3\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx &= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{6\int x^2\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}\\
&= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{24\int x\tan}{}\\
&= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{16x\tanh^{-}}{}\\
&= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{16x\tanh^{-}}{}\\
&= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{16x\tanh^{-}}{}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-198b^2x^2\tanh^{-1}(\tanh(a+bx))+88bx\tanh^{-1}(\tanh(a+bx))^2-16\tanh^{-1}(\tanh(a+\right.}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(231*b^3*x^3 - 198*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 88*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(1
155*b^4)
```

fricas [A] time = 0.48, size = 64, normalized size = 0.80

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

giac [B] time = 0.20, size = 205, normalized size = 2.56

$$\frac{\sqrt{2}\left(\frac{99\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2}{b^3}+\frac{22\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{b\right.}{b^3}\right.}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*sqrt(2)*(99*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(
b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^3 + 22*sqrt(2)*(35*(b*x +
a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^
(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/b^3 + 5*sqrt(2)*(63*(b*x + a)^(11/2) -
 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3
 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)/b^3)/b
```

maple [A] time = 0.14, size = 124, normalized size = 1.55

$$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2(-3\,\mathrm{arctanh}(\tanh(bx+a))+3bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2\left((bx-\mathrm{arctanh}(\tanh(bx+a)))(-2\,\mathrm{arctanh}(\tanh(bx+a))+2bx\right.}{7}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^4*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-3*arctanh(tanh(b*x+a))+3*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tan
h(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(7/2)+1
/5*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(5/2))
```

maxima [A] time = 0.53, size = 53, normalized size = 0.66

$$\frac{2\left(105\,b^4x^4+35\,ab^3x^3-30\,a^2b^2x^2+24\,a^3bx-16\,a^4\right)(bx+a)^{\frac{3}{2}}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*b^4*x^4 + 35*a*b^3*x^3 - 30*a^2*b^2*x^2 + 24*a^3*b*x - 16*a^4)*
(b*x + a)^(3/2)/b^4
```

mupad [B] time = 1.12, size = 1483, normalized size = 18.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (2*b*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 + (x^4*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
(20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(9*b)
 + (x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (8*((20*
b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))/2 + b*x))/(9*b)))/(7*b) + (16*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2/2 + (8*((20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp
(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
+ b*x)^3)/(35*b^4) + (6*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2/2 + (8*((20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) +
1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(3
5*b^2) + (8*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (8
*((20*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(35*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**3*atanh(tanh(a + b*x))**(3/2), x)
```

3.122 $\int x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

```
[Out] 2/5*x^2*arctanh(tanh(b*x+a))^(5/2)/b-8/35*x*arctanh(tanh(b*x+a))^(7/2)/b^2+
16/315*arctanh(tanh(b*x+a))^(9/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(7
/2))/(35*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(9/2))/(315*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^2\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx=\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\int x\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}$$
$$=\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{8\int\tanh^{-1}(\text{t}$$
$$=\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{8\,\text{Subst}\left(\int x\right.}{}$$
$$=\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}+\frac{16\tanh^{-1}(\text{ta}}{3}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-36bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2+63b^2x^2\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(63*b^2*x^2 - 36*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(315*b^3)
```

fricas [A] time = 0.42, size = 53, normalized size = 0.90

$$\frac{2\left(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4\right)\sqrt{bx+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

giac [B] time = 0.16, size = 168, normalized size = 2.85

$$\frac{\sqrt{2}\left(\frac{21\,\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^2}{b^2}+\frac{18\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a}{b^2}+\frac{\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(\right.}{}\right.}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(21*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sq
rt(b*x + a)*a^2)*a^2/b^2 + 18*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/
2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^2 + sqrt(2)*(35*(
b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x
 + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^2)/b
```

maple [A] time = 0.14, size = 69, normalized size = 1.17

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))^2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^3*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x
+a))^(5/2))
```

maxima [A] time = 0.53, size = 42, normalized size = 0.71

$$\frac{2\left(35\,b^3x^3+15\,ab^2x^2-12\,a^2bx+8\,a^3\right)(bx+a)^{\frac{3}{2}}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^3*x^3 + 15*a*b^2*x^2 - 12*a^2*b*x + 8*a^3)*(b*x + a)^(3/2)/b^3
```

mupad [B] time = 1.12, size = 1153, normalized size = 19.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (2*b*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/9 + (x^3*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((
16*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(7*b) +
 (x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (6*((16*b*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2
*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 + b*x))/(7*b)))/(5*b) + (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^2/2 + (6*((16*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b))*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
)^2)/(15*b^3) + (4*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
/2 + (6*((16*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/9 - 2*b*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x))/(7*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(15*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**2*atanh(tanh(a + b*x))**(3/2), x)
```

3.123 $\int x \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}$$

```
[Out] 2/5*x*arctanh(tanh(b*x+a))^(5/2)/b-4/35*arctanh(tanh(b*x+a))^(7/2)/b^2
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b) - (4*ArcTanh[Tanh[a + b*x]]^(7/2))
/(35*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\int\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx}{5b}\\ &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{2\,\text{Subst}\left(\int x^{5/2}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{5b^2}\\ &= \frac{2x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^2}\end{aligned}$$

Mathematica [A] time = 0.08, size = 32, normalized size = 0.84

$$\frac{2\left(7bx-2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*(7*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*b^2
)
```

fricas [A] time = 0.53, size = 41, normalized size = 1.08

$$\frac{2\left(5\,b^{3}x^{3}+8\,ab^{2}x^{2}+a^{2}bx-2\,a^{3}\right)\sqrt{bx+a}}{35\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

giac [B] time = 0.55, size = 131, normalized size = 3.45

$$\frac{\sqrt{2}\left(\frac{35\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^{2}}{b}+\frac{14\,\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)a}{b}+\frac{3\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/105*sqrt(2)*(35*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2/b + 14*
sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a
/b + 3*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)/b)/b
```

maple [A] time = 0.14, size = 42, normalized size = 1.11

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^2*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(5/2))
```

maxima [A] time = 0.54, size = 31, normalized size = 0.82

$$\frac{2\left(5\,b^{2}x^{2}+3\,abx-2\,a^{2}\right)(bx+a)^{\frac{3}{2}}}{35\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^2*x^2 + 3*a*b*x - 2*a^2)*(b*x + a)^(3/2)/b^2
```

mupad [B] time = 1.08, size = 823, normalized size = 21.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (2*b*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/7 + (x^2*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((
12*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 + b*x))/7 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(5*b) +
 (x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/2 + (4*((12*b*(l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1))/2 + b*x))/7 - 2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a
)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 + b*x))/(5*b)))/(3*b) + (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^2/2 + (4*((12*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/7 - 2*b*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b))*(log(2/(exp(2*a)*exp(2*b*x)
 + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))
/(3*b^2)
```

sympy [A] time = 26.90, size = 49, normalized size = 1.29

$$\begin{cases} \frac{2x\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{5b} - \frac{4\operatorname{atanh}^{\frac{7}{2}}(\tanh(a+bx))}{35b^2} & \text{for } b \neq 0 \\ \frac{x^2\operatorname{atanh}^{\frac{3}{2}}(\tanh(a))}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*x*atanh(tanh(a + b*x))**(5/2)/(5*b) - 4*atanh(tanh(a + b*x))**
(7/2)/(35*b**2), Ne(b, 0)), (x**2*atanh(tanh(a))**(3/2)/2, True))
```

3.124 $\int \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{2157, 30\}$

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int x^{3/2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b)
```

fricas [A] time = 0.52, size = 28, normalized size = 1.56

$$\frac{2\left(b^2x^2 + 2\,abx + a^2\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

giac [B] time = 0.20, size = 84, normalized size = 4.67

$$\frac{\sqrt{2}\left(15\sqrt{2}\sqrt{bx+a}\,a^2+10\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a+\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*sqrt(2)*sqrt(b*x + a)*a^2 + 10*sqrt(2)*((b*x + a)^(3/2) -
3*sqrt(b*x + a)*a)*a + sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a +
15*sqrt(b*x + a)*a^2))/b
```

maple [A] time = 0.03, size = 15, normalized size = 0.83

$$\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)/b
```

maxima [A] time = 0.49, size = 12, normalized size = 0.67

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b
```

mupad [B] time = 1.16, size = 97, normalized size = 5.39

$$\frac{\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\right)^2\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] ((log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)))^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(10*b)
```

sympy [A] time = 11.53, size = 26, normalized size = 1.44

$$\begin{cases}\frac{2\,\mathrm{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{5b} & \text{for } b\neq 0\\ x\,\mathrm{atanh}^{\frac{3}{2}}\left(\tanh\left(a\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*atanh(tanh(a + b*x))**(5/2)/(5*b), Ne(b, 0)), (x*atanh(tanh(a)
)**(3/2), True))
```

3.125 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x} dx$

Optimal. Leaf size=91

$$-2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}+2\left(bx-\tanh^{-1}(\tanh(a+\right.$$

```
[Out] 2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-
arctanh(tanh(b*x+a)))^(3/2)+2/3*arctanh(tanh(b*x+a))^(3/2)-2*(b*x-arctanh(t
anh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2159, 2161}

$$-2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}+2\left(bx-\tanh^{-1}(\tanh(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x,x]
```

```
[Out] 2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(
b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt
[ArcTanh[Tanh[a + b*x]]] + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}dx = \frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}-\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\sqrt{\tanh^{-1}(\tanh}}{x}$$

$$= -2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+\frac{2}{3}\tanh^{-1}(\tanh(a$$

$$= 2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}-2\left(\right.$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.88

$$-\frac{2}{3}\left(-4\tanh^{-1}(\tanh(a+bx))^{3/2}+3bx\sqrt{\tanh^{-1}(\tanh(a+bx))}+3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)(\tanh$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x,x]
```

```
[Out] (-2*(3*b*x*Sqrt[ArcTanh[Tanh[a + b*x]]] - 4*ArcTanh[Tanh[a + b*x]]^(3/2) +
3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)))/3
```

fricas [A] time = 0.64, size = 88, normalized size = 0.97

$$\left[a^{\frac{3}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{3}(bx+4a)\sqrt{bx+a}, 2\sqrt{-a}\,a\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{3}(bx+4a)\sqrt{bx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sqr
t(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

giac [A] time = 0.22, size = 57, normalized size = 0.63

$$\frac{1}{3}\sqrt{2}\left(\frac{3\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{2}\,(bx+a)^{\frac{3}{2}}+3\sqrt{2}\sqrt{bx+a}\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(3*sqrt(2)*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(2
)*(b*x + a)^(3/2) + 3*sqrt(2)*sqrt(b*x + a)*a)
```

maple [A] time = 0.14, size = 131, normalized size = 1.44

$$\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}+2a\sqrt{\operatorname{arctanh}(\tanh(bx+a))}+2(\operatorname{arctanh}(\tanh(bx+a))-bx-a)\sqrt{\operatorname{arctanh}(\tanh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x,x)
```

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)+2*a*arctanh(tanh(b*x+a))^(1/2)+2*(arctanh(ta
nh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-2*(a^2+2*a*(arctanh(tanh(b*x+a
))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*
arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x, x)
```

mupad [B] time = 6.00, size = 501, normalized size = 5.51

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4\,b\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)}{3}-2\,b\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x,x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((4*b*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/3 - 2*b*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)))/b + (2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x
)*4i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)*1i
)/4 + (2*b*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x, x)
```

3.126 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2} dx$

Optimal. Leaf size=81

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}+3b\sqrt{\tanh^{-1}(\tanh(a+bx))}-3b\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh}}{\sqrt{bx-\tanh^{-1}(\text{ta}}}\right.$$

```
[Out] -arctanh(tanh(b*x+a))^(3/2)/x-3*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-ar
ctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)+3*b*arctanh(tan
h(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2159, 2161}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}+3b\sqrt{\tanh^{-1}(\tanh(a+bx))}-3b\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh}}{\sqrt{bx-\tanh^{-1}(\text{ta}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^2,x]
```

```
[Out] -3*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]] + 3*b*Sqrt[ArcTanh[Tanh[a + b*x]]] - A
rcTanh[Tanh[a + b*x]]^(3/2)/x
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x} + \frac{1}{2}(3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\,dx$$
$$= 3b\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x} - \frac{1}{2}\Big(3b\Big(bx - \tanh^{-1}($$
$$= -3b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))} + 3$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.98

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x} + 3b\sqrt{\tanh^{-1}(\tanh(a+bx))} - 3b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)\sqrt{\tanh^{-1}(t$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[ArcTanh[Tanh[a + b*x]]] - ArcTanh[Tanh[a + b*x]]^(3/2)/x - 3*b*Arc
Tanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sq
rt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

fricas [A] time = 0.68, size = 102, normalized size = 1.26

$$\left[\frac{3\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(2bx-a)\sqrt{bx+a}}{2x}, \frac{3\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(2bx-a)\sqrt{bx+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*x
 - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

giac [A] time = 0.40, size = 69, normalized size = 0.85

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{2}\sqrt{bx+a}\,b^2 - \frac{\sqrt{2}\sqrt{bx+a}\,ab}{x}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(3*sqrt(2)*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sq
rt(2)*sqrt(b*x + a)*b^2 - sqrt(2)*sqrt(b*x + a)*a*b/x)/b
```

maple [A] time = 0.15, size = 85, normalized size = 1.05

$$2b\left(\sqrt{\operatorname{arctanh}(\tanh(bx+a))} + \frac{\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{2}+\frac{bx}{2}\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{bx} - \frac{3\sqrt{\operatorname{arctanh}(\tanh(bx}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^2,x)
```

```
[Out] 2*b*(arctanh(tanh(b*x+a))^(1/2)+(-1/2*arctanh(tanh(b*x+a))+1/2*b*x)*arctanh
(tanh(b*x+a))^(1/2)/b/x-3/2*(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctan
h(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^2, x)
```

mupad [B] time = 2.22, size = 459, normalized size = 5.67

$$3\,b\,\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}+\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{2\,x}-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\sqrt{\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} \cdots}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^2,x)
```

```
[Out] 3*b*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2) + (log(1/(exp(2*a)*exp(2*b*x) + 1))*(log((exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x)
+ 1))/2)^(1/2))/(2*x) - (log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/(2*x) + b*log(-(4*2^(1/2)*log(1/(exp(2*a)*exp(
2*b*x) + 1)) - 4*2^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 8*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) - log(1/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2) + 4*2^(1/2)*b*
x)/(x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(1/(exp(2*
a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2)))*((9*log((exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)))/8 - (9*log(1/(exp(2*a)*exp(2*b*x) + 1)))/8 - (9*b*x)/4)
^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**2, x)
```

3.127 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3} dx$

Optimal. Leaf size=92

$$\frac{3b^2 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2}-\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x}$$

```
[Out] -1/2*arctanh(tanh(b*x+a))^(3/2)/x^2+3/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/
2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arctanh(tanh(b*x+a)))^(1/2)-3/4*b
*arctanh(tanh(b*x+a))^(1/2)/x
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2168, 2161}

$$\frac{3b^2 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2}-\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^3,x]
```

```
[Out] (3*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(4*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - (3*b*Sqrt[ArcTanh[Tanh[a + b*
x]]])/(4*x) - ArcTanh[Tanh[a + b*x]]^(3/2)/(2*x^2)
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2} + \frac{1}{4}(3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2}\,dx$$
$$= -\frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{2x^2} + \frac{1}{8}\left(3b^2\right)\int \frac{}{x\sqrt{\tanh^{-}}}$$
$$= \frac{3b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x} - \frac{\tanh^{-1}(\tanh(}{2x^2}$$

Mathematica [A] time = 0.07, size = 88, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}} - \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2} - \frac{3b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^3,x]
```

```
[Out] ((-3*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/x - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/x
^2 - (3*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh
[a + b*x]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/4
```

fricas [A] time = 0.57, size = 124, normalized size = 1.35

$$\left[\frac{3\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(5abx+2a^2\right)\sqrt{bx+a}}{8ax^2}, \frac{3\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\left(5abx+2a^2\right)\sqrt{bx}}{4ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(5
*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sqrt
(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

giac [A] time = 0.19, size = 73, normalized size = 0.79

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{2}\left(5\,(bx+a)^{\frac{3}{2}}b^3-3\sqrt{bx+a}\,ab^3\right)}{b^2x^2}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*sqrt(2)*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(2
)*(5*(b*x + a)^(3/2)*b^3 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

maple [A] time = 0.15, size = 91, normalized size = 0.99

$$2b^2\left(\frac{-\frac{5\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8}+\left(\frac{3\,\mathrm{arctanh}(\tanh(bx+a))}{8}-\frac{3bx}{8}\right)\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b^2x^2}-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctanh}(\tanh(b}}{\sqrt{\mathrm{arctanh}(\tanh(bx}}\right.}{8\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^3,x)
```

```
[Out] 2*b^2*((-5/8*arctanh(tanh(b*x+a))^(3/2)+(3/8*arctanh(tanh(b*x+a))-3/8*b*x)*
arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-3/8/(arctanh(tanh(b*x+a))-b*x)^(1/2)*ar
ctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^3, x)
```

mupad [B] time = 6.03, size = 609, normalized size = 6.62

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2\,x^2\,\left(2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x\right)}-\frac{b\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{5\,\ln}{}\right.}{x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^3,x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^2*(2*log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 4*b*x)) + (2^(1/2)*b^2*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(
2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x) + 2^(1/2)*b*x)*16i)/x)*3i)/(8*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2
)) - (b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((5*log(2/(exp(2*a)*exp(2*b*x) + 1)))/4
- (5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/4 + (5*b*x)/2)
)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**3, x)
```

3.128 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4} \, dx$

Optimal. Leaf size=146

$$\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh}}$$

```
[Out] 1/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(3/2)-1/3*arctanh(tanh(b*x+a))^(3/2)/x^3-1/8*b^
2/x/arctanh(tanh(b*x+a))^(1/2)+1/8*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(t
anh(b*x+a))^(1/2)-1/4*b*arctanh(tanh(b*x+a))^(1/2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2163, 2161}

$$\frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh}}{\sqrt{bx-\tanh^{-1}(\tan}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^4,x]
```

```
[Out] (b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - b^2/(8*x*Sqrt[ArcTanh[Tanh[a
+ b*x]]]) + b^3/(8*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x
]]]) - (b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*x^2) - ArcTanh[Tanh[a + b*x]]^(3
/2)/(3*x^3)
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^3} + \frac{1}{2}b\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^3}\,dx$$
$$= -\frac{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^3} + \frac{1}{8}b^2\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$
$$= -\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4x^2} - \frac{\tanh^{-1}(\tanh(a+b}{3x^3}$$
$$= -\frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(}}$$
$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{b^2}{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{8\left(bx-\tanh\right.}$$

Mathematica [A] time = 0.10, size = 117, normalized size = 0.80

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{8\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}+\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-\frac{b^2}{8x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}-\frac{\tanh^{-1}(\tanh(a}{3x^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^4,x]
```

```
[Out] (b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) + Sqrt[ArcTanh[Tanh[a +
b*x]]]*((-7*b)/(12*x^2) - b^2/(8*x*(-(b*x) + ArcTanh[Tanh[a + b*x]])) - (-(
b*x) + ArcTanh[Tanh[a + b*x]])/(3*x^3))
```

fricas [A] time = 0.46, size = 145, normalized size = 0.99

$$\left[\frac{3\sqrt{a}\,b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3ab^2x^2+14a^2bx+8a^3\right)\sqrt{bx+a}}{48a^2x^3}, -\frac{3\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3a\right.}{24a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x + 8
*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

giac [A] time = 0.21, size = 93, normalized size = 0.64

$$-\frac{\sqrt{2}\left(\frac{3\sqrt{2}\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{2}\left(3(bx+a)^{\frac{5}{2}}b^4+8(bx+a)^{\frac{3}{2}}ab^4-3\sqrt{bx+a}\,a^2b^4\right)}{ab^3x^3}\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(2)*(3*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) +
sqrt(2)*(3*(b*x + a)^(5/2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*
a^2*b^4)/(a*b^3*x^3))/b
```

maple [A] time = 0.15, size = 116, normalized size = 0.79

$$2b^3\left(\frac{-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{6}+\left(\frac{\operatorname{arctanh}(\tanh(bx+a))}{16}-\frac{bx}{16}\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b^3x^3}+\frac{\operatorname{arc}}{16(\operatorname{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^4,x)
```

```
[Out] 2*b^3*((-1/16/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(5/2)-1/6*arc
tanh(tanh(b*x+a))^(3/2)+(1/16*arctanh(tanh(b*x+a))-1/16*b*x)*arctanh(tanh(b
*x+a))^(1/2))/b^3/x^3+1/16/(arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arctanh
(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^4, x)
```

mupad [B] time = 5.42, size = 1019, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^4,x)
```

```
[Out] (11*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(
exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(12*x*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + ((log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp
(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^3*(3*log(2/(exp(2*a
)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 6*b*x)) - (2*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(3*log(2/(exp(2*a)*exp
(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
6*b*x)) + (2^(1/2)*b^3*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(
1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
```

```
*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*16i)/(x*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(1/2)))*1i)/(8*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)) - (3*b*(log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2
*b*x) + 1))/2)^(1/2)*((7*log(2/(exp(2*a)*exp(2*b*x) + 1)))/18 - (7*log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/18 + (7*b*x)/9))/(x^2*(2*lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 4*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a + bx))}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**4,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**4, x)
```

3.129 $\int x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=101

$$\frac{256\tanh^{-1}(\tanh(a+bx))^{15/2}}{45045b^5}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{13/2}}{3003b^4}+\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{16x^3\tanh^{-1}(\text{t}$$

```
[Out] 2/7*x^4*arctanh(tanh(b*x+a))^(7/2)/b-16/63*x^3*arctanh(tanh(b*x+a))^(9/2)/b
^2+32/231*x^2*arctanh(tanh(b*x+a))^(11/2)/b^3-128/3003*x*arctanh(tanh(b*x+a
))^(13/2)/b^4+256/45045*arctanh(tanh(b*x+a))^(15/2)/b^5
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{16x^3\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{256\tanh^{-1}(\tanh(a+bx))^{15/2}}{45045b^5}-\frac{128x\tanh^{-1}(\text{ta}}{300}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^4*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (16*x^3*ArcTanh[Tanh[a + b*x]]
^(9/2))/(63*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(11/2))/(231*b^3) - (128*
x*ArcTanh[Tanh[a + b*x]]^(13/2))/(3003*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(
15/2))/(45045*b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{8\int x^3 \tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{16\int x^2 \tanh^{-1}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2 \tanh^{-1}(}{2}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2 \tanh^{-1}(}{2}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2 \tanh^{-1}(}{2}$$
$$= \frac{2x^4 \tanh^{-1}(\tanh(a+bx))^{7/2}}{7b} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2} + \frac{32x^2 \tanh^{-1}(}{2}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-5720b^3x^3\tanh^{-1}(\tanh(a+bx)) + 3120b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 960bx\tanh^{-1}(\right.}{45045b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(6435*b^4*x^4 - 5720*b^3*x^3*ArcTanh[Tanh[a
 + b*x]] + 3120*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 960*b*x*ArcTanh[Tanh[a +
 b*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(45045*b^5)
```

fricas [A] time = 0.52, size = 86, normalized size = 0.85

$$\frac{2\left(3003\,b^7x^7 + 7161\,ab^6x^6 + 4473\,a^2b^5x^5 + 35\,a^3b^4x^4 - 40\,a^4b^3x^3 + 48\,a^5b^2x^2 - 64\,a^6bx + 128\,a^7\right)\sqrt{bx+a}}{45045\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 7161*a*b^6*x^6 + 4473*a^2*b^5*x^5 + 35*a^3*b^4*x^4
- 40*a^4*b^3*x^3 + 48*a^5*b^2*x^2 - 64*a^6*b*x + 128*a^7)*sqrt(b*x + a)/b^5
```

giac [B] time = 0.17, size = 344, normalized size = 3.41

$$\sqrt{2}\left(\frac{143\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a)^{\frac{5}{2}}a^2 - 420\,(bx+a)^{\frac{3}{2}}a^3 + 315\,\sqrt{bx+a}\,a^4\right)a^3}{b^4} + \frac{195\sqrt{2}\left(63\,(bx+a)^{\frac{11}{2}} - 385\,(bx+a)^{\frac{9}{2}}a + 990\,(bx+a)^{\frac{7}{2}}a^2\right.}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/45045*sqrt(2)*(143*sqrt(2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a +
378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*
a^3/b^4 + 195*sqrt(2)*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b
*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 6
93*sqrt(b*x + a)*a^5)*a^2/b^4 + 45*sqrt(2)*(231*(b*x + a)^(13/2) - 1638*(b*
x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 900
9*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*
```

```
a/b^4 + 7*sqrt(2)*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(
b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4
 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x +
a)*a^7)/b^4)/b
```

maple [A] time = 0.14, size = 154, normalized size = 1.52

$$\frac{\frac{2 \operatorname{arctanh}(\tanh(bx+a))^{\frac{15}{2}}}{15}+\frac{2(-4 \operatorname{arctanh}(\tanh(bx+a))+4bx) \operatorname{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13}+\frac{2\left(2(bx-\operatorname{arctanh}(\tanh(bx+a)))^2+(-2 \operatorname{arctanh}(\tanh(bx+\right.}{11}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^5*(1/15*arctanh(tanh(b*x+a))^(15/2)+1/13*(-4*arctanh(tanh(b*x+a))+4*b*x
)*arctanh(tanh(b*x+a))^(13/2)+1/11*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arct
anh(tanh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(11/2)+2/9*(b*x-arctanh(tan
h(b*x+a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(9/2)+1/7
*(b*x-arctanh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(7/2))
```

maxima [A] time = 0.53, size = 64, normalized size = 0.63

$$\frac{2\left(3003\, b^5x^5+1155\, ab^4x^4-840\, a^2b^3x^3+560\, a^3b^2x^2-320\, a^4bx+128\, a^5\right)(bx+a)^{\frac{5}{2}}}{45045\, b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^5*x^5 + 1155*a*b^4*x^4 - 840*a^2*b^3*x^3 + 560*a^3*b^2*x^2
- 320*a^4*b*x + 128*a^5)*(b*x + a)^(5/2)/b^5
```

mupad [B] time = 1.28, size = 2681, normalized size = 26.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (2*b^2*x^7*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/15 + (x^5*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
- (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(13*b
)))/(11*b) - (x^6*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (28*b^2*(log(2/(exp(2*a
)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 + b*x))/15)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
- log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(13*b) - (x^4*(log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (10*((3*b*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (28*b^2*(log(2/(exp(2*a)*ex
p(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
```

```
 + b*x))/15)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(13*b))*(log(2/(exp(2*a)*exp(2*b*x
) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)
)/(11*b)))/(9*b) - (128*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^3/4 - (10*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2
*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
b*x))/(13*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
)^4)/(315*b^5) - (8*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^3/4 - (10*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2
*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
b*x))/(13*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
))/(63*b^2) - (64*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/
4 - (10*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2*b*x
) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)
)/(13*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x) +
 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^3)
/(315*b^4) - (16*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3
/4 - (10*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (12*(3*b^2*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x) - (28*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/15)*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
))/(13*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x)
+ 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2
)/(105*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.130 $\int x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=80

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{13/2}}{3003b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}+\frac{2x^3\tanh^{-1}(\tanh(a}{7b}$$

```
[Out] 2/7*x^3*arctanh(tanh(b*x+a))^(7/2)/b-4/21*x^2*arctanh(tanh(b*x+a))^(9/2)/b^
2+16/231*x*arctanh(tanh(b*x+a))^(11/2)/b^3-32/3003*arctanh(tanh(b*x+a))^(13
/2)/b^4
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{13/2}}{3003b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{11/2}}{231b^3}+\frac{2x^3\tanh^{-1}(\tanh(a}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*x^2*ArcTanh[Tanh[a + b*x]]^
(9/2))/(21*b^2) + (16*x*ArcTanh[Tanh[a + b*x]]^(11/2))/(231*b^3) - (32*ArcT
anh[Tanh[a + b*x]]^(13/2))/(3003*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^3\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx &= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{6\int x^2\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}\\
&= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}+\frac{8\int x\tanh^{-1}(\mathrm{t}}{}\\
&= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}+\frac{16x\tanh^{-1}(\mathrm{ta}}{23}\\
&= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}+\frac{16x\tanh^{-1}(\mathrm{ta}}{23}\\
&= \frac{2x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{9/2}}{21b^2}+\frac{16x\tanh^{-1}(\mathrm{ta}}{23}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-286b^2x^2\tanh^{-1}(\tanh(a+bx))+104bx\tanh^{-1}(\tanh(a+bx))^2-16\tanh^{-1}(\tanh(a+bx))^3+429b^3x^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(429*b^3*x^3 - 286*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 104*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(
3003*b^4)
```

fricas [A] time = 0.45, size = 75, normalized size = 0.94

$$\frac{2\left(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6\right)\sqrt{bx+a}}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a
^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

giac [B] time = 0.13, size = 296, normalized size = 3.70

$$\sqrt{2}\left(\frac{429\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^3}{b^3}+\frac{143\,\sqrt{2}\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+\cdots\right.}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/15015*sqrt(2)*(429*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^3/b^3 + 143*sqrt(2)*(35*(b*x
 + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2/b^3 + 65*sqrt(2)*(63*(b*x + a)^(1
1/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/
2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a/b^3 + 5*sqrt(2
)*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^
2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3
/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^3)/b
```

maple [A] time = 0.14, size = 124, normalized size = 1.55

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{13}{2}}}{13}+\frac{2(-3\operatorname{arctanh}(\tanh(bx+a))+3bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2\left((bx-\operatorname{arctanh}(\tanh(bx+a)))(-2\operatorname{arctanh}(\tanh(bx+a))\cdots\right.}{\cdots}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^4*(1/13*arctanh(tanh(b*x+a))^(13/2)+1/11*(-3*arctanh(tanh(b*x+a))+3*b*x
)*arctanh(tanh(b*x+a))^(11/2)+1/9*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(t
anh(b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(9/2)
+1/7*(b*x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(7/2))
```

maxima [A] time = 0.54, size = 53, normalized size = 0.66

$$\frac{2\left(231\,b^{4}x^{4}+105\,ab^{3}x^{3}-70\,a^{2}b^{2}x^{2}+40\,a^{3}bx-16\,a^{4}\right)(bx+a)^{\frac{5}{2}}}{3003\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*b^4*x^4 + 105*a*b^3*x^3 - 70*a^2*b^2*x^2 + 40*a^3*b*x - 16*a^4)
*(b*x + a)^(5/2)/b^4
```

mupad [B] time = 1.15, size = 2235, normalized size = 27.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (2*b^2*x^6*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/13 + (x^4*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
- (24*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b
)))/(9*b) - (x^5*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b^2*(log(2/(exp(2*a)
*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
)/2 + b*x))/13)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(11*b) - (x^3*(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))
/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (8*((3*b*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b^2*(log(2/(exp(2*a)*exp(
2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 +
 b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(log(2/(exp(2*a)*exp(2*b*x)
+ 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/
(9*b)))/(7*b) - (16*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/
4 - (8*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x) - (24*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))
/(11*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1
))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^3)/(
35*b^4) - (6*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
- log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 -
 (8*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2
```

```
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x) - (24*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x) +
1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(1
1*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/
2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(35*b^
2) - (8*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2
/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (8*((3
*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (10*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (2
4*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))/2 + b*x))/13)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(11*b))*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(35*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.131 $\int x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=59

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{11/2}}{693b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

```
[Out] 2/7*x^2*arctanh(tanh(b*x+a))^(7/2)/b-8/63*x*arctanh(tanh(b*x+a))^(9/2)/b^2+
16/693*arctanh(tanh(b*x+a))^(11/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{11/2}}{693b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (8*x*ArcTanh[Tanh[a + b*x]]^(9
/2))/(63*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(11/2))/(693*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\int x\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{8\int \tanh^{-1}(\tan\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{8\,\text{Subst}\left(\int x^{9/2}\\ &= \frac{2x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}+\frac{16\tanh^{-1}(\tanh\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.83

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-44bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2+99b^2x^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(99*b^2*x^2 - 44*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(693*b^3)
```

fricas [A] time = 0.53, size = 64, normalized size = 1.08

$$\frac{2\left(63\,b^{5}x^{5}+161\,ab^{4}x^{4}+113\,a^{2}b^{3}x^{3}+3\,a^{3}b^{2}x^{2}-4\,a^{4}bx+8\,a^{5}\right)\sqrt{bx+a}}{693\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^4
*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

giac [B] time = 1.14, size = 248, normalized size = 4.20

$$\sqrt{2}\left(\frac{231\,\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)a^{3}}{b^{2}}+\frac{297\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a^{2}}{b^{2}}+\frac{33\,\sqrt{2}\left(35\,(bx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3465*sqrt(2)*(231*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*
sqrt(b*x + a)*a^2)*a^3/b^2 + 297*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^2 + 33*sqrt(
2)*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 -
420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a/b^2 + 5*sqrt(2)*(63*(b*x
 + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x
+ a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)/b^2)/b
```

maple [A] time = 0.14, size = 69, normalized size = 1.17

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{11}{2}}}{11}+\frac{2(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))^{2}\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^3*(1/11*arctanh(tanh(b*x+a))^(11/2)+1/9*(-2*arctanh(tanh(b*x+a))+2*b*x)
*arctanh(tanh(b*x+a))^(9/2)+1/7*(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b
*x+a))^(7/2))
```

maxima [A] time = 0.54, size = 42, normalized size = 0.71

$$\frac{2\left(63\,b^{3}x^{3}+35\,ab^{2}x^{2}-20\,a^{2}bx+8\,a^{3}\right)(bx+a)^{\frac{5}{2}}}{693\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*b^3*x^3 + 35*a*b^2*x^2 - 20*a^2*b*x + 8*a^3)*(b*x + a)^(5/2)/b^3
```

mupad [B] time = 1.14, size = 1789, normalized size = 30.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (2*b^2*x^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/11 + (x^3*(log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) -
 (20*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/
2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))
)/(7*b) - (x^4*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (20*b^2*(log(2/(exp(2*a)*e
xp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/
2 + b*x))/11)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(9*b) - (x^2*(log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)
^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (6*((3*b*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (20*b^2*(log(2/(exp(2*a)*exp(2*b*
x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x
))/11)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))
/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b)
))/(5*b) - (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (6
*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) -
 (20*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/11)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/
2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))
*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 + b*x))/(7*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)^2)/(15*b^3)
- (4*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/4 - (6*((3*b*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2)/2 - (8*(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (20*b^
2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x))/11)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(9*b))*(log(2
/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))/2 + b*x))/(7*b))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(15*b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.132 $\int x \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=38

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}$$

```
[Out] 2/7*x*arctanh(tanh(b*x+a))^(7/2)/b-4/63*arctanh(tanh(b*x+a))^(9/2)/b^2
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b) - (4*ArcTanh[Tanh[a + b*x]]^(9/2))
/(63*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx&=\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\int\tanh^{-1}(\tanh(a+bx))^{7/2}\,dx}{7b}\\&=\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{2\,\text{Subst}\left(\int x^{7/2}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{7b^2}\\&=\frac{2x\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}-\frac{4\tanh^{-1}(\tanh(a+bx))^{9/2}}{63b^2}\end{aligned}$$

Mathematica [A] time = 0.08, size = 32, normalized size = 0.84

$$\frac{2\left(9bx-2\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{7/2}}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*(9*b*x - 2*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*b^2
)
```

fricas [A] time = 0.60, size = 52, normalized size = 1.37

$$\frac{2\left(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4\right)\sqrt{bx+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x
 + a)/b^2
```

giac [B] time = 0.18, size = 197, normalized size = 5.18

$$\frac{\sqrt{2}\left(\frac{105\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^3}{b}+\frac{63\,\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^2}{b}+\frac{27\,\sqrt{2}\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-3\right)}{b}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/315*sqrt(2)*(105*sqrt(2)*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3/b + 63
*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
a^2/b + 27*sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)
^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b + sqrt(2)*(35*(b*x + a)^(9/2) - 180*
(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315
*sqrt(b*x + a)*a^4)/b)/b
```

maple [A] time = 0.14, size = 42, normalized size = 1.11

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^2*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(b*x-arctanh(tanh(b*x+a)))*arctan
h(tanh(b*x+a))^(7/2))
```

maxima [A] time = 0.52, size = 31, normalized size = 0.82

$$\frac{2\left(7\,b^2x^2+5\,abx-2\,a^2\right)(bx+a)^{\frac{5}{2}}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*b^2*x^2 + 5*a*b*x - 2*a^2)*(b*x + a)^(5/2)/b^2
```

mupad [B] time = 1.11, size = 773, normalized size = 20.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(63*b^2) - (log((exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))^4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(252*b^2) - (log(
1/(exp(2*a)*exp(2*b*x) + 1))^4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(252*b^2) + (log(1
/(exp(2*a)*exp(2*b*x) + 1))^3*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(63*b^2) - (x*log(1/(exp(2*a)*exp(2*b*x)
+ 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) + (x*log((exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) - (log(1/(exp(2*
a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^
2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2))/(42*b^2) - (3*x*log(1/(exp(2*a)*exp(2*b*x) + 1)
)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2*(log((exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)
^(1/2))/(28*b) + (3*x*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.133 $\int \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx$

Optimal. Leaf size=18

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

```
[Out] 2/7*arctanh(tanh(b*x+a))^(7/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2157, 30}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int x^{5/2}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*b)
```

fricas [B] time = 0.80, size = 39, normalized size = 2.17

$$\frac{2\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{bx+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

giac [B] time = 0.15, size = 136, normalized size = 7.56

$$\frac{\sqrt{2}\left(35\,\sqrt{2}\,\sqrt{bx+a}\,a^3+35\,\sqrt{2}\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^2+7\,\sqrt{2}\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)\right.}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/35*sqrt(2)*(35*sqrt(2)*sqrt(b*x + a)*a^3 + 35*sqrt(2)*((b*x + a)^(3/2) -
3*sqrt(b*x + a)*a)*a^2 + 7*sqrt(2)*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*
a + 15*sqrt(b*x + a)*a^2)*a + sqrt(2)*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/
2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3))/b
```

maple [A] time = 0.03, size = 15, normalized size = 0.83

$$\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/7*arctanh(tanh(b*x+a))^(7/2)/b
```

maxima [A] time = 0.50, size = 12, normalized size = 0.67

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b
```

mupad [B] time = 1.12, size = 337, normalized size = 18.72

$$\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{28\,b}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)^3\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{28\,b}-\frac{3\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^
(1/2))/(28*b) - (log(1/(exp(2*a)*exp(2*b*x) + 1))^3*(log((exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
))/(28*b) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))^2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1))/2 - log(1/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b) + (3*log(1/(ex
p(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1))*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2))/(28*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.134 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x} dx$

Optimal. Leaf size=121

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+$$

```
[Out] -2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x
-arctanh(tanh(b*x+a)))^(5/2)-2/3*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*
x+a))^(3/2)+2/5*arctanh(tanh(b*x+a))^(5/2)+2*(b*x-arctanh(tanh(b*x+a)))^2*a
rctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2159, 2161}

$$2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2-\frac{2}{3}\tanh^{-1}(\tanh(a+bx))^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x,x]
```

```
[Out] -2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*
(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) + 2*(b*x - ArcTanh[Tanh[a + b*x]])^2*S
qrt[ArcTanh[Tanh[a + b*x]]] - (2*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tan
h[a + b*x]]^(3/2))/3 + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/5
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x} dx = \frac{2}{5}\tanh^{-1}(\tanh(a+bx))^{5/2}-\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\tanh^{-1}(\tanh(a+$$

$$= -\frac{2}{3}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}+\frac{2}{5}\tanh^{-1}(\tanh(a+$$

$$= 2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{2}{3}\left(bx-\tanh^{-1}(\tanh$$

$$= -2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}+2\left(b$$

Mathematica [A] time = 0.10, size = 99, normalized size = 0.82

$$\frac{2}{15}\left(15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}+23\tanh^{-1}(\tanh(a+bx))^{5/2}-35bx\tanh^{-1}(\tanh(a+bx))^{3/2}-15\tanh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x,x]
```

```
[Out] (2*(15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] - 35*b*x*ArcTanh[Tanh[a + b*x]]
^(3/2) + 23*ArcTanh[Tanh[a + b*x]]^(5/2) - 15*ArcTanh[Sqrt[ArcTanh[Tanh[a +
 b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b
*x]])^(5/2)))/15
```

fricas [A] time = 0.72, size = 114, normalized size = 0.94

$$\left[a^{\frac{5}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{15}\left(3b^2x^2+11abx+23a^2\right)\sqrt{bx+a}, 2\sqrt{-a}a^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 + 1
1*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt(-
a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

giac [A] time = 0.19, size = 73, normalized size = 0.60

$$\frac{1}{15}\sqrt{2}\left(\frac{15\sqrt{2}a^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+3\sqrt{2}(bx+a)^{\frac{5}{2}}+5\sqrt{2}(bx+a)^{\frac{3}{2}}a+15\sqrt{2}\sqrt{bx+a}\,a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*sqrt(2)*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 3*sq
rt(2)*(b*x + a)^(5/2) + 5*sqrt(2)*(b*x + a)^(3/2)*a + 15*sqrt(2)*sqrt(b*x +
 a)*a^2)
```

maple [B] time = 0.14, size = 222, normalized size = 1.83

$$\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}+\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}a}{3}+\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x,x)
```

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)+2/3*arctanh(tanh(b*x+a))^(3/2)*a+2/3*arctanh
(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+2*a^2*arctanh(tanh(b*x+a))
^(1/2)+4*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+2*(arcta
nh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-2*(a^3+3*a^2*(arctanh(t
anh(b*x+a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a))
-b*x-a)^3)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1
/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x, x)
```

mupad [B] time = 4.82, size = 789, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x,x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 - (2*(3*b^2
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x) - (8*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/5)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 + b*x))/(3*b)))/b + (2*b^2*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/5 + (
2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i
+ 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*16i)/(x*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)*1i)/8 - (x*(3*b^2*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x) - (8*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/5)*(log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)
^(1/2))/(3*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x, x)
```

3.135 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}+\frac{5}{3}b\tanh^{-1}(\tanh(a+bx))^{3/2}-5b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx$$

```
[Out] 5*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*
x-arctanh(tanh(b*x+a)))^(3/2)+5/3*b*arctanh(tanh(b*x+a))^(3/2)-arctanh(tanh
(b*x+a))^(5/2)/x-5*b*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2159, 2161}

$$-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x}+\frac{5}{3}b\tanh^{-1}(\tanh(a+bx))^{3/2}-5b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^2,x]
```

```
[Out] 5*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 5*b*(b*x - ArcTanh[Tanh[a + b*x]])*
Sqrt[ArcTanh[Tanh[a + b*x]]] + (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/3 - ArcTa
nh[Tanh[a + b*x]]^(5/2)/x
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^2}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x} + \frac{1}{2}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x}\,dx$$

$$= \frac{5}{3}b\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x} - \frac{1}{2}\left(5b\left(bx-\tanh^{-1}(\text{ta}\right.\right.$$

$$= -5b\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{5}{3}b\tanh^{-1}(\tanh(a$$

$$= 5b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2} - 5b\left(\right.$$

Mathematica [A] time = 0.07, size = 106, normalized size = 0.96

$$\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\frac{14}{3}b\left(\tanh^{-1}(\tanh(a+bx))-bx\right) - \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{x} + \frac{2b^2x}{3}\right) - 5b\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^2,x]
```

```
[Out] -5*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) + Sqrt[ArcTanh[Tanh[a + b*x]]
]*((2*b^2*x)/3 + (14*b*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/3 - (-(b*x) + Arc
Tanh[Tanh[a + b*x]])^2/x)
```

fricas [A] time = 0.62, size = 126, normalized size = 1.15

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(2\,b^2x^2+14\,abx-3\,a^2\right)\sqrt{bx+a}}{6\,x},\ \frac{15\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(2\,b^2x^2+\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b^
2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan(s
qrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x]
```

giac [A] time = 0.19, size = 89, normalized size = 0.81

$$\frac{\sqrt{2}\left(\frac{15\sqrt{2}a^2b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{2}(bx+a)^{\frac{3}{2}}b^2 + 12\sqrt{2}\sqrt{bx+a}\,ab^2 - \frac{3\sqrt{2}\sqrt{bx+a}\,a^2b}{x}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(15*sqrt(2)*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2
*sqrt(2)*(b*x + a)^(3/2)*b^2 + 12*sqrt(2)*sqrt(b*x + a)*a*b^2 - 3*sqrt(2)*s
qrt(b*x + a)*a^2*b/x)/b
```

maple [A] time = 0.16, size = 193, normalized size = 1.75

$$2b\left(\frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{3}+2a\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}+2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\sqrt{\operatorname{arctanh}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^2,x)
```

```
[Out] 2*b*(1/3*arctanh(tanh(b*x+a))^(3/2)+2*a*arctanh(tanh(b*x+a))^(1/2)+2*(arcta
nh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+(-1/2*a^2-a*(arctanh(tanh
(b*x+a))-b*x-a)-1/2*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1
/2)/b/x-5/2*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x
-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/
(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^2, x)
```

mupad [B] time = 4.97, size = 616, normalized size = 5.60

$$\frac{2\,b^2\,x\,\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}}{3}-\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{4\,x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^2,x)
```

```
[Out] (2*b^2*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/
(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/3 - ((log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^2)/(4*x) - ((3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (4*b^2*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 + b*x))/3)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/b + (2^(1/2)*b*log(
(((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*16i)/(x*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2
)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)*5i)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**2, x)
```

3.136 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^3} dx$

Optimal. Leaf size=110

$$\frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{15}{4}b^2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)-\text{tanh}$$

```
[Out] -5/4*b*arctanh(tanh(b*x+a))^(3/2)/x-1/2*arctanh(tanh(b*x+a))^(5/2)/x^2-15/4
*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b
*x-arctanh(tanh(b*x+a)))^(1/2)+15/4*b^2*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2159, 2161}

$$\frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{15}{4}b^2\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)-\text{tanh}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^3,x]
```

```
[Out] (-15*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/4 + (15*b^2*Sqrt[ArcTanh[Tanh[a +
 b*x]]])/4 - (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(4*x) - ArcTanh[Tanh[a + b*
x]]^(5/2)/(2*x^2)
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^3}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{2x^2} + \frac{1}{4}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^2}\,dx$$

$$= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{4x} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{2x^2} + \frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{\tanh^{-1}}}{}$$

$$= \frac{15}{4}b^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{4x} - \frac{\tanh^{-1}(\tanh(a+}{2x^2}$$

$$= -\frac{15}{4}b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))} + \frac{15}{4}$$

Mathematica [A] time = 0.05, size = 108, normalized size = 0.98

$$-\frac{-15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))} + 15b^2x^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)\sqrt{\tanh^{-1}(\tanh(a+bx))-bx} + 2\tanh}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^3,x]
```

```
[Out] -1/4*(-15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] + 5*b*x*ArcTanh[Tanh[a + b*x
]]^(3/2) + 2*ArcTanh[Tanh[a + b*x]]^(5/2) + 15*b^2*x^2*ArcTanh[Sqrt[ArcTanh
[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTa
nh[Tanh[a + b*x]]])/x^2
```

fricas [A] time = 0.96, size = 133, normalized size = 1.21

$$\left[\frac{15\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + 2\left(8b^2x^2-9abx-2a^2\right)\sqrt{bx+a}}{8x^2}, \frac{15\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \left(8b^2x^2\right.}{4x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
a))/x^2]
```

giac [A] time = 0.19, size = 92, normalized size = 0.84

$$\frac{\sqrt{2}\left(\frac{15\sqrt{2}ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 8\sqrt{2}\sqrt{bx+a}\,b^3 - \frac{\sqrt{2}\left(9(bx+a)^{\frac{3}{2}}ab^3-7\sqrt{bx+a}\,a^2b^3\right)}{b^2x^2}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(15*sqrt(2)*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*s
qrt(2)*sqrt(b*x + a)*b^3 - sqrt(2)*(9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x +
a)*a^2*b^3)/(b^2*x^2))/b
```

maple [A] time = 0.16, size = 142, normalized size = 1.29

$$2b^2\left(\sqrt{\operatorname{arctanh}(\tanh(bx+a))}+\frac{\left(-\frac{9\operatorname{arctanh}(\tanh(bx+a))}{8}+\frac{9bx}{8}\right)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}+\left(\frac{7a^2}{8}+\frac{7a(\operatorname{arctanh}(\tanh(}{4}\right.}{b^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(arctanh(tanh(b*x+a))^(1/2)+((-9/8*arctanh(tanh(b*x+a))+9/8*b*x)*arct
anh(tanh(b*x+a))^(3/2)+(7/8*a^2+7/4*a*(arctanh(tanh(b*x+a))-b*x-a)+7/8*(arc
tanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-15/8*(arcta
nh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh
(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^3, x)
```

mupad [B] time = 2.07, size = 614, normalized size = 5.58

$$2\,b^2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}+b^2\,\ln\left(\frac{64\left(2\sqrt{2}\,a-2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\sqrt{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}\right.}{x\sqrt{\ln\left(\frac{2}{e}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2) + b^2*log((64*(2*2^(1/2)*a - 2*(log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x)
 + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2) - 2^(1/2)*(2*a - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x) + 2^(1/2)*b*x))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*b*x)^(1/2)))*((225*log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/128 - (225*log(2/(exp(2
*a)*exp(2*b*x) + 1)))/128 - (225*b*x)/64)^(1/2) - ((log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (9*b*
(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(8*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**3, x)
```

3.137 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^4} dx$

Optimal. Leaf size=113

$$\frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x}-\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^3}-\frac{5b\tanh^{-1}(\tanh(a-}{12x^2}$$

```
[Out] -5/12*b*arctanh(tanh(b*x+a))^(3/2)/x^2-1/3*arctanh(tanh(b*x+a))^(5/2)/x^3+5
/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/
(b*x-arctanh(tanh(b*x+a)))^(1/2)-5/8*b^2*arctanh(tanh(b*x+a))^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 2161}

$$-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x}+\frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2}-\frac{\tanh^{-1}(\tanh(a}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^4,x]
```

```
[Out] (5*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(8*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - (5*b^2*Sqrt[ArcTanh[Tanh[a +
b*x]]])/(8*x) - (5*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(12*x^2) - ArcTanh[Tanh[
a + b*x]]^(5/2)/(3*x^3)
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^4}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^3} + \frac{1}{6}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^3}\,dx$$

$$= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^3} + \frac{1}{8}\left(5b^2\right)\int \frac{\sqrt{\tanh^{-1}}}{}$$

$$= -\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x} - \frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{12x^2} - \frac{\tanh^{-1}(\tanh(a+}{3x^3}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{8x} - \frac{5b\tanh^{-1}(\text{ta}}{1}$$

Mathematica [A] time = 0.08, size = 107, normalized size = 0.95

$$\frac{1}{24}\left(-\frac{15b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}} - \frac{15b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x} - \frac{8\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^3} - \frac{10b\tan}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^4,x]
```

```
[Out] ((-15*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/x - (10*b*ArcTanh[Tanh[a + b*x]]^(3
/2))/x^2 - (8*ArcTanh[Tanh[a + b*x]]^(5/2))/x^3 - (15*b^3*ArcTanh[Sqrt[ArcT
anh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/Sqrt[-(b*x) + A
rcTanh[Tanh[a + b*x]]])/24
```

fricas [A] time = 0.66, size = 146, normalized size = 1.29

$$\left[\frac{15\,\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\left(33\,ab^2x^2+26\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{48\,ax^3}, \frac{15\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(}{24\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

giac [A] time = 0.19, size = 88, normalized size = 0.78

$$\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{2}\left(33\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+15\,\sqrt{bx+a}\,a^2b^4\right)}{b^3x^3}\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*(15*sqrt(2)*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt
(2)*(33*(b*x + a)^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)*a
^2*b^4)/(b^3*x^3))/b
```

maple [A] time = 0.15, size = 144, normalized size = 1.27

$$2b^3\left(\frac{-\frac{11\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16}+\left(\frac{5\,\mathrm{arctanh}(\tanh(bx+a))}{6}-\frac{5bx}{6}\right)\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}+\left(-\frac{5a^2}{16}-\frac{5a(\mathrm{arctanh}(\tanh(bx+a))}{8}\right.}{b^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^4,x)
```

```
[Out] 2*b^3*((-11/16*arctanh(tanh(b*x+a))^(5/2)+(5/6*arctanh(tanh(b*x+a))-5/6*b*x
)*arctanh(tanh(b*x+a))^(3/2)+(-5/16*a^2-5/8*a*(arctanh(tanh(b*x+a))-b*x-a)-
5/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^3/x^3-5/
16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arc
tanh(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^4, x)
```

mupad [B] time = 5.68, size = 669, normalized size = 5.92

$$\frac{13\,b\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{12\,x^2\,\left(2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x\right)}-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\right.}{4\,x^3\,\left(3\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^4,x)
```

```
[Out] (2^(1/2)*b^3*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^
(1/2)*b*x)*64i)/x)*5i)/(16*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) - ((log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) +
 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^3*(3*log(2/(exp(2*a)*exp(2*b
*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*
x)) - (11*b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - l
og(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(8*x) + (13*b*(log((2*exp(2*a)*ex
```

```
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2
)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(12*x^2*(2*log(2/(exp(2*a)*exp(2*b*x) +
 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**4,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**4, x)
```

3.138 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^5}\,dx$

Optimal. Leaf size=167

$$\frac{5b^4}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5b^4\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{5b}{64x\sqrt{\tanh^{-1}(t}}$$

```
[Out] 5/64*b^4*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(3/2)-5/24*b*arctanh(tanh(b*x+a))^(3/2)/x^3-1/
4*arctanh(tanh(b*x+a))^(5/2)/x^4-5/64*b^3/x/arctanh(tanh(b*x+a))^(1/2)+5/64
*b^4/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)-5/32*b^2*arctanh
(tanh(b*x+a))^(1/2)/x^2
```

Rubi [A] time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2163, 2161}

$$-\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2}+\frac{5b^4}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^5,x]
```

```
[Out] (5*b^4*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(64*(b*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - (5*b^3)/(64*x*Sqrt[ArcTanh
[Tanh[a + b*x]]]) + (5*b^4)/(64*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh
[Tanh[a + b*x]]]) - (5*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(32*x^2) - (5*b*Ar
cTanh[Tanh[a + b*x]]^(3/2))/(24*x^3) - ArcTanh[Tanh[a + b*x]]^(5/2)/(4*x^4)
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^5}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{4x^4} + \frac{1}{8}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^4}\,dx$$

$$= -\frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{24x^3} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{4x^4} + \frac{1}{16}\left(5b^2\right)\int \frac{\sqrt{\tanh^{-1}\cdots}}{\cdots}$$

$$= -\frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2} - \frac{5b\tanh^{-1}(\tanh(a+bx))^{3/2}}{24x^3} - \frac{\tanh^{-1}(\tanh(a+\cdots}{4x^4}$$

$$= -\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{5b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{32x^2} - \frac{5b\tanh^{-1}(\tanh\cdots}{24x^3\cdots}$$

$$= -\frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5b^4}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tan\cdots}}$$

$$= \frac{5b^4\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{64\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{5b^3}{64x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{\cdots}{64\left(bx-\tan\cdots\right.}$$

Mathematica [A] time = 0.12, size = 134, normalized size = 0.80

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{64\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(10b^2x^2\tanh^{-1}(\tanh(a+bx)) + 8bx\tanh^{-1}(\tanh(a\cdots\right.}{192x^4\left(bx-\tanh^{-1}(\tanh(a+bx\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^5,x]
```

```
[Out] (5*b^4*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]])/(64*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) - (Sqrt[ArcTanh[Tanh[
a + b*x]]]*(15*b^3*x^3 + 10*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 8*b*x*ArcTanh[
Tanh[a + b*x]]^2 - 48*ArcTanh[Tanh[a + b*x]]^3))/(192*x^4*(b*x - ArcTanh[Ta
nh[a + b*x]]))
```

fricas [A] time = 0.62, size = 167, normalized size = 1.00

$$\left[\frac{15\sqrt{a}\,b^4x^4\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\left(15ab^3x^3 + 118a^2b^2x^2 + 136a^3bx + 48a^4\right)\sqrt{bx+a}}{384a^2x^4}, -\frac{15\sqrt{-a}\,b^4x^4\arctan\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^
2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*
a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x^4
)]
```

giac [A] time = 0.39, size = 108, normalized size = 0.65

$$-\frac{\sqrt{2}\left(\frac{15\,\sqrt{2}b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{2}\left(15\,(bx+a)^{\frac{7}{2}}b^5+73\,(bx+a)^{\frac{5}{2}}ab^5-55\,(bx+a)^{\frac{3}{2}}a^2b^5+15\,\sqrt{bx+a}\,a^3b^5\right)}{ab^4x^4}\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/384*sqrt(2)*(15*sqrt(2)*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a)
+ sqrt(2)*(15*(b*x + a)^(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)
^(3/2)*a^2*b^5 + 15*sqrt(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

maple [A] time = 0.15, size = 169, normalized size = 1.01

$$2b^4\left(\frac{-\frac{5\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{128(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{73\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{384}+\left(\frac{55\operatorname{arctanh}(\tanh(bx+a))}{384}-\frac{55bx}{384}\right)\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}+}{b^4x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^5,x)
```

```
[Out] 2*b^4*((-5/128/(arctanh(tanh(b*x+a))-b*x)*arctanh(tanh(b*x+a))^(7/2)-73/384
*arctanh(tanh(b*x+a))^(5/2)+(55/384*arctanh(tanh(b*x+a))-55/384*b*x)*arctan
h(tanh(b*x+a))^(3/2)+(-5/128*a^2-5/64*a*(arctanh(tanh(b*x+a))-b*x-a)-5/128*
(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^4/x^4+5/128/(
arctanh(tanh(b*x+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh
(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^5, x)
```

mupad [B] time = 5.85, size = 1069, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^5,x)
```

```
[Out] (5*b^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(32*x*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - ((log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(
2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^4*(4*log(2/(exp(2*a)
*exp(2*b*x) + 1)) - 4*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 8*b*x)) + (2^(1/2)*b^4*log(((2*2^(1/2)*a + (log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
```

```
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) +
 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1024i)/(x*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^(1/2)))*5i)/(64*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)) - (59*
b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(48*x^2*(2*log(2/(
exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 4*b*x)) + (17*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)^2)/(16*x^3*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**5,x)
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**5, x)
```

3.139 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^6} \, dx$

Optimal. Leaf size=221

$$\frac{3b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+$$

```
[Out] 3/128*b^5*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/128*b^4/x/arctanh(tanh(b*x+a))^(3/2)-
1/128*b^5/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/8*b*arcta
nh(tanh(b*x+a))^(3/2)/x^4-1/5*arctanh(tanh(b*x+a))^(5/2)/x^5-1/64*b^3/x^2/a
rctanh(tanh(b*x+a))^(1/2)+3/128*b^5/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(ta
nh(b*x+a))^(1/2)-1/16*b^2*arctanh(tanh(b*x+a))^(1/2)/x^3
```

Rubi [A] time = 0.17, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$-\frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3}+\frac{3b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tan$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^6,x]
```

```
[Out] (3*b^5*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(128*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b^4/(128*x*ArcTanh[Tanh[a
 + b*x]]^(3/2)) - b^5/(128*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
b*x]]^(3/2)) - b^3/(64*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3*b^5)/(128*(b*
x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) - (b^2*Sqrt[Arc
Tanh[Tanh[a + b*x]]])/(16*x^3) - (b*ArcTanh[Tanh[a + b*x]]^(3/2))/(8*x^4) -
 ArcTanh[Tanh[a + b*x]]^(5/2)/(5*x^5)
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^6}\,dx = -\frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^5} + \frac{1}{2}b\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^5}\,dx$$

$$= -\frac{b\tanh^{-1}(\tanh(a+bx))^{3/2}}{8x^4} - \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^5} + \frac{1}{16}\left(3b^2\right)\int \frac{\sqrt{\tanh^{-1}}}{}$$

$$= -\frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3} - \frac{b\tanh^{-1}(\tanh(a+bx))^{3/2}}{8x^4} - \frac{\tanh^{-1}(\tanh(a+b}{5x^5}$$

$$= -\frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{16x^3} - \frac{b\tanh^{-1}(\tanh(}{8x^4}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^3}{64x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{b^2\sqrt{\tanh^{-1}(ta}}{16x}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^5}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= \frac{3b^5\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{128\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b^4}{128x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{}{128\left(bx\right.}$$

Mathematica [A] time = 0.13, size = 150, normalized size = 0.68

$$\frac{1}{640}\left(-\frac{15b^5\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(10b^3x^3\tanh^{-1}(\tanh(a+bx))+8b^2x^2\tan\right.}{x^5\left(\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^6,x]
```

```
[Out] ((-15*b^5*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - (Sqrt[ArcTanh[Tanh[a
+ b*x]]]*(15*b^4*x^4 + 10*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 8*b^2*x^2*ArcTan
h[Tanh[a + b*x]]^2 - 176*b*x*ArcTanh[Tanh[a + b*x]]^3 + 128*ArcTanh[Tanh[a
+ b*x]]^4))/(x^5*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2))/640
```

fricas [A] time = 0.84, size = 189, normalized size = 0.86

$$\left[\frac{15\sqrt{a}\,b^5x^5\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\left(15ab^4x^4-10a^2b^3x^3-248a^3b^2x^2-336a^4bx-128a^5\right)\sqrt{bx+a}}{1280a^3x^5}, \frac{15\sqrt{-a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="fricas")

[Out] [1/1280*(15*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b*x - 128*a^5) *sqrt(b*x + a))/(a^3*x^5), 1/640*(15*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x + a)* sqrt(-a)/a) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 - 248*a^3*b^2*x^2 - 336*a^4*b* x - 128*a^5)*sqrt(b*x + a))/(a^3*x^5)]

giac [A] time = 0.31, size = 123, normalized size = 0.56

$$\frac{\sqrt{2}\left(\frac{15\sqrt{2}\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{\sqrt{2}\left(15\,(bx+a)^{\frac{9}{2}}b^6-70\,(bx+a)^{\frac{7}{2}}ab^6-128\,(bx+a)^{\frac{5}{2}}a^2b^6+70\,(bx+a)^{\frac{3}{2}}a^3b^6-15\,\sqrt{bx+a}\,a^4b^6\right)}{a^2b^5x^5}\right)}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="giac")

[Out] 1/1280*sqrt(2)*(15*sqrt(2)*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + sqrt(2)*(15*(b*x + a)^(9/2)*b^6 - 70*(b*x + a)^(7/2)*a*b^6 - 128*(b*x + a)^(5/2)*a^2*b^6 + 70*(b*x + a)^(3/2)*a^3*b^6 - 15*sqrt(b*x + a)*a^4*b^6)/ (a^2*b^5*x^5))/b

maple [A] time = 0.18, size = 262, normalized size = 1.19

$$2b^5\left(\frac{\frac{3\operatorname{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{256\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\right)}-\frac{7\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{128(\operatorname{arctanh}(\tanh(bx+a))-bx)}-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{10}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(tanh(b*x+a))^(5/2)/x^6,x)

[Out] 2*b^5*((3/256/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b *x-a)^2)*arctanh(tanh(b*x+a))^(9/2)-7/128/(arctanh(tanh(b*x+a))-b*x)*arctan h(tanh(b*x+a))^(7/2)-1/10*arctanh(tanh(b*x+a))^(5/2)+(7/128*arctanh(tanh(b* x+a))-7/128*b*x)*arctanh(tanh(b*x+a))^(3/2)+(-3/256*a^2-3/128*a*(arctanh(ta nh(b*x+a))-b*x-a)-3/256*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^5/x^5-3/256/(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x +a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^6,x, algorithm="maxima")

[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^6, x)

mupad [B] time = 6.51, size = 1292, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^6,x)
```

```
[Out] (3*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(32*x*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - ((lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*ex
p(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*x^5*(5*log(2/(exp(2*
a)*exp(2*b*x) + 1)) - 5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 10*b*x)) + (b^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(16*x^2*(2*log(2/(exp(2*a
)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 4*b*x)) + (2^(1/2)*b^5*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80
*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^4 + 80*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1024i)/(x*(log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^(1/2)))*3i)/(64*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)) - (93*b^2*(log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp
(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(80*x^3*(3*log(2/(exp(2*a)
*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 6*b*x)) + (21*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/
(20*x^4*(4*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 4*log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 8*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a + bx))}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**6,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(5/2)/x**6, x)
```

3.140 $\int \frac{x^4}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} \, dx$

Optimal. Leaf size=99

$$\frac{256 \tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^5} - \frac{128x \tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^4} + \frac{32x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3} - \frac{16x^3 \tanh^{-1}(\tan}{3b^2}$$

```
[Out] -16/3*x^3*arctanh(tanh(b*x+a))^(3/2)/b^2+32/5*x^2*arctanh(tanh(b*x+a))^(5/2
)/b^3-128/35*x*arctanh(tanh(b*x+a))^(7/2)/b^4+256/315*arctanh(tanh(b*x+a))^
(9/2)/b^5+2*x^4*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{32x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3} - \frac{16x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{256 \tanh^{-1}(\tanh(a+bx))^{9/2}}{315b^5} - \frac{128x \tanh^{-1}(\tan}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^4*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (16*x^3*ArcTanh[Tanh[a + b*x]]^(3/
2))/(3*b^2) + (32*x^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b^3) - (128*x*ArcTan
h[Tanh[a + b*x]]^(7/2))/(35*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(9/2))/(315*
b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8\int x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{16\int x^2\tanh^{-1}(}{}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\tanh^{-1}(\tan}{5b}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\tanh^{-1}(\tan}{5b}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\tanh^{-1}(\tan}{5b}$$

$$= \frac{2x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{16x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{32x^2\tanh^{-1}(\tan}{5b}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.84

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-840b^3x^3\tanh^{-1}(\tanh(a+bx)) + 1008b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 576bx\tanh^{-1}(\tan\right.}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(315*b^4*x^4 - 840*b^3*x^3*ArcTanh[Tanh[a +
 b*x]] + 1008*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 576*b*x*ArcTanh[Tanh[a + b
*x]]^3 + 128*ArcTanh[Tanh[a + b*x]]^4))/(315*b^5)
```

fricas [A] time = 0.65, size = 53, normalized size = 0.54

$$\frac{2\left(35\,b^4x^4 - 40\,ab^3x^3 + 48\,a^2b^2x^2 - 64\,a^3bx + 128\,a^4\right)\sqrt{bx+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*s
qrt(b*x + a)/b^5
```

giac [A] time = 0.16, size = 61, normalized size = 0.62

$$\frac{2\left(35\,(bx+a)^{\frac{9}{2}} - 180\,(bx+a)^{\frac{7}{2}}a + 378\,(bx+a)^{\frac{5}{2}}a^2 - 420\,(bx+a)^{\frac{3}{2}}a^3 + 315\,\sqrt{bx+a}\,a^4\right)}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

maple [A] time = 0.17, size = 153, normalized size = 1.55

$$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{9}{2}}}{9}+\frac{2(-4\,\mathrm{arctanh}(\tanh(bx+a))+4bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}+\frac{2\left(2(bx-\mathrm{arctanh}(\tanh(bx+a)))^{2}+(-2\,\mathrm{arctanh}(\tanh(bx+a\right.}{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^5*(1/9*arctanh(tanh(b*x+a))^(9/2)+1/7*(-4*arctanh(tanh(b*x+a))+4*b*x)*a
rctanh(tanh(b*x+a))^(7/2)+1/5*(2*(b*x-arctanh(tanh(b*x+a)))^2+(-2*arctanh(t
anh(b*x+a))+2*b*x)^2)*arctanh(tanh(b*x+a))^(5/2)+2/3*(b*x-arctanh(tanh(b*x+
a)))^2*(-2*arctanh(tanh(b*x+a))+2*b*x)*arctanh(tanh(b*x+a))^(3/2)+(b*x-arct
anh(tanh(b*x+a)))^4*arctanh(tanh(b*x+a))^(1/2))
```

maxima [A] time = 0.53, size = 64, normalized size = 0.65

$$\frac{2\left(35\,b^{5}x^{5}-5\,ab^{4}x^{4}+8\,a^{2}b^{3}x^{3}-16\,a^{3}b^{2}x^{2}+64\,a^{4}bx+128\,a^{5}\right)}{315\,\sqrt{bx+a}\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^5*x^5 - 5*a*b^4*x^4 + 8*a^2*b^3*x^3 - 16*a^3*b^2*x^2 + 64*a^4*b
*x + 128*a^5)/(sqrt(b*x + a)*b^5)
```

mupad [B] time = 1.09, size = 496, normalized size = 5.01

$$\frac{2\,x^4\,\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}}{9\,b}+\frac{256\,\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,{\left(\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}+b\,x\right)}^4}{315\,b^5}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(9*b) + (256*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x)^4)/(315*b^5) + (16*x^3*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x))/(63*b^2) + (128*x*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x)^3)/(315*b^4) + (32*x^2*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 + b*x)^2)/(105*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\sqrt{\operatorname{atanh}{\left(\tanh{\left(a + bx \right)} \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(atanh(tanh(a + b*x))), x)
```

3.141 $\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} dx$

Optimal. Leaf size=76

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3}-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2}+\frac{2x^3\sqrt{\tanh^{-1}(\tanh(a}{b}$$

```
[Out] -4*x^2*arctanh(tanh(b*x+a))^(3/2)/b^2+16/5*x*arctanh(tanh(b*x+a))^(5/2)/b^3
-32/35*arctanh(tanh(b*x+a))^(7/2)/b^4+2*x^3*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$-\frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2}-\frac{32\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^4}+\frac{16x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^3}+\frac{2x^3\sqrt{\tanh^{-1}(\tanh(a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*x^2*ArcTanh[Tanh[a + b*x]]^(3/2
))/b^2 + (16*x*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*b^3) - (32*ArcTanh[Tanh[a +
 b*x]]^(7/2))/(35*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{6\int x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{8\int x\tanh^{-1}(}{}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-1}(\mathrm{ta}}{5}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-1}(\mathrm{ta}}{5}$$

$$= \frac{2x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^2} + \frac{16x\tanh^{-1}(\mathrm{ta}}{5}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.87

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-70b^2x^2\tanh^{-1}(\tanh(a+bx)) + 56bx\tanh^{-1}(\tanh(a+bx))^2 - 16\tanh^{-1}(\tanh(a\right.}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(35*b^3*x^3 - 70*b^2*x^2*ArcTanh[Tanh[a + b
*x]] + 56*b*x*ArcTanh[Tanh[a + b*x]]^2 - 16*ArcTanh[Tanh[a + b*x]]^3))/(35*
b^4)
```

fricas [A] time = 0.47, size = 42, normalized size = 0.55

$$\frac{2\left(5\,b^3x^3 - 6\,ab^2x^2 + 8\,a^2bx - 16\,a^3\right)\sqrt{bx+a}}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

giac [A] time = 0.13, size = 49, normalized size = 0.64

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}\,a^3\right)}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)/b^4
```

maple [A] time = 0.18, size = 123, normalized size = 1.62

$$\frac{\frac{2\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7} + \frac{2(-3\,\mathrm{arctanh}(\tanh(bx+a))+3bx)\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5} + \frac{2\left((bx-\mathrm{arctanh}(\tanh(bx+a)))(-2\,\mathrm{arctanh}(\tanh(bx+a))+2\right.}{}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^4*(1/7*arctanh(tanh(b*x+a))^(7/2)+1/5*(-3*arctanh(tanh(b*x+a))+3*b*x)*a
rctanh(tanh(b*x+a))^(5/2)+1/3*((b*x-arctanh(tanh(b*x+a)))*(-2*arctanh(tanh(
b*x+a))+2*b*x)+(b*x-arctanh(tanh(b*x+a)))^2)*arctanh(tanh(b*x+a))^(3/2)+(b*
x-arctanh(tanh(b*x+a)))^3*arctanh(tanh(b*x+a))^(1/2))
```

maxima [A] time = 0.52, size = 53, normalized size = 0.70

$$\frac{2\left(5\,b^4x^4 - ab^3x^3 + 2\,a^2b^2x^2 - 8\,a^3bx - 16\,a^4\right)}{35\,\sqrt{bx+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x
 + a)*b^4)
```

mupad [B] time = 1.07, size = 385, normalized size = 5.07

$$\frac{2\,x^3\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{7\,b}+\frac{32\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}+b\,x\right)^3}{35\,b^4}+\frac{12\,x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(7*b) + (32*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 + b*x)^3)/(35*b^4) + (12*x^2*(log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1))/2 + b*x))/(35*b^2) + (16*x*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 + b*x)^2)/(35*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(atanh(tanh(a + b*x))), x)
```

3.142 $\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=57

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}+\frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] -8/3*x*arctanh(tanh(b*x+a))^(3/2)/b^2+16/15*arctanh(tanh(b*x+a))^(5/2)/b^3+
2*x^2*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{16\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^3}-\frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}+\frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (8*x*ArcTanh[Tanh[a + b*x]]^(3/2))
/(3*b^2) + (16*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4\int x\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{8\int \tanh^{-1}(\tanh(}{3b^2}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{8\,\text{Subst}\left(\int x^{3/2}\,dx\right.}{}$$

$$= \frac{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{8x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} + \frac{16\tanh^{-1}(\tanh(a}{15b^3}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.86

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-20bx\tanh^{-1}(\tanh(a+bx)) + 8\tanh^{-1}(\tanh(a+bx))^2 + 15b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 20*b*x*ArcTanh[Tanh[a + b*x]]
 + 8*ArcTanh[Tanh[a + b*x]]^2))/(15*b^3)
```

fricas [A] time = 0.47, size = 31, normalized size = 0.54

$$\frac{2\left(3\,b^2x^2 - 4\,abx + 8\,a^2\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

giac [A] time = 0.16, size = 37, normalized size = 0.65

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}\,a^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

maple [A] time = 0.17, size = 68, normalized size = 1.19

$$\frac{\frac{2\,\text{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5} + \frac{2(-2\,\text{arctanh}(\tanh(bx+a))+2bx)\,\text{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3} + 2\left(bx - \text{arctanh}\left(\tanh\left(bx+a\right)\right)\right)^2\sqrt{\text{arctanh}\,(\text{t}}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2/b^3*(1/5*arctanh(tanh(b*x+a))^(5/2)+1/3*(-2*arctanh(tanh(b*x+a))+2*b*x)*a
rctanh(tanh(b*x+a))^(3/2)+(b*x-arctanh(tanh(b*x+a)))^2*arctanh(tanh(b*x+a))
^(1/2))
```

maxima [A] time = 0.53, size = 42, normalized size = 0.74

$$\frac{2\left(3\,b^3x^3 - ab^2x^2 + 4\,a^2bx + 8\,a^3\right)}{15\,\sqrt{bx+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

mupad [B] time = 1.14, size = 211, normalized size = 3.70

$$\frac{2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(15\,b^2\,x^2-10\,b\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+10\,b\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2-4\cdots\right.}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp(2*b
*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 15
*b^2*x^2 - 10*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
10*b*x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(15*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(atanh(tanh(a + b*x))), x)
```

3.143 $\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} \, dx$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}$$

```
[Out] -4/3*arctanh(tanh(b*x+a))^(3/2)/b^2+2*x*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - (4*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*
b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} \, dx &= \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{2\int \sqrt{\tanh^{-1}(\tanh(a+bx))} \, dx}{b} \\ &= \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{2\,\text{Subst}\left(\int \sqrt{x} \, dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b^2} \\ &= \frac{2x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b} - \frac{4\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^2} \end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.89

$$\frac{2\left(3bx-2\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
[Out] (2*(3*b*x - 2*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^2)
```

fricas [A] time = 0.50, size = 19, normalized size = 0.53

$$\frac{2\sqrt{bx+a}\,(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

giac [A] time = 0.39, size = 23, normalized size = 0.64

$$\frac{2\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

maple [A] time = 0.17, size = 56, normalized size = 1.56

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3}-2a\sqrt{\operatorname{arctanh}(\tanh(bx+a))}-2\left(\operatorname{arctanh}(\tanh(bx+a))-bx-a\right)\sqrt{\operatorname{arctanh}(\tanh(bx}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(1/2),x)
[Out] 2/b^2*(1/3*arctanh(tanh(b*x+a))^(3/2)-a*arctanh(tanh(b*x+a))^(1/2)-(arctanh
(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2))
```

maxima [A] time = 0.54, size = 30, normalized size = 0.83

$$\frac{2\left(b^2x^2-abx-2\,a^2\right)}{3\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

mupad [B] time = 1.21, size = 105, normalized size = 2.92

$$\frac{2\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+3\,b\,x\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 3*b*x))/(3*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{atanh}(\tanh(a+bx))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x/sqrt(atanh(tanh(a + b*x))), x)
```

3.144 $\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=16

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2157, 30}

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

fricas [A] time = 0.44, size = 12, normalized size = 0.75

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

giac [A] time = 0.19, size = 12, normalized size = 0.75

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

maple [A] time = 0.03, size = 15, normalized size = 0.94

$$\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)/b
```

maxima [A] time = 0.53, size = 12, normalized size = 0.75

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

mupad [B] time = 1.18, size = 52, normalized size = 3.25

$$\frac{2\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] (2*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/b
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Exception raised: TypeError
```

3.145 $\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] 2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2161}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 0.96

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
```

fricas [A] time = 0.53, size = 56, normalized size = 1.14

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.17, size = 21, normalized size = 0.43

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.17, size = 42, normalized size = 0.86

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{\mathrm{arctanh}(\tanh(bx+a))}}{\sqrt{\mathrm{arctanh}(\tanh(bx+a))-bx}}\right)}{\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arc
tanh(tanh(b*x+a))-b*x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*sqrt(arctanh(tanh(b*x + a)))), x)
```

mupad [B] time = 7.19, size = 285, normalized size = 5.82

$$\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\left(\frac{\sqrt{2}\,b\,x}{2}-\frac{\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{2}+\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}\;\sqrt{\ln\left(\frac{}{e^{2a}\cdots}\right.}\right.}{x}\right.}{\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2^(1/2)*log(((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*((log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^(1/2)*1i - (2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/2 + (2
^(1/2)*b*x)/2)*1i)/x)*1i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(atanh(tanh(a + b*x)))), x)
```

3.146 $\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=94

$$\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

```
[Out] b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(3/2)-1/x/arctanh(tanh(b*x+a))^(1/2)+b/(b*x-arctanh(t
anh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 1/(x*Sqrt[ArcTanh[Tanh[a + b*x]]])
+ b/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{2}b\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$

$$= -\frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh}}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{\left(bx-\tanh\right.}$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.83

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}} - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]
]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) - Sqrt[ArcTanh[Tanh[a + b*x]]]
/(x*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

fricas [A] time = 0.58, size = 93, normalized size = 0.99

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2a^2x}, -\frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

giac [A] time = 0.24, size = 47, normalized size = 0.50

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx+a}\,b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

maple [A] time = 0.17, size = 95, normalized size = 1.01

$$2b\left(\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)\,bx} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*b*(2*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b/x-2/(-4
*arctanh(tanh(b*x+a))+4*b*x)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arcta
nh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*sqrt(arctanh(tanh(b*x + a)))), x)
```

mupad [B] time = 7.02, size = 570, normalized size = 6.06

$$\frac{2\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}}{x\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{\sqrt{2}\,b\,\ln\left(\frac{\left(\sqrt{2}\,b\,x-\sqrt{2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)+\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\cdots\right.}{\cdots}}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2^(1/2)*b*lo
g((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 -
 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*1i)/(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(atanh(tanh(a + b*x)))), x)
```

3.147 $\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=158

$$\frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)$$

```
[Out] 3/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(5/2)+1/4*b/x/arctanh(tanh(b*x+a))^(3/2)-1/4*b^
2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-1/2/x^2/arctanh(tan
h(b*x+a))^(1/2)+3/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(
1/2)
```

Rubi [A] time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$\frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (3*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + b/(4*x*ArcTanh[Tanh[a + b*x
]]^(3/2)) - b^2/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3
/2)) - 1/(2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3*b^2)/(4*(b*x - ArcTanh[T
anh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{4}b\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$
$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{1}{8}\left(3b^2\right)\int \frac{}{x\tan}$$
$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$
$$= \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a}$$
$$= \frac{3b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{b}{4x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{}{4\left(bx-\tan\right.}$$

Mathematica [A] time = 0.10, size = 98, normalized size = 0.62

$$\frac{1}{4}\left(\frac{\left(5bx-2\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] ((-3*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + ((5*b*x - 2*ArcTanh[Ta
nh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(x^2*(-(b*x) + ArcTanh[Tanh[a +
 b*x]])^2))/4
```

fricas [A] time = 0.58, size = 123, normalized size = 0.78

$$\left[\frac{3\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3abx-2a^2\right)\sqrt{bx+a}}{8a^3x^2}, \frac{3\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx-2a^2\right)\sqrt{bx}}{4a^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

giac [A] time = 0.18, size = 69, normalized size = 0.44

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3(bx+a)^{\frac{3}{2}}b^3-5\sqrt{bx+a}\,ab^3}{a^2b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

maple [A] time = 0.17, size = 148, normalized size = 0.94

$$2b^2\left(\frac{\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\left(-4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)+4bx\right)b^2x^2}+\frac{\frac{6\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)bx}-\frac{6\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-b}}\right.}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)\sqrt{\operatorname{arctanh}(\mathrm{ta}}}}{-4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)+4bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*b^2*(arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b^2/x^2+3
/(-4*arctanh(tanh(b*x+a))+4*b*x)*(2*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(
tanh(b*x+a))+4*b*x)/b/x-2/(-4*arctanh(tanh(b*x+a))+4*b*x)/(arctanh(tanh(b*x
+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*
x)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*sqrt(arctanh(tanh(b*x + a)))), x)
```

mupad [B] time = 6.01, size = 802, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(x^2*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2
*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x)) + (3*b*(l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*e
xp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/2)*b^2*log
((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 +
40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^4*(2*a - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))*1i)/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
```

```
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*3i)/(2*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^(5/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(atanh(tanh(a + b*x)))), x)
```

3.148 $\int \frac{1}{x^4 \sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=212

$$\frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+b}$$

```
[Out] 5/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
/(b*x-arctanh(tanh(b*x+a)))^(7/2)-1/8*b^2/x/arctanh(tanh(b*x+a))^(5/2)+1/8*
b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)+1/12*b/x^2/arctan
h(tanh(b*x+a))^(3/2)-5/24*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x
+a))^(3/2)-1/3/x^3/arctanh(tanh(b*x+a))^(1/2)+5/8*b^3/(b*x-arctanh(tanh(b*x
+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$\frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{5b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (5*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]
]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) - b^2/(8*x*ArcTanh[Tanh[a + b
*x]]^(5/2)) + b^3/(8*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^
(5/2)) + b/(12*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - (5*b^3)/(24*(b*x - ArcTa
nh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - 1/(3*x^3*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (5*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh
[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
```

```
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = -\frac{1}{3x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{1}{6}b\int \frac{1}{x^3\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$$

$$= \frac{b}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{3x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{1}{8}b^2\int \frac{}{x^2\tan}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{}{3x^3\sqrt{\tanh^{-1}}}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= -\frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{b^2}{8x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{}{8\left(bx-\tan\right.}$$

Mathematica [A] time = 0.13, size = 117, normalized size = 0.55

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{8\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}} + \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-26bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+\right.}{24x^3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (5*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)) + (Sqrt[ArcTanh[Tanh[a
 + b*x]]]*(33*b^2*x^2 - 26*b*x*ArcTanh[Tanh[a + b*x]] + 8*ArcTanh[Tanh[a +
b*x]]^2))/(24*x^3*(b*x - ArcTanh[Tanh[a + b*x]])^3)
```

fricas [A] time = 0.56, size = 145, normalized size = 0.68

$$\left[\frac{15\sqrt{a}\,b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(15ab^2x^2-10a^2bx+8a^3\right)\sqrt{bx+a}}{48a^4x^3}, -\frac{15\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(}{24a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*sqr
t(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*b*x
 + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

giac [A] time = 0.21, size = 84, normalized size = 0.40

$$-\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{15\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+33\,\sqrt{bx+a}\,a^2b^4}{a^3b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/b
```

maple [A] time = 0.18, size = 200, normalized size = 0.94

$$2b^3\left(\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{3\left(-4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)+4bx\right)b^3x^3}+\frac{\frac{10\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)b^2x^2}+\frac{10\left(\frac{6\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{(-4\operatorname{arctanh}(\tanh(bx+a))+4bx)bx}-\frac{\cdots}{(-4\,\mathrm{ar}\cdots}\right)}{3(-4\,\mathrm{arctanh}\cdots}}{-4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)+\cdots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*b^3*(2/3*arctanh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b^3/x
^3+10/3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(arctanh(tanh(b*x+a))^(1/2)/(-4*arc
tanh(tanh(b*x+a))+4*b*x)/b^2/x^2+3/(-4*arctanh(tanh(b*x+a))+4*b*x)*(2*arcta
nh(tanh(b*x+a))^(1/2)/(-4*arctanh(tanh(b*x+a))+4*b*x)/b/x-2/(-4*arctanh(tan
h(b*x+a))+4*b*x)/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+
a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*sqrt(arctanh(tanh(b*x + a)))), x)
```

mupad [B] time = 5.75, size = 1086, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(x^3*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3
*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x)) + (5*b^2*
```

```
(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)
*exp(2*b*x) + 1))/2)^(1/2))/(x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (2^(1/2)*b^3*l
og((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7
+ 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 672*a^5*(2*a - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 44
8*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(x*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)
))*5i)/(2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2)) + (10*b*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2
))/(3*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(atanh(tanh(a + b*x)))), x)
```

3.149 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{256\tanh^{-1}(\tanh(a+bx))^{7/2}}{35b^5}+\frac{128x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4}-\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{16x^3\sqrt{\tanh^{-1}(}}{b}$$

```
[Out] -32*x^2*arctanh(tanh(b*x+a))^(3/2)/b^3+128/5*x*arctanh(tanh(b*x+a))^(5/2)/b
^4-256/35*arctanh(tanh(b*x+a))^(7/2)/b^5-2*x^4/b/arctanh(tanh(b*x+a))^(1/2)
+16*x^3*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{32x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{128x\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4}-\frac{256\tanh^{-1}(\tan}{35b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x^4)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (16*x^3*Sqrt[ArcTanh[Tanh[a + b
*x]]])/b^2 - (32*x^2*ArcTanh[Tanh[a + b*x]]^(3/2))/b^3 + (128*x*ArcTanh[Tan
h[a + b*x]]^(5/2))/(5*b^4) - (256*ArcTanh[Tanh[a + b*x]]^(7/2))/(35*b^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{x^3}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{48\int x^2\sqrt{\tanh^{-1}}}{b}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tan}{b^3}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tan}{b^3}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tan}{b^3}$$

$$= -\frac{2x^4}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{32x^2\tanh^{-1}(\tan}{b^3}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 0.87

$$-\frac{2\left(-280b^3x^3\tanh^{-1}(\tanh(a+bx)) + 560b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 448bx\tanh^{-1}(\tanh(a+bx))^3 + 128\tan\right.}{35b^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*(35*b^4*x^4 - 280*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 560*b^2*x^2*ArcTanh[
Tanh[a + b*x]]^2 - 448*b*x*ArcTanh[Tanh[a + b*x]]^3 + 128*ArcTanh[Tanh[a +
b*x]]^4))/(35*b^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

fricas [A] time = 0.52, size = 63, normalized size = 0.66

$$\frac{2\left(5\,b^4x^4 - 8\,ab^3x^3 + 16\,a^2b^2x^2 - 64\,a^3bx - 128\,a^4\right)\sqrt{bx+a}}{35\left(b^6x + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqrt
(b*x + a)/(b^6*x + a*b^5)
```

giac [A] time = 0.19, size = 77, normalized size = 0.81

$$-\frac{2\,a^4}{\sqrt{bx+a}\,b^5} + \frac{2\left(5\,(bx+a)^{\frac{7}{2}}b^{30} - 28\,(bx+a)^{\frac{5}{2}}ab^{30} + 70\,(bx+a)^{\frac{3}{2}}a^2b^{30} - 140\,\sqrt{bx+a}\,a^3b^{30}\right)}{35\,b^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(5
/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^35
```

maple [B] time = 0.15, size = 319, normalized size = 3.36

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{7}-\frac{8\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}a}{5}-\frac{8\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{5}+4\operatorname{arctanh}(\tanh(bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^5*(1/7*arctanh(tanh(b*x+a))^(7/2)-4/5*arctanh(tanh(b*x+a))^(5/2)*a-4/5*
arctanh(tanh(b*x+a))^(5/2)*(arctanh(tanh(b*x+a))-b*x-a)+2*arctanh(tanh(b*x+
a))^(3/2)*a^2+4*arctanh(tanh(b*x+a))^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a)+2
*arctanh(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-4*arctanh(tanh(b
*x+a))^(1/2)*a^3-12*a^2*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(
1/2)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-4*(arct
anh(tanh(b*x+a))-b*x-a)^3*arctanh(tanh(b*x+a))^(1/2)-(a^4+4*a^3*(arctanh(ta
nh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(b*
x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/arctanh(tanh(b*x+a))^(1/2))
```

maxima [A] time = 0.54, size = 64, normalized size = 0.67

$$\frac{2\left(5\,b^{5}x^{5}-3\,ab^{4}x^{4}+8\,a^{2}b^{3}x^{3}-48\,a^{3}b^{2}x^{2}-192\,a^{4}bx-128\,a^{5}\right)}{35\,(bx+a)^{\frac{3}{2}}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^5*x^5 - 3*a*b^4*x^4 + 8*a^2*b^3*x^3 - 48*a^3*b^2*x^2 - 192*a^4*b*
x - 128*a^5)/((b*x + a)^(3/2)*b^5)
```

mupad [B] time = 1.45, size = 1057, normalized size = 11.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/(4*b^4) + (2*((log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^2/(2*b^3) + (4*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)/b^2 + (12*(log
(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1))/2 + b*x))/(7*b^2))*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b))*(log(2/
(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 + b*x))/(3*b)))/b + (2*x^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(7*b^2)
 + (x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(ex
p(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(2*b^3) + (4*(
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)/b^2 + (12*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
```

```
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b^2))*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1))/2 + b*x))/(5*b)))/(3*b) - ((log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^4)/(4*b^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) + (x^2*((log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)/b^2 + (12*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(7*b^2))*(log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2))/(5*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**(3/2), x)
```

3.150 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{32\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4}-\frac{16x\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a$$

```
[Out] -16*x*arctanh(tanh(b*x+a))^(3/2)/b^3+32/5*arctanh(tanh(b*x+a))^(5/2)/b^4-2*
x^3/b/arctanh(tanh(b*x+a))^(1/2)+12*x^2*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{16x\tanh^{-1}(\tanh(a+bx))^{3/2}}{b^3}+\frac{32\tanh^{-1}(\tanh(a+bx))^{5/2}}{5b^4}-\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x^3)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (12*x^2*Sqrt[ArcTanh[Tanh[a + b
*x]]])/b^2 - (16*x*ArcTanh[Tanh[a + b*x]]^(3/2))/b^3 + (32*ArcTanh[Tanh[a +
 b*x]]^(5/2))/(5*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{6\int \frac{x^2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{24\int x\sqrt{\tanh^{-1}(\text{t}}}{b}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{-1}(\text{tanh}}{b^3}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{-1}(\text{tanh}}{b^3}$$

$$= -\frac{2x^3}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{12x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16x\tanh^{-1}(\text{tanh}}{b^3}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.89

$$\frac{2\left(30b^2x^2\tanh^{-1}(\tanh(a+bx)) - 40bx\tanh^{-1}(\tanh(a+bx))^2 + 16\tanh^{-1}(\tanh(a+bx))^3 - 5b^3x^3\right)}{5b^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*(-5*b^3*x^3 + 30*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 40*b*x*ArcTanh[Tanh[a
+ b*x]]^2 + 16*ArcTanh[Tanh[a + b*x]]^3))/(5*b^4*Sqrt[ArcTanh[Tanh[a + b*x]
]])
```

fricas [A] time = 0.52, size = 51, normalized size = 0.69

$$\frac{2\left(b^3x^3 - 2\,ab^2x^2 + 8\,a^2bx + 16\,a^3\right)\sqrt{bx+a}}{5\left(b^5x + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*b
^4)
```

giac [A] time = 0.18, size = 61, normalized size = 0.82

$$\frac{2\,a^3}{\sqrt{bx+a}\,b^4} + \frac{2\left((bx+a)^{\frac{5}{2}}b^{16} - 5\,(bx+a)^{\frac{3}{2}}ab^{16} + 15\,\sqrt{bx+a}\,a^2b^{16}\right)}{5\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a
*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

maple [B] time = 0.15, size = 201, normalized size = 2.72

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5} - 2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}} a - 2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}} \left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^4*(1/5*arctanh(tanh(b*x+a))^(5/2)-arctanh(tanh(b*x+a))^(3/2)*a-arctanh(
tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+3*a^2*arctanh(tanh(b*x+a))^
(1/2)+6*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+3*(arctan
h(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-(-a^3-3*a^2*(arctanh(tan
h(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b*x-a)^2-(arctanh(tanh(b*x+a))-b
*x-a)^3)/arctanh(tanh(b*x+a))^(1/2))
```

maxima [A] time = 0.53, size = 52, normalized size = 0.70

$$\frac{2\left(b^{4}x^{4} - ab^{3}x^{3} + 6\,a^{2}b^{2}x^{2} + 24\,a^{3}bx + 16\,a^{4}\right)}{5\left(bx+a\right)^{\frac{3}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^4*x^4 - a*b^3*x^3 + 6*a^2*b^2*x^2 + 24*a^3*b*x + 16*a^4)/((b*x + a)^
(3/2)*b^4)
```

mupad [B] time = 1.26, size = 660, normalized size = 8.92

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2}}\left(\frac{\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) + 2\,b\,x\right)^{2}}{2\,b^{3}} + \frac{2\left(\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) + 2\,b\,x}{b^{2}} + \frac{8\left(\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2} \cdots\right.}{\cdots}\right.}{\cdots}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(2*b^3) + (2*((log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)/b^2 + (8*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b^2))*(log(2/(e
xp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1))/2 + b*x))/(3*b)))/b + (2*x^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(5*b^2) +
 (x*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)/b^2 + (8*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2
```

```
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b^2))
*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/(3*b) - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^3)/(2*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**3/atanh(tanh(a + b*x))**(3/2), x)
```

3.151 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{16\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^3}+\frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -16/3*arctanh(tanh(b*x+a))^(3/2)/b^3-2*x^2/b/arctanh(tanh(b*x+a))^(1/2)+8*x
*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$\frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{16\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^3}-\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x^2)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (8*x*Sqrt[ArcTanh[Tanh[a + b*x]
]])/b^2 - (16*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{4\int \frac{x}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{8\int \sqrt{\tanh^{-1}(\tanh}{b^2}$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{8\,\text{Subst}\left(\int \sqrt{x}\,dx,\right.}{}$$

$$= -\frac{2x^2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{16\tanh^{-1}(\tanh(a}{3b^3}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.89

$$-\frac{2\left(-12bx\tanh^{-1}(\tanh(a+bx))+8\tanh^{-1}(\tanh(a+bx))^2+3b^2x^2\right)}{3b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*(3*b^2*x^2 - 12*b*x*ArcTanh[Tanh[a + b*x]] + 8*ArcTanh[Tanh[a + b*x]]^2
))/(3*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

fricas [A] time = 0.74, size = 40, normalized size = 0.73

$$\frac{2\left(b^2x^2-4\,abx-8\,a^2\right)\sqrt{bx+a}}{3\left(b^4x+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

giac [A] time = 0.15, size = 46, normalized size = 0.84

$$-\frac{2\,a^2}{\sqrt{bx+a}\,b^3} + \frac{2\left((bx+a)^{\frac{3}{2}}b^6 - 6\,\sqrt{bx+a}\,ab^6\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

maple [B] time = 0.18, size = 106, normalized size = 1.93

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3} - 4a\sqrt{\operatorname{arctanh}(\tanh(bx+a))} - 4\left(\operatorname{arctanh}(\tanh(bx+a)) - bx - a\right)\sqrt{\operatorname{arctanh}(\tanh(bx+a}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/b^3*(1/3*arctanh(tanh(b*x+a))^(3/2)-2*a*arctanh(tanh(b*x+a))^(1/2)-2*(arc
tanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-(a^2+2*a*(arctanh(tanh(
b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(1/2))
```

maxima [A] time = 0.53, size = 41, normalized size = 0.75

$$\frac{2\left(b^3x^3 - 3\,ab^2x^2 - 12\,a^2bx - 8\,a^3\right)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^3*x^3 - 3*a*b^2*x^2 - 12*a^2*b*x - 8*a^3)/((b*x + a)^(3/2)*b^3)
```

mupad [B] time = 1.29, size = 259, normalized size = 4.71

$$-\frac{4\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(3\,b^2\,x^2-6\,b\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+6\,b\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2-4\ \cdots}{3\,b^3\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] -(4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 + 3
*b^2*x^2 - 6*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6
*b*x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(3*b^3*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**2/atanh(tanh(a + b*x))**(3/2), x)
```

3.152 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2*x/b/arctanh(tanh(b*x+a))^(1/2)+4*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$\frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (4*Sqrt[ArcTanh[Tanh[a + b*x]]])/
b^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx &= -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2\int \frac{1}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}\\
&= -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{b^2}\\
&= -\frac{2x}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{4\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 29, normalized size = 0.85

$$\frac{4\tanh^{-1}(\tanh(a+bx))-2bx}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^(3/2),x]
[Out] (-2*b*x + 4*ArcTanh[Tanh[a + b*x]])/(b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

fricas [A] time = 0.46, size = 29, normalized size = 0.85

$$\frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

giac [A] time = 0.19, size = 29, normalized size = 0.85

$$\frac{2\left(\frac{\sqrt{bx+a}}{b}+\frac{a}{\sqrt{bx+a}\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

maple [A] time = 0.14, size = 40, normalized size = 1.18

$$\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}-\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(3/2),x)
[Out] 2/b^2*(arctanh(tanh(b*x+a))^(1/2)-(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b
*x+a))^(1/2))
```

maxima [A] time = 0.53, size = 30, normalized size = 0.88

$$\frac{2\left(b^2x^2+3\,abx+2\,a^2\right)}{(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
[Out] 2*(b^2*x^2 + 3*a*b*x + 2*a^2)/((b*x + a)^(3/2)*b^2)
```

mupad [B] time = 1.33, size = 152, normalized size = 4.47

$$-\frac{4\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+b\,x\right)}{b^2\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] -(4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + b*x))/(b^2*(log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))
))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

3.153 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}} dx$

Optimal. Leaf size=16

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2/b/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{2157, 30\}$

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

fricas [A] time = 0.42, size = 20, normalized size = 1.25

$$-\frac{2\sqrt{bx+a}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(b^2*x + a*b)
```

giac [A] time = 0.18, size = 12, normalized size = 0.75

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

maple [A] time = 0.03, size = 15, normalized size = 0.94

$$-\frac{2}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2/b/arctanh(tanh(b*x+a))^(1/2)
```

maxima [A] time = 0.50, size = 12, normalized size = 0.75

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

mupad [B] time = 1.31, size = 97, normalized size = 6.06

$$\frac{4\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{b\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] (4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/(b*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - log((ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: TypeError
```

3.154 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^{3/2}}\, dx$

Optimal. Leaf size=78

$$-\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

```
[Out] -2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x
-arctanh(tanh(b*x+a)))^(3/2)-2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+
a))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2163, 2161}

$$-\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]
)/(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) - 2/((b*x - ArcTanh[Tanh[a + b*x]])*
Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx &= -\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{\int \frac{1}{x\sqrt{\tanh^{-1}(\tan}}}{-bx+\tanh^{-1}(\text{ta}}\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 75, normalized size = 0.96

$$\frac{2}{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[ArcTanh[Tanh[a + b*
x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

fricas [A] time = 0.63, size = 110, normalized size = 1.41

$$\left[\frac{(bx+a)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,a}{a^2bx+a^3},\ \frac{2\left((bx+a)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a\right)}{a^2bx+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b
*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

giac [A] time = 0.27, size = 37, normalized size = 0.47

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

maple [A] time = 0.15, size = 68, normalized size = 0.87

$$\frac{2}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)-2/(arctanh(tanh(b*x
+a))-b*x)^(3/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*
x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arctanh(tanh(b*x + a))^(3/2)), x)
```

mupad [B] time = 6.14, size = 614, normalized size = 7.87

$$-\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\cdots}{e^{2\,a}\,e^{2\cdots}}\right.\right.\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] (2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i
 + 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^3 - 8*a^3 - 6*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 12*a^2*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*2i)/(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^(3/2) - (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/((log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\text{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**(3/2)), x)
```

3.155 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}-$$

```
[Out] -3*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b
*x-arctanh(tanh(b*x+a)))^(5/2)-1/x/arctanh(tanh(b*x+a))^(3/2)+b/(b*x-arctan
h(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)-3*b/(b*x-arctanh(tanh(b*x+a)))^2
/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$-\frac{3b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-3*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]
]])/(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 1/(x*ArcTanh[Tanh[a + b*x]]^(3/2
)) + b/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)) - (3*b
)/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2}(3b)\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= -\frac{3b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{(bx-\text{ta}}$$

Mathematica [A] time = 0.10, size = 91, normalized size = 0.73

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}} - \frac{\tanh^{-1}(\tanh(a+bx))+2bx}{x\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (3*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - (2*b*x + ArcTanh[Tanh[a +
b*x]])/(x*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.54, size = 151, normalized size = 1.22

$$\left[\frac{3\left(b^2x^2+abx\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3abx+a^2\right)\sqrt{bx+a}}{2\left(a^3bx^2+a^4x\right)}, -\frac{3\left(b^2x^2+abx\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a^3bx^2+a^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2 +
a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(b*x
 + a))/(a^3*b*x^2 + a^4*x)]
```

giac [A] time = 0.22, size = 64, normalized size = 0.52

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} - \frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

maple [A] time = 0.16, size = 105, normalized size = 0.85

$$2b\left(-\frac{1}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2bx}-\frac{3\operatorname{arctanh}\left(\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}\right)}{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))-bx}}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2*b*(-1/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b*x+a))^(1/2)-1/(arctanh(
tanh(b*x+a))-b*x)^2*(1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-3/2/(arctanh(tanh(b
*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-
b*x)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arctanh(tanh(b*x + a))^(3/2)), x)
```

mupad [B] time = 5.86, size = 807, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b*x)/(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^2))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2^(1/2)*b*lo
g((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i + 2^(1/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 +
 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^4*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*6i)/(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \operatorname{atanh}^{\frac{3}{2}}{\left(\tanh{\left(a+bx \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**(3/2)), x)
```

3.156 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=191

$$-\frac{15b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx)}$$

```
[Out] -15/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(7/2)+3/4*b/x/arctanh(tanh(b*x+a))^(5/2)-3/4*
b^2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)-1/2/x^2/arctanh(t
anh(b*x+a))^(3/2)+5/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))
^(3/2)-15/4*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2163, 2161}

$$-\frac{15b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-15*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) + (3*b)/(4*x*ArcTanh[Tanh[a
 + b*x]]^(5/2)) - (3*b^2)/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(5/2)) - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*b^2)/(4*(b*x -
 ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) - (15*b^2)/(4*(b*x
 - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{1}{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{4}(3b)\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$$
$$= \frac{3b}{4x\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{2x^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{1}{8}\left(15b^2\right)\int$$
$$= \frac{3b}{4x\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh}$$
$$= \frac{3b}{4x\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh}$$
$$= \frac{3b}{4x\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{3b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh}$$
$$= -\frac{15b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{3b}{4x\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{}{4\left(bx\right.}$$

Mathematica [A] time = 0.13, size = 115, normalized size = 0.60

$$\frac{1}{4}\left(\frac{9bx\tanh^{-1}(\tanh(a+bx)) - 2\tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2}{x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3} - \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] ((-15*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (8*b^2*x^2 + 9*b*x*Ar
cTanh[Tanh[a + b*x]] - 2*ArcTanh[Tanh[a + b*x]]^2)/(x^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3))/4
```

fricas [A] time = 0.63, size = 189, normalized size = 0.99

$$\left[\frac{15\left(b^3x^3 + ab^2x^2\right)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + 2\left(15ab^2x^2 + 5a^2bx - 2a^3\right)\sqrt{bx+a}}{8\left(a^4bx^3 + a^5x^2\right)}, \frac{15\left(b^3x^3 + ab^2x^2\right)\sqrt{-a}\ a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
 a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2)]
```

giac [A] time = 0.18, size = 80, normalized size = 0.42

$$\frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^3}+\frac{2\,b^2}{\sqrt{bx+a}\,a^3}+\frac{7\,(bx+a)^{\frac{3}{2}}b^2-9\,\sqrt{bx+a}\,ab^2}{4\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

maple [A] time = 0.16, size = 131, normalized size = 0.69

$$2b^2\left(\frac{1}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^3\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{\frac{\frac{7\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{8}+\left(-\frac{9\operatorname{arctanh}(\tanh(bx+a))}{8}+\frac{9bx}{8}\right)\sqrt{\operatorname{arctanh}(}}{b^2x^2}}{(\operatorname{arctanh}(\tanh(bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2*b^2*(1/(arctanh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(1/2)+1/(arctanh
(tanh(b*x+a))-b*x)^3*((7/8*arctanh(tanh(b*x+a))^(3/2)+(-9/8*arctanh(tanh(b*
x+a))+9/8*b*x)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-15/8/(arctanh(tanh(b*x+a
))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)
^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*arctanh(tanh(b*x + a))^(3/2)), x)
```

mupad [B] time = 6.07, size = 1028, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] (2^(1/2)*b^2*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/
2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2
)*2i + 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^7 + 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 672*a^
```

```
5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^6 + 448*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^(1/2)))*15i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2) - (2*(log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1
))/2)^(1/2))/(x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + ((log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)
*((14*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - (60*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3
))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(
2*a)*exp(2*b*x) + 1))))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**(3/2)), x)
```

3.157 $\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=245

$$-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx$$

```
[Out] -35/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(9/2)-5/8*b^2/x/arctanh(tanh(b*x+a))^(7/2)+5/
8*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(7/2)+1/4*b/x^2/arcta
nh(tanh(b*x+a))^(5/2)-7/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x
+a))^(5/2)-1/3/x^3/arctanh(tanh(b*x+a))^(3/2)+35/24*b^3/(b*x-arctanh(tanh(b
*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)-35/8*b^3/(b*x-arctanh(tanh(b*x+a)))^4/
arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2163, 2161}

$$-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-35*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) - (5*b^2)/(8*x*ArcTanh[Tanh
[a + b*x]]^(7/2)) + (5*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[
a + b*x]]^(7/2)) + b/(4*x^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - (7*b^3)/(8*(b*x
 - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - 1/(3*x^3*ArcTa
nh[Tanh[a + b*x]]^(3/2)) + (35*b^3)/(24*(b*x - ArcTanh[Tanh[a + b*x]])^3*Ar
cTanh[Tanh[a + b*x]]^(3/2)) - (35*b^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^4*
Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
```

```
&&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{1}{3x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{1}{2}b\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$$

$$= \frac{b}{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{3x^3 \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{1}{8}\left(5b^2\right)\int$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{b}{4x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{}{3x^3 \tanh^{-1}}$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan}$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan}$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan}$$

$$= -\frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{5b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tan}$$

$$= -\frac{35b^3 \tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} - \frac{5b^2}{8x \tanh^{-1}(\tanh(a+bx))^{7/2}} + \frac{}{8(bx}$$

Mathematica [A] time = 0.15, size = 133, normalized size = 0.54

$$\frac{35b^3 \tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{8\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{9/2}} - \frac{87b^2x^2 \tanh^{-1}(\tanh(a+bx)) - 38bx \tanh^{-1}(\tanh(a+bx))^2 + 8\tanh^{-1}(}{24x^3\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx)) -\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (35*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a +
 b*x]]]])/(8*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2)) - (48*b^3*x^3 + 87*b^
2*x^2*ArcTanh[Tanh[a + b*x]] - 38*b*x*ArcTanh[Tanh[a + b*x]]^2 + 8*ArcTanh[
Tanh[a + b*x]]^3)/(24*x^3*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^4)
```

fricas [A] time = 0.58, size = 211, normalized size = 0.86

$$\left[\frac{105\left(b^4x^4 + ab^3x^3\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\left(105ab^3x^3 + 35a^2b^2x^2 - 14a^3bx + 8a^4\right)\sqrt{bx+a}}{48\left(a^5bx^4 + a^6x^3\right)}, -\frac{105\left(b^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^4*x^4 + a*b^3*x^3)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a)
 + 2*a)/x) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b
*x + a))/(a^5*b*x^4 + a^6*x^3), -1/24*(105*(b^4*x^4 + a*b^3*x^3)*sqrt(-a)*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*
b*x + 8*a^4)*sqrt(b*x + a))/(a^5*b*x^4 + a^6*x^3)]
```

giac [A] time = 0.21, size = 95, normalized size = 0.39

$$-\frac{35\, b^{3} \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\, \sqrt{-a}\, a^{4}} - \frac{2\, b^{3}}{\sqrt{bx + a}\, a^{4}} - \frac{57\, (bx + a)^{\frac{5}{2}} b^{3} - 136\, (bx + a)^{\frac{3}{2}} a b^{3} + 87\, \sqrt{bx + a}\, a^{2} b^{3}}{24\, a^{4} b^{3} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) - 2*b^3/(sqrt(b*x +
 a)*a^4) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/2)*a*b^3 + 87*sq
rt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3)
```

maple [A] time = 0.16, size = 186, normalized size = 0.76

$$2b^{3}\left(-\frac{1}{(\text{arctanh}\,(\tanh\,(bx+a))-bx)^{4}\sqrt{\text{arctanh}\,(\tanh\,(bx+a))}}-\frac{\frac{\frac{19\,\text{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{16}+\left(-\frac{17\,\text{arctanh}(\tanh(bx+a))}{6}+\frac{17bx}{6}\right)\text{arcta}}{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 2*b^3*(-1/(arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(1/2)-1/(arctan
h(tanh(b*x+a))-b*x)^4*((19/16*arctanh(tanh(b*x+a))^(5/2)+(-17/6*arctanh(tan
h(b*x+a))+17/6*b*x)*arctanh(tanh(b*x+a))^(3/2)+(29/16*a^2+29/8*a*(arctanh(t
anh(b*x+a))-b*x-a)+29/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a
))^(1/2))/b^3/x^3-35/16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(ta
nh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\, \text{artanh}\left(\tanh\left(bx + a\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*arctanh(tanh(b*x + a))^(3/2)), x)
```

mupad [B] time = 5.09, size = 1258, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] (((38*b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - (140*b^3*x)/(log(2/(exp(2*a)*exp(2*
```

```
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^4)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - (4*(log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1)
)/2)^(1/2))/(3*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2^(1/2)*b^3*log((((log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*
b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i + 2^(1/2)*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x) - 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^9 + 144*a^2*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 672*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 +
 2016*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + l
og(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 4032*a^5*(2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^4 + 5376*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 4608*a^7*(2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^2 - 512*a^9 - 18*a*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^8
+ 2304*a^8*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)^(1/2)))*35i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(9/2) - (22*b*(log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^
(1/2))/(3*x^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**4*atanh(tanh(a + b*x))**(3/2)), x)
```

3.158 $\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{256\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^5}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}+\frac{32x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2/3*x^4/b/arctanh(tanh(b*x+a))^(3/2)-128/3*x*arctanh(tanh(b*x+a))^(3/2)/b^
4+256/15*arctanh(tanh(b*x+a))^(5/2)/b^5-16/3*x^3/b^2/arctanh(tanh(b*x+a))^(
1/2)+32*x^2*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$-\frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{32x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{128x\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}+\frac{256\tanh^{-1}(\tanh(a+bx))^{5/2}}{15b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^4)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (16*x^3)/(3*b^2*Sqrt[ArcTanh[
Tanh[a + b*x]]]) + (32*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^3 - (128*x*ArcTa
nh[Tanh[a + b*x]]^(3/2))/(3*b^4) + (256*ArcTanh[Tanh[a + b*x]]^(5/2))/(15*b
^5)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^4}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{8\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16\int \frac{x}{\sqrt{\tanh^{-1}(t}}}{b^2}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tanh^{-1}}}{}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tanh^{-1}}}{}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tanh^{-1}}}{}$$

$$= -\frac{2x^4}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{16x^3}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{32x^2\sqrt{\tanh^{-1}}}{}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.84

$$-\frac{2\left(40b^3x^3\tanh^{-1}(\tanh(a+bx)) - 240b^2x^2\tanh^{-1}(\tanh(a+bx))^2 + 320bx\tanh^{-1}(\tanh(a+bx))^3 - 128\tan\right.}{15b^5\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(5*b^4*x^4 + 40*b^3*x^3*ArcTanh[Tanh[a + b*x]] - 240*b^2*x^2*ArcTanh[Ta
nh[a + b*x]]^2 + 320*b*x*ArcTanh[Tanh[a + b*x]]^3 - 128*ArcTanh[Tanh[a + b*
x]]^4))/(15*b^5*ArcTanh[Tanh[a + b*x]]^(3/2))
```

fricas [A] time = 0.49, size = 74, normalized size = 0.75

$$\frac{2\left(3\,b^4x^4 - 8\,ab^3x^3 + 48\,a^2b^2x^2 + 192\,a^3bx + 128\,a^4\right)\sqrt{bx+a}}{15\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sqr
t(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

giac [A] time = 0.25, size = 75, normalized size = 0.76

$$\frac{2\left(12\,(bx+a)a^3 - a^4\right)}{3\,(bx+a)^{\frac{3}{2}}b^5} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}}b^{20} - 20\,(bx+a)^{\frac{3}{2}}ab^{20} + 90\,\sqrt{bx+a}\,a^2b^{20}\right)}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2
)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

maple [B] time = 0.15, size = 295, normalized size = 2.98

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{5}-\frac{8\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}a}{3}-\frac{8\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3}+12a^2\sqrt{\operatorname{arctanh}(\tanh(bx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^5*(1/5*arctanh(tanh(b*x+a))^(5/2)-4/3*arctanh(tanh(b*x+a))^(3/2)*a-4/3*
arctanh(tanh(b*x+a))^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+6*a^2*arctanh(tanh(
b*x+a))^(1/2)+12*a*(arctanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)+
6*(arctanh(tanh(b*x+a))-b*x-a)^2*arctanh(tanh(b*x+a))^(1/2)-(-4*a^3-12*a^2*
(arctanh(tanh(b*x+a))-b*x-a)-12*a*(arctanh(tanh(b*x+a))-b*x-a)^2-4*(arctanh
(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(1/2)-1/3*(a^4+4*a^3*(arctanh(
tanh(b*x+a))-b*x-a)+6*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+4*a*(arctanh(tanh(
b*x+a))-b*x-a)^3+(arctanh(tanh(b*x+a))-b*x-a)^4)/arctanh(tanh(b*x+a))^(3/2)
)
```

maxima [A] time = 0.52, size = 64, normalized size = 0.65

$$\frac{2\left(3\,b^5x^5-5\,ab^4x^4+40\,a^2b^3x^3+240\,a^3b^2x^2+320\,a^4bx+128\,a^5\right)}{15\,(bx+a)^{\frac{5}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^5*x^5 - 5*a*b^4*x^4 + 40*a^2*b^3*x^3 + 240*a^3*b^2*x^2 + 320*a^4*
b*x + 128*a^5)/((b*x + a)^(5/2)*b^5)
```

mupad [B] time = 1.42, size = 817, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^4) + (2*((
2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (8*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(5*b^3))*(
log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1))/2 + b*x))/(3*b)))/b + (2*x^2*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(
5*b^3) + (x*((2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (8*(log(2/(exp(2*a)*exp(2*b
*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*
x))/(5*b^3))*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - lo
g(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b) - (2*(log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1
/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(b^5*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - ((log((2*exp(2*a)*
```

```
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))
/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/(6*b^5*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(x**4/atanh(tanh(a + b*x))**(5/2), x)
```

3.159 $\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{5/2}} dx$

Optimal. Leaf size=76

$$-\frac{32\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}+\frac{16x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/3*x^3/b/arctanh(tanh(b*x+a))^(3/2)-32/3*arctanh(tanh(b*x+a))^(3/2)/b^4-4
*x^2/b^2/arctanh(tanh(b*x+a))^(1/2)+16*x*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$-\frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16x\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{32\tanh^{-1}(\tanh(a+bx))^{3/2}}{3b^4}-\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^3)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (4*x^2)/(b^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]) + (16*x*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^3 - (32*ArcTanh[Tanh[
a + b*x]]^(3/2))/(3*b^4)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^3}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{2\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{b}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{x}{\sqrt{\tanh^{-1}(\tanh}}}{b^2}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh^{-1}(t}}{b^3}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh^{-1}(t}}{b^3}$$

$$= -\frac{2x^3}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{4x^2}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16x\sqrt{\tanh^{-1}(t}}{b^3}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.86

$$-\frac{2\left(6b^2x^2\tanh^{-1}(\tanh(a+bx)) - 24bx\tanh^{-1}(\tanh(a+bx))^2 + 16\tanh^{-1}(\tanh(a+bx))^3 + b^3x^3\right)}{3b^4\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(b^3*x^3 + 6*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 24*b*x*ArcTanh[Tanh[a + b
*x]]^2 + 16*ArcTanh[Tanh[a + b*x]]^3))/(3*b^4*ArcTanh[Tanh[a + b*x]]^(3/2))
```

fricas [A] time = 0.60, size = 62, normalized size = 0.82

$$\frac{2\left(b^3x^3 - 6\,ab^2x^2 - 24\,a^2bx - 16\,a^3\right)\sqrt{bx+a}}{3\left(b^6x^2 + 2\,ab^5x + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
2*a*b^5*x + a^2*b^4)
```

giac [A] time = 0.24, size = 59, normalized size = 0.78

$$-\frac{2\left(9\,(bx+a)a^2 - a^3\right)}{3\,(bx+a)^{\frac{3}{2}}b^4} + \frac{2\left((bx+a)^{\frac{3}{2}}b^8 - 9\,\sqrt{bx+a}\,ab^8\right)}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*b
^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

maple [B] time = 0.15, size = 186, normalized size = 2.45

$$\frac{\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3} - 6a\sqrt{\operatorname{arctanh}(\tanh(bx+a))} - 6(\operatorname{arctanh}(\tanh(bx+a)) - bx - a)\sqrt{\operatorname{arctanh}(\tanh(bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^4*(1/3*arctanh(tanh(b*x+a))^(3/2)-3*a*arctanh(tanh(b*x+a))^(1/2)-3*(arc
tanh(tanh(b*x+a))-b*x-a)*arctanh(tanh(b*x+a))^(1/2)-(3*a^2+6*a*(arctanh(tan
h(b*x+a))-b*x-a)+3*(arctanh(tanh(b*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(1/
2)-1/3*(-a^3-3*a^2*(arctanh(tanh(b*x+a))-b*x-a)-3*a*(arctanh(tanh(b*x+a))-b
*x-a)^2-(arctanh(tanh(b*x+a))-b*x-a)^3)/arctanh(tanh(b*x+a))^(3/2))
```

maxima [A] time = 0.53, size = 52, normalized size = 0.68

$$\frac{2\left(b^4x^4 - 5\,ab^3x^3 - 30\,a^2b^2x^2 - 40\,a^3bx - 16\,a^4\right)}{3\,(bx+a)^{\frac{5}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^4*x^4 - 5*a*b^3*x^3 - 30*a^2*b^2*x^2 - 40*a^3*b*x - 16*a^4)/((b*x +
a)^(5/2)*b^4)
```

mupad [B] time = 1.30, size = 533, normalized size = 7.01

$$\frac{2x\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}}{3b^3} + \frac{\left(\frac{2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right) - \ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right) + 2bx\right)}{b^3} + \frac{4\left(\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2} + bx\right)}{3b^3}\right)\sqrt{\frac{\ln\left(\frac{2e^{2a}e}{e^{2a}e}\right.}{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (2*x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b^3) + (((2*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b
^3 + (4*(log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 + b*x))/(3*b^3))*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/b
- (3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp
(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(b^4*(log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x)
+ 1)))) - ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(
2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(3*b^4*(l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp
(2*b*x) + 1)))^2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

3.160 $\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=59

$$\frac{16\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^3} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/3*x^2/b/arctanh(tanh(b*x+a))^(3/2)-8/3*x/b^2/arctanh(tanh(b*x+a))^(1/2)+
16/3*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2157, 30}

$$-\frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^3} - \frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^2)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (8*x)/(3*b^2*Sqrt[ArcTanh[Tan
h[a + b*x]]]) + (16*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^2}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{4\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\int \frac{1}{\sqrt{\tanh^{-1}(\mathrm{ta}\ldots}}}{3b^2 \ldots}$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{8\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{\ldots}}\ldots\right.}{\ldots}$$

$$= -\frac{2x^2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{8x}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{16\sqrt{\tanh^{-1}(\mathrm{t}\ldots}}{3b\ldots}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.81

$$-\frac{2\left(4bx\tanh^{-1}(\tanh(a+bx)) - 8\tanh^{-1}(\tanh(a+bx))^2 + b^2x^2\right)}{3b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*(b^2*x^2 + 4*b*x*ArcTanh[Tanh[a + b*x]] - 8*ArcTanh[Tanh[a + b*x]]^2))/
(3*b^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

fricas [A] time = 0.42, size = 52, normalized size = 0.88

$$\frac{2\left(3\,b^2x^2 + 12\,abx + 8\,a^2\right)\sqrt{bx+a}}{3\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^2
*b^3)
```

giac [A] time = 0.32, size = 39, normalized size = 0.66

$$\frac{2\sqrt{bx+a}}{b^3} + \frac{2\left(6\,(bx+a)a - a^2\right)}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

maple [A] time = 0.15, size = 91, normalized size = 1.54

$$\frac{2\sqrt{\operatorname{arctanh}(\tanh(bx+a))} - \frac{2(-2\operatorname{arctanh}(\tanh(bx+a))+2bx)}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}} - \frac{2\left(a^2+2a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)+(\operatorname{arctanh}(\tanh(bx+a))-bx-a)^2\ldots\right.}{3\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\ldots}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/b^3*(arctanh(tanh(b*x+a))^(1/2)-(-2*arctanh(tanh(b*x+a))+2*b*x)/arctanh(t
anh(b*x+a))^(1/2)-1/3*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b
*x+a))-b*x-a)^2)/arctanh(tanh(b*x+a))^(3/2))
```

maxima [A] time = 0.54, size = 42, normalized size = 0.71

$$\frac{2\left(3\,b^3x^3 + 15\,ab^2x^2 + 20\,a^2bx + 8\,a^3\right)}{3\,(bx+a)^{\frac{5}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*b^3*x^3 + 15*a*b^2*x^2 + 20*a^2*b*x + 8*a^3)/((b*x + a)^(5/2)*b^3)
```

mupad [B] time = 1.26, size = 259, normalized size = 4.39

$$\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(-b^2\,x^2-2\,b\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^2-4\,\ln\left(\cdots\right.\right.}{3\,b^3\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2)*(2*log(2/(exp(2*a)*exp(2*b*x) + 1))^2 - 4*log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*log(2/(exp(2*a)*exp(2*b
*x) + 1)) + 2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2 - b^
2*x^2 - 2*b*x*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x*log(2/(exp(2*a)*exp(2*b*x) + 1))))/(3*b^3*(log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

3.161 $\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{5/2}} dx$

Optimal. Leaf size=38

$$-\frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/3*x/b/arctanh(tanh(b*x+a))^(3/2)-4/3/b^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2168, 2157, 30}

$$-\frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x)/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - 4/(3*b^2*Sqrt[ArcTanh[Tanh[a +
b*x]]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\tanh^{-1}(\tanh(a+bx))^{5/2}} dx &= -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{3/2}} dx}{3b}\\
&= -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{3b^2}\\
&= -\frac{2x}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}-\frac{4}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 31, normalized size = 0.82

$$-\frac{2\left(2\tanh^{-1}(\tanh(a+bx))+bx\right)}{3b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/ArcTanh[Tanh[a + b*x]]^(5/2),x]
[Out] (-2*(b*x + 2*ArcTanh[Tanh[a + b*x]]))/(3*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))
```

fricas [A] time = 0.59, size = 41, normalized size = 1.08

$$-\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
[Out] -2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

giac [A] time = 0.23, size = 20, normalized size = 0.53

$$-\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
[Out] -2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

maple [A] time = 0.15, size = 42, normalized size = 1.11

$$\frac{-\frac{2}{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{2(bx-\operatorname{arctanh}(\tanh(bx+a)))}{3\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctanh(tanh(b*x+a))^(5/2),x)
[Out] 2/b^2*(-1/arctanh(tanh(b*x+a))^(1/2)-1/3*(b*x-arctanh(tanh(b*x+a)))/arctanh
(tanh(b*x+a))^(3/2))
```

maxima [A] time = 0.53, size = 31, normalized size = 0.82

$$-\frac{2\left(3\,b^2x^2+5\,abx+2\,a^2\right)}{3\,(bx+a)^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -2/3*(3*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x + a)^(5/2)*b^2)
```

mupad [B] time = 1.38, size = 152, normalized size = 4.00

$$-\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+b\,x\right)}{3\,b^2\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] -(8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)) + b*x))/(3*b^2*(log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1
)))^2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: TypeError
```

3.162 $\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/3/b/arctanh(tanh(b*x+a))^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{2157, 30\}$

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(-5/2),x]
```

```
[Out] -2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= -\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{2}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(-5/2),x]
```

```
[Out] -2/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2))
```

fricas [B] time = 0.52, size = 31, normalized size = 1.72

$$-\frac{2\sqrt{bx+a}}{3\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

giac [A] time = 0.23, size = 12, normalized size = 0.67

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
[Out] -2/3/((b*x + a)^(3/2)*b)
```

maple [A] time = 0.03, size = 15, normalized size = 0.83

$$-\frac{2}{3b\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^(5/2),x)
[Out] -2/3/b/arctanh(tanh(b*x+a))^(3/2)
```

maxima [A] time = 0.50, size = 12, normalized size = 0.67

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
[Out] -2/3/((b*x + a)^(3/2)*b)
```

mupad [B] time = 1.37, size = 103, normalized size = 5.72

$$-\frac{8\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atanh(tanh(a + b*x))^(5/2),x)
[Out] -(8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(
2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**(5/2),x)
[Out] Exception raised: TypeError
```

3.163 $\int \frac{1}{x \tanh^{-1}(\tanh(a+bx))^{5/2}} dx$

Optimal. Leaf size=108

$$\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}-$$

```
[Out] 2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-
arctanh(tanh(b*x+a)))^(5/2)-2/3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x
+a))^(3/2)+2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2163, 2161}

$$\frac{2}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 2/(3*(b*x - ArcTanh[Tanh[a + b*x]])
*ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[A
rcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{\int \frac{1}{x\tanh^{-1}(\tanh}{bx-\tanh^{-1}(\mathrm{ta}}$$

$$= -\frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\left(bx-\tanh^{-1}(\mathrm{t}\right.}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{2}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\mathrm{ta}}$$

Mathematica [A] time = 0.17, size = 91, normalized size = 0.84

$$\frac{2\left(4\tanh^{-1}(\tanh(a+bx))-bx\right)}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{2\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (2*(-(b*x) + 4*ArcTanh[Tanh
[a + b*x]]))/(3*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x
]])^2)
```

fricas [A] time = 0.51, size = 177, normalized size = 1.64

$$\left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(3\,abx+4\,a^2\right)\sqrt{bx+a}}{3\left(a^3b^2x^2+2\,a^4bx+a^5\right)}, \frac{2\left(3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-a}\,\mathrm{arc}\right.}{3\left(a^3b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x +
 a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(
-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)]
```

giac [A] time = 0.24, size = 45, normalized size = 0.42

$$\frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} + \frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x +
 a)^(3/2)*a^2)
```

maple [A] time = 0.15, size = 93, normalized size = 0.86

$$\frac{2}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{2}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\operatorname{arctanh}\left(\tanh\left(bx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b*x+a))^(1/2)+2/3/(arctanh(tanh
(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)-2/(arctanh(tanh(b*x+a))-b*x)^(5/2)
*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arctanh(tanh(b*x + a))^(5/2)), x)
```

mupad [B] time = 6.26, size = 886, normalized size = 8.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] (2^(1/2)*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 -
log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*2i
 - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^5 + 40*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 80*a^3*(2*a - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^2 - 32*a^5 - 10*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 80*a^
4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))
*4i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2) + (16*(log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/((log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2
*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (16*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/
(3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a
)*exp(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(x*atanh(tanh(a + b*x))**(5/2)), x)
```

3.164 $\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}} dx$

Optimal. Leaf size=155

$$\frac{5b}{(bx-\tanh^{-1}(\tanh(a+bx)))^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{5b}{3(bx-\tanh^{-1}(\tanh(a+bx)))^2 \tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] 5*b*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*
x-arctanh(tanh(b*x+a)))^(7/2)-1/x/arctanh(tanh(b*x+a))^(5/2)+b/(b*x-arctanh
(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(5/2)-5/3*b/(b*x-arctanh(tanh(b*x+a)))^
2/arctanh(tanh(b*x+a))^(3/2)+5*b/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(
b*x+a))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$\frac{5b}{(bx-\tanh^{-1}(\tanh(a+bx)))^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{5b}{3(bx-\tanh^{-1}(\tanh(a+bx)))^2 \tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (5*b*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]
])/(b*x - ArcTanh[Tanh[a + b*x]])^(7/2) - 1/(x*ArcTanh[Tanh[a + b*x]]^(5/2)
) + b/((b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2)) - (5*b)
/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*b)/
((b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^2 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{2}(5b)\int \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{7/2}}\,dx$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= -\frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{b}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(}$$

$$= \frac{5b\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{1}{x\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{}{\left(bx-\tan\right.}$$

Mathematica [A] time = 0.15, size = 113, normalized size = 0.73

$$\frac{14bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2 - 2b^2x^2}{3x\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{5b\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (5*b*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (-2*b^2*x^2 + 14*b*x*ArcTa
nh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)/(3*x*(b*x - ArcTanh[Tanh[a
+ b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

fricas [A] time = 0.60, size = 221, normalized size = 1.43

$$\left[\frac{15\left(b^3x^3 + 2ab^2x^2 + a^2bx\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\left(15ab^2x^2 + 20a^2bx + 3a^3\right)\sqrt{bx+a}}{6\left(a^4b^2x^3 + 2a^5bx^2 + a^6x\right)}, -\frac{15\left(b^3x^3 + 2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a))
/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2
*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^2*b*
x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

giac [A] time = 0.22, size = 65, normalized size = 0.42

$$-\frac{5b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3} - \frac{2\left(6(bx+a)b + ab\right)}{3(bx+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx+a}}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b + a
*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

maple [A] time = 0.16, size = 130, normalized size = 0.84

$$2b\left(-\frac{1}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}-\frac{2}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}\sqrt{\operatorname{arctanh}\left(\mathrm{ta}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2*b*(-1/3/(arctanh(tanh(b*x+a))-b*x)^2/arctanh(tanh(b*x+a))^(3/2)-2/(arctan
h(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(1/2)-1/(arctanh(tanh(b*x+a))-b*
x)^3*(1/2*arctanh(tanh(b*x+a))^(1/2)/b/x-5/2/(arctanh(tanh(b*x+a))-b*x)^(1/
2)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*arctanh(tanh(b*x + a))^(5/2)), x)
```

mupad [B] time = 7.34, size = 1230, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] (2^(1/2)*b*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2
- log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*
2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^7 + 84*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 280*a^3*(2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^4 + 560*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 672*a^5*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 128*a^7 - 14*a*(2*a - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^6 + 448*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)^(1/2)))*20i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2) - (32*b*(log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) +
1))/2)^(1/2))/(3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
```

```
log(2/(exp(2*a)*exp(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + ((log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b
*x) + 1))/2)^(1/2)*(x*((32*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + (6*b*(8*log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - 8*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 16*b*x))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4) - (8*log(2/(exp(2*a)*exp(2*
b*x) + 1)) - 8*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 16*
b*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(x**2*atanh(tanh(a + b*x))**(5/2)), x)
```

3.165 $\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=224

$$\frac{35b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^2}{12\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] 35/4*b^2*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
)/(b*x-arctanh(tanh(b*x+a)))^(9/2)+5/4*b/x/arctanh(tanh(b*x+a))^(7/2)-5/4*b
^2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(7/2)-1/2/x^2/arctanh(ta
nh(b*x+a))^(5/2)+7/4*b^2/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^
(5/2)-35/12*b^2/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)+35/
4*b^2/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$\frac{35b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^2}{12\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (35*b^2*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*x
]]]])/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) + (5*b)/(4*x*ArcTanh[Tanh[a
+ b*x]]^(7/2)) - (5*b^2)/(4*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a +
 b*x]]^(7/2)) - 1/(2*x^2*ArcTanh[Tanh[a + b*x]]^(5/2)) + (7*b^2)/(4*(b*x -
ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(5/2)) - (35*b^2)/(12*(b*x
 - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2)) + (35*b^2)/(4*(b
*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^3 \tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{1}{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{4}(5b)\int \frac{1}{x^2\tanh^{-1}(\tanh(a+bx))^{7/2}}\,dx$$

$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{1}{2x^2\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{1}{8}\left(35b^2\right)\int$$

$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh}$$

$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh}$$

$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh}$$

$$= \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{5b^2}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh}$$

$$= \frac{35b^2\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} + \frac{5b}{4x\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{}{4\left(bx\right.}$$

Mathematica [A] time = 0.13, size = 133, normalized size = 0.59

$$\frac{80b^2x^2\tanh^{-1}(\tanh(a+bx))+39bx\tanh^{-1}(\tanh(a+bx))^2-6\tanh^{-1}(\tanh(a+bx))^3-8b^3x^3}{12x^2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4} - \frac{35b^2\tanh^{-1}}{4\left(\tanh^{-1}(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-35*b^2*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2)) + (-8*b^3*x^3 + 80*b
^2*x^2*ArcTanh[Tanh[a + b*x]] + 39*b*x*ArcTanh[Tanh[a + b*x]]^2 - 6*ArcTanh
[Tanh[a + b*x]]^3)/(12*x^2*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[T
anh[a + b*x]])^4)
```

fricas [A] time = 0.57, size = 255, normalized size = 1.14

$$\left[\frac{105\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\sqrt{a}\,\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4\right)\sqrt{bx+}}{24\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*
```

```
x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(105*
(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a
)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a)
)/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

giac [A] time = 0.18, size = 93, normalized size = 0.42

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^4}+\frac{2\left(9\,(bx+a)b^2+ab^2\right)}{3\,(bx+a)^{\frac{3}{2}}a^4}+\frac{11\,(bx+a)^{\frac{3}{2}}b^2-13\,\sqrt{bx+a}\,ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*b
^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt(b
*x + a)*a*b^2)/(a^4*b^2*x^2)
```

maple [A] time = 0.15, size = 157, normalized size = 0.70

$$2b^2\left(\frac{3}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{1}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^3\operatorname{arctanh}\left(\tan\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2*b^2*(3/(arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(1/2)+1/3/(arcta
nh(tanh(b*x+a))-b*x)^3/arctanh(tanh(b*x+a))^(3/2)+1/(arctanh(tanh(b*x+a))-b
*x)^4*((11/8*arctanh(tanh(b*x+a))^(3/2)+(-13/8*arctanh(tanh(b*x+a))+13/8*b*
x)*arctanh(tanh(b*x+a))^(1/2))/b^2/x^2-35/8/(arctanh(tanh(b*x+a))-b*x)^(1/2
)*arctanh(arctanh(tanh(b*x+a))^(1/2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*arctanh(tanh(b*x + a))^(5/2)), x)
```

mupad [B] time = 8.47, size = 1514, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((4*(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) -
2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)))/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(
```

```
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)) + (56*b^2*x)/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))^2 -
 ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((140*b)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (280
*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
 + (2^(1/2)*b^2*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(
1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*((2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^9 + 144*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 672*a^3*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^6 + 2016*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 4
032*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log
(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 5376*a^6*(2*a - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^3 - 4608*a^7*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 512*a^9 - 18*a*(2*a
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^8 + 2304*a^8*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)
/(2*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*70i)/(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(9/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(x**3*atanh(tanh(a + b*x))**(5/2)), x)
```

3.166 $\int \frac{1}{x^4 \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=278

$$\frac{105b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^3}$$

```
[Out] 105/8*b^3*arctan(arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2
))/(b*x-arctanh(tanh(b*x+a)))^(11/2)-35/24*b^2/x/arctanh(tanh(b*x+a))^(9/2)
+35/24*b^3/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(9/2)+5/12*b/x^2
/arctanh(tanh(b*x+a))^(7/2)-15/8*b^3/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(t
anh(b*x+a))^(7/2)-1/3/x^3/arctanh(tanh(b*x+a))^(5/2)+21/8*b^3/(b*x-arctanh(
tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(5/2)-35/8*b^3/(b*x-arctanh(tanh(b*x+a
)))^4/arctanh(tanh(b*x+a))^(3/2)+105/8*b^3/(b*x-arctanh(tanh(b*x+a)))^5/arc
tanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2161}

$$\frac{105b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{35b^3}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (105*b^3*ArcTan[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]])/(8*(b*x - ArcTanh[Tanh[a + b*x]])^(11/2)) - (35*b^2)/(24*x*ArcTanh[T
anh[a + b*x]]^(9/2)) + (35*b^3)/(24*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[
Tanh[a + b*x]]^(9/2)) + (5*b)/(12*x^2*ArcTanh[Tanh[a + b*x]]^(7/2)) - (15*b
^3)/(8*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(7/2)) - 1/(
3*x^3*ArcTanh[Tanh[a + b*x]]^(5/2)) + (21*b^3)/(8*(b*x - ArcTanh[Tanh[a + b
*x]])^3*ArcTanh[Tanh[a + b*x]]^(5/2)) - (35*b^3)/(8*(b*x - ArcTanh[Tanh[a +
 b*x]])^4*ArcTanh[Tanh[a + b*x]]^(3/2)) + (105*b^3)/(8*(b*x - ArcTanh[Tanh[
a + b*x]])^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2161

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(2*ArcTan[Sqrt[v]/Rt[(b*u - a*v)/a, 2]])/(a*Rt[(b*u - a*
v)/a, 2]), x] /; NeQ[b*u - a*v, 0] && PosQ[(b*u - a*v)/a]] /; PiecewiseLine
arQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
```

```
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^4\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{1}{3x^3\tanh^{-1}(\tanh(a+bx))^{5/2}} - \frac{1}{6}(5b)\int \frac{1}{x^3\tanh^{-1}(\tanh(a+bx))^{7/2}}\,dx$$

$$= \frac{5b}{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{1}{3x^3\tanh^{-1}(\tanh(a+bx))^{5/2}} + \frac{1}{24}\left(35b^2\right)$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{5b}{12x^2\tanh^{-1}(\tanh(a+bx))^{7/2}} - \frac{}{3x^3\tan}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= -\frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{35b^3}{24\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(}$$

$$= \frac{105b^3\tan^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{8\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} - \frac{35b^2}{24x\tanh^{-1}(\tanh(a+bx))^{9/2}} + \frac{}{24(}$$

Mathematica [A] time = 0.22, size = 150, normalized size = 0.54

$$\frac{1}{24}\left(\frac{315b^3\tanh^{-1}\left(\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{11/2}} + \frac{208b^3x^3\tanh^{-1}(\tanh(a+bx))+165b^2x^2\tanh^{-1}(\tanh(a+bx))^2 -}{x^3\left(bx-\tanh^{-1}(\tanh(a+bx\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] ((315*b^3*ArcTanh[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(11/2) + (-16*b^4*x^4 + 208*b
^3*x^3*ArcTanh[Tanh[a + b*x]] + 165*b^2*x^2*ArcTanh[Tanh[a + b*x]]^2 - 50*b
*x*ArcTanh[Tanh[a + b*x]]^3 + 8*ArcTanh[Tanh[a + b*x]]^4)/(x^3*(b*x - ArcTa
nh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]^(3/2)))/24
```

fricas [A] time = 0.51, size = 277, normalized size = 1.00

$$\left[\frac{315\left(b^5x^5+2\,ab^4x^4+a^2b^3x^3\right)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\left(315\,ab^4x^4+420\,a^2b^3x^3+63\,a^3b^2x^2-18\,a^4bx+8\right.}{48\left(a^6b^2x^5+2\,a^7bx^4+a^8x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(315*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*sqrt(a)*log((b*x + 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) - 2*(315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*b^
2*x^2 - 18*a^4*b*x + 8*a^5)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8
*x^3), -1/24*(315*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*sqrt(-a)*arctan(sqr
t(b*x + a)*sqrt(-a)/a) + (315*a*b^4*x^4 + 420*a^2*b^3*x^3 + 63*a^3*b^2*x^2
- 18*a^4*b*x + 8*a^5)*sqrt(b*x + a))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)]
```

giac [A] time = 0.22, size = 115, normalized size = 0.41

$$-\frac{105\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^5}-\frac{315\,(bx+a)^4b^3-840\,(bx+a)^3ab^3+693\,(bx+a)^2a^2b^3-144\,(bx+a)a^3b^3-16\,a^4b^3}{24\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\,a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -105/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/24*(315*(b*x +
 a)^4*b^3 - 840*(b*x + a)^3*a*b^3 + 693*(b*x + a)^2*a^2*b^3 - 144*(b*x + a)
*a^3*b^3 - 16*a^4*b^3)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)^3*a^5)
```

maple [A] time = 0.16, size = 211, normalized size = 0.76

$$2b^3\left(-\frac{1}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^4\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}-\frac{4}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^5\sqrt{\operatorname{arctanh}(\mathrm{t}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2*b^3*(-1/3/(arctanh(tanh(b*x+a))-b*x)^4/arctanh(tanh(b*x+a))^(3/2)-4/(arct
anh(tanh(b*x+a))-b*x)^5/arctanh(tanh(b*x+a))^(1/2)-1/(arctanh(tanh(b*x+a))-
b*x)^5*((41/16*arctanh(tanh(b*x+a))^(5/2)+(-35/6*arctanh(tanh(b*x+a))+35/6*
b*x)*arctanh(tanh(b*x+a))^(3/2)+(55/16*a^2+55/8*a*(arctanh(tanh(b*x+a))-b*x
-a)+55/16*(arctanh(tanh(b*x+a))-b*x-a)^2)*arctanh(tanh(b*x+a))^(1/2))/b^3/x
^3-105/16/(arctanh(tanh(b*x+a))-b*x)^(1/2)*arctanh(arctanh(tanh(b*x+a))^(1/
2)/(arctanh(tanh(b*x+a))-b*x)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^4\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4*arctanh(tanh(b*x + a))^(5/2)), x)
```

mupad [B] time = 6.42, size = 2359, normalized size = 8.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] (8*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (((140*
b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - (840*b^3*x)/(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)*(
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*
exp(2*b*x) + 1))/2)^(1/2))/(x*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - ((log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/
2)*(x*((((1232*b^4)/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^3) - (80*b^3*(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)))/(9*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^4))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b) - (952*b^3)/(9*(2*a*b
 - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (128*b^2*
(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(9*(2*a*
b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)) - (68*b*
(2*b*(2*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 2*log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 4*b*x) - 7*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(9*(2*a*
b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^2*(l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp
(2*b*x) + 1)))^2) + (2^(1/2)*b^3*log((((log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^(1/2)*2i - 2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 2^(1/2)*b*x)*
((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^11 + 220*a^2*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^9 - 1320*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^8 + 5280*a^4*(2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^7 - 14784*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 29568*a^6*(2
*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*
```

```
a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 42240*a^7*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4
 + 42240*a^8*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 28160*a^9*(2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^2 - 2048*a^11 - 22*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^10 + 1126
4*a^10*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*1i)/(2*x*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1
/2)))*210i)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(11/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(x**4*atanh(tanh(a + b*x))**(5/2)), x)
```

3.167 $\int x^{7/2} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{99}bx^{11/2}$$

```
[Out] -4/99*b*x^(11/2)+2/9*x^(9/2)*arctanh(tanh(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{99}bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(11/2))/99 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/9
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx)) - \frac{1}{9}(2b)\int x^{9/2}\,dx\\ &= -\frac{4}{99}bx^{11/2} + \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.05, size = 23, normalized size = 0.85

$$\frac{2}{99}x^{9/2}\left(11\tanh^{-1}(\tanh(a+bx)) - 2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(9/2)*(-2*b*x + 11*ArcTanh[Tanh[a + b*x]]))/99
```

fricas [A] time = 0.47, size = 18, normalized size = 0.67

$$\frac{2}{99}\left(9\,bx^5 + 11\,ax^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2/99*(9*b*x^5 + 11*a*x^4)*sqrt(x)
```

giac [A] time = 0.18, size = 13, normalized size = 0.48

$$\frac{2}{11} b x^{\frac{11}{2}} + \frac{2}{9} a x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2/11*b*x^(11/2) + 2/9*a*x^(9/2)
```

maple [A] time = 0.24, size = 20, normalized size = 0.74

$$-\frac{4 b x^{\frac{11}{2}}}{99} + \frac{2 x^{\frac{9}{2}} \operatorname{arctanh}\left(\tanh\left(b x + a\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a)),x)
```

```
[Out] -4/99*b*x^(11/2)+2/9*x^(9/2)*arctanh(tanh(b*x+a))
```

maxima [A] time = 0.33, size = 19, normalized size = 0.70

$$-\frac{4}{99} b x^{\frac{11}{2}} + \frac{2}{9} x^{\frac{9}{2}} \operatorname{artanh}\left(\tanh\left(b x + a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4/99*b*x^(11/2) + 2/9*x^(9/2)*arctanh(tanh(b*x + a))
```

mupad [B] time = 1.24, size = 57, normalized size = 2.11

$$\frac{x^{9/2} \ln\left(\frac{e^{2 a} e^{2 b x}}{e^{2 a} e^{2 b x}+1}\right)}{9} - \frac{4 b x^{11/2}}{99} - \frac{x^{9/2} \ln\left(\frac{1}{e^{2 a} e^{2 b x}+1}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atanh(tanh(a + b*x)),x)
```

```
[Out] (x^(9/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/9 - (4*b*x^(
11/2))/99 - (x^(9/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/9
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a)),x)
```

```
[Out] Timed out
```

3.168 $\int x^{5/2} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{63}bx^{9/2}$$

```
[Out] -4/63*b*x^(9/2)+2/7*x^(7/2)*arctanh(tanh(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{63}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]],x]
[Out] (-4*b*x^(9/2))/63 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{1}{7}(2b)\int x^{7/2}\,dx\\ &= -\frac{4}{63}bx^{9/2}+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.85

$$\frac{2}{63}x^{7/2}\left(9\tanh^{-1}(\tanh(a+bx))-2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]],x]
[Out] (2*x^(7/2)*(-2*b*x + 9*ArcTanh[Tanh[a + b*x]]))/63
```

fricas [A] time = 0.52, size = 18, normalized size = 0.67

$$\frac{2}{63}\left(7\,bx^{4}+9\,ax^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

giac [A] time = 0.25, size = 13, normalized size = 0.48

$$\frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

maple [A] time = 0.24, size = 20, normalized size = 0.74

$$-\frac{4b\,x^{\frac{9}{2}}}{63} + \frac{2x^{\frac{7}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a)),x)
```

```
[Out] -4/63*b*x^(9/2)+2/7*x^(7/2)*arctanh(tanh(b*x+a))
```

maxima [A] time = 0.33, size = 19, normalized size = 0.70

$$-\frac{4}{63}\,bx^{\frac{9}{2}} + \frac{2}{7}\,x^{\frac{7}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4/63*b*x^(9/2) + 2/7*x^(7/2)*arctanh(tanh(b*x + a))
```

mupad [B] time = 1.09, size = 57, normalized size = 2.11

$$\frac{x^{7/2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7} - \frac{4\,b\,x^{9/2}}{63} - \frac{x^{7/2}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atanh(tanh(a + b*x)),x)
```

```
[Out] (x^(7/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/7 - (4*b*x^(
9/2))/63 - (x^(7/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a)),x)
```

```
[Out] Timed out
```

3.169 $\int x^{3/2} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{35}bx^{7/2}$$

```
[Out] -4/35*b*x^(7/2)+2/5*x^(5/2)*arctanh(tanh(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2168, 30}

$$\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{35}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (-4*b*x^(7/2))/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx)) - \frac{1}{5}(2b)\int x^{5/2}\,dx\\ &= -\frac{4}{35}bx^{7/2} + \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.85

$$\frac{2}{35}x^{5/2}\left(7\tanh^{-1}(\tanh(a+bx)) - 2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(5/2)*(-2*b*x + 7*ArcTanh[Tanh[a + b*x]]))/35
```

fricas [A] time = 0.45, size = 18, normalized size = 0.67

$$\frac{2}{35}\left(5\,bx^{3} + 7\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 13, normalized size = 0.48

$$\frac{2}{7}bx^{\frac{7}{2}} + \frac{2}{5}ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

maple [A] time = 0.24, size = 20, normalized size = 0.74

$$-\frac{4b\,x^{\frac{7}{2}}}{35} + \frac{2x^{\frac{5}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a)),x)
```

```
[Out] -4/35*b*x^(7/2)+2/5*x^(5/2)*arctanh(tanh(b*x+a))
```

maxima [A] time = 0.33, size = 19, normalized size = 0.70

$$-\frac{4}{35}bx^{\frac{7}{2}} + \frac{2}{5}x^{\frac{5}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -4/35*b*x^(7/2) + 2/5*x^(5/2)*arctanh(tanh(b*x + a))
```

mupad [B] time = 1.10, size = 57, normalized size = 2.11

$$\frac{x^{5/2}\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{5} - \frac{4\,b\,x^{7/2}}{35} - \frac{x^{5/2}\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x)),x)
```

```
[Out] (x^(5/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/5 - (4*b*x^(
7/2))/35 - (x^(5/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/5
```

sympy [A] time = 14.46, size = 26, normalized size = 0.96

$$-\frac{4bx^{\frac{7}{2}}}{35} + \frac{2x^{\frac{5}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a)),x)
```

```
[Out] -4*b*x**(7/2)/35 + 2*x**(5/2)*atanh(tanh(a + b*x))/5
```

3.170 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=27

$$\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{15}bx^{5/2}$$

```
[Out] -4/15*b*x^(5/2)+2/3*x^(3/2)*arctanh(tanh(b*x+a))
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{15}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]],x]
[Out] (-4*b*x^(5/2))/15 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))\,dx &= \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))-\frac{1}{3}(2b)\int x^{3/2}\,dx\\ &= -\frac{4}{15}bx^{5/2}+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.85

$$\frac{2}{15}x^{3/2}\left(5\tanh^{-1}(\tanh(a+bx))-2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]],x]
[Out] (2*x^(3/2)*(-2*b*x + 5*ArcTanh[Tanh[a + b*x]]))/15
```

fricas [A] time = 0.57, size = 16, normalized size = 0.59

$$\frac{2}{15}\left(3\,bx^2+5\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

giac [A] time = 1.50, size = 13, normalized size = 0.48

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

maple [A] time = 0.24, size = 20, normalized size = 0.74

$$-\frac{4b\,x^{\frac{5}{2}}}{15}+\frac{2x^{\frac{3}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))*x^(1/2),x)
```

```
[Out] -4/15*b*x^(5/2)+2/3*x^(3/2)*arctanh(tanh(b*x+a))
```

maxima [A] time = 0.33, size = 19, normalized size = 0.70

$$-\frac{4}{15}\,bx^{\frac{5}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))*x^(1/2),x, algorithm="maxima")
```

```
[Out] -4/15*b*x^(5/2) + 2/3*x^(3/2)*arctanh(tanh(b*x + a))
```

mupad [B] time = 1.09, size = 57, normalized size = 2.11

$$\frac{x^{3/2}\,\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{3}-\frac{4\,b\,x^{5/2}}{15}-\frac{x^{3/2}\,\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x)),x)
```

```
[Out] (x^(3/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/3 - (4*b*x^(
5/2))/15 - (x^(3/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x)), x)
```

3.171 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}} dx$

Optimal. Leaf size=25

$$2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{3}bx^{3/2}$$

```
[Out] -4/3*b*x^(3/2)+2*arctanh(tanh(b*x+a))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2168, 30}

$$2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{4}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/Sqrt[x],x]
[Out] (-4*b*x^(3/2))/3 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\,dx &= 2\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - (2b)\int \sqrt{x}\,dx\\ &= -\frac{4}{3}bx^{3/2} + 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.92

$$\frac{2}{3}\sqrt{x}\left(3\tanh^{-1}(\tanh(a+bx)) - 2bx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/Sqrt[x],x]
[Out] (2*Sqrt[x]*(-2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/3
```

fricas [A] time = 0.58, size = 12, normalized size = 0.48

$$\frac{2}{3}(bx+3a)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + 3*a)*sqrt(x)
```

giac [A] time = 0.21, size = 13, normalized size = 0.52

$$\frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

maple [A] time = 0.24, size = 20, normalized size = 0.80

$$-\frac{4b\,x^{\frac{3}{2}}}{3} + 2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(1/2),x)
```

```
[Out] -4/3*b*x^(3/2)+2*arctanh(tanh(b*x+a))*x^(1/2)
```

maxima [A] time = 0.33, size = 19, normalized size = 0.76

$$-\frac{4}{3}\,bx^{\frac{3}{2}} + 2\,\sqrt{x}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -4/3*b*x^(3/2) + 2*sqrt(x)*arctanh(tanh(b*x + a))
```

mupad [B] time = 1.13, size = 56, normalized size = 2.24

$$\sqrt{x}\,\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) - \frac{4\,b\,x^{3/2}}{3} - \sqrt{x}\,\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^(1/2),x)
```

```
[Out] x^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - (4*b*x^(3/2)
)/3 - x^(1/2)*log(1/(exp(2*a)*exp(2*b*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))/sqrt(x), x)
```

3.172 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}} \, dx$

Optimal. Leaf size=23

$$4b\sqrt{x} - \frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

```
[Out] -2*arctanh(tanh(b*x+a))/x^(1/2)+4*b*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$4b\sqrt{x} - \frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(3/2),x]
```

```
[Out] 4*b*Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}} \, dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}} + (2b)\int \frac{1}{\sqrt{x}} \, dx\\ &= 4b\sqrt{x} - \frac{2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 20, normalized size = 0.87

$$\frac{4bx - 2\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(3/2),x]
```

```
[Out] (4*b*x - 2*ArcTanh[Tanh[a + b*x]])/Sqrt[x]
```

fricas [A] time = 0.40, size = 12, normalized size = 0.52

$$\frac{2(bx-a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x - a)/sqrt(x)
```

giac [A] time = 0.19, size = 13, normalized size = 0.57

$$2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

maple [A] time = 0.24, size = 20, normalized size = 0.87

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}{\sqrt{x}} + 4b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(3/2),x)
```

```
[Out] -2*arctanh(tanh(b*x+a))/x^(1/2)+4*b*x^(1/2)
```

maxima [A] time = 0.33, size = 19, normalized size = 0.83

$$4\,b\sqrt{x} - \frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(3/2),x, algorithm="maxima")
```

```
[Out] 4*b*sqrt(x) - 2*arctanh(tanh(b*x + a))/sqrt(x)
```

mupad [B] time = 1.12, size = 56, normalized size = 2.43

$$\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{\sqrt{x}} + 4\,b\,\sqrt{x} - \frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^(3/2),x)
```

```
[Out] log(1/(exp(2*a)*exp(2*b*x) + 1))/x^(1/2) + 4*b*x^(1/2) - log((exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/x^(1/2)
```

sympy [A] time = 1.05, size = 22, normalized size = 0.96

$$4b\sqrt{x} - \frac{2\,\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(3/2),x)
```

```
[Out] 4*b*sqrt(x) - 2*atanh(tanh(a + b*x))/sqrt(x)
```

3.173 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}} \, dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}-\frac{4b}{3\sqrt{x}}$$

```
[Out] -2/3*arctanh(tanh(b*x+a))/x^(3/2)-4/3*b/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}-\frac{4b}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(5/2),x]
```

```
[Out] (-4*b)/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]])/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}} \, dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}+\frac{1}{3}(2b)\int \frac{1}{x^{3/2}} \, dx\\ &= -\frac{4b}{3\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 0.78

$$-\frac{2\left(\tanh^{-1}(\tanh(a+bx))+2bx\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(5/2),x]
```

```
[Out] (-2*(2*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3/2))
```

fricas [A] time = 0.57, size = 11, normalized size = 0.41

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

giac [A] time = 0.17, size = 11, normalized size = 0.41

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

maple [A] time = 0.24, size = 20, normalized size = 0.74

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}{3x^{\frac{3}{2}}}-\frac{4b}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(5/2),x)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))/x^(3/2)-4/3*b/x^(1/2)
```

maxima [A] time = 0.33, size = 19, normalized size = 0.70

$$-\frac{4\,b}{3\,\sqrt{x}}-\frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*b/sqrt(x) - 2/3*arctanh(tanh(b*x + a))/x^(3/2)
```

mupad [B] time = 1.24, size = 52, normalized size = 1.93

$$-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+4\,b\,x}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^(5/2),x)
```

```
[Out] -(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(1/(exp(2*a)*ex
p(2*b*x) + 1)) + 4*b*x)/(3*x^(3/2))
```

sympy [A] time = 10.54, size = 27, normalized size = 1.00

$$-\frac{4b}{3\sqrt{x}}-\frac{2\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(5/2),x)
```

```
[Out] -4*b/(3*sqrt(x)) - 2*atanh(tanh(a + b*x))/(3*x**(3/2))
```

3.174 $\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{7/2}} dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}-\frac{4b}{15x^{3/2}}$$

```
[Out] -4/15*b/x^(3/2)-2/5*arctanh(tanh(b*x+a))/x^(5/2)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}-\frac{4b}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]/x^(7/2),x]
```

```
[Out] (-4*b)/(15*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]])/(5*x^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{7/2}} dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}+\frac{1}{5}(2b)\int \frac{1}{x^{5/2}} dx\\ &= -\frac{4b}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))}{5x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.85

$$-\frac{2\left(3\tanh^{-1}(\tanh(a+bx))+2bx\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]/x^(7/2),x]
```

```
[Out] (-2*(2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/(15*x^(5/2))
```

fricas [A] time = 0.49, size = 13, normalized size = 0.48

$$-\frac{2\,(5\,bx+3\,a)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(5*b*x + 3*a)/x^(5/2)
```

giac [A] time = 0.22, size = 13, normalized size = 0.48

$$-\frac{2\,(5\,bx+3\,a)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*b*x + 3*a)/x^(5/2)
```

maple [A] time = 0.25, size = 20, normalized size = 0.74

$$-\frac{4b}{15x^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\,(\tanh\,(bx+a))}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))/x^(7/2),x)
```

```
[Out] -4/15*b/x^(3/2)-2/5*arctanh(tanh(b*x+a))/x^(5/2)
```

maxima [A] time = 0.33, size = 19, normalized size = 0.70

$$-\frac{4\,b}{15\,x^{\frac{3}{2}}}-\frac{2\,\mathrm{artanh}\,(\tanh\,(bx+a))}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))/x^(7/2),x, algorithm="maxima")
```

```
[Out] -4/15*b/x^(3/2) - 2/5*arctanh(tanh(b*x + a))/x^(5/2)
```

mupad [B] time = 1.28, size = 57, normalized size = 2.11

$$\frac{\ln\left(\frac{1}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{5\,x^{5/2}}-\frac{4\,b}{15\,x^{3/2}}-\frac{\ln\left(\frac{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)}{5\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))/x^(7/2),x)
```

```
[Out] log(1/(exp(2*a)*exp(2*b*x) + 1))/(5*x^(5/2)) - (4*b)/(15*x^(3/2)) - log((ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/(5*x^(5/2))
```

sympy [A] time = 155.35, size = 27, normalized size = 1.00

$$-\frac{4b}{15x^{\frac{3}{2}}}-\frac{2\,\mathrm{atanh}\,(\tanh\,(a+bx))}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))/x**(7/2),x)
```

```
[Out] -4*b/(15*x**(3/2)) - 2*atanh(tanh(a + b*x))/(5*x**(5/2))
```

3.175 $\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\,dx$

Optimal. Leaf size=48

$$-\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16b^2x^{13/2}}{1287}$$

```
[Out] 16/1287*b^2*x^(13/2)-8/99*b*x^(11/2)*arctanh(tanh(b*x+a))+2/9*x^(9/2)*arcta
nh(tanh(b*x+a))^2
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16b^2x^{13/2}}{1287}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(13/2))/1287 - (8*b*x^(11/2)*ArcTanh[Tanh[a + b*x]])/99 + (2*x^(9
/2)*ArcTanh[Tanh[a + b*x]]^2)/9
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{9}(4b)\int x^{9/2}\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{99}\left(8b^2\right.\\ &= \frac{16b^2x^{13/2}}{1287}-\frac{8}{99}bx^{11/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.06, size = 40, normalized size = 0.83

$$\frac{2x^{9/2}\left(-52bx\tanh^{-1}(\tanh(a+bx))+143\tanh^{-1}(\tanh(a+bx))^2+8b^2x^2\right)}{1287}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(9/2)*(8*b^2*x^2 - 52*b*x*ArcTanh[Tanh[a + b*x]] + 143*ArcTanh[Tanh[a
+ b*x]]^2))/1287
```

fricas [A] time = 0.61, size = 29, normalized size = 0.60

$$\frac{2}{1287}\left(99\,b^2x^6 + 234\,abx^5 + 143\,a^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2/1287*(99*b^2*x^6 + 234*a*b*x^5 + 143*a^2*x^4)*sqrt(x)
```

giac [A] time = 0.18, size = 24, normalized size = 0.50

$$\frac{2}{13}\,b^2x^{\frac{13}{2}} + \frac{4}{11}\,abx^{\frac{11}{2}} + \frac{2}{9}\,a^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2/13*b^2*x^(13/2) + 4/11*a*b*x^(11/2) + 2/9*a^2*x^(9/2)
```

maple [A] time = 0.28, size = 38, normalized size = 0.79

$$\frac{2x^{\frac{9}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{9} - \frac{8b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))}{11} - \frac{2x^{\frac{13}{2}}b}{143}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/9*x^(9/2)*arctanh(tanh(b*x+a))^2-8/9*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a)
)-2/143*x^(13/2)*b)
```

maxima [A] time = 0.35, size = 36, normalized size = 0.75

$$\frac{16}{1287}\,b^2x^{\frac{13}{2}} - \frac{8}{99}\,bx^{\frac{11}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{2}{9}\,x^{\frac{9}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 16/1287*b^2*x^(13/2) - 8/99*b*x^(11/2)*arctanh(tanh(b*x + a)) + 2/9*x^(9/2)
*arctanh(tanh(b*x + a))^2
```

mupad [B] time = 1.15, size = 122, normalized size = 2.54

$$\frac{x^{9/2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}{18}+\frac{2\,b^2\,x^{13/2}}{13}-\frac{2\,b\,x^{11/2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^(9/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/18 + (2*b^2*x^(13/2))/13 - (2*b*x^(11/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x))/11
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Timed out
```

3.176 $\int x^{5/2} \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=48

$$-\frac{8}{63}bx^{9/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{693}b^2x^{11/2}$$

```
[Out] 16/693*b^2*x^(11/2)-8/63*b*x^(9/2)*arctanh(tanh(b*x+a))+2/7*x^(7/2)*arctanh
(tanh(b*x+a))^2
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8}{63}bx^{9/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{693}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(11/2))/693 - (8*b*x^(9/2)*ArcTanh[Tanh[a + b*x]])/63 + (2*x^(7/2
)*ArcTanh[Tanh[a + b*x]]^2)/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\tanh^{-1}(\tanh(a+bx))^2\,dx &= \frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2-\frac{1}{7}(4b)\int x^{7/2}\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{8}{63}bx^{9/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{63}\left(8b^2\right)\int\\ &= \frac{16}{693}b^2x^{11/2}-\frac{8}{63}bx^{9/2}\tanh^{-1}(\tanh(a+bx))+\frac{2}{7}x^{7/2}\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.83

$$\frac{2}{693}x^{7/2}\left(-44bx\tanh^{-1}(\tanh(a+bx))+99\tanh^{-1}(\tanh(a+bx))^2+8b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(7/2)*(8*b^2*x^2 - 44*b*x*ArcTanh[Tanh[a + b*x]] + 99*ArcTanh[Tanh[a +
 b*x]]^2))/693
```

fricas [A] time = 0.41, size = 29, normalized size = 0.60

$$\frac{2}{693}\left(63\,b^{2}x^{5}+154\,abx^{4}+99\,a^{2}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
[Out] 2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

giac [A] time = 0.22, size = 24, normalized size = 0.50

$$\frac{2}{11}\,b^{2}x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^{2}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

maple [A] time = 0.25, size = 38, normalized size = 0.79

$$\frac{2x^{\frac{7}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{2}}{7}-\frac{8b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9}-\frac{2bx^{\frac{11}{2}}}{99}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a))^2,x)
[Out] 2/7*x^(7/2)*arctanh(tanh(b*x+a))^2-8/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))-
2/99*b*x^(11/2))
```

maxima [A] time = 0.34, size = 36, normalized size = 0.75

$$\frac{16}{693}\,b^{2}x^{\frac{11}{2}}-\frac{8}{63}\,bx^{\frac{9}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{2}{7}\,x^{\frac{7}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] 16/693*b^2*x^(11/2) - 8/63*b*x^(9/2)*arctanh(tanh(b*x + a)) + 2/7*x^(7/2)*a
rctanh(tanh(b*x + a))^2
```

mupad [B] time = 1.13, size = 122, normalized size = 2.54

$$\frac{x^{7/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{2}}{14}+\frac{2\,b^{2}\,x^{11/2}}{11}-\frac{2\,b\,x^{9/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atanh(tanh(a + b*x))^2,x)
[Out] (x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/14 + (2*b^2*x^(11/2))/11 - (2*b*x^(9/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x))/9
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Timed out
```

3.177 $\int x^{3/2} \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=48

$$-\frac{8}{35} b x^{7/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{5} x^{5/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{16}{315} b^2 x^{9/2}$$

```
[Out] 16/315*b^2*x^(9/2)-8/35*b*x^(7/2)*arctanh(tanh(b*x+a))+2/5*x^(5/2)*arctanh(
tanh(b*x+a))^2
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2168, 30}

$$-\frac{8}{35} b x^{7/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{5} x^{5/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{16}{315} b^2 x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(9/2))/315 - (8*b*x^(7/2)*ArcTanh[Tanh[a + b*x]])/35 + (2*x^(5/2)
*ArcTanh[Tanh[a + b*x]]^2)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^{3/2} \tanh^{-1}(\tanh(a+bx))^2 \, dx &= \frac{2}{5} x^{5/2} \tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{5}(4b) \int x^{5/2} \tanh^{-1}(\tanh(a+bx)) \, dx \\
&= -\frac{8}{35} b x^{7/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{5} x^{5/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{35} \left(8b^2\right) \\
&= \frac{16}{315} b^2 x^{9/2} - \frac{8}{35} b x^{7/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{5} x^{5/2} \tanh^{-1}(\tanh(a+bx))^2
\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.83

$$\frac{2}{315} x^{5/2} \left(-36 b x \tanh^{-1}(\tanh(a+bx)) + 63 \tanh^{-1}(\tanh(a+bx))^2 + 8 b^2 x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(5/2)*(8*b^2*x^2 - 36*b*x*ArcTanh[Tanh[a + b*x]] + 63*ArcTanh[Tanh[a +
 b*x]]^2))/315
```

fricas [A] time = 0.47, size = 29, normalized size = 0.60

$$\frac{2}{315}\left(35\,b^2x^4 + 90\,abx^3 + 63\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

giac [A] time = 0.15, size = 24, normalized size = 0.50

$$\frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{7}\,abx^{\frac{7}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

maple [A] time = 0.25, size = 38, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{5} - \frac{8b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7} - \frac{2b\,x^{\frac{9}{2}}}{63}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/5*x^(5/2)*arctanh(tanh(b*x+a))^2-8/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))-
2/63*b*x^(9/2))
```

maxima [A] time = 0.35, size = 36, normalized size = 0.75

$$\frac{16}{315}\,b^2x^{\frac{9}{2}} - \frac{8}{35}\,bx^{\frac{7}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right) + \frac{2}{5}\,x^{\frac{5}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 16/315*b^2*x^(9/2) - 8/35*b*x^(7/2)*arctanh(tanh(b*x + a)) + 2/5*x^(5/2)*ar
ctanh(tanh(b*x + a))^2
```

mupad [B] time = 1.14, size = 122, normalized size = 2.54

$$\frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{10}+\frac{2\,b^2\,x^{9/2}}{9}-\frac{2\,b\,x^{7/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/10 + (2*b^2*x^(9/2))/9 - (2*b*x^(7/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x))/7
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**(3/2)*atanh(tanh(a + b*x))**2, x)
```

3.178 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^2 \, dx$

Optimal. Leaf size=48

$$-\frac{8}{15} b x^{5/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{3} x^{3/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{16}{105} b^2 x^{7/2}$$

```
[Out] 16/105*b^2*x^(7/2)-8/15*b*x^(5/2)*arctanh(tanh(b*x+a))+2/3*x^(3/2)*arctanh(
tanh(b*x+a))^2
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{2168, 30\}$

$$-\frac{8}{15} b x^{5/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{3} x^{3/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{16}{105} b^2 x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (16*b^2*x^(7/2))/105 - (8*b*x^(5/2)*ArcTanh[Tanh[a + b*x]])/15 + (2*x^(3/2)
*ArcTanh[Tanh[a + b*x]]^2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^2 \, dx &= \frac{2}{3} x^{3/2} \tanh^{-1}(\tanh(a+bx))^2 - \frac{1}{3}(4b) \int x^{3/2} \tanh^{-1}(\tanh(a+bx)) \, dx \\
&= -\frac{8}{15} b x^{5/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{3} x^{3/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{1}{15} \left(8b^2\right) \int \\
&= \frac{16}{105} b^2 x^{7/2} - \frac{8}{15} b x^{5/2} \tanh^{-1}(\tanh(a+bx)) + \frac{2}{3} x^{3/2} \tanh^{-1}(\tanh(a+bx))^2
\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.83

$$\frac{2}{105} x^{3/2} \left(-28 b x \tanh^{-1}(\tanh(a+bx)) + 35 \tanh^{-1}(\tanh(a+bx))^2 + 8 b^2 x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(3/2)*(8*b^2*x^2 - 28*b*x*ArcTanh[Tanh[a + b*x]] + 35*ArcTanh[Tanh[a +
 b*x]]^2))/105
```

fricas [A] time = 0.54, size = 27, normalized size = 0.56

$$\frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

giac [A] time = 0.74, size = 24, normalized size = 0.50

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

maple [A] time = 0.25, size = 38, normalized size = 0.79

$$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{3}-\frac{8b\left(\frac{x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))}{5}-\frac{2b\,x^{\frac{7}{2}}}{35}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2*x^(1/2),x)
```

```
[Out] 2/3*x^(3/2)*arctanh(tanh(b*x+a))^2-8/3*b*(1/5*x^(5/2)*arctanh(tanh(b*x+a))-
2/35*b*x^(7/2))
```

maxima [A] time = 0.34, size = 36, normalized size = 0.75

$$\frac{16}{105}\,b^2x^{\frac{7}{2}}-\frac{8}{15}\,bx^{\frac{5}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)+\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 16/105*b^2*x^(7/2) - 8/15*b*x^(5/2)*arctanh(tanh(b*x + a)) + 2/3*x^(3/2)*ar
ctanh(tanh(b*x + a))^2
```

mupad [B] time = 1.12, size = 122, normalized size = 2.54

$$\frac{x^{3/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{6}+\frac{2\,b^2\,x^{7/2}}{7}-\frac{2\,b\,x^{5/2}\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^2,x)
```

```
[Out] (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/6 + (2*b^2*x^(7/2))/7 - (2*b*x^(5/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\,\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**2, x)
```

3.179 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=46

$$-\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx))+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{15}b^2x^{5/2}$$

```
[Out] 16/15*b^2*x^(5/2)-8/3*b*x^(3/2)*arctanh(tanh(b*x+a))+2*arctanh(tanh(b*x+a))
^2*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx))+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2+\frac{16}{15}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/Sqrt[x],x]
```

```
[Out] (16*b^2*x^(5/2))/15 - (8*b*x^(3/2)*ArcTanh[Tanh[a + b*x]])/3 + 2*Sqrt[x]*Ar
cTanh[Tanh[a + b*x]]^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}\,dx &= 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-(4b)\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))\,dx\\ &= -\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx))+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2+\frac{1}{3}\left(8b^2\right)\int x^{3/2}\\ &= \frac{16}{15}b^2x^{5/2}-\frac{8}{3}bx^{3/2}\tanh^{-1}(\tanh(a+bx))+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.87

$$\frac{2}{15}\sqrt{x}\left(-20bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2+8b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(8*b^2*x^2 - 20*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[a +
 b*x]]^2))/15
```

fricas [A] time = 0.49, size = 24, normalized size = 0.52

$$\frac{2}{15}\left(3\,b^2x^2 + 10\,abx + 15\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

giac [A] time = 0.16, size = 24, normalized size = 0.52

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

maple [A] time = 0.26, size = 47, normalized size = 1.02

$$\frac{2b^2x^{\frac{5}{2}}}{5} + \frac{4\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx\right)b\,x^{\frac{3}{2}}}{3} + 2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx\right)^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(1/2),x)
```

```
[Out] 2/5*b^2*x^(5/2)+4/3*(arctanh(tanh(b*x+a))-b*x)*b*x^(3/2)+2*(arctanh(tanh(b*
x+a))-b*x)^2*x^(1/2)
```

maxima [A] time = 0.34, size = 36, normalized size = 0.78

$$\frac{16}{15}\,b^2x^{\frac{5}{2}} - \frac{8}{3}\,bx^{\frac{3}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right) + 2\sqrt{x}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 16/15*b^2*x^(5/2) - 8/3*b*x^(3/2)*arctanh(tanh(b*x + a)) + 2*sqrt(x)*arctan
h(tanh(b*x + a))^2
```

mupad [B] time = 1.15, size = 122, normalized size = 2.65

$$\frac{\sqrt{x}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{2} + \frac{2\,b^2\,x^{5/2}}{5} - \frac{2\,b\,x^{3/2}\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right) + 2\,b\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^(1/2),x)
```

```
[Out] (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/2 + (2*b^2*x^(5/2))/5 - (2*b*x^(3/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x))/3
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/sqrt(x), x)
```

3.180 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}} \, dx$

Optimal. Leaf size=44

$$8b\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{16}{3}b^2x^{3/2}$$

```
[Out] -16/3*b^2*x^(3/2)-2*arctanh(tanh(b*x+a))^2/x^(1/2)+8*b*arctanh(tanh(b*x+a))
*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$8b\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \frac{16}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(3/2),x]
```

```
[Out] (-16*b^2*x^(3/2))/3 + 8*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]] - (2*ArcTanh[Tanh[
a + b*x]]^2)/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}} \, dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} + (4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{\sqrt{x}} \, dx \\
&= 8b\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}} - \left(8b^2\right)\int \sqrt{x} \, dx \\
&= -\frac{16}{3}b^2x^{3/2} + 8b\sqrt{x}\tanh^{-1}(\tanh(a+bx)) - \frac{2\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.91

$$-\frac{2\left(-12bx\tanh^{-1}(\tanh(a+bx)) + 3\tanh^{-1}(\tanh(a+bx))^2 + 8b^2x^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(3/2),x]
```

```
[Out] (-2*(8*b^2*x^2 - 12*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2
))/(3*Sqrt[x])
```

fricas [A] time = 0.48, size = 23, normalized size = 0.52

$$\frac{2\left(b^2x^2+6\,abx-3\,a^2\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

giac [A] time = 0.21, size = 24, normalized size = 0.55

$$\frac{2}{3}\,b^2x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

maple [A] time = 0.25, size = 37, normalized size = 0.84

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{\sqrt{x}}+8b\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}-\frac{2b\,x^{\frac{3}{2}}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(3/2),x)
```

```
[Out] -2*arctanh(tanh(b*x+a))^2/x^(1/2)+8*b*(arctanh(tanh(b*x+a))*x^(1/2)-2/3*b*x
^(3/2))
```

maxima [A] time = 0.34, size = 36, normalized size = 0.82

$$-\frac{16}{3}\,b^2x^{\frac{3}{2}}+8\,b\sqrt{x}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)-\frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] -16/3*b^2*x^(3/2) + 8*b*sqrt(x)*arctanh(tanh(b*x + a)) - 2*arctanh(tanh(b*x
 + a))^2/sqrt(x)
```

mupad [B] time = 1.16, size = 122, normalized size = 2.77

$$\frac{2\,b^2\,x^{3/2}}{3}-\frac{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2\,\sqrt{x}}-2\,b\,\sqrt{x}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^(3/2),x)
```

```
[Out] (2*b^2*x^(3/2))/3 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(2*x^(1/2)) - 2*b*x^(1/2)*(l
```

```
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^2(\tanh(a+bx))}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(3/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**2/x**(3/2), x)
```

3.181 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}-\frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}}+\frac{16b^2\sqrt{x}}{3}$$

```
[Out] -2/3*arctanh(tanh(b*x+a))^2/x^(3/2)-8/3*b*arctanh(tanh(b*x+a))/x^(1/2)+16/3
*b^2*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}-\frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}}+\frac{16b^2\sqrt{x}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(5/2),x]
```

```
[Out] (16*b^2*Sqrt[x])/3 - (8*b*ArcTanh[Tanh[a + b*x]])/(3*Sqrt[x]) - (2*ArcTanh[
Tanh[a + b*x]]^2)/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}}\,dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}+\frac{1}{3}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{3/2}}\,dx\\
&= -\frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}+\frac{1}{3}\left(8b^2\right)\int \frac{1}{\sqrt{x}}\,dx\\
&= \frac{16b^2\sqrt{x}}{3}-\frac{8b\tanh^{-1}(\tanh(a+bx))}{3\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{3x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.83

$$\frac{2\left(-4bx\tanh^{-1}(\tanh(a+bx))-\tanh^{-1}(\tanh(a+bx))^2+8b^2x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(5/2),x]
```

```
[Out] (2*(8*b^2*x^2 - 4*b*x*ArcTanh[Tanh[a + b*x]] - ArcTanh[Tanh[a + b*x]]^2))/(
3*x^(3/2))
```

fricas [A] time = 0.59, size = 24, normalized size = 0.50

$$\frac{2\left(3\,b^2x^2-6\,abx-a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

giac [A] time = 0.17, size = 23, normalized size = 0.48

$$2\,b^2\sqrt{x}-\frac{2\left(6\,abx+a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

maple [A] time = 0.25, size = 38, normalized size = 0.79

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{3x^{\frac{3}{2}}}+\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{\sqrt{x}}+2b\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(5/2),x)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))^2/x^(3/2)+8/3*b*(-arctanh(tanh(b*x+a))/x^(1/2)+2*
b*x^(1/2))
```

maxima [A] time = 0.34, size = 36, normalized size = 0.75

$$\frac{16}{3}\,b^2\sqrt{x}-\frac{8\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{3\,\sqrt{x}}-\frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 16/3*b^2*sqrt(x) - 8/3*b*arctanh(tanh(b*x + a))/sqrt(x) - 2/3*arctanh(tanh(
b*x + a))^2/x^(3/2)
```

mupad [B] time = 1.13, size = 122, normalized size = 2.54

$$2\,b^2\,\sqrt{x}-\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{6\,x^{3/2}}+\frac{2\,b\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^(5/2),x)
```

```
[Out] 2*b^2*x^(1/2) - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(6*x^(3/2)) + (2*b*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))/x^(1/2)
```

sympy [A] time = 10.50, size = 48, normalized size = 1.00

$$\frac{16b^2\sqrt{x}}{3} - \frac{8b\,\text{atanh}\,(\tanh\,(a+bx))}{3\sqrt{x}} - \frac{2\,\text{atanh}^2\,(\tanh\,(a+bx))}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(5/2),x)
```

```
[Out] 16*b**2*sqrt(x)/3 - 8*b*atanh(tanh(a + b*x))/(3*sqrt(x)) - 2*atanh(tanh(a +
 b*x))**2/(3*x**(3/2))
```

3.182 $\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{7/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}-\frac{16b^2}{15\sqrt{x}}$$

```
[Out] -8/15*b*arctanh(tanh(b*x+a))/x^(3/2)-2/5*arctanh(tanh(b*x+a))^2/x^(5/2)-16/
15*b^2/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2168, 30}

$$-\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}-\frac{16b^2}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^2/x^(7/2),x]
```

```
[Out] (-16*b^2)/(15*Sqrt[x]) - (8*b*ArcTanh[Tanh[a + b*x]])/(15*x^(3/2)) - (2*Arc
Tanh[Tanh[a + b*x]]^2)/(5*x^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{7/2}}\,dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}+\frac{1}{5}(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))}{x^{5/2}}\,dx\\
&= -\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}+\frac{1}{15}\left(8b^2\right)\int \frac{1}{x^{3/2}}\,dx\\
&= -\frac{16b^2}{15\sqrt{x}}-\frac{8b\tanh^{-1}(\tanh(a+bx))}{15x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^2}{5x^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.83

$$-\frac{2\left(4bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2+8b^2x^2\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^2/x^(7/2),x]
```

```
[Out] (-2*(8*b^2*x^2 + 4*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)
)/(15*x^(5/2))
```

fricas [A] time = 0.53, size = 24, normalized size = 0.50

$$-\frac{2\left(15\,b^2x^2+10\,abx+3\,a^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(15*b^2*x^2 + 10*a*b*x + 3*a^2)/x^(5/2)
```

giac [A] time = 0.25, size = 24, normalized size = 0.50

$$-\frac{2\left(15\,b^2x^2+10\,abx+3\,a^2\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*b^2*x^2 + 10*a*b*x + 3*a^2)/x^(5/2)
```

maple [A] time = 0.25, size = 38, normalized size = 0.79

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{5x^{\frac{5}{2}}}+\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{3x^{\frac{3}{2}}}-\frac{2b}{3\sqrt{x}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^2/x^(7/2),x)
```

```
[Out] -2/5*arctanh(tanh(b*x+a))^2/x^(5/2)+8/5*b*(-1/3*arctanh(tanh(b*x+a))/x^(3/2
)-2/3*b/x^(1/2))
```

maxima [A] time = 0.35, size = 36, normalized size = 0.75

$$-\frac{16\,b^2}{15\,\sqrt{x}}-\frac{8\,b\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{15\,x^{\frac{3}{2}}}-\frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] -16/15*b^2/sqrt(x) - 8/15*b*arctanh(tanh(b*x + a))/x^(3/2) - 2/5*arctanh(ta
nh(b*x + a))^2/x^(5/2)
```

mupad [B] time = 1.13, size = 122, normalized size = 2.54

$$\frac{2\,b\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3\,x^{3/2}}-\frac{2\,b^2}{\sqrt{x}}-\frac{{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{10\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^2/x^(7/2),x)
```

```
[Out] (2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*x^(3/2)) - (2*b^2)/x^(1/2) - (log(2/(exp(
```

```
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^2/(10*x^(5/2))
```

sympy [A] time = 154.70, size = 49, normalized size = 1.02

$$-\frac{16b^2}{15\sqrt{x}} - \frac{8b\,\operatorname{atanh}(\tanh(a+bx))}{15x^{\frac{3}{2}}} - \frac{2\,\operatorname{atanh}^2(\tanh(a+bx))}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**2/x**(7/2),x)
```

```
[Out] -16*b**2/(15*sqrt(x)) - 8*b*atanh(tanh(a + b*x))/(15*x**(3/2)) - 2*atanh(ta
nh(a + b*x))**2/(5*x**(5/2))
```

3.183 $\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^3\,dx$

Optimal. Leaf size=69

$$\frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{15/2}}{6435}$$

```
[Out] -32/6435*b^3*x^(15/2)+16/429*b^2*x^(13/2)*arctanh(tanh(b*x+a))-4/33*b*x^(11
/2)*arctanh(tanh(b*x+a))^2+2/9*x^(9/2)*arctanh(tanh(b*x+a))^3
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$\frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{15/2}}{6435}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(15/2))/6435 + (16*b^2*x^(13/2)*ArcTanh[Tanh[a + b*x]])/429 - (4
*b*x^(11/2)*ArcTanh[Tanh[a + b*x]]^2)/33 + (2*x^(9/2)*ArcTanh[Tanh[a + b*x]
]^3)/9
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{1}{3}(2b)\int x^{9/2}\tanh^{-1}(\tanh(a+bx))^2\,dx\\ &= -\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{9}x^{9/2}\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{33}\left(8b\right.\\ &= \frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{9}x^{9}\\ &= -\frac{32b^3x^{15/2}}{6435}+\frac{16}{429}b^2x^{13/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{33}bx^{11/2}\tanh^{-1}(\tanh(a\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.83

$$-\frac{2x^{9/2}\left(-120b^2x^2\tanh^{-1}(\tanh(a+bx))+390bx\tanh^{-1}(\tanh(a+bx))^2-715\tanh^{-1}(\tanh(a+bx))^3+16b^3x\right.}{6435}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*x^(9/2)*(16*b^3*x^3 - 120*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 390*b*x*ArcT
anh[Tanh[a + b*x]]^2 - 715*ArcTanh[Tanh[a + b*x]]^3))/6435
```

fricas [A] time = 0.43, size = 40, normalized size = 0.58

$$\frac{2}{6435}\left(429\,b^3x^7+1485\,ab^2x^6+1755\,a^2bx^5+715\,a^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/6435*(429*b^3*x^7 + 1485*a*b^2*x^6 + 1755*a^2*b*x^5 + 715*a^3*x^4)*sqrt(x
)
```

giac [A] time = 0.42, size = 35, normalized size = 0.51

$$\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{9}\,a^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/15*b^3*x^(15/2) + 6/13*a*b^2*x^(13/2) + 6/11*a^2*b*x^(11/2) + 2/9*a^3*x^(
9/2)
```

maple [A] time = 0.29, size = 56, normalized size = 0.81

$$\frac{2x^{\frac{9}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{9}-\frac{4b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{11}-\frac{4b\left(\frac{x^{\frac{13}{2}}\operatorname{arctanh}(\tanh(bx+a))}{13}-\frac{2x^{\frac{15}{2}}b}{195}\right)}{11}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/9*x^(9/2)*arctanh(tanh(b*x+a))^3-4/3*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a)
)^2-4/11*b*(1/13*x^(13/2)*arctanh(tanh(b*x+a))-2/195*x^(15/2)*b))
```

maxima [A] time = 0.36, size = 55, normalized size = 0.80

$$-\frac{4}{33}\,bx^{\frac{11}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\frac{2}{9}\,x^{\frac{9}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{16}{6435}\left(2\,b^2x^{\frac{15}{2}}-15\,bx^{\frac{13}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -4/33*b*x^(11/2)*arctanh(tanh(b*x + a))^2 + 2/9*x^(9/2)*arctanh(tanh(b*x +
a))^3 - 16/6435*(2*b^2*x^(15/2) - 15*b*x^(13/2)*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 1.17, size = 182, normalized size = 2.64

$$\frac{2\,b^3\,x^{15/2}}{15}-\frac{x^{9/2}\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{36}+\frac{3\,b\,x^{11/2}\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{22}-\frac{3\,b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*b^3*x^(15/2))/15 - (x^(9/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/36 + (3*b*x^(11/
2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^2)/22 - (3*b^2*x^(13/2)*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
))/13
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

3.184 $\int x^{5/2} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$\frac{16}{231} b^2 x^{11/2} \tanh^{-1}(\tanh(a+bx)) - \frac{4}{21} b x^{9/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+bx))^3 - \frac{32 b^3 x^{13/2}}{3003}$$

```
[Out] -32/3003*b^3*x^(13/2)+16/231*b^2*x^(11/2)*arctanh(tanh(b*x+a))-4/21*b*x^(9/
2)*arctanh(tanh(b*x+a))^2+2/7*x^(7/2)*arctanh(tanh(b*x+a))^3
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2168, 30}

$$\frac{16}{231} b^2 x^{11/2} \tanh^{-1}(\tanh(a+bx)) - \frac{4}{21} b x^{9/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+bx))^3 - \frac{32 b^3 x^{13/2}}{3003}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(13/2))/3003 + (16*b^2*x^(11/2)*ArcTanh[Tanh[a + b*x]])/231 - (4
*b*x^(9/2)*ArcTanh[Tanh[a + b*x]]^2)/21 + (2*x^(7/2)*ArcTanh[Tanh[a + b*x]]
^3)/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int x^{5/2} \tanh^{-1}(\tanh(a+bx))^3 \, dx &= \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+bx))^3 - \frac{1}{7}(6b) \int x^{7/2} \tanh^{-1}(\tanh(a+bx))^2 \, dx \\
&= -\frac{4}{21} b x^{9/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7} x^{7/2} \tanh^{-1}(\tanh(a+bx))^3 + \frac{1}{21}\left(8b^2\right) \int \\
&= \frac{16}{231} b^2 x^{11/2} \tanh^{-1}(\tanh(a+bx)) - \frac{4}{21} b x^{9/2} \tanh^{-1}(\tanh(a+bx))^2 + \frac{2}{7} x^{7/2} \mathrm{ta} \\
&= -\frac{32 b^3 x^{13/2}}{3003} + \frac{16}{231} b^2 x^{11/2} \tanh^{-1}(\tanh(a+bx)) - \frac{4}{21} b x^{9/2} \tanh^{-1}(\tanh(a+b
\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.83

$$\frac{2 x^{7/2} \left(104 b^2 x^2 \tanh^{-1}(\tanh(a+bx)) - 286 b x \tanh^{-1}(\tanh(a+bx))^2 + 429 \tanh^{-1}(\tanh(a+bx))^3 - 16 b^3 x^3\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (2*x^(7/2)*(-16*b^3*x^3 + 104*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 286*b*x*ArcT
anh[Tanh[a + b*x]]^2 + 429*ArcTanh[Tanh[a + b*x]]^3))/3003
```

fricas [A] time = 0.49, size = 40, normalized size = 0.58

$$\frac{2}{3003}\left(231\,b^3x^6+819\,ab^2x^5+1001\,a^2bx^4+429\,a^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x)
```

giac [A] time = 0.20, size = 35, normalized size = 0.51

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

maple [A] time = 0.25, size = 56, normalized size = 0.81

$$\frac{2x^{\frac{7}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{7}-\frac{12b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{9}-\frac{4b\left(\frac{x^{\frac{11}{2}}\operatorname{arctanh}(\tanh(bx+a))}{11}-\frac{2x^{\frac{13}{2}}b}{143}\right)}{9}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/7*x^(7/2)*arctanh(tanh(b*x+a))^3-12/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))
^2-4/9*b*(1/11*x^(11/2)*arctanh(tanh(b*x+a))-2/143*x^(13/2)*b))
```

maxima [A] time = 0.35, size = 55, normalized size = 0.80

$$-\frac{4}{21}\,bx^{\frac{9}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\frac{2}{7}\,x^{\frac{7}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{16}{3003}\left(2\,b^2x^{\frac{13}{2}}-13\,bx^{\frac{11}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -4/21*b*x^(9/2)*arctanh(tanh(b*x + a))^2 + 2/7*x^(7/2)*arctanh(tanh(b*x + a
))^3 - 16/3003*(2*b^2*x^(13/2) - 13*b*x^(11/2)*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 1.18, size = 182, normalized size = 2.64

$$\frac{2\,b^3\,x^{13/2}}{13}-\frac{x^{7/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{28}+\frac{b\,x^{9/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{6}-\frac{3\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*b^3*x^(13/2))/13 - (x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/28 + (b*x^(9/2)*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2)/6 - (3*b^2*x^(11/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/1
1
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

3.185 $\int x^{3/2} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$\frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{11/2}}{1155}$$

```
[Out] -32/1155*b^3*x^(11/2)+16/105*b^2*x^(9/2)*arctanh(tanh(b*x+a))-12/35*b*x^(7/
2)*arctanh(tanh(b*x+a))^2+2/5*x^(5/2)*arctanh(tanh(b*x+a))^3
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$\frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32b^3x^{11/2}}{1155}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(11/2))/1155 + (16*b^2*x^(9/2)*ArcTanh[Tanh[a + b*x]])/105 - (12
*b*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2)/35 + (2*x^(5/2)*ArcTanh[Tanh[a + b*x]]
^3)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{1}{5}(6b)\int x^{5/2}\tanh^{-1}(\tanh(a+bx))^2\,dx\\ &= -\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{35}\left(24b\right.\\ &= \frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{5}x^{5/2}\\ &= -\frac{32b^3x^{11/2}}{1155}+\frac{16}{105}b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))-\frac{12}{35}bx^{7/2}\tanh^{-1}(\tanh(a+\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.83

$$-\frac{2x^{5/2}\left(-88b^2x^2\tanh^{-1}(\tanh(a+bx))+198bx\tanh^{-1}(\tanh(a+bx))^2-231\tanh^{-1}(\tanh(a+bx))^3+16b^3x^3\right.}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*x^(5/2)*(16*b^3*x^3 - 88*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 198*b*x*ArcTa
nh[Tanh[a + b*x]]^2 - 231*ArcTanh[Tanh[a + b*x]]^3))/1155
```

fricas [A] time = 0.62, size = 40, normalized size = 0.58

$$\frac{2}{1155}\left(105\,b^3x^5 + 385\,ab^2x^4 + 495\,a^2bx^3 + 231\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

giac [A] time = 0.18, size = 35, normalized size = 0.51

$$\frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

maple [A] time = 0.25, size = 56, normalized size = 0.81

$$\frac{2x^{\frac{5}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{5} - \frac{12b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{7} - \frac{4b\left(\frac{x^{\frac{9}{2}}\operatorname{arctanh}(\tanh(bx+a))}{9} - \frac{2bx^{\frac{11}{2}}}{99}\right)}{7}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/5*x^(5/2)*arctanh(tanh(b*x+a))^3-12/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))
^2-4/7*b*(1/9*x^(9/2)*arctanh(tanh(b*x+a))-2/99*b*x^(11/2)))
```

maxima [A] time = 0.35, size = 55, normalized size = 0.80

$$-\frac{12}{35}\,bx^{\frac{7}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2+\frac{2}{5}\,x^{\frac{5}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3-\frac{16}{1155}\left(2\,b^2x^{\frac{11}{2}} - 11\,bx^{\frac{9}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -12/35*b*x^(7/2)*arctanh(tanh(b*x + a))^2 + 2/5*x^(5/2)*arctanh(tanh(b*x +
a))^3 - 16/1155*(2*b^2*x^(11/2) - 11*b*x^(9/2)*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 1.17, size = 182, normalized size = 2.64

$$\frac{2\,b^3\,x^{11/2}}{11}-\frac{x^{5/2}\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{20}+\frac{3\,b\,x^{7/2}\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{14}-\frac{b^2\,x^9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*b^3*x^(11/2))/11 - (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/20 + (3*b*x^(7/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2)/14 - (b^2*x^(9/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \operatorname{atanh}^3(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**(3/2)*atanh(tanh(a + b*x))**3, x)
```

3.186 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^3 \, dx$

Optimal. Leaf size=69

$$\frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32}{315}b^3x^{9/2}$$

```
[Out] -32/315*b^3*x^(9/2)+16/35*b^2*x^(7/2)*arctanh(tanh(b*x+a))-4/5*b*x^(5/2)*ar
ctanh(tanh(b*x+a))^2+2/3*x^(3/2)*arctanh(tanh(b*x+a))^3
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$\frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3-\frac{32}{315}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-32*b^3*x^(9/2))/315 + (16*b^2*x^(7/2)*ArcTanh[Tanh[a + b*x]])/35 - (4*b*x
^(5/2)*ArcTanh[Tanh[a + b*x]]^2)/5 + (2*x^(3/2)*ArcTanh[Tanh[a + b*x]]^3)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^3\,dx &= \frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3-(2b)\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^2\,dx\\ &= -\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^3+\frac{1}{5}\left(8b^2\right)\int x\\ &= \frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2+\frac{2}{3}x^{3/2}\tanh\\ &= -\frac{32}{315}b^3x^{9/2}+\frac{16}{35}b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))-\frac{4}{5}bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.83

$$-\frac{2}{315}x^{3/2}\left(-72b^2x^2\tanh^{-1}(\tanh(a+bx))+126bx\tanh^{-1}(\tanh(a+bx))^2-105\tanh^{-1}(\tanh(a+bx))^3+16b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3,x]
```

[Out] (-2*x^(3/2)*(16*b^3*x^3 - 72*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 126*b*x*ArcTanh[Tanh[a + b*x]]^2 - 105*ArcTanh[Tanh[a + b*x]]^3))/315

fricas [A] time = 0.68, size = 38, normalized size = 0.55

$$\frac{2}{315}\left(35\,b^3x^4+135\,ab^2x^3+189\,a^2bx^2+105\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="fricas")

[Out] 2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)

giac [A] time = 0.16, size = 35, normalized size = 0.51

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="giac")

[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)

maple [A] time = 0.26, size = 56, normalized size = 0.81

$$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^3}{3}-4b\left(\frac{x^{\frac{5}{2}}\operatorname{arctanh}(\tanh(bx+a))^2}{5}-\frac{4b\left(\frac{x^{\frac{7}{2}}\operatorname{arctanh}(\tanh(bx+a))}{7}-\frac{2bx^{\frac{9}{2}}}{63}\right)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(tanh(b*x+a))^3*x^(1/2),x)

[Out] 2/3*x^(3/2)*arctanh(tanh(b*x+a))^3-4*b*(1/5*x^(5/2)*arctanh(tanh(b*x+a))^2-4/5*b*(1/7*x^(7/2)*arctanh(tanh(b*x+a))-2/63*b*x^(9/2)))

maxima [A] time = 0.36, size = 55, normalized size = 0.80

$$-\frac{4}{5}\,bx^{\frac{5}{2}}\operatorname{artanh}(\tanh(bx+a))^2+\frac{2}{3}\,x^{\frac{3}{2}}\operatorname{artanh}(\tanh(bx+a))^3-\frac{16}{315}\left(2\,b^2x^{\frac{9}{2}}-9\,bx^{\frac{7}{2}}\operatorname{artanh}(\tanh(bx+a))\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tanh(b*x+a))^3*x^(1/2),x, algorithm="maxima")

[Out] -4/5*b*x^(5/2)*arctanh(tanh(b*x + a))^2 + 2/3*x^(3/2)*arctanh(tanh(b*x + a))^3 - 16/315*(2*b^2*x^(9/2) - 9*b*x^(7/2)*arctanh(tanh(b*x + a)))*b

mupad [B] time = 1.16, size = 182, normalized size = 2.64

$$\frac{2\,b^3\,x^{9/2}}{9}-\frac{x^{3/2}\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{12}+\frac{3\,b\,x^{5/2}\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{10}-\frac{3\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)*atanh(tanh(a + b*x))^3,x)

[Out] (2*b^3*x^(9/2))/9 - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/12 + (3*b*x^(5/2)*

```
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2)/10 - (3*b^2*x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/7
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \operatorname{atanh}^3(\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**3, x)
```

3.187 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}} dx$

Optimal. Leaf size=65

$$\frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))-4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3-\frac{32}{35}b^3x^{7/2}$$

```
[Out] -32/35*b^3*x^(7/2)+16/5*b^2*x^(5/2)*arctanh(tanh(b*x+a))-4*b*x^(3/2)*arctan
h(tanh(b*x+a))^2+2*arctanh(tanh(b*x+a))^3*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2168, 30}

$$\frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))-4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3-\frac{32}{35}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/Sqrt[x],x]
```

```
[Out] (-32*b^3*x^(7/2))/35 + (16*b^2*x^(5/2)*ArcTanh[Tanh[a + b*x]])/5 - 4*b*x^(3
/2)*ArcTanh[Tanh[a + b*x]]^2 + 2*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}\,dx &= 2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3-(6b)\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^2\,dx\\
&= -4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2+2\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3+\left(8b^2\right)\int x^{3/2}\,\mathrm{t}\\
&= \frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))-4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2+2\sqrt{x}\tanh^{-1}\\
&= -\frac{32}{35}b^3x^{7/2}+\frac{16}{5}b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))-4bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2+
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.88

$$\frac{2}{35}\sqrt{x}\left(56b^2x^2\tanh^{-1}(\tanh(a+bx))-70bx\tanh^{-1}(\tanh(a+bx))^2+35\tanh^{-1}(\tanh(a+bx))^3-16b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(-16*b^3*x^3 + 56*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 70*b*x*ArcTan
h[Tanh[a + b*x]]^2 + 35*ArcTanh[Tanh[a + b*x]]^3))/35
```

fricas [A] time = 0.53, size = 35, normalized size = 0.54

$$\frac{2}{35}\left(5\,b^{3}x^{3}+21\,ab^{2}x^{2}+35\,a^{2}bx+35\,a^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

giac [A] time = 0.29, size = 35, normalized size = 0.54

$$\frac{2}{7}\,b^{3}x^{\frac{7}{2}}+\frac{6}{5}\,ab^{2}x^{\frac{5}{2}}+2\,a^{2}bx^{\frac{3}{2}}+2\,a^{3}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

maple [A] time = 0.25, size = 69, normalized size = 1.06

$$\frac{2b^{3}x^{\frac{7}{2}}}{7}+\frac{6\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b^{2}x^{\frac{5}{2}}}{5}+2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}b\,x^{\frac{3}{2}}+2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(1/2),x)
```

```
[Out] 2/7*b^3*x^(7/2)+6/5*(arctanh(tanh(b*x+a))-b*x)*b^2*x^(5/2)+2*(arctanh(tanh(
b*x+a))-b*x)^2*b*x^(3/2)+2*(arctanh(tanh(b*x+a))-b*x)^3*x^(1/2)
```

maxima [A] time = 0.36, size = 55, normalized size = 0.85

$$-4\,bx^{\frac{3}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}+2\,\sqrt{x}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}-\frac{16}{35}\left(2\,b^{2}x^{\frac{7}{2}}-7\,bx^{\frac{5}{2}}\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -4*b*x^(3/2)*arctanh(tanh(b*x + a))^2 + 2*sqrt(x)*arctanh(tanh(b*x + a))^3
- 16/35*(2*b^2*x^(7/2) - 7*b*x^(5/2)*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 1.19, size = 182, normalized size = 2.80

$$\frac{2\,b^{3}\,x^{7/2}}{7}-\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^{3}}{4}+\frac{b\,x^{3/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^{2}}{2}-\frac{3\,b^{2}\,x^{5/}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^(1/2),x)
```

```
[Out] (2*b^3*x^(7/2))/7 - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/4 + (b*x^(3/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
```

```
2*b*x) + 1)) + 2*b*x)^2)/2 - (3*b^2*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/5
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/sqrt(x), x)
```

3.188 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{3/2}}\,dx$

Optimal. Leaf size=63

$$-16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))+12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}+\frac{32}{5}b^3x^{5/2}$$

```
[Out] 32/5*b^3*x^(5/2)-16*b^2*x^(3/2)*arctanh(tanh(b*x+a))-2*arctanh(tanh(b*x+a))
^3/x^(1/2)+12*b*arctanh(tanh(b*x+a))^2*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$-16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))+12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}+\frac{32}{5}b^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(3/2),x]
```

```
[Out] (32*b^3*x^(5/2))/5 - 16*b^2*x^(3/2)*ArcTanh[Tanh[a + b*x]] + 12*b*Sqrt[x]*A
rcTanh[Tanh[a + b*x]]^2 - (2*ArcTanh[Tanh[a + b*x]]^3)/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{3/2}}\,dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}+(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}\,dx\\ &= 12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{\sqrt{x}}-\left(24b^2\right)\int \sqrt{x}\tanh^{-1}\\ &= -16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))+12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\tanh^{-1}(\tan}{\sqrt{}}\\ &= \frac{32}{5}b^3x^{5/2}-16b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))+12b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-\frac{2\,\mathrm{ta}}{}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.90

$$\frac{2\left(-40b^2x^2\tanh^{-1}(\tanh(a+bx))+30bx\tanh^{-1}(\tanh(a+bx))^2-5\tanh^{-1}(\tanh(a+bx))^3+16b^3x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(3/2),x]
```

```
[Out] (2*(16*b^3*x^3 - 40*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 30*b*x*ArcTanh[Tanh[a
+ b*x]]^2 - 5*ArcTanh[Tanh[a + b*x]]^3))/(5*Sqrt[x])
```

fricas [A] time = 0.63, size = 34, normalized size = 0.54

$$\frac{2\left(b^3x^3 + 5\,ab^2x^2 + 15\,a^2bx - 5\,a^3\right)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

giac [A] time = 0.42, size = 35, normalized size = 0.56

$$\frac{2}{5}\,b^3x^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

maple [A] time = 0.25, size = 64, normalized size = 1.02

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{\sqrt{x}}+12b\left(\frac{b^2x^{\frac{5}{2}}}{5}+\frac{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b\,x^{\frac{3}{2}}}{3}+\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(3/2),x)
```

```
[Out] -2*arctanh(tanh(b*x+a))^3/x^(1/2)+12*b*(1/5*b^2*x^(5/2)+2/3*(arctanh(tanh(b
*x+a))-b*x)*b*x^(3/2)+(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2))
```

maxima [A] time = 0.35, size = 55, normalized size = 0.87

$$12\,b\sqrt{x}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^2-\frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^3}{\sqrt{x}}+\frac{16}{5}\left(2\,b^2x^{\frac{5}{2}}-5\,bx^{\frac{3}{2}}\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 12*b*sqrt(x)*arctanh(tanh(b*x + a))^2 - 2*arctanh(tanh(b*x + a))^3/sqrt(x)
+ 16/5*(2*b^2*x^(5/2) - 5*b*x^(3/2)*arctanh(tanh(b*x + a)))*b
```

mupad [B] time = 1.21, size = 182, normalized size = 2.89

$$\frac{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,\sqrt{x}}+\frac{2\,b^3\,x^{5/2}}{5}+\frac{3\,b\,\sqrt{x}\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2}-b^2\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^(3/2),x)
```

```
[Out] (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^3/(4*x^(1/2)) + (2*b^3*x^(5/2))/5 + (3*b*x^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2)/2 - b^2*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^3(\tanh(a+bx))}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(3/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**3/x**(3/2), x)
```

3.189 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{5/2}}\,dx$

Optimal. Leaf size=65

$$16b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}}-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}-\frac{32}{3}b^3x^{3/2}$$

```
[Out] -32/3*b^3*x^(3/2)-2/3*arctanh(tanh(b*x+a))^3/x^(3/2)-4*b*arctanh(tanh(b*x+a
))^2/x^(1/2)+16*b^2*arctanh(tanh(b*x+a))*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2168, 30}

$$16b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}}-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}-\frac{32}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(5/2),x]
```

```
[Out] (-32*b^3*x^(3/2))/3 + 16*b^2*Sqrt[x]*ArcTanh[Tanh[a + b*x]] - (4*b*ArcTanh[
Tanh[a + b*x]]^2)/Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]]^3)/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{5/2}}\,dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}}+(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{3/2}}\,dx\\ &= -\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))^3}{3x^{3/2}}+\left(8b^2\right)\int \frac{\tanh^{-1}(\tan\\ &= 16b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(\\ &= -\frac{32}{3}b^3x^{3/2}+16b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))-\frac{4b\tanh^{-1}(\tanh(a+bx))^2}{\sqrt{x}}-2\,t\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.85

$$-\frac{2\left(-24b^2x^2\tanh^{-1}(\tanh(a+bx))+6bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3+16b^3x^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(5/2),x]
```

```
[Out] (-2*(16*b^3*x^3 - 24*b^2*x^2*ArcTanh[Tanh[a + b*x]] + 6*b*x*ArcTanh[Tanh[a
+ b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(3*x^(3/2))
```

fricas [A] time = 0.52, size = 34, normalized size = 0.52

$$\frac{2\left(b^3x^3 + 9\,ab^2x^2 - 9\,a^2bx - a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

giac [A] time = 0.16, size = 34, normalized size = 0.52

$$\frac{2}{3}\,b^3x^{\frac{3}{2}} + 6\,ab^2\sqrt{x} - \frac{2\left(9\,a^2bx + a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

maple [A] time = 0.26, size = 55, normalized size = 0.85

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^3}{3x^{\frac{3}{2}}}+4b\left(-\frac{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^2}{\sqrt{x}}+4b\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)\sqrt{x}-\frac{2b\,x^{\frac{3}{2}}}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(5/2),x)
```

```
[Out] -2/3*arctanh(tanh(b*x+a))^3/x^(3/2)+4*b*(-arctanh(tanh(b*x+a))^2/x^(1/2)+4*
b*(arctanh(tanh(b*x+a))*x^(1/2)-2/3*b*x^(3/2)))
```

maxima [A] time = 0.35, size = 55, normalized size = 0.85

$$-\frac{4\,b\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^2}{\sqrt{x}}-\frac{16}{3}\left(2\,b^2x^{\frac{3}{2}}-3\,b\sqrt{x}\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)\right)b-\frac{2\,\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] -4*b*arctanh(tanh(b*x + a))^2/sqrt(x) - 16/3*(2*b^2*x^(3/2) - 3*b*sqrt(x)*a
rctanh(tanh(b*x + a)))*b - 2/3*arctanh(tanh(b*x + a))^3/x^(3/2)
```

mupad [B] time = 1.22, size = 182, normalized size = 2.80

$$\frac{\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^3}{12\,x^{3/2}}+\frac{2\,b^3\,x^{3/2}}{3}-\frac{3\,b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)^2}{2\,\sqrt{x}}-3\,b^2\,\sqrt{x}\,\left(\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^(5/2),x)
```

```
[Out] (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^3/(12*x^(3/2)) + (2*b^3*x^(3/2))/3 - (3*b*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^2)/(2*x^(1/2)) - 3*b^2*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
```

sympy [**A**] time = 10.58, size = 66, normalized size = 1.02

$$-\frac{32b^3x^{\frac{3}{2}}}{3}+16b^2\sqrt{x}\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)-\frac{4b\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}-\frac{2\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(5/2),x)
```

```
[Out] -32*b**3*x**(3/2)/3 + 16*b**2*sqrt(x)*atanh(tanh(a + b*x)) - 4*b*atanh(tanh
(a + b*x))**2/sqrt(x) - 2*atanh(tanh(a + b*x))**3/(3*x**(3/2))
```

3.190 $\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{7/2}} \, dx$

Optimal. Leaf size=69

$$-\frac{16b^2 \tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b \tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}} + \frac{32b^3\sqrt{x}}{5}$$

```
[Out] -4/5*b*arctanh(tanh(b*x+a))^2/x^(3/2)-2/5*arctanh(tanh(b*x+a))^3/x^(5/2)-16
/5*b^2*arctanh(tanh(b*x+a))/x^(1/2)+32/5*b^3*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2168, 30}

$$-\frac{16b^2 \tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b \tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}} + \frac{32b^3\sqrt{x}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^3/x^(7/2),x]
```

```
[Out] (32*b^3*Sqrt[x])/5 - (16*b^2*ArcTanh[Tanh[a + b*x]])/(5*Sqrt[x]) - (4*b*Arc
Tanh[Tanh[a + b*x]]^2)/(5*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]]^3)/(5*x^(5/2
))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^3}{x^{7/2}} \, dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}} + \frac{1}{5}(6b) \int \frac{\tanh^{-1}(\tanh(a+bx))^2}{x^{5/2}} \, dx \\
&= -\frac{4b \tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+bx))^3}{5x^{5/2}} + \frac{1}{5}\left(8b^2\right) \int \frac{\tanh^{-1}(\tan}{x^{3/}} \\
&= -\frac{16b^2 \tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b \tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(\tanh(a+b}{5x^{5/2}} \\
&= \frac{32b^3\sqrt{x}}{5} - \frac{16b^2 \tanh^{-1}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b \tanh^{-1}(\tanh(a+bx))^2}{5x^{3/2}} - \frac{2\tanh^{-1}(t}{5}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.83

$$\frac{2\left(-8b^2x^2 \tanh^{-1}(\tanh(a+bx)) - 2bx \tanh^{-1}(\tanh(a+bx))^2 - \tanh^{-1}(\tanh(a+bx))^3 + 16b^3x^3\right)}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^3/x^(7/2),x]
[Out] (2*(16*b^3*x^3 - 8*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 2*b*x*ArcTanh[Tanh[a + b*x]]^2 - ArcTanh[Tanh[a + b*x]]^3))/(5*x^(5/2))
```

fricas [A] time = 0.54, size = 35, normalized size = 0.51

$$\frac{2\left(5\,b^{3}x^{3}-15\,ab^{2}x^{2}-5\,a^{2}bx-a^{3}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="fricas")
[Out] 2/5*(5*b^3*x^3 - 15*a*b^2*x^2 - 5*a^2*b*x - a^3)/x^(5/2)
```

giac [A] time = 0.17, size = 34, normalized size = 0.49

$$2\,b^{3}\sqrt{x}-\frac{2\left(15\,ab^{2}x^{2}+5\,a^{2}bx+a^{3}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="giac")
[Out] 2*b^3*sqrt(x) - 2/5*(15*a*b^2*x^2 + 5*a^2*b*x + a^3)/x^(5/2)
```

maple [A] time = 0.26, size = 56, normalized size = 0.81

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{3}}{5x^{\frac{5}{2}}}+\frac{12b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{2}}{3x^{\frac{3}{2}}}+\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))}{\sqrt{x}}+2b\sqrt{x}\right)}{3}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^3/x^(7/2),x)
[Out] -2/5*arctanh(tanh(b*x+a))^3/x^(5/2)+12/5*b*(-1/3*arctanh(tanh(b*x+a))^2/x^(3/2)+4/3*b*(-arctanh(tanh(b*x+a))/x^(1/2)+2*b*x^(1/2)))
```

maxima [A] time = 0.36, size = 55, normalized size = 0.80

$$\frac{16}{5}\left(2\,b^{2}\sqrt{x}-\frac{b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}{\sqrt{x}}\right)b-\frac{4\,b\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{2}}{5\,x^{\frac{3}{2}}}-\frac{2\,\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{3}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^3/x^(7/2),x, algorithm="maxima")
[Out] 16/5*(2*b^2*sqrt(x) - b*arctanh(tanh(b*x + a))/sqrt(x))*b - 4/5*b*arctanh(tanh(b*x + a))^2/x^(3/2) - 2/5*arctanh(tanh(b*x + a))^3/x^(5/2)
```

mupad [B] time = 1.18, size = 182, normalized size = 2.64

$$\frac{\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)^{3}}{20\,x^{5/2}}+2\,b^{3}\,\sqrt{x}+\frac{3\,b^{2}\,\left(\ln\left(\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{\sqrt{x}}-\frac{b\,\left(\ln\left(\frac{\ }{\mathrm{e}^{2\,a}\,\mathrm{e}\ldots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^3/x^(7/2),x)
```

```
[Out] (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^3/(20*x^(5/2)) + 2*b^3*x^(1/2) + (3*b^2*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x))/x^(1/2) - (b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*x^(3/2))
```

sympy [**A**] time = 155.33, size = 70, normalized size = 1.01

$$\frac{32b^3\sqrt{x}}{5} - \frac{16b^2\operatorname{atanh}(\tanh(a+bx))}{5\sqrt{x}} - \frac{4b\operatorname{atanh}^2(\tanh(a+bx))}{5x^{\frac{3}{2}}} - \frac{2\operatorname{atanh}^3(\tanh(a+bx))}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**3/x**(7/2),x)
```

```
[Out] 32*b**3*sqrt(x)/5 - 16*b**2*atanh(tanh(a + b*x))/(5*sqrt(x)) - 4*b*atanh(ta
nh(a + b*x))**2/(5*x**(3/2)) - 2*atanh(tanh(a + b*x))**3/(5*x**(5/2))
```

3.191 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=143

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}{b^{9/2}}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{b^4}+\frac{2x^{3/2}}{}$$

```
[Out] 2/7*x^(7/2)/b+2/5*x^(5/2)*(b*x-arctanh(tanh(b*x+a)))/b^2+2/3*x^(3/2)*(b*x-a
rctanh(tanh(b*x+a)))^2/b^3-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+
a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(7/2)/b^(9/2)+2*(b*x-arctanh(tanh(b*
x+a)))^3*x^(1/2)/b^4
```

Rubi [A] time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2162}

$$\frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{b^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(7/2))/(7*b) + (2*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(5*b^2) + (2
*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2)/(3*b^3) + (2*Sqrt[x]*(b*x - ArcT
anh[Tanh[a + b*x]])^3)/b^4 - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTan
h[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2))/b^(9/2)
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2x^{7/2}}{7b} - \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{b}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\int \frac{}{\text{ta}}}{b^2}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3}$$

$$= \frac{2x^{7/2}}{7b} + \frac{2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{5b^2} + \frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{3b^3}$$

Mathematica [A] time = 0.07, size = 129, normalized size = 0.90

$$\frac{2\left(-406b^{5/2}x^{5/2}\tanh^{-1}(\tanh(a+bx)) + 350b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2 - 105\sqrt{b}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 + \right.}{105b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*(176*b^(7/2)*x^(7/2) - 406*b^(5/2)*x^(5/2)*ArcTanh[Tanh[a + b*x]] + 350*
b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2 - 105*Sqrt[b]*Sqrt[x]*ArcTanh[Tanh
[a + b*x]]^3 + 105*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)))/(105*b^(9/2))
```

fricas [A] time = 0.50, size = 153, normalized size = 1.07

$$\left[\frac{105\,a^3\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right) + 2\left(15\,b^3x^3 - 21\,ab^2x^2 + 35\,a^2bx - 105\,a^3\right)\sqrt{x}}{105\,b^4}, \frac{2\left(105\,a^3\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/105*(105*a^3*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)
) + 2*(15*b^3*x^3 - 21*a*b^2*x^2 + 35*a^2*b*x - 105*a^3)*sqrt(x))/b^4, 2/10
5*(105*a^3*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (15*b^3*x^3 - 21*a*b^2
*x^2 + 35*a^2*b*x - 105*a^3)*sqrt(x))/b^4]
```

giac [A] time = 0.14, size = 70, normalized size = 0.49

$$\frac{2\,a^4\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{2\left(15\,b^6x^{\frac{7}{2}} - 21\,ab^5x^{\frac{5}{2}} + 35\,a^2b^4x^{\frac{3}{2}} - 105\,a^3b^3\sqrt{x}\right)}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2*a^4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(15*b^6*x^(7/2) -
 21*a*b^5*x^(5/2) + 35*a^2*b^4*x^(3/2) - 105*a^3*b^3*sqrt(x))/b^7
```

maple [B] time = 0.27, size = 481, normalized size = 3.36

$$\frac{2x^{\frac{7}{2}}}{7b}-\frac{2x^{\frac{5}{2}}a}{5b^2}-\frac{2x^{\frac{5}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{5b^2}+\frac{2a^2x^{\frac{3}{2}}}{3b^3}+\frac{4x^{\frac{3}{2}}a(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{3b^3}+\frac{2x^{\frac{3}{2}}(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2/7*x^(7/2)/b-2/5/b^2*x^(5/2)*a-2/5/b^2*x^(5/2)*(arctanh(tanh(b*x+a))-b*x-a
)+2/3/b^3*a^2*x^(3/2)+4/3/b^3*x^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a)+2/3/b^
3*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-2/b^4*x^(1/2)*a^3-6/b^4*a^2*(arcta
nh(tanh(b*x+a))-b*x-a)*x^(1/2)-6/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/
2)-2/b^4*(arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)+2/b^4/((arctanh(tanh(b*x+a)
)-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^4+
8/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2))*a^3*(arctanh(tanh(b*x+a))-b*x-a)+12/b^4/((arctanh(ta
nh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1
/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)^2+8/b^4/((arctanh(tanh(b*x+a))-b*x)*b
)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(t
anh(b*x+a))-b*x-a)^3+2/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^
(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^4
```

maxima [A] time = 0.43, size = 65, normalized size = 0.45

$$\frac{2\,a^4\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}+\frac{2\left(15\,b^3x^{\frac{7}{2}}-21\,ab^2x^{\frac{5}{2}}+35\,a^2bx^{\frac{3}{2}}-105\,a^3\sqrt{x}\right)}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*a^4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/105*(15*b^3*x^(7/2) -
 21*a*b^2*x^(5/2) + 35*a^2*b*x^(3/2) - 105*a^3*sqrt(x))/b^4
```

mupad [B] time = 1.72, size = 475, normalized size = 3.32

$$\frac{2\,x^{7/2}}{7\,b}+\frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{5\,b^2}+\frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{6\,b^3}+\frac{\sqrt{x}\,\Big(\ln\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x)),x)
```

```
[Out] (2*x^(7/2))/(7*b) + (x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(5*b^2) + (x^(3/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2)/(6*b^3) + (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(
4*b^4) + (2^(1/2)*log((64*b^(19/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^
(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) +
```

```
 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2))/(16*
b^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\operatorname{atanh}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**(7/2)/atanh(tanh(a + b*x)), x)
```

3.192 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=116

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}{b^{7/2}}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}+\frac{2x^{3/2}}{}$$

```
[Out] 2/5*x^(5/2)/b+2/3*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))/b^2-2*arctanh(b^(1/2)*
x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(5/2)/
b^(7/2)+2*(b*x-arctanh(tanh(b*x+a)))^2*x^(1/2)/b^3
```

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2159, 2162}

$$\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^3}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)(bx}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(5/2))/(5*b) + (2*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(3*b^2) + (2
*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/b^3 - (2*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)
)/b^(7/2)
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))} \, dx = \frac{2x^{5/2}}{5b}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))} \, dx}{b}$$

$$=\frac{2x^{5/2}}{5b}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}+\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2 \int}{b^2}$$

$$=\frac{2x^{5/2}}{5b}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}$$

$$=\frac{2x^{5/2}}{5b}+\frac{2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^2}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}$$

Mathematica [A] time = 0.13, size = 108, normalized size = 0.93

$$\frac{2\left(-35b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx))+15\sqrt{b}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2-15\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}\tan\right.}{15b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*(23*b^(5/2)*x^(5/2) - 35*b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]] + 15*Sqr
t[b]*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2 - 15*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(
b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2)))/(
15*b^(7/2))
```

fricas [A] time = 0.48, size = 132, normalized size = 1.14

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(3\,b^2x^2-5\,abx+15\,a^2\right)\sqrt{x}}{15\,b^3},-\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\left(3\,b^2x^2-5\,ab\right.\right.}{15\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

giac [A] time = 0.15, size = 59, normalized size = 0.51

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^4x^{\frac{5}{2}}-5\,ab^3x^{\frac{3}{2}}+15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

maple [B] time = 0.27, size = 330, normalized size = 2.84

$$\frac{2x^{\frac{5}{2}}}{5b}-\frac{2x^{\frac{3}{2}}a}{3b^2}-\frac{2x^{\frac{3}{2}}\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{3b^2}+\frac{2a^2\sqrt{x}}{b^3}+\frac{4a\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\sqrt{x}}{b^3}+\frac{2\,(\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2/5*x^(5/2)/b-2/3/b^2*x^(3/2)*a-2/3/b^2*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a
)+2/b^3*a^2*x^(1/2)+4/b^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+2/b^3*(arc
tanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)-2/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/
2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^3-6/b^3/((arcta
nh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*
b)^(1/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)-6/b^3/((arctanh(tanh(b*x+a))-b*x
)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctan
h(tanh(b*x+a))-b*x-a)^2-2/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b
```

```
*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)
^3
```

maxima [A] time = 0.42, size = 54, normalized size = 0.47

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}-5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

mupad [B] time = 1.39, size = 415, normalized size = 3.58

$$\frac{2\,x^{5/2}}{5\,b}+\frac{x^{3/2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3\,b^2}+\frac{\sqrt{x}\,{\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{2\,b^3}+\frac{\sqrt{2}\,\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x)),x)
```

```
[Out] (2*x^(5/2))/(5*b) + (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*b^2) + (x^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3) + (2^(1/2)*log((16*b^(15/2)*(2^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) +
2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) -
log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^(5/2))/(8*b^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{5}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**(5/2)/atanh(tanh(a + b*x)), x)
```

3.193 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=89

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{5/2}}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{2x^{3/2}}{3b}$$

```
[Out] 2/3*x^(3/2)/b-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(
b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(5/2)+2*(b*x-arctanh(tanh(b*x+a)))*x^(1/2
)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2162}

$$\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{5/2}}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(3/2))/(3*b) + (2*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/b^2 - (2*Arc
Tanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^(3/2))/b^(5/2)
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))} \, dx = \frac{2x^{3/2}}{3b}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))} \, dx}{b}$$

$$= \frac{2x^{3/2}}{3b}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}+\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)^2\int \frac{}{\sqrt{}}}{b^2}$$

$$= \frac{2x^{3/2}}{3b}+\frac{2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^2}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)(bx}{b^{5/2}}$$

Mathematica [A] time = 0.09, size = 86, normalized size = 0.97

$$\frac{2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*x^(3/2))/(3*b) - (2*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^2 + (2*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + A
rcTanh[Tanh[a + b*x]])^(3/2))/b^(5/2)
```

fricas [A] time = 0.39, size = 103, normalized size = 1.16

$$\left[\frac{3\,a\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

giac [A] time = 0.18, size = 45, normalized size = 0.51

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(b^2x^{\frac{3}{2}}-3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

maple [B] time = 0.27, size = 207, normalized size = 2.33

$$\frac{2x^{\frac{3}{2}}}{3b}-\frac{2a\sqrt{x}}{b^2}-\frac{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\sqrt{x}}{b^2}+\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)a^2}{b^2\sqrt{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}+\frac{4\arctan\left(\frac{}{\sqrt{(\mathrm{arc}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2/3*x^(3/2)/b-2/b^2*a*x^(1/2)-2/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+2/
b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*
x+a))-b*x)*b)^(1/2))*a^2+4/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(
b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(tanh(b*x+a))-b*x
-a)+2/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(t
anh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

maxima [A] time = 0.42, size = 42, normalized size = 0.47

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}-3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*sq
rt(x))/b^2
```

mupad [B] time = 1.85, size = 354, normalized size = 3.98

$$\frac{2\,x^{3/2}}{3\,b}+\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^2}+\frac{\sqrt{2}\,\ln\left(\frac{4\,b^{11/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\cdots}\right.}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\cdots}}\right.}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x)),x)
```

```
[Out] (2*x^(3/2))/(3*b) + (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^2 + (2^(1/2)*log((
4*b^(11/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*e
xp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(4*b^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x)), x)
```

3.194 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=64

$$\frac{2\sqrt{x}}{b}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}$$

```
[Out] 2*x^(1/2)/b-2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*
x-arctanh(tanh(b*x+a)))^(1/2)/b^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2159, 2162}

$$\frac{2\sqrt{x}}{b}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*Sqrt[x])/b - (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*
x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{2\sqrt{x}}{b}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx}{b}\\ &= \frac{2\sqrt{x}}{b}-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.97

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]],x]
```

```
[Out] (2*Sqrt[x])/b - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/b^(3/2)
```

fricas [A] time = 0.52, size = 85, normalized size = 1.33

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b}, -\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

giac [A] time = 0.14, size = 31, normalized size = 0.48

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

maple [B] time = 0.29, size = 112, normalized size = 1.75

$$\frac{2\sqrt{x}}{b}-\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)a}{b\sqrt{(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx)\,b}}-\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{b\sqrt{(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] 2*x^(1/2)/b-2/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arc
tanh(tanh(b*x+a))-b*x)*b)^(1/2))*a-2/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a
))-b*x-a)
```

maxima [A] time = 0.42, size = 31, normalized size = 0.48

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

mupad [B] time = 2.23, size = 296, normalized size = 4.62

$$\frac{2\sqrt{x}}{b}+\frac{\sqrt{2}\,\ln\left(\frac{b^{7/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}+2\sqrt{2}\,bx\right)}{\left(\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)\right)\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}\right)\sqrt{\ln\left(\cdots\right.}}{2b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x)),x)
```

```
[Out] (2*x^(1/2))/b + (2^(1/2)*log((b^(7/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4
*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x
) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))/(
2*b^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x)), x)
```

3.195 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=53

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(1/2)/(b*x-a
rctanh(tanh(b*x+a)))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2162}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(Sqrt[b]
*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx=-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.96

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/(Sqrt[b
]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])
```

fricas [A] time = 0.62, size = 68, normalized size = 1.28

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

giac [A] time = 0.16, size = 18, normalized size = 0.34

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.26, size = 41, normalized size = 0.77

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))/x^(1/2),x)
```

```
[Out] 2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+
a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.42, size = 18, normalized size = 0.34

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 4.01, size = 347, normalized size = 6.55

$$\frac{\sqrt{2}\,\ln\left(\frac{b^2\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)\left(2\,\sqrt{2}\,a+4\,\sqrt{x}\,\sqrt{b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}-\sqrt{2}\,\left(2\,a-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+\ln\left(\cdots\right.\right.\right.}{2\,\sqrt{b\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}\,\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)}\right)}{\sqrt{b\,\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-b\,\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b^2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))),x)
```

```
[Out] (2^(1/2)*log((b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(2*2^(1/2)*a + 4*x^(1/2)*(b*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x))^(1/2) - 2^(1/2)*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 2*2^
(1/2)*b*x))/(2*(b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))^(1/2)*(log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(b*lo
g(1/(exp(2*a)*exp(2*b*x) + 1)) - b*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b^2*x)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\ \mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))), x)
```

3.196 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=76

$$\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*b^(1/2)/(b*x-a
rctanh(tanh(b*x+a)))^(3/2)+2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2163, 2162}

$$\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(3/2) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]]))
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{b\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))}\,dx}{bx-\tanh^{-1}(\tanh(a+bx))}\\ &= -\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}+\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.96

$$-\frac{2}{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}-\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2) - 2/(Sqrt[x]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]]))
```

fricas [A] time = 0.58, size = 93, normalized size = 1.22

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\sqrt{x}}{ax}, \frac{2\left(x\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(x
))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)]
```

giac [A] time = 0.16, size = 31, normalized size = 0.41

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

maple [A] time = 0.26, size = 76, normalized size = 1.00

$$-\frac{2}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{x}}-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}\right)}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)/((arct
anh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)
*b)^(1/2))
```

maxima [A] time = 0.42, size = 31, normalized size = 0.41

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

mupad [B] time = 1.97, size = 464, normalized size = 6.11

$$\frac{4}{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}+\frac{2\,\sqrt{2}\,\sqrt{b}\,\ln\left(\frac{\sqrt{b}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right.\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))),x)
```

```
[Out] 4/(x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (2*2^(1/2)*b^(1/2)*log((b^(1/2)*(log
(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^
(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^2 - 4*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*a^2))/(2*(log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) +
1))))))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\,\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))), x)
```

3.197 $\int \frac{1}{x^{5/2} \tanh^{-1}(\tanh(a+bx))} \, dx$

Optimal. Leaf size=101

$$-\frac{2b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] -2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-a
rctanh(tanh(b*x+a)))^(5/2)+2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+2*b/(b*x-
arctanh(tanh(b*x+a)))^2/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2163, 2162}

$$-\frac{2b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(5/2) + (2*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a
 + b*x]])^2) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx &= \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{b\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}\\
&= \frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{}{(bx}\\
&= -\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\frac{2b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+
\end{aligned}$$

Mathematica [A] time = 0.20, size = 89, normalized size = 0.88

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}}+\frac{2\left(4bx-\tanh^{-1}(\tanh(a+bx))\right)}{3x^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]),x]
[Out] (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])
/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) + (2*(4*b*x - ArcTanh[Tanh[a + b*x
]]))/(3*x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.54, size = 118, normalized size = 1.17

$$\left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},\ -\frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
[Out] [1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
+ 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

giac [A] time = 0.19, size = 41, normalized size = 0.41

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

maple [A] time = 0.26, size = 98, normalized size = 0.97

$$\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)\,b}}-\frac{2}{3\,(\operatorname{arctanh}(\tanh(bx+a))-bx)\,x^{\frac{3}{2}}}+\frac{}{(\mathrm{arc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a)),x)
[Out] 2*b^2/(arctanh(tanh(b*x+a))-b*x)^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arc
tan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/3/(arctanh(tanh(b*x+a
))-b*x)/x^(3/2)+2/(arctanh(tanh(b*x+a))-b*x)^2*b/x^(1/2)
```

maxima [A] time = 0.43, size = 41, normalized size = 0.41

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

mupad [B] time = 1.83, size = 642, normalized size = 6.36

$$\frac{4}{3\,x^{3/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{8\,b}{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}+\frac{4\,\sqrt{2}\,b^{3/2}\,\ln\left(\frac{\sqrt{b}\,\dots}{\dots}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))),x)
```

```
[Out] 4/(3*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (8*b)/(x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2) + (4*2^(1/2)*b^(3/2)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(
2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 + 24*a^2*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2 + 16*a^4 - 8*a*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3
- 32*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))), x)
```

3.198 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=128

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{2b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] -2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-a
rctanh(tanh(b*x+a)))^(7/2)+2/3*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5/x
^(5/2)/(b*x-arctanh(tanh(b*x+a)))+2*b^2/(b*x-arctanh(tanh(b*x+a)))^3/x^(1/2
)
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2163, 2162}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}+\frac{2b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/
(b*x - ArcTanh[Tanh[a + b*x]])^(7/2) + (2*b^2)/(Sqrt[x]*(b*x - ArcTanh[Tanh
[a + b*x]])^3) + (2*b)/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 2/(5*
x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{b\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{-bx+\tanh^{-1}(\tanh(a+bx))}$$

$$= \frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \cdots$$

$$= \frac{2b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{2b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \cdots$$

$$= -\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{2b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \cdots$$

Mathematica [A] time = 0.16, size = 107, normalized size = 0.84

$$\frac{2\left(-11bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2+23b^2x^2\right)}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]),x]
```

```
[Out] (-2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (2*(23*b^2*x^2 - 11*b*x*ArcTanh
[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(b*x - ArcTanh[T
anh[a + b*x]])^3)
```

fricas [A] time = 0.57, size = 144, normalized size = 1.12

$$\left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2-5\,abx+3\,a^2\right)\sqrt{x}}{15\,a^3x^3}, \frac{2\left(15\,b^2x^3\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^2x^2-5\,ab\cdots\right.\right.}{15\,a^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

giac [A] time = 0.17, size = 52, normalized size = 0.41

$$-\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} - \frac{2\left(15\,b^2x^2-5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

maple [A] time = 0.26, size = 120, normalized size = 0.94

$$-\frac{2}{5\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{5}{2}}}-\frac{2b^{2}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}\sqrt{x}}+\frac{2b}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a)),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)-2/(arctanh(tanh(b*x+a))-b*x)^3*b^2/
x^(1/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*b/x^(3/2)-2*b^3/(arctanh(tanh(b*x+
a))-b*x)^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(
tanh(b*x+a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.42, size = 52, normalized size = 0.41

$$-\frac{2\,b^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}-\frac{2\left(15\,b^{2}x^{2}-5\,abx+3\,a^{2}\right)}{15\,a^{3}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

mupad [B] time = 1.63, size = 822, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))),x)
```

```
[Out] 4/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (8*b)/(3*x^(3/2)*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x)^2) + (16*b^2)/(x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (8*2^(1/2)*b^(
5/2)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*
exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)
*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
4 - 160*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
 log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 240*a^4*(2*a - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^2 + 64*a^6 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 192*a^5*(2*
a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}} \operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a)),x)
```

```
[Out] Integral(1/(x**(7/2)*atanh(tanh(a + b*x))), x)
```

3.199 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=135

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}{b^{9/2}}+\frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}+\frac{7x^{3/2}}{}$$

```
[Out] 7/5*x^(5/2)/b^2+7/3*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))/b^3-7*arctanh(b^(1/2
)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(5/2
)/b^(9/2)-x^(7/2)/b/arctanh(tanh(b*x+a))+7*(b*x-arctanh(tanh(b*x+a)))^2*x^(
1/2)/b^4
```

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2159, 2162}

$$\frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3}+\frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)(bx}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (7*x^(5/2))/(5*b^2) + (7*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))/(3*b^3) +
(7*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/b^4 - (7*ArcTanh[(Sqrt[b]*Sqrt
[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/
2))/b^(9/2) - x^(7/2)/(b*ArcTanh[Tanh[a + b*x]])
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= \frac{7x^{5/2}}{5b^2} - \frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(7\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{x^{3/2}}{\tanh^{-1}(\tanh}}{2b^2}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} - \frac{x^{7/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(7\left(-bx\right.\right.}{}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} + \frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}$$

$$= \frac{7x^{5/2}}{5b^2} + \frac{7x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{3b^3} + \frac{7\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{b^4}$$

Mathematica [A] time = 0.21, size = 144, normalized size = 1.07

$$-\frac{7\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{9/2}} + \frac{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3}{b^4\tanh^{-1}(\tanh(a+bx))} + \frac{6\sqrt{x}\left(\tanh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(5/2))/(5*b^2) - (4*x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/(3*b^3)
 + (6*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)/b^4 - (7*ArcTan[(Sqrt[b]
*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) + ArcTanh[Tanh[a +
 b*x]])^(5/2))/b^(9/2) + (Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)/(b^4
*ArcTanh[Tanh[a + b*x]])
```

fricas [A] time = 0.41, size = 188, normalized size = 1.39

$$\left[\frac{105\left(a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(6\,b^3x^3-14\,ab^2x^2+70\,a^2bx+105\,a^3\right)\sqrt{x}}{30\left(b^5x+ab^4\right)}, -\frac{105\left(a^2bx+a^3\right)\sqrt{\frac{a}{b}}\ a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/30*(105*(a^2*b*x + a^3)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a
)/(b*x + a)) + 2*(6*b^3*x^3 - 14*a*b^2*x^2 + 70*a^2*b*x + 105*a^3)*sqrt(x))
/(b^5*x + a*b^4), -1/15*(105*(a^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqr
t(a/b)/a) - (6*b^3*x^3 - 14*a*b^2*x^2 + 70*a^2*b*x + 105*a^3)*sqrt(x))/(b^5
*x + a*b^4)]
```

giac [A] time = 0.16, size = 76, normalized size = 0.56

$$-\frac{7\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{a^3\sqrt{x}}{(bx+a)b^4} + \frac{2\left(3\,b^8x^{\frac{5}{2}}-10\,ab^7x^{\frac{3}{2}}+45\,a^2b^6\sqrt{x}\right)}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -7*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + a^3*sqrt(x)/((b*x + a)
*b^4) + 2/15*(3*b^8*x^(5/2) - 10*a*b^7*x^(3/2) + 45*a^2*b^6*sqrt(x))/b^10
```

maple [B] time = 0.27, size = 452, normalized size = 3.35

$$\frac{2x^{\frac{5}{2}}}{5b^2}-\frac{4x^{\frac{3}{2}}a}{3b^3}-\frac{4x^{\frac{3}{2}}\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{3b^3}+\frac{6a^2\sqrt{x}}{b^4}+\frac{12a\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\sqrt{x}}{b^4}+\frac{6(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/5*x^(5/2)/b^2-4/3/b^3*x^(3/2)*a-4/3/b^3*x^(3/2)*(arctanh(tanh(b*x+a))-b*x
-a)+6/b^4*a^2*x^(1/2)+12/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)+6/b^4*(
arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)+1/b^4*x^(1/2)/arctanh(tanh(b*x+a))*a^
3+3/b^4*x^(1/2)/arctanh(tanh(b*x+a))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+3/b^4
*x^(1/2)/arctanh(tanh(b*x+a))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+1/b^4*x^(1/2
)/arctanh(tanh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)^3-7/b^4/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
)*a^3-21/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctan
h(tanh(b*x+a))-b*x)*b)^(1/2))*a^2*(arctanh(tanh(b*x+a))-b*x-a)-21/b^4/((arc
tanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x
)*b)^(1/2))*a*(arctanh(tanh(b*x+a))-b*x-a)^2-7/b^4/((arctanh(tanh(b*x+a))-b
*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctan
h(tanh(b*x+a))-b*x-a)^3
```

maxima [A] time = 0.42, size = 75, normalized size = 0.56

$$\frac{6\,b^3x^{\frac{7}{2}}-14\,ab^2x^{\frac{5}{2}}+70\,a^2bx^{\frac{3}{2}}+105\,a^3\sqrt{x}}{15\left(b^5x+ab^4\right)}-\frac{7\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/15*(6*b^3*x^(7/2) - 14*a*b^2*x^(5/2) + 70*a^2*b*x^(3/2) + 105*a^3*sqrt(x)
)/(b^5*x + a*b^4) - 7*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4)
```

mupad [B] time = 1.61, size = 523, normalized size = 3.87

$$\frac{2\,x^{5/2}}{5\,b^2}+\frac{2\,x^{3/2}\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{3\,b^3}+\frac{3\,\sqrt{x}\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{2\,b^4}+\frac{7\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x))^2,x)
```

```
[Out] (2*x^(5/2))/(5*b^2) + (2*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(3*b^3) + (3*x^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^4) + (7*2^(1/2)*log((64*b^(19/2)*(2^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
```

```
*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^(5/2))/(16*b^(9/2)) - (x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^3)/(4*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - l
og(2/(exp(2*a)*exp(2*b*x) + 1))))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Timed out
```

3.200 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=108

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{7/2}}+\frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{}{b\tanh}$$

```
[Out] 5/3*x^(3/2)/b^2-5*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))
*(b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(7/2)-x^(5/2)/b/arctanh(tanh(b*x+a))+5*
(b*x-arctanh(tanh(b*x+a)))*x^(1/2)/b^3
```

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2159, 2162}

$$\frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{b^{7/2}}-\frac{}{b\tanh^{-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (5*x^(3/2))/(3*b^2) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/b^3 - (5*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh
[Tanh[a + b*x]])^(3/2))/b^(7/2) - x^(5/2)/(b*ArcTanh[Tanh[a + b*x]])
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$
$$= \frac{5x^{3/2}}{3b^2} - \frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(5\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{\sqrt{x}}{\tanh^{-1}(\tan}$$
$$= \frac{5x^{3/2}}{3b^2} + \frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{x^{5/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{\left(5\left(-bx\right.\right.}{}$$
$$= \frac{5x^{3/2}}{3b^2} + \frac{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^3} - \frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)(bx}{b^{7/2}}$$

Mathematica [A] time = 0.16, size = 119, normalized size = 1.10

$$\frac{5\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{7/2}} - \frac{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}{b^3\tanh^{-1}(\tanh(a+bx))} - \frac{4\sqrt{x}\left(\tanh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (2*x^(3/2))/(3*b^2) - (4*Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]))/b^3 + (
5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*(-(b*x) +
 ArcTanh[Tanh[a + b*x]])^(3/2))/b^(7/2) - (Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a
 + b*x]])^2)/(b^3*ArcTanh[Tanh[a + b*x]])
```

fricas [A] time = 0.52, size = 161, normalized size = 1.49

$$\left[\frac{15\left(abx+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(2b^2x^2-10abx-15a^2\right)\sqrt{x}}{6\left(b^4x+ab^3\right)}, \frac{15\left(abx+a^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+\left(\right.}{3\left(b^4x+ab^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*
(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

giac [A] time = 0.19, size = 65, normalized size = 0.60

$$\frac{5a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} - \frac{a^2\sqrt{x}}{(bx+a)b^3} + \frac{2\left(b^4x^{\frac{3}{2}}-6ab^3\sqrt{x}\right)}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*
b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

maple [B] time = 0.27, size = 294, normalized size = 2.72

$$\frac{2x^{\frac{3}{2}}}{3b^2}-\frac{4a\sqrt{x}}{b^3}-\frac{4\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\sqrt{x}}{b^3}-\frac{\sqrt{x}\,a^2}{b^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}-\frac{2\sqrt{x}\,a\left(\operatorname{arctanh}\left(\tanh\left(bx\right.\right.\right.}{b^3\operatorname{arctanh}\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2/3*x^(3/2)/b^2-4/b^3*a*x^(1/2)-4/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)-
1/b^3*x^(1/2)/arctanh(tanh(b*x+a))*a^2-2/b^3*x^(1/2)/arctanh(tanh(b*x+a))*a
*(arctanh(tanh(b*x+a))-b*x-a)-1/b^3*x^(1/2)/arctanh(tanh(b*x+a))*(arctanh(t
anh(b*x+a))-b*x-a)^2+5/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^
(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a^2+10/b^3/((arctanh(tanh(b*x+a
))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(
arctanh(tanh(b*x+a))-b*x-a)+5/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arct
an(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^2
```

maxima [A] time = 0.42, size = 64, normalized size = 0.59

$$\frac{2\,b^2x^{\frac{5}{2}}-10\,abx^{\frac{3}{2}}-15\,a^2\sqrt{x}}{3\left(b^4x+ab^3\right)}+\frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(2*b^2*x^(5/2) - 10*a*b*x^(3/2) - 15*a^2*sqrt(x))/(b^4*x + a*b^3) + 5*a
^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
```

mupad [B] time = 1.53, size = 463, normalized size = 4.29

$$\frac{2\,x^{3/2}}{3\,b^2}+\frac{2\,\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^3}+\frac{5\,\sqrt{2}\,\ln\left(\frac{16\,b^{15/2}\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{\right.}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x))^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) + (2*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^3 + (5*2^(1/2)
*log((16*b^(15/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(8*b^(7/2)) - (x^(1/2
)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**(5/2)/atanh(tanh(a + b*x))**2, x)
```

3.201 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}}-\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{x}}{b^2}$$

```
[Out] -x^(3/2)/b/arctanh(tanh(b*x+a))+3*x^(1/2)/b^2-3*arctanh(b^(1/2)*x^(1/2)/(b*
x-arctanh(tanh(b*x+a)))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)/b^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2159, 2162}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}}-\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a +
b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/b^(5/2) - x^(3/2)/(b*ArcTanh[Ta
nh[a + b*x]])
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= \frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} - \frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{1}{\sqrt{x}\tanh^{-1}(\cdots}}{2b^2}$$

$$= \frac{3\sqrt{x}}{b^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{b^{5/2}} - \frac{\cdots}{b\tanh^{-}\cdots}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.98

$$-\frac{3\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{5/2}} - \frac{x^{3/2}}{b\tanh^{-1}(\tanh(a+bx))} + \frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*ArcTanh[Tanh[a + b*x]]) - (3*ArcTan[(Sqrt[b]*S
qrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a
 + b*x]]])/b^(5/2)
```

fricas [A] time = 0.53, size = 134, normalized size = 1.61

$$\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\left(b^3x+ab^2\right)}, -\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

giac [A] time = 0.52, size = 46, normalized size = 0.55

$$-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{a\sqrt{x}}{(bx+a)b^2} + \frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

maple [B] time = 0.26, size = 160, normalized size = 1.93

$$\frac{2\sqrt{x}}{b^2} + \frac{\sqrt{x}\,a}{b^2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)} + \frac{\sqrt{x}\,\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)}{b^2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)} - \frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)a\cdots}{b^2\sqrt{(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx)\,b}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] 2*x^(1/2)/b^2+1/b^2*x^(1/2)/arctanh(tanh(b*x+a))*a+1/b^2*x^(1/2)/arctanh(ta
nh(b*x+a))*(arctanh(tanh(b*x+a))-b*x-a)-3/b^2/((arctanh(tanh(b*x+a))-b*x)*b
)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a-3/b^2/((ar
ctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*
x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)
```

maxima [A] time = 0.42, size = 50, normalized size = 0.60

$$\frac{2\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{b^{3}x+ab^{2}}-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] (2*b*x^(3/2) + 3*a*sqrt(x))/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*b
))/(sqrt(a*b)*b^2)
```

mupad [B] time = 1.77, size = 403, normalized size = 4.86

$$\frac{2\sqrt{x}}{b^{2}}-\frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}{b^{2}\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)}+\frac{3\sqrt{2}\,\ln\left(\frac{4\,b^{11/2}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{}\right.}{\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)\sqrt{\ln\left(\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^2,x)
```

```
[Out] (2*x^(1/2))/b^2 - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(b^2*(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
 + (3*2^(1/2)*log((4*b^(11/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)
*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))/(4*b^(5/2
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**2, x)
```

3.202 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=73

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] -x^(1/2)/b/arctanh(tanh(b*x+a))-arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b
*x+a)))^(1/2))/b^(3/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2168, 2162}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*Sq
rt[b*x - ArcTanh[Tanh[a + b*x]]])) - Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2} \, dx &= -\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}+\frac{\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))} \, dx}{2b}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}-\frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^2,x]
```

```
[Out] -(Sqrt[x]/(b*ArcTanh[Tanh[a + b*x]])) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x
) + ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]
)
```

fricas [A] time = 0.43, size = 115, normalized size = 1.58

$$\left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\left(ab^{3}x+a^{2}b^{2}\right)},-\frac{ab\sqrt{x}+\sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^{3}x+a^{2}b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

giac [A] time = 0.18, size = 36, normalized size = 0.49

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\sqrt{x}}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

maple [A] time = 0.26, size = 61, normalized size = 0.84

$$-\frac{\sqrt{x}}{b\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{b\sqrt{\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -x^(1/2)/b/arctanh(tanh(b*x+a))+1/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*ar
ctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.42, size = 37, normalized size = 0.51

$$-\frac{\sqrt{x}}{b^{2}x+ab}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 1.77, size = 344, normalized size = 4.71

$$\frac{\sqrt{2}\,\ln\left(\frac{b^{7/2}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}\left(\sqrt{2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)-4\sqrt{b}\sqrt{x}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+\cdots}\right.}{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}\right.}{2b^{3/2}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^2,x)
```

```
[Out] (2^(1/2)*log((b^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*
b*x))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(
2*a)*exp(2*b*x) + 1)))))/(2*b^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) - (2*x^
(1/2))/(b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(
exp(2*a)*exp(2*b*x) + 1))))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^{2}\left(\tanh\left(a+bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**2, x)
```

3.203 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))}$$

```
[Out] arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arctanh(tanh
(b*x+a)))^(3/2)/b^(1/2)-1/b/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)-1/b/arctanh(
tanh(b*x+a))/x^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2162}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(Sqrt[b]*(b*x
 - ArcTanh[Tanh[a + b*x]])^(3/2)) - 1/(b*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])) - 1/(b*Sqrt[x]*ArcTanh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{b\sqrt{x}\tanh^{-1}(\tanh(a+bx))} - \frac{\int \frac{}{\sqrt{x}}}{2\left(bx-\right.}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{b}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.82

$$\frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\sqrt{b}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]/(Sqrt[b]*(-
(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2)) + Sqrt[x]/(ArcTanh[Tanh[a + b*x]]*(-
(b*x) + ArcTanh[Tanh[a + b*x]]))
```

fricas [A] time = 0.39, size = 116, normalized size = 1.20

$$\left[\frac{2\,ab\sqrt{x} - \sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{2\left(a^2b^2x+a^3b\right)}, \frac{ab\sqrt{x} - \sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

giac [A] time = 0.25, size = 35, normalized size = 0.36

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} + \frac{\sqrt{x}}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

maple [A] time = 0.27, size = 82, normalized size = 0.85

$$\frac{\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{(\operatorname{arctanh}(\tanh(bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^2/x^(1/2),x)
```

```
[Out] x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))+1/(arctanh(tanh(b*x
+a))-b*x)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(t
anh(b*x+a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.42, size = 35, normalized size = 0.36

$$\frac{\sqrt{x}}{abx+a^2}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 2.18, size = 516, normalized size = 5.32

$$\frac{\sqrt{2}\,\ln\left(-\frac{\sqrt{b}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)+4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2a}\,e^{2}\cdots}{e^{2a}\,e^{2b}\cdots}\right.}\cdots\right.}{2\,\left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)-\cdots\right.}\cdots\right.}{\sqrt{b}\,\left(\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{2\,e^{2}\cdots}{e^{2a}\cdots}\right.\right.\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^2),x)
```

```
[Out] (2^(1/2)*log(-(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)
*b*x)*(4*a^2*b + b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - 4*a*b*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*
x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^
(3/2)) - (4*x^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x}\,\operatorname{atanh}^2\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**2/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**2), x)
```

3.204 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=120

$$-\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{}{\sqrt{x}\,(bx}$$

```
[Out] -1/b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))-1/b/x^(3/2)/arctanh(tanh(b*x+a))+3*
arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*b^(1/2)/(b*x-arct
anh(tanh(b*x+a)))^(5/2)-3/(b*x-arctanh(tanh(b*x+a)))^2/x^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2162}

$$-\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))}+\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}-\frac{}{\sqrt{x}\,(bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(5/2) - 3/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^2) - 1/(b*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(b*x^(3/2)*ArcTa
nh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))} - \frac{3\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{3/2}\tanh^{-1}(\tanh(a+bx))} + \frac{3}{2\left(-\right.}$$

$$= -\frac{3}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} - \frac{3}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Mathematica [A] time = 0.12, size = 104, normalized size = 0.87

$$-\frac{b\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{2}{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{}{\sqrt{\tanh}}\right)}{\left(\tanh^{-1}(\tanh(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2) - 2/(Sqrt[x]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2) - (b*Sqrt[x])/(ArcTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2)
```

fricas [A] time = 0.46, size = 147, normalized size = 1.22

$$\left[\frac{3\left(bx^2+ax\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(3bx+2a\right)\sqrt{x}}{2\left(a^2bx^2+a^3x\right)}, \frac{3\left(bx^2+ax\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(3bx+2a\right)\sqrt{x}}{a^2bx^2+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*
x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2
 + a^3*x)]
```

giac [A] time = 0.16, size = 49, normalized size = 0.41

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} - \frac{3\,bx+2\,a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/2
) + a*sqrt(x))*a^2)
```

maple [A] time = 0.28, size = 105, normalized size = 0.88

$$-\frac{2}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\sqrt{x}}-\frac{b\sqrt{x}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}-\frac{3}{(\operatorname{arctanh}(\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^2/x^(1/2)-1/(arctanh(tanh(b*x+a))-b*x)^2*b*x^
(1/2)/arctanh(tanh(b*x+a))-3/(arctanh(tanh(b*x+a))-b*x)^2*b/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)
)
```

maxima [A] time = 0.42, size = 51, normalized size = 0.42

$$-\frac{3\,bx+2\,a}{a^{2}bx^{\frac{3}{2}}+a^{3}\sqrt{x}}-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x + 2*a)/(a^2*b*x^(3/2) + a^3*sqrt(x)) - 3*b*arctan(b*sqrt(x)/sqrt(a*
b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 1.83, size = 705, normalized size = 5.88

$$\frac{\sqrt{x}\left(\frac{8}{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx}-\frac{24bx}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^{2}}\right)}{2bx^{2}-x\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+6\sqrt{2}\sqrt{b}\,\ln\left(-\frac{\sqrt{b}\sqrt{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e}{e^{2a}e^{2}}\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^2),x)
```

```
[Out] (x^(1/2)*(8/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - (24*b*x)/(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
)/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (6*2^(1/2)*b^(1/2)*log(-(b^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*b^(1/
2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^4 + 24*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 16*a^4 - 8*a*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/
(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a
```

```
)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} \operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**2,x)
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**2), x)
```

3.205 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{}{bx}$$

```
[Out] 5*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-ar
ctanh(tanh(b*x+a)))^(7/2)-5/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2-1/b/x^(5
/2)/(b*x-arctanh(tanh(b*x+a)))-1/b/x^(5/2)/arctanh(tanh(b*x+a))-5*b/(b*x-ar
ctanh(tanh(b*x+a)))^3/x^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2162}

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}}-\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (5*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(7/2) - (5*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a
+ b*x]])^3) - 5/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) - 1/(b*x^(5/2)
*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(b*x^(5/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{5/2}\tanh^{-1}(\tanh(a+bx))} - \frac{5\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{5/2}\tanh^{-1}(\tanh(a+bx))} + \frac{5}{2\left(-\right.}$$

$$= -\frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= -\frac{5b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{5}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

$$= \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} - \frac{5b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Mathematica [A] time = 0.22, size = 120, normalized size = 0.83

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{7/2}} + \frac{b^2\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3} + \frac{2\left(\tanh^{-1}(\tanh(a\right.}{3x^{3/2}\left(bx-\tanh^{-1}(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (2*(-7*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*
x]])^3) + (5*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a
+ b*x]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2) + (b^2*Sqrt[x])/(ArcTanh
[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3)
```

fricas [A] time = 0.57, size = 184, normalized size = 1.27

$$\left[\frac{15\left(b^2x^3+abx^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(15b^2x^2+10abx-2a^2\right)\sqrt{x}}{6\left(a^3bx^3+a^4x^2\right)}, -\frac{15\left(b^2x^3+abx^2\right)\sqrt{\frac{b}{a}}\arctan\left(\right.}{3\left(a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) -
 a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^
4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

giac [A] time = 0.18, size = 58, normalized size = 0.40

$$\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{b^2\sqrt{x}}{(bx+a)a^3} + \frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] 5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)*
a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

maple [A] time = 0.33, size = 128, normalized size = 0.88

$$-\frac{2}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}x^{\frac{3}{2}}}+\frac{4b}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}\sqrt{x}}+\frac{b^{2}\sqrt{x}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}\operatorname{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)+4/(arctanh(tanh(b*x+a))-b*x)^3*b/
x^(1/2)+1/(arctanh(tanh(b*x+a))-b*x)^3*b^2*x^(1/2)/arctanh(tanh(b*x+a))+5/(
arctanh(tanh(b*x+a))-b*x)^3*b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan
(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.42, size = 64, normalized size = 0.44

$$\frac{15\,b^{2}x^{2}+10\,abx-2\,a^{2}}{3\left(a^{3}bx^{\frac{5}{2}}+a^{4}x^{\frac{3}{2}}\right)}+\frac{5\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(15*b^2*x^2 + 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) + a^4*x^(3/2)) + 5*b^2*a
rctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 2.16, size = 871, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^2),x)
```

```
[Out] ((32*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - (80*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)
/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(
exp(2*a)*exp(2*b*x) + 1)))) - 8/(3*x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (2
0*2^(1/2)*b^(3/2)*log(-(b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) +
2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^4 - 160*a^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 240*a^4*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2 + 64*a^6 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5
 - 192*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) +
log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))
```

```
/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} \operatorname{atanh}^2 (\tanh (a + bx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**2), x)
```

3.206 $\int \frac{1}{x^{7/2} \tanh^{-1}(\tanh(a+bx))^2} \, dx$

Optimal. Leaf size=172

$$\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}-\frac{}{5x}$$

```
[Out] 7*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-ar
ctanh(tanh(b*x+a)))^(9/2)-7/3*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3-7/5/x^
(5/2)/(b*x-arctanh(tanh(b*x+a)))^2-1/b/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))-1
/b/x^(7/2)/arctanh(tanh(b*x+a))-7*b^2/(b*x-arctanh(tanh(b*x+a)))^4/x^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2163, 2162}

$$\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}-\frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}-\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}-\frac{}{5x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (7*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(
b*x - ArcTanh[Tanh[a + b*x]])^(9/2) - (7*b^2)/(Sqrt[x]*(b*x - ArcTanh[Tanh[
a + b*x]])^4) - (7*b)/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) - 7/(5*x
^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) - 1/(b*x^(7/2)*(b*x - ArcTanh[Tanh
[a + b*x]])) - 1/(b*x^(7/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx = -\frac{1}{bx^{7/2}\tanh^{-1}(\tanh(a+bx))} - \frac{7\int \frac{1}{x^{9/2}\tanh^{-1}(\tanh(a+bx))}\,dx}{2b}$$

$$= -\frac{1}{bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{bx^{7/2}\tanh^{-1}(\tanh(a+bx))} + \frac{7}{2\left(-\right.}$$

$$= -\frac{7}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} - \frac{1}{bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= -\frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} - \frac{7}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= -\frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} - \frac{7b}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

$$= \frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} - \frac{7b^2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Mathematica [A] time = 0.28, size = 139, normalized size = 0.81

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{9/2}} - \frac{b^3\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4} - \frac{2\left(-16bx\tanh^{-1}(\mathrm{t}\right.}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^2),x]
```

```
[Out] (-7*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]]
)/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2) - (b^3*Sqrt[x])/(ArcTanh[Tanh[a +
 b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) - (2*(58*b^2*x^2 - 16*b*x*ArcTa
nh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(-(b*x) + ArcT
anh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.61, size = 210, normalized size = 1.22

$$\left[\frac{105\left(b^3x^4+ab^2x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(105b^3x^3+70ab^2x^2-14a^2bx+6a^3\right)\sqrt{x}}{30\left(a^4bx^4+a^5x^3\right)}, \frac{105\left(b^3x^4+ab^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] [1/30*(105*(b^3*x^4 + a*b^2*x^3)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/
a) - a)/(b*x + a)) - 2*(105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6*a^3)*sq
rt(x))/(a^4*b*x^4 + a^5*x^3), 1/15*(105*(b^3*x^4 + a*b^2*x^3)*sqrt(b/a)*arc
tan(a*sqrt(b/a)/(b*sqrt(x))) - (105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6
*a^3)*sqrt(x))/(a^4*b*x^4 + a^5*x^3)]
```

giac [A] time = 0.51, size = 70, normalized size = 0.41

$$-\frac{7\,b^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{4}}-\frac{b^{3}\sqrt{x}}{(bx+a)a^{4}}-\frac{2\left(45\,b^{2}x^{2}-10\,abx+3\,a^{2}\right)}{15\,a^{4}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] -7*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - b^3*sqrt(x)/((b*x + a)
*a^4) - 2/15*(45*b^2*x^2 - 10*a*b*x + 3*a^2)/(a^4*x^(5/2))
```

maple [A] time = 0.27, size = 151, normalized size = 0.88

$$-\frac{2}{5\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}x^{\frac{5}{2}}}-\frac{6b^{2}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{4}\sqrt{x}}+\frac{4b}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)-6/(arctanh(tanh(b*x+a))-b*x)^4*b^
2/x^(1/2)+4/3/(arctanh(tanh(b*x+a))-b*x)^3*b/x^(3/2)-1/(arctanh(tanh(b*x+a)
)-b*x)^4*b^3*x^(1/2)/arctanh(tanh(b*x+a))-7/(arctanh(tanh(b*x+a))-b*x)^4*b^
3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+
a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.42, size = 75, normalized size = 0.44

$$-\frac{105\,b^{3}x^{3}+70\,ab^{2}x^{2}-14\,a^{2}bx+6\,a^{3}}{15\left(a^{4}bx^{\frac{7}{2}}+a^{5}x^{\frac{5}{2}}\right)}-\frac{7\,b^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(105*b^3*x^3 + 70*a*b^2*x^2 - 14*a^2*b*x + 6*a^3)/(a^4*b*x^(7/2) + a^
5*x^(5/2)) - 7*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 2.14, size = 1051, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^2),x)
```

```
[Out] ((96*b^2)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - (224*b^3*x)/(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^4)/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(
2/(exp(2*a)*exp(2*b*x) + 1)))) - (32*b)/(3*x^(3/2)*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^3) - 8/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (56*2^(1/2)*b^(5/2)*log(-(
b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) +
```

```
4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^8 + 112*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 - 448*a
^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 + 1120*a^4*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^4 - 1792*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 1792*a^6*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2 + 256*a^8 - 16*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 1024*
a^7*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x)^(9/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}} \operatorname{atanh}^2(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**2,x)
```

```
[Out] Integral(1/(x**(7/2)*atanh(tanh(a + b*x))**2), x)
```

3.207 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=135

$$-\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}{4b^{9/2}}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^4}-\frac{}{4b^2\tan}$$

```
[Out] 35/12*x^(3/2)/b^3-35/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(
1/2))*(b*x-arctanh(tanh(b*x+a)))^(3/2)/b^(9/2)-1/2*x^(7/2)/b/arctanh(tanh(b
*x+a))^2-7/4*x^(5/2)/b^2/arctanh(tanh(b*x+a))+35/4*(b*x-arctanh(tanh(b*x+a)
))*x^(1/2)/b^4
```

Rubi [A] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2159, 2162}

$$-\frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^4}-\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}\right.}{4b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (35*x^(3/2))/(12*b^3) + (35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))/(4*b^4)
 - (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]*(b*x -
 ArcTanh[Tanh[a + b*x]])^(3/2))/(4*b^(9/2)) - x^(7/2)/(2*b*ArcTanh[Tanh[a +
 b*x]]^2) - (7*x^(5/2))/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{35\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+b}{8b^2}$$

$$= \frac{35x^{3/2}}{12b^3} - \frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{7x^{5/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(35\left(-bx+\right.\right.}{}$$

$$= \frac{35x^{3/2}}{12b^3} + \frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^4} - \frac{x^{7/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - $$

$$= \frac{35x^{3/2}}{12b^3} + \frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{4b^4} - \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+b}}\right.}{}$$

Mathematica [A] time = 0.11, size = 147, normalized size = 1.09

$$-\frac{21b^{5/2}x^{5/2}\tanh^{-1}(\tanh(a+bx)) - 140b^{3/2}x^{3/2}\tanh^{-1}(\tanh(a+bx))^2 + 105\sqrt{b}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3 -}{12b^{9/2}\tanh^{-1}(\tanh}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/12*(6*b^(7/2)*x^(7/2) + 21*b^(5/2)*x^(5/2)*ArcTanh[Tanh[a + b*x]] - 140*
b^(3/2)*x^(3/2)*ArcTanh[Tanh[a + b*x]]^2 + 105*Sqrt[b]*Sqrt[x]*ArcTanh[Tanh
[a + b*x]]^3 - 105*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a +
b*x]]]]*ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(3/2))/(
b^(9/2)*ArcTanh[Tanh[a + b*x]]^2)
```

fricas [A] time = 0.57, size = 227, normalized size = 1.68

$$\left[\frac{105\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8b^3x^3-56ab^2x^2-175a^2bx-105a^3\right)\sqrt{x}}{24\left(b^6x^2+2ab^5x+a^2b^4\right)}, \frac{105\left(ab^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*
sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x - 10
5*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 + 2*a
^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56*a*b
^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

giac [A] time = 0.16, size = 77, normalized size = 0.57

$$\frac{35a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^4} - \frac{13a^2bx^{\frac{3}{2}} + 11a^3\sqrt{x}}{4(bx+a)^2b^4} + \frac{2\left(b^6x^{\frac{3}{2}} - 9ab^5\sqrt{x}\right)}{3b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/2
) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x))
/b^9
```

maple [B] time = 0.27, size = 418, normalized size = 3.10

$$\frac{2x^{\frac{3}{2}}}{3b^3}-\frac{6a\sqrt{x}}{b^4}-\frac{6\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx-a\right)\sqrt{x}}{b^4}-\frac{13a^2x^{\frac{3}{2}}}{4b^3\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}-\frac{13x^{\frac{3}{2}}a\left(\operatorname{arctanh}\left(\tanh\left(bx\right.\right.\right.}{2b^3\operatorname{arctanh}\left(\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2/3*x^(3/2)/b^3-6/b^4*a*x^(1/2)-6/b^4*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)-
13/4/b^3/arctanh(tanh(b*x+a))^2*a^2*x^(3/2)-13/2/b^3/arctanh(tanh(b*x+a))^2
*x^(3/2)*a*(arctanh(tanh(b*x+a))-b*x-a)-13/4/b^3/arctanh(tanh(b*x+a))^2*x^(
3/2)*(arctanh(tanh(b*x+a))-b*x-a)^2-11/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)
*a^3-33/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*a^2*(arctanh(tanh(b*x+a))-b*x-
a)-33/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*a*(arctanh(tanh(b*x+a))-b*x-a)^2
-11/4/b^4/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)^3+35/
4/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(
b*x+a))-b*x)*b)^(1/2))*a^2+35/2/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*ar
ctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*a*(arctanh(tanh(b*x+a)
)-b*x-a)+35/4/b^4/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((a
rctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

maxima [A] time = 0.44, size = 86, normalized size = 0.64

$$\frac{8\,b^3x^{\frac{7}{2}}-56\,ab^2x^{\frac{5}{2}}-175\,a^2bx^{\frac{3}{2}}-105\,a^3\sqrt{x}}{12\left(b^6x^2+2\,ab^5x+a^2b^4\right)}+\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(8*b^3*x^(7/2) - 56*a*b^2*x^(5/2) - 175*a^2*b*x^(3/2) - 105*a^3*sqrt(x
))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(
sqrt(a*b)*b^4)
```

mupad [B] time = 1.98, size = 571, normalized size = 4.23

$$\frac{2\,x^{3/2}}{3\,b^3}+\frac{3\,\sqrt{x}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b^4}+\frac{35\,\sqrt{2}\,\ln\left(\frac{256\,b^{19/2}\left(\sqrt{2}\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\right.}{\left(\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)\right)\sqrt{\ln}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*x^(3/2))/(3*b^3) + (3*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b^4 + (35*2^(1/2
)*log((256*b^(19/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(
```

```
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2))/(32*b^(9/2)) - (13*
x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(8*b^4*(log((2*exp(2*a)*exp(2*b*x))/(ex
p(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) - (x^(1/2)*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^3)/(4*b^4*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Timed out
```

3.208 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^3} \, dx$

Optimal. Leaf size=110

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{4b^{7/2}}-\frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+\cdots}$$

```
[Out] -1/2*x^(5/2)/b/arctanh(tanh(b*x+a))^2-5/4*x^(3/2)/b^2/arctanh(tanh(b*x+a))+
15/4*x^(1/2)/b^3-15/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1
/2))*(b*x-arctanh(tanh(b*x+a)))^(1/2)/b^(7/2)
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2159, 2162}

$$-\frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{4b^{7/2}}-\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - (15*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tan
h[a + b*x]]]]*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]])/(4*b^(7/2)) - x^(5/2)/(2*
b*ArcTanh[Tanh[a + b*x]]^2) - (5*x^(3/2))/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2159

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^n/(a*n), x] - Dist[(b*u - a*v)/a, Int[v^(n - 1)/u, x], x]
 /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && GtQ[n, 0] && NeQ[n,
1]
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$
$$= -\frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{15\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+b}}{8b^2}$$
$$= \frac{15\sqrt{x}}{4b^3} - \frac{x^{5/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{5x^{3/2}}{4b^2\tanh^{-1}(\tanh(a+bx))} - \frac{\left(15\left(-bx+\right.\right.}{}$$
$$= \frac{15\sqrt{x}}{4b^3} - \frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}{4b^{7/2}} - \frac{}{2b}$$

Mathematica [A] time = 0.10, size = 104, normalized size = 0.95

$$\frac{1}{4}\left(-\frac{15\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{7/2}} - \frac{5x^{3/2}}{b^2\tanh^{-1}(\tanh(a+bx))} - \frac{2x^{5/}}{b\tanh^{-1}(\tanl}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ((15*Sqrt[x])/b^3 - (2*x^(5/2))/(b*ArcTanh[Tanh[a + b*x]]^2) - (5*x^(3/2))/
(b^2*ArcTanh[Tanh[a + b*x]]) - (15*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + A
rcTanh[Tanh[a + b*x]]]]*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])/b^(7/2))/4
```

fricas [A] time = 0.53, size = 200, normalized size = 1.82

$$\left[\frac{15\left(b^2x^2+2\,abx+a^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8\,b^2x^2+25\,abx+15\,a^2\right)\sqrt{x}}{8\left(b^5x^2+2\,ab^4x+a^2b^3\right)}, -\frac{15\left(b^2x^2+2\,abx+a^2\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

giac [A] time = 0.21, size = 59, normalized size = 0.54

$$-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^3} + \frac{2\,\sqrt{x}}{b^3} + \frac{9\,abx^{\frac{3}{2}} + 7\,a^2\sqrt{x}}{4\,(bx+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

maple [B] time = 0.26, size = 249, normalized size = 2.26

$$\frac{2\sqrt{x}}{b^3}+\frac{9x^{\frac{3}{2}}a}{4b^2 \operatorname{arctanh}(\tanh(bx+a))^2}+\frac{9x^{\frac{3}{2}}(\operatorname{arctanh}(\tanh(bx+a))-bx-a)}{4b^2 \operatorname{arctanh}(\tanh(bx+a))^2}+\frac{7a^2\sqrt{x}}{4b^3 \operatorname{arctanh}(\tanh(bx+a))^2}+\frac{7\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2*x^(1/2)/b^3+9/4/b^2/arctanh(tanh(b*x+a))^2*x^(3/2)*a+9/4/b^2/arctanh(tanh
(b*x+a))^2*x^(3/2)*(arctanh(tanh(b*x+a))-b*x-a)+7/4/b^3/arctanh(tanh(b*x+a)
)^2*a^2*x^(1/2)+7/2/b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*a*(arctanh(tanh(b*x+
a))-b*x-a)+7/4/b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x
-a)^2-15/4/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arct
anh(tanh(b*x+a))-b*x)*b)^(1/2))*a-15/4/b^3/((arctanh(tanh(b*x+a))-b*x)*b)^(
1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)
```

maxima [A] time = 0.44, size = 73, normalized size = 0.66

$$\frac{8\,b^2x^{\frac{5}{2}}+25\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}}{4\left(b^5x^2+2\,ab^4x+a^2b^3\right)}-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(8*b^2*x^(5/2) + 25*a*b*x^(3/2) + 15*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x
+ a^2*b^3) - 15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
```

mupad [B] time = 1.69, size = 511, normalized size = 4.65

$$\frac{2\,\sqrt{x}}{b^3}-\frac{9\,\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{4\,b^3\,\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)}+\frac{15\,\sqrt{2}\,\ln\left(\frac{64\,b^{15/2}\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\cdots}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\sqrt{\ln(\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x))^3,x)
```

```
[Out] (2*x^(1/2))/b^3 - (9*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(4*b^3*(log((2*exp(2
*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1
)))) + (15*2^(1/2)*log((64*b^(15/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b
^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x))/((log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x)
+ 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2))/(16
*b^(7/2)) - (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(2*b^3*(log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**(5/2)/atanh(tanh(a + b*x))**3, x)
```

3.209 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=98

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

```
[Out] -1/2*x^(3/2)/b/arctanh(tanh(b*x+a))^2-3/4*x^(1/2)/b^2/arctanh(tanh(b*x+a))-
3/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(5/2)/(b*x-
arctanh(tanh(b*x+a)))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2168, 2162}

$$-\frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}-\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] (-3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(4*b^(5/
2)*Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]) - x^(3/2)/(2*b*ArcTanh[Tanh[a + b*x]
]^2) - (3*Sqrt[x])/(4*b^2*ArcTanh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{3\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a\cdots}}{8b^2}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{5/2}\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}} - \frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{\cdots}{4b^2\tanh^{\cdots}}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 0.98

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{4b^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}} - \frac{3\sqrt{x}}{4b^2\tanh^{-1}(\tanh(a+bx))} - \frac{x^{3/2}}{2b\tanh^{-1}(\tanh(a+bx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/2*x^(3/2)/(b*ArcTanh[Tanh[a + b*x]]^2) - (3*Sqrt[x])/(4*b^2*ArcTanh[Tanh
[a + b*x]]) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*
x]]]])/(4*b^(5/2)*Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]])
```

fricas [A] time = 0.50, size = 185, normalized size = 1.89

$$\left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)+2\left(5\,ab^2x+3\,a^2b\right)\sqrt{x}}{8\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)}, -\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\,\mathrm{arctan}\cdots}{4\left(ab^5x^2+2\,a^2\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

giac [A] time = 0.17, size = 47, normalized size = 0.48

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^2} - \frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

maple [A] time = 0.27, size = 85, normalized size = 0.87

$$\frac{-\frac{5x^{\frac{3}{2}}}{4b} - \frac{3(\mathrm{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}{4b^2}}{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^2} + \frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\mathrm{arctanh}(\tanh(bx+a))-bx)b}}\right)}{4b^2\sqrt{\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] 2*(-5/8*x^(3/2)/b-3/8*(arctanh(tanh(b*x+a))-b*x)/b^2*x^(1/2))/arctanh(tanh(
b*x+a))^2+3/4/b^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((a
rctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.43, size = 61, normalized size = 0.62

$$-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\left(b^4x^2+2\,ab^3x+a^2b^2\right)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [B] time = 1.87, size = 667, normalized size = 6.81

$$\frac{3\,\sqrt{2}\,\ln\left(\frac{16\,b^{11/2}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)-4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2}}{e^{2\,a}\,e^{2\,b}}\right.}\right.}{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}\right.}{8\,b^{5/2}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^3,x)
```

```
[Out] (3*2^(1/2)*log((16*b^(11/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^
(1/2)*b*x))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2
/(exp(2*a)*exp(2*b*x) + 1)))))/(8*b^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)) -
 (x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(b^2*(log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) - (x^(1/2)*(1/
(b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)) + (8*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 8*log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 16*b*x)/(2*b*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp
(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b*(log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\operatorname{atanh}^3\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**3, x)
```

3.210 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{4b^2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{4b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))}-\frac{}{2b}$$

```
[Out] 1/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/b^(3/2)/(b*x-
arctanh(tanh(b*x+a)))^(3/2)-1/4/b^2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)-1/4/
b^2/arctanh(tanh(b*x+a))/x^(1/2)-1/2*x^(1/2)/b/arctanh(tanh(b*x+a))^2
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2162}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}}-\frac{1}{4b^2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{1}{4b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))}-\frac{}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]]/(4*b^(3/2)*(b
*x - ArcTanh[Tanh[a + b*x]])^(3/2)) - 1/(4*b^2*Sqrt[x]*(b*x - ArcTanh[Tanh[
a + b*x]])) - Sqrt[x]/(2*b*ArcTanh[Tanh[a + b*x]]^2) - 1/(4*b^2*Sqrt[x]*Arc
Tanh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} + \frac{\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{1}{4b^2\sqrt{x}\tanh^{-1}(\tanh(a+bx))} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+}}{8b^2}$$

$$= -\frac{1}{4b^2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{\sqrt{x}}{2b\tanh^{-1}(\tanh(a+bx))^2} - \frac{}{4b^2\sqrt{x}\tan}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4b^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{3/2}} - \frac{1}{4b^2\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{}{2b}$$

Mathematica [**A**] time = 0.15, size = 107, normalized size = 0.86

$$\frac{1}{4}\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{b^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{3/2}} + \frac{\sqrt{x}}{b\tanh^{-1}(\tanh(a+bx))^2 - b^2x\tanh^{-1}(\tanh(a+bx))} - \frac{2\sqrt{x}}{b\tanh^{-1}(\tanh(}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^3,x]
```

```
[Out] ((-2*Sqrt[x])/(b*ArcTanh[Tanh[a + b*x]]^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
-(b*x) + ArcTanh[Tanh[a + b*x]]]]/(b^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]]
)^(3/2)) + Sqrt[x]/(-(b^2*x*ArcTanh[Tanh[a + b*x]]) + b*ArcTanh[Tanh[a + b*
x]]^2))/4
```

fricas [**A**] time = 0.52, size = 186, normalized size = 1.49

$$\left[-\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\left(ab^2x-a^2b\right)\sqrt{x}}{8\left(a^2b^4x^2+2\,a^3b^3x+a^4b^2\right)}, -\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-}{4\left(a^2b^4x^2+2\,a^3b^3x+a^4b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

giac [**A**] time = 0.21, size = 52, normalized size = 0.42

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab} + \frac{bx^{\frac{3}{2}} - a\sqrt{x}}{4\,(bx+a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x))/((b*x + a)^2*a*b)

maple [A] time = 0.27, size = 98, normalized size = 0.78

$$\frac{\frac{2x^{\frac{3}{2}}}{8\operatorname{arctanh}(\tanh(bx+a))-8bx}-\frac{\sqrt{x}}{4b}}{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{(\operatorname{arctanh}(\tanh(bx+a))-bx)b}}\right)}{4\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b\sqrt{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/arctanh(tanh(b*x+a))^3,x)

[Out] 2*(1/8/(arctanh(tanh(b*x+a))-b*x)*x^(3/2)-1/8*x^(1/2)/b)/arctanh(tanh(b*x+a))^2+1/4/(arctanh(tanh(b*x+a))-b*x)/b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))

maxima [A] time = 0.43, size = 64, normalized size = 0.51

$$\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\left(ab^{3}x^{2}+2\,a^{2}b^{2}x+a^{3}b\right)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")

[Out] 1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)

mupad [B] time = 1.84, size = 580, normalized size = 4.64

$$\frac{\sqrt{2}\,\ln\left(-\frac{4\,\sqrt{b}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x}\,\left(\sqrt{2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)+4\,\sqrt{b}\,\sqrt{x}\,\sqrt{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\cdots}\cdots\right)}{\cdots\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\cdots}\right)}{4\,b^{3/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\cdots\right)\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/atanh(tanh(a + b*x))^3,x)

[Out] (2^(1/2)*log(-(4*b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*(b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 + 4*a^2*b^3 - 4*a*b^3*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))))/(4*b^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(3/2)) - (2*x^(1/2))/(b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) - x^(1/2)/(b*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**3, x)
```

3.211 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=152

$$\frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{1}{4b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\ldots$$

```
[Out] 1/4/b^2/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+1/4/b^2/x^(3/2)/arctanh(tanh(b*x
+a))-3/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*x-arc
tanh(tanh(b*x+a)))^(5/2)/b^(1/2)+3/4/b/(b*x-arctanh(tanh(b*x+a)))^2/x^(1/2)
-1/2/b/arctanh(tanh(b*x+a))^2/x^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2168, 2163, 2162}

$$\frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{1}{4b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])/(4*Sqrt[
b]*(b*x - ArcTanh[Tanh[a + b*x]])^(5/2)) + 3/(4*b*Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + 1/(4*b^2*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(2*
b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^2) + 1/(4*b^2*x^(3/2)*ArcTanh[Tanh[a + b*x
]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^3}dx = -\frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2} - \frac{\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^2}dx}{4b}$$

$$= -\frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2} + \frac{1}{4b^2x^{3/2}\tanh^{-1}(\tanh(a+bx))} + \frac{3\int \frac{1}{x^{5/2}\tan}$$

$$= \frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{4b^2x}$$

$$= \frac{3}{4b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{1}{4b^2x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\sqrt{b}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{5/2}} + \frac{3}{4b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Mathematica [A] time = 0.09, size = 118, normalized size = 0.78

$$\frac{\sqrt{x}}{2\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)} + \frac{3\sqrt{x}}{4\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} + \frac{}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]]])/(4*Sqrt
[b]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(5/2)) + (3*Sqrt[x])/(4*ArcTanh[Tanh[
a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2) + Sqrt[x]/(2*ArcTanh[Tanh[a
+ b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]]))
```

fricas [A] time = 0.51, size = 186, normalized size = 1.22

$$\left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\left(3\,ab^2x+5\,a^2b\right)\sqrt{x}}{8\left(a^3b^3x^2+2\,a^4b^2x+a^5b\right)}, -\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\,\arctan\left(\right.}{4\left(a^3b^3x^2+2\,a^4b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]
```

giac [A] time = 0.18, size = 47, normalized size = 0.31

$$\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^2} + \frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(bx+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)
```

maple [A] time = 0.27, size = 112, normalized size = 0.74

$$\frac{\sqrt{x}}{2\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{2}}+\frac{3\sqrt{x}}{4\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\operatorname{arctanh}\left(\tanh\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctanh(tanh(b*x+a))^3/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^2+3/4/(arctanh(
tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))+3/4/(arctanh(tanh(b*x+a))-
b*x)^2/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh
(b*x+a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.43, size = 60, normalized size = 0.39

$$\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\left(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4}\right)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctanh(tanh(b*x+a))^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 1.90, size = 741, normalized size = 4.88

$$\frac{6\,\sqrt{x}}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{2}}-\frac{}{\left(\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^3),x)
```

```
[Out] (6*x^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(
2/(exp(2*a)*exp(2*b*x) + 1)))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (4*x^(1/2))/((l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp
(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (3*2^(1/2)*log((b^(1/2)*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1
/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*(16*a^4*b + b*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^4 - 8*a*b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 - 32*a^3*b
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x) + 24*a^2*b*(2*a - log((2*exp(2*a)*exp(2*b*x
```

```
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
))/(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a
)*exp(2*b*x) + 1)))))/(2*b^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\operatorname{atanh}^3(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atanh(tanh(b*x+a))**3/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**3), x)
```

3.212 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=176

$$\frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{3}{4b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))}+\frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\text{2}$$

```
[Out] 5/4/b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+3/4/b^2/x^(5/2)/(b*x-arctanh(tan
h(b*x+a)))-1/2/b/x^(3/2)/arctanh(tanh(b*x+a))^2+3/4/b^2/x^(5/2)/arctanh(tan
h(b*x+a))-15/4*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))*b^
(1/2)/(b*x-arctanh(tanh(b*x+a)))^(7/2)+15/4/(b*x-arctanh(tanh(b*x+a)))^3/x^
(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2162}

$$\frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{3}{4b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))}+\frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}-\text{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(7/2)) + 15/(4*Sqrt[x]*(b*x - ArcTanh[Ta
nh[a + b*x]])^3) + 5/(4*b*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 3/(4*
b^2*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])) - 1/(2*b*x^(3/2)*ArcTanh[Tanh[a
 + b*x]]^2) + 3/(4*b^2*x^(5/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2} - \frac{3\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{3}{4b^2x^{5/2}\tanh^{-1}(\tanh(a+bx))} + \frac{15\int \frac{1}{x^{7/2}\text{ta}}$$

$$= \frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2bx^{3/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{4b^2x}$$

$$= \frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{3}{4b^2x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{15}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{5}{4bx^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} +$$

$$= -\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{7/2}} + \frac{15}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Mathematica [A] time = 0.18, size = 141, normalized size = 0.80

$$-\frac{b\sqrt{x}}{2\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2} - \frac{7b\sqrt{x}}{4\tanh^{-1}(\tanh(a+bx))\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3} -$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
])/(4*(-(b*x) + ArcTanh[Tanh[a + b*x]])^(7/2)) - 2/(Sqrt[x]*(-(b*x) + ArcTa
nh[Tanh[a + b*x]])^3) - (7*b*Sqrt[x])/(4*ArcTanh[Tanh[a + b*x]]*(-(b*x) + A
rcTanh[Tanh[a + b*x]])^3) - (b*Sqrt[x])/(2*ArcTanh[Tanh[a + b*x]]^2*(-(b*x)
 + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.47, size = 214, normalized size = 1.22

$$\left[\frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2+25\,abx+8\,a^2\right)\sqrt{x}}{8\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)},\ \frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{}}{4(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(
b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

giac [A] time = 0.30, size = 59, normalized size = 0.34

$$-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}-\frac{2}{a^{3}\sqrt{x}}-\frac{7\,b^{2}x^{\frac{3}{2}}+9\,ab\sqrt{x}}{4\,(bx+a)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/4
*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

maple [A] time = 0.28, size = 181, normalized size = 1.03

$$-\frac{2}{(\operatorname{arctanh}(\tanh(bx+a))-bx)^{3}\sqrt{x}}-\frac{7b^{2}x^{\frac{3}{2}}}{4\,(\operatorname{arctanh}(\tanh(bx+a))-bx)^{3}\operatorname{arctanh}(\tanh(bx+a))^{2}}-\frac{}{4\,(\operatorname{arctanh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)^3/x^(1/2)-7/4/(arctanh(tanh(b*x+a))-b*x)^3*b^
2/arctanh(tanh(b*x+a))^2*x^(3/2)-9/4/(arctanh(tanh(b*x+a))-b*x)^3*b/arctanh
(tanh(b*x+a))^2*a*x^(1/2)-9/4/(arctanh(tanh(b*x+a))-b*x)^3*b/arctanh(tanh(b
*x+a))^2*x^(1/2)*(arctanh(tanh(b*x+a))-b*x-a)-15/4/(arctanh(tanh(b*x+a))-b*
x)^3*b/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh
(b*x+a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.43, size = 73, normalized size = 0.41

$$-\frac{15\,b^{2}x^{2}+25\,abx+8\,a^{2}}{4\left(a^{3}b^{2}x^{\frac{5}{2}}+2\,a^{4}bx^{\frac{3}{2}}+a^{5}\sqrt{x}\right)}-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(15*b^2*x^2 + 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) + 2*a^4*b*x^(3/2) + a
^5*sqrt(x)) - 15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 2.17, size = 1077, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^3),x)
```

```
[Out] (x*((12*b)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + (3*b*(16*log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - 16*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 32*b*x
))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^4) - (16*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 16*
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 32*b*x)/(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)^3)/(x^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))) + (15*2^(1/2)*b^(1/2)*log((b^(
1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) +
```

```
1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b
^(1/2)*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*
x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^6 + 60*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - 160*a^3*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 240*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
+ 64*a^6 - 12*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5 - 192*a^5*(2*a - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(7/2) - (8
*b*x^(1/2))/((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(
2/(exp(2*a)*exp(2*b*x) + 1)))^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*
exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} \operatorname{atanh}^3(\tanh(a + bx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**3), x)
```

3.213 $\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx$

Optimal. Leaf size=201

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}+\frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{5}{4b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))}+$$

```
[Out] -35/4*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*
x-arctanh(tanh(b*x+a)))^(9/2)+35/12/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3+7/
4/b/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+5/4/b^2/x^(7/2)/(b*x-arctanh(tanh(
b*x+a)))-1/2/b/x^(5/2)/arctanh(tanh(b*x+a))^2+5/4/b^2/x^(7/2)/arctanh(tanh(
b*x+a))+35/4*b/(b*x-arctanh(tanh(b*x+a)))^4/x^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2162}

$$\frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{5}{4b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}}+-1$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(9/2)) + (35*b)/(4*Sqrt[x]*(b*x - ArcTan
h[Tanh[a + b*x]])^4) + 35/(12*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) + 7
/(4*b*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2) + 5/(4*b^2*x^(7/2)*(b*x - A
rcTanh[Tanh[a + b*x]])) - 1/(2*b*x^(5/2)*ArcTanh[Tanh[a + b*x]]^2) + 5/(4*b
^2*x^(7/2)*ArcTanh[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} - \frac{5\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{5}{4b^2x^{7/2}\tanh^{-1}(\tanh(a+bx))} + \frac{35\int \frac{1}{x^{9/2}\text{ta}}$$

$$= \frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2bx^{5/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{4b^2x}$$

$$= \frac{7}{4bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{5}{4b^2x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{35}{12x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{7}{4bx^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

$$= \frac{35b}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{35}{12x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} +$$

$$= -\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{9/2}} + \frac{35b}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}$$

Mathematica [A] time = 0.28, size = 156, normalized size = 0.78

$$\frac{1}{12}\left(\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{9/2}} + \frac{6b^2\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^2\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3} + \frac{}{\tanh^{-1}(\tanh}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] ((105*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x]]
]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(9/2) + (80*b*x - 8*ArcTanh[Tanh[a +
b*x]])/(x^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (33*b^2*Sqrt[x])/(Ar
cTanh[Tanh[a + b*x]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (6*b^2*Sqrt[x])
/(ArcTanh[Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^3))/12
```

fricas [A] time = 0.54, size = 250, normalized size = 1.24

$$\left[\frac{105\left(b^3x^4+2\,ab^2x^3+a^2bx^2\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3\right)\sqrt{x}}{24\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}, -\frac{105\,(b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sq
rt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*
b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/12*(105*(b^
3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x)))
- (105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4
+ 2*a^5*b*x^3 + a^6*x^2)]
```

giac [A] time = 0.43, size = 71, normalized size = 0.35

$$\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^4}+\frac{2\,(9\,bx-a)}{3\,a^4x^{\frac{3}{2}}}+\frac{11\,b^3x^{\frac{3}{2}}+13\,ab^2\sqrt{x}}{4\,(bx+a)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^4
*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

maple [A] time = 0.29, size = 207, normalized size = 1.03

$$-\frac{2}{3\,(\mathrm{arctanh}\,(\tanh\,(bx+a))-bx)^3\,x^{\frac{3}{2}}}+\frac{6b}{(\mathrm{arctanh}\,(\tanh\,(bx+a))-bx)^4\,\sqrt{x}}+\frac{11b^3x^{\frac{3}{2}}}{4\,(\mathrm{arctanh}\,(\tanh\,(bx+a))-bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)^3/x^(3/2)+6/(arctanh(tanh(b*x+a))-b*x)^4*b/
x^(1/2)+11/4/(arctanh(tanh(b*x+a))-b*x)^4*b^3/arctanh(tanh(b*x+a))^2*x^(3/2
)+13/4/(arctanh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x+a))^2*a*x^(1/2)+13
/4/(arctanh(tanh(b*x+a))-b*x)^4*b^2/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh
(tanh(b*x+a))-b*x-a)+35/4/(arctanh(tanh(b*x+a))-b*x)^4*b^2/((arctanh(tanh(b
*x+a))-b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))
```

maxima [A] time = 0.43, size = 86, normalized size = 0.43

$$\frac{105\,b^3x^3+175\,ab^2x^2+56\,a^2bx-8\,a^3}{12\left(a^4b^2x^{\frac{7}{2}}+2\,a^5bx^{\frac{5}{2}}+a^6x^{\frac{3}{2}}\right)}+\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)/(a^4*b^2*x^(7/2) +
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

mupad [B] time = 2.50, size = 1362, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^3),x)
```

```
[Out] (x^(1/2)*((2*(2*b*(3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x) - 14*b*(log(2/(exp(2*a)*exp(
2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b
```

```
*x)))/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*
b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x
)) + (56*b^2*x)/(3*(2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))))/(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))^2 - (x^(1/2)*((280*b)
/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (280*b^2*x)/(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))/
(2*b*x^2 - x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (70*2^(1/2)*b^(3/2)*log((b^(1/2)*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)
*x^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x)^8 + 112*a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 - 448*a^3*(2*a -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^5 + 1120*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(e
xp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - 17
92*a^5*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(
2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 1792*a^6*(2*a - log((2*exp(2*a)*e
xp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) +
2*b*x)^2 + 256*a^8 - 16*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^7 - 1024*a^7*(2*a
- log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2
*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^(9/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} \operatorname{atanh}^{3}(\tanh(a + bx))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**3), x)
```

3.214 $\int \frac{1}{x^{7/2} \tanh^{-1}(\tanh(a+bx))^3} dx$

Optimal. Leaf size=228

$$-\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}}+\frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{7}{4b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))}+$$

```
[Out] -63/4*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))^(1/2))/(b*
x-arctanh(tanh(b*x+a)))^(11/2)+21/4*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^4+
63/20/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^3+9/4/b/x^(7/2)/(b*x-arctanh(tanh(
b*x+a)))^2+7/4/b^2/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))-1/2/b/x^(7/2)/arctanh
(tanh(b*x+a))^2+7/4/b^2/x^(9/2)/arctanh(tanh(b*x+a))+63/4*b^2/(b*x-arctanh(
tanh(b*x+a)))^5/x^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2168, 2163, 2162}

$$\frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{7}{4b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))}-\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}}+-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] (-63*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[b*x - ArcTanh[Tanh[a + b*x]]]])
/(4*(b*x - ArcTanh[Tanh[a + b*x]])^(11/2)) + (63*b^2)/(4*Sqrt[x]*(b*x - Arc
Tanh[Tanh[a + b*x]])^5) + (21*b)/(4*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^
4) + 63/(20*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3) + 9/(4*b*x^(7/2)*(b*x
 - ArcTanh[Tanh[a + b*x]])^2) + 7/(4*b^2*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*
x]])) - 1/(2*b*x^(7/2)*ArcTanh[Tanh[a + b*x]]^2) + 7/(4*b^2*x^(9/2)*ArcTanh
[Tanh[a + b*x]])
```

Rule 2162

```
Int[1/((u_)*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simpli
fy[D[v, x]]}, Simp[(-2*ArcTanh[Sqrt[v]/Rt[-((b*u - a*v)/a), 2]])/(a*Rt[-((b
*u - a*v)/a), 2]), x] /; NeQ[b*u - a*v, 0] && NegQ[(b*u - a*v)/a]] /; Piece
wiseLinearQ[u, v, x]
```

Rule 2163

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[v^(n + 1)/((n + 1)*(b*u - a*v)), x] - Dist[(a*(n + 1))/((n
+ 1)*(b*u - a*v)), Int[v^(n + 1)/u, x], x] /; NeQ[b*u - a*v, 0]] /; Piecewi
seLinearQ[u, v, x] && LtQ[n, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
```

```
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^3}\,dx = -\frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2} - \frac{7\int \frac{1}{x^{9/2}\tanh^{-1}(\tanh(a+bx))^2}\,dx}{4b}$$

$$= -\frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{7}{4b^2x^{9/2}\tanh^{-1}(\tanh(a+bx))} + \frac{63\int \frac{1}{x^{11/2}\,\mathrm{t}}$$

$$= \frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} - \frac{1}{2bx^{7/2}\tanh^{-1}(\tanh(a+bx))^2} + \frac{}{4b^2x}$$

$$= \frac{9}{4bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{7}{4b^2x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{63}{20x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{9}{4bx^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

$$= \frac{21b}{4x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{63}{20x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

$$= \frac{63b^2}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5} + \frac{21b}{4x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} +$$

$$= -\frac{63b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{bx-\tanh^{-1}(\tanh(a+bx))}}\right)}{4\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^{11/2}} + \frac{63b^2}{4\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5}$$

Mathematica [A] time = 0.35, size = 174, normalized size = 0.76

$$\frac{1}{20}\left(-\frac{315b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))-bx}}\right)}{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^{11/2}} + \frac{75b^3\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh(a+bx))} - \frac{}{\tanh^{-1}(\tanl}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^3),x]
```

```
[Out] ((75*b^3*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]])
 - (315*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[-(b*x) + ArcTanh[Tanh[a + b*x
]]]])/(-(b*x) + ArcTanh[Tanh[a + b*x]])^(11/2) - (10*b^3*Sqrt[x])/(ArcTanh[
Tanh[a + b*x]]^2*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4) + (8*(36*b^2*x^2 - 7*
b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2))/(x^(5/2)*(b*x - Arc
Tanh[Tanh[a + b*x]])^5))/20
```

fricas [A] time = 0.65, size = 276, normalized size = 1.21

$$\left[\frac{315\left(b^4x^5+2\,ab^3x^4+a^2b^2x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right) - 2\left(315\,b^4x^4+525\,ab^3x^3+168\,a^2b^2x^2-24\,a^3bx+8\,a\right.}{40\left(a^5b^2x^5+2\,a^6bx^4+a^7x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(-b/a)*log((b*x - 2*a*
sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*(315*b^4*x^4 + 525*a*b^3*x^3 + 168*a
^2*b^2*x^2 - 24*a^3*b*x + 8*a^4)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*
x^3), 1/20*(315*(b^4*x^5 + 2*a*b^3*x^4 + a^2*b^2*x^3)*sqrt(b/a)*arctan(a*sq
rt(b/a)/(b*sqrt(x))) - (315*b^4*x^4 + 525*a*b^3*x^3 + 168*a^2*b^2*x^2 - 24*
a^3*b*x + 8*a^4)*sqrt(x))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)]
```

giac [A] time = 0.20, size = 80, normalized size = 0.35

$$-\frac{63\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^5}-\frac{15\,b^4x^{\frac{3}{2}}+17\,ab^3\sqrt{x}}{4\,(bx+a)^2a^5}-\frac{2\left(30\,b^2x^2-5\,abx+a^2\right)}{5\,a^5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="giac")
```

```
[Out] -63/4*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/4*(15*b^4*x^(3/2)
 + 17*a*b^3*sqrt(x))/((b*x + a)^2*a^5) - 2/5*(30*b^2*x^2 - 5*a*b*x + a^2)/(
a^5*x^(5/2))
```

maple [A] time = 0.33, size = 229, normalized size = 1.00

$$-\frac{15b^4x^{\frac{3}{2}}}{4\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^5\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^2}-\frac{17b^3a\sqrt{x}}{4\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^5\operatorname{arctanh}\left(\tanh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x)
```

```
[Out] -15/4/(arctanh(tanh(b*x+a))-b*x)^5*b^4/arctanh(tanh(b*x+a))^2*x^(3/2)-17/4/
(arctanh(tanh(b*x+a))-b*x)^5*b^3/arctanh(tanh(b*x+a))^2*a*x^(1/2)-17/4/(arc
tanh(tanh(b*x+a))-b*x)^5*b^3/arctanh(tanh(b*x+a))^2*x^(1/2)*(arctanh(tanh(b
*x+a))-b*x-a)-63/4/(arctanh(tanh(b*x+a))-b*x)^5*b^3/((arctanh(tanh(b*x+a))-
b*x)*b)^(1/2)*arctan(b*x^(1/2)/((arctanh(tanh(b*x+a))-b*x)*b)^(1/2))-2/5/(a
rctanh(tanh(b*x+a))-b*x)^3/x^(5/2)-12/(arctanh(tanh(b*x+a))-b*x)^5*b^2/x^(1
/2)+2/(arctanh(tanh(b*x+a))-b*x)^4*b/x^(3/2)
```

maxima [A] time = 0.43, size = 97, normalized size = 0.43

$$-\frac{315\,b^4x^4+525\,ab^3x^3+168\,a^2b^2x^2-24\,a^3bx+8\,a^4}{20\left(a^5b^2x^{\frac{9}{2}}+2\,a^6bx^{\frac{7}{2}}+a^7x^{\frac{5}{2}}\right)}-\frac{63\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^3,x, algorithm="maxima")
```

```
[Out] -1/20*(315*b^4*x^4 + 525*a*b^3*x^3 + 168*a^2*b^2*x^2 - 24*a^3*b*x + 8*a^4)/
(a^5*b^2*x^(9/2) + 2*a^6*b*x^(7/2) + a^7*x^(5/2)) - 63/4*b^3*arctan(b*sqrt(
x)/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 1.95, size = 2151, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^3),x)
```

```
[Out] 16/(5*x^(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (x*((((448*b^4)/(3*(2*a*b - b*(
2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (16*b^3*(2*b*(
3*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 6*b*x) - 14*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)))/(3*(2*a*b - b
*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4))*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x))/(2*b) - (112*b^3)/((2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*
(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) + 2*b*x)^2) + (8*b^2*(2*b*(3*log(2/(exp(2*a)*exp(2*b*x) + 1
)) - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x) - 14
*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)))/((2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(
log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*ex
p(2*b*x) + 1)) + 2*b*x)^3)) - (4*b*(2*b*(3*log(2/(exp(2*a)*exp(2*b*x) + 1))
 - 3*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x) - 14*b
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)))/((2*a*b - b*(2*a - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))*(lo
g(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)^2))/(x^(3/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*e
xp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^2) - ((336*b^2)/(log(2/
(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + 2*b*x)^4 - (1008*b^3*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log(
(2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)/(x^(1/2)*(lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(
2*b*x) + 1)))) + (252*2^(1/2)*b^(5/2)*log((b^(1/2)*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^(1/2)*(2^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x) - 4*b^(1/2)*x^(1/2)*(log(2/(exp(2*
a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)
) + 2*b*x)^(1/2) + 2*2^(1/2)*b*x)*((2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^10 + 180*
a^2*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^8 - 960*a^3*(2*a - log((2*exp(2*a)*exp(2
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^7 + 3360*a^4*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^6 - 8064*a^5*(2*a - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^5 + 13440*a^6*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4 - 15360*a^7*
(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^3 + 11520*a^8*(2*a - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2 + 1024*a^10 - 20*a*(2*a - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*
x) + 1)) + log(2/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^9 - 5120*a^9*(2*a - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + log(2/(exp(2*a)*exp(
2*b*x) + 1)) + 2*b*x)))/(2*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1))))))/(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^(11
/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}} \operatorname{atanh}^{3}\left(\tanh\left(a+bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**3,x)
```

```
[Out] Integral(1/(x**(7/2)*atanh(tanh(a + b*x))**3), x)
```

3.215 $\int x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=142

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{5/2}}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+\right.}{8b^2}$$

```
[Out] -1/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^3/b^(5/2)+1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)-1/12*x^(3/2)*(b*x
-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)/b-1/8*(b*x-arctanh(tanh(b
*x+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2169, 2165}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{5/2}}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+\right.}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Ta
nh[a + b*x]])^3)/(8*b^(5/2)) + (x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/3 - (
x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(12*b)
 - (Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/
(8*b^2)
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx = \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{1}{6}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{}{\sqrt{\tanh}}$$

$$= \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh}}{12b}$$

$$= \frac{1}{3}x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh}}{12b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{5/2}} + \frac{1}{3}x^{5/2}\sqrt{\tan}$$

Mathematica [A] time = 0.09, size = 104, normalized size = 0.73

$$\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^3\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{8b^{5/2}}+\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(8bx\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(3*b^2*x^2 + 8*b*x*ArcTanh[Tanh[a + b
*x]] - 3*ArcTanh[Tanh[a + b*x]]^2))/(24*b^2) + ((-(b*x) + ArcTanh[Tanh[a +
b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(5/2))
```

fricas [A] time = 0.67, size = 141, normalized size = 0.99

$$\left[\frac{3a^3\sqrt{b}\log\left(2bx+2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8b^3x^2+2ab^2x-3a^2b\right)\sqrt{bx+a}\sqrt{x}}{48b^3}, -\frac{3a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+}}{b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^2*x
 - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

giac [A] time = 0.17, size = 60, normalized size = 0.42

$$\frac{1}{24}\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}-\frac{a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 1/8*a^3*log(abs(
-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2)
```

maple [B] time = 0.29, size = 304, normalized size = 2.14

$$\frac{x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{3b}-\frac{a\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{4b^2}+\frac{a^2\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8b^2}+\frac{\ln\left(\sqrt{b}\sqrt{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*arctanh(tanh(b*x+a))^(3/2)/b-1/4/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(3/2)+1/8/b^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/8/b^(5/2)*ln(b^
(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+3/8/b^(5/2)*a^2*ln(b^(1/2)*x^
(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+1/4/b^2*a*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3/8/b^(5/2)*a
*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a
)^2-1/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)
+1/8/b^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+
1/8/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*
x+a))-b*x-a)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \sqrt{\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)*sqrt(arctanh(tanh(b*x + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\, \sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] int(x^(3/2)*atanh(tanh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \sqrt{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(atanh(tanh(a + b*x))), x)
```

3.216 $\int \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx$

Optimal. Leaf size=104

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tan}}{}$$

```
[Out] -1/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2/b^(3/2)+1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)-1/4*(b*x-arctanh(
tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tan}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Ta
nh[a + b*x]])^2)/(4*b^(3/2)) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/2 - (
Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b)
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,dx=\frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{1}{4}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int\frac{}{\sqrt{\tanh}}$$

$$=\frac{1}{2}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh}}{4b}$$

$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{\tan}$$

Mathematica [A] time = 0.07, size = 84, normalized size = 0.81

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))+bx\right)}{4b}-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh\right.}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(b*x + ArcTanh[Tanh[a + b*x]]))/(4*b)
 - ((-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTan
h[Tanh[a + b*x]]]])/(4*b^(3/2))
```

fricas [A] time = 0.55, size = 114, normalized size = 1.10

$$\left[\frac{a^2\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(2\,b^2x+ab\right)\sqrt{bx+a}\,\sqrt{x}}{8\,b^2},\frac{a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(2\,b^2x+ab\right.}{4\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b
^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x +
 a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

giac [A] time = 0.18, size = 48, normalized size = 0.46

$$\frac{1}{4}\sqrt{bx+a}\left(2\,x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + 1/4*a^2*log(abs(-sqrt(b)*sqrt(x) +
sqrt(b*x + a)))/b^(3/2)
```

maple [B] time = 0.27, size = 174, normalized size = 1.67

$$\frac{\sqrt{x}\,\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{2b}-\frac{a\sqrt{x}\,\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{4b}-\frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)a^2}{4b^{\frac{3}{2}}}-\frac{a\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)/b-1/4/b*a*x^(1/2)*arctanh(tanh(b*x+a
))^(1/2)-1/4/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2-1/2
/b^(3/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x
+a))-b*x-a)-1/4/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))
^(1/2)-1/4/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(
tanh(b*x+a))-b*x-a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\,\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)*sqrt(arctanh(tanh(b*x + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\sqrt{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^(1/2),x)
[Out] int(x^(1/2)*atanh(tanh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{\mathrm{atanh}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(atanh(tanh(a + b*x))), x)
```

3.217 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} \, dx$

Optimal. Leaf size=61

$$\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}}$$

```
[Out] -arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x+
a)))/b^(1/2)+x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[x],x]
```

```
[Out] -((ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/Sqrt[b]) + Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}} \, dx &= \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\mathrm{t}} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{b}} + \sqrt{x}\sqrt{\tanh^{-1}(\mathrm{tanh}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 1.02

$$\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] + ((-(b*x) + ArcTanh[Tanh[a + b*x]])*L
og[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

fricas [A] time = 0.49, size = 93, normalized size = 1.52

$$\left[\frac{a\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b},\,-\frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)
)) - sqrt(b*x + a)*b*sqrt(x))/b]
```

giac [A] time = 0.46, size = 36, normalized size = 0.59

$$-\frac{a\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}+\sqrt{bx+a}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b) + sqrt(b*x + a)*sqrt(
x)
```

maple [A] time = 0.27, size = 75, normalized size = 1.23

$$\sqrt{x}\,\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}+\frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)a}{\sqrt{b}}+\frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\mathrm{arctanh}\left(\tanh\left(b\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+1/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tan
h(b*x+a))^(1/2))*a+1/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
*(arctanh(tanh(b*x+a))-b*x-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(tanh(b*x + a)))/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(1/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(1/2)/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/sqrt(x), x)
```

3.218 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}}\,dx$

Optimal. Leaf size=49

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*b^(1/2)-2*arctanh(tan
h(b*x+a))^(1/2)/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(3/2),x]
```

```
[Out] 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*Sqrt
[ArcTanh[Tanh[a + b*x]]])/Sqrt[x]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}}\,dx &= -\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}+b\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx\\ &= 2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 1.06

$$2\sqrt{b}\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] + 2*Sqrt[b]*Log[b*Sqrt[x] + Sqrt[
b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]
```

fricas [A] time = 0.56, size = 89, normalized size = 1.82

$$\left[\frac{\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x +
 a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
 sqrt(b*x + a)*sqrt(x))/x]
```

giac [A] time = 0.23, size = 57, normalized size = 1.16

$$-\sqrt{b}\,\log\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2\right)+\frac{4\,a\sqrt{b}}{\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(b)*log((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2) + 4*a*sqrt(b)/((sqrt(b)*s
qrt(x) - sqrt(b*x + a))^2 - a)
```

maple [B] time = 0.26, size = 149, normalized size = 3.04

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{x}}+\frac{2b\sqrt{x}\,\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}+\frac{2\sqrt{b}\,\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx\right.\right.}\right.}{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b/(arcta
nh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(1/2)/(arctanh(
tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a+2*b^(1/2
)/(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
*(arctanh(tanh(b*x+a))-b*x-a)
```

maxima [A] time = 0.46, size = 40, normalized size = 0.82

$$2\,\sqrt{b}\,\log\left(\frac{b\sqrt{x}}{\sqrt{ab}}+\sqrt{\frac{bx}{a}+1}\right)-\frac{2\,\sqrt{bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*log(b*sqrt(x)/sqrt(a*b) + sqrt(b*x/a + 1)) - 2*sqrt(b*x + a)/sqrt
(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(3/2),x)
[Out] int(atanh(tanh(a + b*x))^(1/2)/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(3/2),x)
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**(3/2), x)
```

3.219 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2/3*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}}\,dx=\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.97

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}\left(3bx-3\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(5/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(x^(3/2)*(3*b*x - 3*ArcTanh[Tanh[a + b*x]]
))
```

fricas [A] time = 0.53, size = 15, normalized size = 0.43

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

giac [B] time = 0.20, size = 59, normalized size = 1.69

$$\frac{4\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{4}+a^{2}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + a^2*b^(3/2))/((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)^3
```

maple [A] time = 0.27, size = 29, normalized size = 0.83

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)
```

maxima [A] time = 0.42, size = 15, normalized size = 0.43

$$-\frac{2\left(bx+a\right)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

mupad [B] time = 1.71, size = 210, normalized size = 6.00

$$\frac{2\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}-2\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\,\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{\sqrt{x}\,\left(3\,x\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-3\,x\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+6\,b\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(5/2),x)
```

```
[Out] (2*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))*(log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))
/2)^(1/2) - 2*log(2/(exp(2*a)*exp(2*b*x) + 1))*(log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/
(x^(1/2)*(3*x*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 3*x*log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 6*b*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(atanh(tanh(a + b*x)))/x**(5/2), x)
```

3.220 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}} \, dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 4/15*b*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5*
arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2171, 2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(7/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}} \, dx &= \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(2b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{5/2}} \, dx}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.67

$$\frac{2\left(5bx-3\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}{15x^{5/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(7/2),x]
```

```
[Out] (2*(5*b*x - 3*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/(15*x^(
5/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.57, size = 34, normalized size = 0.47

$$\frac{2\left(2\,b^2x^2-abx-3\,a^2\right)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

giac [A] time = 0.19, size = 112, normalized size = 1.56

$$\frac{8\left(15\,b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^6+5\,ab^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4+5\,a^2b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a^3b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(15*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 5*a*b^(5/2)*(sqrt(b)
*sqrt(x) - sqrt(b*x + a))^4 + 5*a^2*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a
))^2 - a^3*b^(5/2))/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

maple [A] time = 0.27, size = 59, normalized size = 0.82

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{5\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{5}{2}}}+\frac{4b\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{15\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(3/2)+4/15*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)
```

maxima [A] time = 0.44, size = 34, normalized size = 0.47

$$\frac{2\left(2\,b^2x^2-abx-3\,a^2\right)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

mupad [B] time = 1.41, size = 174, normalized size = 2.42

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,b^2\,x^2}{15\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}+\frac{4\,b\,x}{15\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(7/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((16*b^2*x^2)/(15*(log(2/(exp(2*a)*exp(2*b*x) +
 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) +
 (4*b*x)/(15*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - 2/5))/x^(5/2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(7/2),x)
```

```
[Out] Timed out
```

3.221 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}} dx$

Optimal. Leaf size=110

$$\frac{16b^2 \tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 16/105*b^2*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3+
8/35*b*arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/7*
arctanh(tanh(b*x+a))^(3/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2171, 2167}

$$\frac{16b^2 \tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(9/2),x]
```

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(105*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(35*x^(5/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(7*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}} dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(4b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{7/2}} dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+ \frac{}{35}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{3/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.60

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-42bx\tanh^{-1}(\tanh(a+bx))+15\tanh^{-1}(\tanh(a+bx))^2+35b^2x^2\right)}{105x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(9/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(35*b^2*x^2 - 42*b*x*ArcTanh[Tanh[a + b*x]]
 + 15*ArcTanh[Tanh[a + b*x]]^2))/(105*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]]
)^3)
```

fricas [A] time = 0.60, size = 45, normalized size = 0.41

$$-\frac{2\left(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3\right)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

giac [A] time = 0.20, size = 138, normalized size = 1.25

$$\frac{32\left(70\,b^{\frac{7}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^8+35\,ab^{\frac{7}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^6+21\,a^2b^{\frac{7}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-7\,a^3b^{\frac{7}{2}}\left(\sqrt{b}\,\sqrt{x}\right.\right.}{105\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 32/105*(70*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 + 35*a*b^(7/2)*(sqrt
(b)*sqrt(x) - sqrt(b*x + a))^6 + 21*a^2*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x
 + a))^4 - 7*a^3*b^(7/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + a^4*b^(7/2))
/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

maple [A] time = 0.28, size = 105, normalized size = 0.95

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{7\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{7}{2}}}-\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\operatorname{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{3}{2}}}\right)}{7\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(3/2)-8/7*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(t
anh(b*x+a))^(3/2)+2/15*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(
b*x+a))^(3/2))
```

maxima [A] time = 0.43, size = 45, normalized size = 0.41

$$-\frac{2\left(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3\right)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

mupad [B] time = 1.59, size = 234, normalized size = 2.13

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{32\,b^2\,x^2}{105\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}+\frac{128\,b^3\,x^3}{105\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}+\frac{\cdots}{35\,(\ln(\cdots}\right.}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(9/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((32*b^2*x^2)/(105*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
+ (128*b^3*x^3)/(105*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (4*b*x)/(35*(log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)) - 2/7))/x^(7/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

3.222 $\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{11/2}}\,dx$

Optimal. Leaf size=148

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 32/315*b^3*arctanh(tanh(b*x+a))^(3/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^4+
16/105*b^2*arctanh(tanh(b*x+a))^(3/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^3+
4/21*b*arctanh(tanh(b*x+a))^(3/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/9*
arctanh(tanh(b*x+a))^(3/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2171, 2167}

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(11/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(3/2))/(315*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(3/2))/(105*x^(5/2)*(b*x - ArcTa
nh[Tanh[a + b*x]])^3) + (4*b*ArcTanh[Tanh[a + b*x]]^(3/2))/(21*x^(7/2)*(b*x
 - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(9*x^(9/2)
*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{11/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(2b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{9/2}}\,dx}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{}{21}$$

$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{4b\tanh^{-1}(\tanh(a+bx))^{3/2}}{21x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} +$$

$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{3/2}}{315x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{3/2}}{105x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Mathematica [A] time = 0.07, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}\left(-189b^2x^2\tanh^{-1}(\tanh(a+bx)) + 135bx\tanh^{-1}(\tanh(a+bx))^2 - 35\tanh^{-1}(\tanh(a\right.}{315x^{9/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[Tanh[a + b*x]]]/x^(11/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(3/2)*(105*b^3*x^3 - 189*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 135*b*x*ArcTanh[Tanh[a + b*x]]^2 - 35*ArcTanh[Tanh[a + b*x]]^3))/(
315*x^(9/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.75, size = 56, normalized size = 0.38

$$\frac{2\left(16\,b^4x^4 - 8\,ab^3x^3 + 6\,a^2b^2x^2 - 5\,a^3bx - 35\,a^4\right)\sqrt{bx+a}}{315\,a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt(
b*x + a)/(a^4*x^(9/2))
```

giac [A] time = 0.20, size = 166, normalized size = 1.12

$$\frac{64\left(315\,b^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{10} + 189\,ab^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{8} + 84\,a^2b^{\frac{9}{2}}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{6} - 36\,a^3b^{\frac{9}{2}}\left(\sqrt{b}\sqrt{\right.\right.}{315\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] 64/315*(315*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^10 + 189*a*b^(9/2)*(s
qrt(b)*sqrt(x) - sqrt(b*x + a))^8 + 84*a^2*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(
b*x + a))^6 - 36*a^3*b^(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 + 9*a^4*b^
(9/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^5*b^(9/2))/((sqrt(b)*sqrt(x)
- sqrt(b*x + a))^2 - a)^9
```

maple [A] time = 0.30, size = 151, normalized size = 1.02

$$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}+\frac{2b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{15(\operatorname{arctanh}(\tanh(bx+a)) \cdots}\right.}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)}\right.}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(3/2)-4/3*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(t
anh(b*x+a))^(3/2)-4/7*b/(arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+
a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(3/2)+2/15*b/(arctanh(tanh(b*x+a))-b*
x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(3/2)))
```

maxima [A] time = 0.43, size = 56, normalized size = 0.38

$$\frac{2\left(16\,b^{4}x^{4}-8\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-5\,a^{3}bx-35\,a^{4}\right)\sqrt{bx+a}}{315\,a^{4}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] 2/315*(16*b^4*x^4 - 8*a*b^3*x^3 + 6*a^2*b^2*x^2 - 5*a^3*b*x - 35*a^4)*sqrt(
b*x + a)/(a^4*x^(9/2))
```

mupad [B] time = 1.52, size = 294, normalized size = 1.99

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{16\,b^2\,x^2}{105\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}+\frac{128\,b^3\,x^3}{315\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}+\cdots\right.}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(1/2)/x^(11/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((16*b^2*x^2)/(105*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
+ (128*b^3*x^3)/(315*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^4*x^4)/(315*(log(
2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 2*b*x)^4) + (4*b*x)/(63*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - 2/9))/x^(9
/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

3.223 $\int x^{3/2} \tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=177

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(a\right.}{64b^2}$$

```
[Out] 3/64*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^4/b^(5/2)+1/4*x^(5/2)*arctanh(tanh(b*x+a))^(3/2)-1/8*x^(5/2)*(b*x-
arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)+1/32*x^(3/2)*(b*x-arctanh(
tanh(b*x+a)))^2*arctanh(tanh(b*x+a))^(1/2)/b+3/64*(b*x-arctanh(tanh(b*x+a))
)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2169, 2165}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(a\right.}{64b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^4)/(64*b^(5/2)) - (x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sq
rt[ArcTanh[Tanh[a + b*x]]])/8 + (x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*S
qrt[ArcTanh[Tanh[a + b*x]]])/(32*b) + (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(64*b^2) + (x^(5/2)*ArcTanh[Tanh[a + b
*x]]^(3/2))/4
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rubi steps

$$\int x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx = \frac{1}{4}x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{1}{8}\left(3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int x^{3}$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{1}{4}x^{5/2}\tan$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{x^{3/2}\left(bx-\right.}{}$$

$$= -\frac{1}{8}x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{x^{3/2}\left(bx-\right.}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{5/2}} - \frac{1}{8}x^{5/2}\left(bx-\right.$$

Mathematica [A] time = 0.10, size = 122, normalized size = 0.69

$$\frac{\sqrt{b}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(11b^2x^2\tanh^{-1}(\tanh(a+bx)) + 11bx\tanh^{-1}(\tanh(a+bx))^2 - 3\tanh^{-1}(\tanh\right.}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b^3*x^3 + 11*b^2*x^2*ArcT
anh[Tanh[a + b*x]] + 11*b*x*ArcTanh[Tanh[a + b*x]]^2 - 3*ArcTanh[Tanh[a + b
*x]]^3) + 3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*Sqrt[x] + Sqrt[b]*Sqr
t[ArcTanh[Tanh[a + b*x]]]])/(64*b^(5/2))
```

fricas [A] time = 0.51, size = 163, normalized size = 0.92

$$\left[\frac{3\,a^4\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{128\,b^3}, -\frac{3\,a^4\sqrt{-b}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/
b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

giac [A] time = 0.72, size = 147, normalized size = 0.83

$$\frac{1}{384}\sqrt{2}\left(8\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4\,x+\frac{a}{b}\right)x-\frac{3\,a^2}{b^2}\right)\sqrt{x}-\frac{3\,a^3\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}\right)a+\sqrt{2}\left(\left(2\left(4\left(6\,x+\frac{a}{b}\right)x\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(2)*(8*sqrt(2)*(sqrt(b*x + a)*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(
x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))*a + sqrt(2)*
((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x + 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 1
5*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2))*b)
```

maple [B] time = 0.24, size = 471, normalized size = 2.66

$$\frac{x^{\frac{3}{2}} \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{4b} - \frac{a\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{8b^2} + \frac{a^2\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{32b^2} + \frac{3a^3\sqrt{x}\ \sqrt{\operatorname{arctar}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 1/4*x^(3/2)*arctanh(tanh(b*x+a))^(5/2)/b-1/8/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(5/2)+1/32/b^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/64/b^2*a^3*x^(
1/2)*arctanh(tanh(b*x+a))^(1/2)+3/64/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tan
h(b*x+a))^(1/2))*a^4+3/16/b^(5/2)*a^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a
))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+9/64/b^2*a^2*(arctanh(tanh(b*x+a))-b
*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+9/32/b^(5/2)*a^2*ln(b^(1/2)*x^(1/2
)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+1/16/b^2*a*(ar
ctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+9/64/b^2*a*(ar
ctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3/16/b^(5/2)
*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x
-a)^3-1/8/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(5/
2)+1/32/b^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/
2)+3/64/b^2*(arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/
2)+3/64/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tan
h(b*x+a))-b*x-a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)*arctanh(tanh(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\, \operatorname{atanh}(\tanh(a+b\,x))^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(3/2)*atanh(tanh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**(3/2)*atanh(tanh(a + b*x))**(3/2), x)
```

3.224 $\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx$

Optimal. Leaf size=139

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{3/2}}-\frac{1}{4}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(a\right.$$

```
[Out] 1/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))^3/b^(3/2)+1/3*x^(3/2)*arctanh(tanh(b*x+a))^(3/2)-1/4*x^(3/2)*(b*x-a
rctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)+1/8*(b*x-arctanh(tanh(b*x+a
)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{3/2}}-\frac{1}{4}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tan
h[a + b*x]])^3)/(8*b^(3/2)) - (x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[
ArcTanh[Tanh[a + b*x]]])/4 + (Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt
[ArcTanh[Tanh[a + b*x]]])/(8*b) + (x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2))/3
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}\,dx &= \frac{1}{3}x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}-\frac{1}{2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\int \sqrt{x}\sqrt{\ } \\
&= -\frac{1}{4}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+\frac{1}{3}x^{3/2}\tan \\
&= -\frac{1}{4}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+\frac{\sqrt{x}\left(bx-\right.}{} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{3/2}}-\frac{1}{4}x^{3/2}\left(bx-\right.
\end{aligned}$$

Mathematica [A] time = 0.08, size = 105, normalized size = 0.76

$$\frac{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{8b^{3/2}}+\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(8bx\tanh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b^2*x^2 + 8*b*x*ArcTanh[Tanh[a + 
b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2))/(24*b) + ((b*x - ArcTanh[Tanh[a + b*x]
])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(3/2))
```

fricas [A] time = 0.48, size = 140, normalized size = 1.01

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8\,b^3x^2+14\,ab^2x+3\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b^2},\ \frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-}}{b\sqrt{x}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b^2*
x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

giac [A] time = 0.32, size = 122, normalized size = 0.88

$$\frac{1}{48}\sqrt{2}\left(6\sqrt{2}\left(\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}\right)a+\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a^2}{b^2}\right)\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*(6*sqrt(2)*(sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + a^2*log(abs(-s
qrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2))*a + sqrt(2)*(sqrt(b*x + a)*(2*(4*
x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
 + a)))/b^(5/2))*b)
```

maple [B] time = 0.24, size = 304, normalized size = 2.19

$$\frac{\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{3b}-\frac{a\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{12b}-\frac{a^2\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{8b}-\frac{\ln\left(\sqrt{b}\,\sqrt{x}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)/b-1/12/b*a*x^(1/2)*arctanh(tanh(b*x+
a))^(3/2)-1/8/b*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-1/8/b^(3/2)*ln(b^(1/
2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3-3/8/b^(3/2)*a^2*ln(b^(1/2)*x^(1/
2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)-1/4/b*a*(arctan
h(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-3/8/b^(3/2)*a*ln(b
^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2-1
/12/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-1/8/b
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-1/8/b^(3
```

```
/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\ \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*arctanh(tanh(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\ \operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(1/2)*atanh(tanh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\ \operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(x)*atanh(tanh(a + b*x))**(3/2), x)
```

3.225 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=101

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4\sqrt{b}}-\frac{3}{4}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(a\right.$$

```
[Out] 3/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))^2/b^(1/2)+1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-3/4*(b*x-arctanh(t
anh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4\sqrt{b}}-\frac{3}{4}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(a\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/Sqrt[x],x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^2)/(4*Sqrt[b]) - (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*S
qrt[ArcTanh[Tanh[a + b*x]]])/4 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2))/2
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}-\frac{1}{4}\left(3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{\sqrt{\tanh^{-1}\ldots}}{\ldots}\\ &= -\frac{3}{4}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+\frac{1}{2}\sqrt{x}\tanh^{-1}(\text{ta}\ldots\\ &= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4\sqrt{b}}-\frac{3}{4}\sqrt{x}\left(bx-\tanh\ldots\right.\end{aligned}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 0.82

$$\frac{1}{4}\left(\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(5\tanh^{-1}(\tanh(a+bx))-3bx\right)+\frac{3\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}\right.}{\sqrt{b}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b*x + 5*ArcTanh[Tanh[a + b*x]]) +
 (3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTan
h[Tanh[a + b*x]]]])/Sqrt[b])/4
```

fricas [A] time = 0.56, size = 119, normalized size = 1.18

$$\left[\frac{3a^2\sqrt{b}\log\left(2bx+2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2b^2x+5ab\right)\sqrt{bx+a}\sqrt{x}}{8b}, -\frac{3a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\right.}{4b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,1]%%%}+%%%{6,[0,2,0
]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%%%},0,%%%{-4,[3,0,0
]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1]%%%}+%%%{-12,[1,2,
2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]%%%}+%%%{-40,[1,1,1
]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{4,[0,3,1]%
%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]%%%}+%%%{4,[0,2,1]%
%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%%},0,%%%{1,[4,0,0]%
%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{6,[2,2,2]%%
%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]%%%}+%%%{4,[2,1,1]%
%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%%%}+%%%{12,[1,3,1]%
%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]%%%}+%%%{4,[1,2,1]%
%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%%%}+%%%{1,[0,4,4]%%
%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%%}+%%%{1,[0,4,0]%%%
}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{-4,[0,3,1]%%%}
+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%%%{-4,[0,1,4]%%%}+%
%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters values [0,85.3561567818,
61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1
,1]%%%}+%%%{-4,[0,1,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1
,1,1]%%%}+%%%{4,[1,1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,
1]%%%}+%%%{6,[0,2,0]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%
```

```
%%},0,%%%{-4,[3,0,0]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1
]%%%}+%%%{-12,[1,2,2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]
%%%}+%%%{-40,[1,1,1]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]
%%%}+%%%{4,[0,3,1]%%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]
%%%}+%%%{4,[0,2,1]%%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%
%},0,%%%{1,[4,0,0]%%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%
%%}+%%%{6,[2,2,2]%%%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]
%%%}+%%%{4,[2,1,1]%%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%
%%}+%%%{12,[1,3,1]%%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]
%%%}+%%%{4,[1,2,1]%%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%
%%}+%%%{1,[0,4,4]%%%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%
%}+%%%{1,[0,4,0]%%%}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}
+%%%{-4,[0,3,1]%%%}+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%
%%{-4,[0,1,4]%%%}+%%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters value
s [0,71.707969239,78.6493344628]sqrt(2)/2/2/abs(b)*b^2/b*(2*(1/sqrt(2)/b*sq
rt(a+b*x)*sqrt(a+b*x)+3*a/2/sqrt(2)/b)*sqrt(a+b*x)*sqrt(-a*b+b*(a+b*x))-6*a
^2/2/sqrt(2)/sqrt(b)*ln(abs(sqrt(-a*b+b*(a+b*x))-sqrt(b)*sqrt(a+b*x))))
```

maple [B] time = 0.24, size = 165, normalized size = 1.63

$$\frac{\sqrt{x}\ \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{2}+\frac{3a\sqrt{x}\ \sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{4}+\frac{3\ln\left(\sqrt{b}\ \sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)a^2}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3/4*a*x^(1/2)*arctanh(tanh(b*x+a))^(
1/2)+3/4/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+3/2*a/b
^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))
-b*x-a)+3/4*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
+3/4/b^(1/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(1/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(3/2)/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(1/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/sqrt(x), x)
```

3.226 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=81

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}\right.$$

[Out] -3*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x+a)))*b^(1/2)-2*arctanh(tanh(b*x+a))^(3/2)/x^(1/2)+3*b*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}\right.$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(3/2),x]

[Out] -3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]]) + 3*b*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/Sqrt[x]

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u - a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n, -2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}} + (3b)\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}\,dx$$
$$= 3b\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}} - \frac{1}{2}\left(3b\left(bx - \text{ta}\right.\right.$$
$$= -3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx - \tanh^{-1}(\tanh(a+bx))\right) + 3b$$

Mathematica [A] time = 0.06, size = 77, normalized size = 0.95

$$\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(3bx - 2\tanh^{-1}(\tanh(a+bx))\right)}{\sqrt{x}} + 3\sqrt{b}\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(3/2),x]
```

```
[Out] ((3*b*x - 2*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] +
 3*Sqrt[b]*(-(b*x) + ArcTanh[Tanh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[A
rcTanh[Tanh[a + b*x]]]]
```

fricas [A] time = 0.50, size = 109, normalized size = 1.35

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(2\,bx + 2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x} + a\right) + 2\,\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{2\,x}, -\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right) - \sqrt{bx+}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sq
rt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,1]%%%}+%%%{6,[0,2,0
]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%%%},0,%%%{-4,[3,0,0
]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1]%%%}+%%%{-12,[1,2,
2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]%%%}+%%%{-40,[1,1,1
]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{4,[0,3,1]%
%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]%%%}+%%%{4,[0,2,1]%
%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%%},0,%%%{1,[4,0,0]%
%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{6,[2,2,2]%%
%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]%%%}+%%%{4,[2,1,1]%
```

```
%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%%%}+%%%{12,[1,3,1]%
%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]%%%}+%%%{4,[1,2,1]%
%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%%%}+%%%{1,[0,4,4]%%
%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%%}+%%%{1,[0,4,0]%%%
}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{-4,[0,3,1]%%%}
+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%%%{-4,[0,1,4]%%%}+%
%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters values [0,85.3561567818,
61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1
,1]%%%}+%%%{-4,[0,1,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1
,1,1]%%%}+%%%{4,[1,1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,
1]%%%}+%%%{6,[0,2,0]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%
%%},0,%%%{-4,[3,0,0]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1
]%%%}+%%%{-12,[1,2,2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]
%%%}+%%%{-40,[1,1,1]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]
%%%}+%%%{4,[0,3,1]%%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]
%%%}+%%%{4,[0,2,1]%%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%
%},0,%%%{1,[4,0,0]%%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%
%%}+%%%{6,[2,2,2]%%%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]
%%%}+%%%{4,[2,1,1]%%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%
%%}+%%%{12,[1,3,1]%%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]
%%%}+%%%{4,[1,2,1]%%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%
%%}+%%%{1,[0,4,4]%%%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%
%}+%%%{1,[0,4,0]%%%}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}
+%%%{-4,[0,3,1]%%%}+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%
%%{-4,[0,1,4]%%%}+%%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters value
s [0,71.707969239,78.6493344628]sqrt(2)/2/2*b/abs(b)*b^2/b*(2*(sqrt(2)*sqrt
(a+b*x)*sqrt(a+b*x)-3*sqrt(2)*a)*sqrt(a+b*x)*sqrt(-a*b+b*(a+b*x))/(-a*b+b*(
a+b*x))-6*a*sqrt(2)/sqrt(b)*ln(abs(sqrt(-a*b+b*(a+b*x))-sqrt(b)*sqrt(a+b*x)
)))
```

maple [B] time = 0.24, size = 280, normalized size = 3.46

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{x}}+\frac{2b\sqrt{x}\;\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}+\frac{3ba\sqrt{x}\;\sqrt{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx}+\frac{3\sqrt{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+2*b/(arcta
nh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+3*b/(arctanh(tanh(b
*x+a))-b*x)*a*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3*b^(1/2)/(arctanh(tanh(b*
x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+6*b^(1/2)/(ar
ctanh(tanh(b*x+a))-b*x)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(a
rctanh(tanh(b*x+a))-b*x-a)+3*b/(arctanh(tanh(b*x+a))-b*x)*(arctanh(tanh(b*x
+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+3*b^(1/2)/(arctanh(tanh(b*x+
a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a
))-b*x-a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(3/2),x)
[Out] int(atanh(tanh(a + b*x))^(3/2)/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(3/2),x)
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**(3/2), x)
```

3.227 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=70

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

```
[Out] 2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))-2/3*arctanh(t
anh(b*x+a))^(3/2)/x^(3/2)-2*b*arctanh(tanh(b*x+a))^(1/2)/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(5/2),x]
```

```
[Out] 2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*b*Sq
rt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] - (2*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*x
^(3/2))
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}}\,dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}+b\int \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}}{x^{3/2}}\,dx\\
&= -\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}+b^2\int \frac{}{\sqrt{x}\sqrt{\tanh^{-1}(}}\\
&= 2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2\tanh}{}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 74, normalized size = 1.06

$$-\frac{2\left(-3b^{3/2}x^{3/2}\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)+3bx\sqrt{\tanh^{-1}(\tanh(a+bx))}+\tanh^{-1}(\tanh(a+bx$$
$$3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(5/2),x]
```

```
[Out] (-2*(3*b*x*Sqrt[ArcTanh[Tanh[a + b*x]]] + ArcTanh[Tanh[a + b*x]]^(3/2) - 3*
b^(3/2)*x^(3/2)*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]))/(3*
x^(3/2))
```

fricas [A] time = 0.58, size = 109, normalized size = 1.56

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,(4\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{3\,x^2},\,-\frac{2\left(3\,\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(4\,b\right.}{3\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(4
*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*
x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,1]%%%}+%%%{6,[0,2,0
]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%%%},0,%%%{-4,[3,0,0
]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1]%%%}+%%%{-12,[1,2,
2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]%%%}+%%%{-40,[1,1,1
]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{4,[0,3,1]%
%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]%%%}+%%%{4,[0,2,1]%
%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%%},0,%%%{1,[4,0,0]%
%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{6,[2,2,2]%%
%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]%%%}+%%%{4,[2,1,1]%
%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%%%}+%%%{12,[1,3,1]%
%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]%%%}+%%%{4,[1,2,1]%
%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%%%}+%%%{1,[0,4,4]%%
%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%%}+%%%{1,[0,4,0]%%%
}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{-4,[0,3,1]%%%}
+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%%%{-4,[0,1,4]%%%}+%
%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters values [0,85.3561567818,
61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1
,1]%%%}+%%%{-4,[0,1,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1
,1,1]%%%}+%%%{4,[1,1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,
1]%%%}+%%%{6,[0,2,0]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%
%%},0,%%%{-4,[3,0,0]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1
]%%%}+%%%{-12,[1,2,2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]
```

```
%%%}+%%%{-40,[1,1,1]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]
%%%}+%%%{4,[0,3,1]%%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]
%%%}+%%%{4,[0,2,1]%%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%
%},0,%%%{1,[4,0,0]%%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%
%%}+%%%{6,[2,2,2]%%%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]
%%%}+%%%{4,[2,1,1]%%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%
%%}+%%%{12,[1,3,1]%%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]
%%%}+%%%{4,[1,2,1]%%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%
%%}+%%%{1,[0,4,4]%%%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%
%}+%%%{1,[0,4,0]%%%}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}
+%%%{-4,[0,3,1]%%%}+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%
%%{-4,[0,1,4]%%%}+%%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters value
s [0,71.707969239,78.6493344628]sqrt(2)/2/2/abs(b)*b^2/b*(2*(-24*sqrt(2)*b^
3*a/9/a*sqrt(a+b*x)*sqrt(a+b*x)+18*sqrt(2)*b^3*a^2/9/a)*sqrt(a+b*x)*sqrt(-a
*b+b*(a+b*x))/(-a*b+b*(a+b*x))^2-4*b^2*sqrt(2)/sqrt(b)*ln(abs(sqrt(-a*b+b*(
a+b*x))-sqrt(b)*sqrt(a+b*x))))
```

maple [B] time = 0.24, size = 315, normalized size = 4.50

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{3\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{3}{2}}}-\frac{4b\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{3\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}\sqrt{x}}+\frac{4b^{2}\sqrt{x}\ \mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{3\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(5/2)-4/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+4/3*b^2/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^2/(arctanh(t
anh(b*x+a))-b*x)^2*a*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(3/2)/(arctanh(
tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+4*b^
(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a)
)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+2*b^2/(arctanh(tanh(b*x+a))-b*x)^2*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(3/2)/(ar
ctanh(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(a
rctanh(tanh(b*x+a))-b*x-a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(3/2)/x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(5/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(3/2)/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(5/2),x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**(3/2)/x**(5/2), x)
```

3.228 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2/5*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(7/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 0.97

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2}\left(5bx-5\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(7/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(x^(5/2)*(5*b*x - 5*ArcTanh[Tanh[a + b*x]]
))
```

fricas [A] time = 0.44, size = 31, normalized size = 0.89

$$-\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{5\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(a*x^(5/2))
```

giac [A] time = 0.15, size = 33, normalized size = 0.94

$$-\frac{2\,(bx+a)^{\frac{5}{2}}b^{6}}{5\,((bx+a)b-ab)^{\frac{5}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="giac")
[Out] -2/5*(b*x + a)^(5/2)*b^6/(((b*x + a)*b - a*b)^(5/2)*a*abs(b))
```

maple [A] time = 0.26, size = 29, normalized size = 0.83

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{5\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x)
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)
```

maxima [A] time = 0.44, size = 15, normalized size = 0.43

$$-\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(7/2),x, algorithm="maxima")
[Out] -2/5*(b*x + a)^(5/2)/(a*x^(5/2))
```

mupad [B] time = 1.51, size = 332, normalized size = 9.49

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{5\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-5\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+10\,b\,x}+\frac{4\,b^2\,x^2}{5\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-5\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+10\,b\,x}-\frac{4\,b\,x\,(\ln\cdots}{5\,\ln(\cdots}\right.}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(7/2),x)
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(5*log(2/(exp(2*a)*
exp(2*b*x) + 1)) - 5*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 10*b*x) + (4*b^2*x^2)/(5*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 5*log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 10*b*x) - (4*b*x*(log(2/(exp
(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
 1)) + 2*b*x))/(5*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 5*log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 10*b*x)))/x^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(7/2),x)
[Out] Timed out
```

3.229 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 4/35*b*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/7*
arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(9/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx &= \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{7/2}}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.67

$$\frac{2\left(7bx-5\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{5/2}}{35x^{7/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(9/2),x]
```

```
[Out] (2*(7*b*x - 5*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(5/2))/(35*x^(
7/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.50, size = 45, normalized size = 0.62

$$\frac{2\left(2\,b^3x^3 - ab^2x^2 - 8\,a^2bx - 5\,a^3\right)\sqrt{bx + a}}{35\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*b^3*x^3 - a*b^2*x^2 - 8*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^2*x^(7/2)
)
```

giac [A] time = 0.16, size = 59, normalized size = 0.82

$$\frac{\sqrt{2}\left(\frac{2\,\sqrt{2}\,(bx+a)b^7}{a^2} - \frac{7\,\sqrt{2}\,b^7}{a}\right)(bx + a)^{\frac{5}{2}}b}{35\,((bx + a)b - ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*sqrt(2)*(2*sqrt(2)*(b*x + a)*b^7/a^2 - 7*sqrt(2)*b^7/a)*(b*x + a)^(5/2
)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

maple [A] time = 0.25, size = 59, normalized size = 0.82

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx + a\right)\right)^{\frac{5}{2}}}{7\left(\mathrm{arctanh}\left(\tanh\left(bx + a\right)\right) - bx\right)x^{\frac{7}{2}}} + \frac{4b\,\mathrm{arctanh}\left(\tanh\left(bx + a\right)\right)^{\frac{5}{2}}}{35\left(\mathrm{arctanh}\left(\tanh\left(bx + a\right)\right) - bx\right)^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(5/2)+4/35*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)
```

maxima [A] time = 0.43, size = 34, normalized size = 0.47

$$\frac{2\left(2\,b^2x^2 - 3\,abx - 5\,a^2\right)(bx + a)^{\frac{3}{2}}}{35\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] 2/35*(2*b^2*x^2 - 3*a*b*x - 5*a^2)*(b*x + a)^(3/2)/(a^2*x^(7/2))
```

mupad [B] time = 1.54, size = 228, normalized size = 3.17

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7} - \frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{7} - \frac{6\,b\,x}{35} + \frac{4\,b^2\,x^2}{35\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)} + \frac{\quad}{35\left(\ln\left(\cdots\right.\right.}\right.}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(9/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/7 - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/7 - (6*b*x)/35 + (4*b^2*x^2)/
(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)) + (16*b^3*x^3)/(35*(log(2/(exp(2*a)*exp(2*b*x
) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2
)))/x^(7/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(9/2),x)
```

```
[Out] Timed out
```

3.230 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

```
[Out] 16/315*b^2*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^3+
8/63*b*arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/9*
arctanh(tanh(b*x+a))^(5/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(11/2),x]
```

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(315*x^(5/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(63*x^(7/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(9*x^(9/2)*(b*x - ArcT
anh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx &= \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{9/2}}\,dx}{9\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\
&= \frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\\
&= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{315x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{5/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.60

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-90bx\tanh^{-1}(\tanh(a+bx))+35\tanh^{-1}(\tanh(a+bx))^2+63b^2x^2\right)}{315x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(11/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(63*b^2*x^2 - 90*b*x*ArcTanh[Tanh[a + b*x]]
 + 35*ArcTanh[Tanh[a + b*x]]^2))/(315*x^(9/2)*(b*x - ArcTanh[Tanh[a + b*x]]
)^3)
```

fricas [A] time = 0.53, size = 56, normalized size = 0.51

$$-\frac{2\left(8\,b^4x^4-4\,ab^3x^3+3\,a^2b^2x^2+50\,a^3bx+35\,a^4\right)\sqrt{bx+a}}{315\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(8*b^4*x^4 - 4*a*b^3*x^3 + 3*a^2*b^2*x^2 + 50*a^3*b*x + 35*a^4)*sqrt
(b*x + a)/(a^3*x^(9/2))
```

giac [A] time = 0.15, size = 78, normalized size = 0.71

$$-\frac{\sqrt{2}\left(\frac{63\sqrt{2}b^9}{a}+4\left(\frac{2\sqrt{2}(bx+a)b^9}{a^3}-\frac{9\sqrt{2}b^9}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{315\left((bx+a)b-ab\right)^{\frac{9}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -1/315*sqrt(2)*(63*sqrt(2)*b^9/a + 4*(2*sqrt(2)*(b*x + a)*b^9/a^3 - 9*sqrt(
2)*b^9/a^2)*(b*x + a))*(b*x + a)^(5/2)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

maple [A] time = 0.27, size = 105, normalized size = 0.95

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{9\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{9}{2}}}-\frac{8b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{2b\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{35(\operatorname{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{5}{2}}}\right)}{9\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(5/2)-8/9*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(t
anh(b*x+a))^(5/2)+2/35*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(5/2)*arctanh(tanh(
b*x+a))^(5/2))
```

maxima [A] time = 0.44, size = 45, normalized size = 0.41

$$-\frac{2\left(8\,b^3x^3-12\,ab^2x^2+15\,a^2bx+35\,a^3\right)(bx+a)^{\frac{3}{2}}}{315\,a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] -2/315*(8*b^3*x^3 - 12*a*b^2*x^2 + 15*a^2*b*x + 35*a^3)*(b*x + a)^(3/2)/(a^
3*x^(9/2))
```

mupad [B] time = 1.64, size = 288, normalized size = 2.62

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{9}-\frac{2\,b\,x}{21}+\frac{4\,b^2\,x^2}{105\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{}{315\,\left(\ln\right.}\right.}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(11/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/9 - log((2*ex
p(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/9 - (2*b*x)/21 + (4*b^2*x^2)/
(105*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)) + (32*b^3*x^3)/(315*(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2) + (128*b^4*x^4)/(315*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)))/x^(9/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(11/2),x)
```

```
[Out] Timed out
```

3.231 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=148

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

```
[Out] 32/1155*b^3*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^4
+16/231*b^2*arctanh(tanh(b*x+a))^(5/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^3
+4/33*b*arctanh(tanh(b*x+a))^(5/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/1
1*arctanh(tanh(b*x+a))^(5/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(13/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(5/2))/(1155*x^(5/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(5/2))/(231*x^(7/2)*(b*x - ArcT
anh[Tanh[a + b*x]])^3) + (4*b*ArcTanh[Tanh[a + b*x]]^(5/2))/(33*x^(9/2)*(b*
x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(11*x^(11
/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{13/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{11/2}}\,dx}{11\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{4b\tanh^{-1}(\tanh(a+bx))^{5/2}}{33x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{5/2}}{1155x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{5/2}}{231x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx)\right.}$$

Mathematica [A] time = 0.08, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}\left(-495b^2x^2\tanh^{-1}(\tanh(a+bx))+385bx\tanh^{-1}(\tanh(a+bx))^2-105\tanh^{-1}(\tan\right.}{1155x^{11/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(3/2)/x^(13/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(5/2)*(231*b^3*x^3 - 495*b^2*x^2*ArcTanh[Tanh[a +
 b*x]] + 385*b*x*ArcTanh[Tanh[a + b*x]]^2 - 105*ArcTanh[Tanh[a + b*x]]^3))/
(1155*x^(11/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.51, size = 67, normalized size = 0.45

$$\frac{2\left(16\,b^5x^5-8\,ab^4x^4+6\,a^2b^3x^3-5\,a^3b^2x^2-140\,a^4bx-105\,a^5\right)\sqrt{bx+a}}{1155\,a^4x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*b^5*x^5 - 8*a*b^4*x^4 + 6*a^2*b^3*x^3 - 5*a^3*b^2*x^2 - 140*a^4*
b*x - 105*a^5)*sqrt(b*x + a)/(a^4*x^(11/2))
```

giac [A] time = 0.17, size = 97, normalized size = 0.66

$$-\frac{\sqrt{2}\left(\frac{231\,\sqrt{2}\,b^{11}}{a}-2\left(\frac{99\,\sqrt{2}\,b^{11}}{a^2}+4\left(\frac{2\,\sqrt{2}\,(bx+a)b^{11}}{a^4}-\frac{11\,\sqrt{2}\,b^{11}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{5}{2}}b}{1155\left((bx+a)b-ab\right)^{\frac{11}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] -1/1155*sqrt(2)*(231*sqrt(2)*b^11/a - 2*(99*sqrt(2)*b^11/a^2 + 4*(2*sqrt(2)
*(b*x + a)*b^11/a^4 - 11*sqrt(2)*b^11/a^3)*(b*x + a))*(b*x + a))*(b*x + a)^
(5/2)*b/(((b*x + a)*b - a*b)^(11/2)*abs(b))
```

maple [A] time = 0.39, size = 151, normalized size = 1.02

$$-\frac{2 \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{11 \left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right) x^{\frac{11}{2}}}-\frac{12b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}-\frac{4b\left(-\frac{\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{7(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{7}{2}}}+\frac{2b\operatorname{arctanh}(\tanh(bx}{35(\operatorname{arctanh}(\tanh(bx+a}\right.}{9(\operatorname{arctanh}(\tanh(bx+a))-bx)}\right.}{11 \left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x)
```

```
[Out] -2/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arctanh(tanh(b*x+a))^(5/2)-12/11*
b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arcta
nh(tanh(b*x+a))^(5/2)-4/9*b/(arctanh(tanh(b*x+a))-b*x)*(-1/7/(arctanh(tanh(
b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(5/2)+2/35*b/(arctanh(tanh(b*x+a)
)-b*x)^2/x^(5/2)*arctanh(tanh(b*x+a))^(5/2)))
```

maxima [A] time = 0.44, size = 56, normalized size = 0.38

$$\frac{2 \left(16 \, b^{4} x^{4} - 24 \, a b^{3} x^{3} + 30 \, a^{2} b^{2} x^{2} - 35 \, a^{3} b x - 105 \, a^{4}\right) (b x + a)^{\frac{3}{2}}}{1155 \, a^{4} x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(16*b^4*x^4 - 24*a*b^3*x^3 + 30*a^2*b^2*x^2 - 35*a^3*b*x - 105*a^4)*
(b*x + a)^(3/2)/(a^4*x^(11/2))
```

mupad [B] time = 1.74, size = 348, normalized size = 2.35

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{11}-\frac{\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)}{11}-\frac{2\,b\,x}{33}+\frac{4\,b^2\,x^2}{231\,\left(\ln\left(\frac{2}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}}{{\mathrm{e}}^{2\,a}\,{\mathrm{e}}^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{\quad}{385\,\left(\ln\left(\frac{\quad}{{\mathrm{e}}^{2}\right.\right.}\right.}{x^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(3/2)/x^(13/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))/11 - log((2*e
xp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/11 - (2*b*x)/33 + (4*b^2*x^2
)/(231*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (16*b^3*x^3)/(385*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^2) + (128*b^4*x^4)/(1155*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^5*x^5)/(11
55*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a
)*exp(2*b*x) + 1)) + 2*b*x)^4)))/x^(11/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(3/2)/x**(13/2),x)
```

```
[Out] Timed out
```

3.232 $\int \sqrt{x} \tanh^{-1}(\tanh(a+bx))^{5/2} \, dx$

Optimal. Leaf size=174

$$-\frac{5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{3/2}}+\frac{5}{32}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\mathrm{ta}\right.$$

```
[Out] -5/64*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh
(b*x+a)))^4/b^(3/2)-5/24*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*
x+a))^(3/2)+1/4*x^(3/2)*arctanh(tanh(b*x+a))^(5/2)+5/32*x^(3/2)*(b*x-arctan
h(tanh(b*x+a)))^2*arctanh(tanh(b*x+a))^(1/2)-5/64*(b*x-arctanh(tanh(b*x+a))
)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2169, 2165}

$$-\frac{5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{3/2}}+\frac{5}{32}x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\mathrm{ta}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^4)/(64*b^(3/2)) + (5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])
^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/32 - (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*
x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(64*b) - (5*x^(3/2)*(b*x - ArcTanh[Tan
h[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2))/24 + (x^(3/2)*ArcTanh[Tanh[a + b
*x]]^(5/2))/4
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}\,dx = \frac{1}{4}x^{3/2}\tanh^{-1}(\tanh(a+bx))^{5/2} - \frac{1}{8}\left(5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int \sqrt{x}\,\mathrm{t}$$

$$= -\frac{5}{24}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2} + \frac{1}{4}x^{3/2}\tanh$$

$$= \frac{5}{32}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5}{24}x^{3/2}\left(bx\right.$$

$$= \frac{5}{32}x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5\sqrt{x}\left(bx-\right.}{}$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}{64b^{3/2}} + \frac{5}{32}x^{3/2}\left(bx\right.$$

Mathematica [A] time = 0.09, size = 121, normalized size = 0.70

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-55b^2x^2\tanh^{-1}(\tanh(a+bx)) + 73bx\tanh^{-1}(\tanh(a+bx))^2 + 15\tanh^{-1}(\tanh(a\right.}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^3*x^3 - 55*b^2*x^2*ArcTanh[Tanh
[a + b*x]] + 73*b*x*ArcTanh[Tanh[a + b*x]]^2 + 15*ArcTanh[Tanh[a + b*x]]^3)
)/(192*b) - (5*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4*Log[b*Sqrt[x] + Sqrt[b]*
Sqrt[ArcTanh[Tanh[a + b*x]]]])/(64*b^(3/2))
```

fricas [A] time = 0.40, size = 162, normalized size = 0.93

$$\left[\frac{15\,a^4\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{384\,b^2}, \frac{15\,a^4\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt
(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

giac [A] time = 0.17, size = 204, normalized size = 1.17

$$\frac{1}{384}\sqrt{2}\left(48\sqrt{2}\left(\sqrt{bx+a}\left(2x+\frac{a}{b}\right)\sqrt{x}+\frac{a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}\right)a^2+16\sqrt{2}\left(\sqrt{bx+a}\left(2\left(4x+\frac{a}{b}\right)x-\frac{3a}{b^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(2)*(48*sqrt(2)*(sqrt(b*x + a)*(2*x + a/b)*sqrt(x) + a^2*log(abs(
-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2))*a^2 + 16*sqrt(2)*(sqrt(b*x + a)
```

```
*(2*(4*x + a/b)*x - 3*a^2/b^2)*sqrt(x) - 3*a^3*log(abs(-sqrt(b)*sqrt(x) + s
qrt(b*x + a)))/b^(5/2))*a*b + sqrt(2)*((2*(4*(6*x + a/b)*x - 5*a^2/b^2)*x +
 15*a^3/b^3)*sqrt(b*x + a)*sqrt(x) + 15*a^4*log(abs(-sqrt(b)*sqrt(x) + sqrt
(b*x + a)))/b^(7/2))*b^2)
```

maple [B] time = 0.24, size = 471, normalized size = 2.71

$$\frac{\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{4b}-\frac{a\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{24b}-\frac{5a^{2}\sqrt{x}\ \operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{96b}-\frac{5a^{3}\sqrt{x}\ \sqrt{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 1/4*x^(1/2)*arctanh(tanh(b*x+a))^(7/2)/b-1/24/b*a*x^(1/2)*arctanh(tanh(b*x+
a))^(5/2)-5/96/b*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-5/64/b*a^3*x^(1/2)*
arctanh(tanh(b*x+a))^(1/2)-5/64/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x
+a))^(1/2))*a^4-5/16/b^(3/2)*a^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1
/2))*(arctanh(tanh(b*x+a))-b*x-a)-15/64/b*a^2*(arctanh(tanh(b*x+a))-b*x-a)*
x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-15/32/b^(3/2)*a^2*ln(b^(1/2)*x^(1/2)+arc
tanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2-5/48/b*a*(arctanh(t
anh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-15/64/b*a*(arctanh(ta
nh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/16/b^(3/2)*a*ln(b^
(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3-1/
24/b*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)-5/96/b
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)-5/64/b*(
arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/64/b^(3/
2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x
-a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\ \operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*arctanh(tanh(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\ \operatorname{atanh}(\tanh(a+b\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] int(x^(1/2)*atanh(tanh(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.233 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=136

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8\sqrt{b}}+\frac{5}{8}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(\right.$$

```
[Out] -5/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^3/b^(1/2)-5/12*(b*x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(3/
2)*x^(1/2)+1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/8*(b*x-arctanh(tanh(b*x
+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8\sqrt{b}}+\frac{5}{8}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\,\left(bx-\tanh^{-1}(\tanh(\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/Sqrt[x],x]
```

```
[Out] (-5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^3)/(8*Sqrt[b]) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^
2*Sqrt[ArcTanh[Tanh[a + b*x]]])/8 - (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]
])*ArcTanh[Tanh[a + b*x]]^(3/2))/12 + (Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)
)/3
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2} - \frac{1}{6}\left(5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{\tanh^{-1}}{}$$

$$= -\frac{5}{12}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2} + \frac{1}{3}\sqrt{x}\tanh^{-1}$$

$$= \frac{5}{8}\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{5}{12}\sqrt{x}\left(bx-\right.$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8\sqrt{b}} + \frac{5}{8}\sqrt{x}\left(bx-\right.$$

Mathematica [A] time = 0.06, size = 101, normalized size = 0.74

$$\frac{1}{24}\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-40bx\tanh^{-1}(\tanh(a+bx)) + 33\tanh^{-1}(\tanh(a+bx))^2 + 15b^2x^2\right) + \frac{5\left(\tanh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 40*b*x*ArcTanh[Tanh[a +
 b*x]] + 33*ArcTanh[Tanh[a + b*x]]^2))/24 + (5*(-(b*x) + ArcTanh[Tanh[a + b
*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*Sqrt[b])
```

fricas [A] time = 0.56, size = 141, normalized size = 1.04

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,bx + 2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x} + a\right) + 2\left(8\,b^3x^2 + 26\,ab^2x + 33\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b}, -\frac{15\,a^3\sqrt{-b}\,\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a*b^
2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,1]%%%}+%%%{6,[0,2,0
]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%%%},0,%%%{-4,[3,0,0
]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1]%%%}+%%%{-12,[1,2,
2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]%%%}+%%%{-40,[1,1,1
```

```
]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{4,[0,3,1]%
%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]%%%}+%%%{4,[0,2,1]%
%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%%},0,%%%{1,[4,0,0]%
%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{6,[2,2,2]%%
%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]%%%}+%%%{4,[2,1,1]%
%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%%%}+%%%{12,[1,3,1]%
%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]%%%}+%%%{4,[1,2,1]%
%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%%%}+%%%{1,[0,4,4]%%
%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%%}+%%%{1,[0,4,0]%%%
}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{-4,[0,3,1]%%%}
+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%%%{-4,[0,1,4]%%%}+%
%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters values [0,85.3561567818,
61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1
,1]%%%}+%%%{-4,[0,1,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1
,1,1]%%%}+%%%{4,[1,1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,
1]%%%}+%%%{6,[0,2,0]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%
%%},0,%%%{-4,[3,0,0]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1
]%%%}+%%%{-12,[1,2,2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]
%%%}+%%%{-40,[1,1,1]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]
%%%}+%%%{4,[0,3,1]%%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]
%%%}+%%%{4,[0,2,1]%%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%
%},0,%%%{1,[4,0,0]%%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%
%%}+%%%{6,[2,2,2]%%%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]
%%%}+%%%{4,[2,1,1]%%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%
%%}+%%%{12,[1,3,1]%%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]
%%%}+%%%{4,[1,2,1]%%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%
%%}+%%%{1,[0,4,4]%%%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%
%}+%%%{1,[0,4,0]%%%}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}
+%%%{-4,[0,3,1]%%%}+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%
%%{-4,[0,1,4]%%%}+%%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters value
s [0,71.707969239,78.6493344628]sqrt(2)/2/abs(b)*b^2/b*(2*((1/3/sqrt(2)/b*s
qrt(a+b*x)*sqrt(a+b*x)+30*a/72/sqrt(2)/b)*sqrt(a+b*x)*sqrt(a+b*x)+45*a^2/72
/sqrt(2)/b)*sqrt(a+b*x)*sqrt(-a*b+b*(a+b*x))-10*a^3/8/sqrt(2)/sqrt(b)*ln(ab
s(sqrt(-a*b+b*(a+b*x))-sqrt(b)*sqrt(a+b*x))))
```

maple [B] time = 0.24, size = 286, normalized size = 2.10

$$\frac{\sqrt{x}\ \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{3}+\frac{5a\sqrt{x}\ \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}{12}+\frac{5a^{2}\sqrt{x}\ \sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{8}+\frac{5\ln\left(\sqrt{b}\ \sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/12*a*x^(1/2)*arctanh(tanh(b*x+a))^
(3/2)+5/8*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5/8/b^(1/2)*ln(b^(1/2)*x^(
1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+15/8*a^2/b^(1/2)*ln(b^(1/2)*x^(1/2)+ar
ctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+5/4*a*(arctanh(tanh(
b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15/8*a/b^(1/2)*ln(b^(1/2)
*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+5/12*(a
rctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5/8*(arctanh(
tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5/8/b^(1/2)*ln(b^(
1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/sqrt(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(1/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(5/2)/x^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(1/2),x)
```

```
[Out] Timed out
```

3.234 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}-\frac{15}{4}b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh}$$

```
[Out] 15/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2*b^(1/2)-2*arctanh(tanh(b*x+a))^(5/2)/x^(1/2)+5/2*b*arctanh(tanh(
b*x+a))^(3/2)*x^(1/2)-15/4*b*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctanh(tan
h(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{3/2}-\frac{15}{4}b\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(3/2),x]
```

```
[Out] (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x -
ArcTanh[Tanh[a + b*x]])^2)/4 - (15*b*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])
*Sqrt[ArcTanh[Tanh[a + b*x]]])/4 + (5*b*Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2
))/2 - (2*ArcTanh[Tanh[a + b*x]]^(5/2))/Sqrt[x]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{3/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}} + (5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{\sqrt{x}}\,dx$$

$$= \frac{5}{2}b\sqrt{x}\,\tanh^{-1}(\tanh(a+bx))^{3/2} - \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{\sqrt{x}} - \frac{1}{4}\left(15b\left(bx - \text{t}\right.\right.$$

$$= -\frac{15}{4}b\sqrt{x}\left(bx - \tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))} + \frac{5}{2}b\sqrt{x}\,\text{tan}$$

$$= \frac{15}{4}\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx - \tanh^{-1}(\tanh(a+bx))\right)^2 - \frac{15}{4}$$

Mathematica [A] time = 0.08, size = 101, normalized size = 0.83

$$\frac{15}{4}\sqrt{b}\left(\tanh^{-1}(\tanh(a+bx)) - bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))} + b\sqrt{x}\right) - \frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-25\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(3/2),x]
```

```
[Out] -1/4*(Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 25*b*x*ArcTanh[Tanh[a + b*
x]] + 8*ArcTanh[Tanh[a + b*x]]^2))/Sqrt[x] + (15*Sqrt[b]*(-(b*x) + ArcTanh[
Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/4
```

fricas [A] time = 0.48, size = 137, normalized size = 1.13

$$\left[\frac{15\,a^2\sqrt{b}\,x\log\left(2\,bx + 2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x} + a\right) + 2\left(2\,b^2x^2 + 9\,abx - 8\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{8\,x}, -\frac{15\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{b}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqrt(
-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x - 8
*a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,1]%%%}+%%%{6,[0,2,0
]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%%%},0,%%%{-4,[3,0,0
]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1]%%%}+%%%{-12,[1,2,
2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]%%%}+%%%{-40,[1,1,1
]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{4,[0,3,1]%
```

```
%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]%%%}+%%%{4,[0,2,1]%
%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%%},0,%%%{1,[4,0,0]%
%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{6,[2,2,2]%%
%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]%%%}+%%%{4,[2,1,1]%
%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%%%}+%%%{12,[1,3,1]%
%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]%%%}+%%%{4,[1,2,1]%
%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%%%}+%%%{1,[0,4,4]%%
%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%%}+%%%{1,[0,4,0]%%%
}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{-4,[0,3,1]%%%}
+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%%%{-4,[0,1,4]%%%}+%
%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters values [0,85.3561567818,
61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1
,1]%%%}+%%%{-4,[0,1,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1
,1,1]%%%}+%%%{4,[1,1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,
1]%%%}+%%%{6,[0,2,0]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%
%%},0,%%%{-4,[3,0,0]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1
]%%%}+%%%{-12,[1,2,2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]
%%%}+%%%{-40,[1,1,1]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]
%%%}+%%%{4,[0,3,1]%%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]
%%%}+%%%{4,[0,2,1]%%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%
%},0,%%%{1,[4,0,0]%%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%
%%}+%%%{6,[2,2,2]%%%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]
%%%}+%%%{4,[2,1,1]%%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%
%%}+%%%{12,[1,3,1]%%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]
%%%}+%%%{4,[1,2,1]%%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%
%%}+%%%{1,[0,4,4]%%%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%
%}+%%%{1,[0,4,0]%%%}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}
+%%%{-4,[0,3,1]%%%}+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%
%%{-4,[0,1,4]%%%}+%%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters value
s [0,71.707969239,78.6493344628]sqrt(2)/2*b/abs(b)*b^2/b*(2*((1/2/sqrt(2)*s
qrt(a+b*x)*sqrt(a+b*x)+5*a/4/sqrt(2))*sqrt(a+b*x)*sqrt(a+b*x)-15*a^2/4/sqrt
(2))*sqrt(a+b*x)*sqrt(-a*b+b*(a+b*x))/(-a*b+b*(a+b*x))-30*a^2/4/sqrt(2)/sqr
t(b)*ln(abs(sqrt(-a*b+b*(a+b*x))-sqrt(b)*sqrt(a+b*x))))
```

maple [B] time = 0.25, size = 460, normalized size = 3.80

$$-\frac{2\operatorname{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{x}}+\frac{2b\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{5}{2}}}{\operatorname{arctanh}(\tanh(bx+a))-bx}+\frac{5ba\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}{2(\operatorname{arctanh}(\tanh(bx+a))-bx)}+\frac{15b\,a}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+2*b/(arcta
nh(tanh(b*x+a))-b*x)*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+5/2*b/(arctanh(tanh
(b*x+a))-b*x)*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+15/4*b/(arctanh(tanh(b*x
+a))-b*x)*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15/4*b^(1/2)/(arctanh(tanh
(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+45/4*b^(1/
2)/(arctanh(tanh(b*x+a))-b*x)*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(
1/2))*(arctanh(tanh(b*x+a))-b*x-a)+15/2*b/(arctanh(tanh(b*x+a))-b*x)*a*(arc
tanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+45/4*b^(1/2)/(a
rctanh(tanh(b*x+a))-b*x)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(
arctanh(tanh(b*x+a))-b*x-a)^2+5/2*b/(arctanh(tanh(b*x+a))-b*x)*(arctanh(tan
h(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+15/4*b/(arctanh(tanh(b*
x+a))-b*x)*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2
)+15/4*b^(1/2)/(arctanh(tanh(b*x+a))-b*x)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b
*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)^{\frac{5}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(3/2),x, algorithm="maxima")
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a + b\,x\right)\right)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(3/2),x)
[Out] int(atanh(tanh(a + b*x))^(5/2)/x^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(3/2),x)
[Out] Timed out
```

3.235 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=106

$$-5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)+5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-\frac{2\tanh^{-}}{}$$

```
[Out] -5*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh
(tanh(b*x+a)))-2/3*arctanh(tanh(b*x+a))^(5/2)/x^(3/2)-10/3*b*arctanh(tanh(b
*x+a))^(3/2)/x^(1/2)+5*b^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2168, 2169, 2165}

$$5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}-5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)-\frac{2\tanh^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(5/2),x]
```

```
[Out] -5*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - A
rcTanh[Tanh[a + b*x]]) + 5*b^2*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]] - (10*b
*ArcTanh[Tanh[a + b*x]]^(3/2))/(3*Sqrt[x]) - (2*ArcTanh[Tanh[a + b*x]]^(5/2
))/(3*x^(3/2))
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{5/2}}\,dx = -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^{3/2}} + \frac{1}{3}(5b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{3/2}}\,dx$$
$$= -\frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{3x^{3/2}} + \left(5b^2\right)\int \frac{\sqrt{\tan}}{}$$
$$= 5b^2\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))} - \frac{10b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3\sqrt{x}} - \frac{2\tanh^{-1}(\tan}{3}$$
$$= -5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right) + 5b^2$$

Mathematica [**A**] time = 0.08, size = 97, normalized size = 0.92

$$5b^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)+\frac{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-10bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 10*b*x*ArcTanh[Tanh[a + b*x]] -
 2*ArcTanh[Tanh[a + b*x]]^2))/(3*x^(3/2)) + 5*b^(3/2)*(-(b*x) + ArcTanh[Tan
h[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]
```

fricas [**A**] time = 0.48, size = 138, normalized size = 1.30

$$\left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(3\,b^2x^2-14\,abx-2\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{6\,x^2}, -\frac{15\,a\sqrt{-b}\,bx^2\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sqrt
(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*a*b
*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,1]%%%}+%%%{6,[0,2,0
]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%%%},0,%%%{-4,[3,0,0
]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1]%%%}+%%%{-12,[1,2,
2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]%%%}+%%%{-40,[1,1,1
]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{4,[0,3,1]%
```

```
%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]%%%}+%%%{4,[0,2,1]%
%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%%},0,%%%{1,[4,0,0]%
%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{6,[2,2,2]%%
%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]%%%}+%%%{4,[2,1,1]%
%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%%%}+%%%{12,[1,3,1]%
%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]%%%}+%%%{4,[1,2,1]%
%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%%%}+%%%{1,[0,4,4]%%
%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%%}+%%%{1,[0,4,0]%%%
}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{-4,[0,3,1]%%%}
+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%%%{-4,[0,1,4]%%%}+%
%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters values [0,85.3561567818,
61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1
,1]%%%}+%%%{-4,[0,1,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1
,1,1]%%%}+%%%{4,[1,1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,
1]%%%}+%%%{6,[0,2,0]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%
%%},0,%%%{-4,[3,0,0]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1
]%%%}+%%%{-12,[1,2,2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]
%%%}+%%%{-40,[1,1,1]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]
%%%}+%%%{4,[0,3,1]%%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]
%%%}+%%%{4,[0,2,1]%%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%
%},0,%%%{1,[4,0,0]%%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%
%%}+%%%{6,[2,2,2]%%%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]
%%%}+%%%{4,[2,1,1]%%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%
%%}+%%%{12,[1,3,1]%%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]
%%%}+%%%{4,[1,2,1]%%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%
%%}+%%%{1,[0,4,4]%%%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%
%}+%%%{1,[0,4,0]%%%}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}
+%%%{-4,[0,3,1]%%%}+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%
%%{-4,[0,1,4]%%%}+%%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters value
s [0,71.707969239,78.6493344628]sqrt(2)/2/abs(b)*b^2/b*(2*((9*b^4*a/9/sqrt(
2)/b/a*sqrt(a+b*x)*sqrt(a+b*x)-60*b^4*a^2/9/sqrt(2)/b/a)*sqrt(a+b*x)*sqrt(a
+b*x)+45*b^4*a^3/9/sqrt(2)/b/a)*sqrt(a+b*x)*sqrt(-a*b+b*(a+b*x))/(-a*b+b*(a
+b*x))^2-10*a*b^2/sqrt(2)/sqrt(b)*ln(abs(sqrt(-a*b+b*(a+b*x))-sqrt(b)*sqrt(
a+b*x))))
```

maple [B] time = 0.25, size = 501, normalized size = 4.73

$$-\frac{2 \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right) x^{\frac{3}{2}}}-\frac{8b \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2} \sqrt{x}}+\frac{8b^{2}\sqrt{x}\ \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(7/2)-8/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+8/3*b^2/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+10/3*b^2/(arctan
h(tanh(b*x+a))-b*x)^2*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5*b^2/(arctanh(t
anh(b*x+a))-b*x)^2*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+5*b^(3/2)/(arctan
h(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+15
*b^(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b
*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+10*b^2/(arctanh(tanh(b*x+a))-b*x
)^2*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+15*b^
(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a)
)^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+10/3*b^2/(arctanh(tanh(b*x+a))-b*x)
^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+5*b^2/(a
rctanh(tanh(b*x+a))-b*x)^2*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(t
anh(b*x+a))^(1/2)+5*b^(3/2)/(arctanh(tanh(b*x+a))-b*x)^2*ln(b^(1/2)*x^(1/2)
+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(5/2),x, algorithm="maxima")
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(5/2),x)
[Out] int(atanh(tanh(a + b*x))^(5/2)/x^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(5/2),x)
[Out] Timed out
```

3.236 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{7/2}} dx$

Optimal. Leaf size=93

$$2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2\tanh^{-1}(\tan$$

```
[Out] 2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))-2/3*b*arctanh
(tanh(b*x+a))^(3/2)/x^(3/2)-2/5*arctanh(tanh(b*x+a))^(5/2)/x^(5/2)-2*b^2*ar
ctanh(tanh(b*x+a))^(1/2)/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2\tanh^{-1}(\tan$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(7/2),x]
```

```
[Out] 2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]] - (2*b^2*
Sqrt[ArcTanh[Tanh[a + b*x]]])/Sqrt[x] - (2*b*ArcTanh[Tanh[a + b*x]]^(3/2))/
(3*x^(3/2)) - (2*ArcTanh[Tanh[a + b*x]]^(5/2))/(5*x^(5/2))
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{7/2}}dx &= -\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}}+b\int \frac{\tanh^{-1}(\tanh(a+bx))^{3/2}}{x^{5/2}}dx\\
&= -\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a+bx))^{5/2}}{5x^{5/2}}+b^2\int \frac{\sqrt{\tanh^{-1}(\tan}}{x^{3/}}\\
&= -\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2b\tanh^{-1}(\tanh(a+bx))^{3/2}}{3x^{3/2}}-\frac{2\tanh^{-1}(\tanh(a}{5x^{5/2}}\\
&= 2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)-\frac{2b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}}-\frac{2b\tan}{}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 95, normalized size = 1.02

$$-\frac{2\left(-15b^{5/2}x^{5/2}\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)+15b^2x^2\sqrt{\tanh^{-1}(\tanh(a+bx))}+5bx\tanh^{-1}(\tan\right.}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(7/2),x]
```

```
[Out] (-2*(15*b^2*x^2*Sqrt[ArcTanh[Tanh[a + b*x]]] + 5*b*x*ArcTanh[Tanh[a + b*x]]
^(3/2) + 3*ArcTanh[Tanh[a + b*x]]^(5/2) - 15*b^(5/2)*x^(5/2)*Log[b*Sqrt[x]
+ Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]]))/(15*x^(5/2))
```

fricas [A] time = 0.90, size = 137, normalized size = 1.47

$$\left[\frac{15\,b^{\frac{5}{2}}x^3\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\left(23\,b^2x^2+11\,abx+3\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{15\,x^3},-\frac{2\left(15\,\sqrt{-b}\,b^2x^3\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^(5/2)*x^3*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*
(23*b^2*x^2 + 11*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3, -2/15*(15*sqrt(
-b)*b^2*x^3*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (23*b^2*x^2 + 11*a
*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x))/x^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,1]%%%}+%%%{6,[0,2,0
]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%%%},0,%%%{-4,[3,0,0
]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1]%%%}+%%%{-12,[1,2,
2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]%%%}+%%%{-40,[1,1,1
]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{4,[0,3,1]%
%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]%%%}+%%%{4,[0,2,1]%
%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%%},0,%%%{1,[4,0,0]%
%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%%%}+%%%{6,[2,2,2]%%
%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]%%%}+%%%{4,[2,1,1]%
%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%%%}+%%%{12,[1,3,1]%
%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]%%%}+%%%{4,[1,2,1]%
%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%%%}+%%%{1,[0,4,4]%%
%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%%}+%%%{1,[0,4,0]%%%
}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}+%%%{-4,[0,3,1]%%%}
+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%%%{-4,[0,1,4]%%%}+%
%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters values [0,85.3561567818,
61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1
,1]%%%}+%%%{-4,[0,1,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1
,1,1]%%%}+%%%{4,[1,1,0]%%%}+%%%{4,[1,0,1]%%%}+%%%{6,[0,2,2]%%%}+%%%{4,[0,2,
1]%%%}+%%%{6,[0,2,0]%%%}+%%%{4,[0,1,2]%%%}+%%%{4,[0,1,1]%%%}+%%%{6,[0,0,2]%
%%},0,%%%{-4,[3,0,0]%%%}+%%%{-12,[2,1,1]%%%}+%%%{4,[2,1,0]%%%}+%%%{4,[2,0,1
```

```
]%%%}+%%%{-12,[1,2,2]%%%}+%%%{8,[1,2,1]%%%}+%%%{4,[1,2,0]%%%}+%%%{8,[1,1,2]
%%%}+%%%{-40,[1,1,1]%%%}+%%%{4,[1,0,2]%%%}+%%%{-4,[0,3,3]%%%}+%%%{4,[0,3,2]
%%%}+%%%{4,[0,3,1]%%%}+%%%{-4,[0,3,0]%%%}+%%%{4,[0,2,3]%%%}+%%%{-40,[0,2,2]
%%%}+%%%{4,[0,2,1]%%%}+%%%{4,[0,1,3]%%%}+%%%{4,[0,1,2]%%%}+%%%{-4,[0,0,3]%%
%},0,%%%{1,[4,0,0]%%%}+%%%{4,[3,1,1]%%%}+%%%{-4,[3,1,0]%%%}+%%%{-4,[3,0,1]%
%%}+%%%{6,[2,2,2]%%%}+%%%{-12,[2,2,1]%%%}+%%%{6,[2,2,0]%%%}+%%%{-12,[2,1,2]
%%%}+%%%{4,[2,1,1]%%%}+%%%{6,[2,0,2]%%%}+%%%{4,[1,3,3]%%%}+%%%{-12,[1,3,2]%
%%}+%%%{12,[1,3,1]%%%}+%%%{-4,[1,3,0]%%%}+%%%{-12,[1,2,3]%%%}+%%%{8,[1,2,2]
%%%}+%%%{4,[1,2,1]%%%}+%%%{12,[1,1,3]%%%}+%%%{4,[1,1,2]%%%}+%%%{-4,[1,0,3]%
%%}+%%%{1,[0,4,4]%%%}+%%%{-4,[0,4,3]%%%}+%%%{6,[0,4,2]%%%}+%%%{-4,[0,4,1]%%
%}+%%%{1,[0,4,0]%%%}+%%%{-4,[0,3,4]%%%}+%%%{4,[0,3,3]%%%}+%%%{4,[0,3,2]%%%}
+%%%{-4,[0,3,1]%%%}+%%%{6,[0,2,4]%%%}+%%%{4,[0,2,3]%%%}+%%%{6,[0,2,2]%%%}+%
%%{-4,[0,1,4]%%%}+%%%{-4,[0,1,3]%%%}+%%%{1,[0,0,4]%%%}] at parameters value
s [0,71.707969239,78.6493344628]sqrt(2)/2/abs(b)*b^2/b*(2*((-345*sqrt(2)*b^
5*a^2/225/a^2*sqrt(a+b*x)*sqrt(a+b*x)+525*sqrt(2)*b^5*a^3/225/a^2)*sqrt(a+b
*x)*sqrt(a+b*x)-225*sqrt(2)*b^5*a^4/225/a^2)*sqrt(a+b*x)*sqrt(-a*b+b*(a+b*x
))/(-a*b+b*(a+b*x))^3-2*b^3*sqrt(2)/sqrt(b)*ln(abs(sqrt(-a*b+b*(a+b*x))-sqr
t(b)*sqrt(a+b*x))))
```

maple [B] time = 0.25, size = 532, normalized size = 5.72

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{5\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{5}{2}}}-\frac{4b\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{15\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{2}x^{\frac{3}{2}}}-\frac{16b^{2}\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{15\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(7/2)-4/15*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(3/2)*arctanh(tanh(b*x+a))^(7/2)-16/15*b^2/(a
rctanh(tanh(b*x+a))-b*x)^3/x^(1/2)*arctanh(tanh(b*x+a))^(7/2)+16/15*b^3/(ar
ctanh(tanh(b*x+a))-b*x)^3*x^(1/2)*arctanh(tanh(b*x+a))^(5/2)+4/3*b^3/(arcta
nh(tanh(b*x+a))-b*x)^3*a*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^3/(arctanh(
tanh(b*x+a))-b*x)^3*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+2*b^(5/2)/(arcta
nh(tanh(b*x+a))-b*x)^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3+6
*b^(5/2)/(arctanh(tanh(b*x+a))-b*x)^3*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b
*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+4*b^3/(arctanh(tanh(b*x+a))-b*x)
^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)+6*b^(5
/2)/(arctanh(tanh(b*x+a))-b*x)^3*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^
(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^2+4/3*b^3/(arctanh(tanh(b*x+a))-b*x)^3*
(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(3/2)+2*b^3/(arct
anh(tanh(b*x+a))-b*x)^3*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh
(b*x+a))^(1/2)+2*b^(5/2)/(arctanh(tanh(b*x+a))-b*x)^3*ln(b^(1/2)*x^(1/2)+ar
ctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^(5/2)/x^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(7/2),x)
```

```
[Out] int(atanh(tanh(a + b*x))^(5/2)/x^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(7/2),x)
```

```
[Out] Timed out
```

3.237 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2/7*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(9/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(7*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 0.97

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{x^{7/2}\left(7bx-7\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(9/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(x^(7/2)*(7*b*x - 7*ArcTanh[Tanh[a + b*x]]
))
```

fricas [A] time = 0.42, size = 42, normalized size = 1.20

$$-\frac{2\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\sqrt{bx+a}}{7\,ax^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/(a*x^(7/2))
```

giac [A] time = 0.18, size = 33, normalized size = 0.94

$$-\frac{2\,(bx+a)^{\frac{7}{2}}b^{8}}{7\,((bx+a)b-ab)^{\frac{7}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="giac")
[Out] -2/7*(b*x + a)^(7/2)*b^8/(((b*x + a)*b - a*b)^(7/2)*a*abs(b))
```

maple [A] time = 0.25, size = 29, normalized size = 0.83

$$-\frac{2\,\mathrm{arctanh}\,(\tanh\,(bx+a))^{\frac{7}{2}}}{7\,(\mathrm{arctanh}\,(\tanh\,(bx+a))-bx)\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x)
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)
```

maxima [A] time = 0.42, size = 15, normalized size = 0.43

$$-\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,ax^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(9/2),x, algorithm="maxima")
[Out] -2/7*(b*x + a)^(7/2)/(a*x^(7/2))
```

mupad [B] time = 1.65, size = 396, normalized size = 11.31

$$-\frac{\sqrt{2}\,\sqrt{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}\,\left(\frac{\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{3}}{14\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-14\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+28\,b\,x}-\frac{\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)^{3}}{14\,\ln\left(\frac{1}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-14\,\ln\left(\frac{e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2}\right.}{2\,x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(9/2),x)
[Out] -(2^(1/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(
exp(2*a)*exp(2*b*x) + 1)))^(1/2)*(log(1/(exp(2*a)*exp(2*b*x) + 1))^3/(14*lo
g(1/(exp(2*a)*exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 28*b*x) - log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1))^3/(14*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 28*b*x) + (3*log(1/(exp(2*a)*exp(2*b*x) + 1))*l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))^2)/(14*log(1/(exp(2*a)*
exp(2*b*x) + 1)) - 14*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 28*b*x) - (3*log(1/(exp(2*a)*exp(2*b*x) + 1))^2*log((exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)))/(14*log(1/(exp(2*a)*exp(2*b*x) + 1)) - 14*log(
(exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 28*b*x)))/(2*x^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(9/2),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}} \, dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 4/63*b*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/9*
arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2171, 2167}

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(11/2),x]
```

```
[Out] (4*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(7/2)*(b*x - ArcTanh[Tanh[a + b*x]
])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(9*x^(9/2)*(b*x - ArcTanh[Tanh[a +
 b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx &= \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(2b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{9/2}}\,dx}{9\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{4b\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{9x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.67

$$\frac{2\left(9bx-7\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{7/2}}{63x^{9/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(11/2),x]
```

```
[Out] (2*(9*b*x - 7*ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(7/2))/(63*x^(
9/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.45, size = 56, normalized size = 0.78

$$\frac{2\left(2\,b^4x^4 - ab^3x^3 - 15\,a^2b^2x^2 - 19\,a^3bx - 7\,a^4\right)\sqrt{bx+a}}{63\,a^2x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*b^4*x^4 - a*b^3*x^3 - 15*a^2*b^2*x^2 - 19*a^3*b*x - 7*a^4)*sqrt(b*x
 + a)/(a^2*x^(9/2))
```

giac [A] time = 0.19, size = 59, normalized size = 0.82

$$\frac{\sqrt{2}\left(\frac{2\,\sqrt{2}\,(bx+a)b^9}{a^2} - \frac{9\,\sqrt{2}\,b^9}{a}\right)(bx+a)^{\frac{7}{2}}b}{63\left((bx+a)b - ab\right)^{\frac{9}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] 1/63*sqrt(2)*(2*sqrt(2)*(b*x + a)*b^9/a^2 - 9*sqrt(2)*b^9/a)*(b*x + a)^(7/2
)*b/(((b*x + a)*b - a*b)^(9/2)*abs(b))
```

maple [A] time = 0.26, size = 59, normalized size = 0.82

$$-\frac{2\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{9\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx\right)x^{\frac{9}{2}}} + \frac{4b\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{63\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx\right)^2 x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x)
```

```
[Out] -2/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(7/2)+4/63*b/(
arctanh(tanh(b*x+a))-b*x)^2/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)
```

maxima [A] time = 0.43, size = 34, normalized size = 0.47

$$\frac{2\left(2\,b^2x^2 - 5\,abx - 7\,a^2\right)(bx+a)^{\frac{5}{2}}}{63\,a^2x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] 2/63*(2*b^2*x^2 - 5*a*b*x - 7*a^2)*(b*x + a)^(5/2)/(a^2*x^(9/2))
```

mupad [B] time = 1.62, size = 293, normalized size = 4.07

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{19\,b\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{63} - \frac{10\,b^2\,x^2}{21} - \frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{18} + \right.}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(11/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((19*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/63 - (10*b^2
*x^2)/21 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/18 + (4*b^3*x^3)/(63*(log(2/(exp(2*a)
*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
+ 2*b*x)) + (16*b^4*x^4)/(63*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp
(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/x^(9/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(11/2),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 16/693*b^2*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^3+
8/99*b*arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/11
*arctanh(tanh(b*x+a))^(7/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(13/2),x]
```

```
[Out] (16*b^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(693*x^(7/2)*(b*x - ArcTanh[Tanh[a +
b*x]])^3) + (8*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(99*x^(9/2)*(b*x - ArcTanh[T
anh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(11*x^(11/2)*(b*x - Ar
cTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx &= \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(4b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{11/2}}\,dx}{11\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{11x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\\ &= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{693x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\tanh^{-1}(\tanh(a+bx))^{7/2}}{99x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.60

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-154bx\tanh^{-1}(\tanh(a+bx))+63\tanh^{-1}(\tanh(a+bx))^2+99b^2x^2\right)}{693x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(13/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(99*b^2*x^2 - 154*b*x*ArcTanh[Tanh[a + b*x]
] + 63*ArcTanh[Tanh[a + b*x]]^2))/(693*x^(11/2)*(b*x - ArcTanh[Tanh[a + b*x
]])^3)
```

fricas [A] time = 0.57, size = 67, normalized size = 0.61

$$-\frac{2\left(8\,b^5x^5-4\,ab^4x^4+3\,a^2b^3x^3+113\,a^3b^2x^2+161\,a^4bx+63\,a^5\right)\sqrt{bx+a}}{693\,a^3x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^5*x^5 - 4*a*b^4*x^4 + 3*a^2*b^3*x^3 + 113*a^3*b^2*x^2 + 161*a^4
*b*x + 63*a^5)*sqrt(b*x + a)/(a^3*x^(11/2))
```

giac [A] time = 0.20, size = 78, normalized size = 0.71

$$-\frac{\sqrt{2}\left(\frac{99\sqrt{2}b^{11}}{a}+4\left(\frac{2\sqrt{2}(bx+a)b^{11}}{a^3}-\frac{11\sqrt{2}b^{11}}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{693\left((bx+a)b-ab\right)^{\frac{11}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="giac")
```

```
[Out] -1/693*sqrt(2)*(99*sqrt(2)*b^11/a + 4*(2*sqrt(2)*(b*x + a)*b^11/a^3 - 11*sq
rt(2)*b^11/a^2)*(b*x + a))*(b*x + a)^(7/2)*b/(((b*x + a)*b - a*b)^(11/2)*ab
s(b))
```

maple [A] time = 0.30, size = 105, normalized size = 0.95

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{11\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{11}{2}}}-\frac{8b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{2b\,\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{63(\mathrm{arctanh}(\tanh(bx+a))-bx)^2x^{\frac{7}{2}}}\right)}{11\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x)
```

```
[Out] -2/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arctanh(tanh(b*x+a))^(7/2)-8/11*b
/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(tanh(b*x+a))-b*x)/x^(9/2)*arctan
h(tanh(b*x+a))^(7/2)+2/63*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(7/2)*arctanh(ta
nh(b*x+a))^(7/2))
```

maxima [A] time = 0.42, size = 45, normalized size = 0.41

$$-\frac{2\left(8\,b^3x^3-20\,ab^2x^2+35\,a^2bx+63\,a^3\right)(bx+a)^{\frac{5}{2}}}{693\,a^3x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] -2/693*(8*b^3*x^3 - 20*a*b^2*x^2 + 35*a^2*b*x + 63*a^3)*(b*x + a)^(5/2)/(a^
3*x^(11/2))
```

mupad [B] time = 1.66, size = 353, normalized size = 3.21

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{23\,b\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{99}-\frac{226\,b^2\,x^2}{693}-\frac{{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}{22}-\right.}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(13/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((23*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/99 - (226*b^
2*x^2)/693 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x)
)/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/22 + (4*b^3*x^3)/(231*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)) + (32*b^4*x^4)/(693*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2
*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (128*b^5*x^5
)/(693*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)))/x^(11/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(13/2),x)
```

```
[Out] Timed out
```

3.240 $\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx$

Optimal. Leaf size=148

$$\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 32/3003*b^3*arctanh(tanh(b*x+a))^(7/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))^4
+16/429*b^2*arctanh(tanh(b*x+a))^(7/2)/x^(9/2)/(b*x-arctanh(tanh(b*x+a)))^3
+12/143*b*arctanh(tanh(b*x+a))^(7/2)/x^(11/2)/(b*x-arctanh(tanh(b*x+a)))^2+
2/13*arctanh(tanh(b*x+a))^(7/2)/x^(13/2)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{32b^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(15/2),x]
```

```
[Out] (32*b^3*ArcTanh[Tanh[a + b*x]]^(7/2))/(3003*x^(7/2)*(b*x - ArcTanh[Tanh[a +
 b*x]])^4) + (16*b^2*ArcTanh[Tanh[a + b*x]]^(7/2))/(429*x^(9/2)*(b*x - ArcT
anh[Tanh[a + b*x]])^3) + (12*b*ArcTanh[Tanh[a + b*x]]^(7/2))/(143*x^(11/2)*
(b*x - ArcTanh[Tanh[a + b*x]])^2) + (2*ArcTanh[Tanh[a + b*x]]^(7/2))/(13*x^
(13/2)*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{15/2}}\,dx = \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{\tanh^{-1}(\tanh(a+bx))^{5/2}}{x^{13/2}}\,dx}{13\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{12b\tanh^{-1}(\tanh(a+bx))^{7/2}}{143x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}}{13x^{13/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$
$$= \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{12b\tanh^{-1}(\tanh(a+bx))^{7/2}}{143x^{11/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$
$$= \frac{32b^3\tanh^{-1}(\tanh(a+bx))^{7/2}}{3003x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\tanh^{-1}(\tanh(a+bx))^{7/2}}{429x^{9/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.09, size = 82, normalized size = 0.55

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{7/2}\left(-1001b^2x^2\tanh^{-1}(\tanh(a+bx))+819bx\tanh^{-1}(\tanh(a+bx))^2-231\tanh^{-1}(\tanh(\right.}{3003x^{13/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^(5/2)/x^(15/2),x]
```

```
[Out] (2*ArcTanh[Tanh[a + b*x]]^(7/2)*(429*b^3*x^3 - 1001*b^2*x^2*ArcTanh[Tanh[a
+ b*x]] + 819*b*x*ArcTanh[Tanh[a + b*x]]^2 - 231*ArcTanh[Tanh[a + b*x]]^3))
/(3003*x^(13/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.49, size = 78, normalized size = 0.53

$$\frac{2\left(16\,b^6x^6-8\,ab^5x^5+6\,a^2b^4x^4-5\,a^3b^3x^3-371\,a^4b^2x^2-567\,a^5bx-231\,a^6\right)\sqrt{bx+a}}{3003\,a^4x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*b^6*x^6 - 8*a*b^5*x^5 + 6*a^2*b^4*x^4 - 5*a^3*b^3*x^3 - 371*a^4*
b^2*x^2 - 567*a^5*b*x - 231*a^6)*sqrt(b*x + a)/(a^4*x^(13/2))
```

giac [A] time = 0.20, size = 97, normalized size = 0.66

$$-\frac{\sqrt{2}\left(\frac{429\,\sqrt{2}\,b^{13}}{a}-2\left(\frac{143\,\sqrt{2}\,b^{13}}{a^2}+4\left(\frac{2\,\sqrt{2}\,(bx+a)b^{13}}{a^4}-\frac{13\,\sqrt{2}\,b^{13}}{a^3}\right)(bx+a)\right)(bx+a)\right)(bx+a)^{\frac{7}{2}}b}{3003\,((bx+a)b-ab)^{\frac{13}{2}}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="giac")
```

```
[Out] -1/3003*sqrt(2)*(429*sqrt(2)*b^13/a - 2*(143*sqrt(2)*b^13/a^2 + 4*(2*sqrt(2
)*(b*x + a)*b^13/a^4 - 13*sqrt(2)*b^13/a^3)*(b*x + a))*(b*x + a))*(b*x + a)
^(7/2)*b/(((b*x + a)*b - a*b)^(13/2)*abs(b))
```

maple [A] time = 0.39, size = 151, normalized size = 1.02

$$-\frac{2\,\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{7}{2}}}{13\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{13}{2}}}-\frac{12b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{11(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{11}{2}}}-\frac{4b\left(-\frac{\mathrm{arctanh}(\tanh(bx+a))^{\frac{7}{2}}}{9(\mathrm{arctanh}(\tanh(bx+a))-bx)x^{\frac{9}{2}}}+\frac{2b\,\mathrm{arctanh}(\cdots}{63(\mathrm{arctanh}(\tan\cdots}\right.}{11(\mathrm{arctanh}(\tanh(bx+a))-bx)\cdots}\right.}{13\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x)
```

```
[Out] -2/13/(arctanh(tanh(b*x+a))-b*x)/x^(13/2)*arctanh(tanh(b*x+a))^(7/2)-12/13*
b/(arctanh(tanh(b*x+a))-b*x)*(-1/11/(arctanh(tanh(b*x+a))-b*x)/x^(11/2)*arc
tanh(tanh(b*x+a))^(7/2)-4/11*b/(arctanh(tanh(b*x+a))-b*x)*(-1/9/(arctanh(ta
nh(b*x+a))-b*x)/x^(9/2)*arctanh(tanh(b*x+a))^(7/2)+2/63*b/(arctanh(tanh(b*x
+a))-b*x)^2/x^(7/2)*arctanh(tanh(b*x+a))^(7/2)))
```

maxima [A] time = 0.42, size = 56, normalized size = 0.38

$$\frac{2\left(16\,b^{4}x^{4}-40\,ab^{3}x^{3}+70\,a^{2}b^{2}x^{2}-105\,a^{3}bx-231\,a^{4}\right)(bx+a)^{\frac{5}{2}}}{3003\,a^{4}x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^(5/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(16*b^4*x^4 - 40*a*b^3*x^3 + 70*a^2*b^2*x^2 - 105*a^3*b*x - 231*a^4)
*(b*x + a)^(5/2)/(a^4*x^(13/2))
```

mupad [B] time = 1.74, size = 413, normalized size = 2.79

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{27\,b\,x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{143}-\frac{106\,b^2\,x^2}{429}-\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\cdots}{26}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^(5/2)/x^(15/2),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((27*b*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/143 - (106*b
^2*x^2)/429 - (log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x
))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/26 + (20*b^3*x^3)/(3003*(log(2/(ex
p(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
+ 1)) + 2*b*x)) + (16*b^4*x^4)/(1001*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (128*b^5
*x^5)/(3003*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))
/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^6*x^6)/(3003*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^4)))/x^(13/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**(5/2)/x**(15/2),x)
```

```
[Out] Timed out
```

3.241 $\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=145

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{7/2}}+\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a\right.}{8b^3}$$

```
[Out] 5/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))^3/b^(7/2)+1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)/b+5/12*x^(3/2)*(b*
x-arctanh(tanh(b*x+a)))*arctanh(tanh(b*x+a))^(1/2)/b^2+5/8*(b*x-arctanh(tan
h(b*x+a)))^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{12b^2}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a\right.}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^3)/(8*b^(7/2)) + (x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b
) + (5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
/(12*b^2) + (5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a
 + b*x]]])/(8*b^3)
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} - \frac{\left(5\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{x}{\sqrt{\tanh^{-1}(}}}{6b}$$

$$= \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} + \frac{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(}}{12b^2}$$

$$= \frac{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b} + \frac{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(}}{12b^2}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{7/2}} + \frac{x^{5/2}\sqrt{\tanh^{-1}(}}{3}$$

Mathematica [A] time = 0.10, size = 105, normalized size = 0.72

$$\frac{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{8b^{7/2}} + \sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-40b\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(33*b^2*x^2 - 40*b*x*ArcTanh[Tanh[a +
 b*x]] + 15*ArcTanh[Tanh[a + b*x]]^2))/(24*b^3) + (5*(b*x - ArcTanh[Tanh[a
+ b*x]])^3*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(7/2
))
```

fricas [A] time = 0.64, size = 140, normalized size = 0.97

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b^4}, \frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*a^
3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

giac [A] time = 0.14, size = 64, normalized size = 0.44

$$\frac{1}{24}\,\sqrt{bx+a}\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x}+\frac{5\,a^3\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*l
og(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

maple [B] time = 0.30, size = 304, normalized size = 2.10

$$\frac{x^{\frac{5}{2}}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{3b}-\frac{5a\,x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{12b^{2}}+\frac{5a^{2}\sqrt{x}\,\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{8b^{3}}-\frac{5\ln\left(\sqrt{b}\,\sqrt{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 1/3*x^(5/2)*arctanh(tanh(b*x+a))^(1/2)/b-5/12/b^2*a*x^(3/2)*arctanh(tanh(b*
x+a))^(1/2)+5/8/b^3*a^2*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-5/8/b^(7/2)*ln(b
^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^3-15/8/b^(7/2)*a^2*ln(b^(1/2)*
x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*x-a)+5/4/b^3*a*
(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)-15/8/b^(7/2
)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b*x+a))-b*
x-a)^2-5/12/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(3/2)*arctanh(tanh(b*x+a))^(
1/2)+5/8/b^3*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)*arctanh(tanh(b*x+a))^(1
/2)-5/8/b^(7/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tan
h(b*x+a))-b*x-a)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(arctanh(tanh(b*x + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] int(x^(5/2)/atanh(tanh(a + b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.242 $\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh\right.}{4b^2}$$

```
[Out] 3/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b
*x+a)))^2/b^(5/2)+1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b+3/4*(b*x-arctanh
(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(bx-\tanh^{-1}(\tanh\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]])^2)/(4*b^(5/2)) + (x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(2*b
) + (3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
/(4*b^2)
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b}-\frac{\left(3\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\right)\int \frac{}{\sqrt{\tanh^{-1}(}}}{4b}$$

$$=\frac{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b}+\frac{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(}}{4b^2}$$

$$=\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{5/2}}+\frac{x^{3/2}\sqrt{\tanh^{-1}(}}{2}$$

Mathematica [A] time = 0.08, size = 88, normalized size = 0.82

$$\frac{\sqrt{b}\sqrt{x}\left(5bx-3\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}+3\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tan\right.}}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(5*b*x - 3*ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b
*x]]] + 3*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[
ArcTanh[Tanh[a + b*x]]]])/(4*b^(5/2))
```

fricas [A] time = 0.59, size = 119, normalized size = 1.11

$$\left[\frac{3a^2\sqrt{b}\log\left(2bx+2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2b^2x-3ab\right)\sqrt{bx+a}\sqrt{x}}{8b^3}, -\frac{3a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(2b^2\right.}{4b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

giac [A] time = 0.16, size = 52, normalized size = 0.49

$$\frac{1}{4}\sqrt{bx+a}\sqrt{x}\left(\frac{2x}{b}-\frac{3a}{b^2}\right)-\frac{3a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqrt
(x) + sqrt(b*x + a)))/b^(5/2)
```

maple [B] time = 0.30, size = 174, normalized size = 1.63

$$\frac{x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{2b}-\frac{3a\sqrt{x}\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{4b^2}+\frac{3\ln\left(\sqrt{b}\sqrt{x}+\sqrt{\operatorname{arctanh}(\tanh(bx+a))}\right)a^2}{4b^{\frac{5}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b-3/4/b^2*a*x^(1/2)*arctanh(tanh(b*x
+a))^(1/2)+3/4/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*a^2+3
/2/b^(5/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arctanh(tanh(b
*x+a))-b*x-a)-3/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)*arctanh(tanh(b*x
+a))^(1/2)+3/4/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))*(arct
anh(tanh(b*x+a))-b*x-a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{\operatorname{artanh}(\tanh(bx+a))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3/2)/sqrt(arctanh(tanh(b*x + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{\mathrm{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^(1/2),x)
[Out] int(x^(3/2)/atanh(tanh(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(1/2),x)
[Out] Integral(x**(3/2)/sqrt(atanh(tanh(a + b*x))), x)
```

3.243 $\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

```
[Out] arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x+a
)))/b^(3/2)+x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2169, 2165}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tan
h[a + b*x]]))/b^(3/2) + (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}} dx &= \frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{\left(-bx+\tanh^{-1}(\tanh(a+bx))\right)\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(}}{2b}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{3/2}}+\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(}}{b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 1.05

$$\frac{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b}-\frac{\left(\tanh^{-1}(\tanh(a+bx))-bx\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[ArcTanh[Tanh[a + b*x]]],x]
```

```
[Out] (Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b - ((-(b*x) + ArcTanh[Tanh[a + b*x]
])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2)
```

fricas [A] time = 0.47, size = 91, normalized size = 1.44

$$\left[\frac{a\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b^2},\ \frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

giac [A] time = 0.16, size = 38, normalized size = 0.60

$$\frac{a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{bx+a}\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x
)/b
```

maple [A] time = 0.29, size = 80, normalized size = 1.27

$$\frac{\sqrt{x}\,\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{b}-\frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)a}{b^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(b\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b-1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(t
anh(b*x+a))^(1/2))*a-1/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2
))*(arctanh(tanh(b*x+a))-b*x-a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\sqrt{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(arctanh(tanh(b*x + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{x}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^(1/2),x)
```

```
[Out] int(x^(1/2)/atanh(tanh(a + b*x))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(atanh(tanh(a + b*x))), x)
```

3.244 $\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=30

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2165}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.10

$$\frac{2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
[Out] (2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/Sqrt[b]
```

fricas [A] time = 0.77, size = 57, normalized size = 1.90

$$\left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arct
an(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

giac [A] time = 0.15, size = 23, normalized size = 0.77

$$-\frac{2 \log\left(\left|-\sqrt{b}\sqrt{x} + \sqrt{bx + a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```

maple [A] time = 0.33, size = 24, normalized size = 0.80

$$\frac{2 \ln\left(\sqrt{b}\,\sqrt{x} + \sqrt{\operatorname{arctanh}\left(\tanh\left(bx + a\right)\right)}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] 2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\sqrt{\operatorname{artanh}\left(\tanh\left(bx + a\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x)*sqrt(arctanh(tanh(b*x + a)))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x}\,\sqrt{\operatorname{atanh}\left(\tanh\left(a + b\,x\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*atanh(tanh(a + b*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\sqrt{\operatorname{atanh}\left(\tanh\left(a + bx\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(atanh(tanh(a + b*x)))), x)
```

3.245 $\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=33

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 2*arctanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2167}

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx=\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 0.97

$$-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{\sqrt{x}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(Sqrt[x]*(-(b*x) + ArcTanh[Tanh[a + b*x]]
))
```

fricas [A] time = 0.49, size = 15, normalized size = 0.45

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

giac [A] time = 0.15, size = 30, normalized size = 0.91

$$\frac{4\sqrt{b}}{\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)
```

maple [A] time = 0.30, size = 29, normalized size = 0.88

$$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

maxima [A] time = 0.43, size = 15, normalized size = 0.45

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

mupad [B] time = 1.62, size = 101, normalized size = 3.06

$$\frac{4\sqrt{\frac{\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)}{2}}}{\sqrt{x}\,\left(\ln\left(\frac{1}{e^{2a}\,e^{2bx}+1}\right)-\ln\left(\frac{e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)+2\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (4*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2))/(x^(1/2)*(log(1/(exp(2*a)*exp(2*b*x) + 1)) - l
og((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(atanh(tanh(a + b*x)))), x)
```

3.246 $\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

```
[Out] 2/3*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))+4/3*b*arc
tanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^2/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2171, 2167}

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (4*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]]
)^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a +
b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx &= \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{4b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 0.64

$$-\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-3bx\right)}{3x^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (-2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(-3*b*x + ArcTanh[Tanh[a + b*x]]))/(3*x^(3
/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.56, size = 23, normalized size = 0.32

$$\frac{2\,(2\,bx-a)\sqrt{bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

giac [A] time = 0.14, size = 55, normalized size = 0.76

$$\frac{8\left(3\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x) -
 sqrt(b*x + a))^2 - a)^3
```

maple [A] time = 0.30, size = 59, normalized size = 0.82

$$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{3}{2}}}+\frac{4b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(1/2)+4/3*b/(a
rctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)
```

maxima [A] time = 0.43, size = 33, normalized size = 0.46

$$\frac{2\left(2\,b^2x^2+abx-a^2\right)}{3\sqrt{bx+a}\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2*x^2 + a*b*x - a^2)/(sqrt(b*x + a)*a^2*x^(3/2))
```

mupad [B] time = 1.57, size = 218, normalized size = 3.03

$$\frac{\sqrt{2}\left(\frac{\frac{4\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{3}-\frac{4\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{3}+\frac{8bx}{3}}{\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}+\frac{16bx}{3\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^2}\right)\sqrt{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}}{2x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] (2^(1/2)*(((4*log(2/(exp(2*a)*exp(2*b*x) + 1)))/3 - (4*log((2*exp(2*a)*exp(
2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/3 + (8*b*x)/3)/(log(2/(exp(2*a)*exp(2*b
*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)
^2 + (16*b*x)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2))*(log((2*exp(2*a)*exp(2*b*x))/
(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)))^(1/2))/(2*x^
(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{\operatorname{atanh}\left(\tanh\left(a+bx\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(atanh(tanh(a + b*x)))), x)
```

3.247 $\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=110

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] 8/15*b*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/5*
arctanh(tanh(b*x+a))^(1/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))+16/15*b^2*arc
tanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^3/x^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (16*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(15*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]])^3) + (8*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(15*x^(3/2)*(b*x - ArcTanh[Ta
nh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(5*x^(5/2)*(b*x - ArcTa
nh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx &= \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}+\frac{(4b)\int \frac{1}{x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}\\ &= \frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}+\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\\ &= \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{8b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.60

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-10bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2+15b^2x^2\right)}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(15*b^2*x^2 - 10*b*x*ArcTanh[Tanh[a + b*x]]
 + 3*ArcTanh[Tanh[a + b*x]]^2))/(15*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^
3)
```

fricas [A] time = 0.47, size = 34, normalized size = 0.31

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

giac [A] time = 0.16, size = 77, normalized size = 0.70

$$\frac{32\left(10\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

maple [A] time = 0.30, size = 105, normalized size = 0.95

$$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{5\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{5}{2}}}-\frac{8b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{2b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\sqrt{x}}\right)}{5\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(tanh(b*x+a))^(1/2)-8/5*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)*arctanh(t
anh(b*x+a))^(1/2)+2/3*b/(arctanh(tanh(b*x+a))-b*x)^2/x^(1/2)*arctanh(tanh(b
*x+a))^(1/2))
```

maxima [A] time = 0.44, size = 45, normalized size = 0.41

$$-\frac{2\left(8\,b^3x^3+4\,ab^2x^2-a^2bx+3\,a^3\right)}{15\,\sqrt{bx+a}\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(8*b^3*x^3 + 4*a*b^2*x^2 - a^2*b*x + 3*a^3)/(sqrt(b*x + a)*a^3*x^(5/2
))
```

mupad [B] time = 1.53, size = 227, normalized size = 2.06

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{5\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{128\,b^2\,x^2}{15\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}+\frac{}{15\left(\ln\left(\frac{}{e^{2\,a}\,e^{\dots}}\right.\right.}\right.}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*b^2*x^2)
/(15*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (32*b*x)/(15*(log(2/(exp(2*a)*exp(2*b*x)
 + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
))/x^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.248 $\int \frac{1}{x^{9/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx$

Optimal. Leaf size=148

$$\frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

```
[Out] 16/35*b^2*arctanh(tanh(b*x+a))^(1/2)/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^3+1
2/35*b*arctanh(tanh(b*x+a))^(1/2)/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))^2+2/7*
arctanh(tanh(b*x+a))^(1/2)/x^(7/2)/(b*x-arctanh(tanh(b*x+a)))+32/35*b^3*arc
tanh(tanh(b*x+a))^(1/2)/(b*x-arctanh(tanh(b*x+a)))^4/x^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}+\frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4}+\frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (32*b^3*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b
*x]])^4) + (16*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*x^(3/2)*(b*x - ArcTanh
[Tanh[a + b*x]])^3) + (12*b*Sqrt[ArcTanh[Tanh[a + b*x]]])/(35*x^(5/2)*(b*x
- ArcTanh[Tanh[a + b*x]])^2) + (2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(7*x^(7/2)*
(b*x - ArcTanh[Tanh[a + b*x]]))
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{9/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx = \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)} + \frac{(6b)\int \frac{1}{x^{7/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{7\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2} + \frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{7x^{7/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

$$= \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3} + \frac{12b\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}$$

$$= \frac{32b^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4} + \frac{16b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.55

$$\frac{2\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(-35b^2x^2\tanh^{-1}(\tanh(a+bx))+21bx\tanh^{-1}(\tanh(a+bx))^2-5\tanh^{-1}(\tanh(a+bx\right.}{35x^{7/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[ArcTanh[Tanh[a + b*x]]]),x]
```

```
[Out] (2*Sqrt[ArcTanh[Tanh[a + b*x]]]*(35*b^3*x^3 - 35*b^2*x^2*ArcTanh[Tanh[a + b
*x]] + 21*b*x*ArcTanh[Tanh[a + b*x]]^2 - 5*ArcTanh[Tanh[a + b*x]]^3))/(35*x
^(7/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.71, size = 45, normalized size = 0.30

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(7
/2))
```

giac [A] time = 0.15, size = 103, normalized size = 0.70

$$\frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4+7\,a^2\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a^3\right)b^{\frac{7}{2}}}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

maple [A] time = 0.30, size = 151, normalized size = 1.02

$$-\frac{2\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}{7\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{7}{2}}}-\frac{12b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}}+\frac{2b\sqrt{\operatorname{arctanh}(\tanh}}{3(\operatorname{arctanh}(\tanh(bx+}\right.}{5(\operatorname{arctanh}(\tanh(bx+a))-bx)}\right.}{7\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x)
```

```
[Out] -2/7/(arctanh(tanh(b*x+a))-b*x)/x^(7/2)*arctanh(tanh(b*x+a))^(1/2)-12/7*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)*arctanh(
tanh(b*x+a))^(1/2)-4/5*b/(arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x
+a))-b*x)/x^(3/2)*arctanh(tanh(b*x+a))^(1/2)+2/3*b/(arctanh(tanh(b*x+a))-b*
x)^2/x^(1/2)*arctanh(tanh(b*x+a))^(1/2)))
```

maxima [A] time = 0.43, size = 55, normalized size = 0.37

$$\frac{2\left(16\,b^{4}x^{4}+8\,ab^{3}x^{3}-2\,a^{2}b^{2}x^{2}+a^{3}bx-5\,a^{4}\right)}{35\,\sqrt{bx+a}\,a^{4}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/arctanh(tanh(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(16*b^4*x^4 + 8*a*b^3*x^3 - 2*a^2*b^2*x^2 + a^3*b*x - 5*a^4)/(sqrt(b*x
 + a)*a^4*x^(7/2))
```

mupad [B] time = 1.64, size = 287, normalized size = 1.94

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{4}{7\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{128\,b^{2}\,x^{2}}{35\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{3}}+\frac{\ }{35\left(\ln\left(-\right.\right.}\right.}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*atanh(tanh(a + b*x))^(1/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(7*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*b^2*x^2)
/(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2
*a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (512*b^3*x^3)/(35*(log(2/(exp(2*a)*exp(2
*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*
x)^4) + (48*b*x)/(35*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*ex
p(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/x^(7/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/atanh(tanh(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.249 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{9/2}}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tan}}{8b^4}$$

```
[Out] 35/8*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^3/b^(9/2)-2*x^(7/2)/b/arctanh(tanh(b*x+a))^(1/2)+7/3*x^(5/2)*arcta
nh(tanh(b*x+a))^(1/2)/b^2+35/12*x^(3/2)*(b*x-arctanh(tanh(b*x+a)))*arctanh(
tanh(b*x+a))^(1/2)/b^3+35/8*(b*x-arctanh(tanh(b*x+a)))^2*x^(1/2)*arctanh(ta
nh(b*x+a))^(1/2)/b^4
```

Rubi [A] time = 0.11, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2168, 2169, 2165}

$$\frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2}+\frac{35x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{12b^3}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^3)/(8*b^(9/2)) - (2*x^(7/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]
]) + (7*x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(3*b^2) + (35*x^(3/2)*(b*x -
ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(12*b^3) + (35*Sqrt[x
]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]]])/(8*b^4)
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7\int \frac{x^{5/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} - \frac{(35\left(-bx+\tanh\right.}{}$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} + \frac{35x^{3/2}\left(bx-\tanh\right.}{}$$

$$= -\frac{2x^{7/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{7x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{3b^2} + \frac{35x^{3/2}\left(bx-\tanh\right.}{}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3}{8b^{9/2}} - \frac{2x}{b\sqrt{\tanh^{-1}(\tanh}}$$

Mathematica [A] time = 0.11, size = 122, normalized size = 0.73

$$\frac{35\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{8b^{9/2}}+\frac{\sqrt{x}\left(231b^2x^2\tanh^{-1}(\tanh(a+bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*(-48*b^3*x^3 + 231*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 280*b*x*ArcTan
h[Tanh[a + b*x]]^2 + 105*ArcTanh[Tanh[a + b*x]]^3))/(24*b^4*Sqrt[ArcTanh[Ta
nh[a + b*x]]]) + (35*(b*x - ArcTanh[Tanh[a + b*x]])^3*Log[b*Sqrt[x] + Sqrt[
b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(8*b^(9/2))
```

fricas [A] time = 0.51, size = 196, normalized size = 1.18

$$\left[\frac{105\left(a^3bx+a^4\right)\sqrt{b}\log\left(2\,bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(8\,b^4x^3-14\,ab^3x^2+35\,a^2b^2x+105\,a^3b\right)\sqrt{bx+a}\sqrt{}}{48\left(b^6x+ab^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(a^3*b*x + a^4)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt
(x) + a) + 2*(8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2*x + 105*a^3*b)*sqrt(b*x
 + a)*sqrt(x))/(b^6*x + a*b^5), 1/24*(105*(a^3*b*x + a^4)*sqrt(-b)*arctan(s
qrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^4*x^3 - 14*a*b^3*x^2 + 35*a^2*b^2
*x + 105*a^3*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x + a*b^5)]
```

giac [A] time = 0.14, size = 75, normalized size = 0.45

$$\frac{\left(\left(2\,x\left(\frac{4\,x}{b}-\frac{7\,a}{b^2}\right)+\frac{35\,a^2}{b^3}\right)x+\frac{105\,a^3}{b^4}\right)\sqrt{x}}{24\,\sqrt{bx+a}}+\frac{35\,a^3\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*x*(4*x/b - 7*a/b^2) + 35*a^2/b^3)*x + 105*a^3/b^4)*sqrt(x)/sqrt(b*
x + a) + 35/8*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2)
```

maple [B] time = 0.25, size = 428, normalized size = 2.58

$$\frac{x^{\frac{7}{2}}}{3b\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}-\frac{7a\,x^{\frac{5}{2}}}{12b^2\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{35a^2x^{\frac{3}{2}}}{24b^3\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}+\frac{35a^3\sqrt{}}{8b^4\sqrt{\operatorname{arctanh}(\tanh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] 1/3*x^(7/2)/b/arctanh(tanh(b*x+a))^(1/2)-7/12/b^2*a*x^(5/2)/arctanh(tanh(b*
x+a))^(1/2)+35/24/b^3*a^2*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+35/8/b^4*a^3*x
^(1/2)/arctanh(tanh(b*x+a))^(1/2)-35/8/b^(9/2)*a^3*ln(b^(1/2)*x^(1/2)+arcta
nh(tanh(b*x+a))^(1/2))+105/8/b^4*a^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/a
rctanh(tanh(b*x+a))^(1/2)-105/8/b^(9/2)*a^2*(arctanh(tanh(b*x+a))-b*x-a)*ln
(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))+35/12/b^3*a*(arctanh(tanh(b*x+
a))-b*x-a)*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+105/8/b^4*a*(arctanh(tanh(b*x
+a))-b*x-a)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-105/8/b^(9/2)*a*(arctanh(t
anh(b*x+a))-b*x-a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-7/12/b^
2*(arctanh(tanh(b*x+a))-b*x-a)*x^(5/2)/arctanh(tanh(b*x+a))^(1/2)+35/24/b^3
*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)+35/8/b^4
*(arctanh(tanh(b*x+a))-b*x-a)^3*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-35/8/b^(
9/2)*(arctanh(tanh(b*x+a))-b*x-a)^3*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{7}{2}}}{\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{7/2}}{\operatorname{atanh}(\tanh(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(7/2)/atanh(tanh(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

3.250 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{7/2}}+\frac{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh}}{4b^3}$$

[Out] 15/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x+a)))^2/b^(7/2)-2*x^(5/2)/b/arctanh(tanh(b*x+a))^(1/2)+5/2*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b^2+15/4*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^3

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$\frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2}+\frac{15\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}{4b^3}+\frac{15\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{\tanh^{-1}(}}\right.}{}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]

[Out] (15*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[Tanh[a + b*x]])^2)/(4*b^(7/2)) - (2*x^(5/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (5*x^(3/2)*Sqrt[ArcTanh[Tanh[a + b*x]]])/(2*b^2) + (15*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b^3)

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m + 1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0] && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ[m, 0] && !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u - a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) && !ILtQ[m + n, -2]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5\int \frac{x^{3/2}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2} - \frac{(15(-bx+\text{ta}}{}$$

$$= -\frac{2x^{5/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{2b^2} + \frac{15\sqrt{x}\,(bx-\text{t}}{}$$

$$= \frac{15\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{7/2}} - \frac{2x}{b\sqrt{\tanh^{-1}(\text{ta}}}$$

Mathematica [A] time = 0.10, size = 104, normalized size = 0.81

$$\frac{15\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{4b^{7/2}} - \frac{\sqrt{x}\left(-25bx\tanh^{-1}(\tanh(a+bx))\right.}{4b^3\sqrt{\tanh^{-}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] -1/4*(Sqrt[x]*(8*b^2*x^2 - 25*b*x*ArcTanh[Tanh[a + b*x]] + 15*ArcTanh[Tanh[
a + b*x]]^2))/(b^3*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (15*(-(b*x) + ArcTanh[Ta
nh[a + b*x]])^2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(4*b
^(7/2))
```

fricas [A] time = 0.57, size = 175, normalized size = 1.37

$$\left[\frac{15\left(a^2bx+a^3\right)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(2\,b^3x^2-5\,ab^2x-15\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{8\left(b^5x+ab^4\right)}, -\frac{15\left(a^2bx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x
) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x
 + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/
(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

giac [A] time = 0.16, size = 63, normalized size = 0.49

$$\frac{\left(x\left(\frac{2\,x}{b}-\frac{5\,a}{b^2}\right)-\frac{15\,a^2}{b^3}\right)\sqrt{x}}{4\,\sqrt{bx+a}} - \frac{15\,a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")

[Out] 1/4*(x*(2*x/b - 5*a/b^2) - 15*a^2/b^3)*sqrt(x)/sqrt(b*x + a) - 15/4*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)

maple [B] time = 0.25, size = 261, normalized size = 2.04

$$\frac{x^{\frac{5}{2}}}{2b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{5a\,x^{\frac{3}{2}}}{4b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{15a^{2}\sqrt{x}}{4b^{3}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{15a^{2}\ln\left(\sqrt{b}\,\sqrt{x}\,+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x)

[Out] 1/2*x^(5/2)/b/arctanh(tanh(b*x+a))^(1/2)-5/4/b^2*a*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)-15/4/b^3*a^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/4/b^(7/2)*a^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-15/2/b^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/2/b^(7/2)*a*(arctanh(tanh(b*x+a))-b*x-a)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-5/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(3/2)/arctanh(tanh(b*x+a))^(1/2)-15/4/b^3*(arctanh(tanh(b*x+a))-b*x-a)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+15/4/b^(7/2)*(arctanh(tanh(b*x+a))-b*x-a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate(x^(5/2)/arctanh(tanh(b*x + a))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)/atanh(tanh(a + b*x))^(3/2),x)

[Out] int(x^(5/2)/atanh(tanh(a + b*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(3/2),x)

[Out] Timed out

3.251 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}-\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a}}$$

```
[Out] 3*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x
+a)))/b^(5/2)-2*x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)+3*x^(1/2)*arctanh(tanh
(b*x+a))^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2168, 2169, 2165}

$$\frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/b^(5/2) - (2*x^(3/2))/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (3
*Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]])/b^2
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = -\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{3\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}$$

$$= -\frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{3\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^2} - \frac{\left(3\left(-bx+\tanh^{-}\right.\right.}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{5/2}} - \frac{2x^{3/2}}{b\sqrt{\tanh^{-1}(\tanh(}}$$

<u>**Mathematica**</u> [**A**] time = 0.09, size = 81, normalized size = 0.94

$$\frac{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{5/2}}+\frac{\sqrt{x}\left(3\tanh^{-1}(\tanh(a+bx))-2bx\right)}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (Sqrt[x]*(-2*b*x + 3*ArcTanh[Tanh[a + b*x]]))/(b^2*Sqrt[ArcTanh[Tanh[a + b*
x]]]) + (3*(b*x - ArcTanh[Tanh[a + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcT
anh[Tanh[a + b*x]]]])/b^(5/2)
```

<u>**fricas**</u> [**A**] time = 0.58, size = 145, normalized size = 1.69

$$\left[\frac{3\left(abx+a^2\right)\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(b^2x+3\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{2\left(b^4x+ab^3\right)}, \frac{3\left(abx+a^2\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}}{b\sqrt{}}\right.}{b^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
 a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x +
 a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a*b)
*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

<u>**giac**</u> [**A**] time = 0.14, size = 48, normalized size = 0.56

$$\frac{\sqrt{x}\left(\frac{x}{b}+\frac{3\,a}{b^2}\right)}{\sqrt{bx+a}}+\frac{3\,a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(x)*(x/b + 3*a/b^2)/sqrt(b*x + a) + 3*a*log(abs(-sqrt(b)*sqrt(x) + sqrt
(b*x + a)))/b^(5/2)
```

maple [A] time = 0.26, size = 130, normalized size = 1.51

$$\frac{x^{\frac{3}{2}}}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{3a\sqrt{x}}{b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{3a\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)}{b^{\frac{5}{2}}}+\frac{3\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] x^(3/2)/b/arctanh(tanh(b*x+a))^(1/2)+3/b^2*a*x^(1/2)/arctanh(tanh(b*x+a))^(
1/2)-3/b^(5/2)*a*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))+3/b^2*(arct
anh(tanh(b*x+a))-b*x-a)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-3/b^(5/2)*(arcta
nh(tanh(b*x+a))-b*x-a)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/arctanh(tanh(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^(3/2),x)
```

```
[Out] int(x^(3/2)/atanh(tanh(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**(3/2), x)
```

3.252 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(3/2)-2*x^(1/2)/b/a
rctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2168, 2165}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2) - (2*Sq
rt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx &= -\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}\,dx}{b}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.06

$$\frac{2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (2*Log[b*Sqrt[x] + Sqrt[b]*
Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(3/2)
```

fricas [A] time = 0.47, size = 119, normalized size = 2.29

$$\left[\frac{(bx+a)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,b\sqrt{x}}{b^3x+ab^2},-\frac{2\left((bx+a)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx\right.}}{b^3x+ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sq
rt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]
```

giac [A] time = 0.14, size = 39, normalized size = 0.75

$$-\frac{2\,\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}-\frac{2\,\sqrt{x}}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) - 2*sqrt(x)/(sqrt(b*x
 + a)*b)
```

maple [A] time = 0.25, size = 42, normalized size = 0.81

$$-\frac{2\sqrt{x}}{b\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2*x^(1/2)/b/arctanh(tanh(b*x+a))^(1/2)+2/b^(3/2)*ln(b^(1/2)*x^(1/2)+arctan
h(tanh(b*x+a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/arctanh(tanh(b*x + a))^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^(3/2),x)
[Out] int(x^(1/2)/atanh(tanh(a + b*x))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(3/2),x)
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**(3/2), x)
```

3.253 $\int \frac{1}{\sqrt{x} \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=33

$$-\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] -2*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2167}

$$-\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x} \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx = -\frac{2\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 0.97

$$\frac{2\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(Sqrt[ArcTanh[Tanh[a + b*x]]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])
)
```

fricas [A] time = 0.51, size = 22, normalized size = 0.67

$$\frac{2\sqrt{bx+a}\sqrt{x}}{abx+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

giac [A] time = 0.14, size = 15, normalized size = 0.45

$$\frac{2\sqrt{x}}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
[Out] 2*sqrt(x)/(sqrt(b*x + a)*a)
```

maple [A] time = 0.25, size = 29, normalized size = 0.88

$$\frac{2\sqrt{x}}{(\operatorname{arctanh}(\tanh(bx+a))-bx)\sqrt{\operatorname{arctanh}(\tanh(bx+a))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x)
[Out] 2*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\operatorname{artanh}(\tanh(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x)*arctanh(tanh(b*x + a))^(3/2)), x)
```

mupad [B] time = 1.74, size = 163, normalized size = 4.94

$$\frac{4x\sqrt{\frac{\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2}}}{\left(\frac{\sqrt{x}\,\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)}{2b}-\frac{\sqrt{x}\,\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2b}\right)\left(b\,\ln\left(\frac{1}{e^{2a}e^{2bx}+1}\right)-b\,\ln\left(\frac{e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2b^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^(3/2)),x)
[Out] (4*x*(log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(1/(exp(2
*a)*exp(2*b*x) + 1))/2)^(1/2))/(((x^(1/2)*log(1/(exp(2*a)*exp(2*b*x) + 1)))
/(2*b) - (x^(1/2)*log((exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)))/(2*
b))*(b*log(1/(exp(2*a)*exp(2*b*x) + 1)) - b*log((exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b^2*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\operatorname{atanh}^{\frac{3}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(3/2),x)
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**(3/2)), x)
```

3.254 $\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{4b\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+\dots}}$$

```
[Out] 2/(b*x-arctanh(tanh(b*x+a)))/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-4*b*x^(1/2)
/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{4b\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+\dots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-4*b*Sqrt[x])/((b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b*x]
]]) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[Tanh[a + b*x]]
])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx &= \frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{(2b)\int \frac{\dots}{\sqrt{x}\,\mathrm{ta}\dots}}{-bx+\mathrm{tan}\dots}\\ &= -\frac{4b\sqrt{x}}{\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{\dots}{\sqrt{x}\left(bx-\mathrm{ta}\dots\right.}\end{aligned}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 0.63

$$-\frac{2\left(\tanh^{-1}(\tanh(a+bx))+bx\right)}{\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*(b*x + ArcTanh[Tanh[a + b*x]]))/(Sqrt[x]*Sqrt[ArcTanh[Tanh[a + b*x]]]*(
-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.43, size = 34, normalized size = 0.50

$$-\frac{2\,(2\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

giac [A] time = 0.16, size = 50, normalized size = 0.74

$$-\frac{2\,b\sqrt{x}}{\sqrt{bx+a}\,a^2}+\frac{4\,\sqrt{b}}{\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) + 4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x
+ a))^2 - a)*a)
```

maple [A] time = 0.25, size = 59, normalized size = 0.87

$$-\frac{2}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)\sqrt{x}\,\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{4b\sqrt{x}}{\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)^2\sqrt{\operatorname{arctanh}(\operatorname{tanh}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-4*b/(arcta
nh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)
```

maxima [A] time = 0.43, size = 32, normalized size = 0.47

$$-\frac{2\left(2\,b^2x^2+3\,abx+a^2\right)}{(bx+a)^{\frac{3}{2}}a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2*(2*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^(3/2)*a^2*sqrt(x))
```

mupad [B] time = 1.49, size = 281, normalized size = 4.13

$$-\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,x}{{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}-\frac{8\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-8\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+16\,b\,x}{2\,b\,{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}^2}\right)}{x^{3/2}-\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] -((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*
a)*exp(2*b*x) + 1))/2)^(1/2)*((16*x)/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - lo
g((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2 - (8*log(2/
(exp(2*a)*exp(2*b*x) + 1)) - 8*log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*
b*x) + 1)) + 16*b*x)/(2*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^(3/2) - (x^(1/2)
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x))/(2*b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} \operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a + bx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**(3/2)), x)
```

3.255 $\int \frac{1}{x^{5/2} \tanh^{-1}(\tanh(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=110

$$-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a-}$$

```
[Out] 2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)+8/3*b/(b*
x-arctanh(tanh(b*x+a)))^2/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-16/3*b^2*x^(1/
2)/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-16*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a +
 b*x]]]) + (8*b)/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[T
anh[a + b*x]]]) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqrt[ArcTanh[
Tanh[a + b*x]]])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx &= \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{(4b)\int\frac{}{x^{3/2}\tan}}{3\left(bx-\tanh\right.}\\
&= \frac{8b}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{}{3x^{3/2}\left(bx-t\right.}\\
&= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{}{3\sqrt{x}\left(bx-\tan\right.}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.58

$$\frac{2\left(-6bx\tanh^{-1}(\tanh(a+bx))+\tanh^{-1}(\tanh(a+bx))^2-3b^2x^2\right)}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (2*(-3*b^2*x^2 - 6*b*x*ArcTanh[Tanh[a + b*x]] + ArcTanh[Tanh[a + b*x]]^2))/
(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

fricas [A] time = 0.58, size = 49, normalized size = 0.45

$$\frac{2\left(8\,b^2x^2+4\,abx-a^2\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^3bx^3+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

giac [A] time = 0.17, size = 107, normalized size = 0.97

$$\frac{2\,b^2\sqrt{x}}{\sqrt{bx+a}\,a^3}-\frac{4\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-12\,ab^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+5\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) - 4/3*(3*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(
b*x + a))^4 - 12*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 5*a^2*b^(3
/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^2)
```

maple [A] time = 0.25, size = 105, normalized size = 0.95

$$-\frac{2}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{8b\left(-\frac{1}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{x}\,\sqrt{\operatorname{arctanh}(\tanh(bx+a}}\right.}{3\left(\operatorname{arctanh}\left(\operatorname{tanh}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(tanh(b*x+a))^(1/2)-8/3*b/(a
rctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tan
h(b*x+a))^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a
))^(1/2))
```

maxima [A] time = 0.42, size = 45, normalized size = 0.41

$$\frac{2\left(8\,b^3x^3+12\,ab^2x^2+3\,a^2bx-a^3\right)}{3\left(bx+a\right)^{\frac{3}{2}}a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

[Out] 2/3*(8*b^3*x^3 + 12*a*b^2*x^2 + 3*a^2*b*x - a^3)/((b*x + a)^(3/2)*a^3*x^(3/2))

mupad [B] time = 1.69, size = 286, normalized size = 2.60

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{32\,x}{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}+\frac{4}{3\,b\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\frac{\cdots}{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2}\cdots}\right.\right.}\right.}{x^{5/2}-\frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^(3/2)),x)

[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2)*((32*x)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + 4/(3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - (128*b*x^2)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)))/(x^(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\operatorname{atanh}^{\frac{3}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(3/2),x)

[Out] Integral(1/(x**(5/2)*atanh(tanh(a + b*x))**(3/2)), x)

3.256 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh}}$$

```
[Out] 4/5*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)+2/5/x
^(5/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(1/2)+16/5*b^2/(b*x-
arctanh(tanh(b*x+a)))^3/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-32/5*b^3*x^(1/2)
/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2171, 2167}

$$-\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-32*b^3*Sqrt[x])/(5*(b*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[ArcTanh[Tanh[a +
 b*x]]]) + (16*b^2)/(5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTan
h[Tanh[a + b*x]]]) + (4*b)/(5*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt
[ArcTanh[Tanh[a + b*x]]]) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])*Sqr
t[ArcTanh[Tanh[a + b*x]]])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{(6b)\int \frac{1}{x^{5/2}\tan}}{5\left(bx-\tanh\right.}$$

$$= \frac{4b}{5x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5x^{5/2}\left(bx-\right.}$$

$$= \frac{16b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5x^{3/2}\left(bx-\right.}$$

$$= -\frac{32b^3\sqrt{x}}{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{}{5\sqrt{x}\left(bx-\tan\right.}$$

Mathematica [A] time = 0.07, size = 80, normalized size = 0.54

$$-\frac{2\left(15b^2x^2\tanh^{-1}(\tanh(a+bx))-5bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3+5b^3x^3\right)}{5x^{5/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(3/2)),x]
```

```
[Out] (-2*(5*b^3*x^3 + 15*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 5*b*x*ArcTanh[Tanh[a +
 b*x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(5*x^(5/2)*Sqrt[ArcTanh[Tanh[a + b*x]
]]*(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.58, size = 58, normalized size = 0.39

$$-\frac{2\left(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3\right)\sqrt{bx+a}\,\sqrt{x}}{5\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b*x^4 + a^5*x^3)
```

giac [A] time = 0.20, size = 161, normalized size = 1.09

$$-\frac{2\,b^3\sqrt{x}}{\sqrt{bx+a}\,a^4}+\frac{4\left(5\,b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^8-30\,ab^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^6+80\,a^2b^{\frac{5}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-50\,a^3\right.}{5\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) + 4/5*(5*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^8 - 30*a*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 + 80*a^2*b^
(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 50*a^3*b^(5/2)*(sqrt(b)*sqrt(x)
 - sqrt(b*x + a))^2 + 11*a^4*b^(5/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2
 - a)^5*a^3)
```

maple [A] time = 0.24, size = 151, normalized size = 1.02

$$-\frac{2}{5\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{5}{2}}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{12b\left(-\frac{1}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\sqrt{\operatorname{arctanh}(\tanh(bx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)/arctanh(tanh(b*x+a))^(1/2)-12/5*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(
tanh(b*x+a))^(1/2)-4/3*b/(arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a
))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-2*b/(arctanh(tanh(b*x+a))-b*x)^2
*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)))
```

maxima [A] time = 0.43, size = 54, normalized size = 0.36

$$-\frac{2\left(16\,b^{4}x^{4}+24\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-a^{3}bx+a^{4}\right)}{5\,(bx+a)^{\frac{3}{2}}a^{4}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(16*b^4*x^4 + 24*a*b^3*x^3 + 6*a^2*b^2*x^2 - a^3*b*x + a^4)/((b*x + a)
^(3/2)*a^4*x^(5/2))
```

mupad [B] time = 1.77, size = 346, normalized size = 2.34

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{16\,x}{5\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^{2}}+\frac{4}{5\,b\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\cdots\right.}{x^{7/2}-\frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{2\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^(3/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((16*x)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + 4/(5*b
*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)) + (128*b*x^2)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3) - (
512*b^2*x^3)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)))/(x^(7/2) - (x^(5/2)*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x))/(2*b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

3.257 $\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{9/2}}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tanh}}{4b^4}$$

```
[Out] 35/4*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(
b*x+a)))^2/b^(9/2)-2/3*x^(7/2)/b/arctanh(tanh(b*x+a))^(3/2)-14/3*x^(5/2)/b^
2/arctanh(tanh(b*x+a))^(1/2)+35/6*x^(3/2)*arctanh(tanh(b*x+a))^(1/2)/b^3+35
/4*(b*x-arctanh(tanh(b*x+a)))*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^4
```

Rubi [A] time = 0.10, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2168, 2169, 2165}

$$-\frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{35x^{3/2}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{6b^3}+\frac{35\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\sqrt{\tanh^{-1}(\tan}}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (35*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[
Tanh[a + b*x]])^2)/(4*b^(9/2)) - (2*x^(7/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3
/2)) - (14*x^(5/2))/(3*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (35*x^(3/2)*Sqrt
[ArcTanh[Tanh[a + b*x]]])/(6*b^3) + (35*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x
]])*Sqrt[ArcTanh[Tanh[a + b*x]]])/(4*b^4)
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{7/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{7\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{35\int \frac{x}{\sqrt{\tanh^{-1}(\text{t}}}}{3b}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{35x^{3/2}\sqrt{\tanh}}{}$$

$$= -\frac{2x^{7/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{14x^{5/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{35x^{3/2}\sqrt{\tanh}}{}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2}{4b^{9/2}} - \frac{2}{3b\tanh^{-1}(\text{ta}}$$

Mathematica [A] time = 0.11, size = 121, normalized size = 0.79

$$\frac{35\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{4b^{9/2}} - \frac{\sqrt{x}\left(56b^2x^2\tanh^{-1}(\tanh(a+bx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] -1/12*(Sqrt[x]*(8*b^3*x^3 + 56*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 175*b*x*Arc
Tanh[Tanh[a + b*x]]^2 + 105*ArcTanh[Tanh[a + b*x]]^3))/(b^4*ArcTanh[Tanh[a
+ b*x]]^(3/2)) + (35*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2*Log[b*Sqrt[x] + Sq
rt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/(4*b^(9/2))
```

fricas [A] time = 0.53, size = 241, normalized size = 1.58

$$\left[\frac{105\left(a^2b^2x^2+2\,a^3bx+a^4\right)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(6\,b^4x^3-21\,ab^3x^2-140\,a^2b^2x-105\,a^3b\right.}{24\left(b^7x^2+2\,ab^6x+a^2b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(b)*log(2*b*x + 2*sqrt(b*x +
 a)*sqrt(b)*sqrt(x) + a) + 2*(6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 10
5*a^3*b)*sqrt(b*x + a)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5), -1/12*(105
*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*
sqrt(x))) - (6*b^4*x^3 - 21*a*b^3*x^2 - 140*a^2*b^2*x - 105*a^3*b)*sqrt(b*x
 + a)*sqrt(x))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)]
```

giac [A] time = 0.15, size = 75, normalized size = 0.49

$$\frac{\left(\left(3\,x\left(\frac{2\,x}{b}-\frac{7\,a}{b^2}\right)-\frac{140\,a^2}{b^3}\right)x-\frac{105\,a^3}{b^4}\right)\sqrt{x}}{12\,(bx+a)^{\frac{3}{2}}} - \frac{35\,a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((3*x*(2*x/b - 7*a/b^2) - 140*a^2/b^3)*x - 105*a^3/b^4)*sqrt(x)/(b*x +
 a)^(3/2) - 35/4*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(9/2)
```

maple [B] time = 0.25, size = 348, normalized size = 2.27

$$\frac{x^{\frac{7}{2}}}{2b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{7a\,x^{\frac{5}{2}}}{4b^{2}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{35a^{2}x^{\frac{3}{2}}}{12b^{3}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{35a^{2}\sqrt{x}}{4b^{4}\sqrt{\operatorname{arctanh}(\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 1/2*x^(7/2)/b/arctanh(tanh(b*x+a))^(3/2)-7/4/b^2*a*x^(5/2)/arctanh(tanh(b*x
+a))^(3/2)-35/12/b^3*a^2*x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-35/4/b^4*a^2*x^
(1/2)/arctanh(tanh(b*x+a))^(1/2)+35/4/b^(9/2)*a^2*ln(b^(1/2)*x^(1/2)+arctan
h(tanh(b*x+a))^(1/2))-35/6/b^3*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(3/2)/arcta
nh(tanh(b*x+a))^(3/2)-35/2/b^4*a*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/arcta
nh(tanh(b*x+a))^(1/2)+35/2/b^(9/2)*a*(arctanh(tanh(b*x+a))-b*x-a)*ln(b^(1/2
)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))-7/4/b^2*(arctanh(tanh(b*x+a))-b*x-a)*
x^(5/2)/arctanh(tanh(b*x+a))^(3/2)-35/12/b^3*(arctanh(tanh(b*x+a))-b*x-a)^2
*x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-35/4/b^4*(arctanh(tanh(b*x+a))-b*x-a)^2
*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)+35/4/b^(9/2)*(arctanh(tanh(b*x+a))-b*x-
a)^2*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{7/2}}{\operatorname{atanh}(\tanh(a+b\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] int(x^(7/2)/atanh(tanh(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.258 $\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=111

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{7/2}}+\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}$$

```
[Out] 5*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))*(b*x-arctanh(tanh(b*x
+a)))/b^(7/2)-2/3*x^(5/2)/b/arctanh(tanh(b*x+a))^(3/2)-10/3*x^(3/2)/b^2/arc
tanh(tanh(b*x+a))^(1/2)+5*x^(1/2)*arctanh(tanh(b*x+a))^(1/2)/b^3
```

Rubi [A] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2168, 2169, 2165}

$$-\frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{5\sqrt{x}\sqrt{\tanh^{-1}(\tanh(a+bx))}}{b^3}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tan\right.}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]]*(b*x - ArcTanh[T
anh[a + b*x]]))/b^(7/2) - (2*x^(5/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) -
(10*x^(3/2))/(3*b^2*Sqrt[ArcTanh[Tanh[a + b*x]]]) + (5*Sqrt[x]*Sqrt[ArcTanh
[Tanh[a + b*x]]])/b^3
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rule 2169

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + n + 1)), x] - Dist[(n*(b*u -
a*v))/(a*(m + n + 1)), Int[u^m*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; P
iecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && GtQ[n, 0] && NeQ[m + n + 1
, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || LtQ[0, m, n])) &&  !ILtQ[m + n,
-2]
```

Rubi steps

$$\int \frac{x^{5/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{5\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{3b}$$

$$= -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5\int \frac{\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(}}}{b^2}$$

$$= -\frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{5\sqrt{x}\sqrt{\tanh^{-1}(\text{ta}}}{b^3}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}{b^{7/2}} - \frac{2x^{5/2}}{3b\tanh^{-1}(\tanh(a}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.91

$$\frac{5\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{7/2}} - \frac{\sqrt{x}\left(10bx\tanh^{-1}(\tanh(a+bx))-15\,\text{t}\right.}{3b^3\tanh^{-1}(\tanh(}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] -1/3*(Sqrt[x]*(2*b^2*x^2 + 10*b*x*ArcTanh[Tanh[a + b*x]] - 15*ArcTanh[Tanh[
a + b*x]]^2))/(b^3*ArcTanh[Tanh[a + b*x]]^(3/2)) + (5*(b*x - ArcTanh[Tanh[a
 + b*x]])*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(7/2)
```

fricas [A] time = 0.80, size = 214, normalized size = 1.93

$$\left[\frac{15\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{b}\log\left(2bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(3b^3x^2+20ab^2x+15a^2b\right)\sqrt{bx+a}\sqrt{x}}{6\left(b^6x^2+2ab^5x+a^2b^4\right)}, \frac{15\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*
sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*
sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x +
a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

giac [A] time = 0.15, size = 61, normalized size = 0.55

$$\frac{\left(x\left(\frac{3x}{b}+\frac{20a}{b^2}\right)+\frac{15a^2}{b^3}\right)\sqrt{x}}{3(bx+a)^{\frac{3}{2}}} + \frac{5a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x*(3*x/b + 20*a/b^2) + 15*a^2/b^3)*sqrt(x)/(b*x + a)^(3/2) + 5*a*log(a
bs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

maple [B] time = 0.25, size = 180, normalized size = 1.62

$$\frac{x^{\frac{5}{2}}}{b\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}+\frac{5a\,x^{\frac{3}{2}}}{3b^{2}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}+\frac{5a\sqrt{x}}{b^{3}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}-\frac{5a\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] x^(5/2)/b/arctanh(tanh(b*x+a))^(3/2)+5/3/b^2*a*x^(3/2)/arctanh(tanh(b*x+a))
^(3/2)+5/b^3*a*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)-5/b^(7/2)*a*ln(b^(1/2)*x^
(1/2)+arctanh(tanh(b*x+a))^(1/2))+5/3/b^2*(arctanh(tanh(b*x+a))-b*x-a)*x^(3
/2)/arctanh(tanh(b*x+a))^(3/2)+5/b^3*(arctanh(tanh(b*x+a))-b*x-a)*x^(1/2)/a
rctanh(tanh(b*x+a))^(1/2)-5/b^(7/2)*(arctanh(tanh(b*x+a))-b*x-a)*ln(b^(1/2)
*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] int(x^(5/2)/atanh(tanh(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.259 $\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))/b^(5/2)-2/3*x^(3/2)/b
/arctanh(tanh(b*x+a))^(3/2)-2*x^(1/2)/b^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2168, 2165}

$$-\frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[ArcTanh[Tanh[a + b*x]]]])/b^(5/2) - (2*x^
(3/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[ArcTanh[T
anh[a + b*x]]])
```

Rule 2165

```
Int[1/(Sqrt[u_]*Sqrt[v_]), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Si
mplify[D[v, x]]}, Simp[(2*ArcTanh[(Rt[a*b, 2]*Sqrt[u])/(a*Sqrt[v])])/Rt[a*b
, 2], x] /; NeQ[b*u - a*v, 0] && PosQ[a*b]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int \frac{x^{3/2}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{3/2}}\,dx}{b}$$

$$= -\frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{\tanh^{-1}(\tan}}}{b^2}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\tanh^{-1}(\tanh(a+bx))}}\right)}{b^{5/2}} - \frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{2\sqrt{}}{b^2\sqrt{\tanh^{-1}(ta}}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 1.04

$$\frac{2\log\left(\sqrt{b}\sqrt{\tanh^{-1}(\tanh(a+bx))}+b\sqrt{x}\right)}{b^{5/2}} - \frac{2\sqrt{x}}{b^2\sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{2x^{3/2}}{3b\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*ArcTanh[Tanh[a + b*x]]^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[Arc
Tanh[Tanh[a + b*x]]]) + (2*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[ArcTanh[Tanh[a + b*
x]]]])/b^(5/2)
```

fricas [A] time = 0.52, size = 186, normalized size = 2.48

$$\left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\left(4\,b^2x+3\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(b^5x^2+2\,ab^4x+a^2b^3\right)}, -\frac{2\left(3\left(b^2x^2+2\,ab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2*a
*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b
*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

giac [A] time = 0.17, size = 49, normalized size = 0.65

$$-\frac{2\,\sqrt{x}\left(\frac{4\,x}{b}+\frac{3\,a}{b^2}\right)}{3\,(bx+a)^{\frac{3}{2}}} - \frac{2\,\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(4*x/b + 3*a/b^2)/(b*x + a)^(3/2) - 2*log(abs(-sqrt(b)*sqrt(x)
 + sqrt(b*x + a)))/b^(5/2)
```

maple [A] time = 0.25, size = 59, normalized size = 0.79

$$-\frac{2x^{\frac{3}{2}}}{3b\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}-\frac{2\sqrt{x}}{b^{2}\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}}+\frac{2\ln\left(\sqrt{b}\,\sqrt{x}+\sqrt{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/3*x^(3/2)/b/arctanh(tanh(b*x+a))^(3/2)-2*x^(1/2)/b^2/arctanh(tanh(b*x+a)
)^(1/2)+2/b^(5/2)*ln(b^(1/2)*x^(1/2)+arctanh(tanh(b*x+a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/arctanh(tanh(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3/2}}{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/atanh(tanh(a + b*x))^(5/2),x)
```

```
[Out] int(x^(3/2)/atanh(tanh(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\operatorname{atanh}^{\frac{5}{2}}\left(\tanh\left(a+bx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(x**(3/2)/atanh(tanh(a + b*x))**(5/2), x)
```

3.260 $\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/3*x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2167}

$$-\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)
)
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2x^{3/2}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Mathematica [A] time = 0.05, size = 34, normalized size = 0.97

$$\frac{2x^{3/2}}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/ArcTanh[Tanh[a + b*x]]^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*ArcTanh[Tanh[a + b*x]]^(3/2)*(-(b*x) + ArcTanh[Tanh[a + b*x]
]))
```

fricas [A] time = 0.55, size = 33, normalized size = 0.94

$$\frac{2\sqrt{bx+a}\,x^{\frac{3}{2}}}{3\left(ab^2x^2+2\,a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

giac [A] time = 0.13, size = 15, normalized size = 0.43

$$\frac{2\,x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

maple [B] time = 0.25, size = 92, normalized size = 2.63

$$-\frac{\sqrt{x}}{b\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{(\operatorname{arctanh}(\tanh(bx+a))-bx)\left(\frac{\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{}{3(\operatorname{arctanh}(}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x)
[Out] -x^(1/2)/b/arctanh(tanh(b*x+a))^(3/2)+(arctanh(tanh(b*x+a))-b*x)/b*(1/3*x^(
1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctanh(tan
h(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/arctanh(tanh(b*x + a))^(5/2), x)
```

mupad [B] time = 1.70, size = 229, normalized size = 6.54

$$-\frac{4\,x^{3/2}\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}}{3\,b^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)\left(\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^2}+x^2-\frac{x\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\right.}{b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/atanh(tanh(a + b*x))^(5/2),x)
[Out] -(4*x^(3/2)*(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log
(2/(exp(2*a)*exp(2*b*x) + 1))/2)^(1/2))/(3*b^2*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*((l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^2/(4*b^2) + x^2 - (x*(log(2/(exp(2*a)*exp(2*b*x) + 1
)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(sqrt(x)/atanh(tanh(a + b*x))**(5/2), x)
```

3.261 $\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{4\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

```
[Out] -2/3*x^(1/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+4/3*x^(1
/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2171, 2167}

$$\frac{4\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3/2)
) + (4*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*Sqrt[ArcTanh[Tanh[a + b
*x]]])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}\tanh^{-1}(}}{3\left(-bx+\tanh\right.}$$

$$= -\frac{2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}}+\frac{}{3\left(bx-\tanh^{-1}(\right.}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.66

$$-\frac{2\sqrt{x}\left(bx-3\tanh^{-1}(\tanh(a+bx))\right)}{3\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*Sqrt[x]*(b*x - 3*ArcTanh[Tanh[a + b*x]]))/(3*ArcTanh[Tanh[a + b*x]]^(3/
2)*(-(b*x) + ArcTanh[Tanh[a + b*x]])^2)
```

fricas [A] time = 0.46, size = 43, normalized size = 0.61

$$\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^2b^2x^2+2\,a^3bx+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

giac [A] time = 0.13, size = 25, normalized size = 0.35

$$\frac{2\,\sqrt{x}\left(\frac{2\,bx}{a^2}+\frac{3}{a}\right)}{3\,(bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x)*(2*b*x/a^2 + 3/a)/(b*x + a)^(3/2)
```

maple [A] time = 0.24, size = 58, normalized size = 0.82

$$\frac{2\sqrt{x}}{3\,(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3\,(\operatorname{arctanh}(\tanh(bx+a))-bx)^2\,\sqrt{\operatorname{arctanh}(\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] 2/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+4/3/(arct
anh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\ \operatorname{artanh}(\tanh(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x)*arctanh(tanh(b*x + a))^(5/2)), x)
```

mupad [B] time = 1.66, size = 346, normalized size = 4.87

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{16\,x^2}{3\,b\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}-\frac{x\left(48\,\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-48\,\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+96\,b\,x\right)}{12\,b^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}\right)}{x^{5/2}-\frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b}+\frac{\sqrt{x}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*((16*x^2)/(3*b*(log(2/(exp(2*a)*exp(2*b*x) + 1)
) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) - (x
*(48*log(2/(exp(2*a)*exp(2*b*x) + 1)) - 48*log((2*exp(2*a)*exp(2*b*x))/(exp
(2*a)*exp(2*b*x) + 1)) + 96*b*x))/(12*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^
(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b + (x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2)/(4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\ \operatorname{atanh}^{\frac{5}{2}}(\tanh(a+bx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(x)*atanh(tanh(a + b*x))**(5/2)), x)
```

3.262 $\int \frac{1}{x^{3/2} \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=106

$$\frac{16b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+b}$$

```
[Out] 2/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-8/3*b*x^(1/
2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)+16/3*b*x^(1/2)/(
b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2171, 2167}

$$\frac{16b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3 \sqrt{\tanh^{-1}(\tanh(a+bx))}} - \frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-8*b*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a + b*x]]^(
3/2)) + 2/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(3
/2)) + (16*b*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*Sqrt[ArcTanh[Tanh
[a + b*x]]])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2} \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx = \frac{2}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right) \tanh^{-1}(\tanh(a+bx))^{3/2}} - \frac{(4b)\int \frac{}{\sqrt{x}\ \text{ta}}}{-bx+\tan}$$

$$= -\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\left(bx-\text{t}\right.}$$

$$= -\frac{8b\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2 \tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\left(bx-\text{t}\right.}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.62

$$\frac{2\left(6bx\tanh^{-1}(\tanh(a+bx))+3\tanh^{-1}(\tanh(a+bx))^2-b^2x^2\right)}{3\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*(-(b^2*x^2) + 6*b*x*ArcTanh[Tanh[a + b*x]] + 3*ArcTanh[Tanh[a + b*x]]^2)
)/(3*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]]^(3/2))
```

fricas [A] time = 0.38, size = 58, normalized size = 0.55

$$-\frac{2\left(8\,b^2x^2+12\,abx+3\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2*
a^4*b*x^2 + a^5*x)
```

giac [A] time = 0.16, size = 62, normalized size = 0.58

$$-\frac{2\sqrt{x}\left(\frac{5\,b^2x}{a^3}+\frac{6\,b}{a^2}\right)}{3\,(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{b}}{\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(5*b^2*x/a^3 + 6*b/a^2)/(b*x + a)^(3/2) + 4*sqrt(b)/(((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)*a^2)
```

maple [A] time = 0.24, size = 104, normalized size = 0.98

$$-\frac{2}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\frac{8b\left(\frac{\sqrt{x}}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}+\frac{\cdots}{3(\mathrm{ar}\cdots}\right.}{\operatorname{arctanh}(\tanh(bx+\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(3/2)-8*b/(arcta
nh(tanh(b*x+a))-b*x)*(1/3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b
*x+a))^(3/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^
(1/2))
```

maxima [A] time = 0.44, size = 45, normalized size = 0.42

$$-\frac{2\left(8\,b^3x^3+20\,ab^2x^2+15\,a^2bx+3\,a^3\right)}{3\,(bx+a)^{\frac{5}{2}}a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(8*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 3*a^3)/((b*x + a)^(5/2)*a^3*s
qrt(x))
```

mupad [B] time = 1.69, size = 348, normalized size = 3.28

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\,\left(\frac{4}{b^2\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}+\frac{128\,x^2}{3\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}-\cdots\right.}{x^{5/2}-\frac{x^{3/2}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b}+\frac{\sqrt{x}\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^2}{4\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log
((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (128*x^2)/(
3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)
*exp(2*b*x) + 1)) + 2*b*x)^3) - (32*x)/(b*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)))/(x^
(5/2) - (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*
b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b + (x^(1/2)*(log(2/(exp(2*a)*ex
p(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2
*b*x)^2)/(4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\operatorname{atanh}^{\frac{5}{2}}{\left(\tanh{\left(a + bx \right)} \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Integral(1/(x**(3/2)*atanh(tanh(a + b*x))**(5/2)), x)
```

3.263 $\int \frac{1}{x^{5/2} \tanh^{-1}(\tanh(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=146

$$\frac{32b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^3}$$

```
[Out] 2/3/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+4*b/(b*x-
arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-16/3*b^2*x^(1/2)
/(b*x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)+32/3*b^2*x^(1/2)/(
b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{32b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-16*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[Tanh[a + b*x]
]^(3/2)) + (4*b)/(Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^2*ArcTanh[Tanh[a +
 b*x]]^(3/2)) + 2/(3*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a
+ b*x]]^(3/2)) + (32*b^2*Sqrt[x])/(3*(b*x - ArcTanh[Tanh[a + b*x]])^4*Sqrt[
ArcTanh[Tanh[a + b*x]]])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = \frac{2}{3x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{(2b)\int \frac{}{x^{3/2}}}{bx-\tan}$$

$$= \frac{4b}{\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{3x^{3/2}\left(bx-\right.}$$

$$= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\left(bx-t\right.}$$

$$= -\frac{16b^2\sqrt{x}}{3\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{\sqrt{x}\left(bx-t\right.}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.54

$$-\frac{2\left(-9b^2x^2\tanh^{-1}(\tanh(a+bx))-9bx\tanh^{-1}(\tanh(a+bx))^2+\tanh^{-1}(\tanh(a+bx))^3+b^3x^3\right)}{3x^{3/2}\tanh^{-1}(\tanh(a+bx))^{3/2}\left(\tanh^{-1}(\tanh(a+bx))-bx\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-2*(b^3*x^3 - 9*b^2*x^2*ArcTanh[Tanh[a + b*x]] - 9*b*x*ArcTanh[Tanh[a + b*
x]]^2 + ArcTanh[Tanh[a + b*x]]^3))/(3*x^(3/2)*ArcTanh[Tanh[a + b*x]]^(3/2)*
(-(b*x) + ArcTanh[Tanh[a + b*x]])^4)
```

fricas [A] time = 0.40, size = 71, normalized size = 0.49

$$\frac{2\left(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

giac [A] time = 0.19, size = 119, normalized size = 0.82

$$\frac{2\sqrt{x}\left(\frac{8\,b^3x}{a^4}+\frac{9\,b^2}{a^3}\right)}{3\,(bx+a)^{\frac{3}{2}}}-\frac{8\left(3\,b^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^4-9\,ab^{\frac{3}{2}}\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2+4\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x)*(8*b^3*x/a^4 + 9*b^2/a^3)/(b*x + a)^(3/2) - 8/3*(3*b^(3/2)*(sqr
t(b)*sqrt(x) - sqrt(b*x + a))^4 - 9*a*b^(3/2)*(sqrt(b)*sqrt(x) - sqrt(b*x +
 a))^2 + 4*a^2*b^(3/2))/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^3*a^3)
```

maple [A] time = 0.27, size = 150, normalized size = 1.03

$$-\frac{2}{3\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{3}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}-\frac{4b\left(-\frac{1}{\left(\operatorname{arctanh}(\tanh(bx+a))-bx\right)\sqrt{x}\,\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}-\cdots\right)}{\operatorname{ar}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(tanh(b*x+a))^(3/2)-4*b/(arc
tanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))-b*x)/x^(1/2)/arctanh(tanh(
b*x+a))^(3/2)-4*b/(arctanh(tanh(b*x+a))-b*x)*(1/3*x^(1/2)/(arctanh(tanh(b*x
+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctanh(tanh(b*x+a))-b*x)^2*x^(1/
2)/arctanh(tanh(b*x+a))^(1/2)))
```

maxima [A] time = 0.43, size = 56, normalized size = 0.38

$$\frac{2\left(16\,b^4x^4+40\,ab^3x^3+30\,a^2b^2x^2+5\,a^3bx-a^4\right)}{3\left(bx+a\right)^{\frac{5}{2}}a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(16*b^4*x^4 + 40*a*b^3*x^3 + 30*a^2*b^2*x^2 + 5*a^3*b*x - a^4)/((b*x +
a)^(5/2)*a^4*x^(3/2))
```

mupad [B] time = 1.76, size = 406, normalized size = 2.78

$$\frac{\sqrt{\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}}\left(\frac{4}{3\,b^2\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}-\frac{128\,x^2}{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}+\frac{\cdots}{b\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)\cdots\right)}\cdots\right)}{x^{7/2}-\frac{x^{5/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)}{b}+\frac{x^{3/2}\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\cdots}{e^{2\,a}\cdots}\right)\cdots\right)}{4\,b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(3*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) - (128*x^2)
/(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*
exp(2*b*x) + 1)) + 2*b*x)^3 + (16*x)/(b*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2) + (512*
b*x^3)/(3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(
exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)))/(x^(7/2) - (x^(5/2)*(log(2/(exp(2*a
)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))
 + 2*b*x))/b + (x^(3/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)
*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.264 $\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{256b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx)}$$

```
[Out] 16/15*b/x^(3/2)/(b*x-arctanh(tanh(b*x+a)))^2/arctanh(tanh(b*x+a))^(3/2)+2/5
/x^(5/2)/(b*x-arctanh(tanh(b*x+a)))/arctanh(tanh(b*x+a))^(3/2)+32/5*b^2/(b*
x-arctanh(tanh(b*x+a)))^3/arctanh(tanh(b*x+a))^(3/2)/x^(1/2)-128/15*b^3*x^(
1/2)/(b*x-arctanh(tanh(b*x+a)))^4/arctanh(tanh(b*x+a))^(3/2)+256/15*b^3*x^(
1/2)/(b*x-arctanh(tanh(b*x+a)))^5/arctanh(tanh(b*x+a))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2171, 2167}

$$\frac{256b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\sqrt{\tanh^{-1}(\tanh(a+bx))}}-\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (-128*b^3*Sqrt[x])/(15*(b*x - ArcTanh[Tanh[a + b*x]])^4*ArcTanh[Tanh[a + b*
x]]^(3/2)) + (32*b^2)/(5*Sqrt[x]*(b*x - ArcTanh[Tanh[a + b*x]])^3*ArcTanh[T
anh[a + b*x]]^(3/2)) + (16*b)/(15*x^(3/2)*(b*x - ArcTanh[Tanh[a + b*x]])^2*
ArcTanh[Tanh[a + b*x]]^(3/2)) + 2/(5*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])
*ArcTanh[Tanh[a + b*x]]^(3/2)) + (256*b^3*Sqrt[x])/(15*(b*x - ArcTanh[Tanh[
a + b*x]])^5*Sqrt[ArcTanh[Tanh[a + b*x]]])
```

Rule 2167

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] /; Ne
Q[b*u - a*v, 0]] /; FreeQ[{m, n}, x] && PiecewiseLinearQ[u, v, x] && EqQ[m
+ n + 2, 0] && NeQ[m, -1]
```

Rule 2171

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, -Simp[(u^(m + 1)*v^(n + 1))/((m + 1)*(b*u - a*v)), x] + Dis
t[(b*(m + n + 2))/((m + 1)*(b*u - a*v)), Int[u^(m + 1)*v^n, x], x] /; NeQ[b
*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x] && NeQ[m + n + 2, 0] && LtQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\tanh^{-1}(\tanh(a+bx))^{5/2}}\,dx = \frac{2}{5x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{(8b)\int \frac{}{x^{5/2}}}{5\left(bx-\text{ta}\right.}$$

$$= \frac{16b}{15x^{3/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^2\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5x^{5/2}\left(b\right.}$$

$$= \frac{32b^2}{5\sqrt{x}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^3\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{15x^{3/2}\left(b\right.}$$

$$= -\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5\sqrt{x}\left(bx-\right.}$$

$$= -\frac{128b^3\sqrt{x}}{15\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^4\tanh^{-1}(\tanh(a+bx))^{3/2}} + \frac{}{5\sqrt{x}\left(bx-\right.}$$

Mathematica [A] time = 0.08, size = 100, normalized size = 0.54

$$\frac{2\left(60b^3x^3\tanh^{-1}(\tanh(a+bx)) + 90b^2x^2\tanh^{-1}(\tanh(a+bx))^2 - 20bx\tanh^{-1}(\tanh(a+bx))^3 + 3\tanh^{-1}(\text{ta}\right.}{15x^{5/2}\left(bx-\tanh^{-1}(\tanh(a+bx))\right)^5\tanh^{-1}(\tanh(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*ArcTanh[Tanh[a + b*x]]^(5/2)),x]
```

```
[Out] (2*(-5*b^4*x^4 + 60*b^3*x^3*ArcTanh[Tanh[a + b*x]] + 90*b^2*x^2*ArcTanh[Tan
h[a + b*x]]^2 - 20*b*x*ArcTanh[Tanh[a + b*x]]^3 + 3*ArcTanh[Tanh[a + b*x]]^
4))/(15*x^(5/2)*(b*x - ArcTanh[Tanh[a + b*x]])^5*ArcTanh[Tanh[a + b*x]]^(3/
2))
```

fricas [A] time = 0.73, size = 82, normalized size = 0.44

$$-\frac{2\left(128\,b^4x^4 + 192\,ab^3x^3 + 48\,a^2b^2x^2 - 8\,a^3bx + 3\,a^4\right)\sqrt{bx+a}\,\sqrt{x}}{15\left(a^5b^2x^5 + 2\,a^6bx^4 + a^7x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*x^4 + 192*a*b^3*x^3 + 48*a^2*b^2*x^2 - 8*a^3*b*x + 3*a^4)*sq
rt(b*x + a)*sqrt(x)/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3)
```

giac [A] time = 0.24, size = 173, normalized size = 0.93

$$-\frac{2\sqrt{x}\left(\frac{11\,b^4x}{a^5} + \frac{12\,b^3}{a^4}\right)}{3\,(bx+a)^{\frac{3}{2}}} + \frac{4\left(45\,b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x} - \sqrt{bx+a}\right)^8 - 240\,ab^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x} - \sqrt{bx+a}\right)^6 + 490\,a^2b^{\frac{5}{2}}\left(\sqrt{b}\sqrt{x} - \sqrt{b}\right.\right.}{15\left(\left(\sqrt{b}\sqrt{x} - \sqrt{bx+a}\right)^2 - a\right)^5 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(x)*(11*b^4*x/a^5 + 12*b^3/a^4)/(b*x + a)^(3/2) + 4/15*(45*b^(5/2)
*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^8 - 240*a*b^(5/2)*(sqrt(b)*sqrt(x) - sqr
```

```
t(b*x + a))^6 + 490*a^2*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 320*a
^3*b^(5/2)*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 + 73*a^4*b^(5/2))/(((sqrt(b)
*sqrt(x) - sqrt(b*x + a))^2 - a)^5*a^4)
```

maple [A] time = 0.26, size = 196, normalized size = 1.05

$$-\frac{2}{5\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^{\frac{5}{2}}\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{\frac{3}{2}}}-\frac{16b\left(-\frac{1}{3(\operatorname{arctanh}(\tanh(bx+a))-bx)x^{\frac{3}{2}}\operatorname{arctanh}(\tanh(bx+a))^{\frac{3}{2}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x)
```

```
[Out] -2/5/(arctanh(tanh(b*x+a))-b*x)/x^(5/2)/arctanh(tanh(b*x+a))^(3/2)-16/5*b/(
arctanh(tanh(b*x+a))-b*x)*(-1/3/(arctanh(tanh(b*x+a))-b*x)/x^(3/2)/arctanh(
tanh(b*x+a))^(3/2)-2*b/(arctanh(tanh(b*x+a))-b*x)*(-1/(arctanh(tanh(b*x+a))
-b*x)/x^(1/2)/arctanh(tanh(b*x+a))^(3/2)-4*b/(arctanh(tanh(b*x+a))-b*x)*(1/
3*x^(1/2)/(arctanh(tanh(b*x+a))-b*x)/arctanh(tanh(b*x+a))^(3/2)+2/3/(arctan
h(tanh(b*x+a))-b*x)^2*x^(1/2)/arctanh(tanh(b*x+a))^(1/2))))
```

maxima [A] time = 0.44, size = 67, normalized size = 0.36

$$-\frac{2\left(128\,b^5x^5+320\,ab^4x^4+240\,a^2b^3x^3+40\,a^3b^2x^2-5\,a^4bx+3\,a^5\right)}{15\left(bx+a\right)^{\frac{5}{2}}a^5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/arctanh(tanh(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(128*b^5*x^5 + 320*a*b^4*x^4 + 240*a^2*b^3*x^3 + 40*a^3*b^2*x^2 - 5*a
^4*b*x + 3*a^5)/((b*x + a)^(5/2)*a^5*x^(5/2))
```

mupad [B] time = 1.86, size = 466, normalized size = 2.51

$$\frac{\sqrt{\frac{\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)}{2}}\left(\frac{4}{5b^2\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}+\frac{256x^2}{5\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)^3}+\frac{}{15b\left(\ln\left(\right.\right.}\right.}{x^{9/2}-\frac{x^{7/2}\left(\ln\left(\frac{2}{e^{2a}e^{2bx}+1}\right)-\ln\left(\frac{2e^{2a}e^{2bx}}{e^{2a}e^{2bx}+1}\right)+2bx\right)}{b}+\frac{x^{5/}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*atanh(tanh(a + b*x))^(5/2)),x)
```

```
[Out] ((log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^(1/2)*(4/(5*b^2*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - l
og((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)) + (256*x^2)
/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*
a)*exp(2*b*x) + 1)) + 2*b*x)^3) + (64*x)/(15*b*(log(2/(exp(2*a)*exp(2*b*x)
+ 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)
- (2048*b*x^3)/(5*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2
```

```
*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4) + (8192*b^2*x^4)/(15*(log(2/(
exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x
) + 1)) + 2*b*x)^5)))/(x^(9/2) - (x^(7/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1))
 - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/b + (x^
(5/2)*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)^2)/(4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/atanh(tanh(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.265 $\int x^m \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=79

$$\frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{n+1} {}_2F_1\left(-m, n+1; n+2; -\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

```
[Out] x^m*arctanh(tanh(b*x+a))^(1+n)*hypergeom([-m, 1+n],[2+n],-arctanh(tanh(b*x+
a))/(b*x-arctanh(tanh(b*x+a))))/b/(1+n)/((b*x/(b*x-arctanh(tanh(b*x+a))))^m
)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2173}

$$\frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{n+1} {}_2F_1\left(-m, n+1; n+2; -\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^m*ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, -(A
rcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(b*(1 + n)*((b*x)/(
b*x - ArcTanh[Tanh[a + b*x]]))^m)
```

Rule 2173

```
Int[(u_)^(m_)*(v_)^(n_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simp
lify[D[v, x]]}, Simp[(u^m*v^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, -((
a*v)/(b*u - a*v))])/(b*(n + 1)*((b*u)/(b*u - a*v))^m), x] /; NeQ[b*u - a*v,
 0]] /; PiecewiseLinearQ[u, v, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int x^m \tanh^{-1}(\tanh(a+bx))^n \, dx = \frac{x^m \left(\frac{bx}{bx-\tanh^{-1}(\tanh(a+bx))}\right)^{-m} \tanh^{-1}(\tanh(a+bx))^{1+n} {}_2F_1\left(-m, 1+n; 2+n; -\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{b(1+n)}$$

Mathematica [A] time = 0.14, size = 71, normalized size = 0.90

$$\frac{x^{m+1} \tanh^{-1}(\tanh(a+bx))^n \left(\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}+1\right)^{-n} {}_2F_1\left(m+1, -n; m+2; -\frac{bx}{\tanh^{-1}(\tanh(a+bx))-bx}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^(1 + m)*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((
b*x)/(-(b*x) + ArcTanh[Tanh[a + b*x]]))])/((1 + m)*(1 + (b*x)/(-(b*x) + Arc
Tanh[Tanh[a + b*x]]))^n)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
[Out] integral(x^m*arctanh(tanh(b*x + a))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
[Out] integrate(x^m*arctanh(tanh(b*x + a))^n, x)
```

maple [F] time = 1.58, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(tanh(b*x+a))^n,x)
[Out] int(x^m*arctanh(tanh(b*x+a))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
[Out] integrate(x^m*arctanh(tanh(b*x + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*atanh(tanh(a + b*x))^n,x)
[Out] int(x^m*atanh(tanh(a + b*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m} \operatorname{atanh}^{n}{\left(\tanh{\left(a+bx \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(tanh(b*x+a))**n,x)
[Out] Integral(x**m*atanh(tanh(a + b*x))**n, x)
```

3.266 $\int x^4 \tanh^{-1}(\tanh(a+bx))^n\,dx$

Optimal. Leaf size=165

$$\frac{24\tanh^{-1}(\tanh(a+bx))^{n+5}}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)}-\frac{24x\tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{4x^3\tanh}{b^2}$$

```
[Out] x^4*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-4*x^3*arctanh(tanh(b*x+a))^(2+n)/b^2
/(1+n)/(2+n)+12*x^2*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-24*x*a
rctanh(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)+24*arctanh(tanh(b*x+a
))^(5+n)/b^5/(n^2+7*n+12)/(n^3+8*n^2+17*n+10)
```

Rubi [A] time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{4x^3\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{12x^2\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{24x\tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{24\tanh^{-1}(}{b^5(n+1)(n+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^4*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (4*x^3*ArcTanh[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (12*x^2*ArcTanh[Tanh[a + b*x]]^(3 +
n))/(b^3*(1 + n)*(2 + n)*(3 + n)) - (24*x*ArcTanh[Tanh[a + b*x]]^(4 + n))/(
b^4*(1 + n)*(2 + n)*(3 + n)*(4 + n)) + (24*ArcTanh[Tanh[a + b*x]]^(5 + n))/
(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\int x^4 \tanh^{-1}(\tanh(a+bx))^n\,dx = \frac{x^4\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4\int x^3\tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^4\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12\int x^2\tanh}{b^2(}$$
$$= \frac{x^4\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2\tanh^{-1}}{b^3(1+n)}$$
$$= \frac{x^4\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2\tanh^{-1}}{b^3(1+n)}$$
$$= \frac{x^4\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2\tanh^{-1}}{b^3(1+n)}$$
$$= \frac{x^4\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{4x^3\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{12x^2\tanh^{-1}}{b^3(1+n)}$$

Mathematica [A] time = 0.11, size = 146, normalized size = 0.88

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\left(-4b^3\left(n^3+12n^2+47n+60\right)x^3\tanh^{-1}(\tanh(a+bx))+12b^2\left(n^2+9n+20\right)x^2\tan}{b^5(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^4*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)*
x^4 - 4*b^3*(60 + 47*n + 12*n^2 + n^3)*x^3*ArcTanh[Tanh[a + b*x]] + 12*b^2*
(20 + 9*n + n^2)*x^2*ArcTanh[Tanh[a + b*x]]^2 - 24*b*(5 + n)*x*ArcTanh[Tanh
[a + b*x]]^3 + 24*ArcTanh[Tanh[a + b*x]]^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*(5 + n))
```

fricas [B] time = 0.40, size = 374, normalized size = 2.27

$$-\frac{\left(24\,a^4bnx-\left(b^5n^4+10\,b^5n^3+35\,b^5n^2+50\,b^5n+24\,b^5\right)x^5-24\,a^5-\left(ab^4n^4+6\,ab^4n^3+11\,ab^4n^2+6\,ab^4n\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] -((24*a^4*b*n*x - (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b^5*n + 24*b^5)*x
^5 - 24*a^5 - (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^4*n^2 + 6*a*b^4*n)*x^4 + 4*
(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x^3 - 12*(a^3*b^2*n^2 + a^3*b^2
*n)*x^2)*cosh(n*log(b*x + a)) + (24*a^4*b*n*x - (b^5*n^4 + 10*b^5*n^3 + 35*
b^5*n^2 + 50*b^5*n + 24*b^5)*x^5 - 24*a^5 - (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a
*b^4*n^2 + 6*a*b^4*n)*x^4 + 4*(a^2*b^3*n^3 + 3*a^2*b^3*n^2 + 2*a^2*b^3*n)*x
^3 - 12*(a^3*b^2*n^2 + a^3*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^5*n^5 + 15*
b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

giac [B] time = 0.14, size = 332, normalized size = 2.01

$$\frac{(bx+a)^n b^5n^4x^5+(bx+a)^n ab^4n^4x^4+10\,(bx+a)^n b^5n^3x^5+6\,(bx+a)^n ab^4n^3x^4+35\,(bx+a)^n b^5n^2x^5-4\,(bx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*n^4*x^5 + (b*x + a)^n*a*b^4*n^4*x^4 + 10*(b*x + a)^n*b^5*n
^3*x^5 + 6*(b*x + a)^n*a*b^4*n^3*x^4 + 35*(b*x + a)^n*b^5*n^2*x^5 - 4*(b*x
+ a)^n*a^2*b^3*n^3*x^3 + 11*(b*x + a)^n*a*b^4*n^2*x^4 + 50*(b*x + a)^n*b^5*
n*x^5 - 12*(b*x + a)^n*a^2*b^3*n^2*x^3 + 6*(b*x + a)^n*a*b^4*n*x^4 + 24*(b*
x + a)^n*b^5*x^5 + 12*(b*x + a)^n*a^3*b^2*n^2*x^2 - 8*(b*x + a)^n*a^2*b^3*n
*x^3 + 12*(b*x + a)^n*a^3*b^2*n*x^2 - 24*(b*x + a)^n*a^4*b*n*x + 24*(b*x +
a)^n*a^5)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 12
0*b^5)
```

maple [B] time = 0.14, size = 654, normalized size = 3.96

$$\frac{x^5 e^{n \ln(\operatorname{arctanh}(\tanh(bx+a)))}}{5+n}+\frac{n\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^4 e^{n \ln(\operatorname{arctanh}(\tanh(bx+a)))}}{b\left(n^2+9n+20\right)}-\frac{4n\left(a^2+2a\left(\operatorname{arctanh}\left(\tanh\left(\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] 1/(5+n)*x^5*exp(n*ln(arctanh(tanh(b*x+a))))+n/b*(arctanh(tanh(b*x+a))-b*x)/
(n^2+9*n+20)*x^4*exp(n*ln(arctanh(tanh(b*x+a))))-4*n*(a^2+2*a*(arctanh(tanh
(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/b^2/(n^3+12*n^2+47*n+60)*x^
3*exp(n*ln(arctanh(tanh(b*x+a))))+24/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*e
xp(n*ln(arctanh(tanh(b*x+a))))*a^5+120/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)
*exp(n*ln(arctanh(tanh(b*x+a))))*a^4*(arctanh(tanh(b*x+a))-b*x-a)+240/b^5/(
n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*a^3*(arctan
h(tanh(b*x+a))-b*x-a)^2+240/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(a
rctanh(tanh(b*x+a))))*a^2*(arctanh(tanh(b*x+a))-b*x-a)^3+120/b^5/(n^3+12*n^
2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*a*(arctanh(tanh(b*x+
a))-b*x-a)^4+24/b^5/(n^3+12*n^2+47*n+60)/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(
b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^5-24*(a^2+2*a*(arctanh(tanh(b*x+a))-
b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)^2*n/b^4/(n^3+12*n^2+47*n+60)/(n^2+3*
n+2)*x*exp(n*ln(arctanh(tanh(b*x+a))))+12/b^3*(arctanh(tanh(b*x+a))-b*x)*(a
^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)*n/(2+n)
/(n^3+12*n^2+47*n+60)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))
```

maxima [A] time = 0.53, size = 139, normalized size = 0.84

$$\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3x^3+12\left(n^2+n\right.\right.}{\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)
*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 -
 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274
*n + 120)*b^5)
```

mupad [B] time = 1.67, size = 546, normalized size = 3.31

$$-\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^5}{4\,b^5\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{x^5\left(n^4+10\,n^3+3\right.}{n^5+15\,n^4+85\,n^3+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*atanh(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^n*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^5)/(4*b^5*(274*n + 225
*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (x^5*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (3*n*x*(log(2/(exp(
2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) +
1)) + 2*b*x)^4)/(2*b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (
n*x^4*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(
2*a)*exp(2*b*x) + 1)) + 2*b*x)*(11*n + 6*n^2 + n^3 + 6))/(2*b*(274*n + 225*
n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (3*n*x^2*(n + 1)*(log(2/(exp(2*a)*exp
(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*
b*x)^3)/(2*b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (n*x^3*(l
og(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp
(2*b*x) + 1)) + 2*b*x)^2*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 +
15*n^4 + n^5 + 120)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**5*atanh(tanh(a))**n/5, Eq(b, 0)), (-x**4/(4*b*atanh(tanh(a +
b*x))**4) - x**3/(3*b**2*atanh(tanh(a + b*x))**3) - x**2/(2*b**3*atanh(tanh
(a + b*x))**2) - x/(b**4*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)))/
b**5, Eq(n, -5)), (Integral(x**4/atanh(tanh(a + b*x))**4, x), Eq(n, -4)), (
Integral(x**4/atanh(tanh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**4/atanh
(tanh(a + b*x))**2, x), Eq(n, -2)), (Integral(x**4/atanh(tanh(a + b*x)), x)
, Eq(n, -1)), (b**4*n**4*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120
*b**5) + 14*b**4*n**3*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b*
*5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b*
*5) + 71*b**4*n**2*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**5*
n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
 + 154*b**4*n*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**5*n**5
+ 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12
0*b**4*x**4*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b*
*5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*b**3*n*
*3*x**3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 48*b**3*n*
*2*x**3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 188*b**3*n
*x**3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 240*b**3*x**
3*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*b**2*n**2*x**
2*atanh(tanh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4
 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 108*b**2*n*x**2*
atanh(tanh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 240*b**2*x**2*atan
h(tanh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*b*n*x*atanh(tanh(a
+ b*x))**4*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 120*b*x*atanh(tanh(a + b*x))**4
*atanh(tanh(a + b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**
5*n**2 + 274*b**5*n + 120*b**5) + 24*atanh(tanh(a + b*x))**5*atanh(tanh(a +
 b*x))**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b*
*5*n + 120*b**5), True))
```

3.267 $\int x^3 \tanh^{-1}(\tanh(a+bx))^n\,dx$

Optimal. Leaf size=121

$$-\frac{6\tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{6x\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{3x^2\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{x^3\tanh^{-1}(\tanh(a}{b(n+1)}$$

```
[Out] x^3*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-3*x^2*arctanh(tanh(b*x+a))^(2+n)/b^2
/(1+n)/(2+n)+6*x*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)-6*arctanh
(tanh(b*x+a))^(4+n)/b^4/(n^2+5*n+4)/(n^2+5*n+6)
```

Rubi [A] time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2168, 2157, 30}

$$-\frac{3x^2\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{6x\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{6\tanh^{-1}(\tanh(a+bx))^{n+4}}{b^4(n+1)(n+2)(n+3)(n+4)}+\frac{x^3\tanh^{-1}(\tanh(a}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^3*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (3*x^2*ArcTanh[Tanh[a +
b*x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (6*x*ArcTanh[Tanh[a + b*x]]^(3 + n))
/(b^3*(1 + n)*(2 + n)*(3 + n)) - (6*ArcTanh[Tanh[a + b*x]]^(4 + n))/(b^4*(1
 + n)*(2 + n)*(3 + n)*(4 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^3 \tanh^{-1}(\tanh(a+bx))^n\,dx = \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3\int x^2 \tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$
$$= \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6\int x \tanh^{-1}}{b^2(1}$$
$$= \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \tanh^{-1}(\mathrm{ta}}{b^3(1+n)(}$$
$$= \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \tanh^{-1}(\mathrm{ta}}{b^3(1+n)(}$$
$$= \frac{x^3 \tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)} - \frac{3x^2 \tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)} + \frac{6x \tanh^{-1}(\mathrm{ta}}{b^3(1+n)(}$$

Mathematica [A] time = 0.10, size = 106, normalized size = 0.88

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\left(-3b^2\left(n^2+7n+12\right)x^2\tanh^{-1}(\tanh(a+bx))+6b(n+4)x\tanh^{-1}(\tanh(a+bx))^2 -}{b^4(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^3*(24 + 26*n + 9*n^2 + n^3)*x^3 - 3*b^2*
(12 + 7*n + n^2)*x^2*ArcTanh[Tanh[a + b*x]] + 6*b*(4 + n)*x*ArcTanh[Tanh[a
+ b*x]]^2 - 6*ArcTanh[Tanh[a + b*x]]^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n))
```

fricas [B] time = 0.51, size = 255, normalized size = 2.11

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] ((6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b
^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*co
sh(n*log(b*x + a)) + (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4
)*x^4 - 6*a^4 + (a*b^3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2
+ a^2*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2
+ 50*b^4*n + 24*b^4)
```

giac [A] time = 0.14, size = 226, normalized size = 1.87

$$\frac{(bx+a)^n b^4n^3x^4+(bx+a)^n ab^3n^3x^3+6\,(bx+a)^n b^4n^2x^4+3\,(bx+a)^n ab^3n^2x^3+11\,(bx+a)^n b^4nx^4-3\,(bx+a}{b^4n^4+10\,b^4n^3+35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n^
2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x + a
)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4 - 3
*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^4)/(
b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

maple [B] time = 0.11, size = 492, normalized size = 4.07

$$\frac{x^4 e^{n \ln(\mathrm{arctanh}(\tanh(bx+a)))}}{4+n}+\frac{n\left(\mathrm{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^3 e^{n\ln(\mathrm{arctanh}(\tanh(bx+a)))}}{b\left(n^2+7n+12\right)}-\frac{6\, e^{n\ln(\mathrm{arctanh}(\tanh(bx+a)))} a}{b^4\left(n^4+10n^3+35n^2+50n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] 1/(4+n)*x^4*exp(n*ln(arctanh(tanh(b*x+a))))+n*(arctanh(tanh(b*x+a))-b*x)/b/
(n^2+7*n+12)*x^3*exp(n*ln(arctanh(tanh(b*x+a))))-6/b^4/(n^4+10*n^3+35*n^2+5
0*n+24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^4-24/b^4/(n^4+10*n^3+35*n^2+50*n+
24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^3*(arctanh(tanh(b*x+a))-b*x-a)-36/b^4
/(n^4+10*n^3+35*n^2+50*n+24)*exp(n*ln(arctanh(tanh(b*x+a))))*a^2*(arctanh(t
anh(b*x+a))-b*x-a)^2-24/b^4/(n^4+10*n^3+35*n^2+50*n+24)*exp(n*ln(arctanh(ta
nh(b*x+a))))*a*(arctanh(tanh(b*x+a))-b*x-a)^3-6/b^4/(n^4+10*n^3+35*n^2+50*n
+24)*exp(n*ln(arctanh(tanh(b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^4-3*n/b^2
*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)/(n^3
+9*n^2+26*n+24)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+6*n*(a^3+3*a^2*(arctanh
(tanh(b*x+a))-b*x-a)+3*a*(arctanh(tanh(b*x+a))-b*x-a)^2+(arctanh(tanh(b*x+a
))-b*x-a)^3)/b^3/(n^4+10*n^3+35*n^2+50*n+24)*x*exp(n*ln(arctanh(tanh(b*x+a)
)))
```

maxima [A] time = 0.54, size = 101, normalized size = 0.83

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4)
```

mupad [B] time = 1.42, size = 418, normalized size = 3.45

$$-\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n\left(\frac{3\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^4}{8\,b^4\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{x^4\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^n*((3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^4)/(8*b^4*(50*n + 35*n
^2 + 10*n^3 + n^4 + 24)) - (x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
10*n^3 + n^4 + 24) + (3*n*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(
2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3)/(4*b^3*(50*n + 35*n
^2 + 10*n^3 + n^4 + 24)) + (n*x^3*(log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((
2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)*(3*n + n^2 + 2))
/(2*b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*n*x^2*(n + 1)*(log(2/(exp(2
*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1
)) + 2*b*x)^2)/(4*b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**4*atanh(tanh(a))**n/4, Eq(b, 0)), (-x**3/(3*b*atanh(tanh(a +
b*x))**3) - x**2/(2*b**2*atanh(tanh(a + b*x))**2) - x/(b**3*atanh(tanh(a +
b*x))) + log(atanh(tanh(a + b*x)))/b**4, Eq(n, -4)), (Integral(x**3/atanh(t
anh(a + b*x))**3, x), Eq(n, -3)), (Integral(x**3/atanh(tanh(a + b*x))**2, x
), Eq(n, -2)), (Integral(x**3/atanh(tanh(a + b*x)), x), Eq(n, -1)), (b**3*n
**3*x**3*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*
n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**3*n**2*x**3*atanh(tanh(a
+ b*x))*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) + 26*b**3*n*x**3*atanh(tanh(a + b*x))*atanh(tanh(a + b
*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2
4*b**3*x**3*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b*
*4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*b**2*n**2*x**2*atanh(tanh
(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) - 21*b**2*n*x**2*atanh(tanh(a + b*x))**2*atanh(t
anh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 36*b**2*x**2*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**4*
n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b*n*x*atanh(t
anh(a + b*x))**3*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**
4*n**2 + 50*b**4*n + 24*b**4) + 24*b*x*atanh(tanh(a + b*x))**3*atanh(tanh(a
 + b*x))**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 - 6*atanh(tanh(a + b*x))**4*atanh(tanh(a + b*x))**n/(b**4*n**4 + 10*b**4*n
**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))
```

3.268 $\int x^2 \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=82

$$\frac{2\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}-\frac{2x\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{x^2\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

```
[Out] x^2*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-2*x*arctanh(tanh(b*x+a))^(2+n)/b^2/(
1+n)/(2+n)+2*arctanh(tanh(b*x+a))^(3+n)/b^3/(3+n)/(n^2+3*n+2)
```

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2168, 2157, 30}

$$-\frac{2x\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}+\frac{2\tanh^{-1}(\tanh(a+bx))^{n+3}}{b^3(n+1)(n+2)(n+3)}+\frac{x^2\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x^2*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - (2*x*ArcTanh[Tanh[a + b*
x]]^(2 + n))/(b^2*(1 + n)*(2 + n)) + (2*ArcTanh[Tanh[a + b*x]]^(3 + n))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\int x^2\tanh^{-1}(\tanh(a+bx))^n\,dx=\frac{x^2\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{2\int x\tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}$$

$$=\frac{x^2\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{2x\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}+\frac{2\int\tanh^{-1}(\tanh}{b^2(1+n}$$

$$=\frac{x^2\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{2x\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}+\frac{2\,\text{Subst}\left(\int x^{2+n}\right.}{b^3}$$

$$=\frac{x^2\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{2x\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}+\frac{2\tanh^{-1}(\tanh(a}{b^3(1+n)(2+n}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.87

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\left(-2b(n+3)x\tanh^{-1}(\tanh(a+bx))+2\tanh^{-1}(\tanh(a+bx))^2+b^2\left(n^2+5n+6\right)x^2\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*(b^2*(6 + 5*n + n^2)*x^2 - 2*b*(3 + n)*x*Ar
cTanh[Tanh[a + b*x]] + 2*ArcTanh[Tanh[a + b*x]]^2))/(b^3*(1 + n)*(2 + n)*(3
 + n))
```

fricas [B] time = 0.67, size = 168, normalized size = 2.05

$$-\frac{\left(2a^2bnx-\left(b^3n^2+3b^3n+2b^3\right)x^3-2a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\cosh\left(n\log\left(bx+a\right)\right)+\left(2a^2bnx-\left(b^3n^2+3\right.\right.}{b^3n^3+6b^3n^2+11b^3n+6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] -((2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b
^2*n)*x^2)*cosh(n*log(b*x + a)) + (2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3
)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^2*n)*x^2)*sinh(n*log(b*x + a)))/(b^3*n^3 +
 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

giac [A] time = 0.14, size = 140, normalized size = 1.71

$$\frac{(bx+a)^nb^3n^2x^3+(bx+a)^nab^2n^2x^2+3\,(bx+a)^nb^3nx^3+(bx+a)^nab^2nx^2+2\,(bx+a)^nb^3x^3-2\,(bx+a)^na^2bn}{b^3n^3+6b^3n^2+11b^3n+6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n*
x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2*b
*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

maple [B] time = 0.10, size = 315, normalized size = 3.84

$$\frac{x^3\mathrm{e}^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{3+n}+\frac{n\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)-bx\right)x^2\mathrm{e}^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{b\left(n^2+5n+6\right)}+\frac{2\,\mathrm{e}^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}a^3}{b^3\left(n^3+6n^2+11n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] 1/(3+n)*x^3*exp(n*ln(arctanh(tanh(b*x+a))))+n/b*(arctanh(tanh(b*x+a))-b*x)/
(n^2+5*n+6)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+2/b^3/(n^3+6*n^2+11*n+6)*ex
p(n*ln(arctanh(tanh(b*x+a))))*a^3+6/b^3/(n^3+6*n^2+11*n+6)*exp(n*ln(arctanh
(tanh(b*x+a))))*a^2*(arctanh(tanh(b*x+a))-b*x-a)+6/b^3/(n^3+6*n^2+11*n+6)*e
xp(n*ln(arctanh(tanh(b*x+a))))*a*(arctanh(tanh(b*x+a))-b*x-a)^2+2/b^3/(n^3+
6*n^2+11*n+6)*exp(n*ln(arctanh(tanh(b*x+a))))*(arctanh(tanh(b*x+a))-b*x-a)^
3-2*n*(a^2+2*a*(arctanh(tanh(b*x+a))-b*x-a)+(arctanh(tanh(b*x+a))-b*x-a)^2)
/b^2/(n^3+6*n^2+11*n+6)*x*exp(n*ln(arctanh(tanh(b*x+a))))
```

maxima [A] time = 0.54, size = 68, normalized size = 0.83

$$\frac{\left(\left(n^2+3\,n+2\right)b^3x^3+\left(n^2+n\right)ab^2x^2-2\,a^2bnx+2\,a^3\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

mupad [B] time = 1.29, size = 304, normalized size = 3.71

$$-\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}-\frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n\,\left(\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)+2\,b\,x\right)^3}{4\,b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{n\,x\,\left(\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^n*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^3/(4*b^3*(11*n + 6*n^2 +
n^3 + 6)) - (x^3*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (n*x*(log(2/(e
xp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x)
 + 1)) + 2*b*x)^2)/(2*b^2*(11*n + 6*n^2 + n^3 + 6)) + (n*x^2*(n + 1)*(log(2
/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b
*x) + 1)) + 2*b*x))/(2*b*(11*n + 6*n^2 + n^3 + 6)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^3\operatorname{atanh}^n(\tanh(a))}{3} \\ -\frac{x^2}{2b\operatorname{atanh}^2(\tanh(a+bx))}-\frac{x}{b^2\operatorname{atanh}(\tanh(a+bx))}+\frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^3} \\ \int\frac{x^2}{\operatorname{atanh}^2(\tanh(a+bx))}\,dx \\ \int\frac{x^2}{\operatorname{atanh}(\tanh(a+bx))}\,dx \\ \frac{b^2n^2x^2\operatorname{atanh}(\tanh(a+bx))\operatorname{atanh}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{5b^2nx^2\operatorname{atanh}(\tanh(a+bx))\operatorname{atanh}^n(\tanh(a+bx))}{b^3n^3+6b^3n^2+11b^3n+6b^3}+\frac{6b^2x^2\operatorname{atanh}(\tanh(a+bx))\operatorname{atanh}^n(\tanh(}{b^3n^3+6b^3n^2+11b^3n+6b^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**3*atanh(tanh(a))**n/3, Eq(b, 0)), (-x**2/(2*b*atanh(tanh(a +
b*x))**2) - x/(b**2*atanh(tanh(a + b*x))) + log(atanh(tanh(a + b*x)))/b**3,
 Eq(n, -3)), (Integral(x**2/atanh(tanh(a + b*x))**2, x), Eq(n, -2)), (Integ
ral(x**2/atanh(tanh(a + b*x)), x), Eq(n, -1)), (b**2*n**2*x**2*atanh(tanh(a
 + b*x))*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b
**3) + 5*b**2*n*x**2*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**2*x**2*atanh(tanh(a + b*x))*at
anh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*b*
n*x*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**
2 + 11*b**3*n + 6*b**3) - 6*b*x*atanh(tanh(a + b*x))**2*atanh(tanh(a + b*x)
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*atanh(tanh(a + b*x)
)**3*atanh(tanh(a + b*x))**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3)
, True))
```

3.269 $\int x \tanh^{-1}(\tanh(a+bx))^n \, dx$

Optimal. Leaf size=48

$$\frac{x\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

```
[Out] x*arctanh(tanh(b*x+a))^(1+n)/b/(1+n)-arctanh(tanh(b*x+a))^(2+n)/b^2/(1+n)/(
2+n)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2168, 2157, 30}

$$\frac{x\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}-\frac{\tanh^{-1}(\tanh(a+bx))^{n+2}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] (x*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b*(1 + n)) - ArcTanh[Tanh[a + b*x]]^(2
+ n)/(b^2*(1 + n)*(2 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\tanh(a+bx))^n\,dx &= \frac{x\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{\int \tanh^{-1}(\tanh(a+bx))^{1+n}\,dx}{b(1+n)}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{\text{Subst}\left(\int x^{1+n}\,dx,x,\tanh^{-1}(\tanh(a+bx))\right)}{b^2(1+n)}\\ &= \frac{x\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}-\frac{\tanh^{-1}(\tanh(a+bx))^{2+n}}{b^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 41, normalized size = 0.85

$$\frac{\left(b(n+2)x-\tanh^{-1}(\tanh(a+bx))\right)\tanh^{-1}(\tanh(a+bx))^{n+1}}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] ((b*(2 + n)*x - ArcTanh[Tanh[a + b*x]])*ArcTanh[Tanh[a + b*x]]^(1 + n))/(b^
2*(1 + n)*(2 + n))
```

fricas [A] time = 0.56, size = 91, normalized size = 1.90

$$\frac{\left(abnx + \left(b^2n + b^2\right)x^2 - a^2\right)\cosh\left(n\log\left(bx + a\right)\right) + \left(abnx + \left(b^2n + b^2\right)x^2 - a^2\right)\sinh\left(n\log\left(bx + a\right)\right)}{b^2n^2 + 3\,b^2n + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] ((a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*cosh(n*log(b*x + a)) + (a*b*n*x + (b^2
*n + b^2)*x^2 - a^2)*sinh(n*log(b*x + a)))/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

giac [A] time = 0.14, size = 76, normalized size = 1.58

$$\frac{(bx + a)^n b^2nx^2 + (bx + a)^n abnx + (bx + a)^n b^2x^2 - (bx + a)^n a^2}{b^2n^2 + 3\,b^2n + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x +
 a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

maple [B] time = 0.10, size = 175, normalized size = 3.65

$$\frac{x^2 e^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{2+n} + \frac{n\left(\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right) - bx\right)x\, e^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}}{b\left(n^2+3n+2\right)} - \frac{e^{n\ln(\operatorname{arctanh}(\tanh(bx+a)))}a^2}{b^2\left(n^2+3n+2\right)} - \frac{2\, e^{n}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(tanh(b*x+a))^n,x)
```

```
[Out] 1/(2+n)*x^2*exp(n*ln(arctanh(tanh(b*x+a))))+n*(arctanh(tanh(b*x+a))-b*x)/b/
(n^2+3*n+2)*x*exp(n*ln(arctanh(tanh(b*x+a))))-1/b^2/(n^2+3*n+2)*exp(n*ln(ar
ctanh(tanh(b*x+a))))*a^2-2/b^2/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+a))))*
a*(arctanh(tanh(b*x+a))-b*x-a)-1/b^2/(n^2+3*n+2)*exp(n*ln(arctanh(tanh(b*x+
a))))*(arctanh(tanh(b*x+a))-b*x-a)^2
```

maxima [A] time = 0.54, size = 42, normalized size = 0.88

$$\frac{\left(b^2(n+1)x^2 + abnx - a^2\right)(bx + a)^n}{\left(n^2 + 3\,n + 2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)
```

mupad [B] time = 1.23, size = 205, normalized size = 4.27

$$-\left(\frac{\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2} - \frac{\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right)}{2}\right)^n \left(\frac{\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b\,x}+1}\right) - \ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}+1}\right) + 2\,b\,x\right)^2}{4\,b^2\,\left(n^2+3\,n+2\right)} - \frac{x^2\,\left(n+1\right)}{n^2+3\,n+2} + \frac{n\,x\,\left(\ln\left(\frac{2}{e^{2\,a}\,e^{2\,b}}\right.\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(tanh(a + b*x))^n,x)
```

```
[Out] -(log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 - log(2/(exp(2*a
)*exp(2*b*x) + 1))/2)^n*((log(2/(exp(2*a)*exp(2*b*x) + 1)) - log((2*exp(2*a
)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x)^2/(4*b^2*(3*n + n^2 + 2))
 - (x^2*(n + 1))/(3*n + n^2 + 2) + (n*x*(log(2/(exp(2*a)*exp(2*b*x) + 1)) -
 log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) + 2*b*x))/(2*b*(3*n
 + n^2 + 2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2 \operatorname{atanh}^n(\tanh(a))}{2} \\ -\frac{x}{b \operatorname{atanh}(\tanh(a+bx))} + \frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b^2} \\ \int \frac{x}{\operatorname{atanh}(\tanh(a+bx))}\, dx \\ \frac{bnx \operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2} + \frac{2bx \operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2} - \frac{\operatorname{atanh}^2(\tanh(a+bx)) \operatorname{atanh}^n(\tanh(a+bx))}{b^2n^2+3b^2n+2b^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x**2*atanh(tanh(a))**n/2, Eq(b, 0)), (-x/(b*atanh(tanh(a + b*x))
) + log(atanh(tanh(a + b*x)))/b**2, Eq(n, -2)), (Integral(x/atanh(tanh(a +
b*x)), x), Eq(n, -1)), (b*n*x*atanh(tanh(a + b*x))*atanh(tanh(a + b*x))**n/
(b**2*n**2 + 3*b**2*n + 2*b**2) + 2*b*x*atanh(tanh(a + b*x))*atanh(tanh(a +
 b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2) - atanh(tanh(a + b*x))**2*atanh(t
anh(a + b*x))**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True))
```

3.270 $\int \tanh^{-1}(\tanh(a+bx))^n\,dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

```
[Out] arctanh(tanh(b*x+a))^(1+n)/b/(1+n)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\tanh(a+bx))^n\,dx &= \frac{\text{Subst}\left(\int x^n\,dx, x, \tanh^{-1}(\tanh(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\tanh(a+bx))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n,x]
```

```
[Out] ArcTanh[Tanh[a + b*x]]^(1 + n)/(b*(1 + n))
```

fricas [A] time = 0.51, size = 39, normalized size = 1.95

$$\frac{(bx+a)\cosh\left(n\log\left(bx+a\right)\right)+(bx+a)\sinh\left(n\log\left(bx+a\right)\right)}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*cosh(n*log(b*x + a)) + (b*x + a)*sinh(n*log(b*x + a)))/(b*n + b)
```

giac [A] time = 0.14, size = 28, normalized size = 1.40

$$\frac{(bx+a)^n bx + (bx+a)^n a}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="giac")
[Out] ((b*x + a)^n*b*x + (b*x + a)^n*a)/(b*n + b)
```

maple [A] time = 0.03, size = 21, normalized size = 1.05

$$\frac{\operatorname{arctanh}(\tanh(bx+a))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n,x)
[Out] arctanh(tanh(b*x+a))^(1+n)/b/(1+n)
```

maxima [A] time = 0.50, size = 21, normalized size = 1.05

$$\frac{(bx+a)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n,x, algorithm="maxima")
[Out] (b*x + a)*(b*x + a)^n/(b*(n + 1))
```

mupad [B] time = 1.18, size = 121, normalized size = 6.05

$$\left(\frac{1}{2}\right)^n \left(\frac{x}{n+1} - \frac{\frac{\ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)}{2} - \frac{\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right)}{2} + b\,x}{b\,(n+1)}\right) \left(\ln\left(\frac{2\,e^{2a}\,e^{2bx}}{e^{2a}\,e^{2bx}+1}\right) - \ln\left(\frac{2}{e^{2a}\,e^{2bx}+1}\right)\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^n,x)
[Out] (1/2)^n*(x/(n + 1) - (log(2/(exp(2*a)*exp(2*b*x) + 1))/2 - log((2*exp(2*a)*
exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1))/2 + b*x)/(b*(n + 1)))*(log((2*exp(2*
a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) + 1)) - log(2/(exp(2*a)*exp(2*b*x) + 1)
))^n
```

sympy [A] time = 0.69, size = 51, normalized size = 2.55

$$\begin{cases} \frac{x}{\operatorname{atanh}(\tanh(a))} & \text{for}\: b = 0 \wedge n = -1 \\ x \operatorname{atanh}^{n}(\tanh(a)) & \text{for}\: b = 0 \\ \frac{\log(\operatorname{atanh}(\tanh(a+bx)))}{b} & \text{for}\: n = -1 \\ \frac{\operatorname{atanh}(\tanh(a+bx)) \operatorname{atanh}^{n}(\tanh(a+bx))}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n,x)
```

```
[Out] Piecewise((x/atanh(tanh(a)), Eq(b, 0) & Eq(n, -1)), (x*atanh(tanh(a))**n, E
q(b, 0)), (log(atanh(tanh(a + b*x)))/b, Eq(n, -1)), (atanh(tanh(a + b*x))*a
tanh(tanh(a + b*x))**n/(b*n + b), True))
```

3.271 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x} dx$

Optimal. Leaf size=64

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

```
[Out] arctanh(tanh(b*x+a))^(1+n)*hypergeom([1, 1+n],[2+n],-arctanh(tanh(b*x+a))/(
b*x-arctanh(tanh(b*x+a))))/(1+n)/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2164}

$$\frac{\tanh^{-1}(\tanh(a+bx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(n+1)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, -(ArcTan
h[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/((1 + n)*(b*x - ArcTanh[
Tanh[a + b*x]]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x}\,dx = \frac{\tanh^{-1}(\tanh(a+bx))^{1+n}\,{}_2F_1\left(1,1+n;2+n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{(1+n)\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 0.94

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\,{}_2F_1\left(-n,-n;1-n;1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[-n, -n, 1 - n, 1 - ArcTanh[Tanh
[a + b*x]]/(b*x)])/(n*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="fricas")
[Out] integral(arctanh(tanh(b*x + a))^n/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="giac")
[Out] integrate(arctanh(tanh(b*x + a))^n/x, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x,x)
[Out] int(arctanh(tanh(b*x+a))^n/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x,x, algorithm="maxima")
[Out] integrate(arctanh(tanh(b*x + a))^n/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^n/x,x)
[Out] int(atanh(tanh(a + b*x))^n/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{n}\left(\tanh\left(a+bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x,x)
[Out] Integral(atanh(tanh(a + b*x))**n/x, x)
```

3.272 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^2}\, dx$

Optimal. Leaf size=71

$$\frac{b\tanh^{-1}(\tanh(a+bx))^n\, {}_2F_1\left(1,n;n+1;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{x}$$

```
[Out] -arctanh(tanh(b*x+a))^n/x+b*arctanh(tanh(b*x+a))^n*hypergeom([1, n],[1+n],-
arctanh(tanh(b*x+a))/(b*x-arctanh(tanh(b*x+a))))/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$\frac{b\tanh^{-1}(\tanh(a+bx))^n\, {}_2F_1\left(1,n;n+1;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{bx-\tanh^{-1}(\tanh(a+bx))}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x^2,x]
```

```
[Out] -(ArcTanh[Tanh[a + b*x]]^n/x) + (b*ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2
F1[1, n, 1 + n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/
(b*x - ArcTanh[Tanh[a + b*x]])
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^2}\, dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^n}{x}+(bn)\int \frac{\tanh^{-1}(\tanh(a+bx))^{-1+n}}{x}\, dx\\ &= -\frac{\tanh^{-1}(\tanh(a+bx))^n}{x}+\frac{b\tanh^{-1}(\tanh(a+bx))^n\, {}_2F_1\left(1,n;1+n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{bx-\tanh^{-1}(\tanh(a+bx))}\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.94

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}\, {}_2F_1\left(1-n,-n;2-n;1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{(n-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x^2,x]
```

```
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[1 - n, -n, 2 - n, 1 - ArcTanh[T
anh[a + b*x]]/(b*x)])/((-1 + n)*x*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

fricas [F] time = 1.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(tanh(b*x + a))^n/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x^2, x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x^2,x)
```

```
[Out] int(arctanh(tanh(b*x+a))^n/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tanh\left(bx+a\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^2,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(tanh(b*x + a))^n/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{atanh}\left(\tanh\left(a+b\,x\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^n/x^2,x)
```

```
[Out] int(atanh(tanh(a + b*x))^n/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{n}\left(\tanh\left(a+bx\right)\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x**2,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**n/x**2, x)
```

3.273 $\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^3}\,dx$

Optimal. Leaf size=101

$$\frac{b^2 n \tanh^{-1}(\tanh(a+bx))^{n-1}\,{}_2F_1\left(1,n-1;n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}-\frac{bn\tanh^{-1}(\tanh(a+}{2x}$$

```
[Out] -1/2*b*n*arctanh(tanh(b*x+a))^(-1+n)/x-1/2*arctanh(tanh(b*x+a))^n/x^2+1/2*b
^2*n*arctanh(tanh(b*x+a))^(-1+n)*hypergeom([1, -1+n],[n],-arctanh(tanh(b*x+
a))/(b*x-arctanh(tanh(b*x+a))))/(b*x-arctanh(tanh(b*x+a)))
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2168, 2164}

$$\frac{b^2 n \tanh^{-1}(\tanh(a+bx))^{n-1}\,{}_2F_1\left(1,n-1;n;-\frac{\tanh^{-1}(\tanh(a+bx))}{bx-\tanh^{-1}(\tanh(a+bx))}\right)}{2\left(bx-\tanh^{-1}(\tanh(a+bx))\right)}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}-\frac{bn\tanh^{-1}(\tanh(a+}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tanh[a + b*x]]^n/x^3,x]
```

```
[Out] -(b*n*ArcTanh[Tanh[a + b*x]]^(-1 + n))/(2*x) - ArcTanh[Tanh[a + b*x]]^n/(2*
x^2) + (b^2*n*ArcTanh[Tanh[a + b*x]]^(-1 + n)*Hypergeometric2F1[1, -1 + n,
n, -(ArcTanh[Tanh[a + b*x]]/(b*x - ArcTanh[Tanh[a + b*x]]))])/(2*(b*x - Arc
Tanh[Tanh[a + b*x]]))
```

Rule 2164

```
Int[(v_)^(n_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[
D[v, x]]}, Simp[(v^(n + 1)*Hypergeometric2F1[1, n + 1, n + 2, -((a*v)/(b*u
- a*v))])/((n + 1)*(b*u - a*v)), x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinea
rQ[u, v, x] && !IntegerQ[n]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\tanh(a+bx))^n}{x^3}\,dx &= -\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}+\frac{1}{2}(bn)\int \frac{\tanh^{-1}(\tanh(a+bx))^{-1+n}}{x^2}\,dx\\ &= -\frac{bn\tanh^{-1}(\tanh(a+bx))^{-1+n}}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}-\frac{1}{2}\left(b^2(1-n)n\right)\int \frac{\tan}{}\\ &= -\frac{bn\tanh^{-1}(\tanh(a+bx))^{-1+n}}{2x}-\frac{\tanh^{-1}(\tanh(a+bx))^n}{2x^2}+\frac{b^2 n\tanh^{-1}(\tanh(a+}{2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.66

$$\frac{\tanh^{-1}(\tanh(a+bx))^n\left(\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)^{-n}{}_2F_1\left(2-n,-n;3-n;1-\frac{\tanh^{-1}(\tanh(a+bx))}{bx}\right)}{(n-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tanh[a + b*x]]^n/x^3,x]
[Out] (ArcTanh[Tanh[a + b*x]]^n*Hypergeometric2F1[2 - n, -n, 3 - n, 1 - ArcTanh[T
anh[a + b*x]]/(b*x)])/((-2 + n)*x^2*(ArcTanh[Tanh[a + b*x]]/(b*x))^n)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="fricas")
[Out] integral(arctanh(tanh(b*x + a))^n/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="giac")
[Out] integrate(arctanh(tanh(b*x + a))^n/x^3, x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\text{arctanh}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tanh(b*x+a))^n/x^3,x)
[Out] int(arctanh(tanh(b*x+a))^n/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(\tanh\left(bx+a\right)\right)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tanh(b*x+a))^n/x^3,x, algorithm="maxima")
[Out] integrate(arctanh(tanh(b*x + a))^n/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\text{atanh}\left(\tanh\left(a+b\,x\right)\right)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tanh(a + b*x))^n/x^3,x)
```

```
[Out] int(atanh(tanh(a + b*x))^n/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}^{n}(\tanh(a+bx))}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tanh(b*x+a))**n/x**3,x)
```

```
[Out] Integral(atanh(tanh(a + b*x))**n/x**3, x)
```

3.274 $\int x^m \coth^{-1}(\tanh(a+bx))\,dx$

Optimal. Leaf size=37

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

```
[Out] -b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arccoth(tanh(b*x+a))/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{x^{m+1}\coth^{-1}(\tanh(a+bx))}{m+1}-\frac{bx^{m+2}}{m^2+3m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] -((b*x^(2 + m))/(2 + 3*m + m^2)) + (x^(1 + m)*ArcCoth[Tanh[a + b*x]])/(1 +
m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^m \coth^{-1}(\tanh(a+bx))\,dx &= \frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}-\frac{b\int x^{1+m}\,dx}{1+m}\\ &= -\frac{bx^{2+m}}{2+3m+m^2}+\frac{x^{1+m}\coth^{-1}(\tanh(a+bx))}{1+m}\end{aligned}$$

Mathematica [A] time = 0.07, size = 34, normalized size = 0.92

$$x^m\left(\frac{x\left(\coth^{-1}(\tanh(a+bx))-bx\right)}{m+1}+\frac{bx^2}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*ArcCoth[Tanh[a + b*x]],x]
```

```
[Out] x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcCoth[Tanh[a + b*x]]))/(1 + m))
```

fricas [A] time = 0.45, size = 33, normalized size = 0.89

$$\frac{\left((bm+b)x^2+(am+2\,a)x\right)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^m*arccoth(tanh(b*x + a)), x)
```

maple [C] time = 0.29, size = 676, normalized size = 18.27

$$\frac{x\,x^m\ln\left(e^{bx+a}\right)}{1+m}-\frac{x\left(-i\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)^2 m+i\pi\operatorname{csgn}\left(\frac{i}{e^{2bx+2a}+1}\right)\operatorname{csgn}\left(ie^{2bx+2a}\right)\operatorname{csgn}\left(\frac{ie^{2bx+2a}}{e^{2bx+2a}+1}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arccoth(tanh(b*x+a)),x)
```

```
[Out] 1/(1+m)*x*x^m*ln(exp(b*x+a))-1/4*x*(-I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I
*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*m+4*I*Pi+I*Pi
*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))*m-I*Pi*csgn(I*exp(2*b*x+2*a))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2*m+2*I*Pi*csgn(I*exp(2*b*x+2*a))
^3+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3*m-2*I*Pi*csgn(I*exp(2*b*x+2*a))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^2
*m+2*I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+I*Pi*csgn(I*exp(2*b*x
+2*a))^3*m-2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2+2*I*Pi*m+I*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3*m+2
*I*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-2*I*Pi*csgn(I*exp(b*x+a))
*csgn(I*exp(2*b*x+2*a))^2*m+2*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*
b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))+4*b*x-4*I*Pi*csgn(I*exp
(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+4*I*Pi*csgn(I/(exp(2*b*x+2*a)+1))^3-4*I*P
i*csgn(I/(exp(2*b*x+2*a)+1))^2)/(1+m)/(2+m)*x^m
```

maxima [A] time = 0.33, size = 38, normalized size = 1.03

$$-\frac{bx^2x^m}{(m+2)(m+1)}+\frac{x^{m+1}\operatorname{arcoth}\left(\tanh\left(bx+a\right)\right)}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arccoth(tanh(b*x + a))/(m + 1)
```

mupad [B] time = 1.60, size = 96, normalized size = 2.59

$$\frac{2\,b\,x^m\,x^2\,\left(m+1\right)}{2\,m^2+6\,m+4}-\frac{x\,x^m\,\left(m+2\right)\,\left(\ln\left(-\frac{2}{e^{2\,a}\,e^{2\,b\,x}-1}\right)-\ln\left(\frac{2\,e^{2\,a}\,e^{2\,b\,x}}{e^{2\,a}\,e^{2\,b\,x}-1}\right)+2\,b\,x\right)}{2\,m^2+6\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*acoth(tanh(a + b*x)),x)
```

```
[Out] (2*b*x^m*x^2*(m + 1))/(6*m + 2*m^2 + 4) - (x*x^m*(m + 2)*(log(-2/(exp(2*a)*
exp(2*b*x) - 1)) - log((2*exp(2*a)*exp(2*b*x))/(exp(2*a)*exp(2*b*x) - 1)) +
 2*b*x))/(6*m + 2*m^2 + 4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} b\log(x) - \frac{\operatorname{acoth}(\tanh(a+bx))}{x} & \text{for } m = -2 \\ \int \frac{\operatorname{acoth}(\tanh(a+bx))}{x}\,dx & \text{for } m = -1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} + \frac{2xx^m \operatorname{acoth}(\tanh(a+bx))}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*acoth(tanh(b*x+a)),x)
```

```
[Out] Piecewise((b*log(x) - acoth(tanh(a + b*x))/x, Eq(m, -2)), (Integral(acoth(t
anh(a + b*x))/x, x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x*x**m*
acoth(tanh(a + b*x))/(m**2 + 3*m + 2) + 2*x*x**m*acoth(tanh(a + b*x))/(m**2
 + 3*m + 2), True))
```

3.275 $\int x^2 \tanh^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))-\frac{bx^4}{12}$$

```
[Out] -1/12*b*x^4+1/3*x^3*arctanh(coth(b*x+a))
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$\frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))-\frac{bx^4}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -(b*x^4)/12 + (x^3*ArcTanh[Coth[a + b*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(\coth(a+bx))\,dx &= \frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))-\frac{1}{3}b\int x^3\,dx\\ &= -\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(\coth(a+bx))\end{aligned}$$

Mathematica [A] time = 0.05, size = 20, normalized size = 0.87

$$-\frac{1}{12}x^3\left(bx-4\tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -1/12*(x^3*(b*x - 4*ArcTanh[Coth[a + b*x]]))
```

fricas [A] time = 0.69, size = 13, normalized size = 0.57

$$\frac{1}{4}bx^4+\frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

giac [B] time = 0.16, size = 71, normalized size = 3.09

$$-\frac{1}{12}\,bx^4+\frac{1}{6}\,x^3\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/12*b*x^4 + 1/6*x^3*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1
)/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

maple [A] time = 0.24, size = 20, normalized size = 0.87

$$-\frac{b\,x^4}{12}+\frac{x^3\,\mathrm{arctanh}\left(\coth\left(bx+a\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(coth(b*x+a)),x)
```

```
[Out] -1/12*b*x^4+1/3*x^3*arctanh(coth(b*x+a))
```

maxima [A] time = 0.39, size = 19, normalized size = 0.83

$$-\frac{1}{12}\,bx^4+\frac{1}{3}\,x^3\,\mathrm{artanh}\left(\coth\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/12*b*x^4 + 1/3*x^3*arctanh(coth(b*x + a))
```

mupad [B] time = 1.10, size = 19, normalized size = 0.83

$$\frac{x^3\,\mathrm{atanh}\left(\coth\left(a+b\,x\right)\right)}{3}-\frac{b\,x^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(coth(a + b*x)),x)
```

```
[Out] (x^3*atanh(coth(a + b*x)))/3 - (b*x^4)/12
```

sympy [A] time = 11.10, size = 49, normalized size = 2.13

$$\begin{cases}\left\langle -\frac{\pi}{6},\frac{\pi}{6}\right\rangle ix^3 & \text{for } a=\log\left(-e^{-bx}\right)\vee a=\log\left(e^{-bx}\right)\\ -\frac{bx^4}{12}+\frac{x^3\,\mathrm{atanh}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(coth(b*x+a)),x)
```

```
[Out] Piecewise((AccumBounds(-pi/6, pi/6)*I*x**3, Eq(a, log(exp(-b*x))) | Eq(a, l
og(-exp(-b*x)))), (-b*x**4/12 + x**3*atanh(1/tanh(a + b*x))/3, True))
```

3.276 $\int x \tanh^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))-\frac{bx^3}{6}$$

[Out] -1/6*b*x^3+1/2*x^2*arctanh(coth(b*x+a))

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6241, 30}

$$\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))-\frac{bx^3}{6}$$

Antiderivative was successfully verified.

[In] Int[x*ArcTanh[Coth[a + b*x]],x]

[Out] -(b*x^3)/6 + (x^2*ArcTanh[Coth[a + b*x]])/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rubi steps

$$\begin{aligned}\int x\tanh^{-1}(\coth(a+bx))\,dx&=\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))-\frac{1}{2}b\int x^2\,dx\\&=-\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(\coth(a+bx))\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 0.87

$$-\frac{1}{6}x^2\left(bx-3\tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcTanh[Coth[a + b*x]],x]

[Out] -1/6*(x^2*(b*x - 3*ArcTanh[Coth[a + b*x]]))

fricas [A] time = 0.60, size = 13, normalized size = 0.57

$$\frac{1}{3}bx^3+\frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="fricas")

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

giac [B] time = 0.15, size = 71, normalized size = 3.09

$$-\frac{1}{6}\,bx^3 + \frac{1}{4}\,x^2\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/6*b*x^3 + 1/4*x^2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)
/((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

maple [A] time = 0.14, size = 20, normalized size = 0.87

$$-\frac{b\,x^3}{6} + \frac{x^2\,\mathrm{arctanh}\left(\coth\left(bx+a\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(coth(b*x+a)),x)
```

```
[Out] -1/6*b*x^3+1/2*x^2*arctanh(coth(b*x+a))
```

maxima [A] time = 0.39, size = 19, normalized size = 0.83

$$-\frac{1}{6}\,bx^3 + \frac{1}{2}\,x^2\,\mathrm{artanh}\left(\coth\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/6*b*x^3 + 1/2*x^2*arctanh(coth(b*x + a))
```

mupad [B] time = 0.06, size = 19, normalized size = 0.83

$$\frac{x^2\,\mathrm{atanh}\left(\coth\left(a+b\,x\right)\right)}{2} - \frac{b\,x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(coth(a + b*x)),x)
```

```
[Out] (x^2*atanh(coth(a + b*x)))/2 - (b*x^3)/6
```

sympy [A] time = 5.55, size = 49, normalized size = 2.13

$$\begin{cases} \left\langle -\frac{\pi}{4}, \frac{\pi}{4}\right\rangle ix^2 & \text{for}\ a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ -\frac{bx^3}{6} + \frac{x^2\,\mathrm{atanh}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(coth(b*x+a)),x)
```

```
[Out] Piecewise((AccumBounds(-pi/4, pi/4)*I*x**2, Eq(a, log(exp(-b*x))) | Eq(a, l
og(-exp(-b*x)))), (-b*x**3/6 + x**2*atanh(1/tanh(a + b*x))/2, True))
```

3.277 $\int \tanh^{-1}(\coth(a+bx))\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\coth(a+bx))^2}{2b}$$

```
[Out] 1/2*arctanh(coth(b*x+a))^2/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2157, 30}

$$\frac{\tanh^{-1}(\coth(a+bx))^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]],x]
```

```
[Out] ArcTanh[Coth[a + b*x]]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\coth(a+bx))\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \tanh^{-1}(\coth(a+bx))\right)}{b}\\ &= \frac{\tanh^{-1}(\coth(a+bx))^2}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.12

$$x\tanh^{-1}(\coth(a+bx))-\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]],x]
```

```
[Out] -1/2*(b*x^2) + x*ArcTanh[Coth[a + b*x]]
```

fricas [A] time = 0.49, size = 10, normalized size = 0.62

$$\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

giac [B] time = 0.14, size = 69, normalized size = 4.31

$$-\frac{1}{2}\,bx^{2}+\frac{1}{2}\,x\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="giac")
```

```
[Out] -1/2*b*x^2 + 1/2*x*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/(
(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))
```

maple [A] time = 0.03, size = 15, normalized size = 0.94

$$\frac{\operatorname{arctanh}\left(\coth\left(bx+a\right)\right)^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a)),x)
```

```
[Out] 1/2*arctanh(coth(b*x+a))^2/b
```

maxima [A] time = 0.38, size = 16, normalized size = 1.00

$$-\frac{1}{2}\,bx^{2}+x\operatorname{artanh}\left(\coth\left(bx+a\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*b*x^2 + x*arctanh(coth(b*x + a))
```

mupad [B] time = 0.02, size = 16, normalized size = 1.00

$$x\operatorname{atanh}\left(\coth\left(a+b\,x\right)\right)-\frac{b\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(coth(a + b*x)),x)
```

```
[Out] x*atanh(coth(a + b*x)) - (b*x^2)/2
```

sympy [A] time = 2.82, size = 46, normalized size = 2.88

$$\begin{cases} x\operatorname{atanh}\left(\coth\left(a\right)\right) & \text{for}\ b=0 \\ \left\langle -\frac{\pi}{2},\frac{\pi}{2}\right\rangle ix & \text{for}\ a=\log\left(-e^{-bx}\right)\vee a=\log\left(e^{-bx}\right) \\ \frac{\operatorname{atanh}^{2}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a)),x)
```

```
[Out] Piecewise((x*atanh(coth(a)), Eq(b, 0)), (AccumBounds(-pi/2, pi/2)*I*x, Eq(a
, log(exp(-b*x))) | Eq(a, log(-exp(-b*x)))), (atanh(1/tanh(a + b*x))**2/(2*
b), True))
```

3.278 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x} \, dx$

Optimal. Leaf size=21

$$bx - \log(x)\left(bx - \tanh^{-1}(\coth(a+bx))\right)$$

```
[Out] b*x-(b*x-arctanh(coth(b*x+a)))*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2158, 29}

$$bx - \log(x)\left(bx - \tanh^{-1}(\coth(a+bx))\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]]/x,x]
```

```
[Out] b*x - (b*x - ArcTanh[Coth[a + b*x]])*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2158

```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v, x]]}, Simp[(b*x)/a, x] - Dist[(b*u - a*v)/a, Int[1/u, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(\coth(a+bx))}{x} \, dx &= bx - \left(bx - \tanh^{-1}(\coth(a+bx))\right) \int \frac{1}{x} \, dx \\ &= bx - \left(bx - \tanh^{-1}(\coth(a+bx))\right) \log(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 0.90

$$\log(x)\left(\tanh^{-1}(\coth(a+bx)) - bx\right) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x,x]
```

```
[Out] b*x + (-(b*x) + ArcTanh[Coth[a + b*x]])*Log[x]
```

fricas [A] time = 0.71, size = 8, normalized size = 0.38

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

giac [C] time = 0.14, size = 15, normalized size = 0.71

$$bx + \frac{1}{2}(i\pi + 2a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="giac")
```

```
[Out] b*x + 1/2*(I*pi + 2*a)*log(x)
```

maple [A] time = 0.14, size = 21, normalized size = 1.00

$$\ln(x)\operatorname{arctanh}(\coth(bx+a)) - \ln(x)xb + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x,x)
```

```
[Out] ln(x)*arctanh(coth(b*x+a))-ln(x)*x*b+b*x
```

maxima [A] time = 0.33, size = 34, normalized size = 1.62

$$-b\left(x+\frac{a}{b}\right)\log(x) + b\left(x+\frac{a\log(x)}{b}\right) + \operatorname{artanh}(\coth(bx+a))\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -b*(x + a/b)*log(x) + b*(x + a*log(x)/b) + arctanh(coth(b*x + a))*log(x)
```

mupad [B] time = 1.20, size = 59, normalized size = 2.81

$$b\,x - \ln(x)\left(\frac{\ln\left(-\frac{2}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)}{2} - \frac{\ln\left(\frac{2\,\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}}{\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1}\right)}{2} + b\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(coth(a + b*x))/x,x)
```

```
[Out] b*x - log(x)*(log(-2/(exp(2*a)*exp(2*b*x) - 1))/2 - log((2*exp(2*a)*exp(2*b
*x))/(exp(2*a)*exp(2*b*x) - 1))/2 + b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(\coth(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x,x)
```

```
[Out] Integral(atanh(coth(a + b*x))/x, x)
```

3.279 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x^2} \, dx$

Optimal. Leaf size=17

$$b\log(x) - \frac{\tanh^{-1}(\coth(a+bx))}{x}$$

```
[Out] -arctanh(coth(b*x+a))/x+b*ln(x)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 29}

$$b\log(x) - \frac{\tanh^{-1}(\coth(a+bx))}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]]/x^2,x]
```

```
[Out] -(ArcTanh[Coth[a + b*x]]/x) + b*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 && !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] && !IntegerQ[m]) || (ILtQ
[m, 0] && !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\coth(a+bx))}{x^2} \, dx &= -\frac{\tanh^{-1}(\coth(a+bx))}{x} + b\int \frac{1}{x} \, dx \\ &= -\frac{\tanh^{-1}(\coth(a+bx))}{x} + b\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.06

$$-\frac{\tanh^{-1}(\coth(a+bx))}{x} + b\log(x) + b$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x^2,x]
```

```
[Out] b - ArcTanh[Coth[a + b*x]]/x + b*Log[x]
```

fricas [A] time = 0.62, size = 13, normalized size = 0.76

$$\frac{bx\log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - a)/x
```

giac [B] time = 0.16, size = 70, normalized size = 4.12

$$b\log\left(|x|\right)-\frac{\log\left(-\frac{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}+1}{\frac{e^{(2\,bx+2\,a)}+1}{e^{(2\,bx+2\,a)}-1}-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - 1/2*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/
((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x
```

maple [A] time = 0.14, size = 18, normalized size = 1.06

$$-\frac{\operatorname{arctanh}\left(\coth\left(bx+a\right)\right)}{x}+b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x^2,x)
```

```
[Out] -arctanh(coth(b*x+a))/x+b*ln(x)
```

maxima [A] time = 0.38, size = 17, normalized size = 1.00

$$b\log(x)-\frac{\operatorname{artanh}\left(\coth\left(bx+a\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*log(x) - arctanh(coth(b*x + a))/x
```

mupad [B] time = 0.08, size = 17, normalized size = 1.00

$$b\,\ln(x)-\frac{\operatorname{atanh}\left(\coth\left(a+b\,x\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(coth(a + b*x))/x^2,x)
```

```
[Out] b*log(x) - atanh(coth(a + b*x))/x
```

sympy [A] time = 5.78, size = 42, normalized size = 2.47

$$\begin{cases}\frac{\left\langle -\frac{\pi}{2},\frac{\pi}{2}\right\rangle i}{x} & \text{for}\ a=\log\left(-e^{-bx}\right)\vee a=\log\left(e^{-bx}\right)\\ b\log\left(x\right)-\frac{\operatorname{atanh}\left(\frac{1}{\tanh\left(a+bx\right)}\right)}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x**2,x)
```

```
[Out] Piecewise((AccumBounds(-pi/2, pi/2)*I/x, Eq(a, log(exp(-b*x))) | Eq(a, log(
-exp(-b*x)))), (b*log(x) - atanh(1/tanh(a + b*x))/x, True))
```

3.280 $\int \frac{\tanh^{-1}(\coth(a+bx))}{x^3} \, dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

```
[Out] -1/2*b/x-1/2*arctanh(coth(b*x+a))/x^2
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2168, 30}

$$-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}-\frac{b}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Coth[a + b*x]]/x^3,x]
```

```
[Out] -b/(2*x) - ArcTanh[Coth[a + b*x]]/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2168

```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[(u^(m + 1)*v^n)/(a*(m + 1)), x] - Dist[(b*n)/(a*(m +
1)), Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n},
 x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0]
 &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ[
n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILtQ
[m, 0] &&  !IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(\coth(a+bx))}{x^3}\,dx&=-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}+\frac{1}{2}b\int \frac{1}{x^2}\,dx\\&=-\frac{b}{2x}-\frac{\tanh^{-1}(\coth(a+bx))}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.78

$$-\frac{\tanh^{-1}(\coth(a+bx))+bx}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Coth[a + b*x]]/x^3,x]
```

```
[Out] -1/2*(b*x + ArcTanh[Coth[a + b*x]])/x^2
```

fricas [A] time = 0.47, size = 11, normalized size = 0.48

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

giac [B] time = 0.16, size = 71, normalized size = 3.09

$$-\frac{b}{2x} - \frac{\log\left(-\frac{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}+1}{\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}-1}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b/x - 1/4*log(-((e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) + 1)/((e^(
2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1) - 1))/x^2
```

maple [A] time = 0.14, size = 20, normalized size = 0.87

$$-\frac{b}{2x} - \frac{\operatorname{arctanh}(\coth(bx+a))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(coth(b*x+a))/x^3,x)
```

```
[Out] -1/2*b/x-1/2*arctanh(coth(b*x+a))/x^2
```

maxima [A] time = 0.39, size = 19, normalized size = 0.83

$$-\frac{b}{2x} - \frac{\operatorname{artanh}(\coth(bx+a))}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(coth(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b/x - 1/2*arctanh(coth(b*x + a))/x^2
```

mupad [B] time = 1.07, size = 16, normalized size = 0.70

$$-\frac{\operatorname{atanh}(\coth(a+bx)) + bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(coth(a + b*x))/x^3,x)
```

```
[Out] -(atanh(coth(a + b*x)) + b*x)/(2*x^2)
```

sympy [A] time = 10.94, size = 49, normalized size = 2.13

$$\begin{cases} \frac{\left\langle -\frac{\pi}{4}, \frac{\pi}{4}\right\rangle i}{x^2} & \text{for } a = \log\left(-e^{-bx}\right) \vee a = \log\left(e^{-bx}\right) \\ -\frac{b}{2x} - \frac{\operatorname{atanh}\left(\frac{1}{\tanh(a+bx)}\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(coth(b*x+a))/x**3,x)
```

```
[Out] Piecewise((AccumBounds(-pi/4, pi/4)*I/x**2, Eq(a, log(exp(-b*x))) | Eq(a, l
og(-exp(-b*x)))), (-b/(2*x) - atanh(1/tanh(a + b*x))/(2*x**2), True))
```

3.281 $\int \tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=27

$$-\text{Li}_2\left(-e^x\right)+\text{Li}_2\left(e^x\right)-2x\tanh^{-1}\left(e^x\right)+x\tanh^{-1}(\cosh(x))$$

```
[Out] -2*x*arctanh(exp(x))+x*arctanh(cosh(x))-polylog(2,-exp(x))+polylog(2,exp(x)
)
```

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {6271, 4182, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^x\right)+\text{PolyLog}\left(2,e^x\right)-2x\tanh^{-1}\left(e^x\right)+x\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Cosh[x]],x]
```

```
[Out] -2*x*ArcTanh[E^x] + x*ArcTanh[Cosh[x]] - PolyLog[2, -E^x] + PolyLog[2, E^x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\cosh(x))\,dx &= x\tanh^{-1}(\cosh(x))+\int x\text{csch}(x)\,dx\\ &= -2x\tanh^{-1}\left(e^x\right)+x\tanh^{-1}(\cosh(x))-\int \log\left(1-e^x\right)\,dx+\int \log\left(1+e^x\right)\,dx\\ &= -2x\tanh^{-1}\left(e^x\right)+x\tanh^{-1}(\cosh(x))-\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^x\right)+\text{Subst}\left(\int \frac{\log(\right.\right.\\ &= -2x\tanh^{-1}\left(e^x\right)+x\tanh^{-1}(\cosh(x))-\text{Li}_2\left(-e^x\right)+\text{Li}_2\left(e^x\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.74

$$\text{Li}_2\left(-e^{-x}\right)-\text{Li}_2\left(e^{-x}\right)+x\left(\log\left(1-e^{-x}\right)-\log\left(e^{-x}+1\right)\right)+x\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Cosh[x]],x]
```

```
[Out] x*ArcTanh[Cosh[x]] + x*(Log[1 - E^(-x)] - Log[1 + E^(-x)]) + PolyLog[2, -E^
(-x)] - PolyLog[2, E^(-x)]
```

fricas [B] time = 0.61, size = 58, normalized size = 2.15

$$\frac{1}{2}x\log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right)-x\log\left(\cosh(x)+\sinh(x)+1\right)+x\log\left(-\cosh(x)-\sinh(x)+1\right)+\mathrm{Li}_2\left(\cosh(x)+\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(-(cosh(x) + 1)/(cosh(x) - 1)) - x*log(cosh(x) + sinh(x) + 1) + x*
log(-cosh(x) - sinh(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - s
inh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}\left(\cosh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(cosh(x)), x)
```

maple [A] time = 0.31, size = 21, normalized size = 0.78

$$x\operatorname{arctanh}\left(\cosh(x)\right)+2\operatorname{dilog}\left(\mathrm{e}^{-x}\right)-\frac{\operatorname{dilog}\left(\mathrm{e}^{-2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cosh(x)),x)
```

```
[Out] x*arctanh(cosh(x))+2*dilog(exp(-x))-1/2*dilog(exp(-2*x))
```

maxima [A] time = 0.37, size = 33, normalized size = 1.22

$$x\operatorname{artanh}\left(\cosh(x)\right)-x\log\left(e^x+1\right)+x\log\left(-e^x+1\right)-\mathrm{Li}_2\left(-e^x\right)+\mathrm{Li}_2\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] x*arctanh(cosh(x)) - x*log(e^x + 1) + x*log(-e^x + 1) - dilog(-e^x) + dilog
(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \operatorname{atanh}\left(\cosh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cosh(x)),x)
```

```
[Out] int(atanh(cosh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cosh(x)),x)
```

```
[Out] Integral(atanh(cosh(x)), x)
```

3.282 $\int x \tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=51

$$-x\text{Li}_2\left(-e^x\right)+x\text{Li}_2\left(e^x\right)+\text{Li}_3\left(-e^x\right)-\text{Li}_3\left(e^x\right)+x^2\left(-\tanh^{-1}\left(e^x\right)\right)+\frac{1}{2}x^2\tanh^{-1}(\cosh(x))$$

```
[Out] -x^2*arctanh(exp(x))+1/2*x^2*arctanh(cosh(x))-x*polylog(2,-exp(x))+x*polylo
g(2,exp(x))+polylog(3,-exp(x))-polylog(3,exp(x))
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6273, 4182, 2531, 2282, 6589}

$$-x\text{PolyLog}\left(2,-e^x\right)+x\text{PolyLog}\left(2,e^x\right)+\text{PolyLog}\left(3,-e^x\right)-\text{PolyLog}\left(3,e^x\right)+x^2\left(-\tanh^{-1}\left(e^x\right)\right)+\frac{1}{2}x^2\tanh^{-1}(\cos$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[Cosh[x]],x]
```

```
[Out] -(x^2*ArcTanh[E^x]) + (x^2*ArcTanh[Cosh[x]])/2 - x*PolyLog[2, -E^x] + x*Pol
yLog[2, E^x] + PolyLog[3, -E^x] - PolyLog[3, E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(\cosh(x))\,dx = \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) + \frac{1}{2}\int x^2\operatorname{csch}(x)\,dx$$
$$= -x^2\tanh^{-1}(e^x) + \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) - \int x\log(1-e^x)\,dx + \int x\log(1+e^x)\,dx$$
$$= -x^2\tanh^{-1}(e^x) + \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) - x\text{Li}_2(-e^x) + x\text{Li}_2(e^x) + \int \text{Li}_2(-e^x)\,dx - \int$$
$$= -x^2\tanh^{-1}(e^x) + \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) - x\text{Li}_2(-e^x) + x\text{Li}_2(e^x) + \text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x}\right.$$
$$= -x^2\tanh^{-1}(e^x) + \frac{1}{2}x^2\tanh^{-1}(\cosh(x)) - x\text{Li}_2(-e^x) + x\text{Li}_2(e^x) + \text{Li}_3(-e^x) - \text{Li}_3(e^x)$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.59

$$\frac{1}{2}\left(2x\text{Li}_2(-e^{-x}) - 2x\text{Li}_2(e^{-x}) + 2\text{Li}_3(-e^{-x}) - 2\text{Li}_3(e^{-x}) + x^2\log(1-e^{-x}) - x^2\log(e^{-x}+1) + x^2\tanh^{-1}(\cosh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[Cosh[x]],x]
```

```
[Out] (x^2*ArcTanh[Cosh[x]] + x^2*Log[1 - E^(-x)] - x^2*Log[1 + E^(-x)] + 2*x*Pol
yLog[2, -E^(-x)] - 2*x*PolyLog[2, E^(-x)] + 2*PolyLog[3, -E^(-x)] - 2*PolyL
og[3, E^(-x)])/2
```

fricas [C] time = 0.41, size = 88, normalized size = 1.73

$$\frac{1}{4}x^2\log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{2}x^2\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}x^2\log(-\cosh(x)-\sinh(x)+1) + x\text{Li}_2(\cosh(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log(-(cosh(x) + 1)/(cosh(x) - 1)) - 1/2*x^2*log(cosh(x) + sinh(x) +
 1) + 1/2*x^2*log(-cosh(x) - sinh(x) + 1) + x*dilog(cosh(x) + sinh(x)) - x*
dilog(-cosh(x) - sinh(x)) - polylog(3, cosh(x) + sinh(x)) + polylog(3, -cos
h(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{artanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(cosh(x)), x)
```

maple [C] time = 0.50, size = 479, normalized size = 9.39

$$-\frac{i\pi\,\text{csgn}(ie^{-x})\,\text{csgn}\left(ie^{-x}(e^x-1)^2\right)^2x^2}{8} - \frac{i\pi\,\text{csgn}\left(i(e^x-1)^2\right)\text{csgn}\left(ie^{-x}(e^x-1)^2\right)^2x^2}{8} + \frac{i\pi\text{csgn}(i(e^x-1))^2\,\text{csgr}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(cosh(x)),x)
```

```
[Out] -1/8*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2-1/8*I*Pi*csgn(
I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2+1/8*I*Pi*csgn(I*(exp(x)-
1))^2*csgn(I*(exp(x)-1)^2)*x^2-1/4*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1
)^2)^2*x^2+1/4*I*Pi*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^2-1/8*I*Pi*csgn(I*(exp
(x)+1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)*x^2-1/8*I*Pi*csgn(I*
(exp(x)+1)^2)^3*x^2+1/8*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2
*x^2+1/8*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1
)^2)*x^2-1/4*I*Pi*x^2-1/8*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^2+1/8*I*Pi*
csgn(I*(exp(x)-1)^2)^3*x^2-1/8*I*Pi*csgn(I*(exp(x)+1))^2*csgn(I*(exp(x)+1)^
2)*x^2+1/8*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^2+1/4
*I*Pi*csgn(I*(exp(x)+1))*csgn(I*(exp(x)+1)^2)^2*x^2+polylog(3,-exp(x))-poly
log(3,exp(x))-1/2*x^2*ln(exp(x)-1)+1/2*x^2*ln(1-exp(x))-1/8*I*Pi*csgn(I*exp
(-x)*(exp(x)-1)^2)^3*x^2+x*polylog(2,exp(x))-x*polylog(2,-exp(x))
```

maxima [A] time = 0.38, size = 56, normalized size = 1.10

$$\frac{1}{2}x^2 \operatorname{artanh}(\cosh(x)) - \frac{1}{2}x^2 \log(e^x + 1) + \frac{1}{2}x^2 \log(-e^x + 1) - x\operatorname{Li}_2(-e^x) + x\operatorname{Li}_2(e^x) + \operatorname{Li}_3(-e^x) - \operatorname{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(cosh(x)) - 1/2*x^2*log(e^x + 1) + 1/2*x^2*log(-e^x + 1) - x
*dilog(-e^x) + x*dilog(e^x) + polylog(3, -e^x) - polylog(3, e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \operatorname{atanh}(\cosh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(cosh(x)),x)
```

```
[Out] int(x*atanh(cosh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}(\cosh(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(cosh(x)),x)
```

```
[Out] Integral(x*atanh(cosh(x)), x)
```

3.283 $\int x^2 \tanh^{-1}(\cosh(x))\,dx$

Optimal. Leaf size=77

$$-x^2\mathrm{Li}_2\left(-e^x\right)+x^2\mathrm{Li}_2\left(e^x\right)+2x\mathrm{Li}_3\left(-e^x\right)-2x\mathrm{Li}_3\left(e^x\right)-2\mathrm{Li}_4\left(-e^x\right)+2\mathrm{Li}_4\left(e^x\right)-\frac{2}{3}x^3\tanh^{-1}\left(e^x\right)+\frac{1}{3}x^3\tanh^{-1}(\cosh(x))$$

```
[Out] -2/3*x^3*arctanh(exp(x))+1/3*x^3*arctanh(cosh(x))-x^2*polylog(2,-exp(x))+x^
2*polylog(2,exp(x))+2*x*polylog(3,-exp(x))-2*x*polylog(3,exp(x))-2*polylog(
4,-exp(x))+2*polylog(4,exp(x))
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6273, 4182, 2531, 6609, 2282, 6589}

$$-x^2\mathrm{PolyLog}\left(2,-e^x\right)+x^2\mathrm{PolyLog}\left(2,e^x\right)+2x\mathrm{PolyLog}\left(3,-e^x\right)-2x\mathrm{PolyLog}\left(3,e^x\right)-2\mathrm{PolyLog}\left(4,-e^x\right)+2\mathrm{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[Cosh[x]],x]
```

```
[Out] (-2*x^3*ArcTanh[E^x])/3 + (x^3*ArcTanh[Cosh[x]])/3 - x^2*PolyLog[2, -E^x] +
 x^2*PolyLog[2, E^x] + 2*x*PolyLog[3, -E^x] - 2*x*PolyLog[3, E^x] - 2*PolyL
og[4, -E^x] + 2*PolyLog[4, E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(\cosh(x))\,dx = \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) + \frac{1}{3}\int x^3\text{csch}(x)\,dx$$
$$= -\frac{2}{3}x^3\tanh^{-1}(e^x) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - \int x^2\log(1-e^x)\,dx + \int x^2\log(1+e$$
$$= -\frac{2}{3}x^3\tanh^{-1}(e^x) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x) + 2\int x\text{Li}_2(-$$
$$= -\frac{2}{3}x^3\tanh^{-1}(e^x) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x) + 2x\text{Li}_3(-e^x)$$
$$= -\frac{2}{3}x^3\tanh^{-1}(e^x) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x) + 2x\text{Li}_3(-e^x)$$
$$= -\frac{2}{3}x^3\tanh^{-1}(e^x) + \frac{1}{3}x^3\tanh^{-1}(\cosh(x)) - x^2\text{Li}_2(-e^x) + x^2\text{Li}_2(e^x) + 2x\text{Li}_3(-e^x)$$

Mathematica [A] time = 0.04, size = 109, normalized size = 1.42

$$\frac{1}{24}\left(24x^2\text{Li}_2(-e^{-x}) + 24x^2\text{Li}_2(e^x) + 48x\text{Li}_3(-e^{-x}) - 48x\text{Li}_3(e^x) + 48\text{Li}_4(-e^{-x}) + 48\text{Li}_4(e^x) - 2x^4 - 8x^3\log(e$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[Cosh[x]],x]
```

```
[Out] (Pi^4 - 2*x^4 + 8*x^3*ArcTanh[Cosh[x]] - 8*x^3*Log[1 + E^(-x)] + 8*x^3*Log[
1 - E^x] + 24*x^2*PolyLog[2, -E^(-x)] + 24*x^2*PolyLog[2, E^x] + 48*x*PolyL
og[3, -E^(-x)] - 48*x*PolyLog[3, E^x] + 48*PolyLog[4, -E^(-x)] + 48*PolyLog
[4, E^x])/24
```

fricas [C] time = 0.83, size = 118, normalized size = 1.53

$$\frac{1}{6}x^3\log\left(-\frac{\cosh(x)+1}{\cosh(x)-1}\right) - \frac{1}{3}x^3\log(\cosh(x)+\sinh(x)+1) + \frac{1}{3}x^3\log(-\cosh(x)-\sinh(x)+1) + x^2\text{Li}_2(\cosh(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log(-(cosh(x) + 1)/(cosh(x) - 1)) - 1/3*x^3*log(cosh(x) + sinh(x) +
 1) + 1/3*x^3*log(-cosh(x) - sinh(x) + 1) + x^2*dilog(cosh(x) + sinh(x)) -
x^2*dilog(-cosh(x) - sinh(x)) - 2*x*polylog(3, cosh(x) + sinh(x)) + 2*x*pol
ylog(3, -cosh(x) - sinh(x)) + 2*polylog(4, cosh(x) + sinh(x)) - 2*polylog(4
, -cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{artanh}(\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(cosh(x)), x)
```

maple [C] time = 0.44, size = 501, normalized size = 6.51

$$-\frac{i\pi \mathrm{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{x}+1\right)^{2}\right)^{3} x^{3}}{12}+\frac{i\pi\,\mathrm{csgn}\left(i\left(\mathrm{e}^{x}+1\right)^{2}\right)\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{x}+1\right)^{2}\right)^{2} x^{3}}{12}-\frac{i\pi\,\mathrm{csgn}\left(i\left(\mathrm{e}^{x}+1\right)^{2}\right)\mathrm{csgn}\left(i\mathrm{e}^{-x}\right)\mathrm{csg}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(cosh(x)),x)
```

```
[Out] -1/12*I*Pi*csgn(I*exp(-x)*(exp(x)+1)^2)^3*x^3+1/12*I*Pi*csgn(I*(exp(x)+1)^2
)*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3-1/12*I*Pi*csgn(I*(exp(x)+1)^2)*csgn(I*
exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)*x^3-1/12*I*Pi*csgn(I*(exp(x)+1))^2*cs
gn(I*(exp(x)+1)^2)*x^3-1/6*I*Pi*csgn(I*(exp(x)-1))*csgn(I*(exp(x)-1)^2)^2*x
^3-1/12*I*Pi*csgn(I*(exp(x)-1)^2)*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3+1/6*I*
Pi*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3+1/12*I*Pi*csgn(I*(exp(x)-1)^2)^3*x^3-
1/6*I*Pi*x^3+1/3*x^3*ln(1-exp(x))-1/3*x^3*ln(exp(x)-1)+2*x*polylog(3,-exp(x
))-2*x*polylog(3,exp(x))-2*polylog(4,-exp(x))+2*polylog(4,exp(x))+1/12*I*Pi
*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+1)^2)^2*x^3-1/12*I*Pi*csgn(I*(exp(x
)+1)^2)^3*x^3+1/6*I*Pi*csgn(I*(exp(x)+1))*csgn(I*(exp(x)+1)^2)^2*x^3-1/12*I
*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)^2*x^3+x^2*polylog(2,exp(x)
)-x^2*polylog(2,-exp(x))+1/12*I*Pi*csgn(I*(exp(x)-1))^2*csgn(I*(exp(x)-1)^2
)*x^3-1/12*I*Pi*csgn(I*exp(-x)*(exp(x)-1)^2)^3*x^3+1/12*I*Pi*csgn(I*(exp(x)
-1)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-1)^2)*x^3
```

maxima [A] time = 0.38, size = 78, normalized size = 1.01

$$\frac{1}{3}x^{3}\operatorname{artanh}(\cosh(x))-\frac{1}{3}x^{3}\log\left(e^{x}+1\right)+\frac{1}{3}x^{3}\log\left(-e^{x}+1\right)-x^{2}\mathrm{Li}_{2}\left(-e^{x}\right)+x^{2}\mathrm{Li}_{2}\left(e^{x}\right)+2\,x\mathrm{Li}_{3}(-e^{x})-2\,x\mathrm{Li}_{3}(e^{x})-2\,\mathrm{Li}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(cosh(x)) - 1/3*x^3*log(e^x + 1) + 1/3*x^3*log(-e^x + 1) - x
^2*dilog(-e^x) + x^2*dilog(e^x) + 2*x*polylog(3, -e^x) - 2*x*polylog(3, e^x
) - 2*polylog(4, -e^x) + 2*polylog(4, e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\,\mathrm{atanh}\left(\cosh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(cosh(x)),x)
```

```
[Out] int(x^2*atanh(cosh(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\operatorname{atanh}\left(\cosh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(cosh(x)),x)
```

```
[Out] Integral(x**2*atanh(cosh(x)), x)
```

3.284 $\int x^2 \tanh^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=307

$$\frac{\text{Li}_4\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{Li}_4\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}-\frac{x\text{Li}_3\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{Li}_3\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(-\frac{(-c-d+1)e^{2a+}}{-c+d+1}\right.}{4b}$$

```
[Out] 1/3*x^3*arctanh(c+d*tanh(b*x+a))+1/6*x^3*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/6*x^3*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,-(1-c-d)*
exp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d)
)/b-1/4*x*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,-(
1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,-(1-c-d)*exp(2*b*x+2*a)/(1
-c+d))/b^3-1/8*polylog(4,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

Rubi [A] time = 0.47, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6243, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,-\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (x^3*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(
4*b) - (x^2*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b)
 - (x*PolyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*b^2) + (
x*PolyLog[3, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b^2) + PolyL
og[4, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^3) - PolyLog[4, -(
((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6243

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*
(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{3}(b(1-c-d))\int \frac{e^{2a+2bx}x^3}{1-c+d+(1-c-d)$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3 1$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3 1$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3 1$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3 1$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x^3 1$$

Mathematica [A] time = 12.69, size = 345, normalized size = 1.12

$$\frac{4b^3x^3\log\left(\frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}+1\right) - 4b^3x^3\log\left(\frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}+1\right) - 6b^2x^2\mathrm{Li}_2\left(\frac{(c-d-}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tanh[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - d)*(Co
sh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] - 4*b^3*x^3*Log[1 + ((1
 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 6*b^2*x^2
*PolyLog[2, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c
```

```
 + d)] + 6*b^2*x^2*PolyLog[2, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a
+ b*x)]))/(1 + c + d)] - 6*b*x*PolyLog[3, ((-1 + c - d)*(-Cosh[2*(a + b*x)]
 + Sinh[2*(a + b*x)]))/(-1 + c + d)] + 6*b*x*PolyLog[3, ((1 + c - d)*(-Cosh
[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1 + c + d)] - 3*PolyLog[4, ((-1 + c -
d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c + d)] + 3*PolyLog[4, (
(1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1 + c + d)])/(24*b^3
)
```

fricas [C] time = 0.47, size = 900, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt(-(c + d + 1)/(c
- d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt(-(c + d -
 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt(-
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c +
d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c
 + d + 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d +
1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) - a^3*log(
2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*s
qrt(-(c + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c
 + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) + 6
*b*x*polylog(3, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x +
a))) + 6*b*x*polylog(3, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + si
nh(b*x + a))) - 6*b*x*polylog(3, sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*b*x*polylog(3, -sqrt(-(c + d - 1)/(c - d - 1))*(c
osh(b*x + a) + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt(-(c + d + 1)/(c -
 d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt(-
(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 +
a^3)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (b^3*x^3 + a^3)*log(-sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - 6*polylog(4, sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x +
 a) + sinh(b*x + a))) - 6*polylog(4, -sqrt(-(c + d + 1)/(c - d + 1))*(cosh(
b*x + a) + sinh(b*x + a))) + 6*polylog(4, sqrt(-(c + d - 1)/(c - d - 1))*(c
osh(b*x + a) + sinh(b*x + a))) + 6*polylog(4, -sqrt(-(c + d - 1)/(c - d - 1
))*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}\left(d \tanh\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tanh(b*x + a) + c), x)
```

maple [C] time = 11.99, size = 5366, normalized size = 17.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.72, size = 281, normalized size = 0.92

$$\frac{1}{3}x^3\operatorname{artanh}(d\tanh(bx+a)+c)-\frac{1}{18}bd\left(\frac{4b^3x^3\log\left(\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}+1\right)+6b^2x^2\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2bx+2a)}}{c-d+1}\right)-6bx\mathrm{Li}_3(-\right.}{b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*tanh(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log((c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog(-(c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d +
 1)) + 3*polylog(4, -(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4
*b^3*x^3*log((c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog
(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, -(c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, -(c + d - 1)*e^(2*b*x + 2*a)/(c
 - d - 1)))/(b^4*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atanh}\left(c+d\tanh\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x^2*atanh(c + d*tanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{atanh}(c+d\tanh(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(c + d*tanh(a + b*x)), x)
```

3.285 $\int x \tanh^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=231

$$-\frac{\text{Li}_3\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{Li}_3\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{4}x^2\log\left(\frac{(-c-d}{-c}\right.$$

```
[Out] 1/2*x^2*arctanh(c+d*tanh(b*x+a))+1/4*x^2*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/4*x^2*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,-(1-c-d)*ex
p(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b-
1/8*polylog(3,-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,-(1+c+d)*e
xp(2*b*x+2*a)/(1+c-d))/b^2
```

Rubi [A] time = 0.37, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6243, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{PolyLog}\left(3,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,-\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tanh[a + b*x]])/2 + (x^2*Log[1 + ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))])/(4*
b) - (x*PolyLog[2, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))])/(4*b) - P
olyLog[3, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(8*b^2) + PolyLog[3
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(8*b^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6243

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
```

```
(m + 1)), x] + (Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^
(2*a + 2*b*x))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[(b*
(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
d + (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &
& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\tanh(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{2}(b(1-c-d))\int \frac{e^{2a+2bx}x^2}{1-c+d+(1-c-d)e\ldots}$$
$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\,\mathrm{lo}\ldots$$
$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\,\mathrm{lo}\ldots$$
$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\,\mathrm{lo}\ldots$$
$$= \frac{1}{2}x^2\tanh^{-1}(c+d\tanh(a+bx)) + \frac{1}{4}x^2\log\left(1+\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\,\mathrm{lo}\ldots$$

Mathematica [A] time = 10.38, size = 259, normalized size = 1.12

$$2b^2x^2\log\left(\frac{(c-d-1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d-1}+1\right) - 2b^2x^2\log\left(\frac{(c-d+1)(\cosh(2(a+bx))-\sinh(2(a+bx)))}{c+d+1}+1\right) - 2bx\mathrm{Li}_2\left(\frac{(c-d-1)\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tanh[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c - d)*(Co
sh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] - 2*b^2*x^2*Log[1 + ((1
 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 2*b*x*Pol
yLog[2, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(-1 + c + d
)] + 2*b*x*PolyLog[2, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)])
)/(1 + c + d)] - PolyLog[3, ((-1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a +
 b*x)]))/(-1 + c + d)] + PolyLog[3, ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh
[2*(a + b*x)]))/(1 + c + d)])/(8*b^2)
```

fricas [C] time = 0.58, size = 746, normalized size = 3.23

$$b^2x^2\log\left(-\frac{(c+1)\cosh(bx+a)+d\sinh(bx+a)}{(c-1)\cosh(bx+a)+d\sinh(bx+a)}\right) - 2bx\mathrm{Li}_2\left(\sqrt{-\frac{c+d+1}{c-d+1}}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 2bx\mathrm{Li}_2\left(-\sqrt{-\frac{c+d+1}{c-d+1}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b
*x + a) + d*sinh(b*x + a))) - 2*b*x*dilog(sqrt(-(c + d + 1)/(c - d + 1))*(c
osh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt(-(c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt(-(c + d - 1)/(c -
 d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x
+ a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d
 + 1))) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a)
 - 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*co
sh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)
/(c - d - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b
*x + a) - 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*l
og(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b
^2*x^2 - a^2)*log(-sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) + (b^2*x^2 - a^2)*log(sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt(-(c + d - 1)/(c - d
- 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt(-(c + d + 1)
/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt(-(c + d
 + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt(-(c
 + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqr
t(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}\left(d \tanh\left(bx + a\right) + c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tanh(b*x + a) + c), x)
```

maple [C] time = 3.94, size = 5062, normalized size = 21.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*tanh(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.72, size = 215, normalized size = 0.93

$$-\frac{1}{8}\, bd\left(\frac{2\, b^2x^2 \log\left(\frac{(c+d+1)e^{(2\, bx+2\, a)}}{c-d+1} + 1\right) + 2\, bx\mathrm{Li}_2\left(-\frac{(c+d+1)e^{(2\, bx+2\, a)}}{c-d+1}\right) - \mathrm{Li}_3(-\frac{(c+d+1)e^{(2\, bx+2\, a)}}{c-d+1})}{b^3d} - \frac{2\, b^2x^2 \log\left(\frac{(c+d-1)e^{(}}{c-d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*b
*x*dilog(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, -(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log((c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)) - polylog(3, -(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) +
1/2*x^2*arctanh(d*tanh(b*x + a) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\, \mathrm{atanh}\left(c + d \tanh\left(a + b\, x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(c + d*tanh(a + b*x)),x)
```

```
[Out] int(x*atanh(c + d*tanh(a + b*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}(c + d \tanh(a + bx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x*atanh(c + d*tanh(a + b*x)), x)
```

3.286 $\int \tanh^{-1}(c + d\tanh(a + bx))\,dx$

Optimal. Leaf size=150

$$\frac{\text{Li}_2\left(-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{Li}_2\left(-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{2}x\log\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}+1\right)$$

```
[Out] x*arctanh(c+d*tanh(b*x+a))+1/2*x*ln(1+(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*x
*ln(1+(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,-(1-c-d)*exp(2*b*x+2*a)
/(1-c+d))/b-1/4*polylog(2,-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

Rubi [A] time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6235, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}+1\right)-\frac{1}{2}x\log\left(\frac{(c+d}{c-}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tanh[a + b*x]] + (x*Log[1 + ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 + ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, -(((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d))]/(4*b) - PolyLog[2
, -(((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d))]/(4*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6235

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + (Dist[b*(1 - c - d), Int[(x*E^(2*a + 2*b*x
))/(1 - c + d + (1 - c - d)*E^(2*a + 2*b*x)), x], x] - Dist[b*(1 + c + d),
Int[(x*E^(2*a + 2*b*x))/(1 + c - d + (1 + c + d)*E^(2*a + 2*b*x)), x], x])
/; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(c + d\tanh(a + bx))\,dx = x\tanh^{-1}(c + d\tanh(a + bx)) + (b(1 - c - d))\int \frac{e^{2a+2bx}x}{1 - c + d + (1 - c - d)e^{2a+2bx}}$$

$$= x\tanh^{-1}(c + d\tanh(a + bx)) + \frac{1}{2}x\log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{2}x\log\left(1 +\right.$$

$$= x\tanh^{-1}(c + d\tanh(a + bx)) + \frac{1}{2}x\log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{2}x\log\left(1 +\right.$$

$$= x\tanh^{-1}(c + d\tanh(a + bx)) + \frac{1}{2}x\log\left(1 + \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{2}x\log\left(1 +\right.$$

Mathematica [A] time = 6.81, size = 131, normalized size = 0.87

$$\frac{\text{Li}_2\left(-\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) - \text{Li}_2\left(-\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right) + 2bx\left(\log\left(\frac{(c+d-1)e^{2(a+bx)}}{c-d-1} + 1\right) - \log\left(\frac{(c+d+1)e^{2(a+bx)}}{c-d+1} + 1\right)\right)}{4b} + x\tanh^{-1}(d\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[c + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tanh[a + b*x]] + (2*b*x*(Log[1 + ((-1 + c + d)*E^(2*(a + b*
x)))/(-1 + c - d)] - Log[1 + ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]) +
PolyLog[2, -(((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d))] - PolyLog[2, -((
(1 + c + d)*E^(2*(a + b*x)))/(1 + c - d))])/(4*b)
```

fricas [B] time = 0.94, size = 552, normalized size = 3.68

$$\frac{bx\log\left(-\frac{(c+1)\cosh(bx+a)+d\sinh(bx+a)}{(c-1)\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(c+d+1)\cosh\,(bx+a) + 2\,(c+d+1)\sinh\,(bx+a) + 2\,(c-d+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-((c + 1)*cosh(b*x + a) + d*sinh(b*x + a))/((c - 1)*cosh(b*x +
 a) + d*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)
*sinh(b*x + a) + 2*(c - d + 1)*sqrt(-(c + d + 1)/(c - d + 1))) + a*log(2*(c
 + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt(
-(c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt(-(c + d - 1)/(c - d - 1))) - a*log(2
*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sq
rt(-(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt(-(c + d + 1)/(c - d + 1)
)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(-(c + d + 1)/(
c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt(-(c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-
sqrt(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog
(sqrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-s
qrt(-(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt
(-(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt(-
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{artanh}\left(d\tanh\left(bx + a\right) + c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + c), x)
```

maple [B] time = 0.30, size = 306, normalized size = 2.04

$$-\frac{\operatorname{arctanh}\left(c+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)-d\right)}{2b}+\frac{\operatorname{arctanh}\left(c+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)+d\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tanh(b*x+a)),x)
```

```
[Out] -1/2/b*arctanh(c+d*tanh(b*x+a))*ln(d*tanh(b*x+a)-d)+1/2/b*arctanh(c+d*tanh(
b*x+a))*ln(d*tanh(b*x+a)+d)+1/4/b*dilog((d*tanh(b*x+a)+c-1)/(c-d-1))+1/4/b*
ln(d*tanh(b*x+a)+d)*ln((d*tanh(b*x+a)+c-1)/(c-d-1))-1/4/b*dilog((d*tanh(b*x
+a)+c+1)/(1+c-d))-1/4/b*ln(d*tanh(b*x+a)+d)*ln((d*tanh(b*x+a)+c+1)/(1+c-d))
+1/4/b*dilog((d*tanh(b*x+a)+c+1)/(1+c+d))+1/4/b*ln(d*tanh(b*x+a)-d)*ln((d*t
anh(b*x+a)+c+1)/(1+c+d))-1/4/b*dilog((d*tanh(b*x+a)+c-1)/(c+d-1))-1/4/b*ln(
d*tanh(b*x+a)-d)*ln((d*tanh(b*x+a)+c-1)/(c+d-1))
```

maxima [A] time = 0.69, size = 142, normalized size = 0.95

$$-\frac{1}{4}\,bd\left(\frac{2\,bx\log\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\operatorname{Li}_2\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\operatorname{Li}_2\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right)}{b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(-
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log((c + d - 1)*
e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog(-(c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arctanh(d*tanh(b*x + a) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(c+d\tanh\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*tanh(a + b*x)),x)
```

```
[Out] int(atanh(c + d*tanh(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(c+d\tanh\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tanh(b*x+a)),x)
```

```
[Out] Integral(atanh(c + d*tanh(a + b*x)), x)
```

3.287 $\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\operatorname{Int}\left(\frac{\tanh^{-1}(d\tanh(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(c+d*tanh(b*x+a))/x,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[c + d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[c + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 14.68, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(c+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[c + d*Tanh[a + b*x]]/x, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctanh(d*tanh(b*x + a) + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctanh(d*tanh(b*x + a) + c)/x, x)
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(c + d\tanh\left(bx + a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*tanh(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(d\tanh\left(bx + a\right) + c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + c)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}\left(c + d\tanh\left(a + b\,x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*tanh(a + b*x))/x,x)
```

```
[Out] int(atanh(c + d*tanh(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(c + d\tanh\left(a + bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(c + d*tanh(a + b*x))/x, x)
```

3.288 $\int x^3 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=155

$$\frac{3\text{Li}_5\left(-\left((d+1)e^{2a+2bx}\right)\right)}{16b^4}-\frac{3x\text{Li}_4\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^3}+\frac{3x^2\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x^3\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{8}x$$

```
[Out] 1/20*b*x^5+1/4*x^4*arctanh(1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x^3*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1+d)*e
xp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(
5,-(1+d)*exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.30, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6239, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 + d + d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 + d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 + d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(1+d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{4}(b(1+d))\int \frac{e^{2a}}{1+(1}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^2\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^2\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^2\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^2\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^2\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1+d)e^2\right.$$

Mathematica [A] time = 4.67, size = 144, normalized size = 0.93

$$\frac{1}{16}\left(\frac{3\text{Li}_5\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{6x\text{Li}_4\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x^2\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{4x^3\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 2x^4\log\left(\frac{e^{-2(a+bx)}}{d+1}+1\right) + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 + d + d*Tanh[a + b*x]] - 2*x^4*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x^2*P
olyLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((1 +
```

```
d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((1 + d)*E^(2*(a + b*x))))])
/b^4)/16
```

fricas [C] time = 0.59, size = 451, normalized size = 2.91

$$\frac{2\,b^{5}x^{5} + 5\,b^{4}x^{4}\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^{3}x^{3}\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right) - 20\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
1)*sinh(b*x + a) + sqrt(-4*d - 4)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*
(d + 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(
-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4
*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*s
qrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqr
t(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*
d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(d*tanh(b*x + a) + d + 1), x)
```

maple [C] time = 5.08, size = 1769, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/8/b^4*d*a^4/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*cs
gn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2
*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/
16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2
*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))+1/16
*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp
(2*b*x+2*a)+1)*exp(2*b*x+2*a))-1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*ex
p(2*b*x+2*a))^3-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+e
xp(2*b*x+2*a)+1))^3+3/16/b^4/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))-1/8/(1+
d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^4-3/8/b^3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+
2*a))*x+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))-1/8*d/(1+d)*ln(1
+(1+d)*exp(2*b*x+2*a))*x^4+1/2/b^4*a^3/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2
))+3/16/b^4*d/(1+d)*polylog(5,-(1+d)*exp(2*b*x+2*a))+1/2/b^4*a^4/(1+d)*ln(1
+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-3
/8/b^4*a^4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))-1/4/b^4*a^3/(1+d)*polylog(2,-(1
+d)*exp(2*b*x+2*a))-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x^3+3/8/b^
```

```
2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x^2+1/2/b^4*d*a^4/(1+d)*ln(1-exp(b
*x+a)*(-d-1)^(1/2))+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/8*I
*x^4*Pi+1/20*b*x^5+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/8*I*x^4*Pi*csgn
(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/2/b^4*d*a^4/(1+d)*ln(1+exp(b*x+
a)*(-d-1)^(1/2))+1/8*x^4*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/8/b^4*a^4/
(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)+1))^3+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2)
)*x-1/2/b^3*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(1+d)*ln
(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2
))*x-3/8/b^4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^4-1/4/b*d/(1+d)*polylog(2
,-(1+d)*exp(2*b*x+2*a))*x^3-1/4/b^4*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a)
)*a^3+3/8/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(1+d)*
polylog(4,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2
*a))*x+1/2/b^4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1/2/b^4*d*a^3/(
1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(d)-
1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn
(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^4*P
i*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp
(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))*cs
gn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^2-1/16*I*x^4*P
i*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/16*I*
x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*
a)+exp(2*b*x+2*a)+1))^2
```

maxima [A] time = 1.13, size = 149, normalized size = 0.96

$$\frac{1}{4}x^4\operatorname{artanh}(d\tanh(bx+a)+d+1)+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left((d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\mathrm{Li}_2\left(-(d+1)e^{(2\,l}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctanh(d*tanh(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d + 1)
*e^(2*b*x + 2*a)) - 3*polylog(5, -(d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\operatorname{atanh}(d+d\tanh(a+b\,x)+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(x^3*atanh(d + d*tanh(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\operatorname{atanh}(d\tanh(a+bx)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**3*atanh(d*tanh(a + b*x) + d + 1), x)
```

3.289 $\int x^2 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=128

$$-\frac{\text{Li}_4\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^3}+\frac{x\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b^2}-\frac{x^2\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{3}x$$

```
[Out] 1/12*b*x^4+1/3*x^3*arctanh(1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1+d)*exp(2*b*x+
2*a))-1/4*x^2*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1+d)*exp
(2*b*x+2*a))/b^2-1/8*polylog(4,-(1+d)*exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.25, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6239, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,-(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((d+1)e\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 + d + d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 + d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 + d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1+d+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{3}(b(1+d))\int \frac{e^{2a}}{1+(1}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^2\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^2\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^2\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^2\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+d)e^2\right.$$

Mathematica [A] time = 5.28, size = 118, normalized size = 0.92

$$\frac{1}{24}\left(\frac{3\text{Li}_4\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{6x^2\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 4x^3\log\left(\frac{e^{-2(a+bx)}}{d+1}+1\right) + 8x^3\tanh^{-1}(d\tanh(a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 + d + d*Tanh[a + b*x]] - 4*x^3*Log[1 + 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))])/b + (6*x*Pol
yLog[3, -(1/((1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((1 + d)*E
^(2*(a + b*x))))])/b^3)/24
```

fricas [C] time = 0.84, size = 382, normalized size = 2.98

$$b^4x^4 + 2b^3x^3\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 6b^2x^2\text{Li}_2\left(\frac{1}{2}\sqrt{-4d-4}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 6b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d - 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*s
inh(b*x + a) + sqrt(-4*d - 4)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d +
 1)*sinh(b*x + a) - sqrt(-4*d - 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d - 4)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d
- 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d - 4
)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d - 4)*(cos
h(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}(d \tanh(bx + a) + d + 1)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tanh(b*x + a) + d + 1), x)
```

maple [C] time = 4.59, size = 1710, normalized size = 13.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/6*x^3*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/12*I*x^3*Pi*csgn(I/(exp(2*b
*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^3-1/2/b^3*a^2/(1+d)*dilog(1
+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2)
)+1/3/b^3/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3-1/4/b/(1+d)*polylog(2,-(1+d)
*exp(2*b*x+2*a))*x^2+1/4/b^3/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a^2+1/4
/b^2/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1+exp(b*
x+a)*(-d-1)^(1/2))-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/12*I*x
^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3-1/6*I*Pi*x^3+1/6/b^3*d*a^
3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/12*b*x^4-1/12*I*x^3*Pi*csgn
(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln
(d)+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/6*d/(1+d)*ln(1+(1+d)*exp(2*b*x
+2*a))*x^3-1/8/b^3*d/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))-1/2/b^2*d*a^2/(
1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2
*a))*x*a^2-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/6/b^3*a^3/
(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/6*I*x^3*Pi*csgn(I*d/(exp(2*b*
x+2*a)+1)*exp(2*b*x+2*a))^2-1/6/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^
3/(1+d)*polylog(4,-(1+d)*exp(2*b*x+2*a))+1/2/b^2/(1+d)*ln(1+(1+d)*exp(2*b*x
+2*a))*x*a^2+1/3/b^3*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^3-1/4/b*d/(1+d)*p
olylog(2,-(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(1+d)*polylog(2,-(1+d)*exp(2*
b*x+2*a))*a^2+1/4/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))*x-1/2/b^2*a^
2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(
-d-1)^(1/2))*x-1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*
a^3/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x
+a)*(-d-1)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))+1/12
*I*x^3*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a
)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*
```

```
d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)
+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1
))^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/6*I*x^3*Pi
*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x
+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))
^2+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I
*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/12*I
*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+
1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))
```

maxima [A] time = 1.11, size = 125, normalized size = 0.98

$$\frac{1}{3}x^3\operatorname{artanh}(d\tanh(bx+a)+d+1)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left((d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-(d+1)e^{(2b}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*tanh(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
((d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d + 1)*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, -(d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d + 1)*e^(2*b*
x + 2*a)))/(b^4*d))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atanh}\left(d+d\tanh\left(a+b\,x\right)+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(x^2*atanh(d + d*tanh(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{atanh}(d\tanh(a+bx)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(d*tanh(a + b*x) + d + 1), x)
```

3.290 $\int x \tanh^{-1}(1 + d + d \tanh(a + bx)) \, dx$

Optimal. Leaf size=101

$$\frac{\text{Li}_3\left(-\left((d+1)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d\tanh(a+bx)+d+1)$$

```
[Out] 1/6*b*x^3+1/2*x^2*arctanh(1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1+d)*exp(2*b*x+2
*a))-1/4*x*polylog(2,-(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1+d)*exp(2*b*
x+2*a))/b^2
```

Rubi [A] time = 0.22, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6239, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((d+1)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d\tanh(a$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 + d + d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 + d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
```

```
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x\tanh^{-1}(1+d+d\tanh(a+bx))\,dx &= \frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(1+d)e^{2a+2bx}}\,dx\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{2}(b(1+d))\int\frac{e^{2a+}}{1+(1+}\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e^{2a}\right.\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e^{2a}\right.\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e^{2a}\right.\\
&= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1+d)e^{2a}\right.
\end{aligned}$$

Mathematica [A] time = 5.27, size = 91, normalized size = 0.90

$$\frac{2b^2x^2\left(2\tanh^{-1}(d\tanh(a+bx)+d+1)-\log\left(\frac{e^{-2(a+bx)}}{d+1}+1\right)\right)+2bx\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d+1}\right)+\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d+1}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 + d + d*Tanh[a + b*x]] - Log[1 + 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, -(1/((1 + d)*E^(2*(a + b*x))))] + PolyLog[3,
 -(1/((1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

fricas [C] time = 0.64, size = 323, normalized size = 3.20

$$2\,b^3x^3+3\,b^2x^2\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{-4\,d-4}\,(\cosh(bx+a)+\sinh(bx+a))\right)-6\,bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d - 4)*(cosh
(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a
) + sinh(b*x + a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) + sqrt(-4*d - 4)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) - sqrt(-4*d - 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d - 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d -
```

```
4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d - 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d - 4)*(cosh(b
*x + a) + sinh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}(d \tanh(bx + a) + d + 1)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tanh(b*x + a) + d + 1), x)
```

maple [C] time = 4.18, size = 1627, normalized size = 16.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3-1/2*x^2*ln(exp(
b*x+a))-1/4*x^2*ln(d)+1/6*b*x^3-1/4*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+
1/8/b^2*d/(1+d)*polylog(3,-(1+d)*exp(2*b*x+2*a))+1/2/b^2*a/(1+d)*dilog(1+ex
p(b*x+a)*(-d-1)^(1/2))+1/2/b^2*d*a/(1+d)*dilog(1+exp(b*x+a)*(-d-1)^(1/2))+1
/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/2/b*d/(1+d)*ln(1+(1+d)*
exp(2*b*x+2*a))*x*a+1/2/b*d*a/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b*d
*a/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))*x-1/2/b/(1+d)*ln(1+(1+d)*exp(2*b*x+2
*a))*x*a-1/4/b^2*d/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polyl
og(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2*d/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*
a))*a+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))*x+1/2/b*a/(1+d)*ln(1-exp(
b*x+a)*(-d-1)^(1/2))*x+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(-d-1)^(1/2))+1/
2/b^2*d*a^2/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))-1/4/b^2/(1+d)*ln(1+(1+d)*ex
p(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*x-1/4/b^2/(1
+d)*polylog(2,-(1+d)*exp(2*b*x+2*a))*a+1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(-
d-1)^(1/2))+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(-d-1)^(1/2))+1/2/b^2*a/(1+d)
*dilog(1-exp(b*x+a)*(-d-1)^(1/2))-1/4*I*Pi*x^2+1/8/b^2/(1+d)*polylog(3,-(1+
d)*exp(2*b*x+2*a))-1/4/(1+d)*ln(1+(1+d)*exp(2*b*x+2*a))*x^2+1/8*I*x^2*Pi*cs
gn(I*exp(2*b*x+2*a))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1
))^3+1/4*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/8*I*x^2*P
i*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^3-1/4/b^2*
a^2/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)+1/4*x^2*ln(d*exp(2*b*x+2*a)
+exp(2*b*x+2*a)+1)-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2
*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))*csgn(
I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*cs
gn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/8*I*x^2*
Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*ex
p(2*b*x+2*a))^2-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/
8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/4/b^2*d*a^2/(1+d)*
ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1
))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b
*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))^
2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp
(2*b*x+2*a)/(exp(2*b*x+2*a)+1))-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2
*b*x+2*a)+exp(2*b*x+2*a)+1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))
```

maxima [A] time = 1.10, size = 101, normalized size = 1.00

$$\frac{1}{24}\left(\frac{4x^3}{d} - \frac{3\left(2b^2x^2\log\left((d+1)e^{(2bx+2a)}+1\right) + 2bx\mathrm{Li}_2\left(-(d+1)e^{(2bx+2a)}\right) - \mathrm{Li}_3(-(d+1)e^{(2bx+2a)})\right)}{b^3d}\right)bd + \frac{1}{2}x^2 \mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d + 1)*e^(2*b*x + 2*a)) - polylog(3, -(d + 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d + 1/2*x^2*arctanh(d*tanh(b*x + a) + d + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \operatorname{atanh}\left(d + d \tanh\left(a + b\,x\right) + 1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(x*atanh(d + d*tanh(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}\left(d \tanh\left(a + bx\right) + d + 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*tanh(a + b*x) + d + 1), x)
```

3.291 $\int \tanh^{-1}(1+d+d\tanh(a+bx))\,dx$

Optimal. Leaf size=69

$$-\frac{\text{Li}_2\left(-\left((d+1)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{2}x\log\left((d+1)e^{2a+2bx}+1\right)+x\tanh^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2+x*arctanh(1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1+d)*exp(2*b*x+2*a))-1/4
*polylog(2,-(1+d)*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6231, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((d+1)e^{2a+2bx}+1\right)+x\tanh^{-1}(d\tanh(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 + d + d*Tanh[a + b*x]] - (x*Log[1 + (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 + d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6231

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+d+d\tanh(a+bx))\,dx = x\tanh^{-1}(1+d+d\tanh(a+bx)) + b\int \frac{x}{1+(1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\tanh(a+bx)) - (b(1+d))\int \frac{e^{2a+2bx}x}{1+(1+d)e^{2a+}}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right)$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right)$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1+d)e^{2a+2bx}\right)$$

Mathematica [B] time = 4.76, size = 201, normalized size = 2.91

$$\frac{-2\text{Li}_2\left(-\sqrt{-d-1}\,e^{a+bx}\right) - 2\text{Li}_2\left(\sqrt{-d-1}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{-d-1}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{-}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + d + d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x) + (1 + d)*
E^(a + b*x)] - 2*b*x*Log[(2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]] - 2*PolyL
og[2, -(Sqrt[-1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 - d]*E^(a + b*x)]
)/(4*b)
```

fricas [B] time = 0.58, size = 239, normalized size = 3.46

$$\frac{b^2x^2 + bx\log\left(-\frac{(d+2)\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(d+1)\cosh\,(bx+a) + 2\,(d+1)\sinh\,(bx+a) + \sqrt{-4\,d-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(-((d + 2)*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) + sqrt(-4*d - 4)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*si
nh(b*x + a) - sqrt(-4*d - 4)) - (b*x + a)*log(1/2*sqrt(-4*d - 4)*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d - 4)*(cosh(b*x +
a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b
*x + a))) - dilog(-1/2*sqrt(-4*d - 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1), x)
```

maple [B] time = 0.34, size = 247, normalized size = 3.58

$$-\frac{\text{arctanh}\left(1+d+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)-d\right)}{2b} + \frac{\text{arctanh}\left(1+d+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] -1/2/b*arctanh(1+d+d*tanh(b*x+a))*ln(d*tanh(b*x+a)-d)+1/2/b*arctanh(1+d+d*t
anh(b*x+a))*ln(d*tanh(b*x+a)+d)+1/8/b*ln(d*tanh(b*x+a)+d)^2-1/4/b*dilog(1+1
/2*d*tanh(b*x+a)+1/2*d)-1/4/b*ln(d*tanh(b*x+a)+d)*ln(1+1/2*d*tanh(b*x+a)+1/
2*d)-1/4/b*dilog(1/2*(d*tanh(b*x+a)+d)/d)-1/4/b*ln(d*tanh(b*x+a)-d)*ln(1/2*
(d*tanh(b*x+a)+d)/d)+1/4/b*dilog((d*tanh(b*x+a)+d+2)/(2*d+2))+1/4/b*ln(d*ta
nh(b*x+a)-d)*ln((d*tanh(b*x+a)+d+2)/(2*d+2))
```

maxima [A] time = 1.11, size = 72, normalized size = 1.04

$$\frac{1}{4}bd\left(\frac{2x^2}{d}-\frac{2bx\log\left((d+1)e^{(2bx+2a)}+1\right)+\mathrm{Li}_2\left(-(d+1)e^{(2bx+2a)}\right)}{b^2d}\right)+x\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d + 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arctanh(d*tanh(b*x + a) + d + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}\left(d+d\tanh\left(a+b\,x\right)+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + d*tanh(a + b*x) + 1),x)
```

```
[Out] int(atanh(d + d*tanh(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*tanh(b*x+a)),x)
```

```
[Out] Integral(atanh(d*tanh(a + b*x) + d + 1), x)
```

3.292 $\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^{-1}(d\tanh(a+bx)+d+1)}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 + d + d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 + d + d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.76, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1+d+d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 + d + d*Tanh[a + b*x]]/x, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

maple [A] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(1+d+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] int(arctanh(1+d+d*tanh(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tanh(b*x + a) + d + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atanh}\left(d+d\tanh\left(a+b\,x\right)+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + d*tanh(a + b*x) + 1)/x,x)
```

```
[Out] int(atanh(d + d*tanh(a + b*x) + 1)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*tanh(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*tanh(a + b*x) + d + 1)/x, x)
```

3.293 $\int x^3 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=168

$$\frac{3\text{Li}_5\left(-\left((1-d)e^{2a+2bx}\right)\right)}{16b^4}-\frac{3x\text{Li}_4\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}+\frac{3x^2\text{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x^3\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-$$

```
[Out] 1/20*b*x^5-1/4*x^4*arctanh(-1+d+d*tanh(b*x+a))-1/8*x^4*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,-(1-d)*
exp(2*b*x+2*a))/b^2-3/8*x*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog
(5,-(1-d)*exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.30, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6239, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,-(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 - d - d*Tanh[a + b*x]])/4 - (x^4*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (3*x^2*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(8*b^2) - (3*x*PolyLog[4,
-((1 - d)*E^(2*a + 2*b*x))])/(8*b^3) + (3*PolyLog[5, -((1 - d)*E^(2*a + 2*b
*x))])/(16*b^4)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{4}(b(1-d))\int \frac{e^{2a+2bx}}{1+(1-d)\ldots}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2b\ldots}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2b\ldots}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2b\ldots}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2b\ldots}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2b\ldots}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{8}x^4\log\left(1+(1-d)e^{2a+2b\ldots}\right.$$

Mathematica [A] time = 4.74, size = 144, normalized size = 0.86

$$\frac{1}{16}\left(\frac{3\text{Li}_5\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{6x\text{Li}_4\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2\text{Li}_3\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{4x^3\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 2x^4\log\left(1-\frac{e^{-2(a+bx)}}{d-1}\right) + 4x^4\tanh\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 - d - d*Tanh[a + b*x]] - 2*x^4*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Po
lyLog[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((-1 + d)*E
```

```
^(2*(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((-1 + d)*E^(2*(a + b*x)))])/b^4)/1
6
```

fricas [C] time = 0.63, size = 424, normalized size = 2.52

$$\frac{2\,b^5x^5 - 5\,b^4x^4\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d-1}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 20\,b^3x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d - 1)*(cosh(b
*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d - 1)*(cosh(b*x + a) +
sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x +
a) + 2*sqrt(d - 1)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) - 2*sqrt(d - 1)) + 60*b^2*x^2*polylog(3, sqrt(d - 1)*(cosh(b*x + a)
+ sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sin
h(b*x + a))) - 120*b*x*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 120*b*x*polylog(4, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*
(b^4*x^4 - a^4)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b
^4*x^4 - a^4)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*p
olylog(5, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sq
rt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x^3\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(d*tanh(b*x + a) + d - 1), x)
```

maple [C] time = 5.00, size = 1773, normalized size = 10.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/16*I*x^4*Pi*csg
n(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)+1))+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))-1/4/b^4*d/(d-1)*polylog(2,
(d-1)*exp(2*b*x+2*a))*a^3+1/2/b^4*d*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))+
1/2/b^4*d*a^4/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/16*I*x^4*Pi*csgn(I/(exp(
2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^3+1/16*I*x^4*Pi*csgn(I*d
/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3-1/8*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a)
)*x^4-3/8/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x^2+3/8/b^3/(d-1)*polyl
og(4,(d-1)*exp(2*b*x+2*a))*x-1/2/b^4*a^4/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))
-1/2/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+3/16/b^4*d/(d-1)*polylog(5,
(d-1)*exp(2*b*x+2*a))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2
/b^4*a^3/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+3/8/b^4/(d-1)*ln(1-(d-1)*exp
(2*b*x+2*a))*a^4+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3+1/4/b^4/(d
-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^3+1/20*b*x^5-1/8*I*x^4*Pi*csgn(I/(exp
(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2-1/8/b^4*d*a^4/(d-1)*l
n(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3
```

```
+1/8*I*x^4*Pi+1/8/b^4*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+1/16*
I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/8/(d-1)*ln(1-(d-1)*e
xp(2*b*x+2*a))*x^4-3/16/b^4/(d-1)*polylog(5,(d-1)*exp(2*b*x+2*a))-1/2/b^3*a
^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-3/8/b^4*d/(d-1)*ln(1-(d-1)*exp(2*b*
x+2*a))*a^4-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^3-1/4*x^4*ln(ex
p(b*x+a))-1/8*x^4*ln(d)-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*
a))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*
d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/8*x^4*ln(d*exp(2*b*x+2*a)-exp(2*b*
x+2*a)-1)+1/2/b^3/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^4*d*a^3/(d-1
)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+3/8/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x
+2*a))*x^2-3/8/b^3*d/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))*x+1/2/b^4*d*a^3/
(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-
1)^(1/2))*x+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)
+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d
/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)+
1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x
+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))+1/16*I*x^4*Pi*csgn(I*(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-ex
p(2*b*x+2*a)-1))^2-1/2/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^3
*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^3*d*a^3/(d-1)*ln(1+exp(b*
x+a)*(d-1)^(1/2))*x
```

maxima [A] time = 1.13, size = 146, normalized size = 0.87

$$-\frac{1}{4}x^4\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left(-(d-1)e^{(2bx+2a)}+1\right)+4b^3x^3\mathrm{Li}_2\left((d-1)e^{(2bx+\cdots}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arctanh(d*tanh(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d - 1)*e^(2*b*x + 2*a))
- 6*b^2*x^2*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d - 1)*
e^(2*b*x + 2*a)) - 3*polylog(5, (d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^3\,\mathrm{atanh}\left(d+d\tanh\left(a+b\,x\right)-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*atanh(d + d*tanh(a + b*x) - 1),x)
```

```
[Out] int(-x^3*atanh(d + d*tanh(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x^{3}\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(x**3*atanh(d*tanh(a + b*x) + d - 1), x)
```

3.294 $\int x^2 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=139

$$-\frac{\mathrm{Li}_4\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^3}+\frac{x\mathrm{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b^2}-\frac{x^2\mathrm{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left((1-d)e^{2a+2bx}+1\right)+$$

```
[Out] 1/12*b*x^4-1/3*x^3*arctanh(-1+d+d*tanh(b*x+a))-1/6*x^3*ln(1+(1-d)*exp(2*b*x
+2*a))-1/4*x^2*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,-(1-d)*ex
p(2*b*x+2*a))/b^2-1/8*polylog(4,-(1-d)*exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6239, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\mathrm{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\mathrm{PolyLog}\left(4,-(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\mathrm{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left((1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 - d - d*Tanh[a + b*x]])/3 - (x^3*Log[1 + (1 - d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b)
+ (x*PolyLog[3, -((1 - d)*E^(2*a + 2*b*x))])/(4*b^2) - PolyLog[4, -((1 - d)
*E^(2*a + 2*b*x))]/(8*b^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) + \frac{1}{3}b\int \frac{x^3}{1+(1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{3}(b(1-d))\int \frac{e^{2a+2bx}}{1+(1-d)}$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2b}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2b}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2b}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2b}\right.$$
$$= \frac{bx^4}{12} + \frac{1}{3}x^3\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{6}x^3\log\left(1+(1-d)e^{2a+2b}\right.$$

Mathematica [A] time = 5.54, size = 119, normalized size = 0.86

$$\frac{1}{24}\left(\frac{3\text{Li}_4\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x\text{Li}_3\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{6x^2\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 4x^3\log\left(1-\frac{e^{-2(a+bx)}}{d-1}\right) + 8x^3\tanh^{-1}(d(-\tanh(a+bx\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 - d - d*Tanh[a + b*x]] - 4*x^3*Log[1 - 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))])/b + (6*x*Poly
Log[3, 1/((-1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((-1 + d)*E^(2*
(a + b*x)))])/b^3)/24
```

fricas [C] time = 0.73, size = 360, normalized size = 2.59

$$b^4x^4 - 2b^3x^3\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) - 6b^2x^2\text{Li}_2\left(\sqrt{d-1}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 6b^2x^2\text{Li}_2\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d - 1)*(cosh(b*x
+ a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh
(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) +
 2*sqrt(d - 1)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x +
a) - 2*sqrt(d - 1)) + 12*b*x*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b
*x + a))) + 12*b*x*polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*(b^3*x^3 + a^3)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
 2*(b^3*x^3 + a^3)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
12*polylog(4, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4,
-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x^2 \operatorname{artanh}\left(d \tanh\left(bx+a\right)+d-1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*tanh(b*x + a) + d - 1), x)
```

maple [C] time = 4.96, size = 1716, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/6/b^3*d*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+1/12*I*x^3*Pi*csg
n(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2
*csgn(I*exp(2*b*x+2*a))-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*
a))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x
+2*a)+1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*
d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/
(exp(2*b*x+2*a)+1))^3+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+
2*a)-exp(2*b*x+2*a)-1))^3+1/8/b^3/(d-1)*polylog(4,(d-1)*exp(2*b*x+2*a))+1/6
/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*x+a)*
(d-1)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/3/b^3/(d-1
)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a)
)*x^2-1/4/b^3/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polyl
og(3,(d-1)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))
+1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))-1/8/b^3*d/(d-1)*polylog(4,(
d-1)*exp(2*b*x+2*a))-1/6*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x^3+1/12*I*x^3*
Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^3+1/6*I*x^3*Pi+1/12*b*x^4-1/
2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^2/(d-1)*dilog
(1+exp(b*x+a)*(d-1)^(1/2))+1/3/b^3*d/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*a^3-1
/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*polylog(2,
(d-1)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))*x
-1/2/b^2/(d-1)*ln(1-(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1-exp(
b*x+a)*(d-1)^(1/2))*x+1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/2/
b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(1+exp(b
*x+a)*(d-1)^(1/2))-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)+1/6*x^3*ln(d*exp(2*
b*x+2*a)-exp(2*b*x+2*a)-1)-1/6*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*
```

```
b*x+2*a)-exp(2*b*x+2*a)-1))^2+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/6/b^
3*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)-1/12*I*x^3*Pi*csgn(I*d)*c
sgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2
-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))*x+1/2/b^2*d/(d-1)*ln(1-(d
-1)*exp(2*b*x+2*a))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*
x-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b
*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))+
1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I*d/
(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)+1))
*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))
```

maxima [A] time = 1.12, size = 123, normalized size = 0.88

$$-\frac{1}{3}x^3 \operatorname{artanh}(d\tanh(bx+a)+d-1)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left(-(d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left((d-1)e^{(2bx+}\right.\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arctanh(d*tanh(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g(-(d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, (d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d - 1)*e^(2*b*x
 + 2*a)))/(b^4*d))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2 \operatorname{atanh}(d+d\tanh(a+b\,x)-1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(d + d*tanh(a + b*x) - 1),x)
```

```
[Out] int(-x^2*atanh(d + d*tanh(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2 \operatorname{atanh}(d\tanh(a+bx)+d-1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(x**2*atanh(d*tanh(a + b*x) + d - 1), x)
```

3.295 $\int x \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=110

$$\frac{\text{Li}_3\left(-\left((1-d)e^{2a+2bx}\right)\right)}{8b^2}-\frac{x\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\tanh(a+bx))\cdot$$

```
[Out] 1/6*b*x^3-1/2*x^2*arctanh(-1+d+d*tanh(b*x+a))-1/4*x^2*ln(1+(1-d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,-(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,-(1-d)*exp(2*b
*x+2*a))/b^2
```

Rubi [A] time = 0.22, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6239, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left((1-d)e^{2a+2bx}+1\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 - d - d*Tanh[a + b*x]])/2 - (x^2*Log[1 + (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))])/(4*b) + P
olyLog[3, -((1 - d)*E^(2*a + 2*b*x))]/(8*b^2)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6239

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tanh[a + b*x]])/(f*
```

```
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-d-d\tanh(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(1-d)e^{2a+2bx}}\,dx$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{2}(b(1-d))\int\frac{e^{2a+2bx}x}{1+(1-d)e^{\cdots}}$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\cdots\right.$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\cdots\right.$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\cdots\right.$$
$$=\frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1-d-d\tanh(a+bx))-\frac{1}{4}x^2\log\left(1+(1-d)e^{2a+2bx}\cdots\right.$$

Mathematica [A] time = 5.37, size = 93, normalized size = 0.85

$$\frac{2b^2x^2\left(2\tanh^{-1}(d(-\tanh(a+bx))-d+1)-\log\left(1-\frac{e^{-2(a+bx)}}{d-1}\right)\right)+2bx\,\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d-1}\right)+\text{Li}_3\left(\frac{e^{-2(a+bx)}}{d-1}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 - d - d*Tanh[a + b*x]] - Log[1 - 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, 1/((-1 + d)*E^(2*(a + b*x)))] + PolyLog[3,
1/((-1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

fricas [C] time = 0.56, size = 306, normalized size = 2.78

$$2\,b^3x^3-3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right)-6\,bx\text{Li}_2\left(\sqrt{d-1}\,(\cosh(bx+a)+\sinh(bx+a))\right)-6\,bx\text{Li}_2\left(-\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d -
2)*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d - 1)*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
 a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) + 2*sqr
t(d - 1)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*x + a) - 2
*sqrt(d - 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x
```

```
+ a)) + 1) + 6*polylog(3, sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
polylog(3, -sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x \operatorname{artanh}\left(d \tanh\left(bx + a\right) + d - 1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*tanh(b*x + a) + d - 1), x)
```

maple [C] time = 4.49, size = 1635, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/4/b^2/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a-1/2/b^2*a^2/(d-1)*ln(1-exp(
b*x+a)*(d-1)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(d-1)^(1/2))+1/8/b^2*
d/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*
(d-1)^(1/2))-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))+1/4/b^2/(d-1)*
ln(1-(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*
x+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1
))^3-1/2*x^2*ln(exp(b*x+a))-1/4*x^2*ln(d)+1/4/(d-1)*ln(1-(d-1)*exp(2*b*x+2*
a))*x^2-1/8/b^2/(d-1)*polylog(3,(d-1)*exp(2*b*x+2*a))+1/4/b^2*a^2/(d-1)*ln(
d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+1/6*b*x^3+1/2/b^2*d*a^2/(d-1)*ln(1-exp(b
*x+a)*(d-1)^(1/2))-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)
-exp(2*b*x+2*a)-1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I/(exp(2
*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+1/2/b^2*d*a^2/(d-1)*ln(
1+exp(b*x+a)*(d-1)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1-exp(b*x+a)*(d-1)^(1/2))
+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(d-1)^(1/2))-1/4/b^2*d/(d-1)*ln(1-(d-
1)*exp(2*b*x+2*a))*a^2-1/4/b*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*x-1/4/
b^2*d/(d-1)*polylog(2,(d-1)*exp(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1-(d-1)*exp(2*
b*x+2*a))*x*a-1/2/b*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/2/b*a/(d-1)*ln
(1+exp(b*x+a)*(d-1)^(1/2))*x+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3+1/8*I*x^
2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))^3+1/4*I*x^2*Pi-1/4*d/(d-1)*l
n(1-(d-1)*exp(2*b*x+2*a))*x^2+1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(
2*b*x+2*a))^3+1/4*x^2*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1)+1/2/b*d*a/(d-1)
*ln(1+exp(b*x+a)*(d-1)^(1/2))*x-1/4/b^2*d*a^2/(d-1)*ln(d*exp(2*b*x+2*a)-exp
(2*b*x+2*a)-1)-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x
+2*a))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b
*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)+1))*csgn(I
*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))^2-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csg
n(I*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*
a))-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)+1))^2+1/8*I*x^2*Pi*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*csgn(
I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))^2+1/8*I*x^2*Pi*cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(
2*b*x+2*a)+1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)
+1))*csgn(I*d/(exp(2*b*x+2*a)+1)*exp(2*b*x+2*a))-1/2/b*d/(d-1)*ln(1-(d-1)*e
xp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1-exp(b*x+a)*(d-1)^(1/2))*x-1/8*I*x^2
*Pi*csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))*
csgn(I/(exp(2*b*x+2*a)+1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-1))
```

maxima [A] time = 1.11, size = 100, normalized size = 0.91

$$\frac{1}{24}\left(\frac{4\,x^3}{d} - \frac{3\left(2\,b^2x^2\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right) + 2\,bx\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right) - \mathrm{Li}_3((d-1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)bd - \frac{1}{2}\,x^2\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d - 1)*e^(2*b*x + 2*a)) - polylog(3, (d - 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d - 1/2*x^2*arctanh(d*tanh(b*x + a) + d - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x \operatorname{atanh}\left(d + d \tanh\left(a + b\,x\right) - 1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(d + d*tanh(a + b*x) - 1),x)
```

```
[Out] int(-x*atanh(d + d*tanh(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x \operatorname{atanh}\left(d \tanh\left(a + bx\right) + d - 1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(x*atanh(d*tanh(a + b*x) + d - 1), x)
```

3.296 $\int \tanh^{-1}(1-d-d\tanh(a+bx))\,dx$

Optimal. Leaf size=76

$$-\frac{\text{Li}_2\left(-\left((1-d)e^{2a+2bx}\right)\right)}{4b}-\frac{1}{2}x\log\left((1-d)e^{2a+2bx}+1\right)+x\tanh^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2-x*arctanh(-1+d+d*tanh(b*x+a))-1/2*x*ln(1+(1-d)*exp(2*b*x+2*a))-1/
4*polylog(2,-(1-d)*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.14, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6231, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left((1-d)e^{2a+2bx}+1\right)+x\tanh^{-1}(d(-\tanh(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 - d - d*Tanh[a + b*x]] - (x*Log[1 + (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, -((1 - d)*E^(2*a + 2*b*x))]/(4*b)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6231

```
Int[ArcTanh[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Tanh[a + b*x]], x] + Dist[b, Int[x/(c - d + c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-d-d\tanh(a+bx))\,dx = x\tanh^{-1}(1-d-d\tanh(a+bx)) + b\int \frac{x}{1+(1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\tanh(a+bx)) - (b(1-d))\int \frac{e^{2a+2bx}x}{1+(1-d)e^{2a+2bx}}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right) + \frac{1}{2}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right) + -$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\tanh(a+bx)) - \frac{1}{2}x\log\left(1+(1-d)e^{2a+2bx}\right) - -$$

S

L

Mathematica [B] time = 4.70, size = 200, normalized size = 2.63

$$-2\text{Li}_2\left(-\sqrt{d-1}\,e^{a+bx}\right) - 2\text{Li}_2\left(\sqrt{d-1}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{d-1}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{d-1}\,e^{a+b}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - d - d*Tanh[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[-1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
 Sqrt[-1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (-1
+ d)*E^(2*(a + b*x)))] - 2*b*x*Log[(-2 + d)*Cosh[a + b*x] + d*Sinh[a + b*x]
] - 2*PolyLog[2, -(Sqrt[-1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[-1 + d]*E
^(a + b*x)])/(4*b)
```

fricas [B] time = 0.50, size = 228, normalized size = 3.00

$$b^2x^2 - bx\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{(d-2)\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(d-1)\cosh\,(bx+a) + 2\,(d-1)\sinh\,(bx+a) + 2\,\sqrt{d-1}\,\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/((d - 2)*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) + 2*sqrt(d - 1)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) - 2*sqrt(d - 1)) - (b*x + a)*log(sqrt(d - 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x +
a)) + 1) - dilog(sqrt(d - 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt
(d - 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\text{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tanh(b*x + a) + d - 1), x)
```

maple [B] time = 0.34, size = 280, normalized size = 3.68

$$\frac{\text{arctanh}\left(-1+d+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+a\right)-d\right)}{2b} - \frac{\text{arctanh}\left(-1+d+d\tanh\left(bx+a\right)\right)\ln\left(d\tanh\left(bx+\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] 1/2/b*arctanh(-1+d+d*tanh(b*x+a))*ln(d*tanh(b*x+a)-d)-1/2/b*arctanh(-1+d+d*
tanh(b*x+a))*ln(d*tanh(b*x+a)+d)+1/8/b*ln(d*tanh(b*x+a)+d)^2-1/4/b*ln(-1/2*
d*tanh(b*x+a)-1/2*d+1)*ln(d*tanh(b*x+a)+d)+1/4/b*ln(-1/2*d*tanh(b*x+a)-1/2*
d+1)*ln(1/2*d*tanh(b*x+a)+1/2*d)+1/4/b*dilog(1/2*d*tanh(b*x+a)+1/2*d)-1/4/b
*dilog(1/2*(d*tanh(b*x+a)+d)/d)-1/4/b*ln(d*tanh(b*x+a)-d)*ln(1/2*(d*tanh(b*
x+a)+d)/d)+1/4/b*dilog((d*tanh(b*x+a)+d-2)/(2*d-2))+1/4/b*ln(d*tanh(b*x+a)-
d)*ln((d*tanh(b*x+a)+d-2)/(2*d-2))
```

maxima [A] time = 1.11, size = 73, normalized size = 0.96

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d-1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left((d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)-x\,\mathrm{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d - 1)*e^(2*b*x + 2*a) + 1) + dilog((d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arctanh(d*tanh(b*x + a) + d - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\mathrm{atanh}\left(d+d\tanh\left(a+b\,x\right)-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d + d*tanh(a + b*x) - 1),x)
```

```
[Out] int(-atanh(d + d*tanh(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \mathrm{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*tanh(b*x+a)),x)
```

```
[Out] -Integral(atanh(d*tanh(a + b*x) + d - 1), x)
```

3.297 $\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\operatorname{Int}\left(\frac{\tanh^{-1}(d(-\tanh(a+bx))-d+1)}{x},x\right)$$

```
[Out] CannotIntegrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 - d - d*Tanh[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 - d - d*Tanh[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Mathematica [A] time = 5.89, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1-d-d\tanh(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 - d - d*Tanh[a + b*x]]/x, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="fricas")
[Out] integral(-arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="giac")
[Out] integrate(-arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

maple [A]   time = 0.94, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arctanh}\left(-1+d+d\tanh\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
[Out] int(-arctanh(-1+d+d*tanh(b*x+a))/x,x)
```

maxima [A]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{artanh}\left(d\tanh\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*tanh(b*x+a))/x,x, algorithm="maxima")
[Out] -integrate(arctanh(d*tanh(b*x + a) + d - 1)/x, x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{\operatorname{atanh}\left(d+d\tanh\left(a+b\,x\right)-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d + d*tanh(a + b*x) - 1)/x,x)
[Out] int(-atanh(d + d*tanh(a + b*x) - 1)/x, x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}\left(d\tanh\left(a+bx\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*tanh(b*x+a))/x,x)
[Out] -Integral(atanh(d*tanh(a + b*x) + d - 1)/x, x)
```

3.298 $\int x^2 \tanh^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=303

$$\frac{\text{Li}_4\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{Li}_4\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^3}-\frac{x\text{Li}_3\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{Li}_3\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{x^2\text{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x^2\text{Li}_2}{}$$

```
[Out] 1/3*x^3*arctanh(c+d*coth(b*x+a))+1/6*x^3*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/6*x^3*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x^2*polylog(2,(1-c-d)*e
xp(2*b*x+2*a)/(1-c+d))/b-1/4*x^2*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/
b-1/4*x*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/4*x*polylog(3,(1+c+
d)*exp(2*b*x+2*a)/(1+c-d))/b^2+1/8*polylog(4,(1-c-d)*exp(2*b*x+2*a)/(1-c+d)
)/b^3-1/8*polylog(4,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b^3
```

Rubi [A] time = 0.46, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6245, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b^2}+\frac{x\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^3}-\frac{\text{PolyLog}\left(4,\frac{(c+d+1)e^{2a+}}{c-d+1}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Coth[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/6 - (x^3*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/6 + (x^2*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b
) - (x^2*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - (x*
PolyLog[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b^2) + (x*PolyLog
[3, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b^2) + PolyLog[4, ((1 -
c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^3) - PolyLog[4, ((1 + c + d)*E^(2
*a + 2*b*x))/(1 + c - d)]/(8*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6245

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
 d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) - \frac{1}{3}(b(1-c-d))\int \frac{e^{2a+2bx}x^{3}}{1-c+d+(-1+c\ldots}$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x\ldots$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x\ldots$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x\ldots$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x\ldots$$
$$= \frac{1}{3}x^3\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{6}x\ldots$$

Mathematica [A] time = 11.27, size = 353, normalized size = 1.17

$$\frac{4b^3x^3\log\left(\frac{2(\cosh(a+bx)-\sinh(a+bx))((c-1)\sinh(a+bx)+d\cosh(a+bx))}{c+d-1}\right) - 4b^3x^3\log\left(\frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}+1\right) - \ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Coth[a + b*x]])/3 + (4*b^3*x^3*Log[(2*(Cosh[a + b*x] - S
inh[a + b*x])*(d*Cosh[a + b*x] + (-1 + c)*Sinh[a + b*x]))/(-1 + c + d)] - 4
*b^3*x^3*Log[1 + ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1
+ c + d)] - 6*b^2*x^2*PolyLog[2, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*
```

```
(a + b*x)]))/(-1 + c + d)] + 6*b^2*x^2*PolyLog[2, ((1 + c - d)*(Cosh[2*(a +
 b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 6*b*x*PolyLog[3, ((-1 + c - d)*
(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] + 6*b*x*PolyLog[3, (
(1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)] - 3*PolyL
og[4, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)]
+ 3*PolyLog[4, ((1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c
 + d)])/(24*b^3)
```

fricas [C] time = 0.92, size = 880, normalized size = 2.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 3*b^2*x^2*dilog(sqrt((c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) - 3*b^2*x^2*dilog(-sqrt((c + d + 1)/(c -
d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(sqrt((c + d - 1)
/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + 3*b^2*x^2*dilog(-sqrt((c +
 d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + a^3*log(2*(c + d +
1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d
+ 1)/(c - d + 1))) + a^3*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*si
nh(b*x + a) - 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) - a^3*log(2*(c +
 d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c
 + d - 1)/(c - d - 1))) - a^3*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d -
1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) + 6*b*x*pol
ylog(3, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + 6*
b*x*polylog(3, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a
))) - 6*b*x*polylog(3, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(
b*x + a))) - 6*b*x*polylog(3, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a)
 + sinh(b*x + a))) - (b^3*x^3 + a^3)*log(sqrt((c + d + 1)/(c - d + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) - (b^3*x^3 + a^3)*log(-sqrt((c + d + 1)/(c
 - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 + a^3)*log(sqrt(
(c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b^3*x^3 +
a^3)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) - 6*polylog(4, sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x +
a))) - 6*polylog(4, -sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*
x + a))) + 6*polylog(4, sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh
(b*x + a))) + 6*polylog(4, -sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) +
sinh(b*x + a))))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*coth(b*x + a) + c), x)
```

maple [C] time = 11.71, size = 5294, normalized size = 17.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*coth(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.70, size = 277, normalized size = 0.91

$$\frac{1}{3}\,x^3\,\text{artanh}\left(d\coth\left(bx+a\right)+c\right)-\frac{1}{18}\,bd\left(\frac{4\,b^3x^3\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+6\,b^2x^2\text{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-6\,bx\text{Li}}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*coth(b*x + a) + c) - 1/18*b*d*((4*b^3*x^3*log(-(c + d + 1
)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 6*b^2*x^2*dilog((c + d + 1)*e^(2*b*x +
 2*a)/(c - d + 1)) - 6*b*x*polylog(3, (c + d + 1)*e^(2*b*x + 2*a)/(c - d +
1)) + 3*polylog(4, (c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^4*d) - (4*b
^3*x^3*log(-(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1) + 1) + 6*b^2*x^2*dilog(
(c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)) - 6*b*x*polylog(3, (c + d - 1)*e^(
2*b*x + 2*a)/(c - d - 1)) + 3*polylog(4, (c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)))/(b^4*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\text{atanh}\left(c+d\coth\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(c + d*coth(a + b*x)),x)
```

```
[Out] int(x^2*atanh(c + d*coth(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{atanh}\left(c+d\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(c + d*coth(a + b*x)), x)
```

3.299 $\int x \tanh^{-1}(c + d \coth(a + bx)) \, dx$

Optimal. Leaf size=229

$$-\frac{\mathrm{Li}_3\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\mathrm{Li}_3\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\mathrm{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\mathrm{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{4}x^2\log\left(1-\frac{(-c-d+1)e}{-c+d+}\right.$$

```
[Out] 1/2*x^2*arctanh(c+d*coth(b*x+a))+1/4*x^2*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d
))-1/4*x^2*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*x*polylog(2,(1-c-d)*exp
(2*b*x+2*a)/(1-c+d))/b-1/4*x*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b-1/
8*polylog(3,(1-c-d)*exp(2*b*x+2*a)/(1-c+d))/b^2+1/8*polylog(3,(1+c+d)*exp(2
*b*x+2*a)/(1+c-d))/b^2
```

Rubi **[A]** time = 0.37, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6245, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{8b^2}+\frac{\text{PolyLog}\left(3,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{8b^2}+\frac{x\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{x\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+}}{c-d+1}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Coth[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - d)*E^(2*a + 2
*b*x))/(1 - c + d)])/4 - (x^2*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c
- d)])/4 + (x*PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)])/(4*b)
- (x*PolyLog[2, ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/(4*b) - PolyLog
[3, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(8*b^2) + PolyLog[3, ((1 + c
 + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(8*b^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6245

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
```

```
(m + 1)), x] + (-Dist[(b*(1 - c - d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E
^(2*a + 2*b*x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[(b
*(1 + c + d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*a + 2*b*x))/(1 + c -
 d - (1 + c + d)*E^(2*a + 2*b*x)), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[(c - d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\coth(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) - \frac{1}{2}(b(1-c-d))\int \frac{e^{2a+2bx}x^2}{1-c+d+(-1+c\cdots}$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\cdots$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\cdots$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\cdots$$

$$= \frac{1}{2}x^2\tanh^{-1}(c+d\coth(a+bx)) + \frac{1}{4}x^2\log\left(1-\frac{(1-c-d)e^{2a+2bx}}{1-c+d}\right) - \frac{1}{4}x^2\cdots$$

Mathematica [A] time = 9.54, size = 267, normalized size = 1.17

$$2b^2x^2\log\left(\frac{2(\cosh(a+bx)-\sinh(a+bx))((c-1)\sinh(a+bx)+d\cosh(a+bx))}{c+d-1}\right) - 2b^2x^2\log\left(\frac{(c-d+1)(\sinh(2(a+bx))-\cosh(2(a+bx)))}{c+d+1}+1\right) - \cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Coth[a + b*x]])/2 + (2*b^2*x^2*Log[(2*(Cosh[a + b*x] - S
inh[a + b*x])*(d*Cosh[a + b*x] + (-1 + c)*Sinh[a + b*x]))/(-1 + c + d)] - 2
*b^2*x^2*Log[1 + ((1 + c - d)*(-Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]))/(1
+ c + d)] - 2*b*x*PolyLog[2, ((-1 + c - d)*(Cosh[2*(a + b*x)] - Sinh[2*(a +
 b*x)]))/(-1 + c + d)] + 2*b*x*PolyLog[2, ((1 + c - d)*(Cosh[2*(a + b*x)] -
 Sinh[2*(a + b*x)]))/(1 + c + d)] - PolyLog[3, ((-1 + c - d)*(Cosh[2*(a + b
*x)] - Sinh[2*(a + b*x)]))/(-1 + c + d)] + PolyLog[3, ((1 + c - d)*(Cosh[2*
(a + b*x)] - Sinh[2*(a + b*x)]))/(1 + c + d)])/(8*b^2)
```

fricas [C] time = 0.78, size = 730, normalized size = 3.19

$$b^2x^2\log\left(-\frac{d\cosh(bx+a)+(c+1)\sinh(bx+a)}{d\cosh(bx+a)+(c-1)\sinh(bx+a)}\right) - 2\,bx\mathrm{Li}_2\left(\sqrt{\frac{c+d+1}{c-d+1}}\left(\cosh(bx+a)+\sinh(bx+a)\right)\right) - 2\,bx\mathrm{Li}_2\left(-\sqrt{\frac{c+d+1}{c-d+1}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a
) + (c - 1)*sinh(b*x + a))) - 2*b*x*dilog(sqrt((c + d + 1)/(c - d + 1))*(co
sh(b*x + a) + sinh(b*x + a))) - 2*b*x*dilog(-sqrt((c + d + 1)/(c - d + 1))*
(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(sqrt((c + d - 1)/(c - d - 1)
)*(cosh(b*x + a) + sinh(b*x + a))) + 2*b*x*dilog(-sqrt((c + d - 1)/(c - d -
 1))*(cosh(b*x + a) + sinh(b*x + a))) - a^2*log(2*(c + d + 1)*cosh(b*x + a)
 + 2*(c + d + 1)*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1)
)) - a^2*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*
(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a^2*log(2*(c + d - 1)*cosh(b*x
 + a) + 2*(c + d - 1)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d
 - 1))) + a^2*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a)
 - 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - (b^2*x^2 - a^2)*log(sqrt(
(c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - (b^2*x^2 -
a^2)*log(-sqrt((c + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1
) + (b^2*x^2 - a^2)*log(sqrt((c + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh
(b*x + a)) + 1) + (b^2*x^2 - a^2)*log(-sqrt((c + d - 1)/(c - d - 1))*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + 2*polylog(3, sqrt((c + d + 1)/(c - d + 1))
*(cosh(b*x + a) + sinh(b*x + a))) + 2*polylog(3, -sqrt((c + d + 1)/(c - d +
 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, sqrt((c + d - 1)/(c -
d - 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*polylog(3, -sqrt((c + d - 1)/(
c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}\left(d \coth\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*coth(b*x + a) + c), x)
```

maple [C] time = 3.60, size = 4990, normalized size = 21.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*coth(b*x+a)),x)
```

```
[Out] -1/4/b^2*a^2/(1+c+d)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-c+
d-1)+1/8/b^2/(1+c+d)*polylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))-1/8*I*Pi*x^2
*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2
*b*x+2*a)-1))^3+1/4*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1
)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^2-1/8*I*Pi*x^2*csgn(I*((exp(2*b*x
+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^3-1/4
/b^2*a^2/(c+d-1)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-c+d+1)
-1/4*I*Pi*x^2-1/4/b^2/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/
b^2/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a+1/2/b^2*a^2/(1+c+d)
*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d
)*(1+c+d))^(1/2))+1/2/b^2*a^2/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d
)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/8/b^2*d/(1+c+d)*pol
ylog(3,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/8/b^2*c/(1+c+d)*polylog(3,(1+c+d)*
exp(2*b*x+2*a)/(1+c-d))+1/2/b^2*a/(1+c+d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d
+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a/(1+
c+d)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/(
(1+c-d)*(1+c+d))^(1/2))-1/4/b/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c
-d))*x-1/4*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2-1/4*c/(1+c+d)
*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2+1/4*x^2*ln((exp(2*b*x+2*a)-1)*c+(
exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)+1/4/b*c/(c+d-1)*polylog(2,(c+d-1)*exp
(2*b*x+2*a)/(c-d-1))*x+1/4/b^2*c/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(
```

```
c-d-1))*a+1/4/b^2*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2+1/4/b*
d/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x+1/4/b^2*d/(c+d-1)*pol
ylog(2,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a-1/2/b/(c+d-1)*ln(1-(c+d-1)*exp(2*b
*x+2*a)/(c-d-1))*x*a+1/2/b*a/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1
)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/4/(1+c+d)*ln(1-(1
+c+d)*exp(2*b*x+2*a)/(1+c-d))*x^2+1/4*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)
/(c-d-1))*x^2-1/8/b^2*c/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))-1
/8/b^2*d/(c+d-1)*polylog(3,(c+d-1)*exp(2*b*x+2*a)/(c-d-1))+1/2/b^2*a/(c+d-1
)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c
-d-1)*(c+d-1))^(1/2))+1/2/b^2*a/(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((
c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/4/b^2/(c+d-1)*
ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2-1/4/b/(c+d-1)*polylog(2,(c+d-1)*ex
p(2*b*x+2*a)/(c-d-1))*x-1/4/b^2/(c+d-1)*polylog(2,(c+d-1)*exp(2*b*x+2*a)/(c
-d-1))*a+1/2/b^2*a^2/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1
))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))+1/2/b^2*a^2/(c+d-1)*ln((c*exp
(b*x+a)+exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^
(1/2))+1/4*c/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2-1/4/(c+d-1)*l
n(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x^2+1/8/b^2/(c+d-1)*polylog(3,(c+d-1)*e
xp(2*b*x+2*a)/(c-d-1))-1/4*x^2*ln((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d
-exp(2*b*x+2*a)+1)-1/2/b*c*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)
*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln
((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(
c+d-1))^(1/2))*x-1/2/b*d*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)*(
c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x+1/2/b*c/(c+d-1)*ln(1-(
c+d-1)*exp(2*b*x+2*a)/(c-d-1))*x*a+1/2/b*d/(c+d-1)*ln(1-(c+d-1)*exp(2*b*x+2
*a)/(c-d-1))*x*a-1/2/b*c*a/(c+d-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*
(c+d-1))^(1/2)+exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x+1/2/b^2*a*c/(1+c+d)*d
ilog((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-
d)*(1+c+d))^(1/2))+1/2/b^2*a*c/(1+c+d)*dilog((c*exp(b*x+a)+exp(b*x+a)*d+((1
+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*d/(1+c+
d)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((
1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a*d/(1+c+d)*dilog((c*exp(b*x+a)+exp(b*x+a)*d
+((1+c-d)*(1+c+d))^(1/2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))-1/2/b/(1+c+d)
*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a+1/2/b*a/(1+c+d)*ln((-c*exp(b*x+a)
-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*
x-1/4/b^2*d/(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2+1/4/b^2*c/(c+d
-1)*ln(1-(c+d-1)*exp(2*b*x+2*a)/(c-d-1))*a^2-1/4/b^2*a^2*d/(1+c+d)*ln(c*exp
(2*b*x+2*a)+d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-c+d-1)-1/4/b^2*a^2*c/(1+c+d)*ln
(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-c+d-1)-1/2/b*d/(1+c+d)*ln
(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*x*a-1/2/b*c/(1+c+d)*ln(1-(1+c+d)*exp(2*b
*x+2*a)/(1+c-d))*x*a+1/2/b*a/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-d-1)
*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))*x-1/2/b^2*c*a^2/(c+d-1
)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-d-
1)*(c+d-1))^(1/2))-1/2/b^2*c*a^2/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((c-
d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a^2/(c+d
-1)*ln((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/((c-
d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a^2/(c+d-1)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((
c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*c*a/(c+d
-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a))/(
(c-d-1)*(c+d-1))^(1/2))-1/2/b^2*c*a/(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x+a)*
d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/
(c+d-1)*dilog((-c*exp(b*x+a)-exp(b*x+a)*d+((c-d-1)*(c+d-1))^(1/2)+exp(b*x+a
))/((c-d-1)*(c+d-1))^(1/2))-1/2/b^2*d*a/(c+d-1)*dilog((c*exp(b*x+a)+exp(b*x
+a)*d+((c-d-1)*(c+d-1))^(1/2)-exp(b*x+a))/((c-d-1)*(c+d-1))^(1/2))-1/8*I*Pi
*x^2*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1))*c
sgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b
*x+2*a)-1))^2-1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*
a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))^2+1/8*I*
Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2
```

```
*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1))^2-1/8*I*Pi*x^2*csgn(I/(exp(2
*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2
*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)
/(exp(2*b*x+2*a)-1))+1/8*I*Pi*x^2*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*((exp(2
*b*x+2*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1))*csgn(I*((exp(2*b*x+2
*a)-1)*c+(exp(2*b*x+2*a)+1)*d-exp(2*b*x+2*a)+1)/(exp(2*b*x+2*a)-1))+1/4/b^2
*d*a^2/(c+d-1)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-c+d+1)+1
/2/b*a/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp(b*
x+a))/((1+c-d)*(1+c+d))^(1/2))*x-1/4/b*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*
x+2*a)/(1+c-d))*x-1/4/b*c/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))
*x-1/4/b^2*d/(1+c+d)*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a-1/4/b^2*c/
(1+c+d)*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a^2-1/4/b^2*c/(1+c+d)*polylog(
2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))*a+1/2/b^2*a^2*c/(1+c+d)*ln((-c*exp(b*x+a)
-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+
1/2/b^2*a^2*c/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)
+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b^2*a^2*d/(1+c+d)*ln((-c*exp(b*x+
a)-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2)
)+1/2/b^2*a^2*d/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/
2)+exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))+1/2/b*a*c/(1+c+d)*ln((-c*exp(b*x+a)
-exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*
x+1/2/b*a*c/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+e
xp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/2/b*a*d/(1+c+d)*ln((-c*exp(b*x+a)-e
xp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)-exp(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+
1/2/b*a*d/(1+c+d)*ln((c*exp(b*x+a)+exp(b*x+a)*d+((1+c-d)*(1+c+d))^(1/2)+exp
(b*x+a))/((1+c-d)*(1+c+d))^(1/2))*x+1/8*I*Pi*x^2*csgn(I*((exp(2*b*x+2*a)-1)
*c+(exp(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1))*csgn(I*((exp(2*b*x+2*a)-1)*c+(ex
p(2*b*x+2*a)+1)*d+exp(2*b*x+2*a)-1)/(exp(2*b*x+2*a)-1))^2+1/4/b^2*c*a^2/(c+
d-1)*ln(c*exp(2*b*x+2*a)+d*exp(2*b*x+2*a)-exp(2*b*x+2*a)-c+d+1)
```

maxima [A] time = 0.72, size = 213, normalized size = 0.93

$$-\frac{1}{8}\,bd\left(\frac{2\,b^2x^2\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+2\,bx\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)-\mathrm{Li}_3(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1})}{b^3d}-\frac{2\,b^2x^2\log\left(-\frac{(c+d-1)e^{(2\,bx\cdot}}{c-d-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*b^2*x^2*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + 2*
b*x*dilog((c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)) - polylog(3, (c + d + 1)
*e^(2*b*x + 2*a)/(c - d + 1)))/(b^3*d) - (2*b^2*x^2*log(-(c + d - 1)*e^(2*b
*x + 2*a)/(c - d - 1) + 1) + 2*b*x*dilog((c + d - 1)*e^(2*b*x + 2*a)/(c - d
 - 1)) - polylog(3, (c + d - 1)*e^(2*b*x + 2*a)/(c - d - 1)))/(b^3*d)) + 1/
2*x^2*arctanh(d*coth(b*x + a) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atanh}\left(c+d\,\mathrm{coth}\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(c + d*coth(a + b*x)),x)
```

```
[Out] int(x*atanh(c + d*coth(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atanh}\left(c+d\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*coth(b*x+a)),x)
[Out] Integral(x*atanh(c + d*coth(a + b*x)), x)
```

3.300 $\int \tanh^{-1}(c + d\coth(a + bx))\, dx$

Optimal. Leaf size=150

$$\frac{\text{Li}_2\left(\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{Li}_2\left(\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)+x\tan$$

```
[Out] x*arctanh(c+d*coth(b*x+a))+1/2*x*ln(1-(1-c-d)*exp(2*b*x+2*a)/(1-c+d))-1/2*x
*ln(1-(1+c+d)*exp(2*b*x+2*a)/(1+c-d))+1/4*polylog(2,(1-c-d)*exp(2*b*x+2*a)/
(1-c+d))/b-1/4*polylog(2,(1+c+d)*exp(2*b*x+2*a)/(1+c-d))/b
```

Rubi [A] time = 0.23, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6237, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)}{4b}-\frac{\text{PolyLog}\left(2,\frac{(c+d+1)e^{2a+2bx}}{c-d+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-d+1)e^{2a+2bx}}{-c+d+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+d+1}{c-d}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Coth[a + b*x]] + (x*Log[1 - ((1 - c - d)*E^(2*a + 2*b*x))/(
1 - c + d)])/2 - (x*Log[1 - ((1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)])/2 +
 PolyLog[2, ((1 - c - d)*E^(2*a + 2*b*x))/(1 - c + d)]/(4*b) - PolyLog[2, (
(1 + c + d)*E^(2*a + 2*b*x))/(1 + c - d)]/(4*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6237

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + (-Dist[b*(1 - c - d), Int[(x*E^(2*a + 2*b*
x))/(1 - c + d - (1 - c - d)*E^(2*a + 2*b*x)), x], x] + Dist[b*(1 + c + d),
 Int[(x*E^(2*a + 2*b*x))/(1 + c - d - (1 + c + d)*E^(2*a + 2*b*x)), x], x])
 /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(c + d\coth(a + bx))\,dx = x\tanh^{-1}(c + d\coth(a + bx)) - (b(1 - c - d))\int \frac{e^{2a+2bx}x}{1 - c + d + (-1 + c + d)e^{2a}}$$

$$= x\tanh^{-1}(c + d\coth(a + bx)) + \frac{1}{2}x\log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{2}x\log\left(1\right.$$

$$= x\tanh^{-1}(c + d\coth(a + bx)) + \frac{1}{2}x\log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{2}x\log\left(1\right.$$

$$= x\tanh^{-1}(c + d\coth(a + bx)) + \frac{1}{2}x\log\left(1 - \frac{(1 - c - d)e^{2a+2bx}}{1 - c + d}\right) - \frac{1}{2}x\log\left(1\right.$$

Mathematica [A] time = 5.42, size = 131, normalized size = 0.87

$$x\tanh^{-1}(d\coth(a+bx)+c) - \frac{-\mathrm{Li}_2\left(\frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) + \mathrm{Li}_2\left(\frac{(c+d+1)e^{2(a+bx)}}{c-d+1}\right) - 2bx\left(\log\left(1 - \frac{(c+d-1)e^{2(a+bx)}}{c-d-1}\right) - \log\left(1 - \right.\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[c + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Coth[a + b*x]] - (-2*b*x*(Log[1 - ((-1 + c + d)*E^(2*(a + b
*x)))/(-1 + c - d)] - Log[1 - ((1 + c + d)*E^(2*(a + b*x)))/(1 + c - d)]) -
 PolyLog[2, ((-1 + c + d)*E^(2*(a + b*x)))/(-1 + c - d)] + PolyLog[2, ((1 +
 c + d)*E^(2*(a + b*x)))/(1 + c - d)])/(4*b)
```

fricas [B] time = 0.47, size = 540, normalized size = 3.60

$$\frac{bx\log\left(-\frac{d\cosh(bx+a)+(c+1)\sinh(bx+a)}{d\cosh(bx+a)+(c-1)\sinh(bx+a)}\right) + a\log\left(2\left(c+d+1\right)\cosh\left(bx+a\right) + 2\left(c+d+1\right)\sinh\left(bx+a\right) + 2\left(c-d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-(d*cosh(b*x + a) + (c + 1)*sinh(b*x + a))/(d*cosh(b*x + a) +
(c - 1)*sinh(b*x + a))) + a*log(2*(c + d + 1)*cosh(b*x + a) + 2*(c + d + 1)
*sinh(b*x + a) + 2*(c - d + 1)*sqrt((c + d + 1)/(c - d + 1))) + a*log(2*(c
+ d + 1)*cosh(b*x + a) + 2*(c + d + 1)*sinh(b*x + a) - 2*(c - d + 1)*sqrt((
c + d + 1)/(c - d + 1))) - a*log(2*(c + d - 1)*cosh(b*x + a) + 2*(c + d - 1
)*sinh(b*x + a) + 2*(c - d - 1)*sqrt((c + d - 1)/(c - d - 1))) - a*log(2*(c
 + d - 1)*cosh(b*x + a) + 2*(c + d - 1)*sinh(b*x + a) - 2*(c - d - 1)*sqrt(
(c + d - 1)/(c - d - 1))) - (b*x + a)*log(sqrt((c + d + 1)/(c - d + 1))*(co
sh(b*x + a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-sqrt((c + d + 1)/(c - d
+ 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(sqrt((c + d - 1)
/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (b*x + a)*log(-sqrt((c
 + d - 1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) - dilog(sqrt((c
 + d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt((c +
d + 1)/(c - d + 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(sqrt((c + d -
1)/(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))) + dilog(-sqrt((c + d - 1)/
(c - d - 1))*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}\left(d\coth\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + c), x)
```

maple [B] time = 0.35, size = 306, normalized size = 2.04

$$-\frac{\operatorname{arctanh}\left(c+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)-d\right)}{2b}+\frac{\operatorname{arctanh}\left(c+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)+d\right)}{2b}+\frac{\mathrm{d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*coth(b*x+a)),x)
```

```
[Out] -1/2/b*arctanh(c+d*coth(b*x+a))*ln(d*coth(b*x+a)-d)+1/2/b*arctanh(c+d*coth(
b*x+a))*ln(d*coth(b*x+a)+d)+1/4/b*dilog((d*coth(b*x+a)+c-1)/(c-d-1))+1/4/b*
ln(d*coth(b*x+a)+d)*ln((d*coth(b*x+a)+c-1)/(c-d-1))-1/4/b*dilog((d*coth(b*x
+a)+c+1)/(1+c-d))-1/4/b*ln(d*coth(b*x+a)+d)*ln((d*coth(b*x+a)+c+1)/(1+c-d))
+1/4/b*dilog((d*coth(b*x+a)+c+1)/(1+c+d))+1/4/b*ln(d*coth(b*x+a)-d)*ln((d*c
oth(b*x+a)+c+1)/(1+c+d))-1/4/b*dilog((d*coth(b*x+a)+c-1)/(c+d-1))-1/4/b*ln(
d*coth(b*x+a)-d)*ln((d*coth(b*x+a)+c-1)/(c+d-1))
```

maxima [A] time = 0.72, size = 142, normalized size = 0.95

$$-\frac{1}{4}\,bd\left(\frac{2\,bx\log\left(-\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d+1)e^{(2\,bx+2\,a)}}{c-d+1}\right)}{b^2d}-\frac{2\,bx\log\left(-\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}+1\right)+\mathrm{Li}_2\left(\frac{(c+d-1)e^{(2\,bx+2\,a)}}{c-d-1}\right.}{b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((2*b*x*log(-(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1) + 1) + dilog(
(c + d + 1)*e^(2*b*x + 2*a)/(c - d + 1)))/(b^2*d) - (2*b*x*log(-(c + d - 1)
*e^(2*b*x + 2*a)/(c - d - 1) + 1) + dilog((c + d - 1)*e^(2*b*x + 2*a)/(c -
d - 1)))/(b^2*d)) + x*arctanh(d*coth(b*x + a) + c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(c+d\coth\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*coth(a + b*x)),x)
```

```
[Out] int(atanh(c + d*coth(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(c+d\coth\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*coth(b*x+a)),x)
```

```
[Out] Integral(atanh(c + d*coth(a + b*x)), x)
```

3.301 $\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(d\coth(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(c+d*coth(b*x+a))/x,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[c + d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[c + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 14.62, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(c+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[c + d*Coth[a + b*x]]/x, x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(d\coth\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctanh(d*coth(b*x + a) + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(d\coth\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctanh(d*coth(b*x + a) + c)/x, x)
```

maple [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(c + d \coth\left(bx + a\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*coth(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*coth(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(d \coth\left(bx + a\right) + c\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + c)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}\left(c + d \coth\left(a + b\,x\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*coth(a + b*x))/x,x)
```

```
[Out] int(atanh(c + d*coth(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(c + d \coth\left(a + bx\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*coth(b*x+a))/x,x)
```

```
[Out] Integral(atanh(c + d*coth(a + b*x))/x, x)
```

3.302 $\int x^3 \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=152

$$\frac{3\text{Li}_5\left((d+1)e^{2a+2bx}\right)}{16b^4}-\frac{3x\text{Li}_4\left((d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3x^2\text{Li}_3\left((d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x^3\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{8}x^4\log\left(1-(d\right.$$

```
[Out] 1/20*b*x^5+1/4*x^4*arctanh(1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x^3*polylog(2,(1+d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1+d)*exp
(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3+3/16*polylog(5,(
1+d)*exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.30, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6241, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(d+1)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 + d + d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 + d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 + d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \tanh^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{4}b\int \frac{x^4}{1+(-1-d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) + \frac{1}{4}(b(1+d))\int \frac{e^{2a+2b}}{1+(-1-}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2b}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2b}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2b}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2b}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2b}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1+d)e^{2a+2b}\right.$$

Mathematica [A] time = 4.65, size = 141, normalized size = 0.93

$$\frac{1}{16}\left(\frac{3\text{Li}_5\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^4} + \frac{6x\text{Li}_4\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3} + \frac{6x^2\text{Li}_3\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2} + \frac{4x^3\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b} - 2x^4\log\left(1-\frac{e^{-2(a+bx)}}{d+1}\right) + 4x^4\tanh\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 + d + d*Coth[a + b*x]] - 2*x^4*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (4*x^3*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x^2*Poly
```

```
Log[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (6*x*PolyLog[4, 1/((1 + d)*E^(2*
(a + b*x)))])/b^3 + (3*PolyLog[5, 1/((1 + d)*E^(2*(a + b*x)))])/b^4)/16
```

fricas [C] time = 1.53, size = 424, normalized size = 2.79

$$2\,b^5x^5 + 5\,b^4x^4\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right) - 20\,b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 + 5*b^4*x^4*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 20*b^3*x^3*dilog(sqrt(d + 1)*(cosh(b
*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-sqrt(d + 1)*(cosh(b*x + a) +
sinh(b*x + a))) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x +
a) + 2*sqrt(d + 1)) - 5*a^4*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*
x + a) - 2*sqrt(d + 1)) + 60*b^2*x^2*polylog(3, sqrt(d + 1)*(cosh(b*x + a)
+ sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sin
h(b*x + a))) - 120*b*x*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a
))) - 120*b*x*polylog(4, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 5*
(b^4*x^4 - a^4)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b
^4*x^4 - a^4)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*p
olylog(5, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -sq
rt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(d*coth(b*x + a) + d + 1), x)
```

maple [C] time = 5.19, size = 1741, normalized size = 11.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/16*I*x^4*Pi*csg
n(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*
Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+e
xp(2*b*x+2*a)-1))^2-1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a
))^3+3/16/b^4/(1+d)*polylog(5,(1+d)*exp(2*b*x+2*a))-1/8/(1+d)*ln(1-(1+d)*ex
p(2*b*x+2*a))*x^4-1/8*I*x^4*Pi+1/2/b^4*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2
))+1/2/b^4*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/8*d/(1+d)*ln(1-(1+d)*ex
p(2*b*x+2*a))*x^4+3/16/b^4*d/(1+d)*polylog(5,(1+d)*exp(2*b*x+2*a))+1/2/b^4*
a^3/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*a^3/(1+d)*dilog(1+exp(b*x
+a)*(1+d)^(1/2))-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3-1/4/b^4/(1
+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^3+3/8/b^2/(1+d)*polylog(3,(1+d)*exp(2
*b*x+2*a))*x^2-3/8/b^3/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x-3/8/b^4/(1+d
)*ln(1-(1+d)*exp(2*b*x+2*a))*a^4+1/20*b*x^5-1/8/b^4*d*a^4/(1+d)*ln(d*exp(2*
b*x+2*a)+exp(2*b*x+2*a)-1)+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1/8*x^4*l
n(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)+1/2/b^3*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d
)^(1/2))*x+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1/2/b^
4*d*a^3/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^3/(1+d)*dilog(1-e
```

```
xp(b*x+a)*(1+d)^(1/2))+1/2/b^4*d*a^4/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2
/b^4*d*a^4/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b^3/(1+d)*ln(1-(1+d)*exp(
2*b*x+2*a))*x*a^3-3/8/b^4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^4-1/4/b*d/(1
+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^3-1/4/b^4*d/(1+d)*polylog(2,(1+d)*exp
(2*b*x+2*a))*a^3+3/8/b^2*d/(1+d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x^2-3/8/b^
3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x+2*a))*x+1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+
a)*(1+d)^(1/2))*x-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x
+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*
x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^2
+1/8*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*c
sgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^3-1/4*x^4*ln(
exp(b*x+a))-1/8*x^4*ln(d)-1/8/b^4*a^4/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2
*a)-1)-1/8*I*x^4*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/16*I*x^4*
Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*ex
p(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b
*x+2*a))^2+1/2/b^3*d*a^3/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^3*d/(1+
d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+
d)^(1/2))*x+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1
))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/16*I*x^4*Pi*csgn(I*exp(2*b
*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1
))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(d*exp(2*b*x+2*a)+exp(2*
b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))
```

maxima [A] time = 1.10, size = 146, normalized size = 0.96

$$\frac{1}{4}x^4 \operatorname{artanh}(d\coth(bx+a)+d+1)+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left(-(d+1)e^{(2bx+2a)}+1\right)+4b^3x^3\mathrm{Li}_2\left((d+1)e^{(2bx+2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctanh(d*coth(b*x + a) + d + 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b^2*x^2*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, (d + 1)*e
^(2*b*x + 2*a)) - 3*polylog(5, (d + 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{atanh}(d+d\coth(a+bx)+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*atanh(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x^3*atanh(d + d*coth(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \operatorname{atanh}(d\coth(a+bx)+d+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**3*atanh(d*coth(a + b*x) + d + 1), x)
```

3.303 $\int x^2 \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=126

$$-\frac{\text{Li}_4\left((d+1)e^{2a+2bx}\right)}{8b^3}+\frac{x\text{Li}_3\left((d+1)e^{2a+2bx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{3}x^3\tanh^{-1}($$

```
[Out] 1/12*b*x^4+1/3*x^3*arctanh(1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1+d)*exp(2*b*x+
2*a))-1/4*x^2*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1+d)*exp(2
*b*x+2*a))/b^2-1/8*polylog(4,(1+d)*exp(2*b*x+2*a))/b^3
```

Rubi [A] time = 0.26, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6241, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(d+1)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(d+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 + d + d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 + d
)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + (
x*PolyLog[3, (1 + d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 + d)*E^(2*a
+ 2*b*x)]/(8*b^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\tanh^{-1}(1+d+d\coth(a+bx))\,dx &= \frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx))+\frac{1}{3}b\int\frac{x^3}{1+(-1-d)e^{2a+2bx}}\,dx\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx))+\frac{1}{3}(b(1+d))\int\frac{e^{2a+2b}}{1+(-1-}\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2b}\right.\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2b}\right.\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2b}\right.\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2b}\right.\\ &= \frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1+d)e^{2a+2b}\right.\end{aligned}$$

Mathematica [A] time = 5.42, size = 116, normalized size = 0.92

$$\frac{1}{24}\left(\frac{3\text{Li}_4\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^3}+\frac{6x\text{Li}_3\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b^2}+\frac{6x^2\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{b}-4x^3\log\left(1-\frac{e^{-2(a+bx)}}{d+1}\right)+8x^3\tanh^{-1}(d\coth(a+bx)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 + d + d*Coth[a + b*x]] - 4*x^3*Log[1 - 1/((1 + d)*E^(2*(a
+ b*x)))] + (6*x^2*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))])/b + (6*x*PolyLo
g[3, 1/((1 + d)*E^(2*(a + b*x)))])/b^2 + (3*PolyLog[4, 1/((1 + d)*E^(2*(a +
 b*x)))])/b^3)/24
```

fricas [C] time = 1.00, size = 360, normalized size = 2.86

$$b^4x^4+2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6\,b^2x^2\text{Li}_2\left(\sqrt{d+1}\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)\right)-6\,b^2x^2\text{Li}_2\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 + 2*b^3*x^3*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d
*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b^2*x^2*dilog(sqrt(d + 1)*(cosh(b*x
+ a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh
(b*x + a))) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) +
 2*sqrt(d + 1)) + 2*a^3*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x +
a) - 2*sqrt(d + 1)) + 12*b*x*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b
*x + a))) + 12*b*x*polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)))
 - 2*(b^3*x^3 + a^3)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
 2*(b^3*x^3 + a^3)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a)) + 1) -
12*polylog(4, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4,
-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}\left(d \coth\left(bx + a\right) + d + 1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*coth(b*x + a) + d + 1), x)
```

maple [C] time = 4.67, size = 1684, normalized size = 13.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/6*x^3*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)+1/6*I*x^3*Pi*csgn(I*d/(exp(2*
b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/6/b^3*a^3/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b
*x+2*a)-1)+1/12*I*x^3*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))-1/6*I*
x^3*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I*d/(
exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/6*I*Pi*x^3+1/3/b^3/(1+d)*ln(1-(1+d)*e
xp(2*b*x+2*a))*a^3-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x^2+1/4/b^3/
(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2/(1+d)*polylog(3,(1+d)*exp
(2*b*x+2*a))*x-1/2/b^3*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^3/(
1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))-1/8/b^3*d/(1+d)*polylog(4,(1+d)*exp(2*b*x
+2*a))-1/2/b^3*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*a^2/(1+d)*
dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/6*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-
1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2
*a)-1))^2+1/12*I*x^3*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/
(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csg
n(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*
x+2*a))^2+1/12*b*x^4-1/3*x^3*ln(exp(b*x+a))-1/6*x^3*ln(d)-1/2/b^2*a^2/(1+d)
*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/
2))*x-1/2/b^3*d*a^3/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^3/(1+d)*
ln(1+exp(b*x+a)*(1+d)^(1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(
1/2))-1/2/b^3*d*a^2/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))+1/12*I*x^3*Pi*csg
n(I*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*e
xp(2*b*x+2*a))^2-1/6/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^3-1/8/b^3/(1+d)*pol
ylog(4,(1+d)*exp(2*b*x+2*a))-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp
(2*b*x+2*a)+exp(2*b*x+2*a)-1))^3+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2
*b*x+2*a)-1))^3+1/2/b^2/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2+1/3/b^3*d/(1
+d)*ln(1-(1+d)*exp(2*b*x+2*a))*a^3-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+
2*a))*x^2+1/4/b^3*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a^2+1/4/b^2*d/(1+
```

```
d)*polylog(3,(1+d)*exp(2*b*x+2*a))*x+1/6/b^3*d*a^3/(1+d)*ln(d*exp(2*b*x+2*a
)+exp(2*b*x+2*a)-1)-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2
*a)/(exp(2*b*x+2*a)-1))^2+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(
exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^2-1/2/b^2*d*a^2/(1+d
)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x-1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)^
(1/2))*x+1/2/b^2*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a^2+1/12*I*x^3*Pi*csg
n(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1
)*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*
a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))-1/12*I*x^3*Pi*csgn(I/(exp(
2*b*x+2*a)-1))*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*
x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))
```

maxima [A] time = 1.12, size = 123, normalized size = 0.98

$$\frac{1}{3}x^3\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left(-(d+1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left((d+1)e^{(2bx+2}\right.\right.}{b^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(d*coth(b*x + a) + d + 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*log
(-(d + 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog((d + 1)*e^(2*b*x + 2*a)) -
 6*b*x*polylog(3, (d + 1)*e^(2*b*x + 2*a)) + 3*polylog(4, (d + 1)*e^(2*b*x
+ 2*a)))/(b^4*d))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\operatorname{atanh}\left(d+d\coth\left(a+b\,x\right)+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x^2*atanh(d + d*coth(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\operatorname{atanh}{\left(d\coth{\left(a+bx\right)}+d+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(d*coth(a + b*x) + d + 1), x)
```

3.304 $\int x \tanh^{-1}(1+d+d\coth(a+bx))\,dx$

Optimal. Leaf size=100

$$\frac{\text{Li}_3\left((d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^3}{6}$$

```
[Out] 1/6*b*x^3+1/2*x^2*arctanh(1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1+d)*exp(2*b*x+2
*a))-1/4*x*polylog(2,(1+d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1+d)*exp(2*b*x+
2*a))/b^2
```

Rubi [A] time = 0.23, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6241, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,(d+1)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(d+1)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\coth(a$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 + d + d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 + d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 + d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 + d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
```

```
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+d+d\coth(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))+\frac{1}{2}b\int\frac{x^2}{1+(-1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))+\frac{1}{2}(b(1+d))\int\frac{e^{2a+2bx}}{1+(-1-d}$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right.$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right.$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right.$$
$$= \frac{bx^3}{6}+\frac{1}{2}x^2\tanh^{-1}(1+d+d\coth(a+bx))-\frac{1}{4}x^2\log\left(1-(1+d)e^{2a+2bx}\right.$$

Mathematica [A] time = 5.22, size = 90, normalized size = 0.90

$$\frac{2b^2x^2\left(2\tanh^{-1}(d\coth(a+bx)+d+1)-\log\left(1-\frac{e^{-2(a+bx)}}{d+1}\right)\right)+2bx\text{Li}_2\left(\frac{e^{-2(a+bx)}}{d+1}\right)+\text{Li}_3\left(\frac{e^{-2(a+bx)}}{d+1}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 + d + d*Coth[a + b*x]] - Log[1 - 1/((1 + d)*E^(2*(a
 + b*x)))]) + 2*b*x*PolyLog[2, 1/((1 + d)*E^(2*(a + b*x)))] + PolyLog[3, 1/
((1 + d)*E^(2*(a + b*x)))])/(8*b^2)
```

fricas [C] time = 0.58, size = 306, normalized size = 3.06

$$2\,b^3x^3+3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right)-6\,bx\text{Li}_2\left(\sqrt{d+1}\,(\cosh(bx+a)+\sinh(bx+a))\right)-6\,bx\text{Li}_2\left(-\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 3*b^2*x^2*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/
(d*cosh(b*x + a) + d*sinh(b*x + a))) - 6*b*x*dilog(sqrt(d + 1)*(cosh(b*x +
a) + sinh(b*x + a))) - 6*b*x*dilog(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
 a))) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) + 2*sqr
t(d + 1)) - 3*a^2*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sinh(b*x + a) - 2
*sqrt(d + 1)) - 3*(b^2*x^2 - a^2)*log(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
 + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x
```

```
+ a)) + 1) + 6*polylog(3, sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) + 6*
polylog(3, -sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*coth(b*x + a) + d + 1), x)
```

maple [C] time = 4.35, size = 1603, normalized size = 16.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] -1/4*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^2+1/8/b^2*d/(1+d)*polylog(3,(1+d)
*exp(2*b*x+2*a))+1/2/b^2*a^2/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a^2
/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*a/(1+d)*dilog(1-exp(b*x+a)*(1+d
)^(1/2))+1/2/b^2*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/4/b^2/(1+d)*ln(1
-(1+d)*exp(2*b*x+2*a))*a^2-1/4/b/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x-1/
4/b^2/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a-1/2*x^2*ln(exp(b*x+a))-1/4*x^
2*ln(d)+1/6*b*x^3-1/4/b^2*d*a^2/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)
-1/4/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x^2+1/8/b^2/(1+d)*polylog(3,(1+d)*exp
(2*b*x+2*a))-1/4/b^2*a^2/(1+d)*ln(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1/4*I*
Pi*x^2-1/2/b/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a-1/4/b^2*d/(1+d)*ln(1-(1+d
)*exp(2*b*x+2*a))*a^2-1/4/b*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*x-1/4/b
^2*d/(1+d)*polylog(2,(1+d)*exp(2*b*x+2*a))*a+1/2/b*a/(1+d)*ln(1-exp(b*x+a)*
(1+d)^(1/2))*x+1/2/b*a/(1+d)*ln(1+exp(b*x+a)*(1+d)^(1/2))*x+1/2/b^2*d*a^2/(
1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))+1/2/b^2*d*a^2/(1+d)*ln(1+exp(b*x+a)*(1+d)
^(1/2))+1/2/b^2*d*a/(1+d)*dilog(1-exp(b*x+a)*(1+d)^(1/2))+1/8*I*x^2*Pi*csgn
(I*exp(2*b*x+2*a))^3+1/4*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a
))^2+1/2/b^2*d*a/(1+d)*dilog(1+exp(b*x+a)*(1+d)^(1/2))-1/8*I*x^2*Pi*csgn(I*
d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(
exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/8*I*x^2*
Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/4*x^2*ln(d*exp
(2*b*x+2*a)+exp(2*b*x+2*a)-1)-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp
(2*b*x+2*a)+exp(2*b*x+2*a)-1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*
b*x+2*a)-1))^3+1/8*I*x^2*Pi*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csg
n(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*
csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/8*
I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2
+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2
*b*x+2*a)+exp(2*b*x+2*a)-1))^2-1/4*I*x^2*Pi*csgn(I*exp(b*x+a))*csgn(I*exp(2
*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/2/b
*d*a/(1+d)*ln(1-exp(b*x+a)*(1+d)^(1/2))*x+1/2/b*d*a/(1+d)*ln(1+exp(b*x+a)*(
1+d)^(1/2))*x-1/2/b*d/(1+d)*ln(1-(1+d)*exp(2*b*x+2*a))*x*a-1/8*I*x^2*Pi*csg
n(I/(exp(2*b*x+2*a)-1))*csgn(I*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))*csgn(I/
(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)+exp(2*b*x+2*a)-1))+1/8*I*x^2*Pi*csgn(I
*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))
*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))
```

maxima [A] time = 1.09, size = 100, normalized size = 1.00

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3((d+1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)bd+\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g((d + 1)*e^(2*b*x + 2*a)) - polylog(3, (d + 1)*e^(2*b*x + 2*a)))/(b^3*d))*
b*d + 1/2*x^2*arctanh(d*coth(b*x + a) + d + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}\left(d+d\coth\left(a+b\,x\right)+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(x*atanh(d + d*coth(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*coth(a + b*x) + d + 1), x)
```

3.305 $\int \tanh^{-1}(1 + d + d\coth(a + bx))\,dx$

Optimal. Leaf size=69

$$-\frac{\text{Li}_2\left((d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(d+1)e^{2a+2bx}\right)+x\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2+x*arctanh(1+d+d*coth(b*x+a))-1/2*x*ln(1-(1+d)*exp(2*b*x+2*a))-1/4
*polylog(2,(1+d)*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6233, 2184, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,(d+1)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(d+1)e^{2a+2bx}\right)+x\tanh^{-1}(d\coth(a+bx)+d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 + d + d*Coth[a + b*x]] - (x*Log[1 - (1 + d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 + d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6233

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+d+d\coth(a+bx))\,dx = x\tanh^{-1}(1+d+d\coth(a+bx)) + b\int \frac{x}{1+(-1-d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\coth(a+bx)) + (b(1+d))\int \frac{e^{2a+2bx}x}{1+(-1-d)e^{2a+2}}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) + \frac{1}{2}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) + -$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1+d+d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1+d)e^{2a+2bx}\right) - -$$

Mathematica [B] time = 3.88, size = 197, normalized size = 2.86

$$-2\text{Li}_2\left(-\sqrt{d+1}\,e^{a+bx}\right) - 2\text{Li}_2\left(\sqrt{d+1}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{d+1}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{d+1}\,e^{a+}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + d + d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 + d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 + d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(-1 + (1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (2 + d)*Sinh[a + b*x]] -
2*PolyLog[2, -(Sqrt[1 + d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 + d]*E^(a +
b*x)])/(4*b)
```

fricas [B] time = 0.49, size = 227, normalized size = 3.29

$$b^2x^2 + bx\log\left(-\frac{d\cosh(bx+a)+(d+2)\sinh(bx+a)}{d\cosh(bx+a)+d\sinh(bx+a)}\right) + a\log\left(2\,(d+1)\cosh\,(bx+a) + 2\,(d+1)\sinh\,(bx+a) + 2\,\sqrt{d+1}\,\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + b*x*log(-(d*cosh(b*x + a) + (d + 2)*sinh(b*x + a))/(d*cosh(b
*x + a) + d*sinh(b*x + a))) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) + 2*sqrt(d + 1)) + a*log(2*(d + 1)*cosh(b*x + a) + 2*(d + 1)*sin
h(b*x + a) - 2*sqrt(d + 1)) - (b*x + a)*log(sqrt(d + 1)*(cosh(b*x + a) + si
nh(b*x + a)) + 1) - (b*x + a)*log(-sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x +
a)) + 1) - dilog(sqrt(d + 1)*(cosh(b*x + a) + sinh(b*x + a))) - dilog(-sqrt
(d + 1)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{artanh}\,(d\coth\,(bx+a) + d + 1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1), x)
```

maple [B] time = 0.43, size = 247, normalized size = 3.58

$$\frac{\text{arctanh}\,(1+d+d\coth\,(bx+a))\ln\,(d\coth\,(bx+a)+d)}{2b} - \frac{\text{arctanh}\,(1+d+d\coth\,(bx+a))\ln\,(d\coth\,(bx+a)-}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b*arctanh(1+d+d*coth(b*x+a))*ln(d*coth(b*x+a)+d)-1/2/b*arctanh(1+d+d*co
th(b*x+a))*ln(d*coth(b*x+a)-d)+1/8/b*ln(d*coth(b*x+a)+d)^2-1/4/b*dilog(1+1/
2*d*coth(b*x+a)+1/2*d)-1/4/b*ln(d*coth(b*x+a)+d)*ln(1+1/2*d*coth(b*x+a)+1/2
*d)-1/4/b*dilog(1/2*(d*coth(b*x+a)+d)/d)-1/4/b*ln(d*coth(b*x+a)-d)*ln(1/2*(
d*coth(b*x+a)+d)/d)+1/4/b*dilog((d*coth(b*x+a)+d+2)/(2*d+2))+1/4/b*ln(d*cot
h(b*x+a)-d)*ln((d*coth(b*x+a)+d+2)/(2*d+2))
```

maxima [A] time = 1.10, size = 72, normalized size = 1.04

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left(-(d+1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left((d+1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)+x\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log(-(d + 1)*e^(2*b*x + 2*a) + 1) + dilog((d + 1)
*e^(2*b*x + 2*a)))/(b^2*d)) + x*arctanh(d*coth(b*x + a) + d + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}\left(d+d\coth\left(a+b\,x\right)+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + d*coth(a + b*x) + 1),x)
```

```
[Out] int(atanh(d + d*coth(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{atanh}\left(d\coth\left(a+bx\right)+d+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*coth(b*x+a)),x)
```

```
[Out] Integral(atanh(d*coth(a + b*x) + d + 1), x)
```

3.306 $\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^{-1}(d\coth(a+bx)+d+1)}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(1+d+d*coth(b*x+a))/x,x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 + d + d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 + d + d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.68, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1+d+d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 + d + d*Coth[a + b*x]]/x, x]
```

fricas [A] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{artanh}(d\coth(bx+a)+d+1)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(d\coth(bx+a)+d+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

maple [A] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(1+d+d\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+d+d*coth(b*x+a))/x,x)
[Out] int(arctanh(1+d+d*coth(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arctanh(d*coth(b*x + a) + d + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{atanh}\left(d+d\coth\left(a+b\,x\right)+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d + d*coth(a + b*x) + 1)/x,x)
[Out] int(atanh(d + d*coth(a + b*x) + 1)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(d\coth\left(a+bx\right)+d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+d+d*coth(b*x+a))/x,x)
[Out] Integral(atanh(d*coth(a + b*x) + d + 1)/x, x)
```

3.307 $\int x^3 \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=165

$$\frac{3\text{Li}_5\left((1-d)e^{2a+2bx}\right)}{16b^4}-\frac{3x\text{Li}_4\left((1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3x^2\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x^3\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{8}x^4\log\left(1-(1-d\right.$$

```
[Out] 1/20*b*x^5-1/4*x^4*arctanh(-1+d+d*coth(b*x+a))-1/8*x^4*ln(1-(1-d)*exp(2*b*x
+2*a))-1/4*x^3*polylog(2,(1-d)*exp(2*b*x+2*a))/b+3/8*x^2*polylog(3,(1-d)*ex
p(2*b*x+2*a))/b^2-3/8*x*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3+3/16*polylog(5,
(1-d)*exp(2*b*x+2*a))/b^4
```

Rubi [A] time = 0.30, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6241, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{3x\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}+\frac{3\text{PolyLog}\left(5,(1-d)e^{2a+2bx}\right)}{16b^4}-\frac{x^3\text{PolyLog}\left(2,(1\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^5)/20 + (x^4*ArcTanh[1 - d - d*Coth[a + b*x]])/4 - (x^4*Log[1 - (1 - d
)*E^(2*a + 2*b*x)])/8 - (x^3*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (
3*x^2*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(8*b^2) - (3*x*PolyLog[4, (1 - d
)*E^(2*a + 2*b*x)])/(8*b^3) + (3*PolyLog[5, (1 - d)*E^(2*a + 2*b*x)])/(16*b
^4)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}b\int\frac{x^4}{1+(-1+d)e^{2a+2bx}}\,dx$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{4}(b(1-d))\int\frac{e^{2a\cdot}}{1+(-1}$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a\cdot}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a\cdot}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a\cdot}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a\cdot}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a\cdot}\right.$$

$$= \frac{bx^5}{20} + \frac{1}{4}x^4\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{8}x^4\log\left(1-(1-d)e^{2a\cdot}\right.$$

Mathematica [A] time = 4.67, size = 147, normalized size = 0.89

$$\frac{1}{16}\left(\frac{3\text{Li}_5\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^4} + \frac{6x\text{Li}_4\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3} + \frac{6x^2\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2} + \frac{4x^3\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b} - 2x^4\log\left(\frac{e^{-2(a+bx)}}{d-1}+1\right) +\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (4*x^4*ArcTanh[1 - d - d*Coth[a + b*x]] - 2*x^4*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (4*x^3*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x^2
*PolyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (6*x*PolyLog[4, -(1/((-
```

```
1 + d)*E^(2*(a + b*x))))])/b^3 + (3*PolyLog[5, -(1/((-1 + d)*E^(2*(a + b*x)
)))])/b^4)/16
```

fricas [C] time = 0.53, size = 451, normalized size = 2.73

$$\frac{2\,b^5x^5 - 5\,b^4x^4\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 20\,b^3x^3\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4\,d+4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 20\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/40*(2*b^5*x^5 - 5*b^4*x^4*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 20*b^3*x^3*dilog(1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) - 20*b^3*x^3*dilog(-1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
1)*sinh(b*x + a) + sqrt(-4*d + 4)) - 5*a^4*log(2*(d - 1)*cosh(b*x + a) + 2*
(d - 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 60*b^2*x^2*polylog(3, 1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 60*b^2*x^2*polylog(3, -1/2*sqrt(
-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, 1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 120*b*x*polylog(4, -1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))) - 5*(b^4*x^4 - a^4)*log(1/2*sqrt(-4
*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 5*(b^4*x^4 - a^4)*log(-1/2*s
qrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 120*polylog(5, 1/2*sqr
t(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))) + 120*polylog(5, -1/2*sqrt(-4*
d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x^3 \operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(d*coth(b*x + a) + d - 1), x)
```

maple [C] time = 4.85, size = 1801, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*arctanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/16*I*x^4*Pi*csgn(I*exp(b*x+a))^2*csgn(I*exp(2*b*x+2*a))+1/2/b^4*d*a^3/(d-
1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^3/(d-1)*dilog(1-exp(b*x+a)*(
1-d)^(1/2))+1/2/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^3-3/8/b^4*d/(d-1)*
ln(1+(d-1)*exp(2*b*x+2*a))*a^4-1/4/b*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a
))*x^3-1/4/b^4*d/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a^3+3/8/b^2*d/(d-1)
*polylog(3,-(d-1)*exp(2*b*x+2*a))*x^2-3/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(
2*b*x+2*a))*x-1/2/b^3*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^3*a^3/
(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*c
sgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/20*b*x^5+1/8/b^4*a^4/(d-1)*ln(
d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))^3+1
/8*I*x^4*Pi+1/2/b^4*d*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^4*d*a^4/
(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/8*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(
d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I*d/(exp(2*b*x+2*a
)-1)*exp(2*b*x+2*a))^3-1/8*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4+3/8/b^3/(
d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))*x+3/8/b^4/(d-1)*ln(1+(d-1)*exp(2*b*x+
2*a))*a^4+1/4/b/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^3+1/4/b^4/(d-1)*po
```

```
lylog(2,-(d-1)*exp(2*b*x+2*a))*a^3-3/8/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x
+2*a))*x^2-1/2/b^4*a^4/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^4*a^4/(d-1)
*ln(1-exp(b*x+a)*(1-d)^(1/2))-1/2/b^4*a^3/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1
/2))-1/2/b^4*a^3/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+3/16/b^4*d/(d-1)*pol
ylog(5,-(d-1)*exp(2*b*x+2*a))+1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*
x+2*a)-1))^3+1/8/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^4-3/16/b^4/(d-1)*polylo
g(5,-(d-1)*exp(2*b*x+2*a))-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1
)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))^3-1/4*x^4*ln(exp(b*x+a))-1/8*x^4*ln(
d)+1/8*x^4*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/8*I*x^4*Pi*csgn(I*exp(b*
x+a))*csgn(I*exp(2*b*x+2*a))^2-1/8/b^4*d*a^4/(d-1)*ln(d*exp(2*b*x+2*a)-exp(
2*b*x+2*a)+1)-1/16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(
I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/16*I*x^4*Pi*csgn(I*(d*exp(2*b*x+
2*a)-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b
*x+2*a)+1))^2+1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*
a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))^2-1/16*I*x^4*Pi*csgn(I*d)*csgn(I
*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/16*I*x^4*Pi*csgn(I/(exp(2*b*x+2*a
)-1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)
*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))+1/16*I*x^4*Pi*csgn(I*d)*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))+1/
16*I*x^4*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*exp(2*
b*x+2*a)/(exp(2*b*x+2*a)-1))+1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2
))*x-1/2/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^3+1/2/b^3*d*a^3/(d-1)*l
n(1+exp(b*x+a)*(1-d)^(1/2))*x
```

maxima [A] time = 1.12, size = 149, normalized size = 0.90

$$-\frac{1}{4}x^4\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)+\frac{1}{40}\left(\frac{2x^5}{d}-\frac{5\left(2b^4x^4\log\left((d-1)e^{(2bx+2a)}+1\right)+4b^3x^3\operatorname{Li}_2\left(-(d-1)e^{(2\right.\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^3*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*x^4*arctanh(d*coth(b*x + a) + d - 1) + 1/40*(2*x^5/d - 5*(2*b^4*x^4*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 4*b^3*x^3*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b^2*x^2*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 6*b*x*polylog(4, -(d - 1
)*e^(2*b*x + 2*a)) - 3*polylog(5, -(d - 1)*e^(2*b*x + 2*a)))/(b^5*d))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^3\operatorname{atanh}\left(d+d\coth\left(a+b\,x\right)-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*atanh(d + d*coth(a + b*x) - 1),x)
```

```
[Out] int(-x^3*atanh(d + d*coth(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x^3\operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**3*atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -Integral(x**3*atanh(d*coth(a + b*x) + d - 1), x)
```

3.308 $\int x^2 \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=137

$$-\frac{\text{Li}_4\left((1-d)e^{2a+2bx}\right)}{8b^3}+\frac{x\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{4b^2}-\frac{x^2\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{3}x^3\tanh^{-1}(d(-$$

[Out] 1/12*b*x^4-1/3*x^3*arctanh(-1+d+d*coth(b*x+a))-1/6*x^3*ln(1-(1-d)*exp(2*b*x+2*a))-1/4*x^2*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/4*x*polylog(3,(1-d)*exp(2*b*x+2*a))/b^2-1/8*polylog(4,(1-d)*exp(2*b*x+2*a))/b^3

Rubi [A] time = 0.26, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6241, 2184, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{4b^2}-\frac{\text{PolyLog}\left(4,(1-d)e^{2a+2bx}\right)}{8b^3}-\frac{x^2\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^4)/12 + (x^3*ArcTanh[1 - d - d*Coth[a + b*x]])/3 - (x^3*Log[1 - (1 - d)*E^(2*a + 2*b*x)])/6 - (x^2*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + (x*PolyLog[3, (1 - d)*E^(2*a + 2*b*x)])/(4*b^2) - PolyLog[4, (1 - d)*E^(2*a + 2*b*x)]/(8*b^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx))+\frac{1}{3}b\int\frac{x^3}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx))+\frac{1}{3}(b(1-d))\int\frac{e^{2a\cdots}}{1+(-1\cdots}$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a\cdots}\right.$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a\cdots}\right.$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a\cdots}\right.$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a\cdots}\right.$$
$$=\frac{bx^4}{12}+\frac{1}{3}x^3\tanh^{-1}(1-d-d\coth(a+bx))-\frac{1}{6}x^3\log\left(1-(1-d)e^{2a\cdots}\right.$$

Mathematica [A] time = 5.35, size = 121, normalized size = 0.88

$$\frac{1}{24}\left(\frac{3\text{Li}_4\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^3}+\frac{6x\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b^2}+\frac{6x^2\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{b}-4x^3\log\left(\frac{e^{-2(a+bx)}}{d-1}+1\right)+8x^3\tanh^{-1}(d(-\coth\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (8*x^3*ArcTanh[1 - d - d*Coth[a + b*x]] - 4*x^3*Log[1 + 1/((-1 + d)*E^(2*(a
 + b*x)))] + (6*x^2*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))])/b + (6*x*P
olyLog[3, -(1/((-1 + d)*E^(2*(a + b*x))))])/b^2 + (3*PolyLog[4, -(1/((-1 +
d)*E^(2*(a + b*x))))])/b^3)/24
```

fricas [C] time = 0.53, size = 382, normalized size = 2.79

$$b^4x^4-2\,b^3x^3\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right)-6\,b^2x^2\text{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh(bx+a)+\sinh(bx+a))\right)-6\,b^{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(b^4*x^4 - 2*b^3*x^3*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(
b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b^2*x^2*dilog(1/2*sqrt(-4*d + 4)*(co
sh(b*x + a) + sinh(b*x + a))) - 6*b^2*x^2*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a))) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*s
inh(b*x + a) + sqrt(-4*d + 4)) + 2*a^3*log(2*(d - 1)*cosh(b*x + a) + 2*(d -
 1)*sinh(b*x + a) - sqrt(-4*d + 4)) + 12*b*x*polylog(3, 1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 12*b*x*polylog(3, -1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a))) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*d + 4)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*d
+ 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - 12*polylog(4, 1/2*sqrt(-4*d + 4
)*(cosh(b*x + a) + sinh(b*x + a))) - 12*polylog(4, -1/2*sqrt(-4*d + 4)*(cos
h(b*x + a) + sinh(b*x + a))))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x^2 \operatorname{artanh}\left(d \coth\left(bx + a\right) + d - 1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*coth(b*x + a) + d - 1), x)
```

maple [C] time = 4.87, size = 1742, normalized size = 12.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2
*b*x+2*a)-exp(2*b*x+2*a)+1))^2+1/6*x^3*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1
)+1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+
1))^3-1/6/b^3*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/6*I*x^3*Pi*
csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))^2+1/12*I*x^3
*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1/6*I*x^3*Pi+1/12*b*x^4+1/1
2*I*x^3*Pi*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*
a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))^2+1/12*I*x^3*Pi*csgn(I*exp(b*x+a
))^2*csgn(I*exp(2*b*x+2*a))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))^3-1/3*x^3*
ln(exp(b*x+a))-1/6*x^3*ln(d)+1/6/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/8/b
^3/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))-1/6*I*x^3*Pi*csgn(I*exp(b*x+a))*c
sgn(I*exp(2*b*x+2*a))^2-1/3/b^3/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3+1/4/b/
(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2-1/4/b^3/(d-1)*polylog(2,-(d-1)*e
xp(2*b*x+2*a))*a^2-1/4/b^2/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x+1/2/b^3
*a^3/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^3/(d-1)*ln(1-exp(b*x+a)*(
1-d)^(1/2))-1/8/b^3*d/(d-1)*polylog(4,-(d-1)*exp(2*b*x+2*a))+1/2/b^3*a^2/(d
-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^3*a^2/(d-1)*dilog(1-exp(b*x+a)*(1
-d)^(1/2))-1/6*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^3+1/12*I*x^3*Pi*csgn(I*
d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(ex
p(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*b*x+
2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1/6/
b^3*d*a^3/(d-1)*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)-1/12*I*x^3*Pi*csgn(I*
exp(2*b*x+2*a))*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/2/b^3*d*a^2/(
d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^2/(d-1)*dilog(1-exp(b*x+a)
*(1-d)^(1/2))+1/3/b^3*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^3-1/4/b*d/(d-1)*
```

```
polylog(2,-(d-1)*exp(2*b*x+2*a))*x^2+1/4/b^3*d/(d-1)*polylog(2,-(d-1)*exp(2
*b*x+2*a))*a^2+1/4/b^2*d/(d-1)*polylog(3,-(d-1)*exp(2*b*x+2*a))*x-1/2/b^2/(
d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a^2+1/2/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-
d)^(1/2))*x+1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/2/b^3*d*a^3/
(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^3*d*a^3/(d-1)*ln(1-exp(b*x+a)*(1-d
)^(1/2))+1/12*I*x^3*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/2/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*
x+2*a))*x*a^2-1/2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))*x-1/12*I*x^3
*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))*
csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))+1/12*I*x^3*P
i*csgn(I*d)*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2
*a)-1)*exp(2*b*x+2*a))-1/2/b^2*d*a^2/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x
```

maxima [A] time = 1.10, size = 125, normalized size = 0.91

$$-\frac{1}{3}x^3 \operatorname{artanh}(d\coth(bx+a)+d-1)+\frac{1}{36}\left(\frac{3x^4}{d}-\frac{2\left(4b^3x^3\log\left((d-1)e^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-(d-1)e^{(2\right.\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*arctanh(d*coth(b*x + a) + d - 1) + 1/36*(3*x^4/d - 2*(4*b^3*x^3*lo
g((d - 1)*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-(d - 1)*e^(2*b*x + 2*a))
- 6*b*x*polylog(3, -(d - 1)*e^(2*b*x + 2*a)) + 3*polylog(4, -(d - 1)*e^(2*b
*x + 2*a)))/(b^4*d))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2 \operatorname{atanh}(d+d\coth(a+bx)-1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(d + d*coth(a + b*x) - 1),x)
```

```
[Out] int(-x^2*atanh(d + d*coth(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2 \operatorname{atanh}(d\coth(a+bx)+d-1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -Integral(x**2*atanh(d*coth(a + b*x) + d - 1), x)
```

3.309 $\int x \tanh^{-1}(1 - d - d \coth(a + bx)) \, dx$

Optimal. Leaf size=109

$$\frac{\text{Li}_3\left((1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^3}{6}$$

```
[Out] 1/6*b*x^3-1/2*x^2*arctanh(-1+d+d*coth(b*x+a))-1/4*x^2*ln(1-(1-d)*exp(2*b*x+
2*a))-1/4*x*polylog(2,(1-d)*exp(2*b*x+2*a))/b+1/8*polylog(3,(1-d)*exp(2*b*x
+2*a))/b^2
```

Rubi [A] time = 0.23, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6241, 2184, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,(1-d)e^{2a+2bx}\right)}{8b^2}-\frac{x\text{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2bx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\coth(a+$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^3)/6 + (x^2*ArcTanh[1 - d - d*Coth[a + b*x]])/2 - (x^2*Log[1 - (1 - d)
*E^(2*a + 2*b*x)])/4 - (x*PolyLog[2, (1 - d)*E^(2*a + 2*b*x)])/(4*b) + Poly
Log[3, (1 - d)*E^(2*a + 2*b*x)]/(8*b^2)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6241

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Coth[a + b*x]])/(f*
```

```
(m + 1)), x] + Dist[b/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a
+ 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
 d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-d-d\coth(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{2}b\int \frac{x^2}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) + \frac{1}{2}(b(1-d))\int \frac{e^{2a+}}{1+(-1+}$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2}\right.$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2}\right.$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2}\right.$$
$$= \frac{bx^3}{6} + \frac{1}{2}x^2\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{4}x^2\log\left(1-(1-d)e^{2a+2}\right.$$

Mathematica [A] time = 5.38, size = 94, normalized size = 0.86

$$\frac{2b^2x^2\left(2\tanh^{-1}(d(-\coth(a+bx))-d+1)-\log\left(\frac{e^{-2(a+bx)}}{d-1}+1\right)\right)+2bx\text{Li}_2\left(-\frac{e^{-2(a+bx)}}{d-1}\right)+\text{Li}_3\left(-\frac{e^{-2(a+bx)}}{d-1}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (2*b^2*x^2*(2*ArcTanh[1 - d - d*Coth[a + b*x]] - Log[1 + 1/((-1 + d)*E^(2*(
a + b*x)))]) + 2*b*x*PolyLog[2, -(1/((-1 + d)*E^(2*(a + b*x))))] + PolyLog[
3, -(1/((-1 + d)*E^(2*(a + b*x))))])/(8*b^2)
```

fricas [C] time = 0.63, size = 323, normalized size = 2.96

$$\frac{2\,b^3x^3 - 3\,b^2x^2\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) - 6\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{-4\,d+4}\,(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right) - 6\,bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 3*b^2*x^2*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cos
h(b*x + a) + (d - 2)*sinh(b*x + a))) - 6*b*x*dilog(1/2*sqrt(-4*d + 4)*(cosh
(b*x + a) + sinh(b*x + a))) - 6*b*x*dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a
) + sinh(b*x + a))) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sinh(b*
x + a) + sqrt(-4*d + 4)) - 3*a^2*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) - sqrt(-4*d + 4)) - 3*(b^2*x^2 - a^2)*log(1/2*sqrt(-4*d + 4)*(c
osh(b*x + a) + sinh(b*x + a)) + 1) - 3*(b^2*x^2 - a^2)*log(-1/2*sqrt(-4*d +
```

```
 4)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + 6*polylog(3, 1/2*sqrt(-4*d + 4)*
(cosh(b*x + a) + sinh(b*x + a))) + 6*polylog(3, -1/2*sqrt(-4*d + 4)*(cosh(b
*x + a) + sinh(b*x + a))))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x \operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*coth(b*x + a) + d - 1), x)
```

maple [C] time = 4.38, size = 1659, normalized size = 15.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))
^3-1/4/b^2*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2-1/8/b^2/(d-1)*polylog(3,-
(d-1)*exp(2*b*x+2*a))+1/4/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2-1/2*x^2*ln(e
xp(b*x+a))-1/4*x^2*ln(d)+1/4*x^2*ln(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1)+1/6*
b*x^3+1/8*I*x^2*Pi*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^3-1/4/b*d/(d
-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*x-1/4/b^2*d/(d-1)*polylog(2,-(d-1)*exp
(2*b*x+2*a))*a+1/2/b/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a-1/2/b*a/(d-1)*ln(
1+exp(b*x+a)*(1-d)^(1/2))*x-1/2/b*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x+1/
2/b^2*d*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a^2/(d-1)*ln(1-exp
(b*x+a)*(1-d)^(1/2))+1/2/b^2*d*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(1/2))+1/2/
b^2*d*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))+1/4/b^2*a^2/(d-1)*ln(d*exp(2*
b*x+2*a)-exp(2*b*x+2*a)+1)+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))^3+1/4*I*x^2*
Pi-1/4*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+
1))^2+1/4/b^2/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*a^2+1/4/b/(d-1)*polylog(2,-(
d-1)*exp(2*b*x+2*a))*x+1/4/b^2/(d-1)*polylog(2,-(d-1)*exp(2*b*x+2*a))*a-1/2
/b^2*a^2/(d-1)*ln(1+exp(b*x+a)*(1-d)^(1/2))-1/2/b^2*a^2/(d-1)*ln(1-exp(b*x+
a)*(1-d)^(1/2))-1/4*d/(d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x^2+1/8/b^2*d/(d-1)*
polylog(3,-(d-1)*exp(2*b*x+2*a))-1/2/b^2*a/(d-1)*dilog(1+exp(b*x+a)*(1-d)^(
1/2))-1/2/b^2*a/(d-1)*dilog(1-exp(b*x+a)*(1-d)^(1/2))-1/8*I*x^2*Pi*csgn(I*e
xp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a
))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))^2+1
/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^3+1/8*I*x^2*Pi*csgn(I
*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*
x+2*a)-exp(2*b*x+2*a)+1))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*cs
gn(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))^2-1/4/b^2*d*a^2/(d-1)*ln(d*exp(2*b*
x+2*a)-exp(2*b*x+2*a)+1)+1/8*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I/(ex
p(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))^2-1/4*I*x^2*Pi*csgn(I*
exp(b*x+a))*csgn(I*exp(2*b*x+2*a))^2+1/8*I*x^2*Pi*csgn(I*exp(b*x+a))^2*csgn
(I*exp(2*b*x+2*a))+1/2/b*d*a/(d-1)*ln(1-exp(b*x+a)*(1-d)^(1/2))*x-1/2/b*d/(
d-1)*ln(1+(d-1)*exp(2*b*x+2*a))*x*a+1/2/b*d*a/(d-1)*ln(1+exp(b*x+a)*(1-d)^(
1/2))*x+1/8*I*x^2*Pi*csgn(I*exp(2*b*x+2*a))*csgn(I/(exp(2*b*x+2*a)-1))*csgn
(I*exp(2*b*x+2*a)/(exp(2*b*x+2*a)-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*b
*x+2*a)/(exp(2*b*x+2*a)-1))*csgn(I*d/(exp(2*b*x+2*a)-1)*exp(2*b*x+2*a))-1/8
*I*x^2*Pi*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a
)+1))*csgn(I/(exp(2*b*x+2*a)-1)*(d*exp(2*b*x+2*a)-exp(2*b*x+2*a)+1))
```

maxima [A] time = 1.10, size = 101, normalized size = 0.93

$$\frac{1}{24}\left(\frac{4\,x^3}{d}-\frac{3\left(2\,b^2x^2\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-(d-1)e^{(2\,bx+2\,a)})\right)}{b^3d}\right)bd-\frac{1}{2}\,x^2\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(4*x^3/d - 3*(2*b^2*x^2*log((d - 1)*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-(d - 1)*e^(2*b*x + 2*a)) - polylog(3, -(d - 1)*e^(2*b*x + 2*a)))/(b^3*d))
*b*d - 1/2*x^2*arctanh(d*coth(b*x + a) + d - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x\,\mathrm{atanh}\left(d+d\coth\left(a+b\,x\right)-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(d + d*coth(a + b*x) - 1),x)
```

```
[Out] int(-x*atanh(d + d*coth(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x\,\mathrm{atanh}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -Integral(x*atanh(d*coth(a + b*x) + d - 1), x)
```

3.310 $\int \tanh^{-1}(1-d-d\coth(a+bx))\,dx$

Optimal. Leaf size=76

$$-\frac{\operatorname{Li}_2\left((1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right)+x\tanh^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2-x*arctanh(-1+d+d*coth(b*x+a))-1/2*x*ln(1-(1-d)*exp(2*b*x+2*a))-1/
4*polylog(2,(1-d)*exp(2*b*x+2*a))/b
```

Rubi [A] time = 0.14, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6233, 2184, 2190, 2279, 2391}

$$-\frac{\operatorname{PolyLog}\left(2,(1-d)e^{2a+2bx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right)+x\tanh^{-1}(d(-\coth(a+bx))-d+1)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] (b*x^2)/2 + x*ArcTanh[1 - d - d*Coth[a + b*x]] - (x*Log[1 - (1 - d)*E^(2*a
+ 2*b*x)])/2 - PolyLog[2, (1 - d)*E^(2*a + 2*b*x)]/(4*b)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6233

```
Int[ArcTanh[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Ar
cTanh[c + d*Coth[a + b*x]], x] + Dist[b, Int[x/(c - d - c*E^(2*a + 2*b*x)),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-d-d\coth(a+bx))\,dx = x\tanh^{-1}(1-d-d\coth(a+bx)) + b\int \frac{x}{1+(-1+d)e^{2a+2bx}}\,dx$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\coth(a+bx)) + (b(1-d))\int \frac{e^{2a+2bx}x}{1+(-1+d)e^{2a\cdot}}$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) +$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) +$$
$$= \frac{bx^2}{2} + x\tanh^{-1}(1-d-d\coth(a+bx)) - \frac{1}{2}x\log\left(1-(1-d)e^{2a+2bx}\right) -$$

Mathematica [B] time = 4.40, size = 208, normalized size = 2.74

$$-2\text{Li}_2\left(-\sqrt{1-d}\,e^{a+bx}\right) - 2\text{Li}_2\left(\sqrt{1-d}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(1-\sqrt{1-d}\,e^{a+bx}\right) - 2\log\left(e^{a+bx}\right)\log\left(\sqrt{1-d}\,e\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - d - d*Coth[a + b*x]] + (b^2*x^2 + Log[E^(a + b*x)]^2 - 2*Log[
E^(a + b*x)]*Log[1 - Sqrt[1 - d]*E^(a + b*x)] - 2*Log[E^(a + b*x)]*Log[1 +
Sqrt[1 - d]*E^(a + b*x)] + 2*Log[E^(a + b*x)]*Log[E^(-a - b*x)*(1 + (-1 + d
)*E^(2*(a + b*x)))] - 2*b*x*Log[d*Cosh[a + b*x] + (-2 + d)*Sinh[a + b*x]] -
 2*PolyLog[2, -(Sqrt[1 - d]*E^(a + b*x))] - 2*PolyLog[2, Sqrt[1 - d]*E^(a +
 b*x)])/(4*b)
```

fricas [B] time = 0.54, size = 240, normalized size = 3.16

$$b^2x^2 - bx\log\left(-\frac{d\cosh(bx+a)+d\sinh(bx+a)}{d\cosh(bx+a)+(d-2)\sinh(bx+a)}\right) + a\log\left(2\,(d-1)\cosh\,(bx+a) + 2\,(d-1)\sinh\,(bx+a) + \sqrt{-4\,d+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - b*x*log(-(d*cosh(b*x + a) + d*sinh(b*x + a))/(d*cosh(b*x + a
) + (d - 2)*sinh(b*x + a))) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*sin
h(b*x + a) + sqrt(-4*d + 4)) + a*log(2*(d - 1)*cosh(b*x + a) + 2*(d - 1)*si
nh(b*x + a) - sqrt(-4*d + 4)) - (b*x + a)*log(1/2*sqrt(-4*d + 4)*(cosh(b*x
+ a) + sinh(b*x + a)) + 1) - (b*x + a)*log(-1/2*sqrt(-4*d + 4)*(cosh(b*x +
a) + sinh(b*x + a)) + 1) - dilog(1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b
*x + a))) - dilog(-1/2*sqrt(-4*d + 4)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*coth(b*x + a) + d - 1), x)
```

maple [B] time = 0.45, size = 280, normalized size = 3.68

$$\frac{\operatorname{arctanh}\left(-1+d+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx+a\right)-d\right)}{2b} - \frac{\operatorname{arctanh}\left(-1+d+d\coth\left(bx+a\right)\right)\ln\left(d\coth\left(bx\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] 1/2/b*arctanh(-1+d+d*coth(b*x+a))*ln(d*coth(b*x+a)-d)-1/2/b*arctanh(-1+d+d*
coth(b*x+a))*ln(d*coth(b*x+a)+d)+1/8/b*ln(d*coth(b*x+a)+d)^2-1/4/b*ln(-1/2*
d*coth(b*x+a)-1/2*d+1)*ln(d*coth(b*x+a)+d)+1/4/b*ln(-1/2*d*coth(b*x+a)-1/2*
d+1)*ln(1/2*d*coth(b*x+a)+1/2*d)+1/4/b*dilog(1/2*d*coth(b*x+a)+1/2*d)+1/4/b
*dilog((d*coth(b*x+a)+d-2)/(2*d-2))+1/4/b*ln(d*coth(b*x+a)-d)*ln((d*coth(b*
x+a)+d-2)/(2*d-2))-1/4/b*dilog(1/2*(d*coth(b*x+a)+d)/d)-1/4/b*ln(d*coth(b*x
+a)-d)*ln(1/2*(d*coth(b*x+a)+d)/d)
```

maxima [A] time = 1.12, size = 73, normalized size = 0.96

$$\frac{1}{4}\,bd\left(\frac{2\,x^2}{d}-\frac{2\,bx\log\left((d-1)e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-(d-1)e^{(2\,bx+2\,a)}\right)}{b^2d}\right)-x\,\mathrm{artanh}\left(d\coth\left(bx+a\right)+d-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*(2*x^2/d - (2*b*x*log((d - 1)*e^(2*b*x + 2*a) + 1) + dilog(-(d - 1)
*e^(2*b*x + 2*a)))/(b^2*d)) - x*arctanh(d*coth(b*x + a) + d - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\mathrm{atanh}\left(d+d\coth\left(a+b\,x\right)-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d + d*coth(a + b*x) - 1),x)
```

```
[Out] int(-atanh(d + d*coth(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \mathrm{atanh}\left(d\coth\left(a+bx\right)+d-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*coth(b*x+a)),x)
```

```
[Out] -Integral(atanh(d*coth(a + b*x) + d - 1), x)
```

3.311 $\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$

Optimal. Leaf size=22

$$\mathrm{Int}\left(\frac{\tanh^{-1}(d(-\coth(a+bx))-d+1)}{x},x\right)$$

```
[Out] CannotIntegrate(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 - d - d*Coth[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 - d - d*Coth[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.75, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1-d-d\coth(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 - d - d*Coth[a + b*x]]/x, x]
```

fricas [A] time = 0.39, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="fricas")
[Out] integral(-arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="giac")
[Out] integrate(-arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

maple [A] time = 0.95, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arctanh}\left(-1+d+d\coth\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+d+d*coth(b*x+a))/x,x)
[Out] int(-arctanh(-1+d+d*coth(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{artanh}\left(d\coth\left(bx+a\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+d+d*coth(b*x+a))/x,x, algorithm="maxima")
[Out] -integrate(arctanh(d*coth(b*x + a) + d - 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int -\frac{\operatorname{atanh}\left(d+d\coth\left(a+b\,x\right)-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d + d*coth(a + b*x) - 1)/x,x)
[Out] int(-atanh(d + d*coth(a + b*x) - 1)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\operatorname{atanh}\left(d\coth\left(a+bx\right)+d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+d+d*coth(b*x+a))/x,x)
[Out] -Integral(atanh(d*coth(a + b*x) + d - 1)/x, x)
```

3.312 $\int (e+fx)^3 \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=302

$$-\frac{3f^3\text{Li}_5\left(-ie^{2i(a+bx)}\right)}{16b^4}+\frac{3f^3\text{Li}_5\left(ie^{2i(a+bx)}\right)}{16b^4}+\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+f}{}$$

```
[Out] 1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)))/f+1/4*(f*x+e)^4*arctanh(tan(b*x+a)
)/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*polylo
g(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))/b^
2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*polyl
og(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b*x+a)
))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp
(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.23, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6251, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^4*ArcTanh[Ta
n[a + b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6251

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3\tanh^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^4\tanh^{-1}(\tan(a+bx))}{4f} - \frac{b\int (e+fx)^4\sec(2a+2bx)\,dx}{4f}$$

$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\tan(a+bx))}{4f} + \frac{1}{2}\int (e+fx)$$

$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\mathrm{Li}}{4}$$

$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\mathrm{Li}}{4}$$

$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\mathrm{Li}}{4}$$

$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\mathrm{Li}}{4}$$

$$= \frac{i(e+fx)^4\tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4\tanh^{-1}(\tan(a+bx))}{4f} - \frac{i(e+fx)^3\mathrm{Li}}{4}$$

Mathematica [B] time = 1.32, size = 654, normalized size = 2.17

$$\frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\tanh^{-1}(\tan(a+bx)) + \frac{-8b^4e^3x\log\left(1-ie^{2i(a+bx)}\right)+8b^4e^3x\log\left(1+ie^{2i(a+bx)}\right)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[Tan[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
```

```
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

fricas [C] time = 1.24, size = 1809, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
a)^2 + 1)) - 3*f^3*polylog(5, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(tan(
b*x + a)^2 + 1)) + 3*f^3*polylog(5, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I
)/(tan(b*x + a)^2 + 1)) - 3*f^3*polylog(5, (-I*tan(b*x + a)^2 - 2*tan(b*x +
 a) + I)/(tan(b*x + a)^2 + 1)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12
*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x +
a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x
^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan
(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 - 12*I*b^3
*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-(-(I - 1)*tan(b*x + a)^
2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - (-4*I*b^3*f^3*x^3 -
 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-(-(I - 1)*tan(
b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^4*f^3
*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^
2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(
b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f
+ 4*a^3*b*e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) +
 I - 1)/(tan(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*
e*f^2 - a^4*f^3)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(t
an(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 +
4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(
((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2
*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e
^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log((-(I - 1)*tan(b*x + a)^
2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^4*f^3*x^4 + 4*b^4*
e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f +
 4*a^3*b*e*f^2 - a^4*f^3)*log((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I
 + 1)/(tan(b*x + a)^2 + 1)) - 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*
f^2 - a^4*f^3)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan
(b*x + a)^2 + 1)) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*
f^3)*log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^
2 + 1)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)
*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - (6*I*b*f^3*x + 6*I*b*e*f^2)*
polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) -
(6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) -
 I)/(tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6*I*b*e*f^2)*polylog(4, (-I*tan
(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - (-6*I*b*f^3*x - 6
*I*b*e*f^2)*polylog(4, (-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x +
a)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (I*tan(
b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b^2*f^3*x^2 + 2
*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I
)/(tan(b*x + a)^2 + 1)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polyl
og(3, (-I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + 6*(b
```

```
^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, (-I*tan(b*x + a)^2 - 2*t
an(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)^3 \operatorname{artanh}(\tan(bx+a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*arctanh(tan(b*x + a)), x)
```

maple [C] time = 43.61, size = 7429, normalized size = 24.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctanh(tan(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\left(f^3x^4+4ef^2x^3+6e^2fx^2+4e^3x\right)\log\left(2\cos(2bx+2a)^2+2\sin(2bx+2a)^2+4\sin(2bx+2a)+2\right)-\frac{1}{16}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}(\tan(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x))*(e + f*x)^3,x)
```

```
[Out] int(atanh(tan(a + b*x))*(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e+fx)^3 \operatorname{atanh}(\tan(a+bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**3*atanh(tan(a + b*x)), x)
```

3.313 $\int (e+fx)^2 \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{if^2\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{f(e+fx)\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{Li}_3\left(ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)^2\text{Li}_2\left(-ie\right.}{4b}$$

```
[Out] 1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)))/f+1/3*(f*x+e)^3*arctanh(tan(b*x+a)
)/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*polylo
g(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-
1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*exp(
2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6251, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/3)*(e + f*x)^3*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^3*ArcTanh[Ta
n[a + b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6251

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
```

```
e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2 \tanh^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{b\int (e+fx)^3\sec(2a+2bx)\,dx}{3f}$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} + \frac{1}{2}\int (e+fx)$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2\mathrm{Li}}{4}$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2\mathrm{Li}}{4}$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2\mathrm{Li}}{4}$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\tan(a+bx))}{3f} - \frac{i(e+fx)^2\mathrm{Li}}{4}$$

Mathematica [A] time = 0.71, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tanh^{-1}(\tan(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\log\left(1+ie^{2i(a+bx)}\right)-12b^3efx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[Tan[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

fricas [C] time = 0.61, size = 1279, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(3*I*f^2*polylog(4, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(tan(b*x +
 a)^2 + 1)) + 3*I*f^2*polylog(4, (I*tan(b*x + a)^2 - 2*tan(b*x + a) - I)/(t
an(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 + 2*tan(b*x + a
) + I)/(tan(b*x + a)^2 + 1)) - 3*I*f^2*polylog(4, (-I*tan(b*x + a)^2 - 2*ta
n(b*x + a) + I)/(tan(b*x + a)^2 + 1)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x +
 6*I*b^2*e^2)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan
(b*x + a)^2 + 1) + 1) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*di
log(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)
 + 1) + (-6*I*b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-(-(I - 1)*
tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) + (-6*I*
b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-(-(I - 1)*tan(b*x + a)^2
 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 4*(b^3*f^2*x^3 + 3*b
^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1
)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a*b
^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x +
 a) + I - 1)/(tan(b*x + a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2
)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 +
 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b
*e*f + a^3*f^2)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(
b*x + a)^2 + 1)) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e
^2 - 3*a^2*b*e*f + a^3*f^2)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) +
 I + 1)/(tan(b*x + a)^2 + 1)) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*
x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log((-(I - 1)*tan(b*x + a)^2 - 2*t
an(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f +
 a^3*f^2)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x
+ a)^2 + 1)) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(((I - 1)*tan(b*x
 + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 8*(b^3*f^2*x^3
+ 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1))
+ 6*(b*f^2*x + b*e*f)*polylog(3, (I*tan(b*x + a)^2 + 2*tan(b*x + a) - I)/(t
an(b*x + a)^2 + 1)) - 6*(b*f^2*x + b*e*f)*polylog(3, (I*tan(b*x + a)^2 - 2*
tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + 6*(b*f^2*x + b*e*f)*polylog(3, (-
I*tan(b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - 6*(b*f^2*x +
 b*e*f)*polylog(3, (-I*tan(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2
 + 1)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)^2 \operatorname{artanh}(\tan(bx+a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arctanh(tan(b*x + a)), x)
```

maple [C] time = 31.86, size = 5543, normalized size = 23.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctanh(tan(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}\left(f^2x^3+3efx^2+3e^2x\right)\log\left(2\cos(2bx+2a)^2+2\sin(2bx+2a)^2+4\sin(2bx+2a)+2\right)-\frac{1}{12}\left(f^2x^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{atanh}\left(\tan\left(a+b\,x\right)\right)\,{\left(e+f\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(atanh(tan(a + b*x))*(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e + f x\right)^{2} \operatorname{atanh}{\left(\tan{\left(a + b x \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atanh(tan(a + b*x)), x)
```

3.314 $\int (e+fx)\tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{f\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+
}\right)}{2f}$$

```
[Out] 1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)))/f+1/2*(f*x+e)^2*arctanh(tan(b*x+a)
)/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(2,
I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylo
g(3,I*exp(2*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6251, 4181, 2531, 2282, 6589}

$$\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie
\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/2)*(e + f*x)^2*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^2*ArcTanh[Ta
n[a + b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6251

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Tan[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\tanh^{-1}(\tan(a+bx))\,dx = \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} + \frac{1}{2}\int (e+fx)$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}_2\big(}{4b}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}_2\big(}{4b}$$

$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\tan(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}_2\big(}{4b}$$

Mathematica [A] time = 0.31, size = 263, normalized size = 1.62

$$-be\left(\frac{i\mathrm{Li}_2\left(-ie^{i(2a+2bx)}\right)}{4b^2} - \frac{i\mathrm{Li}_2\left(ie^{i(2a+2bx)}\right)}{4b^2} - \frac{ix\tan^{-1}\left(e^{2ia+2ibx}\right)}{b}\right) + \frac{f\left(4ib^2x^2\tan^{-1}(\cos(2(a+bx))+i\sin(2(a+bx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*ArcTanh[Tan[a + b*x]],x]
```

```
[Out] e*x*ArcTanh[Tan[a + b*x]] + (f*x^2*ArcTanh[Tan[a + b*x]])/2 - b*e*(((-I)*x*
ArcTan[E^((2*I)*a + (2*I)*b*x)])/b + ((I/4)*PolyLog[2, (-I)*E^(I*(2*a + 2*b
*x))])/b^2 - ((I/4)*PolyLog[2, I*E^(I*(2*a + 2*b*x))])/b^2) + (f*((4*I)*b^2
*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I
*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a
+ b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*
x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

fricas [C] time = 0.77, size = 831, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*I*b*f*x + 2*I*b*e)*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a)
 - I + 1)/(tan(b*x + a)^2 + 1) + 1) + (2*I*b*f*x + 2*I*b*e)*dilog(-((I + 1)
*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1) + 1) + (-2*I
*b*f*x - 2*I*b*e)*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I + 1)
/(tan(b*x + a)^2 + 1) + 1) + (-2*I*b*f*x - 2*I*b*e)*dilog(-(-(I - 1)*tan(b*
x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - 2*(b^2*f*x^2
 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a
) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*(2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x
 + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^
2*f)*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^
2 + 1)) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(((I + 1)*tan(b*x
```

```
+ a)^2 - 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*f*x^2 + 2*b
^2*e*x + 2*a*b*e - a^2*f)*log((-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x + a) + I
 + 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*l
og((-(I - 1)*tan(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1))
 + 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I +
 1)/(tan(b*x + a)^2 + 1)) - 2*(2*a*b*e - a^2*f)*log(((I - 1)*tan(b*x + a)^2
 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 4*(b^2*f*x^2 + 2*b^2*e
*x)*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) + f*polylog(3, (I*tan(b*x +
 a)^2 + 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) - f*polylog(3, (I*tan(b*x
 + a)^2 - 2*tan(b*x + a) - I)/(tan(b*x + a)^2 + 1)) + f*polylog(3, (-I*tan(
b*x + a)^2 + 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)) - f*polylog(3, (-I*t
an(b*x + a)^2 - 2*tan(b*x + a) + I)/(tan(b*x + a)^2 + 1)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right) \operatorname{artanh}\left(\tan\left(bx+a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arctanh(tan(b*x + a)), x)
```

maple [C] time = 4.04, size = 2543, normalized size = 15.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctanh(tan(b*x+a)),x)
```

```
[Out] 1/2*(1/2*f*x^2+e*x)*ln(exp(2*I*(b*x+a))+I)+1/4/b^2*f*(I*b*x+I*a)^2*ln(1-I*e
xp(2*I*(b*x+a)))+1/4/b^2*f*(I*b*x+I*a)*polylog(2,I*exp(2*I*(b*x+a)))-1/4*ln
(exp(2*I*(b*x+a))-I)*x^2*f-1/2*ln(exp(2*I*(b*x+a))-I)*x*e+1/8*f*polylog(3,-
I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^2-1/2*I/b*e*d
ilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b*e*dilog(1-exp(I*(b*x+a))*(-1)^(3/
4))-1/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I))*
csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))+1/2*I/b*e*dilog(((-I)^(1/
2)-exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/
(-I)^(1/2))-1/2/b*a*e*ln(-exp(2*I*(b*x+a))+I)+1/2/b*a*e*ln(exp(2*I*(b*x+a))
+I)-1/4/b^2*a^2*f*ln(exp(2*I*(b*x+a))+I)-1/4/b^2*f*(I*b*x+I*a)^2*ln(1+I*exp
(2*I*(b*x+a)))-1/4/b^2*f*(I*b*x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))+1/4/b^2
*a^2*f*ln(-exp(2*I*(b*x+a))+I)+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(ex
p(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/
4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(ex
p(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a
))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a
))+1))*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*c
sgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2-1/4*I*Pi*x*e*csgn(I/
(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1
/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*
f*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3*x^2+1/4*I*Pi*x*e*
csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3-1/4*I*Pi*x*e*csgn(I*(ex
p(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3-1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))^3*x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))+1))^3*x^2+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2
*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2
-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2
*I*(b*x+a))+1))^2*x^2+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3
/4))+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b*e*(I
*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/8*I*Pi*f*csgn((1+I)*
```

```
(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/2*I/b^2*f*a*dilog(1-exp(
I*(b*x+a))*(-1)^(3/4))+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))
/(-I)^(1/2))+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a
))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*x^2+1/4*I*Pi*x*e*c
sgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I))*c
sgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*csgn(I*(exp(2
*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/8
*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((1-I)*(exp(2
*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))+1))*csgn((1+I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*
x+a))+1))+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn
((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*x^2+1/2*I/b^2*f*a*dilog(1
+exp(I*(b*x+a))*(-1)^(3/4))-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*
(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))*x^2
-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*
(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*(I*b*x
+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))-1/4*I*Pi*x*e*csgn((1+I)*(e
xp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^3+1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(
b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*
x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn((1+I)*(exp(2*I*(b*x+a))+
I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*Pi*f*x^2-1/4*I*Pi*x*e-1/2*I/b^2*f*a*dilog(
((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b*e*(I*b*x+I*a)*ln(1+exp(I*(b
*x+a))*(-1)^(3/4))-1/2*I/b*e*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/
4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*
I*(b*x+a))+1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*
(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))+1))
*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x^2+1/8*I*Pi*f*csgn(I*
(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))+1))^2*x
^2-1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)*co
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}\left(\tan\left(a+b\,x\right)\right)\,\left(e+f\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x))*(e + f*x),x)
```

```
[Out] int(atanh(tan(a + b*x))*(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)\operatorname{atanh}\left(\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atanh(tan(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atanh(tan(a + b*x)), x)
```

3.315 $\int \tanh^{-1}(\tan(a+bx))\,dx$

Optimal. Leaf size=79

$$-\frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\tan(a+bx))$$

```
[Out] I*x*arctan(exp(2*I*(b*x+a)))+x*arctanh(tan(b*x+a))-1/4*I*polylog(2,-I*exp(2
*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6247, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\tan(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Tan[a + b*x]],x]
```

```
[Out] I*x*ArcTan[E^((2*I)*(a + b*x))] + x*ArcTanh[Tan[a + b*x]] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6247

```
Int[ArcTanh[Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[Tan[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \tanh^{-1}(\tan(a+bx))\,dx &= x\tanh^{-1}(\tan(a+bx))-b\int x\sec(2a+2bx)\,dx\\
&= ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\tan(a+bx))+\frac{1}{2}\int \log\left(1-ie^{i(2a+2bx)}\right)dx-\frac{1}{2}\int \log\\
&= ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\tan(a+bx))-\frac{i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{i(2a+2bx)}\right)}{4b}+\\
&= ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\tan(a+bx))-\frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 0.94

$$\frac{i\left(-\text{Li}_2\left(-ie^{2i(a+bx)}\right)+\text{Li}_2\left(ie^{2i(a+bx)}\right)+4bx\left(\tan^{-1}\left(e^{2i(a+bx)}\right)-i\tanh^{-1}(\tan(a+bx))\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Tan[a + b*x]],x]
```

```
[Out] ((I/4)*(4*b*x*(ArcTan[E^((2*I)*(a + b*x))] - I*ArcTanh[Tan[a + b*x]]) - Pol
yLog[2, (-I)*E^((2*I)*(a + b*x))] + PolyLog[2, I*E^((2*I)*(a + b*x))]))/b
```

fricas [B] time = 0.69, size = 499, normalized size = 6.32

$$4\,bx\log\left(-\frac{\tan(bx+a)+1}{\tan(bx+a)-1}\right)-2\,(bx+a)\log\left(\frac{(i+1)\,\tan(bx+a)^2+2\,\tan(bx+a)-i+1}{\tan(bx+a)^2+1}\right)+2\,a\log\left(\frac{(i+1)\,\tan(bx+a)^2+2i\,\tan(bx+a)+i-1}{\tan(bx+a)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(tan(b*x + a) + 1)/(tan(b*x + a) - 1)) - 2*(b*x + a)*log(((
I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x + a)^2 + 1)) + 2*a
*log(((I + 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I - 1)/(tan(b*x + a)^2 +
1)) - 2*a*log(((I + 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I - 1)/(tan(b*x
+ a)^2 + 1)) + 2*(b*x + a)*log(((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I
 + 1)/(tan(b*x + a)^2 + 1)) - 2*(b*x + a)*log((-(I - 1)*tan(b*x + a)^2 + 2*
tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*(b*x + a)*log((-(I - 1)*tan
(b*x + a)^2 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I -
 1)*tan(b*x + a)^2 + 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1)) - 2*a*
log(((I - 1)*tan(b*x + a)^2 - 2*I*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1
)) + I*dilog(-((I + 1)*tan(b*x + a)^2 + 2*tan(b*x + a) - I + 1)/(tan(b*x +
a)^2 + 1) + 1) + I*dilog(-((I + 1)*tan(b*x + a)^2 - 2*tan(b*x + a) - I + 1)
/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2 + 2*tan(b*x
+ a) + I + 1)/(tan(b*x + a)^2 + 1) + 1) - I*dilog(-(-(I - 1)*tan(b*x + a)^2
 - 2*tan(b*x + a) + I + 1)/(tan(b*x + a)^2 + 1) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}\left(\tan\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(tan(b*x + a)), x)
```

maple [B] time = 0.50, size = 180, normalized size = 2.28

$$\frac{\arctan\left(\tan\left(bx+a\right)\right)\operatorname{arctanh}\left(\tan\left(bx+a\right)\right)}{b}+\frac{\arctan\left(\tan\left(bx+a\right)\right)\ln\left(1+\frac{i(1+i\tan(bx+a))^2}{1+\tan^2(bx+a)}\right)}{2b}-\frac{\arctan\left(\tan\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arctanh(tan(b*x+a))+1/2/b*arctan(tan(b*x+a))*ln(1+I*
(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))-1/2/b*arctan(tan(b*x+a))*ln(1-I*(1+I*t
an(b*x+a))^2/(1+tan(b*x+a)^2))-1/4*I/b*dilog(1+I*(1+I*tan(b*x+a))^2/(1+tan(
b*x+a)^2))+1/4*I/b*dilog(1-I*(1+I*tan(b*x+a))^2/(1+tan(b*x+a)^2))
```

maxima [B] time = 0.48, size = 182, normalized size = 2.30

$$\frac{4\,(bx+a)\,\mathrm{artanh}\left(\tan\left(bx+a\right)\right)+\left(\arctan\left(\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2},\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2}\right)-\arctan\left(\frac{1}{2}\,\tan\left(bx+a\right)-\frac{1}{2},\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2, 1
/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a)
+ 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(b*
x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) - I*
dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*ta
n(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I + 1/
2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}\left(\tan\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x)),x)
```

```
[Out] int(atanh(tan(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{atanh}\left(\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tan(b*x+a)),x)
```

```
[Out] Integral(atanh(tan(a + b*x)), x)
```

3.316 $\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx} \, dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(\tan(a+bx))}{e+fx},x\right)$$

[Out] CannotIntegrate(arctanh(tan(b*x+a))/(f*x+e),x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx} \, dx$$

Verification is Not applicable to the result.

[In] Int[ArcTanh[Tan[a + b*x]]/(e + f*x),x]

[Out] Defer[Int][ArcTanh[Tan[a + b*x]]/(e + f*x), x]

Rubi steps

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx} \, dx = \int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx} \, dx$$

Mathematica [A] time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(\tan(a+bx))}{e+fx} \, dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcTanh[Tan[a + b*x]]/(e + f*x),x]

[Out] Integrate[ArcTanh[Tan[a + b*x]]/(e + f*x), x]

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(\tan\left(bx+a\right)\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="fricas")

[Out] integral(arctanh(tan(b*x + a))/(f*x + e), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(\tan\left(bx+a\right)\right)}{fx+e} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="giac")

[Out] integrate(arctanh(tan(b*x + a))/(f*x + e), x)

maple [A] time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\tan \left(bx + a\right)\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(tan(b*x+a))/(f*x+e),x)
[Out] int(arctanh(tan(b*x+a))/(f*x+e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\tan \left(bx + a\right)\right)}{fx + e}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(tan(b*x+a))/(f*x+e),x, algorithm="maxima")
[Out] integrate(arctanh(tan(b*x + a))/(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}\left(\tan \left(a + b\,x\right)\right)}{e + f\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(tan(a + b*x))/(e + f*x),x)
[Out] int(atanh(tan(a + b*x))/(e + f*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\tan \left(a + bx\right)\right)}{e + fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(tan(b*x+a))/(f*x+e),x)
[Out] Integral(atanh(tan(a + b*x))/(e + f*x), x)
```

3.317 $\int x^2 \tanh^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=395

$$\frac{i\text{Li}_4\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3}-\frac{i\text{Li}_4\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^3}+\frac{x\text{Li}_3\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{x\text{Li}_3\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(-\frac{(-c-\cdots}{\cdots}\right)}{4\cdots}$$

```
[Out] 1/3*x^3*arctanh(c+d*tan(b*x+a))+1/6*x^3*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(
1-c-I*d))-1/6*x^3*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x^2*po
lylog(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x^2*polylog(2,-(1+
c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/4*x*polylog(3,-(1-c+I*d)*exp(2*I*a
+2*I*b*x)/(1-c-I*d))/b^2-1/4*x*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c
+I*d))/b^2+1/8*I*polylog(4,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b^3-1/8
*I*polylog(4,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b^3
```

Rubi [A] time = 0.50, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6267, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tan[a + b*x]])/3 + (x^3*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/6 - (x^3*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/6 - ((I/4)*x^2*PolyLog[2, -(((1 - c + I*d)*E^
((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x^2*PolyLog[2, -(((1 + c
 - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + (x*PolyLog[3, -(((1 -
 c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/(4*b^2) - (x*PolyLog[3,
 -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/(4*b^2) + ((I/8
)*PolyLog[4, -(((1 - c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b^3
 - ((I/8)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d
))])/b^3
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6267

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{3}(b(i(1-c)-d))\int \frac{e^{2ia+2ibx}x^3}{1-c-id+(1-c+i$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{6}x^3\log$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{6}x^3\log$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{6}x^3\log$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{6}x^3\log$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\tan(a+bx)) + \frac{1}{6}x^3\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right) - \frac{1}{6}x^3\log$$

Mathematica [A] time = 0.93, size = 346, normalized size = 0.88

$$\frac{1}{3}x^3\tanh^{-1}(d\tan(a+bx)+c)+\frac{4b^3x^3\log\left(1+\frac{(c-id-1)e^{2i(a+bx)}}{c+id-1}\right)-4b^3x^3\log\left(1+\frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right)-6ib^2x^2\text{Li}_2\left(\frac{(-c+id+1)}{c+id}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Tan[a + b*x]])/3 + (4*b^3*x^3*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 4*b^3*x^3*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((1 - c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] + (6*I)*b^2*x^2*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*(a + b*x)))/(1 + c + I*d))] + 6*b*x*PolyLog[3, ((1 - c + I*d)*
E^((2*I)*(a + b*x)))/(-1 + c + I*d)] - 6*b*x*PolyLog[3, -(((1 + c - I*d)*E^
((2*I)*(a + b*x)))/(1 + c + I*d))] + (3*I)*PolyLog[4, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] - (3*I)*PolyLog[4, -(((1 + c - I*d)*E^((2*I)
*(a + b*x)))/(1 + c + I*d))])/(24*b^3)
```

fricas [C] time = 0.75, size = 2161, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) + 6
*I*b^2*x^2*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c
+ 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c
 + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - 6
*I*b^2*x^2*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c
 + 1)*d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4
*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) +
 6*I*b^2*x^2*dilog((2*(I*(c - 1)*d - d^2)*tan(b*x + a)^2 - 2*c^2 - 2*I*(c -
 1)*d - (-2*I*c^2 + 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 4*c
 - 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 6
*I*b^2*x^2*dilog((2*(-I*(c - 1)*d - d^2)*tan(b*x + a)^2 - 2*c^2 + 2*I*(c -
1)*d - (2*I*c^2 + 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 4*c -
 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) + 4*a
^3*log(((I*(c + 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I
*d^2 + 2*I*c + I)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) + 4*a^3*log
(((I*(c + 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 +
 2*I*c + I)*tan(b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(
c - 1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c
 + I)*tan(b*x + a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) - 4*a^3*log(((I*(c - 1)
*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*
tan(b*x + a) - 2*c + 1)/(tan(b*x + a)^2 + 1)) - 6*b*x*polylog(3, ((c^2 + 2*
I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 + (
2*I*c^2 - 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1)/((c^
2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 6*b*x*polylog(3
, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)
*d + d^2 + (-2*I*c^2 - 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) -
2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 6*
b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2
- 2*I*(c - 1)*d + d^2 + (2*I*c^2 - 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan
(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*
c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x +
 a)^2 - c^2 + 2*I*(c - 1)*d + d^2 + (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2 + 4*I
*c - 2*I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c
^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(((2*I*(c + 1)*d + 2*d^2)*tan(b
*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*
c + 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^
2 + d^2 + 2*c + 1)) - 4*(b^3*x^3 + a^3)*log(((-2*I*(c + 1)*d + 2*d^2)*tan(b
*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I
*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c
^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(-(2*(I*(c - 1)*d - d^2)*tan(b*
x + a)^2 - 2*c^2 - 2*I*(c - 1)*d - (-2*I*c^2 + 4*(c - 1)*d + 2*I*d^2 + 4*I*
c - 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^
2 + d^2 - 2*c + 1)) + 4*(b^3*x^3 + a^3)*log(-(2*(-I*(c - 1)*d - d^2)*tan(b*
```

```
x + a)^2 - 2*c^2 + 2*I*(c - 1)*d - (2*I*c^2 + 4*(c - 1)*d - 2*I*d^2 - 4*I*c
 + 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2
 + d^2 - 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*
tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 + (2*I*c^2 - 4*(c + 1)*d - 2*I*d
^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b*x +
a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d - d^2 +
 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 + (-2*I*c^2 - 4*(c + 1
)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)
*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 3*I*polylog(4, ((c^2 + 2*I*(c - 1
)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 + (2*I*c^2
- 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2
- 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 3*I*polylog(4, ((c^2 -
2*I*(c - 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 +
 (-2*I*c^2 - 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 2*c - 1)/(
(c^2 + d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}(d\tan(bx+a)+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tan(b*x + a) + c), x)
```

maple [C] time = 44.58, size = 6967, normalized size = 17.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}x^3\log\left(\left(c^2+d^2+2c+1\right)\cos(2bx+2a)^2+4(c+1)d\sin(2bx+2a)+\left(c^2+d^2+2c+1\right)\sin(2bx+2a)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(-1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*
a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^
2 + (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) - (c
^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
 + 2*a) + (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^
4 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 +
(c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
 2*(3*c^2 - 1)*d^2 - 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4
*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^
```

```
4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (
c^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*si
n(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \operatorname{atanh}(c + d \tan(a + b x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(c + d*tan(a + b*x)),x)
```

```
[Out] int(x^2*atanh(c + d*tan(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}(c + d \tan(a + bx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*tan(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(c + d*tan(a + b*x)), x)
```

3.318 $\int x \tanh^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=295

$$\frac{\text{Li}_3\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2}-\frac{\text{Li}_3\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2}-\frac{ix\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{4}x^2\log\left(1+\frac{(-\right.$$

```
[Out] 1/2*x^2*arctanh(c+d*tan(b*x+a))+1/4*x^2*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(
1-c-I*d))-1/4*x^2*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*x*poly
log(2,-(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*x*polylog(2,-(1+c-I*
d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b+1/8*polylog(3,-(1-c+I*d)*exp(2*I*a+2*I*b
*x)/(1-c-I*d))/b^2-1/8*polylog(3,-(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))/b
^2
```

Rubi [A] time = 0.40, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6267, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,-\right.}{4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tan[a + b*x]])/2 + (x^2*Log[1 + ((1 - c + I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c - I*d)])/4 - (x^2*Log[1 + ((1 + c - I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c + I*d)])/4 - ((I/4)*x*PolyLog[2, -(((1 - c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c - I*d))])/b + ((I/4)*x*PolyLog[2, -(((1 + c - I
*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))])/b + PolyLog[3, -(((1 - c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 - c - I*d))]/(8*b^2) - PolyLog[3, -(((1 + c
- I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c + I*d))]/(8*b^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6267

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 + c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 - c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(c+d\tan(a+bx))\,dx=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{2}(b(i(1-c)-d))\int\frac{e^{2ia+2ibx}x^2}{1-c-id+(1-c+$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{4}x^2$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{4}x^2$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{4}x^2$$

$$=\frac{1}{2}x^2\tanh^{-1}(c+d\tan(a+bx))+\frac{1}{4}x^2\log\left(1+\frac{(1-c+id)e^{2ia+2ibx}}{1-c-id}\right)-\frac{1}{4}x^2$$

Mathematica [A] time = 0.60, size = 257, normalized size = 0.87

$$\frac{1}{2}x^2\tanh^{-1}(d\tan(a+bx)+c)+\frac{2b^2x^2\log\left(1+\frac{(c-id-1)e^{2i(a+bx)}}{c+id-1}\right)-2b^2x^2\log\left(1+\frac{(c-id+1)e^{2i(a+bx)}}{c+id+1}\right)-2ibx\mathrm{Li}_2\left(\frac{(-c+id+1}{c+i}\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Tan[a + b*x]])/2 + (2*b^2*x^2*Log[1 + ((-1 + c - I*d)*E^
((2*I)*(a + b*x)))/(-1 + c + I*d)] - 2*b^2*x^2*Log[1 + ((1 + c - I*d)*E^((2
*I)*(a + b*x)))/(1 + c + I*d)] - (2*I)*b*x*PolyLog[2, ((1 - c + I*d)*E^((2*
I)*(a + b*x)))/(-1 + c + I*d)] + (2*I)*b*x*PolyLog[2, -(((1 + c - I*d)*E^((
2*I)*(a + b*x)))/(1 + c + I*d))] + PolyLog[3, ((1 - c + I*d)*E^((2*I)*(a +
b*x)))/(-1 + c + I*d)] - PolyLog[3, -(((1 + c - I*d)*E^((2*I)*(a + b*x)))/(
1 + c + I*d))])/(8*b^2)
```

fricas [C] time = 1.06, size = 1689, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) + 2
*I*b*x*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)
*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2
)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) - 2*I*b
*x*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d
+ (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/
((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + 2*I*b*x
*dilog((2*(I*(c - 1)*d - d^2)*tan(b*x + a)^2 - 2*c^2 - 2*I*(c - 1)*d - (-2*
I*c^2 + 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 4*c - 2)/((c^2
+ d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 2*I*b*x*dilog
((2*(-I*(c - 1)*d - d^2)*tan(b*x + a)^2 - 2*c^2 + 2*I*(c - 1)*d - (2*I*c^2
+ 4*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2
- 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(((I*(c +
1)*d + d^2)*tan(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I
)*tan(b*x + a) - 2*c - 1)/(tan(b*x + a)^2 + 1)) - 2*a^2*log(((I*(c + 1)*d -
 d^2)*tan(b*x + a)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(
b*x + a) + 2*c + 1)/(tan(b*x + a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d + d^2)*
tan(b*x + a)^2 - c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x +
a) + 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a^2*log(((I*(c - 1)*d - d^2)*tan(b*
x + a)^2 + c^2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2
*c + 1)/(tan(b*x + a)^2 + 1)) - 2*(b^2*x^2 - a^2)*log(((2*I*(c + 1)*d + 2*d
^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d
^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x +
a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b^2*x^2 - a^2)*log(((-2*I*(c + 1)*d + 2*d
^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I*c^2 + 4*(c + 1)*d + 2*I*
d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x +
 a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(-(2*(I*(c - 1)*d - d^
2)*tan(b*x + a)^2 - 2*c^2 - 2*I*(c - 1)*d - (-2*I*c^2 + 4*(c - 1)*d + 2*I*d
^2 + 4*I*c - 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x +
a)^2 + c^2 + d^2 - 2*c + 1)) + 2*(b^2*x^2 - a^2)*log(-(2*(-I*(c - 1)*d - d^
2)*tan(b*x + a)^2 - 2*c^2 + 2*I*(c - 1)*d - (2*I*c^2 + 4*(c - 1)*d - 2*I*d^
2 - 4*I*c + 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2 - 2*c + 1)*tan(b*x + a
)^2 + c^2 + d^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c
+ 1)*tan(b*x + a)^2 - c^2 - 2*I*(c + 1)*d + d^2 + (2*I*c^2 - 4*(c + 1)*d -
2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1)*tan(b
*x + a)^2 + c^2 + d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2
+ 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c + 1)*d + d^2 + (-2*I*c^2 - 4*(c +
1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) - 2*c - 1)/((c^2 + d^2 + 2*c + 1
)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 + 2*I*(c - 1)*d
 - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 - 2*I*(c - 1)*d + d^2 + (2*I*c^2 - 4
*(c - 1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 2*c - 1)/((c^2 + d^2 - 2
*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^2 - 2*I*(c
- 1)*d - d^2 - 2*c + 1)*tan(b*x + a)^2 - c^2 + 2*I*(c - 1)*d + d^2 + (-2*I*
c^2 - 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 2*c - 1)/((c^2 +
d^2 - 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}\left(d \tan\left(bx+a\right)+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tan(b*x + a) + c), x)
```

maple [C] time = 4.95, size = 6593, normalized size = 22.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*tan(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\,bd\int-\frac{}{c^4+d^4+2\left(c^2+1\right)d^2+\left(c^4+d^4+2\left(c^2+1\right)d^2-2\,c^2+1\right)\cos\left(4\,bx+4\,a\right)^2+4\left(c^4+d^4+2\left(c^2-1\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate(-(2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin
(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 + (c^2 - d^2 - 1)*
x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) - (c^2 - d^2 - 1)*x^2*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) + (c^2 - d^2
 - 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2
+ (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d
^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2
+ 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
 2*c^2 + 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*
d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2
*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 - c)*d - 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*
x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(
c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2
*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*
c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 + 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
 2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4
*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^
2 + d^2 + 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atanh}\left(c+d\tan\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(c + d*tan(a + b*x)),x)
```

```
[Out] int(x*atanh(c + d*tan(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atanh}\left(c+d\tan\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*tan(b*x+a)),x)
```

```
[Out] Integral(x*atanh(c + d*tan(a + b*x)), x)
```

3.319 $\int \tanh^{-1}(c + d\tan(a + bx))\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{Li}_2\left(-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{i\text{Li}_2\left(-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{2}x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{2}x\log\left(1+\frac{(c-id+1)e^{2ia}}{c+id+1}\right.$$

```
[Out] x*arctanh(c+d*tan(b*x+a))+1/2*x*ln(1+(1-c+I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d)
)-1/2*x*ln(1+(1+c-I*d)*exp(2*I*a+2*I*b*x)/(1+c+I*d))-1/4*I*polylog(2,-(1-c+
I*d)*exp(2*I*a+2*I*b*x)/(1-c-I*d))/b+1/4*I*polylog(2,-(1+c-I*d)*exp(2*I*a+2
*I*b*x)/(1+c+I*d))/b
```

Rubi [A] time = 0.24, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6259, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,-\frac{(c-id+1)e^{2ia+2ibx}}{c+id+1}\right)}{4b}+\frac{1}{2}x\log\left(1+\frac{(-c+id+1)e^{2ia+2ibx}}{-c-id+1}\right)-\frac{1}{2}x\log\left(1+\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tan[a + b*x]] + (x*Log[1 + ((1 - c + I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c - I*d)])/2 - (x*Log[1 + ((1 + c - I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c + I*d)])/2 - ((I/4)*PolyLog[2, -(((1 - c + I*d)*E^((2*I)*a + (2*
I)*b*x))/(1 - c - I*d))])/b + ((I/4)*PolyLog[2, -(((1 + c - I*d)*E^((2*I)*a
 + (2*I)*b*x))/(1 + c + I*d))])/b
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6259

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + (-Dist[I*b*(1 + c - I*d), Int[(x*E^(2*I*a +
2*I*b*x))/(1 + c + I*d + (1 + c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[
I*b*(1 - c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 - c - I*d + (1 - c + I*d)
*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2
, 1]
```

Rubi steps

$$\int \tanh^{-1}(c + d\tan(a + bx))\,dx = x\tanh^{-1}(c + d\tan(a + bx)) + (b(i(1 - c) - d))\int \frac{e^{2ia+2ibx}x}{1 - c - id + (1 - c + id)e^{2i}}$$

$$= x\tanh^{-1}(c + d\tan(a + bx)) + \frac{1}{2}x\log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{2}x\log\Bigg(1$$

$$= x\tanh^{-1}(c + d\tan(a + bx)) + \frac{1}{2}x\log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{2}x\log\Bigg(1$$

$$= x\tanh^{-1}(c + d\tan(a + bx)) + \frac{1}{2}x\log\left(1 + \frac{(1 - c + id)e^{2ia+2ibx}}{1 - c - id}\right) - \frac{1}{2}x\log\Bigg(1$$

Mathematica [B] time = 32.98, size = 4654, normalized size = 23.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[c + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Tan[a + b*x]] + (d*(-(a*Log[-(Sec[(a + b*x)/2]^2*((-1 + c)*
Cos[a + b*x] + d*Sin[a + b*x]))]) + a*Log[Sec[(a + b*x)/2]^2*(Cos[a + b*x]
+ c*Cos[a + b*x] + d*Sin[a + b*x])] + (a + b*x)*Log[(-d + Sqrt[1 - 2*c + c^
2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(1 + I*Tan[(a + b*
x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*c + c^2 + d^2])]*Log[(-d + Sqrt[1 - 2*
c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] - I*Log[-(((-1 + c)*(I + Tan[(
a + b*x)/2]))/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Log[(-d + Sqrt[1
- 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]] + (a + b*x)*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] + I*Log[((-1 + c)*(-I + T
an[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - I*Log[((-1 + c)*(I + Ta
n[(a + b*x)/2]))/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[(d + Sqrt[
1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - (a + b*x)*Log[-((d + Sq
rt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - I*Log[((1 + c)*(-I
+ Tan[(a + b*x)/2]))/(-I - I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d +
 Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] + I*Log[((1 + c)*(
I + Tan[(a + b*x)/2]))/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((d
+ Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - (a + b*x)*Log[(
-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + I*Log
[((1 + c)*(1 - I*Tan[(a + b*x)/2]))/(1 + c - I*d + I*Sqrt[1 + 2*c + c^2 + d
^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 +
c)] - I*Log[((1 + c)*(1 + I*Tan[(a + b*x)/2]))/(1 + c + I*d - I*Sqrt[1 + 2*
c + c^2 + d^2])]*Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x
)/2])/(1 + c)] + I*PolyLog[2, (d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan
[(a + b*x)/2])/(I - I*c + d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d
 + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c + d + S
qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] +
 (-1 + c)*Tan[(a + b*x)/2])/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] + I*
PolyLog[2, (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2])/(-I
 + I*c - d + Sqrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (d + Sqrt[1 + 2*c +
 c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(-I - I*c + d + Sqrt[1 + 2*c + c^2
+ d^2])] + I*PolyLog[2, (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b
*x)/2])/(I + I*c + d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (-d + Sqr
t[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(-I - I*c - d + Sqrt[1 +
 2*c + c^2 + d^2])] - I*PolyLog[2, (-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c
)*Tan[(a + b*x)/2])/(I + I*c - d + Sqrt[1 + 2*c + c^2 + d^2])])*((-2*a)/(b*
```

```
(-1 + c^2 + d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a +
b*x)] + 2*c*d*Sin[2*(a + b*x)])) + (2*(a + b*x))/(b*(-1 + c^2 + d^2 - Cos[2
*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] + 2*c*d*Sin[2*(a
+ b*x)]))))/(Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/
2]] + Log[(d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]] - Log
[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]] - Log[(-d +
 Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)] + (Log[(-d
+ Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(1 + c)]*Sec[(a + b
*x)/2]^2)/(2*(1 - I*Tan[(a + b*x)/2])) - (Log[(-d + Sqrt[1 - 2*c + c^2 + d^
2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x
)/2])) + (Log[(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(
1 + c)]*Sec[(a + b*x)/2]^2)/(2*(1 + I*Tan[(a + b*x)/2])) + ((I/2)*Log[(d +
Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/
(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 +
 c)) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/
2)*Log[(-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]]*Sec[(a
 + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((I/2)*Log[(d + Sqrt[1 - 2*c + c^2 +
 d^2])/(1 - c) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2
]) + ((I/2)*Log[-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/
2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/2
]^2)/(2*((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2])) + (
(I/2)*Log[((-1 + c)*(1 + I*Tan[(a + b*x)/2]))/(-1 + c + I*d - I*Sqrt[1 - 2*
c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[1 - 2*c + c^2 + d^2])/(-1
+ c) + Tan[(a + b*x)/2]) - ((I/2)*Log[-(((-1 + c)*(I + Tan[(a + b*x)/2]))/(
I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2]))]*Sec[(a + b*x)/2]^2)/((-d + Sqrt[
1 - 2*c + c^2 + d^2])/(-1 + c) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*
x)/2]^2)/(2*((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2])) +
 ((I/2)*Log[((-1 + c)*(-I + Tan[(a + b*x)/2]))/(I - I*c + d + Sqrt[1 - 2*c
+ c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((d + Sqrt[1 - 2*c + c^2 + d^2])/(1 - c)
 + Tan[(a + b*x)/2]) - ((I/2)*Log[((-1 + c)*(I + Tan[(a + b*x)/2]))/(-I + I
*c + d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/((d + Sqrt[1 - 2*c
 + c^2 + d^2])/(1 - c) + Tan[(a + b*x)/2]) - ((a + b*x)*Sec[(a + b*x)/2]^2)
/(2*(-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2])) - ((I/
2)*Log[((1 + c)*(-I + Tan[(a + b*x)/2]))/(-I - I*c + d + Sqrt[1 + 2*c + c^2
 + d^2])]*Sec[(a + b*x)/2]^2)/(-((d + Sqrt[1 + 2*c + c^2 + d^2])/(1 + c)) +
 Tan[(a + b*x)/2]) + ((I/2)*Log[((1 + c)*(I + Tan[(a + b*x)/2]))/(I + I*c +
 d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((d + Sqrt[1 + 2*c +
 c^2 + d^2])/(1 + c)) + Tan[(a + b*x)/2]) + ((I/2)*(-1 + c)*Log[1 - (d + Sq
rt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(I - I*c + d + Sqrt[1
- 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 - 2*c + c^2 + d^2] - (
-1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(-1 + c)*Log[1 - (d + Sqrt[1 - 2*c + c^2
 + d^2] - (-1 + c)*Tan[(a + b*x)/2])/(-I + I*c + d + Sqrt[1 - 2*c + c^2 + d
^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 - 2*c + c^2 + d^2] - (-1 + c)*Tan[(a
+ b*x)/2]) + ((I/2)*(-1 + c)*Log[1 - (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1
+ c)*Tan[(a + b*x)/2])/(I - I*c - d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a +
b*x)/2]^2)/(-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2]) - (
(I/2)*(-1 + c)*Log[1 - (-d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a +
b*x)/2])/(-I + I*c - d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-
d + Sqrt[1 - 2*c + c^2 + d^2] + (-1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(1 + c)
*Log[1 - (d + Sqrt[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(-I - I
*c + d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 + 2*c
+ c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2]) + ((I/2)*(1 + c)*Log[1 - (d + Sqrt
[1 + 2*c + c^2 + d^2] - (1 + c)*Tan[(a + b*x)/2])/(I + I*c + d + Sqrt[1 + 2
*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(d + Sqrt[1 + 2*c + c^2 + d^2] - (1 +
 c)*Tan[(a + b*x)/2]) - ((1 + c)*(a + b*x)*Sec[(a + b*x)/2]^2)/(2*(-d + Sqr
t[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])) + ((I/2)*(1 + c)*Log[((
1 + c)*(1 - I*Tan[(a + b*x)/2]))/(1 + c - I*d + I*Sqrt[1 + 2*c + c^2 + d^2]
)]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b
*x)/2]) - ((I/2)*(1 + c)*Log[((1 + c)*(1 + I*Tan[(a + b*x)/2]))/(1 + c + I*
```

```
d - I*Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c +
c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2]) - ((I/2)*(1 + c)*Log[1 - (-d + Sqrt[
1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x)/2])/(-I - I*c - d + Sqrt[1 + 2
*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1
+ c)*Tan[(a + b*x)/2]) + ((I/2)*(1 + c)*Log[1 - (-d + Sqrt[1 + 2*c + c^2 +
d^2] + (1 + c)*Tan[(a + b*x)/2])/(I + I*c - d + Sqrt[1 + 2*c + c^2 + d^2])]
*Sec[(a + b*x)/2]^2)/(-d + Sqrt[1 + 2*c + c^2 + d^2] + (1 + c)*Tan[(a + b*x
)/2]) + (a*Cos[(a + b*x)/2]^2*(-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] - (-1 +
 c)*Sin[a + b*x])) - Sec[(a + b*x)/2]^2*((-1 + c)*Cos[a + b*x] + d*Sin[a +
b*x])*Tan[(a + b*x)/2]))/((-1 + c)*Cos[a + b*x] + d*Sin[a + b*x]) + (a*Cos[
(a + b*x)/2]^2*(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] - Sin[a + b*x] - c*Sin[a
 + b*x]) + Sec[(a + b*x)/2]^2*(Cos[a + b*x] + c*Cos[a + b*x] + d*Sin[a + b*
x])*Tan[(a + b*x)/2]))/(Cos[a + b*x] + c*Cos[a + b*x] + d*Sin[a + b*x]))
```

fricas [B] time = 0.65, size = 1189, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(d*tan(b*x + a) + c + 1)/(d*tan(b*x + a) + c - 1)) - 2*(b*x
 + a)*log(((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(c + 1)*d +
 (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4*c + 2)/((
c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) - 2*(b*x + a)*l
og(((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d + (-2*I
*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/((c^2 +
 d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log(-(
2*(I*(c - 1)*d - d^2)*tan(b*x + a)^2 - 2*c^2 - 2*I*(c - 1)*d - (-2*I*c^2 +
4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2 -
2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*(b*x + a)*log(-(2*(-I*(
c - 1)*d - d^2)*tan(b*x + a)^2 - 2*c^2 + 2*I*(c - 1)*d - (2*I*c^2 + 4*(c -
1)*d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2 - 2*c + 1
)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1)) + 2*a*log(((I*(c + 1)*d + d^2)*tan
(b*x + a)^2 - c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a)
- 2*c - 1)/(tan(b*x + a)^2 + 1)) + 2*a*log(((I*(c + 1)*d - d^2)*tan(b*x + a
)^2 + c^2 + I*(c + 1)*d + (I*c^2 + I*d^2 + 2*I*c + I)*tan(b*x + a) + 2*c +
1)/(tan(b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d + d^2)*tan(b*x + a)^2 - c^
2 + I*(c - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) + 2*c - 1)/(tan(
b*x + a)^2 + 1)) - 2*a*log(((I*(c - 1)*d - d^2)*tan(b*x + a)^2 + c^2 + I*(c
 - 1)*d + (I*c^2 + I*d^2 - 2*I*c + I)*tan(b*x + a) - 2*c + 1)/(tan(b*x + a)
^2 + 1)) + I*dilog(-((2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 - 2*I*(
c + 1)*d + (2*I*c^2 + 4*(c + 1)*d - 2*I*d^2 + 4*I*c + 2*I)*tan(b*x + a) + 4
*c + 2)/((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) -
 I*dilog(-((-2*I*(c + 1)*d + 2*d^2)*tan(b*x + a)^2 + 2*c^2 + 2*I*(c + 1)*d
+ (-2*I*c^2 + 4*(c + 1)*d + 2*I*d^2 - 4*I*c - 2*I)*tan(b*x + a) + 4*c + 2)/
((c^2 + d^2 + 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*c + 1) + 1) + I*dilog
((2*(I*(c - 1)*d - d^2)*tan(b*x + a)^2 - 2*c^2 - 2*I*(c - 1)*d - (-2*I*c^2
+ 4*(c - 1)*d + 2*I*d^2 + 4*I*c - 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2
- 2*c + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1) - I*dilog((2*(-I*(c -
 1)*d - d^2)*tan(b*x + a)^2 - 2*c^2 + 2*I*(c - 1)*d - (2*I*c^2 + 4*(c - 1)*
d - 2*I*d^2 - 4*I*c + 2*I)*tan(b*x + a) + 4*c - 2)/((c^2 + d^2 - 2*c + 1)*t
an(b*x + a)^2 + c^2 + d^2 - 2*c + 1) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}\left(d \tan\left(bx + a\right) + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) + c), x)
```

maple [B] time = 0.26, size = 612, normalized size = 3.15

$$\frac{\arctan(\tan(bx+a))\,\mathrm{arctanh}(c+d\tan(bx+a))}{b}-\frac{\arctan\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)\ln\left(d\left(\frac{c+d\tan(bx+a)}{d}-\frac{c}{d}\right)+c+1\right)}{2b}+\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tan(b*x+a)),x)
```

```
[Out] 1/b*arctan(tan(b*x+a))*arctanh(c+d*tan(b*x+a))-1/2/b*arctan((c+d*tan(b*x+a)
)/d-c/d)*ln(d*((c+d*tan(b*x+a))/d-c/d)+c+1)+1/2/b*arctan((c+d*tan(b*x+a))/d
-c/d)*ln(d*((c+d*tan(b*x+a))/d-c/d)+c-1)-1/4*I/b*ln(d*((c+d*tan(b*x+a))/d-c
/d)+c+1)*ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))+1/4*I/b*ln(d*((c+d*
tan(b*x+a))/d-c/d)+c+1)*ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1))-1/4*
I/b*dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(1+c+I*d))+1/4*I/b*dilog((I*d+d*
((c+d*tan(b*x+a))/d-c/d))/(I*d-c-1))+1/4*I/b*ln(d*((c+d*tan(b*x+a))/d-c/d)+
c-1)*ln((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-1/4*I/b*ln(d*((c+d*tan(
b*x+a))/d-c/d)+c-1)*ln((I*d+d*((c+d*tan(b*x+a))/d-c/d))/(1-c+I*d))+1/4*I/b*
dilog((I*d-d*((c+d*tan(b*x+a))/d-c/d))/(I*d+c-1))-1/4*I/b*dilog((I*d+d*((c+
d*tan(b*x+a))/d-c/d))/(1-c+I*d))
```

maxima [B] time = 0.52, size = 372, normalized size = 1.92

$$4\,(bx+a)\,\mathrm{artanh}\,(d\tan(bx+a)+c)+\left(\arctan\left(\frac{d^2\tan(bx+a)+(c+1)d}{c^2+d^2+2c+1},\frac{(c+1)d\tan(bx+a)+c^2+2c+1}{c^2+d^2+2c+1}\right)-\arctan\left(\frac{d^2\tan(bx+a)+(c}{c^2+d^2-2c+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(d*tan(b*x + a) + c) + (arctan2((d^2*tan(b*x + a) +
 (c + 1)*d)/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + c^2 + 2*c + 1)
/(c^2 + d^2 + 2*c + 1)) - arctan2((d^2*tan(b*x + a) + (c - 1)*d)/(c^2 + d^2
 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((d^2*tan(b*x + a)^2 + 2*(c + 1)*
d*tan(b*x + a) + c^2 + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*log((d^2
*tan(b*x + a)^2 + 2*(c - 1)*d*tan(b*x + a) + c^2 - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) - I*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + I)) + I*dilog(-(I*d*ta
n(b*x + a) - d)/(I*c + d - I)) - I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d +
 I)) + I*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}\left(c+d\tan\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*tan(a + b*x)),x)
```

```
[Out] int(atanh(c + d*tan(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{atanh}\,(c+d\tan(a+bx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tan(b*x+a)),x)
[Out] Integral(atanh(c + d*tan(a + b*x)), x)
```

3.320 $\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=18

$$\text{Int}\left(\frac{\tanh^{-1}(d\tan(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(c+d*tan(b*x+a))/x,x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[c + d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[c + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 4.80, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(c+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[c + d*Tan[a + b*x]]/x, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{artanh}\left(d\tan\left(bx+a\right)+c\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctanh(d*tan(b*x + a) + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{artanh}\left(d\tan\left(bx+a\right)+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctanh(d*tan(b*x + a) + c)/x, x)
```

maple [A] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(c + d \tan\left(bx + a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctanh(c+d*tan(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(d \tan\left(bx + a\right) + c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) + c)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}\left(c + d \tan\left(a + b\,x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*tan(a + b*x))/x,x)
```

```
[Out] int(atanh(c + d*tan(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(c + d \tan\left(a + bx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(atanh(c + d*tan(a + b*x))/x, x)
```

3.321 $\int x^2 \tanh^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=170

$$-\frac{i\mathrm{Li}_4\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^3}-\frac{x\mathrm{Li}_3\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b^2}+\frac{ix^2\mathrm{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left(1+(1-id)e^{2ia+2i\right.$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arctanh(1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*pol
ylog(3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1-I*d)*exp(2*I*a+
2*I*b*x))/b^3
```

Rubi [A] time = 0.29, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6263, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\mathrm{PolyLog}\left(3,-(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\mathrm{PolyLog}\left(4,-(1-id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\mathrm{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 - I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6263

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(1-id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{3}(b(i+d))\int \frac{e^{2\cdots}}{1+(1\cdots}$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\Big(1+(1-id)\cdots$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\Big(1+(1-id)\cdots$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\Big(1+(1-id)\cdots$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\Big(1+(1-id)\cdots$$

$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{6}x^3\log\Big(1+(1-id)\cdots$$

Mathematica [A] time = 0.44, size = 155, normalized size = 0.91

$$\frac{1}{3}x^3\tanh^{-1}(d\tan(a+bx)-id+1)-\frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{d+i}\right)+6ib^2x^2\text{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{d+i}\right)+6bx\text{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{d+i}\right)-3i\text{Li}\cdots}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 - I*d + d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*Pol
yLog[4, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

fricas [C] time = 0.63, size = 349, normalized size = 2.05

$$\frac{i\,b^4x^4 + 2\,b^3x^3\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right) + 6i\,b^2x^2\text{Li}_2\left(\frac{1}{2}\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}\right) + 6i\,b^2x^2\text{Li}_2\left(-\frac{1}{2}\sqrt{4i\,d-4}\,e^{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 + 2*b^3*x^3*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*
b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) +
6*I*b^2*x^2*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*log
(((2*d + 2*I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) + 2*a^3*log
(((2*d + 2*I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 12*b*x*po
lylog(3, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*sqrt
(4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(4*I*d - 4)*e^
(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x +
I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 12*I*pol
ylog(4, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}(d\tan(bx+a) - i\,d + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*tan(b*x + a) - I*d + 1), x)
```

maple [C] time = 6.10, size = 2346, normalized size = 13.80

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] -1/3*x^3*ln(exp(I*(b*x+a)))-1/6*I*Pi*x^3-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b
*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))
+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))+1/12*I*b*x^4+1/8/b^3/(I+d)*pol
ylog(4,I*(I+d)*exp(2*I*(b*x+a)))+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))
*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))+1/6
*x^3*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/6*x^3*ln(d)-1/2/b^3*a^2/
(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1+
I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/12*I*x^3*Pi*csgn((I*exp(2*I*(b*x+a))+e
xp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(d/(exp(2*I*
(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/2/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a
)))*x*a^2-1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2
/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/4/b/(I+d)*poly
log(2,I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*polylog(2,I*(I+d)*exp(2*I
*(b*x+a)))*a^2-1/6*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/6*I/(I+d)*l
n(1-I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+
1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*
(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csg
n((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3-1/12*I*
x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I
*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/12*I*x
^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp
(2*I*(b*x+a))+1))^3+1/12*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*
I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/12*I*x^3*P
i*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi
*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1
)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x
+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a
))+1))^2-1/2/b^3*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/3/b^
```

```
3*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/2*I/b^3*a^3/(I+d)*ln(1-I*exp
(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I
*(I+d))^(1/2))+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d
+I)-1/4/b^2*d/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))*x+1/12*I*x^3*Pi*csg
n(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/
(exp(2*I*(b*x+a))+1))^2-1/8*I/b^3*d/(I+d)*polylog(4,I*(I+d)*exp(2*I*(b*x+a)
))+1/6*I/b^3*a^3/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+1/3*I/b^
3/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^3-1/4*I/b^2/(I+d)*polylog(3,I*(I+d
)*exp(2*I*(b*x+a)))*x+1/2*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a^2+
1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2*I/b^3*
a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/2*I/b^2*a^2/(I+d)*
ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*
(b*x+a))*(-I*(I+d))^(1/2))*x+1/4*I/b*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x
+a)))*x^2-1/4*I/b^3*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/12*I*
x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x
+a))+1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I
*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x
+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/2/b^3*a^3*d/(I+d)*l
n(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*
csgn(I*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(
I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp
(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b
*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))
+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi*csgn(I*exp(I*(
b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))
```

maxima [B] time = 0.37, size = 341, normalized size = 2.01

$$\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{artanh}(d\tan(bx+a)-i\,d+1)}{b^2}-\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*tan(b*
x + a) - I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*ar
ctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*
x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
 - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d - 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atanh}\left(d\tan\left(a+b\,x\right)+1-d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d*tan(a + b*x) - d*1i + 1),x)
```

```
[Out] int(x^2*atanh(d*tan(a + b*x) - d*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1-I*d+d*tan(b*x+a)),x)
[Out] Integral(x**2*atanh(d*tan(a + b*x) - I*d + 1), x)
```

3.322 $\int x \tanh^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=133

$$-\frac{\text{Li}_3\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{8b^2}+\frac{ix\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\tan(a+b$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctanh(1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1-I*d)*exp(2*
I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(
3,-(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] time = 0.25, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6263, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,-(1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 - I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 - I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6263

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-id+d\tan(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1-id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{2}(b(i+d))\int\frac{e^{2ia+2ib}}{1+(1-id)}$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2}\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2}\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2}\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id+d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1-id)e^{2ia+2}\right.$$

Mathematica [A] time = 0.31, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\tanh^{-1}(d\tan(a+bx)-id+1)-\frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{d+i}\right)+2ibx\text{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{d+i}\right)+\text{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{d+i}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 - I*d + d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((I + d)*E
^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, (-I)/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

fricas [C] time = 0.50, size = 297, normalized size = 2.23

$$2ib^3x^3+3b^2x^2\log\left(-\frac{\left((d+i)e^{(2ibx+2ia)}+i\right)e^{(-2ibx-2ia)}}{d}\right)+2ia^3+6ibx\text{Li}_2\left(\frac{1}{2}\sqrt{4id-4}\,e^{(ibx+ia)}\right)+6ibx\text{Li}_2\left(-\frac{1}{2}\sqrt{4id-4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 + 3*b^2*x^2*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*d - 4)*e^(I*b*x +
I*a)) + 6*I*b*x*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(((2
*d + 2*I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 3*a^2*log(((2
*d + 2*I)*e^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) - 3*(b^2*x^2 -
```

```
a^2)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*log(-
1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(4*I*d - 4)
*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}\left(d \tan\left(bx + a\right) - i\,d + 1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*tan(b*x + a) - I*d + 1), x)
```

maple [C] time = 4.41, size = 2256, normalized size = 16.96

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/6*I*b*x^3+1/
8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/4*x^2*ln(d)-1/8*I*x^2*Pi*csgn(I*exp
(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(
b*x+a)))^2-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)+
1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*
I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2
*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3-1/8/b^2*d/(I+d)*polylog(3,I*(I+d)*
exp(2*I*(b*x+a)))+1/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2)
)-1/8*I/b^2/(I+d)*polylog(3,I*(I+d)*exp(2*I*(b*x+a)))-1/4*I/(I+d)*ln(1-I*(I
+d)*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*
(b*x+a)))^3+1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(ex
p(2*I*(b*x+a))+1))^2+1/4*x^2*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)-1/
8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a
))+1))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+
a)))^2-1/4*I*Pi*x^2-1/2*x^2*ln(exp(I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*d/(exp(2
*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+
exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(
b*x+a))/(exp(2*I*(b*x+a))+1))^3-1/4/b^2*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a
)))*a^2+1/2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/2/b^2
*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-1/4/b/(I+d)*polylog(2,I
*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog(2,I*(I+d)*exp(2*I*(b*x+a))
)*a-1/4*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*csgn(I*d/(e
xp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b
*x+a)))^2+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(ex
p(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*
(b*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I*exp(2*I*
(b*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I*
(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2
*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))^2-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*I*(
b*x+a))+exp(2*I*(b*x+a))*d+I)+1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I
*(I+d))^(1/2))+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))-
1/4*I/b^2/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/2*I/b^2*a^2/(I+d)*ln(1
-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*c
sgn(I*exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*
(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a
))/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*
(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a
))*d+I)/(exp(2*I*(b*x+a))+1))+1/4*I/b^2*d/(I+d)*polylog(2,I*(I+d)*exp(2*I*(
b*x+a)))*a-1/2*I/b/(I+d)*ln(1-I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/4*I/b*d/(I+d)
```

```
*polylog(2,I*(I+d)*exp(2*I*(b*x+a)))*x-1/2*I/b^2*a*d/(I+d)*dilog(1+I*exp(I*
(b*x+a))*(-I*(I+d))^(1/2))+1/2*I/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))
^(1/2))*x-1/2*I/b^2*a*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))+1/
2*I/b*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/2/b*a*d/(I+d)*ln(
1+I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1-I*(I+d)*exp(2*I*(
b*x+a)))*x*a+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(-I*(I+d))^(1/2))*x+1/8*
I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(
exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))
+1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d+I))*csgn(I*(I*exp(2*I*(b
*x+a))+exp(2*I*(b*x+a))*d+I)/(exp(2*I*(b*x+a))+1))+1/8*I*x^2*Pi*csgn(I/(exp
(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I
*(b*x+a))+1))
```

maxima [B] time = 0.36, size = 247, normalized size = 1.86

$$\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{artanh}(d\tan(bx+a)-i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((i\,d-1)e^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\operatorname{arctan}(-d\cos}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*tan(b*x + a) - I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d - 1)*e^(2*I*b
*x + 2*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x +
2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*((b
*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin
(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*polylo
g(3, (I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\mathrm{atanh}\left(d\tan\left(a+b\,x\right)+1-d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d*tan(a + b*x) - d*1i + 1),x)
```

```
[Out] int(x*atanh(d*tan(a + b*x) - d*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*tan(a + b*x) - I*d + 1), x)
```

3.323 $\int \tanh^{-1}(1 - id + d\tan(a + bx))\,dx$

Optimal. Leaf size=93

$$\frac{i\text{Li}_2\left(-\left((1-id)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d\tan(a+bx)-id+1)+\frac{1}{2}ibx^2$$

```
[Out] 1/2*I*b*x^2+x*arctanh(1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1-I*d)*exp(2*I*a+2*I*
b*x))+1/4*I*polylog(2,-(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6255, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d\tan(a+bx)-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 - I*d + d*Tan[a + b*x]] - (x*Log[1 + (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 - I*d)*E^((2*I)*a + (2*I
)*b*x))])/b
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6255

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-id+d\tan(a+bx))\,dx = x\tanh^{-1}(1-id+d\tan(a+bx)) + (ib)\int \frac{x}{1+(1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id+d\tan(a+bx)) - (b(i+d))\int \frac{e^{2ia+2ibx}x}{1+(1-id)e^{2ia+2i}}$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right) -$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id+d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1-id)e^{2ia+2ibx}\right) +$$

Mathematica [B] time = 14.37, size = 766, normalized size = 8.24

$$\frac{x\sec^2(a+bx)(\cos(bx)+i\sin(bx))(\sin(bx)+i\cos(bx))(d\sin(a+bx)+(2-id)\cos(a+bx))\left(\text{Li}_2\left(-\frac{1}{2}(\cos(a)+i\,\text{s}\right.\right.}{(\tan(a+bx)-i)(d\tan(a+bx)-id+2)(id\sin(a+bx)+(d+2i)\cos(a+bx))\left(\frac{\sec^2(bx)\log\left(\frac{\sec(bx)(d\sin(a+bx}{2\cos(a)}\right.}{\tan(bx)-i}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - I*d + d*Tan[a + b*x]] + (x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*x
] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x
] + I*d*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]*((2
 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[1 +
I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog[2, (Sec[b*x]
*(d*Cos[a] + I*(2*I + d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(I + d
))] + PolyLog[2, -1/2*((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*Sin[a])*
(-I + Tan[b*x]))])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + Sin
[b*x])*((2 - I*d)*Cos[a + b*x] + d*Sin[a + b*x]))/(((2*I + d)*Cos[a + b*x]
+ I*d*Sin[a + b*x])*((I*Log[1 + I*Tan[b*x]]*Sec[b*x]*(d*Cos[a] + I*(2*I + d
)*Sin[a]))/((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) + (Log[1 - I*Tan[b*x
]]*Sec[b*x]*((-I)*d*Cos[a] + (2*I + d)*Sin[a]))/((2*I + d)*Cos[a + b*x] + I
*d*Sin[a + b*x]) + 2*b*x*(1 - I*Tan[b*x]) + (Log[(Sec[b*x]*((2 - I*d)*Cos[a
 + b*x] + d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I + Tan
[b*x]) - (Log[1 + ((Cos[a] + I*Sin[a])*(d*Cos[a] + I*(2*I + d)*Sin[a])*(-I
+ Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + Log[(Sec[b*x]*(Cos[a] - I*Sin
[a])*((2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]))/(2*(I + d))]*(-I + Tan[b*
x]) - (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((2*I + d)*Cos[a + b*x] + I*d*Sin[
a + b*x]))/(2*(I + d))]*Sec[b*x]^2)/(I + Tan[b*x]))*(-I + Tan[a + b*x])*(2
- I*d + d*Tan[a + b*x]))
```

fricas [B] time = 1.01, size = 222, normalized size = 2.39

$$i\,b^2x^2 + bx\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right) - i\,a^2 - (bx+a)\log\left(\frac{1}{2}\sqrt{4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right) - (bx+a)\log\left(-\frac{1}{2}\sqrt{4i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 + b*x*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2
*I*a)/d) - I*a^2 - (b*x + a)*log(1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) -
 (b*x + a)*log(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(((2*d + 2*
```

```
I)*e^(I*b*x + I*a) + I*sqrt(4*I*d - 4))/(2*d + 2*I)) + a*log(((2*d + 2*I)*e
^(I*b*x + I*a) - I*sqrt(4*I*d - 4))/(2*d + 2*I)) + I*dilog(1/2*sqrt(4*I*d -
 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(4*I*d - 4)*e^(I*b*x + I*a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) - I*d + 1), x)
```

maple [B] time = 0.48, size = 292, normalized size = 3.14

$$\frac{i\operatorname{arctanh}\left(1-id+d\tan\left(bx+a\right)\right)\ln\left(id+d\tan\left(bx+a\right)\right)}{2b}-\frac{i\operatorname{arctanh}\left(1-id+d\tan\left(bx+a\right)\right)\ln\left(-id+d\tan\left(b\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/2*I/b*arctanh(1-I*d+d*tan(b*x+a))*ln(I*d+d*tan(b*x+a))-1/2*I/b*arctanh(1-
I*d+d*tan(b*x+a))*ln(-I*d+d*tan(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d+d*tan(b*x
+a))/d)+1/4*I/b*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/4*I/
b*dilog((2-I*d+d*tan(b*x+a))/(-2*I*d+2))-1/4*I/b*ln(I*d+d*tan(b*x+a))*ln((2
-I*d+d*tan(b*x+a))/(-2*I*d+2))-1/8*I/b*ln(-I*d+d*tan(b*x+a))^2+1/4*I/b*dilo
g(1-1/2*I*d+1/2*d*tan(b*x+a))+1/4*I/b*ln(-I*d+d*tan(b*x+a))*ln(1-1/2*I*d+1/
2*d*tan(b*x+a))
```

maxima [B] time = 0.43, size = 263, normalized size = 2.83

$$-\frac{4\left(bx+a\right)d\left(\frac{\log\left(d\tan\left(bx+a\right)-i\,d+2\right)}{d}-\frac{\log\left(\tan\left(bx+a\right)-i\right)}{d}\right)+d\left(-\frac{2i\left(\log\left(d\tan\left(bx+a\right)-i\,d+2\right)\log\left(-\frac{i\,d\tan\left(bx+a\right)+d+2i}{2\,d+2i}+1\right)+\operatorname{Li}_2\left(\frac{i\,d\tan\left(bx+a\right)+d}{2\,d+2i}\right.\right.}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d + 2)/d - log(tan(b*x + a) - I
)/d) + d*(-2*I*(log(d*tan(b*x + a) - I*d + 2)*log(-(I*d*tan(b*x + a) + d +
2*I)/(2*d + 2*I) + 1) + dilog((I*d*tan(b*x + a) + d + 2*I)/(2*d + 2*I)))/d
+ (2*I*log(d*tan(b*x + a) - I*d + 2)*log(tan(b*x + a) - I) - I*log(tan(b*x
+ a) - I)^2)/d - 2*I*(log(1/2*d*tan(b*x + a) - 1/2*I*d + 1)*log(tan(b*x + a
) - I) + dilog(-1/2*d*tan(b*x + a) + 1/2*I*d))/d + 2*I*(log(tan(b*x + a) -
I)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) - 8
*(b*x + a)*arctanh(d*tan(b*x + a) - I*d + 1))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(d\tan\left(a+b\,x\right)+1-d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*tan(a + b*x) - d*1i + 1),x)
```

```
[Out] int(atanh(d*tan(a + b*x) - d*1i + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1-I*d+d*tan(b*x+a)),x)
[Out] Integral(atanh(d*tan(a + b*x) - I*d + 1), x)
```

3.324 $\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{\tanh^{-1}(d\tan(a+bx)-id+1)}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 - I*d + d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1-id+d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 - I*d + d*Tan[a + b*x]]/x, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\log\left(-\frac{\left((d+i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(1/2*log(-((d + I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*tan(b*x + a) - I*d + 1)/x, x)
```

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(1-id+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] int(arctanh(1-I*d+d*tan(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-i\,bx+\frac{1}{4}\left(i\,\pi-4i\,a-2\,\log(d)\right)\log(x)-\frac{1}{2}i\int\frac{\arctan\left(-d\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right),-d\sin\left(2\,bx+2\,a\right)-c\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d*
cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*si
n(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atanh}\left(d\tan\left(a+b\,x\right)+1-d\,1{}\mathrm{i}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*tan(a + b*x) - d*1i + 1)/x,x)
```

```
[Out] int(atanh(d*tan(a + b*x) - d*1i + 1)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(d\tan\left(a+bx\right)-id+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*tan(a + b*x) - I*d + 1)/x, x)
```

3.325 $\int x^2 \tanh^{-1}(1 + id - d\tan(a + bx))\,dx$

Optimal. Leaf size=171

$$-\frac{i\text{Li}_4\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{8b^3}-\frac{x\text{Li}_3\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b^2}+\frac{ix^2\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{6}x^3\log\left(1+(1+id)e^{2i\right.$$

```
[Out] 1/12*I*b*x^4-1/3*x^3*arctanh(-1-I*d+d*tan(b*x+a))-1/6*x^3*ln(1+(1+I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*po
lylog(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,-(1+I*d)*exp(2*I*a
+2*I*b*x))/b^3
```

Rubi [A] time = 0.29, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6263, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,-(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\text{l}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 - (x^3*Log[1 + (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, -((1 + I*d)*E^((
2*I)*a + (2*I)*b*x))])/b - (x*PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x
))])/(4*b^2) - ((I/8)*PolyLog[4, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b^3
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6263

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{3}(b(i-d))\int \frac{e^{2ia+2\cdots}}{1+(1+i\cdots}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia\cdots}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia\cdots}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia\cdots}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia\cdots}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{6}x^3\log\left(1+(1+id)e^{2ia\cdots}\right.$$

Mathematica [A] time = 0.41, size = 156, normalized size = 0.91

$$\frac{1}{3}x^3\tanh^{-1}(d(-\tan(a+bx))+id+1) - \frac{4b^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{d-i}\right) + 6ib^2x^2\mathrm{Li}_2\left(\frac{ie^{-2i(a+bx)}}{d-i}\right) + 6bx\mathrm{Li}_3\left(\frac{ie^{-2i(a+bx)}}{d-i}\right) - 3i\mathrm{Li}_4\left(\cdots\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 + I*d - d*Tan[a + b*x]])/3 - (4*b^3*x^3*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((-I + d)*E^((2*I)*(a +
b*x)))] + 6*b*x*PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
g[4, I/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

fricas [C] time = 0.58, size = 349, normalized size = 2.04

$$i\,b^4x^4 - 2\,b^3x^3\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right) - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(I*b^4*x^4 - 2*b^3*x^3*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x
+ 2*I*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) +
6*I*b^2*x^2*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - I*a^4 + 2*a^3*lo
g(((2*d - 2*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + 2*a^3*l
og(((2*d - 2*I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 12*b*x
*polylog(3, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 12*b*x*polylog(3, -1/2*
sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 2*(b^3*x^3 + a^3)*log(1/2*sqrt(-4*I*d -
 4)*e^(I*b*x + I*a) + 1) - 2*(b^3*x^3 + a^3)*log(-1/2*sqrt(-4*I*d - 4)*e^(I
*b*x + I*a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) -
12*I*polylog(4, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x^2 \operatorname{artanh}\left(d \tan\left(bx+a\right) - i\,d - 1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*tan(b*x + a) - I*d - 1), x)
```

maple [C] time = 5.69, size = 2456, normalized size = 14.36

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] -1/3*x^3*ln(exp(I*(b*x+a)))+1/12*I*b*x^4+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x
+a))+1)*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^3+1/6*I*x^3*Pi-1/6*x^3*ln(d)-1/6*I/(I
-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^3+1/8/b^3/(I-d)*polylog(4,I*(I-d)*exp(
2*I*(b*x+a)))+1/6*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x^3-1/12*I*x^3*Pi*
csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)
*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+
a))-I)/(exp(2*I*(b*x+a))+1))^3-1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*ex
p(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*
I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))+1))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(
2*I*(b*x+a))+1))^3-1/2/b^3*a^2/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1
/2))-1/2/b^3*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/4/b/(I-
d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I-d)*polylog(2,I*(I-d)*
exp(2*I*(b*x+a)))*a^2+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(
b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn((exp
(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/12*I*x^3*Pi
*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*
(b*x+a))+1)*exp(2*I*(b*x+a)))-1/2*I/b^3*a^2*d/(I-d)*dilog(1-I*exp(I*(b*x+a)
)*(-I*(I-d))^(1/2))+1/4*I/b^3*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*a
^2-1/2/b^2*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x*a^2+1/2/b^2*a^2*d/(I-d)
*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*
x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2-1/12*I*x^3*Pi*csgn(I
*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I*
(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2
*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/6*x^3*ln(exp(
2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))
+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2+1/2/b^2*a^2*d/(I-d)*ln
```

```
(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x-1/2*I/b^2*a^2/(I-d)*ln(1-I*exp(I*(b
*x+a))*(-I*(I-d))^(1/2))*x-1/4*I/b*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a
)))*x^2-1/2*I/b^2*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x-1/2*I
/b^3*a^2*d/(I-d)*dilog(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2/b^3*a^3*d/(
I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(
b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)
))^2+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/6*I*
x^3*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn
(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))+1)*exp
(2*I*(b*x+a)))-1/2*I/b^3*a^3/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+
1/6*I/b^3*a^3/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/3*I/b^3/(
I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+
a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-
I)/(exp(2*I*(b*x+a))+1))^2-1/4*I/b^2/(I-d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a
)))*x-1/2*I/b^3*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/12*I*x^
3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(
b*x+a))/(exp(2*I*(b*x+a))+1))+1/2*I/b^2/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a))
)*x*a^2-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp
(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(
b*x+a))+1))+1/8*I/b^3*d/(I-d)*polylog(4,I*(I-d)*exp(2*I*(b*x+a)))+1/12*I*x^
3*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x
+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/2/b^3*a^3*d/(I-d)*ln(1-I*exp(I*(b*x+a))*(
-I*(I-d))^(1/2))-1/3/b^3*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^3-1/6/b^3
*a^3*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/4/b^2*d/(I-d)*po
lylog(3,I*(I-d)*exp(2*I*(b*x+a)))*x
```

maxima [B] time = 0.37, size = 340, normalized size = 1.99

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{artanh}(d\tan(bx+a)-i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(8i\,(bx+a)^3-18i\,(bx+a)^2a+18i\,(bx+a)\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*tan(b
*x + a) - I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 + (8*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + 18*I*(b*x + a)*a^2)*a
rctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b
*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((-I
*d - 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x +
a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2
*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (-
I*d - 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d - 1)*e^(2*I*b*x + 2*I*
a)))/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\operatorname{atanh}\left(1-d\tan\left(a+b\,x\right)+d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d*1i - d*tan(a + b*x) + 1),x)
```

```
[Out] int(x^2*atanh(d*1i - d*tan(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x^{2}\operatorname{atanh}\left(d\tan\left(a+bx\right)-id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1-I*d+d*tan(b*x+a)),x)
[Out] -Integral(x**2*atanh(d*tan(a + b*x) - I*d - 1), x)
```

3.326 $\int x \tanh^{-1}(1 + id - d \tan(a + bx))\, dx$

Optimal. Leaf size=134

$$-\frac{\text{Li}_3\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{8b^2}+\frac{ix\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\tan(a+b$$

[Out] 1/6*I*b*x^3-1/2*x^2*arctanh(-1-I*d+d*tan(b*x+a))-1/4*x^2*ln(1+(1+I*d)*exp(2*I*a+2*I*b*x))+1/4*I*x*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(3,-(1+I*d)*exp(2*I*a+2*I*b*x))/b^2

Rubi [A] time = 0.24, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6263, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,-(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1+(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 - (x^2*Log[1 + (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, -((1 + I*d)*E^((2*I
)*a + (2*I)*b*x))])/b - PolyLog[3, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))]/(8
*b^2)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6263

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Tan[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c + I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+id-d\tan(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(1+id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{2}(b(i-d))\int\frac{e^{2ia+}}{1+(1+}$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2}\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2}\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2}\right.$$

$$= \frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id-d\tan(a+bx))-\frac{1}{4}x^2\log\left(1+(1+id)e^{2}\right.$$

Mathematica [A] time = 0.32, size = 120, normalized size = 0.90

$$\frac{1}{2}x^2\tanh^{-1}(d(-\tan(a+bx))+id+1)-\frac{2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{d-i}\right)+2ibx\text{Li}_2\left(\frac{ie^{-2i(a+bx)}}{d-i}\right)+\text{Li}_3\left(\frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 + I*d - d*Tan[a + b*x]])/2 - (2*b^2*x^2*Log[1 - I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((-I + d)*E^((2*I)*(a + b*x)
))] + PolyLog[3, I/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

fricas [C] time = 0.47, size = 297, normalized size = 2.22

$$2i\,b^3x^3-3\,b^2x^2\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)+2i\,a^3+6i\,bx\text{Li}_2\left(\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}\right)+6i\,bx\text{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*I*b^3*x^3 - 3*b^2*x^2*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*
x + 2*I*a) - I)) + 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*d - 4)*e^(I*b*x +
I*a)) + 6*I*b*x*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)) - 3*a^2*log(((
2*d - 2*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 3*a^2*log((
(2*d - 2*I)*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) - 3*(b^2*x^2
```

```
 - a^2)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 3*(b^2*x^2 - a^2)*l
og(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) - 6*polylog(3, 1/2*sqrt(-4*I*
d - 4)*e^(I*b*x + I*a)) - 6*polylog(3, -1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a
)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x \operatorname{artanh}\left(d \tan\left(bx + a\right) - i\, d - 1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*tan(b*x + a) - I*d - 1), x)
```

maple [C] time = 4.47, size = 2358, normalized size = 17.60

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] 1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3-1/4*x^2*ln(d)-1/8*I*x^2
*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))*csgn(I*d/(exp(2*I*(b*x+a)
)+1)*exp(2*I*(b*x+a)))^2-1/4*I/b^2/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^2
+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2
*I*(b*x+a))+1)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*e
xp(2*I*(b*x+a)))^3-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a
))-I)/(exp(2*I*(b*x+a))+1))^3-1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2
*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2+1/4*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*
x+a)))*x^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(e
xp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))+1)*
exp(2*I*(b*x+a)))^2-1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1
/2))+1/4*I*x^2*Pi-1/2*x^2*ln(exp(I*(b*x+a)))+1/2*I/b^2*a*d/(I-d)*dilog(1+I*
exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/4*I/b*d/(I-d)*polylog(2,I*(I-d)*exp(2*I*
(b*x+a)))*x+1/8*I*x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(ex
p(2*I*(b*x+a))+1))^3-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+
a)))^2+1/2/b*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*x*a+1/2/b^2*a/(I-d)*dil
og(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x
+a))+1)*exp(2*I*(b*x+a)))^3+1/2/b^2*a/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I
-d))^(1/2))-1/4/b/(I-d)*polylog(2,I*(I-d)*exp(2*I*(b*x+a)))*x-1/8*I/b^2/(I-
d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))-1/4*I/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*
x+a)))*x^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I
)/(exp(2*I*(b*x+a))+1))+1/4*x^2*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)
+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^3+1/8/b^2*d/(I-
d)*polylog(3,I*(I-d)*exp(2*I*(b*x+a)))-1/4/b^2/(I-d)*polylog(2,I*(I-d)*exp(
2*I*(b*x+a)))*a-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a))
)*csgn(d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*(exp(
2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b
*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(I*
(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2
*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))-1/2/b*a*d/(I-d)*l
n(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x-1/2/b*a*d/(I-d)*ln(1-I*exp(I*(b*x+
a))*(-I*(I-d))^(1/2))*x+1/8*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*
I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)))^2+
1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(
2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a
))+1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1))^2+1/8*I*x^2*Pi*csgn(I*
(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp
```

```
(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))+1))^2-1/2/b^2*a^2*d/(I-d)*ln(1-I*exp(I*(
b*x+a))*(-I*(I-d))^(1/2))+1/4/b^2*d/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+a)))*a^
2+1/4/b^2*a^2*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d+I)+1/2*I/b^2
*a^2/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))+1/2*I/b^2*a^2/(I-d)*ln(1
-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/4*I/b^2*a^2/(I-d)*ln(I*exp(2*I*(b*x+a
))-exp(2*I*(b*x+a))*d+I)+1/2*I/b*a/(I-d)*ln(1+I*exp(I*(b*x+a))*(-I*(I-d))^(
1/2))*x+1/2*I/b*a/(I-d)*ln(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))*x+1/2*I/b^2
*a*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(-I*(I-d))^(1/2))-1/4*I/b^2*d/(I-d)*pol
ylog(2,I*(I-d)*exp(2*I*(b*x+a)))*a-1/2*I/b/(I-d)*ln(1-I*(I-d)*exp(2*I*(b*x+
a)))*x*a+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))+1
))*csgn(I*d/(exp(2*I*(b*x+a))+1)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I/(exp
(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I
*(b*x+a))+1))
```

maxima [B] time = 0.36, size = 246, normalized size = 1.84

$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{artanh}(d\tan(bx+a)-i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left((-i\,d-1)e^{(2i\,bx+2i\,a)}\right)+\left(6i\,(bx+a)^2-12i\,(bx+a)a\right)\operatorname{arctar}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*tan(b*x + a) - I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d - 1)*e^(2*I
*b*x + 2*I*a)) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1) + 3*(
(b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1) + 3*poly
log(3, (-I*d - 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\operatorname{atanh}\left(1-d\tan\left(a+b\,x\right)+d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d*1i - d*tan(a + b*x) + 1),x)
```

```
[Out] int(x*atanh(d*1i - d*tan(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x\operatorname{atanh}\left(d\tan\left(a+bx\right)-id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] -Integral(x*atanh(d*tan(a + b*x) - I*d - 1), x)
```

3.327 $\int \tanh^{-1}(1 + id - d\tan(a + bx))\,dx$

Optimal. Leaf size=94

$$\frac{i\text{Li}_2\left(-\left((id+1)e^{2ia+2ibx}\right)\right)}{4b}-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{2}ibx^2$$

[Out] 1/2*I*b*x^2-x*arctanh(-1-I*d+d*tan(b*x+a))-1/2*x*ln(1+(1+I*d)*exp(2*I*a+2*I*b*x))+1/4*I*polylog(2,-(1+I*d)*exp(2*I*a+2*I*b*x))/b

Rubi [A] time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6255, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,-(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\tan(a+bx))+id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[1 + I*d - d*Tan[a + b*x]],x]

[Out] (I/2)*b*x^2 + x*ArcTanh[1 + I*d - d*Tan[a + b*x]] - (x*Log[1 + (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, -((1 + I*d)*E^((2*I)*a + (2*I)*b*x))])/b

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6255

```
Int[ArcTanh[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[c + d*Tan[a + b*x]], x] + Dist[I*b, Int[x/(c + I*d + c*E^(2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+id-d\tan(a+bx))\,dx = x\tanh^{-1}(1+id-d\tan(a+bx)) + (ib)\int \frac{x}{1+(1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - (b(i-d))\int \frac{e^{2ia+2ibx}x}{1+(1+id)e^{2ia}}$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right.$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right.$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id-d\tan(a+bx)) - \frac{1}{2}x\log\left(1+(1+id)e^{2ia+2ibx}\right.$$

Mathematica [B] time = 15.54, size = 723, normalized size = 7.69

$$x\tanh^{-1}(d(-\tan(a+bx))+id+1) - \frac{x\sec(a+bx)(\cos(bx)+i\sin(bx))(\sin(bx)+i\cos(bx))\left(-\mathrm{Li}_2\left(\frac{1}{2}(\cos(a)+i\sin\right.\right.}{(\tan(a+bx)-i)(id\sin(a+bx)+(d-2i)\cos(a+bx))\left(-\frac{\sec^2(bx)\log\left(\frac{\sec(bx)(-}{}\right.}{\tan}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + I*d - d*Tan[a + b*x]] - (x*((-2*I)*b*x*Log[2*Cos[b*x]*(Cos[b*
x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a + b
*x] + I*d*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*x]*
((2 + I*d)*Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Log[1
 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2, (Sec[b
*x]*(d*Cos[a] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(-I +
 d))] - PolyLog[2, ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a])*(-I +
 Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[b*x] + Sin[b*x]
))/(((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x])*((I*Log[1 - I*Tan[b*x]]*Se
c[b*x]*(d*Cos[a] + (2 + I*d)*Sin[a]))/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a
+ b*x]) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((-I)*d*Cos[a] + (-2*I + d)*Sin[a])
)/((-2*I + d)*Cos[a + b*x] + I*d*Sin[a + b*x]) - (Log[(Sec[b*x]*((2 + I*d)*
Cos[a + b*x] - d*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*x]^2)/(-I
+ Tan[b*x]) + (Log[1 - ((Cos[a] + I*Sin[a])*(d*Cos[a] + (2 + I*d)*Sin[a])*(
-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) - Log[1 - (Sec[b*x]*(d*Cos[a
] + (2 + I*d)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*(-I + d))]*(-I +
Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-2*I + d)*Cos[a + b*x] + I
*d*Sin[a + b*x]))/(2*(-I + d))]*Sec[b*x]^2)/(I + Tan[b*x]) + (2*I)*b*x*(I +
 Tan[b*x]))*(-I + Tan[a + b*x]))
```

fricas [B] time = 0.71, size = 223, normalized size = 2.37

$$\frac{i\,b^2x^2 - bx\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right) - i\,a^2 - (bx+a)\log\left(\frac{1}{2}\sqrt{-4i\,d-4}\,e^{(i\,bx+i\,a)}+1\right) - (bx+a)\log\left(-\frac{1}{2}\sqrt{-4i\,d-4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*b^2*x^2 - b*x*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a
) - I)) - I*a^2 - (b*x + a)*log(1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) -
 (b*x + a)*log(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a) + 1) + a*log(((2*d - 2
*I)*e^(I*b*x + I*a) + I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + a*log(((2*d - 2*I)
```

```
*e^(I*b*x + I*a) - I*sqrt(-4*I*d - 4))/(2*d - 2*I)) + I*dilog(1/2*sqrt(-4*I
*d - 4)*e^(I*b*x + I*a)) + I*dilog(-1/2*sqrt(-4*I*d - 4)*e^(I*b*x + I*a)))/
b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tan(b*x + a) - I*d - 1), x)
```

maple [B] time = 0.50, size = 328, normalized size = 3.49

$$-\frac{i\operatorname{arctanh}\left(-1-id+d\tan\left(bx+a\right)\right)\ln\left(id+d\tan\left(bx+a\right)\right)}{2b}+\frac{i\operatorname{arctanh}\left(-1-id+d\tan\left(bx+a\right)\right)\ln\left(-id+d\tan\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] -1/2*I/b*arctanh(-1-I*d+d*tan(b*x+a))*ln(I*d+d*tan(b*x+a))+1/2*I/b*arctanh(
-1-I*d+d*tan(b*x+a))*ln(-I*d+d*tan(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d+d*tan(
b*x+a))/d)+1/4*I/b*ln(I*d+d*tan(b*x+a))*ln(1/2*I*(-I*d+d*tan(b*x+a))/d)-1/4
*I/b*dilog((-2-I*d+d*tan(b*x+a))/(-2*I*d-2))-1/4*I/b*ln(I*d+d*tan(b*x+a))*l
n((-2-I*d+d*tan(b*x+a))/(-2*I*d-2))-1/8*I/b*ln(-I*d+d*tan(b*x+a))^2-1/4*I/b
*ln(1+1/2*I*d-1/2*d*tan(b*x+a))*ln(-1/2*I*d+1/2*d*tan(b*x+a))+1/4*I/b*ln(1+
1/2*I*d-1/2*d*tan(b*x+a))*ln(-I*d+d*tan(b*x+a))-1/4*I/b*dilog(-1/2*I*d+1/2*
d*tan(b*x+a))
```

maxima [B] time = 0.43, size = 265, normalized size = 2.82

$$-\frac{4\left(bx+a\right)d\left(\frac{\log\left(d\tan\left(bx+a\right)-i\,d-2\right)}{d}-\frac{\log\left(\tan\left(bx+a\right)-i\right)}{d}\right)-d\left(\frac{2i\left(\log\left(d\tan\left(bx+a\right)-i\,d-2\right)\log\left(-\frac{i\,d\tan\left(bx+a\right)+d-2i}{2\,d-2i}+1\right)+\operatorname{Li}_2\left(\frac{i\,d\tan\left(bx+a\right)+d-2i}{2\,d-2i}\right)\right)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log(d*tan(b*x + a) - I*d - 2)/d - log(tan(b*x + a) - I
)/d) - d*(2*I*(log(d*tan(b*x + a) - I*d - 2)*log(-(I*d*tan(b*x + a) + d - 2
*I)/(2*d - 2*I) + 1) + dilog((I*d*tan(b*x + a) + d - 2*I)/(2*d - 2*I)))/d -
 (2*I*log(d*tan(b*x + a) - I*d - 2)*log(tan(b*x + a) - I) - I*log(tan(b*x +
 a) - I)^2)/d + 2*I*(log(-1/2*d*tan(b*x + a) + 1/2*I*d + 1)*log(tan(b*x + a
) - I) + dilog(1/2*d*tan(b*x + a) - 1/2*I*d))/d - 2*I*(log(tan(b*x + a) - I
)*log(-1/2*I*tan(b*x + a) + 1/2) + dilog(1/2*I*tan(b*x + a) + 1/2))/d) + 8*
(b*x + a)*arctanh(d*tan(b*x + a) - I*d - 1))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(1-d\tan\left(a+b\,x\right)+d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*1i - d*tan(a + b*x) + 1),x)
```

```
[Out] int(atanh(d*1i - d*tan(a + b*x) + 1), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{atanh}\left(d\tan\left(a+bx\right)-id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1-I*d+d*tan(b*x+a)),x)
```

```
[Out] -Integral(atanh(d*tan(a + b*x) - I*d - 1), x)
```

3.328 $\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\tanh^{-1}(d(-\tan(a+bx))+id+1)}{x},x\right)$$

```
[Out] CannotIntegrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 + I*d - d*Tan[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Mathematica [A] time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1+id-d\tan(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 + I*d - d*Tan[a + b*x]]/x, x]
```

fricas [A] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d-i)e^{(2i\,bx+2i\,a)}-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="fricas")
[Out] integral(-1/2*log(-d*e^(2*I*b*x + 2*I*a)/((d - I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}\left(d\tan\left(bx+a\right)-i\,d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*tan(b*x + a) - I*d - 1)/x, x)
```

maple [A] time = 1.65, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arctanh}\left(-1-id+d\tan\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] int(-arctanh(-1-I*d+d*tan(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-i\,bx+\frac{1}{4}\left(-i\,\pi-4i\,a-2\,\log\left(-d\right)\right)\log(x)+\frac{1}{2}i\int\frac{\arctan\left(d\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right),-d\sin\left(2\,bx+2\right.\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1-I*d+d*tan(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(-d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) + 2*cos(2*b*x + 2*a) + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\operatorname{atanh}\left(1-d\tan\left(a+b\,x\right)+d\,1\mathrm{i}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*1i - d*tan(a + b*x) + 1)/x,x)
```

```
[Out] int(atanh(d*1i - d*tan(a + b*x) + 1)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{atanh}\left(d\tan\left(a+bx\right)-id-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1-I*d+d*tan(b*x+a))/x,x)
```

```
[Out] -Integral(atanh(d*tan(a + b*x) - I*d - 1)/x, x)
```

3.329 $\int (e+fx)^3 \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=302

$$-\frac{3f^3\text{Li}_5\left(-ie^{2i(a+bx)}\right)}{16b^4}+\frac{3f^3\text{Li}_5\left(ie^{2i(a+bx)}\right)}{16b^4}+\frac{3if^2(e+fx)\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2}{}$$

```
[Out] 1/4*I*(f*x+e)^4*arctan(exp(2*I*(b*x+a)))/f+1/4*(f*x+e)^4*arctanh(cot(b*x+a)
)/f-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^3*polylo
g(2,I*exp(2*I*(b*x+a)))/b+3/8*f*(f*x+e)^2*polylog(3,-I*exp(2*I*(b*x+a)))/b^
2-3/8*f*(f*x+e)^2*polylog(3,I*exp(2*I*(b*x+a)))/b^2+3/8*I*f^2*(f*x+e)*polyl
og(4,-I*exp(2*I*(b*x+a)))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*I*(b*x+a)
))/b^3-3/16*f^3*polylog(5,-I*exp(2*I*(b*x+a)))/b^4+3/16*f^3*polylog(5,I*exp
(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.23, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6253, 4181, 2531, 6609, 2282, 6589}

$$\frac{3if^2(e+fx)\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{3if^2(e+fx)\text{PolyLog}\left(4,ie^{2i(a+bx)}\right)}{8b^3}+\frac{3f(e+fx)^2\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^3*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] ((I/4)*(e + f*x)^4*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^4*ArcTanh[Co
t[a + b*x]])/(4*f) - ((I/4)*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (3*f*(e +
f*x)^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (3*f*(e + f*x)^2*Pol
yLog[3, I*E^((2*I)*(a + b*x))])/(8*b^2) + (((3*I)/8)*f^2*(e + f*x)*PolyLog[
4, (-I)*E^((2*I)*(a + b*x))])/b^3 - (((3*I)/8)*f^2*(e + f*x)*PolyLog[4, I*E
^((2*I)*(a + b*x))])/b^3 - (3*f^3*PolyLog[5, (-I)*E^((2*I)*(a + b*x))])/(16
*b^4) + (3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6253

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^3 \tanh^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{b\int (e+fx)^4 \sec(2a+2bx)\,dx}{4f}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} + \frac{1}{2}\int (e+$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3}{}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3}{}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3}{}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3}{}$$

$$= \frac{i(e+fx)^4 \tan^{-1}\left(e^{2i(a+bx)}\right)}{4f} + \frac{(e+fx)^4 \tanh^{-1}(\cot(a+bx))}{4f} - \frac{i(e+fx)^3}{}$$

Mathematica [B] time = 0.30, size = 654, normalized size = 2.17

$$\frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\tanh^{-1}(\cot(a+bx)) + \frac{-8b^4e^3x\log\left(1-ie^{2i(a+bx)}\right)+8b^4e^3x\log\left(1+ie^{2i(a+bx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^3*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTanh[Cot[a + b*x]])/4 + (
-8*b^4*e^3*x*Log[1 - I*E^((2*I)*(a + b*x))] - 12*b^4*e^2*f*x^2*Log[1 - I*E^
((2*I)*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] - 2*b^4
*f^3*x^4*Log[1 - I*E^((2*I)*(a + b*x))] + 8*b^4*e^3*x*Log[1 + I*E^((2*I)*(a
 + b*x))] + 12*b^4*e^2*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 8*b^4*e*f^2*x
^3*Log[1 + I*E^((2*I)*(a + b*x))] + 2*b^4*f^3*x^4*Log[1 + I*E^((2*I)*(a + b
*x))] - (4*I)*b^3*(e + f*x)^3*PolyLog[2, (-I)*E^((2*I)*(a + b*x))] + (4*I)*
```

```
b^3*(e + f*x)^3*PolyLog[2, I*E^((2*I)*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))] + 12*b^2*e*f^2*x*PolyLog[3, (-I)*E^((2*I)*(a + b*
x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] - 6*b^2*e^2*f*Pol
yLog[3, I*E^((2*I)*(a + b*x))] - 12*b^2*e*f^2*x*PolyLog[3, I*E^((2*I)*(a +
b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^((2*I)*(a + b*x))] + (6*I)*b*e*f^2*Po
lyLog[4, (-I)*E^((2*I)*(a + b*x))] + (6*I)*b*f^3*x*PolyLog[4, (-I)*E^((2*I)
*(a + b*x))] - (6*I)*b*e*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))] - (6*I)*b*f^
3*x*PolyLog[4, I*E^((2*I)*(a + b*x))] - 3*f^3*PolyLog[5, (-I)*E^((2*I)*(a +
 b*x))] + 3*f^3*PolyLog[5, I*E^((2*I)*(a + b*x))])/(16*b^4)
```

fricas [C] time = 0.67, size = 1567, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/32*(3*f^3*polylog(5, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*poly
log(5, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*f^3*polylog(5, -I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a)) - 3*f^3*polylog(5, -I*cos(2*b*x + 2*a) - sin(
2*b*x + 2*a)) - (4*I*b^3*f^3*x^3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x +
4*I*b^3*e^3)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - (4*I*b^3*f^3*x^
3 + 12*I*b^3*e*f^2*x^2 + 12*I*b^3*e^2*f*x + 4*I*b^3*e^3)*dilog(I*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a)) - (-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*
b^3*e^2*f*x - 4*I*b^3*e^3)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) -
(-4*I*b^3*f^3*x^3 - 12*I*b^3*e*f^2*x^2 - 12*I*b^3*e^2*f*x - 4*I*b^3*e^3)*di
log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 4*(b^4*f^3*x^4 + 4*b^4*e*f^2*
x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2
*a) + 1)/(cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1)) - 2*(4*a*b^3*e^3 - 6*a^
2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(cos(2*b*x + 2*a) + I*sin(2*b*x +
 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*lo
g(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2
*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^
3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^
4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 -
 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(I*cos(2*b*x + 2*a) - sin(2*
b*x + 2*a) + 1) + 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^
4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-I*c
os(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) - 2*(b^4*f^3*x^4 + 4*b^4*e*f^2*x^3
+ 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x + 4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e
*f^2 - a^4*f^3)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(4*a*b^
3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2 - a^4*f^3)*log(-cos(2*b*x + 2*a) +
I*sin(2*b*x + 2*a) + I) + 2*(4*a*b^3*e^3 - 6*a^2*b^2*e^2*f + 4*a^3*b*e*f^2
- a^4*f^3)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - (-6*I*b*f^3*x
- 6*I*b*e*f^2)*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - (-6*I*b*
f^3*x - 6*I*b*e*f^2)*polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - (6
*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, -I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
) - (6*I*b*f^3*x + 6*I*b*e*f^2)*polylog(4, -I*cos(2*b*x + 2*a) - sin(2*b*x
+ 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, I*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a)) + 6*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*
polylog(3, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - 6*(b^2*f^3*x^2 + 2*b^2*
e*f^2*x + b^2*e^2*f)*polylog(3, -I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 6
*(b^2*f^3*x^2 + 2*b^2*e*f^2*x + b^2*e^2*f)*polylog(3, -I*cos(2*b*x + 2*a) -
 sin(2*b*x + 2*a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)^3 \operatorname{artanh}(\cot(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="giac")
[Out] integrate((f*x + e)^3*arctanh(cot(b*x + a)), x)
```

maple [C] time = 41.71, size = 7429, normalized size = 24.60

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*arctanh(cot(b*x+a)),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\left(f^3x^4+4\,ef^2x^3+6\,e^2fx^2+4\,e^3x\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*arctanh(cot(b*x+a)),x, algorithm="maxima")
[Out] 1/16*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)
^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/16*(f^3*x^4 + 4*e*f
^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*
a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(1/2*((b*f^3*x^4 + 4*b*e*f^2*x^3
+ 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f^3*x^4
 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + (b*f^3*x^4 + 4*b*e*f^2*x^3 + 6*b*e^2*f*x^2 + 4*b*e^3*x)*cos(2*b*x +
2*a))/(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atanh}\left(\cot\left(a+b\,x\right)\right)\,{\left(e+f\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x))*(e + f*x)^3,x)
[Out] int(atanh(cot(a + b*x))*(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)^{3}\operatorname{atanh}{\left(\cot{\left(a+bx\right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*atanh(cot(b*x+a)),x)
[Out] Integral((e + f*x)**3*atanh(cot(a + b*x)), x)
```

3.330 $\int (e+fx)^2 \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=234

$$\frac{if^2\text{Li}_4\left(-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{Li}_4\left(ie^{2i(a+bx)}\right)}{8b^3}+\frac{f(e+fx)\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{Li}_3\left(ie^{2i(a+bx)}\right)}{4b^2}-\frac{i(e+fx)^2\text{Li}_2\left(-ie^{2i(a\right.}}{4b}$$

```
[Out] 1/3*I*(f*x+e)^3*arctan(exp(2*I*(b*x+a)))/f+1/3*(f*x+e)^3*arctanh(cot(b*x+a)
)/f-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)^2*polylo
g(2,I*exp(2*I*(b*x+a)))/b+1/4*f*(f*x+e)*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-
1/4*f*(f*x+e)*polylog(3,I*exp(2*I*(b*x+a)))/b^2+1/8*I*f^2*polylog(4,-I*exp(
2*I*(b*x+a)))/b^3-1/8*I*f^2*polylog(4,I*exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6253, 4181, 2531, 6609, 2282, 6589}

$$\frac{f(e+fx)\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{4b^2}-\frac{f(e+fx)\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{4b^2}+\frac{if^2\text{PolyLog}\left(4,-ie^{2i(a+bx)}\right)}{8b^3}-\frac{if^2\text{PolyLog}\left(4,\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^2*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] ((I/3)*(e + f*x)^3*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^3*ArcTanh[Co
t[a + b*x]])/(3*f) - ((I/4)*(e + f*x)^2*PolyLog[2, (-I)*E^((2*I)*(a + b*x))
])/b + ((I/4)*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*(e + f*
x)*PolyLog[3, (-I)*E^((2*I)*(a + b*x))])/(4*b^2) - (f*(e + f*x)*PolyLog[3,
I*E^((2*I)*(a + b*x))])/(4*b^2) + ((I/8)*f^2*PolyLog[4, (-I)*E^((2*I)*(a +
b*x))])/b^3 - ((I/8)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6253

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
```

```
e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (e+fx)^2\tanh^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^3\tanh^{-1}(\cot(a+bx))}{3f} - \frac{b\int (e+fx)^3\sec(2a+2bx)\,dx}{3f}$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\cot(a+bx))}{3f} + \frac{1}{2}\int (e+$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+fx)^2}{}$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+fx)^2}{}$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+fx)^2}{}$$

$$= \frac{i(e+fx)^3\tan^{-1}\left(e^{2i(a+bx)}\right)}{3f} + \frac{(e+fx)^3\tanh^{-1}(\cot(a+bx))}{3f} - \frac{i(e+fx)^2}{}$$

Mathematica [A] time = 0.18, size = 409, normalized size = 1.75

$$\frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\tanh^{-1}(\cot(a+bx)) + \frac{-12b^3e^2x\log\left(1-ie^{2i(a+bx)}\right)+12b^3e^2x\log\left(1+ie^{2i(a+bx)}\right)-12b^3e}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^2*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTanh[Cot[a + b*x]])/3 + (-12*b^3*e^2*x*Lo
g[1 - I*E^((2*I)*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 - I*E^((2*I)*(a + b*x))
] - 4*b^3*f^2*x^3*Log[1 - I*E^((2*I)*(a + b*x))] + 12*b^3*e^2*x*Log[1 + I*E
^((2*I)*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 + I*E^((2*I)*(a + b*x))] + 4*b^3
*f^2*x^3*Log[1 + I*E^((2*I)*(a + b*x))] - (6*I)*b^2*(e + f*x)^2*PolyLog[2,
(-I)*E^((2*I)*(a + b*x))] + (6*I)*b^2*(e + f*x)^2*PolyLog[2, I*E^((2*I)*(a
+ b*x))] + 6*b*e*f*PolyLog[3, (-I)*E^((2*I)*(a + b*x))] + 6*b*f^2*x*PolyLog
[3, (-I)*E^((2*I)*(a + b*x))] - 6*b*e*f*PolyLog[3, I*E^((2*I)*(a + b*x))] -
 6*b*f^2*x*PolyLog[3, I*E^((2*I)*(a + b*x))] + (3*I)*f^2*PolyLog[4, (-I)*E^
((2*I)*(a + b*x))] - (3*I)*f^2*PolyLog[4, I*E^((2*I)*(a + b*x))])/(24*b^3)
```

fricas [C] time = 0.97, size = 1081, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(-3*I*f^2*polylog(4, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - 3*I*f^2*
polylog(4, I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*c
os(2*b*x + 2*a) + sin(2*b*x + 2*a)) + 3*I*f^2*polylog(4, -I*cos(2*b*x + 2*a
) - sin(2*b*x + 2*a)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*f*x + 6*I*b^2*e^2)*di
log(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (6*I*b^2*f^2*x^2 + 12*I*b^2*e*
f*x + 6*I*b^2*e^2)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (-6*I*b^2
*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-I*cos(2*b*x + 2*a) + sin(2*
b*x + 2*a)) + (-6*I*b^2*f^2*x^2 - 12*I*b^2*e*f*x - 6*I*b^2*e^2)*dilog(-I*co
s(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 8*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3
*e^2*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a) -
sin(2*b*x + 2*a) + 1)) + 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(cos(2*
b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f
^2)*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 4*(b^3*f^2*x^3 + 3*b^3
*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b
*x + 2*a) + sin(2*b*x + 2*a) + 1) + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*
e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(I*cos(2*b*x + 2*a) - sin(2
*b*x + 2*a) + 1) - 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e
^2 - 3*a^2*b*e*f + a^3*f^2)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)
 + 4*(b^3*f^2*x^3 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x + 3*a*b^2*e^2 - 3*a^2*b*e*f
 + a^3*f^2)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) + 4*(3*a*b^2*e^
2 - 3*a^2*b*e*f + a^3*f^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I)
- 4*(3*a*b^2*e^2 - 3*a^2*b*e*f + a^3*f^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a) + I) + 6*(b*f^2*x + b*e*f)*polylog(3, I*cos(2*b*x + 2*a) + sin(2*
b*x + 2*a)) - 6*(b*f^2*x + b*e*f)*polylog(3, I*cos(2*b*x + 2*a) - sin(2*b*x
 + 2*a)) + 6*(b*f^2*x + b*e*f)*polylog(3, -I*cos(2*b*x + 2*a) + sin(2*b*x +
 2*a)) - 6*(b*f^2*x + b*e*f)*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2
*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)^2 \operatorname{artanh}(\cot(bx+a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*arctanh(cot(b*x + a)), x)
```

maple [C] time = 32.12, size = 5543, normalized size = 23.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*arctanh(cot(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}\left(f^2x^3 + 3\,efx^2 + 3\,e^2x\right)\log\left(2\,\cos\left(2\,bx + 2\,a\right)^2 + 2\,\sin\left(2\,bx + 2\,a\right)^2 + 4\,\sin\left(2\,bx + 2\,a\right) + 2\right) - \frac{1}{12}\left(f^2x^3 + 3\,ef\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x
 + 2*a)^2 + 4*sin(2*b*x + 2*a) + 2) - 1/12*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*
log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) -
 integrate(2/3*((b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(4*b*x + 4*a)*cos(
2*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + (b*f^2*x^3 + 3*b*e*f*x^2 + 3*b*e^2*x)*cos(2*b*x + 2*a))/(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{atanh}\left(\cot\left(a+b\,x\right)\right)\,{\left(e+f\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x))*(e + f*x)^2,x)
```

```
[Out] int(atanh(cot(a + b*x))*(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e + fx\right)^{2} \operatorname{atanh}{\left(\cot{\left(a + bx \right)} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*atanh(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)**2*atanh(cot(a + b*x)), x)
```

3.331 $\int (e+fx)\tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=162

$$\frac{f\text{Li}_3\left(-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{Li}_3\left(ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f}$$

```
[Out] 1/2*I*(f*x+e)^2*arctan(exp(2*I*(b*x+a)))/f+1/2*(f*x+e)^2*arctanh(cot(b*x+a)
)/f-1/4*I*(f*x+e)*polylog(2,-I*exp(2*I*(b*x+a)))/b+1/4*I*(f*x+e)*polylog(2,
I*exp(2*I*(b*x+a)))/b+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1/8*f*polylo
g(3,I*exp(2*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6253, 4181, 2531, 2282, 6589}

$$\frac{f\text{PolyLog}\left(3,-ie^{2i(a+bx)}\right)}{8b^2}-\frac{f\text{PolyLog}\left(3,ie^{2i(a+bx)}\right)}{8b^2}-\frac{i(e+fx)\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i(e+fx)\text{PolyLog}\left(2,ie^{2i(a}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] ((I/2)*(e + f*x)^2*ArcTan[E^((2*I)*(a + b*x))])/f + ((e + f*x)^2*ArcTanh[Co
t[a + b*x]])/(2*f) - ((I/4)*(e + f*x)*PolyLog[2, (-I)*E^((2*I)*(a + b*x))])
/b + ((I/4)*(e + f*x)*PolyLog[2, I*E^((2*I)*(a + b*x))])/b + (f*PolyLog[3,
(-I)*E^((2*I)*(a + b*x))])/(8*b^2) - (f*PolyLog[3, I*E^((2*I)*(a + b*x))])/
(8*b^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6253

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[((e + f*x)^(m + 1)*ArcTanh[Cot[a + b*x]])/(f*(m + 1)), x] - Dist[b/
(f*(m + 1)), Int[(e + f*x)^(m + 1)*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b,
 e, f}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (e+fx)\tanh^{-1}(\cot(a+bx))\,dx = \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{b\int (e+fx)^2\sec(2a+2bx)\,dx}{2f}$$
$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} + \frac{1}{2}\int (e+f$$
$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}$$
$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}$$
$$= \frac{i(e+fx)^2\tan^{-1}\left(e^{2i(a+bx)}\right)}{2f} + \frac{(e+fx)^2\tanh^{-1}(\cot(a+bx))}{2f} - \frac{i(e+fx)\mathrm{Li}$$

Mathematica [A] time = 0.13, size = 263, normalized size = 1.62

$$-be\left(\frac{i\mathrm{Li}_2\left(-ie^{i(2a+2bx)}\right)}{4b^2} - \frac{i\mathrm{Li}_2\left(ie^{i(2a+2bx)}\right)}{4b^2} - \frac{ix\tan^{-1}\left(e^{2ia+2ibx}\right)}{b}\right) + \frac{f\left(4ib^2x^2\tan^{-1}(\cos(2(a+bx))+i\sin(2(a+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)*ArcTanh[Cot[a + b*x]],x]
```

```
[Out] e*x*ArcTanh[Cot[a + b*x]] + (f*x^2*ArcTanh[Cot[a + b*x]])/2 - b*e*(((-I)*x*
ArcTan[E^((2*I)*a + (2*I)*b*x)])/b + ((I/4)*PolyLog[2, (-I)*E^(I*(2*a + 2*b
*x))])/b^2 - ((I/4)*PolyLog[2, I*E^(I*(2*a + 2*b*x))])/b^2) + (f*((4*I)*b^2
*x^2*ArcTan[Cos[2*(a + b*x)] + I*Sin[2*(a + b*x)]] + (2*I)*b*x*PolyLog[2, I
*Cos[2*(a + b*x)] - Sin[2*(a + b*x)]] - (2*I)*b*x*PolyLog[2, (-I)*Cos[2*(a
+ b*x)] + Sin[2*(a + b*x)]] - PolyLog[3, I*Cos[2*(a + b*x)] - Sin[2*(a + b*
x)]] + PolyLog[3, (-I)*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]]))/(8*b^2)
```

fricas [C] time = 0.74, size = 677, normalized size = 4.18

$$\frac{\left(2i\,bfx+2i\,be\right)\mathrm{Li}_2\left(i\,\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right)\right)+\left(2i\,bfx+2i\,be\right)\mathrm{Li}_2\left(i\,\cos\left(2\,bx+2\,a\right)-\sin\left(2\,bx+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*I*b*f*x + 2*I*b*e)*dilog(I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) +
(2*I*b*f*x + 2*I*b*e)*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + (-2*I*
b*f*x - 2*I*b*e)*dilog(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) + (-2*I*b*f*
x - 2*I*b*e)*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + 4*(b^2*f*x^2 +
 2*b^2*e*x)*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a
) - sin(2*b*x + 2*a) + 1)) + 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) + I*s
in(2*b*x + 2*a) + I) - 2*(2*a*b*e - a^2*f)*log(cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a) + I) - 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(I*cos(2*b*
x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2
```

```
*f)*log(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) + 1) - 2*(b^2*f*x^2 + 2*b^2*e
*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(
b^2*f*x^2 + 2*b^2*e*x + 2*a*b*e - a^2*f)*log(-I*cos(2*b*x + 2*a) - sin(2*b*
x + 2*a) + 1) + 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2
*a) + I) - 2*(2*a*b*e - a^2*f)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) +
 I) + f*polylog(3, I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)) - f*polylog(3, I*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) + f*polylog(3, -I*cos(2*b*x + 2*a) + s
in(2*b*x + 2*a)) - f*polylog(3, -I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)))/b^
2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx + e) \operatorname{artanh}(\cot(bx + a))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*arctanh(cot(b*x + a)), x)
```

maple [C] time = 3.71, size = 2544, normalized size = 15.70

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*arctanh(cot(b*x+a)),x)
```

```
[Out] 1/4*I*Pi*x*e+1/8*I*Pi*f*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2
*I*(b*x+a))-1))^3+1/4/b^2*f*(I*b*x+I*a)^2*ln(1-I*exp(2*I*(b*x+a)))+1/4/b^2*
f*(I*b*x+I*a)*polylog(2,I*exp(2*I*(b*x+a)))-1/4*ln(exp(2*I*(b*x+a))-I)*x^2*
f-1/2*ln(exp(2*I*(b*x+a))-I)*x*e+1/8*f*polylog(3,-I*exp(2*I*(b*x+a)))/b^2-1
/8*f*polylog(3,I*exp(2*I*(b*x+a)))/b^2-1/2*I/b*e*dilog(1+exp(I*(b*x+a))*(-1
)^(3/4))-1/2*I/b*e*dilog(1-exp(I*(b*x+a))*(-1)^(3/4))+1/4*I*Pi*x*e*csgn(I*(
exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x
+a))+I))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/8*I*Pi*f
*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))
-1))^2*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a
))+I)/(exp(2*I*(b*x+a))-1))^2+1/2*I/b*e*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(
-I)^(1/2))+1/2*I/b*e*dilog(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/4*I*Pi
*x*e*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3+1/8*I*Pi*f*csg
n((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3*x^2-1/2/b*a*e*ln(-exp(
2*I*(b*x+a))+I)+1/2/b*a*e*ln(exp(2*I*(b*x+a))+I)-1/4/b^2*a^2*f*ln(exp(2*I*(
b*x+a))+I)-1/4/b^2*f*(I*b*x+I*a)^2*ln(1+I*exp(2*I*(b*x+a)))-1/4/b^2*f*(I*b*
x+I*a)*polylog(2,-I*exp(2*I*(b*x+a)))-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))-1))^3*x^2-1/2*(-1/2*f*x^2-e*x)*ln(exp(2*I*(b*x+a))+I)+1
/4*I*Pi*x*e*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2
*I*(b*x+a))-1))^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a)
)-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*Pi*f*cs
gn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a)
)+I)/(exp(2*I*(b*x+a))-1))^2*x^2+1/4/b^2*a^2*f*ln(-exp(2*I*(b*x+a))+I)-1/8*
I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2*x^2-1/4*I*Pi
*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*Pi*f*csg
n((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2-1/4*I*Pi*x*e*csgn(
(1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/4*I*Pi*x*e*csgn(I*(exp
(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp
(2*I*(b*x+a))-1))-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))+I))*csgn(I/(exp(2*I
*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))-1/8*I*Pi*f*
csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((1-I)*(exp(2*I*(b*x+
a))+I)/(exp(2*I*(b*x+a))-1))*x^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))+I))*cs
gn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1)
```

```
)*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*x+a))-1)
)*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))+1/8*I*Pi*f*csgn(I*(exp(
2*I*(b*x+a))-I))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(
exp(2*I*(b*x+a))-1))*x^2+1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*
(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))+1/8*I*Pi
*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b
*x+a))-I)/(exp(2*I*(b*x+a))-1))*x^2-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I
))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*Pi*f*csgn(I*(e
xp(2*I*(b*x+a))-I))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2
-1/4*I*Pi*x*e*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*
(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2-1/8*I*Pi*f*csgn(I*(exp(2*I*(b*
x+a))-I)/(exp(2*I*(b*x+a))-1))*csgn((1+I)*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*
x+a))-1))^2*x^2+1/2*I/b^2*f*a*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))+1
/2*I/b^2*f*a*(I*b*x+I*a)*ln(1-exp(I*(b*x+a))*(-1)^(3/4))+1/2*I/b*e*(I*b*x+I
*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)))/(-I)^(1/2))+1/2*I/b^2*f*a*dilog(1-exp(I*
(b*x+a))*(-1)^(3/4))+1/2*I/b*e*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(b*x+a)))/(
-I)^(1/2))+1/4*I*Pi*x*e*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1
))^3+1/8*I*Pi*f*csgn((1-I)*(exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3*x^2
+1/8*I*Pi*f*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^3*x^2+1/2*I/b
^2*f*a*dilog(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)+e
xp(I*(b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)-exp(I*(
b*x+a)))/(-I)^(1/2))-1/2*I/b^2*f*a*(I*b*x+I*a)*ln(((-I)^(1/2)+exp(I*(b*x+a)
))/(-I)^(1/2))-1/2*I/b^2*f*a*dilog(((-I)^(1/2)-exp(I*(b*x+a)))/(-I)^(1/2))-
1/2*I/b*e*(I*b*x+I*a)*ln(1+exp(I*(b*x+a))*(-1)^(3/4))-1/2*I/b*e*(I*b*x+I*a)
*ln(1-exp(I*(b*x+a))*(-1)^(3/4))-1/8*I*Pi*f*csgn(I/(exp(2*I*(b*x+a))-1))*cs
gn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2*x^2-1/4*I*Pi*x*e*csgn(I/(
exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))-I)/(exp(2*I*(b*x+a))-1))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\cos\left(2\,bx+2\,a\right)^2+2\,\sin\left(2\,bx+2\,a\right)^2+4\,\sin\left(2\,bx+2\,a\right)+2\right)-\frac{1}{8}\left(fx^2+2\,ex\right)\log\left(2\,\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2*b*x + 2*a)^2 + 4*sin
(2*b*x + 2*a) + 2) - 1/8*(f*x^2 + 2*e*x)*log(2*cos(2*b*x + 2*a)^2 + 2*sin(2
*b*x + 2*a)^2 - 4*sin(2*b*x + 2*a) + 2) - integrate(((b*f*x^2 + 2*b*e*x)*co
s(4*b*x + 4*a)*cos(2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) + (b*f*x^2 + 2*b*e*x)*cos(2*b*x + 2*a))/(cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{atanh}\left(\cot\left(a+b\,x\right)\right)\,\left(e+f\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x))*(e + f*x),x)
```

```
[Out] int(atanh(cot(a + b*x))*(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e+fx\right)\operatorname{atanh}\left(\cot\left(a+bx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*atanh(cot(b*x+a)),x)
```

```
[Out] Integral((e + f*x)*atanh(cot(a + b*x)), x)
```

3.332 $\int \tanh^{-1}(\cot(a+bx))\,dx$

Optimal. Leaf size=79

$$-\frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\cot(a+bx))$$

```
[Out] I*x*arctan(exp(2*I*(b*x+a)))+x*arctanh(cot(b*x+a))-1/4*I*polylog(2,-I*exp(2
*I*(b*x+a)))/b+1/4*I*polylog(2,I*exp(2*I*(b*x+a)))/b
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6249, 4181, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{PolyLog}\left(2,ie^{2i(a+bx)}\right)}{4b}+ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\cot(a+bx))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[Cot[a + b*x]],x]
```

```
[Out] I*x*ArcTan[E^((2*I)*(a + b*x))] + x*ArcTanh[Cot[a + b*x]] - ((I/4)*PolyLog[
2, (-I)*E^((2*I)*(a + b*x))])/b + ((I/4)*PolyLog[2, I*E^((2*I)*(a + b*x))])
/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6249

```
Int[ArcTanh[Cot[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTanh[Cot[a + b
*x]], x] - Dist[b, Int[x*Sec[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(\cot(a+bx))\,dx &= x\tanh^{-1}(\cot(a+bx))-b\int x\sec(2a+2bx)\,dx\\ &= ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\cot(a+bx))+\frac{1}{2}\int\log\left(1-ie^{i(2a+2bx)}\right)dx-\frac{1}{2}\int\log\\ &= ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\cot(a+bx))-\frac{i\,\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{i(2a+2bx)}\right)}{4b}+-\ i\\ &= ix\tan^{-1}\left(e^{2i(a+bx)}\right)+x\tanh^{-1}(\cot(a+bx))-\frac{i\text{Li}_2\left(-ie^{2i(a+bx)}\right)}{4b}+\frac{i\text{Li}_2\left(ie^{2i(a+bx)}\right)}{4b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.94

$$\frac{i\left(-\text{Li}_2\left(-ie^{2i(a+bx)}\right)+\text{Li}_2\left(ie^{2i(a+bx)}\right)+4bx\left(\tan^{-1}\left(e^{2i(a+bx)}\right)-i\tanh^{-1}(\cot(a+bx))\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[Cot[a + b*x]],x]
```

```
[Out] ((I/4)*(4*b*x*(ArcTan[E^((2*I)*(a + b*x))] - I*ArcTanh[Cot[a + b*x]]) - Pol
yLog[2, (-I)*E^((2*I)*(a + b*x))] + PolyLog[2, I*E^((2*I)*(a + b*x))]))/b
```

fricas [B] time = 0.61, size = 389, normalized size = 4.92

$$4\,bx\log\left(-\frac{\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a)+1}{\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a)+1}\right)+2\,a\log\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)+i\right)-2\,a\log\left(\cos\left(2\,bx+2\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1)/(cos(2*b*x + 2*a)
 - sin(2*b*x + 2*a) + 1)) + 2*a*log(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) +
 I) - 2*a*log(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + I) - 2*(b*x + a)*log(
I*cos(2*b*x + 2*a) + sin(2*b*x + 2*a) + 1) + 2*(b*x + a)*log(I*cos(2*b*x +
2*a) - sin(2*b*x + 2*a) + 1) - 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) + sin(2*
b*x + 2*a) + 1) + 2*(b*x + a)*log(-I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a) +
1) + 2*a*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + I) - 2*a*log(-cos(2*b
*x + 2*a) - I*sin(2*b*x + 2*a) + I) + I*dilog(I*cos(2*b*x + 2*a) + sin(2*b*
x + 2*a)) + I*dilog(I*cos(2*b*x + 2*a) - sin(2*b*x + 2*a)) - I*dilog(-I*cos
(2*b*x + 2*a) + sin(2*b*x + 2*a)) - I*dilog(-I*cos(2*b*x + 2*a) - sin(2*b*x
 + 2*a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \text{artanh}\left(\cot\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(cot(b*x + a)), x)
```

maple [B] time = 0.42, size = 160, normalized size = 2.03

$$-\frac{i\,\text{arctanh}\left(\cot\left(bx+a\right)\right)\ln\left(1-\frac{i(\cot(bx+a)+1)^2}{-\left(\cot^2(bx+a)\right)+1}\right)}{2b}+\frac{i\,\text{arctanh}\left(\cot\left(bx+a\right)\right)\ln\left(1+\frac{i(\cot(bx+a)+1)^2}{-\left(\cot^2(bx+a)\right)+1}\right)}{2b}+\frac{i\,\text{dilog}\left(1+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cot(b*x+a)),x)
```

```
[Out] -1/2*I/b*arctanh(cot(b*x+a))*ln(1-I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1))+1/2
*I/b*arctanh(cot(b*x+a))*ln(1+I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1))+1/4*I/b
*dilog(1+I*(cot(b*x+a)+1)^2/(-cot(b*x+a)^2+1))-1/4*I/b*dilog(1-I*(cot(b*x+a
)+1)^2/(-cot(b*x+a)^2+1))
```

maxima [B] time = 0.47, size = 184, normalized size = 2.33

$$4\left(bx+a\right)\text{artanh}\left(\frac{1}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2},\frac{1}{2}\,\tan\left(bx+a\right)+\frac{1}{2}\right)-\arctan\left(\frac{1}{2}\,\tan\left(bx+a\right)-\frac{1}{2},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(1/tan(b*x + a)) + (arctan2(1/2*tan(b*x + a) + 1/2,
 1/2*tan(b*x + a) + 1/2) - arctan2(1/2*tan(b*x + a) - 1/2, -1/2*tan(b*x + a
) + 1/2))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log(1/2*tan(b*x + a)^2 + tan(
b*x + a) + 1/2) + (b*x + a)*log(1/2*tan(b*x + a)^2 - tan(b*x + a) + 1/2) -
I*dilog((1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2) + I*dilog(-(1/2*I - 1/2)*
tan(b*x + a) + 1/2*I + 1/2) + I*dilog((1/2*I - 1/2)*tan(b*x + a) + 1/2*I +
1/2) - I*dilog(-(1/2*I + 1/2)*tan(b*x + a) - 1/2*I + 1/2))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(\cot\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x)),x)
```

```
[Out] int(atanh(cot(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(\cot\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cot(b*x+a)),x)
```

```
[Out] Integral(atanh(cot(a + b*x)), x)
```

3.333 $\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx}\,dx$

Optimal. Leaf size=18

$$\operatorname{Int}\left(\frac{\tanh^{-1}(\cot(a+bx))}{e+fx},x\right)$$

```
[Out] CannotIntegrate(arctanh(cot(b*x+a))/(f*x+e),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[Cot[a + b*x]]/(e + f*x),x]
[Out] Defer[Int][ArcTanh[Cot[a + b*x]]/(e + f*x), x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx}\,dx=\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Mathematica [A] time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(\cot(a+bx))}{e+fx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[Cot[a + b*x]]/(e + f*x),x]
[Out] Integrate[ArcTanh[Cot[a + b*x]]/(e + f*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{artanh}\left(\cot\left(bx+a\right)\right)}{fx+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="fricas")
[Out] integral(arctanh(cot(b*x + a))/(f*x + e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="giac")
[Out] integrate(arctanh(cot(b*x + a))/(f*x + e), x)
```

maple [A] time = 3.62, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(cot(b*x+a))/(f*x+e),x)
```

```
[Out] int(arctanh(cot(b*x+a))/(f*x+e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(\cot\left(bx+a\right)\right)}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(cot(b*x+a))/(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(cot(b*x + a))/(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}\left(\cot\left(a+b\,x\right)\right)}{e+f\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(cot(a + b*x))/(e + f*x),x)
```

```
[Out] int(atanh(cot(a + b*x))/(e + f*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}\left(\cot\left(a+bx\right)\right)}{e+fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(cot(b*x+a))/(f*x+e),x)
```

```
[Out] Integral(atanh(cot(a + b*x))/(e + f*x), x)
```

3.334 $\int x^2 \tanh^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=391

$$\frac{i\text{Li}_4\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3}-\frac{i\text{Li}_4\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^3}+\frac{x\text{Li}_3\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{x\text{Li}_3\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}-\frac{ix^2\text{Li}_2\left(\frac{(-c-id+1)e^{2ia}}{-c+id+1}\right.}{4b}$$

```
[Out] 1/3*x^3*arctanh(c+d*cot(b*x+a))+1/6*x^3*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(
1-c+I*d))-1/6*x^3*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x^2*po
lylog(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x^2*polylog(2,(1+c+
I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/4*x*polylog(3,(1-c-I*d)*exp(2*I*a+2*
I*b*x)/(1-c+I*d))/b^2-1/4*x*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d
))/b^2+1/8*I*polylog(4,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b^3-1/8*I*po
lylog(4,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^3
```

Rubi [A] time = 0.49, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6269, 2190, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b^2}-\frac{x\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b^2}+\frac{i\text{PolyLog}\left(4,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^3}-\frac{i\text{PolyLog}\left(4,\frac{(c+}{}\right.}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Cot[a + b*x]])/3 + (x^3*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/6 - (x^3*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/6 - ((I/4)*x^2*PolyLog[2, ((1 - c - I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + (x*PolyLog[3, ((1 - c - I*d
)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/(4*b^2) - (x*PolyLog[3, ((1 + c
+ I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/(4*b^2) + ((I/8)*PolyLog[4,
 ((1 - c - I*d)*E^((2*I)*a + (2*I)*b*x))/(1 - c + I*d)])/b^3 - ((I/8)*PolyL
og[4, ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b^3
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6269

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(c+d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{3}(b(i(1+c)-d))\int \frac{e^{2ia+2ibx}x^3}{1+c-id+(-1-c-$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log$$

$$= \frac{1}{3}x^3\tanh^{-1}(c+d\cot(a+bx)) + \frac{1}{6}x^3\log\left(1-\frac{(1-c-id)e^{2ia+2ibx}}{1-c+id}\right) - \frac{1}{6}x^3\log$$

Mathematica [A] time = 0.97, size = 339, normalized size = 0.87

$$\frac{1}{3}x^3\tanh^{-1}(d\cot(a+bx)+c)+\frac{4b^3x^3\log\left(1-\frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right)-4b^3x^3\log\left(1-\frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right)-6ib^2x^2\text{Li}_2\left(\frac{(c+id-1)e^{2i}}{c-id-}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[c + d*Cot[a + b*x]])/3 + (4*b^3*x^3*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 4*b^3*x^3*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (6*I)*b^2*x^2*PolyLog[2, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] + (6*I)*b^2*x^2*PolyLog[2, ((1 + c + I*
d)*E^((2*I)*(a + b*x)))/(1 + c - I*d)] + 6*b*x*PolyLog[3, ((-1 + c + I*d)*E
^((2*I)*(a + b*x)))/(-1 + c - I*d)] - 6*b*x*PolyLog[3, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + (3*I)*PolyLog[4, ((-1 + c + I*d)*E^((2*I)*
(a + b*x)))/(-1 + c - I*d)] - (3*I)*PolyLog[4, ((1 + c + I*d)*E^((2*I)*(a +
 b*x)))/(1 + c - I*d)])/(24*b^3)
```

fricas [C] time = 1.02, size = 1799, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^3*x^3*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 6*I*b^2*x^2*dilog(-(c^
2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2
+ 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 +
 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2
 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*si
n(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 6*I*b^2*x^2*dilog(-(
c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^
2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2
 - 2*c + 1) + 1) + 6*I*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d
^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*
sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) + 4*a^3*log(1/2*c^2
+ I*(c + 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*
(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 4*a^3*log(1/2*c^2
 + I*(c - 1)*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2
*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 4*a^3*log(-1/2*c
^2 + I*(c + 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1
/2*(I*c^2 + I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 4*a^3*log(-1/2
*c^2 + I*(c - 1)*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) +
 1/2*(I*c^2 + I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 6*b*x*polylo
g(3, ((c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(
c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - 6*
b*x*polylog(3, ((c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-
I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c
 + 1)) + 6*b*x*polylog(3, ((c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x
+ 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 +
 d^2 - 2*c + 1)) + 6*b*x*polylog(3, ((c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*
cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2
*a))/(c^2 + d^2 - 2*c + 1)) - 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I
*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^
2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) - 4*(b^3*
x^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x
 + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c
+ 1)/(c^2 + d^2 + 2*c + 1)) + 4*(b^3*x^3 + a^3)*log((c^2 + d^2 - (c^2 + 2*I
*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^
2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 4*(b^3*
x^3 + a^3)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x
 + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c
+ 1)/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2 + 2*I*(c + 1)*d - d^2 +
2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2
*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 - 2*I*(c + 1)*d
- d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2 - 2*I*c -
 I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + 3*I*polylog(4, ((c^2 + 2*I*(
```

```
c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*d^2 -
 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) - 3*I*polylog(4, ((c^2
 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d
+ I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}\left(d \cot\left(bx + a\right) + c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*cot(b*x + a) + c), x)
```

maple [C] time = 51.09, size = 6775, normalized size = 17.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}\, x^3 \log\left(\left(c^2 + d^2 + 2\, c + 1\right) \cos\left(2\, bx + 2\, a\right)^2 + 4\, (c+1) d \sin\left(2\, bx + 2\, a\right) + \left(c^2 + d^2 + 2\, c + 1\right) \sin\left(2\, bx + 2\, a\right)^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/12*x^3*log((c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b
*x + 2*a) + (c^2 + d^2 + 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 -
 d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 2*c + 1) - 1/12*x^3*log((c^2 + d^2 - 2*c
 + 1)*cos(2*b*x + 2*a)^2 + 4*(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c
+ 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2
*a) - 2*c + 1) - 4*b*d*integrate(1/3*(2*(c^2 + d^2 - 1)*x^3*cos(2*b*x + 2*a
)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^3*sin(2*b*x + 2*a)^2
 - (c^2 - d^2 - 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*x + 2*a) + (c^
2 - d^2 - 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^3*cos(2*b*x
+ 2*a) - (c^2 - d^2 - 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4
 + 2*(c^2 + 1)*d^2 + (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x +
4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (
c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4
+ 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 -
2*(3*c^2 - 1)*d^2 - 2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*
(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4
 - 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c
^3 - c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin
(4*b*x + 4*a) + 8*(c*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\, \mathrm{atanh}\left(c + d \cot\left(a + b\, x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(c + d*cot(a + b*x)),x)
```

```
[Out] int(x^2*atanh(c + d*cot(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}(c + d \cot(a + bx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(c+d*cot(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(c + d*cot(a + b*x)), x)
```

3.335 $\int x \tanh^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=293

$$\frac{\text{Li}_3\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2}-\frac{\text{Li}_3\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2}-\frac{ix\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{4}x^2\log\left(1-\frac{(-c-id}{-c}\right.$$

```
[Out] 1/2*x^2*arctanh(c+d*cot(b*x+a))+1/4*x^2*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(
1-c+I*d))-1/4*x^2*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*x*poly
log(2,(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*x*polylog(2,(1+c+I*d)
*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b+1/8*polylog(3,(1-c-I*d)*exp(2*I*a+2*I*b*x)
/(1-c+I*d))/b^2-1/8*polylog(3,(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))/b^2
```

Rubi [A] time = 0.40, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {6269, 2190, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{8b^2}-\frac{\text{PolyLog}\left(3,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{8b^2}-\frac{ix\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{ix\text{PolyLog}\left(2,\frac{(c+id+}{c-}\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Cot[a + b*x]])/2 + (x^2*Log[1 - ((1 - c - I*d)*E^((2*I)*
a + (2*I)*b*x))/(1 - c + I*d)])/4 - (x^2*Log[1 - ((1 + c + I*d)*E^((2*I)*a
+ (2*I)*b*x))/(1 + c - I*d)])/4 - ((I/4)*x*PolyLog[2, ((1 - c - I*d)*E^((2*
I)*a + (2*I)*b*x))/(1 - c + I*d)])/b + ((I/4)*x*PolyLog[2, ((1 + c + I*d)*E
^((2*I)*a + (2*I)*b*x))/(1 + c - I*d)])/b + PolyLog[3, ((1 - c - I*d)*E^((2
*I)*a + (2*I)*b*x))/(1 - c + I*d)]/(8*b^2) - PolyLog[3, ((1 + c + I*d)*E^((
2*I)*a + (2*I)*b*x))/(1 + c - I*d)]/(8*b^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6269

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
```

```
 + 1)), x] + (-Dist[(I*b*(1 - c - I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)
*E^(2*I*a + 2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x]
, x] + Dist[(I*b*(1 + c + I*d))/(f*(m + 1)), Int[((e + f*x)^(m + 1)*E^(2*I*
a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)*E^(2*I*a + 2*I*b*x)), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x \tanh^{-1}(c + d\cot(a + bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(c + d\cot(a + bx)) + \frac{1}{2}(b(i(1 + c) - d))\int \frac{e^{2ia+2ibx}x}{1 + c - id + (-1 - c \cdots}$$

$$= \frac{1}{2}x^2\tanh^{-1}(c + d\cot(a + bx)) + \frac{1}{4}x^2\log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \mathrm{l}\cdots$$

$$= \frac{1}{2}x^2\tanh^{-1}(c + d\cot(a + bx)) + \frac{1}{4}x^2\log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \mathrm{l}\cdots$$

$$= \frac{1}{2}x^2\tanh^{-1}(c + d\cot(a + bx)) + \frac{1}{4}x^2\log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \mathrm{l}\cdots$$

$$= \frac{1}{2}x^2\tanh^{-1}(c + d\cot(a + bx)) + \frac{1}{4}x^2\log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{4}x^2 \mathrm{l}\cdots$$

Mathematica [A] time = 0.57, size = 253, normalized size = 0.86

$$\frac{1}{2}x^2\tanh^{-1}(d\cot(a+bx)+c)+\frac{2b^2x^2\log\left(1 - \frac{(c+id-1)e^{2i(a+bx)}}{c-id-1}\right) - 2b^2x^2\log\left(1 - \frac{(c+id+1)e^{2i(a+bx)}}{c-id+1}\right) - 2ibx\mathrm{Li}_2\left(\frac{(c+id-1)e^{\cdots}}{c-id-\cdots}\right.\cdots}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[c + d*Cot[a + b*x]])/2 + (2*b^2*x^2*Log[1 - ((-1 + c + I*d)*E^
((2*I)*(a + b*x)))/(-1 + c - I*d)] - 2*b^2*x^2*Log[1 - ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] - (2*I)*b*x*PolyLog[2, ((-1 + c + I*d)*E^((2
*I)*(a + b*x)))/(-1 + c - I*d)] + (2*I)*b*x*PolyLog[2, ((1 + c + I*d)*E^((2
*I)*(a + b*x)))/(1 + c - I*d)] + PolyLog[3, ((-1 + c + I*d)*E^((2*I)*(a + b
*x)))/(-1 + c - I*d)] - PolyLog[3, ((1 + c + I*d)*E^((2*I)*(a + b*x)))/(1 +
 c - I*d)])/(8*b^2)
```

fricas [C] time = 0.64, size = 1463, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b^2*x^2*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d
*cos(2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*I*b*x*dilog(-(c^2 +
```

```
d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*
(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c
 + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c +
 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x
+ 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - 2*I*b*x*dilog(-(c^2 + d^2 -
(c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1
)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)
+ 1) + 2*I*b*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*co
s(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a)
 - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1) - 2*a^2*log(1/2*c^2 + I*(c + 1)*d -
1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 +
 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) + 2*a^2*log(1/2*c^2 + I*(c - 1)*d -
 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2
- 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) - 2*a^2*log(-1/2*c^2 + I*(c + 1)*d
 + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^
2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) + 2*a^2*log(-1/2*c^2 + I*(c - 1)
*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*
d^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b^2*x^2 - a^2)*log((c^2 +
 d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2
*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*
c + 1)) - 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*
b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 +
 d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2
*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*
c + 1)) + 2*(b^2*x^2 - a^2)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2
*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*
b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) - polylog(3, ((c^2 + 2*I*(c +
1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c + 1)*d - I*d^2 + 2*I
*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) - polylog(3, ((c^2 - 2*I*(
c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c + 1)*d + I*d^2
- 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*c + 1)) + polylog(3, ((c^2 +
2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*(c - 1)*d - I*
d^2 - 2*I*c + I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)) + polylog(3, ((c^
2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 - 2*(c - 1)*d
 + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a))/(c^2 + d^2 - 2*c + 1)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*cot(b*x + a) + c), x)
```

maple [C] time = 4.97, size = 6425, normalized size = 21.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(c+d*cot(b*x+a)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2\,bd\int \frac{}{c^4+d^4+2\left(c^2+1\right)d^2+\left(c^4+d^4+2\left(c^2+1\right)d^2-2\,c^2+1\right)\cos\left(4\,bx+4\,a\right)^2+4\left(c^4+d^4+2\left(c^2-1\right)d^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -2*b*d*integrate((2*(c^2 + d^2 - 1)*x^2*cos(2*b*x + 2*a)^2 + 2*c*d*x^2*sin(
2*b*x + 2*a) + 2*(c^2 + d^2 - 1)*x^2*sin(2*b*x + 2*a)^2 - (c^2 - d^2 - 1)*x
^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x + 2*a) + (c^2 - d^2 - 1)*x^2*cos
(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2*cos(2*b*x + 2*a) - (c^2 - d^2
- 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a))/(c^4 + d^4 + 2*(c^2 + 1)*d^2 +
 (c^4 + d^4 + 2*(c^2 + 1)*d^2 - 2*c^2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^
4 + 2*(c^2 - 1)*d^2 - 2*c^2 + 1)*cos(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 +
 1)*d^2 - 2*c^2 + 1)*sin(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 - 1)*d^2 -
2*c^2 + 1)*sin(2*b*x + 2*a)^2 - 2*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 - 1)*d^2 -
2*c^2 - 2*(c^4 - d^4 - 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 - c)*d
)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) - 4*(c^4 - d^4 - 2*c^2 + 1)*cos(2*
b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 - c)*d + 2*(c*d^3 + (c^3 - c)*d)*cos(2*b*x
 + 2*a) - (c^4 - d^4 - 2*c^2 + 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c
*d^3 + (c^3 - c)*d)*sin(2*b*x + 2*a) + 1), x) + 1/8*x^2*log((c^2 + d^2 + 2*
c + 1)*cos(2*b*x + 2*a)^2 + 4*(c + 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 + 2*c
 + 1)*sin(2*b*x + 2*a)^2 + c^2 + d^2 - 2*(c^2 - d^2 + 2*c + 1)*cos(2*b*x +
2*a) + 2*c + 1) - 1/8*x^2*log((c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a)^2 + 4*
(c - 1)*d*sin(2*b*x + 2*a) + (c^2 + d^2 - 2*c + 1)*sin(2*b*x + 2*a)^2 + c^2
 + d^2 - 2*(c^2 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) - 2*c + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \operatorname{atanh}\left(c+d\cot\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(c + d*cot(a + b*x)),x)
```

```
[Out] int(x*atanh(c + d*cot(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atanh}\left(c+d\cot\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(c+d*cot(b*x+a)),x)
```

```
[Out] Integral(x*atanh(c + d*cot(a + b*x)), x)
```

3.336 $\int \tanh^{-1}(c + d\cot(a + bx))\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{Li}_2\left(\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{i\text{Li}_2\left(\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c+id+1)e^{2ia+2ib}}{c-id+1}\right.$$

```
[Out] x*arctanh(c+d*cot(b*x+a))+1/2*x*ln(1-(1-c-I*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d)
)-1/2*x*ln(1-(1+c+I*d)*exp(2*I*a+2*I*b*x)/(1+c-I*d))-1/4*I*polylog(2,(1-c-I
*d)*exp(2*I*a+2*I*b*x)/(1-c+I*d))/b+1/4*I*polylog(2,(1+c+I*d)*exp(2*I*a+2*I
*b*x)/(1+c-I*d))/b
```

Rubi [A] time = 0.24, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {6261, 2190, 2279, 2391}

$$-\frac{i\text{PolyLog}\left(2,\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)}{4b}+\frac{i\text{PolyLog}\left(2,\frac{(c+id+1)e^{2ia+2ibx}}{c-id+1}\right)}{4b}+\frac{1}{2}x\log\left(1-\frac{(-c-id+1)e^{2ia+2ibx}}{-c+id+1}\right)-\frac{1}{2}x\log\left(1-\frac{(c}{}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Cot[a + b*x]] + (x*Log[1 - ((1 - c - I*d)*E^((2*I)*a + (2*I
)*b*x))/(1 - c + I*d)])/2 - (x*Log[1 - ((1 + c + I*d)*E^((2*I)*a + (2*I)*b*
x))/(1 + c - I*d)])/2 - ((I/4)*PolyLog[2, ((1 - c - I*d)*E^((2*I)*a + (2*I)
*b*x))/(1 - c + I*d)])/b + ((I/4)*PolyLog[2, ((1 + c + I*d)*E^((2*I)*a + (2
*I)*b*x))/(1 + c - I*d)])/b
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6261

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + (-Dist[I*b*(1 - c - I*d), Int[(x*E^(2*I*a +
2*I*b*x))/(1 - c + I*d - (1 - c - I*d)*E^(2*I*a + 2*I*b*x)), x], x] + Dist[
I*b*(1 + c + I*d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + c - I*d - (1 + c + I*d)
*E^(2*I*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - I*d)^2
, 1]
```

Rubi steps

$$\int \tanh^{-1}(c + d\cot(a + bx))\,dx = x\tanh^{-1}(c + d\cot(a + bx)) + (b(i(1 + c) - d))\int \frac{e^{2ia+2ibx}x}{1 + c - id + (-1 - c - id)e^{2}}$$

$$= x\tanh^{-1}(c + d\cot(a + bx)) + \frac{1}{2}x\log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{2}x\log\left(1 -\right.$$

$$= x\tanh^{-1}(c + d\cot(a + bx)) + \frac{1}{2}x\log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{2}x\log\left(1 -\right.$$

$$= x\tanh^{-1}(c + d\cot(a + bx)) + \frac{1}{2}x\log\left(1 - \frac{(1 - c - id)e^{2ia+2ibx}}{1 - c + id}\right) - \frac{1}{2}x\log\left(1 -\right.$$

Mathematica [B] time = 31.71, size = 4463, normalized size = 23.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[c + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[c + d*Cot[a + b*x]] - (d*(a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b
*x] + (-1 + c)*Sin[a + b*x]))] - a*Log[-(Sec[(a + b*x)/2]^2*(d*Cos[a + b*x]
 + Sin[a + b*x] + c*Sin[a + b*x]))] - (a + b*x)*Log[-((-1 + c + Sqrt[1 - 2*
c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(-I + Tan[(a + b*x)/2]))/
(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c +
c^2 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(I + Tan[(a + b*x)/2]))/(-1 +
 c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 +
 d^2])/d) + Tan[(a + b*x)/2]] + (a + b*x)*Log[-((1 + c + Sqrt[1 + 2*c + c^2
 + d^2])/d) + Tan[(a + b*x)/2]] + I*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c
- I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2
])/d) + Tan[(a + b*x)/2]] - I*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d +
 Sqrt[1 + 2*c + c^2 + d^2])]*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) +
 Tan[(a + b*x)/2]] - (a + b*x)*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d] - I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(1 - c + I*d + Sq
rt[1 - 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(
a + b*x)/2])/d] + I*Log[-((d*(I + Tan[(a + b*x)/2]))/(1 - c - I*d + Sqrt[1
- 2*c + c^2 + d^2]))]*Log[(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b
*x)/2])/d] + (a + b*x)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a +
 b*x)/2])/d] + I*Log[-((d*(-I + Tan[(a + b*x)/2]))/(-1 - c + I*d + Sqrt[1 +
 2*c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b
*x)/2])/d] - I*Log[-((d*(I + Tan[(a + b*x)/2]))/(-1 - c - I*d + Sqrt[1 + 2*
c + c^2 + d^2]))]*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)
/2])/d] - I*PolyLog[2, (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x
)/2])/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*PolyLog[2, (-1 + c +
Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c + I*d + Sqrt[1 - 2*
c + c^2 + d^2])] - I*PolyLog[2, (1 + c - Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[
(a + b*x)/2])/(1 + c + I*d - Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2, (1
+ c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c - I*d + Sqrt[1
 + 2*c + c^2 + d^2])] - I*PolyLog[2, (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d
*Tan[(a + b*x)/2])/(1 + c + I*d + Sqrt[1 + 2*c + c^2 + d^2])] + I*PolyLog[2
, (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c - I*d + S
qrt[1 - 2*c + c^2 + d^2])] - I*PolyLog[2, (1 - c + Sqrt[1 - 2*c + c^2 + d^2
] + d*Tan[(a + b*x)/2])/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2])] + I*Poly
Log[2, (-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(-1 - c +
I*d + Sqrt[1 + 2*c + c^2 + d^2])])*((2*a)/(b*(1 - c^2 - d^2 - Cos[2*(a + b*
x)] + c^2*Cos[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])
```

```
) - (2*(a + b*x))/(b*(1 - c^2 - d^2 - Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x
)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)]))))/(-Log[-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] + Log[-((1 + c + Sqrt[1 + 2
*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]] - Log[(1 - c + Sqrt[1 - 2*c + c^2 +
 d^2] + d*Tan[(a + b*x)/2])/d] + Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] +
d*Tan[(a + b*x)/2])/d] - ((I/2)*Log[-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/
d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)
*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a +
b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) - ((I/2)*Log[(1 - c + Sqrt[1 - 2*c + c^2
 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]
) + ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]
*Sec[(a + b*x)/2]^2)/(-I + Tan[(a + b*x)/2]) + ((I/2)*Log[-((-1 + c + Sqrt[
1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(
a + b*x)/2]) - ((I/2)*Log[-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2]]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) + ((I/2)*Log[(1 - c
+ Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I
 + Tan[(a + b*x)/2]) - ((I/2)*Log[(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/d]*Sec[(a + b*x)/2]^2)/(I + Tan[(a + b*x)/2]) - ((a + b*x)
*Sec[(a + b*x)/2]^2)/(2*(-((-1 + c + Sqrt[1 - 2*c + c^2 + d^2])/d) + Tan[(a
 + b*x)/2])) - ((I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(-1 + c - I*d + Sqrt[
1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sqrt[1 - 2*c + c^2
+ d^2])/d) + Tan[(a + b*x)/2]) + ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(-1
+ c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((-1 + c + Sq
rt[1 - 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2]) + ((a + b*x)*Sec[(a + b*x)/
2]^2)/(2*(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(a + b*x)/2])) + (
(I/2)*Log[(d*(-I + Tan[(a + b*x)/2]))/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d
^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d^2])/d) + Tan[(
a + b*x)/2]) - ((I/2)*Log[(d*(I + Tan[(a + b*x)/2]))/(1 + c + I*d + Sqrt[1
+ 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-((1 + c + Sqrt[1 + 2*c + c^2 + d
^2])/d) + Tan[(a + b*x)/2]) - ((I/2)*d*Log[1 - (-1 + c + Sqrt[1 - 2*c + c^2
 + d^2] - d*Tan[(a + b*x)/2])/(-1 + c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*S
ec[(a + b*x)/2]^2)/(-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]
) + ((I/2)*d*Log[1 - (-1 + c + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/
2])/(-1 + c + I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(-1 + c
 + Sqrt[1 - 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - ((I/2)*d*Log[1 - (1 +
c - Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c + I*d - Sqrt[1 +
 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 + c - Sqrt[1 + 2*c + c^2 + d^2]
- d*Tan[(a + b*x)/2]) + ((I/2)*d*Log[1 - (1 + c + Sqrt[1 + 2*c + c^2 + d^2]
 - d*Tan[(a + b*x)/2])/(1 + c - I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a +
b*x)/2]^2)/(1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - ((I/2
)*d*Log[1 - (1 + c + Sqrt[1 + 2*c + c^2 + d^2] - d*Tan[(a + b*x)/2])/(1 + c
 + I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 + c + Sqrt[1 +
2*c + c^2 + d^2] - d*Tan[(a + b*x)/2]) - (d*(a + b*x)*Sec[(a + b*x)/2]^2)/(
2*(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])) - ((I/2)*d*Log[
-((d*(-I + Tan[(a + b*x)/2]))/(1 - c + I*d + Sqrt[1 - 2*c + c^2 + d^2]))]*S
ec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])
 + ((I/2)*d*Log[-((d*(I + Tan[(a + b*x)/2]))/(1 - c - I*d + Sqrt[1 - 2*c +
c^2 + d^2]))]*Sec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Ta
n[(a + b*x)/2]) - ((I/2)*d*Log[1 - (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/(1 - c - I*d + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2
]^2)/(1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) + ((I/2)*d*Lo
g[1 - (1 - c + Sqrt[1 - 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])/(1 - c + I*d
 + Sqrt[1 - 2*c + c^2 + d^2])]*Sec[(a + b*x)/2]^2)/(1 - c + Sqrt[1 - 2*c +
c^2 + d^2] + d*Tan[(a + b*x)/2]) + (d*(a + b*x)*Sec[(a + b*x)/2]^2)/(2*(-1
- c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2])) + ((I/2)*d*Log[-((d*
(-I + Tan[(a + b*x)/2]))/(-1 - c + I*d + Sqrt[1 + 2*c + c^2 + d^2]))]*Sec[(
a + b*x)/2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) -
((I/2)*d*Log[-((d*(I + Tan[(a + b*x)/2]))/(-1 - c - I*d + Sqrt[1 + 2*c + c^
2 + d^2]))]*Sec[(a + b*x)/2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan
```

```
[(a + b*x)/2]) - ((I/2)*d*Log[1 - (-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*T
an[(a + b*x)/2])/(-1 - c + I*d + Sqrt[1 + 2*c + c^2 + d^2])]*Sec[(a + b*x)/
2]^2)/(-1 - c + Sqrt[1 + 2*c + c^2 + d^2] + d*Tan[(a + b*x)/2]) - (a*Cos[(a
 + b*x)/2]^2*(-(Sec[(a + b*x)/2]^2*((-1 + c)*Cos[a + b*x] - d*Sin[a + b*x])
) - Sec[(a + b*x)/2]^2*(d*Cos[a + b*x] + (-1 + c)*Sin[a + b*x])*Tan[(a + b*
x)/2]))/(d*Cos[a + b*x] + (-1 + c)*Sin[a + b*x]) + (a*Cos[(a + b*x)/2]^2*(-
(Sec[(a + b*x)/2]^2*(Cos[a + b*x] + c*Cos[a + b*x] - d*Sin[a + b*x])) - Sec
[(a + b*x)/2]^2*(d*Cos[a + b*x] + Sin[a + b*x] + c*Sin[a + b*x])*Tan[(a + b
*x)/2]))/(d*Cos[a + b*x] + Sin[a + b*x] + c*Sin[a + b*x]))
```

fricas [B] time = 1.00, size = 1099, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(-(d*cos(2*b*x + 2*a) + (c + 1)*sin(2*b*x + 2*a) + d)/(d*cos(
2*b*x + 2*a) + (c - 1)*sin(2*b*x + 2*a) + d)) + 2*a*log(1/2*c^2 + I*(c + 1)
*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*
d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + c + 1/2) - 2*a*log(1/2*c^2 + I*(c - 1)*
d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a) - c + 1/2) + 2*a*log(-1/2*c^2 + I*(c + 1)*
d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 + 2*I*c + I)*sin(2*b*x + 2*a) - c - 1/2) - 2*a*log(-1/2*c^2 + I*(c - 1)*
d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*c + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d
^2 - 2*I*c + I)*sin(2*b*x + 2*a) + c - 1/2) - 2*(b*x + a)*log((c^2 + d^2 -
(c^2 + 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1
)*d + I*d^2 - 2*I*c - I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1))
 - 2*(b*x + a)*log((c^2 + d^2 - (c^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2
*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) +
2*c + 1)/(c^2 + d^2 + 2*c + 1)) + 2*(b*x + a)*log((c^2 + d^2 - (c^2 + 2*I*(
c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2
+ 2*I*c - I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1)) + 2*(b*x +
a)*log((c^2 + d^2 - (c^2 - 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x + 2*a)
+ (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c + I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^
2 + d^2 - 2*c + 1)) + I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c + 1)*d - d^2 + 2*
c + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*(c + 1)*d + I*d^2 - 2*I*c - I)*sin(2*
b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1) - I*dilog(-(c^2 + d^2 - (c
^2 - 2*I*(c + 1)*d - d^2 + 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c + 1)*d
 - I*d^2 + 2*I*c + I)*sin(2*b*x + 2*a) + 2*c + 1)/(c^2 + d^2 + 2*c + 1) + 1
) - I*dilog(-(c^2 + d^2 - (c^2 + 2*I*(c - 1)*d - d^2 - 2*c + 1)*cos(2*b*x +
 2*a) + (-I*c^2 + 2*(c - 1)*d + I*d^2 + 2*I*c - I)*sin(2*b*x + 2*a) - 2*c +
 1)/(c^2 + d^2 - 2*c + 1) + 1) + I*dilog(-(c^2 + d^2 - (c^2 - 2*I*(c - 1)*d
 - d^2 - 2*c + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*(c - 1)*d - I*d^2 - 2*I*c +
 I)*sin(2*b*x + 2*a) - 2*c + 1)/(c^2 + d^2 - 2*c + 1) + 1))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}\left(d\cot\left(bx+a\right)+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + c), x)
```

maple [B] time = 0.47, size = 629, normalized size = 3.24

$$-\frac{\operatorname{arctanh}\left(c+d\cot\left(bx+a\right)\right)\pi}{2b}+\frac{\operatorname{arctanh}\left(c+d\cot\left(bx+a\right)\right)\operatorname{arccot}\left(\cot\left(bx+a\right)\right)}{b}+\frac{\arctan\left(\frac{c+d\cot(bx+a)}{d}-\frac{c}{d}\right)1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*cot(b*x+a)),x)
```

```
[Out] -1/2/b*arctanh(c+d*cot(b*x+a))*Pi+1/b*arctanh(c+d*cot(b*x+a))*arccot(cot(b*
x+a))+1/2/b*arctan((c+d*cot(b*x+a))/d-c/d)*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+
1)-1/2/b*arctan((c+d*cot(b*x+a))/d-c/d)*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)+
1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*ln((I*d-d*((c+d*cot(b*x+a))/d-c/
d))/(1+c+I*d))-1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c+1)*ln((I*d+d*((c+d*c
ot(b*x+a))/d-c/d))/(I*d-c-1))+1/4*I/b*dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d)
)/(1+c+I*d))-1/4*I/b*dilog((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(I*d-c-1))-1/4*
I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)*ln((I*d-d*((c+d*cot(b*x+a))/d-c/d))/
(I*d+c-1))+1/4*I/b*ln(d*((c+d*cot(b*x+a))/d-c/d)+c-1)*ln((I*d+d*((c+d*cot(b
*x+a))/d-c/d))/(1-c+I*d))-1/4*I/b*dilog((I*d-d*((c+d*cot(b*x+a))/d-c/d))/(I
*d+c-1))+1/4*I/b*dilog((I*d+d*((c+d*cot(b*x+a))/d-c/d))/(1-c+I*d))
```

maxima [B] time = 0.53, size = 392, normalized size = 2.02

$$4\,(bx+a)\,\operatorname{artanh}\left(c+\frac{d}{\tan(bx+a)}\right)+\left(\arctan\left(\frac{(c+1)d+\left(c^2+2\,c+1\right)\tan(bx+a)}{c^2+d^2+2\,c+1},\frac{(c+1)d\tan(bx+a)+d^2}{c^2+d^2+2\,c+1}\right)-\arctan\left(\frac{(c-1)d+\left(c^2-2\,c+1\right)\mathrm{ta}}{c^2+d^2-2\,c+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(b*x + a)*arctanh(c + d/tan(b*x + a)) + (arctan2(((c + 1)*d + (c^2 +
 2*c + 1)*tan(b*x + a))/(c^2 + d^2 + 2*c + 1), ((c + 1)*d*tan(b*x + a) + d^
2)/(c^2 + d^2 + 2*c + 1)) - arctan2(((c - 1)*d + (c^2 - 2*c + 1)*tan(b*x +
a))/(c^2 + d^2 - 2*c + 1), ((c - 1)*d*tan(b*x + a) + d^2)/(c^2 + d^2 - 2*c
+ 1)))*log(tan(b*x + a)^2 + 1) - (b*x + a)*log((2*(c + 1)*d*tan(b*x + a) +
(c^2 + 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 + d^2 + 2*c + 1)) + (b*x + a)*lo
g((2*(c - 1)*d*tan(b*x + a) + (c^2 - 2*c + 1)*tan(b*x + a)^2 + d^2)/(c^2 +
d^2 - 2*c + 1)) + I*dilog(-((c + 1)*tan(b*x + a) - I*c - I)/(I*c + d + I))
- I*dilog(-((c - 1)*tan(b*x + a) - I*c + I)/(I*c + d - I)) + I*dilog(-((c -
 1)*tan(b*x + a) + I*c - I)/(-I*c + d + I)) - I*dilog(-((c + 1)*tan(b*x + a
) + I*c + I)/(-I*c + d - I)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(c+d\cot\left(a+b\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*cot(a + b*x)),x)
```

```
[Out] int(atanh(c + d*cot(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(c+d\cot\left(a+bx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*cot(b*x+a)),x)
```

```
[Out] Integral(atanh(c + d*cot(a + b*x)), x)
```

3.337 $\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x} \, dx$

Optimal. Leaf size=18

$$\mathrm{Int}\left(\frac{\tanh^{-1}(d\cot(a+bx)+c)}{x},x\right)$$

```
[Out] CannotIntegrate(arctanh(c+d*cot(b*x+a))/x,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[c + d*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[c + d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x} \, dx = \int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x} \, dx$$

Mathematica [A] time = 5.26, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(c+d\cot(a+bx))}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[c + d*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[c + d*Cot[a + b*x]]/x, x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\,(d\cot\,(bx+a)+c)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(arctanh(d*cot(b*x + a) + c)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{artanh}\,(d\cot\,(bx+a)+c)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="giac")
[Out] integrate(arctanh(d*cot(b*x + a) + c)/x, x)
```

maple [A] time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(c+d\cot(bx+a))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c+d*cot(b*x+a))/x,x)
[Out] int(arctanh(c+d*cot(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}(d\cot(bx+a)+c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c+d*cot(b*x+a))/x,x, algorithm="maxima")
[Out] integrate(arctanh(d*cot(b*x + a) + c)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\operatorname{atanh}(c+d\cot(a+b\,x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(c + d*cot(a + b*x))/x,x)
[Out] int(atanh(c + d*cot(a + b*x))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(c+d\cot(a+bx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c+d*cot(b*x+a))/x,x)
[Out] Integral(atanh(c + d*cot(a + b*x))/x, x)
```

3.338 $\int x^2 \tanh^{-1}(1 + id + d\cot(a + bx))\,dx$

Optimal. Leaf size=168

$$-\frac{i\text{Li}_4\left((id+1)e^{2ia+2ibx}\right)}{8b^3}-\frac{x\text{Li}_3\left((id+1)e^{2ia+2ibx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3$$

```
[Out] 1/12*I*b*x^4+1/3*x^3*arctanh(1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1+I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*poly
log(3,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1+I*d)*exp(2*I*a+2*I
*b*x))/b^3
```

Rubi [A] time = 0.30, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {6265, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1+id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
+ I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 + I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6265

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(-1-id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) + \frac{1}{3}(b(i-d))\int \frac{e^{2ia+}}{1+(-1-}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+id)e^{2ia}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+id)e^{2ia}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+id)e^{2ia}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+id)e^{2ia}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1+id)e^{2ia}\right.$$

Mathematica [A] time = 0.46, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\tanh^{-1}(d\cot(a+bx)+id+1) - \frac{4b^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{d-i}\right) + 6ib^2x^2\mathrm{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right) + 6bx\mathrm{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right) - 3i\mathrm{Li}_4\left(\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 + I*d + d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a
 + b*x)))] + 6*b*x*PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))] - (3*I)*
PolyLog[4, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

fricas [C] time = 0.46, size = 180, normalized size = 1.07

$$\frac{2i\,b^4x^4 + 4\,b^3x^3\log\left(-\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right) - 2i\,a^4 + 4\,a^3\log\left(\frac{(d-i)e^{(2i\,bx+2i}}{d-i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 + 4*b^3*x^3*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 6*I*b^2*x^2*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*
I*a^4 + 4*a^3*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*b*x*polylo
g(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((-I*d - 1)*e^(2
*I*b*x + 2*I*a) + 1) - 3*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}\left(d \cot\left(bx + a\right) + i\, d + 1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(d*cot(b*x + a) + I*d + 1), x)
```

maple [C] time = 5.64, size = 2456, normalized size = 14.62

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-
1/2/b^2*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a^2+1/2/b^2*a^2*d/(I-d)*ln
(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x+1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*
x+a))*(I*(I-d))^(1/2))*x-1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x
^2+1/4/b^3/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/3*x^3*ln(exp(I*
(b*x+a)))+1/12*I*b*x^4+1/6*I*x^3*Pi-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(
exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/6*
x^3*ln(d)+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/1
2*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+
a))-1)*exp(2*I*(b*x+a)))^3+1/12*I*x^3*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*
I*(b*x+a)))^3-1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I
)/(exp(2*I*(b*x+a))-1))^3-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2
*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/2/b^3*a^3*d/
(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/8/b^3/(I-d)*polylog(4,-I*(I-
d)*exp(2*I*(b*x+a)))-1/2/b^3*a^2/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(
1/2))-1/2/b^3*a^2/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/6*d/(I-
d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x^3+1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*
d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/12*I*x^3*Pi*csgn(d/(exp(2
*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/6*I/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)
))*x^3-1/12*I*x^3*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*
I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x
+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csg
n(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*Pi*csgn(I*(exp(2*I*
(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a
))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*(exp(
2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*
(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/6*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))*csg
n(I*exp(2*I*(b*x+a)))^2+1/6*x^3*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)
+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*ex
p(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/3*I/b^3/(I-d)*ln(1+I*(I-d)*exp(
2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a
))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/
(exp(2*I*(b*x+a))-1))^2-1/2*I/b^3*a^3/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))
```

```
^(1/2))-1/2*I/b^3*a^3/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/6*I/b^
3*a^3/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)+1/8*I/b^3*d/(I-d)*p
olylog(4,-I*(I-d)*exp(2*I*(b*x+a)))-1/4*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(
2*I*(b*x+a)))*x+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)
))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-1/4*I/b*d/(I-d)*polylog(2,
-I*(I-d)*exp(2*I*(b*x+a)))*x^2+1/4*I/b^3*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I
*(b*x+a)))*a^2+1/2*I/b^2/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a^2-1/2*I/b
^2*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^2*a^2/(I-d)*l
n(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))*x-1/2*I/b^3*a^2*d/(I-d)*dilog(1-I*exp
(I*(b*x+a))*(I*(I-d))^(1/2))-1/2*I/b^3*a^2*d/(I-d)*dilog(1+I*exp(I*(b*x+a))
*(I*(I-d))^(1/2))-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*
I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b
*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn
(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1/12
*I*x^3*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/
(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/2/b^3*a^3*d/(I-d)*ln(1+I*exp(I*(b*
x+a))*(I*(I-d))^(1/2))-1/6/b^3*a^3*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b
*x+a))*d-I)-1/3/b^3*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^3+1/4/b^2*d/(I
-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))*x+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+
a)))^2*csgn(I*exp(2*I*(b*x+a)))
```

maxima [B] time = 0.37, size = 342, normalized size = 2.04

$$\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{artanh}(d\cot(bx+a)+i\,d+1)}{b^2}-\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(bx+a)a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*cot(b*
x + a) + I*d + 1)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
+ a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*a
rctan2(d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*
x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((I*d
 + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)
*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 + 2*d
*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, (I*d
 + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (I*d + 1)*e^(2*I*b*x + 2*I*a)))
/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\mathrm{atanh}\left(d\cot\left(a+b\,x\right)+1+d\,1{}\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(d*1i + d*cot(a + b*x) + 1),x)
```

```
[Out] int(x^2*atanh(d*1i + d*cot(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{atanh}(d\cot(a+bx)+id+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Integral(x**2*atanh(d*cot(a + b*x) + I*d + 1), x)
```

3.339 $\int x \tanh^{-1}(1 + id + d \cot(a + bx)) \, dx$

Optimal. Leaf size=132

$$-\frac{\text{Li}_3\left((id+1)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d\cot(a+bx)+id+$$

```
[Out] 1/6*I*b*x^3+1/2*x^2*arctanh(1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1+I*d)*exp(2*
I*a+2*I*b*x))+1/4*I*x*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(3
,(1+I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] time = 0.25, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6265, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,(1+id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 +
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 + I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 + I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6265

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
```

```
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1+id+d\cot(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1-id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))+\frac{1}{2}(b(i-d))\int\frac{e^{2ia+2il}}{1+(-1-ia}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1+id+d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1+id)e^{2ia+2}\right.$$

Mathematica [A] time = 0.32, size = 119, normalized size = 0.90

$$\frac{1}{2}x^2\tanh^{-1}(d\cot(a+bx)+id+1)-\frac{2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{d-i}\right)+2ibx\text{Li}_2\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right)+\text{Li}_3\left(-\frac{ie^{-2i(a+bx)}}{d-i}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 + I*d + d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + I/((-I + d)*
E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/((-I + d)*E^((2*I)*(a + b
*x)))] + PolyLog[3, (-I)/((-I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

fricas [C] time = 1.12, size = 157, normalized size = 1.19

$$\frac{4i\,b^3x^3+6\,b^2x^2\log\left(-\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)+4i\,a^3+6i\,bx\text{Li}_2\left(-(-i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\,a^2\log\left(\frac{(d-i)e^{(2i\,bx+2i\,a)}+}{d-i}\right.}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 + 6*b^2*x^2*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*
I*b*x - 2*I*a)/d) + 4*I*a^3 + 6*I*b*x*dilog(-(-I*d - 1)*e^(2*I*b*x + 2*I*a)
) - 6*a^2*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) - 6*(b^2*x^2 - a^2
)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (I*d + 1)*e^(2*I*b
*x + 2*I*a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}(d\cot(bx+a) + i\,d + 1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(d*cot(b*x + a) + I*d + 1), x)
```

maple [C] time = 4.46, size = 2358, normalized size = 17.86

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b^2*a*d/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2*I/b^2*a*d
/(I-d)*dilog(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/4*I/b*d/(I-d)*polylog(2,
-I*(I-d)*exp(2*I*(b*x+a)))*x-1/4*I/b^2*d/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(
b*x+a)))*a-1/2*I/b/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a+1/8*I*x^2*Pi*cs
gn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^3-1/8*I*
x^2*Pi*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))
^2+1/6*I*b*x^3+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a
))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*d)
*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1
)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/4/b^2*d/(I-d)
*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2-1/4*x^2*ln(d)-1/2/b^2*a^2*d/(I-d)*ln(1-
I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/2/b^2*a^2*d/(I-d)*ln(1+I*exp(I*(b*x+a))
*(I*(I-d))^(1/2))-1/4/b/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*x-1/4/b^
2/(I-d)*polylog(2,-I*(I-d)*exp(2*I*(b*x+a)))*a+1/4*d/(I-d)*ln(1+I*(I-d)*exp
(2*I*(b*x+a)))*x^2+1/2/b^2*a/(I-d)*dilog(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2)
)-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b
*x+a))-1))^3+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1
/8*I/b^2/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+a)))-1/4*I/(I-d)*ln(1+I*(I-d
)*exp(2*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b
*x+a)))^2-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a)
)*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)
/(exp(2*I*(b*x+a))-1))+1/4*I*x^2*Pi-1/2*x^2*ln(exp(I*(b*x+a)))+1/8*I*x^2*Pi
*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1
)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x
+a)))^3-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a
)))^2+1/4*x^2*ln(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I)+1/8*I*x^2*Pi*csgn
(I*(exp(2*I*(b*x+a))*d-I*exp(2*I*(b*x+a))+I))*csgn(I*(exp(2*I*(b*x+a))*d-I*
exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2+1/2/b^2*a/(I-d)*dilog(1+I*exp(I
*(b*x+a))*(I*(I-d))^(1/2))+1/8/b^2*d/(I-d)*polylog(3,-I*(I-d)*exp(2*I*(b*x+
a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^3+1/4/b^2*a
^2*d/(I-d)*ln(I*exp(2*I*(b*x+a))-exp(2*I*(b*x+a))*d-I)-1/8*I*x^2*Pi*csgn(I*
exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))^2-1/8*I*x^2
*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-
1))^2-1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(
exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)
)/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1
/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2
*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/2/b*a*d/(I-d)*ln(1-I*exp(I*(b*x+a)
)*(I*(I-d))^(1/2))*x-1/2/b*a*d/(I-d)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))
*x+1/2/b*d/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*x*a+1/8*I*x^2*Pi*csgn(I*exp
(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))
*csgn(I*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+a))*d-I*exp(2
```

```
*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-I*exp(2*I*(b*
x+a))+I)/(exp(2*I*(b*x+a))-1))^2-1/4*I/b^2*a^2/(I-d)*ln(I*exp(2*I*(b*x+a))-
exp(2*I*(b*x+a))*d-I)-1/4*I/b^2/(I-d)*ln(1+I*(I-d)*exp(2*I*(b*x+a)))*a^2+1/
2*I/b^2*a^2/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(1/2))+1/2*I/b^2*a^2/(I-d
)*ln(1+I*exp(I*(b*x+a))*(I*(I-d))^(1/2))-1/8*I*x^2*Pi*csgn(I*(exp(2*I*(b*x+
a))*d-I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*d-
I*exp(2*I*(b*x+a))+I)/(exp(2*I*(b*x+a))-1))+1/2*I/b*a/(I-d)*ln(1+I*exp(I*(b
*x+a))*(I*(I-d))^(1/2))*x+1/2*I/b*a/(I-d)*ln(1-I*exp(I*(b*x+a))*(I*(I-d))^(
1/2))*x
```

maxima [B] time = 0.35, size = 248, normalized size = 1.88

$$\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{artanh}(d\cot(bx+a)+i\,d+1)}{b}-\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((i\,d+1)e^{(2i\,bx+2i\,a)}\right)+\left(-6i\,(bx+a)^2+12i\,(bx+a)a\right)\arctan(d\cos\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*cot(b*x + a) + I*d + 1)/b
- (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((I*d + 1)*e^(2*I*b
*x + 2*I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(d*cos(2*b*x +
2*a) + sin(2*b*x + 2*a), d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3*((b
*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin
(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*polylo
g(3, (I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\operatorname{atanh}\left(d\cot\left(a+b\,x\right)+1+d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(d*1i + d*cot(a + b*x) + 1),x)
```

```
[Out] int(x*atanh(d*1i + d*cot(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{atanh}\left(d\cot\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Integral(x*atanh(d*cot(a + b*x) + I*d + 1), x)
```

3.340 $\int \tanh^{-1}(1 + id + d\cot(a + bx))\,dx$

Optimal. Leaf size=93

$$\frac{i\text{Li}_2\left((id+1)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) + x\tanh^{-1}(d\cot(a+bx)+id+1) + \frac{1}{2}ibx^2$$

```
[Out] 1/2*I*b*x^2+x*arctanh(1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1+I*d)*exp(2*I*a+2*I*
b*x))+1/4*I*polylog(2,(1+I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6257, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,(1+id)e^{2ia+2ibx}\right)}{4b} - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) + x\tanh^{-1}(d\cot(a+bx)+id+1) + \frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 + I*d + d*Cot[a + b*x]] - (x*Log[1 - (1 + I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 + I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6257

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1+id+d\cot(a+bx))\,dx = x\tanh^{-1}(1+id+d\cot(a+bx)) + (ib)\int \frac{x}{1+(-1-id)e^{2ia+2ibx}}\,dx$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) + (b(i-d))\int \frac{e^{2ia+2ibx}x}{1+(-1-id)e^{2ia+2}}$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) +$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) -$$

$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1+id+d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1+id)e^{2ia+2ibx}\right) +$$

Mathematica [B] time = 39.25, size = 709, normalized size = 7.62

$$\frac{x\csc^2(a+bx)(\cos(bx)-i\sin(bx))(\cos(bx)+i\sin(bx))\Big(i}{(\cot(a+bx)+i)(d\cot(a+bx)+id+2)\left(\frac{(d-2i)\cos(a+bx)(\log(1-i\tan(bx))-\log(1+i\tan(bx)))}{d\cos(a+bx)+(2+id)\sin(a+bx)}+\frac{d\sin(a+bx)(\log(1-i\tan(bx))-\log(1+i}{(d-2i)\sin(a+bx)-id\cos(a+bx}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 + I*d + d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*
x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a
+ b*x] + (2 + I*d)*Sin[a + b*x]))/(2*(-I + d))]*Log[1 - I*Tan[b*x]] - I*Log
[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x])
)/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - I*Pol
yLog[2, (Sec[b*x]*((2 + I*d)*Cos[a] - d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b
*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((-2 - I*d)*Cos[a] + d*Sin[a])
*(I + Tan[b*x]))/(2*(-I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*Sin[b
*x]))/((I + Cot[a + b*x])*(2 + I*d + d*Cot[a + b*x])*((2*I)*b*x + Log[1 + (
Sec[b*x]*((-2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]
 - Log[(Sec[b*x]*((-I)*Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a +
 b*x]))/2] + ((-2*I + d)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[
b*x]]))/(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]) + (d*(Log[1 - I*Tan[b*x]]
 - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-I)*d*Cos[a + b*x] + (-2*I + d)*Sin
[a + b*x]) + 2*b*x*Tan[b*x] - I*Log[1 + (Sec[b*x]*((-2 - I*d)*Cos[a] + d*Si
n[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] + I*Log[(Sec[b*x]*((-I)*
Cos[a] + Sin[a])*(d*Cos[a + b*x] + (2 + I*d)*Sin[a + b*x]))/2]*Tan[b*x] - I
*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Tan[b*x]))
```

fricas [A] time = 0.45, size = 122, normalized size = 1.31

$$\frac{2i\,b^2x^2 + 2\,bx\log\left(-\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right) - 2i\,a^2 - 2\,(bx+a)\log\left((-i\,d-1)e^{(2i\,bx+2i\,a)}+1\right) + 2\,a\log\left(\frac{(d-i)e^{(2i\,b}}{d-}\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 + 2*b*x*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
 - 2*I*a)/d) - 2*I*a^2 - 2*(b*x + a)*log((-I*d - 1)*e^(2*I*b*x + 2*I*a) + 1
) + 2*a*log(((d - I)*e^(2*I*b*x + 2*I*a) + I)/(d - I)) + I*dilog(-(-I*d - 1
)*e^(2*I*b*x + 2*I*a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + I*d + 1), x)
```

maple [B] time = 0.61, size = 299, normalized size = 3.22

$$\frac{i\operatorname{arctanh}\left(1+id+d\cot\left(bx+a\right)\right)\ln\left(id-d\cot\left(bx+a\right)\right)}{2b}-\frac{i\operatorname{arctanh}\left(1+id+d\cot\left(bx+a\right)\right)\ln\left(id+d\cot\left(bx+\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b*arctanh(1+I*d+d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))-1/2*I/b*arctanh(1+
I*d+d*cot(b*x+a))*ln(I*d+d*cot(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d-d*cot(b*x+
a))/d)+1/4*I/b*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/4*I/b
*dilog((-2-I*d-d*cot(b*x+a))/(-2*I*d-2))-1/4*I/b*ln(I*d-d*cot(b*x+a))*ln((-
2-I*d-d*cot(b*x+a))/(-2*I*d-2))-1/8*I/b*ln(I*d+d*cot(b*x+a))^2+1/4*I/b*dilo
g(1+1/2*I*d+1/2*d*cot(b*x+a))+1/4*I/b*ln(I*d+d*cot(b*x+a))*ln(1+1/2*I*d+1/2
*d*cot(b*x+a))
```

maxima [B] time = 0.49, size = 288, normalized size = 3.10

$$-\frac{4\left(bx+a\right)d\left(\frac{\log\left(\left(i\,d+2\right)\tan\left(bx+a\right)+d\right)}{d}-\frac{\log\left(i\,\tan\left(bx+a\right)+1\right)}{d}\right)-d\left(\frac{2i\left(\log\left(\left(i\,d+2\right)\tan\left(bx+a\right)+d\right)\log\left(\frac{\left(d-2i\right)\tan\left(bx+a\right)-i\,d}{2i\,d+2}+1\right)+\operatorname{Li}_2\left(-\frac{\left(d-2i\right)\tan}{2i}\right.\right.}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d + 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) - d*(2*I*(log((I*d + 2)*tan(b*x + a) + d)*log(((d - 2*I)*tan(b*x +
 a) - I*d)/(2*I*d + 2) + 1) + dilog(-((d - 2*I)*tan(b*x + a) - I*d)/(2*I*d
+ 2)))/d + 2*I*(log(1/2*(d - 2*I)*tan(b*x + a) - 1/2*I*d)*log(I*tan(b*x + a
) + 1) + dilog(-1/2*(d - 2*I)*tan(b*x + a) + 1/2*I*d + 1))/d - (2*I*log((I*
d + 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) + 1
)^2)/d - 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dilo
g(1/2*I*tan(b*x + a) + 1/2))/d) - 8*(b*x + a)*arctanh(I*d + d/tan(b*x + a)
+ 1))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(d\cot\left(a+b\,x\right)+1+d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*1i + d*cot(a + b*x) + 1),x)
```

```
[Out] int(atanh(d*1i + d*cot(a + b*x) + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(d\cot\left(a+bx\right)+id+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+I*d+d*cot(b*x+a)),x)
```

```
[Out] Integral(atanh(d*cot(a + b*x) + I*d + 1), x)
```

3.341 $\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\tanh^{-1}(d\cot(a+bx)+id+1)}{x},x\right)$$

[Out] CannotIntegrate(arctanh(1+I*d+d*cot(b*x+a))/x,x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x,x]

[Out] Defer[Int][ArcTanh[1 + I*d + d*Cot[a + b*x]]/x, x]

Rubi steps

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx = \int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1+id+d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x,x]

[Out] Integrate[ArcTanh[1 + I*d + d*Cot[a + b*x]]/x, x]

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\log\left(-\frac{\left((d-i)e^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{d}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")

[Out] integral(1/2*log(-((d - I)*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/d)
/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(arctanh(d*cot(b*x + a) + I*d + 1)/x, x)
```

maple [A] time = 1.84, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arctanh}(1 + id + d\cot(bx + a))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] int(arctanh(1+I*d+d*cot(b*x+a))/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-i\,bx+\frac{1}{4}\left(-i\,\pi-4i\,a-2\,\log(-d)\right)\log(x)+\frac{1}{2}i\int \frac{\arctan\left(d\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a),-d\sin(2\,bx+2\,a)\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] -I*b*x + 1/4*(-I*pi - 4*I*a - 2*log(-d))*log(x) + 1/2*I*integrate(arctan2(d
*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) + cos(2*b*x + 2*a
) - 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*s
in(2*b*x + 2*a)^2 + 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\operatorname{atanh}\left(d\cot(a+b\,x)+1+d\,1\mathrm{i}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(d*1i + d*cot(a + b*x) + 1)/x,x)
```

```
[Out] int(atanh(d*1i + d*cot(a + b*x) + 1)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atanh}(d\cot(a + bx) + id + 1)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(1+I*d+d*cot(b*x+a))/x,x)
```

```
[Out] Integral(atanh(d*cot(a + b*x) + I*d + 1)/x, x)
```

3.342 $\int x^2 \tanh^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal. Leaf size=169

$$-\frac{i\text{Li}_4\left((1-id)e^{2ia+2ibx}\right)}{8b^3}-\frac{x\text{Li}_3\left((1-id)e^{2ia+2ibx}\right)}{4b^2}+\frac{ix^2\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{3}x^3$$

```
[Out] 1/12*I*b*x^4-1/3*x^3*arctanh(-1+I*d+d*cot(b*x+a))-1/6*x^3*ln(1-(1-I*d)*exp(
2*I*a+2*I*b*x))+1/4*I*x^2*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/4*x*pol
ylog(3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2-1/8*I*polylog(4,(1-I*d)*exp(2*I*a+2*
I*b*x))/b^3
```

Rubi [A] time = 0.30, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6265, 2184, 2190, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{4b^2}-\frac{i\text{PolyLog}\left(4,(1-id)e^{2ia+2ibx}\right)}{8b^3}+\frac{ix^2\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{6}x^3\log\left(1\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/12)*b*x^4 + (x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 - (x^3*Log[1 - (1
- I*d)*E^((2*I)*a + (2*I)*b*x)])/6 + ((I/4)*x^2*PolyLog[2, (1 - I*d)*E^((2*
I)*a + (2*I)*b*x)])/b - (x*PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/(
4*b^2) - ((I/8)*PolyLog[4, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)])/b^3
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6265

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) + \frac{1}{3}(ib)\int \frac{x^3}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) + \frac{1}{3}(b(i+d))\int \frac{e^{2ia+}}{1+(-1+}$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+}\right.$$
$$= \frac{1}{12}ibx^4 + \frac{1}{3}x^3\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{6}x^3\log\left(1-(1-id)e^{2ia+}\right.$$

Mathematica [A] time = 0.43, size = 155, normalized size = 0.92

$$\frac{1}{3}x^3\tanh^{-1}(d(-\cot(a+bx))-id+1) - \frac{4b^3x^3\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+6ib^2x^2\text{Li}_2\left(\frac{ie^{-2i(a+bx)}}{d+i}\right)+6bx\text{Li}_3\left(\frac{ie^{-2i(a+bx)}}{d+i}\right)-3i\text{Li}_4\left(\frac{i}{}\right.}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^3*ArcTanh[1 - I*d - d*Cot[a + b*x]])/3 - (4*b^3*x^3*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/((I + d)*E^((2*I)*(a +
 b*x)))] + 6*b*x*PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))] - (3*I)*PolyLo
g[4, I/((I + d)*E^((2*I)*(a + b*x)))])/(24*b^3)
```

fricas [C] time = 0.60, size = 180, normalized size = 1.07

$$\frac{2i\,b^4x^4 - 4\,b^3x^3\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right) + 6i\,b^2x^2\text{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right) - 2i\,a^4 + 4\,a^3\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right) - 6\,bx}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(2*I*b^4*x^4 - 4*b^3*x^3*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 6*I*b^2*x^2*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a)) - 2*I*
a^4 + 4*a^3*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*b*x*polylog(
3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)) - 4*(b^3*x^3 + a^3)*log((I*d - 1)*e^(2*I
*b*x + 2*I*a) + 1) - 3*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x^2 \operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(d*cot(b*x + a) + I*d - 1), x)
```

maple [C] time = 5.56, size = 2346, normalized size = 13.88

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2-1/3*x^3*ln(exp(I*(
b*x+a)))-1/6*I*Pi*x^3-1/12*I*x^3*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)
*exp(2*I*(b*x+a)))^2+1/12*I*b*x^4+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(ex
p(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b
*x+a)))^3+1/2/b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a^2-1/12*I*x^3*P
i*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/6*x^3*ln(d)-1/2*I/b^3*a
^3/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I/b^3*d/(I+d)*polylog(4
,-I*(I+d)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*
(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/8/b^3/(I+d
)*polylog(4,-I*(I+d)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x
+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3+1/12*I*x^3*Pi*csgn(d/(ex
p(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a))
)^3-1/4/b/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4/b^3/(I+d)*poly
log(2,-I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/2/b^3*a^2/(I+d)*dilog(1+I*exp(I*(b*x
+a))*(I*(I+d))^(1/2))-1/2/b^3*a^2/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^
(1/2))-1/6*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3+1/12*I*x^3*Pi*csgn((I
*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/12*I*x^3*
Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^3-1
/6*I/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^3-1/6*I*x^3*Pi*csgn(I*exp(I*(b*
x+a)))*csgn(I*exp(2*I*(b*x+a)))^2-1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1)
)*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(b*x+
a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/3*I/b^3/(I+d)*ln(1+I*(I+d
)*exp(2*I*(b*x+a)))*a^3+1/12*I*x^3*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(
I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2-1/4/b^2*d/
(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/4*I/b^3*d/(I+d)*polylog(2,-I
*(I+d)*exp(2*I*(b*x+a)))*a^2-1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(
I+d))^(1/2))*x-1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x
+1/2*I/b^3*a^2*d/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/12*I*x^3
*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-
1))^2+1/2*I/b^3*a^2*d/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/12*
I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))
-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2
+1/12*I*x^3*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*
```

```
exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/12*I*x^3*P
i*csgn(I*d)*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I
*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/6*x^3*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a
))*d-I)+1/2*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a^2-1/2*I/b^2*a^2/
(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2/b^2*a^2*d/(I+d)*ln(1-I*e
xp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1
)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2-1/2/b^3
*a^3*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2/b^3*a^3*d/(I+d)*ln(
1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1/3/b^3*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b
*x+a)))*a^3+1/6/b^3*a^3*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)
+1/12*I*x^3*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+1/12*I*x^3
*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b
*x+a))/(exp(2*I*(b*x+a))-1))+1/12*I*x^3*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*ex
p(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))-1/12*I*x^3*Pi
*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csg
n((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))-1/4*I/b^2
/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))*x+1/6*I/b^3*a^3/(I+d)*ln(I*exp(
2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1/2*I/b^3*a^3/(I+d)*ln(1+I*exp(I*(b*x+a)
)*(I*(I+d))^(1/2))-1/12*I*x^3*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b
*x+a))/(exp(2*I*(b*x+a))-1))^2
```

maxima [B] time = 0.38, size = 343, normalized size = 2.03

$$-\frac{\frac{12\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\operatorname{artanh}(d\cot(bx+a)+i\,d-1)}{b^2}+\frac{-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2+\left(-8i\,(bx+a)^3+18i\,(bx+a)^2a-18i\,(bx+a\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/36*(12*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctanh(d*cot(b
*x + a) + I*d - 1)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*x
 + a)^2*a^2 + (-8*I*(b*x + a)^3 + 18*I*(b*x + a)^2*a - 18*I*(b*x + a)*a^2)*
arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2
*b*x + 2*a) + 1) + (-12*I*(b*x + a)^2 + 18*I*(b*x + a)*a - 9*I*a^2)*dilog((
-I*d + 1)*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)^3 - 9*(b*x + a)^2*a + 9*(b*x
+ a)*a^2)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 -
 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3,
(-I*d + 1)*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, (-I*d + 1)*e^(2*I*b*x + 2*
I*a)))/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x^2\,\mathrm{atanh}\left(d\cot\left(a+b\,x\right)-1+d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*atanh(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-x^2*atanh(d*1i + d*cot(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x^2\operatorname{atanh}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2*atanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -Integral(x**2*atanh(d*cot(a + b*x) + I*d - 1), x)
```

3.343 $\int x \tanh^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal. Leaf size=133

$$-\frac{\text{Li}_3\left((1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d(-\cot(a+bx))-id$$

```
[Out] 1/6*I*b*x^3-1/2*x^2*arctanh(-1+I*d+d*cot(b*x+a))-1/4*x^2*ln(1-(1-I*d)*exp(2
*I*a+2*I*b*x))+1/4*I*x*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b-1/8*polylog(
3,(1-I*d)*exp(2*I*a+2*I*b*x))/b^2
```

Rubi [A] time = 0.25, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6265, 2184, 2190, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,(1-id)e^{2ia+2ibx}\right)}{8b^2}+\frac{ix\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2ibx}\right)+\frac{1}{2}x^2\tanh^{-1}(d$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/6)*b*x^3 + (x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 - (x^2*Log[1 - (1 -
 I*d)*E^((2*I)*a + (2*I)*b*x)])/4 + ((I/4)*x*PolyLog[2, (1 - I*d)*E^((2*I)*
a + (2*I)*b*x)])/b - PolyLog[3, (1 - I*d)*E^((2*I)*a + (2*I)*b*x)]/(8*b^2)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6265

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^(m + 1)*ArcTanh[c + d*Cot[a + b*x]])/(f*(m
```

```
 + 1)), x] + Dist[(I*b)/(f*(m + 1)), Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(
2*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &&
EqQ[(c - I*d)^2, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(1-id-d\cot(a+bx))\,dx = \frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(ib)\int\frac{x^2}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))+\frac{1}{2}(b(i+d))\int\frac{e^{2ia+2ib}}{1+(-1+id}$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2il}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2il}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2il}\right.$$
$$=\frac{1}{6}ibx^3+\frac{1}{2}x^2\tanh^{-1}(1-id-d\cot(a+bx))-\frac{1}{4}x^2\log\left(1-(1-id)e^{2ia+2il}\right.$$

Mathematica [A] time = 0.31, size = 119, normalized size = 0.89

$$\frac{1}{2}x^2\tanh^{-1}(d(-\cot(a+bx))-id+1)-\frac{2b^2x^2\log\left(1+\frac{e^{-2i(a+bx)}}{-1+id}\right)+2ibx\text{Li}_2\left(\frac{ie^{-2i(a+bx)}}{d+i}\right)+\text{Li}_3\left(\frac{ie^{-2i(a+bx)}}{d+i}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (x^2*ArcTanh[1 - I*d - d*Cot[a + b*x]])/2 - (2*b^2*x^2*Log[1 + 1/((-1 + I*d
)*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/((I + d)*E^((2*I)*(a + b*x
)))] + PolyLog[3, I/((I + d)*E^((2*I)*(a + b*x)))])/(8*b^2)
```

fricas [C] time = 0.65, size = 157, normalized size = 1.18

$$\frac{4i\,b^3x^3-6\,b^2x^2\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)+4i\,a^3+6i\,bx\text{Li}_2\left(-(i\,d-1)e^{(2i\,bx+2i\,a)}\right)-6\,a^2\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right)-6\left(b^2x^2\right.}{24\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/24*(4*I*b^3*x^3 - 6*b^2*x^2*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*
x + 2*I*a) - I)) + 4*I*a^3 + 6*I*b*x*dilog(-(I*d - 1)*e^(2*I*b*x + 2*I*a))
- 6*a^2*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) - 6*(b^2*x^2 - a^2)*
log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1) - 3*polylog(3, (-I*d + 1)*e^(2*I*b*x
 + 2*I*a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -x\,\text{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(d*cot(b*x + a) + I*d - 1), x)
```

maple [C] time = 4.34, size = 2256, normalized size = 16.96

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*arctanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/4*I/b*d/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(b*x+a)))*x+1/6*I*b*x^3+1/8*I*x^
2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(
b*x+a))/(exp(2*I*(b*x+a))-1))+1/8*I*x^2*Pi*csgn(I*d)*csgn(I*exp(2*I*(b*x+a)
)/(exp(2*I*(b*x+a))-1))*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))+1/8
*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))^3+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a)
)+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(
2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))^2+1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x
+a))-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))
-1))^2-1/4*x^2*ln(d)+1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a
))*d-I))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))
-1))^2-1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I
*(b*x+a))-1))^3+1/2/b^2*a/(I+d)*dilog(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))+1
/2/b^2*a/(I+d)*dilog(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/4/b/(I+d)*polylo
g(2,-I*(I+d)*exp(2*I*(b*x+a)))*x-1/4/b^2/(I+d)*polylog(2,-I*(I+d)*exp(2*I*(
b*x+a)))*a-1/4*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x^2+1/4*x^2*ln(I*exp(
2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)+1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)
*exp(2*I*(b*x+a)))^2-1/4*I*Pi*x^2-1/2*x^2*ln(exp(I*(b*x+a)))+1/8*I*x^2*Pi*c
sgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*
exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a
)))^3-1/8*I*x^2*Pi*csgn(d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^3-1/8*I/b^
2/(I+d)*polylog(3,-I*(I+d)*exp(2*I*(b*x+a)))-1/4*I/(I+d)*ln(1+I*(I+d)*exp(2
*I*(b*x+a)))*x^2-1/8*I*x^2*Pi*csgn(I*d)*csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2
*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I
)/(exp(2*I*(b*x+a))-1))^3+1/2/b^2*a^2*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d
))^(1/2))-1/4/b^2*a^2*d/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)-1
/4/b^2*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2-1/8/b^2*d/(I+d)*polylog(3
,-I*(I+d)*exp(2*I*(b*x+a)))+1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(
b*x+a))-1))^3+1/8*I*x^2*Pi*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I)/(
exp(2*I*(b*x+a))-1))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I
*(b*x+a))/(exp(2*I*(b*x+a))-1))^2+1/2*I/b^2*a^2/(I+d)*ln(1-I*exp(I*(b*x+a))
*(I*(I+d))^(1/2))-1/4*I/b^2*a^2/(I+d)*ln(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a)
)*d-I)-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))/(e
xp(2*I*(b*x+a))-1))^2-1/4*I/b^2/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*a^2+1/
2*I/b^2*a^2/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I*x^2*Pi*csgn(
I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))*csgn(d/(exp(2*I*(b*x+a))-1)*exp(
2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*exp(2*I*(b*x+a))/(exp(2*I*(b*x+a))-1))*
csgn(I*d/(exp(2*I*(b*x+a))-1)*exp(2*I*(b*x+a)))^2+1/8*I*x^2*Pi*csgn(I*exp(I
*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-1/4*I*x^2*Pi*csgn(I*exp(I*(b*x+a)))*c
sgn(I*exp(2*I*(b*x+a)))^2-1/8*I*x^2*Pi*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(
b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))*csgn((I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a)
)*d-I)/(exp(2*I*(b*x+a))-1))+1/2/b^2*a^2*d/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(
I+d))^(1/2))+1/2*I/b*a/(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x+1/2*I
/b*a/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2*I/b^2*a*d/(I+d)*dil
og(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/8*I*x^2*Pi*csgn(I/(exp(2*I*(b*x+a)
)-1))*csgn(I*(I*exp(2*I*(b*x+a))+exp(2*I*(b*x+a))*d-I))*csgn(I*(I*exp(2*I*(
b*x+a))+exp(2*I*(b*x+a))*d-I)/(exp(2*I*(b*x+a))-1))+1/4*I/b^2*d/(I+d)*polyl
og(2,-I*(I+d)*exp(2*I*(b*x+a)))*a+1/2/b*a*d/(I+d)*ln(1-I*exp(I*(b*x+a))*(I*
```

```
(I+d))^(1/2))*x-1/2/b*d/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a)))*x*a+1/2/b*a*d/
(I+d)*ln(1+I*exp(I*(b*x+a))*(I*(I+d))^(1/2))*x-1/2*I/b^2*a*d/(I+d)*dilog(1-
I*exp(I*(b*x+a))*(I*(I+d))^(1/2))-1/2*I/b/(I+d)*ln(1+I*(I+d)*exp(2*I*(b*x+a
)))*x*a
```

maxima [B] time = 0.36, size = 249, normalized size = 1.87

$$-\frac{\frac{12\left((bx+a)^2-2\,(bx+a)a\right)\operatorname{artanh}(d\cot(bx+a)+i\,d-1)}{b}+\frac{-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\operatorname{Li}_2\left((-i\,d+1)e^{(2i\,bx+2i\,a)}\right)+\left(-6i\,(bx+a)^2+12i\,(bx+a)a\right)\arctan(-}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/24*(12*((b*x + a)^2 - 2*(b*x + a)*a)*arctanh(d*cot(b*x + a) + I*d - 1)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog((-I*d + 1)*e^(2*I
*b*x + 2*I*a)) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*arctan2(-d*cos(2*b*x
 + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1) + 3
*((b*x + a)^2 - 2*(b*x + a)*a)*log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)
*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1) + 3*po
lylog(3, (-I*d + 1)*e^(2*I*b*x + 2*I*a)))/b)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -x\operatorname{atanh}\left(d\cot\left(a+b\,x\right)-1+d\,1\mathrm{i}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*atanh(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-x*atanh(d*1i + d*cot(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int x\operatorname{atanh}\left(d\cot\left(a+bx\right)+id-1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x*atanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -Integral(x*atanh(d*cot(a + b*x) + I*d - 1), x)
```

3.344 $\int \tanh^{-1}(1 - id - d\cot(a + bx))\,dx$

Optimal. Leaf size=94

$$\frac{i\text{Li}_2\left((1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{2}ibx^2$$

```
[Out] 1/2*I*b*x^2-x*arctanh(-1+I*d+d*cot(b*x+a))-1/2*x*ln(1-(1-I*d)*exp(2*I*a+2*I
*b*x))+1/4*I*polylog(2,(1-I*d)*exp(2*I*a+2*I*b*x))/b
```

Rubi [A] time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6257, 2184, 2190, 2279, 2391}

$$\frac{i\text{PolyLog}\left(2,(1-id)e^{2ia+2ibx}\right)}{4b}-\frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right)+x\tanh^{-1}(d(-\cot(a+bx))-id+1)+\frac{1}{2}ibx^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] (I/2)*b*x^2 + x*ArcTanh[1 - I*d - d*Cot[a + b*x]] - (x*Log[1 - (1 - I*d)*E^
((2*I)*a + (2*I)*b*x)])/2 + ((I/4)*PolyLog[2, (1 - I*d)*E^((2*I)*a + (2*I)*
b*x)])/b
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6257

```
Int[ArcTanh[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tanh[c + d*Cot[a + b*x]], x] + Dist[I*b, Int[x/(c - I*d - c*E^(2*I*a + 2*I*
b*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, 1]
```

Rubi steps

$$\int \tanh^{-1}(1-id-d\cot(a+bx))\,dx = x\tanh^{-1}(1-id-d\cot(a+bx)) + (ib)\int \frac{x}{1+(-1+id)e^{2ia+2ibx}}\,dx$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) + (b(i+d))\int \frac{e^{2ia+2ibx}x}{1+(-1+id)e^{2ia+2}}$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) +$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) -$$
$$= \frac{1}{2}ibx^2 + x\tanh^{-1}(1-id-d\cot(a+bx)) - \frac{1}{2}x\log\left(1-(1-id)e^{2ia+2ibx}\right) +$$

Mathematica [B] time = 30.01, size = 605, normalized size = 6.44

$$\frac{x\csc^2(a+bx)(\cos(bx)-i\sin(bx))(\cos(bx)+i\sin(bx))\left(i\text{Li}_2\left(\frac{(\cos(a)-i\sin(a))((2-id)\cos(a)+d\sin(a))(\tan(bx)+i)}{2(d+i)}\right)-i\text{Li}_2\left(\frac{1}{2}\text{s}\right.\right.}{(\cot(a+bx)+i)(d\cot(a+bx)+id-2)\left(\frac{\sec^2(bx)\log\left(\frac{i\sec(bx)(d}{}\right.}{1+i}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]],x]
```

```
[Out] x*ArcTanh[1 - I*d - d*Cot[a + b*x]] + (x*Csc[a + b*x]^2*(2*b*x*Log[2*Cos[b*
x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*(d*Cos[a
+ b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*(I + d))]*Log[1 - I*Tan[b*x]] - I*Lo
g[(I*Sec[b*x]*(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I
)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] -
 I*PolyLog[2, (Sec[b*x]*((2 - I*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin
[a + b*x]))/2] + I*PolyLog[2, ((Cos[a] - I*Sin[a])*((2 - I*d)*Cos[a] + d*Si
n[a])*(I + Tan[b*x]))/(2*(I + d))])*(Cos[b*x] - I*Sin[b*x])*(Cos[b*x] + I*S
in[b*x]))/((I + Cot[a + b*x])*(-2 + I*d + d*Cot[a + b*x])*(-((Log[1 - I*Tan
[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*Sin[a]))/(d*Cos[a + b*x] + I*(2*I +
 d)*Sin[a + b*x])) + (Log[1 + I*Tan[b*x]]*Sec[b*x]*((2*I + d)*Cos[a] + I*d*
Sin[a]))/(d*Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]) + (Log[(I*Sec[b*x]*(d*
Cos[a + b*x] + I*(2*I + d)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a])]*Sec[b*
x]^2)/(1 + I*Tan[b*x]) - 2*b*x*(I + Tan[b*x]) + I*Log[1 - (Sec[b*x]*((2 - I
*d)*Cos[a] + d*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*(I + Tan[b*x])))
```

fricas [A] time = 0.72, size = 122, normalized size = 1.30

$$\frac{2i\,b^2x^2 - 2\,bx\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right) - 2i\,a^2 - 2\,(bx+a)\log\left((i\,d-1)e^{(2i\,bx+2i\,a)}+1\right) + 2\,a\log\left(\frac{(d+i)e^{(2i\,bx+2i\,a)}-i}{d+i}\right) + i\,\text{I}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*I*b^2*x^2 - 2*b*x*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2
*I*a) - I)) - 2*I*a^2 - 2*(b*x + a)*log((I*d - 1)*e^(2*I*b*x + 2*I*a) + 1)
+ 2*a*log(((d + I)*e^(2*I*b*x + 2*I*a) - I)/(d + I)) + I*dilog(-(I*d - 1)*e
^(2*I*b*x + 2*I*a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\text{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(d*cot(b*x + a) + I*d - 1), x)
```

maple [B] time = 0.55, size = 335, normalized size = 3.56

$$\frac{i\operatorname{arctanh}(-1+id+d\cot(bx+a))\ln(id+d\cot(bx+a))}{2b}-\frac{i\operatorname{arctanh}(-1+id+d\cot(bx+a))\ln(id-d\cot(\ldots}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] 1/2*I/b*arctanh(-1+I*d+d*cot(b*x+a))*ln(I*d+d*cot(b*x+a))-1/2*I/b*arctanh(-
1+I*d+d*cot(b*x+a))*ln(I*d-d*cot(b*x+a))+1/4*I/b*dilog(1/2*I*(-I*d-d*cot(b*
x+a))/d)+1/4*I/b*ln(I*d-d*cot(b*x+a))*ln(1/2*I*(-I*d-d*cot(b*x+a))/d)-1/4*I
/b*dilog((2-I*d-d*cot(b*x+a))/(-2*I*d+2))-1/4*I/b*ln(I*d-d*cot(b*x+a))*ln((
2-I*d-d*cot(b*x+a))/(-2*I*d+2))-1/8*I/b*ln(I*d+d*cot(b*x+a))^2+1/4*I/b*ln(1
-1/2*I*d-1/2*d*cot(b*x+a))*ln(I*d+d*cot(b*x+a))-1/4*I/b*ln(1-1/2*I*d-1/2*d*
cot(b*x+a))*ln(1/2*I*d+1/2*d*cot(b*x+a))-1/4*I/b*dilog(1/2*I*d+1/2*d*cot(b*
x+a))
```

maxima [B] time = 0.43, size = 286, normalized size = 3.04

$$-\frac{4(bx+a)d\left(\frac{\log((id-2)\tan(bx+a)+d)}{d}-\frac{\log(i\tan(bx+a)+1)}{d}\right)+d\left(-\frac{2i\left(\log((id-2)\tan(bx+a)+d)\log\left(\frac{(d+2i)\tan(bx+a)-id}{2id-2}+1\right)+\operatorname{Li}_2\left(-\frac{(d+2i)\ldots}{}\right.\right.}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(b*x + a)*d*(log((I*d - 2)*tan(b*x + a) + d)/d - log(I*tan(b*x + a)
 + 1)/d) + d*(-2*I*(log((I*d - 2)*tan(b*x + a) + d)*log(((d + 2*I)*tan(b*x
+ a) - I*d)/(2*I*d - 2) + 1) + dilog(-((d + 2*I)*tan(b*x + a) - I*d)/(2*I*d
 - 2)))/d - 2*I*(log(-1/2*(d + 2*I)*tan(b*x + a) + 1/2*I*d)*log(I*tan(b*x +
 a) + 1) + dilog(1/2*(d + 2*I)*tan(b*x + a) - 1/2*I*d + 1))/d + (2*I*log((I
*d - 2)*tan(b*x + a) + d)*log(I*tan(b*x + a) + 1) - I*log(I*tan(b*x + a) +
1)^2)/d + 2*I*(log(I*tan(b*x + a) + 1)*log(-1/2*I*tan(b*x + a) + 1/2) + dil
og(1/2*I*tan(b*x + a) + 1/2))/d) + 8*(b*x + a)*arctanh(I*d + d/tan(b*x + a)
 - 1))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\operatorname{atanh}(d\cot(a+b\,x)-1+d\,1\mathrm{i})\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-atanh(d*1i + d*cot(a + b*x) - 1),x)
```

```
[Out] int(-atanh(d*1i + d*cot(a + b*x) - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{atanh}(d\cot(a+bx)+id-1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atanh(-1+I*d+d*cot(b*x+a)),x)
```

```
[Out] -Integral(atanh(d*cot(a + b*x) + I*d - 1), x)
```

3.345 $\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\tanh^{-1}(d(-\cot(a+bx))-id+1)}{x},x\right)$$

```
[Out] CannotIntegrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x,x]
[Out] Defer[Int][ArcTanh[1 - I*d - d*Cot[a + b*x]]/x, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx=\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Mathematica [A] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{-1}(1-id-d\cot(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x,x]
[Out] Integrate[ArcTanh[1 - I*d - d*Cot[a + b*x]]/x, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\log\left(-\frac{de^{(2i\,bx+2i\,a)}}{(d+i)e^{(2i\,bx+2i\,a)}-i}\right)}{2\,x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="fricas")
[Out] integral(-1/2*log(-d*e^(2*I*b*x + 2*I*a)/((d + I)*e^(2*I*b*x + 2*I*a) - I))
/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{artanh}\left(d\cot\left(bx+a\right)+i\,d-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="giac")
```

[Out] integrate(-arctanh(d*cot(b*x + a) + I*d - 1)/x, x)

maple [A] time = 1.53, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arctanh}\left(-1+id+d\cot\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)

[Out] int(-arctanh(-1+I*d+d*cot(b*x+a))/x,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-i\,bx+\frac{1}{4}\left(i\,\pi-4i\,a-2\,\log(d)\right)\log(x)-\frac{1}{2}i\int\frac{\arctan\left(-d\cos\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right),-d\sin\left(2\,bx+2\,a\right)\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-arctanh(-1+I*d+d*cot(b*x+a))/x,x, algorithm="maxima")

[Out] -I*b*x + 1/4*(I*pi - 4*I*a - 2*log(d))*log(x) - 1/2*I*integrate(arctan2(-d*cos(2*b*x + 2*a) + sin(2*b*x + 2*a), -d*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)/x, x) + 1/4*integrate(log((d^2 + 1)*cos(2*b*x + 2*a)^2 + (d^2 + 1)*sin(2*b*x + 2*a)^2 - 2*d*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a) + 1)/x, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{\operatorname{atanh}\left(d\cot\left(a+b\,x\right)-1+d\,1\mathrm{i}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-atanh(d*1i + d*cot(a + b*x) - 1)/x,x)

[Out] int(-atanh(d*1i + d*cot(a + b*x) - 1)/x, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\operatorname{atanh}\left(d\cot\left(a+bx\right)+id-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-atanh(-1+I*d+d*cot(b*x+a))/x,x)

[Out] -Integral(atanh(d*cot(a + b*x) + I*d - 1)/x, x)

3.346 $\int \tanh^{-1}(e^x)\,dx$

Optimal. Leaf size=21

$$\frac{\text{Li}_2(e^x)}{2}-\frac{\text{Li}_2(-e^x)}{2}$$

[Out] -1/2*polylog(2,-exp(x))+1/2*polylog(2,exp(x))

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2282, 5912}

$$\frac{1}{2}\text{PolyLog}(2,e^x)-\frac{1}{2}\text{PolyLog}(2,-e^x)$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[E^x],x]

[Out] -PolyLog[2, -E^x]/2 + PolyLog[2, E^x]/2

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}(e^x)\,dx&=\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x}\,dx,x,e^x\right)\\&=-\frac{\text{Li}_2(-e^x)}{2}+\frac{\text{Li}_2(e^x)}{2}\end{aligned}$$

Mathematica [B] time = 0.03, size = 51, normalized size = 2.43

$$-\frac{\text{Li}_2(-e^x)}{2}+\frac{\text{Li}_2(e^x)}{2}+\frac{1}{2}x\log(1-e^x)-\frac{1}{2}x\log(e^x+1)+x\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[E^x],x]

[Out] x*ArcTanh[E^x] + (x*Log[1 - E^x])/2 - (x*Log[1 + E^x])/2 - PolyLog[2, -E^x]
/2 + PolyLog[2, E^x]/2

fricas [B] time = 0.84, size = 65, normalized size = 3.10

$$\frac{1}{2}x\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{2}x\log(\cosh(x)+\sinh(x)+1)+\frac{1}{2}x\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}\text{Li}_2(\text{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/2*x*log(cos
h(x) + sinh(x) + 1) + 1/2*x*log(-cosh(x) - sinh(x) + 1) + 1/2*dilog(cosh(x)
 + sinh(x)) - 1/2*dilog(-cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}(e^x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(e^x), x)
```

maple [A] time = 0.05, size = 31, normalized size = 1.48

$$\ln(\mathrm{e}^x)\operatorname{arctanh}(\mathrm{e}^x) - \frac{\operatorname{dilog}(\mathrm{e}^x)}{2} - \frac{\operatorname{dilog}(\mathrm{e}^x + 1)}{2} - \frac{\ln(\mathrm{e}^x)\ln(\mathrm{e}^x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(exp(x)),x)
```

```
[Out] ln(exp(x))*arctanh(exp(x))-1/2*dilog(exp(x))-1/2*dilog(exp(x)+1)-1/2*ln(exp
(x))*ln(exp(x)+1)
```

maxima [B] time = 0.31, size = 58, normalized size = 2.76

$$-\frac{1}{2}x\big(\log(e^x + 1) - \log(e^x - 1)\big) + x\operatorname{artanh}(e^x) + \frac{1}{2}\log(-e^x)\log(e^x + 1) - \frac{1}{2}x\log(e^x - 1) + \frac{1}{2}\operatorname{Li}_2(e^x + 1) - \frac{1}{2}\operatorname{Li}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(log(e^x + 1) - log(e^x - 1)) + x*arctanh(e^x) + 1/2*log(-e^x)*log(e
^x + 1) - 1/2*x*log(e^x - 1) + 1/2*dilog(e^x + 1) - 1/2*dilog(-e^x + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{atanh}(\mathrm{e}^x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(exp(x)),x)
```

```
[Out] int(atanh(exp(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}(e^x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(exp(x)),x)
```

```
[Out] Integral(atanh(exp(x)), x)
```

3.347 $\int x \tanh^{-1}(e^x)\, dx$

Optimal. Leaf size=43

$$-\frac{1}{2}x\mathrm{Li}_2(-e^x)+\frac{x\mathrm{Li}_2(e^x)}{2}+\frac{\mathrm{Li}_3(-e^x)}{2}-\frac{\mathrm{Li}_3(e^x)}{2}$$

```
[Out] -1/2*x*polylog(2,-exp(x))+1/2*x*polylog(2,exp(x))+1/2*polylog(3,-exp(x))-1/
2*polylog(3,exp(x))
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6213, 2531, 2282, 6589}

$$-\frac{1}{2}x\mathrm{PolyLog}(2,-e^x)+\frac{1}{2}x\mathrm{PolyLog}(2,e^x)+\frac{1}{2}\mathrm{PolyLog}(3,-e^x)-\frac{1}{2}\mathrm{PolyLog}(3,e^x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[E^x],x]
```

```
[Out] -(x*PolyLog[2, -E^x])/2 + (x*PolyLog[2, E^x])/2 + PolyLog[3, -E^x]/2 - Poly
Log[3, E^x]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}(e^x)\,dx = -\left(\frac{1}{2}\int x\log(1-e^x)\,dx\right)+\frac{1}{2}\int x\log(1+e^x)\,dx$$
$$= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{1}{2}\int \text{Li}_2(-e^x)\,dx-\frac{1}{2}\int \text{Li}_2(e^x)\,dx$$
$$= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(-x)}{x}dx,x,e^x\right)-\frac{1}{2}\text{Subst}\left(\int\frac{\text{Li}_2(x)}{x}dx,x,e^x\right)$$
$$= -\frac{1}{2}x\text{Li}_2(-e^x)+\frac{x\text{Li}_2(e^x)}{2}+\frac{\text{Li}_3(-e^x)}{2}-\frac{\text{Li}_3(e^x)}{2}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.65

$$\frac{1}{4}\left(-2x\text{Li}_2(-e^x)+2x\text{Li}_2(e^x)+2\text{Li}_3(-e^x)-2\text{Li}_3(e^x)+x^2\log(1-e^x)-x^2\log(e^x+1)+2x^2\tanh^{-1}(e^x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[E^x],x]
```

```
[Out] (2*x^2*ArcTanh[E^x] + x^2*Log[1 - E^x] - x^2*Log[1 + E^x] - 2*x*PolyLog[2,
-E^x] + 2*x*PolyLog[2, E^x] + 2*PolyLog[3, -E^x] - 2*PolyLog[3, E^x])/4
```

fricas [C] time = 0.54, size = 95, normalized size = 2.21

$$\frac{1}{4}x^2\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right)-\frac{1}{4}x^2\log(\cosh(x)+\sinh(x)+1)+\frac{1}{4}x^2\log(-\cosh(x)-\sinh(x)+1)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/4*x^2*log
(cosh(x) + sinh(x) + 1) + 1/4*x^2*log(-cosh(x) - sinh(x) + 1) + 1/2*x*dilog
(cosh(x) + sinh(x)) - 1/2*x*dilog(-cosh(x) - sinh(x)) - 1/2*polylog(3, cosh
(x) + sinh(x)) + 1/2*polylog(3, -cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{artanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(e^x), x)
```

maple [A] time = 0.04, size = 62, normalized size = 1.44

$$\frac{x^2\,\text{arctanh}(e^x)}{2}-\frac{x^2\ln(e^x+1)}{4}-\frac{x\,\text{polylog}(2,-e^x)}{2}+\frac{\text{polylog}(3,-e^x)}{2}+\frac{x^2\ln(1-e^x)}{4}+\frac{x\,\text{polylog}(2,e^x)}{2}-\frac{\text{poly}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(exp(x)),x)
```

```
[Out] 1/2*x^2*arctanh(exp(x))-1/4*x^2*ln(exp(x)+1)-1/2*x*polylog(2,-exp(x))+1/2*p
olylog(3,-exp(x))+1/4*x^2*ln(1-exp(x))+1/2*x*polylog(2,exp(x))-1/2*polylog(
3,exp(x))
```

maxima [B] time = 0.31, size = 59, normalized size = 1.37

$$\frac{1}{2}\,x^2\,\mathrm{artanh}\left(e^x\right)-\frac{1}{4}\,x^2\log\left(e^x+1\right)+\frac{1}{4}\,x^2\log\left(-e^x+1\right)-\frac{1}{2}\,x\mathrm{Li}_2\left(-e^x\right)+\frac{1}{2}\,x\mathrm{Li}_2\left(e^x\right)+\frac{1}{2}\,\mathrm{Li}_3(-e^x)-\frac{1}{2}\,\mathrm{Li}_3(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(e^x) - 1/4*x^2*log(e^x + 1) + 1/4*x^2*log(-e^x + 1) - 1/2*x
*dilog(-e^x) + 1/2*x*dilog(e^x) + 1/2*polylog(3, -e^x) - 1/2*polylog(3, e^x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\mathrm{atanh}\left(\mathrm{e}^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(exp(x)),x)
```

```
[Out] int(x*atanh(exp(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\mathrm{atanh}\left(e^x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(exp(x)),x)
```

```
[Out] Integral(x*atanh(exp(x)), x)
```

3.348 $\int x^2 \tanh^{-1}(e^x)\, dx$

Optimal. Leaf size=58

$$-\frac{1}{2}x^2\text{Li}_2\left(-e^x\right)+\frac{1}{2}x^2\text{Li}_2\left(e^x\right)+x\text{Li}_3\left(-e^x\right)-x\text{Li}_3\left(e^x\right)-\text{Li}_4\left(-e^x\right)+\text{Li}_4\left(e^x\right)$$

```
[Out] -1/2*x^2*polylog(2,-exp(x))+1/2*x^2*polylog(2,exp(x))+x*polylog(3,-exp(x))-
x*polylog(3,exp(x))-polylog(4,-exp(x))+polylog(4,exp(x))
```

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6213, 2531, 6609, 2282, 6589}

$$-\frac{1}{2}x^2\text{PolyLog}\left(2,-e^x\right)+\frac{1}{2}x^2\text{PolyLog}\left(2,e^x\right)+x\text{PolyLog}\left(3,-e^x\right)-x\text{PolyLog}\left(3,e^x\right)-\text{PolyLog}\left(4,-e^x\right)+\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[E^x],x]
```

```
[Out] -(x^2*PolyLog[2, -E^x])/2 + (x^2*PolyLog[2, E^x])/2 + x*PolyLog[3, -E^x] -
x*PolyLog[3, E^x] - PolyLog[4, -E^x] + PolyLog[4, E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}(e^x)\,dx = -\left(\frac{1}{2}\int x^2 \log(1-e^x)\,dx\right) + \frac{1}{2}\int x^2 \log(1+e^x)\,dx$$
$$= -\frac{1}{2}x^2\text{Li}_2(-e^x) + \frac{1}{2}x^2\text{Li}_2(e^x) + \int x\text{Li}_2(-e^x)\,dx - \int x\text{Li}_2(e^x)\,dx$$
$$= -\frac{1}{2}x^2\text{Li}_2(-e^x) + \frac{1}{2}x^2\text{Li}_2(e^x) + x\text{Li}_3(-e^x) - x\text{Li}_3(e^x) - \int \text{Li}_3(-e^x)\,dx + \int \text{Li}_3(e^x)\,dx$$
$$= -\frac{1}{2}x^2\text{Li}_2(-e^x) + \frac{1}{2}x^2\text{Li}_2(e^x) + x\text{Li}_3(-e^x) - x\text{Li}_3(e^x) - \text{Subst}\left(\int \frac{\text{Li}_3(-x)}{x}\,dx, x, e^x\right) + \text{Sub}$$
$$= -\frac{1}{2}x^2\text{Li}_2(-e^x) + \frac{1}{2}x^2\text{Li}_2(e^x) + x\text{Li}_3(-e^x) - x\text{Li}_3(e^x) - \text{Li}_4(-e^x) + \text{Li}_4(e^x)$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.60

$$\frac{1}{6}\left(-3x^2\text{Li}_2(-e^x) + 3x^2\text{Li}_2(e^x) + 6x\text{Li}_3(-e^x) - 6x\text{Li}_3(e^x) - 6\text{Li}_4(-e^x) + 6\text{Li}_4(e^x) + x^3\log(1-e^x) - x^3\log(e^x +$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[E^x],x]
```

```
[Out] (2*x^3*ArcTanh[E^x] + x^3*Log[1 - E^x] - x^3*Log[1 + E^x] - 3*x^2*PolyLog[2
, -E^x] + 3*x^2*PolyLog[2, E^x] + 6*x*PolyLog[3, -E^x] - 6*x*PolyLog[3, E^x
] - 6*PolyLog[4, -E^x] + 6*PolyLog[4, E^x])/6
```

fricas [C] time = 0.53, size = 120, normalized size = 2.07

$$\frac{1}{6}x^3\log\left(-\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)-1}\right) - \frac{1}{6}x^3\log(\cosh(x)+\sinh(x)+1) + \frac{1}{6}x^3\log(-\cosh(x)-\sinh(x)+1) + \frac{1}{2}x^2\text{L}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3*log(-(cosh(x) + sinh(x) + 1)/(cosh(x) + sinh(x) - 1)) - 1/6*x^3*log
(cosh(x) + sinh(x) + 1) + 1/6*x^3*log(-cosh(x) - sinh(x) + 1) + 1/2*x^2*dil
og(cosh(x) + sinh(x)) - 1/2*x^2*dilog(-cosh(x) - sinh(x)) - x*polylog(3, co
sh(x) + sinh(x)) + x*polylog(3, -cosh(x) - sinh(x)) + polylog(4, cosh(x) +
sinh(x)) - polylog(4, -cosh(x) - sinh(x))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(e^x), x)
```

maple [A] time = 0.04, size = 79, normalized size = 1.36

$$\frac{x^3\operatorname{arctanh}(e^x)}{3} - \frac{x^3\ln(e^x+1)}{6} - \frac{x^2\operatorname{polylog}(2,-e^x)}{2} + x\operatorname{polylog}(3,-e^x) - \operatorname{polylog}(4,-e^x) + \frac{x^3\ln(1-e^x)}{6} + \frac{x^2\operatorname{pol}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(exp(x)),x)
```

```
[Out] 1/3*x^3*arctanh(exp(x))-1/6*x^3*ln(exp(x)+1)-1/2*x^2*polylog(2,-exp(x))+x*p
olylog(3,-exp(x))-polylog(4,-exp(x))+1/6*x^3*ln(1-exp(x))+1/2*x^2*polylog(2
,exp(x))-x*polylog(3,exp(x))+polylog(4,exp(x))
```

maxima [A] time = 0.31, size = 76, normalized size = 1.31

$$\frac{1}{3}x^3\operatorname{artanh}(e^x)-\frac{1}{6}x^3\log(e^x+1)+\frac{1}{6}x^3\log(-e^x+1)-\frac{1}{2}x^2\mathrm{Li}_2(-e^x)+\frac{1}{2}x^2\mathrm{Li}_2(e^x)+x\mathrm{Li}_3(-e^x)-x\mathrm{Li}_3(e^x)-\mathrm{Li}_4(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(e^x) - 1/6*x^3*log(e^x + 1) + 1/6*x^3*log(-e^x + 1) - 1/2*x
^2*dilog(-e^x) + 1/2*x^2*dilog(e^x) + x*polylog(3, -e^x) - x*polylog(3, e^x
) - polylog(4, -e^x) + polylog(4, e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\operatorname{atanh}(\mathrm{e}^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(exp(x)),x)
```

```
[Out] int(x^2*atanh(exp(x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\operatorname{atanh}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(exp(x)),x)
```

```
[Out] Integral(x**2*atanh(exp(x)), x)
```

3.349 $\int \tanh^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=35

$$\frac{\text{Li}_2\left(e^{a+bx}\right)}{2b}-\frac{\text{Li}_2\left(-e^{a+bx}\right)}{2b}$$

```
[Out] -1/2*polylog(2,-exp(b*x+a))/b+1/2*polylog(2,exp(b*x+a))/b
```

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2282, 5912}

$$\frac{\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}-\frac{\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[E^(a + b*x)],x]
[Out] -PolyLog[2, -E^(a + b*x)]/(2*b) + PolyLog[2, E^(a + b*x)]/(2*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \tanh^{-1}\left(e^{a+bx}\right) dx &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{x}\, dx, x, e^{a+bx}\right)}{b}\\ &= -\frac{\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{\text{Li}_2\left(e^{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 1.94

$$\frac{-\text{Li}_2\left(-e^{a+bx}\right)+\text{Li}_2\left(e^{a+bx}\right)+bx\left(\log\left(1-e^{a+bx}\right)-\log\left(e^{a+bx}+1\right)+2\tanh^{-1}\left(e^{a+bx}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[E^(a + b*x)],x]
[Out] (b*x*(2*ArcTanh[E^(a + b*x)] + Log[1 - E^(a + b*x)] - Log[1 + E^(a + b*x)])
 - PolyLog[2, -E^(a + b*x)] + PolyLog[2, E^(a + b*x)])/(2*b)
```

fricas [B] time = 0.67, size = 138, normalized size = 3.94

$$bx\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-bx\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)-a\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh(b*x
 + a) - 1)) - b*x*log(cosh(b*x + a) + sinh(b*x + a) + 1) - a*log(cosh(b*x +
 a) + sinh(b*x + a) - 1) + (b*x + a)*log(-cosh(b*x + a) - sinh(b*x + a) + 1
) + dilog(cosh(b*x + a) + sinh(b*x + a)) - dilog(-cosh(b*x + a) - sinh(b*x
+ a)))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{artanh}\left(e^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(arctanh(e^(b*x + a)), x)
```

maple [B] time = 0.05, size = 67, normalized size = 1.91

$$\frac{\ln\left(e^{bx+a}\right)\operatorname{arctanh}\left(e^{bx+a}\right)}{b}-\frac{\operatorname{dilog}\left(e^{bx+a}\right)}{2b}-\frac{\operatorname{dilog}\left(e^{bx+a}+1\right)}{2b}-\frac{\ln\left(e^{bx+a}\right)\ln\left(e^{bx+a}+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(exp(b*x+a)),x)
```

```
[Out] 1/b*ln(exp(b*x+a))*arctanh(exp(b*x+a))-1/2/b*dilog(exp(b*x+a))-1/2/b*dilog(
exp(b*x+a)+1)-1/2/b*ln(exp(b*x+a))*ln(exp(b*x+a)+1)
```

maxima [B] time = 0.31, size = 107, normalized size = 3.06

$$\frac{(bx+a)\operatorname{artanh}\left(e^{(bx+a)}\right)}{b}-\frac{(bx+a)\left(\log\left(e^{(bx+a)}+1\right)-\log\left(e^{(bx+a)}-1\right)\right)-\log\left(-e^{(bx+a)}\right)\log\left(e^{(bx+a)}+1\right)+(bx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] (b*x + a)*arctanh(e^(b*x + a))/b - 1/2*((b*x + a)*(log(e^(b*x + a) + 1) - l
og(e^(b*x + a) - 1)) - log(-e^(b*x + a))*log(e^(b*x + a) + 1) + (b*x + a)*l
og(e^(b*x + a) - 1) - dilog(e^(b*x + a) + 1) + dilog(-e^(b*x + a) + 1))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{atanh}\left(e^{a+b\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(exp(a + b*x)),x)
```

```
[Out] int(atanh(exp(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atanh}\left(e^{a+bx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(exp(b*x+a)),x)
```

```
[Out] Integral(atanh(exp(a + b*x)), x)
```

3.350 $\int x \tanh^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=71

$$\frac{\text{Li}_3\left(-e^{a+bx}\right)}{2b^2}-\frac{\text{Li}_3\left(e^{a+bx}\right)}{2b^2}-\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}$$

```
[Out] -1/2*x*polylog(2,-exp(b*x+a))/b+1/2*x*polylog(2,exp(b*x+a))/b+1/2*polylog(3
,-exp(b*x+a))/b^2-1/2*polylog(3,exp(b*x+a))/b^2
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {6213, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-e^{a+bx}\right)}{2b^2}-\frac{\text{PolyLog}\left(3,e^{a+bx}\right)}{2b^2}-\frac{x\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}+\frac{x\text{PolyLog}\left(2,e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[E^(a + b*x)],x]
```

```
[Out] -(x*PolyLog[2, -E^(a + b*x)])/(2*b) + (x*PolyLog[2, E^(a + b*x)])/(2*b) + P
olyLog[3, -E^(a + b*x)]/(2*b^2) - PolyLog[3, E^(a + b*x)]/(2*b^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}\left(e^{a+bx}\right)dx = -\left(\frac{1}{2}\int x\log\left(1-e^{a+bx}\right)dx\right)+\frac{1}{2}\int x\log\left(1+e^{a+bx}\right)dx$$
$$= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\int \text{Li}_2\left(-e^{a+bx}\right)dx}{2b}-\frac{\int \text{Li}_2\left(e^{a+bx}\right)dx}{2b}$$
$$= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x}dx, x, e^{a+bx}\right)}{2b^2}-\frac{\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}dx, x\right.}{2b^2}$$
$$= -\frac{x\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x\text{Li}_2\left(e^{a+bx}\right)}{2b}+\frac{\text{Li}_3\left(-e^{a+bx}\right)}{2b^2}-\frac{\text{Li}_3\left(e^{a+bx}\right)}{2b^2}$$

Mathematica [A] time = 0.05, size = 113, normalized size = 1.59

$$\frac{b^2x^2\log\left(1-e^{a+bx}\right)-b^2x^2\log\left(e^{a+bx}+1\right)+2b^2x^2\tanh^{-1}\left(e^{a+bx}\right)-2bx\text{Li}_2\left(-e^{a+bx}\right)+2bx\text{Li}_2\left(e^{a+bx}\right)+2\text{Li}_3\left(\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[E^(a + b*x)],x]
```

```
[Out] (2*b^2*x^2*ArcTanh[E^(a + b*x)] + b^2*x^2*Log[1 - E^(a + b*x)] - b^2*x^2*Lo
g[1 + E^(a + b*x)] - 2*b*x*PolyLog[2, -E^(a + b*x)] + 2*b*x*PolyLog[2, E^(a
 + b*x)] + 2*PolyLog[3, -E^(a + b*x)] - 2*PolyLog[3, E^(a + b*x)])/(4*b^2)
```

fricas [C] time = 0.90, size = 199, normalized size = 2.80

$$\frac{b^2x^2\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right)-b^2x^2\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right)+2\,bx\text{Li}_2\left(\cosh\left(bx+a\right)+\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^2*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^2*x^2*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 2*b*x*di
log(cosh(b*x + a) + sinh(b*x + a)) - 2*b*x*dilog(-cosh(b*x + a) - sinh(b*x
+ a)) + a^2*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b^2*x^2 - a^2)*log(-c
osh(b*x + a) - sinh(b*x + a) + 1) - 2*polylog(3, cosh(b*x + a) + sinh(b*x +
 a)) + 2*polylog(3, -cosh(b*x + a) - sinh(b*x + a)))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,\text{artanh}\left(e^{(bx+a)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(e^(b*x + a)), x)
```

maple [B] time = 0.05, size = 153, normalized size = 2.15

$$\frac{x^2\,\text{arctanh}\left(e^{bx+a}\right)}{2}-\frac{\ln\left(e^{bx+a}+1\right)x^2}{4}+\frac{\ln\left(e^{bx+a}+1\right)a^2}{4b^2}-\frac{x\,\text{polylog}\left(2,-e^{bx+a}\right)}{2b}+\frac{\text{polylog}\left(3,-e^{bx+a}\right)}{2b^2}+\frac{\ln\left(1-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(exp(b*x+a)),x)
```

```
[Out] 1/2*x^2*arctanh(exp(b*x+a))-1/4*ln(exp(b*x+a)+1)*x^2+1/4/b^2*ln(exp(b*x+a)+
1)*a^2-1/2*x*polylog(2,-exp(b*x+a))/b+1/2*polylog(3,-exp(b*x+a))/b^2+1/4*ln
(1-exp(b*x+a))*x^2-1/4/b^2*ln(1-exp(b*x+a))*a^2+1/2*x*polylog(2,exp(b*x+a))
/b-1/2*polylog(3,exp(b*x+a))/b^2-1/2/b^2*a^2*arctanh(exp(b*x+a))
```

maxima [A] time = 0.33, size = 108, normalized size = 1.52

$$\frac{1}{2}\,x^2\,\operatorname{artanh}\left(e^{(bx+a)}\right)-\frac{1}{4}\,b\left(\frac{b^2x^2\log\left(e^{(bx+a)}+1\right)+2\,bx\operatorname{Li}_2\left(-e^{(bx+a)}\right)-2\operatorname{Li}_3(-e^{(bx+a)})}{b^3}-\frac{b^2x^2\log\left(-e^{(bx+a)}+1\right)+2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctanh(e^(b*x + a)) - 1/4*b*((b^2*x^2*log(e^(b*x + a) + 1) + 2*b*x
*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))/b^3 - (b^2*x^2*log(-e^(b
*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x + a)))/b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\operatorname{atanh}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(exp(a + b*x)),x)
```

```
[Out] int(x*atanh(exp(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{atanh}\left(e^{a}e^{bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(exp(b*x+a)),x)
```

```
[Out] Integral(x*atanh(exp(a)*exp(b*x)), x)
```

3.351 $\int x^2 \tanh^{-1}\left(e^{a+bx}\right) dx$

Optimal. Leaf size=101

$$-\frac{\text{Li}_4\left(-e^{a+bx}\right)}{b^3}+\frac{\text{Li}_4\left(e^{a+bx}\right)}{b^3}+\frac{x\text{Li}_3\left(-e^{a+bx}\right)}{b^2}-\frac{x\text{Li}_3\left(e^{a+bx}\right)}{b^2}-\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b}+\frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b}$$

```
[Out] -1/2*x^2*polylog(2,-exp(b*x+a))/b+1/2*x^2*polylog(2,exp(b*x+a))/b+x*polylog
(3,-exp(b*x+a))/b^2-x*polylog(3,exp(b*x+a))/b^2-polylog(4,-exp(b*x+a))/b^3+
polylog(4,exp(b*x+a))/b^3
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6213, 2531, 6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(3,-e^{a+bx}\right)}{b^2}-\frac{x\text{PolyLog}\left(3,e^{a+bx}\right)}{b^2}-\frac{\text{PolyLog}\left(4,-e^{a+bx}\right)}{b^3}+\frac{\text{PolyLog}\left(4,e^{a+bx}\right)}{b^3}-\frac{x^2\text{PolyLog}\left(2,-e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[E^(a + b*x)],x]
```

```
[Out] -(x^2*PolyLog[2, -E^(a + b*x)])/(2*b) + (x^2*PolyLog[2, E^(a + b*x)])/(2*b)
 + (x*PolyLog[3, -E^(a + b*x)])/b^2 - (x*PolyLog[3, E^(a + b*x)])/b^2 - Pol
yLog[4, -E^(a + b*x)]/b^3 + PolyLog[4, E^(a + b*x)]/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \tanh^{-1}\left(e^{a+bx}\right) dx = -\left(\frac{1}{2}\int x^2 \log\left(1-e^{a+bx}\right) dx\right) + \frac{1}{2}\int x^2 \log\left(1+e^{a+bx}\right) dx$$
$$= -\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b} + \frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b} + \frac{\int x\text{Li}_2\left(-e^{a+bx}\right) dx}{b} - \frac{\int x\text{Li}_2\left(e^{a+bx}\right) dx}{b}$$
$$= -\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b} + \frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b} + \frac{x\text{Li}_3\left(-e^{a+bx}\right)}{b^2} - \frac{x\text{Li}_3\left(e^{a+bx}\right)}{b^2} - \frac{\int \text{Li}_3\left(-e^{a+bx}\right) dx}{b^2} +$$
$$= -\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b} + \frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b} + \frac{x\text{Li}_3\left(-e^{a+bx}\right)}{b^2} - \frac{x\text{Li}_3\left(e^{a+bx}\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{\text{Li}_3(-x)}{x} dx,\right.}{b^3}$$
$$= -\frac{x^2\text{Li}_2\left(-e^{a+bx}\right)}{2b} + \frac{x^2\text{Li}_2\left(e^{a+bx}\right)}{2b} + \frac{x\text{Li}_3\left(-e^{a+bx}\right)}{b^2} - \frac{x\text{Li}_3\left(e^{a+bx}\right)}{b^2} - \frac{\text{Li}_4\left(-e^{a+bx}\right)}{b^3} + \frac{\text{Li}_4\left(\right.}{}$$

Mathematica [A] time = 0.05, size = 149, normalized size = 1.48

$$\frac{b^3x^3\log\left(1-e^{a+bx}\right) - b^3x^3\log\left(e^{a+bx}+1\right) + 2b^3x^3\tanh^{-1}\left(e^{a+bx}\right) - 3b^2x^2\text{Li}_2\left(-e^{a+bx}\right) + 3b^2x^2\text{Li}_2\left(e^{a+bx}\right) + 6bx\text{Li}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[E^(a + b*x)],x]
```

```
[Out] (2*b^3*x^3*ArcTanh[E^(a + b*x)] + b^3*x^3*Log[1 - E^(a + b*x)] - b^3*x^3*Lo
g[1 + E^(a + b*x)] - 3*b^2*x^2*PolyLog[2, -E^(a + b*x)] + 3*b^2*x^2*PolyLog
[2, E^(a + b*x)] + 6*b*x*PolyLog[3, -E^(a + b*x)] - 6*b*x*PolyLog[3, E^(a +
 b*x)] - 6*PolyLog[4, -E^(a + b*x)] + 6*PolyLog[4, E^(a + b*x)])/(6*b^3)
```

fricas [C] time = 0.60, size = 248, normalized size = 2.46

$$\frac{b^3x^3\log\left(-\frac{\cosh(bx+a)+\sinh(bx+a)+1}{\cosh(bx+a)+\sinh(bx+a)-1}\right) - b^3x^3\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+1\right) + 3\,b^2x^2\text{Li}_2\left(\cosh\left(bx+a\right)+\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^3*log(-(cosh(b*x + a) + sinh(b*x + a) + 1)/(cosh(b*x + a) + sinh
(b*x + a) - 1)) - b^3*x^3*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*b^2*x^
2*dilog(cosh(b*x + a) + sinh(b*x + a)) - 3*b^2*x^2*dilog(-cosh(b*x + a) - s
inh(b*x + a)) - a^3*log(cosh(b*x + a) + sinh(b*x + a) - 1) - 6*b*x*polylog(
3, cosh(b*x + a) + sinh(b*x + a)) + 6*b*x*polylog(3, -cosh(b*x + a) - sinh(
b*x + a)) + (b^3*x^3 + a^3)*log(-cosh(b*x + a) - sinh(b*x + a) + 1) + 6*pol
ylog(4, cosh(b*x + a) + sinh(b*x + a)) - 6*polylog(4, -cosh(b*x + a) - sinh
(b*x + a)))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}\left(e^{(bx+a)}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="giac")
```

[Out] integrate(x^2*arctanh(e^(b*x + a)), x)

maple [B] time = 0.05, size = 185, normalized size = 1.83

$$\frac{x^3 \operatorname{arctanh}\left(e^{bx+a}\right)}{3}-\frac{\operatorname{polylog}\left(4,-e^{bx+a}\right)}{b^3}+\frac{\operatorname{polylog}\left(4,e^{bx+a}\right)}{b^3}-\frac{\ln\left(e^{bx+a}+1\right)x^3}{6}-\frac{x^2\operatorname{polylog}\left(2,-e^{bx+a}\right)}{2b}+\frac{x\,\mathrm{pc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arctanh(exp(b*x+a)),x)

[Out] 1/3*x^3*arctanh(exp(b*x+a))-polylog(4,-exp(b*x+a))/b^3+polylog(4,exp(b*x+a))/b^3-1/6*ln(exp(b*x+a)+1)*x^3-1/2*x^2*polylog(2,-exp(b*x+a))/b+x*polylog(3,-exp(b*x+a))/b^2+1/6*ln(1-exp(b*x+a))*x^3+1/2*x^2*polylog(2,exp(b*x+a))/b-x*polylog(3,exp(b*x+a))/b^2+1/3/b^3*a^3*arctanh(exp(b*x+a))-1/6/b^3*ln(exp(b*x+a)+1)*a^3+1/6/b^3*ln(1-exp(b*x+a))*a^3

maxima [A] time = 0.33, size = 142, normalized size = 1.41

$$\frac{1}{3}x^3 \operatorname{artanh}\left(e^{(bx+a)}\right)-\frac{1}{6}b\left(\frac{b^3x^3\log\left(e^{(bx+a)}+1\right)+3\,b^2x^2\mathrm{Li}_2\left(-e^{(bx+a)}\right)-6\,bx\mathrm{Li}_3(-e^{(bx+a)})+6\,\mathrm{Li}_4(-e^{(bx+a)})}{b^4}-\frac{b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arctanh(exp(b*x+a)),x, algorithm="maxima")

[Out] 1/3*x^3*arctanh(e^(b*x + a)) - 1/6*b*((b^3*x^3*log(e^(b*x + a) + 1) + 3*b^2*x^2*dilog(-e^(b*x + a)) - 6*b*x*polylog(3, -e^(b*x + a)) + 6*polylog(4, -e^(b*x + a)))/b^4 - (b^3*x^3*log(-e^(b*x + a) + 1) + 3*b^2*x^2*dilog(e^(b*x + a)) - 6*b*x*polylog(3, e^(b*x + a)) + 6*polylog(4, e^(b*x + a)))/b^4)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{atanh}\left(e^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*atanh(exp(a + b*x)),x)

[Out] int(x^2*atanh(exp(a + b*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{atanh}\left(e^a e^{bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*atanh(exp(b*x+a)),x)

[Out] Integral(x**2*atanh(exp(a)*exp(b*x)), x)

3.352 $\int \tanh^{-1}\left(a+bf^{c+dx}\right)\,dx$

Optimal. Leaf size=168

$$\frac{\text{Li}_2\left(1-\frac{2}{bf^{c+dx}+a+1}\right)}{2d\log(f)}-\frac{\text{Li}_2\left(1-\frac{2bf^{c+dx}}{(1-a)\left(bf^{c+dx}+a+1\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\tanh^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{2bf^{c+dx}}{(1-a)\left(a+bf^{c+dx}+1\right)}\right)\text{tanh}}{d\log(f)}$$

```
[Out] -arctanh(a+b*f^(d*x+c))*ln(2/(1+a+b*f^(d*x+c)))/d/ln(f)+arctanh(a+b*f^(d*x+
c))*ln(2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c)))/d/ln(f)+1/2*polylog(2,1-2/(1+
a+b*f^(d*x+c)))/d/ln(f)-1/2*polylog(2,1-2*b*f^(d*x+c)/(1-a)/(1+a+b*f^(d*x+c
)))/d/ln(f)
```

Rubi [A] time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2282, 6111, 5920, 2402, 2315, 2447}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{a+bf^{c+dx}+1}\right)}{2d\log(f)}-\frac{\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(1-a)\left(a+bf^{c+dx}+1\right)}\right)}{2d\log(f)}-\frac{\log\left(\frac{2}{a+bf^{c+dx}+1}\right)\tanh^{-1}\left(a+bf^{c+dx}\right)}{d\log(f)}+\frac{\log\left(\frac{}{(1-a)(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] -((ArcTanh[a + b*f^(c + d*x)]*Log[2/(1 + a + b*f^(c + d*x))])/(d*Log[f])) +
 (ArcTanh[a + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c
 + d*x)))])/(d*Log[f]) + PolyLog[2, 1 - 2/(1 + a + b*f^(c + d*x))]/(2*d*Log
[f]) - PolyLog[2, 1 - (2*b*f^(c + d*x))/((1 - a)*(1 + a + b*f^(c + d*x)))]/
(2*d*Log[f])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
```

```
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6111

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcTanh[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGt
Q[p, 0]
```

Rubi steps

$$\int \tanh^{-1}\left(a+bf^{c+dx}\right)dx = \frac{\operatorname{Subst}\left(\int \frac{\tanh^{-1}(a+bx)}{x}\,dx, x, f^{c+dx}\right)}{d\log(f)}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\tanh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bf^{c+dx}\right)}{bd\log(f)}$$

$$= -\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right.}\right.}{d\log(f)}$$

$$= -\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right.}\right.}{d\log(f)}$$

$$= -\frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1+a+bf^{c+dx}}\right)}{d\log(f)} + \frac{\tanh^{-1}\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(1-a)\left(1+a+bf^{c+dx}\right.}\right.}{d\log(f)}$$

Mathematica [A] time = 0.08, size = 108, normalized size = 0.64

$$\frac{\operatorname{Li}_2\left(-\frac{bf^{c+dx}}{a-1}\right)-\operatorname{Li}_2\left(-\frac{bf^{c+dx}}{a+1}\right)+dx\log(f)\left(\log\left(\frac{a+bf^{c+dx}-1}{a-1}\right)-\log\left(\frac{a+bf^{c+dx}+1}{a+1}\right)+2\tanh^{-1}\left(a+bf^{c+dx}\right)\right)}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (d*x*Log[f]*(2*ArcTanh[a + b*f^(c + d*x)] + Log[(-1 + a + b*f^(c + d*x))/(-
1 + a)] - Log[(1 + a + b*f^(c + d*x))/(1 + a)]) + PolyLog[2, -((b*f^(c + d*
x))/(-1 + a))] - PolyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f])
```

fricas [A] time = 0.49, size = 284, normalized size = 1.69

$$dx\log(f)\log\left(-\frac{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a+1}{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a-1}\right)+c\log\left(b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\,\mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(d*x*log(f)*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)) + c*
log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f) - c
*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f) -
(d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(a + 1)) + (d*x + c)*log(f)*log((b*cosh((d*x + c)*log(f)) + b*sinh(
(d*x + c)*log(f)) + a - 1)/(a - 1)) - dilog(-(b*cosh((d*x + c)*log(f)) + b*
sinh((d*x + c)*log(f)) + a + 1)/(a + 1) + 1) + dilog(-(b*cosh((d*x + c)*log
(f)) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1))/(d*log(f))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.8Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,2,0,0,0]%%%}+%%%{2,[0,1,1,1,1,0]%%%}+%%%{-2,[0,1,1,0,0
,0]%%%}+%%%{1,[0,1,0,2,0,1]%%%}+%%%{-2,[0,1,0,1,1,0]%%%}+%%%{1,[0,1,0,0,0,0
]%%%} / %%%{4,[0,0,0,2,0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.08, size = 164, normalized size = 0.98

$$\frac{\ln\left(b\,f^{dx+c}\right)\operatorname{arctanh}\left(a+b\,f^{dx+c}\right)}{d\ln(f)}-\frac{\operatorname{dilog}\left(\frac{1+a+b\,f^{dx+c}}{1+a}\right)}{2d\ln(f)}-\frac{\ln\left(b\,f^{dx+c}\right)\ln\left(\frac{1+a+b\,f^{dx+c}}{1+a}\right)}{2d\ln(f)}+\frac{\operatorname{dilog}\left(\frac{b\,f^{dx+c}+a-1}{a-1}\right)}{2d\ln(f)}+\frac{\ln\left(b\,f^{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a+b*f^(d*x+c)),x)
```

```
[Out] 1/d/ln(f)*ln(b*f^(d*x+c))*arctanh(a+b*f^(d*x+c))-1/2/d/ln(f)*dilog((1+a+b*f
^(d*x+c))/(1+a))-1/2/d/ln(f)*ln(b*f^(d*x+c))*ln((1+a+b*f^(d*x+c))/(1+a))+1/
2/d/ln(f)*dilog((b*f^(d*x+c)+a-1)/(a-1))+1/2/d/ln(f)*ln(b*f^(d*x+c))*ln((b*
f^(d*x+c)+a-1)/(a-1))
```

maxima [A] time = 0.33, size = 202, normalized size = 1.20

$$\frac{(dx+c)\operatorname{artanh}\left(bf^{dx+c}+a\right)}{d}-\frac{(dx+c)b\left(\frac{\log\left(bf^{dx+c}+a+1\right)}{b}-\frac{\log\left(bf^{dx+c}+a-1\right)}{b}\right)\log(f)-b\left(\frac{\log\left(bf^{dx+c}+a+1\right)\log\left(-\frac{bf^{dx+c}+a+1}{a+1}+1\right.}{b}\right.}{2\,d\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)*arctanh(b*f^(d*x + c) + a)/d - 1/2*((d*x + c)*b*(log(b*f^(d*x + c
) + a + 1)/b - log(b*f^(d*x + c) + a - 1)/b)*log(f) - b*((log(b*f^(d*x + c)
 + a + 1)*log(-(b*f^(d*x + c) + a + 1)/(a + 1) + 1) + dilog((b*f^(d*x + c)
+ a + 1)/(a + 1)))/b - (log(b*f^(d*x + c) + a - 1)*log(-(b*f^(d*x + c) + a
- 1)/(a - 1) + 1) + dilog((b*f^(d*x + c) + a - 1)/(a - 1)))/b))/(d*log(f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{atanh}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(a + b*f^(c + d*x)),x)
[Out] int(atanh(a + b*f^(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a+b*f**(d*x+c)),x)
[Out] Timed out
```

3.353 $\int x \tanh^{-1}\left(a + bf^{c+dx}\right) dx$

Optimal. Leaf size=211

$$-\frac{\text{Li}_3\left(\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{Li}_3\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d^2\log^2(f)}+\frac{x\text{Li}_2\left(\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\text{Li}_2\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}-\frac{1}{4}x^2\log\left(-a-bf^{c+dx}+1\right)+\frac{1}{4}x^2\log\left(a+bf^{c+dx}+1\right)$$

```
[Out] -1/4*x^2*ln(1-a-b*f^(d*x+c))+1/4*x^2*ln(1+a+b*f^(d*x+c))+1/4*x^2*ln(1-b*f^(
d*x+c)/(1-a))-1/4*x^2*ln(1+b*f^(d*x+c)/(1+a))+1/2*x*polylog(2,b*f^(d*x+c)/(
1-a))/d/ln(f)-1/2*x*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-1/2*polylog(3,b*f
^(d*x+c)/(1-a))/d^2/ln(f)^2+1/2*polylog(3,-b*f^(d*x+c)/(1+a))/d^2/ln(f)^2
```

Rubi [A] time = 0.15, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6213, 2532, 2531, 2282, 6589}

$$-\frac{\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{2d^2\log^2(f)}+\frac{\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{2d^2\log^2(f)}+\frac{x\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}-\frac{1}{4}x^2\log\left(-a-bf^{c+dx}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] -(x^2*Log[1 - a - b*f^(c + d*x)])/4 + (x^2*Log[1 + a + b*f^(c + d*x)])/4 +
(x^2*Log[1 - (b*f^(c + d*x))/(1 - a)])/4 - (x^2*Log[1 + (b*f^(c + d*x))/(1
+ a)])/4 + (x*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x*PolyLo
g[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - PolyLog[3, (b*f^(c + d*x))
/(1 - a)]/(2*d^2*Log[f]^2) + PolyLog[3, -((b*f^(c + d*x))/(1 + a))]/(2*d^2*
Log[f]^2)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
```

```
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\tanh^{-1}\left(a+bf^{c+dx}\right)dx = -\left(\frac{1}{2}\int x\log\left(1-a-bf^{c+dx}\right)dx\right)+\frac{1}{2}\int x\log\left(1+a+bf^{c+dx}\right)dx$$

$$= -\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-$$

$$= -\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-$$

$$= -\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-$$

$$= -\frac{1}{4}x^2\log\left(1-a-bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1+a+bf^{c+dx}\right)+\frac{1}{4}x^2\log\left(1-\frac{bf^{c+dx}}{1-a}\right)-$$

Mathematica [A] time = 0.12, size = 177, normalized size = 0.84

$$\frac{d^2x^2\log^2(f)\log\left(\frac{bf^{c+dx}}{a-1}+1\right)-d^2x^2\log^2(f)\log\left(\frac{bf^{c+dx}}{a+1}+1\right)+2d^2x^2\log^2(f)\tanh^{-1}\left(a+bf^{c+dx}\right)-2\text{Li}_3\left(-\frac{bf^{c}}{a-}\right.}{4d^2\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^2*x^2*ArcTanh[a + b*f^(c + d*x)]*Log[f]^2 + d^2*x^2*Log[f]^2*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 2*d*x*Log[f]*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 2*d*x*Log[f]*Pol
yLog[2, -((b*f^(c + d*x))/(1 + a))] - 2*PolyLog[3, -((b*f^(c + d*x))/(-1 +
a))] + 2*PolyLog[3, -((b*f^(c + d*x))/(1 + a))])/(4*d^2*Log[f]^2)
```

fricas [C] time = 0.68, size = 396, normalized size = 1.88

$$\frac{d^2x^2\log(f)^2\log\left(-\frac{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a+1}{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a-1}\right)-c^2\log\left(b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(d^2*x^2*log(f)^2*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)
) - c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*lo
g(f)^2 + c^2*log(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a -
```

```
1)*log(f)^2 - 2*d*x*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(a + 1) + 1)*log(f) + 2*d*x*dilog(-(b*cosh((d*x + c)*log(f))
+ b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f) - (d^2*x^2 - c^2)*l
og(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)/(
a + 1)) + (d^2*x^2 - c^2)*log(f)^2*log((b*cosh((d*x + c)*log(f)) + b*sinh((
d*x + c)*log(f)) + a - 1)/(a - 1)) + 2*polylog(3, -(b*cosh((d*x + c)*log(f)
) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 2*polylog(3, -(b*cosh((d*x + c)*lo
g(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)))/(d^2*log(f)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{artanh}\left(bf^{dx+c} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(b*f^(d*x + c) + a), x)
```

maple [B] time = 0.13, size = 596, normalized size = 2.82

$$\frac{x^2 \ln\left(1 + a + b\, f^{dx+c}\right)}{4} - \frac{x^2 \ln\left(1 - a - b\, f^{dx+c}\right)}{4} + \frac{\ln\left(1 - \frac{b\, f^{dx} f^{c}}{1-a}\right) x^2}{4} + \frac{\ln\left(1 - \frac{b\, f^{dx} f^{c}}{1-a}\right) xc}{2d} + \frac{\ln\left(1 - \frac{b\, f^{dx} f^{c}}{1-a}\right) c^2}{4d^2} + \frac{\text{polylo}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a+b*f^(d*x+c)),x)
```

```
[Out] 1/4*x^2*ln(1+a+b*f^(d*x+c))-1/4*x^2*ln(1-a-b*f^(d*x+c))+1/4*ln(1-b*f^(d*x)*
f^c/(1-a))*x^2+1/2/d*ln(1-b*f^(d*x)*f^c/(1-a))*x*c+1/4/d^2*ln(1-b*f^(d*x)*f
^c/(1-a))*c^2+1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x+1/2/ln(f)/d^2*po
lylog(2,b*f^(d*x)*f^c/(1-a))*c-1/2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(1-a
))+1/4/d^2*c^2*ln(1-a-f^c*f^(d*x)*b)-1/2/ln(f)/d^2*c*dilog((f^c*f^(d*x)*b+a
-1)/(a-1))-1/2/d*c*ln((f^c*f^(d*x)*b+a-1)/(a-1))*x-1/2/d^2*c^2*ln((f^c*f^(d
*x)*b+a-1)/(a-1))-1/4*ln(1-b*f^(d*x)*f^c/(-1-a))*x^2-1/2/d*ln(1-b*f^(d*x)*f
^c/(-1-a))*x*c-1/4/d^2*ln(1-b*f^(d*x)*f^c/(-1-a))*c^2-1/2/ln(f)/d*polylog(2
,b*f^(d*x)*f^c/(-1-a))*x-1/2/ln(f)/d^2*polylog(2,b*f^(d*x)*f^c/(-1-a))*c+1/
2/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^c/(-1-a))-1/4/d^2*c^2*ln(1+a+f^c*f^(d*x
)*b)+1/2/ln(f)/d^2*c*dilog((1+a+f^c*f^(d*x)*b)/(1+a))+1/2/d*c*ln((1+a+f^c*f
^(d*x)*b)/(1+a))*x+1/2/d^2*c^2*ln((1+a+f^c*f^(d*x)*b)/(1+a))
```

maxima [A] time = 0.36, size = 194, normalized size = 0.92

$$-\frac{1}{4}bd\left(\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log(f)^2+2\,dx\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log(f)-2\,\mathrm{Li}_3(-\frac{bf^{dx}f^c}{a+1})}{bd^3\log(f)^3}-\frac{d^2x^2\log\left(\frac{bf^{dx}f^c}{a-1}+1\right)\log(f)^2+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*b*d*((d^2*x^2*log(b*f^(d*x)*f^c/(a + 1) + 1)*log(f)^2 + 2*d*x*dilog(-b
*f^(d*x)*f^c/(a + 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a + 1)))/(b*d^3
*log(f)^3) - (d^2*x^2*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f)^2 + 2*d*x*dilog
(-b*f^(d*x)*f^c/(a - 1))*log(f) - 2*polylog(3, -b*f^(d*x)*f^c/(a - 1)))/(b*
d^3*log(f)^3))*log(f) + 1/2*x^2*arctanh(b*f^(d*x + c) + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\mathrm{atanh}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atanh(a + b*f^(c + d*x)),x)
```

```
[Out] int(x*atanh(a + b*f^(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

3.354 $\int x^2 \tanh^{-1}\left(a + bf^{c+dx}\right) dx$

Optimal. Leaf size=264

$$\frac{\text{Li}_4\left(\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{Li}_4\left(-\frac{bf^{c+dx}}{a+1}\right)}{d^3\log^3(f)}-\frac{x\text{Li}_3\left(\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\text{Li}_3\left(-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}+\frac{x^2\text{Li}_2\left(\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}-\frac{x^2\text{Li}_2\left(-\frac{bf^{c+dx}}{a+1}\right)}{2d\log(f)}-\frac{1}{6}x^3\log\left(-a-bf^{c+dx}\right.$$

```
[Out] -1/6*x^3*ln(1-a-b*f^(d*x+c))+1/6*x^3*ln(1+a+b*f^(d*x+c))+1/6*x^3*ln(1-b*f^(
d*x+c)/(1-a))-1/6*x^3*ln(1+b*f^(d*x+c)/(1+a))+1/2*x^2*polylog(2,b*f^(d*x+c)
/(1-a))/d/ln(f)-1/2*x^2*polylog(2,-b*f^(d*x+c)/(1+a))/d/ln(f)-x*polylog(3,b
*f^(d*x+c)/(1-a))/d^2/ln(f)^2+x*polylog(3,-b*f^(d*x+c)/(1+a))/d^2/ln(f)^2+p
olylog(4,b*f^(d*x+c)/(1-a))/d^3/ln(f)^3-polylog(4,-b*f^(d*x+c)/(1+a))/d^3/l
n(f)^3
```

Rubi [A] time = 0.20, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6213, 2532, 2531, 6609, 2282, 6589}

$$-\frac{x\text{PolyLog}\left(3,\frac{bf^{c+dx}}{1-a}\right)}{d^2\log^2(f)}+\frac{x\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+1}\right)}{d^2\log^2(f)}+\frac{\text{PolyLog}\left(4,\frac{bf^{c+dx}}{1-a}\right)}{d^3\log^3(f)}-\frac{\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+1}\right)}{d^3\log^3(f)}+\frac{x^2\text{PolyLog}\left(2,\frac{bf^{c+dx}}{1-a}\right)}{2d\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] -(x^3*Log[1 - a - b*f^(c + d*x)])/6 + (x^3*Log[1 + a + b*f^(c + d*x)])/6 +
(x^3*Log[1 - (b*f^(c + d*x))/(1 - a)])/6 - (x^3*Log[1 + (b*f^(c + d*x))/(1
+ a)])/6 + (x^2*PolyLog[2, (b*f^(c + d*x))/(1 - a)])/(2*d*Log[f]) - (x^2*Po
lyLog[2, -((b*f^(c + d*x))/(1 + a))])/(2*d*Log[f]) - (x*PolyLog[3, (b*f^(c
+ d*x))/(1 - a)])/(d^2*Log[f]^2) + (x*PolyLog[3, -((b*f^(c + d*x))/(1 + a))
])/(d^2*Log[f]^2) + PolyLog[4, (b*f^(c + d*x))/(1 - a)]/(d^3*Log[f]^3) - Po
lyLog[4, -((b*f^(c + d*x))/(1 + a))]/(d^3*Log[f]^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6213

```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Dist[1/2, Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Dist[1/2, Int[x^m
*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[m,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2\tanh^{-1}\left(a+bf^{c+dx}\right)dx=-\left(\frac{1}{2}\int x^2\log\left(1-a-bf^{c+dx}\right)dx\right)+\frac{1}{2}\int x^2\log\left(1+a+bf^{c+dx}\right)dx$$

$$=-\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)$$

$$=-\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)$$

$$=-\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)$$

$$=-\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)$$

$$=-\frac{1}{6}x^3\log\left(1-a-bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1+a+bf^{c+dx}\right)+\frac{1}{6}x^3\log\left(1-\frac{bf^{c+dx}}{1-a}\right)$$

Mathematica [A] time = 0.09, size = 235, normalized size = 0.89

$$d^3x^3\log^3(f)\log\left(\frac{bf^{c+dx}}{a-1}+1\right)-d^3x^3\log^3(f)\log\left(\frac{bf^{c+dx}}{a+1}+1\right)+2d^3x^3\log^3(f)\tanh^{-1}\left(a+bf^{c+dx}\right)+3d^2x^2\log^2$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTanh[a + b*f^(c + d*x)],x]
```

```
[Out] (2*d^3*x^3*ArcTanh[a + b*f^(c + d*x)]*Log[f]^3 + d^3*x^3*Log[f]^3*Log[1 + (
b*f^(c + d*x))/(-1 + a)] - d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(1 + a)
] + 3*d^2*x^2*Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(-1 + a))] - 3*d^2*x^2*
Log[f]^2*PolyLog[2, -((b*f^(c + d*x))/(1 + a))] - 6*d*x*Log[f]*PolyLog[3, -
((b*f^(c + d*x))/(-1 + a))] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x))/(1
```

```
+ a))] + 6*PolyLog[4, -((b*f^(c + d*x))/(-1 + a))] - 6*PolyLog[4, -((b*f^(c
 + d*x))/(1 + a))])/(6*d^3*Log[f]^3)
```

fricas [C] time = 0.53, size = 480, normalized size = 1.82

$$\frac{d^3x^3\log(f)^3\log\left(-\frac{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a+1}{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a-1}\right)-3\,d^2x^2\mathrm{Li}_2\left(-\frac{b\cosh\left((dx+c)\log(f)\right)+b\sinh\left((dx+c)\log(f)\right)+a+1}{a+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(d^3*x^3*log(f)^3*log(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log
(f)) + a + 1)/(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)
) - 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) +
 a + 1)/(a + 1) + 1)*log(f)^2 + 3*d^2*x^2*dilog(-(b*cosh((d*x + c)*log(f))
+ b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1) + 1)*log(f)^2 + c^3*log(b*cosh(
(d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a + 1)*log(f)^3 - c^3*log(b*
cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)) + a - 1)*log(f)^3 - (d^3*
x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)
) + a + 1)/(a + 1)) + (d^3*x^3 + c^3)*log(f)^3*log((b*cosh((d*x + c)*log(f)
) + b*sinh((d*x + c)*log(f)) + a - 1)/(a - 1)) + 6*d*x*log(f)*polylog(3, -(
b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a + 1)) - 6*d*x*log(f
)*polylog(3, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a - 1)
) - 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))/(a
+ 1)) + 6*polylog(4, -(b*cosh((d*x + c)*log(f)) + b*sinh((d*x + c)*log(f)))
/(a - 1)))/(d^3*log(f)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \operatorname{artanh}\left(bf^{dx+c}+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(b*f^(d*x + c) + a), x)
```

maple [B] time = 0.13, size = 672, normalized size = 2.55

$$\frac{x^3\ln\left(1+a+b\,f^{dx+c}\right)}{6}-\frac{x^3\ln\left(1-a-b\,f^{dx+c}\right)}{6}+\frac{\ln\left(1-\frac{b\,f^{dx}f^{c}}{1-a}\right)x^3}{6}-\frac{\ln\left(1-\frac{b\,f^{dx}f^{c}}{1-a}\right)x\,c^2}{2d^2}-\frac{\ln\left(1-\frac{b\,f^{dx}f^{c}}{1-a}\right)c^3}{3d^3}+\frac{\mathrm{polyl}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a+b*f^(d*x+c)),x)
```

```
[Out] 1/6*x^3*ln(1+a+b*f^(d*x+c))-1/6*x^3*ln(1-a-b*f^(d*x+c))+1/6*ln(1-b*f^(d*x)*
f^c/(1-a))*x^3-1/2/d^2*ln(1-b*f^(d*x)*f^c/(1-a))*x*c^2-1/3/d^3*ln(1-b*f^(d*
x)*f^c/(1-a))*c^3+1/2/ln(f)/d*polylog(2,b*f^(d*x)*f^c/(1-a))*x^2-1/2/ln(f)/
d^3*polylog(2,b*f^(d*x)*f^c/(1-a))*c^2-1/ln(f)^2/d^2*polylog(3,b*f^(d*x)*f^
c/(1-a))*x+1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c/(1-a))-1/6/d^3*c^3*ln(1-a-
f^c*f^(d*x)*b)+1/2/ln(f)/d^3*c^2*dilog((f^c*f^(d*x)*b+a-1)/(a-1))+1/2/d^2*c
^2*ln((f^c*f^(d*x)*b+a-1)/(a-1))*x+1/2/d^3*c^3*ln((f^c*f^(d*x)*b+a-1)/(a-1)
)-1/6*ln(1-b*f^(d*x)*f^c/(-1-a))*x^3+1/2/d^2*ln(1-b*f^(d*x)*f^c/(-1-a))*x*c
^2+1/3/d^3*ln(1-b*f^(d*x)*f^c/(-1-a))*c^3-1/2/ln(f)/d*polylog(2,b*f^(d*x)*f
^c/(-1-a))*x^2+1/2/ln(f)/d^3*polylog(2,b*f^(d*x)*f^c/(-1-a))*c^2+1/ln(f)^2/
d^2*polylog(3,b*f^(d*x)*f^c/(-1-a))*x-1/ln(f)^3/d^3*polylog(4,b*f^(d*x)*f^c
/(-1-a))+1/6/d^3*c^3*ln(1+a+f^c*f^(d*x)*b)-1/2/ln(f)/d^3*c^2*dilog((1+a+f^c
```

```
*f^(d*x)*b)/(1+a))-1/2/d^2*c^2*ln((1+a+f^c*f^(d*x)*b)/(1+a))*x-1/2/d^3*c^3*
ln((1+a+f^c*f^(d*x)*b)/(1+a))
```

maxima [A] time = 0.37, size = 254, normalized size = 0.96

$$\frac{1}{3}\,x^3\,\mathrm{artanh}\left(bf^{dx+c}+a\right)-\frac{1}{6}\,bd\left(\frac{d^3x^3\log\left(\frac{bf^{dx}f^c}{a+1}+1\right)\log(f)^3+3\,d^2x^2\mathrm{Li}_2\left(-\frac{bf^{dx}f^c}{a+1}\right)\log(f)^2-6\,dx\log(f)\mathrm{Li}_3(-}{bd^4\log(f)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a+b*f^(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctanh(b*f^(d*x + c) + a) - 1/6*b*d*((d^3*x^3*log(b*f^(d*x)*f^c/(a
 + 1) + 1)*log(f)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a + 1))*log(f)^2 - 6*
d*x*log(f)*polylog(3, -b*f^(d*x)*f^c/(a + 1)) + 6*polylog(4, -b*f^(d*x)*f^c
/(a + 1)))/(b*d^4*log(f)^4) - (d^3*x^3*log(b*f^(d*x)*f^c/(a - 1) + 1)*log(f
)^3 + 3*d^2*x^2*dilog(-b*f^(d*x)*f^c/(a - 1))*log(f)^2 - 6*d*x*log(f)*polyl
og(3, -b*f^(d*x)*f^c/(a - 1)) + 6*polylog(4, -b*f^(d*x)*f^c/(a - 1)))/(b*d^
4*log(f)^4))*log(f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\mathrm{atanh}\left(a+b\,f^{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atanh(a + b*f^(c + d*x)),x)
```

```
[Out] int(x^2*atanh(a + b*f^(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a+b*f**(d*x+c)),x)
```

```
[Out] Timed out
```

3.355 $\int e^{c(a+bx)} \tanh^{-1}(\sinh(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctanh(sinh(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A] time = 0.17, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2194, 6275, 2282, 12, 1247, 632, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Sinh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Sinh[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\sinh(ac+bcx))\,dx = \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\sinh(x))\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{e^x\cosh(x)}{1-\sinh^2(x)}\,dx, x, ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{2x(-1-x^2)}{1-6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{2\,\text{Subst}\left(\int \frac{x(-1-x^2)}{1-6x^2+x^4}\,dx, x, e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} - \frac{\text{Subst}\left(\int \frac{-1-x}{1-6x+x^2}\,dx, x, e^{2ac+2bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\,dx, x, e\right.}{2bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\sinh(c(a+bx)))}{bc} + \frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac+2bcx}\right)}{2bc}$$

Mathematica [A] time = 0.17, size = 153, normalized size = 1.43

$$\frac{\log\left(-2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)+\log\left(2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)-2e^{c(a+bx)}\tanh^{-1}\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)-2\sqrt{2}}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Sinh[a*c + b*c*x]],x]
```

```
[Out] (-2*E^(c*(a + b*x))*ArcTanh[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] - 2*
Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1 + E^
(c*(a + b*x)))/Sqrt[2]] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))] +
Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

fricas [B] time = 0.48, size = 234, normalized size = 2.19

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\,\text{si}}{\cosh(bcx+ac)^2+\text{sinh}}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(sinh(b*c*x + a*c) + 1)/(
sinh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^
2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) +
3)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
 a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh
(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)
^2)))/(b*c)
```

giac [A] time = 0.43, size = 157, normalized size = 1.47

$$\frac{e^{((bx+a)c)}\log\left(-\frac{e^{(bcx+ac)}-e^{(-bcx-ac)}+2}{e^{(bcx+ac)}-e^{(-bcx-ac)}-2}\right)}{2\,bc}+\frac{\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(e^(b*c*x + a*c) - e^(-b*c*x - a*c) + 2)/(e^(b*c*x
 + a*c) - e^(-b*c*x - a*c) - 2))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sqrt(2) +
2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)) + lo
g(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

maple [C] time = 0.69, size = 868, normalized size = 8.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x)
```

```
[Out] 1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*Pi
*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*exp(-c*(b*x+a)))*csg
n(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*exp(c*(b*x+a))+
1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I*exp(-c*(
b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/2*I/b/c*
Pi*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(
b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*Pi+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2
*exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a
))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a
))+2*exp(c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)
))*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b
*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*
c*(b*x+a))+2*exp(c*(b*x+a))+1))^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c
*(b*x+a))*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))^3*exp(c*(b*x+a))-1/4*I/b/
c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I*exp(-c*(b*x+a)))*
csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a)
)-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))-1)+1/2/b/c*ln
(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/
2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)+1/2/b/c*l
n(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

maxima [B] time = 0.42, size = 184, normalized size = 1.72

$$\frac{\operatorname{artanh}\left(\sinh\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc}+\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+\cdots}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

mupad [B] time = 1.59, size = 179, normalized size = 1.67

$$\frac{e^{a\,c+b\,c\,x}\,\ln\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}+1\right)}{2\,b\,c}-\frac{e^{a\,c+b\,c\,x}\,\ln\left(\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}-\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+1\right)}{2\,b\,c}+\frac{\ln\left(6\,\sqrt{2}\,e^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-8\,e^{2\,c\,(a}\right.}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atanh(sinh(a*c + b*c*x)),x)
```

```
[Out] (exp(a*c + b*c*x)*log((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2 +
 1))/(2*b*c) - (exp(a*c + b*c*x)*log((exp(-b*c*x)*exp(-a*c))/2 - (exp(b*c*x
)*exp(a*c))/2 + 1))/(2*b*c) + (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2)
 - 8*exp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*
c*(a + b*x)) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(sinh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

3.356 $\int e^{c(a+bx)} \tanh^{-1}(\cosh(ac + bcx))\,dx$

Optimal. Leaf size=49

$$\frac{\log\left(1 - e^{2c(a+bx)}\right)}{bc} + \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctanh(cosh(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2194, 6275, 2282, 12, 260}

$$\frac{\log\left(1 - e^{2c(a+bx)}\right)}{bc} + \frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Cosh[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\cosh(ac+bcx))\,dx=\frac{\text{Subst}\left(\int e^x\tanh^{-1}(\cosh(x))\,dx,x,ac+bcx\right)}{bc}$$
$$=\frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int e^x\text{csch}(x)\,dx,x,ac+bcx\right)}{bc}$$
$$=\frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\text{Subst}\left(\int\frac{2x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$=\frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{2\,\text{Subst}\left(\int\frac{x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$=\frac{e^{ac+bcx}\tanh^{-1}(\cosh(c(a+bx)))}{bc}+\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 1.22

$$\frac{\log\left(1-e^{2c(a+bx)}\right)+e^{c(a+bx)}\tanh^{-1}\left(\frac{1}{2}e^{-c(a+bx)}\left(e^{2c(a+bx)}+1\right)\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Cosh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTanh[(1 + E^(2*c*(a + b*x)))/(2*E^(c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

fricas [A] time = 0.46, size = 93, normalized size = 1.90

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(-\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\,\log\left(\frac{2\,\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log(-(cosh(b*c*x + a*c) + 1)/(
cosh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - si
nh(b*c*x + a*c))))/(b*c)
```

giac [B] time = 0.21, size = 147, normalized size = 3.00

$$\frac{\left(e^{(bcx)}\log\left(-\frac{e^{(2\,bcx+2\,ac)}}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}-\frac{2\,e^{(bcx+ac)}}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}-\frac{1}{e^{(2\,bcx+2\,ac)}-2\,e^{(bcx+ac)}+1}\right)+2\,e^{(-ac)}\log\left(\left|e^{(2\,bcx+2\,ac)}-1\right|\right)\right)e}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x +
 a*c) + 1) - 2*e^(b*c*x + a*c)/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1
) - 1/(e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) + 1)) + 2*e^(-a*c)*log(abs(e
^(2*b*c*x + 2*a*c) - 1)))*e^(a*c)/(b*c)
```

maple [C] time = 0.55, size = 887, normalized size = 18.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x)
```

```
[Out] 1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+1)+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a
))-1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(
c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)*exp(c*(b*x+a)
)-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(
b*x+a))-1)^2)^2*exp(c*(b*x+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn
(I*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a
))*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a
)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c
*Pi*csgn(I*(exp(c*(b*x+a))+1))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a)
)-1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*ex
p(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(e
xp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)
^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)^2*exp(c*(b*x+a))-1/2*I/b/c
*exp(c*(b*x+a))*Pi-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2
)^3*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*(exp(c*(b*x+
a))+1)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/4*I
/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*(b*
x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/2*I/b/c*
Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/b/c*exp(
c*(b*x+a))*ln(exp(c*(b*x+a))-1)-2*a/b+1/b/c*ln(exp(2*c*(b*x+a))-1)
```

maxima [A] time = 0.33, size = 64, normalized size = 1.31

$$\frac{\operatorname{artanh}\left(\cosh\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}+1\right)}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

mupad [B] time = 1.52, size = 111, normalized size = 2.27

$$\frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}-1\right)}{b\,c}-\frac{e^{b\,c\,x}\,e^{a\,c}\,\ln\left(1-\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}-\frac{e^{b\,c\,x}\,e^{a\,c}}{2}\right)}{2\,b\,c}+\frac{e^{b\,c\,x}\,e^{a\,c}\,\ln\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}+1\right)}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atanh(cosh(a*c + b*c*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(b*c*x)*exp(a*c)*log(1 - (exp(
-b*c*x)*exp(-a*c))/2 - (exp(b*c*x)*exp(a*c))/2))/(2*b*c) + (exp(b*c*x)*exp(
a*c)*log((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2 + 1))/(2*b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(cosh(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

3.357 $\int e^{c(a+bx)} \tanh^{-1}(\tanh(ac+bcx))\,dx$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

```
[Out] -exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arctanh(tanh(c*(b*x+a)))/b/c
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2194, 6275}

$$\frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Tanh[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcTanh[Tanh[c*(a + b*x)]])/(b*
c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{c(a+bx)}\tanh^{-1}(\tanh(ac+bcx))\,dx &= \frac{\text{Subst}\left(\int e^x\tanh^{-1}(\tanh(x))\,dx,x,ac+bcx\right)}{bc}\\
&= \frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}-\frac{\text{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}\\
&= -\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\tanh(c(a+bx)))}{bc}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(\tanh^{-1}\left(\frac{e^{2c(a+bx)}-1}{e^{2c(a+bx)}+1}\right)-1\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Tanh[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcTanh[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

fricas [A] time = 0.47, size = 46, normalized size = 1.02

$$\frac{(bcx + ac - 1)\cosh(bcx + ac) + (bcx + ac - 1)\sinh(bcx + ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="fricas")
[Out] ((b*c*x + a*c - 1)*cosh(b*c*x + a*c) + (b*c*x + a*c - 1)*sinh(b*c*x + a*c))
/(b*c)
```

giac [A] time = 0.14, size = 35, normalized size = 0.78

$$\frac{\left(b^2c^2x + abc^2 - bc\right)e^{(bcx+ac)}}{b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="giac")
[Out] (b^2*c^2*x + a*b*c^2 - b*c)*e^(b*c*x + a*c)/(b^2*c^2)
```

maple [A] time = 0.17, size = 68, normalized size = 1.51

$$\frac{(xbc + ac)\,e^{xbc+ac} - e^{xbc+ac} + e^{xbc+ac}\left(\operatorname{arctanh}(\tanh(xbc + ac)) - xbc - ac\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x)
[Out] 1/b/c*((b*c*x+a*c)*exp(b*c*x+a*c)-exp(b*c*x+a*c)+exp(b*c*x+a*c)*(arctanh(ta
nh(b*c*x+a*c))-x*b*c-a*c))
```

maxima [A] time = 0.33, size = 43, normalized size = 0.96

$$\frac{\operatorname{artanh}(\tanh(bcx + ac))\,e^{((bx+a)c)}}{bc} - \frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(tanh(b*c*x+a*c)),x, algorithm="maxima")
[Out] arctanh(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

mupad [B] time = 0.16, size = 28, normalized size = 0.62

$$\frac{e^{a\,c+b\,c\,x}\,\left(\operatorname{atanh}(\tanh(a\,c + b\,c\,x)) - 1\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atanh(tanh(a*c + b*c*x)),x)
[Out] (exp(a*c + b*c*x)*(atanh(tanh(a*c + b*c*x)) - 1))/(b*c)
```

sympy [A] time = 3.31, size = 56, normalized size = 1.24

$$\begin{cases} 0 & \text{for}\ b = 0 \wedge c = 0 \\ xe^{ac}\operatorname{atanh}(\tanh(ac)) & \text{for}\ b = 0 \\ 0 & \text{for}\ c = 0 \\ \frac{e^{ac}e^{bcx}\operatorname{atanh}(\tanh(ac+bcx))}{bc} - \frac{e^{ac}e^{bcx}}{bc} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(tanh(b*c*x+a*c)),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0)), (x*exp(a*c)*atanh(tanh(a*c)), Eq(b, 0))
, (0, Eq(c, 0)), (exp(a*c)*exp(b*c*x)*atanh(tanh(a*c + b*c*x))/(b*c) - exp(
a*c)*exp(b*c*x)/(b*c), True))
```

3.358 $\int e^{c(a+bx)} \tanh^{-1}(\coth(ac+bcx))\,dx$

Optimal. Leaf size=45

$$\frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

```
[Out] -exp(b*c*x+a*c)/b/c+exp(b*c*x+a*c)*arctanh(coth(c*(b*x+a)))/b/c
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2194, 6275}

$$\frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}-\frac{e^{ac+bcx}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Coth[a*c + b*c*x]],x]
```

```
[Out] -(E^(a*c + b*c*x)/(b*c)) + (E^(a*c + b*c*x)*ArcTanh[Coth[c*(a + b*x)]])/(b*
c)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\begin{aligned}\int e^{c(a+bx)}\tanh^{-1}(\coth(ac+bcx))\,dx &= \frac{\operatorname{Subst}\left(\int e^x\tanh^{-1}(\coth(x))\,dx,x,ac+bcx\right)}{bc}\\ &= \frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}-\frac{\operatorname{Subst}\left(\int e^x\,dx,x,ac+bcx\right)}{bc}\\ &= -\frac{e^{ac+bcx}}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\coth(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 1.02

$$\frac{e^{c(a+bx)}\left(\tanh^{-1}\left(\frac{e^{2c(a+bx)}+1}{e^{2c(a+bx)}-1}\right)-1\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Coth[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*(-1 + ArcTanh[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*
x)))]))/(b*c)
```

fricas [A] time = 0.60, size = 25, normalized size = 0.56

$$\frac{(bcx + ac - 1)e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="fricas")
[Out] (b*c*x + a*c - 1)*e^(b*c*x + a*c)/(b*c)
```

giac [A] time = 0.13, size = 40, normalized size = 0.89

$$\frac{\left(e^{(bcx)}\log\left(-e^{(2\,bcx+2\,ac)}\right) - 2\,e^{(bcx)}\right)e^{(ac)}}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="giac")
[Out] 1/2*(e^(b*c*x)*log(-e^(2*b*c*x + 2*a*c)) - 2*e^(b*c*x))*e^(a*c)/(b*c)
```

maple [C] time = 0.31, size = 351, normalized size = 7.80

$$\frac{e^{c(bx+a)}\ln\left(e^{c(bx+a)}\right)}{bc} + \frac{i\left(2\pi\operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}-1}\right)^{2} - 2\pi\operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}-1}\right)^{3} - \pi\operatorname{csgn}\left(\frac{i}{e^{2c(bx+a)}-1}\right)\operatorname{csgn}\left(ie^{2c(bx+a)}\right)\operatorname{csgr}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x)
[Out] 1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a)))+1/4*I*(2*Pi*csgn(I/(exp(2*c*(b*x+a)
)-1))^2-2*Pi*csgn(I/(exp(2*c*(b*x+a))-1))^3-Pi*csgn(I/(exp(2*c*(b*x+a))-1))
*csgn(I*exp(2*c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))+Pi*
csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^
2-Pi*csgn(I*exp(c*(b*x+a)))^2*csgn(I*exp(2*c*(b*x+a)))+2*Pi*csgn(I*exp(c*(b
*x+a)))*csgn(I*exp(2*c*(b*x+a)))^2-Pi*csgn(I*exp(2*c*(b*x+a)))^3+Pi*csgn(I*
exp(2*c*(b*x+a)))*csgn(I*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^2-Pi*csgn(I
*exp(2*c*(b*x+a))/(exp(2*c*(b*x+a))-1))^3+4*I-2*Pi)/b/c*exp(c*(b*x+a))
```

maxima [A] time = 0.33, size = 43, normalized size = 0.96

$$\frac{\operatorname{artanh}\left(\coth\left(bcx + ac\right)\right)e^{((bx+a)c)}}{bc} - \frac{e^{(bcx+ac)}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(coth(b*c*x+a*c)),x, algorithm="maxima")
[Out] arctanh(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - e^(b*c*x + a*c)/(b*c)
```

mupad [B] time = 0.07, size = 28, normalized size = 0.62

$$\frac{e^{a\,c+b\,c\,x}\,\left(\mathrm{atanh}\left(\mathrm{coth}\left(a\,c+b\,c\,x\right)\right)-1\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(a + b*x))*atanh(coth(a*c + b*c*x)),x)
[Out] (exp(a*c + b*c*x)*(atanh(coth(a*c + b*c*x)) - 1))/(b*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atanh}\left(\coth\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(coth(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atanh(coth(a*c + b*c*x)), x)
```

3.359 $\int e^{c(a+bx)} \tanh^{-1}(\mathbf{sech}(ac+bcx))\,dx$

Optimal. Leaf size=49

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctanh(sech(c*(b*x+a)))/b/c+ln(1-exp(2*c*(b*x+a)))/b/c
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2194, 6275, 2282, 12, 260}

$$\frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}+\frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Sech[c*(a + b*x)]])/(b*c) + Log[1 - E^(2*c*(a + b*
x))]/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\operatorname{sech}(ac+bcx))\,dx = \frac{\operatorname{Subst}\left(\int e^x\tanh^{-1}(\operatorname{sech}(x))\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int e^x\operatorname{csch}(x)\,dx,x,ac+bcx\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\operatorname{Subst}\left(\int \frac{2x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{2\operatorname{Subst}\left(\int \frac{x}{-1+x^2}\,dx,x,e^{ac+bcx}\right)}{bc}$$
$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{sech}(c(a+bx)))}{bc} + \frac{\log\left(1-e^{2c(a+bx)}\right)}{bc}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 1.20

$$\frac{\log\left(1-e^{2c(a+bx)}\right)+e^{c(a+bx)}\tanh^{-1}\left(\frac{2e^{c(a+bx)}}{e^{2c(a+bx)}+1}\right)}{bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Sech[a*c + b*c*x]],x]
```

```
[Out] (E^(c*(a + b*x))*ArcTanh[(2*E^(c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))] + Log
[1 - E^(2*c*(a + b*x))])/(b*c)
```

fricas [A] time = 0.48, size = 92, normalized size = 1.88

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\cosh(bcx+ac)+1}{\cosh(bcx+ac)-1}\right)+2\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((cosh(b*c*x + a*c) + 1)/(c
osh(b*c*x + a*c) - 1)) + 2*log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))))/(b*c)
```

giac [B] time = 0.32, size = 98, normalized size = 2.00

$$\frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}-1}\right)}{2bc}+\frac{\log\left(\left|e^{(2bcx+2ac)}-1\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)) + 1)/(2/(e
^(b*c*x + a*c) + e^(-b*c*x - a*c)) - 1))/(b*c) + log(abs(e^(2*b*c*x + 2*a*c
) - 1))/(b*c)
```

maple [C] time = 0.38, size = 872, normalized size = 17.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x)
```

```
[Out] -1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I/(exp(2*c*(b*x+a))+1))*csg
n(I*(exp(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))*exp(c*(b*x+a))-1/4*I/b/c*Pi*
csgn(I*(exp(c*(b*x+a))+1))^2*csgn(I*(exp(c*(b*x+a))+1)^2)*exp(c*(b*x+a))+1/
4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1))^2*csgn(I*(exp(c*(b*x+a))-1)^2)*exp(c*
(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^3*e
xp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+
a))+1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/2*I/b/c*Pi*csgn(I*(exp(c*
(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))-1)^2)^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csg
n(I*(exp(c*(b*x+a))+1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+
a))-1)^2)^3*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I
/(exp(2*c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))*ex
p(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))-1)^2)*csgn(I*(exp(c*(b*x+a
))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c
*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))-1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*
x+a))+1/2*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1))*csgn(I*(exp(c*(b*x+a))+1)^2)^
2*exp(c*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(exp(c*(b*x+a))+1)^2)*csgn(I*(exp(c*(b
*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(exp
(c*(b*x+a))+1)^2/(exp(2*c*(b*x+a))+1))^3*exp(c*(b*x+a))+1/b/c*exp(c*(b*x+a)
)*ln(exp(c*(b*x+a))+1)-1/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-1)-2*a/b+1/b/
c*ln(exp(2*c*(b*x+a))-1)
```

maxima [A] time = 0.33, size = 64, normalized size = 1.31

$$\frac{\operatorname{artanh}\left(\operatorname{sech}\left(bcx+ac\right)\right)e^{\left(\left(bx+a\right)c\right)}}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}+1\right)}{bc}+\frac{\log\left(e^{\left(bcx+ac\right)}-1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(sech(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(b*c*x + a*c) + 1)
/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

mupad [B] time = 1.71, size = 119, normalized size = 2.43

$$\frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c}-1\right)}{b\,c}-\frac{e^{a\,c+b\,c\,x}\,\ln\left(1-\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}\right)}{2\,b\,c}+\frac{\ln\left(\frac{1}{\frac{e^{b\,c\,x}\,e^{a\,c}}{2}+\frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}}+1\right)\,e^{a\,c+b\,c\,x}}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] log(exp(2*b*c*x)*exp(2*a*c) - 1)/(b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(
b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) + (log(1/((exp(b*
c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*b*c
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(sech(b*c*x+a*c)),x)
```

```
[Out] Timed out
```

3.360 $\int e^{c(a+bx)} \tanh^{-1}(\text{csch}(ac+bcx))\,dx$

Optimal. Leaf size=107

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\text{csch}(c(a+bx)))}{bc}$$

```
[Out] exp(b*c*x+a*c)*arctanh(csch(c*(b*x+a)))/b/c+1/2*ln(3-exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c+1/2*ln(3-exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

Rubi [A] time = 0.17, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2194, 6275, 2282, 12, 1247, 632, 31}

$$\frac{\left(1-\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3-2\sqrt{2}\right)}{2bc}+\frac{\left(1+\sqrt{2}\right)\log\left(-e^{2c(a+bx)}+3+2\sqrt{2}\right)}{2bc}+\frac{e^{ac+bcx}\tanh^{-1}(\text{csch}(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(c*(a + b*x))*ArcTanh[Csch[a*c + b*c*x]],x]
```

```
[Out] (E^(a*c + b*c*x)*ArcTanh[Csch[c*(a + b*x)]])/(b*c) + ((1 - Sqrt[2])*Log[3 -
 2*Sqrt[2] - E^(2*c*(a + b*x))])/(2*b*c) + ((1 + Sqrt[2])*Log[3 + 2*Sqrt[2]
 - E^(2*c*(a + b*x))])/(2*b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6275

```
Int[((a_.) + ArcTanh[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]}
, Dist[a + b*ArcTanh[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/
(1 - u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] &&
 InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ
[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTanh[u]), x]]
```

Rubi steps

$$\int e^{c(a+bx)}\tanh^{-1}(\operatorname{csch}(ac+bcx))\,dx = \frac{\operatorname{Subst}\left(\int e^x\tanh^{-1}(\operatorname{csch}(x))\,dx,x,ac+bcx\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int \frac{e^x\coth(x)\operatorname{csch}(x)}{1-\operatorname{csch}^2(x)}\,dx,x,ac+b\right.}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int \frac{2x(1+x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{2\operatorname{Subst}\left(\int \frac{x(1+x^2)}{1-6x^2+x^4}\,dx,x,e^{ac+bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\operatorname{Subst}\left(\int \frac{1+x}{1-6x+x^2}\,dx,x,e^{2ac+2bcx}\right)}{bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\operatorname{Subst}\left(\int \frac{1}{-3+2\sqrt{2}+x}\,dx,x,e\right.}{2bc}$$

$$= \frac{e^{ac+bcx}\tanh^{-1}(\operatorname{csch}(c(a+bx)))}{bc}+\frac{\left(1-\sqrt{2}\right)\log\left(3-2\sqrt{2}-e^{2ac+2bcx}\right)}{2bc}$$

Mathematica [A] time = 0.17, size = 150, normalized size = 1.40

$$\frac{\log\left(-2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)+\log\left(2e^{c(a+bx)}-e^{2c(a+bx)}+1\right)-2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(a+bx)}-1}{\sqrt{2}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{e^{c(a+bx)}+}{\sqrt{2}}\right.}{2bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^(c*(a + b*x))*ArcTanh[Csch[a*c + b*c*x]],x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(-1 + E^(c*(a + b*x)))/Sqrt[2]] + 2*Sqrt[2]*ArcTanh[(1
+ E^(c*(a + b*x)))/Sqrt[2]] + 2*E^(c*(a + b*x))*ArcTanh[(2*E^(c*(a + b*x)))
/(-1 + E^(2*c*(a + b*x)))] + Log[1 - 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))]
 + Log[1 + 2*E^(c*(a + b*x)) - E^(2*c*(a + b*x))])/(2*b*c)
```

fricas [B] time = 1.47, size = 233, normalized size = 2.18

$$\frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\log\left(\frac{\sinh(bcx+ac)+1}{\sinh(bcx+ac)-1}\right)+\sqrt{2}\log\left(\frac{3\left(2\sqrt{2}+3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}+4\right)\cosh(bcx+ac)\sinh}{\cosh(bcx+ac)^2+\sinh(}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*log((sinh(b*c*x + a*c) + 1)/(s
inh(b*c*x + a*c) - 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh(b*c*x + a*c)^2
 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) + 3
)*sinh(b*c*x + a*c)^2 - 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
a*c)^2 - 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 - 3)/(cosh(
b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^
2)))/(b*c)
```

giac [A] time = 0.43, size = 167, normalized size = 1.56

$$\frac{e^{((bx+a)c)}\log\left(-\frac{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}+1}{\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}-1}\right)}{2\,bc}+\frac{\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,bcx+2\,ac)}-6\right|}\right)+\log\left(\left|e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1\right|\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="giac")
```

```
[Out] 1/2*e^((b*x + a)*c)*log(-(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1)/(2/(e
^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 1))/(b*c) + 1/2*(sqrt(2)*log(abs(-4*sq
rt(2) + 2*e^(2*b*c*x + 2*a*c) - 6)/abs(4*sqrt(2) + 2*e^(2*b*c*x + 2*a*c) -
6)) + log(abs(e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)))/(b*c)
```

maple [C] time = 0.56, size = 842, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x)
```

```
[Out] 1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1)+1/4*I/b/c*Pi
*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(I/(exp(2*c*(b*x+a))-1)
)*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))*exp(c
*(b*x+a))+1/4*I/b/c*Pi*csgn(I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1))*csgn(
I*(-exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x
+a))-1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(-exp(2*c*(b*x+a))+2*
exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(
I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(
I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*exp(c*(b*x+a)
)+1/4*I/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I/(exp(2*c*(b*x+a))-1)*(ex
p(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/b/c*Pi*csgn(I*(-
exp(2*c*(b*x+a))+2*exp(c*(b*x+a))+1)/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))
+1/4*I/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))*csgn(I/(exp(2*c
*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4*I/
b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1)*(exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-1))^3
*exp(c*(b*x+a))-1/2/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))-2*exp(c*(b*x+a))
-1)+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2))^2)*2^(1/2)-1/2/b/c*ln(exp(2*c*(
b*x+a))-(2^(1/2)-1)^2)*2^(1/2)-2*a/b+1/2/b/c*ln(exp(2*c*(b*x+a))-(1+2^(1/2)
)^2)+1/2/b/c*ln(exp(2*c*(b*x+a))-(2^(1/2)-1)^2)
```

maxima [B] time = 0.42, size = 184, normalized size = 1.72

$$\frac{\operatorname{artanh}\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc}+\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}+1}{\sqrt{2}+e^{(bcx+ac)}-1}\right)}{2\,bc}-\frac{\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(bcx+ac)}-1}{\sqrt{2}+e^{(bcx+ac)}+1}\right)}{2\,bc}+\frac{\log\left(e^{(2\,bcx+2\,ac)}+2\,e^{(bcx+}\right.}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*arctanh(csch(b*c*x+a*c)),x, algorithm="maxima")
```

```
[Out] arctanh(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*log(-(sqrt(2
) - e^(b*c*x + a*c) + 1)/(sqrt(2) + e^(b*c*x + a*c) - 1))/(b*c) - 1/2*sqrt(
```

```
2)*log(-(sqrt(2) - e^(b*c*x + a*c) - 1)/(sqrt(2) + e^(b*c*x + a*c) + 1))/(b
*c) + 1/2*log(e^(2*b*c*x + 2*a*c) + 2*e^(b*c*x + a*c) - 1)/(b*c) + 1/2*log(
e^(2*b*c*x + 2*a*c) - 2*e^(b*c*x + a*c) - 1)/(b*c)
```

mupad [B] time = 1.62, size = 187, normalized size = 1.75

$$\frac{\ln\left(6\sqrt{2}\,e^{2c(a+bx)}-2\sqrt{2}-8\,e^{2c(a+bx)}\right)\left(\sqrt{2}+1\right)}{2bc}-\frac{e^{ac+bcx}\ln\left(1-\frac{1}{\frac{e^{bcx}e^{ac}}{2}-\frac{e^{-bcx}e^{-ac}}{2}}\right)}{2bc}-\frac{\ln\left(2\sqrt{2}-8\,e^{2c(a+bx)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atanh(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

```
[Out] (log(6*2^(1/2)*exp(2*c*(a + b*x)) - 2*2^(1/2) - 8*exp(2*c*(a + b*x)))*(2^(1
/2) + 1))/(2*b*c) - (exp(a*c + b*c*x)*log(1 - 1/((exp(b*c*x)*exp(a*c))/2 -
(exp(-b*c*x)*exp(-a*c))/2)))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)
) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c) + (log(1/((exp(b*c
*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2) + 1)*exp(a*c + b*c*x))/(2*b*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^{ac}\int e^{bcx}\operatorname{atanh}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(c*(b*x+a))*atanh(csch(b*c*x+a*c)),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)*atanh(csch(a*c + b*c*x)), x)
```

3.361 $\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx$

Optimal. Leaf size=136

$$ad\log(x)+\frac{ae\log^2\left(fx^m\right)}{2m}-\frac{bd\mathrm{Li}_2\left(-cx^n\right)}{2n}+\frac{bd\mathrm{Li}_2\left(cx^n\right)}{2n}-\frac{be\mathrm{Li}_2\left(-cx^n\right)\log\left(fx^m\right)}{2n}+\frac{be\mathrm{Li}_2\left(cx^n\right)\log\left(fx^m\right)}{2n}+\frac{bem\mathrm{Li}_3\left(-\right.}{2n^2}$$

```
[Out] a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m-1/2*b*d*polylog(2,-c*x^n)/n-1/2*b*e*ln(f*x^
m)*polylog(2,-c*x^n)/n+1/2*b*d*polylog(2,c*x^n)/n+1/2*b*e*ln(f*x^m)*polylog
(2,c*x^n)/n+1/2*b*e*m*polylog(3,-c*x^n)/n^2-1/2*b*e*m*polylog(3,c*x^n)/n^2
```

Rubi [A] time = 0.54, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2301, 6742, 6095, 5912, 6071, 6069, 2374, 6589}

$$-\frac{bd\text{PolyLog}\left(2,-cx^n\right)}{2n}+\frac{bd\text{PolyLog}\left(2,cx^n\right)}{2n}-\frac{be\log\left(fx^m\right)\text{PolyLog}\left(2,-cx^n\right)}{2n}+\frac{be\log\left(fx^m\right)\text{PolyLog}\left(2,cx^n\right)}{2n}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) - (b*d*PolyLog[2, -(c*x^n)])/(2*n) -
(b*e*Log[f*x^m]*PolyLog[2, -(c*x^n)])/(2*n) + (b*d*PolyLog[2, c*x^n])/(2*n)
 + (b*e*Log[f*x^m]*PolyLog[2, c*x^n])/(2*n) + (b*e*m*PolyLog[3, -(c*x^n)])/
(2*n^2) - (b*e*m*PolyLog[3, c*x^n])/(2*n^2)
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6069

```
Int[(ArcTanh[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Di
st[1/2, Int[(Log[d*x^m]*Log[1 + c*x^n])/x, x], x] - Dist[1/2, Int[(Log[d*x^
m]*Log[1 - c*x^n])/x, x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 6071

```
Int[(Log[(d_.)*(x_)^(m_.)]*(ArcTanh[(c_.)*(x_)^(n_.)]*(b_.) + (a_)))/(x_),
x_Symbol] :> Dist[a, Int[Log[d*x^m]/x, x], x] + Dist[b, Int[(Log[d*x^m]*Arc
Tanh[c*x^n])/x, x], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[
1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
 n}, x] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)\left(d+e\log\left(fx^m\right)\right)}{x}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx^n)\right)}{x}+\frac{e\left(a+b\tanh^{-1}(cx^n)\right)\log\left(fx^m\right)}{x}\right)$$

$$= d\int \frac{a+b\tanh^{-1}(cx^n)}{x}\,dx + e\int \frac{\left(a+b\tanh^{-1}(cx^n)\right)\log\left(fx^m\right)}{x}$$

$$= (ae)\int \frac{\log\left(fx^m\right)}{x}\,dx + (be)\int \frac{\tanh^{-1}(cx^n)\log\left(fx^m\right)}{x}\,dx + \frac{dS}{}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\mathrm{Li}_2(-cx^n)}{2n} + \frac{bd\mathrm{Li}_2(cx^n)}{2n} - \frac{1}{2}(b$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\mathrm{Li}_2(-cx^n)}{2n} - \frac{be\log\left(fx^m\right)\mathrm{Li}_2(}{2n}$$

$$= ad\log(x) + \frac{ae\log^2\left(fx^m\right)}{2m} - \frac{bd\mathrm{Li}_2(-cx^n)}{2n} - \frac{be\log\left(fx^m\right)\mathrm{Li}_2(}{2n}$$

Mathematica [C] time = 0.29, size = 114, normalized size = 0.84

$$\frac{bcx^n\left(d+e\log\left(fx^m\right)\right){}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n}-\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n^2}+\frac{1}{2}a\log(x)\left(2d+2e\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

```
[Out] -((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, c^2*x
^(2*n)])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x
^(2*n)]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*Log[f*x^m
]))/2
```

fricas [C] time = 0.55, size = 327, normalized size = 2.40

$$\frac{2\,aemn^2\log(x)^2 - 2\,bem\mathrm{polylog}\left(3, c\cosh\left(n\log(x)\right) + c\sinh\left(n\log(x)\right)\right) + 2\,bem\mathrm{polylog}\left(3, -c\cosh\left(n\log(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*e*m*n^2*log(x)^2 - 2*b*e*m*polylog(3, c*cosh(n*log(x)) + c*sinh(n*
log(x))) + 2*b*e*m*polylog(3, -c*cosh(n*log(x)) - c*sinh(n*log(x))) + 2*(b*
e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(c*cosh(n*log(x)) + c*sinh(n*log(
x))) - 2*(b*e*m*n*log(x) + b*e*n*log(f) + b*d*n)*dilog(-c*cosh(n*log(x)) -
c*sinh(n*log(x))) - (b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*log(
x))*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1) + (b*e*m*n^2*log(x)^2 + 2*
(b*e*n^2*log(f) + b*d*n^2)*log(x))*log(-c*cosh(n*log(x)) - c*sinh(n*log(x))
 + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x) + (b*e*m*n^2*log(x)^2 + 2*(b*e*
n^2*log(f) + b*d*n^2)*log(x))*log(-(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1
)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)))/n^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{artanh}\,(cx^n)+a)\big(e\log\big(fx^m\big)+d\big)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

maple [C] time = 0.40, size = 668, normalized size = 4.91

$$-\frac{ebm\ln(x)\,\mathrm{polylog}\,(2,-c\,x^n)}{2n}+\frac{eb\,\mathrm{dilog}\,(c\,x^n+1)\,m\ln(x)}{2n}+\frac{ebm\ln(x)\,\mathrm{polylog}\,(2,c\,x^n)}{2n}+\frac{eb\,\mathrm{dilog}\,(c\,x^n)\,m\ln(x)}{2n}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^n))*(d+e*ln(f*x^m))/x,x)
```

```
[Out] -1/2*e*b/n*m*ln(x)*polylog(2,-c*x^n)+1/2*e*b/n*dilog(c*x^n+1)*m*ln(x)-1/2/n
*dilog(c*x^n+1)*b*d+1/n*ln(x^n)*a*d+1/2/n*dilog(1-c*x^n)*b*d+1/2*e*a/m*ln(x
^m)^2-1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I/n*di
log(c*x^n+1)*Pi*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)-1/4*I/n*dilog(1-c*x
^n)*Pi*b*e*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I/n*dilog(1-c*x^n)*Pi*b*
e*csgn(I*f)*csgn(I*f*x^m)^2-1/4*I/n*dilog(c*x^n+1)*Pi*b*e*csgn(I*f)*csgn(I*
f*x^m)^2+1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*f)*csgn(I*f*x^m)^2+1/4*I/n*dilog(1-c
*x^n)*Pi*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*x^m)
*csgn(I*f*x^m)^2+1/2*b*e*m*polylog(3,-c*x^n)/n^2-1/2*b*e*m*polylog(3,c*x^n)
/n^2-1/4*I/n*dilog(c*x^n+1)*Pi*b*e*csgn(I*x^m)*csgn(I*f*x^m)^2+1/2*e*b/n*ln
(1-c*x^n)*ln(c*x^n)*m*ln(x)-1/2*e*b/n*dilog(c*x^n+1)*ln(x^m)-1/2*e*b/n*dilo
g(c*x^n)*ln(x^m)+1/n*ln(x^n)*ln(f)*a*e+1/2/n*dilog(1-c*x^n)*ln(f)*b*e-1/4*I
/n*dilog(1-c*x^n)*Pi*b*e*csgn(I*f*x^m)^3+1/4*I/n*dilog(c*x^n+1)*Pi*b*e*csgn
(I*f*x^m)^3-1/2*I/n*ln(x^n)*Pi*a*e*csgn(I*f*x^m)^3+1/2*e*b/n*m*ln(x)*polylo
g(2,c*x^n)-1/2*e*b/n*ln(1-c*x^n)*ln(c*x^n)*ln(x^m)+1/2*e*b/n*dilog(c*x^n)*m
*ln(x)-1/2/n*dilog(c*x^n+1)*ln(f)*b*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae\log\big(fx^m\big)^2}{2\,m}+ad\log(x)-\frac{1}{4}\Big(bem\log(x)^2-2\,be\log(x)\log\left(x^m\right)-2\left(e\log(f)+d\right)b\log(x)\Big)\log\left(cx^n+1\right)+\frac{1}{4}\Big(bem$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/4*(b*e*m*log(x)^2 - 2*b*e*log(x)*lo
g(x^m) - 2*(e*log(f) + d)*b*log(x))*log(c*x^n + 1) + 1/4*(b*e*m*log(x)^2 -
2*b*e*log(x)*log(x^m) - 2*(e*log(f) + d)*b*log(x))*log(-c*x^n + 1) + integr
```

```
ate(1/2*(2*b*c*e*n*x^n*log(x)*log(x^m) - (b*c*e*m*n*log(x)^2 - 2*(e*n*log(f
) + d*n)*b*c*log(x))*x^n)/(c^2*x*x^(2*n) - x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atanh}\,(c\,x^n))\,\left(d + e\,\ln\left(f\,x^m\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*atanh(c*x^n))*(d + e*log(f*x^m)))/x,x)
[Out] int(((a + b*atanh(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))*(d+e*ln(f*x**m))/x,x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```