Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/7.2.5-Inverse-hyperbolic-cosine-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [296]. This is test number [191].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.66 (295)	% 0.34 (1)
Mathematica	% 97.30 (288)	% 2.70 (8)
Maple	% 62.50 (185)	% 37.50 (111)
Maxima	% 27.36 (81)	% 72.64 (215)
Fricas	% 41.22 (122)	% 58.78 (174)
Sympy	% 27.03 (80)	% 72.97 (216)
Giac	% 28.38 (84)	% 71.62 (212)
Mupad	% 21.62 (64)	% 78.38 (232)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

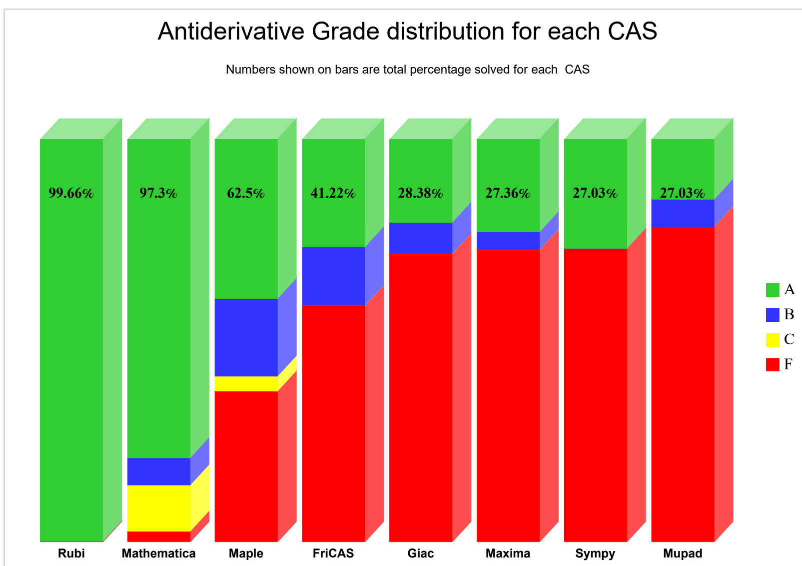
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

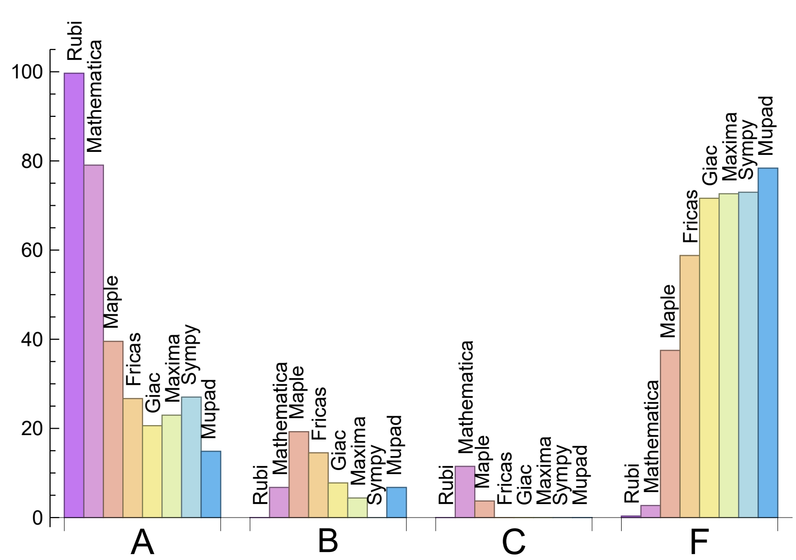
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.66	0.00	0.00	0.34
Mathematica	79.05	6.76	11.49	2.70
Maple	39.53	19.26	3.72	37.50
Maxima	22.97	4.39	0.00	72.64
Fricas	26.69	14.53	0.00	58.78
Sympy	27.03	0.00	0.00	72.97
Giac	20.61	7.77	0.00	71.62
Mupad	14.86	6.76	0.00	78.38

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	8	75.00 %	25.00 %	0.00 %
Maple	111	63.96 %	0.00 %	36.04 %
Maxima	215	83.26 %	5.12 %	11.63 %
Fricas	174	64.37 %	0.00 %	35.63 %
Sympy	216	89.81 %	10.19 %	0.00 %
Giac	212	63.21 %	7.08 %	29.72 %
Mupad	232	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

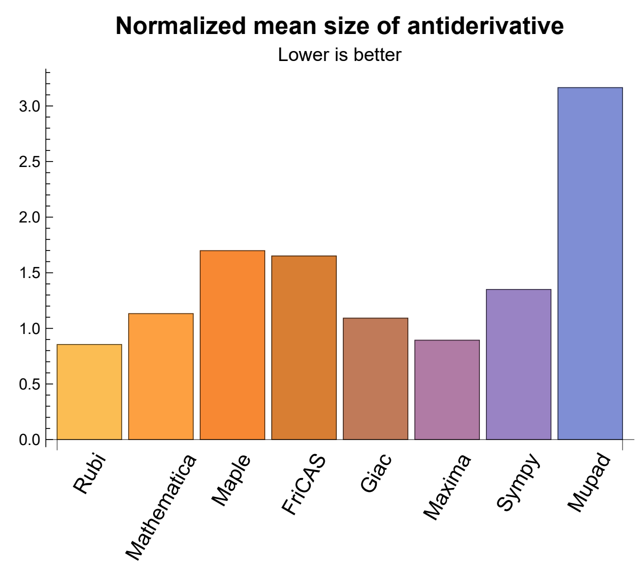
1.3 Performance

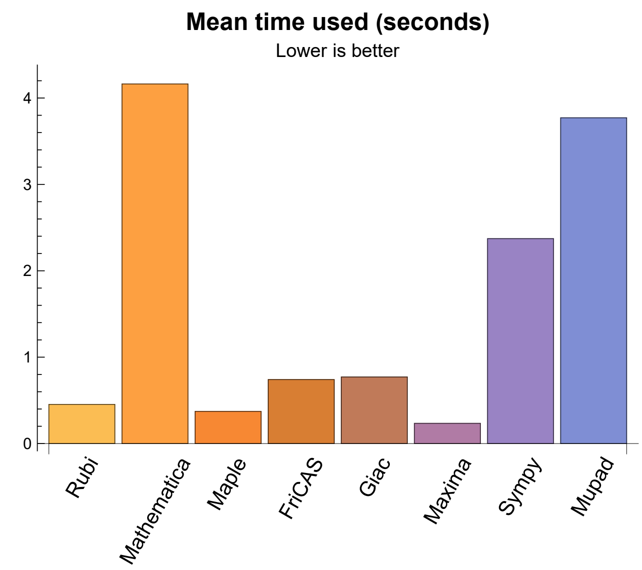
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.45	199.76	0.85	161.00	1.00
Mathematica	4.16	282.10	1.13	161.50	0.98
Maple	0.37	399.90	1.70	214.00	1.55
Maxima	0.23	94.93	0.89	0.00	0.00
Fricas	0.74	247.02	1.65	114.50	1.20
Sympy	2.37	281.32	1.35	48.50	0.86
Giac	0.77	126.38	1.09	57.50	0.81
Mupad	3.77	337.23	3.16	-1.00	-0.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{30, 31, 34, 35, 36, 37, 39, 40, 47, 48, 78, 82, 135, 141, 147, 153, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 268, 272, 273, 288, 289}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {76, 77, 98, 109, 118, 120, 126, 128, 269, 270, 271}

Mathematica {3, 12, 13, 25, 26, 32, 33, 38, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 91, 92, 94, 96, 98, 109, 110, 112, 118, 119, 120, 121, 126, 127, 128, 129, 136, 137, 138, 139, 140, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 231, 232, 233, 234, 235, 239, 244, 245, 251, 252, 253, 256, 257, 258, 260, 261, 262, 265, 266, 267, 275, 276, 277, 278, 279, 280, 281, 282, 283, 290, 294}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `sleleivre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

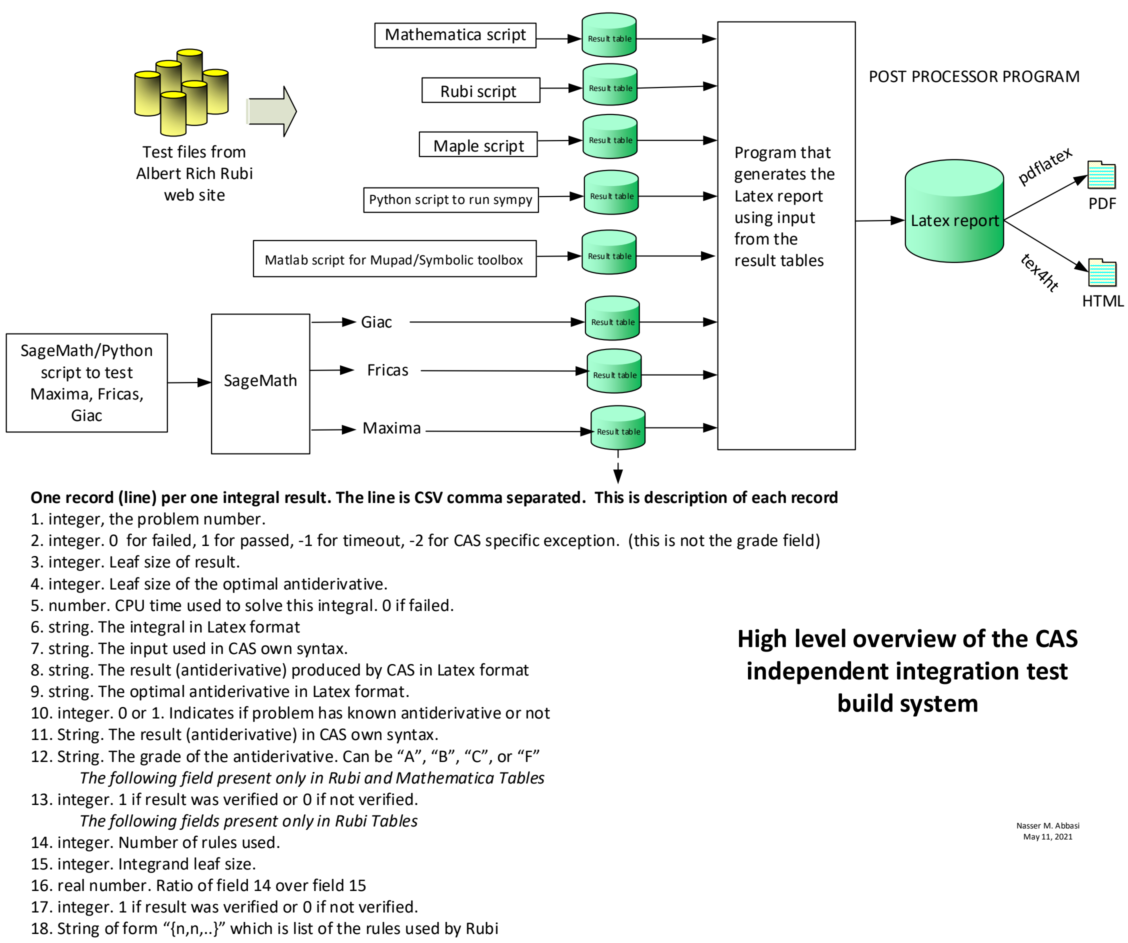
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296 }

B grade: { }

C grade: { }

F grade: { 61 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 53, 54, 55, 58, 59, 60, 62, 63, 64, 66, 67, 68, 71, 72, 73, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 163, 164, 169, 171, 173, 174, 175, 176, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 196, 197, 206, 207, 208, 209, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 272, 273, 275, 276, 277, 284, 285, 286, 287, 288, 289, 290, 292, 293, 295, 296 }

B grade: { 125, 127, 128, 129, 157, 162, 165, 166, 167, 168, 170, 172, 177, 183, 189, 195, 239, 278, 291, 294 }

C grade: { 7, 12, 13, 20, 25, 26, 46, 49, 50, 51, 52, 56, 57, 61, 65, 69, 70, 74, 88, 89, 90, 198, 199, 200, 201, 202, 203, 204, 205, 279, 280, 281, 282, 283 }

F grade: { 79, 80, 215, 221, 269, 270, 271, 274 }

2.1.3 Maple

A grade: { 3, 4, 5, 8, 9, 10, 12, 16, 17, 18, 23, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 53, 56, 58, 59, 60, 61, 62, 63, 64, 68, 69, 78, 82, 85, 86, 88, 93, 95, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 110, 112, 113, 115, 117, 123, 130, 131, 132, 133, 134, 135, 139, 140, 141, 145, 146, 147, 151, 152, 153, 159, 164, 169, 173, 179, 185, 191, 197, 199, 201, 203, 205, 226, 227, 230, 232, 233, 234, 235, 236, 237, 238, 239, 243, 250, 268, 270, 271, 272, 273, 274, 288, 289, 290, 292, 294 }

B grade: { 1, 2, 6, 7, 13, 14, 15, 19, 20, 21, 22, 26, 54, 55, 57, 65, 66, 67, 70, 71, 72, 73, 74, 83, 84, 87, 89, 90, 94, 96, 105, 107, 111, 114, 116, 118, 120, 122, 124, 125, 126, 128, 136, 137, 138, 142, 143, 144, 148, 149, 150, 269, 278, 281, 282, 283, 291 }

C grade: { 45, 46, 198, 200, 202, 204, 275, 276, 277, 279, 280 }

F grade: { 11, 24, 38, 49, 50, 51, 52, 75, 76, 77, 79, 80, 81, 91, 92, 119, 121, 127, 129, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 231, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 284, 285, 286, 287, 293, 295, 296 }

2.1.4 Maxima

A grade: { 1, 2, 3, 14, 15, 16, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 78, 82, 86, 97, 99, 135, 141, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 232, 233, 234, 236, 237, 242, 243, 249, 250, 268, 272, 273, 276, 277, 288, 289, 292, 293 }

B grade: { 83, 84, 85, 93, 94, 95, 96, 100, 102, 111, 238, 275, 278 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 87, 88, 89, 90, 91, 92, 98, 101, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 235, 239, 240, 241, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 274, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 294, 295, 296 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 8, 9, 10, 14, 15, 16, 21, 22, 23, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 49, 78, 82, 83, 84, 85, 86, 96, 97, 135, 141, 147, 153, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 232, 233, 234, 236, 237, 241, 242, 243, 249, 268, 272, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 288, 289, 292, 296 }

B grade: { 5, 6, 7, 18, 19, 20, 50, 51, 52, 88, 89, 90, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 111, 113, 114, 115, 116, 117, 122, 123, 124, 125, 238, 240, 247, 248, 250, 293, 294, 295 }

C grade: { }

F grade: { 4, 11, 12, 13, 17, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 87, 91, 92, 98, 109, 110, 112, 118, 119, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 235, 239, 244, 245, 246, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 274, 284, 285, 286, 287, 290, 291 }

2.1.6 Sympy

A grade: { 1, 2, 3, 8, 9, 10, 14, 15, 16, 21, 22, 23, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 82, 83, 84, 85, 86, 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 113, 114, 115, 116, 117, 122, 123, 124, 125, 135, 141, 147, 153, 159, 164, 169, 179, 185, 191, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 234, 288, 289, 292, 293 }

B grade: { }

C grade: { }

F grade: { 4, 5, 6, 7, 11, 12, 13, 17, 18, 19, 20, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 87, 88, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 118, 119, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 290, 291, 294, 295, 296 }

2.1.7 Giac

A grade: { 1, 2, 3, 16, 30, 31, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 82, 83, 84, 85, 88, 89, 135, 141, 147, 153, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 219, 220, 221, 222, 223, 225, 226, 227, 230, 232, 233, 234, 236, 237, 243, 279, 288, 289 }

B grade: { 5, 18, 52, 86, 90, 93, 94, 95, 96, 97, 238, 250, 275, 276, 277, 278, 280, 281, 282, 283, 292, 293, 294 }

C grade: { }

F grade: { 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 38, 45, 46, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 87, 91, 92, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 224, 228, 229, 231, 235, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 284, 285, 286, 287, 290, 291, 295, 296 }

2.1.8 Mupad

A grade: { 30, 31, 34, 35, 36, 37, 39, 40, 47, 48, 78, 82, 135, 141, 147, 153, 159, 164, 169, 173, 179, 185, 191, 197, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 268, 272, 273, 288, 289 }

B grade: { 3, 16, 86, 97, 234, 238, 243, 250, 275, 276, 277, 278, 279, 280, 281, 282, 283, 292, 293, 294 }

C grade: { }

F grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 38, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 160, 161, 162, 163, 165, 166, 167, 168, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 228, 229, 231, 232, 233, 235, 236, 237, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 274, 284, 285, 286, 287, 290, 291, 295, 296 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	153	351	229	153	258	167	-1
normalized size	1	1.00	0.84	1.92	1.25	0.84	1.41	0.91	-0.01
time (sec)	N/A	0.153	0.267	0.033	0.312	0.548	1.277	0.367	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	113	233	140	106	155	129	-1
normalized size	1	1.00	0.92	1.89	1.14	0.86	1.26	1.05	-0.01
time (sec)	N/A	0.103	0.206	0.011	0.488	0.529	0.575	0.350	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	107	82	65	80	87	68
normalized size	1	1.00	0.75	1.10	0.85	0.67	0.82	0.90	0.70
time (sec)	N/A	0.042	0.078	0.013	0.460	0.494	0.261	0.285	0.706

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	176	295	0	0	0	0	-1
normalized size	1	1.00	0.99	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.271	0.011	0.164	0.000	0.689	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	92	126	0	454	0	215	-1
normalized size	1	1.00	1.11	1.52	0.00	5.47	0.00	2.59	-0.01
time (sec)	N/A	0.088	0.100	0.030	0.000	0.562	0.000	1.149	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	190	338	0	1044	0	0	-1
normalized size	1	1.00	1.44	2.56	0.00	7.91	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.193	0.023	0.000	0.869	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	244	1108	0	1799	0	0	-1
normalized size	1	1.00	1.25	5.68	0.00	9.23	0.00	0.00	-0.01
time (sec)	N/A	0.237	0.557	0.025	0.000	1.494	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	191	329	0	237	371	0	-1
normalized size	1	1.00	0.57	0.99	0.00	0.71	1.11	0.00	-0.00
time (sec)	N/A	1.464	0.271	0.127	0.000	0.740	2.458	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	131	207	0	163	223	0	-1
normalized size	1	1.00	0.61	0.96	0.00	0.76	1.04	0.00	-0.00
time (sec)	N/A	0.996	0.190	0.113	0.000	0.609	1.210	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	105	100	0	98	110	0	-1
normalized size	1	1.00	0.86	0.82	0.00	0.80	0.90	0.00	-0.01
time (sec)	N/A	0.651	0.076	0.097	0.000	0.616	0.529	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	252	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.426	0.163	0.265	0.000	0.617	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	848	374	0	0	0	0	-1
normalized size	1	1.00	3.27	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.582	3.431	0.355	0.000	0.770	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	936	766	0	0	0	0	-1
normalized size	1	1.00	2.66	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.694	4.527	0.516	0.000	0.647	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	193	408	265	213	323	0	-1
normalized size	1	1.00	1.01	2.14	1.39	1.12	1.69	0.00	-0.01
time (sec)	N/A	0.145	0.296	0.006	0.389	0.732	1.525	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	142	274	167	147	197	0	-1
normalized size	1	1.00	1.08	2.08	1.27	1.11	1.49	0.00	-0.01
time (sec)	N/A	0.105	0.210	0.006	0.438	0.622	0.683	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	117	123	99	88	105	126	83
normalized size	1	1.00	1.10	1.16	0.93	0.83	0.99	1.19	0.78
time (sec)	N/A	0.044	0.101	0.006	0.440	0.639	0.341	0.485	1.029

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	183	314	0	0	0	0	-1
normalized size	1	1.00	0.94	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	0.141	0.043	0.000	0.708	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	121	145	0	507	0	231	-1
normalized size	1	1.00	1.38	1.65	0.00	5.76	0.00	2.62	-0.01
time (sec)	N/A	0.056	0.205	0.006	0.000	0.821	0.000	1.078	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	184	361	0	1132	0	0	-1
normalized size	1	1.00	1.33	2.62	0.00	8.20	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.392	0.006	0.000	0.908	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	259	1137	0	1963	0	0	-1
normalized size	1	1.00	1.28	5.63	0.00	9.72	0.00	0.00	-0.00
time (sec)	N/A	0.164	0.981	0.005	0.000	1.551	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	386	791	0	472	750	0	-1
normalized size	1	1.00	0.97	1.99	0.00	1.19	1.88	0.00	-0.00
time (sec)	N/A	1.689	0.851	0.084	0.000	0.605	3.702	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	360	517	0	319	461	0	-1
normalized size	1	1.00	1.39	2.00	0.00	1.23	1.78	0.00	-0.00
time (sec)	N/A	1.148	0.661	0.074	0.000	0.689	1.700	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	174	245	0	185	240	0	-1
normalized size	1	1.00	1.16	1.63	0.00	1.23	1.60	0.00	-0.01
time (sec)	N/A	0.756	0.432	0.099	0.000	0.694	0.706	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	285	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	0.279	0.033	0.000	0.732	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	961	536	0	0	0	0	-1
normalized size	1	1.00	3.44	1.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	4.601	0.072	0.000	0.982	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	1093	1170	0	0	0	0	-1
normalized size	1	1.00	2.88	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.747	7.780	0.161	0.000	0.547	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	287	394	0	0	0	0	-1
normalized size	1	1.00	0.73	1.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.170	0.614	0.565	0.000	0.490	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	187	254	0	0	0	0	-1
normalized size	1	1.00	0.76	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.704	0.330	0.421	0.000	0.545	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	98	120	0	0	0	0	-1
normalized size	1	1.00	0.84	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.338	0.142	0.286	0.000	0.481	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.034	0.215	0.285	0.000	0.521	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	0.403	0.458	0.000	0.596	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	366	530	649	0	0	0	0	-1
normalized size	1	0.98	1.42	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.751	1.464	0.494	0.000	0.557	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	186	268	285	0	0	0	0	-1
normalized size	1	0.98	1.41	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.475	0.795	0.327	0.000	0.578	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	10.171	0.284	0.000	0.559	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.032	103.177	0.446	0.000	0.520	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.383	6.703	4.036	0.000	0.746	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	0.197	4.226	0.000	1.003	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	177	0	0	0	0	0	-1
normalized size	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.235	4.839	0.000	0.615	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.030	0.387	1.755	0.000	0.639	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	0.838	1.498	0.000	0.691	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	216	255	385	250	503	316	-1
normalized size	1	1.00	0.58	0.69	1.04	0.68	1.36	0.85	-0.00
time (sec)	N/A	0.459	0.306	0.031	0.325	0.898	16.256	0.333	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	154	176	257	179	328	214	-1
normalized size	1	1.00	0.58	0.66	0.96	0.67	1.23	0.80	-0.00
time (sec)	N/A	0.350	0.208	0.013	0.407	0.463	6.163	0.380	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	103	113	154	121	199	134	-1
normalized size	1	1.00	0.57	0.62	0.85	0.67	1.10	0.74	-0.01
time (sec)	N/A	0.189	0.162	0.011	0.310	1.319	2.104	0.308	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	62	74	71	90	70	-1
normalized size	1	1.00	0.71	0.74	0.88	0.85	1.07	0.83	-0.01
time (sec)	N/A	0.071	0.064	0.010	0.312	0.568	0.509	0.310	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	375	214	0	0	0	0	-1
normalized size	1	1.00	0.78	0.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.758	0.401	1.503	0.000	0.947	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	687	1632	0	0	0	0	-1
normalized size	1	1.00	0.89	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.101	1.377	3.402	0.000	0.571	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	3.051	0.312	0.000	0.522	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	1.974	0.386	0.000	0.683	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	551	0	0	296	0	82	-1
normalized size	1	1.00	5.74	0.00	0.00	3.08	0.00	0.85	-0.01
time (sec)	N/A	0.193	3.228	0.326	0.000	0.593	0.000	0.617	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	609	0	0	613	0	190	-1
normalized size	1	1.00	3.38	0.00	0.00	3.41	0.00	1.06	-0.01
time (sec)	N/A	0.176	2.214	0.322	0.000	0.718	0.000	0.596	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	655	0	0	1098	0	411	-1
normalized size	1	1.00	2.43	0.00	0.00	4.08	0.00	1.53	-0.00
time (sec)	N/A	0.805	3.515	0.346	0.000	0.897	0.000	0.761	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	844	0	0	1752	0	876	-1
normalized size	1	1.00	2.29	0.00	0.00	4.75	0.00	2.37	-0.00
time (sec)	N/A	1.014	6.273	0.323	0.000	1.264	0.000	1.192	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	713	713	491	1213	0	0	0	0	-1
normalized size	1	1.00	0.69	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.484	1.971	1.085	0.000	0.606	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	356	855	0	0	0	0	-1
normalized size	1	1.00	0.74	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.176	1.207	0.943	0.000	0.679	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	251	461	0	0	0	0	-1
normalized size	1	1.00	0.98	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.546	1.052	0.728	0.000	0.624	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	785	785	1121	1072	0	0	0	0	-1
normalized size	1	1.00	1.43	1.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.379	4.027	0.623	0.000	0.712	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	918	918	1139	1956	0	0	0	0	-1
normalized size	1	1.00	1.24	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.561	7.311	0.933	0.000	0.626	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1029	1029	901	1638	0	0	0	0	-1
normalized size	1	1.00	0.88	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.057	4.448	1.149	0.000	0.956	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	725	725	623	1177	0	0	0	0	-1
normalized size	1	1.00	0.86	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.693	2.800	1.039	0.000	0.606	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	432	656	0	0	0	0	-1
normalized size	1	1.00	1.09	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.757	1.612	0.768	0.000	0.704	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	F(-2)	F	F	F(-2)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1270	0	3068	1965	0	0	0	0	-1
normalized size	1	0.00	2.42	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.853	11.693	0.637	0.000	0.528	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1385	1385	1802	2116	0	0	0	0	-1
normalized size	1	1.00	1.30	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.518	7.976	1.233	0.000	0.763	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1015	1015	1282	1540	0	0	0	0	-1
normalized size	1	1.00	1.26	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.106	7.391	1.130	0.000	0.576	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	644	877	0	0	0	0	-1
normalized size	1	1.00	1.13	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.905	6.206	0.896	0.000	0.611	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1744	1744	6244	4234	0	0	0	0	-1
normalized size	1	1.00	3.58	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.779	18.358	0.749	0.000	0.657	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	405	859	0	0	0	0	-1
normalized size	1	1.00	0.85	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.288	2.094	1.043	0.000	0.628	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	267	559	0	0	0	0	-1
normalized size	1	1.00	0.93	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.914	1.588	0.820	0.000	1.077	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	172	239	0	0	0	0	-1
normalized size	1	1.00	1.26	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.472	0.696	0.655	0.000	0.580	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	932	754	0	0	0	0	-1
normalized size	1	1.00	2.55	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.703	1.866	0.398	0.000	0.670	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	1115	1978	0	0	0	0	-1
normalized size	1	1.00	2.13	3.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.836	5.844	0.785	0.000	0.659	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	549	549	353	1238	0	0	0	0	-1
normalized size	1	1.00	0.64	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.599	1.952	0.921	0.000	1.591	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	281	879	0	0	0	0	-1
normalized size	1	1.00	0.61	1.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.267	1.082	0.867	0.000	1.516	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	178	123	498	0	0	0	0	-1
normalized size	1	1.25	0.87	3.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.307	0.386	0.664	0.000	2.246	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	773	773	1203	1926	0	0	0	0	-1
normalized size	1	1.00	1.56	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.857	9.735	0.626	0.000	0.794	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	204	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	0.661	0.736	0.000	0.682	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	242	204	0	0	0	0	0	-1
normalized size	1	1.21	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.625	0.257	0.861	0.000	0.753	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	248	219	0	0	0	0	0	-1
normalized size	1	0.95	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.695	2.167	1.170	0.000	1.171	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.955	0.182	4.481	0.000	0.724	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.599	4.500	4.522	0.000	0.499	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.129	1.940	4.146	0.000	0.485	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	246	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.021	0.216	0.000	0.596	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.063	0.316	2.562	0.000	0.699	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	121	308	321	110	255	163	-1
normalized size	1	1.00	0.80	2.03	2.11	0.72	1.68	1.07	-0.01
time (sec)	N/A	0.187	0.190	0.031	0.335	0.523	1.385	0.812	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	207	212	91	170	132	-1
normalized size	1	1.00	0.97	1.99	2.04	0.88	1.63	1.27	-0.01
time (sec)	N/A	0.119	0.127	0.013	0.498	0.633	0.673	1.846	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	120	151	75	104	112	-1
normalized size	1	1.00	0.97	1.33	1.68	0.83	1.16	1.24	-0.01
time (sec)	N/A	0.064	0.076	0.011	0.335	0.739	0.290	1.911	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	56	36	30	57	46	93	266
normalized size	1	1.00	1.37	0.88	0.73	1.39	1.12	2.27	6.49
time (sec)	N/A	0.016	0.039	0.002	0.301	0.540	0.153	5.936	3.998

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	153	436	0	0	0	0	-1
normalized size	1	1.00	1.17	3.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.014	0.232	0.000	0.717	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	83	97	0	322	0	73	-1
normalized size	1	1.00	1.30	1.52	0.00	5.03	0.00	1.14	-0.02
time (sec)	N/A	0.083	0.116	0.018	0.000	0.695	0.000	0.402	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	136	181	0	460	0	170	-1
normalized size	1	1.00	1.28	1.71	0.00	4.34	0.00	1.60	-0.01
time (sec)	N/A	0.102	0.286	0.019	0.000	0.679	0.000	0.526	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	162	397	0	566	0	340	-1
normalized size	1	1.00	1.05	2.58	0.00	3.68	0.00	2.21	-0.01
time (sec)	N/A	0.172	0.316	0.021	0.000	0.638	0.000	1.945	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	110	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.169	0.158	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	111	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.107	0.151	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	103	78	1241	279	527	822	-1
normalized size	1	1.00	0.76	0.58	9.19	2.07	3.90	6.09	-0.01
time (sec)	N/A	0.086	0.126	0.025	0.388	0.514	3.251	5.612	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	115	359	797	226	394	598	-1
normalized size	1	1.00	0.97	3.02	6.70	1.90	3.31	5.03	-0.01
time (sec)	N/A	0.071	0.133	0.013	0.440	0.867	1.599	4.266	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	67	449	168	258	406	-1
normalized size	1	1.00	0.73	0.69	4.63	1.73	2.66	4.19	-0.01
time (sec)	N/A	0.063	0.076	0.007	0.500	0.515	0.729	3.733	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	81	162	203	110	148	245	-1
normalized size	1	1.00	1.08	2.16	2.71	1.47	1.97	3.27	-0.01
time (sec)	N/A	0.039	0.164	0.006	0.308	0.492	0.315	3.723	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	61	41	35	65	51	100	272
normalized size	1	1.00	1.33	0.89	0.76	1.41	1.11	2.17	5.91
time (sec)	N/A	0.024	0.051	0.002	0.341	0.601	0.151	1.903	3.962

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	69	111	0	0	0	0	-1
normalized size	1	1.00	0.85	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.069	0.090	0.000	0.666	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	78	88	80	133	0	0	-1
normalized size	1	1.00	1.39	1.57	1.43	2.38	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.137	0.007	0.545	0.824	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	65	118	117	0	0	-1
normalized size	1	1.00	0.83	0.98	1.79	1.77	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.061	0.006	0.618	0.470	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	101	120	0	276	0	0	-1
normalized size	1	1.00	1.02	1.21	0.00	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.234	0.009	0.000	0.984	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	76	260	208	0	0	-1
normalized size	1	1.00	0.83	0.73	2.50	2.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.081	0.009	0.562	0.508	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	136	152	0	416	0	0	-1
normalized size	1	1.00	0.99	1.11	0.00	3.04	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.233	0.006	0.000	0.779	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	220	218	0	618	1268	0	-1
normalized size	1	1.00	1.01	1.00	0.00	2.83	5.82	0.00	-0.00
time (sec)	N/A	0.485	0.335	0.044	0.000	0.660	7.033	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	212	822	0	481	916	0	-1
normalized size	1	1.00	1.14	4.42	0.00	2.59	4.92	0.00	-0.01
time (sec)	N/A	0.436	0.274	0.042	0.000	0.590	4.317	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	168	167	0	358	610	0	-1
normalized size	1	1.00	1.12	1.11	0.00	2.39	4.07	0.00	-0.01
time (sec)	N/A	0.323	0.237	0.040	0.000	0.525	1.778	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	167	334	0	233	335	0	-1
normalized size	1	1.00	1.52	3.04	0.00	2.12	3.05	0.00	-0.01
time (sec)	N/A	0.251	0.229	0.036	0.000	0.661	0.821	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	105	100	0	141	143	0	-1
normalized size	1	1.00	1.64	1.56	0.00	2.20	2.23	0.00	-0.02
time (sec)	N/A	0.125	0.088	0.053	0.000	0.629	0.305	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	117	140	263	0	0	0	0	-1
normalized size	1	0.99	1.19	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.422	0.054	0.000	1.595	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	161	290	0	0	0	0	-1
normalized size	1	1.00	1.46	2.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	0.771	0.170	0.000	0.622	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	194	229	320	0	0	-1
normalized size	1	1.00	0.88	2.11	2.49	3.48	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.207	0.273	0.694	0.679	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	251	381	0	0	0	0	-1
normalized size	1	1.00	1.35	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.387	1.024	0.312	0.000	0.636	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	404	450	0	1074	2518	0	-1
normalized size	1	1.00	1.06	1.18	0.00	2.81	6.59	0.00	-0.00
time (sec)	N/A	0.723	0.611	0.051	0.000	0.723	16.245	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	359	1554	0	828	1828	0	-1
normalized size	1	1.00	1.17	5.06	0.00	2.70	5.95	0.00	-0.00
time (sec)	N/A	0.625	0.560	0.051	0.000	0.698	9.573	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	296	326	0	607	1173	0	-1
normalized size	1	1.00	1.13	1.24	0.00	2.32	4.48	0.00	-0.00
time (sec)	N/A	0.472	0.426	0.046	0.000	2.082	4.723	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	244	605	0	395	685	0	-1
normalized size	1	1.00	1.39	3.46	0.00	2.26	3.91	0.00	-0.01
time (sec)	N/A	0.359	0.319	0.044	0.000	0.856	1.997	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	168	180	0	239	282	0	-1
normalized size	1	1.00	1.47	1.58	0.00	2.10	2.47	0.00	-0.01
time (sec)	N/A	0.184	0.186	0.056	0.000	0.572	0.766	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	217	471	0	0	0	0	-1
normalized size	1	1.00	1.36	2.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.572	0.073	0.000	1.017	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	327	0	0	0	0	0	-1
normalized size	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.362	1.184	0.225	0.000	0.624	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	266	375	0	0	0	0	-1
normalized size	1	1.00	1.62	2.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.369	1.113	0.291	0.000	0.809	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	504	0	0	0	0	0	-1
normalized size	1	1.00	1.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	2.384	0.507	0.000	2.161	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	562	2465	0	1236	2876	0	-1
normalized size	1	1.00	1.49	6.54	0.00	3.28	7.63	0.00	-0.00
time (sec)	N/A	1.174	0.888	0.053	0.000	0.746	19.271	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	475	517	0	890	1889	0	-1
normalized size	1	1.00	1.54	1.67	0.00	2.88	6.11	0.00	-0.00
time (sec)	N/A	0.835	0.637	0.069	0.000	0.541	9.094	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	360	933	0	579	1027	0	-1
normalized size	1	1.00	1.72	4.46	0.00	2.77	4.91	0.00	-0.00
time (sec)	N/A	0.562	0.492	0.041	0.000	0.799	4.516	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	261	275	0	344	444	0	-1
normalized size	1	1.00	2.02	2.13	0.00	2.67	3.44	0.00	-0.01
time (sec)	N/A	0.284	0.272	0.058	0.000	0.687	1.598	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	308	727	0	0	0	0	-1
normalized size	1	1.00	1.60	3.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	0.800	0.074	0.000	0.641	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	872	0	0	0	0	0	-1
normalized size	1	1.00	3.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	2.522	0.221	0.000	0.753	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	398	605	0	0	0	0	-1
normalized size	1	1.00	2.04	3.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.426	2.308	0.294	0.000	1.343	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	1374	0	0	0	0	0	-1
normalized size	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.842	9.356	0.478	0.000	0.681	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	209	151	194	0	0	0	0	-1
normalized size	1	0.98	0.71	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	0.248	0.401	0.000	0.604	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	109	134	0	0	0	0	-1
normalized size	1	1.00	0.75	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.334	0.184	0.339	0.000	1.590	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	137	102	130	0	0	0	0	-1
normalized size	1	0.97	0.72	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.157	0.192	0.000	0.676	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	66	0	0	0	0	-1
normalized size	1	1.00	0.88	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.083	0.034	0.000	0.747	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	60	0	0	0	0	-1
normalized size	1	1.00	0.84	1.03	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.115	0.030	0.000	0.541	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	0.878	0.259	0.000	0.685	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	259	293	665	0	0	0	0	-1
normalized size	1	0.98	1.11	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.394	1.972	0.408	0.000	0.728	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	230	418	0	0	0	0	-1
normalized size	1	1.00	1.18	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.298	2.580	0.372	0.000	0.806	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	187	150	374	0	0	0	0	-1
normalized size	1	0.98	0.79	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	1.957	0.217	0.000	0.773	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	108	170	0	0	0	0	-1
normalized size	1	1.00	0.98	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.489	0.050	0.000	0.603	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	94	143	139	0	0	0	0	-1
normalized size	1	0.96	1.46	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	1.029	0.042	0.000	0.796	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	7.766	0.232	0.000	0.632	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	323	323	993	0	0	0	0	-1
normalized size	1	0.99	0.99	3.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.111	1.281	0.450	0.000	0.712	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	186	624	0	0	0	0	-1
normalized size	1	1.00	0.73	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.899	0.559	0.399	0.000	0.876	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	311	223	557	0	0	0	0	-1
normalized size	1	1.23	0.88	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.799	0.692	0.237	0.000	0.640	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	127	254	0	0	0	0	-1
normalized size	1	1.00	0.78	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.509	0.326	0.061	0.000	0.605	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	109	207	0	0	0	0	-1
normalized size	1	1.00	0.83	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.260	0.420	0.064	0.000	1.465	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	1.744	0.240	0.000	0.754	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	427	424	1375	0	0	0	0	-1
normalized size	1	0.99	0.98	3.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.104	2.013	0.471	0.000	0.920	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	330	860	0	0	0	0	-1
normalized size	1	1.00	0.92	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.887	1.304	0.419	0.000	0.788	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	348	272	777	0	0	0	0	-1
normalized size	1	0.99	0.77	2.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.965	1.326	0.261	0.000	0.861	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	195	353	0	0	0	0	-1
normalized size	1	1.00	0.89	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	1.032	0.070	0.000	0.797	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	170	144	295	0	0	0	0	-1
normalized size	1	0.98	0.83	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.445	0.673	0.074	0.000	0.776	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	14.554	0.256	0.000	0.796	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	342	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.951	0.740	180.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	223	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	0.536	180.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	237	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.759	0.511	180.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	437	0	0	0	0	0	-1
normalized size	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.575	2.517	0.158	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	110	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.350	0.225	0.140	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.087	2.268	0.255	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	558	0	0	0	0	0	-1
normalized size	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.412	3.953	180.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	592	0	0	0	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.110	2.509	180.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	1144	0	0	0	0	0	-1
normalized size	1	1.00	5.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.687	7.940	0.161	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	290	0	0	0	0	0	-1
normalized size	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.363	0.760	0.144	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.102	1.553	0.275	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	968	0	0	0	0	0	-1
normalized size	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.239	11.219	180.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	1008	0	0	0	0	0	-1
normalized size	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.751	9.307	180.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	1846	0	0	0	0	0	-1
normalized size	1	1.00	6.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.108	9.263	0.164	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	494	0	0	0	0	0	-1
normalized size	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.609	3.557	0.145	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.101	1.139	0.291	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F(-2)	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	1523	0	0	0	0	0	-1
normalized size	1	1.00	2.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.142	14.065	180.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	288	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.276	5.750	0.203	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	765	0	0	0	0	0	-1
normalized size	1	1.00	3.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.642	7.508	0.143	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.101	1.209	0.270	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	319	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.622	0.640	180.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	205	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	0.412	180.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	216	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	0.380	180.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	306	0	0	0	0	0	-1
normalized size	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.237	1.429	0.165	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	110	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.110	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.094	0.081	0.282	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	396	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.644	1.536	180.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	265	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	1.015	180.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	265	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	1.628	180.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	314	0	0	0	0	0	-1
normalized size	1	1.00	2.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	6.712	0.146	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	145	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.182	0.147	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.104	0.090	0.301	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	615	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.765	3.656	180.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	391	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.441	2.465	180.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	391	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.237	3.039	180.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	687	0	0	0	0	0	-1
normalized size	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.780	5.484	0.146	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	219	0	0	0	0	0	-1
normalized size	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.409	1.206	0.141	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.108	0.100	0.284	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	654	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.776	5.073	180.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	445	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.406	3.104	180.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	452	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.475	2.920	180.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	916	0	0	0	0	0	-1
normalized size	1	1.00	3.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.784	4.769	0.148	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	243	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.627	0.807	0.143	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.103	0.099	0.286	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	150	277	0	0	0	0	-1
normalized size	1	1.00	0.79	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.371	0.053	0.000	0.606	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	180	218	0	0	0	0	-1
normalized size	1	1.00	1.07	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.276	0.026	0.000	1.441	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	109	254	0	0	0	0	-1
normalized size	1	1.00	0.75	1.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.487	0.021	0.000	0.618	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	131	194	0	0	0	0	-1
normalized size	1	1.00	1.03	1.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.462	0.018	0.000	0.656	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	138	0	0	0	0	-1
normalized size	1	1.00	0.90	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.191	0.020	0.000	0.675	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	119	0	0	0	0	-1
normalized size	1	1.00	1.10	1.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.165	0.018	0.000	0.628	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	94	269	0	0	0	0	-1
normalized size	1	1.00	0.63	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.146	0.029	0.000	0.755	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	94	201	0	0	0	0	-1
normalized size	1	1.00	0.72	1.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.159	0.029	0.000	0.531	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0	-1
normalized size	1	1.08	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.321	0.542	180.000	0.000	0.783	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0	-1
normalized size	1	1.08	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.328	0.482	180.000	0.000	0.805	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0	-1
normalized size	1	1.08	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.326	0.489	180.000	0.000	0.768	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0	-1
normalized size	1	1.08	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.316	0.434	180.000	0.000	0.726	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	163	140	0	0	0	0	0	-1
normalized size	1	1.08	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.300	180.000	0.000	0.703	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	161	140	0	0	0	0	0	-1
normalized size	1	1.08	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	0.302	180.000	0.000	0.900	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0	-1
normalized size	1	1.08	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.312	0.343	180.000	0.000	0.548	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	140	0	0	0	0	0	-1
normalized size	1	1.08	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.317	0.364	180.000	0.000	1.558	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	107.335	180.000	0.000	0.884	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.311	180.001	180.000	0.000	0.777	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	24.046	180.000	0.000	0.545	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.303	32.213	180.000	0.000	0.594	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	36.884	180.000	0.000	0.773	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.324	123.320	180.000	0.000	1.601	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	75.451	180.000	0.000	0.732	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.309	180.000	180.000	0.000	0.568	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.280	15.193	180.000	0.000	0.645	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.293	39.126	180.000	0.000	1.672	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	39.123	180.000	0.000	0.725	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.314	175.703	180.000	0.000	0.929	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	4.116	2.653	0.000	0.595	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	1.984	2.543	0.000	0.644	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	218	178	0	0	0	0	0	-1
normalized size	1	1.06	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.427	2.997	0.000	0.517	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	124	106	0	0	0	0	0	-1
normalized size	1	1.05	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.202	2.694	0.000	0.759	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	1.252	1.182	0.000	0.700	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	0.047	0.080	0.000	1.009	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	75	56	40	0	60	-1
normalized size	1	1.00	0.68	0.64	0.48	0.34	0.00	0.51	-0.01
time (sec)	N/A	0.070	0.058	0.033	0.505	0.590	0.000	1.211	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	65	46	35	0	55	-1
normalized size	1	1.00	0.86	0.76	0.53	0.41	0.00	0.64	-0.01
time (sec)	N/A	0.050	0.042	0.004	0.377	0.618	0.000	0.640	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	49	33	28	29	47	40
normalized size	1	1.00	1.28	0.98	0.66	0.56	0.58	0.94	0.80
time (sec)	N/A	0.031	0.025	0.005	0.620	0.830	0.257	1.463	1.429

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	65	0	0	0	0	-1
normalized size	1	1.00	1.00	1.41	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.038	0.090	0.000	1.327	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	29	19	26	0	45	-1
normalized size	1	1.00	1.00	0.72	0.48	0.65	0.00	1.12	-0.02
time (sec)	N/A	0.026	0.016	0.003	1.300	0.580	0.000	0.688	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	49	35	30	32	0	62	-1
normalized size	1	1.00	0.64	0.46	0.39	0.42	0.00	0.82	-0.01
time (sec)	N/A	0.041	0.029	0.003	0.805	0.622	0.000	1.534	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	46	38	17	72	0	22	23
normalized size	1	1.00	1.92	1.58	0.71	3.00	0.00	0.92	0.96
time (sec)	N/A	0.009	0.049	0.022	0.799	1.015	0.000	0.230	0.369

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	179	91	0	0	0	0	-1
normalized size	1	1.00	2.98	1.52	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.511	0.021	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	264	0	0	298	0	0	-1
normalized size	1	1.00	1.82	0.00	0.00	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.233	0.141	0.000	0.593	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	171	0	0	210	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	1.68	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.128	0.115	0.000	0.725	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	104	0	128	131	0	0	-1
normalized size	1	1.00	1.44	0.00	1.78	1.82	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.067	0.118	0.744	0.689	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	37	37	44	63	0	62	32
normalized size	1	1.00	0.76	0.76	0.90	1.29	0.00	1.27	0.65
time (sec)	N/A	0.039	0.066	0.008	0.949	0.583	0.000	0.201	0.591

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	118	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.139	0.069	0.000	0.758	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	130	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.928	0.087	0.000	0.605	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	152	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.477	0.069	0.000	0.618	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	264	0	0	298	0	0	-1
normalized size	1	1.00	1.80	0.00	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.240	0.120	0.000	0.607	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	171	0	0	210	0	0	-1
normalized size	1	1.00	1.55	0.00	0.00	1.91	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.131	0.117	0.000	0.684	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	104	0	128	131	0	0	-1
normalized size	1	1.00	1.42	0.00	1.75	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.066	0.117	0.588	0.480	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	37	44	63	0	67	32
normalized size	1	1.00	1.00	1.12	1.33	1.91	0.00	2.03	0.97
time (sec)	N/A	0.017	0.027	0.005	0.649	1.216	0.000	0.239	1.525

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.143	0.074	0.000	0.694	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	141	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.746	0.071	0.000	0.561	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	168	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.688	0.069	0.000	0.843	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	311	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.109	3.607	0.069	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	254	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	0.723	0.069	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	210	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.026	0.307	0.069	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	166	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.314	0.072	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	242	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.043	1.110	0.069	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	273	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.072	1.042	0.069	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	291	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.077	1.339	0.069	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	277	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.059	1.676	0.071	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	221	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.054	0.652	0.072	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	178	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.026	0.308	0.071	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	134	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.280	0.072	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	209	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.027	1.053	0.069	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	238	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.055	0.905	0.070	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	260	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.060	1.084	0.069	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.145	0.585	0.000	0.717	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F(-2)	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	0	870	0	0	0	0	-1
normalized size	1	1.00	0.00	3.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.396	0.903	0.000	0.525	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F(-2)	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	197	0	491	0	0	0	0	-1
normalized size	1	1.01	0.00	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	1.065	0.010	0.000	0.622	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F	F	F(-2)	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	0	207	0	0	0	0	-1
normalized size	1	1.00	0.00	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	3.163	0.008	0.000	0.458	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.139	0.373	0.000	0.522	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	5.335	0.353	0.000	0.558	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	A	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	115	0	0	0	0	-1
normalized size	1	1.00	0.00	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.720	0.007	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	138	376	495	133	0	564	1408
normalized size	1	1.00	0.84	2.28	3.00	0.81	0.00	3.42	8.53
time (sec)	N/A	0.161	0.099	0.045	0.326	0.778	0.000	0.249	69.812

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	119	288	275	112	0	417	1067
normalized size	1	1.00	1.03	2.50	2.39	0.97	0.00	3.63	9.28
time (sec)	N/A	0.121	0.139	0.012	0.319	0.551	0.000	0.247	33.479

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	93	194	177	88	0	280	852
normalized size	1	1.00	1.39	2.90	2.64	1.31	0.00	4.18	12.72
time (sec)	N/A	0.070	0.168	0.009	0.310	0.625	0.000	0.510	16.716

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	69	147	143	66	0	151	79
normalized size	1	1.00	2.23	4.74	4.61	2.13	0.00	4.87	2.55
time (sec)	N/A	0.017	0.031	0.015	0.309	0.527	0.000	0.260	0.175

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	141	156	0	246	0	103	8883
normalized size	1	1.00	1.41	1.56	0.00	2.46	0.00	1.03	88.83
time (sec)	N/A	0.092	0.102	0.013	0.000	0.980	0.000	0.232	23.408

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	140	237	0	334	0	200	3029
normalized size	1	1.00	1.28	2.17	0.00	3.06	0.00	1.83	27.79
time (sec)	N/A	0.074	0.167	0.016	0.000	0.496	0.000	0.247	15.820

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	142	236	0	338	0	326	958
normalized size	1	1.00	1.03	1.71	0.00	2.45	0.00	2.36	6.94
time (sec)	N/A	0.109	0.266	0.016	0.000	0.468	0.000	0.333	14.149

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	179	374	0	431	0	487	1537
normalized size	1	1.00	0.95	1.98	0.00	2.28	0.00	2.58	8.13
time (sec)	N/A	0.156	0.168	0.014	0.000	2.073	0.000	0.699	18.889

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	198	603	0	569	0	817	2347
normalized size	1	1.00	0.83	2.53	0.00	2.39	0.00	3.43	9.86
time (sec)	N/A	0.201	0.279	0.016	0.000	0.947	0.000	1.001	28.031

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	198	0	0	0	0	0	-1
normalized size	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.738	0.398	0.010	0.000	0.687	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	136	0	0	0	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	0.228	0.008	0.000	0.672	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	76	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.278	0.139	0.007	0.000	0.645	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.038	0.003	0.000	0.518	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.040	0.152	0.007	0.000	0.623	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.039	0.473	0.007	0.000	0.539	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	85	0	0	0	0	-1
normalized size	1	1.00	0.88	1.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.045	0.065	0.000	0.745	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	19	78	0	0	0	0	-1
normalized size	1	1.00	6.33	26.00	0.00	0.00	0.00	0.00	-0.33
time (sec)	N/A	0.121	0.063	0.422	0.000	0.609	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	45	37	66	61	106	295
normalized size	1	1.00	0.93	0.83	0.69	1.22	1.13	1.96	5.46
time (sec)	N/A	0.052	0.036	0.003	0.304	0.507	0.788	0.192	4.683

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	0	39	152	76	124	303
normalized size	1	1.00	0.91	0.00	0.71	2.76	1.38	2.25	5.51
time (sec)	N/A	0.054	0.048	0.043	0.303	0.710	58.148	0.732	1.068

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	180	91	0	276	0	119	53
normalized size	1	1.00	3.10	1.57	0.00	4.76	0.00	2.05	0.91
time (sec)	N/A	0.093	0.557	0.049	0.000	0.623	0.000	13.041	1.512

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	108	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.74	0.00	0.00	-0.02
time (sec)	N/A	0.119	0.200	0.304	0.000	0.654	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	33	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.61	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.095	0.278	0.000	1.591	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [65] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	12	0.417
2	A	4	4	1.00	12	0.333
3	A	4	4	1.00	10	0.400
4	A	8	5	1.00	12	0.417
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	6	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
8	A	18	7	1.00	14	0.500
9	A	13	7	1.00	14	0.500
10	A	9	7	1.00	12	0.583
11	A	10	6	1.00	14	0.429
12	A	10	7	1.00	14	0.500
13	A	13	10	1.00	14	0.714
14	A	5	5	1.00	16	0.312
15	A	4	4	1.00	16	0.250
16	A	4	4	1.00	14	0.286
17	A	8	5	1.00	16	0.312
18	A	3	3	1.00	16	0.188
19	A	4	4	1.00	16	0.250
20	A	6	6	1.00	16	0.375
21	A	18	7	1.00	18	0.389
22	A	13	7	1.00	18	0.389
23	A	9	7	1.00	16	0.438
24	A	10	6	1.00	18	0.333
25	A	10	7	1.00	18	0.389
26	A	13	10	1.00	18	0.556
27	A	27	7	1.00	18	0.389
28	A	17	6	1.00	18	0.333
29	A	11	7	1.00	16	0.438
30	A	0	0	0.00	0	0.000
31	A	0	0	0.00	0	0.000
32	A	19	7	0.98	18	0.389
33	A	11	7	0.98	16	0.438
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	3	3	1.00	16	0.188
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	6	6	1.00	14	0.429
42	A	6	6	1.00	14	0.429
43	A	6	6	1.00	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
44	A	3	3	1.00	12	0.250
45	A	18	6	1.00	14	0.429
46	A	26	9	1.00	14	0.643
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	7	8	1.00	16	0.500
50	A	8	10	1.00	16	0.625
51	A	9	11	1.00	16	0.688
52	A	10	12	1.00	16	0.750
53	A	16	12	1.00	31	0.387
54	A	13	8	1.00	31	0.258
55	A	8	6	1.00	29	0.207
56	A	23	22	1.00	31	0.710
57	A	38	22	1.00	31	0.710
58	A	24	17	1.00	31	0.548
59	A	20	12	1.00	31	0.387
60	A	12	9	1.00	29	0.310
61	F	0	0	N/A	0	N/A
62	A	30	20	1.00	31	0.645
63	A	26	15	1.00	31	0.484
64	A	14	10	1.00	29	0.345
65	A	38	31	1.00	31	1.000
66	A	13	7	1.00	31	0.226
67	A	9	7	1.00	31	0.226
68	A	6	5	1.00	29	0.172
69	A	10	7	1.00	31	0.226
70	A	13	10	1.00	31	0.323
71	A	19	14	1.00	31	0.452
72	A	17	12	1.00	31	0.387
73	A	5	6	1.25	29	0.207
74	A	25	17	1.00	31	0.548
75	A	7	5	1.00	30	0.167
76	A	7	5	1.21	35	0.143
77	A	7	5	0.95	37	0.135
78	A	0	0	0.00	0	0.000
79	A	14	11	1.00	35	0.314

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	12	10	1.00	33	0.303
81	A	9	7	1.00	25	0.280
82	A	0	0	0.00	0	0.000
83	A	6	6	1.00	10	0.600
84	A	5	5	1.00	10	0.500
85	A	5	5	1.00	8	0.625
86	A	3	3	1.00	6	0.500
87	A	9	6	1.00	10	0.600
88	A	4	4	1.00	10	0.400
89	A	5	5	1.00	10	0.500
90	A	7	7	1.00	10	0.700
91	A	7	6	1.00	14	0.429
92	A	7	6	1.00	15	0.400
93	A	8	5	1.00	21	0.238
94	A	7	6	1.00	21	0.286
95	A	6	5	1.00	21	0.238
96	A	5	5	1.00	19	0.263
97	A	4	3	1.00	10	0.300
98	A	7	7	1.00	21	0.333
99	A	5	5	1.00	21	0.238
100	A	4	4	1.00	21	0.190
101	A	6	6	1.00	21	0.286
102	A	6	5	1.00	21	0.238
103	A	8	6	1.00	21	0.286
104	A	9	7	1.00	23	0.304
105	A	8	6	1.00	23	0.261
106	A	7	7	1.00	23	0.304
107	A	6	6	1.00	21	0.286
108	A	4	4	1.00	12	0.333
109	A	8	8	0.99	23	0.348
110	A	9	7	1.00	23	0.304
111	A	5	5	1.00	23	0.217
112	A	11	9	1.00	23	0.391
113	A	19	8	1.00	23	0.348
114	A	14	8	1.00	23	0.348
115	A	12	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	8	7	1.00	21	0.333
117	A	6	4	1.00	12	0.333
118	A	9	9	1.00	23	0.391
119	A	11	8	1.00	23	0.348
120	A	9	9	1.00	23	0.391
121	A	15	11	1.00	23	0.478
122	A	16	6	1.00	23	0.261
123	A	13	8	1.00	23	0.348
124	A	9	6	1.00	21	0.286
125	A	6	4	1.00	12	0.333
126	A	10	9	1.00	23	0.391
127	A	13	9	1.00	23	0.391
128	A	10	10	1.00	23	0.435
129	A	21	12	1.00	23	0.522
130	A	14	7	0.98	23	0.304
131	A	11	7	1.00	23	0.304
132	A	11	7	0.97	23	0.304
133	A	8	7	1.00	21	0.333
134	A	5	5	1.00	12	0.417
135	A	0	0	0.00	0	0.000
136	A	13	6	0.98	23	0.261
137	A	10	6	1.00	23	0.261
138	A	10	6	0.98	23	0.261
139	A	6	6	1.00	21	0.286
140	A	6	6	0.96	12	0.500
141	A	0	0	0.00	0	0.000
142	A	26	9	0.99	23	0.391
143	A	20	9	1.00	23	0.391
144	A	18	10	1.23	23	0.435
145	A	11	10	1.00	21	0.476
146	A	7	7	1.00	12	0.583
147	A	0	0	0.00	0	0.000
148	A	24	8	0.99	23	0.348
149	A	17	8	1.00	23	0.348
150	A	18	10	0.99	23	0.435
151	A	9	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	8	7	0.98	12	0.583
153	A	0	0	0.00	0	0.000
154	A	21	9	1.00	25	0.360
155	A	16	9	1.00	25	0.360
156	A	16	9	1.00	25	0.360
157	A	11	9	1.00	23	0.391
158	A	8	7	1.00	14	0.500
159	A	0	0	0.00	0	0.000
160	A	27	11	1.00	25	0.440
161	A	24	12	1.00	25	0.480
162	A	13	11	1.00	23	0.478
163	A	9	8	1.00	14	0.571
164	A	0	0	0.00	0	0.000
165	A	29	11	1.00	25	0.440
166	A	26	12	1.00	25	0.480
167	A	14	11	1.00	23	0.478
168	A	10	8	1.00	14	0.571
169	A	0	0	0.00	0	0.000
170	A	35	13	1.00	25	0.520
171	A	16	11	1.00	23	0.478
172	A	11	8	1.00	14	0.571
173	A	0	0	0.00	0	0.000
174	A	20	8	1.00	25	0.320
175	A	15	8	1.00	25	0.320
176	A	15	8	1.00	25	0.320
177	A	10	8	1.00	23	0.348
178	A	7	6	1.00	14	0.429
179	A	0	0	0.00	0	0.000
180	A	19	7	1.00	25	0.280
181	A	14	7	1.00	25	0.280
182	A	14	7	1.00	25	0.280
183	A	8	7	1.00	23	0.304
184	A	8	7	1.00	14	0.500
185	A	0	0	0.00	0	0.000
186	A	36	10	1.00	25	0.400
187	A	26	10	1.00	25	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	A	24	11	1.00	25	0.440
189	A	13	11	1.00	23	0.478
190	A	9	8	1.00	14	0.571
191	A	0	0	0.00	0	0.000
192	A	34	9	1.00	25	0.360
193	A	23	9	1.00	25	0.360
194	A	24	11	1.00	25	0.440
195	A	11	10	1.00	23	0.435
196	A	10	8	1.00	14	0.571
197	A	0	0	0.00	0	0.000
198	A	8	6	1.00	23	0.261
199	A	8	6	1.00	23	0.261
200	A	6	6	1.00	23	0.261
201	A	6	6	1.00	23	0.261
202	A	4	4	1.00	23	0.174
203	A	4	4	1.00	23	0.174
204	A	7	7	1.00	23	0.304
205	A	7	7	1.00	23	0.304
206	A	3	3	1.08	25	0.120
207	A	3	3	1.08	25	0.120
208	A	3	3	1.08	25	0.120
209	A	3	3	1.08	25	0.120
210	A	3	3	1.08	25	0.120
211	A	3	3	1.08	25	0.120
212	A	3	3	1.08	25	0.120
213	A	3	3	1.08	25	0.120
214	A	0	0	0.00	0	0.000
215	A	0	0	0.00	0	0.000
216	A	0	0	0.00	0	0.000
217	A	0	0	0.00	0	0.000
218	A	0	0	0.00	0	0.000
219	A	0	0	0.00	0	0.000
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	0	0	0.00	0	0.000
223	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	0	0	0.00	0	0.000
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000
228	A	3	3	1.06	23	0.130
229	A	5	5	1.05	21	0.238
230	A	0	0	0.00	0	0.000
231	A	5	5	1.00	10	0.500
232	A	7	5	1.00	10	0.500
233	A	6	5	1.00	8	0.625
234	A	5	5	1.00	6	0.833
235	A	5	5	1.00	10	0.500
236	A	3	3	1.00	10	0.300
237	A	4	4	1.00	10	0.400
238	A	3	3	1.00	4	0.750
239	A	5	5	1.00	10	0.500
240	A	3	2	1.00	14	0.143
241	A	6	5	1.00	14	0.357
242	A	2	2	1.00	14	0.143
243	A	5	4	1.00	12	0.333
244	A	1	1	1.00	14	0.071
245	A	1	1	1.00	14	0.071
246	A	2	2	1.00	14	0.143
247	A	3	2	1.00	14	0.143
248	A	5	4	1.00	14	0.286
249	A	2	2	1.00	14	0.143
250	A	4	3	1.00	12	0.250
251	A	1	1	1.00	14	0.071
252	A	1	1	1.00	14	0.071
253	A	2	2	1.00	14	0.143
254	A	2	2	1.00	16	0.125
255	A	2	2	1.00	16	0.125
256	A	1	1	1.00	16	0.062
257	A	1	1	1.00	16	0.062
258	A	1	1	1.00	16	0.062
259	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	2	2	1.00	16	0.125
261	A	2	2	1.00	16	0.125
262	A	2	2	1.00	16	0.125
263	A	1	1	1.00	16	0.062
264	A	1	1	1.00	16	0.062
265	A	1	1	1.00	16	0.062
266	A	2	2	1.00	16	0.125
267	A	2	2	1.00	16	0.125
268	A	0	0	0.00	0	0.000
269	A	8	8	1.00	40	0.200
270	A	7	7	1.01	40	0.175
271	A	6	7	1.00	38	0.184
272	A	0	0	0.00	0	0.000
273	A	0	0	0.00	0	0.000
274	A	6	6	1.00	10	0.600
275	A	5	4	1.00	12	0.333
276	A	5	4	1.00	12	0.333
277	A	5	4	1.00	10	0.400
278	A	5	4	1.00	8	0.500
279	A	9	8	1.00	12	0.667
280	A	9	8	1.00	12	0.667
281	A	7	5	1.00	12	0.417
282	A	8	6	1.00	12	0.500
283	A	10	7	1.00	12	0.583
284	A	37	8	1.00	14	0.571
285	A	27	8	1.00	14	0.571
286	A	17	8	1.00	12	0.667
287	A	7	4	1.00	10	0.400
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	7	7	1.00	19	0.368
291	A	2	2	1.00	20	0.100
292	A	4	4	1.00	12	0.333
293	A	4	4	1.00	14	0.286
294	A	5	5	1.00	10	0.500
295	A	2	2	1.00	26	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
296	A	2	2	1.00	26	0.077

Chapter 3

Listing of integrals

3.1 $\int (d+ex)^3 \cosh^{-1}(cx)\,dx$

Optimal. Leaf size=183

$$-\frac{\left(8c^4d^4+24c^2d^2e^2+3e^4\right)\cosh^{-1}(cx)}{32c^4e}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(ex\left(26c^2d^2+9e^2\right)+4d\left(19c^2d^2+16e^2\right)\right)}{96c^3}-\frac{\sqrt{cx-1}\,\sqrt{}}{}$$

```
[Out] -1/32*(8*c^4*d^4+24*c^2*d^2*e^2+3*e^4)*arccosh(c*x)/c^4/e+1/4*(e*x+d)^4*arc
cosh(c*x)/e-7/48*d*(e*x+d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/16*(e*x+d)^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/96*(4*d*(19*c^2*d^2+16*e^2)+e*(26*c^2*d^2+9*
e^2)*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5802, 100, 153, 147, 52}

$$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(ex\left(26c^2d^2+9e^2\right)+4d\left(19c^2d^2+16e^2\right)\right)}{96c^3}-\frac{\left(24c^2d^2e^2+8c^4d^4+3e^4\right)\cosh^{-1}(cx)}{32c^4e}-\frac{\sqrt{cx-1}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*ArcCosh[c*x],x]
```

```
[Out] (-7*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(48*c) - (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(d + e*x)^3)/(16*c) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d*(19*c^
2*d^2 + 16*e^2) + e*(26*c^2*d^2 + 9*e^2)*x))/(96*c^3) - ((8*c^4*d^4 + 24*c^
2*d^2*e^2 + 3*e^4)*ArcCosh[c*x])/(32*c^4*e) + ((d + e*x)^4*ArcCosh[c*x])/(4
*e)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (d+ex)^3 \cosh^{-1}(cx)\,dx &= \frac{(d+ex)^4\cosh^{-1}(cx)}{4e} - \frac{c\int \frac{(d+ex)^4}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{4e} \\
&= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} + \frac{(d+ex)^4\cosh^{-1}(cx)}{4e} - \frac{\int \frac{(d+ex)^2\left(4c^2d^2+3e^2+7c^2dex\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{16ce} \\
&= -\frac{7d\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} + \frac{(d+ex)^4\cosh^{-1}(c\ldots}{4e} \\
&= -\frac{7d\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\ldots}{\ldots} \\
&= -\frac{7d\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}{48c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}{16c} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\ldots}{\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.27, size = 153, normalized size = 0.84

$$
-\frac{-24c^4x\cosh^{-1}(cx)\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)+9\left(8c^2d^2e+e^3\right)\log\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)+c\sqrt{cx-1}\,\sqrt{c\ldots}}{96c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*ArcCosh[c*x],x]
```

```
[Out] -1/96*(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 7
2*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3)) - 24*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*
e^2*x^2 + e^3*x^3)*ArcCosh[c*x] + 9*(8*c^2*d^2*e + e^3)*Log[c*x + Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]])/c^4
```

fricas [A] time = 0.55, size = 153, normalized size = 0.84

$$\frac{3\left(8\,c^4e^3x^4+32\,c^4de^2x^3+48\,c^4d^2ex^2+32\,c^4d^3x-24\,c^2d^2e-3\,e^3\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(6\,c^3e^3x^3+32\,c^3de\right.}{96\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*c^4*e^3*x^4 + 32*c^4*d*e^2*x^3 + 48*c^4*d^2*e*x^2 + 32*c^4*d^3*x
 - 24*c^2*d^2*e - 3*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (6*c^3*e^3*x^3 + 32
*c^3*d*e^2*x^2 + 96*c^3*d^3 + 64*c*d*e^2 + 9*(8*c^3*d^2*e + c*e^3)*x)*sqrt(
c^2*x^2 - 1))/c^4
```

giac [A] time = 0.37, size = 167, normalized size = 0.91

$$\frac{1}{4}\,(xe+d)^4e^{(-1)}\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{1}{96}\left(\sqrt{c^2x^2-1}\left(\left(2\,x\left(\frac{3\,xe^4}{c}+\frac{16\,de^3}{c}\right)+\frac{9\left(8\,c^5d^2e^2+c^3e^4\right)}{c^6}\right)x+\frac{32\left(3\,c^5\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/4*(x*e + d)^4*e^(-1)*log(c*x + sqrt(c^2*x^2 - 1)) - 1/96*(sqrt(c^2*x^2 -
1)*((2*x*(3*x*e^4/c + 16*d*e^3/c) + 9*(8*c^5*d^2*e^2 + c^3*e^4)/c^6)*x + 32
*(3*c^5*d^3*e + 2*c^3*d*e^3)/c^6) - 3*(8*c^4*d^4 + 24*c^2*d^2*e^2 + 3*e^4)*
log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^3*abs(c)))*e^(-1)
```

maple [B] time = 0.03, size = 351, normalized size = 1.92

$$\frac{e^3\mathrm{arccosh}\,(cx)\,x^4}{4}+e^2\mathrm{arccosh}\,(cx)\,x^3d+\frac{3e\,\mathrm{arccosh}\,(cx)\,x^2d^2}{2}+\mathrm{arccosh}\,(cx)\,x\,d^3+\frac{\mathrm{arccosh}\,(cx)\,d^4}{4e}-\frac{e^3\sqrt{cx-1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*arccosh(c*x),x)
```

```
[Out] 1/4*e^3*arccosh(c*x)*x^4+e^2*arccosh(c*x)*x^3*d+3/2*e*arccosh(c*x)*x^2*d^2+
arccosh(c*x)*x*d^3+1/4/e*arccosh(c*x)*d^4-1/16/c*e^3*(c*x-1)^(1/2)*(c*x+1)^
(1/2)*x^3-1/3/c*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*d-1/4/e*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2*x^2-1)^(1/2))-3/4/c*e*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*d^2*x-1/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^3-3/4/c^2*e*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-
3/32/c^3*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x-2/3/c^3*e^2*(c*x-1)^(1/2)*(c*x+1
)^(1/2)*d-3/32/c^4*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x
+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.31, size = 229, normalized size = 1.25

$$-\frac{1}{96}\left(\frac{6\sqrt{c^2x^2-1}\,e^3x^3}{c^2}+\frac{32\sqrt{c^2x^2-1}\,de^2x^2}{c^2}+\frac{72\sqrt{c^2x^2-1}\,d^2ex}{c^2}+\frac{96\sqrt{c^2x^2-1}\,d^3}{c^2}+\frac{72\,d^2e\log\left(2\,c^2x+2\right.}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/96*(6*sqrt(c^2*x^2 - 1)*e^3*x^3/c^2 + 32*sqrt(c^2*x^2 - 1)*d*e^2*x^2/c^2
 + 72*sqrt(c^2*x^2 - 1)*d^2*e*x/c^2 + 96*sqrt(c^2*x^2 - 1)*d^3/c^2 + 72*d^2
*e*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3 + 9*sqrt(c^2*x^2 - 1)*e^3*x/c^4
 + 64*sqrt(c^2*x^2 - 1)*d*e^2/c^4 + 9*e^3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)
*c)/c^5)*c + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*arccosh(c*
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(c\,x)\,\left(d+e\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)*(d + e*x)^3,x)
```

```
[Out] int(acosh(c*x)*(d + e*x)^3, x)
```

sympy [A] time = 1.28, size = 258, normalized size = 1.41

$$\begin{cases} d^3x\operatorname{acosh}(cx) + \frac{3d^2ex^2\operatorname{acosh}(cx)}{2} + de^2x^3\operatorname{acosh}(cx) + \frac{e^3x^4\operatorname{acosh}(cx)}{4} - \frac{d^3\sqrt{c^2x^2-1}}{c} - \frac{3d^2ex\sqrt{c^2x^2-1}}{4c} - \frac{de^2x^2\sqrt{c^2x^2-1}}{3c} - \frac{e^3x^3 \cdots}{} \\ \frac{i\pi\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*acosh(c*x),x)
```

```
[Out] Piecewise((d**3*x*acosh(c*x) + 3*d**2*e*x**2*acosh(c*x)/2 + d*e**2*x**3*aco
sh(c*x) + e**3*x**4*acosh(c*x)/4 - d**3*sqrt(c**2*x**2 - 1)/c - 3*d**2*e*x*
sqrt(c**2*x**2 - 1)/(4*c) - d*e**2*x**2*sqrt(c**2*x**2 - 1)/(3*c) - e**3*x*
*3*sqrt(c**2*x**2 - 1)/(16*c) - 3*d**2*e*acosh(c*x)/(4*c**2) - 2*d*e**2*sqr
t(c**2*x**2 - 1)/(3*c**3) - 3*e**3*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 3*e**3
*acosh(c*x)/(32*c**4), Ne(c, 0)), (I*pi*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*
x**3 + e**3*x**4/4)/2, True))
```

3.2 $\int (d+ex)^2 \cosh^{-1}(cx)\,dx$

Optimal. Leaf size=123

$$-\frac{1}{6}d\left(\frac{3e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)^2}{9c}+\frac{\cosh^{-1}(}{}$$

```
[Out] -1/6*d*(2*d^2/e+3*e/c^2)*arccosh(c*x)+1/3*(e*x+d)^3*arccosh(c*x)/e-1/9*(e*x
+d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/18*(5*c^2*d*e*x+16*c^2*d^2+4*e^2)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5802, 100, 147, 52}

$$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3}-\frac{1}{6}d\left(\frac{3e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)^2}{9c}+\frac{\cosh^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*ArcCosh[c*x],x]
```

```
[Out] -(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(9*c) - (Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(4*(4*c^2*d^2 + e^2) + 5*c^2*d*e*x))/(18*c^3) - (d*((2*d^2)/e + (3*e
)/c^2)*ArcCosh[c*x])/6 + ((d + e*x)^3*ArcCosh[c*x])/(3*e)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2\cosh^{-1}(cx)\,dx = \frac{(d+ex)^3\cosh^{-1}(cx)}{3e} - \frac{c\int \frac{(d+ex)^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3e}$$
$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}\,(d+ex)^2}{9c} + \frac{(d+ex)^3\cosh^{-1}(cx)}{3e} - \frac{\int \frac{(d+ex)\left(3c^2d^2+2e^2+5c^2dex\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{9ce}$$
$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}\,(d+ex)^2}{9c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3} + \frac{(d+}{}$$
$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}\,(d+ex)^2}{9c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3} - \frac{1}{6}d\Big($$

Mathematica [A] time = 0.21, size = 113, normalized size = 0.92

$$-\frac{-6c^3x\cosh^{-1}(cx)\left(3d^2+3dex+e^2x^2\right)+\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)+4e^2\right)+9cde\log\left(cx+\sqrt{c}\right.}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*ArcCosh[c*x],x]
```

```
[Out] -1/18*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*
x^2)) - 6*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCosh[c*x] + 9*c*d*e*Log[c*x
+ Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/c^3
```

fricas [A] time = 0.53, size = 106, normalized size = 0.86

$$\frac{3\left(2\,c^3e^2x^3+6\,c^3dex^2+6\,c^3d^2x-3\,cde\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(2\,c^2e^2x^2+9\,c^2dex+18\,c^2d^2+4\,e^2\right)\sqrt{c^2x^2-1}}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*c^3*e^2*x^3 + 6*c^3*d*e*x^2 + 6*c^3*d^2*x - 3*c*d*e)*log(c*x + s
qrt(c^2*x^2 - 1)) - (2*c^2*e^2*x^2 + 9*c^2*d*e*x + 18*c^2*d^2 + 4*e^2)*sqrt
(c^2*x^2 - 1))/c^3
```

giac [A] time = 0.35, size = 129, normalized size = 1.05

$$\frac{1}{3}(xe+d)^3e^{(-1)}\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{1}{18}\left(\sqrt{c^2x^2-1}\left(x\left(\frac{2\,xe^3}{c}+\frac{9\,de^2}{c}\right)+\frac{2\left(9\,c^3d^2e+2\,ce^3\right)}{c^4}\right)-\frac{3\left(2\,c^2d^3+3\,de^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/3*(x*e + d)^3*e^(-1)*log(c*x + sqrt(c^2*x^2 - 1)) - 1/18*(sqrt(c^2*x^2 -
1)*(x*(2*x*e^3/c + 9*d*e^2/c) + 2*(9*c^3*d^2*e + 2*c*e^3)/c^4) - 3*(2*c^2*d
^3 + 3*d*e^2)*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c*abs(c)))*e^(-1)
```

maple [B] time = 0.01, size = 233, normalized size = 1.89

$$\frac{e^{2}\operatorname{arccosh}(cx)\,x^{3}}{3}+e\operatorname{arccosh}(cx)\,x^{2}d+\operatorname{arccosh}(cx)\,x\,d^{2}+\frac{\operatorname{arccosh}(cx)\,d^{3}}{3e}-\frac{e^{2}\sqrt{cx-1}\,\sqrt{cx+1}\,x^{2}}{9c}-\frac{\sqrt{cx-1}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*arccosh(c*x),x)
```

```
[Out] 1/3*e^2*arccosh(c*x)*x^3+e*arccosh(c*x)*x^2*d+arccosh(c*x)*x*d^2+1/3/e*arcc
osh(c*x)*d^3-1/9/c*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-1/3/e*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-1/2/c*e*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*d*x-1/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2-1/2/c^2*e*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))-2/9
/c^3*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

maxima [A] time = 0.49, size = 140, normalized size = 1.14

$$-\frac{1}{18}\left(\frac{2\sqrt{c^{2}x^{2}-1}\,e^{2}x^{2}}{c^{2}}+\frac{9\sqrt{c^{2}x^{2}-1}\,dex}{c^{2}}+\frac{18\sqrt{c^{2}x^{2}-1}\,d^{2}}{c^{2}}+\frac{9\,de\log\left(2\,c^{2}x+2\sqrt{c^{2}x^{2}-1}\,c\right)}{c^{3}}+\frac{4\sqrt{c^{2}x^{2}-1}\,e}{c^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/18*(2*sqrt(c^2*x^2 - 1)*e^2*x^2/c^2 + 9*sqrt(c^2*x^2 - 1)*d*e*x/c^2 + 18
*sqrt(c^2*x^2 - 1)*d^2/c^2 + 9*d*e*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3
 + 4*sqrt(c^2*x^2 - 1)*e^2/c^4)*c + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*arc
cosh(c*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(c\,x)\,\left(d+e\,x\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)*(d + e*x)^2,x)
```

```
[Out] int(acosh(c*x)*(d + e*x)^2, x)
```

sympy [A] time = 0.58, size = 155, normalized size = 1.26

$$\begin{cases} d^{2}x\operatorname{acosh}(cx)+dex^{2}\operatorname{acosh}(cx)+\frac{e^{2}x^{3}\operatorname{acosh}(cx)}{3}-\frac{d^{2}\sqrt{c^{2}x^{2}-1}}{c}-\frac{dex\sqrt{c^{2}x^{2}-1}}{2c}-\frac{e^{2}x^{2}\sqrt{c^{2}x^{2}-1}}{9c}-\frac{de\operatorname{acosh}(cx)}{2c^{2}}-\frac{2e^{2}\sqrt{c^{2}x^{2}}}{9c^{3}} \\ \frac{i\pi\left(d^{2}x+dex^{2}+\frac{e^{2}x^{3}}{3}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*acosh(c*x),x)
```

```
[Out] Piecewise((d**2*x*acosh(c*x) + d*e*x**2*acosh(c*x) + e**2*x**3*acosh(c*x)/3
 - d**2*sqrt(c**2*x**2 - 1)/c - d*e*x*sqrt(c**2*x**2 - 1)/(2*c) - e**2*x**2
*sqrt(c**2*x**2 - 1)/(9*c) - d*e*acosh(c*x)/(2*c**2) - 2*e**2*sqrt(c**2*x**
2 - 1)/(9*c**3), Ne(c, 0)), (I*pi*(d**2*x + d*e*x**2 + e**2*x**3/3)/2, True
))
```

3.3 $\int (d+ex)\cosh^{-1}(cx)\,dx$

Optimal. Leaf size=97

$$-\frac{1}{4}\left(\frac{e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)}{4c}+\frac{\cosh^{-1}(cx)(d+ex)^2}{2e}-\frac{3d\sqrt{cx-1}\,\sqrt{cx+1}}{4c}$$

```
[Out] -1/4*(2*d^2/e+e/c^2)*arccosh(c*x)+1/2*(e*x+d)^2*arccosh(c*x)/e-3/4*d*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c-1/4*(e*x+d)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5802, 90, 80, 52}

$$-\frac{1}{4}\left(\frac{e}{c^2}+\frac{2d^2}{e}\right)\cosh^{-1}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)}{4c}+\frac{\cosh^{-1}(cx)(d+ex)^2}{2e}-\frac{3d\sqrt{cx-1}\,\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*ArcCosh[c*x],x]
```

```
[Out] (-3*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
d + e*x))/(4*c) - (((2*d^2)/e + e/c^2)*ArcCosh[c*x])/4 + ((d + e*x)^2*ArcCo
sh[c*x])/(2*e)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\cosh^{-1}(cx)\,dx = \frac{(d+ex)^2\cosh^{-1}(cx)}{2e} - \frac{c\int \frac{(d+ex)^2}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{2e}$$
$$= -\frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} + \frac{(d+ex)^2\cosh^{-1}(cx)}{2e} - \frac{\int \frac{2c^2d^2+e^2+3c^2dex}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{4ce}$$
$$= -\frac{3d\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} + \frac{(d+ex)^2\cosh^{-1}(cx)}{2e} - \frac{1}{4}\Big($$
$$= -\frac{3d\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} - \frac{1}{4}\left(\frac{2d^2}{e} + \frac{e}{c^2}\right)\cosh^{-1}(cx) +$$

Mathematica [A] time = 0.08, size = 73, normalized size = 0.75

$$-\frac{-2c^2x\cosh^{-1}(cx)(2d+ex) + c\sqrt{cx-1}\sqrt{cx+1}(4d+ex) + 2e\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)}{4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)*ArcCosh[c*x],x]
```

```
[Out] -1/4*(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d + e*x) - 2*c^2*x*(2*d + e*x)*ArcC
osh[c*x] + 2*e*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/c^2
```

fricas [A] time = 0.49, size = 65, normalized size = 0.67

$$\frac{\left(2c^2ex^2 + 4c^2dx - e\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \sqrt{c^2x^2-1}(cex+4cd)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="fricas")
```

```
[Out] 1/4*((2*c^2*e*x^2 + 4*c^2*d*x - e)*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*
x^2 - 1)*(c*e*x + 4*c*d))/c^2
```

giac [A] time = 0.29, size = 87, normalized size = 0.90

$$\frac{1}{2}\left(x^2e + 2dx\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \frac{1}{4}\sqrt{c^2x^2-1}\left(\frac{xe}{c} + \frac{4d}{c}\right) + \frac{e\log\left(\left|-x|c| + \sqrt{c^2x^2-1}\right|\right)}{4c|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 1/4*sqrt(c^2*x^2 - 1)*(x
*e/c + 4*d/c) + 1/4*e*log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c*abs(c))
```

maple [A] time = 0.01, size = 107, normalized size = 1.10

$$\frac{\operatorname{arccosh}(cx)\,x^2e}{2} + \operatorname{arccosh}(cx)\,xd - \frac{\sqrt{cx-1}\sqrt{cx+1}\,ex}{4c} - \frac{d\sqrt{cx-1}\sqrt{cx+1}}{c} - \frac{\sqrt{cx-1}\sqrt{cx+1}\,e\ln\left(cx+\sqrt{c^2\right.}}{4c^2\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*arccosh(c*x),x)
```

```
[Out] 1/2*arccosh(c*x)*x^2*e+arccosh(c*x)*x*d-1/4/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e
*x-d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/c^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2
*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.46, size = 82, normalized size = 0.85

$$-\frac{1}{4}c\left(\frac{\sqrt{c^2x^2-1}\,ex}{c^2}+\frac{4\sqrt{c^2x^2-1}\,d}{c^2}+\frac{e\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)+\frac{1}{2}\left(ex^2+2\,dx\right)\operatorname{arcosh}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x),x, algorithm="maxima")
```

```
[Out] -1/4*c*(sqrt(c^2*x^2 - 1)*e*x/c^2 + 4*sqrt(c^2*x^2 - 1)*d/c^2 + e*log(2*c^2
*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3) + 1/2*(e*x^2 + 2*d*x)*arccosh(c*x)
```

mupad [B] time = 0.71, size = 68, normalized size = 0.70

$$d\,x\,\operatorname{acosh}(c\,x)+e\,x\,\operatorname{acosh}(c\,x)\left(\frac{x}{2}-\frac{1}{4\,c^2\,x}\right)-\frac{d\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c}-\frac{e\,x\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)*(d + e*x),x)
```

```
[Out] d*x*acosh(c*x) + e*x*acosh(c*x)*(x/2 - 1/(4*c^2*x)) - (d*(c*x - 1)^(1/2)*(c
*x + 1)^(1/2))/c - (e*x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/(4*c)
```

sympy [A] time = 0.26, size = 80, normalized size = 0.82

$$\begin{cases} dx\operatorname{acosh}(cx)+\frac{ex^2\operatorname{acosh}(cx)}{2}-\frac{d\sqrt{c^2x^2-1}}{c}-\frac{ex\sqrt{c^2x^2-1}}{4c}-\frac{e\operatorname{acosh}(cx)}{4c^2} & \text{for } c\neq 0\\ \frac{i\pi\left(dx+\frac{ex^2}{2}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*acosh(c*x),x)
```

```
[Out] Piecewise((d*x*acosh(c*x) + e*x**2*acosh(c*x)/2 - d*sqrt(c**2*x**2 - 1)/c -
 e*x*sqrt(c**2*x**2 - 1)/(4*c) - e*acosh(c*x)/(4*c**2), Ne(c, 0)), (I*pi*(d
*x + e*x**2/2)/2, True))
```

3.4 $\int \frac{\cosh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=178

$$\frac{\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e}-$$

```
[Out] -1/2*arccosh(c*x)^2/e+arccosh(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/(c*d-(c^2*d^2-e^2)^(1/2)))/e+arccosh(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e
```

Rubi [A] time = 0.27, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5800, 5562, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\cosh^{-1}(cx)\log\left(\cdots\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x),x]
```

```
[Out] -ArcCosh[c*x]^2/(2*e) + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqr
t[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqr
t[c^2*d^2 - e^2])])/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2
 - e^2]))]/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))
]/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{x\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$
$$= -\frac{\cosh^{-1}(cx)^2}{2e} + \text{Subst}\left(\int \frac{e^x x}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{e^x}{cd+\sqrt{c^2d^2}}\right.$$
$$= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} - \frac{\text{Sub}}{}$$
$$= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} - \frac{\text{Sub}}{}$$
$$= -\frac{\cosh^{-1}(cx)^2}{2e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\text{Li}_2}{}$$

Mathematica [A] time = 0.01, size = 176, normalized size = 0.99

$$\frac{\text{Li}_2\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right)}{e} + \frac{\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e} + \frac{\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e} - \frac{\text{cos}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x),x]
```

```
[Out] -1/2*ArcCosh[c*x]^2/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqr
t[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqr
t[c^2*d^2 - e^2])])/e + PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^
2 - e^2])]/e + PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))
]/e
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcosh}(cx)}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="giac")
```

[Out] integrate(arccosh(c*x)/(e*x + d), x)

maple [A]   time = 0.16, size = 295, normalized size = 1.66

$$-\frac{\operatorname{arccosh}(cx)^2}{2e}+\frac{\operatorname{arccosh}(cx)\ln\left(\frac{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)e+cd+\sqrt{c^2d^2-e^2}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\operatorname{arccosh}(cx)\ln\left(\frac{-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)e-cd+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(c*x)/(e*x+d),x)

[Out] -1/2*arccosh(c*x)^2/e+1/e*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+1/e*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))+1/e*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))+1/e*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(c*x)/(e*x+d),x, algorithm="maxima")

[Out] integrate(arccosh(c*x)/(e*x + d), x)

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(c\,x)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(c*x)/(d + e*x),x)

[Out] int(acosh(c*x)/(d + e*x), x)

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(c*x)/(e*x+d),x)

[Out] Integral(acosh(c*x)/(d + e*x), x)

3.5 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^2} dx$

Optimal. Leaf size=83

$$\frac{2c\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\sqrt{cd+e}}-\frac{\cosh^{-1}(cx)}{e(d+ex)}$$

```
[Out] -arccosh(c*x)/e/(e*x+d)+2*c*arctanh((c*d+e)^(1/2)*(c*x+1)^(1/2)/(c*d-e)^(1/
2)/(c*x-1)^(1/2))/e/(c*d-e)^(1/2)/(c*d+e)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5802, 93, 208}

$$\frac{2c\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\sqrt{cd+e}}-\frac{\cosh^{-1}(cx)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^2,x]
```

```
[Out] -(ArcCosh[c*x]/(e*(d + e*x))) + (2*c*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 + c*x])/
(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(Sqrt[c*d - e]*e*Sqrt[c*d + e])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{\cosh^{-1}(cx)}{e(d+ex)}+\frac{c\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{e}\\
&= -\frac{\cosh^{-1}(cx)}{e(d+ex)}+\frac{(2c)\,\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e}\\
&= -\frac{\cosh^{-1}(cx)}{e(d+ex)}+\frac{2c\tanh^{-1}\left(\frac{\sqrt{cd+e}\sqrt{1+cx}}{\sqrt{cd-e}\sqrt{-1+cx}}\right)}{\sqrt{cd-e}\,e\sqrt{cd+e}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 92, normalized size = 1.11

$$\frac{\frac{c\left(\log(d+ex)-\log\left(-\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d^2-e^2}+c^2dx+e\right)\right)}{\sqrt{c^2d^2-e^2}}-\frac{\cosh^{-1}(cx)}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^2,x]
```

```
[Out] (-(ArcCosh[c*x]/(d + e*x)) + (c*(Log[d + e*x] - Log[e + c^2*d*x - Sqrt[c^2*
d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]]))/Sqrt[c^2*d^2 - e^2])/e
```

fricas [B] time = 0.56, size = 454, normalized size = 5.47

$$\left[\frac{\left(c^2d^2e-e^3\right)x\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2d^2-e^2}\left(cdex+cd^2\right)\log\left(\frac{c^3d^2x+cde+\sqrt{c^2d^2-e^2}\left(c^2dx+e\right)+\left(c^2d^2+\sqrt{c^2d^2-e^2}\,cd-e^2\right.}{ex+d}\right.}{c^2d^4e-d^2e^3+\left(c^2d^3e^2-de^4\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [((c^2*d^2*e - e^3)*x*log(c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*d^2 - e^2)*(c
*d*e*x + c*d^2)*log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e)
+ (c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) +
 (c^2*d^3 - d*e^2 + (c^2*d^2*e - e^3)*x)*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^
2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), ((c^2*d^2*e - e^3)*x*log(c*x
+ sqrt(c^2*x^2 - 1)) - 2*sqrt(-c^2*d^2 + e^2)*(c*d*e*x + c*d^2)*arctan(-(sq
rt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d)
)/(c^2*d^2 - e^2)) + (c^2*d^3 - d*e^2 + (c^2*d^2*e - e^3)*x)*log(-c*x + sqr
t(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

giac [B] time = 1.15, size = 215, normalized size = 2.59

$$\left(\frac{ce^3\log\left(\left|c^2d-\sqrt{c^2d^2-e^2}\,|c|\right|\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{\sqrt{c^2d^2-e^2}}-\frac{ce^3\log\left(\left|c^2d-\sqrt{c^2d^2-e^2}\left(\sqrt{c^2-\frac{2c^2d}{xe+d}+\frac{c^2d^2}{(xe+d)^2}-\frac{e^2}{(xe+d)^2}}+\frac{\sqrt{c^2d^2}}{}\right.\right.\right.}{\sqrt{c^2d^2-e^2}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (c*e^3*log(abs(c^2*d - sqrt(c^2*d^2 - e^2)*abs(c)))*sgn(1/(x*e + d))/sqrt(c
^2*d^2 - e^2) - c*e^3*log(abs(c^2*d - sqrt(c^2*d^2 - e^2)*(sqrt(c^2 - 2*c^2
*d/(x*e + d) + c^2*d^2/(x*e + d)^2 - e^2/(x*e + d)^2) + sqrt(c^2*d^2*e^2 -
e^4)*e^(-1)/(x*e + d))))/(sqrt(c^2*d^2 - e^2)*sgn(1/(x*e + d))))*e^(-4) - e
^(-1)*log(c*x + sqrt(c^2*x^2 - 1))/(x*e + d)
```

maple [A] time = 0.03, size = 126, normalized size = 1.52

$$-\frac{c\operatorname{arccosh}(cx)}{(cxe+cd)e}-\frac{c\sqrt{cx-1}\sqrt{cx+1}\,\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{cxe+cd}\right)}{e^2\sqrt{\frac{c^2d^2-e^2}{e^2}}\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^2,x)
```

```
[Out] -c/(c*e*x+c*d)/e*arccosh(c*x)-c/e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-2*(c^2*
d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))/((c^2*d^2
-e^2)/e^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more d
etails)Is e-c*d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{acosh}\left(c\,x\right)}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)/(d + e*x)^2,x)
```

```
[Out] int(acosh(c*x)/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}{\left(cx \right)}}{\left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**2,x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x)**2, x)
```

3.6 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=132

$$\frac{c^3 d \tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}-\frac{c\sqrt{cx-1}\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}$$

```
[Out] -1/2*arccosh(c*x)/e/(e*x+d)^2+c^3*d*arctanh((c*d+e)^(1/2)*(c*x+1)^(1/2)/(c*
d-e)^(1/2)/(c*x-1)^(1/2))/(c*d-e)^(3/2)/e/(c*d+e)^(3/2)-1/2*c*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A] time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5802, 96, 93, 208}

$$-\frac{c\sqrt{cx-1}\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}+\frac{c^3 d \tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}-\frac{\cosh^{-1}(cx)}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^3,x]
```

```
[Out] -(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c^2*d^2 - e^2)*(d + e*x)) - ArcCosh[c
*x]/(2*e*(d + e*x)^2) + (c^3*d*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 + c*x])/(Sqrt[
c*d - e]*Sqrt[-1 + c*x])])/((c*d - e)^(3/2)*e*(c*d + e)^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{\cosh^{-1}(cx)}{2e(d+ex)^2} + \frac{c\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}\,dx}{2e}$$
$$= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(c^3d\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}\,dx}{2e\left(c^2d^2-e^2\right)}$$
$$= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(c^3d\right)\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx, x, \frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e\left(c^2d^2-e^2\right)}$$
$$= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)}{2e(d+ex)^2} + \frac{c^3d\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{(cd-e)^{3/2}e(cd+e)^{3/2}}$$

Mathematica [A] time = 0.19, size = 190, normalized size = 1.44

$$\frac{c(d+ex)\left(-e\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{c^2d^2-e^2} - c^2d(d+ex)\log\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{c^2d^2-e^2} + c^2dx + e\right) + c^2d(d+\ldots}{2e(cd-e)(cd+e)\sqrt{c^2d^2-e^2}\,(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^3,x]
```

```
[Out] (-((c^2*d^2 - e^2)^(3/2)*ArcCosh[c*x]) + c*(d + e*x)*(-(e*Sqrt[c^2*d^2 - e^
2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + c^2*d*(d + e*x)*Log[d + e*x] - c^2*d*(d
+ e*x)*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
)/(2*(c*d - e)*e*(c*d + e)*Sqrt[c^2*d^2 - e^2]*(d + e*x)^2)
```

fricas [B] time = 0.87, size = 1044, normalized size = 7.91

$$\left[-\frac{c^4d^6 - c^2d^4e^2 + \left(c^4d^4e^2 - c^2d^2e^4\right)x^2 + \left(c^3d^3e^2x^2 + 2\,c^3d^4ex + c^3d^5\right)\sqrt{c^2d^2-e^2}\,\log\left(\frac{c^3d^2x+cde-\sqrt{c^2d^2-e^2}\left(c^2dx+e\right)+\left(c^2\ldots\right.}{ex+d\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*d^2*e^4)*x^2 + (c^3*d^3*e
^2*x^2 + 2*c^3*d^4*e*x + c^3*d^5)*sqrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*
e - sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 - sqrt(c^2*d^2 - e^2)*c*d
- e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + 2*(c^4*d^5*e - c^2*d^3*e^3)*x - ((c^
4*d^4*e^2 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^5
)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (c^4*d^6 - 2*c^2*d^4*e^2 + d^2*e^4 + (c
^4*d^4*e^2 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^
5)*x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (c^3*d^5*e - c*d^3*e^3 + (c^3*d^4*e^2
 - c*d^2*e^4)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 +
(c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^
4 + d^3*e^6)*x), -1/2*(c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*d^2*e^4)*
x^2 + 2*(c^3*d^3*e^2*x^2 + 2*c^3*d^4*e*x + c^3*d^5)*sqrt(-c^2*d^2 + e^2)*ar
ctan(-(sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e
*x + c*d))/(c^2*d^2 - e^2)) + 2*(c^4*d^5*e - c^2*d^3*e^3)*x - ((c^4*d^4*e^2
 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^5)*x)*log(
```

```
c*x + sqrt(c^2*x^2 - 1)) - (c^4*d^6 - 2*c^2*d^4*e^2 + d^2*e^4 + (c^4*d^4*e^
2 - 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e - 2*c^2*d^3*e^3 + d*e^5)*x)*log
(-c*x + sqrt(c^2*x^2 - 1)) + (c^3*d^5*e - c*d^3*e^3 + (c^3*d^4*e^2 - c*d^2*
e^4)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*
e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e
^6)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep)]Undef/Unsigned Inf encountered in limitLimit: Max order reached
or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 338, normalized size = 2.56

$$-\frac{c^{2}\operatorname{arccosh}(cx)}{2(cxe+cd)^{2}e}-\frac{c^{4}\sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^{2}dx-\sqrt{c^{2}x^{2}-1}\,\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}\,e+e\right)}{cxe+cd}\right)xd}{2e\sqrt{c^{2}x^{2}-1}\,(cd+e)(cd-e)(cxe+cd)\sqrt{\frac{c^{2}d^{2}-e^{2}}{e^{2}}}}-\frac{c^{4}\sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^{2}dx-\sqrt{c^{2}\cdots}\right.}{\cdots}\right.}{2e^{2}\sqrt{c^{2}x^{2}-1}\,(cd+e)(cd-e)(c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2/(c*e*x+c*d)^2/e*arccosh(c*x)-1/2*c^4/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)
/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*ln
(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*
x*d-1/2*c^4/e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-
e)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*(
(c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^2-1/2*c^2*(c*x+1)^(1/2)*(c*x-1
)^(1/2)/(c*d+e)/(c*d-e)/(c*e*x+c*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more d
etails)Is e-c*d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(c\,x)}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)/(d + e*x)^3,x)
```

```
[Out] int(acosh(c*x)/(d + e*x)^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(cx)}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**3,x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x)**3, x)
```

3.7 $\int \frac{\cosh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=195

$$-\frac{c^3d\sqrt{cx-1}\,\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{c\sqrt{cx-1}\,\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{c^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}$$

```
[Out] -1/3*arccosh(c*x)/e/(e*x+d)^3+1/3*c^3*(2*c^2*d^2+e^2)*arctanh((c*d+e)^(1/2)
*(c*x+1)^(1/2)/(c*d-e)^(1/2)/(c*x-1)^(1/2))/(c*d-e)^(5/2)/e/(c*d+e)^(5/2)-1
/6*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)/(e*x+d)^2-1/2*c^3*d*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)^2/(e*x+d)
```

Rubi [A] time = 0.24, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5802, 103, 151, 12, 93, 208}

$$-\frac{c\sqrt{cx-1}\,\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{c^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}-\frac{c^3d\sqrt{cx-1}\,\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]/(d + e*x)^4,x]
```

```
[Out] -(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*(c^2*d^2 - e^2)*(d + e*x)^2) - (c^3*d*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c*d - e)^2*(c*d + e)^2*(d + e*x)) - ArcCo
sh[c*x]/(3*e*(d + e*x)^3) + (c^3*(2*c^2*d^2 + e^2)*ArcTanh[(Sqrt[c*d + e]*S
qrt[1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(3*(c*d - e)^(5/2)*e*(c*d +
e)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{c\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}\,dx}{3e}$$

$$= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}-\frac{c\int \frac{-2c^2d+c^2ex}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}\,dx}{6e\left(c^2d^2-e^2\right)}$$

$$= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{c^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{c\int \frac{c^2\left(2c^2d^2+e^2\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}\,dx}{6e\left(c^2d^2-e^2\right)^2}$$

$$= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{c^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{\left(c^3\left(2c^2d^2+e^2\right)\right)\int \frac{}{\sqrt{-1+cx}}}{6e\left(c^2d^2-e^2\right)}$$

$$= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{c^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{\left(c^3\left(2c^2d^2+e^2\right)\right)\text{Subst}\left(\int\right.}{3e\left(c^2d\right.}$$

$$= -\frac{c\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{c^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{c^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{c}}\right.}{3(cd-e)^{5/2}e(cd+e}$$

Mathematica [C] time = 0.56, size = 244, normalized size = 1.25

$$\frac{1}{6}\left(\frac{c\sqrt{cx-1}\,\sqrt{cx+1}\,\left(e^2-c^2d(4d+3ex)\right)}{\left(e^2-c^2d^2\right)^2(d+ex)^2}-\frac{ic^3\left(2c^2d^2+e^2\right)\log\left(\frac{12e^2(e-cd)^2(cd+e)^2\left(\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{e^2-c^2d^2}-ic^2dx-ie\right)}{c^3\sqrt{e^2-c^2d^2}\left(2c^2d^2+e^2\right)(d+ex)}\right)}{e(e-cd)^2(cd+e)^2\sqrt{e^2-c^2d^2}}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]/(d + e*x)^4,x]
```

```
[Out] ((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2 - c^2*d*(4*d + 3*e*x)))/((-(c^2*d^2)
+ e^2)^2*(d + e*x)^2) - (2*ArcCosh[c*x])/(e*(d + e*x)^3) - (I*c^3*(2*c^2*d^
```

```
2 + e^2)*Log[(12*e^2*(-(c*d) + e)^2*(c*d + e)^2*((-I)*e - I*c^2*d*x + Sqrt[
-(c^2*d^2) + e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*Sqrt[-(c^2*d^2) + e^2
]*(2*c^2*d^2 + e^2)*(d + e*x))])/(e*(-(c*d) + e)^2*(c*d + e)^2*Sqrt[-(c^2*d
^2) + e^2]))/6
```

fricas [B] time = 1.49, size = 1799, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [-1/6*(3*c^6*d^9 - 3*c^4*d^7*e^2 + 3*(c^6*d^6*e^3 - c^4*d^4*e^5)*x^3 + 9*(c
^6*d^7*e^2 - c^4*d^5*e^4)*x^2 - (2*c^5*d^8 + c^3*d^6*e^2 + (2*c^5*d^5*e^3 +
 c^3*d^3*e^5)*x^3 + 3*(2*c^5*d^6*e^2 + c^3*d^4*e^4)*x^2 + 3*(2*c^5*d^7*e +
c^3*d^5*e^3)*x)*sqrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2 -
 e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^
2 - 1))/(e*x + d)) + 9*(c^6*d^8*e - c^4*d^6*e^3)*x - 2*((c^6*d^6*e^3 - 3*c^
4*d^4*e^5 + 3*c^2*d^2*e^7 - e^9)*x^3 + 3*(c^6*d^7*e^2 - 3*c^4*d^5*e^4 + 3*c
^2*d^3*e^6 - d*e^8)*x^2 + 3*(c^6*d^8*e - 3*c^4*d^6*e^3 + 3*c^2*d^4*e^5 - d^
2*e^7)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(c^6*d^9 - 3*c^4*d^7*e^2 + 3*c^2
*d^5*e^4 - d^3*e^6 + (c^6*d^6*e^3 - 3*c^4*d^4*e^5 + 3*c^2*d^2*e^7 - e^9)*x^
3 + 3*(c^6*d^7*e^2 - 3*c^4*d^5*e^4 + 3*c^2*d^3*e^6 - d*e^8)*x^2 + 3*(c^6*d^
8*e - 3*c^4*d^6*e^3 + 3*c^2*d^4*e^5 - d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 -
 1)) + (4*c^5*d^8*e - 5*c^3*d^6*e^3 + c*d^4*e^5 + 3*(c^5*d^6*e^3 - c^3*d^4*
e^5)*x^2 + (7*c^5*d^7*e^2 - 8*c^3*d^5*e^4 + c*d^3*e^6)*x)*sqrt(c^2*x^2 - 1)
)/(c^6*d^12*e - 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 - d^6*e^7 + (c^6*d^9*e^4 - 3
*c^4*d^7*e^6 + 3*c^2*d^5*e^8 - d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 - 3*c^4*d^8*
e^5 + 3*c^2*d^6*e^7 - d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 - 3*c^4*d^9*e^4 + 3*c^
2*d^7*e^6 - d^5*e^8)*x), -1/6*(3*c^6*d^9 - 3*c^4*d^7*e^2 + 3*(c^6*d^6*e^3 -
 c^4*d^4*e^5)*x^3 + 9*(c^6*d^7*e^2 - c^4*d^5*e^4)*x^2 + 2*(2*c^5*d^8 + c^3*
d^6*e^2 + (2*c^5*d^5*e^3 + c^3*d^3*e^5)*x^3 + 3*(2*c^5*d^6*e^2 + c^3*d^4*e^
4)*x^2 + 3*(2*c^5*d^7*e + c^3*d^5*e^3)*x)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqr
t(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))
/(c^2*d^2 - e^2)) + 9*(c^6*d^8*e - c^4*d^6*e^3)*x - 2*((c^6*d^6*e^3 - 3*c^4
*d^4*e^5 + 3*c^2*d^2*e^7 - e^9)*x^3 + 3*(c^6*d^7*e^2 - 3*c^4*d^5*e^4 + 3*c^
2*d^3*e^6 - d*e^8)*x^2 + 3*(c^6*d^8*e - 3*c^4*d^6*e^3 + 3*c^2*d^4*e^5 - d^2
*e^7)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(c^6*d^9 - 3*c^4*d^7*e^2 + 3*c^2*
d^5*e^4 - d^3*e^6 + (c^6*d^6*e^3 - 3*c^4*d^4*e^5 + 3*c^2*d^2*e^7 - e^9)*x^3
 + 3*(c^6*d^7*e^2 - 3*c^4*d^5*e^4 + 3*c^2*d^3*e^6 - d*e^8)*x^2 + 3*(c^6*d^8
*e - 3*c^4*d^6*e^3 + 3*c^2*d^4*e^5 - d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 -
1)) + (4*c^5*d^8*e - 5*c^3*d^6*e^3 + c*d^4*e^5 + 3*(c^5*d^6*e^3 - c^3*d^4*e
^5)*x^2 + (7*c^5*d^7*e^2 - 8*c^3*d^5*e^4 + c*d^3*e^6)*x)*sqrt(c^2*x^2 - 1))
/(c^6*d^12*e - 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 - d^6*e^7 + (c^6*d^9*e^4 - 3*
c^4*d^7*e^6 + 3*c^2*d^5*e^8 - d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 - 3*c^4*d^8*e
^5 + 3*c^2*d^6*e^7 - d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 - 3*c^4*d^9*e^4 + 3*c^2
*d^7*e^6 - d^5*e^8)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
```

```
(t_nostep)]Undef/Unsigned Inf encountered in limitLimit: Max order reached
or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.02, size = 1108, normalized size = 5.68

$$-\frac{c^3 \operatorname{arccosh}(cx)}{3(cxe+cd)^3 e}-\frac{c^7\sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{cxe+cd}\right)x^2d^2}{3\sqrt{c^2x^2-1}\,(cd+e)(cd-e)\left(c^2d^2-e^2\right)(cxe+cd)^2\sqrt{\frac{c^2d^2-e^2}{e^2}}}-\frac{2c^7\sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^2d\right.}{}\right.}{3e\sqrt{c^2x^2-1}\,(cd+e)(cd-e)\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)/(e*x+d)^4,x)
```

```
[Out] -1/3*c^3/(c*e*x+c*d)^3/e*arccosh(c*x)-1/3*c^7*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(
c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)
/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)
/(c*e*x+c*d))*x^2*d^2-2/3*c^7/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/
2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln
(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*
x*d^3-1/2*c^5*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(
c*e*x+c*d)^2*x*d-1/6*c^5*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/
(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2
*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x^2
-1/3*c^7/e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/
(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x
^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^4-2/3*c^5*(c*x+1)
^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*d^2-1/3*c^
5*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-
e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/
2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x*d-1/6*c^5*(c*x+1)^(1/2)*(c
*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/(
(c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^
2)^(1/2)*e+e)/(c*e*x+c*d))*d^2+1/6*c^3*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d
+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more d
etails)Is e-c*d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(c\,x)}{(d+e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)/(d + e*x)^4,x)
```

```
[Out] int(acosh(c*x)/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(cx)}{(d+ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)/(e*x+d)**4,x)
```

```
[Out] Integral(acosh(c*x)/(d + e*x)**4, x)
```

3.8 $\int (d+ex)^3 \cosh^{-1}(cx)^2\,dx$

Optimal. Leaf size=334

$$-\frac{3e^3\cosh^{-1}(cx)^2}{32c^4}-\frac{4de^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{3c^3}-\frac{3e^3x\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{16c^3}-\frac{3d^2e\cosh^{-1}(cx)^2}{4c^2}+\frac{4de^2}{3c^2}$$

```
[Out] 2*d^3*x+4/3*d*e^2*x/c^2+3/4*d^2*e*x^2+3/32*e^3*x^2/c^2+2/9*d*e^2*x^3+1/32*e
^3*x^4-1/4*d^4*arccosh(c*x)^2/e-3/4*d^2*e*arccosh(c*x)^2/c^2-3/32*e^3*arcco
sh(c*x)^2/c^4+1/4*(e*x+d)^4*arccosh(c*x)^2/e-2*d^3*arccosh(c*x)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c-4/3*d*e^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-3
/2*d^2*e*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/16*e^3*x*arccosh(c*
x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-2/3*d*e^2*x^2*arccosh(c*x)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c-1/8*e^3*x^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 1.46, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{3d^2e\cosh^{-1}(cx)^2}{4c^2}+\frac{4de^2x}{3c^2}-\frac{4de^2\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{3c^3}+\frac{3e^3x^2}{32c^2}-\frac{3e^3\cosh^{-1}(cx)^2}{32c^4}-\frac{3e^3x\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{co}}{16c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d^3*x + (4*d*e^2*x)/(3*c^2) + (3*d^2*e*x^2)/4 + (3*e^3*x^2)/(32*c^2) + (2
*d*e^2*x^3)/9 + (e^3*x^4)/32 - (2*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[
c*x])/c - (4*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^3) - (3*
d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) - (3*e^3*x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(16*c^3) - (2*d*e^2*x^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c) - (e^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*A
rcCosh[c*x])/(8*c) - (d^4*ArcCosh[c*x]^2)/(4*e) - (3*d^2*e*ArcCosh[c*x]^2)/
(4*c^2) - (3*e^3*ArcCosh[c*x]^2)/(32*c^4) + ((d + e*x)^4*ArcCosh[c*x]^2)/(4
*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
```

```
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^3\cosh^{-1}(cx)^2\,dx = \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \frac{c\int \frac{(d+ex)^4\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2e}$$

$$= \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \frac{c\int \left(\frac{d^4\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4d^3ex\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{6d^2e^2x^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \ldots\right.}{2e}$$

$$= \frac{(d+ex)^4\cosh^{-1}(cx)^2}{4e} - \left(2cd^3\right)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{\left(cd^4\right)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}}{2e}$$

$$= -\frac{2d^3\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{3d^2ex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c} - \frac{2de}{}$$

$$= 2d^3x + \frac{3}{4}d^2ex^2 + \frac{2}{9}de^2x^3 + \frac{e^3x^4}{32} - \frac{2d^3\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{4de^2\sqrt{-1}}{}$$

$$= 2d^3x + \frac{4de^2x}{3c^2} + \frac{3}{4}d^2ex^2 + \frac{3e^3x^2}{32c^2} + \frac{2}{9}de^2x^3 + \frac{e^3x^4}{32} - \frac{2d^3\sqrt{-1+cx}\sqrt{1+cx}\cosh}{c}$$

Mathematica [A] time = 0.27, size = 191, normalized size = 0.57

$$c^2x\left(c^2\left(576d^3+216d^2ex+64de^2x^2+9e^3x^3\right)+3e^2(128d+9ex)\right)-6c\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)\left(c^2\left(96d^3+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*ArcCosh[c*x]^2,x]
```

```
[Out] (c^2*x*(3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 216*d^2*e*x + 64*d*e^2*x^2 +
 9*e^3*x^3)) - 6*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(
96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3))*ArcCosh[c*x] + 9*(-24*c^2*
d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*ArcCos
h[c*x]^2)/(288*c^4)
```

fricas [A] time = 0.74, size = 237, normalized size = 0.71

$$\frac{9\,c^4e^3x^4 + 64\,c^4de^2x^3 + 27\left(8\,c^4d^2e + c^2e^3\right)x^2 + 9\left(8\,c^4e^3x^4 + 32\,c^4de^2x^3 + 48\,c^4d^2ex^2 + 32\,c^4d^3x - 24\,c^2d^2e - 3\,e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*c^4*e^3*x^4 + 64*c^4*d*e^2*x^3 + 27*(8*c^4*d^2*e + c^2*e^3)*x^2 +
9*(8*c^4*e^3*x^4 + 32*c^4*d*e^2*x^3 + 48*c^4*d^2*e*x^2 + 32*c^4*d^3*x - 24*
c^2*d^2*e - 3*e^3)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 6*(6*c^3*e^3*x^3 + 32*c
^3*d*e^2*x^2 + 96*c^3*d^3 + 64*c*d*e^2 + 9*(8*c^3*d^2*e + c*e^3)*x)*sqrt(c^
2*x^2 - 1)*log(c*x + sqrt(c^2*x^2 - 1)) + 192*(3*c^4*d^3 + 2*c^2*d*e^2)*x)/
c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.13, size = 329, normalized size = 0.99

$$\frac{72\mathrm{arccosh}\left(cx\right)^2c^4x^4e^3 + 288\mathrm{arccosh}\left(cx\right)^2c^4x^3d\,e^2 + 432\mathrm{arccosh}\left(cx\right)^2c^4x^2d^2e + 288\mathrm{arccosh}\left(cx\right)^2c^4x\,d^3 - 36\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*arccosh(c*x)^2,x)
```

```
[Out] 1/288/c^4*(72*arccosh(c*x)^2*c^4*x^4*e^3+288*arccosh(c*x)^2*c^4*x^3*d*e^2+4
32*arccosh(c*x)^2*c^4*x^2*d^2*e+288*arccosh(c*x)^2*c^4*x*d^3-36*arccosh(c*x
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*x^3*e^3-192*arccosh(c*x)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*c^3*x^2*d*e^2-432*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*
x*d^2*e-576*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3*d^3-216*arccosh(c*
x)^2*c^2*d^2*e-54*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*x*e^3-384*arcc
osh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c*d*e^2+9*c^4*e^3*x^4+64*c^4*d*e^2*x^3
+216*c^4*d^2*e*x^2+576*x*c^4*d^3-27*arccosh(c*x)^2*e^3+27*c^2*x^2*e^3+384*x
*c^2*d*e^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\left(e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x\right)\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)^2 - \int \frac{\left(c^3e^3x^6 + 4\,c^3de^2x^5 - 6\,cd^2ex^2 - 4\,cd^3x + \left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))^2 - integrate(1/2*(c^3*e^3*x^6 + 4*c^3*d*e^2*x^5 - 6*c*d^2*
e*x^2 - 4*c*d^3*x + (6*c^3*d^2*e - c*e^3)*x^4 + 4*(c^3*d^3 - c*d*e^2)*x^3 +
 (c^2*e^3*x^5 + 4*c^2*d*e^2*x^4 + 6*c^2*d^2*e*x^3 + 4*c^2*d^3*x^2)*sqrt(c*x
 + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c^2
*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(c\,x)^2\,(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2*(d + e*x)^3,x)
```

```
[Out] int(acosh(c*x)^2*(d + e*x)^3, x)
```

sympy [A] time = 2.46, size = 371, normalized size = 1.11

$$\begin{cases} d^3x\operatorname{acosh}^2(cx) + 2d^3x + \frac{3d^2ex^2\operatorname{acosh}^2(cx)}{2} + \frac{3d^2ex^2}{4} + de^2x^3\operatorname{acosh}^2(cx) + \frac{2de^2x^3}{9} + \frac{e^3x^4\operatorname{acosh}^2(cx)}{4} + \frac{e^3x^4}{32} - \frac{2d^3\sqrt{c^2x}}{} \\ -\frac{\pi^2\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d**3*x*acosh(c*x)**2 + 2*d**3*x + 3*d**2*e*x**2*acosh(c*x)**2/2
+ 3*d**2*e*x**2/4 + d*e**2*x**3*acosh(c*x)**2 + 2*d*e**2*x**3/9 + e**3*x**4
*acosh(c*x)**2/4 + e**3*x**4/32 - 2*d**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/c -
 3*d**2*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(2*c) - 2*d*e**2*x**2*sqrt(c**2*
x**2 - 1)*acosh(c*x)/(3*c) - e**3*x**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(8*c)
 - 3*d**2*e*acosh(c*x)**2/(4*c**2) + 4*d*e**2*x/(3*c**2) + 3*e**3*x**2/(32*
c**2) - 4*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 3*e**3*x*sqrt(c*
*2*x**2 - 1)*acosh(c*x)/(16*c**3) - 3*e**3*acosh(c*x)**2/(32*c**4), Ne(c, 0
)), (-pi**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)/4, True)
)
```

3.9 $\int (d+ex)^2 \cosh^{-1}(cx)^2 \, dx$

Optimal. Leaf size=215

$$-\frac{4e^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{9c^3}-\frac{de\cosh^{-1}(cx)^2}{2c^2}+\frac{4e^2x}{9c^2}-\frac{d^3\cosh^{-1}(cx)^2}{3e}-\frac{2d^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c}-\frac{dex\cdots}{\cdots}$$

```
[Out] 2*d^2*x+4/9*e^2*x/c^2+1/2*d*e*x^2+2/27*e^2*x^3-1/3*d^3*arccosh(c*x)^2/e-1/2
*d*e*arccosh(c*x)^2/c^2+1/3*(e*x+d)^3*arccosh(c*x)^2/e-2*d^2*arccosh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c-4/9*e^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c^3-d*e*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-2/9*e^2*x^2*arccosh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 1.00, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{de\cosh^{-1}(cx)^2}{2c^2}+\frac{4e^2x}{9c^2}-\frac{4e^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{9c^3}-\frac{d^3\cosh^{-1}(cx)^2}{3e}-\frac{2d^2\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c}-\frac{dex\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d^2*x + (4*e^2*x)/(9*c^2) + (d*e*x^2)/2 + (2*e^2*x^3)/27 - (2*d^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (4*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*ArcCosh[c*x])/(9*c^3) - (d*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/
c - (2*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(9*c) - (d^3*ArcC
osh[c*x]^2)/(3*e) - (d*e*ArcCosh[c*x]^2)/(2*c^2) + ((d + e*x)^3*ArcCosh[c*x
]^2)/(3*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^2 \cosh^{-1}(cx)^2\,dx = \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - \frac{(2c)\int \frac{(d+ex)^3\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3e}$$

$$= \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - \frac{(2c)\int \left(\frac{d^3\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3d^2ex\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3de^2x^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \right.}{3e}$$

$$= \frac{(d+ex)^3\cosh^{-1}(cx)^2}{3e} - \left(2cd^2\right)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx - \frac{\left(2cd^3\right)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}}{3e}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{dex\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{2e^2x^2}{}$$

$$= 2d^2x + \frac{1}{2}dex^2 + \frac{2e^2x^3}{27} - \frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{4e^2\sqrt{-1+cx}\,\sqrt{1+}}{9c^3}$$

$$= 2d^2x + \frac{4e^2x}{9c^2} + \frac{1}{2}dex^2 + \frac{2e^2x^3}{27} - \frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c} - \frac{4e^2\sqrt{-1+}}{}$$

Mathematica [A] time = 0.19, size = 131, normalized size = 0.61

$$\frac{9\cosh^{-1}(cx)^2\left(2c^3x\left(3d^2+3dex+e^2x^2\right)-3cde\right)+cx\left(c^2\left(108d^2+27dex+4e^2x^2\right)+24e^2\right)-6\sqrt{cx-1}\,\sqrt{cx+}}{54c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*ArcCosh[c*x]^2,x]
```

```
[Out] (c*x*(24*e^2 + c^2*(108*d^2 + 27*d*e*x + 4*e^2*x^2)) - 6*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))*ArcCosh[c*x] + 9*(-
3*c*d*e + 2*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcCosh[c*x]^2)/(54*c^3)
```

fricas [A] time = 0.61, size = 163, normalized size = 0.76

$$\frac{4\,c^3e^2x^3+27\,c^3dex^2+9\left(2\,c^3e^2x^3+6\,c^3dex^2+6\,c^3d^2x-3\,cde\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\left(2\,c^2e^2x^2+9\,c^2dex+1\right.}{54\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(4*c^3*e^2*x^3 + 27*c^3*d*e*x^2 + 9*(2*c^3*e^2*x^3 + 6*c^3*d*e*x^2 + 6
*c^3*d^2*x - 3*c*d*e)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 6*(2*c^2*e^2*x^2 + 9
*c^2*d*e*x + 18*c^2*d^2 + 4*e^2)*sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2*x^2 -
 1)) + 12*(9*c^3*d^2 + 2*c*e^2)*x)/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.11, size = 207, normalized size = 0.96

$$18\text{arccosh}\,(cx)^2\,c^3x^3e^2+54\text{arccosh}\,(cx)^2\,c^3x^2de+54\text{arccosh}\,(cx)^2\,c^3x\,d^2-12\,\text{arccosh}\,(cx)\,\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*arccosh(c*x)^2,x)
```

```
[Out] 1/54/c^3*(18*arccosh(c*x)^2*c^3*x^3*e^2+54*arccosh(c*x)^2*c^3*x^2*d*e+54*ar
ccosh(c*x)^2*c^3*x*d^2-12*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2*
e^2-54*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x*d*e-108*arccosh(c*x)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*d^2-27*arccosh(c*x)^2*c*d*e-24*arccosh(c*x)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2+4*e^2*c^3*x^3+27*c^3*x^2*d*e+108*x*c^3*d^2
+24*c*x*e^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(e^2x^3+3\,dex^2+3\,d^2x\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2-\int\frac{2\left(c^3e^2x^5+3\,c^3dex^4-3\,cdex^2-3\,cd^2x+\left(3\,c^3d^2-ce^2\right.\right.}{3\left(c^3x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^
2 - integrate(2/3*(c^3*e^2*x^5 + 3*c^3*d*e*x^4 - 3*c*d*e*x^2 - 3*c*d^2*x +
(3*c^3*d^2 - c*e^2)*x^3 + (c^2*e^2*x^4 + 3*c^2*d*e*x^3 + 3*c^2*d^2*x^2)*sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 +
 (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(c\,x)^2\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2*(d + e*x)^2,x)
```

```
[Out] int(acosh(c*x)^2*(d + e*x)^2, x)
```

sympy [A] time = 1.21, size = 223, normalized size = 1.04

$$\begin{cases} d^2x\operatorname{acosh}^2(cx) + 2d^2x + dex^2\operatorname{acosh}^2(cx) + \frac{dex^2}{2} + \frac{e^2x^3\operatorname{acosh}^2(cx)}{3} + \frac{2e^2x^3}{27} - \frac{2d^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} - \frac{dex\sqrt{c^2x^2-1}\operatorname{aco} \\ -\frac{\pi^2\left(d^2x+dex^2+\frac{e^2x^3}{3}\right)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d**2*x*acosh(c*x)**2 + 2*d**2*x + d*e*x**2*acosh(c*x)**2 + d*e*x
**2/2 + e**2*x**3*acosh(c*x)**2/3 + 2*e**2*x**3/27 - 2*d**2*sqrt(c**2*x**2
- 1)*acosh(c*x)/c - d*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 2*e**2*x**2*sq
rt(c**2*x**2 - 1)*acosh(c*x)/(9*c) - d*e*acosh(c*x)**2/(2*c**2) + 4*e**2*x/
(9*c**2) - 4*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3), Ne(c, 0)), (-pi*
*2*(d**2*x + d*e*x**2 + e**2*x**3/3)/4, True))
```

3.10 $\int (d+ex)\cosh^{-1}(cx)^2\,dx$

Optimal. Leaf size=122

$$-\frac{e\cosh^{-1}(cx)^2}{4c^2}-\frac{d^2\cosh^{-1}(cx)^2}{2e}+\frac{\cosh^{-1}(cx)^2(d+ex)^2}{2e}-\frac{2d\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{c}-\frac{ex\sqrt{cx-1}\,\sqrt{cx+1}\,\cos}{2c}$$

```
[Out] 2*d*x+1/4*e*x^2-1/2*d^2*arccosh(c*x)^2/e-1/4*e*arccosh(c*x)^2/c^2+1/2*(e*x+
d)^2*arccosh(c*x)^2/e-2*d*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2*e*
x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.65, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{e\cosh^{-1}(cx)^2}{4c^2}-\frac{d^2\cosh^{-1}(cx)^2}{2e}+\frac{\cosh^{-1}(cx)^2(d+ex)^2}{2e}-\frac{2d\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)}{c}-\frac{ex\sqrt{cx-1}\,\sqrt{cx+1}\,\cos}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d*x + (e*x^2)/4 - (2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (e*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) - (d^2*ArcCosh[c*x]^2)/(
2*e) - (e*ArcCosh[c*x]^2)/(4*c^2) + ((d + e*x)^2*ArcCosh[c*x]^2)/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
```

```
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\begin{aligned}\int (d+ex)\cosh^{-1}(cx)^2\,dx &= \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - \frac{c\int \frac{(d+ex)^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e}\\
&= \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - \frac{c\int \left(\frac{d^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2dex\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{e^2x^2\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\right)dx}{e}\\
&= \frac{(d+ex)^2\cosh^{-1}(cx)^2}{2e} - (2cd)\int \frac{x\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{\left(cd^2\right)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e}\\
&= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c} - \frac{d^2\cosh^{-}}{2e}\\
&= 2dx + \frac{ex^2}{4} - \frac{2d\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{c} - \frac{ex\sqrt{-1+cx}\sqrt{1+cx}\cosh^{-1}(cx)}{2c}\end{aligned}$$

Mathematica [A] time = 0.08, size = 105, normalized size = 0.86

$$\frac{e\left(2c^2x^2-1\right)\cosh^{-1}(cx)^2}{4c^2}+dx\cosh^{-1}(cx)^2-\frac{2d\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{c}-\frac{ex\sqrt{cx-1}\sqrt{cx+1}\cosh^{-1}(cx)}{2c}+2$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*ArcCosh[c*x]^2,x]
```

```
[Out] 2*d*x + (e*x^2)/4 - (2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/c - (e*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(2*c) + d*x*ArcCosh[c*x]^2 + (
e*(-1 + 2*c^2*x^2)*ArcCosh[c*x]^2)/(4*c^2)
```

fricas [A] time = 0.62, size = 98, normalized size = 0.80

$$\frac{c^2ex^2+8\,c^2dx+\left(2\,c^2ex^2+4\,c^2dx-e\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2-2\,\sqrt{c^2x^2-1}\,(cex+4\,cd)\log\left(cx+\sqrt{c^2x^2-1}\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*e*x^2 + 8*c^2*d*x + (2*c^2*e*x^2 + 4*c^2*d*x - e)*log(c*x + sqrt(c
^2*x^2 - 1))^2 - 2*sqrt(c^2*x^2 - 1)*(c*e*x + 4*c*d)*log(c*x + sqrt(c^2*x^2
 - 1)))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.10, size = 100, normalized size = 0.82

$$\frac{\frac{e\left(2\operatorname{arccosh}(cx)^2c^2x^2-2\operatorname{arccosh}(cx)cx\sqrt{cx-1}\,\sqrt{cx+1}-\operatorname{arccosh}(cx)^2+c^2x^2\right)}{4c}+d\left(\operatorname{arccosh}(cx)^2cx-2\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*arccosh(c*x)^2,x)
```

```
[Out] 1/c*(1/4*e*(2*arccosh(c*x)^2*c^2*x^2-2*arccosh(c*x)*c*x*(c*x-1)^(1/2)*(c*x+
1)^(1/2)-arccosh(c*x)^2+c^2*x^2)/c+d*(arccosh(c*x)^2*c*x-2*arccosh(c*x)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(ex^2+2\,dx\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2-\int\frac{\left(c^3ex^4+2\,c^3dx^3-cex^2-2\,cdx+\left(c^2ex^3+2\,c^2dx^2\right)\sqrt{cx+1}\,\sqrt{cx-}\right.}{c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\,\sqrt{cx-1}\,-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*arccosh(c*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x^2 + 2*d*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - integrate((c
^3*e*x^4 + 2*c^3*d*x^3 - c*e*x^2 - 2*c*d*x + (c^2*e*x^3 + 2*c^2*d*x^2)*sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 +
(c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(c\,x)^2\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2*(d + e*x),x)
```

```
[Out] int(acosh(c*x)^2*(d + e*x), x)
```

sympy [A] time = 0.53, size = 110, normalized size = 0.90

$$\begin{cases} dx\operatorname{acosh}^2(cx)+2dx+\frac{ex^2\operatorname{acosh}^2(cx)}{2}+\frac{ex^2}{4}-\frac{2d\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c}-\frac{ex\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{2c}-\frac{e\operatorname{acosh}^2(cx)}{4c^2} & \text{for } c\neq 0\\ -\frac{\pi^2\left(dx+\frac{ex^2}{2}\right)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*acosh(c*x)**2,x)
```

```
[Out] Piecewise((d*x*acosh(c*x)**2 + 2*d*x + e*x**2*acosh(c*x)**2/2 + e*x**2/4 -
2*d*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(
2*c) - e*acosh(c*x)**2/(4*c**2), Ne(c, 0)), (-pi**2*(d*x + e*x**2/2)/4, Tru
e))
```

3.11 $\int \frac{\cosh^{-1}(cx)^2}{d+ex}\,dx$

Optimal. Leaf size=272

$$\frac{2\cosh^{-1}(cx)\mathrm{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2\cosh^{-1}(cx)\mathrm{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}-\frac{2\mathrm{Li}_3\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}-\frac{2\mathrm{Li}_3\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\mathrm{co}}{}$$

```
[Out] -1/3*arccosh(c*x)^3/e+arccosh(c*x)^2*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+arccosh(c*x)^2*ln(1+e*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e+2*arccosh(c*x)*polylog(2,-e*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+2*arccosh(c*x)*po
lylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e-2
*polylog(3,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/
e-2*polylog(3,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)
))/e
```

Rubi [A] time = 0.43, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5800, 5562, 2190, 2531, 2282, 6589}

$$\frac{2\cosh^{-1}(cx)\mathrm{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2\cosh^{-1}(cx)\mathrm{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}-\frac{2\mathrm{PolyLog}\left(3,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x),x]
```

```
[Out] -ArcCosh[c*x]^3/(3*e) + (ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d - S
qrt[c^2*d^2 - e^2])])/e + (ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d +
 Sqrt[c^2*d^2 - e^2])])/e + (2*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCosh[c*x])
/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (2*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCo
sh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/e - (2*PolyLog[3, -((e*E^ArcCosh[c*
x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e - (2*PolyLog[3, -((e*E^ArcCosh[c*x])/(
c*d + Sqrt[c^2*d^2 - e^2]))])/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{x^2\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$
$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \text{Subst}\left(\int \frac{e^x x^2}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int \frac{}{cd+\sqrt{}}\right.$$
$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$
$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$
$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$
$$= -\frac{\cosh^{-1}(cx)^3}{3e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\cosh^{-1}(cx)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}$$

Mathematica [A] time = 0.16, size = 252, normalized size = 0.93

$$-\frac{-6\cosh^{-1}(cx)\text{Li}_2\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right) - 6\cosh^{-1}(cx)\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right) + 6\text{Li}_3\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right) + 6\text{Li}_3\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x),x]
```

```
[Out] -1/3*(ArcCosh[c*x]^3 - 3*ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d - S
qrt[c^2*d^2 - e^2])] - 3*ArcCosh[c*x]^2*Log[1 + (e*E^ArcCosh[c*x])/(c*d + S
qrt[c^2*d^2 - e^2])] - 6*ArcCosh[c*x]*PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d)
```

```
 + Sqrt[c^2*d^2 - e^2])] - 6*ArcCosh[c*x]*PolyLog[2, -((e*E^ArcCosh[c*x])/(
c*d + Sqrt[c^2*d^2 - e^2]))] + 6*PolyLog[3, (e*E^ArcCosh[c*x])/(-(c*d) + Sq
rt[c^2*d^2 - e^2])] + 6*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
 - e^2]))])/e
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arcosh}(cx)^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)^2/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(cx)^2}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(arccosh(c*x)^2/(e*x + d), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(cx)^2}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d),x)
```

```
[Out] int(arccosh(c*x)^2/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(cx)^2}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(c*x)^2/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(c\,x)^2}{d+e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2/(d + e*x),x)
```

```
[Out] int(acosh(c*x)^2/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(cx)}{d+ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d),x)
```

```
[Out] Integral(acosh(c*x)**2/(d + e*x), x)
```

3.12 $\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^2} dx$

Optimal. Leaf size=259

$$\frac{2c\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2c\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right.}{e\sqrt{c^2d^2-e^2}}$$

```
[Out] -arccosh(c*x)^2/e/(e*x+d)+2*c*arccosh(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)-2*c*arccosh(c*x)*
ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*
d^2-e^2)^(1/2)+2*c*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2
*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)-2*c*polylog(2,-e*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5802, 5832, 3320, 2264, 2190, 2279, 2391}

$$\frac{2c\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2c\cosh^{-1}(cx)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\cosh^{-1}(cx)}{e\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x)^2,x]
```

```
[Out] -(ArcCosh[c*x]^2/(e*(d + e*x))) + (2*c*ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*
x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2]) - (2*c*ArcCosh[c*
x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2
 - e^2]) + (2*c*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])
)])/(e*Sqrt[c^2*d^2 - e^2]) - (2*c*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + S
qrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2 - e^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^2}\,dx = -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(2c)\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}\,dx}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(2c)\,\mathrm{Subst}\left(\int \frac{x}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{e^x x}{e+2cde^x+ee^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{(4c)\,\mathrm{Subst}\left(\int \frac{e^x x}{2cd-2\sqrt{c^2d^2-e^2}+2ee^x}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{c^2d^2-e^2}} - \frac{(4c)\,\mathrm{Subst}\left(\int \frac{}{2cd+2}\right.}{}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right.}{e\sqrt{c^2d^2-e^2}}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right.}{e\sqrt{c^2d^2-e^2}}$$

$$= -\frac{\cosh^{-1}(cx)^2}{e(d+ex)} + \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2c\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right.}{e\sqrt{c^2d^2-e^2}}$$

Mathematica [C] time = 3.43, size = 848, normalized size = 3.27

$$-\frac{c\left(\frac{\cosh^{-1}(cx)^2}{cd+cex}+\frac{2\left(2\cosh^{-1}(cx)\tan^{-1}\left(\frac{(cd+e)\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{e^2-c^2d^2}}\right)-2i\cos^{-1}\left(-\frac{cd}{e}\right)\tan^{-1}\left(\frac{(e-cd)\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{e^2-c^2d^2}}\right)+\left(\cos^{-1}\left(-\frac{cd}{e}\right)+2\left(\tan^{-1}\left(\frac{(cd+e)\coth}{\sqrt{e^2}}\right.\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x)^2,x]
```

```
[Out] -((c*(ArcCosh[c*x]^2/(c*d + c*e*x) + (2*(2*ArcCosh[c*x]*ArcTan[((c*d + e)*C
oth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (2*I)*ArcCos[-((c*d)/e)]*Arc
Tan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcCos[-
((c*d)/e)] + 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e
^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))
*Log[Sqrt[-(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(ArcCosh[c*x]/2)*Sqrt[c*d +
c*e*x])] + (ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])
/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[
-(c^2*d^2) + e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[
2]*Sqrt[e]*Sqrt[c*d + c*e*x])] - (ArcCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) +
e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(c*d - e +
 I*Sqrt[-(c^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqr
t[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTa
n[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d +
e)*(-(c*d) + e + I*Sqrt[-(c^2*d^2) + e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(
c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2,
((c*d - I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[
ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/
2]))] - PolyLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^
2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]
*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[-(c^2*d^2) + e^2]))/e)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arcosh}(cx)^2}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)^2/(e^2*x^2 + 2*d*e*x + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(t_nostep)]Evaluation time:
2.95index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 0.36, size = 374, normalized size = 1.44

$$-\frac{c\operatorname{arccosh}(cx)^2}{e(cxe+cd)}+\frac{2c\operatorname{arccosh}(cx)\ln\left(\frac{-\left(cx+\sqrt{cx-1}\ \sqrt{cx+1}\right)e-cd+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\operatorname{arccosh}(cx)\ln\left(\frac{\left(cx+\sqrt{cx-1}\ \sqrt{cx+1}\right)e+cd}{cd+\sqrt{c^2d^2-e^2}}\right.}{e\sqrt{c^2d^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d)^2,x)
```

```
[Out] -c*arccosh(c*x)^2/e/(c*e*x+c*d)+2*c/e*arccosh(c*x)/(c^2*d^2-e^2)^(1/2)*ln((
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^
2-e^2)^(1/2)))-2*c/e*arccosh(c*x)/(c^2*d^2-e^2)^(1/2)*ln(((c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+2*c/
e/(c^2*d^2-e^2)^(1/2)*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*
d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))-2*c/e/(c^2*d^2-e^2)^(1/2)*dilog
(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^
2-e^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more d
etails)Is e-c*d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(c\,x)^2}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2/(d + e*x)^2,x)
```

```
[Out] int(acosh(c*x)^2/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d)**2,x)
```

```
[Out] Integral(acosh(c*x)**2/(d + e*x)**2, x)
```

3.13 $\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^3} \, dx$

Optimal. Leaf size=352

$$\frac{c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)}-\frac{c\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{c^3d\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{c^3d\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}+\frac{c^3d\cosh^{-1}(cx)\log}{e\left(c^2d^2-\right.}$$

```
[Out] -1/2*arccosh(c*x)^2/e/(e*x+d)^2+c^2*ln(e*x+d)/e/(c^2*d^2-e^2)+c^3*d*arccosh
(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e
/(c^2*d^2-e^2)^(3/2)-c^3*d*arccosh(c*x)*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)+c^3*d*polylog(2,-e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)
^(3/2)-c^3*d*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e
^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)-c*(c*x+1)*arccosh(c*x)*((c*x-1)/(c*x+1))^
(1/2)/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A] time = 0.69, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5802, 5832, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{c^3d\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{c^3d\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}+\frac{c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)}-\frac{c\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{c^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*x]^2/(d + e*x)^3,x]
```

```
[Out] -((c*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*ArcCosh[c*x])/((c^2*d^2 - e^2)*
(d + e*x))) - ArcCosh[c*x]^2/(2*e*(d + e*x)^2) + (c^3*d*ArcCosh[c*x]*Log[1
+ (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*(c^2*d^2 - e^2)^(3/2)
) - (c^3*d*ArcCosh[c*x]*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^
2])])/(e*(c^2*d^2 - e^2)^(3/2)) + (c^2*Log[d + e*x])/(e*(c^2*d^2 - e^2)) +
(c^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*(c
^2*d^2 - e^2)^(3/2)) - (c^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c
^2*d^2 - e^2]))])/(e*(c^2*d^2 - e^2)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(cx)^2}{(d+ex)^3}\,dx = -\frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c\int \frac{\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}\,dx}{e}$$

$$= -\frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\,\text{Subst}\left(\int \frac{x}{(cd+e\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\,\text{Subst}\left(\int \frac{\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{c^2d^2-e^2} +$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{cd+x}\,dx, x, cex\right)}{e\left(c^2d^2-e^2\right)} + \frac{\left(2c^3d\right)\text{Subs}}{}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)} + \frac{\left(2c^3d\right)\text{Subst}\left(\int \frac{e^x}{2cd-2\sqrt{c^2d}}\right.}{\left(c^2d^2\right.}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}} - \frac{c^3d}{}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}} - \frac{c^3d}{}$$

$$= -\frac{c\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)\cosh^{-1}(cx)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\cosh^{-1}(cx)^2}{2e(d+ex)^2} + \frac{c^3d\cosh^{-1}(cx)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}} - \frac{c^3d}{}$$

Mathematica [C] time = 4.53, size = 936, normalized size = 2.66

$$c^2\left(-\frac{\cosh^{-1}(cx)^2}{2e(cd+cex)^2} - \frac{\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\cosh^{-1}(cx)}{(cd-e)(cd+e)(cd+cex)} + \frac{\log\left(\frac{ex}{d}+1\right)}{c^2d^2e-e^3} + \frac{cd\left(2\cosh^{-1}(cx)\tan^{-1}\left(\frac{(cd+e)\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{e^2-c^2d^2}}\right)-\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[c*x]^2/(d + e*x)^3,x]
```

```
[Out] c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/((c*d - e)*(c*d
+ e)*(c*d + c*e*x))) - ArcCosh[c*x]^2/(2*e*(c*d + c*e*x)^2) + Log[1 + (e*x)
/d]/(c^2*d^2*e - e^3) + (c*d*(2*ArcCosh[c*x]*ArcTan[((c*d + e)*Coth[ArcCosh
[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (2*I)*ArcCos[-((c*d)/e)]*ArcTan[((-(c*d
) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcCos[-((c*d)/e)]
+ 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcT
an[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[Sqrt[-
(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(ArcCosh[c*x]/2)*Sqrt[c*d + c*e*x])] +
(ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2
*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2)
+ e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*
Sqrt[c*d + c*e*x])] - (ArcCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) + e)*Tanh[Arc
Cosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(c*d - e + I*Sqrt[-(c
^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2
```

```
) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTan[((-(c*d)
+ e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(-(c*d)
+ e + I*Sqrt[-(c^2*d^2) + e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I
*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*d - I*S
qrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x
]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - Pol
yLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^
2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCo
sh[c*x]/2]))])))/(e*(-(c^2*d^2) + e^2)^(3/2)))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arcosh}(cx)^2}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(arccosh(c*x)^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(t_nostep)]Undef/Unsigned Inf encountered in limitLimit: Max order reached
or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.52, size = 766, normalized size = 2.18

$$-\frac{c^3\operatorname{arccosh}(cx)\,e\sqrt{cx+1}\,\sqrt{cx-1}\,x}{(cxe+cd)^2\left(c^2d^2-e^2\right)}+\frac{c^4\operatorname{arccosh}(cx)\,e\,x^2}{(cxe+cd)^2\left(c^2d^2-e^2\right)}-\frac{c^4\operatorname{arccosh}(cx)^2\,d^2}{2e\,(cxe+cd)^2\left(c^2d^2-e^2\right)}-\frac{c^3\operatorname{arccosh}(cx)\,\sqrt{cx}}{(cxe+cd)^2\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*x)^2/(e*x+d)^3,x)
```

```
[Out] -c^3*arccosh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x+c^4*arccosh(c*x)*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*x^2-1/2*c^4*arccosh(c*x)^
2/e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*d^2-c^3*arccosh(c*x)/(c*e*x+c*d)^2/(c^2*d^2
-e^2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*d+2*c^4*arccosh(c*x)/(c*e*x+c*d)^2/(c^2*d
^2-e^2)*d*x+c^4*arccosh(c*x)/e/(c*e*x+c*d)^2/(c^2*d^2-e^2)*d^2+1/2*c^2*arcc
osh(c*x)^2*e/(c*e*x+c*d)^2/(c^2*d^2-e^2)+c^3/(c^2*d^2-e^2)^(3/2)/e*arccosh(
c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*
d+(c^2*d^2-e^2)^(1/2)))*d-c^3/(c^2*d^2-e^2)^(3/2)/e*arccosh(c*x)*ln(((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(
1/2)))*d+c^3/(c^2*d^2-e^2)^(3/2)/e*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))*d-c^3/(c^2*d^2-e^2
)^(3/2)/e*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2
))/(c*d+(c^2*d^2-e^2)^(1/2)))*d+c^2/(c^2*d^2-e^2)/e*ln(2*c*d*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+e)-2*c^2/(c^2*d^2
-e^2)/e*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*x)^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more d
etails)Is e-c*d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}\left(c\,x\right)^2}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*x)^2/(d + e*x)^3,x)
```

```
[Out] int(acosh(c*x)^2/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{2}{\left(cx \right)}}{\left(d + ex\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*x)**2/(e*x+d)**3,x)
```

```
[Out] Integral(acosh(c*x)**2/(d + e*x)**3, x)
```

3.14 $\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=191

$$\frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{4e}-\frac{b\left(8c^4d^4+24c^2d^2e^2+3e^4\right)\cosh^{-1}(cx)}{32c^4e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(ex\left(26c^2d^2+9e^2\right)+\cdots\right.}{96c^3}$$

```
[Out] -1/32*b*(8*c^4*d^4+24*c^2*d^2*e^2+3*e^4)*arccosh(c*x)/c^4/e+1/4*(e*x+d)^4*(
a+b*arccosh(c*x))/e-7/48*b*d*(e*x+d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/16*b
*(e*x+d)^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/96*b*(4*d*(19*c^2*d^2+16*e^2)+e*
(26*c^2*d^2+9*e^2)*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] time = 0.14, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5802, 100, 153, 147, 52}

$$\frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{4e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(ex\left(26c^2d^2+9e^2\right)+4d\left(19c^2d^2+16e^2\right)\right)}{96c^3}-\frac{b\left(24c^2d^2e^2+8c^4d\cdots\right.}{32\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-7*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(48*c) - (b*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(d + e*x)^3)/(16*c) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d*
(19*c^2*d^2 + 16*e^2) + e*(26*c^2*d^2 + 9*e^2)*x))/(96*c^3) - (b*(8*c^4*d^4
 + 24*c^2*d^2*e^2 + 3*e^4)*ArcCosh[c*x])/(32*c^4*e) + ((d + e*x)^4*(a + b*A
rcCosh[c*x]))/(4*e)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{(d+ex)^4 \left(a+b\cosh^{-1}(cx)\right)}{4e} - \frac{(bc)\int \frac{(d+ex)^4}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{4e}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} + \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{4e} - \frac{b\int \frac{(d+ex)^2(4\cdots}{\sqrt{-1\cdots}}}{\cdots}$$

$$= -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} + \frac{(d+ex)^4\cdots}{\cdots}$$

$$= -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} - \frac{b\sqrt{-1+\cdots}}{\cdots}$$

$$= -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{48c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^3}{16c} - \frac{b\sqrt{-1+\cdots}}{\cdots}$$

Mathematica [A] time = 0.30, size = 193, normalized size = 1.01

$$\frac{24ac^4x\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)+24bc^4x\cosh^{-1}(cx)\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)-9be\left(8c^2d^2+e^2\right)\log\cdots}{96c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (24*a*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - b*c*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x
^2 + 6*e^3*x^3)) + 24*b*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*A
rcCosh[c*x] - 9*b*e*(8*c^2*d^2 + e^2)*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x
]])/(96*c^4)
```

fricas [A] time = 0.73, size = 213, normalized size = 1.12

$$\frac{24\,ac^4e^3x^4+96\,ac^4de^2x^3+144\,ac^4d^2ex^2+96\,ac^4d^3x+3\left(8\,bc^4e^3x^4+32\,bc^4de^2x^3+48\,bc^4d^2ex^2+32\,bc^4d^3x-2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/96*(24*a*c^4*e^3*x^4 + 96*a*c^4*d*e^2*x^3 + 144*a*c^4*d^2*e*x^2 + 96*a*c^
4*d^3*x + 3*(8*b*c^4*e^3*x^4 + 32*b*c^4*d*e^2*x^3 + 48*b*c^4*d^2*e*x^2 + 32
```

```
*b*c^4*d^3*x - 24*b*c^2*d^2*e - 3*b*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (6*
b*c^3*e^3*x^3 + 32*b*c^3*d*e^2*x^2 + 96*b*c^3*d^3 + 64*b*c*d*e^2 + 9*(8*b*c
^3*d^2*e + b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 408, normalized size = 2.14

$$\frac{a\,x^{4}e^{3}}{4}+ad\,e^{2}x^{3}+\frac{3a\,d^{2}e\,x^{2}}{2}+ax\,d^{3}+\frac{a\,d^{4}}{4e}+\frac{b\,\mathrm{arccosh}\,(cx)\,x^{4}e^{3}}{4}+b\,\mathrm{arccosh}\,(cx)\,d\,e^{2}x^{3}+\frac{3b\,\mathrm{arccosh}\,(cx)\,d^{2}e\,x^{2}}{2}+b\,\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/4*a*x^4*e^3+a*d*e^2*x^3+3/2*a*d^2*e*x^2+a*x*d^3+1/4*a/e*d^4+1/4*b*arccosh
(c*x)*x^4*e^3+b*arccosh(c*x)*d*e^2*x^3+3/2*b*arccosh(c*x)*d^2*e*x^2+b*arcco
sh(c*x)*x*d^3+1/4*b/e*arccosh(c*x)*d^4-1/4*b/e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2*x^2-1)^(1/2))-1/16/c*b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*e^3*x^3-1/3*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*d*e^2-3/4*b*d^2*e
*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-b*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/4/c^
2*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^
(1/2))-3/32/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x-2/3*b/c^3*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*d*e^2-3/32/c^4*b*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)
^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.39, size = 265, normalized size = 1.39

$$\frac{1}{4}\,ae^{3}x^{4}+ade^{2}x^{3}+\frac{3}{2}\,ad^{2}ex^{2}+\frac{3}{4}\left(2\,x^{2}\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^{2}x^{2}-1}\,x}{c^{2}}+\frac{\log\left(2\,c^{2}x+2\,\sqrt{c^{2}x^{2}-1}\,c\right)}{c^{3}}\right)\right)bd^{2}e+\frac{1}{3}\left(3\,x^{3}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/4*(2*x^2*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^
2*e + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x
^2 - 1)/c^4))*b*d*e^2 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3
/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c
^5)*c)*b*e^3 + a*d^3*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}\,(c\,x))\,(d+e\,x)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x)^3, x)
```

sympy [A] time = 1.53, size = 323, normalized size = 1.69

$$\begin{cases} ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{acosh}(cx) + \frac{3bd^2ex^2\operatorname{acosh}(cx)}{2} + bde^2x^3\operatorname{acosh}(cx) + \frac{be^3x^4\operatorname{acosh}(cx)}{4} - \frac{bd^3\sqrt{c^2x}}{c} \\ \left(a + \frac{i\pi b}{2}\right)\left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b
*d**3*x*acosh(c*x) + 3*b*d**2*e*x**2*acosh(c*x)/2 + b*d*e**2*x**3*acosh(c*x
) + b*e**3*x**4*acosh(c*x)/4 - b*d**3*sqrt(c**2*x**2 - 1)/c - 3*b*d**2*e*x*
sqrt(c**2*x**2 - 1)/(4*c) - b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(3*c) - b*e**
3*x**3*sqrt(c**2*x**2 - 1)/(16*c) - 3*b*d**2*e*acosh(c*x)/(4*c**2) - 2*b*d*
e**2*sqrt(c**2*x**2 - 1)/(3*c**3) - 3*b*e**3*x*sqrt(c**2*x**2 - 1)/(32*c**3
) - 3*b*e**3*acosh(c*x)/(32*c**4), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x + 3*d
**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4), True))
```

3.15 $\int (d+ex)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=132

$$\frac{(d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{bd\left(\frac{3e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{6e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3}-\frac{b\sqrt{cx\cdot}}{}$$

```
[Out] -1/6*b*d*(2*d^2+3*e^2/c^2)*arccosh(c*x)/e+1/3*(e*x+d)^3*(a+b*arccosh(c*x))/
e-1/9*b*(e*x+d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/18*b*(5*c^2*d*e*x+16*c^2*
d^2+4*e^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{5802, 100, 147, 52\}$

$$\frac{(d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(4\left(4c^2d^2+e^2\right)+5c^2dex\right)}{18c^3}-\frac{bd\left(\frac{3e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{6e}-\frac{b\sqrt{cx\cdot}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)^2)/(9*c) - (b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*(4*c^2*d^2 + e^2) + 5*c^2*d*e*x))/(18*c^3) - (b*d*(2*d^2 + (3
*e^2)/c^2)*ArcCosh[c*x])/(6*e) + ((d + e*x)^3*(a + b*ArcCosh[c*x]))/(3*e)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
```

```
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{(d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)}{3e} - \frac{(bc)\int \frac{(d+ex)^3}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{3e}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} + \frac{(d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)}{3e} - \frac{b\int \frac{(d+ex)(3c^{\cdots}}{\sqrt{-1}\cdots}}{\cdots}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(4\left(4c^2d^2+e^2\right)+5c^2de\cdots\right)}{18c^3}$$

$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}{9c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(4\left(4c^2d^2+e^2\right)+5c^2de\cdots\right)}{18c^3}$$

Mathematica [A] time = 0.21, size = 142, normalized size = 1.08

$$ad^2x+adex^2+\frac{1}{3}ae^2x^3-\frac{bde\log\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)}{2c^2}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(18d^2+9dex+2e^2x^2\right)+4e^2\right)}{18c^3}+\frac{1}{3}bx\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*d^2*x + a*d*e*x^2 + (a*e^2*x^3)/3 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*e^
2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)))/(18*c^3) + (b*x*(3*d^2 + 3*d*e*x +
 e^2*x^2)*ArcCosh[c*x])/3 - (b*d*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])
/(2*c^2)
```

fricas [A] time = 0.62, size = 147, normalized size = 1.11

$$\frac{6\,ac^3e^2x^3+18\,ac^3dex^2+18\,ac^3d^2x+3\left(2\,bc^3e^2x^3+6\,bc^3dex^2+6\,bc^3d^2x-3\,bcde\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(2\,bc^2\cdots\right.}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*e^2*x^3 + 18*a*c^3*d*e*x^2 + 18*a*c^3*d^2*x + 3*(2*b*c^3*e^2*
x^3 + 6*b*c^3*d*e*x^2 + 6*b*c^3*d^2*x - 3*b*c*d*e)*log(c*x + sqrt(c^2*x^2 -
 1)) - (2*b*c^2*e^2*x^2 + 9*b*c^2*d*e*x + 18*b*c^2*d^2 + 4*b*e^2)*sqrt(c^2*
x^2 - 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.01, size = 274, normalized size = 2.08

$$\frac{a\,x^3e^2}{3}+ade\,x^2+ax\,d^2+\frac{a\,d^3}{3e}+\frac{b\,\mathrm{arccosh}\,(cx)\,x^3e^2}{3}+b\,\mathrm{arccosh}\,(cx)\,de\,x^2+b\,\mathrm{arccosh}\,(cx)\,x\,d^2+\frac{b\,\mathrm{arccosh}\,(cx)\,d^3}{3e}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/3*a*x^3*e^2+a*d*e*x^2+a*x*d^2+1/3*a/e*d^3+1/3*b*arccosh(c*x)*x^3*e^2+b*ar
ccosh(c*x)*d*e*x^2+b*arccosh(c*x)*x*d^2+1/3*b/e*arccosh(c*x)*d^3-1/3*b/e*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-1
/9*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*e^2-1/2*b*d*e*x*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c-b*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2/c^2*b*e*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))-2/9*b/c^3*(c*x-1)^
(1/2)*(c*x+1)^(1/2)*e^2
```

maxima [A] time = 0.44, size = 167, normalized size = 1.27

$$\frac{1}{3}\,ae^2x^3+adex^2+\frac{1}{2}\left(2\,x^2\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bde+\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 + a*d*e*x^2 + 1/2*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*
x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d*e + 1/9*(3*x^3*arcco
sh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*e^2 +
a*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}\,(c\,x))\,\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x)^2,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x)^2, x)
```

sympy [A] time = 0.68, size = 197, normalized size = 1.49

$$\begin{cases} ad^2x+adex^2+\frac{ae^2x^3}{3}+bd^2x\,\mathrm{acosh}\,(cx)+bdex^2\,\mathrm{acosh}\,(cx)+\frac{be^2x^3\,\mathrm{acosh}\,(cx)}{3}-\frac{bd^2\sqrt{c^2x^2-1}}{c}-\frac{bdex\sqrt{c^2x^2-1}}{2c}-\frac{be^2x^2\sqrt{}}{9} \\ \left(a+\frac{i\pi b}{2}\right)\left(d^2x+dex^2+\frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*acosh(c*x) + b*
d*e*x**2*acosh(c*x) + b*e**2*x**3*acosh(c*x)/3 - b*d**2*sqrt(c**2*x**2 - 1)
/c - b*d*e*x*sqrt(c**2*x**2 - 1)/(2*c) - b*e**2*x**2*sqrt(c**2*x**2 - 1)/(9
*c) - b*d*e*acosh(c*x)/(2*c**2) - 2*b*e**2*sqrt(c**2*x**2 - 1)/(9*c**3), Ne
(c, 0)), ((a + I*pi*b/2)*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

3.16 $\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=106

$$\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}-\frac{b\left(\frac{e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{4e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\,(d+ex)}{4c}-\frac{3bd\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

```
[Out] -1/4*b*(2*d^2+e^2/c^2)*arccosh(c*x)/e+1/2*(e*x+d)^2*(a+b*arccosh(c*x))/e-3/
4*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*(e*x+d)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c
```

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5802, 90, 80, 52}

$$\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e}-\frac{b\left(\frac{e^2}{c^2}+2d^2\right)\cosh^{-1}(cx)}{4e}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\,(d+ex)}{4c}-\frac{3bd\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(d + e*x))/(4*c) - (b*(2*d^2 + e^2/c^2)*ArcCosh[c*x])/(4*e) + ((d + e*x)
^2*(a + b*ArcCosh[c*x]))/(2*e)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{(d+ex)^2}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{2e}$$
$$= -\frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} + \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{2e} - \frac{b\int \frac{2c^2d^2+e^2}{\sqrt{-1+cx}}}{4c}$$
$$= -\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} + \frac{(d+ex)^2\left(a+b\right.}{2e}$$
$$= -\frac{3bd\sqrt{-1+cx}\sqrt{1+cx}}{4c} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}(d+ex)}{4c} - \frac{b\left(2d^2+\frac{e^2}{c^2}\right)\cosh}{4e}$$

Mathematica [A] time = 0.10, size = 117, normalized size = 1.10

$$adx+\frac{1}{2}aex^2-\frac{be\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{2c^2}-\frac{bd\sqrt{cx-1}\sqrt{cx+1}}{c}+bdx\cosh^{-1}(cx)+\frac{1}{2}bex^2\cosh^{-1}(cx)-\frac{bex\sqrt{cx-1}\sqrt{cx+1}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 - (b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (b*e*x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])/(4*c) + b*d*x*ArcCosh[c*x] + (b*e*x^2*ArcCosh[c*x])/
2 - (b*e*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(2*c^2)
```

fricas [A] time = 0.64, size = 88, normalized size = 0.83

$$\frac{2\,ac^2ex^2+4\,ac^2dx+\left(2\,bc^2ex^2+4\,bc^2dx-be\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-(bcex+4\,bcd)\sqrt{c^2x^2-1}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*e*x^2 + 4*a*c^2*d*x + (2*b*c^2*e*x^2 + 4*b*c^2*d*x - b*e)*log(
c*x + sqrt(c^2*x^2 - 1)) - (b*c*e*x + 4*b*c*d)*sqrt(c^2*x^2 - 1))/c^2
```

giac [A] time = 0.48, size = 126, normalized size = 1.19

$$\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{\sqrt{c^2x^2-1}}{c}\right)bd+adx+\frac{1}{4}\left(2\,ax^2+\left(2\,x^2\log\left(cx+\sqrt{c^2x^2-1}\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}-\frac{\log(}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] (x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*b*d + a*d*x + 1/4*(2
*a*x^2 + (2*x^2*log(c*x + sqrt(c^2*x^2 - 1)) - c*(sqrt(c^2*x^2 - 1)*x/c^2 -
 log(abs(-x*abs(c) + sqrt(c^2*x^2 - 1)))/(c^2*abs(c))))*b)*e
```

maple [A] time = 0.01, size = 123, normalized size = 1.16

$$\frac{a\,x^2e}{2}+adx+\frac{b\,\mathrm{arccosh}\,(cx)\,x^2e}{2}+b\,\mathrm{arccosh}\,(cx)\,xd-\frac{bex\sqrt{cx-1}\,\sqrt{cx+1}}{4c}-\frac{bd\sqrt{cx-1}\,\sqrt{cx+1}}{c}-\frac{b\sqrt{cx-1}\,\sqrt{cx}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/2*a*x^2*e+a*d*x+1/2*b*arccosh(c*x)*x^2*e+b*arccosh(c*x)*x*d-1/4*b*e*x*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c-b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/c^2*b*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.44, size = 99, normalized size = 0.93

$$\frac{1}{2}aex^2+\frac{1}{4}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)be+adx+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*
c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*e + a*d*x + (c*x*arccosh(c*x) - sqrt
(c^2*x^2 - 1))*b*d/c
```

mupad [B] time = 1.03, size = 83, normalized size = 0.78

$$\frac{a\,x\,(2\,d+e\,x)}{2}+b\,d\,x\operatorname{acosh}(c\,x)+b\,e\,x\operatorname{acosh}(c\,x)\left(\frac{x}{2}-\frac{1}{4\,c^2\,x}\right)-\frac{b\,d\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c}-\frac{b\,e\,x\,\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x),x)
```

```
[Out] (a*x*(2*d + e*x))/2 + b*d*x*acosh(c*x) + b*e*x*acosh(c*x)*(x/2 - 1/(4*c^2*x
)) - (b*d*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/c - (b*e*x*(c*x - 1)^(1/2)*(c*x
+ 1)^(1/2))/(4*c)
```

sympy [A] time = 0.34, size = 105, normalized size = 0.99

$$\begin{cases} adx+\frac{aex^2}{2}+bdx\operatorname{acosh}(cx)+\frac{bex^2\operatorname{acosh}(cx)}{2}-\frac{bd\sqrt{c^2x^2-1}}{c}-\frac{bex\sqrt{c^2x^2-1}}{4c}-\frac{be\operatorname{acosh}(cx)}{4c^2} & \text{for } c\neq 0 \\ \left(a+\frac{i\pi b}{2}\right)\left(dx+\frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**2/2 + b*d*x*acosh(c*x) + b*e*x**2*acosh(c*x)/2 -
b*d*sqrt(c**2*x**2 - 1)/c - b*e*x*sqrt(c**2*x**2 - 1)/(4*c) - b*e*acosh(c*x
)/(4*c**2), Ne(c, 0)), ((a + I*pi*b/2)*(d*x + e*x**2/2), True))
```

3.17 $\int \frac{a+b\cosh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=195

$$\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{e e^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{e e^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{b\mathrm{Li}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^2/b/e+(a+b*arccosh(c*x))*ln(1+e*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+(a+b*arccosh(c*x))*ln(1+e*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e+b*polylog(2,-e*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+b*polylog(2,-e
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e
```

Rubi [A] time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{e e^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{b\text{PolyLog}\left(2,-\frac{e e^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{e e^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e}+\frac{\left(a+bc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x),x]
```

```
[Out] -(a + b*ArcCosh[c*x])^2/(2*b*e) + ((a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCos
h[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/e + ((a + b*ArcCosh[c*x])*Log[1 + (e*
E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/e + (b*PolyLog[2, -((e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (b*PolyLog[2, -((e*E^ArcCosh[c
*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \text{Subst}\left(\int \frac{e^x(a+bx)}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}\left(\int\right.$$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)}{}$$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)}{}$$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(cx)\right)}{}$$

Mathematica [A] time = 0.14, size = 183, normalized size = 0.94

$$\frac{-\left(a+b\cosh^{-1}(cx)\right)\left(a-2b\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)-2b\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)+b\cosh^{-1}(cx)\right)+2b^2\text{Li}_2\left(\frac{ee^{\cosh^{-1}(cx}}{\sqrt{c^2d^2-e^2}-}\right.}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 + (e*E^ArcCosh[c*x]
)/(c*d - Sqrt[c^2*d^2 - e^2])] - 2*b*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt
[c^2*d^2 - e^2])])) + 2*b^2*PolyLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^
2*d^2 - e^2])] + 2*b^2*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
- e^2]))])/(2*b*e)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="giac")
```

[Out] integrate((b*arccosh(c*x) + a)/(e*x + d), x)

maple [A] time = 0.04, size = 314, normalized size = 1.61

$$\frac{a\ln(cxe+cd)}{e}-\frac{b\operatorname{arccosh}(cx)^2}{2e}+\frac{b\operatorname{arccosh}(cx)\ln\left(\frac{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)e+cd+\sqrt{c^2d^2-e^2}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{b\operatorname{arccosh}(cx)\ln\left(\frac{-\left(cx+\sqrt{cx}\right.}{}\right.}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))/(e*x+d),x)

[Out] a*ln(c*e*x+c*d)/e-1/2*b*arccosh(c*x)^2/e+b/e*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))+b/e*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))+b/e*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))+b/e*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex+d}\,dx+\frac{a\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/(e*x+d),x, algorithm="maxima")

[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x) + a*log(e*x + d)/e

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\operatorname{acosh}(c\,x)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c*x))/(d + e*x),x)

[Out] int((a + b*acosh(c*x))/(d + e*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{acosh}(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acosh(c*x))/(e*x+d),x)

[Out] Integral((a + b*acosh(c*x))/(d + e*x), x)

3.18 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=88

$$\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\,\sqrt{cd+e}}-\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}$$

```
[Out] (-a-b*arccosh(c*x))/e/(e*x+d)+2*b*c*arctanh((c*d+e)^(1/2)*(c*x+1)^(1/2)/(c*
d-e)^(1/2)/(c*x-1)^(1/2))/e/(c*d-e)^(1/2)/(c*d+e)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5802, 93, 208}

$$\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{e\sqrt{cd-e}\,\sqrt{cd+e}}-\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(e*(d + e*x))) + (2*b*c*ArcTanh[(Sqrt[c*d + e]*Sqrt[
1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(Sqrt[c*d - e]*e*Sqrt[c*d + e])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^2}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}\,dx}{e}\\
&= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{(2bc)\,\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right)}{e}\\
&= -\frac{a+b\cosh^{-1}(cx)}{e(d+ex)}+\frac{2bc\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{\sqrt{cd-e}\,e\sqrt{cd+e}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 121, normalized size = 1.38

$$-\frac{\frac{a}{d+ex}-\frac{bc\log(d+ex)}{\sqrt{c^2d^2-e^2}}+\frac{bc\log\left(-\sqrt{cx-1}\sqrt{cx+1}\sqrt{c^2d^2-e^2}+c^2dx+e\right)}{\sqrt{c^2d^2-e^2}}+\frac{b\cosh^{-1}(cx)}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^2,x]
```

```
[Out] -((a/(d + e*x) + (b*ArcCosh[c*x])/(d + e*x) - (b*c*Log[d + e*x])/Sqrt[c^2*d
^2 - e^2] + (b*c*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]])/Sqrt[c^2*d^2 - e^2])/e)
```

fricas [B] time = 0.82, size = 507, normalized size = 5.76

$$\left[-\frac{ac^2d^3-ade^2-\left(bc^2d^2e-be^3\right)x\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bcdex+bcd^2\right)\sqrt{c^2d^2-e^2}\log\left(\frac{c^3d^2x+cde+\sqrt{c^2d^2-e^2}\left(c^2dx\right.}{\ldots}\right.}{c^2d^4e-d^2e^3+\left(c^2d^3e^2-\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*e^2 - (b*c^2*d^2*e - b*e^3)*x*log(c*x + sqrt(c^2*x^2 - 1
)) - (b*c*d*e*x + b*c*d^2)*sqrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*e + sqr
t(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*
sqrt(c^2*x^2 - 1))/(e*x + d)) - (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3
)*x)*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d
*e^4)*x), -(a*c^2*d^3 - a*d*e^2 - (b*c^2*d^2*e - b*e^3)*x*log(c*x + sqrt(c^
2*x^2 - 1)) + 2*(b*c*d*e*x + b*c*d^2)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqrt(-c
^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^
2*d^2 - e^2)) - (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*log(-c*x +
sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

giac [B] time = 1.08, size = 231, normalized size = 2.62

$$\left(\left(\frac{ce^3\log\left(\left|c^2d-\sqrt{c^2d^2-e^2}|c|\right|\right)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{\sqrt{c^2d^2-e^2}}-\frac{ce^3\log\left(\left|c^2d-\sqrt{c^2d^2-e^2}\left(\sqrt{c^2-\frac{2c^2d}{xe+d}+\frac{c^2d^2}{(xe+d)^2}-\frac{e^2}{(xe+d)^2}}+\frac{\sqrt{c^2\ldots}}{\ldots}\right.\right.\right.}{\sqrt{c^2d^2-e^2}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] ((c*e^3*log(abs(c^2*d - sqrt(c^2*d^2 - e^2)*abs(c)))*sgn(1/(x*e + d))/sqrt(
c^2*d^2 - e^2) - c*e^3*log(abs(c^2*d - sqrt(c^2*d^2 - e^2)*(sqrt(c^2 - 2*c^
2*d/(x*e + d) + c^2*d^2/(x*e + d)^2 - e^2/(x*e + d)^2) + sqrt(c^2*d^2*e^2 -
 e^4)*e^(-1)/(x*e + d))))/(sqrt(c^2*d^2 - e^2)*sgn(1/(x*e + d))))*e^(-4) -
e^(-1)*log(c*x + sqrt(c^2*x^2 - 1))/(x*e + d))*b - a*e^(-1)/(x*e + d)
```

maple [A] time = 0.01, size = 145, normalized size = 1.65

$$-\frac{ca}{(cxe+cd)\,e}-\frac{cb\,\mathrm{arccosh}\,(cx)}{(cxe+cd)\,e}-\frac{cb\sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(-\frac{2\left(c^2dx-\sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e+e\right)}{cxe+cd}\right)}{e^2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))/(e*x+d)^2,x)

[Out] -c*a/(c*e*x+c*d)/e-c*b/(c*e*x+c*d)/e*arccosh(c*x)-c*b/e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*x^2-1)^(1/2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more details)Is e-c*d positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c*x))/(d + e*x)^2,x)

[Out] int((a + b*acosh(c*x))/(d + e*x)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(cx \right)}}{\left(d + ex\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acosh(c*x))/(e*x+d)**2,x)

[Out] Integral((a + b*acosh(c*x))/(d + e*x)**2, x)

3.19 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^3}\,dx$

Optimal. Leaf size=138

$$-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/e/(e*x+d)^2+b*c^3*d*arctanh((c*d+e)^(1/2)*(c*x+1)^(
1/2)/(c*d-e)^(1/2)/(c*x-1)^(1/2))/(c*d-e)^(3/2)/e/(c*d+e)^(3/2)-1/2*b*c*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5802, 96, 93, 208}

$$-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{2\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cx+1}\sqrt{cd+e}}{\sqrt{cx-1}\sqrt{cd-e}}\right)}{e(cd-e)^{3/2}(cd+e)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^3,x]
```

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c^2*d^2 - e^2)*(d + e*x)) - (a + b*
ArcCosh[c*x])/(2*e*(d + e*x)^2) + (b*c^3*d*ArcTanh[(Sqrt[c*d + e]*Sqrt[1 +
c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/((c*d - e)^(3/2)*e*(c*d + e)^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}\,dx}{2e}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(bc^3d\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}\,dx}{2e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{\left(bc^3d\right)\text{Subst}\left(\int \frac{1}{cd-e-(cd+e)x^2}\,dx,x,\frac{\sqrt{1+cx}}{\sqrt{-1+cx}}\right.}{e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{2e(d+ex)^2}+\frac{bc^3d\tanh^{-1}\left(\frac{\sqrt{cd+e}\,\sqrt{1+cx}}{\sqrt{cd-e}\,\sqrt{-1+cx}}\right)}{(cd-e)^{3/2}e(cd+e)^{3/2}}$$

Mathematica [A] time = 0.39, size = 184, normalized size = 1.33

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\log(d+ex)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{bc^3d\log\left(-\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{c^2d^2-e^2}+c^2dx+e\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d^2 - e^2)
*(d + e*x)) - (b*ArcCosh[c*x])/(e*(d + e*x)^2) + (b*c^3*d*Log[d + e*x])/(e*
(c^2*d^2 - e^2)^(3/2)) - (b*c^3*d*Log[e + c^2*d*x - Sqrt[c^2*d^2 - e^2]*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]])/(e*(c^2*d^2 - e^2)^(3/2)))/2
```

fricas [B] time = 0.91, size = 1132, normalized size = 8.20

$$\left[-\frac{(a+b)c^4d^6-(2a+b)c^2d^4e^2+ad^2e^4+\left(bc^4d^4e^2-bc^2d^2e^4\right)x^2+\left(bc^3d^3e^2x^2+2\,bc^3d^4ex+bc^3d^5\right)\sqrt{c^2d^2-e^2}\,\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*((a + b)*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + a*d^2*e^4 + (b*c^4*d^4*e^2
 - b*c^2*d^2*e^4)*x^2 + (b*c^3*d^3*e^2*x^2 + 2*b*c^3*d^4*e*x + b*c^3*d^5)*s
qrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*e - sqrt(c^2*d^2 - e^2)*(c^2*d*x +
e) + (c^2*d^2 - sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)
) + 2*(b*c^4*d^5*e - b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 +
 b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x + sqrt
(c^2*x^2 - 1)) - (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2
- 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5
)*x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^3*d^5*e - b*c*d^3*e^3 + (b*c^3*d^
4*e^2 - b*c*d^2*e^4)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4
*e^5 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2
*d^5*e^4 + d^3*e^6)*x), -1/2*((a + b)*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + a*d
^2*e^4 + (b*c^4*d^4*e^2 - b*c^2*d^2*e^4)*x^2 + 2*(b*c^3*d^3*e^2*x^2 + 2*b*c
^3*d^4*e*x + b*c^3*d^5)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqrt(-c^2*d^2 + e^2)*
sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2))
```

```
+ 2*(b*c^4*d^5*e - b*c^2*d^3*e^3)*x - ((b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b
*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x + sqrt(c
^2*x^2 - 1)) - (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 -
2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*
x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^3*d^5*e - b*c*d^3*e^3 + (b*c^3*d^4*
e^2 - b*c*d^2*e^4)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e
^5 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d
^5*e^4 + d^3*e^6)*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x + d)^3, x)
```

maple [B] time = 0.01, size = 361, normalized size = 2.62

$$-\frac{c^2 a}{2(cxe+cd)^2 e} - \frac{c^2 b \operatorname{arccosh}(cx)}{2(cxe+cd)^2 e} - \frac{c^4 b\sqrt{cx+1}\,\sqrt{cx-1}\,\ln\left(-\frac{2\left(c^2 dx - \sqrt{c^2x^2-1}\,\sqrt{\frac{c^2d^2-e^2}{e^2}}\,e + e\right)}{cxe+cd}\right) xd}{2e\sqrt{c^2x^2-1}\,(cd+e)(cd-e)(cxe+cd)\sqrt{\frac{c^2d^2-e^2}{e^2}}} - \frac{c^4 b\sqrt{cx+1}\,\sqrt{cx \cdots}}{2e^2\sqrt{c^2x^2-1 \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arccosh(c*x)-1/2*c^4*b
/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c*e*x+c*d
)/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)
/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x*d-1/2*c^4*b/e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)
/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*ln
(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*
d^2-1/2*c^2*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c*e*x+c*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more d
etails)Is e-c*d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}(c\,x)}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x)^3, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x)**3, x)
```

3.20 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^4}\,dx$

Optimal. Leaf size=202

$$-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc^3d\sqrt{cx-1}\,\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{bc^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/e/(e*x+d)^3+1/3*b*c^3*(2*c^2*d^2+e^2)*arctanh((c*d+
e)^(1/2)*(c*x+1)^(1/2)/(c*d-e)^(1/2)/(c*x-1)^(1/2))/(c*d-e)^(5/2)/e/(c*d+e)
^(5/2)-1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)/(e*x+d)^2-1/2*b*c^
3*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*d^2-e^2)^2/(e*x+d)
```

Rubi [A] time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5802, 103, 151, 12, 93, 208}

$$-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}-\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6\left(c^2d^2-e^2\right)(d+ex)^2}+\frac{bc^3\left(2c^2d^2+e^2\right)\tanh^{-1}\left(\frac{\sqrt{cx+1}\,\sqrt{cd+e}}{\sqrt{cx-1}\,\sqrt{cd-e}}\right)}{3e(cd-e)^{5/2}(cd+e)^{5/2}}-\frac{bc^3d\sqrt{cx-1}\,\sqrt{cx+1}}{2(cd-e)^2(cd+e)^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x)^4,x]
```

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*(c^2*d^2 - e^2)*(d + e*x)^2) - (b*c^
3*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*(c*d - e)^2*(c*d + e)^2*(d + e*x)) - (
a + b*ArcCosh[c*x])/(3*e*(d + e*x)^3) + (b*c^3*(2*c^2*d^2 + e^2)*ArcTanh[(S
qrt[c*d + e]*Sqrt[1 + c*x])/(Sqrt[c*d - e]*Sqrt[-1 + c*x])])/(3*(c*d - e)^(
5/2)*e*(c*d + e)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(d+ex)^4}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^3}\,dx}{3e}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}-\frac{(bc)\int \frac{-2c^2d+c^2ex}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)^2}\,dx}{6e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{(bc)\int \frac{c^2(2c}{\sqrt{-1+cx}\,\sqrt{}}}{6e\left(c^2d^2\right.}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{\left(bc^3\left(2c^2d^2+\right.\right.}{6}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{\left(bc^3\left(2c^2d^2+\right.\right.}{}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6\left(c^2d^2-e^2\right)(d+ex)^2}-\frac{bc^3d\sqrt{-1+cx}\,\sqrt{1+cx}}{2\left(c^2d^2-e^2\right)^2(d+ex)}-\frac{a+b\cosh^{-1}(cx)}{3e(d+ex)^3}+\frac{bc^3\left(2c^2d^2+e^2\right.}{3(cd-}$$

Mathematica [C] time = 0.98, size = 259, normalized size = 1.28

$$-\frac{\frac{2a+\frac{bce\sqrt{cx-1}\,\sqrt{cx+1}\,(d+ex)\left(c^2d(4d+3ex)-e^2\right)}{\left(e^2-c^2d^2\right)^2}}{(d+ex)^3}+\frac{ibc^3\left(2c^2d^2+e^2\right)\log\left(\frac{12e^2(e-cd)^2(cd+e)^2\left(\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{e^2-c^2d^2}-ic^2dx-ie\right)}{bc^3\sqrt{e^2-c^2d^2}\left(2c^2d^2+e^2\right)(d+ex)}\right)}{(e-cd)^2(cd+e)^2\sqrt{e^2-c^2d^2}}+\frac{2b\cosh^{-1}(cx)}{(d+ex)^3}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x)^4,x]
```

```
[Out] -1/6*((2*a + (b*c*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x)*(-e^2 + c^2*d*(4
*d + 3*e*x)))/(-(c^2*d^2) + e^2)^2)/(d + e*x)^3 + (2*b*ArcCosh[c*x])/(d + e
*x)^3 + (I*b*c^3*(2*c^2*d^2 + e^2)*Log[(12*e^2*(-(c*d) + e)^2*(c*d + e)^2*(
```

```
(-I)*e - I*c^2*d*x + Sqrt[-(c^2*d^2) + e^2]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/
(b*c^3*Sqrt[-(c^2*d^2) + e^2]*(2*c^2*d^2 + e^2)*(d + e*x))])/((-(c*d) + e)^
2*(c*d + e)^2*Sqrt[-(c^2*d^2) + e^2]))/e
```

fricas [B] time = 1.55, size = 1963, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [-1/6*((2*a + 3*b)*c^6*d^9 - 3*(2*a + b)*c^4*d^7*e^2 + 6*a*c^2*d^5*e^4 - 2*
a*d^3*e^6 + 3*(b*c^6*d^6*e^3 - b*c^4*d^4*e^5)*x^3 + 9*(b*c^6*d^7*e^2 - b*c^
4*d^5*e^4)*x^2 - (2*b*c^5*d^8 + b*c^3*d^6*e^2 + (2*b*c^5*d^5*e^3 + b*c^3*d^
3*e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 + b*c^3*d^4*e^4)*x^2 + 3*(2*b*c^5*d^7*e + b
*c^3*d^5*e^3)*x)*sqrt(c^2*d^2 - e^2)*log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2
- e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x
^2 - 1))/(e*x + d)) + 9*(b*c^6*d^8*e - b*c^4*d^6*e^3)*x - 2*((b*c^6*d^6*e^3
 - 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 - b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 - 3*b*
c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 - b*d*e^8)*x^2 + 3*(b*c^6*d^8*e - 3*b*c^4*d^6
*e^3 + 3*b*c^2*d^4*e^5 - b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*
c^6*d^9 - 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 - b*d^3*e^6 + (b*c^6*d^6*e^3 -
3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 - b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 - 3*b*c^4
*d^5*e^4 + 3*b*c^2*d^3*e^6 - b*d*e^8)*x^2 + 3*(b*c^6*d^8*e - 3*b*c^4*d^6*e^
3 + 3*b*c^2*d^4*e^5 - b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (4*b*c^
5*d^8*e - 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 - b*c^3*d^4*e^5)
*x^2 + (7*b*c^5*d^7*e^2 - 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)*sqrt(c^2*x^2 -
1))/(c^6*d^12*e - 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 - d^6*e^7 + (c^6*d^9*e^4 -
 3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 - d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 - 3*c^4*d^
8*e^5 + 3*c^2*d^6*e^7 - d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 - 3*c^4*d^9*e^4 + 3*
c^2*d^7*e^6 - d^5*e^8)*x), -1/6*((2*a + 3*b)*c^6*d^9 - 3*(2*a + b)*c^4*d^7*
e^2 + 6*a*c^2*d^5*e^4 - 2*a*d^3*e^6 + 3*(b*c^6*d^6*e^3 - b*c^4*d^4*e^5)*x^3
 + 9*(b*c^6*d^7*e^2 - b*c^4*d^5*e^4)*x^2 + 2*(2*b*c^5*d^8 + b*c^3*d^6*e^2 +
 (2*b*c^5*d^5*e^3 + b*c^3*d^3*e^5)*x^3 + 3*(2*b*c^5*d^6*e^2 + b*c^3*d^4*e^4
)*x^2 + 3*(2*b*c^5*d^7*e + b*c^3*d^5*e^3)*x)*sqrt(-c^2*d^2 + e^2)*arctan(-(
sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*
d))/(c^2*d^2 - e^2)) + 9*(b*c^6*d^8*e - b*c^4*d^6*e^3)*x - 2*((b*c^6*d^6*e^
3 - 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 - b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 - 3*b
*c^4*d^5*e^4 + 3*b*c^2*d^3*e^6 - b*d*e^8)*x^2 + 3*(b*c^6*d^8*e - 3*b*c^4*d^
6*e^3 + 3*b*c^2*d^4*e^5 - b*d^2*e^7)*x)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b
*c^6*d^9 - 3*b*c^4*d^7*e^2 + 3*b*c^2*d^5*e^4 - b*d^3*e^6 + (b*c^6*d^6*e^3 -
 3*b*c^4*d^4*e^5 + 3*b*c^2*d^2*e^7 - b*e^9)*x^3 + 3*(b*c^6*d^7*e^2 - 3*b*c^
4*d^5*e^4 + 3*b*c^2*d^3*e^6 - b*d*e^8)*x^2 + 3*(b*c^6*d^8*e - 3*b*c^4*d^6*e
^3 + 3*b*c^2*d^4*e^5 - b*d^2*e^7)*x)*log(-c*x + sqrt(c^2*x^2 - 1)) + (4*b*c
^5*d^8*e - 5*b*c^3*d^6*e^3 + b*c*d^4*e^5 + 3*(b*c^5*d^6*e^3 - b*c^3*d^4*e^5
)*x^2 + (7*b*c^5*d^7*e^2 - 8*b*c^3*d^5*e^4 + b*c*d^3*e^6)*x)*sqrt(c^2*x^2 -
 1))/(c^6*d^12*e - 3*c^4*d^10*e^3 + 3*c^2*d^8*e^5 - d^6*e^7 + (c^6*d^9*e^4
- 3*c^4*d^7*e^6 + 3*c^2*d^5*e^8 - d^3*e^10)*x^3 + 3*(c^6*d^10*e^3 - 3*c^4*d
^8*e^5 + 3*c^2*d^6*e^7 - d^4*e^9)*x^2 + 3*(c^6*d^11*e^2 - 3*c^4*d^9*e^4 + 3
*c^2*d^7*e^6 - d^5*e^8)*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{(ex + d)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x + d)^4, x)
```

maple [B] time = 0.00, size = 1137, normalized size = 5.63

$$-\frac{c^{3} a}{3 \left(c x e+c d\right)^{3} e}-\frac{c^{3} b \,\mathrm{arccosh}\left(c x\right)}{3 \left(c x e+c d\right)^{3} e}-\frac{c^{7} b \sqrt{c x+1}\, \sqrt{c x-1}\, \ln \left(-\frac{2 \left(c^{2} d x-\sqrt{c^{2} x^{2}-1}\, \sqrt{\frac{c^{2} d^{2}-e^{2}}{e^{2}}}\, e+e\right)}{c x e+c d}\right) x^{2} d^{2}}{3 \sqrt{c^{2} x^{2}-1}\, \left(c d+e\right) \left(c d-e\right) \left(c^{2} d^{2}-e^{2}\right) \left(c x e+c d\right)^{2} \sqrt{\frac{c^{2} d^{2}-e^{2}}{e^{2}}}}-\frac{2 c^{7} b \sqrt{c x+1}\,}{3 e \sqrt{c^{2} x^{2}-1}\, (}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x+d)^4,x)
```

```
[Out] -1/3*c^3*a/(c*e*x+c*d)^3/e-1/3*c^3*b/(c*e*x+c*d)^3/e*arccosh(c*x)-1/3*c^7*b
*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2
)/(c*e*x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*
((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x^2*d^2-2/3*c^7*b/e*(c*x+1)^(1/
2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d
)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^
2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x*d^3-1/3*c^7*b/e^2*(c*x+1)^(1/2)*(c*x-1)^(
1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^
2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2
)*e+e)/(c*e*x+c*d))*d^4-1/2*c^5*b*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*
d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*x*d-2/3*c^5*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)/
(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*d^2-1/6*c^5*b*e^2*(c*x+1)^(1/2)
*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^
2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)
/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x^2-1/3*c^5*b*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(
c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)
/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)
/(c*e*x+c*d))*x*d-1/6*c^5*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(
c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*e*x+c*d)^2/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*
(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*d^2+
1/6*c^3*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c^2*d^2-e^2)/(c*
e*x+c*d)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6} \left(6 c \int \frac{1}{3 \left(c^{3} e^{4} x^{6}+3 c^{3} d e^{3} x^{5}-3 c d^{2} e^{2} x^{2}-c d^{3} e x+\left(3 c^{3} d^{2} e^{2}-c e^{4}\right) x^{4}+\left(c^{3} d^{3} e-3 c d e^{3}\right) x^{3}+\left(c^{2} e^{4} x^{5}+3 c^{2} d e^{3}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(6*c*integrate(1/3/(c^3*e^4*x^6 + 3*c^3*d*e^3*x^5 - 3*c*d^2*e^2*x^2 -
c*d^3*e*x + (3*c^3*d^2*e^2 - c*e^4)*x^4 + (c^3*d^3*e - 3*c*d*e^3)*x^3 + (c^
2*e^4*x^5 + 3*c^2*d*e^3*x^4 - 3*d^2*e^2*x - d^3*e + (3*c^2*d^2*e^2 - e^4)*x
^3 + (c^2*d^3*e - 3*d*e^3)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x
) + 2*(c^6*d^3 + 3*c^4*d*e^2)*log(e*x + d)/(c^6*d^6*e - 3*c^4*d^4*e^3 + 3*c
^2*d^2*e^5 - e^7) - (3*c^6*d^6 - 2*c^4*d^4*e^2 - c^2*d^2*e^4 + 2*(c^6*d^4*e
^2 - c^2*e^6)*x^2 + (5*c^6*d^5*e - 2*c^4*d^3*e^3 - 3*c^2*d*e^5)*x - 2*(c^6*
d^6 - 3*c^4*d^4*e^2 + 3*c^2*d^2*e^4 - e^6)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)) + (c^6*d^6 + 3*c^5*d^5*e + 3*c^4*d^4*e^2 + c^3*d^3*e^3 + (c^6*d^3*e^
3 + 3*c^5*d^2*e^4 + 3*c^4*d*e^5 + c^3*e^6)*x^3 + 3*(c^6*d^4*e^2 + 3*c^5*d^3
*e^3 + 3*c^4*d^2*e^4 + c^3*d*e^5)*x^2 + 3*(c^6*d^5*e + 3*c^5*d^4*e^2 + 3*c^
4*d^3*e^3 + c^3*d^2*e^4)*x)*log(c*x + 1) + (c^6*d^6 - 3*c^5*d^5*e + 3*c^4*d
^4*e^2 - c^3*d^3*e^3 + (c^6*d^3*e^3 - 3*c^5*d^2*e^4 + 3*c^4*d*e^5 - c^3*e^6
)*x^3 + 3*(c^6*d^4*e^2 - 3*c^5*d^3*e^3 + 3*c^4*d^2*e^4 - c^3*d*e^5)*x^2 + 3
```

```
*(c^6*d^5*e - 3*c^5*d^4*e^2 + 3*c^4*d^3*e^3 - c^3*d^2*e^4)*x)*log(c*x - 1))
/(c^6*d^9*e - 3*c^4*d^7*e^3 + 3*c^2*d^5*e^5 - d^3*e^7 + (c^6*d^6*e^4 - 3*c^
4*d^4*e^6 + 3*c^2*d^2*e^8 - e^10)*x^3 + 3*(c^6*d^7*e^3 - 3*c^4*d^5*e^5 + 3*
c^2*d^3*e^7 - d*e^9)*x^2 + 3*(c^6*d^8*e^2 - 3*c^4*d^6*e^4 + 3*c^2*d^4*e^6 -
 d^2*e^8)*x))*b - 1/3*a/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{{\left(d+e\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x)^4,x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(cx\right)}{\left(d+ex\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x+d)**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x)**4, x)
```

3.21 $\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=398

$$-\frac{3e^3\left(a+b\cosh^{-1}(cx)\right)^2}{32c^4}-\frac{4bde^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{3be^3x\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{16c^3}-$$

```
[Out] 2*b^2*d^3*x+4/3*b^2*d*e^2*x/c^2+3/4*b^2*d^2*e*x^2+3/32*b^2*e^3*x^2/c^2+2/9*
b^2*d*e^2*x^3+1/32*b^2*e^3*x^4-1/4*d^4*(a+b*arccosh(c*x))^2/e-3/4*d^2*e*(a+
b*arccosh(c*x))^2/c^2-3/32*e^3*(a+b*arccosh(c*x))^2/c^4+1/4*(e*x+d)^4*(a+b*
arccosh(c*x))^2/e-2*b*d^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-
4/3*b*d*e^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-3/2*b*d^2*e*
x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/16*b*e^3*x*(a+b*arccos
h(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-2/3*b*d*e^2*x^2*(a+b*arccosh(c*x))*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/8*b*e^3*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 1.69, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{4bde^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{3e^3\left(a+b\cosh^{-1}(cx)\right)^2}{32c^4}-\frac{3be^3x\sqrt{cx-1}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x + (4*b^2*d*e^2*x)/(3*c^2) + (3*b^2*d^2*e*x^2)/4 + (3*b^2*e^3*x^
2)/(32*c^2) + (2*b^2*d*e^2*x^3)/9 + (b^2*e^3*x^4)/32 - (2*b*d^3*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*d*e^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^3) - (3*b*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(2*c) - (3*b*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*(a + b*ArcCosh[c*x]))/(16*c^3) - (2*b*d*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(3*c) - (b*e^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(8*c) - (d^4*(a + b*ArcCosh[c*x])^2)/(4*e) - (3*d^2*e
*(a + b*ArcCosh[c*x])^2)/(4*c^2) - (3*e^3*(a + b*ArcCosh[c*x])^2)/(32*c^4)
+ ((d + e*x)^4*(a + b*ArcCosh[c*x])^2)/(4*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
```

```
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{(d+ex)^4 \left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \frac{(d+ex)^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2e}$$

$$= \frac{(d+ex)^4 \left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \frac{(bc)\int \left(\frac{d^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4d^3ex\left(a+b\cosh^{-1}(\ldots\right.}{\sqrt{-1+cx}\sqrt{1+c\ldots}}\right.}{}$$

$$= \frac{(d+ex)^4 \left(a+b\cosh^{-1}(cx)\right)^2}{4e} - \left(2bcd^3\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \ldots$$

$$= -\frac{2bd^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}\ldots}{2c}$$

$$= 2b^2d^3x + \frac{3}{4}b^2d^2ex^2 + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd^3\sqrt{-1+cx}\sqrt{1+cx}\,(a\ldots}{c}$$

$$= 2b^2d^3x + \frac{4b^2de^2x}{3c^2} + \frac{3}{4}b^2d^2ex^2 + \frac{3b^2e^3x^2}{32c^2} + \frac{2}{9}b^2de^2x^3 + \frac{1}{32}b^2e^3x^4 - \frac{2bd\ldots}{}$$

Mathematica [A] time = 0.85, size = 386, normalized size = 0.97

$$\frac{c\left(72a^2c^3x\left(4d^3+6d^2ex+4de^2x^2+e^3x^3\right)-6ab\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(96d^3+72d^2ex+32de^2x^2+6e^3x^3\right)+e^2(64d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (c*(72*a^2*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) - 6*a*b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d + 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*
d*e^2*x^2 + 6*e^3*x^3)) + b^2*c*x*(3*e^2*(128*d + 9*e*x) + c^2*(576*d^3 + 2
16*d^2*e*x + 64*d*e^2*x^2 + 9*e^3*x^3))) - 6*b*c*(-24*a*c^3*x*(4*d^3 + 6*d^
2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(e^2*(64*d
+ 9*e*x) + c^2*(96*d^3 + 72*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^3)))*ArcCosh[c
*x] + 9*b^2*(-24*c^2*d^2*e - 3*e^3 + 8*c^4*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x
^2 + e^3*x^3))*ArcCosh[c*x]^2 - 54*a*b*e*(8*c^2*d^2 + e^2)*Log[c*x + Sqrt[-
1 + c*x]*Sqrt[1 + c*x]])/(288*c^4)
```

fricas [A] time = 0.61, size = 472, normalized size = 1.19

$$\frac{9\left(8a^2+b^2\right)c^4e^3x^4+32\left(9a^2+2b^2\right)c^4de^2x^3+27\left(8\left(2a^2+b^2\right)c^4d^2e+b^2c^2e^3\right)x^2+9\left(8b^2c^4e^3x^4+32b^2c^4de^2x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/288*(9*(8*a^2 + b^2)*c^4*e^3*x^4 + 32*(9*a^2 + 2*b^2)*c^4*d*e^2*x^3 + 27*
(8*(2*a^2 + b^2)*c^4*d^2*e + b^2*c^2*e^3)*x^2 + 9*(8*b^2*c^4*e^3*x^4 + 32*b
^2*c^4*d*e^2*x^3 + 48*b^2*c^4*d^2*e*x^2 + 32*b^2*c^4*d^3*x - 24*b^2*c^2*d^2
*e - 3*b^2*e^3)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 96*(3*(a^2 + 2*b^2)*c^4*d^
3 + 4*b^2*c^2*d*e^2)*x + 6*(24*a*b*c^4*e^3*x^4 + 96*a*b*c^4*d*e^2*x^3 + 144
*a*b*c^4*d^2*e*x^2 + 96*a*b*c^4*d^3*x - 72*a*b*c^2*d^2*e - 9*a*b*e^3 - (6*b
^2*c^3*e^3*x^3 + 32*b^2*c^3*d*e^2*x^2 + 96*b^2*c^3*d^3 + 64*b^2*c*d*e^2 + 9
*(8*b^2*c^3*d^2*e + b^2*c*e^3)*x)*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2
 - 1)) - 6*(6*a*b*c^3*e^3*x^3 + 32*a*b*c^3*d*e^2*x^2 + 96*a*b*c^3*d^3 + 64*
a*b*c*d*e^2 + 9*(8*a*b*c^3*d^2*e + a*b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.08, size = 791, normalized size = 1.99

$$2b^2d^3x+\frac{a^2d^4}{4e}-\frac{3b^2\mathrm{arccosh}\,(cx)^2\,e^3}{32c^4}+b^2\mathrm{arccosh}\,(cx)^2\,x\,d^3+\frac{b^2\mathrm{arccosh}\,(cx)^2\,x^4e^3}{4}+a^2e^2x^3d+\frac{3a^2e\,x^2d^2}{2}-\frac{3abe\sqrt{cx-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 2*b^2*d^3*x+1/4*a^2/e*d^4-3/32/c^4*b^2*arccosh(c*x)^2*e^3+b^2*arccosh(c*x)^
2*x*d^3+1/4*b^2*arccosh(c*x)^2*x^4*e^3+a^2*e^2*x^3*d+3/2*a^2*e*x^2*d^2-3/2/
c^2*a*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^2*ln(c*x+(c^2*x^2
-1)^(1/2))-2/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^3-2/c*a*b*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*d^3+2*a*b*e^2*arccosh(c*x)*x^3*d+3*a*b*e*arccosh(c
*x)*x^2*d^2-2/3/c*a*b*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*d-1/2*a*b/e*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^4*ln(c*x+(c^2*x^2-1)^(1/2))-2/3/
c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*d*e^2-3/2/c*a*b*e*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*d^2*x-3/2/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*x*d^2*e-3/16/c^4*a*b*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*
ln(c*x+(c^2*x^2-1)^(1/2))+1/2*a*b/e*arccosh(c*x)*d^4+b^2*arccosh(c*x)^2*x^3
*d*e^2+3/2*b^2*arccosh(c*x)^2*x^2*d^2*e+1/2*a*b*e^3*arccosh(c*x)*x^4+2*a*b*
arccosh(c*x)*x*d^3+3/4*b^2*d^2*e*x^2+3/32*b^2*e^3*x^2/c^2+2/9*b^2*d*e^2*x^3
-4/3/c^3*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e^2-1/8/c*a*b*e^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-4/3/c^3*a*b*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
d-3/16/c^3*a*b*e^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x-1/8/c*b^2*arccosh(c*x)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*x^3*e^3-3/16/c^3*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*x*e^3+1/32*b^2*e^3*x^4-3/4/c^2*b^2*arccosh(c*x)^2*d^2*e+a^2*x*d
^3+1/4*a^2*e^3*x^4+4/3*b^2*d*e^2*x/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}a^2e^3x^4+a^2de^2x^3+b^2d^3x\operatorname{arcosh}(cx)^2+\frac{3}{2}a^2d^2ex^2+\frac{3}{2}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x\right.}}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*e^3*x^4 + a^2*d*e^2*x^3 + b^2*d^3*x*arccosh(c*x)^2 + 3/2*a^2*d^2*e*
x^2 + 3/2*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x +
2*sqrt(c^2*x^2 - 1)*c)/c^3))*a*b*d^2*e + 2/3*(3*x^3*arccosh(c*x) - c*(sqrt(
c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*d*e^2 + 1/16*(8*x^4*ar
ccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*a*b*e^3 + 2*b^2*d^3*(x - sqrt(c
^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d^3*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^
2 - 1))*a*b*d^3/c + 1/4*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - integrate(1/2*(b^2*c^3*e^3*x^6 +
4*b^2*c^3*d*e^2*x^5 - 4*b^2*c*d*e^2*x^3 - 6*b^2*c*d^2*e*x^2 + (6*c^3*d^2*e
- c*e^3)*b^2*x^4 + (b^2*c^2*e^3*x^5 + 4*b^2*c^2*d*e^2*x^4 + 6*b^2*c^2*d^2*e
*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(
c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{acosh}(c\,x))^2\,(d+e\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x)^3, x)
```

sympy [A] time = 3.70, size = 750, normalized size = 1.88

$$\begin{cases} a^2d^3x+\frac{3a^2d^2ex^2}{2}+a^2de^2x^3+\frac{a^2e^3x^4}{4}+2abd^3x\operatorname{acosh}(cx)+3abd^2ex^2\operatorname{acosh}(cx)+2abde^2x^3\operatorname{acosh}(cx)+\frac{abe^3x^{\cdots}}{\cdots} \\ \left(a+\frac{i\pi b}{2}\right)^2\left(d^3x+\frac{3d^2ex^2}{2}+de^2x^3+\frac{e^3x^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e**
3*x**4/4 + 2*a*b*d**3*x*acosh(c*x) + 3*a*b*d**2*e*x**2*acosh(c*x) + 2*a*b*d
*e**2*x**3*acosh(c*x) + a*b*e**3*x**4*acosh(c*x)/2 - 2*a*b*d**3*sqrt(c**2*x
**2 - 1)/c - 3*a*b*d**2*e*x*sqrt(c**2*x**2 - 1)/(2*c) - 2*a*b*d*e**2*x**2*s
qrt(c**2*x**2 - 1)/(3*c) - a*b*e**3*x**3*sqrt(c**2*x**2 - 1)/(8*c) - 3*a*b*
d**2*e*acosh(c*x)/(2*c**2) - 4*a*b*d*e**2*sqrt(c**2*x**2 - 1)/(3*c**3) - 3*
a*b*e**3*x*sqrt(c**2*x**2 - 1)/(16*c**3) - 3*a*b*e**3*acosh(c*x)/(16*c**4)
+ b**2*d**3*x*acosh(c*x)**2 + 2*b**2*d**3*x + 3*b**2*d**2*e*x**2*acosh(c*x)
**2/2 + 3*b**2*d**2*e*x**2/4 + b**2*d*e**2*x**3*acosh(c*x)**2 + 2*b**2*d*e*
*2*x**3/9 + b**2*e**3*x**4*acosh(c*x)**2/4 + b**2*e**3*x**4/32 - 2*b**2*d**
3*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 3*b**2*d**2*e*x*sqrt(c**2*x**2 - 1)*ac
osh(c*x)/(2*c) - 2*b**2*d*e**2*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c) -
b**2*e**3*x**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(8*c) - 3*b**2*d**2*e*acosh(c
*x)**2/(4*c**2) + 4*b**2*d*e**2*x/(3*c**2) + 3*b**2*e**3*x**2/(32*c**2) - 4
*b**2*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 3*b**2*e**3*x*sqrt(c
**2*x**2 - 1)*acosh(c*x)/(16*c**3) - 3*b**2*e**3*acosh(c*x)**2/(32*c**4), N
e(c, 0)), ((a + I*pi*b/2)**2*(d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3
*x**4/4), True))
```

3.22 $\int (d+ex)^2 \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=259

$$-\frac{4be^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{de\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e}-\frac{2bd^2\sqrt{cx-1}\sqrt{cx}}{}$$

```
[Out] 2*b^2*d^2*x+4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+2/27*b^2*e^2*x^3-1/3*d^3*(a+b
*arccosh(c*x))^2/e-1/2*d*e*(a+b*arccosh(c*x))^2/c^2+1/3*(e*x+d)^3*(a+b*arcc
osh(c*x))^2/e-2*b*d^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-4/9*
b*e^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-b*d*e*x*(a+b*arcco
sh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-2/9*b*e^2*x^2*(a+b*arccosh(c*x))*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 1.15, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{de\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2}-\frac{4be^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e}-\frac{2bd^2\sqrt{cx-1}\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x + (4*b^2*e^2*x)/(9*c^2) + (b^2*d*e*x^2)/2 + (2*b^2*e^2*x^3)/27
 - (2*b*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*e^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3) - (b*d*e*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (2*b*e^2*x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c) - (d^3*(a + b*ArcCosh[c*x])^2)
/(3*e) - (d*e*(a + b*ArcCosh[c*x])^2)/(2*c^2) + ((d + e*x)^3*(a + b*ArcCosh
[c*x])^2)/(3*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
```

```
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3e}$$

$$= \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \frac{(2bc)\int\left(\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3d^2ex\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\sqrt{1+cx}}\right.}{}$$

$$= \frac{(d+ex)^3\left(a+b\cosh^{-1}(cx)\right)^2}{3e} - \left(2bcd^2\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx - \frac{(2bcd}{}$$

$$= -\frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bdex\sqrt{-1+cx}\sqrt{1+cx}\left(a+\right.}{c}$$

$$= 2b^2d^2x + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{c}$$

$$= 2b^2d^2x + \frac{4b^2e^2x}{9c^2} + \frac{1}{2}b^2dex^2 + \frac{2}{27}b^2e^2x^3 - \frac{2bd^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\,\mathrm{co}\right.}{c}$$

Mathematica [A] time = 0.66, size = 360, normalized size = 1.39

$$a^2d^2x+a^2dex^2+\frac{1}{3}a^2e^2x^3-\frac{4abe^2\sqrt{cx-1}\sqrt{cx+1}}{9c^3}-\frac{abde\log\left(cx+\sqrt{cx-1}\sqrt{cx+1}\right)}{c^2}-\frac{b\cosh^{-1}(cx)\left(b\sqrt{cx-1}\sqrt{cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] a^2*d^2*x + 2*b^2*d^2*x + (4*b^2*e^2*x)/(9*c^2) + a^2*d*e*x^2 + (b^2*d*e*x^
2)/2 + (a^2*e^2*x^3)/3 + (2*b^2*e^2*x^3)/27 - (2*a*b*d^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/c - (4*a*b*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (a*b*d*e
*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (2*a*b*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])/(9*c) - (b*(-6*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) + b*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(4*e^2 + c^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2)))*ArcCosh[c*x])
/(9*c^3) + (b^2*((-3*d*e)/c^2 + 2*x*(3*d^2 + 3*d*e*x + e^2*x^2))*ArcCosh[c*
x]^2)/6 - (a*b*d*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/c^2
```

fricas [A] time = 0.69, size = 319, normalized size = 1.23

$$\frac{2\left(9\,a^2+2\,b^2\right)c^3e^2x^3+27\left(2\,a^2+b^2\right)c^3dex^2+9\left(2\,b^2c^3e^2x^3+6\,b^2c^3dex^2+6\,b^2c^3d^2x-3\,b^2cde\right)\log\left(cx+\sqrt{c^2\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(9*a^2 + 2*b^2)*c^3*e^2*x^3 + 27*(2*a^2 + b^2)*c^3*d*e*x^2 + 9*(2*b
^2*c^3*e^2*x^3 + 6*b^2*c^3*d*e*x^2 + 6*b^2*c^3*d^2*x - 3*b^2*c*d*e)*log(c*x
 + sqrt(c^2*x^2 - 1))^2 + 6*(9*(a^2 + 2*b^2)*c^3*d^2 + 4*b^2*c*e^2)*x + 6*(
6*a*b*c^3*e^2*x^3 + 18*a*b*c^3*d*e*x^2 + 18*a*b*c^3*d^2*x - 9*a*b*c*d*e - (
2*b^2*c^2*e^2*x^2 + 9*b^2*c^2*d*e*x + 18*b^2*c^2*d^2 + 4*b^2*e^2)*sqrt(c^2*
x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1)) - 6*(2*a*b*c^2*e^2*x^2 + 9*a*b*c^2*d
*e*x + 18*a*b*c^2*d^2 + 4*a*b*e^2)*sqrt(c^2*x^2 - 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.07, size = 517, normalized size = 2.00

$$2b^2d^2x-\frac{2ab\,e^2\sqrt{cx-1}\,\sqrt{cx+1}\,x^2}{9c}-\frac{2b^2\mathrm{arccosh}\,(cx)\,\sqrt{cx-1}\,\sqrt{cx+1}\,x^2e^2}{9c}-\frac{abe\sqrt{cx-1}\,\sqrt{cx+1}\,d\ln\left(cx+\sqrt{\cdots}\right.}{c^2\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 2*b^2*d^2*x-2/9/c*a*b*e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-2/9/c*b^2*arccosh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*e^2-1/c^2*a*b*e*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1)^(1/2))-4/9/c^3*a*b*e^2*(c*x-1
)^(1/2)*(c*x+1)^(1/2)-4/9/c^3*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
e^2+2*a*b*e*arccosh(c*x)*x^2*d-2/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*d^2-2/c*a*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2+b^2*arccosh(c*x)^2*x*d^2+a
^2*e*x^2*d+1/3*a^2/e*d^3+1/3*b^2*arccosh(c*x)^2*x^3*e^2+2/27*b^2*e^2*x^3-1/
c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d*e-2/3*a*b/e*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*ln(c*x+(c^2*x^2-1)^(1/2))-1/c*a*b*e*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*d*x+4/9*b^2*e^2*x/c^2+1/2*b^2*d*e*x^2+b^2*arccos
```

```
h(c*x)^2*x^2*d*e-1/2/c^2*b^2*arccosh(c*x)^2*d*e+2*a*b*arccosh(c*x)*x*d^2+2/
3*a*b/e*arccosh(c*x)*d^3+2/3*a*b*e^2*arccosh(c*x)*x^3+a^2*x*d^2+1/3*a^2*e^2
*x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a^2e^2x^3+b^2d^2x\operatorname{arcosh}(cx)^2+a^2dex^2+\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)abde+\frac{2}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*e^2*x^3 + b^2*d^2*x*arccosh(c*x)^2 + a^2*d*e*x^2 + (2*x^2*arccosh(c
*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3
))*a*b*d*e + 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqr
t(c^2*x^2 - 1)/c^4))*a*b*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x
)/c) + a^2*d^2*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^2/c + 1/3
*(b^2*e^2*x^3 + 3*b^2*d*e*x^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - i
ntegrate(2/3*(b^2*c^3*e^2*x^5 + 3*b^2*c^3*d*e*x^4 - b^2*c*e^2*x^3 - 3*b^2*c
*d*e*x^2 + (b^2*c^2*e^2*x^4 + 3*b^2*c^2*d*e*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1
))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x
 + 1)*sqrt(c*x - 1) - c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{acosh}(c\,x))^2\,(d+e\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x)^2, x)
```

sympy [A] time = 1.70, size = 461, normalized size = 1.78

$$\begin{cases} a^2d^2x+a^2dex^2+\frac{a^2e^2x^3}{3}+2abd^2x\operatorname{acosh}(cx)+2abdex^2\operatorname{acosh}(cx)+\frac{2abe^2x^3\operatorname{acosh}(cx)}{3}-\frac{2abd^2\sqrt{c^2x^2-1}}{c}-\frac{abdex\sqrt{c^2x^2-1}}{c} \\ \left(a+\frac{i\pi b}{2}\right)^2\left(d^2x+dex^2+\frac{e^2x^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*b*d**2*x*ac
osh(c*x) + 2*a*b*d*e*x**2*acosh(c*x) + 2*a*b*e**2*x**3*acosh(c*x)/3 - 2*a*b
*d**2*sqrt(c**2*x**2 - 1)/c - a*b*d*e*x*sqrt(c**2*x**2 - 1)/c - 2*a*b*e**2*
x**2*sqrt(c**2*x**2 - 1)/(9*c) - a*b*d*e*acosh(c*x)/c**2 - 4*a*b*e**2*sqrt(
c**2*x**2 - 1)/(9*c**3) + b**2*d**2*x*acosh(c*x)**2 + 2*b**2*d**2*x + b**2*
d*e*x**2*acosh(c*x)**2 + b**2*d*e*x**2/2 + b**2*e**2*x**3*acosh(c*x)**2/3 +
 2*b**2*e**2*x**3/27 - 2*b**2*d**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - b**2*
d*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*e**2*x**2*sqrt(c**2*x**2 -
1)*acosh(c*x)/(9*c) - b**2*d*e*acosh(c*x)**2/(2*c**2) + 4*b**2*e**2*x/(9*c*
*2) - 4*b**2*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3), Ne(c, 0)), ((a +
 I*pi*b/2)**2*(d**2*x + d*e*x**2 + e**2*x**3/3), True))
```

3.23 $\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=150

$$-\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}+\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}-\frac{2bd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}$$

```
[Out] 2*b^2*d*x+1/4*b^2*e*x^2-1/2*d^2*(a+b*arccosh(c*x))^2/e-1/4*e*(a+b*arccosh(c
*x))^2/c^2+1/2*(e*x+d)^2*(a+b*arccosh(c*x))^2/e-2*b*d*(a+b*arccosh(c*x))*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2*b*e*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c
```

Rubi [A] time = 0.76, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5802, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}+\frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e}-\frac{2bd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x + (b^2*e*x^2)/4 - (2*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x]))/c - (b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2
*c) - (d^2*(a + b*ArcCosh[c*x])^2)/(2*e) - (e*(a + b*ArcCosh[c*x])^2)/(4*c^
2) + ((d + e*x)^2*(a + b*ArcCosh[c*x])^2)/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rubi steps

$$\int (d+ex)\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{e}$$

$$= \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - \frac{(bc)\int \left(\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2dex\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \right.}{e}$$

$$= \frac{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}{2e} - (2bcd)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx - \frac{\left(bcd^2\right)\int}{}$$

$$= -\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\,c\right.}{2c}$$

$$= 2b^2dx + \frac{1}{4}b^2ex^2 - \frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{bex\sqrt{-1+cx}}{}$$

Mathematica [A] time = 0.43, size = 174, normalized size = 1.16

$$\frac{c\left(2a^2cx(2d+ex) - 2ab\sqrt{cx-1}\,\sqrt{cx+1}\,(4d+ex) + b^2cx(8d+ex)\right) - 2bc\cosh^{-1}(cx)\left(b\sqrt{cx-1}\,\sqrt{cx+1}\,(4d+ex)\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (c*(2*a^2*c*x*(2*d + e*x) - 2*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4*d + e*x)
+ b^2*c*x*(8*d + e*x)) - 2*b*c*(-2*a*c*x*(2*d + e*x) + b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(4*d + e*x))*ArcCosh[c*x] + b^2*(4*c^2*d*x + e*(-1 + 2*c^2*x^2))
*ArcCosh[c*x]^2 - 2*a*b*e*Log[c*x + Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(4*c^2)
```

fricas [A] time = 0.69, size = 185, normalized size = 1.23

$$\frac{\left(2\,a^2+b^2\right)c^2ex^2+4\left(a^2+2\,b^2\right)c^2dx+\left(2\,b^2c^2ex^2+4\,b^2c^2dx-b^2e\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\left(2\,abc^2ex^2+4\,a\right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*c^2*e*x^2 + 4*(a^2 + 2*b^2)*c^2*d*x + (2*b^2*c^2*e*x^2 +
 4*b^2*c^2*d*x - b^2*e)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*(2*a*b*c^2*e*x^2
 + 4*a*b*c^2*d*x - a*b*e - (b^2*c*e*x + 4*b^2*c*d)*sqrt(c^2*x^2 - 1))*log(c
*x + sqrt(c^2*x^2 - 1)) - 2*(a*b*c*e*x + 4*a*b*c*d)*sqrt(c^2*x^2 - 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.10, size = 245, normalized size = 1.63

$$\frac{a^2x^2e}{2}+a^2dx+\frac{b^2\text{arccosh}\left(cx\right)^2x^2e}{2}-\frac{b^2\text{arccosh}\left(cx\right)\sqrt{cx-1}\,\sqrt{cx+1}\,xe}{2c}+\frac{b^2e\,x^2}{4}-\frac{b^2\text{arccosh}\left(cx\right)^2e}{4c^2}+b^2\text{arccosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/2*a^2*x^2*e+a^2*d*x+1/2*b^2*arccosh(c*x)^2*x^2*e-1/2/c*b^2*arccosh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*x*e+1/4*b^2*e*x^2-1/4/c^2*b^2*arccosh(c*x)^2*e+b
^2*arccosh(c*x)^2*x*d-2/c*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d+2*
b^2*d*x+a*b*arccosh(c*x)*x^2*e+2*a*b*arccosh(c*x)*x*d-1/2/c*a*b*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*e*x-2/c*a*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d-1/2/c^2*a*b*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2dx\,\text{arcosh}\left(cx\right)^2+\frac{1}{2}\,a^2ex^2+\frac{1}{2}\left(2\,x^2\,\text{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)abe+\frac{1}{2}\left(x^2\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d*x*arccosh(c*x)^2 + 1/2*a^2*e*x^2 + 1/2*(2*x^2*arccosh(c*x) - c*(sqrt(
c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*a*b*e + 1/2
*(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 - 2*integrate((c^3*x^4 + sqr
t(c*x + 1)*sqrt(c*x - 1)*c^2*x^3 - c*x^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1))/(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x), x))*b^2*
e + 2*b^2*d*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d*x + 2*(c*x*arcco
sh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{acosh}\,(c\,x))^2\ (d + e\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x), x)
```

sympy [A] time = 0.71, size = 240, normalized size = 1.60

$$\begin{cases} a^2dx + \frac{a^2ex^2}{2} + 2abdx\,\mathrm{acosh}\,(cx) + abex^2\,\mathrm{acosh}\,(cx) - \frac{2abd\sqrt{c^2x^2-1}}{c} - \frac{abex\sqrt{c^2x^2-1}}{2c} - \frac{abe\,\mathrm{acosh}\,(cx)}{2c^2} + b^2dx\,\mathrm{acosh}^2\,(cx) \\ \left(a + \frac{i\pi b}{2}\right)^2 \left(dx + \frac{ex^2}{2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x*acosh(c*x) + a*b*e*x**2*aco
sh(c*x) - 2*a*b*d*sqrt(c**2*x**2 - 1)/c - a*b*e*x*sqrt(c**2*x**2 - 1)/(2*c)
 - a*b*e*acosh(c*x)/(2*c**2) + b**2*d*x*acosh(c*x)**2 + 2*b**2*d*x + b**2*e
*x**2*acosh(c*x)**2/2 + b**2*e*x**2/4 - 2*b**2*d*sqrt(c**2*x**2 - 1)*acosh(
c*x)/c - b**2*e*x*sqrt(c**2*x**2 - 1)*acosh(c*x)/(2*c) - b**2*e*acosh(c*x)*
*2/(4*c**2), Ne(c, 0)), ((a + I*pi*b/2)**2*(d*x + e*x**2/2), True))
```

3.24 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=303

$$\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\Big(}{e}$$

```
[Out] -1/3*(a+b*arccosh(c*x))^3/b/e+(a+b*arccosh(c*x))^2*ln(1+e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e+(a+b*arccosh(c*x))^2*ln(1+e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e+2*b*(a+b*arcc
osh(c*x))*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)
^(1/2)))/e+2*b*(a+b*arccosh(c*x))*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e-2*b^2*polylog(3,-e*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e-2*b^2*polylog(3,-e*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e
```

Rubi [A] time = 0.46, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5800, 5562, 2190, 2531, 2282, 6589}

$$\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e}+\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e}-\frac{2b^2\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x),x]
```

```
[Out] -(a + b*ArcCosh[c*x])^3/(3*b*e) + ((a + b*ArcCosh[c*x])^2*Log[1 + (e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/e + ((a + b*ArcCosh[c*x])^2*Log[1 +
 (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/e + (2*b*(a + b*ArcCosh[c
*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/e + (2*
b*(a + b*ArcCosh[c*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2
- e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 -
 e^2]))])/e - (2*b^2*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 -
e^2]))])/e
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex}\,dx = \text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \text{Subst}\left(\int \frac{e^x(a+bx)^2}{cd-\sqrt{c^2d^2-e^2}+ee^x}\,dx, x, \cosh^{-1}(cx)\right) + \text{Subst}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(c\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(c\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(c\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^3}{3be} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e} + \frac{\left(a+b\cosh^{-1}(c\right.}{}$$

Mathematica [A] time = 0.28, size = 285, normalized size = 0.94

$$\frac{6b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}-cd}\right)+6b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(-\frac{ee^{\cosh^{-1}(cx)}}{cd+\sqrt{c^2d^2-e^2}}\right)+3\left(a+b\cosh^{-1}(cx)\right)^2\log\left(\frac{}{cc}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])^3/b) + 3*(a + b*ArcCosh[c*x])^2*Log[1 + (e*E^ArcCos
h[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])] + 3*(a + b*ArcCosh[c*x])^2*Log[1 + (e*
```

```
E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])] + 6*b*(a + b*ArcCosh[c*x])*Pol
yLog[2, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^2 - e^2])] + 6*b*(a + b*Arc
Cosh[c*x])*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))] -
6*b^2*PolyLog[3, (e*E^ArcCosh[c*x])/(-(c*d) + Sqrt[c^2*d^2 - e^2])] - 6*b^2
*PolyLog[3, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(3*e)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arcosh}(cx)^2 + 2\,ab\operatorname{arcosh}(cx) + a^2}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(e*x + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\log(ex+d)}{e} + \int \frac{b^2\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{ex+d} + \frac{2\,ab\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2
/(e*x + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^2}{d + ex} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x), x)
```

3.25 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx$

Optimal. Leaf size=279

$$\frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}+1\right)}{e\sqrt{c^2d^2-e^2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)}{e(d+ex)}$$

```
[Out] -(a+b*arccosh(c*x))^2/e/(e*x+d)+2*b*c*(a+b*arccosh(c*x))*ln(1+e*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)-2*b
*c*(a+b*arccosh(c*x))*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^
2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)+2*b^2*c*polylog(2,-e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/(c*d-(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(1/2)-2*b^2*c*p
olylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/
(c^2*d^2-e^2)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5802, 5832, 3320, 2264, 2190, 2279, 2391}

$$\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2b^2c\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\sqrt{c^2d^2-e^2}}+\frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\sqrt{c^2d^2-e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c*x])^2/(e*(d + e*x))) + (2*b*c*(a + b*ArcCosh[c*x])*Log[1
 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2])
 - (2*b*c*(a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d
^2 - e^2])])/(e*Sqrt[c^2*d^2 - e^2]) + (2*b^2*c*PolyLog[2, -((e*E^ArcCosh[c
*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2 - e^2]) - (2*b^2*c*Poly
Log[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(e*Sqrt[c^2*d^2
- e^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^2}\,dx = -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(d+ex)}\,dx}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(2bc)\,\mathrm{Subst}\left(\int \frac{a+bx}{cd+e\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\,\mathrm{Subst}\left(\int \frac{e^x(a+bx)}{e+2cde^x+ee^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{(4bc)\,\mathrm{Subst}\left(\int \frac{e^x(a+bx)}{2cd-2\sqrt{c^2d^2-e^2}+2ee^x}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{c^2d^2-e^2}} - \frac{(4bc)}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2bc\left(a+b\,\mathrm{co}\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2bc\left(a+b\,\mathrm{co}\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{e(d+ex)} + \frac{2bc\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}} - \frac{2bc\left(a+b\,\mathrm{co}\right.}{}$$

Mathematica [C] time = 4.60, size = 961, normalized size = 3.44

$$-\frac{a^2}{e(d+ex)}+2bc\left(\frac{2\tanh^{-1}\left(\frac{\sqrt{(cd-e)e}\sqrt{\frac{cx-1}{cx+1}}}{\sqrt{e(cd+e)}}\right)}{\sqrt{(cd-e)e}\sqrt{e(cd+e)}}-\frac{\cosh^{-1}(cx)}{e(cd+cex)}\right)a-\frac{b^2c\left(\frac{\cosh^{-1}(cx)^2}{cd+cex}+\frac{2\left(2\cosh^{-1}(cx)\tan^{-1}\left(\frac{(cd+e)\coth\left(\frac{1}{2}\cosh^{-1}(c\right.}{\sqrt{e^2-c^2d^2}}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x)^2,x]
```

```
[Out] -(a^2/(e*(d + e*x))) + 2*a*b*c*(-(ArcCosh[c*x]/(e*(c*d + c*e*x))) + (2*ArcT
anh[(Sqrt[(c*d - e)*e]*Sqrt[(-1 + c*x)/(1 + c*x)])/Sqrt[e*(c*d + e)]])/(Sqr
t[(c*d - e)*e]*Sqrt[e*(c*d + e)])) - (b^2*c*(ArcCosh[c*x]^2/(c*d + c*e*x) +
 (2*(2*ArcCosh[c*x]*ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2)
 + e^2]] - (2*I)*ArcCos[-((c*d)/e)]*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/
2])/Sqrt[-(c^2*d^2) + e^2]] + (ArcCos[-((c*d)/e)] + 2*(ArcTan[((c*d + e)*Co
th[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[Arc
Cosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[Sqrt[-(c^2*d^2) + e^2]/(Sqrt[2]
*Sqrt[e]*E^(ArcCosh[c*x]/2)*Sqrt[c*d + c*e*x])] + (ArcCos[-((c*d)/e)] - 2*(
ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((
-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[(Sqrt[-(c^2
*d^2) + e^2]*E^(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*Sqrt[c*d + c*e*x])] - (Ar
cCos[-((c*d)/e)] + 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*
d^2) + e^2]])*Log[((c*d + e)*(c*d - e + I*Sqrt[-(c^2*d^2) + e^2])*(-1 + Tan
h[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x
]/2]))] - (ArcCos[-((c*d)/e)] - 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2]
)/Sqrt[-(c^2*d^2) + e^2]])*Log[((c*d + e)*(-(c*d) + e + I*Sqrt[-(c^2*d^2) +
 e^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*T
anh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*d - I*Sqrt[-(c^2*d^2) + e^2])*(c
*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sq
rt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*d + I*Sqrt[-(
c^2*d^2) + e^2])*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))
/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))]))))/Sqrt[-(c
^2*d^2) + e^2]))/e
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{arcosh}\,(cx)^2+2\,ab\,\mathrm{arcosh}\,(cx)+a^2}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(e^2*x^2 + 2*d*e*x
 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\mathrm{arcosh}\,(cx)+a)^2}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d)^2, x)
```

maple [A] time = 0.07, size = 536, normalized size = 1.92

$$-\frac{c\,a^2}{(cxe+cd)\,e}-\frac{c\,b^2\mathrm{arccosh}\,(cx)^2}{e\,(cxe+cd)}+\frac{2c\,b^2\mathrm{arccosh}\,(cx)\ln\left(\frac{-\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)e-cd+\sqrt{c^2d^2-e^2}}{-cd+\sqrt{c^2d^2-e^2}}\right)}{e\sqrt{c^2d^2-e^2}}-\frac{2c\,b^2\mathrm{arccosh}\,(cx)\ln\left(\frac{(cx}{}\right.}{e\sqrt{c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d)^2,x)
```

```
[Out] -c*a^2/(c*e*x+c*d)/e-c*b^2*arccosh(c*x)^2/e/(c*e*x+c*d)+2*c*b^2/e*arccosh(c
*x)/(c^2*d^2-e^2)^(1/2)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d
^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))-2*c*b^2/e*arccosh(c*x)/(c^2*d^2-
e^2)^(1/2)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))
/(c*d+(c^2*d^2-e^2)^(1/2)))+2*c*b^2/e/(c^2*d^2-e^2)^(1/2)*dilog((-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/
2)))-2*c*b^2/e/(c^2*d^2-e^2)^(1/2)*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))-2*c*a*b/(c*e*x+c*d)/
e*arccosh(c*x)-2*c*a*b/e^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(-2*(c^2*d*x-(c^2*
x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))/((c^2*d^2-e^2)/e^2
)^(1/2)/(c^2*x^2-1)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more d
etails)Is e-c*d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}{\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\,(cx)\right)^2}{\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x)**2, x)
```

3.26 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx$

Optimal. Leaf size=380

$$-\frac{bc\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc^3d\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}+1\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{bc^3d\left(a+b\cosh^{-1}(cx\right.}{e\left(c^2d^2\right.}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^2/e/(e*x+d)^2+b^2*c^2*ln(e*x+d)/e/(c^2*d^2-e^2)+b*c
^3*d*(a+b*arccosh(c*x))*ln(1+e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*
d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)-b*c^3*d*(a+b*arccosh(c*x))*ln(1+e*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^
(3/2)+b^2*c^3*d*polylog(2,-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(c*d-(c^2*d^
2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)-b^2*c^3*d*polylog(2,-e*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))/e/(c^2*d^2-e^2)^(3/2)-b*c*(c*
x+1)*(a+b*arccosh(c*x))*((c*x-1)/(c*x+1))^(1/2)/(c^2*d^2-e^2)/(e*x+d)
```

Rubi [A] time = 0.75, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5802, 5832, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{b^2c^3d\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{cd-\sqrt{c^2d^2-e^2}}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{b^2c^3d\text{PolyLog}\left(2,-\frac{ee^{\cosh^{-1}(cx)}}{\sqrt{c^2d^2-e^2}+cd}\right)}{e\left(c^2d^2-e^2\right)^{3/2}}-\frac{bc\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)}+\frac{bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -((b*c*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*(a + b*ArcCosh[c*x]))/((c^2*d
^2 - e^2)*(d + e*x))) - (a + b*ArcCosh[c*x])^2/(2*e*(d + e*x)^2) + (b*c^3*d
*(a + b*ArcCosh[c*x])*Log[1 + (e*E^ArcCosh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]
)])/(e*(c^2*d^2 - e^2)^(3/2)) - (b*c^3*d*(a + b*ArcCosh[c*x])*Log[1 + (e*E^
ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2])])/(e*(c^2*d^2 - e^2)^(3/2)) + (b^
2*c^2*Log[d + e*x])/(e*(c^2*d^2 - e^2)) + (b^2*c^3*d*PolyLog[2, -((e*E^ArcC
osh[c*x])/(c*d - Sqrt[c^2*d^2 - e^2]))])/(e*(c^2*d^2 - e^2)^(3/2)) - (b^2*c
^3*d*PolyLog[2, -((e*E^ArcCosh[c*x])/(c*d + Sqrt[c^2*d^2 - e^2]))])/(e*(c^2
*d^2 - e^2)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
```

```
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(d+ex)^3}\,dx = -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{(bc)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}(d+ex)^2}\,dx}{e}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{a+bx}{(cd+e\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{e}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(b^2c^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{\left(b^2c^2\right)\text{Subst}\left(\int\right.}{e\left(c^2d^2\right.}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{b^2c^2\log(d+ex)}{e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\cosh\right.}{e}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\cosh\right.}{e}$$

$$= -\frac{bc\sqrt{-\frac{1-cx}{1+cx}}(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2d^2-e^2\right)(d+ex)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2e(d+ex)^2} + \frac{bc^3d\left(a+b\cosh\right.}{e}$$

Mathematica [C] time = 7.78, size = 1093, normalized size = 2.88

$$-\frac{a^2}{2e(d+ex)^2}+bc^2\left(-\frac{\cosh^{-1}(cx)}{e(cd+cex)^2}+\frac{2cd\tanh^{-1}\left(\frac{\sqrt{cd-e}\sqrt{\frac{cx-1}{cx+1}}}{\sqrt{cd+e}}\right)}{(cd-e)^{3/2}e(cd+e)^{3/2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{(cd-e)(cd+e)(cd+cex)}\right)a+b^2c^2\left(-\frac{\cosh^{-1}(}{2e(cd+}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x)^3,x]
```

```
[Out] -1/2*a^2/(e*(d + e*x)^2) + a*b*c^2*(-((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c*d
- e)*(c*d + e)*(c*d + c*e*x))) - ArcCosh[c*x]/(e*(c*d + c*e*x)^2) + (2*c*d*
ArcTanh[(Sqrt[c*d - e]*Sqrt[(-1 + c*x)/(1 + c*x)])/Sqrt[c*d + e]])/((c*d -
e)^(3/2)*e*(c*d + e)^(3/2))) + b^2*c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*ArcCosh[c*x])/((c*d - e)*(c*d + e)*(c*d + c*e*x))) - ArcCosh[c*x]^2/(2
*e*(c*d + c*e*x)^2) + Log[1 + (e*x)/d]/(c^2*d^2*e - e^3) + (c*d*(2*ArcCosh[
c*x]*ArcTan[((c*d + e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] - (2*I
)*ArcCos[-((c*d)/e)]*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*
d^2) + e^2]] + (ArcCos[-((c*d)/e)] + 2*(ArcTan[((c*d + e)*Coth[ArcCosh[c*x]
/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/S
qrt[-(c^2*d^2) + e^2]]))*Log[Sqrt[-(c^2*d^2) + e^2]/(Sqrt[2]*Sqrt[e]*E^(Arc
Cosh[c*x]/2)*Sqrt[c*(d + e*x)])] + (ArcCos[-((c*d)/e)] - 2*(ArcTan[((c*d +
e)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]] + ArcTan[((-(c*d) + e)*Tan
h[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]]))*Log[(Sqrt[-(c^2*d^2) + e^2]*E^
```

```
(ArcCosh[c*x]/2))/(Sqrt[2]*Sqrt[e]*Sqrt[c*(d + e*x)])] - (ArcCos[-((c*d)/e)
] + 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^2) + e^2]])*L
og[((c*d + e)*(c*d - e + I*Sqrt[-(c^2*d^2) + e^2])*(-1 + Tanh[ArcCosh[c*x]/
2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcC
os[-((c*d)/e)] - 2*ArcTan[((-(c*d) + e)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*d^
2) + e^2]])*Log[((c*d + e)*(-(c*d) + e + I*Sqrt[-(c^2*d^2) + e^2])*(1 + Tan
h[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x
]/2]))] + I*(PolyLog[2, ((c*d - I*Sqrt[-(c^2*d^2) + e^2])*(c*d + e - I*Sqrt
[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e + I*Sqrt[-(c^2*d^2) +
 e^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*d + I*Sqrt[-(c^2*d^2) + e^2]
)*(c*d + e - I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))/(e*(c*d + e +
I*Sqrt[-(c^2*d^2) + e^2]*Tanh[ArcCosh[c*x]/2]))])))/(e*(-(c^2*d^2) + e^2)^(
3/2)))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arcosh}(cx)^2 + 2\,ab\operatorname{arcosh}(cx) + a^2}{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(e^3*x^3 + 3*d*e^2
*x^2 + 3*d^2*e*x + d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2}{(ex + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x + d)^3, x)
```

maple [B] time = 0.16, size = 1170, normalized size = 3.08

$$-\frac{c^2a^2}{2\,(cxe+cd)^2\,e}-\frac{c^4b^2\operatorname{arccosh}(cx)^2\,d^2}{2e\left(c^2d^2-e^2\right)(cxe+cd)^2}-\frac{c^3b^2\operatorname{arccosh}(cx)\,e\sqrt{cx+1}\,\sqrt{cx-1}\,x}{\left(c^2d^2-e^2\right)(cxe+cd)^2}-\frac{c^3b^2\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{c}}{\left(c^2d^2-e^2\right)(cxe+cd)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a^2/(c*e*x+c*d)^2/e-1/2*c^4*b^2*arccosh(c*x)^2/e/(c^2*d^2-e^2)/(c*
e*x+c*d)^2*d^2-c^3*b^2*arccosh(c*x)*e/(c^2*d^2-e^2)/(c*e*x+c*d)^2*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x-c^3*b^2*arccosh(c*x)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*d+c^4*b^2*arccosh(c*x)*e/(c^2*d^2-e^2)/(c*e*x+c*d)^2
*x^2+2*c^4*b^2*arccosh(c*x)/(c^2*d^2-e^2)/(c*e*x+c*d)^2*x*d+c^4*b^2*arccosh
(c*x)/e/(c^2*d^2-e^2)/(c*e*x+c*d)^2*d^2+1/2*c^2*b^2*arccosh(c*x)^2*e/(c^2*d
^2-e^2)/(c*e*x+c*d)^2+c^3*b^2/(c^2*d^2-e^2)^(3/2)/e*arccosh(c*x)*ln((-(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)
^(1/2)))*d-c^3*b^2/(c^2*d^2-e^2)^(3/2)/e*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/(c*d+(c^2*d^2-e^2)^(1/2)))*d+c^
3*b^2/(c^2*d^2-e^2)^(3/2)/e*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e-c*d
+(c^2*d^2-e^2)^(1/2))/(-c*d+(c^2*d^2-e^2)^(1/2)))*d-c^3*b^2/(c^2*d^2-e^2)^(
```

```
3/2)/e*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e+c*d+(c^2*d^2-e^2)^(1/2))/
(c*d+(c^2*d^2-e^2)^(1/2)))*d+c^2*b^2/(c^2*d^2-e^2)/e*ln(2*c*d*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+e)-2*c^2*b^2/(c^
2*d^2-e^2)/e*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-c^2*a*b/(c*e*x+c*d)^2/e*ar
ccosh(c*x)-c^4*a*b/e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/
(c*d-e)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1
/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+e)/(c*e*x+c*d))*x*d-c^4*a*b/e^2*(c*x+1)^(1/
2)*(c*x-1)^(1/2)/(c^2*x^2-1)^(1/2)/(c*d+e)/(c*d-e)/(c*e*x+c*d)/((c^2*d^2-e^
2)/e^2)^(1/2)*ln(-2*(c^2*d*x-(c^2*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*e+
e)/(c*e*x+c*d))*d^2-c^2*a*b*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*d+e)/(c*d-e)/(c*
e*x+c*d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c*d>0)', see `assume?` for more d
etails)Is e-c*d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2}{(d+e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x)^3,x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d + e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x)**3, x)
```

3.27 $\int \frac{(d+ex)^3}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=394

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{4bc^4}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^4}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx\right.}{4bc^4}$$

```
[Out] d^3*cosh(a/b)*Shi(a/b+arccosh(c*x))/b/c+3/4*d*e^2*cosh(a/b)*Shi(a/b+arccosh
(c*x))/b/c^3+3/2*d^2*e*cosh(2*a/b)*Shi(2*a/b+2*arccosh(c*x))/b/c^2+1/4*e^3*
cosh(2*a/b)*Shi(2*a/b+2*arccosh(c*x))/b/c^4+3/4*d*e^2*cosh(3*a/b)*Shi(3*a/b
+3*arccosh(c*x))/b/c^3+1/8*e^3*cosh(4*a/b)*Shi(4*a/b+4*arccosh(c*x))/b/c^4-
d^3*Chi(a/b+arccosh(c*x))*sinh(a/b)/b/c-3/4*d*e^2*Chi(a/b+arccosh(c*x))*sin
h(a/b)/b/c^3-3/2*d^2*e*Chi(2*a/b+2*arccosh(c*x))*sinh(2*a/b)/b/c^2-1/4*e^3*
Chi(2*a/b+2*arccosh(c*x))*sinh(2*a/b)/b/c^4-3/4*d*e^2*Chi(3*a/b+3*arccosh(c
*x))*sinh(3*a/b)/b/c^3-1/8*e^3*Chi(4*a/b+4*arccosh(c*x))*sinh(4*a/b)/b/c^4
```

Rubi [A] time = 1.17, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5806, 6742, 3303, 3298, 3301, 5448, 12}

$$-\frac{3d^2e\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{3de^2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{3de^2\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}\right.}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d^3*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (3*d*e^2*CoshIn
tegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (3*d^2*e*CoshIntegral[(2*
a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) - (e^3*CoshIntegral[(2*a)/b
 + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(4*b*c^4) - (3*d*e^2*CoshIntegral[(3*a)/b
 + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) - (e^3*CoshIntegral[(4*a)/b + 4
*ArcCosh[c*x]]*Sinh[(4*a)/b])/(8*b*c^4) + (d^3*Cosh[a/b]*SinhIntegral[a/b +
 ArcCosh[c*x]])/(b*c) + (3*d*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
)/(4*b*c^3) + (3*d^2*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]]
)/(2*b*c^2) + (e^3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(4
*b*c^4) + (3*d*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4
*b*c^3) + (e^3*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c
^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*(c*d + e*Cosh[x])^m*Sin
h[x], x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+b\cosh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{(cd+e\cosh(x))^3\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^3d^3\sinh(x)}{a+bx} + \frac{3c^2d^2e\cosh(x)\sinh(x)}{a+bx} + \frac{3cde^2\cosh^2(x)\sinh(x)}{a+bx} + \frac{e^3\cosh^3(x)\sinh(x)}{a+bx}\right) dx, \ldots}{c^4}$$

$$= \frac{d^3\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} + \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c\ldots\right)}{c^2}$$

$$= \frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \frac{\left(3de^2\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)} + \frac{\sinh(3x)}{4(a+bx)}\right) dx, \ldots\right)}{c^3}$$

$$= -\frac{d^3\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc} + \frac{\left(3d^2e\right)\text{Su}\ldots}{\ldots}$$

$$= -\frac{d^3\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc} + \frac{\left(3de^2\cosh\ldots\right.}{\ldots}$$

$$= -\frac{d^3\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{3de^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{3d^2e\text{Chi}\ldots}{\ldots}$$

Mathematica [A] time = 0.61, size = 287, normalized size = 0.73

$$\frac{8c^3d^3\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) - 2cd\sinh\left(\frac{a}{b}\right)\left(4c^2d^2+3e^2\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) - 2e\sinh\left(\frac{2a}{b}\right)\left(6c^2d^2\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*c*d*(4*c^2*d^2 + 3*e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - 2*
e*(6*c^2*d^2 + e^2)*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - 6*
c*d*e^2*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - e^3*CoshIntegr
al[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] + 8*c^3*d^3*Cosh[a/b]*SinhIntegral
[a/b + ArcCosh[c*x]] + 6*c*d*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
 + 12*c^2*d^2*e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 2*e^3*
Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 6*c*d*e^2*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + e^3*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])])/(8*b*c^4)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^3x^3 + 3\,de^2x^2 + 3\,d^2ex + d^3}{b\,\text{arcosh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3}{b\,\text{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.56, size = 394, normalized size = 1.00

$$-\frac{e^3\mathrm{e}^{-\frac{4a}{b}}\,\mathrm{Ei}\left(1,-4\,\mathrm{arccosh}(cx)-\frac{4a}{b}\right)}{16c^3b}+\frac{e^3\mathrm{e}^{\frac{4a}{b}}\,\mathrm{Ei}\left(1,4\,\mathrm{arccosh}(cx)+\frac{4a}{b}\right)}{16c^3b}+\frac{3e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arccosh}(cx)+\frac{2a}{b}\right)d^2}{4cb}+\frac{e^3\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arccosh}(cx)+\frac{2a}{b}\right)}{8c^3b}-\frac{3e\,\mathrm{e}^{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-1/16/c^3*e^3/b*exp(-4*a/b)*Ei(1,-4*arccosh(c*x)-4*a/b)+1/16/c^3*e^3/b
*exp(4*a/b)*Ei(1,4*arccosh(c*x)+4*a/b)+3/4/c*e/b*exp(2*a/b)*Ei(1,2*arccosh(
c*x)+2*a/b)*d^2+1/8/c^3*e^3/b*exp(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b)-3/4/c*e
/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)*d^2-1/8/c^3*e^3/b*exp(-2*a/b)*Ei
(1,-2*arccosh(c*x)-2*a/b)-3/8/c^2*d*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-
3*a/b)+1/2*d^3/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)+3/8/c^2*d/b*exp(a/b)*Ei(1,
arccosh(c*x)+a/b)*e^2-1/2*d^3/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)-3/8/c^2*d
/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e^2+3/8/c^2*d*e^2/b*exp(3*a/b)*Ei(1,3*
arccosh(c*x)+3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^3}{b\,\text{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^3/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{a+b\,\mathrm{acosh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*acosh(c*x)),x)
[Out] int((d + e*x)^3/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*acosh(c*x)),x)
[Out] Integral((d + e*x)**3/(a + b*acosh(c*x)), x)
```

3.28 $\int \frac{(d+ex)^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=245

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}+\frac{e^2\cdots}{}$$

```
[Out] d^2*cosh(a/b)*Shi(a/b+arccosh(c*x))/b/c+1/4*e^2*cosh(a/b)*Shi(a/b+arccosh(c
*x))/b/c^3+d*e*cosh(2*a/b)*Shi(2*a/b+2*arccosh(c*x))/b/c^2+1/4*e^2*cosh(3*a
/b)*Shi(3*a/b+3*arccosh(c*x))/b/c^3-d^2*Chi(a/b+arccosh(c*x))*sinh(a/b)/b/c
-1/4*e^2*Chi(a/b+arccosh(c*x))*sinh(a/b)/b/c^3-d*e*Chi(2*a/b+2*arccosh(c*x)
)*sinh(2*a/b)/b/c^2-1/4*e^2*Chi(3*a/b+3*arccosh(c*x))*sinh(3*a/b)/b/c^3
```

Rubi [A] time = 0.70, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5806, 6742, 3303, 3298, 3301, 5448}

$$-\frac{de\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{bc^2}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d^2*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (e^2*CoshIntegr
al[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (d*e*CoshIntegral[(2*a)/b + 2
*ArcCosh[c*x]]*Sinh[(2*a)/b])/(b*c^2) - (e^2*CoshIntegral[(3*a)/b + 3*ArcCo
sh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) + (d^2*Cosh[a/b]*SinhIntegral[a/b + ArcCo
sh[c*x]])/(b*c) + (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b*c^3
) + (d*e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b*c^2) + (e
^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*(c*d + e*Cosh[x])^m*Sin
h[x], x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b\cosh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{(cd+e\cosh(x))^2\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c^2d^2\sinh(x)}{a+bx}+\frac{e^2\cosh^2(x)\sinh(x)}{a+bx}+\frac{cde\sinh(2x)}{a+bx}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3}$$

$$= \frac{d^2\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} + \frac{(de)\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \ldots$$

$$= \frac{e^2\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3} + \frac{\left(d^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right.}{a+bx}\ldots\right.}{c}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{de\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} + \frac{d^2\cosh\ldots}{\ldots}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{de\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{bc^2} + \frac{d^2\cosh\ldots}{\ldots}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{e^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{de\text{Chi}\left(\frac{2a}{b}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.33, size = 187, normalized size = 0.76

$$\frac{-\sinh\left(\frac{a}{b}\right)\left(4c^2d^2+e^2\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4c^2d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-4cde\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-((4*c^2*d^2 + e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b]) - 4*c*d*e*
CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - e^2*CoshIntegral[3*(a/
b + ArcCosh[c*x])]*Sinh[(3*a)/b] + 4*c^2*d^2*Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c*x]] + e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 4*c*d*e*Cos
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + e^2*Cosh[(3*a)/b]*SinhInt
egral[3*(a/b + ArcCosh[c*x])])/(4*b*c^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^2+2\,dex+d^2}{b\,\text{arcosh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{b \operatorname{arcosh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.42, size = 254, normalized size = 1.04

$$\frac{-\frac{e^2 e^{-\frac{3a}{b}} \operatorname{Ei}\left(1,-3\operatorname{arccosh}(cx)-\frac{3a}{b}\right)}{8c^2 b} + \frac{e^2 e^{\frac{3a}{b}} \operatorname{Ei}\left(1,3\operatorname{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2 b} + \frac{e^{\frac{a}{b}} \operatorname{Ei}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right) d^2}{2b} + \frac{e^{\frac{a}{b}} \operatorname{Ei}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right) e^2}{8c^2 b} - \frac{e^{-\frac{a}{b}} \operatorname{Ei}\left(1,-\operatorname{arc}\right.}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-1/8/c^2*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/8/c^2*e^2/b*e
xp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*
d^2+1/8/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arc
cosh(c*x)-a/b)*d^2-1/8/c^2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e^2-1/2/c*d*
e/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)+1/2/c*d*e/b*exp(2*a/b)*Ei(1,2*a
rccosh(c*x)+2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{b \operatorname{arcosh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^2/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{a + b \operatorname{acosh}(c\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*acosh(c*x)),x)
```

```
[Out] int((d + e*x)^2/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{a + b \operatorname{acosh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)**2/(a + b*acosh(c*x)), x)
```

3.29 $\int \frac{d+ex}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=116

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}-$$

```
[Out] d*cosh(a/b)*Shi(a/b+arccosh(c*x))/b/c+1/2*e*cosh(2*a/b)*Shi(2*a/b+2*arccosh
(c*x))/b/c^2-d*Chi(a/b+arccosh(c*x))*sinh(a/b)/b/c-1/2*e*Chi(2*a/b+2*arccos
h(c*x))*sinh(2*a/b)/b/c^2
```

Rubi [A] time = 0.34, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5806, 6742, 3303, 3298, 3301, 5448, 12}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^2}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(b*c)) - (e*CoshIntegral[(
2*a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(2*b*c^2) + (d*Cosh[a/b]*SinhIntegr
al[a/b + ArcCosh[c*x]])/(b*c) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*A
rcCosh[c*x]])/(2*b*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5806

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Dist[1/c^(m + 1), Subst[Int[(a + b*x)^n*(c*d + e*Cosh[x])^m*Sin
h[x], x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[m, 0
]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{d+ex}{a+b\cosh^{-1}(cx)}\,dx = \frac{\text{Subst}\left(\int \frac{(cd+e\cosh(x))\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{cd\sinh(x)}{a+bx}+\frac{e\cosh(x)\sinh(x)}{a+bx}\right)dx, x, \cosh^{-1}(cx)\right)}{c^2}$$

$$= \frac{d\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(cx)\right)}{c^2} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{c}$$

$$= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\sinh(}{a+b}\right.}{}$$

$$= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc} + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{S}}{}$$

$$= -\frac{d\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{e\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2bc^2} + \frac{d\cosh\left(\frac{a}{b}\right)\text{S}}{}$$

Mathematica [A] time = 0.14, size = 98, normalized size = 0.84

$$\frac{-2cd\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) - e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + 2cd\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{2bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*c*d*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - e*CoshIntegral[2*(a/b
+ ArcCosh[c*x])]*Sinh[(2*a)/b] + 2*c*d*Cosh[a/b]*SinhIntegral[a/b + ArcCosh
[c*x]] + e*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])])/(2*b*c^2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex+d}{b\,\text{arcosh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex + d}{b \operatorname{arcosh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.29, size = 120, normalized size = 1.03

$$\frac{\frac{e^{\frac{a}{b}} \operatorname{Ei}\left(1, \operatorname{arccosh}(cx)+\frac{a}{b}\right) d}{2b} - \frac{e^{-\frac{a}{b}} \operatorname{Ei}\left(1, -\operatorname{arccosh}(cx)-\frac{a}{b}\right) d}{2b} - \frac{e\, e^{-\frac{2a}{b}} \operatorname{Ei}\left(1, -2\operatorname{arccosh}(cx)-\frac{2a}{b}\right)}{4cb} + \frac{e\, e^{\frac{2a}{b}} \operatorname{Ei}\left(1, 2\operatorname{arccosh}(cx)+\frac{2a}{b}\right)}{4cb}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d-1/2/b*exp(-a/b)*Ei(1,-arccosh(
c*x)-a/b)*d-1/4/c*e/b*exp(-2*a/b)*Ei(1,-2*arccosh(c*x)-2*a/b)+1/4/c*e/b*exp
(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex + d}{b \operatorname{arcosh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d + e\,x}{a + b\,\operatorname{acosh}(c\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*acosh(c*x)),x)
```

```
[Out] int((d + e*x)/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{a + b \operatorname{acosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)/(a + b*acosh(c*x)), x)
```

3.30 $\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx = \int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{aex+ad+(bex+bd)\,\text{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)(b\,\text{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x + d)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\,(a+b\,\mathrm{arccosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x+d)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + d)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(c\,x))\,(d+e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))*(d + e*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(cx))\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x)), x)
```

3.31 $\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ae^2x^2+2\,adex+ad^2+\left(be^2x^2+2\,bdex+bd^2\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e^2*x^2 + 2*a*d*e*x + a*d^2 + (b*e^2*x^2 + 2*b*d*e*x + b*d^2)
*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2 (a + b \operatorname{arccosh}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arccosh(c*x)),x)
[Out] int(1/(e*x+d)^2/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2 (b \operatorname{arcosh}(cx) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x + d)^2*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(a + b \operatorname{acosh}(c\,x))\,(d + e\,x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x)^2),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \operatorname{acosh}(cx)) (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*acosh(c*x)),x)
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x)**2), x)
```

3.32 $\int \frac{(d+ex)^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=374

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e^2\sinh\left(\frac{3a}{b}\right.}{}$$

```
[Out] d^2*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c+1/4*e^2*Chi((a+b*arccosh(c*x)
)/b)*cosh(a/b)/b^2/c^3+2*d*e*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)/b^2/c^
2+3/4*e^2*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)/b^2/c^3-d^2*Shi((a+b*arcc
osh(c*x))/b)*sinh(a/b)/b^2/c-1/4*e^2*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^
2/c^3-2*d*e*Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c^2-3/4*e^2*Shi(3*(
a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/
c/(a+b*arccosh(c*x))-2*d*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c
*x))-e^2*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 0.75, antiderivative size = 366, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5804, 5656, 5781, 3303, 3298, 3301, 5666}

$$\frac{2de\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^2}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^3}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx\right.}{4b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (2*d*e*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) - (e^2*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d^2*Cosh[a/b]*CoshInt
egral[a/b + ArcCosh[c*x]])/(b^2*c) + (e^2*Cosh[a/b]*CoshIntegral[a/b + ArcC
osh[c*x]])/(4*b^2*c^3) + (2*d*e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcC
osh[c*x]])/(b^2*c^2) + (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCos
h[c*x]])/(4*b^2*c^3) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(b^
2*c) - (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b^2*c^3) - (2*d*
e*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c^2) - (3*e^2*
Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b^2*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5804

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcCosh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d^2}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{2dex}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{e^2x^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx$$

$$= d^2\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx+(2de)\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx+e^2\int \frac{x}{\left(a+b\cos\right.}$$

$$= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{(c}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d^2}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{2d}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{2dex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{e^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d^2}{}$$

Mathematica [A] time = 1.46, size = 530, normalized size = 1.42

$$-\frac{-\cosh\left(\frac{a}{b}\right)\left(4c^2d^2+e^2\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4ac^2d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4bc^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/4*(4*b*c^2*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*b*c^3*d^2*x*Sqrt[(-1 + c*x
)/(1 + c*x)] + 8*b*c^2*d*e*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*b*c^3*d*e*x^2*S
qrt[(-1 + c*x)/(1 + c*x)] + 4*b*c^2*e^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*
b*c^3*e^2*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] - (4*c^2*d^2 + e^2)*(a + b*ArcCosh
[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] - 8*c*d*e*(a + b*ArcCosh[
c*x])*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] - 3*a*e^2*Cosh[(3*
a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*b*e^2*ArcCosh[c*x]*Cosh[(3*a
)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + 4*a*c^2*d^2*Sinh[a/b]*SinhInteg
ral[a/b + ArcCosh[c*x]] + a*e^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
+ 4*b*c^2*d^2*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + b*e
^2*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*a*c*d*e*Sinh
[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 8*b*c*d*e*ArcCosh[c*x]*Sin
h[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 3*a*e^2*Sinh[(3*a)/b]*Sin
hIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*e^2*ArcCosh[c*x]*Sinh[(3*a)/b]*Sinh
Integral[3*(a/b + ArcCosh[c*x])])/(b^2*c^3*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^2+2\,dex+d^2}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^2 + 2*d*e*x + d^2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x)
 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex+d)^2}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2/(b*arccosh(c*x) + a)^2, x)
```

maple [A] time = 0.49, size = 649, normalized size = 1.74

$$\frac{\frac{\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2+\sqrt{cx-1}\,\sqrt{cx+1}+4c^3x^3-3cx\right)e^2}{8c^2b(a+b\operatorname{arccosh}(cx))}-\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\operatorname{Ei}\left(1,3\operatorname{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e^2\left(4c^3x^3-3cx+4\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-\sqrt{cx-1}\,\sqrt{cx+1}\right)}{8b\,c^2(a+b\operatorname{arccosh}(cx))}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/8*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c^3*x^3-3*c*x)*e^2/c^2/b/(a+b*arccosh(c*x))-3/8*e^2/c^2/b^2*exp(3*a/b)*
```

```
Ei(1,3*arccosh(c*x)+3*a/b)-1/8/b*e^2/c^2*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-3/8/b^
2*e^2/c^2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+
1)^(1/2)+c*x)*d^2/b/(a+b*arccosh(c*x))-1/2*d^2/b^2*exp(a/b)*Ei(1,arccosh(c*
x)+a/b)+1/8*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e^2/c^2/b/(a+b*arccosh(c*x))
-1/8/c^2*e^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*d^2*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arccosh(c*
x)-a/b)-1/8/c^2/b*e^2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-
1/8/c^2/b^2*e^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)+1/2*(-2*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+2*c^2*x^2-1)*d*e/c/(a+b*arccosh(c*x))/b-e*d/c/b^2*exp(2*a/b)
*Ei(1,2*arccosh(c*x)+2*a/b)-1/2/b*e*d/c*(2*c^2*x^2-1+2*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c)/(a+b*arccosh(c*x))-1/b^2*e*d/c*exp(-2*a/b)*Ei(1,-2*arccosh(c*x
)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3e^2x^5 + 2\,c^3dex^4 - 2\,cdex^2 - cd^2x + \left(c^3d^2 - ce^2\right)x^3 + \left(c^2e^2x^4 + 2\,c^2dex^3 - 2\,dex + \left(c^2d^2 - e^2\right)x^2 - d^2\right)\sqrt{cx +}}{abc^3x^2 + \sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x - abc + \left(b^2c^3x^2 + \sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x - b^2c\right)\log\left(cx + \sqrt{cx+1}\,\sqrt{cx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^5 + 2*c^3*d*e*x^4 - 2*c*d*e*x^2 - c*d^2*x + (c^3*d^2 - c*e^2)*x
^3 + (c^2*e^2*x^4 + 2*c^2*d*e*x^3 - 2*d*e*x + (c^2*d^2 - e^2)*x^2 - d^2)*sq
rt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c
^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*c^5*e^2*x^6 + 4*c^
5*d*e*x^5 - 8*c^3*d*e*x^3 + (c^5*d^2 - 6*c^3*e^2)*x^4 + 4*c*d*e*x + (3*c^3*
e^2*x^4 + 4*c^3*d*e*x^3 + c*d^2 + (c^3*d^2 - c*e^2)*x^2)*(c*x + 1)*(c*x - 1
) + c*d^2 - (2*c^3*d^2 - 3*c*e^2)*x^2 + (6*c^4*e^2*x^5 + 8*c^4*d*e*x^4 - 8*
c^2*d*e*x^2 + (2*c^4*d^2 - 7*c^2*e^2)*x^3 + 2*d*e - (c^2*d^2 - 2*e^2)*x)*sq
rt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 -
 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x
 - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^
2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d+e\,x\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x)^2/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex\right)^{2}}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**2/(a + b*acosh(c*x))**2, x)
```

3.33 $\int \frac{d+ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=190

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{d\sinh\left(\frac{a}{b}\right)\text{Sh}}{}$$

```
[Out] d*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c+e*Chi(2*(a+b*arccosh(c*x))/b)*c
osh(2*a/b)/b^2/c^2-d*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c-e*Shi(2*(a+b
*arccosh(c*x))/b)*sinh(2*a/b)/b^2/c^2-d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+
b*arccosh(c*x))-e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 0.47, antiderivative size = 186, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5804, 5656, 5781, 3303, 3298, 3301, 5666}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^2}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^2}+\frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}-\frac{d\,\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (e*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d*Cosh[a/b]*CoshInte
gral[a/b + ArcCosh[c*x]])/(b^2*c) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b +
 2*ArcCosh[c*x]])/(b^2*c^2) - (d*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
)/(b^2*c) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c
^2)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5804

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*ArcCosh[c*x])^n, x], x] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{ex}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx\\
&= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx+e\int \frac{x}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx\\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{(cd)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx\right.}}{b}\\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d\,\mathrm{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\cosh^{-1}\right.}{bc}\\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{e\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}\right.}{b^2c^2}\\
&= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}-\frac{ex\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{d\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx\right.}{b^2c}
\end{aligned}$$

Mathematica [A] time = 0.79, size = 268, normalized size = 1.41

$$-\frac{-cd\cosh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-e\cosh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((b*c*d*Sqrt[(-1 + c*x)/(1 + c*x)] + b*c^2*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]
+ b*c*e*x*Sqrt[(-1 + c*x)/(1 + c*x)] + b*c^2*e*x^2*Sqrt[(-1 + c*x)/(1 + c*x
)] - c*d*(a + b*ArcCosh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] -
e*(a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] +
 a*c*d*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + b*c*d*ArcCosh[c*x]*Sinh
[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + a*e*Sinh[(2*a)/b]*SinhIntegral[2*(
a/b + ArcCosh[c*x])] + b*e*ArcCosh[c*x]*Sinh[(2*a)/b]*SinhIntegral[2*(a/b +
 ArcCosh[c*x])])/(b^2*c^2*(a + b*ArcCosh[c*x])))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ex+d}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ex+d}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)/(b*arccosh(c*x) + a)^2, x)
```

maple [A] time = 0.33, size = 285, normalized size = 1.50

$$\frac{\frac{\left(-\sqrt{cx-1}\,\sqrt{cx+1}+cx\right)d}{2b(a+b\operatorname{arccosh}(cx))}-\frac{e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right)d}{2b^2}-\frac{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)d}{2b(a+b\operatorname{arccosh}(cx))}-\frac{e^{-\frac{a}{b}}\operatorname{Ei}\left(1,-\operatorname{arccosh}(cx)-\frac{a}{b}\right)d}{2b^2}+\frac{\left(-2\sqrt{cx+1}\,\sqrt{cx-1}\,xc+2c^2x^2-1\right)e}{4c(a+b\operatorname{arccosh}(cx))b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*d/b/(a+b*arccosh(c*x))-1/2/b^2*
exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(
a+b*arccosh(c*x))*d-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d+1/4*(-2*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c+2*c^2*x^2-1)*e/c/(a+b*arccosh(c*x))/b-1/2/c*e/
b^2*exp(2*a/b)*Ei(1,2*arccosh(c*x)+2*a/b)-1/4/c*e/b*(2*c^2*x^2-1+2*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x*c)/(a+b*arccosh(c*x))-1/2/c*e/b^2*exp(-2*a/b)*Ei(1,-2
*arccosh(c*x)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3ex^4+c^3dx^3-cex^2-cdx+\left(c^2ex^3+c^2dx^2-ex-d\right)\sqrt{cx+1}\,\sqrt{cx-1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^4 + c^3*d*x^3 - c*e*x^2 - c*d*x + (c^2*e*x^3 + c^2*d*x^2 - e*x -
d)*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*
a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x -
b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((2*c^5*e*x^5 + c
```

```
^5*d*x^4 - 4*c^3*e*x^3 - 2*c^3*d*x^2 + (2*c^3*e*x^3 + c^3*d*x^2 + c*d)*(c*x
 + 1)*(c*x - 1) + 2*c*e*x + (4*c^4*e*x^4 + 2*c^4*d*x^3 - 4*c^2*e*x^2 - c^2*
d*x + e)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*x^4 + (c*x + 1)*(c*x -
 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(
c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2
*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x -
 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x)/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)/(a + b*acosh(c*x))**2, x)
```

3.34 $\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 10.17, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/((d + e*x)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2ex+a^2d+\left(b^2ex+b^2d\right)\text{arcosh}\left(cx\right)^2+2\left(abex+abd\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arccosh(c*x)^2 + 2*(a*b*e*x
 + a*b*d)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)(b\,\text{arcosh}\left(cx\right)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x + d)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)\,(a+b\,\mathrm{arccosh}\,(cx))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arccosh(c*x))^2,x)
[Out] int(1/(e*x+d)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\,\sqrt{cx-1}\,-cx}{abc^3ex^3+abc^3dx^2-abcex-abcd+\left(abc^2ex^2+abc^2dx\right)\sqrt{cx+1}\,\sqrt{cx-1}\,+\left(b^2c^3ex^3+b^2c^3dx^2-b^2cex-b^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*e*x^3
 + a*b*c^3*d*x^2 - a*b*c*e*x - a*b*c*d + (a*b*c^2*e*x^2 + a*b*c^2*d*x)*sqrt
(c*x + 1)*sqrt(c*x - 1) + (b^2*c^3*e*x^3 + b^2*c^3*d*x^2 - b^2*c*e*x - b^2*
c*d + (b^2*c^2*e*x^2 + b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((c^5*d*x^4 - 2*c^3*d*x^2 + (c^3*d
*x^2 + 2*c*e*x + c*d)*(c*x + 1)*(c*x - 1) + (2*c^4*d*x^3 + 2*c^2*e*x^2 - c^
2*d*x - e)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*e^2*x^6 + 2*a*b*c^5*
d*e*x^5 - 4*a*b*c^3*d*e*x^3 + (c^5*d^2 - 2*c^3*e^2)*a*b*x^4 + 2*a*b*c*d*e*x
 + a*b*c*d^2 - (2*c^3*d^2 - c*e^2)*a*b*x^2 + (a*b*c^3*e^2*x^4 + 2*a*b*c^3*d
*e*x^3 + a*b*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*b*c^4*e^2*x^5 + 2*a*b*
c^4*d*e*x^4 - 2*a*b*c^2*d*e*x^2 - a*b*c^2*d^2*x + (c^4*d^2 - c^2*e^2)*a*b*x
^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e^2*x^6 + 2*b^2*c^5*d*e*x^5 - 4*
b^2*c^3*d*e*x^3 + (c^5*d^2 - 2*c^3*e^2)*b^2*x^4 + 2*b^2*c*d*e*x + b^2*c*d^2
 - (2*c^3*d^2 - c*e^2)*b^2*x^2 + (b^2*c^3*e^2*x^4 + 2*b^2*c^3*d*e*x^3 + b^2
*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^2*x^5 + 2*b^2*c^4*d*e*x^4
- 2*b^2*c^2*d*e*x^2 - b^2*c^2*d^2*x + (c^4*d^2 - c^2*e^2)*b^2*x^3)*sqrt(c*x
 + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x)),x)
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\,(cx)\right)^{2}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*acosh(c*x))**2,x)
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x)), x)
```

3.35 $\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 103.18, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/((d + e*x)^2*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2e^2x^2+2\,a^2dex+a^2d^2+\left(b^2e^2x^2+2\,b^2dex+b^2d^2\right)\operatorname{arcosh}(cx)^2+2\left(abe^2x^2+2\,abdex+abd^2\right)\operatorname{arcosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e^2*x^2 + 2*a^2*d*e*x + a^2*d^2 + (b^2*e^2*x^2 + 2*b^2*d*e*
x + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*e^2*x^2 + 2*a*b*d*e*x + a*b*d^2)*arcco
sh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+d)^2(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)^2*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex+d\right)^{2} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{abc^{3}e^{2}x^{4}+2\,abc^{3}dex^{3}-2\,abcdex-abcd^{2}+\left(c^{3}d^{2}-ce^{2}\right)abx^{2}+\left(abc^{2}e^{2}x^{3}+2\,abc^{2}dex^{2}+abc^{2}d^{2}x\right)\sqrt{cx+1}\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*e^2*x
^4 + 2*a*b*c^3*d*e*x^3 - 2*a*b*c*d*e*x - a*b*c*d^2 + (c^3*d^2 - c*e^2)*a*b*
x^2 + (a*b*c^2*e^2*x^3 + 2*a*b*c^2*d*e*x^2 + a*b*c^2*d^2*x)*sqrt(c*x + 1)*s
qrt(c*x - 1) + (b^2*c^3*e^2*x^4 + 2*b^2*c^3*d*e*x^3 - 2*b^2*c*d*e*x - b^2*c
*d^2 + (c^3*d^2 - c*e^2)*b^2*x^2 + (b^2*c^2*e^2*x^3 + 2*b^2*c^2*d*e*x^2 + b
^2*c^2*d^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1))) - integrate((c^5*e*x^5 - c^5*d*x^4 - 2*c^3*e*x^3 + 2*c^3*d*x^2 + (c
^3*e*x^3 - c^3*d*x^2 - 3*c*e*x - c*d)*(c*x + 1)*(c*x - 1) + c*e*x + (2*c^4*
e*x^4 - 2*c^4*d*x^3 - 5*c^2*e*x^2 + c^2*d*x + 2*e)*sqrt(c*x + 1)*sqrt(c*x -
 1) - c*d)/(a*b*c^5*e^3*x^7 + 3*a*b*c^5*d*e^2*x^6 + (3*c^5*d^2*e - 2*c^3*e^
3)*a*b*x^5 + 3*a*b*c*d^2*e*x + (c^5*d^3 - 6*c^3*d*e^2)*a*b*x^4 + a*b*c*d^3
- (6*c^3*d^2*e - c*e^3)*a*b*x^3 - (2*c^3*d^3 - 3*c*d*e^2)*a*b*x^2 + (a*b*c^
3*e^3*x^5 + 3*a*b*c^3*d*e^2*x^4 + 3*a*b*c^3*d^2*e*x^3 + a*b*c^3*d^3*x^2)*(c
*x + 1)*(c*x - 1) + 2*(a*b*c^4*e^3*x^6 + 3*a*b*c^4*d*e^2*x^5 - 3*a*b*c^2*d^
2*e*x^2 - a*b*c^2*d^3*x + (3*c^4*d^2*e - c^2*e^3)*a*b*x^4 + (c^4*d^3 - 3*c^
2*d*e^2)*a*b*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e^3*x^7 + 3*b^2*c^
5*d*e^2*x^6 + (3*c^5*d^2*e - 2*c^3*e^3)*b^2*x^5 + 3*b^2*c*d^2*e*x + (c^5*d^
3 - 6*c^3*d*e^2)*b^2*x^4 + b^2*c*d^3 - (6*c^3*d^2*e - c*e^3)*b^2*x^3 - (2*c
^3*d^3 - 3*c*d*e^2)*b^2*x^2 + (b^2*c^3*e^3*x^5 + 3*b^2*c^3*d*e^2*x^4 + 3*b^
2*c^3*d^2*e*x^3 + b^2*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^3*x^6
 + 3*b^2*c^4*d*e^2*x^5 - 3*b^2*c^2*d^2*e*x^2 - b^2*c^2*d^3*x + (3*c^4*d^2*e
 - c^2*e^3)*b^2*x^4 + (c^4*d^3 - 3*c^2*d*e^2)*b^2*x^3)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d+e\,x\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x)^2),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x)^2), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x)**2), x)
```

3.36 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)^3}{e(m+1)}-\frac{3bc\text{Int}\left(\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}},x\right)}{e(m+1)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arccosh(c*x))^3/e/(1+m)-3*b*c*Unintegrable((e*x+d)^(1+m)
*(a+b*arccosh(c*x))^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)/e/(1+m)
```

Rubi [A] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x])^3,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^3)/(e*(1 + m)) - (3*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
x])/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx=\frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^3}{e(1+m)}-\frac{(3bc)\int \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} \, dx}{e(1+m)}$$

Mathematica [A] time = 6.70, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^3,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^3, x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\,\text{arcosh}\,(cx)^3+3\,ab^2\,\text{arcosh}\,(cx)^2+3\,a^2b\,\text{arcosh}\,(cx)+a^3\right)(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(c*x)^3 + 3*a*b^2*arccosh(c*x)^2 + 3*a^2*b*arccosh(c*x
) + a^3)*(e*x + d)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arcosh}\,(cx)+a)^3(ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(e*x + d)^m, x)
```

maple [A] time = 4.04, size = 0, normalized size = 0.00

$$\int (ex+d)^m (a+b\,\text{arccosh}\,(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x))^3,x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3ex+b^3d\right)(ex+d)^m \log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^3}{e(m+1)}+\frac{(ex+d)^{m+1}a^3}{e(m+1)}+\int -\frac{3\left(\left(\left(b^3c^2dx+ab^2e(m+1)-\left(ab^2c^2e(m\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^3,x, algorithm="maxima")
```

```
[Out] (b^3*e*x + b^3*d)*(e*x + d)^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^3/(e*(
m + 1)) + (e*x + d)^(m + 1)*a^3/(e*(m + 1)) + integrate(-3*(((b^3*c^2*d*x +
 a*b^2*e*(m + 1) - (a*b^2*c^2*e*(m + 1) - b^3*c^2*e)*x^2)*sqrt(c*x + 1)*sqr
t(c*x - 1)*(e*x + d)^m + (b^3*c^3*d*x^2 - b^3*c*d - (a*b^2*c^3*e*(m + 1) -
b^3*c^3*e)*x^3 + (a*b^2*c*e*(m + 1) - b^3*c*e)*x)*(e*x + d)^m)*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))^2 - ((a^2*b*c^2*e*(m + 1)*x^2 - a^2*b*e*(m + 1))
*sqrt(c*x + 1)*sqrt(c*x - 1)*(e*x + d)^m + (a^2*b*c^3*e*(m + 1)*x^3 - a^2*b
*c*e*(m + 1)*x)*(e*x + d)^m)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^3*e
*(m + 1)*x^3 - c*e*(m + 1)*x + (c^2*e*(m + 1)*x^2 - e*(m + 1))*sqrt(c*x + 1
)*sqrt(c*x - 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\text{acosh}\,(c\,x))^3\,(d+e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^3*(d + e*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^3*(d + e*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\text{acosh}\,(cx))^3\,(d+ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x))**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))**3*(d + e*x)**m, x)
```

3.37 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=80

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)^2}{e(m+1)}-\frac{2bc\text{Int}\left(\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}},x\right)}{e(m+1)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arccosh(c*x))^2/e/(1+m)-2*b*c*Unintegrable((e*x+d)^(1+m)
*(a+b*arccosh(c*x))/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)/e/(1+m)
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x])^2)/(e*(1 + m)) - (2*b*c*Defer[Int][
((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
)/(e*(1 + m))
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)^2}{e(1+m)}-\frac{(2bc)\int \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} \, dx}{e(1+m)}$$

Mathematica [A] time = 0.20, size = 0, normalized size = 0.00

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{arcosh}(cx)^2 + 2\,ab \operatorname{arcosh}(cx) + a^2\right)(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*(e*x + d)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(cx) + a)^2 (ex+d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(e*x + d)^m, x)
```

maple [A] time = 4.23, size = 0, normalized size = 0.00

$$\int \left(ex+d\right)^{m} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2ex+b^2d\right)(ex+d)^m\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{e(m+1)}+\frac{(ex+d)^{m+1}a^2}{e(m+1)}+\int -\frac{2\left(\left(b^2c^2dx+abe(m+1)-\left(abc^2e(m+1\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (b^2*e*x + b^2*d)*(e*x + d)^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(e*(
m + 1)) + (e*x + d)^(m + 1)*a^2/(e*(m + 1)) + integrate(-2*((b^2*c^2*d*x +
a*b*e*(m + 1) - (a*b*c^2*e*(m + 1) - b^2*c^2*e)*x^2)*sqrt(c*x + 1)*sqrt(c*x
 - 1)*(e*x + d)^m + (b^2*c^3*d*x^2 - b^2*c*d - (a*b*c^3*e*(m + 1) - b^2*c^3
*e)*x^3 + (a*b*c*e*(m + 1) - b^2*c*e)*x)*(e*x + d)^m)*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))/(c^3*e*(m + 1)*x^3 - c*e*(m + 1)*x + (c^2*e*(m + 1)*x^2 -
 e*(m + 1))*sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d+e\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{2}\left(d+ex\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**2*(d + e*x)**m, x)
```

3.38 $\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=125

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{e(m+1)}-\frac{\sqrt{2}b\sqrt{cx-1}\,(cd+e)(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-cx),\frac{e(1-cx)}{cd+e}\right)}{ce(m+1)}$$

```
[Out] (e*x+d)^(1+m)*(a+b*arccosh(c*x))/e/(1+m)-b*(c*d+e)*(e*x+d)^m*AppellF1(1/2,-
1-m,1/2,3/2,e*(-c*x+1)/(c*d+e),-1/2*c*x+1/2)*2^(1/2)*(c*x-1)^(1/2)/c/e/(1+m
)/((c*(e*x+d)/(c*d+e))^m)
```

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5802, 139, 138}

$$\frac{(d+ex)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{e(m+1)}-\frac{\sqrt{2}b\sqrt{cx-1}\,(cd+e)(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-cx),\frac{e(1-cx)}{cd+e}\right)}{ce(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((Sqrt[2]*b*(c*d + e)*Sqrt[-1 + c*x]*(d + e*x)^m*AppellF1[1/2, 1/2, -1 - m
, 3/2, (1 - c*x)/2, (e*(1 - c*x))/(c*d + e)])/(c*e*(1 + m)*((c*(d + e*x))/(
c*d + e))^m)) + ((d + e*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(e*(1 + m))
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^m \left(a+b\cosh^{-1}(cx)\right) dx = \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{e(1+m)} - \frac{(bc)\int \frac{(d+ex)^{1+m}}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{e(1+m)}$$
$$= \frac{(d+ex)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{e(1+m)} - \frac{\left(b(cd+e)(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}\right)\int \frac{\left(\frac{cd}{cd+e}+\frac{c}{cd}\right.}{\sqrt{-1+cx}}}{e(1+m)}$$
$$= -\frac{\sqrt{2}\,b(cd+e)\sqrt{-1+cx}\,(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m} F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-cx),\right.}{ce(1+m)}$$

Mathematica [A] time = 0.23, size = 177, normalized size = 1.42

$$\frac{(d+ex)^m\left(\frac{c(d+ex)}{cd+e}\right)^{-m}\left(c(d+ex)\left(a+b\cosh^{-1}(cx)\right)\left(\frac{c(d+ex)}{cd+e}\right)^m - 2be\sqrt{2cx-2}\,F_1\left(\frac{1}{2};-\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}-\frac{cx}{2},\frac{e-cex}{cd+e}\right)+b\sqrt{\right.}}{ce(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)^m*(a + b*ArcCosh[c*x]),x]
```

```
[Out] ((d + e*x)^m*(-2*b*e*Sqrt[-2 + 2*c*x]*AppellF1[1/2, -1/2, -m, 3/2, 1/2 - (c
*x)/2, (e - c*e*x)/(c*d + e)] + b*(-(c*d) + e)*Sqrt[-2 + 2*c*x]*AppellF1[1/
2, 1/2, -m, 3/2, 1/2 - (c*x)/2, (e - c*e*x)/(c*d + e)] + c*(d + e*x)*((c*(d
 + e*x))/(c*d + e))^m*(a + b*ArcCosh[c*x])))/(c*e*(1 + m)*((c*(d + e*x))/(c
*d + e))^m)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((b\operatorname{arcosh}(cx)+a)(ex+d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)*(e*x + d)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arcosh}(cx)+a)(ex+d)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(e*x + d)^m, x)
```

maple [F] time = 4.84, size = 0, normalized size = 0.00

$$\int (ex+d)^m\,(a+b\operatorname{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(a+b*arccosh(c*x)),x)
```

```
[Out] int((e*x+d)^m*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{(ex+d)(ex+d)^m \log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e(m+1)} - \int \frac{\left(c^2ex^2+c^2dx\right)(ex+d)^m}{c^2e(m+1)x^2-e(m+1)}\,dx + \int \frac{}{c^3e(m+1)x^3-ce(m+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] b*((e*x + d)*(e*x + d)^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*(m + 1))
 - integrate((c^2*e*x^2 + c^2*d*x)*(e*x + d)^m/(c^2*e*(m + 1)*x^2 - e*(m +
1)), x) + integrate((c*e*x + c*d)*(e*x + d)^m/(c^3*e*(m + 1)*x^3 - c*e*(m +
 1)*x + (c^2*e*(m + 1)*x^2 - e*(m + 1))*sqrt(c*x + 1)*sqrt(c*x - 1)), x)) +
 (e*x + d)^(m + 1)*a/(e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}\,(c\,x))\,\,(d+e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{acosh}\,(cx))\,(d+ex)^m\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x)**m, x)
```

3.39 $\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(d+ex)^m}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcCosh[c*x]),x]
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx=\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Mathematica [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x]),x]
[Out] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x]), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m}{b\,\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((e*x + d)^m/(b*arccosh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^m/(b*arccosh(c*x) + a), x)
```

maple [A] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{a+b\,\mathrm{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arccosh(c*x)),x)
```

```
[Out] int((e*x+d)^m/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)^m/(b*arccosh(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d+e\,x)^m}{a+b\,\mathrm{acosh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*acosh(c*x)),x)
```

```
[Out] int((d + e*x)^m/(a + b*acosh(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x)**m/(a + b*acosh(c*x)), x)
```

3.40 $\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((e*x+d)^m/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d + e*x)^m/(a + b*ArcCosh[c*x])^2,x]
[Out] Defer[Int][(d + e*x)^m/(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x])^2,x]
[Out] Integrate[(d + e*x)^m/(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex+d)^m}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral((e*x + d)^m/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m/(b*arccosh(c*x) + a)^2, x)
```

maple [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^m}{(a+b\,\mathrm{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((e*x+d)^m/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2x^2-1\right)\sqrt{cx+1}\,\sqrt{cx-1}\,(ex+d)^m+\left(c^3x^3-cx\right)(ex+d)^m}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1)*(e*x + d)^m + (c^3*x^3 - c*x)*(
e*x + d)^m)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c +
(b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))) + integrate((((c^3*e*(m + 1)*x^3 + c^3*d*x^2 - c*
e*(m - 1)*x + c*d)*(c*x + 1)*(c*x - 1)*(e*x + d)^m + (2*c^4*e*(m + 1)*x^4 +
 2*c^4*d*x^3 - c^2*e*(3*m + 1)*x^2 - c^2*d*x + e*m)*sqrt(c*x + 1)*sqrt(c*x
- 1)*(e*x + d)^m + (c^5*e*(m + 1)*x^5 + c^5*d*x^4 - 2*c^3*e*(m + 1)*x^3 - 2
*c^3*d*x^2 + c*e*(m + 1)*x + c*d)*(e*x + d)^m)/(a*b*c^5*e*x^5 + a*b*c^5*d*x
^4 - 2*a*b*c^3*e*x^3 - 2*a*b*c^3*d*x^2 + a*b*c*e*x + a*b*c*d + (a*b*c^3*e*x
^3 + a*b*c^3*d*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*b*c^4*e*x^4 + a*b*c^4*d*x^3
- a*b*c^2*e*x^2 - a*b*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e*x^5
 + b^2*c^5*d*x^4 - 2*b^2*c^3*e*x^3 - 2*b^2*c^3*d*x^2 + b^2*c*e*x + b^2*c*d
+ (b^2*c^3*e*x^3 + b^2*c^3*d*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e*x^4 +
b^2*c^4*d*x^3 - b^2*c^2*e*x^2 - b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d+e\,x)^m}{(a+b\,\mathrm{acosh}\,(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x)^m/(a + b*acosh(c*x))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^m}{(a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x)**m/(a + b*acosh(c*x))**2, x)
```

3.41 $\int (c+dx^2)^4 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=370

$$-\frac{4d^3(1-a^2x^2)^4(9a^2c+7d)}{441a^9\sqrt{ax-1}\sqrt{ax+1}}+\frac{d^4(1-a^2x^2)^5}{81a^9\sqrt{ax-1}\sqrt{ax+1}}+\frac{2d^2(1-a^2x^2)^3(63a^4c^2+90a^2cd+35d^2)}{525a^9\sqrt{ax-1}\sqrt{ax+1}}-\frac{4d(1-a^2x^2)^2}{}$$

```
[Out] c^4*x*arccosh(a*x)+4/3*c^3*d*x^3*arccosh(a*x)+6/5*c^2*d^2*x^5*arccosh(a*x)+
4/7*c*d^3*x^7*arccosh(a*x)+1/9*d^4*x^9*arccosh(a*x)+1/315*(315*a^8*c^4+420*
a^6*c^3*d+378*a^4*c^2*d^2+180*a^2*c*d^3+35*d^4)*(-a^2*x^2+1)/a^9/(a*x-1)^(1
/2)/(a*x+1)^(1/2)-4/945*d*(105*a^6*c^3+189*a^4*c^2*d+135*a^2*c*d^2+35*d^3)*
(-a^2*x^2+1)^2/a^9/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/525*d^2*(63*a^4*c^2+90*a^2
*c*d+35*d^2)*(-a^2*x^2+1)^3/a^9/(a*x-1)^(1/2)/(a*x+1)^(1/2)-4/441*d^3*(9*a^
2*c+7*d)*(-a^2*x^2+1)^4/a^9/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/81*d^4*(-a^2*x^2+
1)^5/a^9/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi **[A]** time = 0.46, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {194, 5705, 12, 1610, 1799, 1850}

$$\frac{2d^2(1-a^2x^2)^3(63a^4c^2+90a^2cd+35d^2)}{525a^9\sqrt{ax-1}\sqrt{ax+1}}-\frac{4d(1-a^2x^2)^2(189a^4c^2d+105a^6c^3+135a^2cd^2+35d^3)}{945a^9\sqrt{ax-1}\sqrt{ax+1}}+\frac{(1-a^2x^2)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcCosh[a*x],x]
```

```
[Out] ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*(
1 - a^2*x^2))/(315*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (4*d*(105*a^6*c^3 +
189*a^4*c^2*d + 135*a^2*c*d^2 + 35*d^3)*(1 - a^2*x^2)^2)/(945*a^9*Sqrt[-1 +
 a*x]*Sqrt[1 + a*x]) + (2*d^2*(63*a^4*c^2 + 90*a^2*c*d + 35*d^2)*(1 - a^2*x
^2)^3)/(525*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (4*d^3*(9*a^2*c + 7*d)*(1 -
 a^2*x^2)^4)/(441*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (d^4*(1 - a^2*x^2)^5)
/(81*a^9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + c^4*x*ArcCosh[a*x] + (4*c^3*d*x^3*
ArcCosh[a*x])/3 + (6*c^2*d^2*x^5*ArcCosh[a*x])/5 + (4*c*d^3*x^7*ArcCosh[a*x
])/7 + (d^4*x^9*ArcCosh[a*x])/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
```

```
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^4 \cosh^{-1}(ax)\,dx = c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(a$$

$$= c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(a$$

$$= c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(a$$

$$= c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(a$$

$$= c^4x\cosh^{-1}(ax)+\frac{4}{3}c^3dx^3\cosh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\cosh^{-1}(ax)+\frac{4}{7}cd^3x^7\cosh^{-1}(a$$

$$= \frac{\left(315a^8c^4+420a^6c^3d+378a^4c^2d^2+180a^2cd^3+35d^4\right)\left(1-a^2x^2\right)}{315a^9\sqrt{-1+ax}\sqrt{1+ax}} - \frac{4d\left(105a^6c\right.}{}$$

Mathematica [A] time = 0.31, size = 216, normalized size = 0.58

$$\frac{1}{315}x\cosh^{-1}(ax)\left(315c^4+420c^3dx^2+378c^2d^2x^4+180cd^3x^6+35d^4x^8\right)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^8\left(99225c^4+441\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcCosh[a*x],x]
```

```
[Out] -1/99225*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(4480*d^4 + 320*a^2*d^3*(81*c + 7*d*
x^2) + 48*a^4*d^2*(1323*c^2 + 270*c*d*x^2 + 35*d^2*x^4) + 8*a^6*d*(11025*c^
3 + 3969*c^2*d*x^2 + 1215*c*d^2*x^4 + 175*d^3*x^6) + a^8*(99225*c^4 + 44100
*c^3*d*x^2 + 23814*c^2*d^2*x^4 + 8100*c*d^3*x^6 + 1225*d^4*x^8)))/a^9 + (x*
(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*d^3*x^6 + 35*d^4*x^8)*Ar
cCosh[a*x])/315
```

fricas [A] time = 0.90, size = 250, normalized size = 0.68

$$\frac{315\left(35a^9d^4x^9+180a^9cd^3x^7+378a^9c^2d^2x^5+420a^9c^3dx^3+315a^9c^4x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(1225a^8d^4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/99225*(315*(35*a^9*d^4*x^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 42
0*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (1225*a^8*d
^4*x^8 + 99225*a^8*c^4 + 88200*a^6*c^3*d + 63504*a^4*c^2*d^2 + 100*(81*a^8*
c*d^3 + 14*a^6*d^4)*x^6 + 25920*a^2*c*d^3 + 6*(3969*a^8*c^2*d^2 + 1620*a^6*
c*d^3 + 280*a^4*d^4)*x^4 + 4480*d^4 + 4*(11025*a^8*c^3*d + 7938*a^6*c^2*d^2
 + 3240*a^4*c*d^3 + 560*a^2*d^4)*x^2)*sqrt(a^2*x^2 - 1))/a^9
```

giac [A] time = 0.33, size = 316, normalized size = 0.85

$$\frac{1}{315}\left(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{\left(315\,a^8c^4+420\,a^6c^3d+37\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c
^4*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/315*(315*a^8*c^4 + 420*a^6*c^3*d + 3
78*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*sqrt(a^2*x^2 - 1)/a^9 - 1/99225*(4
4100*(a^2*x^2 - 1)^(3/2)*a^6*c^3*d + 23814*(a^2*x^2 - 1)^(5/2)*a^4*c^2*d^2
+ 79380*(a^2*x^2 - 1)^(3/2)*a^4*c^2*d^2 + 8100*(a^2*x^2 - 1)^(7/2)*a^2*c*d^
3 + 34020*(a^2*x^2 - 1)^(5/2)*a^2*c*d^3 + 1225*(a^2*x^2 - 1)^(9/2)*d^4 + 56
700*(a^2*x^2 - 1)^(3/2)*a^2*c*d^3 + 6300*(a^2*x^2 - 1)^(7/2)*d^4 + 13230*(a
^2*x^2 - 1)^(5/2)*d^4 + 14700*(a^2*x^2 - 1)^(3/2)*d^4)/a^9
```

maple [A] time = 0.03, size = 255, normalized size = 0.69

$$\frac{\frac{a\,\mathrm{arccosh}(ax)d^4x^9}{9}+\frac{4a\,\mathrm{arccosh}(ax)c\,d^3x^7}{7}+\frac{6a\,\mathrm{arccosh}(ax)c^2d^2x^5}{5}+\frac{4a\,\mathrm{arccosh}(ax)c^3d\,x^3}{3}+\mathrm{arccosh}\left(ax\right)c^4ax-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(1225a^8\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arccosh(a*x),x)
```

```
[Out] 1/a*(1/9*a*arccosh(a*x)*d^4*x^9+4/7*a*arccosh(a*x)*c*d^3*x^7+6/5*a*arccosh(
a*x)*c^2*d^2*x^5+4/3*a*arccosh(a*x)*c^3*d*x^3+arccosh(a*x)*c^4*a*x-1/99225/
a^8*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(1225*a^8*d^4*x^8+8100*a^8*c*d^3*x^6+23814*
a^8*c^2*d^2*x^4+1400*a^6*d^4*x^6+44100*a^8*c^3*d*x^2+9720*a^6*c*d^3*x^4+992
25*a^8*c^4+31752*a^6*c^2*d^2*x^2+1680*a^4*d^4*x^4+88200*a^6*c^3*d+12960*a^4
*c*d^3*x^2+63504*a^4*c^2*d^2+2240*a^2*d^4*x^2+25920*a^2*c*d^3+4480*d^4))
```

maxima [A] time = 0.32, size = 385, normalized size = 1.04

$$-\frac{1}{99225}\left(\frac{1225\,\sqrt{a^2x^2-1}\,d^4x^8}{a^2}+\frac{8100\,\sqrt{a^2x^2-1}\,cd^3x^6}{a^2}+\frac{23814\,\sqrt{a^2x^2-1}\,c^2d^2x^4}{a^2}+\frac{1400\,\sqrt{a^2x^2-1}\,d^4x^6}{a^4}+4410\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/99225*(1225*sqrt(a^2*x^2 - 1)*d^4*x^8/a^2 + 8100*sqrt(a^2*x^2 - 1)*c*d^3
*x^6/a^2 + 23814*sqrt(a^2*x^2 - 1)*c^2*d^2*x^4/a^2 + 1400*sqrt(a^2*x^2 - 1)
*d^4*x^6/a^4 + 44100*sqrt(a^2*x^2 - 1)*c^3*d*x^2/a^2 + 9720*sqrt(a^2*x^2 -
1)*c*d^3*x^4/a^4 + 99225*sqrt(a^2*x^2 - 1)*c^4/a^2 + 31752*sqrt(a^2*x^2 - 1
)*c^2*d^2*x^2/a^4 + 1680*sqrt(a^2*x^2 - 1)*d^4*x^4/a^6 + 88200*sqrt(a^2*x^2
 - 1)*c^3*d/a^4 + 12960*sqrt(a^2*x^2 - 1)*c*d^3*x^2/a^6 + 63504*sqrt(a^2*x^
2 - 1)*c^2*d^2/a^6 + 2240*sqrt(a^2*x^2 - 1)*d^4*x^2/a^8 + 25920*sqrt(a^2*x^
```

```
2 - 1)*c*d^3/a^8 + 4480*sqrt(a^2*x^2 - 1)*d^4/a^10)*a + 1/315*(35*d^4*x^9 +
 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c^4*x)*arccosh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)\,\left(d\,x^2+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2)^4,x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2)^4, x)
```

sympy [A] time = 16.26, size = 503, normalized size = 1.36

$$\begin{cases} c^4x\operatorname{acosh}(ax) + \frac{4c^3dx^3\operatorname{acosh}(ax)}{3} + \frac{6c^2d^2x^5\operatorname{acosh}(ax)}{5} + \frac{4cd^3x^7\operatorname{acosh}(ax)}{7} + \frac{d^4x^9\operatorname{acosh}(ax)}{9} - \frac{c^4\sqrt{a^2x^2-1}}{a} - \frac{4c^3dx^2\sqrt{a^2x^2-1}}{9a} - \\ \frac{i\pi\left(c^4x+\frac{4c^3dx^3}{3}+\frac{6c^2d^2x^5}{5}+\frac{4cd^3x^7}{7}+\frac{d^4x^9}{9}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*acosh(a*x),x)
```

```
[Out] Piecewise((c**4*x*acosh(a*x) + 4*c**3*d*x**3*acosh(a*x)/3 + 6*c**2*d**2*x**
5*acosh(a*x)/5 + 4*c*d**3*x**7*acosh(a*x)/7 + d**4*x**9*acosh(a*x)/9 - c**4
*sqrt(a**2*x**2 - 1)/a - 4*c**3*d*x**2*sqrt(a**2*x**2 - 1)/(9*a) - 6*c**2*d
**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - 4*c*d**3*x**6*sqrt(a**2*x**2 - 1)/(49
*a) - d**4*x**8*sqrt(a**2*x**2 - 1)/(81*a) - 8*c**3*d*sqrt(a**2*x**2 - 1)/(
9*a**3) - 8*c**2*d**2*x**2*sqrt(a**2*x**2 - 1)/(25*a**3) - 24*c*d**3*x**4*s
qrt(a**2*x**2 - 1)/(245*a**3) - 8*d**4*x**6*sqrt(a**2*x**2 - 1)/(567*a**3)
- 16*c**2*d**2*sqrt(a**2*x**2 - 1)/(25*a**5) - 32*c*d**3*x**2*sqrt(a**2*x**
2 - 1)/(245*a**5) - 16*d**4*x**4*sqrt(a**2*x**2 - 1)/(945*a**5) - 64*c*d**3
*sqrt(a**2*x**2 - 1)/(245*a**7) - 64*d**4*x**2*sqrt(a**2*x**2 - 1)/(2835*a*
*7) - 128*d**4*sqrt(a**2*x**2 - 1)/(2835*a**9), Ne(a, 0)), (I*pi*(c**4*x +
4*c**3*d*x**3/3 + 6*c**2*d**2*x**5/5 + 4*c*d**3*x**7/7 + d**4*x**9/9)/2, Tr
ue))
```

3.42 $\int \left(c+dx^2\right)^3 \cosh^{-1}(ax)\, dx$

Optimal. Leaf size=267

$$\frac{3d^2\left(1-a^2x^2\right)^3\left(7a^2c+5d\right)}{175a^7\sqrt{ax-1}\sqrt{ax+1}}-\frac{d^3\left(1-a^2x^2\right)^4}{49a^7\sqrt{ax-1}\sqrt{ax+1}}-\frac{d\left(1-a^2x^2\right)^2\left(35a^4c^2+42a^2cd+15d^2\right)}{105a^7\sqrt{ax-1}\sqrt{ax+1}}+\frac{\left(1-a^2x^2\right)\left(35a^6c^3\right.}{35a^7\sqrt{}}$$

```
[Out] c^3*x*arccosh(a*x)+c^2*d*x^3*arccosh(a*x)+3/5*c*d^2*x^5*arccosh(a*x)+1/7*d^
3*x^7*arccosh(a*x)+1/35*(35*a^6*c^3+35*a^4*c^2*d+21*a^2*c*d^2+5*d^3)*(-a^2*
x^2+1)/a^7/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/105*d*(35*a^4*c^2+42*a^2*c*d+15*d^
2)*(-a^2*x^2+1)^2/a^7/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/175*d^2*(7*a^2*c+5*d)*(
-a^2*x^2+1)^3/a^7/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/49*d^3*(-a^2*x^2+1)^4/a^7/(
a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {194, 5705, 12, 1610, 1799, 1850}

$$-\frac{d\left(1-a^2x^2\right)^2\left(35a^4c^2+42a^2cd+15d^2\right)}{105a^7\sqrt{ax-1}\sqrt{ax+1}}+\frac{\left(1-a^2x^2\right)\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)}{35a^7\sqrt{ax-1}\sqrt{ax+1}}+\frac{3d^2\left(1-a^2x^2\right)^3\left(7a^2c\right.}{175a^7\sqrt{ax-1}\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcCosh[a*x],x]
```

```
[Out] ((35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*(1 - a^2*x^2))/(35*a^7*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (d*(35*a^4*c^2 + 42*a^2*c*d + 15*d^2)*(1 -
a^2*x^2)^2)/(105*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*d^2*(7*a^2*c + 5*d)
*(1 - a^2*x^2)^3)/(175*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (d^3*(1 - a^2*x^
2)^4)/(49*a^7*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + c^3*x*ArcCosh[a*x] + c^2*d*x^
3*ArcCosh[a*x] + (3*c*d^2*x^5*ArcCosh[a*x])/5 + (d^3*x^7*ArcCosh[a*x])/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^3 \cosh^{-1}(ax)\,dx = c^3 x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax) -$$

$$= c^3 x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax) -$$

$$= c^3 x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax) -$$

$$= c^3 x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax) -$$

$$= c^3 x\cosh^{-1}(ax) + c^2dx^3\cosh^{-1}(ax) + \frac{3}{5}cd^2x^5\cosh^{-1}(ax) + \frac{1}{7}d^3x^7\cosh^{-1}(ax) -$$

$$= \frac{\left(35a^6c^3+35a^4c^2d+21a^2cd^2+5d^3\right)\left(1-a^2x^2\right)}{35a^7\sqrt{-1+ax}\sqrt{1+ax}} - \frac{d\left(35a^4c^2+42a^2cd+15d^2\right)\left(\right.}{105a^7\sqrt{-1+ax}\sqrt{1+}}$$

Mathematica [A] time = 0.21, size = 154, normalized size = 0.58

$$\frac{1}{35}x\cosh^{-1}(ax)\left(35c^3+35c^2dx^2+21cd^2x^4+5d^3x^6\right) - \frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^6\left(3675c^3+1225c^2dx^2+441cd^2x^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcCosh[a*x],x]
```

```
[Out] -1/3675*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(240*d^3 + 24*a^2*d^2*(49*c + 5*d*x^2
) + 2*a^4*d*(1225*c^2 + 294*c*d*x^2 + 45*d^2*x^4) + a^6*(3675*c^3 + 1225*c^
2*d*x^2 + 441*c*d^2*x^4 + 75*d^3*x^6)))/a^7 + (x*(35*c^3 + 35*c^2*d*x^2 + 2
1*c*d^2*x^4 + 5*d^3*x^6)*ArcCosh[a*x])/35
```

fricas [A] time = 0.46, size = 179, normalized size = 0.67

$$\frac{105\left(5a^7d^3x^7+21a^7cd^2x^5+35a^7c^2dx^3+35a^7c^3x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(75a^6d^3x^6+3675a^6c^3+2450a\right.}{3675}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/3675*(105*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7*c^2*d*x^3 + 35*a^7*c
^3*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (75*a^6*d^3*x^6 + 3675*a^6*c^3 + 2450*
a^4*c^2*d + 1176*a^2*c*d^2 + 9*(49*a^6*c*d^2 + 10*a^4*d^3)*x^4 + 240*d^3 +
(1225*a^6*c^2*d + 588*a^4*c*d^2 + 120*a^2*d^3)*x^2)*sqrt(a^2*x^2 - 1))/a^7
```

giac [A] time = 0.38, size = 214, normalized size = 0.80

$$\frac{1}{35}\left(5\,d^3x^7+21\,cd^2x^5+35\,c^2dx^3+35\,c^3x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{\left(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d^3\right)\sqrt{a^2x^2-\cdots}}{35\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*x^3 + 35*c^3*x)*log(a*x + sqrt(a^
2*x^2 - 1)) - 1/35*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*sqrt(
a^2*x^2 - 1)/a^7 - 1/3675*(1225*(a^2*x^2 - 1)^(3/2)*a^4*c^2*d + 441*(a^2*x^
2 - 1)^(5/2)*a^2*c*d^2 + 1470*(a^2*x^2 - 1)^(3/2)*a^2*c*d^2 + 75*(a^2*x^2 -
 1)^(7/2)*d^3 + 315*(a^2*x^2 - 1)^(5/2)*d^3 + 525*(a^2*x^2 - 1)^(3/2)*d^3)/
a^7
```

maple [A] time = 0.01, size = 176, normalized size = 0.66

$$\frac{\frac{a\,\mathrm{arccosh}(ax)d^3x^7}{7}+\frac{3a\,\mathrm{arccosh}(ax)c\,d^2x^5}{5}+a\,\mathrm{arccosh}\,(ax)\,c^2d\,x^3+\mathrm{arccosh}\,(ax)\,c^3ax-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(75a^6d^3x^6+441a^6c\,d^2x^4+12\cdots\right.}{\cdots}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arccosh(a*x),x)
```

```
[Out] 1/a*(1/7*a*arccosh(a*x)*d^3*x^7+3/5*a*arccosh(a*x)*c*d^2*x^5+a*arccosh(a*x)
*c^2*d*x^3+arccosh(a*x)*c^3*a*x-1/3675/a^6*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(75*
a^6*d^3*x^6+441*a^6*c*d^2*x^4+1225*a^6*c^2*d*x^2+90*a^4*d^3*x^4+3675*a^6*c^
3+588*a^4*c*d^2*x^2+2450*a^4*c^2*d+120*a^2*d^3*x^2+1176*a^2*c*d^2+240*d^3))
```

maxima [A] time = 0.41, size = 257, normalized size = 0.96

$$-\frac{1}{3675}\left(\frac{75\,\sqrt{a^2x^2-1}\,d^3x^6}{a^2}+\frac{441\,\sqrt{a^2x^2-1}\,cd^2x^4}{a^2}+\frac{1225\,\sqrt{a^2x^2-1}\,c^2dx^2}{a^2}+\frac{90\,\sqrt{a^2x^2-1}\,d^3x^4}{a^4}+\frac{3675\,\sqrt{a^2x^2-\cdots}}{a^2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/3675*(75*sqrt(a^2*x^2 - 1)*d^3*x^6/a^2 + 441*sqrt(a^2*x^2 - 1)*c*d^2*x^4
/a^2 + 1225*sqrt(a^2*x^2 - 1)*c^2*d*x^2/a^2 + 90*sqrt(a^2*x^2 - 1)*d^3*x^4/
a^4 + 3675*sqrt(a^2*x^2 - 1)*c^3/a^2 + 588*sqrt(a^2*x^2 - 1)*c*d^2*x^2/a^4
+ 2450*sqrt(a^2*x^2 - 1)*c^2*d/a^4 + 120*sqrt(a^2*x^2 - 1)*d^3*x^2/a^6 + 11
76*sqrt(a^2*x^2 - 1)*c*d^2/a^6 + 240*sqrt(a^2*x^2 - 1)*d^3/a^8)*a + 1/35*(5
*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*x^3 + 35*c^3*x)*arccosh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}\,(a\,x)\,\left(d\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2)^3,x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2)^3, x)
```

<u>**sympy**</u> **[A]** time = 6.16, size = 328, normalized size = 1.23

$$\begin{cases} c^3x\operatorname{acosh}(ax) + c^2dx^3\operatorname{acosh}(ax) + \frac{3cd^2x^5\operatorname{acosh}(ax)}{5} + \frac{d^3x^7\operatorname{acosh}(ax)}{7} - \frac{c^3\sqrt{a^2x^2-1}}{a} - \frac{c^2dx^2\sqrt{a^2x^2-1}}{3a} - \frac{3cd^2x^4\sqrt{a^2x^2-1}}{25a} \\ \frac{i\pi\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*acosh(a*x),x)
```

```
[Out] Piecewise((c**3*x*acosh(a*x) + c**2*d*x**3*acosh(a*x) + 3*c*d**2*x**5*acosh
(a*x)/5 + d**3*x**7*acosh(a*x)/7 - c**3*sqrt(a**2*x**2 - 1)/a - c**2*d*x**2
*sqrt(a**2*x**2 - 1)/(3*a) - 3*c*d**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - d**
3*x**6*sqrt(a**2*x**2 - 1)/(49*a) - 2*c**2*d*sqrt(a**2*x**2 - 1)/(3*a**3) -
 4*c*d**2*x**2*sqrt(a**2*x**2 - 1)/(25*a**3) - 6*d**3*x**4*sqrt(a**2*x**2 -
 1)/(245*a**3) - 8*c*d**2*sqrt(a**2*x**2 - 1)/(25*a**5) - 8*d**3*x**2*sqrt(
a**2*x**2 - 1)/(245*a**5) - 16*d**3*sqrt(a**2*x**2 - 1)/(245*a**7), Ne(a, 0
)), (I*pi*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7)/2, True))
```

3.43 $\int \left(c+dx^2\right)^2 \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=181

$$-\frac{2d\left(1-a^2x^2\right)^2\left(5a^2c+3d\right)}{45a^5\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{d^2\left(1-a^2x^2\right)^3}{25a^5\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\left(1-a^2x^2\right)\left(15a^4c^2+10a^2cd+3d^2\right)}{15a^5\sqrt{ax-1}\,\sqrt{ax+1}}+c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx$$

```
[Out] c^2*x*arccosh(a*x)+2/3*c*d*x^3*arccosh(a*x)+1/5*d^2*x^5*arccosh(a*x)+1/15*(
15*a^4*c^2+10*a^2*c*d+3*d^2)*(-a^2*x^2+1)/a^5/(a*x-1)^(1/2)/(a*x+1)^(1/2)-2
/45*d*(5*a^2*c+3*d)*(-a^2*x^2+1)^2/a^5/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/25*d^2
*(-a^2*x^2+1)^3/a^5/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {194, 5705, 12, 520, 1247, 698}

$$\frac{\left(1-a^2x^2\right)\left(15a^4c^2+10a^2cd+3d^2\right)}{15a^5\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{2d\left(1-a^2x^2\right)^2\left(5a^2c+3d\right)}{45a^5\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{d^2\left(1-a^2x^2\right)^3}{25a^5\sqrt{ax-1}\,\sqrt{ax+1}}+c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcCosh[a*x],x]
```

```
[Out] ((15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*(1 - a^2*x^2))/(15*a^5*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]) - (2*d*(5*a^2*c + 3*d)*(1 - a^2*x^2)^2)/(45*a^5*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]) + (d^2*(1 - a^2*x^2)^3)/(25*a^5*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]) + c^2*x*ArcCosh[a*x] + (2*c*d*x^3*ArcCosh[a*x])/3 + (d^2*x^5*ArcCosh[a*x
])/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)^2 \cosh^{-1}(ax)\,dx = c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-a\int \frac{x\left(15c^2+10cdx\right.}{15\sqrt{-1+ax}}$$

$$= c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{1}{15}a\int \frac{x\left(15c^2+10\right.}{\sqrt{-1+a}}$$

$$= c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x(}{}}{15\sqrt{-1+a}}$$

$$= c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Sub}}{30\sqrt{-}}$$

$$= c^2x\cosh^{-1}(ax)+\frac{2}{3}cdx^3\cosh^{-1}(ax)+\frac{1}{5}d^2x^5\cosh^{-1}(ax)-\frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Sub}}{}$$

$$= \frac{\left(15a^4c^2+10a^2cd+3d^2\right)\left(1-a^2x^2\right)}{15a^5\sqrt{-1+ax}\sqrt{1+ax}}-\frac{2d\left(5a^2c+3d\right)\left(1-a^2x^2\right)^2}{45a^5\sqrt{-1+ax}\sqrt{1+ax}}+\frac{d^2\left(1-\right.}{25a^5\sqrt{-1+}}$$

Mathematica [A] time = 0.16, size = 103, normalized size = 0.57

$$\cosh^{-1}(ax)\left(c^2x+\frac{2}{3}cdx^3+\frac{d^2x^5}{5}\right)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^4\left(225c^2+50cdx^2+9d^2x^4\right)+4a^2d\left(25c+3dx^2\right)+24d\right.}{225a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcCosh[a*x],x]
```

```
[Out] -1/225*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(24*d^2 + 4*a^2*d*(25*c + 3*d*x^2) + a
^4*(225*c^2 + 50*c*d*x^2 + 9*d^2*x^4)))/a^5 + (c^2*x + (2*c*d*x^3)/3 + (d^2
*x^5)/5)*ArcCosh[a*x]
```

fricas [A] time = 1.32, size = 121, normalized size = 0.67

$$\frac{15\left(3a^5d^2x^5+10a^5cdx^3+15a^5c^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(9a^4d^2x^4+225a^4c^2+100a^2cd+2\left(25a^4cd+6\right.\right.}{225a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/225*(15*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15*a^5*c^2*x)*log(a*x + sqrt(a^
2*x^2 - 1)) - (9*a^4*d^2*x^4 + 225*a^4*c^2 + 100*a^2*c*d + 2*(25*a^4*c*d +
6*a^2*d^2)*x^2 + 24*d^2)*sqrt(a^2*x^2 - 1))/a^5
```

giac [A] time = 0.31, size = 134, normalized size = 0.74

$$\frac{1}{15}\left(3\,d^2x^5+10\,cdx^3+15\,c^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{\left(15\,a^4c^2+10\,a^2cd+3\,d^2\right)\sqrt{a^2x^2-1}}{15\,a^5}-\frac{50\left(a^2x^2-1\right)^{\frac{3}{2}}a^2cd+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/1
5*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*sqrt(a^2*x^2 - 1)/a^5 - 1/225*(50*(a^2*
x^2 - 1)^(3/2)*a^2*c*d + 9*(a^2*x^2 - 1)^(5/2)*d^2 + 30*(a^2*x^2 - 1)^(3/2)
*d^2)/a^5
```

maple [A] time = 0.01, size = 113, normalized size = 0.62

$$\frac{\frac{a\,\mathrm{arccosh}(ax)d^2x^5}{5}+\frac{2a\,\mathrm{arccosh}(ax)cd\,x^3}{3}+\mathrm{arccosh}\left(ax\right)c^2ax-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(9a^4d^2x^4+50a^4cd\,x^2+225a^4c^2+12a^2d^2x^2+100a^2cd+24d^2\right)}{225a^4}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arccosh(a*x),x)
```

```
[Out] 1/a*(1/5*a*arccosh(a*x)*d^2*x^5+2/3*a*arccosh(a*x)*c*d*x^3+arccosh(a*x)*c^2
*a*x-1/225/a^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)*(9*a^4*d^2*x^4+50*a^4*c*d*x^2+22
5*a^4*c^2+12*a^2*d^2*x^2+100*a^2*c*d+24*d^2))
```

maxima [A] time = 0.31, size = 154, normalized size = 0.85

$$-\frac{1}{225}\left(\frac{9\,\sqrt{a^2x^2-1}\,d^2x^4}{a^2}+\frac{50\,\sqrt{a^2x^2-1}\,cdx^2}{a^2}+\frac{225\,\sqrt{a^2x^2-1}\,c^2}{a^2}+\frac{12\,\sqrt{a^2x^2-1}\,d^2x^2}{a^4}+\frac{100\,\sqrt{a^2x^2-1}\,cd}{a^4}+\frac{24\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/225*(9*sqrt(a^2*x^2 - 1)*d^2*x^4/a^2 + 50*sqrt(a^2*x^2 - 1)*c*d*x^2/a^2
+ 225*sqrt(a^2*x^2 - 1)*c^2/a^2 + 12*sqrt(a^2*x^2 - 1)*d^2*x^2/a^4 + 100*sq
rt(a^2*x^2 - 1)*c*d/a^4 + 24*sqrt(a^2*x^2 - 1)*d^2/a^6)*a + 1/15*(3*d^2*x^5
 + 10*c*d*x^3 + 15*c^2*x)*arccosh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acosh}\left(a\,x\right)\,{\left(d\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2)^2,x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2)^2, x)
```

sympy [A] time = 2.10, size = 199, normalized size = 1.10

$$\begin{cases} c^2x\,\mathrm{acosh}\left(ax\right)+\frac{2cdx^3\,\mathrm{acosh}\left(ax\right)}{3}+\frac{d^2x^5\,\mathrm{acosh}\left(ax\right)}{5}-\frac{c^2\sqrt{a^2x^2-1}}{a}-\frac{2cdx^2\sqrt{a^2x^2-1}}{9a}-\frac{d^2x^4\sqrt{a^2x^2-1}}{25a}-\frac{4cd\sqrt{a^2x^2-1}}{9a^3}-\frac{4d^2x^2\sqrt{a^2x^2-\cdots}}{75a^3} \\ \frac{i\pi\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*acosh(a*x),x)
```

```
[Out] Piecewise((c**2*x*acosh(a*x) + 2*c*d*x**3*acosh(a*x)/3 + d**2*x**5*acosh(a*
x)/5 - c**2*sqrt(a**2*x**2 - 1)/a - 2*c*d*x**2*sqrt(a**2*x**2 - 1)/(9*a) -
d**2*x**4*sqrt(a**2*x**2 - 1)/(25*a) - 4*c*d*sqrt(a**2*x**2 - 1)/(9*a**3) -
 4*d**2*x**2*sqrt(a**2*x**2 - 1)/(75*a**3) - 8*d**2*sqrt(a**2*x**2 - 1)/(75
*a**5), Ne(a, 0)), (I*pi*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/2, True))
```

3.44 $\int \left(c+dx^2\right) \cosh^{-1}(ax)\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(9a^2c+2d\right)}{9a^3}+cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)-\frac{dx^2\sqrt{ax-1}\,\sqrt{ax+1}}{9a}$$

```
[Out] c*x*arccosh(a*x)+1/3*d*x^3*arccosh(a*x)-1/9*(9*a^2*c+2*d)*(a*x-1)^(1/2)*(a*
x+1)^(1/2)/a^3-1/9*d*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A] time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5705, 460, 74}

$$-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(9a^2c+2d\right)}{9a^3}+cx\cosh^{-1}(ax)-\frac{dx^2\sqrt{ax-1}\,\sqrt{ax+1}}{9a}+\frac{1}{3}dx^3\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcCosh[a*x],x]
```

```
[Out] -((9*a^2*c + 2*d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3) - (d*x^2*Sqrt[-1 +
a*x]*Sqrt[1 + a*x])/(9*a) + c*x*ArcCosh[a*x] + (d*x^3*ArcCosh[a*x])/3
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(c+dx^2\right)\cosh^{-1}(ax)\,dx &= cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)-a\int \frac{x\left(c+\frac{dx^2}{3}\right)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx\\ &= -\frac{dx^2\sqrt{-1+ax}\,\sqrt{1+ax}}{9a}+cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\cosh^{-1}(ax)+\frac{1}{9}\left(a\left(-9c-\frac{2d}{a^2}\right)\right)\int\\ &= -\frac{\left(9a^2c+2d\right)\sqrt{-1+ax}\,\sqrt{1+ax}}{9a^3}-\frac{dx^2\sqrt{-1+ax}\,\sqrt{1+ax}}{9a}+cx\cosh^{-1}(ax)+\frac{1}{3}dx^3\end{aligned}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.71

$$\cosh^{-1}(ax)\left(cx+\frac{dx^3}{3}\right)-\frac{\sqrt{ax-1}\sqrt{ax+1}\left(a^2\left(9c+dx^2\right)+2d\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcCosh[a*x],x]
```

```
[Out] -1/9*(Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(2*d + a^2*(9*c + d*x^2)))/a^3 + (c*x +
(d*x^3)/3)*ArcCosh[a*x]
```

fricas [A] time = 0.57, size = 71, normalized size = 0.85

$$\frac{3\left(a^3dx^3+3\,a^3cx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(a^2dx^2+9\,a^2c+2\,d\right)\sqrt{a^2x^2-1}}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(a^3*d*x^3 + 3*a^3*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - (a^2*d*x^2 +
9*a^2*c + 2*d)*sqrt(a^2*x^2 - 1))/a^3
```

giac [A] time = 0.31, size = 70, normalized size = 0.83

$$\frac{1}{3}\left(dx^3+3\,cx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-\frac{\left(a^2x^2-1\right)^{\frac{3}{2}}d}{9\,a^3}-\frac{\sqrt{a^2x^2-1}\left(3\,a^2c+d\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="giac")
```

```
[Out] 1/3*(d*x^3 + 3*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 1/9*(a^2*x^2 - 1)^(3/2)*
d/a^3 - 1/3*sqrt(a^2*x^2 - 1)*(3*a^2*c + d)/a^3
```

maple [A] time = 0.01, size = 62, normalized size = 0.74

$$\frac{\frac{a\,\mathrm{arccosh}(ax)d\,x^3}{3}+\mathrm{arccosh}\,(ax)\,cax-\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2d\,x^2+9a^2c+2d\right)}{9a^2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arccosh(a*x),x)
```

```
[Out] 1/a*(1/3*a*arccosh(a*x)*d*x^3+arccosh(a*x)*c*a*x-1/9/a^2*(a*x-1)^(1/2)*(a*x
+1)^(1/2)*(a^2*d*x^2+9*a^2*c+2*d))
```

maxima [A] time = 0.31, size = 74, normalized size = 0.88

$$-\frac{1}{9}\left(\frac{\sqrt{a^2x^2-1}\,dx^2}{a^2}+\frac{9\,\sqrt{a^2x^2-1}\,c}{a^2}+\frac{2\,\sqrt{a^2x^2-1}\,d}{a^4}\right)a+\frac{1}{3}\left(dx^3+3\,cx\right)\mathrm{arcosh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -1/9*(sqrt(a^2*x^2 - 1)*d*x^2/a^2 + 9*sqrt(a^2*x^2 - 1)*c/a^2 + 2*sqrt(a^2*
x^2 - 1)*d/a^4)*a + 1/3*(d*x^3 + 3*c*x)*arccosh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(a\,x)\,\left(d\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2),x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2), x)
```

sympy [A] time = 0.51, size = 90, normalized size = 1.07

$$\begin{cases} cx\operatorname{acosh}(ax) + \frac{dx^3\operatorname{acosh}(ax)}{3} - \frac{c\sqrt{a^2x^2-1}}{a} - \frac{dx^2\sqrt{a^2x^2-1}}{9a} - \frac{2d\sqrt{a^2x^2-1}}{9a^3} & \text{for } a \neq 0 \\ \frac{i\pi\left(cx+\frac{dx^3}{3}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*acosh(a*x),x)
```

```
[Out] Piecewise((c*x*acosh(a*x) + d*x**3*acosh(a*x)/3 - c*sqrt(a**2*x**2 - 1)/a -
 d*x**2*sqrt(a**2*x**2 - 1)/(9*a) - 2*d*sqrt(a**2*x**2 - 1)/(9*a**3), Ne(a,
 0)), (I*pi*(c*x + d*x**3/3)/2, True))
```

3.45 $\int \frac{\cosh^{-1}(ax)}{c+dx^2}\,dx$

Optimal. Leaf size=481

$$-\frac{\text{Li}_2\left(-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\text{Li}_2\left(-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{\sqrt{-c}\,a+\sqrt{-ca^2-d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{Li}_2\left(\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{\sqrt{-c}\,a+\sqrt{-ca^2-d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\cosh^{-1}(ax)\log\left(1\right.}{2\sqrt{-c}}$$

```
[Out] 1/2*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/
2)-(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)-1/2*arccosh(a*x)*ln(1+(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d
^(1/2)+1/2*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(
-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)-1/2*arccosh(a*x)*ln(1+(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^
(1/2)/d^(1/2)-1/2*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(
-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)+1/2*polylog(2,(a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^
(1/2)-1/2*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2
)+(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)+1/2*polylog(2,(a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^(1/2)/d^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\right.}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2),x]
```

```
[Out] (ArcCosh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c)
- d])])/(2*Sqrt[-c]*Sqrt[d]) - (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x
])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d]) + (ArcCosh[a*x]
*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(2*Sq
rt[-c]*Sqrt[d]) - (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c
] + Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*E^Ar
cCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d]))]/(2*Sqrt[-c]*Sqrt[d]) + Poly
Log[2, (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])]/(2*Sqrt[
-c]*Sqrt[d]) - PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a
^2*c) - d]))]/(2*Sqrt[-c]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(a
*Sqrt[-c] + Sqrt[-(a^2*c) - d])]/(2*Sqrt[-c]*Sqrt[d])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{c+dx^2}\,dx = \int \left(\frac{\sqrt{-c}\,\cosh^{-1}(ax)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\cosh^{-1}(ax)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}} - \frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}$$

$$= -\frac{\text{Subst}\left(\int \frac{x\sinh(x)}{a\sqrt{-c}-\sqrt{d}\,\cosh(x)}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}} - \frac{\text{Subst}\left(\int \frac{x\sinh(x)}{a\sqrt{-c}+\sqrt{d}\,\cosh(x)}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x x}{a\sqrt{-c}-\sqrt{-a^2c-d}-\sqrt{d}\,e^x}\,dx, x, \cosh^{-1}(ax)\right)}{2\sqrt{-c}} - \frac{\text{Subst}\left(\int \frac{e^x x}{a\sqrt{-c}+\sqrt{-a^2c-d}-\sqrt{d}\,e^x}\,dx, x, \cosh\right.}{2\sqrt{-c}}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} + \frac{\cosh^{-1}(ax)\log}{2\surd}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} + \frac{\cosh^{-1}(ax)\log}{2\surd}$$

$$= \frac{\cosh^{-1}(ax)\log\left(1-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} - \frac{\cosh^{-1}(ax)\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-a^2c-d}}\right)}{2\sqrt{-c}\,\sqrt{d}} + \frac{\cosh^{-1}(ax)\log}{2\surd}$$

Mathematica [A] time = 0.40, size = 375, normalized size = 0.78

$$\text{Li}_2\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)-\text{Li}_2\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{-ca^2-d}-a\sqrt{-c}}\right)-\text{Li}_2\left(-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{-c}a+\sqrt{-ca^2-d}}\right)+\text{Li}_2\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{-c}a+\sqrt{-ca^2-d}}\right)-\cosh^{-1}(ax)\log\left(\frac{\sqrt{}}{a\sqrt{-}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c
) - d])]) + ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(-(a*Sqrt[-c]) +
Sqrt[-(a^2*c) - d])] + ArcCosh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqr
t[-c] + Sqrt[-(a^2*c) - d])] - ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x]
)/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])] + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])] - PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(-
(a*Sqrt[-c]) + Sqrt[-(a^2*c) - d])] - PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])
/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d]))] + PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(2*Sqrt[-c]*Sqrt[d])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcosh}(ax)}{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/(d*x^2 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}(ax)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(d*x^2 + c), x)
```

maple [C] time = 1.50, size = 214, normalized size = 0.44

$$\frac{a\left(\sum_{\_R1=\text{RootOf}\left(d\_Z^4+\left(4a^2c+2d\right)\_Z^2+d\right)}\frac{\_R1\left(\text{arccosh}(ax)\ln\left(\frac{\_R1-ax-\sqrt{ax-1}\,\sqrt{ax+1}}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-ax-\sqrt{ax-1}\,\sqrt{ax+1}}{\_R1}\right)\right)}{\_R1^2d+2a^2c+d}\right)}{2}-\frac{a\left(\sum_{\_R1=\text{RootOf}\left(d\_Z\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c),x)
```

```
[Out] 1/2*a*sum(_R1/(_R1^2*d+2*a^2*c+d)*(arccosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(
a*x+1)^(1/2))/_R1)+dilog((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=Ro
otOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2+d))-1/2*a*sum(1/_R1/(_R1^2*d+2*a^2*c+d)*(arc
cosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)+dilog((_R1-a*x-(a*x
-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=RootOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2+d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}(ax)}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)/(d*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2),x)
[Out] int(acosh(a*x)/(c + d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c),x)
[Out] Integral(acosh(a*x)/(c + d*x**2), x)
```

3.46 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=774

$$\frac{\text{Li}_2\left(-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{Li}_2\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{-ca^2-d}}\right)}{4(-c)^{3/2}\sqrt{d}}+\frac{\text{Li}_2\left(-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{-c}a+\sqrt{-ca^2-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{Li}_2\left(\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{-c}a+\sqrt{-ca^2-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\cosh^{-1}(ax)\log\left(1-\right.}{4(-c)^{3/2}}$$

```
[Out] -1/4*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1
/2)-(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)+1/4*arccosh(a*x)*ln(1+(a*x+(a*x-1
)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(3/2)/
d^(1/2)-1/4*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*
(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)+1/4*arccosh(a*x)*ln(1+(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)
^(3/2)/d^(1/2)+1/4*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*
(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)-1/4*polylog(2,(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)-(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d
^(1/2)+1/4*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/
2)+(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)-1/4*polylog(2,(a*x+(a*x-1)^(1/2)*(
a*x+1)^(1/2))*d^(1/2)/(a*(-c)^(1/2)+(-a^2*c-d)^(1/2)))/(-c)^(3/2)/d^(1/2)-1
/4*arccosh(a*x)/c/d^(1/2)/((-c)^(1/2)-x*d^(1/2))+1/4*arccosh(a*x)/c/d^(1/2)
/((-c)^(1/2)+x*d^(1/2))+1/2*a*arctanh((a*x+1)^(1/2)*(a*(-c)^(1/2)-d^(1/2))^
(1/2)/(a*x-1)^(1/2)/(a*(-c)^(1/2)+d^(1/2))^(1/2))/c/d^(1/2)/(a*(-c)^(1/2)-d
^(1/2))^(1/2)/(a*(-c)^(1/2)+d^(1/2))^(1/2)-1/2*a*arctanh((a*x+1)^(1/2)*(a*(
-c)^(1/2)+d^(1/2))^(1/2)/(a*x-1)^(1/2)/(a*(-c)^(1/2)-d^(1/2))^(1/2))/c/d^(1
/2)/(a*(-c)^(1/2)-d^(1/2))^(1/2)/(a*(-c)^(1/2)+d^(1/2))^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{a\sqrt{-c}-\sqrt{a^2(-c)-d}}\right)}{4(-c)^{3/2}\sqrt{d}}+\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}e^{\cosh^{-1}(ax)}}{\sqrt{a^2(-c)-d}+a\sqrt{-c}}\right)}{4(-c)^{3/2}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\right.}{4(-c)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] -ArcCosh[a*x]/(4*c*Sqrt[d]*(Sqrt[-c] - Sqrt[d]*x)) + ArcCosh[a*x]/(4*c*Sqrt
[d]*(Sqrt[-c] + Sqrt[d]*x)) + (a*ArcTanh[(Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[1
 + a*x])/(Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[-1 + a*x])])/(2*c*Sqrt[a*Sqrt[-c]
 - Sqrt[d]]*Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[d]) - (a*ArcTanh[(Sqrt[a*Sqrt[-
c] + Sqrt[d]]*Sqrt[1 + a*x])/(Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[-1 + a*x])])/
(2*c*Sqrt[a*Sqrt[-c] - Sqrt[d]]*Sqrt[a*Sqrt[-c] + Sqrt[d]]*Sqrt[d]) - (ArcC
osh[a*x]*Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])
])/(4*(-c)^(3/2)*Sqrt[d]) + (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/
(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])])/(4*(-c)^(3/2)*Sqrt[d]) - (ArcCosh[a*x]*
Log[1 - (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])])/(4*(-c
)^(3/2)*Sqrt[d]) + (ArcCosh[a*x]*Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-
c] + Sqrt[-(a^2*c) - d])])/(4*(-c)^(3/2)*Sqrt[d]) + PolyLog[2, -((Sqrt[d]*E
^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d]))]/(4*(-c)^(3/2)*Sqrt[d]) -
 PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] - Sqrt[-(a^2*c) - d])]/(4*
(-c)^(3/2)*Sqrt[d]) + PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(a*Sqrt[-c] + S
qrt[-(a^2*c) - d]))]/(4*(-c)^(3/2)*Sqrt[d]) - PolyLog[2, (Sqrt[d]*E^ArcCosh
[a*x])/(a*Sqrt[-c] + Sqrt[-(a^2*c) - d])]/(4*(-c)^(3/2)*Sqrt[d])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx = \int \left(-\frac{d\cosh^{-1}(ax)}{4c\left(\sqrt{-c}\sqrt{d}-dx\right)^2}-\frac{d\cosh^{-1}(ax)}{4c\left(\sqrt{-c}\sqrt{d}+dx\right)^2}-\frac{d\cosh^{-1}(ax)}{2c\left(-cd-d^2x^2\right)}\right)dx$$

$$=-\frac{d\int \frac{\cosh^{-1}(ax)}{\left(\sqrt{-c}\sqrt{d}-dx\right)^2}\,dx}{4c}-\frac{d\int \frac{\cosh^{-1}(ax)}{\left(\sqrt{-c}\sqrt{d}+dx\right)^2}\,dx}{4c}-\frac{d\int \frac{\cosh^{-1}(ax)}{-cd-d^2x^2}\,dx}{2c}$$

$$=-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}\,x\right)}+\frac{a\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\left(\sqrt{-c}\sqrt{d}-dx\right)}\,dx}{4c}-\frac{a\int \frac{}{\sqrt{-}}$$

$$=-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}\,x\right)}+\frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{4(-c)^{3/2}}+\frac{\int \frac{\cosh^{-1}(ax)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{4(-c)^{3/2}}+\frac{a\,\mathrm{Sub}}{}$$

$$=-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}\,x\right)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}}-\frac{}{2}$$

$$=-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}\,x\right)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}}-\frac{}{2}$$

$$=-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}\,x\right)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}}-\frac{}{2}$$

$$=-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}\,x\right)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}}-\frac{}{2}$$

$$=-\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\cosh^{-1}(ax)}{4c\sqrt{d}\left(\sqrt{-c}+\sqrt{d}\,x\right)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{1+ax}}{\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{-1+ax}}\right)}{2c\sqrt{a\sqrt{-c}-\sqrt{d}}\sqrt{a\sqrt{-c}+\sqrt{d}}\sqrt{d}}-\frac{}{2}$$

Mathematica [C] time = 1.38, size = 687, normalized size = 0.89

$$i\left(2\mathrm{Li}_2\left(\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{\sqrt{-ca^2-d}-ia\sqrt{c}}\right)+2\mathrm{Li}_2\left(-\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{i\sqrt{c}\,a+\sqrt{-ca^2-d}}\right)+\cosh^{-1}(ax)\left(-\cosh^{-1}(ax)+2\left(\log\left(1+\frac{\sqrt{d}\,e^{\cosh^{-1}(ax)}}{-\sqrt{a^2(-c)-d}+ia\sqrt{c}}\right)+1\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*(ArcCosh[a*x]/((-I)*Sqrt[c] + Sqrt[d]*x) + (a*Log[(2*d*(I*Sqrt[d
] + a^2*Sqrt[c]*x - I*Sqrt[-(a^2*c) - d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x]))/(a*
Sqrt[-(a^2*c) - d]*(Sqrt[c] + I*Sqrt[d]*x))])/Sqrt[-(a^2*c) - d]) - 2*Sqrt[
c]*(-(ArcCosh[a*x]/(I*Sqrt[c] + Sqrt[d]*x)) - (a*Log[(2*d*(-Sqrt[d] - I*a^2
*Sqrt[c]*x + Sqrt[-(a^2*c) - d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x]))/(a*Sqrt[-(a^
2*c) - d]*(I*Sqrt[c] + Sqrt[d]*x))])/Sqrt[-(a^2*c) - d]) + I*(ArcCosh[a*x]*
(-ArcCosh[a*x] + 2*(Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] - Sqrt[-(
a^2*c) - d])] + Log[1 + (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*
c) - d])])) + 2*PolyLog[2, (Sqrt[d]*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[
-(a^2*c) - d])] + 2*PolyLog[2, -((Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sq
rt[-(a^2*c) - d]))]) - I*(ArcCosh[a*x]*(-ArcCosh[a*x] + 2*(Log[1 + (Sqrt[d]
*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[-(a^2*c) - d])] + Log[1 - (Sqrt[d]*
E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*c) - d])])) + 2*PolyLog[2, -((Sqr
t[d]*E^ArcCosh[a*x])/((-I)*a*Sqrt[c] + Sqrt[-(a^2*c) - d]))] + 2*PolyLog[2,
 (Sqrt[d]*E^ArcCosh[a*x])/(I*a*Sqrt[c] + Sqrt[-(a^2*c) - d])]))/(8*c^(3/2)*
Sqrt[d])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arcosh}(ax)}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(d*x^2 + c)^2, x)
```

maple [C] time = 3.40, size = 1632, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*a^2*arccosh(a*x)*x/c/(a^2*d*x^2+a^2*c)+1/4*a/c*sum(_R1/(_R1^2*d+2*a^2*c
+d)*(arccosh(a*x)*ln((_R1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)+dilog((_R1-
a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)),_R1=RootOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2
+d))+a^5*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(d*(a*x+(a*x
-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2))*c
/(a^2*c+d)/d^3-a^3*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(d
*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d
)^(1/2))/(a^2*c+d)/d^3*(a^2*c*(a^2*c+d))^(1/2)+a^3*((2*a^2*c+2*(a^2*c*(a^2*
c+d))^(1/2)+d)*d)^(1/2)*arctan(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*
c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2))/(a^2*c+d)/d^2-1/2*a*((2*a^2*c+2*(a
^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)
)/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2))/c/(a^2*c+d)/d^2*(a^2*c*(
a^2*c+d))^(1/2)-a^3*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(
d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*
```

```
d)^(1/2))/d^3+a*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctan(d*(a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(
1/2))/c/d^3*(a^2*c*(a^2*c+d))^(1/2)-1/2*a*((2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/
2)+d)*d)^(1/2)*arctan(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((2*a^2*c+2*(a^2*
c*(a^2*c+d))^(1/2)+d)*d)^(1/2))/c/d^2+a^5*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1
/2)+d)*d)^(1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a
^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2))*c/(a^2*c+d)/d^3+a^3*(-(2*a^2*c-2*(a^2*c*
(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((
-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2))/(a^2*c+d)/d^3*(a^2*c*(a^2*c
+d))^(1/2)+a^3*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(d*(
a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)
^(1/2))/(a^2*c+d)/d^2+1/2*a*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2
)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d)
)^(1/2)-d)*d)^(1/2))/c/(a^2*c+d)/d^2*(a^2*c*(a^2*c+d))^(1/2)-a^3*(-(2*a^2*c
-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2))/d^3-a*(-(2*a^2*c-
2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(d*(a*x+(a*x-1)^(1/2)*(a*x+1)^
(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)*d)^(1/2))/c/d^3*(a^2*c*(a^2*
c+d))^(1/2)-1/2*a*(-(2*a^2*c-2*(a^2*c*(a^2*c+d))^(1/2)+d)*d)^(1/2)*arctanh(
d*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/((-2*a^2*c+2*(a^2*c*(a^2*c+d))^(1/2)-d)
*d)^(1/2))/c/d^2-1/4*a/c*sum(1/_R1/(_R1^2*d+2*a^2*c+d)*(arccosh(a*x)*ln((_R
1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/_R1)+dilog((_R1-a*x-(a*x-1)^(1/2)*(a*x+1
)^(1/2))/_R1)),_R1=RootOf(d*_Z^4+(4*a^2*c+2*d)*_Z^2+d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/(d*x^2 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)}{\left(d\,x^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^2,x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**2, x)
```

3.47 $\int \sqrt{c+dx^2} \cosh^{-1}(ax) \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\cosh^{-1}(ax)\sqrt{c+dx^2},x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)*arccosh(a*x),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{c+dx^2} \cosh^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcCosh[a*x],x]
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcCosh[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2} \cosh^{-1}(ax) \, dx = \int \sqrt{c+dx^2} \cosh^{-1}(ax) \, dx$$

Mathematica [A] time = 3.05, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^2} \cosh^{-1}(ax) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcCosh[a*x],x]
[Out] Integrate[Sqrt[c + d*x^2]*ArcCosh[a*x], x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{dx^2+c} \operatorname{arcosh}(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="fricas")
[Out] integral(sqrt(d*x^2 + c)*arccosh(a*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2+c} \operatorname{arcosh}(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*arccosh(a*x), x)
```

maple [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \sqrt{d\,x^2+c}\, \operatorname{arccosh}(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arccosh(a*x),x)
```

```
[Out] int((d*x^2+c)^(1/2)*arccosh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dx^2 + c} \operatorname{arcosh}(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*arccosh(a*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \operatorname{acosh}(a\,x)\,\sqrt{d\,x^2 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)*(c + d*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx^2} \operatorname{acosh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*acosh(a*x),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*acosh(a*x), x)
```

3.48 $\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable(arccosh(a*x)/(d*x^2+c)^(1/2),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]/Sqrt[c + d*x^2],x]
[Out] Defer[Int][ArcCosh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx = \int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Mathematica [A] time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]/Sqrt[c + d*x^2],x]
[Out] Integrate[ArcCosh[a*x]/Sqrt[c + d*x^2], x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\operatorname{arcosh}(ax)}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] integral(arccosh(a*x)/sqrt(d*x^2 + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)/sqrt(d*x^2 + c), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(1/2),x)
[Out] int(arccosh(a*x)/(d*x^2+c)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)/sqrt(d*x^2 + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{acosh}(a\,x)}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(1/2),x)
[Out] int(acosh(a*x)/(c + d*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(1/2),x)
[Out] Integral(acosh(a*x)/sqrt(c + d*x**2), x)
```

3.49 $\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}$$

```
[Out] -arctanh(d^(1/2)*(a^2*x^2-1)^(1/2)/a/(d*x^2+c)^(1/2))*(a^2*x^2-1)^(1/2)/c/d
^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+x*arccosh(a*x)/c/(d*x^2+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {191, 5705, 12, 519, 444, 63, 217, 206}

$$\frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x])/(c*Sqrt[c + d*x^2]) - (Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]
*Sqrt[-1 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(c*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1
 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}}\,dx \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}}\,dx}{c} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x}{\sqrt{-1+a^2x^2}\,\sqrt{c+dx^2}}\,dx}{c\sqrt{-1+ax}\,\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+a^2x}\,\sqrt{c+dx}}\,dx,x,x^2\right)}{2c\sqrt{-1+ax}\,\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\,\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+\frac{d}{a^2}+\frac{dx^2}{a^2}}}\,dx,x,\sqrt{-1+a^2x^2}\right)}{ac\sqrt{-1+ax}\,\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\,\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{a^2}}\,dx,x,\frac{\sqrt{-1+a^2x^2}}{\sqrt{c+dx^2}}\right)}{ac\sqrt{-1+ax}\,\sqrt{1+ax}} \\
&= \frac{x\cosh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\sqrt{-1+a^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{-1+a^2x^2}}{a\sqrt{c+dx^2}}\right)}{c\sqrt{d}\,\sqrt{-1+ax}\,\sqrt{1+ax}}
\end{aligned}
$$

Mathematica [C] time = 3.23, size = 551, normalized size = 5.74

$$x\cosh^{-1}(ax)+\frac{2(ax-1)^{3/2}\sqrt{\frac{(ax+1)\left(a\sqrt{c}-i\sqrt{d}\right)}{(ax-1)\left(a\sqrt{c}+i\sqrt{d}\right)}}\left(a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right)\sqrt{\frac{\left(a^2c+d\right)\left(c+dx^2\right)}{cd(ax-1)^2}}\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}\,x}{\sqrt{c}}-1}{1-ax}}\,\Pi\left(\frac{2a\sqrt{c}}{\sqrt{c}\,a+i\sqrt{d}};\sin^{-1}\left(\sqrt{-\frac{\frac{i\sqrt{d}\,x}{\sqrt{c}}+a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right.}{2-2ax}}\right.\right.\right.}{a\sqrt{ax+1}\left(a^2c+d\right)\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right.}{1}}}}{c\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x] + (2*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1 + a*
x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*((a*((-I)*a*Sqrt[c] + Sqrt[d])*(I
*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] - a*x + (I*Sqrt[d]*x)
/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] +
a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*S
qrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt[c]) + I*Sqrt[d])*S
qrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-((-1 + (I*Sqrt[d]*x
)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*EllipticPi[(2*a*Sqrt[c
])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*(
(I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*Sqrt
[c] + I*Sqrt[d])^2]))/(a*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[-((-1 + (I*Sqrt[d]*
x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]))/(c*Sqrt[c + d*x^2])
```

fricas [A] time = 0.59, size = 296, normalized size = 3.08

$$\left[\frac{4\sqrt{dx^2+c}\,dx\log\left(ax+\sqrt{a^2x^2-1}\right)+\left(dx^2+c\right)\sqrt{d}\,\log\left(8\,a^4d^2x^4+a^4c^2-6\,a^2cd+8\left(a^4cd-a^2d^2\right)x^2-4\left(2\,a^3d\right.\right.}{4\left(cd^2x^2+c^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(d*x^2 + c)*d*x*log(a*x + sqrt(a^2*x^2 - 1)) + (d*x^2 + c)*sqrt
(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2)*x^2 - 4
*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(d) + d^
2))/(c*d^2*x^2 + c^2*d), 1/2*(2*sqrt(d*x^2 + c)*d*x*log(a*x + sqrt(a^2*x^2
- 1)) + (d*x^2 + c)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c - d)*sqrt(a^2*
x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c*d - a*d^2)*
x^2)))/(c*d^2*x^2 + c^2*d)]
```

giac [A] time = 0.62, size = 82, normalized size = 0.85

$$\frac{x\log\left(ax+\sqrt{a^2x^2-1}\right)}{\sqrt{dx^2+c}\,c}+\frac{a\log\left(\left|-\sqrt{a^2x^2-1}\,\sqrt{d}+\sqrt{a^2c+\left(a^2x^2-1\right)d+d}\right|\right)}{c\sqrt{d}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 - 1))/(sqrt(d*x^2 + c)*c) + a*log(abs(-sqrt(a^2*x^
2 - 1)*sqrt(d) + sqrt(a^2*c + (a^2*x^2 - 1)*d + d)))/(c*sqrt(d)*abs(a))
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 details)Is d-a^2*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(a\,x\right)}{{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}{\left(ax \right)}}{\left(c + dx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**(3/2), x)
```

3.50 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{2\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{3c^2\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{a\left(1-a^2x^2\right)}{3c\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2c+d\right)\sqrt{c+dx^2}}+\frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}$$

```
[Out] 1/3*x*arccosh(a*x)/c/(d*x^2+c)^(3/2)-2/3*arctanh(d^(1/2)*(a^2*x^2-1)^(1/2)/
a/(d*x^2+c)^(1/2))*(a^2*x^2-1)^(1/2)/c^2/d^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2
)+2/3*x*arccosh(a*x)/c^2/(d*x^2+c)^(1/2)+1/3*a*(-a^2*x^2+1)/c/(a^2*c+d)/(a*
x-1)^(1/2)/(a*x+1)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {192, 191, 5705, 12, 519, 571, 78, 63, 217, 206}

$$-\frac{2\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{3c^2\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{a\left(1-a^2x^2\right)}{3c\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2c+d\right)\sqrt{c+dx^2}}+\frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] (a*(1 - a^2*x^2))/(3*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*
x^2]) + (x*ArcCosh[a*x])/(3*c*(c + d*x^2)^(3/2)) + (2*x*ArcCosh[a*x])/(3*c^
2*Sqrt[c + d*x^2]) - (2*Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]*Sqrt[-1 + a^2*x
^2])/(a*Sqrt[c + d*x^2])])/(3*c^2*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{x\left(3c+2dx^2\right)}{3c^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{3/2}}\,dx$$

$$= \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(3c+2dx^2\right)}{\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x\left(3c+2dx^2\right)}{\sqrt{-1+a^2x^2}\,\left(c+dx^2\right)^{3/2}}\,dx}{3c^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\left(\int \frac{3c+2dx}{\sqrt{-1+a^2x}\,(c+dx)^{3/2}}\,dx,x,x^2\right)}{6c^2\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{a\left(1-a^2x^2\right)}{3c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x\right.}{}$$

$$= \frac{a\left(1-a^2x^2\right)}{3c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(2\sqrt{-1+a^2x\right.}{}$$

$$= \frac{a\left(1-a^2x^2\right)}{3c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(2\sqrt{-1+a^2x\right.}{3}$$

$$= \frac{a\left(1-a^2x^2\right)}{3c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\sqrt{c+dx^2}} + \frac{x\cosh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\cosh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{2\sqrt{-1+a^2x^2}}{3c^2\sqrt{d}}$$

Mathematica [C] time = 2.21, size = 609, normalized size = 3.38

$$\frac{-\frac{ac\sqrt{ax+1}\,\sqrt{ax-1}\,\left(c+dx^2\right)}{a^2c+d} + \frac{4(ax-1)^{3/2}\left(c+dx^2\right)\sqrt{\frac{(ax+1)\left(a\sqrt{c}-i\sqrt{d}\right)}{(ax-1)\left(a\sqrt{c}+i\sqrt{d}\right)}}\left(a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right)\sqrt{\frac{\left(a^2c+d\right)\left(c+dx^2\right)}{cd(ax-1)^2}}\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}\,x}{\sqrt{c}}-1}{1-ax}}\,\Pi\left(\frac{2a\sqrt{c}}{\sqrt{c}\,a+i\sqrt{d}};\sin^{-1}\left(\right.\right.\right.}{a\sqrt{ax+1}\left(a^2c+\right.}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] (-((a*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2))/(a^2*c + d)) + x*(3*c + 2
*d*x^2)*ArcCosh[a*x] + (4*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1
 + a*x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*(c + d*x^2)*((a*((-I)*a*Sqrt
[c] + Sqrt[d])*(I*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] - a*
x + (I*Sqrt[d]*x)/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt
[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt
```

```
[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt[c
]) + I*Sqrt[d])*Sqrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-((
-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*Ellip
ticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[d]
*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[c]
*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2]))/(a*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[-(
(-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]))/(3
*c^2*(c + d*x^2)^(3/2))
```

fricas [B] time = 0.72, size = 613, normalized size = 3.41

$$\left[\frac{\left(a^2c^3 + \left(a^2cd^2 + d^3\right)x^4 + c^2d + 2\left(a^2c^2d + cd^2\right)x^2\right)\sqrt{d}\,\log\left(8\,a^4d^2x^4 + a^4c^2 - 6\,a^2cd + 8\left(a^4cd - a^2d^2\right)x^2 - 4\right.}{6\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((a^2*c^3 + (a^2*c*d^2 + d^3)*x^4 + c^2*d + 2*(a^2*c^2*d + c*d^2)*x^2)
*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2)*x^
2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(d)
 + d^2) + 2*(2*(a^2*c*d^2 + d^3)*x^3 + 3*(a^2*c^2*d + c*d^2)*x)*sqrt(d*x^2
+ c)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(a*c*d^2*x^2 + a*c^2*d)*sqrt(a^2*x^2
- 1)*sqrt(d*x^2 + c))/(a^2*c^5*d + c^4*d^2 + (a^2*c^3*d^3 + c^2*d^4)*x^4 +
2*(a^2*c^4*d^2 + c^3*d^3)*x^2), 1/3*((a^2*c^3 + (a^2*c*d^2 + d^3)*x^4 + c^2
*d + 2*(a^2*c^2*d + c*d^2)*x^2)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c -
d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c
*d - a*d^2)*x^2)) + (2*(a^2*c*d^2 + d^3)*x^3 + 3*(a^2*c^2*d + c*d^2)*x)*sqr
t(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (a*c*d^2*x^2 + a*c^2*d)*sqrt(a^
2*x^2 - 1)*sqrt(d*x^2 + c))/(a^2*c^5*d + c^4*d^2 + (a^2*c^3*d^3 + c^2*d^4)*
x^4 + 2*(a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

giac [A] time = 0.60, size = 190, normalized size = 1.06

$$-\frac{1}{3}\,a\left(\frac{2\,a^2|d|}{\left(a^2cd + \left(\sqrt{a^2d}\,\sqrt{dx^2+c} - \sqrt{\left(dx^2+c\right)a^2d - a^2cd - d^2}\right)^2 + d^2\right)c\sqrt{d}\,|a|} - \frac{|d|\log\left(\left(\sqrt{a^2d}\,\sqrt{dx^2+c} - \sqrt{(a\right.\right.}{c^2d^{\frac{3}{2}}|a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*a*(2*a^2*abs(d)/((a^2*c*d + (sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2
 + c)*a^2*d - a^2*c*d - d^2))^2 + d^2)*c*sqrt(d)*abs(a)) - abs(d)*log((sqrt
(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2)/(c^2*
d^(3/2)*abs(a))) + 1/3*x*(2*d*x^2/c^2 + 3/c)*log(a*x + sqrt(a^2*x^2 - 1))/(
d*x^2 + c)^(3/2)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{\left(d\,x^2 + c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(5/2),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 details)Is d-a^2*c zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(5/2),x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^(5/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**(5/2), x)
```

3.51 $\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{8\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{15c^3\sqrt{d}\sqrt{ax-1}\sqrt{ax+1}}+\frac{2a\left(1-a^2x^2\right)\left(3a^2c+2d\right)}{15c^2\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)^2\sqrt{c+dx^2}}+\frac{a\left(1-a^2x^2\right)}{15c\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right.}$$

```
[Out] 1/5*x*arccosh(a*x)/c/(d*x^2+c)^(5/2)+4/15*x*arccosh(a*x)/c^2/(d*x^2+c)^(3/2
)+1/15*a*(-a^2*x^2+1)/c/(a^2*c+d)/(d*x^2+c)^(3/2)/(a*x-1)^(1/2)/(a*x+1)^(1/
2)-8/15*arctanh(d^(1/2)*(a^2*x^2-1)^(1/2)/a/(d*x^2+c)^(1/2))*(a^2*x^2-1)^(1
/2)/c^3/d^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+8/15*x*arccosh(a*x)/c^3/(d*x^2+
c)^(1/2)+2/15*a*(3*a^2*c+2*d)*(-a^2*x^2+1)/c^2/(a^2*c+d)^2/(a*x-1)^(1/2)/(a
*x+1)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {192, 191, 5705, 12, 519, 6715, 949, 78, 63, 217, 206}

$$\frac{2a\left(1-a^2x^2\right)\left(3a^2c+2d\right)}{15c^2\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)^2\sqrt{c+dx^2}}-\frac{8\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{15c^3\sqrt{d}\sqrt{ax-1}\sqrt{ax+1}}+\frac{a\left(1-a^2x^2\right)}{15c\sqrt{ax-1}\sqrt{ax+1}\left(a^2c+d\right)(}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (a*(1 - a^2*x^2))/(15*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2
)^(3/2)) + (2*a*(3*a^2*c + 2*d)*(1 - a^2*x^2))/(15*c^2*(a^2*c + d)^2*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*x^2]) + (x*ArcCosh[a*x])/(5*c*(c + d*x^2)
^(5/2)) + (4*x*ArcCosh[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcCosh[a*x]
)/(15*c^3*Sqrt[c + d*x^2]) - (8*Sqrt[-1 + a^2*x^2]*ArcTanh[(Sqrt[d]*Sqrt[-1
 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(15*c^3*Sqrt[d]*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{7/2}}\,dx = \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x\left(15c^2+20cdx^2+8d^2x^4 \cdots\right.}{15c^3\sqrt{-1+ax}\sqrt{1+ax}\left(c+d \cdots\right.}$$

$$= \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(15c^2+20cdx^2+8d^2x^4\right)}{\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{5/2}}\,dx}{15c^3}$$

$$= \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\int \frac{x\left(15c^2+20cdx^2+8d^2 \cdots\right.}{\sqrt{-1+a^2x^2}\left(c+dx^2\right)^{\cdots}}}{15c^3\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)\text{Subst}\left(\int \frac{15c^2+20c \cdots}{\sqrt{-1+a^2x} \cdots}\right.}{30c^3\sqrt{-1+ax}\sqrt{1+ \cdots}}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{x\cosh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\cosh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x \cdots}{15c \cdots}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{2a\left(3a^2c+2d\right)\left(1-a^2x^2\right)}{15c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+d \cdots}}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{2a\left(3a^2c+2d\right)\left(1-a^2x^2\right)}{15c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+d \cdots}}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{2a\left(3a^2c+2d\right)\left(1-a^2x^2\right)}{15c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+d \cdots}}$$

$$= \frac{a\left(1-a^2x^2\right)}{15c\left(a^2c+d\right)\sqrt{-1+ax}\sqrt{1+ax}\left(c+dx^2\right)^{3/2}} + \frac{2a\left(3a^2c+2d\right)\left(1-a^2x^2\right)}{15c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{c+d \cdots}}$$

Mathematica [C] time = 3.51, size = 655, normalized size = 2.43

$$\frac{16(ax-1)^{3/2}\left(c+dx^2\right)^2\sqrt{\frac{(ax+1)\left(a\sqrt{c}-i\sqrt{d}\right)}{(ax-1)\left(a\sqrt{c}+i\sqrt{d}\right)}}\left(a\sqrt{c}\left(-a\sqrt{c}+i\sqrt{d}\right)\sqrt{\frac{\left(a^2c+d\right)\left(c+dx^2\right)}{cd(ax-1)^2}}\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{1-ax}}\,\Pi\left(\frac{2a\sqrt{c}}{\sqrt{c}a+i\sqrt{d}};\sin^{-1}\left(\sqrt{-\frac{\frac{i\sqrt{d}x}{\sqrt{c}}+a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)-1}{2-2ax}}\right)\middle| \cdots\right) \cdots\right)}{ac^3\sqrt{ax+1}\left(a^2c+d\right)\sqrt{-\frac{a\left(x+\frac{i\sqrt{c}}{\sqrt{d}}\right)+\frac{i\sqrt{d}x}{\sqrt{c}}-1}{1-ax}} \cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (-((a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(
7*c + 6*d*x^2)))/(c^2*(a^2*c + d)^2)) + (x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4
```

```
)*ArcCosh[a*x])/c^3 + (16*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])*(1
 + a*x))/((a*Sqrt[c] + I*Sqrt[d])*(-1 + a*x))]*(c + d*x^2)^2*((a*((-I)*a*Sq
rt[c] + Sqrt[d])*(I*Sqrt[c] + Sqrt[d]*x)*Sqrt[(1 + (I*a*Sqrt[c])/Sqrt[d] -
a*x + (I*Sqrt[d]*x)/Sqrt[c])/(1 - a*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sq
rt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sq
rt[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2])/(-1 + a*x) + a*Sqrt[c]*(-(a*Sqrt
[c]) + I*Sqrt[d])*Sqrt[((a^2*c + d)*(c + d*x^2))/(c*d*(-1 + a*x)^2)]*Sqrt[-
((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))]*Ell
ipticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqrt[-((-1 + (I*Sqrt[
d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(2 - 2*a*x))]], ((4*I)*a*Sqrt[
c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d])^2]))/(a*c^3*(a^2*c + d)*Sqrt[1 + a*x]*S
qrt[-((-1 + (I*Sqrt[d]*x)/Sqrt[c] + a*((I*Sqrt[c])/Sqrt[d] + x))/(1 - a*x))
]))/(15*(c + d*x^2)^(5/2))
```

fricas [B] time = 0.90, size = 1098, normalized size = 4.08

$$\left[\frac{2\left(a^4c^5+2\,a^2c^4d+\left(a^4c^2d^3+2\,a^2cd^4+d^5\right)x^6+c^3d^2+3\left(a^4c^3d^2+2\,a^2c^2d^3+cd^4\right)x^4+3\left(a^4c^4d+2\,a^2c^3d^2+c^2d\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(2*(a^4*c^5 + 2*a^2*c^4*d + (a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^6 + c
^3*d^2 + 3*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^4 + 3*(a^4*c^4*d + 2*a^2
*c^3*d^2 + c^2*d^3)*x^2)*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d +
8*(a^4*c*d - a^2*d^2)*x^2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)
*sqrt(d*x^2 + c)*sqrt(d) + d^2) + (8*(a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^5
+ 20*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^4*c^4*d + 2*a^2*c^3*
d^2 + c^2*d^3)*x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (7*a^3*c^4
*d + 5*a*c^3*d^2 + 2*(3*a^3*c^2*d^3 + 2*a*c*d^4)*x^4 + (13*a^3*c^3*d^2 + 9*
a*c^2*d^3)*x^2)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^4*c^8*d + 2*a^2*c^7*d
^2 + c^6*d^3 + (a^4*c^5*d^4 + 2*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^4*c^6*d^3
 + 2*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^4*c^7*d^2 + 2*a^2*c^6*d^3 + c^5*d^4)
*x^2), 1/15*(4*(a^4*c^5 + 2*a^2*c^4*d + (a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x
^6 + c^3*d^2 + 3*(a^4*c^3*d^2 + 2*a^2*c^2*d^3 + c*d^4)*x^4 + 3*(a^4*c^4*d +
 2*a^2*c^3*d^2 + c^2*d^3)*x^2)*sqrt(-d)*arctan(1/2*(2*a^2*d*x^2 + a^2*c - d
)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*x^4 - a*c*d + (a^3*c*
d - a*d^2)*x^2)) + (8*(a^4*c^2*d^3 + 2*a^2*c*d^4 + d^5)*x^5 + 20*(a^4*c^3*d
^2 + 2*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^4*c^4*d + 2*a^2*c^3*d^2 + c^2*d^3)*
x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (7*a^3*c^4*d + 5*a*c^3*d^
2 + 2*(3*a^3*c^2*d^3 + 2*a*c*d^4)*x^4 + (13*a^3*c^3*d^2 + 9*a*c^2*d^3)*x^2)
*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^4*c^8*d + 2*a^2*c^7*d^2 + c^6*d^3 +
(a^4*c^5*d^4 + 2*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^4*c^6*d^3 + 2*a^2*c^5*d^
4 + c^4*d^5)*x^4 + 3*(a^4*c^7*d^2 + 2*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

giac [A] time = 0.76, size = 411, normalized size = 1.53

$$\frac{4}{15}\,a\left(\frac{|d|\log\left(\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{\left(dx^2+c\right)a^2d-a^2cd-d^2}\right)^2\right)}{c^3d^{\frac{3}{2}}|a|}-\frac{3\,a^6c^2d^{\frac{5}{2}}|d|+7\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{\left(dx^2+c\right)a\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 4/15*a*(abs(d)*log((sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d -
a^2*c*d - d^2))^2)/(c^3*d^(3/2)*abs(a)) - (3*a^6*c^2*d^(5/2)*abs(d) + 7*(sq
```

```
rt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2*a^4*
c*d^(3/2)*abs(d) + 5*a^4*c*d^(7/2)*abs(d) + 2*(sqrt(a^2*d)*sqrt(d*x^2 + c)
- sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^4*a^2*sqrt(d)*abs(d) + 4*(sqrt(a
^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2*a^2*d^(5
/2)*abs(d) + 2*a^2*d^(9/2)*abs(d))/((a^2*c*d + (sqrt(a^2*d)*sqrt(d*x^2 + c)
 - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2 + d^2)^3*c^2*d*abs(a))) + 1/1
5*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c)*x*log(a*x + sqrt(a^2*x^2 - 1))/(
d*x^2 + c)^(5/2)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{\left(d\,x^{2}+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(7/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 details)Is d-a^2*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}\left(a\,x\right)}{{\left(d\,x^2+c\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(7/2),x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}{\left(ax \right)}}{\left(c + dx^{2}\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Integral(acosh(a*x)/(c + d*x**2)**(7/2), x)
```

3.52 $\int \frac{\cosh^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=369

$$-\frac{16\sqrt{a^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a^2x^2-1}}{a\sqrt{c+dx^2}}\right)}{35c^4\sqrt{d}\,\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2a\left(1-a^2x^2\right)\left(5a^2c+3d\right)}{105c^2\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}+\frac{a\left(1-a^2x^2\right)}{35c\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2c+\right.}$$

```
[Out] 1/7*x*arccosh(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arccosh(a*x)/c^2/(d*x^2+c)^(5/2
)+8/35*x*arccosh(a*x)/c^3/(d*x^2+c)^(3/2)+1/35*a*(-a^2*x^2+1)/c/(a^2*c+d)/(
d*x^2+c)^(5/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/105*a*(5*a^2*c+3*d)*(-a^2*x^2+
1)/c^2/(a^2*c+d)^2/(d*x^2+c)^(3/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)-16/35*arctan
h(d^(1/2)*(a^2*x^2-1)^(1/2)/a/(d*x^2+c)^(1/2))*(a^2*x^2-1)^(1/2)/c^4/d^(1/2
)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+16/35*x*arccosh(a*x)/c^4/(d*x^2+c)^(1/2)+4/10
5*a*(11*a^4*c^2+15*a^2*c*d+6*d^2)*(-a^2*x^2+1)/c^3/(a^2*c+d)^3/(a*x-1)^(1/2
)/(a*x+1)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] time = 1.01, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {192, 191, 5705, 12, 519, 6715, 1622, 949, 78, 63, 217, 206}

$$\frac{4a\left(1-a^2x^2\right)\left(11a^4c^2+15a^2cd+6d^2\right)}{105c^3\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2c+d\right)^3\sqrt{c+dx^2}}+\frac{2a\left(1-a^2x^2\right)\left(5a^2c+3d\right)}{105c^2\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}-\frac{16\sqrt{a^2x^2-1}\,\tanh^{-}}{35c^4\sqrt{d}\,\sqrt{ax-}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (a*(1 - a^2*x^2))/(35*c*(a^2*c + d)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2
)^(5/2)) + (2*a*(5*a^2*c + 3*d)*(1 - a^2*x^2))/(105*c^2*(a^2*c + d)^2*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*(c + d*x^2)^(3/2)) + (4*a*(11*a^4*c^2 + 15*a^2*c*d
+ 6*d^2)*(1 - a^2*x^2))/(105*c^3*(a^2*c + d)^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*Sqrt[c + d*x^2]) + (x*ArcCosh[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcCosh
[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcCosh[a*x])/(35*c^3*(c + d*x^2)^
(3/2)) + (16*x*ArcCosh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - (16*Sqrt[-1 + a^2*x
^2]*ArcTanh[(Sqrt[d]*Sqrt[-1 + a^2*x^2])/(a*Sqrt[c + d*x^2])])/(35*c^4*Sqrt
[d]*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1622

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && GtQ[Expon[Px, x], 2
]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c+dx^2\right)^{9/2}}\,dx = \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x\left(35c^3+7\right.}{35c^4\sqrt{-1}}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(35c^3+70c^2dx\right.}{\sqrt{-1+ax}\,\sqrt{}}}{3}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)}{35c}$$

$$= \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\cosh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{\left(a\sqrt{-1+a^2x^2}\right)}{}$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{5/2}} + \frac{x\cosh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\cosh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\cosh}{35c^3\left(\right.}$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right.}$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right.}$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right.}$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right.}$$

$$= \frac{a\left(1-a^2x^2\right)}{35c\left(a^2c+d\right)\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right)^{5/2}} + \frac{2a\left(5a^2c+3d\right)\left(1-a^2x^2\right)}{105c^2\left(a^2c+d\right)^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\left(c+dx^2\right.}$$

Mathematica [C] time = 6.27, size = 844, normalized size = 2.29

$$\frac{32\sqrt{\frac{\left(a\sqrt{c}-i\sqrt{d}\right)\left(1+\frac{2}{ax-1}\right)}{\sqrt{c}a+i\sqrt{d}}}\left(\left(\sqrt{c}a+i\sqrt{d}\right)\sqrt{-\frac{i\left(\frac{ca^2}{ax-1}+i\sqrt{c}\sqrt{d}a+d+\frac{d}{ax-1}\right)}{a\sqrt{c}\sqrt{d}}}\left(\frac{a\sqrt{c}}{ax-1}-i\sqrt{d}\left(1+\frac{1}{ax-1}\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{i\left(\frac{ca^2}{ax-1}-i\sqrt{c}\sqrt{\cdots}\right)}{a\sqrt{c}\cdots}}}{\sqrt{2}}\cdots\right.\right.\right.}{35ac^4\left(ca\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[c + d*x^2]*(-1/35*a/(c*(a^2*c + d)*(c + d
*x^2)^3) - (2*a*(5*a^2*c + 3*d))/(105*c^2*(a^2*c + d)^2*(c + d*x^2)^2) - (4
*a*(11*a^4*c^2 + 15*a^2*c*d + 6*d^2))/(105*c^3*(a^2*c + d)^3*(c + d*x^2)))
+ (x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^6)*ArcCosh[a*x])/(35*
c^4*(c + d*x^2)^(7/2)) + (32*(-1 + a*x)^(3/2)*Sqrt[((a*Sqrt[c] - I*Sqrt[d])
*(1 + 2/(-1 + a*x)))/(a*Sqrt[c] + I*Sqrt[d])]*((a*Sqrt[c] + I*Sqrt[d])*Sqrt
[((-I)*(I*a*Sqrt[c]*Sqrt[d] + d + (a^2*c)/(-1 + a*x) + d/(-1 + a*x)))/(a*Sq
rt[c]*Sqrt[d])]*((a*Sqrt[c])/(-1 + a*x) - I*Sqrt[d]*(1 + (-1 + a*x)^(-1)))*
EllipticF[ArcSin[Sqrt[(I*((-I)*a*Sqrt[c]*Sqrt[d] + d + (a^2*c)/(-1 + a*x) +
 d/(-1 + a*x)))/(a*Sqrt[c]*Sqrt[d])]/Sqrt[2]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a
*Sqrt[c] + I*Sqrt[d])^2] + a*Sqrt[c]*(-(a*Sqrt[c]) + I*Sqrt[d])*Sqrt[(I*((-
I)*a*Sqrt[c]*Sqrt[d] + d + (a^2*c)/(-1 + a*x) + d/(-1 + a*x)))/(a*Sqrt[c]*S
qrt[d])]*Sqrt[((a^2*c + d)*((a^2*c)/(-1 + a*x)^2 + d*(1 + (-1 + a*x)^(-1))^
2))/(a^2*c*d)]*EllipticPi[(2*a*Sqrt[c])/(a*Sqrt[c] + I*Sqrt[d]), ArcSin[Sqr
t[(I*((-I)*a*Sqrt[c]*Sqrt[d] + d + (a^2*c)/(-1 + a*x) + d/(-1 + a*x)))/(a*S
qrt[c]*Sqrt[d])]/Sqrt[2]], ((4*I)*a*Sqrt[c]*Sqrt[d])/(a*Sqrt[c] + I*Sqrt[d]
)^2]))/(35*a*c^4*(a^2*c + d)*Sqrt[1 + a*x]*Sqrt[(I*((-I)*a*Sqrt[c]*Sqrt[d]
+ d + (a^2*c)/(-1 + a*x) + d/(-1 + a*x)))/(a*Sqrt[c]*Sqrt[d])]*Sqrt[c + (d*
(-1 + a*x)^2*(1 + (-1 + a*x)^(-1))^2)/a^2])
```

fricas [B] time = 1.26, size = 1752, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/105*(12*(a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 + (a^6*c^3*d^4 + 3*a^4*c^
2*d^5 + 3*a^2*c*d^6 + d^7)*x^8 + c^4*d^3 + 4*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 +
 3*a^2*c^2*d^5 + c*d^6)*x^6 + 6*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^
4 + c^2*d^5)*x^4 + 4*(a^6*c^6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*
x^2)*sqrt(d)*log(8*a^4*d^2*x^4 + a^4*c^2 - 6*a^2*c*d + 8*(a^4*c*d - a^2*d^2
)*x^2 - 4*(2*a^3*d*x^2 + a^3*c - a*d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqr
t(d) + d^2) + 3*(16*(a^6*c^3*d^4 + 3*a^4*c^2*d^5 + 3*a^2*c*d^6 + d^7)*x^7 +
 56*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5 + c*d^6)*x^5 + 70*(a^6*c^5
*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x^3 + 35*(a^6*c^6*d + 3*a^4
*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x)*sqrt(d*x^2 + c)*log(a*x + sqrt(a^2*x
^2 - 1)) - (57*a^5*c^6*d + 82*a^3*c^5*d^2 + 33*a*c^4*d^3 + 4*(11*a^5*c^3*d^
4 + 15*a^3*c^2*d^5 + 6*a*c*d^6)*x^6 + 2*(71*a^5*c^4*d^3 + 98*a^3*c^3*d^4 +
39*a*c^2*d^5)*x^4 + (155*a^5*c^5*d^2 + 218*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2)
*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^6*c^11*d + 3*a^4*c^10*d^2 + 3*a^2*c^
9*d^3 + c^8*d^4 + (a^6*c^7*d^5 + 3*a^4*c^6*d^6 + 3*a^2*c^5*d^7 + c^4*d^8)*x
^8 + 4*(a^6*c^8*d^4 + 3*a^4*c^7*d^5 + 3*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^6
*c^9*d^3 + 3*a^4*c^8*d^4 + 3*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^6*c^10*d^2 +
 3*a^4*c^9*d^3 + 3*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/105*(24*(a^6*c^7 + 3*a^4*
c^6*d + 3*a^2*c^5*d^2 + (a^6*c^3*d^4 + 3*a^4*c^2*d^5 + 3*a^2*c*d^6 + d^7)*x
```

```
^8 + c^4*d^3 + 4*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5 + c*d^6)*x^6
+ 6*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x^4 + 4*(a^6*c^
6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x^2)*sqrt(-d)*arctan(1/2*(2*
a^2*d*x^2 + a^2*c - d)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c)*sqrt(-d)/(a^3*d^2*
x^4 - a*c*d + (a^3*c*d - a*d^2)*x^2)) + 3*(16*(a^6*c^3*d^4 + 3*a^4*c^2*d^5
+ 3*a^2*c*d^6 + d^7)*x^7 + 56*(a^6*c^4*d^3 + 3*a^4*c^3*d^4 + 3*a^2*c^2*d^5
+ c*d^6)*x^5 + 70*(a^6*c^5*d^2 + 3*a^4*c^4*d^3 + 3*a^2*c^3*d^4 + c^2*d^5)*x
^3 + 35*(a^6*c^6*d + 3*a^4*c^5*d^2 + 3*a^2*c^4*d^3 + c^3*d^4)*x)*sqrt(d*x^2
 + c)*log(a*x + sqrt(a^2*x^2 - 1)) - (57*a^5*c^6*d + 82*a^3*c^5*d^2 + 33*a*
c^4*d^3 + 4*(11*a^5*c^3*d^4 + 15*a^3*c^2*d^5 + 6*a*c*d^6)*x^6 + 2*(71*a^5*c
^4*d^3 + 98*a^3*c^3*d^4 + 39*a*c^2*d^5)*x^4 + (155*a^5*c^5*d^2 + 218*a^3*c^
4*d^3 + 87*a*c^3*d^4)*x^2)*sqrt(a^2*x^2 - 1)*sqrt(d*x^2 + c))/(a^6*c^11*d +
 3*a^4*c^10*d^2 + 3*a^2*c^9*d^3 + c^8*d^4 + (a^6*c^7*d^5 + 3*a^4*c^6*d^6 +
3*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^6*c^8*d^4 + 3*a^4*c^7*d^5 + 3*a^2*c^6*d
^6 + c^5*d^7)*x^6 + 6*(a^6*c^9*d^3 + 3*a^4*c^8*d^4 + 3*a^2*c^7*d^5 + c^6*d^
6)*x^4 + 4*(a^6*c^10*d^2 + 3*a^4*c^9*d^3 + 3*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

giac [B] time = 1.19, size = 876, normalized size = 2.37

$$\frac{8}{105}a\left(\frac{3\,|d|\log\left(\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{\left(dx^2+c\right)a^2d-a^2cd-d^2}\right)^2\right)}{c^4d^{\frac{3}{2}}|a|}-\frac{11\,a^{10}c^4d^{\frac{9}{2}}|d|+49\left(\sqrt{a^2d}\,\sqrt{dx^2+c}-\sqrt{\left(dx^2\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 8/105*a*(3*abs(d)*log((sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d
 - a^2*c*d - d^2))^2)/(c^4*d^(3/2)*abs(a)) - (11*a^10*c^4*d^(9/2)*abs(d) +
49*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^
2*a^8*c^3*d^(7/2)*abs(d) + 37*a^8*c^3*d^(11/2)*abs(d) + 77*(sqrt(a^2*d)*sqr
t(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^4*a^6*c^2*d^(5/2)*a
bs(d) + 112*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d
 - d^2))^2*a^6*c^2*d^(9/2)*abs(d) + 47*a^6*c^2*d^(13/2)*abs(d) + 33*(sqrt(a
^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^6*a^4*c*d^
(3/2)*abs(d) + 93*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a
^2*c*d - d^2))^4*a^4*c*d^(7/2)*abs(d) + 87*(sqrt(a^2*d)*sqrt(d*x^2 + c) - s
qrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2*a^4*c*d^(11/2)*abs(d) + 27*a^4*c*
d^(15/2)*abs(d) + 6*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d -
 a^2*c*d - d^2))^8*a^2*sqrt(d)*abs(d) + 24*(sqrt(a^2*d)*sqrt(d*x^2 + c) - s
qrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^6*a^2*d^(5/2)*abs(d) + 36*(sqrt(a^2
*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^4*a^2*d^(9/2
)*abs(d) + 24*(sqrt(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c
*d - d^2))^2*a^2*d^(13/2)*abs(d) + 6*a^2*d^(17/2)*abs(d))/((a^2*c*d + (sqrt
(a^2*d)*sqrt(d*x^2 + c) - sqrt((d*x^2 + c)*a^2*d - a^2*c*d - d^2))^2 + d^2)
^5*c^3*d*abs(a))) + 1/35*(2*(4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*
x^2 + 35/c)*x*log(a*x + sqrt(a^2*x^2 - 1))/(d*x^2 + c)^(7/2)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)}{\left(d\,x^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arccosh(a*x)/(d*x^2+c)^(9/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for more
 details)Is d-a^2*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}\left(a\,x\right)}{{\left(d\,x^2+c\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c + d*x^2)^(9/2),x)
```

```
[Out] int(acosh(a*x)/(c + d*x^2)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

3.53 $\int (f+gx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=713

$$\frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{f^2g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\text{c}\right.}{c^2}$$

```
[Out] 1/2*f^3*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-3/8*f*g^2*x*(a+b*arccosh(
c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+3/4*f*g^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2
+d)^(1/2)-f^2*g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^
2-2/15*g^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-1/5
*g^3*x^2*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+b*f^2
*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/15*b*g^3*x*(-c^2*
d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c*f^3*x^2*(-c^2*d*x^2+
d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/16*b*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/
c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b*c*f^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+1/45*b*g^3*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-3/16*b*c*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-1/25*b*c*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4
*f^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-3/16*f*g^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2
)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.48, antiderivative size = 713, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5836, 5822, 5683, 5676, 30, 5718, 5743, 5759, 100, 12, 74, 5733}

$$-\frac{f^2g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2}+\frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^3\sqrt{d-c^2dx^2}\left(a+b\right.}{4bc\sqrt{cx-1}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*f^2*g*x*Sqrt[d - c^2*d*x^2])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*g^3
*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f^3*x^
2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*f*g^2*x^2*Sq
rt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f^2*g*x^3*Sqr
t[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*g^3*x^3*Sqrt[d - c^
2*d*x^2])/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*f*g^2*x^4*Sqrt[d - c
^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g^3*x^5*Sqrt[d - c^2*d*
x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f^3*x*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/2 - (3*f*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8
*c^2) + (3*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (f^2*g*(
1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/c^2 - (2*g^3*(
1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*c^4) - (g^
3*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2)
 - (f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (3*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int\left(f^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)+3f\right.}{}$$

$$= \frac{\left(f^3\sqrt{d-c^2dx^2}\right)\int\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{2}f^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}fg^2x^3\sqrt{d-c^2dx^2}\,(a$$

$$= \frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^2gx^3\sqrt{d-c^2}}{3\sqrt{-1+cx}\sqrt{1}}$$

$$= \frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^3x^2\sqrt{d-}}{4\sqrt{-1+cx}}$$

Mathematica [A] time = 1.97, size = 491, normalized size = 0.69

$$\frac{-3600ac\sqrt{d}f\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(4c^2f^2+3g^2\right)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+240a\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(6c^4x\left(10f^3+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (240*a*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-16*g^3 -
c^2*g*(120*f^2 + 45*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*
f*g^2*x^2 + 4*g^3*x^3)) - 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 + 3*g^2)*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 +
c^2*x^2))] + 2400*b*c^2*f^2*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(
1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 3600*b*c
^3*f^3*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[
c*x] - Sinh[2*ArcCosh[c*x]])) - 675*b*c*f*g^2*Sqrt[d - c^2*d*x^2]*(8*ArcCos
h[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + 8*
b*g^3*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*Arc
Cosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(28
800*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ag^3x^3+3afg^2x^2+3af^2gx+af^3+\left(bg^3x^3+3bfg^2x^2+3bf^2gx+bf^3\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 1.08, size = 1213, normalized size = 1.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -2/15*a*g^3/d/c^4*(-c^2*d*x^2+d)^(3/2)+3/4*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(
c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(c*x
+1)/c^2/(c*x-1)*arccosh(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2)*g/(c*x+1)*c^2/(c*x-
1)*arccosh(c*x)*x^4*f^2+1/2*a*f^3*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*f^3*d/(c^2*d
)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3/8*a*f*g^2/c^2*x*(-c^
2*d*x^2+d)^(1/2)-a*f^2*g/c^2/d*(-c^2*d*x^2+d)^(3/2)-1/3*b*(-d*(c^2*x^2-1))^
(1/2)*g/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3*f^2+b*(-d*(c^2*x^2-1))^(1/2)*g/(c
*x+1)^(1/2)/c/(c*x-1)^(1/2)*x*f^2-3/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(c*x+
1)^(1/2)*c/(c*x-1)^(1/2)*x^4+3/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(c*x+1)^(1
/2)/c/(c*x-1)^(1/2)*x^2-9/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(c*x+1)/(c*x-1)*
arccosh(c*x)*x^3-2*b*(-d*(c^2*x^2-1))^(1/2)*g/(c*x+1)/(c*x-1)*arccosh(c*x)*
x^2*f^2-3/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*f*arc
cosh(c*x)^2*g^2+1/5*b*(-d*(c^2*x^2-1))^(1/2)*g^3/(c*x+1)*c^2/(c*x-1)*arccos
h(c*x)*x^6-1/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/(c*x+1)/c^2/(c*x-1)*arccosh(c*
x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)*g/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*f^2+1/2*
b*(-d*(c^2*x^2-1))^(1/2)*f^3/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3-1/5*a*g^3
*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*b*(-d*(c^2*x^2-1))^(1/2)*f^3/(c*x+1)^(1
/2)/c/(c*x-1)^(1/2)-3/4*a*f*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+3/8*a*f*g^2/c^
2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/45*b*(-d*(
c^2*x^2-1))^(1/2)*g^3/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^3+2/15*b*(-d*(c^2*x^2
-1))^(1/2)*g^3/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*x-1/25*b*(-d*(c^2*x^2-1))^(1
/2)*g^3/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^5-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f^3/
(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2-3/128*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/(c*x
+1)^(1/2)/c^3/(c*x-1)^(1/2)-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)/c*f^3*arccosh(c*x)^2+2/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/(c*x+1)/c^
4/(c*x-1)*arccosh(c*x)-4/15*b*(-d*(c^2*x^2-1))^(1/2)*g^3/(c*x+1)/(c*x-1)*ar
ccosh(c*x)*x^4-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^3/(c*x+1)/(c*x-1)*arccosh(c*x
)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{-c^2dx^2+d}\,x+\frac{\sqrt{d}\,\arcsin(cx)}{c}\right)af^3-\frac{1}{15}\,ag^3\left(\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)+\frac{3}{8}\,afg^2\left(\frac{\sqrt{-c^2dx^2+d}}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f^3 - 1/15*a*g^3*(3*
(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 3/
8*a*f*g^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d)
+ sqrt(d)*arcsin(c*x)/c^3) - (-c^2*d*x^2 + d)^(3/2)*a*f^2*g/(c^2*d) + integ
```

```
rate(sqrt(-c^2*d*x^2 + d)*b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
+ 3*sqrt(-c^2*d*x^2 + d)*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
 + 3*sqrt(-c^2*d*x^2 + d)*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
+ sqrt(-c^2*d*x^2 + d)*b*f^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\,\mathrm{acosh}\left(cx\right)\right)\left(f+gx\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x)**3, x)
```

3.54 $\int (f+gx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=479

$$\frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{2fg(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{3c^2}$$

```
[Out] 1/2*f^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/8*g^2*x*(a+b*arccosh(c*
x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/4*g^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^
(1/2)-2/3*f*g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+
2/3*b*f*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c*f^2*
x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*g^2*x^2*(-c^2*d
*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/9*b*c*f*g*x^3*(-c^2*d*x^2+d)^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*b*c*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-1/4*f^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/
b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*g^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d
)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.18, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {5836, 5822, 5683, 5676, 30, 5718, 5743, 5759}

$$\frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{2fg(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*f*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f
^2*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*g^2*x^2*S
qrt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*f*g*x^3*Sq
rt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g^2*x^4*Sqrt[d -
 c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f^2*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/2 - (g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(8*c^2) + (g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (2*f*g*(1
 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c^2) - (f^2*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
```

```
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int\left(f^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)+2fgx\right.}{}$$

$$= \frac{\left(f^2\sqrt{d-c^2dx^2}\right)\int\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}+$$

$$= \frac{1}{2}f^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}g^2x^3\sqrt{d-c^2dx^2}\left(a+b\right.$$

$$= \frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcfgx^3\sqrt{d-c^2d}}{9\sqrt{-1+cx}\sqrt{1+}}$$

$$= \frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bg^2x^2\sqrt{d-c^2d}}{16c\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 1.21, size = 356, normalized size = 0.74

$$\frac{48ac\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(12c^2f^2x+16fg\left(c^2x^2-1\right)+3g^2x\left(2c^2x^2-1\right)\right)-144a\sqrt{d}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(4c^2f^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(12*c^2*f^
2*x + 16*f*g*(-1 + c^2*x^2) + 3*g^2*x*(-1 + 2*c^2*x^2)) - 144*a*Sqrt[d]*(4*
c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^
2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 64*b*c*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x
 + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCos
h[c*x]]) - 144*b*c^2*f^2*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcC
osh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) - 9*b*g^2*Sqrt[d - c^2*d*x^
2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh
[c*x]]))/(1152*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\text{arcosh}\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.94, size = 855, normalized size = 1.78

$$-\frac{a g^2 x \left(-c^2 d x^2+d\right)^{\frac{3}{2}}}{4c^2 d}+\frac{a g^2 x \sqrt{-c^2 d x^2+d}}{8c^2}+\frac{a g^2 d \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d x^2+d}}\right)}{8c^2 \sqrt{c^2 d}}-\frac{2 a f g \left(-c^2 d x^2+d\right)^{\frac{3}{2}}}{3c^2 d}+\frac{a f^2 x \sqrt{-c^2 d x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a*g^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a*g^2/c^2*x*(-c^2*d*x^2+d)^(1/2
)+1/8*a*g^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2)
)-2/3*a*f*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f^2*x*(-c^2*d*x^2+d)^(1/2)+1/2
*a*f^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/4*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*f^2-1/16
*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*g^
2+1/4*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5-3/8
*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/8*b*(-d*(c
^2*x^2-1))^(1/2)*g^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x-1/128*b*(-d*(c^2*x^
2-1))^(1/2)*g^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)+2/3*b*(-d*(c^2*x^2-1))^(1/2
)*f*g/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)+1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^2/(c*
x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f^2/(c*x+1)/
(c*x-1)*arccosh(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^(1/2)*f^2/(c*x+1)^(1/2)/c/(c*
x-1)^(1/2)-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x
^2-1/16*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+1/16
*b*(-d*(c^2*x^2-1))^(1/2)*g^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+2/3*b*(-d*(
c^2*x^2-1))^(1/2)*f*g/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4-4/3*b*(-d*(c^2*x
^2-1))^(1/2)*f*g/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2-2/9*b*(-d*(c^2*x^2-1))^(1
/2)*f*g/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3+2/3*b*(-d*(c^2*x^2-1))^(1/2)*f*g/
(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{-c^2dx^2+d}\,x+\frac{\sqrt{d}\,\arcsin\left(cx\right)}{c}\right)af^2+\frac{1}{8}\,ag^2\left(\frac{\sqrt{-c^2dx^2+d}\,x}{c^2}-\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}{c^2d}+\frac{\sqrt{d}\,\arcsin\left(cx\right)}{c^3}\right)-\frac{2\,(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f^2 + 1/8*a*g^2*(sqr
t(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + sqrt(d)*arcs
in(c*x)/c^3) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*f*g/(c^2*d) + integrate(sqrt(-c
^2*d*x^2 + d)*b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*sqrt(-c^
2*d*x^2 + d)*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(-c^2*d*x
^2 + d)*b*f^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
[Out] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\,(cx-1)\,(cx+1)}\,\,(a+b\operatorname{acosh}{(cx)})\left(f+gx\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x)**2, x)
```

3.55 $\int (f+gx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=255

$$\frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{3c^2}$$

```
[Out] 1/2*f*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/3*g*(-c*x+1)*(c*x+1)*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/3*b*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c*f*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-1/9*b*c*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-1/4*f*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5836, 5822, 5683, 5676, 30, 5718}

$$\frac{1}{2}fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{f\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{g(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*g*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*f*x^2
*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*g*x^3*Sqrt[d
- c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (f*x*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x]))/2 - (g*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(3*c^2) - (f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*
b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
```

```
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\,\sqrt{1+cx}\,(f+gx)\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{d-c^2dx^2}\int \left(f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)+gx\sqrt{-1\ldots\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(f\sqrt{d-c^2dx^2}\right)\int \sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(g\ldots}{}$$

$$= \frac{1}{2}fx\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right) - \frac{g(1-cx)(1+cx)\sqrt{d-c^2dx\ldots}}{3c^2}$$

$$= \frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+c\ldots}}$$

Mathematica [A] time = 1.05, size = 251, normalized size = 0.98

$$\frac{12a\sqrt{d-c^2dx^2}\left(3c^2fx+2g\left(c^2x^2-1\right)\right)-36ac\sqrt{d}\,f\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-\frac{9bcf\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-\ldots\right.}{\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)}}{72c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (12*a*Sqrt[d - c^2*d*x^2]*(3*c^2*f*x + 2*g*(-1 + c^2*x^2)) - 36*a*c*Sqrt[d]
*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*b*g*Sqrt
[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCos
```

```
h[c*x] - Cosh[3*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (9
*b*c*f*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*Arc
Cosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/(7
2*c^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\text{arcosh}\left(cx\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x)), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [B] time = 0.73, size = 461, normalized size = 1.81

$$-\frac{ag\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{3c^2d}+\frac{afx\sqrt{-c^2d\,x^2+d}}{2}+\frac{afd\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,g\,\text{arccosh}\left(cx\right)}{3\left(cx+1\right)c^2\left(cx-1\right)}+\frac{b\sqrt{-d\left(c\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a*g/c^2/d*(-c^2*d*x^2+d)^(3/2)+1/2*a*f*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*f*
d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b*(-d*(c^2
*x^2-1))^(1/2)*g/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)+1/3*b*(-d*(c^2*x^2-1))^(1
/2)*g/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4-2/3*b*(-d*(c^2*x^2-1))^(1/2)*g/(
c*x+1)/(c*x-1)*arccosh(c*x)*x^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)*f/(c*x+1)/(c*x
-1)*c^2*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*f/(c*x+1)/(c*x-1)*arc
cosh(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^(1/2)*f/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c-1/
9*b*(-d*(c^2*x^2-1))^(1/2)*g/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3+1/3*b*(-d*(c
^2*x^2-1))^(1/2)*g/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x-1/4*b*(-d*(c^2*x^2-1))^(
1/2)*f/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)/c*f*arccosh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{-c^2dx^2+d}\,x+\frac{\sqrt{d}\,\arcsin\left(cx\right)}{c}\right)af-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}ag}{3\,c^2d}+\int\sqrt{-c^2dx^2+d}\,bgx\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f - 1/3*(-c^2*d*x^2
+ d)^(3/2)*a*g/(c^2*d) + integrate(sqrt(-c^2*d*x^2 + d)*b*g*x*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + sqrt(-c^2*d*x^2 + d)*b*f*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
[Out] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\,\mathrm{acosh}\,(cx)\right)\left(f+gx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))*(f + g*x), x)
```

3.56 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=785

$$\frac{\sqrt{d-c^2dx^2}\left(1-\frac{c^2f^2}{g^2}\right)\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{cx-1}\sqrt{cx+1}\,(f+gx)}-\frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{cx-1}\sqrt{cx+1}\,(f+gx)}-\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{2bg\sqrt{cx-1}\sqrt{c}}$$

```
[Out] a*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/g/(-c*x+1)/(c*x+1)+b*arccosh(c*x)*(-c^2
*d*x^2+d)^(1/2)/g-b*c*x*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-
1/2*c*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g/(c*x-1)^(1/2)/(c*x+1)
^(1/2)+1/2*(1-c^2*f^2/g^2)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(g
*x+f)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(-c^2*x^2+1)*(a+b*arccosh(c*x))^2*(-c
^2*d*x^2+d)^(1/2)/b/c/(g*x+f)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*arccosh(c*x)*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-
g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*arccosh(c
*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^
2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*pol
ylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2
*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*poly
log(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*
f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-a*arcta
nh((c^2*f*x+g)/(c^2*f^2-g^2)^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*f^2-g^2)^(1/2)*(
c^2*x^2-1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c*x+1)/(c*x+1)
```

Rubi [A] time = 3.38, antiderivative size = 785, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.710, Rules used = {5836, 5824, 683, 5816, 6742, 93, 208, 1610, 1654, 12, 725, 206, 5860, 5858, 5718, 8, 5832, 3320, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\sqrt{d-c^2dx^2}\sqrt{c^2f^2-g^2}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{g^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] -((b*c*x*Sqrt[d - c^2*d*x^2])/(g*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (a*(1 - c
^2*x^2)*Sqrt[d - c^2*d*x^2])/(g*(1 - c*x)*(1 + c*x)) + (b*Sqrt[d - c^2*d*x^
2]*ArcCosh[c*x])/g - (c*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*
g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - (c^2*f^2)/g^2)*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) - ((
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(f + g*x)) - (a*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*x^2]*Sq
rt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*
x^2])])/(g^2*(1 - c*x)*(1 + c*x)) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(
g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(
g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*Pol
yLog[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5816

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*
Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[
n, 0] && IGtQ[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5824

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[((f + g*
x)^m*(d1*d2 + e1*e2*x^2)*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*
(n + 1)), x] - Dist[1/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
```

```
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5858

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5860

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(f+gx)} - \frac{\sqrt{d-c^2dx^2}\int \frac{\left(g+2c^2fx+\right.}{}}{2bc\sqrt{-1+c}}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{cc}\right.}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(f}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{cc}\right.}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(f}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{cc}\right.}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(f}$$

$$= -\frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{cc}\right.}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}\,(f}$$

$$= \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} - \frac{cx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bg\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(1-\right.}{}$$

$$= \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g} - \frac{cx\sqrt{d-c^2dx^2}\,(}{2bg\sqrt{-1}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\,\cosh}{g}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\,\cosh}{g}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\,\cosh}{g}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\,\cosh}{g}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{a\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{g(1-cx)(1+cx)} + \frac{b\sqrt{d-c^2dx^2}\,\cosh}{g}$$

Mathematica [C] time = 4.03, size = 1121, normalized size = 1.43

$$2a\sqrt{d-c^2dx^2}\,g-2ac\sqrt{d}\,f\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+2a\sqrt{d}\,\sqrt{g^2-c^2f^2}\,\log(f+gx)-2a\sqrt{d}\,\sqrt{g^2-c^2f^2}\,\log\left(d\left(fxc^2+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] (2*a*g*Sqrt[d - c^2*d*x^2] - 2*a*c*Sqrt[d]*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2
])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[f + g
*x] - 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt
[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]] + b*Sqrt[d - c^2*d*x^2]*((2*c*g*x*S
qrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + 2*g*ArcCosh[c*x] + (c*f*Sqrt[(-1 + c
*x)/(1 + c*x)]*ArcCosh[c*x]^2)/(1 - c*x) + (2*(-(c*f) + g)*(c*f + g)*(2*Arc
Cosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] -
 (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-
(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh
[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/
2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCos
h[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c
*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g
)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*S
qrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*
f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2
]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[
c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] -
(ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c
^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])*(1
+ Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCos
h[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I
*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f
^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) +
 g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f +
 g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(Sqrt[-(c^2*f^2) +
 g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(2*g^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{gx+f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(g*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.62, size = 1072, normalized size = 1.37

$$\frac{a\sqrt{-\left(x+\frac{f}{g}\right)^2 c^2 d+\frac{2c^2 d f\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}{g}+\frac{a c^2 d f \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-\left(x+\frac{f}{g}\right)^2 c^2 d+\frac{2c^2 d f\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}\right)}{g^2\sqrt{c^2 d}}+\frac{a d \ln\left(-\frac{2d\left(c^2f^2-g^2\right)}{g^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x)
```

```
[Out] a/g*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+a/g^2*
c^2*d*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*
(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^3*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln
((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))
*c^2*f^2-a/g*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^
2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/
g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-1/2*b*(-d*(c^2*x^2-1))^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f*arccosh(c*x)^2*c/g^2+b*(-d*(c^2*x^2-1))^(1/2
)/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^2*c^2-b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)/g*c*x-b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)/(c*x+1)/g*arccosh
(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)/g^2*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-
g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g
^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+b*(
-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*d
ilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(
c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)/g^2*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*
f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)Is g-c*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x),x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\,(a+b\,\mathrm{acosh}\,(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f),x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/(f + g*x), x)
```

3.57 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{(f+gx)^2}\,dx$

Optimal. Leaf size=918

$$\frac{bf^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)^2c^3}{2g^2\left(c^2f^2-g^2\right)\sqrt{cx-1}\sqrt{cx+1}}+\frac{af^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)c^3}{g^2\left(c^2f^2-g^2\right)\sqrt{cx-1}\sqrt{cx+1}}-\frac{2af\sqrt{d-c^2dx^2}\tanh^{-1}\left(\frac{\sqrt{cf+g}\sqrt{cx+1}}{\sqrt{cf-g}\sqrt{cx-1}}\right)c^2}{\sqrt{cf-g}\,g^2\sqrt{cf+g}\sqrt{cx-1}\sqrt{cx+1}}-$$

```
[Out] -a*(-c^2*d*x^2+d)^(1/2)/g/(g*x+f)+a*c^3*f^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1/
2)/g^2/(c^2*f^2-g^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*b*c^3*f^2*arccosh(c*x)
^2*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(
c^2*f*x+g)^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c^2*f^2-g^2)/(g
*x+f)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*(-c^2*x^2+1)*(a+b*arccosh(c*x))^2*(
-c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*ln(g*x+f)
*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*a*c^2*f*arctanh((c*
f+g)^(1/2)*(c*x+1)^(1/2)/(c*f-g)^(1/2)/(c*x-1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/
g^2/(c*f-g)^(1/2)/(c*f+g)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^2*f*arccosh
(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(
-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c
^2*f*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^
2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-b*c^2*f*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f
^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+b*c^2*f*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(
c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-b*arccosh(c*x)*((c*x-1)/(c*x+1))^(1/2)*(c*x+1)^(1/2)*(-c
^2*d*x^2+d)^(1/2)/g/(g*x+f)/(c*x-1)^(1/2)
```

Rubi [A] time = 3.56, antiderivative size = 918, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 22, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.710, Rules used = {5836, 5824, 37, 5814, 12, 180, 52, 96, 93, 208, 5860, 5858, 5676, 5832, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bf^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)^2c^3}{2g^2\left(c^2f^2-g^2\right)\sqrt{cx-1}\sqrt{cx+1}}+\frac{af^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)c^3}{g^2\left(c^2f^2-g^2\right)\sqrt{cx-1}\sqrt{cx+1}}-\frac{2af\sqrt{d-c^2dx^2}\tanh^{-1}\left(\frac{\sqrt{cf+g}\sqrt{cx+1}}{\sqrt{cf-g}\sqrt{cx-1}}\right)c^2}{\sqrt{cf-g}\,g^2\sqrt{cf+g}\sqrt{cx-1}\sqrt{cx+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x)^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/(g*(f + g*x))) + (a*c^3*f^2*Sqrt[d - c^2*d*x^2]*A
rcCosh[c*x])/(g^2*(c^2*f^2 - g^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-
((1 - c*x)/(1 + c*x))]*Sqrt[1 + c*x]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(g*S
qrt[-1 + c*x]*(f + g*x)) + (b*c^3*f^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2)/(
2*g^2*(c^2*f^2 - g^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((g + c^2*f*x)^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*(c^2*f^2 - g^2)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(f + g*x)^2) - ((1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)^2) - (2*a*c^2
*f*Sqrt[d - c^2*d*x^2]*ArcTanh[(Sqrt[c*f + g]*Sqrt[1 + c*x])/(Sqrt[c*f - g]
*Sqrt[-1 + c*x])])/(Sqrt[c*f - g]*g^2*Sqrt[c*f + g]*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*
g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcCosh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^2*d*x^2]*Log[f + g*x])/(g^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[
```

```
c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (b*c^2*f*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c
*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(g^2*Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5814

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.
) + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)
^m, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*c*n, Int[SimplifyInteg
rand[(u*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x
], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && IL
tQ[m, 0] && LtQ[m + p + 1, 0]
```

Rule 5824

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[((f + g*
x)^m*(d1*d2 + e1*e2*x^2)*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*
(n + 1)), x] - Dist[1/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5858

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5860

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{(f+gx)^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{(f+gx)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\sqrt{d-c^2dx^2}\int \frac{\left(2g+2c^2fx\right.}{}}{2bc\sqrt{-1+cx}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{2bc\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{2bc\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{2bc\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{2bc\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{2bc\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{\left(g+c^2fx\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}(f+gx)^2} - \frac{\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{2bc\sqrt{-1+cx}\sqrt{}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3f^2\sqrt{d-c^2}}{2g^2\left(c^2f^2-g^2\right)}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+c}}{g\sqrt{-}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+c}}{g\sqrt{-}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+c}}{g\sqrt{-}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+c}}{g\sqrt{-}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+c}}{g\sqrt{-}}$$

$$= -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} + \frac{ac^3f^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{g^2\left(c^2f^2-g^2\right)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-\frac{1-cx}{1+cx}}\sqrt{1+c}}{g\sqrt{-}}$$

Mathematica [C] time = 7.31, size = 1139, normalized size = 1.24

$$\frac{\frac{2a\sqrt{d}f\log(f+gx)c^2}{\sqrt{g^2-c^2f^2}} - \frac{2a\sqrt{d}f\log\left(d\left(fxc^2+g\right)+\sqrt{d}\sqrt{g^2-c^2f^2}\sqrt{d-c^2dx^2}\right)c^2}{\sqrt{g^2-c^2f^2}} + 2a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)c + b\sqrt{d-c^2dx^2}\left(\frac{\cosh^{-1}(c}{\sqrt{\frac{cx-1}{cx+1}}(cx}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f + g*x)^2,x]
```

```
[Out] ((-2*a*g*Sqrt[d - c^2*d*x^2])/(f + g*x) + 2*a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d
 - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*a*c^2*Sqrt[d]*f*Log[f + g*x])/
Sqrt[-(c^2*f^2) + g^2] - (2*a*c^2*Sqrt[d]*f*Log[d*(g + c^2*f*x) + Sqrt[d]*S
qrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[-(c^2*f^2) + g^2] + b*c*Sq
rt[d - c^2*d*x^2]*((-2*g*ArcCosh[c*x])/(c*f + c*g*x) + ArcCosh[c*x]^2/(Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (2*Log[1 + (g*x)/f])/(Sqrt[(-1 + c*x)/(
1 + c*x)]*(1 + c*x)) + (2*c*f*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCos
h[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*
f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)]
 + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + Arc
Tan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[
-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] +
 (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2)
 + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]
*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[Ar
cCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(
c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^
2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f)
 + g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g +
I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*
Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*
x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - Po
lyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g
^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcC
osh[c*x]/2]))])))/(Sqrt[-(c^2*f^2) + g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x))))/(2*g^2)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)}{g^2x^2+2\,fgx+f^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2
), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.93, size = 1956, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(3/2)-a/g*c^2*f/(c^2*f^2-g^2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/
g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-a/g^2*c^4*f^2/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*a
rctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)
/g^2)^(1/2))-a/g^3*c^4*f^3/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln(
(-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*
(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+
a/g*c^2*f/(c^2*f^2-g^2)*d/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^
2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d
+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))+a*c^2/(c^2*f^2-g^
2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+a*c^2
/(c^2*f^2-g^2)*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c
^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c/g^2+b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)/g^2/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x-1
)/(c*x+1)/g^2/(g*x+f)*x^3*c^4*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)/g/(g*x+f)*x*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/
(c*x-1)/(c*x+1)/g/(g*x+f)*x^2*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(g*x+f)*c*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c
*x)/(c*x-1)/(c*x+1)/g^2/(g*x+f)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*
x)/(c*x-1)/(c*x+1)/g/(g*x+f)-b*(-d*(c^2*x^2-1))^(1/2)*c^2/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*f*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*
(c^2*x^2-1))^(1/2)*c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*
f*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1
/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-2*b*(-d*(c^2*x^2-1))^(1/2)*c^3/(c*x-1)^(1/2
)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2+b
*(-d*(c^2*x^2-1))^(1/2)*c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)*l
n((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))+g)*f^2-b*(-d*(c^2*x^2-1))^(1/2)*c^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c
^2*f^2-g^2)^(1/2)*f*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^
2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*c^2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)*f*dilog(((c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+2*b*(
-d*(c^2*x^2-1))^(1/2)*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(c^2*f^2-g^2)*ln(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))-b*(-d*(c^2*x^2-1))^(1/2)*c/(c*x-1)^(1/2)/(c*x+1)
^(1/2)/(c^2*f^2-g^2)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))+g)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)Is g-c*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\sqrt{d - c^2\,d\,x^2}}{\left(f + g\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a + b\,\mathrm{acosh}\,(cx)\right)}{\left(f + gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/(f + g*x)**2, x)
```

3.58 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=1029

$$\frac{bc^3dg^3\sqrt{d-c^2dx^2}\,x^7}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dfg^2\sqrt{d-c^2dx^2}\,x^6}{12\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{8bcdg^3\sqrt{d-c^2dx^2}\,x^5}{175\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bc^3df^2g\sqrt{d-c^2dx^2}\,x^5}{25\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3df^3\sqrt{d-c^2}}{16\sqrt{cx-1}\,\sqrt{c}}$$

```
[Out] 3/8*d*f^3*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-3/16*d*f*g^2*x*(a+b*arc
cosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+3/8*d*f*g^2*x^3*(a+b*arccosh(c*x))*(-c^
2*d*x^2+d)^(1/2)+1/4*d*f^3*x*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^
2+d)^(1/2)+1/2*d*f*g^2*x^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+
d)^(1/2)-3/5*d*f^2*g*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)
^(1/2)/c^2-2/35*d*g^3*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d
)^(1/2)/c^4-1/7*d*g^3*x^2*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x
^2+d)^(1/2)/c^2+3/5*b*d*f^2*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+2/35*b*d*g^3*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-5/16*b*c*d*f^3*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/32*b
*d*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/5*b*c*d*f
^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/105*b*d*g^3*x^3
*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*c^3*d*f^3*x^4*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/32*b*c*d*f*g^2*x^4*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/25*b*c^3*d*f^2*g*x^5*(-c^2*d*x^
2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-8/175*b*c*d*g^3*x^5*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/12*b*c^3*d*f*g^2*x^6*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/49*b*c^3*d*g^3*x^7*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-3/16*d*f^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/
2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/32*d*f*g^2*(a+b*arccosh(c*x))^2*(-c^2*
d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 2.06, antiderivative size = 1029, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 5718, 194, 5745, 5743, 5759, 100, 12, 74, 5733, 373}

$$\frac{bc^3dg^3\sqrt{d-c^2dx^2}\,x^7}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dfg^2\sqrt{d-c^2dx^2}\,x^6}{12\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{8bcdg^3\sqrt{d-c^2dx^2}\,x^5}{175\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3bc^3df^2g\sqrt{d-c^2dx^2}\,x^5}{25\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3df^3\sqrt{d-c^2}}{16\sqrt{cx-1}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d*f^2*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2
*b*d*g^3*x*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*
b*c*d*f^3*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b
*d*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*
b*c*d*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
d*g^3*x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^
3*d*f^3*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c
*d*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*
c^3*d*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8
*b*c*d*g^3*x^5*Sqrt[d - c^2*d*x^2])/(175*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*c^3*d*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(12*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*c^3*d*g^3*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3
*d*f^3*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (3*d*f*g^2*x*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c^2) + (3*d*f*g^2*x^3*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*f^3*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/4 + (d*f*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (3*d*f^2*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt
```

```
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2) - (2*d*g^3*(1 - c*x)^2*(1 + c
*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*c^4) - (d*g^3*x^2*(1 -
c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (3*d
*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (3*d*f*g^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
```

```
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)^3\left(a+b\,\mathrm{c}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(f^3(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}\right.\right.}{}$$

$$= -\frac{\left(df^3\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}df^3x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d$$

$$= \frac{3}{8}df^3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{8}dfg^2x^3\sqrt{d-c^2d}$$

$$= \frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcdf^2gx^3}{5\sqrt{-1+}}$$

$$= \frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bdg^3x\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^3x}{16\sqrt{-1}}$$

Mathematica [A] time = 4.45, size = 901, normalized size = 0.88

$$\frac{-352800bc^3d\sqrt{d-c^2dx^2}\left(\cosh\left(2\cosh^{-1}(cx)\right)+2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-\sinh\left(2\cosh^{-1}(cx)\right)\right)\right)f^3+2205}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5040*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(32*g^3
 + c^2*g*(336*f^2 + 105*f*g*x + 16*g^2*x^2) + 4*c^6*x^3*(35*f^3 + 84*f^2*g*
x + 70*f*g^2*x^2 + 20*g^3*x^3) - 2*c^4*x*(175*f^3 + 336*f^2*g*x + 245*f*g^2
*x^2 + 64*g^3*x^3)) - 529200*a*c*d^(3/2)*f*(2*c^2*f^2 + g^2)*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*
x^2))] + 235200*b*c^2*d*f^2*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(
1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 352800*b
*c^3*d*f^3*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcC
osh[c*x] - Sinh[2*ArcCosh[c*x]])) + 22050*b*c^3*d*f^3*Sqrt[d - c^2*d*x^2]*(
8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x
]]) - 66150*b*c*d*f*g^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcC
osh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 2352*b*c^2*d*f^2*g*Sqrt[
d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[
c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*S
inh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]) + 784*b*d*g^3*S
qrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*
x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]) - 3675*b*c*
d*f*g^2*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] -
 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh
[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])) - 4*b*d*
g^3*Sqrt[d - c^2*d*x^2]*(55125*c*x - 55125*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*ArcCosh[c*x] - 1225*Cosh[3*ArcCosh[c*x]] - 1323*Cosh[5*ArcCosh[c*x]] -
```

```
225*Cosh[7*ArcCosh[c*x]] + 3675*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 6615*A
rcCosh[c*x]*Sinh[5*ArcCosh[c*x]] + 1575*ArcCosh[c*x]*Sinh[7*ArcCosh[c*x]]))
/(2822400*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2dg^3x^5 + 3\,ac^2dfg^2x^4 - 3\,adf^2gx - adf^3 + \left(3\,ac^2df^2g - adg^3\right)x^3 + \left(ac^2df^3 - 3\,adfg^2\right)x^2 + \left(bc^2dg\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 - 3*a*d*f^2*g*x - a*d*f^3
+ (3*a*c^2*d*f^2*g - a*d*g^3)*x^3 + (a*c^2*d*f^3 - 3*a*d*f*g^2)*x^2 + (b*c^
2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 - 3*b*d*f^2*g*x - b*d*f^3 + (3*b*c^2*d*f^
2*g - b*d*g^3)*x^3 + (b*c^2*d*f^3 - 3*b*d*f*g^2)*x^2)*arccosh(c*x))*sqrt(-c
^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 1.15, size = 1638, normalized size = 1.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/2*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+
11/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+
3/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x-3/
5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^6*f^2+9/5
*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4*f^2+3/8*
a*f^3*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2/35*a
*g^3/d/c^4*(-c^2*d*x^2+d)^(5/2)-3/5*a*f^2*g/c^2/d*(-c^2*d*x^2+d)^(5/2)-1/7*
a*g^3*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/8*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(3/2)
+13/35*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^6-
1/35*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x^2+3/
5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*f^2-1/4*b*(
-d*(c^2*x^2-1))^(1/2)*f^3*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^5+7/8*b*(-d*
(c^2*x^2-1))^(1/2)*f^3*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3+1/12*b*(-d*(c
^2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-7/32*b*(-d*(c^
2*x^2-1))^(1/2)*f*g^2*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+3/32*b*(-d*(c^2*x
^2-1))^(1/2)*f*g^2*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+3/25*b*(-d*(c^2*x^2-
1))^(1/2)*g*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^5*f^2-2/5*b*(-d*(c^2*x^2-1)
)^(1/2)*g*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3*f^2+3/5*b*(-d*(c^2*x^2-1))^(1
/2)*g*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x*f^2-9/5*b*(-d*(c^2*x^2-1))^(1/2)*g*
d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2*f^2-17/16*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2
*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3-3/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)/c^3*f*arccosh(c*x)^2*d*g^2+3/8*a*f^3*d*x*(-c^2*d*x^2+d)^
```

```
(1/2)+1/16*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x
^4-5/16*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2+1/
49*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^7-8/175
*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^5-3/16*b*(-
d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*f^3*arccosh(c*x)^2*d+2/3
5*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)-5/8*b*(-d
*(c^2*x^2-1))^(1/2)*f^3*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x-9/35*b*(-d*(c^2*x^
2-1))^(1/2)*g^3*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^4+1/105*b*(-d*(c^2*x^2-1))
^(1/2)*g^3*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^3+2/35*b*(-d*(c^2*x^2-1))^(1/2
)*g^3*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*x+7/768*b*(-d*(c^2*x^2-1))^(1/2)*f*
g^2*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)+1/4*a*f^3*x*(-c^2*d*x^2+d)^(3/2)-1/7*
b*(-d*(c^2*x^2-1))^(1/2)*g^3*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^8-1/2*a*f
*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+3/16*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)
+3/16*a*f*g^2/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(
1/2))+17/128*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x+3\sqrt{-c^2dx^2+d}\,dx+\frac{3d^{\frac{3}{2}}\arcsin(cx)}{c}\right)af^3-\frac{1}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f^3 - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d
*x^2 + d)^(5/2)/(c^4*d))*a*g^3 + 1/16*a*f*g^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c
^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 +
3*d^(3/2)*arcsin(c*x)/c^3) - 3/5*(-c^2*d*x^2 + d)^(5/2)*a*f^2*g/(c^2*d) + i
ntegrate((-c^2*d*x^2 + d)^(3/2)*b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1)) + 3*(-c^2*d*x^2 + d)^(3/2)*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)) + 3*(-c^2*d*x^2 + d)^(3/2)*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)) + (-c^2*d*x^2 + d)^(3/2)*b*f^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(c x \right)}\right) \left(f + g x\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))*(f + g*x)**3, x
)
```

3.59 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=725

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}df^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df^2\sqrt{d-c^2dx^2}\left(a+\right.}{16bc\sqrt{cx-1}}$$

```
[Out] 3/8*d*f^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/16*d*g^2*x*(a+b*arcco
sh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/8*d*g^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)+1/4*d*f^2*x*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)
^(1/2)+1/6*d*g^2*x^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/
2)-2/5*d*f*g*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c
^2+2/5*b*d*f*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/16*b*
c*d*f^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/32*b*d*g^2*x
^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/15*b*c*d*f*g*x^3*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*c^3*d*f^2*x^4*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/96*b*c*d*g^2*x^4*(-c^2*d*x^2+d)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/25*b*c^3*d*f*g*x^5*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*c^3*d*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-3/16*d*f^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/
2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*d*g^2*(a+b*arccosh(c*x))^2*(-c^2*d*
x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.69, antiderivative size = 725, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 5718, 194, 5745, 5743, 5759}

$$\frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}df^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df^2\sqrt{d-c^2dx^2}\left(a+\right.}{16bc\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d*f*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b
*c*d*f^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*
g^2*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c*d
*f*g*x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*
f^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c*d*g
^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d*
f*g*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*g
^2*x^6*Sqrt[d - c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*f^2*x*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (d*g^2*x*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(16*c^2) + (d*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/8 + (d*f^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/4 + (d*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/6 - (2*d*f*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/(5*c^2) - (3*d*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*g^2*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
```

```
2 + c*d2, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)^2\left(a+b\cosh\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(f^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}\right.\right.}{}$$

$$= -\frac{\left(df^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}df^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}d$$

$$= \frac{3}{8}df^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dg^2x^3\sqrt{d-c^2dx^2}$$

$$= \frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}-\frac{4bcdfgx^3}{15\sqrt{-1+}}$$

$$= \frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bdg^2x^2\sqrt{}}{32c\sqrt{-1+}}$$

Mathematica [A] time = 2.80, size = 623, normalized size = 0.86

$$\frac{-3600ad^{3/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(6c^2f^2+g^2\right)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-240acd\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(30c^2f^2x\left(2c^2x\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-240*a*c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(96*f*
g*(-1 + c^2*x^2)^2 + 30*c^2*f^2*x*(-5 + 2*c^2*x^2) + 5*g^2*x*(3 - 14*c^2*x^
2 + 8*c^4*x^4)) - 3600*a*d^(3/2)*(6*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 3
200*b*c*d*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*
(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 7200*b*c^2*d*f^2*Sqrt[d
- c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*
ArcCosh[c*x]])) + 450*b*c^2*d*f^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + C
osh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 450*b*d*g^2*Sq
rt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]
*Sinh[4*ArcCosh[c*x]]) - 32*b*c*d*f*g*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] -
 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh
[c*x]*Sinh[5*ArcCosh[c*x]]) - 25*b*d*g^2*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c
*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh
[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]]
+ Sinh[6*ArcCosh[c*x]])))/(57600*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2dg^2x^4+2\,ac^2dfgx^3-2\,adfgx-adf^2+\left(ac^2df^2-adg^2\right)x^2+\left(bc^2dg^2x^4+2\,bc^2dfgx^3-2\,bdf\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 - 2*a*d*f*g*x - a*d*f^2 + (a
*c^2*d*f^2 - a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 - 2*b*d*f*
g*x - b*d*f^2 + (b*c^2*d*f^2 - b*d*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2
+ d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 1.04, size = 1177, normalized size = 1.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/24*a*g^2/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/4*a*f^2*x*(-c^2*d*x^2+d)^(3/2)+2/5*
b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)-1/4*b*(-d*(
c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^5*f^2+7/8*b*(-d*(c^2
*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3*f^2-1/6*b*(-d*(c^2*x^
2-1))^(1/2)*g^2*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+11/24*b*(-d*(c^2*x^2
-1))^(1/2)*g^2*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+1/16*b*(-d*(c^2*x^2-1
))^(1/2)*g^2*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x-6/5*b*(-d*(c^2*x^2-1))^(1
/2)*f*g*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+2/25*b*(-d*(c^2*x^2-1))^(1/2)*f*
g*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^5-4/15*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d
/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^3+2/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/(c*x+
1)^(1/2)/c/(c*x-1)^(1/2)*x-5/8*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*a
rccosh(c*x)*x*f^2-17/48*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)/(c*x-1)*arcc
osh(c*x)*x^3-3/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ar
ccosh(c*x)^2*d*f^2-1/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)/c^3*arccosh(c*x)^2*d*g^2+1/32*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)^(1/2
)/c/(c*x-1)^(1/2)*x^2+1/16*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c^3/(c*
x-1)^(1/2)*x^4*f^2-5/16*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c/(c*x-1)^
(1/2)*x^2*f^2+1/36*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)^(1/2)*c^3/(c*x-1)
^(1/2)*x^6-7/96*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2
)*x^4-1/6*a*g^2*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/16*a*g^2/c^2*d*x*(-c^2*d*x^2
+d)^(1/2)+1/16*a*g^2/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x
^2+d)^(1/2))-2/5*a*f*g/c^2/d*(-c^2*d*x^2+d)^(5/2)+3/8*a*f^2*d^2/(c^2*d)^(1/
2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3/8*a*f^2*d*x*(-c^2*d*x^2+d
)^(1/2)-2/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)
*x^6+6/5*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^
4+17/128*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*f^2+7/230
4*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x+3\sqrt{-c^2dx^2+d}\,dx+\frac{3\,d^{\frac{3}{2}}\arcsin\left(cx\right)}{c}\right)af^2+\frac{1}{48}\,ag^2\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}{c^2}-\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}{c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f^2 + 1/48*a*g^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d
*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcs
in(c*x)/c^3) - 2/5*(-c^2*d*x^2 + d)^(5/2)*a*f*g/(c^2*d) + integrate((-c^2*d
*x^2 + d)^(3/2)*b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*(-c^2*
d*x^2 + d)^(3/2)*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (-c^2*d*x
^2 + d)^(3/2)*b*f^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(c x \right)}\right) \left(f + g x\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))*(f + g*x)**2, x
)
```

3.60 $\int (f+gx)\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=398

$$\frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dfx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df\sqrt{d-c^2dx^2}\left(a+b\,c\right.}{16bc\sqrt{cx-1}\sqrt{c\right.}}$$

```
[Out] 3/8*d*f*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+1/4*d*f*x*(-c*x+1)*(c*x+1
)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/5*d*g*(-c*x+1)^2*(c*x+1)^2*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/5*b*d*g*x*(-c^2*d*x^2+d)^(1/2)/c/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/16*b*c*d*f*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-2/15*b*c*d*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+1/16*b*c^3*d*f*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+1/25*b*c^3*d*g*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/16
*d*f*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Rubi [A] time = 0.76, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 5718, 194}

$$\frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dfx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3df\sqrt{d-c^2dx^2}\left(a+b\,c\right.}{16bc\sqrt{cx-1}\sqrt{c\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*g*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d
*f*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*g*
x^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*f*x^4
*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*g*x^5*Sq
rt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*f*x*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*f*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/4 - (d*g*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2) - (3*d*f*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
```

```
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)\left(a+b\cosh^{-1}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(f(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\right.}{\sqrt{-1+cx}\sqrt{1}}$$

$$= -\frac{\left(df\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}dfx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{dg(1-c}{}$$

$$= \frac{3}{8}dfx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{4}dfx(1-cx)(1+cx)\sqrt{}$$

$$= \frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcdgx^3\sqrt{d-}}{15\sqrt{-1+cx}\sqrt{}}$$

Mathematica [A] time = 1.61, size = 432, normalized size = 1.09

$$-10800acd^{3/2}f\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 720ad\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(5c^2fx\left(2c^2x^2-5\right)+8g\left(c^2x\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-720*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(8*g*(-1
 + c^2*x^2)^2 + 5*c^2*f*x*(-5 + 2*c^2*x^2)) - 10800*a*c*d^(3/2)*f*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 +
 c^2*x^2))] + 800*b*d*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*
x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]) - 3600*b*c*d*f*S
qrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - S
inh[2*ArcCosh[c*x]])) + 225*b*c*d*f*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 +
 Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 8*b*d*g*Sqrt
[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh
[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]] + 75*ArcCosh[c*x]*
Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(28800*c^2*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2dgx^3+ac^2dfx^2-adgx-adf+\left(bc^2dgx^3+bc^2dfx^2-bdgx-bdf\right)\text{arcosh}(cx)\right)\sqrt{-c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*g*x^3 + a*c^2*d*f*x^2 - a*d*g*x - a*d*f + (b*c^2*d*g*x^3
 + b*c^2*d*f*x^2 - b*d*g*x - b*d*f)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.77, size = 656, normalized size = 1.65

$$-\frac{ag\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{5c^2 d}+\frac{afx\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{4}+\frac{3afdx\sqrt{-c^2 d\,x^2+d}}{8}+\frac{3af\,d^2\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{8\sqrt{c^2 d}}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}}{5\sqrt{cx+1}\,c\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/5*a*g/c^2/d*(-c^2*d*x^2+d)^(5/2)+1/4*a*f*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f*
d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*f*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*x^2+d)^(1/2))+1/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)^(1/2)/c/(c*x-
1)^(1/2)*x+1/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^
3*x^4-5/16*b*(-d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2-1
/4*b*(-d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)*x^5+7/8*b*
(-d*(c^2*x^2-1))^(1/2)*f*d/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-5/8*b*(-d*(
c^2*x^2-1))^(1/2)*f*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x+17/128*b*(-d*(c^2*x^2-
1))^(1/2)*f*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c-1/5*b*(-d*(c^2*x^2-1))^(1/2)*g*
d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^6+3/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*
x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4-3/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)/
(c*x-1)*arccosh(c*x)*x^2+1/25*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)^(1/2)*c^
3/(c*x-1)^(1/2)*x^5-2/15*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)^(1/2)*c/(c*x-
1)^(1/2)*x^3-3/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*f*
arccosh(c*x)^2*d+1/5*b*(-d*(c^2*x^2-1))^(1/2)*g*d/(c*x+1)/c^2/(c*x-1)*arcco
sh(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x+3\sqrt{-c^2dx^2+d}\,dx+\frac{3\,d^{\frac{3}{2}}\arcsin\left(cx\right)}{c}\right)af-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}ag}{5\,c^2d}+\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}bgx\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a*f - 1/5*(-c^2*d*x^2 + d)^(5/2)*a*g/(c^2*d) + integrate((-c^2
*d*x^2 + d)^(3/2)*b*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (-c^2*d*x^
2 + d)^(3/2)*b*f*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\mathrm{acosh}(cx))\left(f+gx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))*(f + g*x), x)
```

3.61 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))}{f+gx}\,dx$

Optimal. Leaf size=1270

$$-\frac{bc^2d(cf-g)\sqrt{d-c^2dx^2}\,x^2}{4g^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{cd(cf-g)(cf+g)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2x}{2bg^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{cd(cf-g)\sqrt{d-c^2dx^2}\,\left(a\right.}{2g^2}$$

```
[Out] -a*d*(c*f-g)*(c*f+g)*(-c^2*d*x^2+d)^(1/2)/g^3+1/6*a*d*(-2*c^2*x^2+3*c*x+2)*
(-c^2*d*x^2+d)^(1/2)/g-b*d*(c*f-g)*(c*f+g)*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2
)/g^3+1/6*b*d*(-2*c^2*x^2+3*c*x+2)*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/g+1/2*
c*d*(c*f-g)*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/g^2+b*c*d*(c*f-g)*(c*
f+g)*x*(-c^2*d*x^2+d)^(1/2)/g^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*c^2*d*(c*
f-g)*x^2*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*c*d*x*
(4*c^2*x^2-9*c*x-12)*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2
*a*d*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*
d*arccosh(c*x)^2*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*d*(
c*f-g)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^2/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+1/2*c*d*(c*f-g)*(c*f+g)*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)
/b/g^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*d*(c*f-g)^2*(c*f+g)^2*(a+b*arccosh(c
*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2
*d*(c*f-g)*(c*f+g)*(-c^2*x^2+1)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b
/c/g^2/(g*x+f)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*a*d*(c*f-g)^(3/2)*(c*f+g)^(3/2
)*arctanh((c*f+g)^(1/2)*(c*x+1)^(1/2)/(c*f-g)^(1/2)/(c*x-1)^(1/2))*(-c^2*d*
x^2+d)^(1/2)/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d*(c*f-g)*(c*f+g)*arccosh(c*
x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2
*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d*(c
*f-g)*(c*f+g)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c
^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^4/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-b*d*(c*f-g)*(c*f+g)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/
2)/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d*(c*f-g)*(c*f+g)*polylog(2,-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(
-c^2*d*x^2+d)^(1/2)/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [F] time = 3.85, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] (b*c*d*(c*f - g)*(c*f + g)*x*Sqrt[d - c^2*d*x^2])/(g^3*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) - (b*c^2*d*(c*f - g)*x^2*Sqrt[d - c^2*d*x^2])/(4*g^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (a*d*(c*f - g)*(c*f + g)*(1 - c^2*x^2)*Sqrt[d - c^2*d*x
^2])/(g^3*(1 - c*x)*(1 + c*x)) - (b*d*(c*f - g)*(c*f + g)*Sqrt[d - c^2*d*x^
2]*ArcCosh[c*x])/g^3 + (c*d*(c*f - g)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(2*g^2) - (d*(c*f - g)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(
4*b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d*(c*f - g)*(c*f + g)*x*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (d*(c*f - g)^2*(c*f + g)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2
*b*c*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) + (d*(c*f - g)*(c*f + g)*(
1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^2*Sqrt[-1
```

```
 + c*x]*Sqrt[1 + c*x]*(f + g*x)) + (a*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 -
g^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x)/(Sqrt[c^2
*f^2 - g^2]*Sqrt[-1 + c^2*x^2])])/(g^4*(1 - c*x)*(1 + c*x)) - (b*d*(c*f - g
)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E
^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) + (b*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*A
rcCosh[c*x]*Log[1 + (E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(g^4*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d*(c*f - g)*(c*f + g)*Sqrt[c^2*f^2 - g^2]
*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 -
g^2]))])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(c*f - g)*(c*f + g)*Sqrt
[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f +
Sqrt[c^2*f^2 - g^2]))])/(g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*d*Sqrt[d -
c^2*d*x^2]*Defer[Int][(-1 + c*x)^(3/2)*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]),
x])/(g*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(-\frac{c(cf-g)\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{g^2}+\frac{c(-1+cx)^{3/2}\sqrt{1}}{}\right.}{\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{\left(cd(cf-g)\sqrt{d-c^2dx^2}\right)\int\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)dx}{g^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{cd(cf-g)x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2g^2}+\frac{d(cf-g)(cf+g)\left(1-\right.}{2bcg^2\sqrt{-}}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{cd(cf-g)x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-}\right.}{2g^2}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{cd(cf-g)x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-}\right.}{2g^2}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{cd(cf-g)x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-}\right.}{2g^2}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{cd(cf-g)x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-}\right.}{2g^2}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{ad(cf-g)(cf+g)\left(1-c^2x^2\right)\sqrt{d-c^{}}}{g^3(1-cx)(1+cx)}$$

$$= -\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{ad(cf-g)(cf+g)\left(1-c^2x^2\right)\sqrt{d-c^{}}}{g^3(1-cx)(1+cx)}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{ad}{}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{ad}{}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{ad}{}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{ad}{}$$

$$= \frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^2d(cf-g)x^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{ad}{}$$

Mathematica [C] time = 11.69, size = 3068, normalized size = 2.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((a*d*(-3*c^2*f^2 + 4*g^2))/(3*g^3) + (a*c^2*d*f*
x)/(2*g^2) - (a*c^2*d*x^2)/(3*g)) + (a*c*d^(3/2)*f*(2*c^2*f^2 - 3*g^2)*ArcT
an[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(2*g^4) + (a*
d^(3/2)*(-(c^2*f^2) + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*d^(3/2)*(-(c^2*f^2)
 + g^2)^(3/2)*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[-(d
*(-1 + c^2*x^2))]])/g^4 + (b*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*((-2*c*g*x)/
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + 2*g*ArcCosh[c*x] - (c*f*ArcCosh[c*
x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (2*(-(c*f) + g)*(c*f + g)*(2
*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2
]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[Arc
Cosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c
*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(Ar
cCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan
[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/
2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])] - (ArcCos[-
((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) +
 g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcC
osh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))
] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt
[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])
*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[Ar
cCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g
 - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c
^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^
2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c
*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(Sqrt[-(c^2*f^
2) + g^2]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(2*g^2) - (b*d*Sqrt[-(d*(
-1 + c*x)*(1 + c*x))]*((-9*(-2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[
c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f)
 + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (ArcCos[-((c*f)/g)] +
 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTa
n[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(
c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] - (
ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*
f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) +
 g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*S
qrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcC
osh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^
2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2)
 + g^2]*Tanh[ArcCosh[c*x]/2]))] + (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) +
 g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) +
 g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*
Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - I*(PolyLog[2, ((c*f - I*Sq
rt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]
/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - Poly
Log[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2
]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCos
h[c*x]/2]))])))/Sqrt[-(c^2*f^2) + g^2] - (-18*c*g*(-4*c^2*f^2 + g^2)*x + 18
*g*(-4*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 1
8*c*f*(2*c^2*f^2 - g^2)*ArcCosh[c*x]^2 - 9*c*f*g^2*Cosh[2*ArcCosh[c*x]] + 2
```

```
*g^3*Cosh[3*ArcCosh[c*x]] + (9*(8*c^4*f^4 - 8*c^2*f^2*g^2 + g^4)*(2*ArcCosh
[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2*
I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2
*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/
Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c*
x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f +
 g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Ta
nh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt[
-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])] - (ArcCos[-((c*f)/g
)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*
Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x]
/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (Arc
Cos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f
^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + Ta
nh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*
x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqr
t[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2)
+ g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^2
])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g +
 I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[-(c^2*f^2) + g^2]
 + 18*c*f*g^2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 6*g^3*ArcCosh[c*x]*Sinh[3
*ArcCosh[c*x]])/g^4))/(72*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/(g*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.64, size = 1965, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x)
```

```
[Out] -1/2*b*(-d*(c^2*x^2-1))^(1/2)*f*c^2*d/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x+1/
2*b*(-d*(c^2*x^2-1))^(1/2)*f*c^4*d/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x^3-b*(
-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)/(c*x+1)/g^3*arccosh(c*x)*x^2*c^4*f^2+1/3*a/
```

```
g*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)+a/g*d*(-
(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-a/g*d^2/(-d*
(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(
-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^
2-g^2)/g^2)^(1/2))/(x+f/g))+1/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)^(1/2)/(c
*x+1)^(1/2)/g*x^3*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)^(1/2)/(c*x+1)^
(1/2)/g*c*x+b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)/g^4*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2
)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1)
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+1/8*b*(-d*(c^
2*x^2-1))^(1/2)*f*c*d/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2+b*(-d*(c^2*x^2-1))^(1
/2)*d/(c*x-1)/(c*x+1)/g^3*arccosh(c*x)*c^2*f^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^3*arccosh(c*x)^2*c^3*d/g^4-3/4*b*(-d*(c^2*x^
2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f*arccosh(c*x)^2*c*d/g^2-1/3*b*(-d*
(c^2*x^2-1))^(1/2)*d/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^4*c^4+5/3*b*(-d*(c^2*
x^2-1))^(1/2)*d/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^2*c^2-b*(c^2*f^2-g^2)^(3/2
)*d*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*arccosh(c*x)*ln(
(-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f
^2-g^2)^(1/2)))-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*c^3*d/(c*x-1)^(1/2)/(c*x+1)^
(1/2)/g^2*x^2+b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^3*x*
c^3*f^2+b*(c^2*f^2-g^2)^(3/2)*d*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)/g^4*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f
^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-a/g^3*d*(-(x+f/g)^2*c^2*d+2*c^2*d
*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*c^2*f^2+2*a/g^3*d^2/(-d*(c^2*f^2-g^
2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2
-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)
^(1/2))/(x+f/g))*c^2*f^2+1/2*a/g^2*c^2*d*f*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x
+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+3/2*a/g^2*c^2*d^2*f/(c^2*d)^(1/2)*arctan
((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)
^(1/2))-a/g^4*d^2*c^4*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*
c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-a/g^5*d^2/(-d*(c^2*f^
2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2
*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/
g^2)^(1/2))/(x+f/g))*c^4*f^4-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x-1)/(c*x+1)
/g*arccosh(c*x)
```

maxima **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)Is g-c*f zero or nonzero?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/(f + g*x),x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/(f + g*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/(g*x+f),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/(f + g*x), x)
```

3.62 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=1385

$$-\frac{bc^5d^2g^3\sqrt{d-c^2dx^2}\,x^9}{81\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3bc^5d^2fg^2\sqrt{d-c^2dx^2}\,x^8}{64\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{19bc^3d^2g^3\sqrt{d-c^2dx^2}\,x^7}{441\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3bc^5d^2f^2g\sqrt{d-c^2dx^2}\,x^7}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{17bc^3d^2}{96\sqrt{c}}$$

```
[Out] 3/7*b*d^2*f^2*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15/256
*b*d^2*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/7*b*c
*d^2*f^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/256*b*c*
d^2*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+9/35*b*c^3*d
^2*f^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/96*b*c^3*d
^2*f*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/49*b*c^5*d^
2*f^2*g*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/64*b*c^5*d^2
*f*g^2*x^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15/256*d^2*f*g^
2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+15/64*d^2*f*g^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+5/16*d^2*f^3
*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+5/16*d^2*f*g^2*x^3*(-c*x+1)*(c*x
+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+3/8*d^2*f*g^2*x^3*(-c*x+1)^2*(c
*x+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-3/7*d^2*f^2*g*(-c*x+1)^3*(c
*x+1)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/9*d^2*g^3*x^2*(-c*x+1
)^3*(c*x+1)^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+2/63*b*d^2*g^3*x*
(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-25/96*b*c*d^2*f^3*x^2*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/189*b*d^2*g^3*x^3*(-c^2*
d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/96*b*c^3*d^2*f^3*x^4*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/21*b*c*d^2*g^3*x^5*(-c^2*d*x^2+
d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+19/441*b*c^3*d^2*g^3*x^7*(-c^2*d*x^2+d
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/81*b*c^5*d^2*g^3*x^9*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*d^2*f^3*(-c^2*x^2+1)^3*(-c^2*d*x^2+
d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/32*d^2*f^3*(a+b*arccosh(c*x))^2*(-
c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15/128*d^2*f*g^2*x*(a+b*
arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+5/24*d^2*f^3*x*(-c*x+1)*(c*x+1)*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+1/6*d^2*f^3*x*(-c*x+1)^2*(c*x+1)^2*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-2/63*d^2*g^3*(-c*x+1)^3*(c*x+1)^3*(a+b*
arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4
```

Rubi [A] time = 2.52, antiderivative size = 1385, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 20, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.645, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 261, 5718, 194, 5745, 5743, 5759, 266, 43, 100, 12, 74, 5733, 373}

$$-\frac{bc^5d^2g^3\sqrt{d-c^2dx^2}\,x^9}{81\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3bc^5d^2fg^2\sqrt{d-c^2dx^2}\,x^8}{64\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{19bc^3d^2g^3\sqrt{d-c^2dx^2}\,x^7}{441\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3bc^5d^2f^2g\sqrt{d-c^2dx^2}\,x^7}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{17bc^3d^2}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d^2*f^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(2*b*d^2*g^3*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (25*b*c*d^2*f^3*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (15*b*d^2*f*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (3*b*c*d^2*f^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*d^2*g^3*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (5*b*c^3*d^2*f^3*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) - (59*b*c*d^2*f*g^2*x^4*Sqrt[d - c^2*d*x^2])/(256*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (9*b*c^3*d^2*f^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b*c*d^2*g^3*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[-1 +
```

```
c*x]*Sqrt[1 + c*x]) + (17*b*c^3*d^2*f*g^2*x^6*Sqrt[d - c^2*d*x^2])/(96*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*d^2*f^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (19*b*c^3*d^2*g^3*x^7*Sqrt[d - c^2*d*x^2])
/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*d^2*f*g^2*x^8*Sqrt[d - c^2*d
*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*g^3*x^9*Sqrt[d - c^2*
d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*f^3*(1 - c^2*x^2)^3*Sqrt
[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*f^3*x*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 - (15*d^2*f*g^2*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(128*c^2) + (15*d^2*f*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/64 + (5*d^2*f^3*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/24 + (5*d^2*f*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (d^2*f^3*x*(1 - c*x)^2*(1 + c*x)^2*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 + (3*d^2*f*g^2*x^3*(1 - c*x)^2*(
1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (3*d^2*f^2*g*(1 -
c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (2*d
^2*g^3*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(6
3*c^4) - (d^2*g^3*x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/(9*c^2) - (5*d^2*f^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^
2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*d^2*f*g^2*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
```

```
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
```

```
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)^3\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)^3\left(a+b\cosh\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(f^3(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx\right.\right.}{}$$

$$= \frac{\left(d^2f^3\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2f^3x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{8}d$$

$$= \frac{bd^2f^3\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5}{24}d^2f^3x(1-cx)(1+cx)\sqrt{d-}$$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3bcd^2f^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{9bc^3d^2f^2g}{35\sqrt{-1+}}$$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{25bcd^2f^3x}{96\sqrt{-1+}}$$

$$= \frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{25bcd^2f^3x}{96\sqrt{-1+}}$$

Mathematica [A] time = 7.98, size = 1802, normalized size = 1.30

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(-1/63*(a*d^2*g*(27*c^2*f^2 + 2*g^2))/c^4 + (a*d^
2*f*(88*c^2*f^2 - 15*g^2)*x)/(128*c^2) - (a*d^2*g*(-81*c^2*f^2 + g^2)*x^2)/
(63*c^2) - (a*d^2*f*(104*c^2*f^2 - 177*g^2)*x^3)/192 + (a*d^2*g*(-27*c^2*f^
2 + 5*g^2)*x^4)/21 + (a*c^2*d^2*f*(8*c^2*f^2 - 51*g^2)*x^5)/48 - (a*c^2*d^2
*g*(-27*c^2*f^2 + 19*g^2)*x^6)/63 + (3*a*c^4*d^2*f*g^2*x^7)/8 + (a*c^4*d^2*
g^3*x^8)/9) - (5*a*d^(5/2)*f*(8*c^2*f^2 + 3*g^2)*ArcTan[(c*x*Sqrt[-(d*(-1 +
 c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(128*c^3) - (b*d^2*f^2*g*Sqrt[-(d*(
-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3
```

```
*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]))/(12*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)) - (b*d^2*f^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(Cosh[2*ArcCosh[c*x]
] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])))/(8*c*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(8*
ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]
))/(64*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (3*b*d^2*f*g^2*Sqrt[-(d*(-
1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c
*x]*Sinh[4*ArcCosh[c*x]]))/(128*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
 (b*d^2*f^2*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-450*c*x + 450*Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3*ArcCosh[c*x]] + 9*Cosh[5*A
rcCosh[c*x]] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 45*ArcCosh[c*x]*Sinh[
5*ArcCosh[c*x]]))/(600*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*g
^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-450*c*x + 450*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3*ArcCosh[c*x]] + 9*Cosh[5*ArcCosh[c*x]
] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c
*x]]))/(3600*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^3*Sqrt[-(
d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]]
- 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcC
osh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*Ar
cCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f*g^2
*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCos
h[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sin
h[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*S
inh[6*ArcCosh[c*x]])))/(384*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*
d^2*f^2*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-55125*c*x + 1225*Cosh[3*ArcCosh
[c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 441*C
osh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c*x]] - 1225*ArcCosh[c*x]*Sinh[3*Ar
cCosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] - 525*ArcCosh[c*x]*Sin
h[7*ArcCosh[c*x]])))/(235200*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b
*d^2*g^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-55125*c*x + 1225*Cosh[3*ArcCosh[
c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 441*Co
sh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c*x]] - 1225*ArcCosh[c*x]*Sinh[3*Arc
Cosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] - 525*ArcCosh[c*x]*Sinh
[7*ArcCosh[c*x]])))/(352800*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*
d^2*f*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(1440*ArcCosh[c*x]^2 - 576*Cosh[2
*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*ArcCosh[c*x]] + 9*Cos
h[8*ArcCosh[c*x]] + 1152*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 576*ArcCosh[c*
x]*Sinh[4*ArcCosh[c*x]] - 384*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]] - 72*ArcCos
h[c*x]*Sinh[8*ArcCosh[c*x]]))/(24576*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)) - (b*d^2*g^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-1389150*c*x + 31752*Cosh
[5*ArcCosh[c*x]] + 5*(2025*Cosh[7*ArcCosh[c*x]] + 245*Cosh[9*ArcCosh[c*x]]
- 63*ArcCosh[c*x]*(-4410*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 504*Sinh[5*
ArcCosh[c*x]] + 225*Sinh[7*ArcCosh[c*x]] + 35*Sinh[9*ArcCosh[c*x]]))))/(254
01600*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2g^3x^7 + 3\,ac^4d^2fg^2x^6 + 3\,ad^2f^2gx + ad^2f^3 + \left(3\,ac^4d^2f^2g - 2\,ac^2d^2g^3\right)x^5 + \left(ac^4d^2f^3 - 6\,ac^2d^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*d
^2*f^3 + (3*a*c^4*d^2*f^2*g - 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 - 6*a*c
^2*d^2*f*g^2)*x^4 - (6*a*c^2*d^2*f^2*g - a*d^2*g^3)*x^3 - (2*a*c^2*d^2*f^3
- 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b*d^2
*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g - 2*b*c^2*d^2*g^3)*x^5 + (b*c^4*d
^2*f^3 - 6*b*c^2*d^2*f*g^2)*x^4 - (6*b*c^2*d^2*f^2*g - b*d^2*g^3)*x^3 - (2*
b*c^2*d^2*f^3 - 3*b*d^2*f*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 1.23, size = 2116, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -2/63*a*g^3/d/c^4*(-c^2*d*x^2+d)^(7/2)-11/16*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d
^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x-16/63*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c
*x+1)/(c*x-1)*arccosh(c*x)*x^4-5/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)/c*f^3*arccosh(c*x)^2*d^2+2/63*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^
2/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)-1/36*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c
*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^6+13/96*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c
*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^4-11/32*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c
*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2-1/81*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c*x+
1)^(1/2)*c^5/(c*x-1)^(1/2)*x^9+19/441*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c*x
+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^7-1/21*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c*x+
1)^(1/2)*c/(c*x-1)^(1/2)*x^5+1/189*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c*x+1)
^(1/2)/c/(c*x-1)^(1/2)*x^3+2/63*b*(-d*(c^2*x^2-1))^(1/2)*g^3*d^2/(c*x+1)^(1
/2)/c^3/(c*x-1)^(1/2)*x+359/24576*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+1
)^(1/2)/c^3/(c*x-1)^(1/2)+1/6*a*f^3*x*(-c^2*d*x^2+d)^(5/2)-1/9*a*g^3*x^2*(-
c^2*d*x^2+d)^(7/2)/c^2/d+1/16*a*f*g^2/c^2*x*(-c^2*d*x^2+d)^(5/2)-3/7*a*f^2*
g/c^2/d*(-c^2*d*x^2+d)^(7/2)+17/96*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+
1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-3/49*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^
(1/2)*c^5/(c*x-1)^(1/2)*x^7*f^2+9/35*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)
^(1/2)*c^3/(c*x-1)^(1/2)*x^5*f^2-3/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)
^(1/2)*c/(c*x-1)^(1/2)*x^3*f^2+3/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(
1/2)/c/(c*x-1)^(1/2)*x*f^2-3/64*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+1)^
(1/2)*c^5/(c*x-1)^(1/2)*x^8-59/256*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x+
1)^(1/2)*c/(c*x-1)^(1/2)*x^4+15/256*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x
+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+5/16*a*f^3*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*
f^3*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+5/24*a*f
^3*d*x*(-c^2*d*x^2+d)^(3/2)-133/128*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2*d^2/(c*x
+1)/(c*x-1)*arccosh(c*x)*x^3-12/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)/(c
*x-1)*arccosh(c*x)*x^2*f^2-17/24*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c*x+1)*c
^4/(c*x-1)*arccosh(c*x)*x^5+59/48*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2/(c*x+1)*
c^2/(c*x-1)*arccosh(c*x)*x^3-15/256*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)/c^3*f*arccosh(c*x)^2*d^2*g^2+1/9*b*(-d*(c^2*x^2-1))^(1/2)*g^3
*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^10-26/63*b*(-d*(c^2*x^2-1))^(1/2)*g
^3*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^8+34/63*b*(-d*(c^2*x^2-1))^(1/2)*
g^3*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^6-1/63*b*(-d*(c^2*x^2-1))^(1/2)*
g^3*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x^2+3/7*b*(-d*(c^2*x^2-1))^(1/2)*g
*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*f^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)*f^3*
d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^7+18/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^
2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4*f^2+3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g
^2*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^9-23/16*b*(-d*(c^2*x^2-1))^(1/2)*
f*g^2*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+127/64*b*(-d*(c^2*x^2-1))^(1
```

```
/2)*f*g^2*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+15/128*b*(-d*(c^2*x^2-1)
)^(1/2)*f*g^2*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+3/7*b*(-d*(c^2*x^2-1))
^(1/2)*g*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^8*f^2-12/7*b*(-d*(c^2*x^2-1
))^(1/2)*g*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^6*f^2-3/8*a*f*g^2*x*(-c^2
*d*x^2+d)^(7/2)/c^2/d+5/64*a*f*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+15/128*a*f*
g^2/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+15/128*a*f*g^2/c^2*d^3/(c^2*d)^(1/2)*arc
tan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+299/2304*b*(-d*(c^2*x^2-1))^(1/2)
*f^3*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}\left(8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x+10\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx+15\sqrt{-c^2dx^2+d}\,d^2x+\frac{15\,d^{\frac{5}{2}}\arcsin\left(cx\right)}{c}\right)af^3+\frac{1}{128}\left(\frac{8\left(-c^2dx^2\right.}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f^3 + 1/128*(8*(-c^2*d*
x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*x^2
 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*arcsin
(c*x)/c^3)*a*f*g^2 - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d
*x^2 + d)^(7/2)/(c^4*d))*a*g^3 - 3/7*(-c^2*d*x^2 + d)^(7/2)*a*f^2*g/(c^2*d)
 + integrate((-c^2*d*x^2 + d)^(5/2)*b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)) + 3*(-c^2*d*x^2 + d)^(5/2)*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sq
rt(c*x - 1)) + 3*(-c^2*d*x^2 + d)^(5/2)*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)) + (-c^2*d*x^2 + d)^(5/2)*b*f^3*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(f+g\,x\right)}^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.63 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=1015

$$-\frac{bc^5d^2g^2\sqrt{d-c^2dx^2}\,x^8}{64\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bc^5d^2fg\sqrt{d-c^2dx^2}\,x^7}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{17bc^3d^2g^2\sqrt{d-c^2dx^2}\,x^6}{288\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{6bc^3d^2fg\sqrt{d-c^2dx^2}\,x^5}{35\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc^3d^2f^2\sqrt{}}{96\sqrt{cx-}}$$

```
[Out] 5/16*d^2*f^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-5/128*d^2*g^2*x*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*g^2*x^3*(a+b*arccosh(c*x))
*(-c^2*d*x^2+d)^(1/2)+5/24*d^2*f^2*x*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-
c^2*d*x^2+d)^(1/2)+5/48*d^2*g^2*x^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)+1/6*d^2*f^2*x*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)+1/8*d^2*g^2*x^3*(-c*x+1)^2*(c*x+1)^2*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)-2/7*d^2*f*g*(-c*x+1)^3*(c*x+1)^3*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)/c^2+2/7*b*d^2*f*g*x*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-25/96*b*c*d^2*f^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+5/256*b*d^2*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-2/7*b*c*d^2*f*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
5/96*b*c^3*d^2*f^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/
768*b*c*d^2*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+6/35*b
*c^3*d^2*f*g*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/288*b*
c^3*d^2*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/49*b*c^5
*d^2*f*g*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/64*b*c^5*d^
2*g^2*x^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*d^2*f^2*(
-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/32*d^2*f
^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-5/256*d^2*g^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/
2)/(c*x+1)^(1/2)
```

Rubi [A] time = 2.11, antiderivative size = 1015, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 261, 5718, 194, 5745, 5743, 5759, 266, 43}

$$-\frac{bc^5d^2g^2\sqrt{d-c^2dx^2}\,x^8}{64\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bc^5d^2fg\sqrt{d-c^2dx^2}\,x^7}{49\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{17bc^3d^2g^2\sqrt{d-c^2dx^2}\,x^6}{288\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{6bc^3d^2fg\sqrt{d-c^2dx^2}\,x^5}{35\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5bc^3d^2f^2\sqrt{}}{96\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d^2*f*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2
5*b*c*d^2*f^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(5*b*d^2*g^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (2*b*c*d^2*f*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (5*b*c^3*d^2*f^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (59*b*c*d^2*g^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (6*b*c^3*d^2*f*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (17*b*c^3*d^2*g^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (2*b*c^5*d^2*f*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (b*c^5*d^2*g^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*d^2*f^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*f^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/16 - (5*d^2*g^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128
*c^2) + (5*d^2*g^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/64 + (5*d^
2*f^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 +
(5*d^2*g^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
)/48 + (d^2*f^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
```

```
h[c*x]))/6 + (d^2*g^2*x^3*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/8 - (2*d^2*f*g*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(7*c^2) - (5*d^2*f^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^2*g^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)^2\left(a+b\,\mathrm{co}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(f^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}\right.\right.}{}$$

$$= \frac{\left(d^2f^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2f^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+$$

$$= \frac{bd^2f^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5}{24}d^2f^2x(1-cx)(1+cx)\sqrt{}$$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2fgx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{6bc^3d^2f}{35\sqrt{-1}}$$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcd^2f}{7\sqrt{-1}}$$

$$= \frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5bd^2g^2}{256c\sqrt{-}}$$

Mathematica [A] time = 7.39, size = 1282, normalized size = 1.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((-2*a*d^2*f*g)/(7*c^2) + (a*d^2*(88*c^2*f^2 - 5*
g^2)*x)/(128*c^2) + (6*a*d^2*f*g*x^2)/7 + (a*d^2*(-104*c^2*f^2 + 59*g^2)*x^
3)/192 - (6*a*c^2*d^2*f*g*x^4)/7 + (a*c^2*d^2*(8*c^2*f^2 - 17*g^2)*x^5)/48
```

```
+ (2*a*c^4*d^2*f*g*x^6)/7 + (a*c^4*d^2*g^2*x^7)/8) - (5*a*d^(5/2)*(8*c^2*f^
2 + g^2)*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/
(128*c^3) - (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*((-1 +
c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]))/(18
*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f^2*Sqrt[-(d*(-1 + c*x)
*(1 + c*x))]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*
ArcCosh[c*x]])))/(8*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f^2*Sq
rt[-(d*(-1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*
ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(64*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(8*ArcCosh[c*x]^2 + Cosh[
4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(128*c^3*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)) + (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-
450*c*x + 450*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3
*ArcCosh[c*x]] + 9*Cosh[5*ArcCosh[c*x]] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*
x]] - 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]]))/(900*c^2*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x)) + (b*d^2*f^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*Ar
cCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCos
h[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*Arc
Cosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cos
h[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*
ArcCosh[c*x]] + 18*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh
[4*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]])))/(1152*c^3*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*f*g*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]
*(-55125*c*x + 1225*Cosh[3*ArcCosh[c*x]] + 3*(18375*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*ArcCosh[c*x] + 441*Cosh[5*ArcCosh[c*x]] + 75*Cosh[7*ArcCosh[c
*x]] - 1225*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 2205*ArcCosh[c*x]*Sinh[5*Ar
cCosh[c*x]] - 525*ArcCosh[c*x]*Sinh[7*ArcCosh[c*x]])))/(352800*c^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*d^2*g^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*
(1440*ArcCosh[c*x]^2 - 576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]]
+ 64*Cosh[6*ArcCosh[c*x]] + 9*Cosh[8*ArcCosh[c*x]] + 1152*ArcCosh[c*x]*Sinh
[2*ArcCosh[c*x]] - 576*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 384*ArcCosh[c*x]
*Sinh[6*ArcCosh[c*x]] - 72*ArcCosh[c*x]*Sinh[8*ArcCosh[c*x]]))/(73728*c^3*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2g^2x^6 + 2\,ac^4d^2fgx^5 - 4\,ac^2d^2fgx^3 + 2\,ad^2fgx + ad^2f^2 + \left(ac^4d^2f^2 - 2\,ac^2d^2g^2\right)x^4 - \left(2\,ac^2d^2f^2 -\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="f
ricas")
```

```
[Out] integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 - 4*a*c^2*d^2*f*g*x^3 + 2
*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 - 2*a*c^2*d^2*g^2)*x^4 - (2*a*c^2
*d^2*f^2 - a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4*b*
c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*d^2*
g^2)*x^4 - (2*b*c^2*d^2*f^2 - b*d^2*g^2)*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2
 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="g
iac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 1.13, size = 1540, normalized size = 1.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/48*a*g^2/c^2*x*(-c^2*d*x^2+d)^(5/2)-133/384*b*(-d*(c^2*x^2-1))^(1/2)*g^2*
d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3-11/16*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*
x+1)/(c*x-1)*arccosh(c*x)*x*f^2-5/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)/c*arccosh(c*x)^2*d^2*f^2-5/256*b*(-d*(c^2*x^2-1))^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d^2*g^2-59/768*b*(-d*(c^2*x^2-1)
)^(1/2)*g^2*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+5/256*b*(-d*(c^2*x^2-1))^
(1/2)*g^2*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2-1/36*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^6*f^2+13/96*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^4*f^2-11/32*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^2*f^2-1/64*b*(-d*(c^2*x^2-1))^(1/2)*
g^2*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^8+17/288*b*(-d*(c^2*x^2-1))^(1/2)
*g^2*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6+5/24*a*f^2*d*x*(-c^2*d*x^2+d)^
(3/2)+5/128*a*g^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+
d)^(1/2))-2/7*a*f*g/c^2/d*(-c^2*d*x^2+d)^(7/2)-1/8*a*g^2*x*(-c^2*d*x^2+d)^(
7/2)/c^2/d+5/192*a*g^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a*g^2/c^2*d^2*x*(
-c^2*d*x^2+d)^(1/2)+1/6*a*f^2*x*(-c^2*d*x^2+d)^(5/2)+5/16*a*f^2*d^2*x*(-c^2
*d*x^2+d)^(1/2)+5/16*a*f^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))+2/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)*c^6/(c*x-1)*arc
cosh(c*x)*x^8-8/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)*c^4/(c*x-1)*arcc
osh(c*x)*x^6+12/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)*c^2/(c*x-1)*arcc
osh(c*x)*x^4+359/73728*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)^(1/2)/c^3/(
c*x-1)^(1/2)+299/2304*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c/(c*x-1)^
(1/2)*f^2-2/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2
)*x^3+2/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x-
2/49*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^7+6
/35*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^5+1/
8*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^9-23/
48*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+12
7/192*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5
+5/128*b*(-d*(c^2*x^2-1))^(1/2)*g^2*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+
2/7*b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)+1/6*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^7*f^2-17/24*
b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^5*f^2+59/48
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^3*f^2-8/7*
b*(-d*(c^2*x^2-1))^(1/2)*f*g*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}\left(8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x+10\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx+15\sqrt{-c^2dx^2+d}\,d^2x+\frac{15\,d^{\frac{5}{2}}\arcsin\left(cx\right)}{c}\right)af^2+\frac{1}{384}\left(\frac{8\left(-c^2dx^2\right.}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="m
axima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f^2 + 1/384*(8*(-c^2*d*
```

```
x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*x^2
 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*arcsin
(c*x)/c^3)*a*g^2 - 2/7*(-c^2*d*x^2 + d)^(7/2)*a*f*g/(c^2*d) + integrate((-c
^2*d*x^2 + d)^(5/2)*b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*(-
c^2*d*x^2 + d)^(5/2)*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (-c^2
*d*x^2 + d)^(5/2)*b*f^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
[Out] int((f + g*x)^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
[Out] Timed out
```

3.64 $\int (f+gx)\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=568

$$\frac{1}{6}d^2fx(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}d^2fx(1-cx)($$

```
[Out] 5/16*d^2*f*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+5/24*d^2*f*x*(-c*x+1)*
(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+1/6*d^2*f*x*(-c*x+1)^2*(c*x
+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/7*d^2*g*(-c*x+1)^3*(c*x+1)^
3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/7*b*d^2*g*x*(-c^2*d*x^2+d)^
(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-25/96*b*c*d^2*f*x^2*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*b*c*d^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+5/96*b*c^3*d^2*f*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+3/35*b*c^3*d^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-1/49*b*c^5*d^2*g*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)+1/36*b*d^2*f*(-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-5/32*d^2*f*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)
^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5836, 5822, 5685, 5683, 5676, 30, 14, 261, 5718, 194}

$$\frac{1}{6}d^2fx(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{24}d^2fx(1-cx)($$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d^2*g*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (25*b*
c*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d
^2*g*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c^3*d
^2*f*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*
d^2*g*x^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
^2*g*x^7*Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*f*
(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(5*d^2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (5*d^2*f*x*(1 - c
*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 + (d^2*f*x*(1 -
c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (d^2*g*(1
- c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2) - (5
*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}(f+gx)\left(a+b\cos\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(f(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(c\right.\right.}{\sqrt{-1+cx}\,\sqrt{}}$$

$$= \frac{\left(d^2f\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2fx(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d^2}{}$$

$$= \frac{bd^2f\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5}{24}d^2fx(1-cx)(1+cx)\sqrt{d-}$$

$$= \frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3bc^3d^2gx^5\sqrt{}}{35\sqrt{-1+cx}}$$

$$= \frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^2gx^3\sqrt{}}{7\sqrt{-1+c}}$$

Mathematica [A] time = 6.21, size = 644, normalized size = 1.13

$$\frac{d^2\left(-882000ac\sqrt{d}f\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+8400a\sqrt{\frac{cx-1}{cx+1}}(cx+1)\sqrt{d-c^2dx^2}\left(48g\left(c^2x^2-1\right)^3+7\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(8400*a*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(48*g
*(-1 + c^2*x^2)^3 + 7*c^2*f*x*(33 - 26*c^2*x^2 + 8*c^4*x^4)) - 882000*a*c*S
qrt[d]*f*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))] + 78400*b*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*(
(-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]
) - 352800*b*c*f*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]
*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + 44100*b*c*f*Sqrt[d - c^2*d*x^2]*(
8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x
]]) - 1568*b*g*Sqrt[d - c^2*d*x^2]*(450*c*x - 450*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x)*ArcCosh[c*x] - 25*Cosh[3*ArcCosh[c*x]] - 9*Cosh[5*ArcCosh[c*x]]
 + 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] + 45*ArcCosh[c*x]*Sinh[5*ArcCosh[c*
x]]) + 1225*b*c*f*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCo
```

```
sh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x
]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])
) + 4*b*g*Sqrt[d - c^2*d*x^2]*(55125*c*x - 55125*Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*ArcCosh[c*x] - 1225*Cosh[3*ArcCosh[c*x]] - 1323*Cosh[5*ArcCosh[c
*x]] - 225*Cosh[7*ArcCosh[c*x]] + 3675*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] +
6615*ArcCosh[c*x]*Sinh[5*ArcCosh[c*x]] + 1575*ArcCosh[c*x]*Sinh[7*ArcCosh[c
*x]])))/(2822400*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2gx^5 + ac^4d^2fx^4 - 2\,ac^2d^2gx^3 - 2\,ac^2d^2fx^2 + ad^2gx + ad^2f + \left(bc^4d^2gx^5 + bc^4d^2fx^4 - 2\,bc^2d^2gx^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 - 2*a*c^2*d^2*g*x^3 - 2*a*c^2*d
^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 - 2*b*c
^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arccosh(c*x))*sqrt(
-c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.90, size = 877, normalized size = 1.54

$$-\frac{ag\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{7c^2d}+\frac{afx\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{6}+\frac{5afdx\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{24}+\frac{5af\,d^2x\sqrt{-c^2d\,x^2+d}}{16}+\frac{5af\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{16\sqrt{c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/7*a*g/c^2/d*(-c^2*d*x^2+d)^(7/2)+1/6*a*f*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*f
*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3/
(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-5/32*b*(-d*(c^2*
x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*f*arccosh(c*x)^2*d^2+1/7*b*(-d*
(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^8-4/7*b*(-d*(c^
2*x^2-1))^(1/2)*g*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^6+6/7*b*(-d*(c^2*x
^2-1))^(1/2)*g*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^4-4/7*b*(-d*(c^2*x^2-
1))^(1/2)*g*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+1/6*b*(-d*(c^2*x^2-1))^(1/
2)*f*d^2/(c*x+1)/(c*x-1)*c^6*arccosh(c*x)*x^7-17/24*b*(-d*(c^2*x^2-1))^(1/2
)*f*d^2/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)*x^5+59/48*b*(-d*(c^2*x^2-1))^(1/2)
*f*d^2/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-11/16*b*(-d*(c^2*x^2-1))^(1/2)*
f*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x+299/2304*b*(-d*(c^2*x^2-1))^(1/2)*f*d^
2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c-1/49*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)
^(1/2)*c^5/(c*x-1)^(1/2)*x^7+3/35*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1
/2)*c^3/(c*x-1)^(1/2)*x^5-1/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)*
c/(c*x-1)^(1/2)*x^3+1/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)^(1/2)/c/(c*x
```

```
-1)^(1/2)*x-1/36*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)
*c^5*x^6+13/96*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c
^3*x^4-11/32*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x
^2+1/7*b*(-d*(c^2*x^2-1))^(1/2)*g*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}\left(8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x+10\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx+15\sqrt{-c^2dx^2+d}\,d^2x+\frac{15\,d^{\frac{5}{2}}\arcsin\left(cx\right)}{c}\right)af-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}ag}{7\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f - 1/7*(-c^2*d*x^2 + d
)^(7/2)*a*g/(c^2*d) + integrate((-c^2*d*x^2 + d)^(5/2)*b*g*x*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1)) + (-c^2*d*x^2 + d)^(5/2)*b*f*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(f+g\,x\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((f + g*x)*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.65 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx$

Optimal. Leaf size=1744

$$-\frac{bd^2x^5\sqrt{d-c^2dx^2}\,c^5}{25g\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bd^2fx^4\sqrt{d-c^2dx^2}\,c^5}{16g^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{d^2fx^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)c^4}{4g^2}-\frac{bd^2\left(c^2f^2-2g^2\right)x^3\sqrt{d-\cdots}}{9g^3\sqrt{cx-1}\,\sqrt{cx+\cdots}}$$

```
[Out] -b*c*d^2*(c^2*f^2-g^2)^2*x*(-c^2*d*x^2+d)^(1/2)/g^5/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-1/16*b*c^3*d^2*f*x^2*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-1/9*b*c^3*d^2*(c^2*f^2-2*g^2)*x^3*(-c^2*d*x^2+d)^(1/2)/g^3/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+1/16*b*c^5*d^2*f*x^4*(-c^2*d*x^2+d)^(1/2)/g^2/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+1/16*c*d^2*f*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^2
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d^2*(c^2*f^2-g^2)^(5/2)*arccosh(c*x)*ln(1+(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)
^(1/2)/g^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d^2*(c^2*f^2-g^2)^(5/2)*arccosh(c*
x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^
2*d*x^2+d)^(1/2)/g^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-a*d^2*(c^2*f^2-g^2)^(5/2)*
arctanh((c^2*f*x+g)/(c^2*f^2-g^2)^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2
)*(-c^2*d*x^2+d)^(1/2)/g^6/(-c*x+1)/(c*x+1)+1/3*b*c*d^2*(c^2*f^2-2*g^2)*x*(
-c^2*d*x^2+d)^(1/2)/g^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d^2*(c^2*f^2-g^2)^2*a
rccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/g^5-2/15*d^2*(-c*x+1)*(c*x+1)*(a+b*arccosh
(c*x))*(-c^2*d*x^2+d)^(1/2)/g-1/2*c*d^2*(c^2*f^2-g^2)^2*x*(a+b*arccosh(c*x)
)^2*(-c^2*d*x^2+d)^(1/2)/b/g^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*d^2*(c^2*f^2
-g^2)^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/g^6/(g*x+f)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+1/4*b*c^3*d^2*f*(c^2*f^2-2*g^2)*x^2*(-c^2*d*x^2+d)^(1/2)
/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/4*c*d^2*f*(c^2*f^2-2*g^2)*(a+b*arccosh(c
*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+a*d^2*(c^2*f^
2-g^2)^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/g^5/(-c*x+1)/(c*x+1)-1/2*c^2*d^2
*f*(c^2*f^2-2*g^2)*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/g^4-1/5*c^2*d^
2*x^2*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/g+2/15*b*c*d
^2*x*(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/45*b*c^3*d^2*x^3*
(-c^2*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/25*b*c^5*d^2*x^5*(-c^2
*d*x^2+d)^(1/2)/g/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d^2*(c^2*f^2-g^2)^(5/2)*pol
ylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^
2*d*x^2+d)^(1/2)/g^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d^2*(c^2*f^2-g^2)^(5/2)*
polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(
-c^2*d*x^2+d)^(1/2)/g^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*c^2*d^2*f*x*(a+b*ar
ccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/g^2-1/4*c^4*d^2*f*x^3*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/g^2-1/3*d^2*(c^2*f^2-2*g^2)*(-c*x+1)*(c*x+1)*(a+b*arcco
sh(c*x))*(-c^2*d*x^2+d)^(1/2)/g^3-1/2*d^2*(c^2*f^2-g^2)^2*(-c^2*x^2+1)*(a+b
*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Rubi [A] time = 4.78, antiderivative size = 1744, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 31, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5836, 5826, 5683, 5676, 30, 5718, 5743, 5759, 100, 12, 74, 5733, 5824, 683, 5816, 6742, 93, 208, 1610, 1654, 725, 206, 5860, 5858, 8, 5832, 3320, 2264, 2190, 2279, 2391}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
```

```
[Out] (2*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
c*d^2*(c^2*f^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2])/(3*g^3*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - (b*c*d^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2])/(g^5*Sqrt[-1 +
```

```
c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*f*x^2*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*f*(c^2*f^2 - 2*g^2)*x^2*Sqrt[d - c^2*d
*x^2])/(4*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*x^3*Sqrt[d - c^2*d
*x^2])/(45*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*(c^2*f^2 - 2*g^2)*x
^3*Sqrt[d - c^2*d*x^2])/(9*g^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^5*d^2*f
*x^4*Sqrt[d - c^2*d*x^2])/(16*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^
2*x^5*Sqrt[d - c^2*d*x^2])/(25*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a*d^2*(c^
2*f^2 - g^2)^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(g^5*(1 - c*x)*(1 + c*x))
 + (b*d^2*(c^2*f^2 - g^2)^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/g^5 + (c^2*d^
2*f*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*g^2) - (c^2*d^2*f*(c^2*f
^2 - 2*g^2)*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*g^4) - (c^4*d^2*
f*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*g^2) - (2*d^2*(1 - c*x)*
(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*g) - (d^2*(c^2*f^2
- 2*g^2)*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*g
^3) - (c^2*d^2*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(5*g) + (c*d^2*f*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*g^
2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d^2*f*(c^2*f^2 - 2*g^2)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*d
^2*(c^2*f^2 - g^2)^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*g^5
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(c^2*f^2 - g^2)^3*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x])^2)/(2*b*c*g^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x))
 - (d^2*(c^2*f^2 - g^2)^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(2*b*c*g^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(f + g*x)) - (a*d^2*(c^2*f
^2 - g^2)^(5/2)*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTanh[(g + c^2*f*x
)/(Sqrt[c^2*f^2 - g^2]*Sqrt[-1 + c^2*x^2])]])/(g^6*(1 - c*x)*(1 + c*x)) + (b
*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (E^ArcC
osh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])]])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*Log[1 + (
E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])]])/(g^6*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))]])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (b*d^2*(c^2*f^2 - g^2)^(5/2)*Sqrt[d - c^2*d*x^2]*PolyLog[2, -((E^A
rcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))]])/(g^6*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
```

```
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5816

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(
x_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*
x + h*x^2)^p/(d + e*x)^2, x]}, Dist[(a + b*ArcCosh[c*x])^n, u, x] - Dist[b*
c*n, Int[SimplifyIntegrand[(u*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[1 + c*x]*
Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[
n, 0] && IGtQ[p, 0] && EqQ[e*g - 2*d*h, 0]
```

Rule 5824

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)]*((f_) + (g_.)*(x_))^(m_), x_Symbol] :> Simp[((f + g*
x)^m*(d1*d2 + e1*e2*x^2)*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*
(n + 1)), x] - Dist[1/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(d1*d2*g*m + 2*e1*e
2*f*x + e1*e2*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && ILtQ[m, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0
]
```

Rule 5826

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n, (f + g*x)
^m*(d1 + e1*x)^(p - 1/2)*(d2 + e2*x)^(p - 1/2), x], x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Integ
erQ[m] && IGtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5858

```
Int[ArcCosh[(c_.)*(x_)]^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(p_)*((d2_) + (e
2_.)*(x_))^(p_), x_Symbol] :> With[{u = ExpandIntegrand[(d1 + e1*x)^p*(d2 +
 e2*x)^p*ArcCosh[c*x]^n, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{c, d1, e
1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*d1,
 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 5860

```
Int[(ArcCosh[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d1_) + (e1_.)*(x_))^(
p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(d1 + e1*x
)^p*(d2 + e2*x)^p, RFx*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d1
, e1, d2, e2}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p - 1/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{f+gx}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int\left(-\frac{c^2f\left(c^2f^2-2g^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{g^4}+\frac{c^2\left(c^2f^2-2g^2\right.}{}\right.}{}$$

$$= -\frac{\left(c^4d^2f\sqrt{d-c^2dx^2}\right)\int x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{g^2\sqrt{-1+cx}\,\sqrt{1+cx}}+$$

$$= -\frac{c^2d^2f\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2g^4}-\frac{c^4d^2fx^3\sqrt{d-c}}{}$$

$$= \frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d^2f\left(c^2f^2-2g^2\right)x^2\sqrt{d-c^2dx^2}}{4g^4\sqrt{-1+cx}\,\sqrt{1+cx}}-$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^3d^2fx^2}{16g^2\sqrt{-1+}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^3d^2fx^2}{16g^2\sqrt{-1+}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^3d^2fx^2}{16g^2\sqrt{-1+}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^3d^2fx^2}{16g^2\sqrt{-1+}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bc^3d^2fx^2}{16g^2\sqrt{-1+}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2\left(c^2f^2\right.}{g^5\sqrt{-1}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2\left(c^2f^2\right.}{g^5\sqrt{-1}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2\left(c^2f^2\right.}{g^5\sqrt{-1}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2\left(c^2f^2\right.}{g^5\sqrt{-1}}$$

$$= \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bcd^2\left(c^2f^2-2g^2\right)x\sqrt{d-c^2dx^2}}{3g^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2\left(c^2f^2\right.}{g^5\sqrt{-1}}$$

Mathematica [C] time = 18.36, size = 6244, normalized size = 3.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/(f + g*x),x]
[Out] Result too large to show
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2 + d}}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="fri
cas")
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(g*x + f), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="gia
c")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.75, size = 4234, normalized size = 2.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x)
[Out] -1/8*b*(-d*(c^2*x^2-1))^(1/2)*f^3*d^2*c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4-1
/25*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g*x^5*c^5+11/4
5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g*x^3*c^3-23/15*
b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g*c*x+33/128*b*(-d
*(c^2*x^2-1))^(1/2)*f*d^2*c/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2+b*d^2*(-d*(c^2*
x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*dilog((-(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-
g^2)^(1/2)))-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)/g^2*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-
g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-a/g*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1
/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/
g))-1/3*a/g^3*d*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^
(3/2)*c^2*f^2+a/g^5*d^2*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^
2)/g^2)^(1/2)*c^4*f^4-2*a/g^3*d^2*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(
c^2*f^2-g^2)/g^2)^(1/2)*c^2*f^2+b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^6*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f
```

```
^4-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)/g^6*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-
g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4-2*b*d^2*(-d*(c^2*x^2-1))^(1/
2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*arccosh(c*x)*ln((-(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g
^2)^(1/2)))*c^2*f^2+2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)/g^4*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^2*f^2+1/5*a/g*(
-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(5/2)-14/15*b*(-d
*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^4*c^4+34/15*b*(-d*
(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)/g*arccosh(c*x)*x^2*c^2+1/3*a/g*d*(-(
x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)+a/g*d^2*(-(x+
f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-1/9*b*(-d*(c^2*
x^2-1))^(1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^3*x^3*c^5*f^2+7/3*b*(-d*(c^
2*x^2-1))^(1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^3*x*c^3*f^2+1/4*b*(-d*(c^
2*x^2-1))^(1/2)*f^3*d^2*c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*x^2-b*(-d*(c^2*
x^2-1))^(1/2)*d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^5*x*c^5*f^4+1/16*b*(-d*(c^2
*x^2-1))^(1/2)*f*d^2*c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*x^4-9/16*b*(-d*(c^
2*x^2-1))^(1/2)*f*d^2*c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*x^2+b*d^2*(-d*(c^
2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*arccosh
(c*x)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c
*f+(c^2*f^2-g^2)^(1/2)))-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)/g^2*arccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2
-1))^(1/2)*d^2/(c*x-1)/(c*x+1)/g^5*arccosh(c*x)*c^4*f^4+7/3*b*(-d*(c^2*x^2-
1))^(1/2)*d^2/(c*x-1)/(c*x+1)/g^3*arccosh(c*x)*c^2*f^2-1/2*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^5*arccosh(c*x)^2*d^2*c^5/g^6+5/4*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f^3*arccosh(c*x)^2*d^2*c^
3/g^4-15/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*f*arccosh(
c*x)^2*d^2*c/g^2+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x-1)/(c*x+1)/g*arccosh
(c*x)*x^6*c^6+3*a/g^3*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^
2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d
+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^2*f^2+a/g^6*d^3
*c^6*f^5/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g
*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+a/g^7*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1
/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/
g))*c^6*f^6-3*a/g^5*d^3/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)
/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2
*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))*c^4*f^4-5/2*a/g^4*d
^3*c^4*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f
/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+1/4*a/g^2*c^2*d*f*(-(x+f/g)^2*c^2*d+
2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)*x+7/8*a/g^2*c^2*d^2*f*(-(x+f
/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x+15/8*a/g^2*c^2
*d^3*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(
x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))-1/2*a/g^4*d^2*c^4*f^3*(-(x+f/g)^2*c^2*d+
2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)*x-23/15*b*(-d*(c^2*x^2-1))^(
1/2)*d^2/(c*x-1)/(c*x+1)/g*arccosh(c*x)-1/4*b*(-d*(c^2*x^2-1))^(1/2)*f*d^2*
c^6/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x^5+11/8*b*(-d*(c^2*x^2-1))^(1/2)*f*d^
2*c^4/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x^3-9/8*b*(-d*(c^2*x^2-1))^(1/2)*f*d
^2*c^2/(c*x-1)/(c*x+1)/g^2*arccosh(c*x)*x+1/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
(c*x-1)/(c*x+1)/g^3*arccosh(c*x)*x^4*c^6*f^2-8/3*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(c*x-1)/(c*x+1)/g^3*arccosh(c*x)*x^2*c^4*f^2-1/2*b*(-d*(c^2*x^2-1))^(1/2
)*f^3*d^2*c^6/(c*x-1)/(c*x+1)/g^4*arccosh(c*x)*x^3+1/2*b*(-d*(c^2*x^2-1))^(
1/2)*f^3*d^2*c^4/(c*x-1)/(c*x+1)/g^4*arccosh(c*x)*x+b*(-d*(c^2*x^2-1))^(1/2
)*d^2/(c*x-1)/(c*x+1)/g^5*arccosh(c*x)*x^2*c^6*f^4+b*d^2*(-d*(c^2*x^2-1))^(
1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^6*dilog((-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2
)))*c^4*f^4-b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/
```

```
(c*x+1)^(1/2)/g^6*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g
^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^4*f^4-2*b*d^2*(-d*(c^2*x^2-1))^(1/2
)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/g^4*dilog((-(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))
*c^2*f^2+2*b*d^2*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)/g^4*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^
2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*c^2*f^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/(g*x+f),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for more d
etails)Is g-c*f zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{f+g\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/(f + g*x),x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/(f + g*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acosh}\,(cx))}{f+gx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/(g*x+f),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/(f + g*x), x)
```

3.66 $\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=478

$$\frac{f^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{3f^2g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{3fg^2x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}}$$

```
[Out] -3*f^2*g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-2/3*g
^3*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^4/(-c^2*d*x^2+d)^(1/2)-3/2*f*g^2*x
*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-1/3*g^3*x^2*(
-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-3*b*f^2*g*x*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-2/3*b*g^3*x*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-3/4*b*f*g^2*x^2*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/9*b*g^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(
-c^2*d*x^2+d)^(1/2)+1/2*f^3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/b/c/(-c^2*d*x^2+d)^(1/2)+3/4*f*g^2*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {5836, 5822, 5676, 5718, 8, 5759, 30}

$$-\frac{3f^2g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{f^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}+\frac{3fg^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\right.}{4bc^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-3*b*f^2*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (2*b*
g^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (3*b*f*g^
2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (b*g^3*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*Sqrt[d - c^2*d*x^2]) - (3*f^2*g*(1 - c*x
)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (2*g^3*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^4*Sqrt[d - c^2*d*x^2]) - (3*f*g^2*
x*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) - (
g^3*x^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^2*Sqrt[d - c^2*d*x^2
]) + (f^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[
d - c^2*d*x^2]) + (3*f*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^3 \left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \left(\frac{f^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3f^2gx\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3fg^2x^2\left(a+b\cos\right.}{\sqrt{-1+cx}\,\sqrt{1}}\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(3f^2g\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x(a}{\sqrt{}}}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{3f^2g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{3fg^2x(1-cx)(1+cx)\left(a+b\,\mathrm{co}\right.}{2c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{3bf^2gx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}} - \frac{bg^3x^3\sqrt{-1+cx}}{9c\sqrt{d-c^2}}$$

$$= -\frac{3bf^2gx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{3bfg^2x^2\sqrt{-1+cx}}{4c\sqrt{d-c^2d}}$$

Mathematica [A] time = 2.09, size = 405, normalized size = 0.85

$$\frac{-\frac{36acf\left(2c^2f^2+3g^2\right)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{\sqrt{d}} - \frac{12a\sqrt{d-c^2dx^2}\left(c^2g\left(18f^2+9fgx+2g^2x^2\right)+4g^3\right)}{d} + \frac{216bc^2f^2g\sqrt{d-c^2dx^2}\left(cx-\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(cx)\right)}{d\sqrt{cx-1}\,\sqrt{cx+1}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-12*a*Sqrt[d - c^2*d*x^2]*(4*g^3 + c^2*g*(18*f^2 + 9*f*g*x + 2*g^2*x^2)))
/d + (36*b*c^3*f^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/Sqr
t[d - c^2*d*x^2] + (216*b*c^2*f^2*g*Sqrt[d - c^2*d*x^2]*(c*x - Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*ArcCosh[c*x]))/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*g^3
*Sqrt[d - c^2*d*x^2]*(c*x*(6 + c^2*x^2) - 3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2
 + c^2*x^2)*ArcCosh[c*x]))/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (36*a*c*f*(2*
c^2*f^2 + 3*g^2)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]
)/Sqrt[d] + (27*b*c*f*g^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-Cosh[2*Arc
Cosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] + Sinh[2*ArcCosh[c*x]])))/Sqrt[d
 - c^2*d*x^2])/(72*c^4)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(ag^3x^3+3\,afg^2x^2+3\,af^2gx+af^3+\left(bg^3x^3+3\,bfg^2x^2+3\,bf^2gx+bf^3\right)\mathrm{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}}{c^2dx^2-d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-(a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(c^2
*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)^3 (b \operatorname{arcosh}(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^3*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 1.04, size = 859, normalized size = 1.80

$$-\frac{a g^3 x^2 \sqrt{-c^2 d x^2+d}}{3 c^2 d}-\frac{2 a g^3 \sqrt{-c^2 d x^2+d}}{3 d c^4}-\frac{3 a f g^2 x \sqrt{-c^2 d x^2+d}}{2 c^2 d}+\frac{3 a f g^2 \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-c^2 d x^2+d}}\right)}{2 c^2 \sqrt{c^2 d}}-\frac{3 a f^2 g \sqrt{-c^2 d}}{c^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a*g^3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3*a*g^3/d/c^4*(-c^2*d*x^2+d)^(1
/2)-3/2*a*f*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+3/2*a*f*g^2/c^2/(c^2*d)^(1/2)*
arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3*a*f^2*g/c^2/d*(-c^2*d*x^2+d)
^(1/2)+a*f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c/(c^2*x^2-1)*f^3*a
rccosh(c*x)^2+1/9*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c/d/(c^2*x^2-1)*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3+2/3*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^3/d/(c^2*x^2-1)*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x-3/2*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/d/(c^2*x^2-1)*
arccosh(c*x)*x^3+3/2*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/d/c^2/(c^2*x^2-1)*arcco
sh(c*x)*x-3/8*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/d/c^3/(c^2*x^2-1)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)-3*b*(-d*(c^2*x^2-1))^(1/2)*g/d/(c^2*x^2-1)*arccosh(c*x)*x^2
*f^2-3/4*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^
2-1)*f*arccosh(c*x)^2*g^2+3/4*b*(-d*(c^2*x^2-1))^(1/2)*f*g^2/d/c/(c^2*x^2-1
)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+3*b*(-d*(c^2*x^2-1))^(1/2)*g/c/d/(c^2*x^2
-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*f^2+2/3*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^4/
d/(c^2*x^2-1)*arccosh(c*x)-1/3*b*(-d*(c^2*x^2-1))^(1/2)*g^3/d/(c^2*x^2-1)*a
rccosh(c*x)*x^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^2/d/(c^2*x^2-1)*arccosh(
c*x)*x^2+3*b*(-d*(c^2*x^2-1))^(1/2)*g/c^2/d/(c^2*x^2-1)*arccosh(c*x)*f^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} a g^3 \left(\frac{\sqrt{-c^2dx^2+d}\, x^2}{c^2 d}+\frac{2\sqrt{-c^2dx^2+d}}{c^4 d}\right)-\frac{3}{2} a f g^2 \left(\frac{\sqrt{-c^2dx^2+d}\, x}{c^2 d}-\frac{\arcsin(cx)}{c^3\sqrt{d}}\right)+\frac{a f^3 \arcsin(cx)}{c\sqrt{d}}-\frac{3\sqrt{-c^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/3*a*g^3*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*
d)) - 3/2*a*f*g^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d
))) + a*f^3*arcsin(c*x)/(c*sqrt(d)) - 3*sqrt(-c^2*d*x^2 + d)*a*f^2*g/(c^2*d
) + integrate(b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*
x^2 + d) + 3*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d
*x^2 + d) + 3*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*
x^2 + d) + b*f^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))\left(f+gx\right)^3}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**3/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.67 $\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=288

$$\frac{f^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{2fg(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{g^2x(1-cx)(cx+1)\left(a+b\,\mathrm{cc}\right.}{2c^2\sqrt{d-c^2dx^2}}$$

```
[Out] -2*f*g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-1/2*g^2
*x*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-2*b*f*g*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/2*f^2*(a+b*arccosh(c*x))^2*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/2)+1/4*g^2*(a+b*arccosh(c*x))^2
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {5836, 5822, 5676, 5718, 8, 5759, 30}

$$\frac{f^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{2fg(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}+\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\,\mathrm{cc}\right.}{4bc^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*b*f*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (b*g^2*
x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (2*f*g*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) - (g^2*x*(1 -
 c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) + (f^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^
2]) + (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sq
rt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
```

```
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \left(\frac{f^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2fgx\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{g^2x^2\left(a+b\cosh^{-1}\right.}{\sqrt{-1+cx}\sqrt{1+}}\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f^2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(2fg\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x(a+}{\sqrt{-}}}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{2fg(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2x(1-cx)(1+cx)\left(a+b\cosh^{-}\right.}{2c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{2bfgx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{bg^2x^2\sqrt{-1+cx}\sqrt{1+cx}}{4c\sqrt{d-c^2dx^2}} - \frac{2fg(1-cx)(1+cx}{c^2\sqrt{d}}$$

Mathematica [A] time = 1.59, size = 267, normalized size = 0.93

$$\frac{-\frac{4a\left(2c^2f^2+g^2\right)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{\sqrt{d}} - \frac{4acg\sqrt{d-c^2dx^2}\,(4f+gx)}{d} + \frac{4bc^2f^2\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\cosh^{-1}(cx)^2}{\sqrt{d-c^2dx^2}} + \frac{16bcfg\sqrt{d-c^2dx^2}\left(\frac{cx}{\sqrt{cx-1}\sqrt{cx+1}}-\cosh^{-1}(cx\right.}{d}}{8c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-4*a*c*g*(4*f + g*x)*Sqrt[d - c^2*d*x^2])/d + (16*b*c*f*g*Sqrt[d - c^2*d*
x^2]*((c*x)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ArcCosh[c*x]))/d + (4*b*c^2*f^
2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/Sqrt[d - c^2*d*x^2]
 - (4*a*(2*c^2*f^2 + g^2)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^
2*x^2))])/Sqrt[d] + (b*g^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-Cosh[2*Ar
cCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] + Sinh[2*ArcCosh[c*x]])))/Sqrt[d
 - c^2*d*x^2])/(8*c^3)
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\text{arcosh}\left(cx\right)\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2
+ 2*b*f*g*x + b*f^2)*arccosh(c*x))/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^2\left(b\,\text{arcosh}\left(cx\right)+a\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^2*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 0.82, size = 559, normalized size = 1.94

$$-\frac{a\,g^2x\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a\,g^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}-\frac{2afg\sqrt{-c^2d\,x^2+d}}{c^2d}+\frac{a\,f^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{-d\left(c^2x^2-\right.}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a*g^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a*g^2/c^2/(c^2*d)^(1/2)*arctan(
(c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-2*a*f*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*
f^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/4*b*(-d*(c
^2*x^2-1))^(1/2)*g^2/d/c/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-2*b*(-
d*(c^2*x^2-1))^(1/2)*f*g/(c^2*x^2-1)/d*arccosh(c*x)*x^2+2*b*(-d*(c^2*x^2-1)
)^(1/2)*f*g/c/(c^2*x^2-1)/d*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-1/2*b*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c/(c^2*x^2-1)*arccosh(c*x)^2*f^2-
1/4*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*
arccosh(c*x)^2*g^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)*g^2/d/(c^2*x^2-1)*arccosh(c
*x)*x^3+1/2*b*(-d*(c^2*x^2-1))^(1/2)*g^2/d/c^2/(c^2*x^2-1)*arccosh(c*x)*x-1
/8*b*(-d*(c^2*x^2-1))^(1/2)*g^2/d/c^3/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)+2*b*(-d*(c^2*x^2-1))^(1/2)*f*g/c^2/(c^2*x^2-1)/d*arccosh(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,ag^2\left(\frac{\sqrt{-c^2dx^2+d}\,x}{c^2d}-\frac{\arcsin\left(cx\right)}{c^3\sqrt{d}}\right)+\frac{af^2\arcsin\left(cx\right)}{c\sqrt{d}}-\frac{2\,\sqrt{-c^2dx^2+d}\,afg}{c^2d}+\int\frac{bg^2x^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -1/2*a*g^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + a
*f^2*arcsin(c*x)/(c*sqrt(d)) - 2*sqrt(-c^2*d*x^2 + d)*a*f*g/(c^2*d) + integ
rate(b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d)
+ 2*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d) + b
*f^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)\left(f+gx\right)^2}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.68 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{f\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bgx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

```
[Out] -g*(-c*x+1)*(c*x+1)*(a+b*arccosh(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-b*g*x*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/2*f*(a+b*arccosh(c*x))^2*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5836, 5822, 5676, 5718, 8}

$$\frac{f\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}-\frac{g(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bgx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -((b*g*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - (g*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2]) + (f*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5822

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, (f + g*x)^m,
x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] &&
 EqQ[e2 + c*d2, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0] && ((EqQ[n, 1] && GtQ[p, -1]) || GtQ[p, 0] || EqQ[m, 1
] || (EqQ[m, 2] && LtQ[p, -2]))
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \left(\frac{f\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{gx\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\right)dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(f\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}} + \frac{\left(g\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh\right.}{\sqrt{-1+cx}\,\cdots}}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} + \frac{f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(c\right.}{2bc\sqrt{d-c^2dx^2}}$$

$$= -\frac{bgx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{g(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}} + \frac{f\sqrt{-1+c}\cdots}{\cdots}$$

Mathematica [A] time = 0.70, size = 172, normalized size = 1.26

$$\frac{-\frac{2acf\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{\sqrt{d}}-\frac{2ag\sqrt{d-c^2dx^2}}{d}+\frac{bcf\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)^2}{\sqrt{d-c^2dx^2}}+\frac{2bg\sqrt{d-c^2dx^2}\left(\frac{cx}{\sqrt{cx-1}\sqrt{cx+1}}-\cosh^{-1}(cx)\right)}{d}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-2*a*g*Sqrt[d - c^2*d*x^2])/d + (2*b*g*Sqrt[d - c^2*d*x^2]*((c*x)/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - ArcCosh[c*x]))/d + (b*c*f*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*ArcCosh[c*x]^2)/Sqrt[d - c^2*d*x^2] - (2*a*c*f*ArcTan[(c*x*Sq
rt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d])/(2*c^2)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\operatorname{arcosh}(cx)\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x))/(
c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(b\operatorname{arcosh}(cx)+a\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

maple [A] time = 0.66, size = 239, normalized size = 1.76

$$-\frac{ag\sqrt{-c^2d\,x^2+d}}{c^2d}+\frac{af\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,f\operatorname{arccosh}(cx)^2}{2dc\left(c^2x^2-1\right)}-\frac{b\sqrt{-d\left(c^2x^2-\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a*g/c^2/d*(-c^2*d*x^2+d)^(1/2)+a*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-
c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/c/(c^2*x^2-1)*f*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*g/d/(c^2*x^2-1)
*arccosh(c*x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)*g/c/d/(c^2*x^2-1)*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x+b*(-d*(c^2*x^2-1))^(1/2)*g/c^2/d/(c^2*x^2-1)*arccosh(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{af\arcsin(cx)}{c\sqrt{d}}-\frac{\sqrt{-c^2dx^2+d}\,ag}{c^2d}+\int\frac{bgx\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}}+\frac{bf\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] a*f*arcsin(c*x)/(c*sqrt(d)) - sqrt(-c^2*d*x^2 + d)*a*g/(c^2*d) + integrate(
b*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d) + b*f*log
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(f+g\,x\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{acosh}(cx)\right)\left(f+gx\right)}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.69 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=365

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}+1\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right.}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}$$

```
[Out] (a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g
^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^
(1/2)-(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2
*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x
^2+d)^(1/2)+b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-
g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)
^(1/2)-b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^
(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^(1/2
)
```

Rubi [A] time = 0.70, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {5836, 5832, 3320, 2264, 2190, 2279, 2391}

$$\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\,\sqrt{c^2f^2-g^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*
g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
- (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x
]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]
) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f -
Sqrt[c^2*f^2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2]) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^
2 - g^2]))])/(Sqrt[c^2*f^2 - g^2]*Sqrt[d - c^2*d*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(f+gx)}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{a+bx}{cf+g\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2ce^xf+g+e^{2x}g}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(2g\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{e^x(a+bx)}{2cf+2e^xg-2\sqrt{c^2f^2-g^2}}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{\left(2g\sqrt{-1+\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\cdots\right.}{\sqrt{c\cdots}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\cdots\right.}{\sqrt{c\cdots}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\,\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\cdots\right.}{\sqrt{c\cdots}}$$

Mathematica [C] time = 1.87, size = 932, normalized size = 2.55

$$\frac{\frac{a\log(f+gx)}{\sqrt{d}} - \frac{a\log\left(d\left(fxc^2+g\right)+\sqrt{d}\,\sqrt{g^2-c^2f^2}\,\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} - \frac{b\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(2\cosh^{-1}(cx)\tan^{-1}\left(\frac{(cf+g)\coth\left(\frac{1}{2}\cosh^{-1}(cx)\right)}{\sqrt{g^2-c^2f^2}}\right)-2i\cos^{-1}\left(-\frac{cf}{g}\right)\tan^{-1}\left(\cdots\right.\right.}{\cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] ((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt[-(c^2*f^2
) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^
2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*
Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[A
rcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[
2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2
*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[
((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCo
sh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (
ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + T
anh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c
*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/
2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2)
 + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]
```

```
*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*
(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*
Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[
-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]
))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/Sqrt[d
 - c^2*d*x^2])/Sqrt[-(c^2*f^2) + g^2]
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)}{c^2dgx^3+c^2dfx^2-dgx-df},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*
x^2 - d*g*x - d*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}\,(gx+f)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

maple [A] time = 0.40, size = 754, normalized size = 2.07

$$-\frac{a\ln\left(\frac{-\frac{2d(c^2f^2-g^2)}{g^2}+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{-\frac{d(c^2f^2-g^2)}{g^2}}\sqrt{-\left(x+\frac{f}{g}\right)^2c^2d+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}-\frac{d(c^2f^2-g^2)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{-\frac{d(c^2f^2-g^2)}{g^2}}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{c^2f^2-g^2}\,\sqrt{cx-}}{d\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/g/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x
+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-
d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)/d/(c^4*f^2*x^2-c^2*g^2*x^2-
c^2*f^2+g^2)*ln((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/
2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*arccosh(c*x)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f
^2+g^2)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c
*f+(c^2*f^2-g^2)^(1/2)))-b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f^2+g^2)*dilog((-(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2
)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d/(c^4*f^2*x^2-c^2*g^2*x^2-c^2*f^2+g^2)*dilog(((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\sqrt{-c^2dx^2 + d}\left(gx + f\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{\left(f + g\,x\right)\sqrt{d - c^2\,d\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/((f + g*x)*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/((f + g*x)*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{\sqrt{-d\,(cx-1)\,(cx+1)}\left(f + gx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)), x)
```

3.70 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=523

$$-\frac{g\sqrt{cx-1}\sqrt{-\frac{1-cx}{cx+1}}\,(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)(f+gx)}+\frac{c^2f\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}+\right.}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}$$

```
[Out] -g*(c*x+1)^(3/2)*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*((c*x-1)/(c*x+1))^(1/2)/(
c^2*f^2-g^2)/(g*x+f)/(-c^2*d*x^2+d)^(1/2)+b*c*ln(g*x+f)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(c^2*f^2-g^2)/(-c^2*d*x^2+d)^(1/2)+c^2*f*(a+b*arccosh(c*x))*ln(1+(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-c^2*f*(a+b*arccosh(
c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)+b*c^2*f
*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-b*c^2*
f*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {5836, 5832, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{bc^2f\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{bc^2f\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{\sqrt{d-c^2dx^2}\left(c^2f^2-g^2\right)^{3/2}}-\frac{g\sqrt{cx-1}}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((g*Sqrt[-1 + c*x]*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)^(3/2)*(a + b*Arc
Cosh[c*x]))/((c^2*f^2 - g^2)*(f + g*x)*Sqrt[d - c^2*d*x^2])) + (c^2*f*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*g)/(c*
f - Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) - (c
^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c
*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*
x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[f + g*x])/((c^2*f^2 - g^2)*Sq
rt[d - c^2*d*x^2]) + (b*c^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[
d - c^2*d*x^2]) - (b*c^2*f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^Arc
Cosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/((c^2*f^2 - g^2)^(3/2)*Sqrt[d -
 c^2*d*x^2])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
 + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
```

```
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(f+gx)^2}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(c\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{a+bx}{(cf+g\cosh(x))^2}\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{\left(c^2f\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{S}}{\left(c^2f^2\right.}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Su}}{\left(c^2f^2-g^2\right)\sqrt{}}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(}{\left(c^2f^2-g^2\right)\sqrt{d-c^2d}}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{\left(c^2f^2\right.}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{\left(c^2f^2\right.}$$

$$= -\frac{g\sqrt{-1+cx}\,\sqrt{-\frac{1-cx}{1+cx}}\,(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\left(c^2f^2-g^2\right)(f+gx)\sqrt{d-c^2dx^2}} + \frac{c^2f\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{\left(c^2f^2\right.}$$

Mathematica [C] time = 5.84, size = 1115, normalized size = 2.13

$$-\frac{af\log(f+gx)c^2}{\sqrt{d}\left(g^2-c^2f^2\right)^{3/2}}-\frac{af\log\left(d\left(fxc^2+g\right)+\sqrt{d}\sqrt{g^2-c^2f^2}\sqrt{d-c^2dx^2}\right)c^2}{\sqrt{d}(cf-g)(cf+g)\sqrt{g^2-c^2f^2}}+\frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(-\frac{g\sqrt{\frac{cx-1}{cx+1}}(cx+1)\text{co}}{(cf-g)(cf+g)(c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -((a*g*Sqrt[d - c^2*d*x^2])/(d*(-(c^2*f^2) + g^2)*(f + g*x))) - (a*c^2*f*Lo
g[f + g*x])/(Sqrt[d]*(-(c^2*f^2) + g^2)^(3/2)) - (a*c^2*f*Log[d*(g + c^2*f*
x) + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/(Sqrt[d]*(c*f - g
)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) + (b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*(-((g*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/((c*f - g)*(c
```

```
*f + g)*(c*f + c*g*x))) + Log[1 + (g*x)/f]/(c^2*f^2 - g^2) + (c*f*(2*ArcCos
h[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - (2
*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^
2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*
x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])
/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^(ArcCosh[c
*x]/2)*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] - 2*(ArcTan[((c*f
+ g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*T
anh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(ArcCosh[c*x]/2)*Sqrt
[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/
g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])
*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-1 + Tanh[ArcCosh[c*x
]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - (Ar
cCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*
f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2*f^2) + g^2])*(1 + T
anh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c
*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sq
rt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2)
 + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt[-(c^2*f^2) + g^
2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g
+ I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(-(c^2*f^2) + g^2)^(3
/2)))/Sqrt[d - c^2*d*x^2]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{c^2dg^2x^4+2\,c^2dfgx^3-2\,dfgx-df^2+\left(c^2df^2-dg^2\right)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*
d*f*g*x^3 - 2*d*f*g*x - d*f^2 + (c^2*d*f^2 - d*g^2)*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}\,(gx+f)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

maple [B] time = 0.78, size = 1978, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a/d/(c^2*f^2-g^2)/(x+f/g)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2
*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(
x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(
-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x
```

```
+1)*(c*x-1)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^
2*f^2-g^2)/(g*x+f)*x^3*c^4*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x
^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c*g+b*(-d*(c^2*x^
2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x^2*c^2*g-b*(-
d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(c^2*x^2
-1)/(c^2*f^2-g^2)/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d/(
c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*g-b*c^2*(-d*(c^2*x^2-1))^(1/2)*(c*x+1)^(1/
2)*(c*x-1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f
^2*g^2-g^4)/d*f*arccosh(c*x)*(c^2*f^2-g^2)^(1/2)*ln((-(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*c^2*(
-d*(c^2*x^2-1))^(1/2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^
2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d*f*arccosh(c*x)*(c^2*f^2-g^2)
^(1/2)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*
f+(c^2*f^2-g^2)^(1/2)))+2*b*c^3*(-d*(c^2*x^2-1))^(1/2)*(c*x+1)^(1/2)*(c*x-1
)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^
4)/d*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2-b*c^3*(-d*(c^2*x^2-1))^(1/2)*(
c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x
^2+2*c^2*f^2*g^2-g^4)/d*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))+g)*f^2-b*c^2*(-d*(c^2*x^2-1))^(1/2)*(c*x+1)^(
1/2)*(c*x-1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2
*f^2*g^2-g^4)/d*f*(c^2*f^2-g^2)^(1/2)*dilog((-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))+b*c^2*(-d*(c^2*
x^2-1))^(1/2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^
4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d*f*(c^2*f^2-g^2)^(1/2)*dilog(((c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1
/2)))-2*b*c*(-d*(c^2*x^2-1))^(1/2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c^6*f^4*x^2
-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d*ln(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*g^2+b*c*(-d*(c^2*x^2-1))^(1/2)*(c*x+1)^(1/2)*(c*x-1)^(
1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/
d*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*g+2*c*f*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))+g)*g^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arcosh}\,(cx) + a}{\sqrt{-c^2dx^2 + d}\left(gx + f\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(c\,x)}{\left(f + g\,x\right)^2\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/((f + g*x)^2*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/((f + g*x)^2*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{\sqrt{-d\,(cx - 1)\,(cx + 1)}\,\left(f + gx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)**2), x)
```

3.71 $\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=549

$$-\frac{(1-cx)(cf-g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{g^3(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}}$$

```
[Out] -1/2*(c*f-g)^3*(-c*x+1)*(a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+1/2*(
c*f+g)^3*(c*x+1)*(a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+g^3*(-c*x+1)
*(c*x+1)*(a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+b*g^3*x*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-3/2*f*g^2*(a+b*arccosh(c*x))^2*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*(c*f-g)^3*ln(
2/(c*x+1))*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/d/(c*x+1)/((c*x-
1)/(c*x+1))^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*b*(c*f+g)^3*ln(2/(c*x+1))*((-c*x
+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/d/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)
/(-c^2*d*x^2+d)^(1/2)+1/2*b*(c*f+g)^3*ln((c*x-1)/(c*x+1))*((-c*x+1)*(c*x+1)
)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/d/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)/(-c^2*d*x^
2+d)^(1/2)
```

Rubi [A] time = 1.60, antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.452, Rules used = {5836, 5834, 37, 5848, 12, 6719, 260, 266, 36, 31, 29, 5676, 5718, 8}

$$-\frac{3fg^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cf-g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b*g^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) - ((c*f
- g)^3*(1 - c*x)*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) + ((c*
f + g)^3*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) + (g
^3*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^4*d*Sqrt[d - c^2*d*x^2]) -
(3*f*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sq
rt[d - c^2*d*x^2]) + (b*(c*f + g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*
x^2]*Log[Sqrt[-((1 - c*x)/(1 + c*x))]])/(c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]
*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*
Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(
1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f + g)^3*Sqrt[(1 - c*x)*(1 + c*x)]*Sq
rt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^4*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(1
+ c*x)*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5834

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5848

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[(b*c*Sqrt[1 - c^2*x^2])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(f+gx)^3\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \left(-\frac{(cf-g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^3\sqrt{-1+cx}\,(1+cx)^{3/2}} + \frac{(cf+g)^3\left(a+b\cosh^{-1}(cx)\right)}{2c^3(-1+cx)^{3/2}\sqrt{1+cx}} + \frac{3fg^2}{c^2}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left((cf-g)^3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,(1+cx)^{3/2}}\,dx}{2c^3d\sqrt{d-c^2dx^2}} - \frac{\left(3fg^2\sqrt{-1+cx}\,\sqrt{1+c\right.}{c^2d\sqrt{d-\right.}$$

$$= -\frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(1+cx)\left(a+b\cosh^{-1}(cx\right.}{2c^4d\sqrt{d-c^2dx^2}}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(}{}$$

$$= \frac{bg^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{(cf-g)^3(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^3(}{}$$

Mathematica [A] time = 1.95, size = 353, normalized size = 0.64

$$\frac{-2\sqrt{d}\left(-a\left(c^4f^3x+c^2g\left(3f^2+3fgx-g^2x^2\right)+2g^3\right)+bcf\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(c^2f^2+3g^2\right)\log\left(\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\right)+bg\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-3*b*c*Sqrt[d]*f*g^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 +
 6*a*c*f*g^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*
(-1 + c^2*x^2))] + b*Sqrt[d]*ArcCosh[c*x]*(3*g^3 + 2*c^4*f^3*x + 6*c^2*f*g*
(f + g*x) - g^3*Cosh[2*ArcCosh[c*x]]) - 2*Sqrt[d]*(-(a*(2*g^3 + c^4*f^3*x +
 c^2*g*(3*f^2 + 3*f*g*x - g^2*x^2))) + b*c*f*(c^2*f^2 + 3*g^2)*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + b*g*(3
```

```
*c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/
2]]) + b*Sqrt[d]*g^3*Sinh[2*ArcCosh[c*x]])/(2*c^4*d^(3/2)*Sqrt[d - c^2*d*x^
2])
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ag^3x^3 + 3\,afg^2x^2 + 3\,af^2gx + af^3 + \left(bg^3x^3 + 3\,bfg^2x^2 + 3\,bf^2gx + bf^3\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3*
b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/(c^4*
d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx + f\right)^3 \left(b \operatorname{arcosh}\left(cx\right) + a\right)}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^3*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.92, size = 1238, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a*g^3*x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*a*g^3/d/c^4/(-c^2*d*x^2+d)^(1/2)+3*
a*f*g^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-3*a*f*g^2/c^2/d/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3*a*f^2*g/c^2/d/(-c^2*d*x^2+d)^(1/2)+a
*f^3/d*x/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^
2*x^2-1)*x*f^3-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^4/d^2/(c^2*x^2-1)*g^3-b*(-d*(c^2*x^2-1))^(
1/2)*g^3/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-b*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*f^3*arccosh(c*x)-3*b
*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x^2*f^2*g+b*(-d*(c^2*x
^2-1))^(1/2)*arccosh(c*x)/c^4/d^2/(c^2*x^2-1)*(c*x+1)*(c*x-1)*g^3-3*b*(-d*(
c^2*x^2-1))^(1/2)*arccosh(c*x)/c^2/d^2/(c^2*x^2-1)*x*f*g^2+b*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2)-1)*f^3+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*g^3+b*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))/c/d^2/(c^2*x^2-1)*f^3+3/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2*f*g^2-3*b*(-d*(c^2*x^2-1))^(1/2)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*f*arccosh(c*x)*g^2+3*b*(-d*
(c^2*x^2-1))^(1/2)*arccosh(c*x)/c^2/d^2/(c^2*x^2-1)*(c*x+1)*(c*x-1)*f^2*g+3
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*l
n(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f^2*g+3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
```

```
2)-1)*f*g^2-3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^2/d^2/(c^2*x^2-1)*f^2*g+3*b*(-d*(c^2*x^2-1)
)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c
^3/d^2/(c^2*x^2-1)*f*g^2-b*(-d*(c^2*x^2-1))^(1/2)*g^3/c^4/d^2/(c^2*x^2-1)*a
rccosh(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcf^3\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)}{2d}-ag^3\left(\frac{x^2}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{2}{\sqrt{-c^2dx^2+d}\,c^4d}\right)+3\,afg^2\left(\frac{x}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{\arcsin(cx)}{c^3d^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] -1/2*b*c*f^3*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d - a*g^3*(x^2/(sqrt(-c^2*d*
x^2 + d)*c^2*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d)) + 3*a*f*g^2*(x/(sqrt(-c^2
*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + b*f^3*x*arccosh(c*x)/(sqr
t(-c^2*d*x^2 + d)*d) + a*f^3*x/(sqrt(-c^2*d*x^2 + d)*d) + 3*a*f^2*g/(sqrt(-
c^2*d*x^2 + d)*c^2*d) + integrate(b*g^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))/(-c^2*d*x^2 + d)^(3/2) + 3*b*f*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1))/(-c^2*d*x^2 + d)^(3/2) + 3*b*f^2*g*x*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(f+g\,x\right)}^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((f + g*x)^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(cx \right)}\right) \left(f + gx\right)^{3}}{\left(- d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**3/(-d*(c*x - 1)*(c*x + 1))**(3/2), x
)
```

3.72 $\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=459

$$-\frac{(1-cx)(cf-g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{2bc^3d\sqrt{d-c^2dx^2}}$$

```
[Out] -1/2*(c*f-g)^2*(-c*x+1)*(a+b*arccosh(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)+1/2*(
c*f+g)^2*(c*x+1)*(a+b*arccosh(c*x))/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/2*g^2*(a+b
*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/d/(-c^2*d*x^2+d)^(1/2)-1
/2*b*(c*f-g)^2*ln(2/(c*x+1))*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^
3/d/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*b*(c*f+g)^2*ln
(2/(c*x+1))*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c*x+1)/((c*x
-1)/(c*x+1))^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/2*b*(c*f+g)^2*ln((c*x-1)/(c*x+1))
*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c*x+1)/((c*x-1)/(c*x+1)
)^(1/2)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 1.27, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {5836, 5834, 37, 5848, 12, 6719, 260, 266, 36, 31, 29, 5676}

$$-\frac{(1-cx)(cf-g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}+\frac{(cx+1)(cf+g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}}-\frac{g^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{2bc^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((c*f - g)^2*(1 - c*x)*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2])
 + ((c*f + g)^2*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2
]) - (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*S
qrt[d - c^2*d*x^2]) + (b*(c*f + g)^2*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2
*x^2]*Log[Sqrt[-((1 - c*x)/(1 + c*x))]])/(c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))
]*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)^2*Sqrt[(1 - c*x)*(1 + c*x)]
*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))]*
(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*(c*f + g)^2*Sqrt[(1 - c*x)*(1 + c*x)]*S
qrt[1 - c^2*x^2]*Log[2/(1 + c*x)])/(2*c^3*d*Sqrt[-((1 - c*x)/(1 + c*x))]*(1
 + c*x)*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5834

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5848

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[(b*c*Sqrt[1 - c^2*x^2])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
```

```
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)^2\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \left(-\frac{(cf-g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{-1+cx}\,(1+cx)^{3/2}} + \frac{(cf+g)^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2(-1+cx)^{3/2}\sqrt{1+cx}} + \cdots\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left((cf-g)^2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,(1+cx)^{3/2}}\,dx}{2c^2d\sqrt{d-c^2dx^2}} - \frac{\left(g^2\sqrt{-1+cx}\sqrt{1+c\cdots\right.}{c^2d\sqrt{d\cdots}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(\cdots\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(\cdots\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(\cdots\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(\cdots\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(\cdots\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(\cdots\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)^2(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{(cf+g)^2(1+cx)\left(a+b\cosh^{-1}(\cdots\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.08, size = 281, normalized size = 0.61

$$2\sqrt{d}\left(ac\left(c^2f^2x+2fg+g^2x\right)-b\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(c^2f^2+g^2\right)\log\left(\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\right)-2bcfg\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\log\left(\cdots\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*b*c*Sqrt[d]*(2*f*g + c^2*f^2*x + g^2*x)*ArcCosh[c*x] - b*Sqrt[d]*g^2*Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 + 2*a*g^2*Sqrt[d - c^2*d*x
^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*Sqrt[d]*
```

```
(a*c*(2*f*g + c^2*f^2*x + g^2*x) - b*(c^2*f^2 + g^2)*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] - 2*b*c*f*g*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]]))/(2*c^3*d^(3/2)*S
qrt[d - c^2*d*x^2])
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(ag^2x^2+2\,afgx+af^2+\left(bg^2x^2+2\,bfgx+bf^2\right)\text{arcosh}\,(cx)\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2 +
 2*b*f*g*x + b*f^2)*arccosh(c*x))/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)^2\left(b\,\text{arcosh}\,(cx)+a\right)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((g*x + f)^2*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.87, size = 879, normalized size = 1.92

$$\frac{a\,g^2x}{c^2d\sqrt{-c^2d\,x^2+d}}-\frac{a\,g^2\arctan\left(\frac{\sqrt{c^2d}\;x}{\sqrt{-c^2d\,x^2+d}}\right)}{c^2d\sqrt{c^2d}}+\frac{2afg}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{a\,f^2x}{d\sqrt{-c^2d\,x^2+d}}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\;\sqrt{cx-1}\;\sqrt{cx}}{2d^2c^3\left(c^2x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*g^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a*g^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)
^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+2*a*f*g/c^2/d/(-c^2*d*x^2+d)^(1/2)+a*f^2/d*x
/(-c^2*d*x^2+d)^(1/2)+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/d^2/c^3/(c^2*x^2-1)*g^2*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*f^2-b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*g^2+2*b*
(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*(c*x+1)*(c*x-1)*f*g
-2*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x^2*f*g-b*(-d*(c^2
*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x*f^2-b*(-d*(c^2*x^2-1))^(1/2)*
arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x*g^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f^
2+2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^2/(c^2*x^2-1
)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
)-1)*g^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^
2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*f^2-2*b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*f*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/
(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcf^2\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)}{2\,d}+ag^2\left(\frac{x}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{\arcsin\left(cx\right)}{c^3d^{\frac{3}{2}}}\right)+\frac{bf^2x\,\mathrm{arcosh}\left(cx\right)}{\sqrt{-c^2dx^2+d}\,d}+\frac{af^2x}{\sqrt{-c^2dx^2+d}\,d}+\frac{2}{\sqrt{-c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] -1/2*b*c*f^2*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + a*g^2*(x/(sqrt(-c^2*d*x^
2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + b*f^2*x*arccosh(c*x)/(sqrt(-c^
2*d*x^2 + d)*d) + a*f^2*x/(sqrt(-c^2*d*x^2 + d)*d) + 2*a*f*g/(sqrt(-c^2*d*x
^2 + d)*c^2*d) + integrate(b*g^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/(-c^2*d*x^2 + d)^(3/2) + 2*b*f*g*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/
(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)\left(f+gx\right)^2}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*(f + g*x)**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x
)
```

3.73 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{\left(c^2fx+g\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\,(cf-g)\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}-\frac{bf\sqrt{cx-1}\sqrt{cx+1}\log(1-cx)}{cd\sqrt{d-c^2dx^2}}$$

```
[Out] (c^2*f*x+g)*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*(c*f-g)*arctanh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*f*ln(-c*x+1)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 178, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5836, 78, 37, 5820, 35, 206}

$$-\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{f(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\,(cf-g)\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}-\frac{bf\sqrt{cx-1}\sqrt{}}{cd\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -(((c*f - g)*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2])) + (f*(1 + c
*x)*(a + b*ArcCosh[c*x]))/(c*d*Sqrt[d - c^2*d*x^2]) - (b*(c*f - g)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) - (b*f*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c*x])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5820

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d1_) + (e1_.)*(x_))^(p_)*((d2_) +
 (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHid
```

```
e[(f + g*x)^m*(d1 + e1*x)^p*(d2 + e2*x)^p, x]}, Dist[a + b*ArcCosh[c*x], u,
 x] - Dist[b*c, Int[Dist[1/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), u, x], x], x]] /
; FreeQ[{a, b, c, d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2
+ c*d2, 0] && IGtQ[m, 0] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] &&
 (LtQ[m, -2*p - 1] || GtQ[m, 3])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}+\frac{\left(bc\sqrt{-1+\right.}{}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}-\frac{bf\sqrt{-1+}{c}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}-\frac{bf\sqrt{-1+}{c}$$

$$= -\frac{(cf-g)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{f(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}-\frac{b(cf-g)}{}$$

Mathematica [A] time = 0.39, size = 123, normalized size = 0.87

$$\frac{b\sqrt{d-c^2dx^2}\left((cf+g)\log(1-cx)+(cf-g)\log(cx+1)\right)}{2c^2d^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(c^2fx+g\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -(((g + c^2*f*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d^2*(-1 + c
^2*x^2))) + (b*Sqrt[d - c^2*d*x^2]*((c*f + g)*Log[1 - c*x] + (c*f - g)*Log[
1 + c*x]))/(2*c^2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [F] time = 2.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(agx+af+\left(bgx+bf\right)\operatorname{arcosh}(cx)\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccosh(c*x))/(c
^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)(b\,\mathrm{arcosh}\,(cx)+a)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.66, size = 498, normalized size = 3.51

$$\frac{ag}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{afx}{d\sqrt{-c^2d\,x^2+d}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,f\,\mathrm{arccosh}\,(cx)}{d^2c\left(c^2x^2-1\right)}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}}{c^2d^2\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*g/c^2/d/(-c^2*d*x^2+d)^(1/2)+a*f/d*x/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*f*arccosh(c*x)+b*(-
d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/c^2/d^2/(c^2*x^2-1)*(c*x-1)*(c*x+1)*g-b*(
-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x^2*g-b*(-d*(c^2*x^2-1))
^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x*f+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2/(c^2*x^2-1)*f
-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*ln(1+c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))/c^2/d^2/(c^2*x^2-1)*g+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*f+
b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*g
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcf\sqrt{-\frac{1}{c^4d}}\,\log\left(x^2-\frac{1}{c^2}\right)}{2\,d}+bg\left(\frac{\frac{\left(c\sqrt{d}\,x+\sqrt{cx+1}\,\sqrt{cx-1}\,\sqrt{d}\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-cx+1}}+\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\sqrt{d}}{\sqrt{-cx+1}}}{\sqrt{cx+1}\,c^3d^2x+(cx+1)\sqrt{cx-1}\,c^2d^2}-\int\frac{}{\sqrt{-cx+1}\left(\left(c^2d^{\frac{3}{2}}x^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] -1/2*b*c*f*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b*g*(((c*sqrt(d)*x + sqrt(
c*x + 1)*sqrt(c*x - 1)*sqrt(d))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt
(-c*x + 1) + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d)/sqrt(-c*x + 1))/(sqrt(c*x
+ 1)*c^3*d^2*x + (c*x + 1)*sqrt(c*x - 1)*c^2*d^2) - integrate((c^2*x^3 + c*
x^2*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)) - x)/(sqrt(-c*x + 1)*((c^2*d^(3
/2)*x^2 - d^(3/2))*e^(3/2*log(c*x + 1) + log(c*x - 1)) + 2*(c^3*d^(3/2)*x^3
 - c*d^(3/2)*x)*e^(log(c*x + 1) + 1/2*log(c*x - 1)) + (c^4*d^(3/2)*x^4 - c^
2*d^(3/2)*x^2)*sqrt(c*x + 1))), x)) + b*f*x*arccosh(c*x)/(sqrt(-c^2*d*x^2 +
 d)*d) + a*f*x/(sqrt(-c^2*d*x^2 + d)*d) + a*g/(sqrt(-c^2*d*x^2 + d)*c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(f+g\,x\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
[Out] int(((f + g*x)*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))\left(f + gx\right)}{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))*(f + g*x)/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.74 $\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=773

$$-\frac{g^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}+1\right)}{d\sqrt{d-c^2dx^2}\,\left(c^2f^2-g^2\right)^{3/2}}+\frac{g^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{ge^{\cosh}}{\sqrt{c^2f^2}}\right.}{d\sqrt{d-c^2dx^2}\,\left(c^2f^2-g^2\right)^{3/2}}$$

```
[Out] -1/2*(-c*x+1)*(a+b*arccosh(c*x))/d/(c*f-g)/(-c^2*d*x^2+d)^(1/2)+1/2*(c*x+1)
*(a+b*arccosh(c*x))/d/(c*f+g)/(-c^2*d*x^2+d)^(1/2)-g^2*(a+b*arccosh(c*x))*l
n(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/d/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)+g^2*(a+b*arc
cosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2
)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-
b*g^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/
2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)
+b*g^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1
/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2
)-1/2*b*ln(2/(c*x+1))*((-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c*f-g)
/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*b*ln(2/(c*x+1))*(
(-c*x+1)*(c*x+1))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c*f+g)/(c*x+1)/((c*x-1)/(c*x+
1))^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/2*b*ln((c*x-1)/(c*x+1))*((-c*x+1)*(c*x+1))
^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c*f+g)/(c*x+1)/((c*x-1)/(c*x+1))^(1/2)/(-c^2*d
*x^2+d)^(1/2)
```

Rubi [A] time = 1.86, antiderivative size = 773, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 17, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {5836, 5834, 37, 5848, 12, 6719, 260, 266, 36, 31, 29, 5832, 3320, 2264, 2190, 2279, 2391}

$$-\frac{bg^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{d\sqrt{d-c^2dx^2}\,\left(c^2f^2-g^2\right)^{3/2}}+\frac{bg^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{d\sqrt{d-c^2dx^2}\,\left(c^2f^2-g^2\right)^{3/2}}-\frac{g^2\sqrt{cx-1}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/((f + g*x)*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((1 - c*x)*(a + b*ArcCosh[c*x]))/(2*d*(c*f - g)*Sqrt[d - c^2*d*x^2]) + ((1
 + c*x)*(a + b*ArcCosh[c*x]))/(2*d*(c*f + g)*Sqrt[d - c^2*d*x^2]) - (g^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCosh[c*x]*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
 + (g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + (E^ArcCos
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c
^2*d*x^2]) + (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[Sqrt[-((1 -
 c*x)/(1 + c*x))]])/(d*(c*f + g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqr
t[d - c^2*d*x^2]) - (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[2/(1
 + c*x)])/(2*d*(c*f - g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqrt[d - c^
2*d*x^2]) - (b*Sqrt[(1 - c*x)*(1 + c*x)]*Sqrt[1 - c^2*x^2]*Log[2/(1 + c*x)]
)/(2*d*(c*f + g)*Sqrt[-((1 - c*x)/(1 + c*x))]*(1 + c*x)*Sqrt[d - c^2*d*x^2]
) - (b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*
f - Sqrt[c^2*f^2 - g^2]))])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2]) +
 (b*g^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -((E^ArcCosh[c*x]*g)/(c*f +
 Sqrt[c^2*f^2 - g^2]))])/(d*(c^2*f^2 - g^2)^(3/2)*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5834

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Int[Expand
Integrand[(a + b*ArcCosh[c*x])^n/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), (f + g*
x)^m*(d1 + e1*x)^(p + 1/2)*(d2 + e2*x)^(p + 1/2), x], x] /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, g}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && Int
egerQ[m] && ILtQ[p + 1/2, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IGtQ[n, 0]
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rule 5848

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCosh[c*x], v, x] - Dist[(b*c*Sqrt[1 - c^2*x^2])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]), Int[SimplifyIntegrand[v/Sqrt[1 - c^2*x^2], x],
 x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{(f+gx)\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int\frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}(f+gx)}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int\left(-\frac{c\left(a+b\cosh^{-1}(cx)\right)}{2(cf-g)\sqrt{-1+cx}\,(1+cx)^{3/2}}+\frac{c\left(a+b\cosh^{-1}(cx)\right)}{2(cf+g)(-1+cx)^{3/2}\sqrt{1+cx}}+\frac{}{(cf-g}\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(c\sqrt{-1+cx}\sqrt{1+cx}\right)\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,(1+cx)^{3/2}}\,dx}{2d(cf-g)\sqrt{d-c^2dx^2}} - \frac{\left(c\sqrt{-1+cx}\sqrt{1+cx}\right)\int\frac{a+b\cosh^{-1}}{(-1+cx)^{3/2}\sqrt{}}}{2d(cf+g)\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{\left(g^2\sqrt{-1+cx}\sqrt{1\right.}{}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{\left(2g^2\sqrt{-1+cx}\sqrt{\right.}{}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{\left(2g^3\sqrt{-1+cx}\sqrt{\right.}{}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1}}{}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1}}{}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1}}{}$$

$$= -\frac{(1-cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf-g)\sqrt{d-c^2dx^2}} + \frac{(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2d(cf+g)\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{-1+cx}\sqrt{1}}{}$$

Mathematica [C] time = 9.73, size = 1203, normalized size = 1.56

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/((f + g*x)*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] ((-(a*g) + a*c^2*f*x)*Sqrt[-(d*(-1 + c^2*x^2))])/(d^2*(-(c^2*f^2) + g^2)*(-
1 + c^2*x^2)) + (a*g^2*Log[f + g*x])/(d^(3/2)*(-(c*f) + g)*(c*f + g)*Sqrt[-
(c^2*f^2) + g^2]) - (a*g^2*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) +
g^2]*Sqrt[-(d*(-1 + c^2*x^2))]])/(d^(3/2)*(-(c*f) + g)*(c*f + g)*Sqrt[-(c^2
```

```
*f^2) + g^2]) - (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((ArcCosh[c*x]*Co
th[ArcCosh[c*x]/2])/(c*f + g)) + (2*c*f*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)])/(c^2*f^2 - g^2) + (2*g*Log[Tanh[ArcCosh[c*x]/2]])/(-(c^2*f^2) + g^2
) + (2*g^2*(2*ArcCosh[c*x]*ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c
^2*f^2) + g^2]] - (2*I)*ArcCos[-((c*f)/g)]*ArcTan[((-(c*f) + g)*Tanh[ArcCos
h[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] + 2*(ArcTan[((c*f
+ g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + ArcTan[((-(c*f) + g)*T
anh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[Sqrt[-(c^2*f^2) + g^2]/(
Sqrt[2]*E^(ArcCosh[c*x]/2)*Sqrt[g]*Sqrt[c*f + c*g*x])] + (ArcCos[-((c*f)/g)
] - 2*(ArcTan[((c*f + g)*Coth[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + Ar
cTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^(
ArcCosh[c*x]/2)*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*f + c*g*x])
] - (ArcCos[-((c*f)/g)] + 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[c*x]/2])/Sqrt
[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(c*f - g + I*Sqrt[-(c^2*f^2) + g^2])*(-
1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcC
osh[c*x]/2]))] - (ArcCos[-((c*f)/g)] - 2*ArcTan[((-(c*f) + g)*Tanh[ArcCosh[
c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(-(c*f) + g + I*Sqrt[-(c^2
*f^2) + g^2])*(1 + Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) +
 g^2]*Tanh[ArcCosh[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g
^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))/(g*(c*f + g
 + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))] - PolyLog[2, ((c*f + I*
Sqrt[-(c^2*f^2) + g^2])*(c*f + g - I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*
x]/2]))/(g*(c*f + g + I*Sqrt[-(c^2*f^2) + g^2]*Tanh[ArcCosh[c*x]/2]))])))/(
(-(c*f) + g)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) - (ArcCosh[c*x]*Tanh[ArcCosh
[c*x]/2])/(c*f - g)))/(2*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{c^4d^2gx^5+c^4d^2fx^4-2\,c^2d^2gx^3-2\,c^2d^2fx^2+d^2gx+d^2f},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*g*x^5 + c^4*d^2
*f*x^4 - 2*c^2*d^2*g*x^3 - 2*c^2*d^2*f*x^2 + d^2*g*x + d^2*f), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(gx+f\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*(g*x + f)), x)
```

maple [B] time = 0.63, size = 1926, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a*g/d/(c^2*f^2-g^2)/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/
g^2)^(1/2)+a*f/(c^2*f^2-g^2)/d/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2
```

```
*f^2-g^2)/g^2)^(1/2)*c^2*x+a*g/d/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)
*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1
/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/
g))-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^2*f^2-g^2)*(c*
x+1)*(c*x-1)*g+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^2*f
^2-g^2)*x^2*c^2*g+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)/(c^
2*f^2-g^2)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccosh
(c*x)/d^2/(c^2*x^2-1)/(c^2*f^2-g^2)*x*c^2*f+b*(c^2*f^2-g^2)^(1/2)*(-d*(c^2*
x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^
4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d^2*arccosh(c*x)*ln((-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*g^
2-b*(c^2*f^2-g^2)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d^2*a
rccosh(c*x)*ln(((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2)
)/(c*f+(c^2*f^2-g^2)^(1/2)))*g^2-2*b*(c^2*f^2-g^2)*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x
^2+2*c^2*f^2*g^2-g^4)/d^2*c*f*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+b*(c^2*f^
2-g^2)*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^6*f^4*x^2-2*c^
4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d^2*ln(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2)-1)*c*f+b*(c^2*f^2-g^2)*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^
2*g^2-g^4)/d^2*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c*f-b*(c^2*f^2-g^2)*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2
*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d^2*ln(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2)-1)*g+b*(c^2*f^2-g^2)*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/
d^2*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-b*(c^2*f^2-g^2)^(1/2)*(-d*(c^2*
x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^6*f^4*x^2-2*c^4*f^2*g^2*x^2-c^
4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d^2*dilog(((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*g^2+b*(c^2*f^2
-g^2)^(1/2)*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^6*f^4*x^2
-2*c^4*f^2*g^2*x^2-c^4*f^4+c^2*g^4*x^2+2*c^2*f^2*g^2-g^4)/d^2*dilog((-(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)
^(1/2)))*g^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(-c^2 d x^2 + d\right)^{\frac{3}{2}} \left(gx + f\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*(g*x + f)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{\left(f + g\,x\right)\,\left(d - c^2\,d\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/((f + g*x)*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/((f + g*x)*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}} \left(f + gx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/((-d*(c*x - 1)*(c*x + 1))**(3/2)*(f + g*x)), x)
```

3.75 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=239

$$\frac{f\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-c^2x^2}}+\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+\right.}{2c^2\sqrt{1-c^2x^2}}$$

```
[Out] f*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(1+n)/(-c^2*x^2+
1)^(1/2)+1/2*g*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*x^2+1)
^(1/2)-1/2*exp(a/b)*g*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(-c^2*x^2+1)^(1/
2)
```

Rubi [A] time = 0.47, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5836, 5832, 3317, 3307, 2181}

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-c^2x^2}}-\frac{ge^{a/b}\sqrt{cx-1}\sqrt{c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (f*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*
Sqrt[1 - c^2*x^2]) + (g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n
*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c^2*x^2]*
(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)*g*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[1 -
c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
```

```
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5836

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d
_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^Fra
cPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f + g*x)^m*(
1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d
, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)\,dx, x, \text{co}\right.}{c^2\sqrt{1-c^2x^2}}$$

$$= \frac{f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{\left(g\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\right.}{c^2}$$

$$= \frac{f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{\left(g\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\right.}{2c^2\sqrt{}}$$

$$= \frac{f\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}} + \frac{e^{-\frac{a}{b}}g\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}{}$$

Mathematica [A] time = 0.66, size = 204, normalized size = 0.85

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2cfe^{a/b}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n+bg(n\right.}{2bc^2(n+1)\sqrt{1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2*c*E^(a/b)*f
*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n - b*E^((2*a)/b)*g*(
1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] + b*g
*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(
2*b*c^2*E^(a/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^n
)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\left(gx+f\right)\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(g*x + f)*(b*arccosh(c*x) + a)^n/(c^2*x^2 - 1)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)(b\,\mathrm{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)\,(a+b\,\mathrm{arccosh}\,(cx))^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(gx+f)(b\,\mathrm{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f+g\,x)\,(a+b\,\mathrm{acosh}\,(c\,x))^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int(((f + g*x)*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^n\,(f+gx)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n*(f + g*x)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.76 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=200

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}-\frac{ge^{a/b}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(\right.}{2c^2\sqrt{}}$$

```
[Out] f*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)/b/c/(1+n)/(-c*x+1)^(1/2)+1/2*g*(a+
b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^2/exp(a/
b)/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)-1/2*exp(a/b)*g*(a+b*arccosh(c
*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^2/(((a+b*arccosh(c*x
))/b)^n)/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 242, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5837, 5832, 3317, 3307, 2181}

$$\frac{ge^{-\frac{a}{b}}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}\sqrt{cx+1}}-\frac{ge^{a/b}\sqrt{1-c^2x^2}\left(a+b\cosh\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (f*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[1 - c*
x]*Sqrt[1 + c*x]) + (g*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c*x]*Sqrt[1 + c*x]*(-
((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)*g*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[1 - c*x]*Sqrt[1 +
 c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
```

```
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5837

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[((-(d
1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(1 - c^2
*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[
c*x])^n, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[e1 -
 c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !(Gt
Q[d1, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-cx}\sqrt{1+cx}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)\,dx, x, \cosh^{-1}(cx\right.}{c^2\sqrt{1-cx}\sqrt{1+cx}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\sqrt{1+cx}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int (a+bx)^n\,\text{co}\right.}{c^2\sqrt{1-cx}\sqrt{1-}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\sqrt{1+cx}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int e^{-x}(a+bx\right.}{2c^2\sqrt{1-cx}\sqrt{1+}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-cx}\sqrt{1+cx}} + \frac{e^{-\frac{a}{b}}g\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{n}}{2c^2\sqrt{1}}$$

Mathematica [A] time = 0.26, size = 204, normalized size = 1.02

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2cfe^{a/b}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n+b\right.}{2bc^2(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2*c*E^(a/b)*f
*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n - b*E^((2*a)/b)*g*(
1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] + b*g
*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(
2*b*c^2*E^(a/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^n
)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{cx+1}\,\sqrt{-cx+1}\,\left(gx+f\right)\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{n}}{c^{2}x^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(-sqrt(c*x + 1)*sqrt(-c*x + 1)*(g*x + f)*(b*arccosh(c*x) + a)^n/(c^
2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Simplification assuming t_nostep near 0Simplification assuming t_nos
tep near 0Simplification assuming a near 0Unable to build a single algebrai
c extension for simplifying.Trying rational simplification only. This might
 return a wrong answer if simplifying 0/0!Simplification assuming t_nostep
near 0Simplification assuming t_nostep near 0Simplification assuming a near
 0Warning: vanishing non integral power expansionWarning: vanishing non int
egral power expansionUnable to build a single algebraic extension for simpl
ifying.Trying rational simplification only. This might return a wrong answe
r if simplifying 0/0!Simplification assuming t_nostep near 0Simplification
assuming t_nostep near 0Simplification assuming a near 0Unable to build a s
ingle algebraic extension for simplifying.Trying rational simplification on
ly. This might return a wrong answer if simplifying 0/0!Simplification assu
ming t_nostep near 0Simplification assuming t_nostep near 0Simplification a
ssuming a near 0Warning: vanishing non integral power expansionWarning: van
ishing non integral power expansionUnable to build a single algebraic exten
sion for simplifying.Trying rational simplification only. This might return
 a wrong answer if simplifying 0/0!Unable to check sign: (8*pi/(sign(t_nost
ep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable
to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_no
step^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_
nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check
sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*
t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)
-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*
pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^
2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-
8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(
t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)U
nable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sig
n(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^
4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to
check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_noste
p^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nos
tep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sig
n: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_n
```

```
ostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)
/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/
(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-
1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*p
i/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_n
ostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unab
le to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t
_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2
*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to che
ck sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4
-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep
^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign:
(8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nost
ep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)
>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(si
gn(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/
2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(
sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nost
ep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable
to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_no
step^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_
nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check
sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*
t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)
-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*
pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^
2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-
8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(
t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)U
nable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sig
n(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^
4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to
check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_noste
p^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nos
tep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sig
n: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_n
ostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)
/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/
(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-
1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*p
i/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_n
ostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unab
le to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t
_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2
*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to che
ck sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4
-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep
^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign:
(8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nost
ep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)
>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(si
gn(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/
2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(
sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nost
ep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable
to check sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_no
step^4-2*t_nostep^2)-1)/2)Unable to check sign: (8*pi/(sign(t_nostep^4-2*t_
nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)Unable to check
sign: (8*pi/(sign(t_nostep^4-2*t_nostep^2)-1)/2)>(-8*pi/(sign(t_nostep^4-2*
```

```
t_nostep^2)-1)/2)Evaluation time: 12.83sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n}}{\sqrt{-cx+1}\,\sqrt{cx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}}{\sqrt{cx+1}\,\sqrt{-cx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(-c*x+1)^(1/2)/(c*x+1)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*x + 1)*sqrt(-c*x + 1)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{n}}{\sqrt{1-c\,x}\,\sqrt{c\,x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x))^n)/((1 - c*x)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] int(((f + g*x)*(a + b*acosh(c*x))^n)/((1 - c*x)^(1/2)*(c*x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{n}\left(f+gx\right)}{\sqrt{-cx+1}\,\sqrt{cx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(-c*x+1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n*(f + g*x)/(sqrt(-c*x + 1)*sqrt(c*x + 1)), x)
```

3.77 $\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{\mathbf{d1}+c\mathbf{d1}x}\,\sqrt{\mathbf{d2}-c\mathbf{d2}x}}\,dx$

Optimal. Leaf size=260

$$\frac{ge^{-\frac{a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{cd1x+d1}\,\sqrt{d2-cd2x}}-\frac{ge^{a/b}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a\right.}{}$$

```
[Out] f*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(1+n)/(c*d1*x+d1
)^(1/2)/(-c*d2*x+d2)^(1/2)+1/2*g*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arcco
sh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/
b)^n)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)-1/2*exp(a/b)*g*(a+b*arccosh(c*x)
)^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/(((a+b*
arccosh(c*x))/b)^n)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 248, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {5837, 5832, 3317, 3307, 2181}

$$\frac{ge^{-\frac{a}{b}}\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{cd1x+d1}\,\sqrt{d2-cd2x}}-\frac{ge^{a/b}\sqrt{1-c^2x^2}\,\left(a+b\,c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]
),x]
```

```
[Out] (f*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[d1 + c
*d1*x]*Sqrt[d2 - c*d2*x]) + (g*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^n*Gam
ma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[d1 + c*d1*x]*Sqrt
[d2 - c*d2*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^(a/b)*g*Sqrt[1 - c^2*x^2]
*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[d
1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 5832

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/(
Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(
c^(m + 1)*Sqrt[-(d1*d2)]), Subst[Int[(a + b*x)^n*(c*f + g*Cosh[x])^m, x], x
, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[
e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && GtQ[d1, 0] && LtQ[d2,
0] && (GtQ[m, 0] || IGtQ[n, 0])
```

Rule 5837

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_)*((f_) + (g_.)*(x_))^(m_.), x_Symbol] :> Dist[((-(d
1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(1 - c^2
*x^2)^FracPart[p], Int[(f + g*x)^m*(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[
c*x])^n, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g, n}, x] && EqQ[e1 -
 c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[m] && IntegerQ[p - 1/2] &&  !(Gt
Q[d1, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d1+cd1x}\sqrt{d2-cd2x}}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d1+cd1x}\sqrt{d2-cd2x}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int (a+bx)^n(cf+g\cosh(x))\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{d1+cd1x}\sqrt{d2-cd2x}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(cf(a+bx)^n+g(a+bx)^n\cosh(x)\right)dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{d1+cd1x}\sqrt{d2-cd2x}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{d1+cd1x}\sqrt{d2-cd2x}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int (a+bx)^n \cosh\right.}{c^2\sqrt{d1+cd1x}\sqrt{d2-}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{d1+cd1x}\sqrt{d2-cd2x}} + \frac{\left(g\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int e^{-x}(a+bx)^n\right.}{2c^2\sqrt{d1+cd1x}\sqrt{d2-}}$$

$$= \frac{f\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{d1+cd1x}\sqrt{d2-cd2x}} + \frac{e^{-\frac{a}{b}}g\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(\right.}{2c^2\sqrt{d1+c}}$$

Mathematica [A] time = 2.17, size = 219, normalized size = 0.84

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{cx-1}{cx+1}}\sqrt{cd1x+d1}\sqrt{d2-cd2x}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(-2cfe^{a/b}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{(a+b}{}\right.\right.}{2bc^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f + g*x)*(a + b*ArcCosh[c*x])^n)/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]),x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcC
osh[c*x])^n*(-2*c*E^(a/b)*f*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/
b^2))^n + b*E^((2*a)/b)*g*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n
, a/b + ArcCosh[c*x]] - b*g*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, -((
a + b*ArcCosh[c*x])/b)]))/(2*b*c^2*d1*d2*E^(a/b)*(1 + n)*(-1 + c*x)*(-((a +
 b*ArcCosh[c*x])^2/b^2))^n)
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{cd_1x + d_1}\,\sqrt{-cd_2x + d_2}\,\left(gx + f\right)(b\,\text{arcosh}\,(cx) + a)^n}{c^2d_1d_2x^2 - d_1d_2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(g*x + f)*(b*arccosh(c*x) +
a)^n/(c^2*d1*d2*x^2 - d1*d2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Simplification assuming d1 near 0Simplification assuming t_nostep ne
ar 0Simplification assuming d1 near 0Simplification assuming t_nostep near
0Simplification assuming d1 near 0Simplification assuming t_nostep near 0Si
mplification assuming d1 near 0Simplification assuming t_nostep near 0Simpl
ification assuming a near 0Simplification assuming d1 near 0Simplification
assuming t_nostep near 0Simplification assuming d1 near 0Simplification ass
uming t_nostep near 0Simplification assuming a near 0Simplification assumin
g d1 near 0Simplification assuming t_nostep near 0Simplification assuming d
1 near 0Simplification assuming t_nostep near 0Simplification assuming d1 n
ear 0Simplification assuming t_nostep near 0Simplification assuming d1 near
 0Simplification assuming t_nostep near 0Simplification assuming a near 0Si
mplification assuming d1 near 0Simplification assuming t_nostep near 0Simpl
ification assuming d1 near 0Simplification assuming t_nostep near 0Simplifi
cation assuming a near 0Unable to check sign: (8*pi/(sign(2*d1*t_nostep^2-t
_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable to ch
eck sign: (8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t
_nostep^2-t_nostep^4)+1)/2)Unable to check sign: (8*pi/(sign(2*d1*t_nostep^
2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable to
 check sign: (8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d
1*t_nostep^2-t_nostep^4)+1)/2)Unable to check sign: (8*pi/(sign(2*d1*t_nost
ep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable
 to check sign: (8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(
2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable to check sign: (8*pi/(sign(2*d1*t_n
ostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)Una
ble to check sign: (8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(si
gn(2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable to check sign: (8*pi/(sign(2*d1*
t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)
Unable to check sign: (8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/
(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable to check sign: (8*pi/(sign(2*
d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)
/2)Unable to check sign: (8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*
pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable to check sign: (8*pi/(sign
(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)
+1)/2)Unable to check sign: (8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(
-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable to check sign: (8*pi/(s
```

```
ign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep
^4)+1)/2)Unable to check sign: (8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2
)>(-8*pi/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)Unable to check sign: (8*pi
/(sign(2*d1*t_nostep^2-t_nostep^4)+1)/2)>(-8*pi/(sign(2*d1*t_nostep^2-t_nos
tep^4)+1)/2)Evaluation time: 17.66sym2poly/r2sym(const gen & e,const index_
m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{n}}{\sqrt{cd1x+d1}\,\sqrt{-cd2x+d2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(gx+f\right)\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{n}}{\sqrt{cd_{1}x+d_{1}}\,\sqrt{-cd_{2}x+d_{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*arccosh(c*x))^n/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)*(b*arccosh(c*x) + a)^n/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x
+ d2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(f+g\,x\right)\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{d_{1}+c\,d_{1}\,x}\,\sqrt{d_{2}-c\,d_{2}\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f + g*x)*(a + b*acosh(c*x))^n)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/
2)),x)
```

```
[Out] int(((f + g*x)*(a + b*acosh(c*x))^n)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/
2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{n}\left(f+gx\right)}{\sqrt{d_{1}\left(cx+1\right)}\,\sqrt{-d_{2}\left(cx-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(a+b*acosh(c*x))**n/(c*d1*x+d1)**(1/2)/(-c*d2*x+d2)**(1/2
),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n*(f + g*x)/(sqrt(d1*(c*x + 1))*sqrt(-d2*(c*x
- 1))), x)
```

3.78 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{cx-1}\,\sqrt{cx+1}},x\right)}{\sqrt{1-c^2x^2}}$$

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*Unintegrable((a+b*arccosh(c*x))^n*ln(h*(g*x+f)^
m)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)/(-c^2*x^2+1)^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((a + b*ArcCosh[c*x])^n*Log[h*(f +
 g*x)^m])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((a + b*ArcCosh[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)^n \log\left(\left(gx+f\right)^m h\right)}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*log((g*x + f)^m*h)/(c^2
*x^2 - 1), x)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 4.48, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{n} \ln\left(h\left(gx+f\right)^{m}\right)}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^{n} \log\left(\left(gx+f\right)^{m}h\right)}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\left(f+g\,x\right)^{m}\right)\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{n}}{\sqrt{1-c^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2 \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=774

$$\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2\text{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-}}{}$$

```
[Out] 1/12*m*(a+b*arccosh(c*x))^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b^2/c/(-c^2*x^2+1)^
(1/2)+1/3*(a+b*arccosh(c*x))^3*ln(h*(g*x+f)^m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
b/c/(-c^2*x^2+1)^(1/2)-1/3*m*(a+b*arccosh(c*x))^3*ln(1+(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/
(-c^2*x^2+1)^(1/2)-1/3*m*(a+b*arccosh(c*x))^3*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^
2*x^2+1)^(1/2)-m*(a+b*arccosh(c*x))^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^
2+1)^(1/2)-m*(a+b*arccosh(c*x))^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)
^(1/2)+2*b*m*(a+b*arccosh(c*x))*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(
1/2)+2*b*m*(a+b*arccosh(c*x))*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/
2)-2*b^2*m*polylog(4,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2
)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/2)-2*b^2*m*polylog(
4,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.60, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {5713, 5676, 5841, 5839, 5800, 5562, 2190, 2531, 6609, 2282, 6589}

$$-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2\text{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2\text{I}}{c\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^4)/(12*b^2*c*Sqrt[1 -
c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[1 +
(E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(3*b*c*Sqrt[1 - c^2*x^2])
- (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[1 + (E^ArcCosh
[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(3*b*c*Sqrt[1 - c^2*x^2]) + (Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3*Log[h*(f + g*x)^m])/(3*b*c*Sqr
t[1 - c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Po
lyLog[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^
2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*PolyLog[2,
 -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2])
+ (2*b*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[3, -((E^
ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (2*b
*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[3, -((E^ArcCos
h[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) - (2*b^2*m*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[4, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2
*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) - (2*b^2*m*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*PolyLog[4, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1
 - c^2*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5839

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_
.))^(n_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol]
 :> Simp[(Log[h*(f + g*x)^m]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d
2)]*(n + 1)), x] - Dist[(g*m)/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(a + b*ArcC
```

```
osh[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g
, h, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0]
```

Rule 5841

```
Int[Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_.
))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d
+ e*x^2)^FracPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[L
og[h*(f + g*x)^m]*(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ
[p - 1/2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc\sqrt{1-c^2x^2}} - \frac{\left(gm\sqrt{-\right.}{}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3\log\left(h(f+gx)^m\right)}{3bc\sqrt{1-c^2x^2}} - \frac{\left(gm\sqrt{-\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+\right.}{3}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^4}{12b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

Mathematica [F] time = 4.50, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
[Out] Integrate[((a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}\left(b^2\,\mathrm{arcosh}\,(cx)^2+2\,ab\,\mathrm{arcosh}\,(cx)+a^2\right)\log\left(\left(gx+f\right)^m h\right)}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2
)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.71sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 4.52, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2 \ln\left(h\left(gx+f\right)^m\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^2 \log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)\,(a+b\,\mathrm{acosh}\,(c\,x))^2}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x))^2)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x))^2)/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2 \log\left(h\left(f+gx\right)^m\right)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1
)), x)
```

3.80 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=600

$$\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\mathrm{Li}_2\left(-\frac{e^{\cosh^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{c}}{}$$

```
[Out] 1/6*m*(a+b*arccosh(c*x))^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b^2/c/(-c^2*x^2+1)^(
1/2)+1/2*(a+b*arccosh(c*x))^2*ln(h*(g*x+f)^m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b
/c/(-c^2*x^2+1)^(1/2)-1/2*m*(a+b*arccosh(c*x))^2*ln(1+(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(
-c^2*x^2+1)^(1/2)-1/2*m*(a+b*arccosh(c*x))^2*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2
*x^2+1)^(1/2)-m*(a+b*arccosh(c*x))*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1
)^(1/2)-m*(a+b*arccosh(c*x))*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g
/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/2
)+b*m*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/
2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*x^2+1)^(1/2)+b*m*polylog(3,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c/(-c^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.13, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {5713, 5676, 5841, 5839, 5800, 5562, 2190, 2531, 2282, 6589}

$$-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-\frac{ge^{\cosh^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c\sqrt{1-c^2x^2}}-\frac{m\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)\mathrm{PolyL}}{c\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b^2*c*Sqrt[1 - c
^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[1 + (
E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/(2*b*c*Sqrt[1 - c^2*x^2]) -
 (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[1 + (E^ArcCosh[
c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/(2*b*c*Sqrt[1 - c^2*x^2]) + (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*Log[h*(f + g*x)^m])/(2*b*c*Sqrt
[1 - c^2*x^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyL
og[2, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x
^2]) - (m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, -((E
^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (b*
m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, -((E^ArcCosh[c*x]*g)/(c*f - Sqrt[
c^2*f^2 - g^2]))])/(c*Sqrt[1 - c^2*x^2]) + (b*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[3, -((E^ArcCosh[c*x]*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/(c*Sqrt[1
- c^2*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5839

```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_
.))^(n_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol]
 :> Simp[(Log[h*(f + g*x)^m]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d
2)]*(n + 1)), x] - Dist[(g*m)/(b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(a + b*ArcC
osh[c*x])^(n + 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, g
, h, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d1, 0] && LtQ[
d2, 0] && IGtQ[n, 0]
```

Rule 5841

```
Int[Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCosh[(c_.)*(x_)]*(b_.
))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d
```

```
+ e*x^2)^FracPart[p])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[L
og[h*(f + g*x)^m]*(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ
[p - 1/2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc\sqrt{1-c^2x^2}} - \frac{\left(gm\sqrt{-1\right.}{}$$

$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\log\left(h(f+gx)^m\right)}{2bc\sqrt{1-c^2x^2}} - \frac{\left(gm\sqrt{-1\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+\right.}{2b}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

$$= \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{6b^2c\sqrt{1-c^2x^2}} - \frac{m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a\right.}{}$$

Mathematica [F] time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
[Out] Integrate[((a + b*ArcCosh[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)\log\left(\left(gx+f\right)^m h\right)}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*log((g*x + f)^m*h)/(c^2*x
^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arcosh}\,(cx)+a)\log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

maple [F] time = 4.15, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\text{arccosh}\,(cx))\ln\left(h\left(gx+f\right)^m\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arcosh}\,(cx)+a)\log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\ln\left(h\left(f+g\,x\right)^m\right)\,\left(a+b\,\text{acosh}\left(c\,x\right)\right)}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x)))/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((log(h*(f + g*x)^m)*(a + b*acosh(c*x)))/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))\log\left(h\left(f + gx\right)^{m}\right)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)),
 x)
```

3.81 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=237

$$\frac{im\text{Li}_2\left(\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{Li}_2\left(\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{ige^{i\sin^{-1}(c}}{\sqrt{c^2f^2-g^2}+}\right.}{c}$$

```
[Out] 1/2*I*m*arcsin(c*x)^2/c+arcsin(c*x)*ln(h*(g*x+f)^m)/c-m*arcsin(c*x)*ln(1-I*
(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c-m*arcsin(c*x)*ln(
1-I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+I*m*polylog(2
,I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c+I*m*polylog(2,
I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c
```

Rubi [A] time = 0.35, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {216, 2404, 4741, 4519, 2190, 2279, 2391}

$$\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{PolyLog}\left(2,\frac{ige^{i\sin^{-1}(cx)}}{\sqrt{c^2f^2-g^2}+cf}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{ige^{i\sin^{-1}(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/
(c*f - Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x
])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f + g*x)^m])/c
+ (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/c +
 (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/c
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
```

```
 b*Log[c*(d + e*x)^n]), x] - Dist[b*e*n, Int[SimplifyIntegrand[u/(d + e*x),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

Rule 4519

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(I*(e + f*x)^(m + 1))/(b*f*(m + 1)
), x] + (Int[((e + f*x)^m*E^(I*(c + d*x)))/(a - Rt[a^2 - b^2, 2] - I*b*E^(I
*(c + d*x))), x] + Int[((e + f*x)^m*E^(I*(c + d*x)))/(a + Rt[a^2 - b^2, 2]
- I*b*E^(I*(c + d*x))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] &
& PosQ[a^2 - b^2]
```

Rule 4741

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[((a + b*x)^n*Cos[x])/(c*d + e*Sin[x]), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\int \frac{\sin^{-1}(cx)}{cf+cgx}\,dx$$

$$= \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\text{Subst}\left(\int \frac{x\cos(x)}{c^2f+cg\sin(x)}\,dx, x, \sin^{-1}(cx)\right)$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} + \frac{\sin^{-1}(cx)\log\left(h(f+gx)^m\right)}{c} - (gm)\,\text{Subst}\left(\int \frac{e^{ix}x}{c^2f-ice^{ix}g-c\sqrt{c^2f^2-}}\right.$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

$$= \frac{im\sin^{-1}(cx)^2}{2c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\sin^{-1}(cx)\log\left(1-\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

Mathematica [A] time = 0.02, size = 246, normalized size = 1.04

$$\frac{im\text{Li}_2\left(\frac{ie^{i\sin^{-1}(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{im\text{Li}_2\left(\frac{ie^{i\sin^{-1}(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{icge^{i\sin^{-1}(cx)}}{c^2f-c\sqrt{c^2f^2-g^2}}\right)}{c}-\frac{m\sin^{-1}(cx)\log\left(1-\frac{icge^{i\sin^{-1}(cx)}}{c\sqrt{c^2f^2-g^2}+}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] ((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*ArcSin[c*x])*g
)/(c^2*f - c*Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*Ar
cSin[c*x])*g)/(c^2*f + c*Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(f +
 g*x)^m])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 -
 g^2])])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 -
g^2])])/c
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\log\left(\left(gx+f\right)^m h\right)}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h\left(gx+f\right)^m\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
[Out] int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/(1 - c^2*x^2)^(1/2),x)
[Out] int(log(h*(f + g*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h\left(f+gx\right)^m\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.82 $\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\text{Int}\left(\frac{\log\left(h(f+gx)^m\right)}{\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)},x\right)}{\sqrt{1-c^2x^2}}$$

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*Unintegrable(ln(h*(g*x+f)^m)/(a+b*arccosh(c*x))
/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)/(-c^2*x^2+1)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][Log[h*(f + g*x)^m]/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,\log\left(\left(gx+f\right)^m h\right)}{ac^2x^2+\left(bc^2x^2-b\right)\text{arcosh}\,(cx)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 - b
)*arccosh(c*x) - a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}\,(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 2.56, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(h\left(gx+f\right)^m\right)}{(a+b\,\mathrm{arccosh}\,(cx))\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(ln(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\left(gx+f\right)^m h\right)}{\sqrt{-c^2x^2+1}\,(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(h*(g*x+f)^m)/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(h\left(f+g\,x\right)^m\right)}{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(h*(f + g*x)^m)/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(log(h*(f + g*x)^m)/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(h\left(f+gx\right)^m\right)}{\sqrt{-(cx-1)(cx+1)}\,\left(a+b\,\mathrm{acosh}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(h*(g*x+f)**m)/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(log(h*(f + g*x)**m)/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))
), x)
```

3.83 $\int x^3 \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(4a\left(19a^2+16\right)-\left(26a^2+9\right)(a+bx)\right)}{96b^4}-\frac{\left(8a^4+24a^2+3\right)\cosh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{a+}}{}$$

```
[Out] -1/32*(8*a^4+24*a^2+3)*arccosh(b*x+a)/b^4+1/4*x^4*arccosh(b*x+a)+7/48*a*x^2
*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^2-1/16*x^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/
2)/b+1/96*(4*a*(19*a^2+16)-(26*a^2+9)*(b*x+a))*(b*x+a-1)^(1/2)*(b*x+a+1)^(1
/2)/b^4
```

Rubi [A] time = 0.19, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5866, 5802, 100, 153, 147, 52}

$$\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(4a\left(19a^2+16\right)-\left(26a^2+9\right)(a+bx)\right)}{96b^4}-\frac{\left(8a^4+24a^2+3\right)\cosh^{-1}(a+bx)}{32b^4}+\frac{7ax^2\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a + b*x],x]
```

```
[Out] (7*a*x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(48*b^2) - (x^3*Sqrt[-1 + a
+ b*x]*Sqrt[1 + a + b*x])/(16*b) + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4
*a*(16 + 19*a^2) - (9 + 26*a^2)*(a + b*x)))/(96*b^4) - ((3 + 24*a^2 + 8*a^4
)*ArcCosh[a + b*x])/(32*b^4) + (x^4*ArcCosh[a + b*x])/4
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^3\cosh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^3\cosh^{-1}(x)\,dx,x,a+bx\right)}{b}$$

$$=\frac{1}{4}x^4\cosh^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,a+bx\right)$$

$$=-\frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b}+\frac{1}{4}x^4\cosh^{-1}(a+bx)-\frac{1}{16}\text{Subst}\left(\int\frac{\left(\frac{3+4a^2}{b^2}-\frac{7ax}{b^2}\right)\left(-\right.}{\sqrt{-1+x}\,\sqrt{1}}\right.$$

$$=\frac{7ax^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{48b^2}-\frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b}+\frac{1}{4}x^4\cosh^{-1}(a+bx)$$

$$=\frac{7ax^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{48b^2}-\frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+}}{}$$

$$=\frac{7ax^2\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{48b^2}-\frac{x^3\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{16b}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+}}{}$$

Mathematica [A] time = 0.19, size = 121, normalized size = 0.80

$$\frac{-3\left(8a^4+24a^2+3\right)\log\left(\sqrt{a+bx-1}\,\sqrt{a+bx+1}+a+bx\right)+\sqrt{a+bx-1}\,\sqrt{a+bx+1}\,\left(50a^3-26a^2bx+14ab^2\right.}{96b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a + b*x],x]
```

```
[Out] (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(55*a + 50*a^3 - 9*b*x - 26*a^2*b*x +
 14*a*b^2*x^2 - 6*b^3*x^3) + 24*b^4*x^4*ArcCosh[a + b*x] - 3*(3 + 24*a^2 +
8*a^4)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(96*b^4)
```

fricas [A] time = 0.52, size = 110, normalized size = 0.72

$$\frac{3\left(8\,b^4x^4-8\,a^4-24\,a^2-3\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\left(6\,b^3x^3-14\,ab^2x^2-50\,a^3+\left(26\,a^2+9\right)\right.}{96\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(8*b^4*x^4 - 8*a^4 - 24*a^2 - 3)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1)) - (6*b^3*x^3 - 14*a*b^2*x^2 - 50*a^3 + (26*a^2 + 9)*b*x - 55
*a)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/b^4
```

giac [A] time = 0.81, size = 163, normalized size = 1.07

$$\frac{1}{4}\,x^4\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)-\frac{1}{96}\left(\sqrt{b^2x^2+2\,abx+a^2-1}\left(\left(2\,x\left(\frac{3\,x}{b^2}-\frac{7\,a}{b^3}\right)+\frac{26\,a^2b^3+9\,b^3}{b^7}\right)x-\frac{5\left(10\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/96*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 - 1)*((2*x*(3*x/b^2 - 7*a/b^3) + (26*a^2*b^3 + 9*b^3)/b^7)*x - 5*(10
*a^3*b^2 + 11*a*b^2)/b^7) - 3*(8*a^4 + 24*a^2 + 3)*log(abs(-a*b - (x*abs(b)
 - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b^4*abs(b)))*b
```

maple [B] time = 0.03, size = 308, normalized size = 2.03

$$\frac{x^4\mathrm{arccosh}\,(bx+a)}{4}-\frac{x^3\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{16b}+\frac{7a\,x^2\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{48b^2}-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\ldots}{4b^4\sqrt{(\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(b*x+a),x)
```

```
[Out] 1/4*x^4*arccosh(b*x+a)-1/16*x^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b+7/48*a*x^
2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^2-1/4/b^4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/
2)/((b*x+a)^2-1)^(1/2)*a^4*ln(b*x+a+((b*x+a)^2-1)^(1/2))-13/48/b^3*(b*x+a-1
)^(1/2)*(b*x+a+1)^(1/2)*x*a^2+25/48/b^4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*a^3
-3/4/b^4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/((b*x+a)^2-1)^(1/2)*a^2*ln(b*x+a+(
(b*x+a)^2-1)^(1/2))-3/32/b^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*x+55/96/b^4*(b
*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*a-3/32/b^4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/((
b*x+a)^2-1)^(1/2)*ln(b*x+a+((b*x+a)^2-1)^(1/2))
```

maxima [B] time = 0.34, size = 321, normalized size = 2.11

$$\frac{1}{4}\,x^4\,\mathrm{arcosh}\,(bx+a)-\frac{1}{96}\left(\frac{6\,\sqrt{b^2x^2+2\,abx+a^2-1}\,x^3}{b^2}-\frac{14\,\sqrt{b^2x^2+2\,abx+a^2-1}\,ax^2}{b^3}+\frac{105\,a^4\log\left(2\,b^2x+\ldots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccosh(b*x + a) - 1/96*(6*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x^3/b^
2 - 14*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a*x^2/b^3 + 105*a^4*log(2*b^2*x +
2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 35*sqrt(b^2*x^2 + 2*a*
```

```
b*x + a^2 - 1)*a^2*x/b^4 - 90*(a^2 - 1)*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^
2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 - 105*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a
^3/b^5 - 9*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*x/b^4 + 9*(a^2 - 1)^
2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 55*sqr
t(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*a/b^5)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \operatorname{acosh}(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a + b*x),x)
```

```
[Out] int(x^3*acosh(a + b*x), x)
```

sympy [A] time = 1.39, size = 255, normalized size = 1.68

$$\begin{cases} -\frac{a^4 \operatorname{acosh}(a+bx)}{4b^4} + \frac{25a^3\sqrt{a^2+2abx+b^2x^2-1}}{48b^4} - \frac{13a^2x\sqrt{a^2+2abx+b^2x^2-1}}{48b^3} - \frac{3a^2\operatorname{acosh}(a+bx)}{4b^4} + \frac{7ax^2\sqrt{a^2+2abx+b^2x^2-1}}{48b^2} + \frac{55a\sqrt{a^2+2abx+b}}{96b^4} \\ \frac{x^4\operatorname{acosh}(a)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(b*x+a),x)
```

```
[Out] Piecewise((-a**4*acosh(a + b*x)/(4*b**4) + 25*a**3*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/(48*b**4) - 13*a**2*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(48
*b**3) - 3*a**2*acosh(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/(48*b**2) + 55*a*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(96*b**4
) + x**4*acosh(a + b*x)/4 - x**3*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(16*b
) - 3*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(32*b**3) - 3*acosh(a + b*x)/(
32*b**4), Ne(b, 0)), (x**4*acosh(a)/4, True))
```

3.84 $\int x^2 \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(11a^2-5abx+4\right)}{18b^3}+\frac{a\left(2a^2+3\right)\cosh^{-1}(a+bx)}{6b^3}+\frac{1}{3}x^3\cosh^{-1}(a+bx)-\frac{x^2\sqrt{a+bx-1}}{9}$$

```
[Out] 1/6*a*(2*a^2+3)*arccosh(b*x+a)/b^3+1/3*x^3*arccosh(b*x+a)-1/9*x^2*(b*x+a-1)
^(1/2)*(b*x+a+1)^(1/2)/b-1/18*(-5*a*b*x+11*a^2+4)*(b*x+a-1)^(1/2)*(b*x+a+1)
^(1/2)/b^3
```

Rubi [A] time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5866, 5802, 100, 147, 52}

$$-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(11a^2-5abx+4\right)}{18b^3}+\frac{a\left(2a^2+3\right)\cosh^{-1}(a+bx)}{6b^3}-\frac{x^2\sqrt{a+bx-1}\sqrt{a+bx+1}}{9b}+\frac{1}{3}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[a + b*x],x]
```

```
[Out] -(x^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(9*b) - (Sqrt[-1 + a + b*x]*Sqr
t[1 + a + b*x]*(4 + 11*a^2 - 5*a*b*x))/(18*b^3) + (a*(3 + 2*a^2)*ArcCosh[a
+ b*x])/(6*b^3) + (x^3*ArcCosh[a + b*x])/3
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
```

```
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x^2\cosh^{-1}(a+bx)\,dx=\frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)^2\cosh^{-1}(x)\,dx,x,a+bx\right)}{b}$$
$$=\frac{1}{3}x^3\cosh^{-1}(a+bx)-\frac{1}{3}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx\right)$$
$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}+\frac{1}{3}x^3\cosh^{-1}(a+bx)-\frac{1}{9}\text{Subst}\left(\int\frac{\left(\frac{2+3a^2}{b^2}-\frac{5ax}{b^2}\right)\left(-\frac{a}{b}\right.}{\sqrt{-1+x}\sqrt{1+}}\right.$$
$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}\left(4+11a^2-5abx\right)}{18b^3}+\frac{1}{3}x^3$$
$$=-\frac{x^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{9b}-\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}\left(4+11a^2-5abx\right)}{18b^3}+\frac{a(3}{}$$

Mathematica [A] time = 0.13, size = 101, normalized size = 0.97

$$\frac{\left(6a^3+9a\right)\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)-\sqrt{a+bx-1}\sqrt{a+bx+1}\left(11a^2-5abx+2b^2x^2+4\right)+6b^3x}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCosh[a + b*x],x]
```

```
[Out] (-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2))
 + 6*b^3*x^3*ArcCosh[a + b*x] + (9*a + 6*a^3)*Log[a + b*x + Sqrt[-1 + a + b
*x]*Sqrt[1 + a + b*x]])/(18*b^3)
```

fricas [A] time = 0.63, size = 91, normalized size = 0.88

$$\frac{3\left(2\,b^3x^3+2\,a^3+3\,a\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\left(2\,b^2x^2-5\,abx+11\,a^2+4\right)\sqrt{b^2x^2+2\,abx+a^2-}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/18*(3*(2*b^3*x^3 + 2*a^3 + 3*a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^
2 - 1)) - (2*b^2*x^2 - 5*a*b*x + 11*a^2 + 4)*sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))/b^3
```

giac [A] time = 1.85, size = 132, normalized size = 1.27

$$\frac{1}{3}x^3\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)-\frac{1}{18}\left(\sqrt{b^2x^2+2\,abx+a^2-1}\left(x\left(\frac{2\,x}{b^2}-\frac{5\,a}{b^3}\right)+\frac{11\,a^2b+4\,b}{b^5}\right)+\frac{3\left(2\,a^3+3\,a\right)\text{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/18*(sqrt(b^2*x^2 + 2*a*b*x
 + a^2 - 1)*(x*(2*x/b^2 - 5*a/b^3) + (11*a^2*b + 4*b)/b^5) + 3*(2*a^3 + 3*a
)*log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b
^3*abs(b)))*b
```

maple [B] time = 0.01, size = 207, normalized size = 1.99

$$\frac{x^3 \operatorname{arccosh}(bx+a)}{3} - \frac{x^2\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{9b} + \frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,a^3\ln\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{3b^3\sqrt{(bx+a)^2-1}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(b*x+a),x)
```

```
[Out] 1/3*x^3*arccosh(b*x+a)-1/9*x^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b+1/3/b^3*(b
*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/((b*x+a)^2-1)^(1/2)*a^3*ln(b*x+a+((b*x+a)^2-1
)^(1/2))+5/18/b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*x*a-11/18/b^3*(b*x+a-1)^(
1/2)*(b*x+a+1)^(1/2)*a^2+1/2/b^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/((b*x+a)^2
-1)^(1/2)*a*ln(b*x+a+((b*x+a)^2-1)^(1/2))-2/9/b^3*(b*x+a-1)^(1/2)*(b*x+a+1)
^(1/2)
```

maxima [B] time = 0.50, size = 212, normalized size = 2.04

$$\frac{1}{3}x^3 \operatorname{arcosh}(bx+a) - \frac{1}{18}b\left(\frac{2\sqrt{b^2x^2+2abx+a^2-1}\,x^2}{b^2} - \frac{15a^3\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccosh(b*x + a) - 1/18*b*(2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x^2/
b^2 - 15*a^3*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b
^4 - 5*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a*x/b^3 + 9*(a^2 - 1)*a*log(2*b^2*
x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^4 + 15*sqrt(b^2*x^2 +
2*a*b*x + a^2 - 1)*a^2/b^4 - 4*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)/
b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \operatorname{acosh}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(a + b*x),x)
```

```
[Out] int(x^2*acosh(a + b*x), x)
```

sympy [A] time = 0.67, size = 170, normalized size = 1.63

$$\begin{cases} \frac{a^3 \operatorname{acosh}(a+bx)}{3b^3} - \frac{11a^2\sqrt{a^2+2abx+b^2x^2-1}}{18b^3} + \frac{5ax\sqrt{a^2+2abx+b^2x^2-1}}{18b^2} + \frac{a \operatorname{acosh}(a+bx)}{2b^3} + \frac{x^3 \operatorname{acosh}(a+bx)}{3} - \frac{x^2\sqrt{a^2+2abx+b^2x^2-1}}{9b} - 2 \\ \frac{x^3 \operatorname{acosh}(a)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(b*x+a),x)

[Out] Piecewise((a**3*acosh(a + b*x)/(3*b**3) - 11*a**2*sqrt(a**2 + 2*a*b*x + b**
2*x**2 - 1)/(18*b**3) + 5*a*x*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(18*b**2
) + a*acosh(a + b*x)/(2*b**3) + x**3*acosh(a + b*x)/3 - x**2*sqrt(a**2 + 2*
a*b*x + b**2*x**2 - 1)/(9*b) - 2*sqrt(a**2 + 2*a*b*x + b**2*x**2 - 1)/(9*b*
*3), Ne(b, 0)), (x**3*acosh(a)/3, True))
```

3.85 $\int x \cosh^{-1}(a + bx) \, dx$

Optimal. Leaf size=90

$$-\frac{(2a^2+1)\cosh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{4b^2}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{x\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{4b}$$

```
[Out] -1/4*(2*a^2+1)*arccosh(b*x+a)/b^2+1/2*x^2*arccosh(b*x+a)+3/4*a*(b*x+a-1)^(1
/2)*(b*x+a+1)^(1/2)/b^2-1/4*x*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5866, 5802, 90, 80, 52}

$$-\frac{(2a^2+1)\cosh^{-1}(a+bx)}{4b^2}+\frac{3a\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{4b^2}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{x\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[a + b*x],x]
```

```
[Out] (3*a*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(4*b^2) - (x*Sqrt[-1 + a + b*x]*
Sqrt[1 + a + b*x])/(4*b) - ((1 + 2*a^2)*ArcCosh[a + b*x])/(4*b^2) + (x^2*Ar
cCosh[a + b*x])/2
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int x\cosh^{-1}(a+bx)\,dx = \frac{\text{Subst}\left(\int\left(-\frac{a}{b}+\frac{x}{b}\right)\cosh^{-1}(x)\,dx,x,a+bx\right)}{b}$$

$$=\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{1}{2}\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,a+bx\right)$$

$$=-\frac{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-\frac{1}{4}\text{Subst}\left(\int\frac{\frac{1+2a^2}{b^2}-\frac{3ax}{b^2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right.$$

$$=\frac{3a\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b^2}-\frac{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b}+\frac{1}{2}x^2\cosh^{-1}(a+bx)-$$

$$=\frac{3a\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b^2}-\frac{x\sqrt{-1+a+bx}\sqrt{1+a+bx}}{4b}-\frac{\left(1+2a^2\right)\cosh^{-1}(a+bx}{4b^2}$$

Mathematica [A] time = 0.08, size = 87, normalized size = 0.97

$$\frac{-\left(2a^2+1\right)\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)+2b^2x^2\cosh^{-1}(a+bx)+(3a-bx)\sqrt{a+bx-1}\sqrt{a+bx+1}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCosh[a + b*x],x]
```

```
[Out] ((3*a - b*x)*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + 2*b^2*x^2*ArcCosh[a + b
*x] - (1 + 2*a^2)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(4*b
^2)
```

fricas [A] time = 0.74, size = 75, normalized size = 0.83

$$\frac{\left(2\,b^2x^2-2\,a^2-1\right)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}\,(bx-3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^2*x^2 - 2*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1
)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(b*x - 3*a))/b^2
```

giac [A] time = 1.91, size = 112, normalized size = 1.24

$$\frac{1}{2}\,x^2\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)-\frac{1}{4}\left(\sqrt{b^2x^2+2\,abx+a^2-1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{\left(2\,a^2+1\right)\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2\right.\right.\right.}{b^2|b|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(b*x + a + sqrt((b*x + a)^2 - 1)) - 1/4*(sqrt(b^2*x^2 + 2*a*b*x
+ a^2 - 1)*(x/b^2 - 3*a/b^3) - (2*a^2 + 1)*log(abs(-a*b - (x*abs(b) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b)))/(b^2*abs(b)))*b
```

maple [A] time = 0.01, size = 120, normalized size = 1.33

$$\frac{x^{2}\operatorname{arccosh}\left(bx+a\right)}{2}-\frac{\operatorname{arccosh}\left(bx+a\right)a^{2}}{2b^{2}}-\frac{x\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{4b}+\frac{3a\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{4b^{2}}-\frac{\sqrt{bx+a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(b*x+a),x)
```

```
[Out] 1/2*x^2*arccosh(b*x+a)-1/2/b^2*arccosh(b*x+a)*a^2-1/4*x*(b*x+a-1)^(1/2)*(b*
x+a+1)^(1/2)/b+3/4*a*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b^2-1/4/b^2*(b*x+a-1)^
(1/2)*(b*x+a+1)^(1/2)/((b*x+a)^2-1)^(1/2)*ln(b*x+a+((b*x+a)^2-1)^(1/2))
```

maxima [B] time = 0.34, size = 151, normalized size = 1.68

$$\frac{1}{2}\,x^{2}\operatorname{arcosh}\left(bx+a\right)-\frac{1}{4}\,b\left(\frac{3\,a^{2}\log\left(2\,b^{2}x+2\,ab+2\,\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}\,b\right)}{b^{3}}+\frac{\sqrt{b^{2}x^{2}+2\,abx+a^{2}-1}\,x}{b^{2}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccosh(b*x + a) - 1/4*b*(3*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*b)/b^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x/b^2 - (
a^2 - 1)*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^3 -
 3*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a/b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\operatorname{acosh}\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(a + b*x),x)
```

```
[Out] int(x*acosh(a + b*x), x)
```

sympy [A] time = 0.29, size = 104, normalized size = 1.16

$$\begin{cases} -\frac{a^{2}\operatorname{acosh}\left(a+bx\right)}{2b^{2}}+\frac{3a\sqrt{a^{2}+2abx+b^{2}x^{2}-1}}{4b^{2}}+\frac{x^{2}\operatorname{acosh}\left(a+bx\right)}{2}-\frac{x\sqrt{a^{2}+2abx+b^{2}x^{2}-1}}{4b}-\frac{\operatorname{acosh}\left(a+bx\right)}{4b^{2}} & \text{for}\: b\neq 0\\ \frac{x^{2}\operatorname{acosh}\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(b*x+a),x)
```

```
[Out] Piecewise((-a**2*acosh(a + b*x)/(2*b**2) + 3*a*sqrt(a**2 + 2*a*b*x + b**2*x
**2 - 1)/(4*b**2) + x**2*acosh(a + b*x)/2 - x*sqrt(a**2 + 2*a*b*x + b**2*x*
*2 - 1)/(4*b) - acosh(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*acosh(a)/2, True)
)
```

3.86 $\int \cosh^{-1}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{(a+bx)\cosh^{-1}(a+bx)}{b}-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{b}$$

```
[Out] (b*x+a)*arccosh(b*x+a)/b-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5864, 5654, 74}

$$\frac{(a+bx)\cosh^{-1}(a+bx)}{b}-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x],x]
```

```
[Out] -((Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/b) + ((a + b*x)*ArcCosh[a + b*x])/
b
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int \cosh^{-1}(x)\,dx,x,a+bx\right)}{b}\\
&= \frac{(a+bx)\cosh^{-1}(a+bx)}{b}-\frac{\operatorname{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,a+bx\right)}{b}\\
&= -\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{b}+\frac{(a+bx)\cosh^{-1}(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 1.37

$$x\cosh^{-1}(a+bx)-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}-2a\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x],x]
```

```
[Out] x*ArcCosh[a + b*x] - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] - 2*a*ArcSinh[Sq
rt[-1 + a + b*x]/Sqrt[2]])/b
```

fricas [A] time = 0.54, size = 57, normalized size = 1.39

$$\frac{(bx+a)\log\left(bx+a+\sqrt{b^2x^2+2\,abx+a^2-1}\right)-\sqrt{b^2x^2+2\,abx+a^2-1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - sqrt(b^2*x^2
+ 2*a*b*x + a^2 - 1))/b
```

giac [B] time = 5.94, size = 93, normalized size = 2.27

$$-b\left(\frac{a\log\left(\left|-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^2+2\,abx+a^2-1}}{b^2}\right)+x\log\left(bx+a+\sqrt{(bx+a)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="giac")
```

```
[Out] -b*(a*log(abs(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*abs(b))
)/(b*abs(b)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)/b^2) + x*log(b*x + a + sqr
t((b*x + a)^2 - 1))
```

maple [A] time = 0.00, size = 36, normalized size = 0.88

$$\frac{(bx+a)\,\mathrm{arccosh}\,(bx+a)-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a),x)
```

```
[Out] 1/b*((b*x+a)*arccosh(b*x+a)-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))
```

maxima [A] time = 0.30, size = 30, normalized size = 0.73

$$\frac{(bx+a)\,\mathrm{arcosh}\,(bx+a)-\sqrt{(bx+a)^2-1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*x + a)*arccosh(b*x + a) - sqrt((b*x + a)^2 - 1))/b
```

mupad [B] time = 4.00, size = 266, normalized size = 6.49

$$x\,\mathrm{acosh}\,(a+b\,x)-\frac{\frac{4\,a\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}{b\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}+\frac{4\,a\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^3}{b\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^3}-\frac{8\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^2\sqrt{a-1}\,\sqrt{a+1}}{b\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^2}}{\frac{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^4}{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^4}-\frac{2\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^2}+1}+\frac{4\,a\,\mathrm{atanh}\left(\frac{\sqrt{a-1}-\sqrt{\cdots}}{\sqrt{a+1}-\sqrt{\cdots}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x),x)
```

```
[Out] x*acosh(a + b*x) - ((4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2)))/(b*((a + 1)
^(1/2) - (a + b*x + 1)^(1/2))) + (4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))
^3)/(b*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) - (8*((a - 1)^(1/2) - (a +
b*x - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a + 1)^(1/2) - (a + b*x
 + 1)^(1/2))^2))/(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4/((a + 1)^(1/2) -
(a + b*x + 1)^(1/2))^4 - (2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^2 + 1) + (4*a*atanh(((a - 1)^(1/2) - (a + b
*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))))/b
```

sympy [A] time = 0.15, size = 46, normalized size = 1.12

$$\begin{cases} \frac{a \operatorname{acosh}{\left(a + bx \right)}}{b} + x \operatorname{acosh}{\left(a + bx \right)} - \frac{\sqrt{a^{2} + 2abx + b^{2}x^{2} - 1}}{b} & \text{for}\: b \neq 0 \\ x \operatorname{acosh}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a),x)
```

```
[Out] Piecewise((a*acosh(a + b*x)/b + x*acosh(a + b*x) - sqrt(a**2 + 2*a*b*x + b*
*2*x**2 - 1)/b, Ne(b, 0)), (x*acosh(a), True))
```

3.87 $\int \frac{\cosh^{-1}(a+bx)}{x} dx$

Optimal. Leaf size=131

$$\text{Li}_2\left(\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{Li}_2\left(\frac{e^{\cosh^{-1}(a+bx)}}{a+\sqrt{a^2-1}}\right)+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a}}{\sqrt{a^2-1}}\right.$$

```
[Out] -1/2*arccosh(b*x+a)^2+arccosh(b*x+a)*ln(1-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^
(1/2))/(a-(a^2-1)^(1/2)))+arccosh(b*x+a)*ln(1-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a
+1)^(1/2))/(a+(a^2-1)^(1/2)))+polylog(2,(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1
/2))/(a-(a^2-1)^(1/2)))+polylog(2,(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(
a+(a^2-1)^(1/2)))
```

Rubi [A] time = 0.25, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5866, 5800, 5562, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\text{PolyLog}\left(2,\frac{e^{\cosh^{-1}(a+bx)}}{\sqrt{a^2-1}+a}\right)+\cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{a^2-1}}\right)+\cosh^{-1}(a+bx)\,\text{l}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x,x]
```

```
[Out] -ArcCosh[a + b*x]^2/2 + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a - Sq
rt[-1 + a^2])] + ArcCosh[a + b*x]*Log[1 - E^ArcCosh[a + b*x]/(a + Sqrt[-1 +
 a^2])] + PolyLog[2, E^ArcCosh[a + b*x]/(a - Sqrt[-1 + a^2])] + PolyLog[2,
E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{x\sinh(x)}{-\frac{a}{b}+\frac{\cosh(x)}{b}}\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \frac{\text{Subst}\left(\int \frac{e^x x}{-\frac{a}{b}-\frac{\sqrt{-1+a^2}}{b}+\frac{e^x}{b}}\,dx, x, \cosh^{-1}(a+bx)\right)}{b} + \frac{\text{Subst}\left(\int \frac{e^x}{-\frac{a}{b}+\frac{\sqrt{-1}}{b}}\right.}{}$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx)\log\left(1-\right.$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx)\log\left(1-\right.$$

$$= -\frac{1}{2}\cosh^{-1}(a+bx)^2 + \cosh^{-1}(a+bx)\log\left(1-\frac{e^{\cosh^{-1}(a+bx)}}{a-\sqrt{-1+a^2}}\right) + \cosh^{-1}(a+bx)\log\left(1-\right.$$

Mathematica [A] time = 0.01, size = 153, normalized size = 1.17

$$\text{Li}_2\left(-\frac{e^{\cosh^{-1}(a+bx)}}{\sqrt{a^2-1}-a}\right)+\text{Li}_2\left(\frac{e^{\cosh^{-1}(a+bx)}}{a+\sqrt{a^2-1}}\right)+\cosh^{-1}(a+bx)\log\left(\frac{e^{\cosh^{-1}(a+bx)}}{b\left(-\frac{\sqrt{a^2-1}}{b}-\frac{a}{b}\right)}+1\right)+\cosh^{-1}(a+bx)\log\left(\frac{e^{\cosh^{-1}(a+}}{b\left(\frac{\sqrt{a^2-1}}{b}\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x,x]
```

```
[Out] -1/2*ArcCosh[a + b*x]^2 + ArcCosh[a + b*x]*Log[1 + E^ArcCosh[a + b*x]/((-(a
/b) - Sqrt[-1 + a^2]/b)*b)] + ArcCosh[a + b*x]*Log[1 + E^ArcCosh[a + b*x]/(
(-(a/b) + Sqrt[-1 + a^2]/b)*b)] + PolyLog[2, -(E^ArcCosh[a + b*x]/(-a + Sqr
t[-1 + a^2]))] + PolyLog[2, E^ArcCosh[a + b*x]/(a + Sqrt[-1 + a^2])]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{arcosh}(bx+a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="fricas")
[Out] integral(arccosh(b*x + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="giac")
[Out] integrate(arccosh(b*x + a)/x, x)
```

maple [B] time = 0.23, size = 436, normalized size = 3.33

$$-\frac{\operatorname{arccosh}(bx+a)^2}{2}+\frac{a\operatorname{arccosh}(bx+a)\ln\left(\frac{\sqrt{a^2-1}-bx-\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{a+\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}-\frac{a\operatorname{arccosh}(bx+a)\ln\left(\frac{\sqrt{a^2-1}+bx+\sqrt{bx+}}{-a+\sqrt{a}}\right)}{\sqrt{a^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x,x)
[Out] -1/2*arccosh(b*x+a)^2+a*arccosh(b*x+a)/(a^2-1)^(1/2)*ln(((a^2-1)^(1/2)-b*x-
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))-a*arccosh(b*x+a)/(a^2-1
)^(1/2)*ln(((a^2-1)^(1/2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^
(1/2)))-(a^2-1+a*(a^2-1)^(1/2))*arccosh(b*x+a)*(2*ln(((a^2-1)^(1/2)-b*x-(b*
x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))*a^2-ln(((a^2-1)^(1/2)-b*x-
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2)))-ln(((a^2-1)^(1/2)+b*x+(
b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^(1/2)))-2*a*(a^2-1)^(1/2)*ln(((
a^2-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^(1/2))))/(a^2-
1)+dilog(((a^2-1)^(1/2)+b*x+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(-a+(a^2-1)^(1
/2)))+dilog(((a^2-1)^(1/2)-b*x-(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/(a+(a^2-1)^
(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(bx+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x,x, algorithm="maxima")
[Out] integrate(arccosh(b*x + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a+b\,x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/x,x)
[Out] int(acosh(a + b*x)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x,x)
```

```
[Out] Integral(acosh(a + b*x)/x, x)
```

3.88 $\int \frac{\cosh^{-1}(a+bx)}{x^2} \, dx$

Optimal. Leaf size=64

$$-\frac{2b \tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\cosh^{-1}(a+bx)}{x}$$

```
[Out] -arccosh(b*x+a)/x-2*b*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a
-1)^(1/2))/(-a^2+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5866, 5802, 93, 205}

$$-\frac{2b \tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\cosh^{-1}(a+bx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(ArcCosh[a + b*x]/x) - (2*b*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1
 + a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x^2}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{\cosh^{-1}(a+bx)}{x} + \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx\right)$$
$$= -\frac{\cosh^{-1}(a+bx)}{x} + 2\,\text{Subst}\left(\int \frac{1}{-\frac{1}{b}-\frac{a}{b}-\left(\frac{1}{b}-\frac{a}{b}\right)x^2}\,dx, x, \frac{\sqrt{1+a+bx}}{\sqrt{-1+a+bx}}\right)$$
$$= -\frac{\cosh^{-1}(a+bx)}{x} - \frac{2b\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{\sqrt{1-a^2}}$$

Mathematica [C] time = 0.12, size = 83, normalized size = 1.30

$$-\frac{\cosh^{-1}(a+bx)}{x} - \frac{ib\log\left(\frac{2\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+\frac{i(a^2+abx-1)}{\sqrt{1-a^2}}\right)}{bx}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(ArcCosh[a + b*x]/x) - (I*b*Log[(2*(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] +
 (I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2]))/(b*x)])/Sqrt[1 - a^2]
```

fricas [B] time = 0.69, size = 322, normalized size = 5.03

$$\left[\frac{\sqrt{a^2-1}\,bx\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}\,a-1\right)-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)+\left(a^2-1\right)x\log\left(-bx-a+\sqrt{b^2x^2+2\,ab}\right.}{\left(a^2-1\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - 1)*b*x*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*
(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + (a^
2 - 1)*x*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - (a^2 - (a^2 -
1)*x - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^2 - 1)*x),
(2*sqrt(-a^2 + 1)*b*x*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x
+ a^2 - 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + (a^2 - 1)*x*log(-b*x - a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 - 1)) - (a^2 - (a^2 - 1)*x - 1)*log(b*x + a + sqrt(b^2
*x^2 + 2*a*b*x + a^2 - 1)))/((a^2 - 1)*x)]
```

giac [A] time = 0.40, size = 73, normalized size = 1.14

$$\frac{2\,b\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}} - \frac{\log\left(bx+a+\sqrt{(bx+a)^2-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/sqrt(-a^2 + 1))/
sqrt(-a^2 + 1) - log(b*x + a + sqrt((b*x + a)^2 - 1))/x
```

maple [A] time = 0.02, size = 97, normalized size = 1.52

$$-\frac{\operatorname{arccosh}(bx+a)}{x}-\frac{b\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\sqrt{a^2-1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)}{\sqrt{(bx+a)^2-1}\,(a-1)(1+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^2,x)
```

```
[Out] -arccosh(b*x+a)/x-b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(a^2-1)^(1/2)*ln(2*((a^
2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*(b*x+a)-1)/b/x)/((b*x+a)^2-1)^(1/2)/(a-1)/
(1+a)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a+b\,x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/x^2,x)
```

```
[Out] int(acosh(a + b*x)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**2,x)
```

```
[Out] Integral(acosh(a + b*x)/x**2, x)
```

3.89 $\int \frac{\cosh^{-1}(a+bx)}{x^3} dx$

Optimal. Leaf size=106

$$-\frac{ab^2 \tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}$$

```
[Out] -1/2*arccosh(b*x+a)/x^2-a*b^2*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2
)/(b*x+a-1)^(1/2))/(-a^2+1)^(3/2)+1/2*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a
^2+1)/x
```

Rubi [A] time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5866, 5802, 96, 93, 205}

$$-\frac{ab^2 \tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x) - ArcCosh[a + b*x]
/(2*x^2) - (a*b^2*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[
-1 + a + b*x])])/(1 - a^2)^(3/2)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x^3}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\cosh^{-1}(a+bx)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx\right)$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, \ldots\right)}{2\left(1-a^2\right)}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{-\frac{1}{b}-\frac{a}{b}-\left(\frac{1}{b}-\frac{a}{b}\right)x^2}\,dx, x, \ldots\right)}{1-a^2}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\cosh^{-1}(a+bx)}{2x^2}-\frac{ab^2\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{1+a+bx}}{\sqrt{1+a}\sqrt{-1+a+bx}}\right)}{\left(1-a^2\right)^{3/2}}$$

Mathematica [C] time = 0.29, size = 136, normalized size = 1.28

$$\frac{-\cosh^{-1}(a+bx)+\frac{bx\left(-\sqrt{a+bx-1}\sqrt{a+bx+1}+\frac{iabx\log\left(\frac{4i\sqrt{1-a^2}\left(-i\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx+1}+a^2+abx-1\right)}{ab^2x}\right)}{\sqrt{1-a^2}}\right)}{a^2-1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (-ArcCosh[a + b*x] + (b*x*(-(Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]) + (I*a*b
*x*Log[((4*I)*Sqrt[1 - a^2]*(-1 + a^2 + a*b*x - I*Sqrt[1 - a^2]*Sqrt[-1 + a
 + b*x]*Sqrt[1 + a + b*x]))/(a*b^2*x)])/Sqrt[1 - a^2]))/(-1 + a^2))/(2*x^2)
```

fricas [B] time = 0.68, size = 460, normalized size = 4.34

$$\left[\frac{\sqrt{a^2-1}\,ab^2x^2\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2+\sqrt{a^2-1}\,a-1\right)+\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-\left(a^2-1\right)b^2x^2+\left(a^4-2\,a^2+1\right)x^2 \ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - 1)*a*b^2*x^2*log((a^2*b*x + a^3 + sqrt(b^2*x^2 + 2*a*b*x +
 a^2 - 1)*(a^2 + sqrt(a^2 - 1)*a - 1) + (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a
```

```
)/x) - (a^2 - 1)*b^2*x^2 + (a^4 - 2*a^2 + 1)*x^2*log(-b*x - a + sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*b*x -
 (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a
*b*x + a^2 - 1)))/((a^4 - 2*a^2 + 1)*x^2), -1/2*(2*sqrt(-a^2 + 1)*a*b^2*x^2
*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*sqrt(-a^2
+ 1))/(a^2 - 1)) + (a^2 - 1)*b^2*x^2 - (a^4 - 2*a^2 + 1)*x^2*log(-b*x - a +
 sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^
2 - 1)*b*x + (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*log(b*x + a + sqrt(b
^2*x^2 + 2*a*b*x + a^2 - 1)))/((a^4 - 2*a^2 + 1)*x^2)]
```

giac [A] time = 0.53, size = 170, normalized size = 1.60

$$-\left(\frac{ab\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{\left(a^2-1\right)\sqrt{-a^2+1}}-\frac{\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)ab+a^2|b|-|b|}{\left(\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)^2-a^2+1\right)\left(a^2-1\right)}\right)b-\frac{\log\left(bx+a+\sqrt{(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -(a*b*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))/sqrt(-a^2 + 1)
)/((a^2 - 1)*sqrt(-a^2 + 1)) - ((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
1))*a*b + a^2*abs(b) - abs(b))/(((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))^2 - a^2 + 1)*(a^2 - 1)))*b - 1/2*log(b*x + a + sqrt((b*x + a)^2 - 1))/
x^2
```

maple [B] time = 0.02, size = 181, normalized size = 1.71

$$-\frac{\operatorname{arccosh}(bx+a)}{2x^2}+\frac{b^2\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)a}{2\sqrt{a^2-1}\,(1+a)(a-1)\sqrt{(bx+a)^2-1}}-\frac{b\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,a}{2x\left(a^2-1\right)(1+a)(a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^3,x)
```

```
[Out] -1/2*arccosh(b*x+a)/x^2+1/2*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(a^2-1)^(1/
2)/(1+a)/(a-1)/((b*x+a)^2-1)^(1/2)*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+
a*(b*x+a)-1)/b/x)*a-1/2*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x/(a^2-1)/(1+a)/(
a-1)*a^2+1/2*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x/(a^2-1)/(1+a)/(a-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a+b\,x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/x^3,x)
[Out] int(acosh(a + b*x)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**3,x)
[Out] Integral(acosh(a + b*x)/x**3, x)
```

3.90 $\int \frac{\cosh^{-1}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=154

$$-\frac{\left(2a^2+1\right)b^3\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{3\left(1-a^2\right)^{5/2}}+\frac{ab^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{2\left(1-a^2\right)^2x}+\frac{b\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{6\left(1-a^2\right)x^2}-\frac{\cosh^{-1}(a+bx)}{3x^3}$$

```
[Out] -1/3*arccosh(b*x+a)/x^3-1/3*(2*a^2+1)*b^3*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2
)/(1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(5/2)+1/6*b*(b*x+a-1)^(1/2)*(b*x+a+
1)^(1/2)/(-a^2+1)/x^2+1/2*a*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)^2/
x
```

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5866, 5802, 103, 151, 12, 93, 205}

$$\frac{ab^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{2\left(1-a^2\right)^2x}-\frac{\left(2a^2+1\right)b^3\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{3\left(1-a^2\right)^{5/2}}+\frac{b\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{6\left(1-a^2\right)x^2}-\frac{\cosh^{-1}(a+bx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/x^4,x]
```

```
[Out] (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(6*(1 - a^2)*x^2) + (a*b^2*Sqrt[-1
 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)^2*x) - ArcCosh[a + b*x]/(3*x^3)
 - ((1 + 2*a^2)*b^3*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqr
t[-1 + a + b*x]])])/(3*(1 - a^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{x^4}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^4}\,dx, x, a+bx\right)}{b}$$

$$= -\frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx\right)$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6\left(1-a^2\right)x^2} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{b^2\,\text{Subst}\left(\int \frac{\frac{2a}{b}+\frac{x}{b}}{\sqrt{-1+x}\sqrt{1+x}\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x \right.}{6\left(1-a^2\right)}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6\left(1-a^2\right)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2 x} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{b^4}{}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6\left(1-a^2\right)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2 x} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{((1}{}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6\left(1-a^2\right)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2 x} - \frac{\cosh^{-1}(a+bx)}{3x^3} + \frac{((1}{}$$

$$= \frac{b\sqrt{-1+a+bx}\sqrt{1+a+bx}}{6\left(1-a^2\right)x^2} + \frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2 x} - \frac{\cosh^{-1}(a+bx)}{3x^3} - \frac{(1}{}$$

Mathematica [C] time = 0.32, size = 162, normalized size = 1.05

$$\frac{1}{6}\left(-\frac{i\left(2a^2+1\right)b^3\log\left(\frac{12\left(1-a^2\right)^{3/2}\left(\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx+1}+ia^2+iabx-i\right)}{b^3\left(2a^2x+x\right)}\right)}{\left(1-a^2\right)^{5/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-a^2+3abx+1\right)}{\left(a^2-1\right)^2x^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a + b*x]/x^4,x]
```

```
[Out] ((b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(1 - a^2 + 3*a*b*x))/((-1 + a^2)^2
*x^2) - (2*ArcCosh[a + b*x])/x^3 - (I*(1 + 2*a^2)*b^3*Log[(12*(1 - a^2)^(3/
2)*(-I + I*a^2 + I*a*b*x + Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*
x]))/(b^3*(x + 2*a^2*x))])/(1 - a^2)^(5/2))/6
```

fricas [B] time = 0.64, size = 566, normalized size = 3.68

$$\left[\frac{\left(2\,a^2+1\right)\sqrt{a^2-1}\,b^3x^3\log\left(\frac{a^2bx+a^3+\sqrt{b^2x^2+2\,abx+a^2-1}\left(a^2-\sqrt{a^2-1}\,a-1\right)-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)+3\left(a^3-a\right)b^3x^3+2\left(a^6-3\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((2*a^2 + 1)*sqrt(a^2 - 1)*b^3*x^3*log((a^2*b*x + a^3 + sqrt(b^2*x^2 +
 2*a*b*x + a^2 - 1)*(a^2 - sqrt(a^2 - 1)*a - 1) - (a*b*x + a^2 - 1)*sqrt(a^
2 - 1) - a)/x) + 3*(a^3 - a)*b^3*x^3 + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*x^3*log(
-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(a^6 - 3*a^4 - (a^6 - 3*a
^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2 + 2*a*b*x + a^2
 - 1)) + (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt(b^2*x^2 + 2*a*b
*x + a^2 - 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3), 1/6*(2*(2*a^2 + 1)*sqrt(-a^
2 + 1)*b^3*x^3*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1)*sqrt(-a^2 + 1))/(a^2 - 1)) + 3*(a^3 - a)*b^3*x^3 + 2*(a^6 - 3*a^4 + 3*a
^2 - 1)*x^3*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)) - 2*(a^6 - 3*
a^4 - (a^6 - 3*a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*log(b*x + a + sqrt(b^2*x^2
 + 2*a*b*x + a^2 - 1)) + (3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt
(b^2*x^2 + 2*a*b*x + a^2 - 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3)]
```

giac [B] time = 1.95, size = 340, normalized size = 2.21

$$\frac{1}{3}\,b\left(\frac{\left(2\,a^2b^2+b^2\right)\arctan\left(-\frac{x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}}{\sqrt{-a^2+1}}\right)}{\left(a^4-2\,a^2+1\right)\sqrt{-a^2+1}}-\frac{2\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2-1}\right)^3a^2b^2-6\left(x|b|-\sqrt{b^2x^2+2\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b*((2*a^2*b^2 + b^2)*arctan(-(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 -
 1))/sqrt(-a^2 + 1))/((a^4 - 2*a^2 + 1)*sqrt(-a^2 + 1)) - (2*(x*abs(b) - sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1))^3*a^2*b^2 - 6*(x*abs(b) - sqrt(b^2*x^2 + 2
*a*b*x + a^2 - 1))*a^4*b^2 - 4*a^5*b*abs(b) + (x*abs(b) - sqrt(b^2*x^2 + 2*
a*b*x + a^2 - 1))^3*b^2 + 7*(x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*
a^2*b^2 + 8*a^3*b*abs(b) - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1))*b
```

```
^2 - 4*a*b*abs(b))/((a^4 - 2*a^2 + 1)*((x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x +
 a^2 - 1))^2 - a^2 + 1)^2)) - 1/3*log(b*x + a + sqrt((b*x + a)^2 - 1))/x^3
```

maple [B] time = 0.02, size = 397, normalized size = 2.58

$$-\frac{\operatorname{arccosh}(bx+a)}{3x^3}-\frac{b^3\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\ln\left(\frac{2\sqrt{a^2-1}\,\sqrt{(bx+a)^2-1}+2a(bx+a)-2}{bx}\right)a^2}{3\left(a^2-1\right)^{\frac{3}{2}}(1+a)(a-1)\sqrt{(bx+a)^2-1}}-\frac{b^3\sqrt{bx+a-1}\,\sqrt{bx+a}}{6\left(a^2-1\right)^{\frac{3}{2}}(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/x^4,x)
```

```
[Out] -1/3*arccosh(b*x+a)/x^3-1/3*b^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(a^2-1)^(3/
2)/(1+a)/(a-1)/((b*x+a)^2-1)^(1/2)*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+
a*(b*x+a)-1)/b/x)*a^2-1/6*b^3*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(a^2-1)^(3/2)
/(1+a)/(a-1)/((b*x+a)^2-1)^(1/2)*ln(2*((a^2-1)^(1/2)*((b*x+a)^2-1)^(1/2)+a*
(b*x+a)-1)/b/x)+1/2*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x/(a^2-1)^2/(1+a)/(
a-1)*a^3-1/6*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x^2/(a^2-1)^2/(1+a)/(a-1)*a^
4-1/2*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x/(a^2-1)^2/(1+a)/(a-1)*a+1/3*b*(
b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/x^2/(a^2-1)^2/(1+a)/(a-1)*a^2-1/6*b*(b*x+a-1
)^(1/2)*(b*x+a+1)^(1/2)/x^2/(a^2-1)^2/(1+a)/(a-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a+b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/x^4,x)
```

```
[Out] int(acosh(a + b*x)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/x**4,x)
```

```
[Out] Integral(acosh(a + b*x)/x**4, x)
```

3.91 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] -1/2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/
2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5864, 5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+\right.}{2bd}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{bd}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Mathematica [A] time = 0.17, size = 110, normalized size = 1.20

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)}{2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]
] + Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x
])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\text{arcosh}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\mathrm{arcosh}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c + d*x)), x)
```

3.92 $\int \frac{1}{\sqrt{a-b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=94

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] -1/2*exp(a/b)*erf((a-b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/
2*erfi((a-b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5864, 5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - b*ArcCosh[c + d*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a - b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a-b\cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a-b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a-b\,\text{c}\right.}{2bd}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a-b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a-b}\right.}{bd}$$

$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a-b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Mathematica [A] time = 0.11, size = 111, normalized size = 1.18

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}-\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}-\cosh^{-1}(c+dx)\right)+\sqrt{\cosh^{-1}(c+dx)-\frac{a}{b}}\,\Gamma\left(\frac{1}{2},\cosh^{-1}(c+dx)-\frac{a}{b}\right)\right)}{2d\sqrt{a-b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a - b*ArcCosh[c + d*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b - ArcCosh[c + d*x]]*Gamma[1/2, a/b - ArcCosh[c + d*x]
] + Sqrt[-(a/b) + ArcCosh[c + d*x]]*Gamma[1/2, -(a/b) + ArcCosh[c + d*x]])/
(2*d*E^(a/b)*Sqrt[a - b*ArcCosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b\,\text{arcosh}\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-b*arccosh(d*x + c) + a), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - b\,\mathrm{arccosh}\,(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*arccosh(d*x+c))^(1/2),x)
[Out] int(1/(a-b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-b\,\mathrm{arcosh}\,(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a - b\,\mathrm{acosh}\,(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*acosh(c + d*x))^(1/2),x)
[Out] int(1/(a - b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - b\,\mathrm{acosh}\,(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*acosh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a - b*acosh(c + d*x)), x)
```

3.93 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=135

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d}-\frac{be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{25d}-\frac{4be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{75d}$$

```
[Out] 1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))/d-8/75*b*e^4*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2)/d-4/75*b*e^4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/25*b*e^
4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5866, 12, 5662, 100, 74}

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d}-\frac{be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^4}{25d}-\frac{4be^4\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{75d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-8*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(75*d) - (4*b*e^4*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(75*d) - (b*e^4*Sqrt[-1 + c + d*x]
*(c + d*x)^4*Sqrt[1 + c + d*x])/(25*d) + (e^4*(c + d*x)^5*(a + b*ArcCosh[c
+ d*x]))/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{5d} - \frac{\left(be^4\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{-1+x}\sqrt{1+x}}\,dx,\right.}{5d}$$
$$= -\frac{be^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{25d} + \frac{e^4(c+dx)^5\left(a+b\cosh\right.}{5d}$$
$$= -\frac{be^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{25d} + \frac{e^4(c+dx)^5\left(a+b\cosh\right.}{5d}$$
$$= -\frac{4be^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{75d} - \frac{be^4\sqrt{-1+c+dx}\,(c+}{25d}$$
$$= -\frac{4be^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{75d} - \frac{be^4\sqrt{-1+c+dx}\,(c+}{25d}$$
$$= -\frac{8be^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{75d} - \frac{4be^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1}}{75d}$$

Mathematica [A] time = 0.13, size = 103, normalized size = 0.76

$$\frac{e^4\left((c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)-\frac{4}{15}b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(c^2+2cdx+d^2x^2+2\right)-\frac{1}{5}b\sqrt{c+dx-1}\,\sqrt{c}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^4*(-1/5*(b*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x]) - (4*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x]*(2 + c^2 + 2*c*d*x + d^2*x^2))/15 + (c + d
*x)^5*(a + b*ArcCosh[c + d*x])))/(5*d)
```

fricas [B] time = 0.51, size = 279, normalized size = 2.07

$$15\,ad^5e^4x^5 + 75\,acd^4e^4x^4 + 150\,ac^2d^3e^4x^3 + 150\,ac^3d^2e^4x^2 + 75\,ac^4de^4x + 15\left(bd^5e^4x^5 + 5\,bcd^4e^4x^4 + 10\,bc^2d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a*d^5*e^4*x^5 + 75*a*c*d^4*e^4*x^4 + 150*a*c^2*d^3*e^4*x^3 + 150*a
*c^3*d^2*e^4*x^2 + 75*a*c^4*d*e^4*x + 15*(b*d^5*e^4*x^5 + 5*b*c*d^4*e^4*x^4
 + 10*b*c^2*d^3*e^4*x^3 + 10*b*c^3*d^2*e^4*x^2 + 5*b*c^4*d*e^4*x + b*c^5*e^
4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (3*b*d^4*e^4*x^4 + 12
*b*c*d^3*e^4*x^3 + 2*(9*b*c^2 + 2*b)*d^2*e^4*x^2 + 4*(3*b*c^3 + 2*b*c)*d*e^
4*x + (3*b*c^4 + 4*b*c^2 + 8*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [B] time = 5.61, size = 822, normalized size = 6.09

$$\frac{1}{600}\left(120\,ad^4x^5+600\,acd^3x^4+1200\,ac^2d^2x^3+1200\,ac^3dx^2-600\left(d\left(\frac{c\log\left(\left|-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2-1}\right.\right.\right.}{d|d|}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/600*(120*a*d^4*x^5 + 600*a*c*d^3*x^4 + 1200*a*c^2*d^2*x^3 + 1200*a*c^3*d*
x^2 - 600*(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c^4 + 600*(2*x^2*log(d*x + c +
 sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x
/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*c^3*d + 200*(6*x^3*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*
(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d + 4*d)/d^5) + 3*(2*c^3 + 3*c)*log(abs(-c
*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^3*abs(d)))*
d)*b*c^2*d^2 + 25*(24*x^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
- (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^
3 + 9*d^3)/d^7)*x - 5*(10*c^3*d^2 + 11*c*d^2)/d^7) - 3*(8*c^4 + 24*c^2 + 3)
*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^
4*abs(d)))*d)*b*c*d^3 + (120*x^5*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*((2*(3*x*(4*x/d^2 - 9*c/d^3) +
(47*c^2*d^5 + 16*d^5)/d^9)*x - 7*(22*c^3*d^4 + 23*c*d^4)/d^9)*x + (274*c^4*
d^3 + 607*c^2*d^3 + 64*d^3)/d^9) + 15*(8*c^5 + 40*c^3 + 15*c)*log(abs(-c*d
- (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^5*abs(d)))*d)*
b*d^4 + 600*a*c^4*x)*e^4
```

maple [A] time = 0.02, size = 78, normalized size = 0.58

$$\frac{\frac{(dx+c)^5e^4a}{5}+e^4b\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)}{5}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(3(dx+c)^4+4(dx+c)^2+8\right)}{75}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a+e^4*b*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)*(3*(d*x+c)^4+4*(d*x+c)^2+8)))
```

maxima [B] time = 0.39, size = 1241, normalized size = 9.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*d^4*e^4*x^5 + a*c*d^3*e^4*x^4 + 2*a*c^2*d^2*e^4*x^3 + 2*a*c^3*d*e^4*x
^2 + (2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (
c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 -
 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*b*c^3*d*e^4 + 1/3*(6*x^3*arcco
sh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2
*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
```

```
*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*b*c^2*d^2*e^
4 + 1/24*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^
3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*
x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 -
 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*b*c*d^3*e^4 + 1/600*
(120*x^5*arccosh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^4/d^2 -
 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^3/d^3 + 126*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*c^2*x^2/d^4 - 945*c^5*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2
*c*d*x + c^2 - 1)*d)/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3*x/d^5
- 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x^2/d^4 + 1050*(c^2 - 1)*c
^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 + 945*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^4/d^6 + 161*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*(c^2 - 1)*c*x/d^5 - 225*(c^2 - 1)^2*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 735*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(
c^2 - 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)^2/d^6)*d)
*b*d^4*e^4 + a*c^4*e^4*x + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 -
 1))*b*c^4*e^4/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^4\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x)), x)
```

sympy [A] time = 3.25, size = 527, normalized size = 3.90

$$\begin{cases} ac^4e^4x + 2ac^3de^4x^2 + 2ac^2d^2e^4x^3 + acd^3e^4x^4 + \frac{ad^4e^4x^5}{5} + \frac{bc^5e^4\operatorname{acosh}(c+dx)}{5d} + bc^4e^4x\operatorname{acosh}(c+dx) - \frac{bc^4e^4\sqrt{c^2+2cd \ldots}}{25d} \\ c^4e^4x\left(a+b\operatorname{acosh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**4*e**4*x + 2*a*c**3*d*e**4*x**2 + 2*a*c**2*d**2*e**4*x**3 +
 a*c*d**3*e**4*x**4 + a*d**4*e**4*x**5/5 + b*c**5*e**4*acosh(c + d*x)/(5*d)
 + b*c**4*e**4*x*acosh(c + d*x) - b*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)/(25*d) + 2*b*c**3*d*e**4*x**2*acosh(c + d*x) - 4*b*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 2*b*c**2*d**2*e**4*x**3*acosh(c + d*x
) - 6*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 4*b*c**2
*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(75*d) + b*c*d**3*e**4*x**4*acos
h(c + d*x) - 4*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 -
 8*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 + b*d**4*e**4*x**5*ac
osh(c + d*x)/5 - b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 -
 4*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 - 8*b*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(
c)), True))
```

3.94 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=119

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)^3}{16d}-\frac{3be^3\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)}{32d}$$

[Out] -3/32*b*e^3*arccosh(d*x+c)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))/d-3/32*b*e^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/16*b*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d

Rubi [A] time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 100, 90, 52}

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{be^3\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)^3}{16d}-\frac{3be^3\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)}{32d}$$

Antiderivative was successfully verified.

[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x]),x]

[Out] (-3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(32*d) - (b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(16*d) - (3*b*e^3*ArcCosh[c + d*x])/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x]))/(4*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{-1+x}\sqrt{1+x}}dx,\right.}{4d}$$

$$= -\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{16d} + \frac{e^3(c+dx)^4\left(a+b\cosh\right.}{4d}$$

$$= -\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{16d} + \frac{e^3(c+dx)^4\left(a+b\cosh\right.}{4d}$$

$$= -\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{32d} - \frac{be^3\sqrt{-1+c+dx}\,(c+d}{16d}$$

$$= -\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{32d} - \frac{be^3\sqrt{-1+c+dx}\,(c+d}{16d}$$

Mathematica [A] time = 0.13, size = 115, normalized size = 0.97

$$\frac{e^3\left((c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right) - \frac{1}{4}b\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)^3 - \frac{3}{8}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+\right.\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^3*(-1/4*(b*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]) + (c + d*x)
^4*(a + b*ArcCosh[c + d*x]) - (3*b*(Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c
 + d*x] + 2*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/8))/(4*d)
```

fricas [B] time = 0.87, size = 226, normalized size = 1.90

$$\frac{8\,ad^4e^3x^4 + 32\,acd^3e^3x^3 + 48\,ac^2d^2e^3x^2 + 32\,ac^3de^3x + \left(8\,bd^4e^3x^4 + 32\,bcd^3e^3x^3 + 48\,bc^2d^2e^3x^2 + 32\,bc^3de^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*d^4*e^3*x^4 + 32*a*c*d^3*e^3*x^3 + 48*a*c^2*d^2*e^3*x^2 + 32*a*c^
3*d*e^3*x + (8*b*d^4*e^3*x^4 + 32*b*c*d^3*e^3*x^3 + 48*b*c^2*d^2*e^3*x^2 +
32*b*c^3*d*e^3*x + (8*b*c^4 - 3*b)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*
```

```
x + c^2 - 1)) - (2*b*d^3*e^3*x^3 + 6*b*c*d^2*e^3*x^2 + 3*(2*b*c^2 + b)*d*e^
3*x + (2*b*c^3 + 3*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [B] time = 4.27, size = 598, normalized size = 5.03

$$\frac{1}{96}\left(24\,ad^3x^4 + 96\,acd^2x^3 + 144\,ac^2dx^2 - 96\left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cd}}{d^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/96*(24*a*d^3*x^4 + 96*a*c*d^2*x^3 + 144*a*c^2*d*x^2 - 96*(d*(c*log(abs(-c
*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d*abs(d)) + s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)))*b*c^3 + 72*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x/d^2 - 3*c/d^3) - (2*c^2 +
1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(
d^2*abs(d)))*d)*b*c^2*d + 16*(6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(x*(2*x/d^2 - 5*c/d^3) + (11
*c^2*d + 4*d)/d^5) + 3*(2*c^3 + 3*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^3*abs(d)))*d)*b*c*d^2 + (24*x^4*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*((2*x*(3*x/d^2 - 7*c/d^3) + (26*c^2*d^3 + 9*d^3)/d^7)*x - 5*(10*c^3*d^2
 + 11*c*d^2)/d^7) - 3*(8*c^4 + 24*c^2 + 3)*log(abs(-c*d - (x*abs(d) - sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^4*abs(d)))*d)*b*d^3 + 96*a*c^3*x)
*e^3
```

maple [B] time = 0.01, size = 359, normalized size = 3.02

$$\frac{d^3x^4a\,e^3}{4} + d^2x^3ac\,e^3 + \frac{3d\,x^2a\,c^2e^3}{2} + xa\,c^3e^3 + \frac{a\,c^4e^3}{4d} + \frac{d^3\text{arccosh}\,(dx+c)\,x^4b\,e^3}{4} + d^2\text{arccosh}\,(dx+c)\,x^3bc\,e^3 + \frac{3d\,\text{arcc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/4*d^3*x^4*a*e^3+d^2*x^3*a*c*e^3+3/2*d*x^2*a*c^2*e^3+x*a*c^3*e^3+1/4/d*a*c
^4*e^3+1/4*d^3*arccosh(d*x+c)*x^4*b*e^3+d^2*arccosh(d*x+c)*x^3*b*c*e^3+3/2*
d*arccosh(d*x+c)*x^2*b*c^2*e^3+arccosh(d*x+c)*x*b*c^3*e^3+1/4/d*arccosh(d*x
+c)*b*c^4*e^3-1/16*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3*b*e^3-3/16*d*(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*b*c*e^3-3/16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)*x*b*c^2*e^3-1/16/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*b*c^3*e^3-3/32*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)*x*b*e^3-3/32/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*b*
c*e^3-3/32/d*e^3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d
*x+c+((d*x+c)^2-1)^(1/2))
```

maxima [B] time = 0.44, size = 797, normalized size = 6.70

$$\frac{1}{4}\,ad^3e^3x^4 + acd^2e^3x^3 + \frac{3}{2}\,ac^2de^3x^2 + \frac{3}{4}\left(2\,x^2\,\text{arcosh}\,(dx+c) - d\left(\frac{3\,c^2\log\left(2\,d^2x + 2\,cd + 2\,\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\,d\right.}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*e^3*x^4 + a*c*d^2*e^3*x^3 + 3/2*a*c^2*d*e^3*x^2 + 3/4*(2*x^2*arcc
osh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d
^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1)*c/d^3))*b*c^2*d*e^3 + 1/6*(6*x^3*arccosh(d*x + c) - d*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d +
 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d
*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*b*c*d^2*e^3 + 1/96*(24*x^4*a
rccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^2 - 14*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x + 2*c*d + 2*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3/d^5 - 9*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^2*log(2*d^2*x
+ 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*b*d^3*e^3 + a*c^3*e^3*x + ((d*x + c)*a
rccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b*c^3*e^3/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^3\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x)), x)
```

sympy [A] time = 1.60, size = 394, normalized size = 3.31

$$\begin{cases} ac^3e^3x + \frac{3ac^2de^3x^2}{2} + acd^2e^3x^3 + \frac{ad^3e^3x^4}{4} + \frac{bc^4e^3\operatorname{acosh}(c+dx)}{4d} + bc^3e^3x\operatorname{acosh}(c+dx) - \frac{bc^3e^3\sqrt{c^2+2cdx+d^2x^2-1}}{16d} + \frac{3bc^2d \\ c^3e^3x\left(a+b\operatorname{acosh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**3*e**3*x + 3*a*c**2*d*e**3*x**2/2 + a*c*d**2*e**3*x**3 + a*
d**3*e**3*x**4/4 + b*c**4*e**3*acosh(c + d*x)/(4*d) + b*c**3*e**3*x*acosh(c
 + d*x) - b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(16*d) + 3*b*c**
2*d*e**3*x**2*acosh(c + d*x)/2 - 3*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)/16 + b*c*d**2*e**3*x**3*acosh(c + d*x) - 3*b*c*d*e**3*x**2*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)/(32*d) + b*d**3*e**3*x**4*acosh(c + d*x)/4 - b*d**2*e**3*x**3*sqr
t(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)/32 - 3*b*e**3*acosh(c + d*x)/(32*d), Ne(d, 0)), (c**3*e**3*x*(a
 + b*acosh(c)), True))
```

3.95 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=97

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d}-\frac{be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{9d}-\frac{2be^2\sqrt{c+dx-1}\sqrt{c+dx+1}}{9d}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))/d-2/9*b*e^2*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)/d-1/9*b*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 12, 5662, 100, 74}

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d}-\frac{be^2\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^2}{9d}-\frac{2be^2\sqrt{c+dx-1}\sqrt{c+dx+1}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-2*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(9*d) - (b*e^2*Sqrt[-1 + c
+ d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(9*d) + (e^2*(c + d*x)^3*(a + b*ArcCo
sh[c + d*x]))/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\sqrt{1+x}}\,dx,\right.}{3d}$$
$$= -\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{9d} + \frac{e^2(c+dx)^3\left(a+b\cosh\right.}{3d}$$
$$= -\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{9d} + \frac{e^2(c+dx)^3\left(a+b\cosh\right.}{3d}$$
$$= -\frac{2be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{9d} - \frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+}}{9d}$$

Mathematica [A] time = 0.08, size = 71, normalized size = 0.73

$$\frac{e^2\left(\frac{1}{3}(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)-\frac{1}{9}b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(c^2+2cdx+d^2x^2+2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^2*(-1/9*(b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(2 + c^2 + 2*c*d*x + d^2
*x^2)) + ((c + d*x)^3*(a + b*ArcCosh[c + d*x]))/3))/d
```

fricas [B] time = 0.52, size = 168, normalized size = 1.73

$$\frac{3\,ad^3e^2x^3+9\,acd^2e^2x^2+9\,ac^2de^2x+3\left(bd^3e^2x^3+3\,bcd^2e^2x^2+3\,bc^2de^2x+bc^3e^2\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cd\right.}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*d^3*e^2*x^3 + 9*a*c*d^2*e^2*x^2 + 9*a*c^2*d*e^2*x + 3*(b*d^3*e^2*x
^3 + 3*b*c*d^2*e^2*x^2 + 3*b*c^2*d*e^2*x + b*c^3*e^2)*log(d*x + c + sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)) - (b*d^2*e^2*x^2 + 2*b*c*d*e^2*x + (b*c^2 + 2*b
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [B] time = 3.73, size = 406, normalized size = 4.19

$$\frac{1}{18}\left(6\,ad^2x^3+18\,acdx^2-18\left(d\left(\frac{c\log\left(\left|-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2-1}\right)|d|\right|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}}{d^2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/18*(6*a*d^2*x^3 + 18*a*c*d*x^2 - 18*(d*(c*log(abs(-c*d - (x*abs(d) - sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)/d^2) - x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c^
```

```
2 + 9*(2*x^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1)*(x/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*a
bs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*c*d
+ (6*x^3*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 +
 2*c*d*x + c^2 - 1)*(x*(2*x/d^2 - 5*c/d^3) + (11*c^2*d + 4*d)/d^5) + 3*(2*c
^3 + 3*c)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs
(d)))/(d^3*abs(d)))*d)*b*d^2 + 18*a*c^2*x)*e^2
```

maple [A] time = 0.01, size = 67, normalized size = 0.69

$$\frac{\frac{(dx+c)^3 e^2 a}{3} + e^2 b \left(\frac{\operatorname{arccosh}(dx+c)(dx+c)^3}{3} - \frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left((dx+c)^2+2\right)}{9}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a+e^2*b*(1/3*arccosh(d*x+c)*(d*x+c)^3-1/9*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2)))
```

maxima [B] time = 0.50, size = 449, normalized size = 4.63

$$\frac{1}{3}\,ad^2e^2x^3+acde^2x^2+\frac{1}{2}\left(2\,x^2\operatorname{arcosh}\left(dx+c\right)-d\left(\frac{3\,c^2\log\left(2\,d^2x+2\,cd+2\,\sqrt{d^2x^2+2\,cdx+c^2-1}\,d\right)}{d^3}+\frac{\sqrt{d^2x^2+}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*e^2*x^3 + a*c*d*e^2*x^2 + 1/2*(2*x^2*arccosh(d*x + c) - d*(3*c^2*
log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2
*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d
^3))*b*c*d*e^2 + 1/18*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 -
 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*(c^2 - 1)/d^4))*b*d^2*e^2 + a*c^2*e^2*x + ((d*x + c)*arccosh(d*x + c)
 - sqrt((d*x + c)^2 - 1))*b*c^2*e^2/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,\left(a+b\operatorname{acosh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x)), x)
```

sympy [A] time = 0.73, size = 258, normalized size = 2.66

$$\begin{cases} ac^2e^2x + acde^2x^2 + \frac{ad^2e^2x^3}{3} + \frac{bc^3e^2\operatorname{acosh}(c+dx)}{3d} + bc^2e^2x\operatorname{acosh}(c+dx) - \frac{bc^2e^2\sqrt{c^2+2cdx+d^2x^2-1}}{9d} + bcde^2x^2\operatorname{acosh}(c+ \\ c^2e^2x\left(a+b\operatorname{acosh}(c)\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c**2*e**2*x + a*c*d*e**2*x**2 + a*d**2*e**2*x**3/3 + b*c**3*e*
*2*acosh(c + d*x)/(3*d) + b*c**2*e**2*x*acosh(c + d*x) - b*c**2*e**2*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + b*c*d*e**2*x**2*acosh(c + d*x) - 2*b
*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 + b*d**2*e**2*x**3*acosh(c
 + d*x)/3 - b*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 - 2*b*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b
*acosh(c)), True))
```

3.96 $\int (ce + dex) \left(a + b \cosh^{-1}(c + dx)\right) dx$

Optimal. Leaf size=75

$$\frac{e(c+dx)^2 \left(a+b\cosh^{-1}(c+dx)\right)}{2d} - \frac{be\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{4d} - \frac{be\cosh^{-1}(c+dx)}{4d}$$

```
[Out] -1/4*b*e*arccosh(d*x+c)/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))/d-1/4*b*e*(d
*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5866, 12, 5662, 90, 52}

$$\frac{e(c+dx)^2 \left(a+b\cosh^{-1}(c+dx)\right)}{2d} - \frac{be\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{4d} - \frac{be\cosh^{-1}(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(4*d) - (b*e*ArcCosh[
c + d*x])/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x]))/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\sqrt{1+x}}dx,x,c\right.}{2d}$$
$$= -\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{4d} + \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c\right.}{2d}$$
$$= -\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{4d} - \frac{be\cosh^{-1}(c+dx)}{4d} + \frac{e(c+}{}$$

Mathematica [A] time = 0.16, size = 81, normalized size = 1.08

$$\frac{e\left(\frac{1}{2}(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)-\frac{1}{4}b\left(\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)+2\tanh^{-1}\left(\sqrt{\frac{c+dx-1}{c+dx+1}}\right)\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e*(((c + d*x)^2*(a + b*ArcCosh[c + d*x]))/2 - (b*(Sqrt[-1 + c + d*x]*(c +
d*x)*Sqrt[1 + c + d*x] + 2*ArcTanh[Sqrt[(-1 + c + d*x)/(1 + c + d*x)]]))/4)
)/d
```

fricas [A] time = 0.49, size = 110, normalized size = 1.47

$$\frac{2\,ad^2ex^2+4\,acdex+\left(2\,bd^2ex^2+4\,bcdex+\left(2\,bc^2-b\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-(bdex+bce)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*e*x^2 + 4*a*c*d*e*x + (2*b*d^2*e*x^2 + 4*b*c*d*e*x + (2*b*c^2
- b)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (b*d*e*x + b*c*e
)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [B] time = 3.72, size = 245, normalized size = 3.27

$$\frac{1}{4}\left(2\,adx^2-4\left(d\left(\frac{c\log\left(\left|-cd-\left(x|d|-\sqrt{d^2x^2+2\,cdx+c^2-1}\right)|d|\right|\right)}{d|d|}+\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}}{d^2}\right)-x\log\left(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x^2 - 4*(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))*abs(d)))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) -
x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))*b*c + (2*x^2*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*(x/d^2 - 3*c/d^3) - (2*c^2 + 1)*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1))*abs(d)))/(d^2*abs(d)))*d)*b*d + 4*a*c*x)*e
```

maple [B] time = 0.01, size = 162, normalized size = 2.16

$$\frac{ade\,x^2}{2}+xace+\frac{a\,c^2e}{2d}+\frac{d\operatorname{arccosh}(dx+c)\,x^2be}{2}+\operatorname{arccosh}(dx+c)\,xbce+\frac{\operatorname{arccosh}(dx+c)\,b\,c^2e}{2d}-\frac{\sqrt{dx+c-1}\,\sqrt{dx+}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/2*a*d*e*x^2+x*a*c*e+1/2/d*a*c^2*e+1/2*d*arccosh(d*x+c)*x^2*b*e+arccosh(d*
x+c)*x*b*c*e+1/2/d*arccosh(d*x+c)*b*c^2*e-1/4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)*x*b*e-1/4/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*b*c*e-1/4/d*e*b*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))
```

maxima [B] time = 0.31, size = 203, normalized size = 2.71

$$\frac{1}{2}\,adex^2+\frac{1}{4}\left(2\,x^2\operatorname{arcosh}(dx+c)-d\left(\frac{3\,c^2\log\left(2\,d^2x+2\,cd+2\,\sqrt{d^2x^2+2\,cdx+c^2-1}\,d\right)}{d^3}+\frac{\sqrt{d^2x^2+2\,cdx+c^2-}}{d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*e*x^2 + 1/4*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d
+ 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*b*d*e + a*c*e*x
 + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b*c*e/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\operatorname{acosh}(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x)), x)
```

sympy [A] time = 0.31, size = 148, normalized size = 1.97

$$\begin{cases} acex+\frac{adex^2}{2}+\frac{bc^2e\operatorname{acosh}(c+dx)}{2d}+bcex\operatorname{acosh}(c+dx)-\frac{bce\sqrt{c^2+2cdx+d^2x^2-1}}{4d}+\frac{bdex^2\operatorname{acosh}(c+dx)}{2}-\frac{bex\sqrt{c^2+2cdx+d^2x^2-1}}{4}- \\ cex\,(a+b\operatorname{acosh}(c)) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Piecewise((a*c*e*x + a*d*e*x**2/2 + b*c**2*e*acosh(c + d*x)/(2*d) + b*c*e*x
*acosh(c + d*x) - b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(4*d) + b*d*e*
x**2*acosh(c + d*x)/2 - b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 - b*e*
acosh(c + d*x)/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c)), True))
```

3.97 $\int \left(a+b\cosh^{-1}(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$ax-\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}$$

```
[Out] a*x+b*(d*x+c)*arccosh(d*x+c)/d-b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5864, 5654, 74}

$$ax-\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[c + d*x],x]
```

```
[Out] a*x - (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (b*(c + d*x)*ArcCosh[c +
 d*x])/d
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}(c+dx)\right)\,dx &= ax+b\int \cosh^{-1}(c+dx)\,dx\\ &= ax+\frac{b\,\text{Subst}\left(\int \cosh^{-1}(x)\,dx,x,c+dx\right)}{d}\\ &= ax+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,c+dx\right)}{d}\\ &= ax-\frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{d}+\frac{b(c+dx)\cosh^{-1}(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.33

$$ax-\frac{b\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}-2c\sinh^{-1}\left(\frac{\sqrt{c+dx-1}}{\sqrt{2}}\right)\right)}{d}+bx\cosh^{-1}(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[c + d*x],x]
```

```
[Out] a*x + b*x*ArcCosh[c + d*x] - (b*(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 2*c
*ArcSinh[Sqrt[-1 + c + d*x]/Sqrt[2]]))/d
```

fricas [A] time = 0.60, size = 65, normalized size = 1.41

$$\frac{adx + (bdx + bc)\log\left(dx + c + \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right) - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + (b*d*x + b*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b)/d
```

giac [B] time = 1.90, size = 100, normalized size = 2.17

$$-\left(d\left(\frac{c\log\left(\left|-cd - \left(x|d| - \sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\right)|d|\right|\right)}{d|d|} + \frac{\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}}{d^2}\right) - x\log\left(dx + c + \sqrt{(dx + c)^2 - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="giac")
```

```
[Out] -(d*(c*log(abs(-c*d - (x*abs(d) - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*abs(d)
))/(d*abs(d)) + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)/d^2) - x*log(d*x + c + sq
rt((d*x + c)^2 - 1)))*b + a*x
```

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$ax + \frac{b\left((dx + c)\,\mathrm{arccosh}\,(dx + c) - \sqrt{dx + c - 1}\,\sqrt{dx + c + 1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x+c),x)
```

```
[Out] a*x+b/d*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
```

maxima [A] time = 0.34, size = 35, normalized size = 0.76

$$ax + \frac{\left((dx + c)\,\mathrm{arcosh}\,(dx + c) - \sqrt{(dx + c)^2 - 1}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + ((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*b/d
```

mupad [B] time = 3.96, size = 272, normalized size = 5.91

$$a\,x+b\,x\,\mathrm{acosh}\,(c + d\,x)-\frac{b\left(\frac{4\,c\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)}{d\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)}+\frac{4\,c\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)^3}{d\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)^3}-\frac{8\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)^2\sqrt{c-1}\,\sqrt{c+1}}{d\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)^2}\right)}{\frac{\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)^4}{\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)^4}-\frac{2\left(\sqrt{c-1}-\sqrt{c+d\,x-1}\right)^2}{\left(\sqrt{c+1}-\sqrt{c+d\,x+1}\right)^2}+1}+\frac{4\,b\,c\,\mathrm{atanh}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acosh(c + d*x),x)
```

```
[Out] a*x + b*x*acosh(c + d*x) - (b*((4*c*((c - 1)^(1/2) - (c + d*x - 1)^(1/2)))/
(d*((c + 1)^(1/2) - (c + d*x + 1)^(1/2))) + (4*c*((c - 1)^(1/2) - (c + d*x
- 1)^(1/2))^3)/(d*((c + 1)^(1/2) - (c + d*x + 1)^(1/2))^3) - (8*((c - 1)^(1
/2) - (c + d*x - 1)^(1/2))^2*(c - 1)^(1/2)*(c + 1)^(1/2))/(d*((c + 1)^(1/2)
 - (c + d*x + 1)^(1/2))^2)))/(((c - 1)^(1/2) - (c + d*x - 1)^(1/2))^4/((c +
 1)^(1/2) - (c + d*x + 1)^(1/2))^4 - (2*((c - 1)^(1/2) - (c + d*x - 1)^(1/2
))^2)/((c + 1)^(1/2) - (c + d*x + 1)^(1/2))^2 + 1) + (4*b*c*atanh(((c - 1)^
(1/2) - (c + d*x - 1)^(1/2))/((c + 1)^(1/2) - (c + d*x + 1)^(1/2))))/d
```

sympy [A] time = 0.15, size = 51, normalized size = 1.11

$$ax + b\left(\begin{cases} \frac{c \operatorname{acosh}(c+dx)}{d} + x \operatorname{acosh}(c+dx) - \frac{\sqrt{c^2+2cdx+d^2x^2-1}}{d} & \text{for } d \neq 0 \\ x \operatorname{acosh}(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((c*acosh(c + d*x)/d + x*acosh(c + d*x) - sqrt(c**2 + 2*c*
d*x + d**2*x**2 - 1)/d, Ne(d, 0)), (x*acosh(c), True))
```

3.98 $\int \frac{a+b\cosh^{-1}(c+dx)}{ce+dex}\,dx$

Optimal. Leaf size=81

$$\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{b\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)}{2de}$$

```
[Out] 1/2*(a+b*arccosh(d*x+c))^2/b/d/e+(a+b*arccosh(d*x+c))*ln(1+1/(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-1/2*b*polylog(2,-1/(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2))^2)/d/e
```

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5866, 12, 5660, 3718, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)}{2de}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\log\left(e^{2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcCosh[c + d*x])^2/(2*b*d*e) + ((a + b*ArcCosh[c + d*x])*Log[1 + E
^(2*ArcCosh[c + d*x])])/(d*e) + (b*PolyLog[2, -E^(2*ArcCosh[c + d*x])])/(2*
d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{ex}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x}\,dx, x, c+dx\right)}{de}\\
&= \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}(c+dx)\right)}{de}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}-\frac{b\,\text{S}}{}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}-\frac{b\,\text{S}}{}\\
&= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2bde}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de}+\frac{b\text{L}}{}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 69, normalized size = 0.85

$$\frac{2a\log(c+dx)-b\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)+b\cosh^{-1}(c+dx)^2+2b\cosh^{-1}(c+dx)\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)}{2de}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x),x]
```

```
[Out] (b*ArcCosh[c + d*x]^2 + 2*b*ArcCosh[c + d*x]*Log[1 + E^(-2*ArcCosh[c + d*x]
)] + 2*a*Log[c + d*x] - b*PolyLog[2, -E^(-2*ArcCosh[c + d*x])])/(2*d*e)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(dx+c)+a}{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(dx+c)+a}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

maple [A] time = 0.09, size = 111, normalized size = 1.37

$$\frac{a\ln(dx+c)}{de}-\frac{b\,\mathrm{arccosh}(dx+c)^2}{2de}+\frac{b\,\mathrm{arccosh}(dx+c)\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{de}+\frac{b\,\mathrm{polylog}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] 1/d*a/e*ln(d*x+c)-1/2/d*b/e*arccosh(d*x+c)^2+1/d*b/e*arccosh(d*x+c)*ln(1+(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+1/2/d*b/e*polylog(2,-(d*x+c+(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)}{dex+ce}\,dx+\frac{a\log(dex+ce)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] b*integrate(log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d*e*x + c*e
), x) + a*log(d*e*x + c*e)/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{acosh}(c+d\,x)}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c+dx}\,dx+\int\frac{b\,\mathrm{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e),x)
```

```
[Out] (Integral(a/(c + d*x), x) + Integral(b*acosh(c + d*x)/(c + d*x), x))/e
```

3.99 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^2}\,dx$

Optimal. Leaf size=56

$$\frac{b\tan^{-1}\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\right)}{de^2}-\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}$$

```
[Out] (-a-b*arccosh(d*x+c))/d/e^2/(d*x+c)+b*arctan((d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))/d/e^2
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 12, 5662, 92, 203}

$$\frac{b\tan^{-1}\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\right)}{de^2}-\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])/(d*e^2*(c + d*x))) + (b*ArcTan[Sqrt[-1 + c + d*x
]*Sqrt[1 + c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^2x^2}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^2}\,dx,x,c+dx\right)}{de^2}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx,x,c+dx\right)}{de^2}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)}{de^2}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{de^2(c+dx)} + \frac{b\tan^{-1}\left(\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\right)}{de^2}$$

Mathematica [A] time = 0.14, size = 78, normalized size = 1.39

$$\frac{\frac{-a-b\cosh^{-1}(c+dx)}{c+dx}+\frac{b\sqrt{(c+dx)^2-1}\,\tan^{-1}\left(\sqrt{(c+dx)^2-1}\right)}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}}{de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^2,x]
```

```
[Out] ((-a - b*ArcCosh[c + d*x])/(c + d*x) + (b*Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqr
t[-1 + (c + d*x)^2]])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(d*e^2)
```

fricas [B] time = 0.82, size = 133, normalized size = 2.38

$$\frac{bdx\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-ac+2\left(bcdx+bc^2\right)\arctan\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+(b}{cd^2e^2x+c^2de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] (b*d*x*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - a*c + 2*(b*c*d*x
+ b*c^2)*arctan(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + (b*d*x + b*
c)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)))/(c*d^2*e^2*x + c^2*d*
e^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.01, size = 88, normalized size = 1.57

$$-\frac{a}{d\,e^{2}\left(dx+c\right)}-\frac{b\,\mathrm{arccosh}\left(dx+c\right)}{d\,e^{2}\left(dx+c\right)}-\frac{b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{\left(dx+c\right)^{2}-1}}\right)}{d\,e^{2}\sqrt{\left(dx+c\right)^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a/e^2/(d*x+c)-1/d*b/e^2/(d*x+c)*arccosh(d*x+c)-1/d*b/e^2*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2))
```

maxima [A] time = 0.55, size = 80, normalized size = 1.43

$$-b\left(\frac{\mathrm{arcosh}\left(dx+c\right)}{d^{2}e^{2}x+cde^{2}}+\frac{\arcsin\left(\frac{de^{2}}{\left|d^{2}e^{2}x+cde^{2}\right|}\right)}{de^{2}}\right)-\frac{a}{d^{2}e^{2}x+cde^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b*(arccosh(d*x + c)/(d^2*e^2*x + c*d*e^2) + arcsin(d*e^2/abs(d^2*e^2*x + c
*d*e^2))/(d*e^2)) - a/(d^2*e^2*x + c*d*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{acosh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^{2}+2cdx+d^{2}x^{2}}\, dx+\int \frac{b\,\mathrm{acosh}\left(c+dx\right)}{c^{2}+2cdx+d^{2}x^{2}}\, dx}{e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b*acosh(c + d*x)/(c
**2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.100 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^3}\,dx$

Optimal. Leaf size=66

$$\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{2de^3(c+dx)}-\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

```
[Out] 1/2*(-a-b*arccosh(d*x+c))/d/e^3/(d*x+c)^2+1/2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)/d/e^3/(d*x+c)
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5866, 12, 5662, 95}

$$\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{2de^3(c+dx)}-\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(2*d*e^3*(c + d*x)) - (a + b*ArcCo
sh[c + d*x])/(2*d*e^3*(c + d*x)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^3x^3}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^3}\,dx, x, c+dx\right)}{de^3}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx, x, c+dx\right)}{2de^3}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{2de^3(c+dx)} - \frac{a+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.83

$$-\frac{a-b\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,+b\cosh^{-1}(c+dx)}{2de^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^3,x]
```

```
[Out] -1/2*(a - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] + b*ArcCosh[c +
d*x])/(d*e^3*(c + d*x)^2)
```

fricas [B] time = 0.47, size = 117, normalized size = 1.77

$$\frac{ad^2x^2+2\,acdx-bc^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\left(bc^2dx+bc^3\right)\sqrt{d^2x^2+2\,cdx+c^2-1}}{2\left(c^2d^3e^3x^2+2\,c^3d^2e^3x+c^4de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*x^2 + 2*a*c*d*x - b*c^2*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)) + (b*c^2*d*x + b*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/(c^2*d^3*
e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arcosh}\,(dx+c)+a}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^3, x)
```

maple [A] time = 0.01, size = 65, normalized size = 0.98

$$\frac{-\frac{a}{2e^3(dx+c)^2}+\frac{b\left(-\frac{\mathrm{arccosh}(dx+c)}{2(dx+c)^2}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2dx+2c}\right)}{e^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x)
```

```
[Out] 1/d*(-1/2*a/e^3/(d*x+c)^2+b/e^3*(-1/2/(d*x+c)^2*arccosh(d*x+c)+1/2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)))
```

maxima [B] time = 0.62, size = 118, normalized size = 1.79

$$\frac{1}{2} b \left(\frac{\sqrt{d^2x^2 + 2\,cdx + c^2 - 1}\,d}{d^3e^3x + cd^2e^3} - \frac{\operatorname{arcosh}(dx + c)}{d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3} \right) - \frac{a}{2\left(d^3e^3x^2 + 2\,cd^2e^3x + c^2de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*e^3*x + c*d^2*e^3) - arccos
h(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 1/2*a/(d^3*e^3*x^2
+ 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\operatorname{acosh}(c + d\,x)}{(c\,e + d\,e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b\operatorname{acosh}(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(
b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.101 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^4}\,dx$

Optimal. Leaf size=99

$$-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{6de^4(c+dx)^2}+\frac{b\tan^{-1}\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\right)}{6de^4}$$

```
[Out] 1/3*(-a-b*arccosh(d*x+c))/d/e^4/(d*x+c)^3+1/6*b*arctan((d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2))/d/e^4+1/6*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 103, 92, 203}

$$-\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{6de^4(c+dx)^2}+\frac{b\tan^{-1}\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\right)}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(6*d*e^4*(c + d*x)^2) - (a + b*Arc
Cosh[c + d*x])/(3*d*e^4*(c + d*x)^3) + (b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]])/(6*d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
```

```
NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^4x^4}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^4}\,dx, x, c+dx\right)}{de^4}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\,dx, x, c+dx\right)}{3de^4}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^4(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx, x\right.}{6de^4}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^4(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\right.}{6de^4}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^4(c+dx)^2} - \frac{a+b\cosh^{-1}(c+dx)}{3de^4(c+dx)^3} + \frac{b\tan^{-1}\left(\sqrt{-1+c+dx}\,\sqrt{1+\right.}{6de^4}$$

Mathematica [A] time = 0.23, size = 101, normalized size = 1.02

$$\frac{\frac{b\left(\frac{(c+dx-1)(c+dx+1)}{(c+dx)^2}+\sqrt{(c+dx)^2-1}\tan^{-1}\left(\sqrt{(c+dx)^2-1}\right)\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{(c+dx)^3}}{6de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*(a + b*ArcCosh[c + d*x]))/(c + d*x)^3 + (b*(((-1 + c + d*x)*(1 + c + d
*x))/(c + d*x)^2 + Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqrt[-1 + (c + d*x)^2]]))/
(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]))/(6*d*e^4)
```

fricas [B] time = 0.98, size = 276, normalized size = 2.79

$$-\frac{2\,ac^3-2\left(bc^3d^3x^3+3\,bc^4d^2x^2+3\,bc^5dx+bc^6\right)\arctan\left(-dx-c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)-2\left(bd^3x^3+3\,bcd^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*c^3 - 2*(b*c^3*d^3*x^3 + 3*b*c^4*d^2*x^2 + 3*b*c^5*d*x + b*c^6)*a
rctan(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2
*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(-d*x - c + sqrt(d^2*
```

```
x^2 + 2*c*d*x + c^2 - 1)) - (b*c^3*d*x + b*c^4)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1))/(c^3*d^4*e^4*x^3 + 3*c^4*d^3*e^4*x^2 + 3*c^5*d^2*e^4*x + c^6*d*e^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(dx+c)+a}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^4, x)
```

maple [A] time = 0.01, size = 120, normalized size = 1.21

$$-\frac{a}{3d\,e^4\,(dx+c)^3}-\frac{b \operatorname{arccosh}(dx+c)}{3d\,e^4\,(dx+c)^3}-\frac{b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{6d\,e^4\sqrt{(dx+c)^2-1}}+\frac{b\sqrt{dx+c-1}\,\sqrt{dx+c}}{6d\,e^4\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a/e^4/(d*x+c)^3-1/3/d*b/e^4/(d*x+c)^3*arccosh(d*x+c)-1/6/d*b/e^4*(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/
2))+1/6*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}b\left(\frac{2\,d^2x^2+4\,cdx+2\,c^2-\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\log(dx+c+1)+\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\mathrm{lo}}{d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*((2*d^2*x^2 + 4*c*d*x + 2*c^2 - (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3)*log(d*x + c + 1) + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*log(d*x +
 c - 1) - 2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^4*e^4*x^
3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 6*integrate(1/3/(d^6*e
^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x
^4 + 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*
x^2 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x + (d^5*e^4*x^5 + 5*c*d^4*e^4*x^4 + c^
5*e^4 - c^3*e^4 + (10*c^2*d^3*e^4 - d^3*e^4)*x^3 + (10*c^3*d^2*e^4 - 3*c*d^
2*e^4)*x^2 + (5*c^4*d*e^4 - 3*c^2*d*e^4)*x)*e^(1/2*log(d*x + c + 1) + 1/2*l
og(d*x + c - 1))), x)) - 1/3*a/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e
^4*x + c^3*d*e^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b\,\mathrm{acosh}\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x*
*4), x) + Integral(b*acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.102 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^5}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{6de^5(c+dx)}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{12de^5(c+dx)^3}$$

```
[Out] 1/4*(-a-b*arccosh(d*x+c))/d/e^5/(d*x+c)^4+1/12*b*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)/d/e^5/(d*x+c)^3+1/6*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^5/(d*x+c)
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 12, 5662, 103, 95}

$$-\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{6de^5(c+dx)}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{12de^5(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(12*d*e^5*(c + d*x)^3) + (b*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(6*d*e^5*(c + d*x)) - (a + b*ArcCosh[c + d*
x])/(4*d*e^5*(c + d*x)^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^5}\,dx &= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^5x^5}\,dx, x, c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^5}\,dx, x, c+dx\right)}{de^5}\\
&= -\frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^4\sqrt{1+x}}\,dx, x, c+dx\right)}{4de^5}\\
&= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{12de^5(c+dx)^3} - \frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx,\right.}{12de^5}\\
&= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{12de^5(c+dx)^3} - \frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx,\right.}{6de^5}\\
&= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{12de^5(c+dx)^3} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{6de^5(c+dx)} - \frac{a+b\cosh^{-1}(c+dx)}{4de^5(c+dx)^4}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 86, normalized size = 0.83

$$\frac{-3a+b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(2c^3+6c^2dx+6cd^2x^2+c+2d^3x^3+dx\right)-3b\cosh^{-1}(c+dx)}{12de^5(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^5,x]
```

```
[Out] (-3*a + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(c + 2*c^3 + d*x + 6*c^2*d*x
 + 6*c*d^2*x^2 + 2*d^3*x^3) - 3*b*ArcCosh[c + d*x])/(12*d*e^5*(c + d*x)^4)
```

fricas [B] time = 0.51, size = 208, normalized size = 2.00

$$\frac{3\,ad^4x^4+12\,acd^3x^3+18\,ac^2d^2x^2+12\,ac^3dx-3\,bc^4\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)+\left(2\,bc^4d^3x^3+6\,bc^5d\right.}{12\left(c^4d^5e^5x^4+4\,c^5d^4e^5x^3+6\,c^6d^3e^5x^2+4\,c^7d^2e^5x+c^8d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*d^4*x^4 + 12*a*c*d^3*x^3 + 18*a*c^2*d^2*x^2 + 12*a*c^3*d*x - 3*b*
c^4*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + (2*b*c^4*d^3*x^3 + 6
*b*c^5*d^2*x^2 + 2*b*c^7 + b*c^5 + (6*b*c^6 + b*c^4)*d*x)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))/(c^4*d^5*e^5*x^4 + 4*c^5*d^4*e^5*x^3 + 6*c^6*d^3*e^5*x^2
+ 4*c^7*d^2*e^5*x + c^8*d*e^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.01, size = 76, normalized size = 0.73

$$\frac{-\frac{a}{4e^5(dx+c)^4}+\frac{b\left(-\frac{\operatorname{arccosh}(dx+c)}{4(dx+c)^4}+\frac{\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\left(2(dx+c)^2+1\right)}{12(dx+c)^3}\right)}{e^5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x)
```

```
[Out] 1/d*(-1/4*a/e^5/(d*x+c)^4+b/e^5*(-1/4/(d*x+c)^4*arccosh(d*x+c)+1/12*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*(2*(d*x+c)^2+1)/(d*x+c)^3))
```

maxima [B] time = 0.56, size = 260, normalized size = 2.50

$$\frac{1}{12}\,b\left(\frac{\left(2\,d^4x^4+8\,cd^3x^3+2\,c^4+\left(12\,c^2d^2-d^2\right)x^2-c^2+2\left(4\,c^3d-cd\right)x-1\right)d}{\left(d^5e^5x^3+3\,cd^4e^5x^2+3\,c^2d^3e^5x+c^3d^2e^5\right)\sqrt{dx+c+1}\,\sqrt{dx+c-1}}-\frac{3\,\operatorname{arcosh}(dx}{d^5e^5x^4+4\,cd^4e^5x^3+6\,c^2d^3e^5x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b*((2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 + (12*c^2*d^2 - d^2)*x^2 - c^2 + 2
*(4*c^3*d - c*d)*x - 1)*d/((d^5*e^5*x^3 + 3*c*d^4*e^5*x^2 + 3*c^2*d^3*e^5*x
 + c^3*d^2*e^5)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)) - 3*arccosh(d*x + c)/(
d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d^2*e^5*x + c^4*d
*e^5)) - 1/4*a/(d^5*e^5*x^4 + 4*c*d^4*e^5*x^3 + 6*c^2*d^3*e^5*x^2 + 4*c^3*d
^2*e^5*x + c^4*d*e^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^5,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx+\int \frac{b\,\mathrm{acosh}\,(c+dx)}{c^5+5c^4dx+10c^3d^2x^2+10c^2d^3x^3+5cd^4x^4+d^5x^5}\,dx}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**5,x)
```

```
[Out] (Integral(a/(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*
c*d**4*x**4 + d**5*x**5), x) + Integral(b*acosh(c + d*x)/(c**5 + 5*c**4*d*x
 + 10*c**3*d**2*x**2 + 10*c**2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5), x))/
e**5
```

3.103 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^6}\,dx$

Optimal. Leaf size=137

$$-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{40de^6(c+dx)^2}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{20de^6(c+dx)^4}+\frac{3b\tan^{-1}\left(\sqrt{c+dx-1}\,\sqrt{c+dx}\right.}{40de^6}$$

```
[Out] 1/5*(-a-b*arccosh(d*x+c))/d/e^6/(d*x+c)^5+3/40*b*arctan((d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2))/d/e^6+1/20*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^4+
3/40*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^2
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 103, 92, 203}

$$-\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5}+\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{40de^6(c+dx)^2}+\frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{20de^6(c+dx)^4}+\frac{3b\tan^{-1}\left(\sqrt{c+dx-1}\,\sqrt{c+dx}\right.}{40de^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(20*d*e^6*(c + d*x)^4) + (3*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x])/(40*d*e^6*(c + d*x)^2) - (a + b*ArcCosh[c
 + d*x])/(5*d*e^6*(c + d*x)^5) + (3*b*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x]])/(40*d*e^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^6}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{e^6x^6}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x^6}\,dx, x, c+dx\right)}{de^6}$$
$$= -\frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^5\sqrt{1+x}}\,dx, x, c+dx\right)}{5de^6}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4} - \frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{b\,\text{Subst}\left(\int \frac{3}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\right.}{20de^6}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4} - \frac{a+b\cosh^{-1}(c+dx)}{5de^6(c+dx)^5} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^3\sqrt{1-}}\right.}{20de^6}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{40de^6(c+dx)^2} - \frac{a+b\cosh^{-1}(c+}{5de^6(c+dx)^5}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{40de^6(c+dx)^2} - \frac{a+b\cosh^{-1}(c+}{5de^6(c+dx)^5}$$
$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{20de^6(c+dx)^4} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{40de^6(c+dx)^2} - \frac{a+b\cosh^{-1}(c+}{5de^6(c+dx)^5}$$

Mathematica [A] time = 0.23, size = 136, normalized size = 0.99

$$\frac{-\frac{a+b\cosh^{-1}(c+dx)}{(c+dx)^5} + \frac{b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{4(c+dx)^4} + \frac{3b\left(\frac{(c+dx-1)(c+dx+1)}{(c+dx)^2} + \sqrt{(c+dx)^2-1}\,\tan^{-1}\left(\sqrt{(c+dx)^2-1}\right)\right)}{8\sqrt{c+dx-1}\,\sqrt{c+dx+1}}}{5de^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^6,x]
```

```
[Out] ((b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(4*(c + d*x)^4) - (a + b*ArcCosh[
c + d*x])/(c + d*x)^5 + (3*b*(((-1 + c + d*x)*(1 + c + d*x))/(c + d*x)^2 +
Sqrt[-1 + (c + d*x)^2]*ArcTan[Sqrt[-1 + (c + d*x)^2]]))/(8*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x]))/(5*d*e^6)
```

fricas [B] time = 0.78, size = 416, normalized size = 3.04

$$-\frac{8\,ac^5 - 6\left(bc^5d^5x^5 + 5\,bc^6d^4x^4 + 10\,bc^7d^3x^3 + 10\,bc^8d^2x^2 + 5\,bc^9dx + bc^{10}\right)\arctan\left(-dx - c + \sqrt{d^2x^2 + 2\,cdx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="fricas")
```

```
[Out] -1/40*(8*a*c^5 - 6*(b*c^5*d^5*x^5 + 5*b*c^6*d^4*x^4 + 10*b*c^7*d^3*x^3 + 10
*b*c^8*d^2*x^2 + 5*b*c^9*d*x + b*c^10)*arctan(-d*x - c + sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)) - 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c
^3*d^2*x^2 + 5*b*c^4*d*x)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))
- 8*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3 + 10*b*c^3*d^2*x^2 + 5*b*
c^4*d*x + b*c^5)*log(-d*x - c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (3*b*c
^5*d^3*x^3 + 9*b*c^6*d^2*x^2 + 3*b*c^8 + 2*b*c^6 + (9*b*c^7 + 2*b*c^5)*d*x)
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/(c^5*d^6*e^6*x^5 + 5*c^6*d^5*e^6*x^4 +
10*c^7*d^4*e^6*x^3 + 10*c^8*d^3*e^6*x^2 + 5*c^9*d^2*e^6*x + c^10*d*e^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(dx+c)+a}{(dex+ce)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^6, x)
```

maple [A] time = 0.01, size = 152, normalized size = 1.11

$$-\frac{a}{5d\,e^6\,(dx+c)^5}-\frac{b\operatorname{arccosh}(dx+c)}{5d\,e^6\,(dx+c)^5}-\frac{3b\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,\arctan\left(\frac{1}{\sqrt{(dx+c)^2-1}}\right)}{40d\,e^6\sqrt{(dx+c)^2-1}}+\frac{3b\sqrt{dx+c-1}\,\sqrt{dx+c}}{40d\,e^6\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x)
```

```
[Out] -1/5/d*a/e^6/(d*x+c)^5-1/5/d*b/e^6/(d*x+c)^5*arccosh(d*x+c)-3/40/d*b/e^6*(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1
/2))+3/40*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^2+1/20*b*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/d/e^6/(d*x+c)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{30}b\left(\frac{6\,d^4x^4+24\,cd^3x^3+6\,c^4+2\left(18\,c^2d^2+d^2\right)x^2+2\,c^2+4\left(6\,c^3d+cd\right)x-3\left(d^5x^5+5\,cd^4x^4+10\,c^2d^3x^3+10\,c\right.}{d^6e^6x^5+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^6,x, algorithm="maxima")
```

```
[Out] 1/30*b*((6*d^4*x^4 + 24*c*d^3*x^3 + 6*c^4 + 2*(18*c^2*d^2 + d^2)*x^2 + 2*c^
2 + 4*(6*c^3*d + c*d)*x - 3*(d^5*x^5 + 5*c*d^4*x^4 + 10*c^2*d^3*x^3 + 10*c^
3*d^2*x^2 + 5*c^4*d*x + c^5)*log(d*x + c + 1) + 3*(d^5*x^5 + 5*c*d^4*x^4 +
10*c^2*d^3*x^3 + 10*c^3*d^2*x^2 + 5*c^4*d*x + c^5)*log(d*x + c - 1) - 6*log
(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^6*e^6*x^5 + 5*c*d^5*e^6
*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^2 + 5*c^4*d^2*e^6*x + c^5*d*e^
6) - 30*integrate(1/5/(d^8*e^6*x^8 + 8*c*d^7*e^6*x^7 + c^8*e^6 - c^6*e^6 +
(28*c^2*d^6*e^6 - d^6*e^6)*x^6 + 2*(28*c^3*d^5*e^6 - 3*c*d^5*e^6)*x^5 + 5*(
14*c^4*d^4*e^6 - 3*c^2*d^4*e^6)*x^4 + 4*(14*c^5*d^3*e^6 - 5*c^3*d^3*e^6)*x^
```

```
3 + (28*c^6*d^2*e^6 - 15*c^4*d^2*e^6)*x^2 + 2*(4*c^7*d*e^6 - 3*c^5*d*e^6)*x
 + (d^7*e^6*x^7 + 7*c*d^6*e^6*x^6 + c^7*e^6 - c^5*e^6 + (21*c^2*d^5*e^6 - d
^5*e^6)*x^5 + 5*(7*c^3*d^4*e^6 - c*d^4*e^6)*x^4 + 5*(7*c^4*d^3*e^6 - 2*c^2*
d^3*e^6)*x^3 + (21*c^5*d^2*e^6 - 10*c^3*d^2*e^6)*x^2 + (7*c^6*d*e^6 - 5*c^4
*d*e^6)*x)*e^(1/2*log(d*x + c + 1) + 1/2*log(d*x + c - 1))), x)) - 1/5*a/(d
^6*e^6*x^5 + 5*c*d^5*e^6*x^4 + 10*c^2*d^4*e^6*x^3 + 10*c^3*d^3*e^6*x^2 + 5*
c^4*d^2*e^6*x + c^5*d*e^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^6,x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^{6} + 6 c^{5} d x + 15 c^{4} d^{2} x^{2} + 20 c^{3} d^{3} x^{3} + 15 c^{2} d^{4} x^{4} + 6 c d^{5} x^{5} + d^{6} x^{6}}\, dx + \int \frac{b \operatorname{acosh}{\left(c + d x \right)}}{c^{6} + 6 c^{5} d x + 15 c^{4} d^{2} x^{2} + 20 c^{3} d^{3} x^{3} + 15 c^{2} d^{4} x^{4} + 6 c d^{5} x^{5} + d^{6} x^{6}}\, dx}{e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**6,x)
```

```
[Out] (Integral(a/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15
*c**2*d**4*x**4 + 6*c*d**5*x**5 + d**6*x**6), x) + Integral(b*acosh(c + d*x
)/(c**6 + 6*c**5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4
*x**4 + 6*c*d**5*x**5 + d**6*x**6), x))/e**6
```

3.104 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=218

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{25d}-\frac{8be^4\sqrt{c+dx}}{}$$

```
[Out] 16/75*b^2*e^4*x+8/225*b^2*e^4*(d*x+c)^3/d+2/125*b^2*e^4*(d*x+c)^5/d+1/5*e^4
*(d*x+c)^5*(a+b*arccosh(d*x+c))^2/d-16/75*b*e^4*(a+b*arccosh(d*x+c))*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/d-8/75*b*e^4*(d*x+c)^2*(a+b*arccosh(d*x+c))*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)/d-2/25*b*e^4*(d*x+c)^4*(a+b*arccosh(d*x+c))*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.49, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5866, 12, 5662, 5759, 5718, 8, 30}

$$\frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^2}{5d}-\frac{2be^4\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{25d}-\frac{8be^4\sqrt{c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (16*b^2*e^4*x)/75 + (8*b^2*e^4*(c + d*x)^3)/(225*d) + (2*b^2*e^4*(c + d*x)^
5)/(125*d) - (16*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[
c + d*x]))/(75*d) - (8*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*
x]*(a + b*ArcCosh[c + d*x]))/(75*d) - (2*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)
^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(25*d) + (e^4*(c + d*x)^5*(a
 + b*ArcCosh[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
```

```
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^2}{5d} - \frac{\left(2be^4\right)\text{Subst}\left(\int \frac{x^5\left(a+b\cosh^{-1}\right.}{\sqrt{-1+x}\,\sqrt{1+}}\right.}{5d}$$

$$= -\frac{2be^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{25d} -$$

$$= \frac{2b^2e^4(c+dx)^5}{125d} - \frac{8be^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\,\mathrm{c}\right.}{75d}$$

$$= \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{-1+c+dx}\,\sqrt{1+c+d}}{75d}$$

$$= \frac{16}{75}b^2e^4x + \frac{8b^2e^4(c+dx)^3}{225d} + \frac{2b^2e^4(c+dx)^5}{125d} - \frac{16be^4\sqrt{-1+c+dx}}{}$$

Mathematica [A] time = 0.34, size = 220, normalized size = 1.01

$$\frac{e^4\left(9\left(25a^2+2b^2\right)(c+dx)^5+30ab\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(-3(c+dx)^4-4(c+dx)^2-8\right)+30b\cosh^{-1}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^4*(240*b^2*(c + d*x) + 40*b^2*(c + d*x)^3 + 9*(25*a^2 + 2*b^2)*(c + d*x)
^5 + 30*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-8 - 4*(c + d*x)^2 - 3*(c
```

```
 + d*x)^4) + 30*b*(15*a*(c + d*x)^5 - 8*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d
*x] - 4*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] - 3*b*Sqrt[-1 +
c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 225*b^2*(c + d*x
)^5*ArcCosh[c + d*x]^2))/(1125*d)
```

fricas [B] time = 0.66, size = 618, normalized size = 2.83

$$9\left(25\,a^2+2\,b^2\right)d^5e^4x^5+45\left(25\,a^2+2\,b^2\right)cd^4e^4x^4+10\left(9\left(25\,a^2+2\,b^2\right)c^2+4\,b^2\right)d^3e^4x^3+30\left(3\left(25\,a^2+2\,b^2\right)c^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(9*(25*a^2 + 2*b^2)*d^5*e^4*x^5 + 45*(25*a^2 + 2*b^2)*c*d^4*e^4*x^4
+ 10*(9*(25*a^2 + 2*b^2)*c^2 + 4*b^2)*d^3*e^4*x^3 + 30*(3*(25*a^2 + 2*b^2)*
c^3 + 4*b^2*c)*d^2*e^4*x^2 + 15*(3*(25*a^2 + 2*b^2)*c^4 + 8*b^2*c^2 + 16*b^
2)*d*e^4*x + 225*(b^2*d^5*e^4*x^5 + 5*b^2*c*d^4*e^4*x^4 + 10*b^2*c^2*d^3*e^
4*x^3 + 10*b^2*c^3*d^2*e^4*x^2 + 5*b^2*c^4*d*e^4*x + b^2*c^5*e^4)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 30*(15*a*b*d^5*e^4*x^5 + 75*a*b
*c*d^4*e^4*x^4 + 150*a*b*c^2*d^3*e^4*x^3 + 150*a*b*c^3*d^2*e^4*x^2 + 75*a*b
*c^4*d*e^4*x + 15*a*b*c^5*e^4 - (3*b^2*d^4*e^4*x^4 + 12*b^2*c*d^3*e^4*x^3 +
 2*(9*b^2*c^2 + 2*b^2)*d^2*e^4*x^2 + 4*(3*b^2*c^3 + 2*b^2*c)*d*e^4*x + (3*b
^2*c^4 + 4*b^2*c^2 + 8*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 30*(3*a*b*d^4*e^4*x^4 + 12*a*b*
c*d^3*e^4*x^3 + 2*(9*a*b*c^2 + 2*a*b)*d^2*e^4*x^2 + 4*(3*a*b*c^3 + 2*a*b*c)
*d*e^4*x + (3*a*b*c^4 + 4*a*b*c^2 + 8*a*b)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^4(b\operatorname{arcosh}(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arccosh(d*x + c) + a)^2, x)
```

maple [A] time = 0.04, size = 218, normalized size = 1.00

$$\frac{(dx+c)^5e^4a^2}{5}+e^4b^2\left(\frac{(dx+c)^5\operatorname{arccosh}(dx+c)^2}{5}-\frac{16\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{75}-\frac{2(dx+c)^4\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{25}-\frac{8\operatorname{arccc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a^2+e^4*b^2*(1/5*(d*x+c)^5*arccosh(d*x+c)^2-16/75*ar
ccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/25*(d*x+c)^4*arccosh(d*x+c)*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-8/75*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2)*(d*x+c)^2+16/75*d*x+16/75*c+2/125*(d*x+c)^5+8/225*(d*x+c)^3)+2*e^4
*a*b*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(3*
(d*x+c)^4+4*(d*x+c)^2+8)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*d^4*e^4*x^5 + a^2*c*d^3*e^4*x^4 + 2*a^2*c^2*d^2*e^4*x^3 + 2*a^2*c^3
*d*e^4*x^2 + 2*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*c^3*d*e^4 + 2/3*
(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 -
15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*
c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*
a*b*c^2*d^2*e^4 + 1/12*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*
c^4*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x +
 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/
d^4 + 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a*b*c*
d^3*e^4 + 1/300*(120*x^5*arccosh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^3/d^3 + 126*sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x^2/d^4 - 945*c^5*log(2*d^2*x + 2*c*d + 2*
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^2
 - 1)*c^3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x^2/d^4 +
1050*(c^2 - 1)*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^6 + 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^4/d^6 + 161*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c*x/d^5 - 225*(c^2 - 1)^2*c*log(2*d^2*x + 2
*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 735*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*(c^2 - 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^
2 - 1)^2/d^6)*d)*a*b*d^4*e^4 + a^2*c^4*e^4*x + 2*((d*x + c)*arccosh(d*x + c
) - sqrt((d*x + c)^2 - 1))*a*b*c^4*e^4/d + 1/5*(b^2*d^4*e^4*x^5 + 5*b^2*c*d
^3*e^4*x^4 + 10*b^2*c^2*d^2*e^4*x^3 + 10*b^2*c^3*d*e^4*x^2 + 5*b^2*c^4*e^4*
x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - integrate(2/5*(b^
2*d^7*e^4*x^7 + 7*b^2*c*d^6*e^4*x^6 + (21*c^2*d^5*e^4 - d^5*e^4)*b^2*x^5 +
5*(7*c^3*d^4*e^4 - c*d^4*e^4)*b^2*x^4 + 5*(7*c^4*d^3*e^4 - 2*c^2*d^3*e^4)*b
^2*x^3 + 10*(2*c^5*d^2*e^4 - c^3*d^2*e^4)*b^2*x^2 + 5*(c^6*d*e^4 - c^4*d*e^
4)*b^2*x + (b^2*d^6*e^4*x^6 + 6*b^2*c*d^5*e^4*x^5 + 15*b^2*c^2*d^4*e^4*x^4
+ 20*b^2*c^3*d^3*e^4*x^3 + 15*b^2*c^4*d^2*e^4*x^2 + 5*b^2*c^5*d*e^4*x)*sqrt
(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^2, x)
```

sympy [A] time = 7.03, size = 1268, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**4*e**4*x + 2*a**2*c**3*d*e**4*x**2 + 2*a**2*c**2*d**2*e*
*4*x**3 + a**2*c*d**3*e**4*x**4 + a**2*d**4*e**4*x**5/5 + 2*a*b*c**5*e**4*a
cosh(c + d*x)/(5*d) + 2*a*b*c**4*e**4*x*acosh(c + d*x) - 2*a*b*c**4*e**4*sq
rt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 4*a*b*c**3*d*e**4*x**2*acosh(c
+ d*x) - 8*a*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 4*a*b*
c**2*d**2*e**4*x**3*acosh(c + d*x) - 12*a*b*c**2*d*e**4*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/25 - 8*a*b*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/(75*d) + 2*a*b*c*d**3*e**4*x**4*acosh(c + d*x) - 8*a*b*c*d**2*e**4*x*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 16*a*b*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/75 + 2*a*b*d**4*e**4*x**5*acosh(c + d*x)/5 - 2*a*b*d
**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 8*a*b*d*e**4*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/75 - 16*a*b*e**4*sqrt(c**2 + 2*c*d*x +
d**2*x**2 - 1)/(75*d) + b**2*c**5*e**4*acosh(c + d*x)**2/(5*d) + b**2*c**4*
e**4*x*acosh(c + d*x)**2 + 2*b**2*c**4*e**4*x/25 - 2*b**2*c**4*e**4*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(25*d) + 2*b**2*c**3*d*e**4*x*
*2*acosh(c + d*x)**2 + 4*b**2*c**3*d*e**4*x**2/25 - 8*b**2*c**3*e**4*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 2*b**2*c**2*d**2*e**4*
x**3*acosh(c + d*x)**2 + 4*b**2*c**2*d**2*e**4*x**3/25 - 12*b**2*c**2*d*e**
4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*b**2*c**2
*e**4*x/75 - 8*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)/(75*d) + b**2*c*d**3*e**4*x**4*acosh(c + d*x)**2 + 2*b**2*c*d**3*e**
4*x**4/25 - 8*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*ac
osh(c + d*x)/25 + 8*b**2*c*d*e**4*x**2/75 - 16*b**2*c*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/75 + b**2*d**4*e**4*x**5*acosh(c + d*
x)**2/5 + 2*b**2*d**4*e**4*x**5/125 - 2*b**2*d**3*e**4*x**4*sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*b**2*d**2*e**4*x**3/225 - 8*b**
2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/75 + 16*b
**2*e**4*x/75 - 16*b**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)/(75*d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(c))**2, True))
```

3.105 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=186

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{8d}-\frac{3be^3\sqrt{c+d}}{}$$

```
[Out] 3/32*b^2*e^3*(d*x+c)^2/d+1/32*b^2*e^3*(d*x+c)^4/d-3/32*e^3*(a+b*arccosh(d*x
+c))^2/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^2/d-3/16*b*e^3*(d*x+c)*(a+b
*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/8*b*e^3*(d*x+c)^3*(a+b
*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.44, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5866, 12, 5662, 5759, 5676, 30}

$$\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}-\frac{be^3\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{8d}-\frac{3be^3\sqrt{c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (3*b^2*e^3*(c + d*x)^2)/(32*d) + (b^2*e^3*(c + d*x)^4)/(32*d) - (3*b*e^3*Sq
rt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(16*
d) - (b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh
[c + d*x]))/(8*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^2)/(32*d) + (e^3*(c + d
*x)^4*(a + b*ArcCosh[c + d*x])^2)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$
$$=\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}-\frac{\left(be^3\right)\text{Subst}\left(\int\frac{x^4\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\,\sqrt{1+x}}\,d\right.}{2d}$$
$$=-\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{8d}+\frac{e^3(}{}$$
$$=\frac{b^2e^3(c+dx)^4}{32d}-\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}\right.}{16d}$$
$$=\frac{3b^2e^3(c+dx)^2}{32d}+\frac{b^2e^3(c+dx)^4}{32d}-\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+}}{16d}$$

Mathematica [A] time = 0.27, size = 212, normalized size = 1.14

$$\frac{e^3\left(\left(8a^2+b^2\right)(c+dx)^4+2ab\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(-2(c+dx)^2-3\right)(c+dx)-6ab\log\left(\sqrt{c+dx-1}\,\sqrt{c+dx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^3*(3*b^2*(c + d*x)^2 + (8*a^2 + b^2)*(c + d*x)^4 + 2*a*b*Sqrt[-1 + c + d
*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3 - 2*(c + d*x)^2) + 2*b*(c + d*x)*(8*a*(
c + d*x)^3 - 3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 2*b*Sqrt[-1 + c + d
*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + b^2*(-3 + 8*(c + d*x)
^4)*ArcCosh[c + d*x]^2 - 6*a*b*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x]]))/(32*d)
```

fricas [B] time = 0.59, size = 481, normalized size = 2.59

$$\frac{\left(8a^2+b^2\right)d^4e^3x^4+4\left(8a^2+b^2\right)cd^3e^3x^3+3\left(2\left(8a^2+b^2\right)c^2+b^2\right)d^2e^3x^2+2\left(2\left(8a^2+b^2\right)c^3+3b^2c\right)de^3x+\left(8b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^2 + b^2)*d^4*e^3*x^4 + 4*(8*a^2 + b^2)*c*d^3*e^3*x^3 + 3*(2*(8*a
^2 + b^2)*c^2 + b^2)*d^2*e^3*x^2 + 2*(2*(8*a^2 + b^2)*c^3 + 3*b^2*c)*d*e^3*
x + (8*b^2*d^4*e^3*x^4 + 32*b^2*c*d^3*e^3*x^3 + 48*b^2*c^2*d^2*e^3*x^2 + 32
*b^2*c^3*d*e^3*x + (8*b^2*c^4 - 3*b^2)*e^3)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))^2 + 2*(8*a*b*d^4*e^3*x^4 + 32*a*b*c*d^3*e^3*x^3 + 48*a*b*
c^2*d^2*e^3*x^2 + 32*a*b*c^3*d*e^3*x + (8*a*b*c^4 - 3*a*b)*e^3 - (2*b^2*d^3
*e^3*x^3 + 6*b^2*c*d^2*e^3*x^2 + 3*(2*b^2*c^2 + b^2)*d*e^3*x + (2*b^2*c^3 +
 3*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)) - 2*(2*a*b*d^3*e^3*x^3 + 6*a*b*c*d^2*e^3*x^2 + 3*(2
*a*b*c^2 + a*b)*d*e^3*x + (2*a*b*c^3 + 3*a*b*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (b \operatorname{arcosh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^2, x)
```

maple [B] time = 0.04, size = 822, normalized size = 4.42

$$\frac{d^3x^4a^2e^3}{4} - \frac{e^3b^2\operatorname{arccosh}(dx+c)\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,c^3}{8d} - \frac{3\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,xab\,c^2e^3}{8} - \frac{3e^3b^2\operatorname{arcco}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/4*d^3*x^4*a^2*e^3-1/8/d*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)*c^3-3/8*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*a*b*c^2*e^3-3/8*e^3*b^2*arc
cosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2+2*d^2*arccosh(d*x+c)*x^3*
a*b*c*e^3-1/8/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a*b*c^3*e^3+3*d*arccosh(d*x
+c)*x^2*a*b*c^2*e^3-1/8*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3*a*b*e^3-1/8
*d^2*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-3/16/d*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)*a*b*c*e^3-3/16/d*e^3*b^2*arccosh(d*x+c)*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*c+1/32/d*e^3*b^2*c^4+3/32/d*e^3*b^2*c^2+1/4/d*a^2*
c^4*e^3+1/32*d^3*e^3*b^2*x^4-3/8*d*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*x^2*c-3/8*d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*a*b*c*e^3-3/
16/d*e^3*a*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+(
(d*x+c)^2-1)^(1/2))+3/2*d*e^3*b^2*arccosh(d*x+c)^2*x^2*c^2+d^2*e^3*b^2*arcc
osh(d*x+c)^2*x^3*c+3/16*e^3*b^2*x*c+1/8*e^3*b^2*x*c^3+x*a^2*c^3*e^3+3/32*d*
e^3*b^2*x^2+1/4*d^3*e^3*b^2*arccosh(d*x+c)^2*x^4+e^3*b^2*arccosh(d*x+c)^2*x
*c^3+1/8*d^2*e^3*b^2*x^3*c+3/16*d*e^3*b^2*x^2*c^2+3/2*d*x^2*a^2*c^2*e^3+1/4
/d*e^3*b^2*arccosh(d*x+c)^2*c^4-3/32/d*e^3*b^2*arccosh(d*x+c)^2+d^2*x^3*a^2
*c*e^3+1/2*d^3*arccosh(d*x+c)*x^4*a*b*e^3-3/16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)*x*a*b*e^3-3/16*e^3*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x
+2*arccosh(d*x+c)*x*a*b*c^3*e^3+1/2/d*arccosh(d*x+c)*a*b*c^4*e^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*e^3*x^4 + a^2*c*d^2*e^3*x^3 + 3/2*a^2*c^2*d*e^3*x^2 + 3/2*(2*x^
2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*l
og(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*c^2*d*e^3 + 1/3*(6*x^3*arccosh(d*x +
 c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x +
 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4
 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a*b*c*d^2*e^3 + 1/48
*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^2 -
14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x + 2*c*
d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3/d
^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^2*lo
g(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a*b*d^3*e^3 + a^2*c^3*e^3*x
+ 2*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b*c^3*e^3/d + 1/
4*(b^2*d^3*e^3*x^4 + 4*b^2*c*d^2*e^3*x^3 + 6*b^2*c^2*d*e^3*x^2 + 4*b^2*c^3*
e^3*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - integrate(1/2
*(b^2*d^6*e^3*x^6 + 6*b^2*c*d^5*e^3*x^5 + (15*c^2*d^4*e^3 - d^4*e^3)*b^2*x^
4 + 4*(5*c^3*d^3*e^3 - c*d^3*e^3)*b^2*x^3 + 2*(7*c^4*d^2*e^3 - 3*c^2*d^2*e^
3)*b^2*x^2 + 4*(c^5*d*e^3 - c^3*d*e^3)*b^2*x + (b^2*d^5*e^3*x^5 + 5*b^2*c*d
^4*e^3*x^4 + 10*b^2*c^2*d^3*e^3*x^3 + 10*b^2*c^3*d^2*e^3*x^2 + 4*b^2*c^4*d*
e^3*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2
 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{acosh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^2, x)
```

sympy [A] time = 4.32, size = 916, normalized size = 4.92

$$\begin{cases} a^2c^3e^3x + \frac{3a^2c^2de^3x^2}{2} + a^2cd^2e^3x^3 + \frac{a^2d^3e^3x^4}{4} + \frac{abc^4e^3\,\mathrm{acosh}\,(c+dx)}{2d} + 2abc^3e^3x\,\mathrm{acosh}\,(c+dx) - \frac{abc^3e^3\sqrt{c^2+2cdx+d^2x^2-1}}{8d} + \\ c^3e^3x\,(a+b\,\mathrm{acosh}\,(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**3*e**3*x + 3*a**2*c**2*d*e**3*x**2/2 + a**2*c*d**2*e**3*
x**3 + a**2*d**3*e**3*x**4/4 + a*b*c**4*e**3*acosh(c + d*x)/(2*d) + 2*a*b*c
**3*e**3*x*acosh(c + d*x) - a*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/(8*d) + 3*a*b*c**2*d*e**3*x**2*acosh(c + d*x) - 3*a*b*c**2*e**3*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)/8 + 2*a*b*c*d**2*e**3*x**3*acosh(c + d*x) -
 3*a*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3*a*b*c*e**3*
sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(16*d) + a*b*d**3*e**3*x**4*acosh(c +
d*x)/2 - a*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3*a*b*
e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 3*a*b*e**3*acosh(c + d*x)/
(16*d) + b**2*c**4*e**3*acosh(c + d*x)**2/(4*d) + b**2*c**3*e**3*x*acosh(c
+ d*x)**2 + b**2*c**3*e**3*x/8 - b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*
```

```
x**2 - 1)*acosh(c + d*x)/(8*d) + 3*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2/
2 + 3*b**2*c**2*d*e**3*x**2/16 - 3*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)*acosh(c + d*x)/8 + b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 +
b**2*c*d**2*e**3*x**3/8 - 3*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)*acosh(c + d*x)/8 + 3*b**2*c*e**3*x/16 - 3*b**2*c*e**3*sqrt(c**2 +
2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(16*d) + b**2*d**3*e**3*x**4*acosh(
c + d*x)**2/4 + b**2*d**3*e**3*x**4/32 - b**2*d**2*e**3*x**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/8 + 3*b**2*d*e**3*x**2/32 - 3*b**2*e*
*3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/16 - 3*b**2*e**3*a
cosh(c + d*x)**2/(32*d), Ne(d, 0)), (c**3*e**3*x*(a + b*acosh(c))**2, True)
)
```

3.106 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=150

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{9d}-\frac{4be^2\sqrt{c+dx}}{}$$

```
[Out] 4/9*b^2*e^2*x+2/27*b^2*e^2*(d*x+c)^3/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c
))^2/d-4/9*b*e^2*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-2/9
*b*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5866, 12, 5662, 5759, 5718, 8, 30}

$$\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{2be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{9d}-\frac{4be^2\sqrt{c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (4*b^2*e^2*x)/9 + (2*b^2*e^2*(c + d*x)^3)/(27*d) - (4*b*e^2*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(9*d) - (2*b*e^2*Sqrt[-1 +
c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(9*d) + (e
^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int e^2x^2\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^2\,\text{Subst}\left(\int x^2\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\frac{\left(2be^2\right)\text{Subst}\left(\int\frac{x^3\left(a+b\cosh^{-1}\right.}{\sqrt{-1+x}\,\sqrt{1+}}\right.}{3d}$$

$$=-\frac{2be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{9d}$$

$$=\frac{2b^2e^2(c+dx)^3}{27d}-\frac{4be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{9d}$$

$$=\frac{4}{9}b^2e^2x+\frac{2b^2e^2(c+dx)^3}{27d}-\frac{4be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\right.}{9d}$$

Mathematica [A] time = 0.24, size = 168, normalized size = 1.12

$$e^2\left(\left(9a^2+2b^2\right)(c+dx)^3+6ab\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(-(c+dx)^2-2\right)+6b\cosh^{-1}(c+dx)\left(3a(c+dx)^3-b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^2*(12*b^2*(c + d*x) + (9*a^2 + 2*b^2)*(c + d*x)^3 + 6*a*b*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x]*(-2 - (c + d*x)^2) + 6*b*(3*a*(c + d*x)^3 - 2*b*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x] - b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
+ c + d*x])*ArcCosh[c + d*x] + 9*b^2*(c + d*x)^3*ArcCosh[c + d*x]^2))/(27*d
)
```

fricas [B] time = 0.53, size = 358, normalized size = 2.39

$$\left(9\,a^2+2\,b^2\right)d^3e^2x^3+3\left(9\,a^2+2\,b^2\right)cd^2e^2x^2+3\left(\left(9\,a^2+2\,b^2\right)c^2+4\,b^2\right)de^2x+9\left(b^2d^3e^2x^3+3\,b^2cd^2e^2x^2+3\,b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*d^3*e^2*x^3 + 3*(9*a^2 + 2*b^2)*c*d^2*e^2*x^2 + 3*((9
*a^2 + 2*b^2)*c^2 + 4*b^2)*d*e^2*x + 9*(b^2*d^3*e^2*x^3 + 3*b^2*c*d^2*e^2*x
^2 + 3*b^2*c^2*d*e^2*x + b^2*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))^2 + 6*(3*a*b*d^3*e^2*x^3 + 9*a*b*c*d^2*e^2*x^2 + 9*a*b*c^2*d*e^
2*x + 3*a*b*c^3*e^2 - (b^2*d^2*e^2*x^2 + 2*b^2*c*d*e^2*x + (b^2*c^2 + 2*b^2
)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)) - 6*(a*b*d^2*e^2*x^2 + 2*a*b*c*d*e^2*x + (a*b*c^2 + 2*a*b)*
e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^2, x)
```

maple [A] time = 0.04, size = 167, normalized size = 1.11

$$\frac{\frac{(dx+c)^3 e^2 a^2}{3} + e^2 b^2 \left(\frac{(dx+c)^3 \operatorname{arccosh}(dx+c)^2}{3} - \frac{4 \operatorname{arccosh}(dx+c) \sqrt{dx+c-1}\, \sqrt{dx+c+1}}{9} - \frac{2 \operatorname{arccosh}(dx+c) \sqrt{dx+c-1}\, \sqrt{dx+c+1}\, (dx+c)^2}{9} + \frac{4dx}{9} + \cdots\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^2+e^2*b^2*(1/3*(d*x+c)^3*arccosh(d*x+c)^2-4/9*arcc
osh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/9*arccosh(d*x+c)*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*(d*x+c)^2+4/9*d*x+4/9*c+2/27*(d*x+c)^3)+2*e^2*a*b*(1/3*a
rccosh(d*x+c)*(d*x+c)^3-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a^2 d^2 e^2 x^3 + a^2 c d e^2 x^2 + \left(2 x^2 \operatorname{arcosh}(dx + c) - d \left(\frac{3 c^2 \log\left(2 d^2 x + 2 cd + 2 \sqrt{d^2 x^2 + 2 cdx + c^2 - 1}\, d\right)}{d^3} + \frac{\sqrt{d^2 x^2 + \cdots}}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*e^2*x^3 + a^2*c*d*e^2*x^2 + (2*x^2*arccosh(d*x + c) - d*(3*c^2*
log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2
*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d
^3))*a*b*c*d*e^2 + 1/9*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2
- 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 1
5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*(c^2 - 1)/d^4))*a*b*d^2*e^2 + a^2*c^2*e^2*x + 2*((d*x + c)*arccosh(d
*x + c) - sqrt((d*x + c)^2 - 1))*a*b*c^2*e^2/d + 1/3*(b^2*d^2*e^2*x^3 + 3*b
^2*c*d*e^2*x^2 + 3*b^2*c^2*e^2*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
- 1) + c)^2 - integrate(2/3*(b^2*d^5*e^2*x^5 + 5*b^2*c*d^4*e^2*x^4 + (10*c^
```

```
2*d^3*e^2 - d^3*e^2)*b^2*x^3 + 3*(3*c^3*d^2*e^2 - c*d^2*e^2)*b^2*x^2 + 3*(c
^4*d*e^2 - c^2*d*e^2)*b^2*x + (b^2*d^4*e^2*x^4 + 4*b^2*c*d^3*e^2*x^3 + 6*b^
2*c^2*d^2*e^2*x^2 + 3*b^2*c^3*d*e^2*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))
*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2
+ c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
 (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^2\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^2, x)
```

sympy [A] time = 1.78, size = 610, normalized size = 4.07

$$\begin{cases} a^2c^2e^2x + a^2cde^2x^2 + \frac{a^2d^2e^2x^3}{3} + \frac{2abc^3e^2\operatorname{acosh}(c+dx)}{3d} + 2abc^2e^2x\operatorname{acosh}(c+dx) - \frac{2abc^2e^2\sqrt{c^2+2cdx+d^2x^2-1}}{9d} + 2abcde^2 \\ c^2e^2x\left(a+b\operatorname{acosh}(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c**2*e**2*x + a**2*c*d*e**2*x**2 + a**2*d**2*e**2*x**3/3 +
2*a*b*c**3*e**2*acosh(c + d*x)/(3*d) + 2*a*b*c**2*e**2*x*acosh(c + d*x) - 2
*a*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + 2*a*b*c*d*e**2*
x**2*acosh(c + d*x) - 4*a*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9
 + 2*a*b*d**2*e**2*x**3*acosh(c + d*x)/3 - 2*a*b*d*e**2*x**2*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/9 - 4*a*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/
(9*d) + b**2*c**3*e**2*acosh(c + d*x)**2/(3*d) + b**2*c**2*e**2*x*acosh(c +
 d*x)**2 + 2*b**2*c**2*e**2*x/9 - 2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)*acosh(c + d*x)/(9*d) + b**2*c*d*e**2*x**2*acosh(c + d*x)**2 +
2*b**2*c*d*e**2*x**2/9 - 4*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
1)*acosh(c + d*x)/9 + b**2*d**2*e**2*x**3*acosh(c + d*x)**2/3 + 2*b**2*d**2
*e**2*x**3/27 - 2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*aco
sh(c + d*x)/9 + 4*b**2*e**2*x/9 - 4*b**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)/(9*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acosh(c))**2,
True))
```

3.107 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=110

$$\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{e\left(a+b\cosh^{-1}(c\right.}{4d}$$

```
[Out] 1/4*b^2*e*(d*x+c)^2/d-1/4*e*(a+b*arccosh(d*x+c))^2/d+1/2*e*(d*x+c)^2*(a+b*a
rccosh(d*x+c))^2/d-1/2*b*e*(d*x+c)*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5662, 5759, 5676, 30}

$$\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)}{2d}-\frac{e\left(a+b\cosh^{-1}(c\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (b^2*e*(c + d*x)^2)/(4*d) - (b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c +
d*x]*(a + b*ArcCosh[c + d*x]))/(2*d) - (e*(a + b*ArcCosh[c + d*x])^2)/(4*d)
 + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
```

```
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{(be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\,d\right.}{d}$$

$$=-\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{2d}+\frac{e(c}{}$$

$$=\frac{b^2e(c+dx)^2}{4d}-\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(\right.}{2d}$$

Mathematica [A] time = 0.23, size = 167, normalized size = 1.52

$$\frac{e\left((c+dx)\left(2a^2(c+dx)-2ab\sqrt{c+dx-1}\sqrt{c+dx+1}+b^2(c+dx)\right)-2ab\log\left(\sqrt{c+dx-1}\sqrt{c+dx+1}+c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e*((c + d*x)*(2*a^2*(c + d*x) + b^2*(c + d*x) - 2*a*b*Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]) - 2*b*(c + d*x)*(-2*a*(c + d*x) + b*Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x])*ArcCosh[c + d*x] + b^2*(-1 + 2*c^2 + 4*c*d*x + 2*d^2*x^2)*
ArcCosh[c + d*x]^2 - 2*a*b*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x]]))/(4*d)
```

fricas [B] time = 0.66, size = 233, normalized size = 2.12

$$\frac{\left(2a^2+b^2\right)d^2ex^2+2\left(2a^2+b^2\right)cdex+\left(2b^2d^2ex^2+4b^2cdex+\left(2b^2c^2-b^2\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+2cdx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*d^2*e*x^2 + 2*(2*a^2 + b^2)*c*d*e*x + (2*b^2*d^2*e*x^2 +
 4*b^2*c*d*e*x + (2*b^2*c^2 - b^2)*e)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))^2 + 2*(2*a*b*d^2*e*x^2 + 4*a*b*c*d*e*x + (2*a*b*c^2 - a*b)*e -
(b^2*d*e*x + b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt
```

```
(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(a*b*d*e*x + a*b*c*e)*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b\operatorname{arcosh}(dx + c) + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2, x)
```

maple [B] time = 0.04, size = 334, normalized size = 3.04

$$\frac{a^2ex^2d}{2}+xa^2ce+\frac{a^2c^2e}{2d}+\frac{deb^2\operatorname{arccosh}(dx+c)^2x^2}{2}+eb^2\operatorname{arccosh}(dx+c)^2xc+\frac{eb^2\operatorname{arccosh}(dx+c)^2c^2}{2d}-\frac{eb^2\sqrt{dx+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*e*x^2*d+x*a^2*c*e+1/2/d*a^2*c^2*e+1/2*d*e*b^2*arccosh(d*x+c)^2*x^2+
e*b^2*arccosh(d*x+c)^2*x*c+1/2/d*e*b^2*arccosh(d*x+c)^2*c^2-1/2*e*b^2*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*arccosh(d*x+c)*x-1/2/d*e*b^2*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)*arccosh(d*x+c)*c-1/4/d*e*b^2*arccosh(d*x+c)^2+1/4*b^2*d*e*x^2+
1/2*e*b^2*x*c+1/4/d*e*b^2*c^2+d*arccosh(d*x+c)*x^2*a*b*e+2*arccosh(d*x+c)*x
*a*b*c*e+1/d*arccosh(d*x+c)*a*b*c^2*e-1/2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x
*a*b*e-1/2/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a*b*c*e-1/2/d*e*a*b*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2dex^2+\frac{1}{2}\left(2x^2\operatorname{arcosh}(dx+c)-d\left(\frac{3c^2\log\left(2d^2x+2cd+2\sqrt{d^2x^2+2cdx+c^2-1}\,d\right)}{d^3}+\frac{\sqrt{d^2x^2+2cdx+c^2}}{d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*e*x^2 + 1/2*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*
d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a*b*d*e + a^2
*c*e*x + 2*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b*c*e/d +
 1/2*(b^2*d*e*x^2 + 2*b^2*c*e*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)^2 - integrate((b^2*d^4*e*x^4 + 4*b^2*c*d^3*e*x^3 + (5*c^2*d^2*e -
d^2*e)*b^2*x^2 + 2*(c^3*d*e - c*d*e)*b^2*x + (b^2*d^3*e*x^3 + 3*b^2*c*d^2*e
*x^2 + 2*b^2*c^2*d*e*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt
(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^
2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*
x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^2, x)
```

sympy [**A**] time = 0.82, size = 335, normalized size = 3.05

$$\begin{cases} a^2cex + \frac{a^2dex^2}{2} + \frac{abc^2e\operatorname{acosh}(c+dx)}{d} + 2abcex\operatorname{acosh}(c+dx) - \frac{abce\sqrt{c^2+2cdx+d^2x^2-1}}{2d} + abdex^2\operatorname{acosh}(c+dx) - \frac{abex\sqrt{}}{} \\ cex\left(a+b\operatorname{acosh}(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*c*e*x + a**2*d*e*x**2/2 + a*b*c**2*e*acosh(c + d*x)/d + 2*a
*b*c*e*x*acosh(c + d*x) - a*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(2*d
) + a*b*d*e*x**2*acosh(c + d*x) - a*b*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/2 - a*b*e*acosh(c + d*x)/(2*d) + b**2*c**2*e*acosh(c + d*x)**2/(2*d) +
b**2*c*e*x*acosh(c + d*x)**2 + b**2*c*e*x/2 - b**2*c*e*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*acosh(c + d*x)/(2*d) + b**2*d*e*x**2*acosh(c + d*x)**2/2 +
 b**2*d*e*x**2/4 - b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)/2 - b**2*e*acosh(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))
**2, True))
```

3.108 $\int \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=64

$$-\frac{2b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

```
[Out] 2*b^2*x+(d*x+c)*(a+b*arccosh(d*x+c))^2/d-2*b*(a+b*arccosh(d*x+c))*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.12, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5864, 5654, 5718, 8}

$$-\frac{2b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]
))/d + ((c + d*x)*(a + b*ArcCosh[c + d*x])^2)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d} - \frac{(2b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\cosh\right.}{d}$$

$$= 2b^2x - \frac{2b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{(c+dx)\left(a+b\right.}{}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 1.64

$$\frac{a^2(c+dx) - 2ab\sqrt{c+dx-1}\sqrt{c+dx+1} - 2b\cosh^{-1}(c+dx)\left(b\sqrt{c+dx-1}\sqrt{c+dx+1} - a(c+dx)\right) + 2b^2(c}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (a^2*(c + d*x) + 2*b^2*(c + d*x) - 2*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x] - 2*b*(-(a*(c + d*x)) + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[
c + d*x] + b^2*(c + d*x)*ArcCosh[c + d*x]^2)/d
```

fricas [B] time = 0.63, size = 141, normalized size = 2.20

$$\frac{\left(a^2+2\,b^2\right)dx + \left(b^2dx+b^2c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2 - 2\,\sqrt{d^2x^2+2\,cdx+c^2-1}\,ab + 2\left(abdx\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 + 2*b^2)*d*x + (b^2*d*x + b^2*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x
 + c^2 - 1))^2 - 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*a*b + 2*(a*b*d*x + a*b
*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b^2)*log(d*x + c + sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2, x)
```

maple [A] time = 0.05, size = 100, normalized size = 1.56

$$\frac{a^2\,(dx+c) + b^2\left((dx+c)\,\text{arccosh}\,(dx+c)^2 - 2\,\text{arccosh}\,(dx+c)\,\sqrt{dx+c-1}\,\sqrt{dx+c+1} + 2dx + 2c\right) + 2ab\,\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(d*x+c)+b^2*((d*x+c)*arccosh(d*x+c)^2-2*arccosh(d*x+c)*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)+2*d*x+2*c)+2*a*b*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\left(x\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^2-\int\frac{2\left(d^3x^3+2\,cd^2x^2+\left(d^2x^2+cdx\right)\sqrt{dx+c+1}\,\sqrt{dx+c-1}+\left(c^2\right.\right.}{d^3x^3+3\,cd^2x^2+c^3+\left(d^2x^2+2\,cdx+c^2-1\right)\sqrt{dx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - integrate(2*(d^3*
x^3 + 2*c*d^2*x^2 + (d^2*x^2 + c*d*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
 (c^2*d - d)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3
 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + (3*c^2*d - d)*x - c), x))*b^2 + a^2*x + 2*((d*x + c)*arccos
h(d*x + c) - sqrt((d*x + c)^2 - 1))*a*b/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (a+b\,\mathrm{acosh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))^2, x)
```

sympy [A] time = 0.30, size = 143, normalized size = 2.23

$$\begin{cases} a^2x+\frac{2abc\,\mathrm{acosh}\,(c+dx)}{d}+2abx\,\mathrm{acosh}\,(c+dx)-\frac{2ab\sqrt{c^2+2cdx+d^2x^2-1}}{d}+\frac{b^2c\,\mathrm{acosh}^2\,(c+dx)}{d}+b^2x\,\mathrm{acosh}^2\,(c+dx)+2b^2x- \\ x\,(a+b\,\mathrm{acosh}\,(c))^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*c*acosh(c + d*x)/d + 2*a*b*x*acosh(c + d*x) - 2*a
*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + b**2*c*acosh(c + d*x)**2/d + b*
*2*x*acosh(c + d*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)*acosh(c + d*x)/d, Ne(d, 0)), (x*(a + b*acosh(c))**2, True))
```

3.109 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{ce+dex}\,dx$

Optimal. Leaf size=118

$$-\frac{b\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}+\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde}+\frac{\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh\right.}{de}$$

```
[Out] 1/3*(a+b*arccosh(d*x+c))^3/b/d/e+(a+b*arccosh(d*x+c))^2*ln(1+1/(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-b*(a+b*arccosh(d*x+c))*polylog(2,-1/(d*x
+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-1/2*b^2*polylog(3,-1/(d*x+c+(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5866, 12, 5660, 3718, 2190, 2531, 2282, 6589}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(c+dx)}\right)}{2de}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcCosh[c + d*x])^3/(3*b*d*e) + ((a + b*ArcCosh[c + d*x])^2*Log[1 +
 E^(2*ArcCosh[c + d*x])])/(d*e) + (b*(a + b*ArcCosh[c + d*x])*PolyLog[2, -E
^(2*ArcCosh[c + d*x])])/(d*e) - (b^2*PolyLog[3, -E^(2*ArcCosh[c + d*x])])/(
2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{ex}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x}\,dx, x, c+dx\right)}{de}$$

$$= \frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \cosh^{-1}(c+dx)\right)}{de}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} - \frac{(2}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{b}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{b}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} + \frac{b}{}$$

Mathematica [A] time = 0.42, size = 140, normalized size = 1.19

$$\frac{a^2\log(c+dx) - b\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right) + ab\cosh^{-1}(c+dx)^2 + 2ab\cosh^{-1}(c+dx)\log\left(e^{-}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x),x]
```

```
[Out] (a*b*ArcCosh[c + d*x]^2 + (b^2*ArcCosh[c + d*x]^3)/3 + 2*a*b*ArcCosh[c + d*
x]*Log[1 + E^(-2*ArcCosh[c + d*x])] + b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*
ArcCosh[c + d*x])] + a^2*Log[c + d*x] - b*(a + b*ArcCosh[c + d*x])*PolyLog[
2, -E^(-2*ArcCosh[c + d*x])] - (b^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])])/2
)/(d*e)
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arcosh}(dx+c)^2+2\,ab\operatorname{arcosh}(dx+c)+a^2}{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d*e*x + c
*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(dx+c)+a)^2}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e), x)
```

maple [A] time = 0.05, size = 263, normalized size = 2.23

$$\frac{a^2\ln(dx+c)}{de}-\frac{b^2\operatorname{arccosh}(dx+c)^3}{3de}+\frac{b^2\operatorname{arccosh}(dx+c)^2\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{de}+\frac{b^2\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^2/e*ln(d*x+c)-1/3/d*b^2/e*arccosh(d*x+c)^3+1/d*b^2/e*arccosh(d*x+c)^2
*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+1/d*b^2/e*arccosh(d*x+c)*p
olylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/2/d*b^2/e*polylog(3,
-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/d*a*b/e*arccosh(d*x+c)^2+2/d*
a*b/e*arccosh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+1/d*a*
b/e*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\log(dex+ce)}{de}+\int\frac{b^2\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^2}{dex+ce}+\frac{2\,ab\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right.}{dex+ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^2*log(d*e*x + c*e)/(d*e) + integrate(b^2*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^2/(d*e*x + c*e) + 2*a*b*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)/(d*e*x + c*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\operatorname{acosh}(c+d\,x))^2}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c+dx}\,dx + \int \frac{b^2 \operatorname{acosh}^2(c+dx)}{c+dx}\,dx + \int \frac{2ab \operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2/(c + d*x), x) + Integral(b**2*acosh(c + d*x)**2/(c + d*x), x
) + Integral(2*a*b*acosh(c + d*x)/(c + d*x), x))/e
```

3.110 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{4b\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{2ib^2\text{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)}{de^2}+\frac{2ib^2\text{Li}_2\left(\right.}{}$$

```
[Out] -(a+b*arccosh(d*x+c))^2/d/e^2/(d*x+c)+4*b*(a+b*arccosh(d*x+c))*arctan(d*x+c
+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-2*I*b^2*polylog(2,-I*(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+2*I*b^2*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)))/d/e^2
```

Rubi [A] time = 0.24, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5662, 5761, 4180, 2279, 2391}

$$-\frac{2ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)}{de^2}+\frac{2ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)}{de^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)}+\frac{4b\tan^{-1}\left(e^{\cosh^{-1}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])^2/(d*e^2*(c + d*x))) + (4*b*(a + b*ArcCosh[c + d
*x])*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((2*I)*b^2*PolyLog[2, (-I)*E^Arc
Cosh[c + d*x]])/(d*e^2) + ((2*I)*b^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e
^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
```

```
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,x\,\sqrt{1+x}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{(2b)\,\text{Subst}\left(\int (a+bx)\text{sech}(x)\,dx, x, \cosh^{-1}(c+dx)\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{(2i}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{(2i}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^2(c+dx)} + \frac{4b\left(a+b\cosh^{-1}(c+dx)\right)\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{2ib}{}$$

Mathematica [A] time = 0.77, size = 161, normalized size = 1.46

$$\frac{-\frac{a^2}{c+dx} + 2ab\left(2\tan^{-1}\left(\tanh\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\right) - \frac{\cosh^{-1}(c+dx)}{c+dx}\right) - ib^2\left(2\text{Li}_2\left(-ie^{-\cosh^{-1}(c+dx)}\right) - 2\text{Li}_2\left(ie^{-\cosh^{-1}(c+dx}\right.\right.}{de^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^2/(c + d*x)) + 2*a*b*(-(ArcCosh[c + d*x]/(c + d*x)) + 2*ArcTan[Tanh[Ar
cCosh[c + d*x]/2]]) - I*b^2*(ArcCosh[c + d*x]*(((-I)*ArcCosh[c + d*x])/(c +
 d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1 + I/E^ArcCosh[c + d*x]])
+ 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[2, I/E^ArcCosh[c + d*x]
]))/(d*e^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arcosh}(dx+c)^2 + 2\,ab \operatorname{arcosh}(dx+c) + a^2}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d^2*e^2*x
^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Er
ror: Bad Argument Value
```

maple [A] time = 0.17, size = 290, normalized size = 2.64

$$-\frac{a^2}{d\,e^2\,(dx+c)} - \frac{b^2\operatorname{arccosh}(dx+c)^2}{d\,e^2\,(dx+c)} - \frac{2ib^2\operatorname{arccosh}(dx+c)\ln\left(1+i\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)\right)}{d\,e^2} + \frac{2ib}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x)
```

```
[Out] -1/d*a^2/e^2/(d*x+c)-1/d*b^2/e^2*arccosh(d*x+c)^2/(d*x+c)-2*I/d*b^2/e^2*arc
cosh(d*x+c)*ln(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+2*I/d*b^2/e^2*a
rccosh(d*x+c)*ln(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-2*I/d*b^2/e^2
*dilog(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))+2*I/d*b^2/e^2*dilog(1-I
*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))-2/d*a*b/e^2/(d*x+c)*arccosh(d*x+c
)-2/d*a*b/e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/
((d*x+c)^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b^2\left(\frac{\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^2}{d^2e^2x+cde^2} - \int \frac{2\left(d^2x^2+2\,cdx+\sqrt{dx+c+1}\,(\right.}{d^4e^2x^4+4\,cd^3e^2x^3+c^4e^2-c^2e^2+\left(6\,c^2d^2e^2-d^2e^2\right)x^2+\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^2*(log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^2*e^2*x + c*d
*e^2) - integrate(2*(d^2*x^2 + 2*c*d*x + sqrt(d*x + c + 1)*(d*x + c)*sqrt(d
*x + c - 1) + c^2 - 1)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(
d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 - c^2*e^2 + (6*c^2*d^2*e^2 - d^2*e^
2)*x^2 + (d^3*e^2*x^3 + 3*c*d^2*e^2*x^2 + c^3*e^2 - c*e^2 + (3*c^2*d*e^2 -
d*e^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(2*c^3*d*e^2 - c*d*e^2)*x
```

```
), x)) - 2*a*b*(arccosh(d*x + c)/(d^2*e^2*x + c*d*e^2) + arcsin(d*e^2/abs(d
^2*e^2*x + c*d*e^2))/(d*e^2)) - a^2/(d^2*e^2*x + c*d*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^2\,\mathrm{acosh}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{2ab\,\mathrm{acosh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**2*acosh(c + d
*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(2*a*b*acosh(c + d*x)/(c*
*2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.111 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^3} \, dx$

Optimal. Leaf size=92

$$\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{b^2\log(c+dx)}{de^3}$$

```
[Out] -1/2*(a+b*arccosh(d*x+c))^2/d/e^3/(d*x+c)^2-b^2*ln(d*x+c)/d/e^3+b*(a+b*arcc
osh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^3/(d*x+c)
```

Rubi [A] time = 0.21, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5866, 12, 5662, 5724, 29}

$$\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2}-\frac{b^2\log(c+dx)}{de^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(d*e^3*(c
 + d*x)) - (a + b*ArcCosh[c + d*x])^2/(2*d*e^3*(c + d*x)^2) - (b^2*Log[c +
d*x])/(d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^3x^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^3}\,dx, x, c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} + \frac{b\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx, x, c+dx\right)}{de^3}$$

$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} -$$

$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)^2} -$$

Mathematica [A] time = 0.21, size = 81, normalized size = 0.88

$$\frac{b\left(\frac{\sqrt{c+dx-1}\,\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{c+dx} - b\log(c+dx)\right) - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{2(c+dx)^2}}{de^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^3,x]
```

```
[Out] (-1/2*(a + b*ArcCosh[c + d*x])^2/(c + d*x)^2 + b*((Sqrt[-1 + c + d*x]*Sqrt[
1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(c + d*x) - b*Log[c + d*x]))/(d*e^3)
```

fricas [B] time = 0.68, size = 320, normalized size = 3.48

$$\frac{2\,abc^2d^2x^2 + 4\,abc^3dx + 2\,abc^4 - b^2c^2\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^2 - a^2c^2 + 2\left(abd^2x^2 + 2\,abcdx + \left(b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2*d^2*x^2 + 4*a*b*c^3*d*x + 2*a*b*c^4 - b^2*c^2*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 - a^2*c^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d
*x + (b^2*c^2*d*x + b^2*c^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c
 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(b^2*c^2*d^2*x^2 + 2*b^2*c^3*d*x
+ b^2*c^4)*log(d*x + c) + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*log(-d*x
- c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) + 2*(a*b*c^2*d*x + a*b*c^3)*sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1))/(c^2*d^3*e^3*x^2 + 2*c^3*d^2*e^3*x + c^4*d*e^3
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^2}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^3, x)
```

maple [B] time = 0.27, size = 194, normalized size = 2.11

$$-\frac{a^2}{2d\,e^3\,(dx+c)^2}+\frac{b^2\mathrm{arccosh}\,(dx+c)}{d\,e^3}+\frac{b^2\mathrm{arccosh}\,(dx+c)\,\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{d\,e^3\,(dx+c)}-\frac{b^2\mathrm{arccosh}\,(dx+c)^2}{2d\,e^3\,(dx+c)^2}-\frac{b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^2/e^3/(d*x+c)^2+1/d*b^2/e^3*arccosh(d*x+c)+1/d*b^2/e^3*arccosh(d*x
+c)/(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)-1/2/d*b^2/e^3*arccosh(d*x+c)^2/
(d*x+c)^2-1/d*b^2/e^3*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/d*a
*b/e^3/(d*x+c)^2*arccosh(d*x+c)+1/d*a*b/e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)
/(d*x+c)
```

maxima [B] time = 0.69, size = 229, normalized size = 2.49

$$\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}\,d\,\mathrm{arcosh}\,(dx+c)}{d^3e^3x+cd^2e^3}-\frac{\log\,(dx+c)}{de^3}\right)b^2+ab\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}\,d}{d^3e^3x+cd^2e^3}-\frac{\mathrm{arcosh}\,(dx}{d^3e^3x^2+2\,cd^2e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] (sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arccosh(d*x + c)/(d^3*e^3*x + c*d^2*e^
3) - log(d*x + c)/(d*e^3))*b^2 + a*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(
d^3*e^3*x + c*d^2*e^3) - arccosh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^
2*d*e^3)) - 1/2*b^2*arccosh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d
*e^3) - 1/2*a^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2}{{\left(c\,e+d\,e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\, dx+\int \frac{b^2\operatorname{acosh}^2\left(c+dx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\, dx+\int \frac{2ab\operatorname{acosh}\left(c+dx\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\, dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*a*b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/e**3
```

3.112 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^4} dx$

Optimal. Leaf size=186

$$\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2}-\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3}+\frac{2b\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}\right.}{3de^4}$$

```
[Out] 1/3*b^2/d/e^4/(d*x+c)-1/3*(a+b*arccosh(d*x+c))^2/d/e^4/(d*x+c)^3+2/3*b*(a+b
*arccosh(d*x+c))*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^4-1/3*I*
b^2*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+1/3*I*b^2*p
olylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+1/3*b*(a+b*arccos
h(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A] time = 0.39, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5662, 5748, 5761, 4180, 2279, 2391, 30}

$$-\frac{ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)}{3de^4}+\frac{ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)}{3de^4}+\frac{b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx\right.}{3de^4(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] b^2/(3*d*e^4*(c + d*x)) + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x]))/(3*d*e^4*(c + d*x)^2) - (a + b*ArcCosh[c + d*x])^2/(3*d*e^
4*(c + d*x)^3) + (2*b*(a + b*ArcCosh[c + d*x])*ArcTan[E^ArcCosh[c + d*x]])/
(3*d*e^4) - ((I/3)*b^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^4) + ((I/3
)*b^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{e^4x^4}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x^4}\,dx,x,c+dx\right)}{de^4}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} + \frac{(2b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\,dx,x,c+dx\right)}{3de^4}$$
$$= \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de^4(c+dx)^3} +$$
$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cos\right.}{3de^4(}$$
$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cos\right.}{3de^4(}$$
$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cos\right.}{3de^4(}$$
$$= \frac{b^2}{3de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{3de^4(c+dx)^2} - \frac{\left(a+b\cos\right.}{3de^4(}$$

Mathematica [A] time = 1.02, size = 251, normalized size = 1.35

$$-\frac{a^2}{(c+dx)^3} + ab\left(\frac{\sqrt{\frac{c+dx-1}{c+dx+1}}\,(c+dx+1)}{(c+dx)^2} - \frac{2\cosh^{-1}(c+dx)}{(c+dx)^3} + 2\tan^{-1}\left(\tanh\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\right)\right) + b^2\left(-i\text{Li}_2\left(-ie^{-\cosh^{-1}(c+dx)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^4,x]
```

```
[Out] (-(a^2/(c + d*x)^3) + a*b*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x
))/(c + d*x)^2 - (2*ArcCosh[c + d*x])/(c + d*x)^3 + 2*ArcTan[Tanh[ArcCosh[c
 + d*x]/2]]) + b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh[c + d*x]^2/(c + d*x)^3
- I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]] + I*ArcCosh[c + d*x]*Log
[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] + I*Poly
Log[2, I/E^ArcCosh[c + d*x]]))/(3*d*e^4)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\operatorname{arcosh}(dx+c)^2 + 2\,ab\operatorname{arcosh}(dx+c) + a^2}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/(d^4*e^4*x
^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^2}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^4, x)
```

maple [A] time = 0.31, size = 381, normalized size = 2.05

$$-\frac{a^2}{3d\,e^4\,(dx+c)^3}+\frac{b^2\operatorname{arccosh}(dx+c)\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{3d\,e^4\,(dx+c)^2}-\frac{b^2\operatorname{arccosh}(dx+c)^2}{3d\,e^4\,(dx+c)^3}+\frac{b^2}{3d\,e^4\,(dx+c)}-\frac{ib^2\operatorname{arcc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x)
```

```
[Out] -1/3/d*a^2/e^4/(d*x+c)^3+1/3/d*b^2/e^4/(d*x+c)^2*arccosh(d*x+c)*(d*x+c+1)^(
1/2)*(d*x+c-1)^(1/2)-1/3/d*b^2/e^4/(d*x+c)^3*arccosh(d*x+c)^2+1/3*b^2/d/e^4
/(d*x+c)-1/3*I/d*b^2/e^4*arccosh(d*x+c)*ln(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)))+1/3*I/d*b^2/e^4*arccosh(d*x+c)*ln(1-I*(d*x+c+(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)))-1/3*I/d*b^2/e^4*dilog(1+I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)))+1/3*I/d*b^2/e^4*dilog(1-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
-2/3/d*a*b/e^4/(d*x+c)^3*arccosh(d*x+c)-1/3/d*a*b/e^4*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*arctan(1/((d*x+c)^2-1)^(1/2))+1/3/d*a*b/e^4*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^2}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}-\frac{a^2}{3\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}+\int\frac{}{3\left(d^7e^4x^7+7\,cd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^4*e^4*x^3
+ 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^2/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*((3*a*b*d^3 +
 b^2*d^3)*x^3 + 3*(c^3 - c)*a*b + (c^3 - c)*b^2 + 3*(3*a*b*c*d^2 + b^2*c*d^
2)*x^2 + (b^2*c^2 + 3*(c^2 - 1)*a*b + (3*a*b*d^2 + b^2*d^2)*x^2 + 2*(3*a*b*
c*d + b^2*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(3*c^2*d - d)*a*
b + (3*c^2*d - d)*b^2)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c
)/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^5*e^4 - d^
5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 - 2*c^2*d
^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^6 + 6*c*d^
5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(5*c^3*d
^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*
d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*c^6*d*e^4
- 5*c^4*d*e^4)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c+d\,x))^2}{(c\,e+d\,e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{b^2\operatorname{acosh}^2(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx + \int \frac{2ab\operatorname{acosh}(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**2*acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(2*a*b*acosh(c + d*x)/(c
**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.113 $\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=382

$$\frac{6b^2e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{125d}+\frac{8b^2e^4(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\,\mathrm{c}\right.}{5d}$$

```
[Out] 16/25*a*b^2*e^4*x+16/25*b^3*e^4*(d*x+c)*arccosh(d*x+c)/d+8/75*b^2*e^4*(d*x+
c)^3*(a+b*arccosh(d*x+c))/d+6/125*b^2*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))/d+
1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))^3/d-4144/5625*b^3*e^4*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)/d-272/5625*b^3*e^4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)/d-6/625*b^3*e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-8/25*b*e^4*
(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-4/25*b*e^4*(d*x+c)
^2*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/25*b*e^4*(d*x
+c)^4*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.72, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5759, 5718, 5654, 74, 100}

$$\frac{6b^2e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{125d}+\frac{8b^2e^4(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{75d}+\frac{16}{25}ab^2e^4x+\frac{e^4(c+dx)^5\left(a+b\,\mathrm{c}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (16*a*b^2*e^4*x)/25 - (4144*b^3*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(
5625*d) - (272*b^3*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5
625*d) - (6*b^3*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(625*
d) + (16*b^3*e^4*(c + d*x)*ArcCosh[c + d*x])/(25*d) + (8*b^2*e^4*(c + d*x)^
3*(a + b*ArcCosh[c + d*x]))/(75*d) + (6*b^2*e^4*(c + d*x)^5*(a + b*ArcCosh[
c + d*x]))/(125*d) - (8*b*e^4*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])^2)/(25*d) - (4*b*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1
 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(25*d) - (3*b*e^4*Sqrt[-1 + c + d*x
]*(c + d*x)^4*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(25*d) + (e^4*(
c + d*x)^5*(a + b*ArcCosh[c + d*x])^3)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4 \left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^4x^4\left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^4\,\text{Subst}\left(\int x^4\left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)^3}{5d} - \frac{\left(3be^4\right)\text{Subst}\left(\int \frac{x^5\left(a+b\cosh^{-1}\right.}{\sqrt{-1+x}\,\sqrt{1+}}\right.}{5d}$$
$$= -\frac{3be^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{25d}$$
$$= \frac{6b^2e^4(c+dx)^5\left(a+b\cosh^{-1}(c+dx)\right)}{125d} - \frac{4be^4\sqrt{-1+c+dx}\,(c+dx}{}$$
$$= -\frac{6b^3e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{625d} + \frac{8b^2e^4(c+dx)^3\left(a+b\right.}{75d}$$
$$= \frac{16}{25}ab^2e^4x - \frac{8b^3e^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{225d} - \frac{6b^3e^4\sqrt{-1}}{}$$
$$= \frac{16}{25}ab^2e^4x - \frac{272b^3e^4\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5625d} - \frac{6b^3e^4\sqrt{}}{}$$
$$= \frac{16}{25}ab^2e^4x - \frac{32b^3e^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{45d} - \frac{272b^3e^4\sqrt{-1+c}}{}$$
$$= \frac{16}{25}ab^2e^4x - \frac{4144b^3e^4\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5625d} - \frac{272b^3e^4\sqrt{-1+}}{}$$

Mathematica [A] time = 0.61, size = 404, normalized size = 1.06

$$\frac{e^4\left(3a\left(25a^2+6b^2\right)(c+dx)^5 + \frac{1}{15}b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(-27\left(25a^2+2b^2\right)(c+dx)^4 - 4\left(225a^2+68b^2\right)(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^4*(240*a*b^2*(c + d*x) + 40*a*b^2*(c + d*x)^3 + 3*a*(25*a^2 + 6*b^2)*(c
+ d*x)^5 + (b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-8*(225*a^2 + 518*b^2)
- 4*(225*a^2 + 68*b^2)*(c + d*x)^2 - 27*(25*a^2 + 2*b^2)*(c + d*x)^4))/15 -
 b*(-240*b^2*(c + d*x) - 40*b^2*(c + d*x)^3 - 225*a^2*(c + d*x)^5 - 18*b^2*
(c + d*x)^5 + 240*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 120*a*b*Sqrt[-
1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] + 90*a*b*Sqrt[-1 + c + d*x]*(c +
 d*x)^4*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 15*b^2*(-15*a*(c + d*x)^5 + 8
*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 4*b*Sqrt[-1 + c + d*x]*(c + d*x)^
2*Sqrt[1 + c + d*x] + 3*b*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])
*ArcCosh[c + d*x]^2 + 75*b^3*(c + d*x)^5*ArcCosh[c + d*x]^3))/(375*d)
```

fricas [B] time = 0.72, size = 1074, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(45*(25*a^3 + 6*a*b^2)*d^5*e^4*x^5 + 225*(25*a^3 + 6*a*b^2)*c*d^4*e^
4*x^4 + 150*(4*a*b^2 + 3*(25*a^3 + 6*a*b^2)*c^2)*d^3*e^4*x^3 + 450*(4*a*b^2
*c + (25*a^3 + 6*a*b^2)*c^3)*d^2*e^4*x^2 + 225*(8*a*b^2*c^2 + (25*a^3 + 6*a
*b^2)*c^4 + 16*a*b^2)*d*e^4*x + 1125*(b^3*d^5*e^4*x^5 + 5*b^3*c*d^4*e^4*x^4
 + 10*b^3*c^2*d^3*e^4*x^3 + 10*b^3*c^3*d^2*e^4*x^2 + 5*b^3*c^4*d*e^4*x + b^
3*c^5*e^4)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 225*(15*a*b
^2*d^5*e^4*x^5 + 75*a*b^2*c*d^4*e^4*x^4 + 150*a*b^2*c^2*d^3*e^4*x^3 + 150*a
*b^2*c^3*d^2*e^4*x^2 + 75*a*b^2*c^4*d*e^4*x + 15*a*b^2*c^5*e^4 - (3*b^3*d^4
*e^4*x^4 + 12*b^3*c*d^3*e^4*x^3 + 2*(9*b^3*c^2 + 2*b^3)*d^2*e^4*x^2 + 4*(3*
b^3*c^3 + 2*b^3*c)*d*e^4*x + (3*b^3*c^4 + 4*b^3*c^2 + 8*b^3)*e^4)*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^
2 + 15*(9*(25*a^2*b + 2*b^3)*d^5*e^4*x^5 + 45*(25*a^2*b + 2*b^3)*c*d^4*e^4*
x^4 + 10*(4*b^3 + 9*(25*a^2*b + 2*b^3)*c^2)*d^3*e^4*x^3 + 30*(4*b^3*c + 3*(
25*a^2*b + 2*b^3)*c^3)*d^2*e^4*x^2 + 15*(8*b^3*c^2 + 3*(25*a^2*b + 2*b^3)*c
^4 + 16*b^3)*d*e^4*x + (40*b^3*c^3 + 9*(25*a^2*b + 2*b^3)*c^5 + 240*b^3*c)*
e^4 - 30*(3*a*b^2*d^4*e^4*x^4 + 12*a*b^2*c*d^3*e^4*x^3 + 2*(9*a*b^2*c^2 + 2
*a*b^2)*d^2*e^4*x^2 + 4*(3*a*b^2*c^3 + 2*a*b^2*c)*d*e^4*x + (3*a*b^2*c^4 +
4*a*b^2*c^2 + 8*a*b^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c
+ sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - (27*(25*a^2*b + 2*b^3)*d^4*e^4*x^4 +
 108*(25*a^2*b + 2*b^3)*c*d^3*e^4*x^3 + 2*(450*a^2*b + 136*b^3 + 81*(25*a^2
*b + 2*b^3)*c^2)*d^2*e^4*x^2 + 4*(27*(25*a^2*b + 2*b^3)*c^3 + 2*(225*a^2*b
+ 68*b^3)*c)*d*e^4*x + (27*(25*a^2*b + 2*b^3)*c^4 + 1800*a^2*b + 4144*b^3 +
 4*(225*a^2*b + 68*b^3)*c^2)*e^4)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^4 (b \operatorname{arcosh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4*(b*arccosh(d*x + c) + a)^3, x)
```

maple [A] time = 0.05, size = 450, normalized size = 1.18

$$\frac{\frac{(dx+c)^5 e^4 a^3}{5} + e^4 b^3 \left(\frac{(dx+c)^5 \operatorname{arccosh}(dx+c)^3}{5} - \frac{8 \operatorname{arccosh}(dx+c)^2 \sqrt{dx+c-1}\, \sqrt{dx+c+1}}{25} - \frac{3(dx+c)^4 \operatorname{arccosh}(dx+c)^2 \sqrt{dx+c-1}\, \sqrt{dx+c+1}}{25} - \frac{4(dx+}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*(d*x+c)^5*e^4*a^3+e^4*b^3*(1/5*(d*x+c)^5*arccosh(d*x+c)^3-8/25*arc
cosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-3/25*(d*x+c)^4*arccosh(d*x+c)
^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4/25*(d*x+c)^2*arccosh(d*x+c)^2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)+16/25*(d*x+c)*arccosh(d*x+c)-4144/5625*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)+6/125*(d*x+c)^5*arccosh(d*x+c)-6/625*(d*x+c)^4*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)-272/5625*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+
8/75*arccosh(d*x+c)*(d*x+c)^3)+3*e^4*a*b^2*(1/5*(d*x+c)^5*arccosh(d*x+c)^2-
16/75*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/25*(d*x+c)^4*arccosh
(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-8/75*arccosh(d*x+c)*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)*(d*x+c)^2+16/75*d*x+16/75*c+2/125*(d*x+c)^5+8/225*(d*x+c)^
3)+3*e^4*a^2*b*(1/5*(d*x+c)^5*arccosh(d*x+c)-1/75*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*(3*(d*x+c)^4+4*(d*x+c)^2+8)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/5*a^3*d^4*e^4*x^5 + a^3*c*d^3*e^4*x^4 + 2*a^3*c^2*d^2*e^4*x^3 + 2*a^3*c^3
*d*e^4*x^2 + 3*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*c^3*d*e^4 + (6
*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15
*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*
d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^
2*b*c^2*d^2*e^4 + 1/8*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*x^3/d^2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c
^4*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sq
rt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x +
2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)*c^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d
^4 + 9*(c^2 - 1)^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1
)*d)/d^5 + 55*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a^2*b*c
*d^3*e^4 + 1/200*(120*x^5*arccosh(d*x + c) - (24*sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*x^4/d^2 - 54*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^3/d^3 + 126*sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2*x^2/d^4 - 945*c^5*log(2*d^2*x + 2*c*d + 2
*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 315*sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1)*c^3*x/d^5 - 32*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x^2/d^4 +
 1050*(c^2 - 1)*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*d)/d^6 + 945*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^4/d^6 + 161*sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c*x/d^5 - 225*(c^2 - 1)^2*c*log(2*d^2*x +
2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^6 - 735*sqrt(d^2*x^2 + 2*c
*d*x + c^2 - 1)*(c^2 - 1)*c^2/d^6 + 64*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c
^2 - 1)^2/d^6)*d)*a^2*b*d^4*e^4 + a^3*c^4*e^4*x + 3*((d*x + c)*arccosh(d*x
+ c) - sqrt((d*x + c)^2 - 1))*a^2*b*c^4*e^4/d + 1/5*(b^3*d^4*e^4*x^5 + 5*b^
3*c*d^3*e^4*x^4 + 10*b^3*c^2*d^2*e^4*x^3 + 10*b^3*c^3*d*e^4*x^2 + 5*b^3*c^4
*e^4*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + integrate(3/
5*((5*a*b^2*d^7*e^4 - b^3*d^7*e^4)*x^7 + 7*(5*a*b^2*c*d^6*e^4 - b^3*c*d^6*e
^4)*x^6 + (5*(21*c^2*d^5*e^4 - d^5*e^4)*a*b^2 - (21*c^2*d^5*e^4 - d^5*e^4)*
b^3)*x^5 + 5*(5*(7*c^3*d^4*e^4 - c*d^4*e^4)*a*b^2 - (7*c^3*d^4*e^4 - c*d^4*
e^4)*b^3)*x^4 + 5*(c^7*e^4 - c^5*e^4)*a*b^2 + 5*(5*(7*c^4*d^3*e^4 - 2*c^2*d
^3*e^4)*a*b^2 - (7*c^4*d^3*e^4 - 2*c^2*d^3*e^4)*b^3)*x^3 + 5*((21*c^5*d^2*e
^4 - 10*c^3*d^2*e^4)*a*b^2 - 2*(2*c^5*d^2*e^4 - c^3*d^2*e^4)*b^3)*x^2 + ((5
*a*b^2*d^6*e^4 - b^3*d^6*e^4)*x^6 + 6*(5*a*b^2*c*d^5*e^4 - b^3*c*d^5*e^4)*x
^5 - 5*(3*b^3*c^2*d^4*e^4 - (15*c^2*d^4*e^4 - d^4*e^4)*a*b^2)*x^4 + 5*(c^6*
e^4 - c^4*e^4)*a*b^2 - 20*(b^3*c^3*d^3*e^4 - (5*c^3*d^3*e^4 - c*d^3*e^4)*a*
b^2)*x^3 - 15*(b^3*c^4*d^2*e^4 - (5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*a*b^2)*x^2
 - 5*(b^3*c^5*d*e^4 - 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*a*b^2)*x)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + 5*((7*c^6*d*e^4 - 5*c^4*d*e^4)*a*b^2 - (c^6*d*e^4
- c^4*d*e^4)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(
d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c +
1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,(a+b\,\mathrm{acosh}\,(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^3, x)
```

sympy [A] time = 16.25, size = 2518, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**4*e**4*x + 2*a**3*c**3*d*e**4*x**2 + 2*a**3*c**2*d**2*e*
*4*x**3 + a**3*c*d**3*e**4*x**4 + a**3*d**4*e**4*x**5/5 + 3*a**2*b*c**5*e**
4*acosh(c + d*x)/(5*d) + 3*a**2*b*c**4*e**4*x*acosh(c + d*x) - 3*a**2*b*c**
4*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 6*a**2*b*c**3*d*e**4*x
**2*acosh(c + d*x) - 12*a**2*b*c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)/25 + 6*a**2*b*c**2*d**2*e**4*x**3*acosh(c + d*x) - 18*a**2*b*c**2*d*e*
*4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 - 4*a**2*b*c**2*e**4*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 3*a**2*b*c*d**3*e**4*x**4*acosh(c +
 d*x) - 12*a**2*b*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25
- 8*a**2*b*c*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/25 + 3*a**2*b*d**4
*e**4*x**5*acosh(c + d*x)/5 - 3*a**2*b*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)/25 - 4*a**2*b*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)/25 - 8*a**2*b*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(25*d) + 3*a*b*
*2*c**5*e**4*acosh(c + d*x)**2/(5*d) + 3*a*b**2*c**4*e**4*x*acosh(c + d*x)*
*2 + 6*a*b**2*c**4*e**4*x/25 - 6*a*b**2*c**4*e**4*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)*acosh(c + d*x)/(25*d) + 6*a*b**2*c**3*d*e**4*x**2*acosh(c + d*x
)**2 + 12*a*b**2*c**3*d*e**4*x**2/25 - 24*a*b**2*c**3*e**4*x*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 6*a*b**2*c**2*d**2*e**4*x**3*aco
sh(c + d*x)**2 + 12*a*b**2*c**2*d**2*e**4*x**3/25 - 36*a*b**2*c**2*d*e**4*x
**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*c**2*
e**4*x/25 - 8*a*b**2*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
 + d*x)/(25*d) + 3*a*b**2*c*d**3*e**4*x**4*acosh(c + d*x)**2 + 6*a*b**2*c*d
**3*e**4*x**4/25 - 24*a*b**2*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*c*d*e**4*x**2/25 - 16*a*b**2*c*e**4*x*
sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 3*a*b**2*d**4*e**4
*x**5*acosh(c + d*x)**2/5 + 6*a*b**2*d**4*e**4*x**5/125 - 6*a*b**2*d**3*e**
4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/25 + 8*a*b**2*d*
*2*e**4*x**3/75 - 8*a*b**2*d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)
*acosh(c + d*x)/25 + 16*a*b**2*e**4*x/25 - 16*a*b**2*e**4*sqrt(c**2 + 2*c*d
*x + d**2*x**2 - 1)*acosh(c + d*x)/(25*d) + b**3*c**5*e**4*acosh(c + d*x)**
3/(5*d) + 6*b**3*c**5*e**4*acosh(c + d*x)/(125*d) + b**3*c**4*e**4*x*acosh(
c + d*x)**3 + 6*b**3*c**4*e**4*x*acosh(c + d*x)/25 - 3*b**3*c**4*e**4*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(25*d) - 6*b**3*c**4*e**4
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(625*d) + 2*b**3*c**3*d*e**4*x**2*aco
sh(c + d*x)**3 + 12*b**3*c**3*d*e**4*x**2*acosh(c + d*x)/25 - 12*b**3*c**3*
e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 24*b**3*
c**3*e**4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/625 + 8*b**3*c**3*e**4*aco
sh(c + d*x)/(75*d) + 2*b**3*c**2*d**2*e**4*x**3*acosh(c + d*x)**3 + 12*b**3
*c**2*d**2*e**4*x**3*acosh(c + d*x)/25 - 18*b**3*c**2*d*e**4*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 36*b**3*c**2*d*e**4*x**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/625 + 8*b**3*c**2*e**4*x*acosh(c + d*
x)/25 - 4*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x
)**2/(25*d) - 272*b**3*c**2*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(5625
*d) + b**3*c*d**3*e**4*x**4*acosh(c + d*x)**3 + 6*b**3*c*d**3*e**4*x**4*aco
sh(c + d*x)/25 - 12*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
 1)*acosh(c + d*x)**2/25 - 24*b**3*c*d**2*e**4*x**3*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)/625 + 8*b**3*c*d*e**4*x**2*acosh(c + d*x)/25 - 8*b**3*c*e**4*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 544*b**3*c*e*
*4*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/5625 + 16*b**3*c*e**4*acosh(c + d
*x)/(25*d) + b**3*d**4*e**4*x**5*acosh(c + d*x)**3/5 + 6*b**3*d**4*e**4*x**
5*acosh(c + d*x)/125 - 3*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d**2*x**
2 - 1)*acosh(c + d*x)**2/25 - 6*b**3*d**3*e**4*x**4*sqrt(c**2 + 2*c*d*x + d
```

```
**2*x**2 - 1)/625 + 8*b**3*d**2*e**4*x**3*acosh(c + d*x)/75 - 4*b**3*d*e**4
*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/25 - 272*b**3*
d*e**4*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/5625 + 16*b**3*e**4*x*acos
h(c + d*x)/25 - 8*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)**2/(25*d) - 4144*b**3*e**4*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(5625*
d), Ne(d, 0)), (c**4*e**4*x*(a + b*acosh(c))**3, True))
```

3.114 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=307

$$\frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{9b^2e^3(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d}$$

```
[Out] -45/256*b^3*e^3*arccosh(d*x+c)/d+9/32*b^2*e^3*(d*x+c)^2*(a+b*arccosh(d*x+c)
)/d+3/32*b^2*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))/d-3/32*e^3*(a+b*arccosh(d*x
+c))^3/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^3/d-45/256*b^3*e^3*(d*x+c)*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/128*b^3*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/d-9/32*b*e^3*(d*x+c)*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)/d-3/16*b*e^3*(d*x+c)^3*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.62, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5759, 5676, 90, 52, 100}

$$\frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{9b^2e^3(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{32d}+\frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (-45*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(256*d) - (3*b
^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(128*d) - (45*b^3*
e^3*ArcCosh[c + d*x])/(256*d) + (9*b^2*e^3*(c + d*x)^2*(a + b*ArcCosh[c + d
*x]))/(32*d) + (3*b^2*e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x]))/(32*d) - (9
*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*
x])^2)/(32*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(
a + b*ArcCosh[c + d*x])^2)/(16*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^3)/(32*
d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^3)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3 \left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)^3}{4d} - \frac{(3be^3)\,\text{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\,\sqrt{1+x}}\right.}{4d}$$

$$= -\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{16d} +$$

$$= \frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)}{32d} - \frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{}}{}$$

$$= -\frac{3b^3e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{128d} + \frac{9b^2e^3(c+dx)^2\left(a+b\,\mathrm{co}\right.}{32d}$$

$$= -\frac{9b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{64d} - \frac{3b^3e^3\sqrt{-1+c+dx}\,(c+}{128d}$$

$$= -\frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{256d} - \frac{3b^3e^3\sqrt{-1+c+dx}\,(c+}{128d}$$

$$= -\frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{256d} - \frac{3b^3e^3\sqrt{-1+c+dx}\,(c+}{128d}$$

Mathematica [A] time = 0.56, size = 359, normalized size = 1.17

$$\frac{e^3\left(8a\left(8a^2+3b^2\right)(c+dx)^4+3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)\left(-2\left(8a^2+b^2\right)(c+dx)^2-3\left(8a^2+5b^2\right)\right)-9b\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^3*(72*a*b^2*(c + d*x)^2 + 8*a*(8*a^2 + 3*b^2)*(c + d*x)^4 + 3*b*Sqrt[-1
+ c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3*(8*a^2 + 5*b^2) - 2*(8*a^2 + b^2
)*(c + d*x)^2) - 24*b*(c + d*x)*(-3*b^2*(c + d*x) - 8*a^2*(c + d*x)^3 - b^2
*(c + d*x)^3 + 6*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 4*a*b*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 24*b^2*(-3*a +
8*a*(c + d*x)^4 - 3*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*
Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 8*b^
3*(-3 + 8*(c + d*x)^4)*ArcCosh[c + d*x]^3 - 9*b*(8*a^2 + 5*b^2)*Log[c + d*x
 + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(256*d)
```

fricas [B] time = 0.70, size = 828, normalized size = 2.70

$$\frac{8\left(8a^3+3ab^2\right)d^4e^3x^4+32\left(8a^3+3ab^2\right)cd^3e^3x^3+24\left(3ab^2+2\left(8a^3+3ab^2\right)c^2\right)d^2e^3x^2+16\left(9ab^2c+2\left(8a^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^3 + 3*a*b^2)*d^4*e^3*x^4 + 32*(8*a^3 + 3*a*b^2)*c*d^3*e^3*x^3
 + 24*(3*a*b^2 + 2*(8*a^3 + 3*a*b^2)*c^2)*d^2*e^3*x^2 + 16*(9*a*b^2*c + 2*(
8*a^3 + 3*a*b^2)*c^3)*d*e^3*x + 8*(8*b^3*d^4*e^3*x^4 + 32*b^3*c*d^3*e^3*x^3
 + 48*b^3*c^2*d^2*e^3*x^2 + 32*b^3*c^3*d*e^3*x + (8*b^3*c^4 - 3*b^3)*e^3)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 24*(8*a*b^2*d^4*e^3*x^4
 + 32*a*b^2*c*d^3*e^3*x^3 + 48*a*b^2*c^2*d^2*e^3*x^2 + 32*a*b^2*c^3*d*e^3*x
 + (8*a*b^2*c^4 - 3*a*b^2)*e^3 - (2*b^3*d^3*e^3*x^3 + 6*b^3*c*d^2*e^3*x^2 +
 3*(2*b^3*c^2 + b^3)*d*e^3*x + (2*b^3*c^3 + 3*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*(8
*(8*a^2*b + b^3)*d^4*e^3*x^4 + 32*(8*a^2*b + b^3)*c*d^3*e^3*x^3 + 24*(b^3 +
 2*(8*a^2*b + b^3)*c^2)*d^2*e^3*x^2 + 16*(3*b^3*c + 2*(8*a^2*b + b^3)*c^3)*
d*e^3*x + (24*b^3*c^2 + 8*(8*a^2*b + b^3)*c^4 - 24*a^2*b - 15*b^3)*e^3 - 16
*(2*a*b^2*d^3*e^3*x^3 + 6*a*b^2*c*d^2*e^3*x^2 + 3*(2*a*b^2*c^2 + a*b^2)*d*e
^3*x + (2*a*b^2*c^3 + 3*a*b^2*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*(2*(8*a^2*b + b^3)*d^3*e
^3*x^3 + 6*(8*a^2*b + b^3)*c*d^2*e^3*x^2 + 3*(8*a^2*b + 5*b^3 + 2*(8*a^2*b
+ b^3)*c^2)*d*e^3*x + (2*(8*a^2*b + b^3)*c^3 + 3*(8*a^2*b + 5*b^3)*c)*e^3)*
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3(b\operatorname{arcosh}(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^3, x)
```

maple [B] time = 0.05, size = 1554, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] -3/16*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3*a^2*b*e^3+3*d^2*arccosh(d*x+c
)*x^3*a^2*b*c*e^3+3*d^2*e^3*a*b^2*arccosh(d*x+c)^2*x^3*c+9/2*d*e^3*a*b^2*ar
ccosh(d*x+c)^2*x^2*c^2-3/16*d^2*e^3*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)*x^3-9/128*d*e^3*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c+9/2
*d*arccosh(d*x+c)*x^2*a^2*b*c^2*e^3-9/32/d*e^3*b^3*arccosh(d*x+c)^2*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)*c-3/16/d*e^3*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)*c^3-9/16*e^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)*x-9/16*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*a^2*b*c^2*e^3-9/16*e^3*b^3*
arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-9/32/d*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)*a^2*b*c*e^3-3/16/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a^2*b
*c^3*e^3-9/8*d*e^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2
*c+1/4/d*a^3*c^4*e^3-3/32/d*e^3*b^3*arccosh(d*x+c)^3+1/4*d^3*x^4*a^3*e^3+x*
a^3*c^3*e^3-9/8*e^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*
c^2-9/16/d*e^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-9/16*
d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*a^2*b*c*e^3-3/8*d^2*e^3*a*b^2*arccosh
(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-9/32/d*e^3*a^2*b*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))-3/8/d*e
^3*a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-9/16*d*e^3*b^3*
arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-9/128*e^3*b^3*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2+3*e^3*a*b^2*arccosh(d*x+c)^2*x*c^3-45/256*b
^3*e^3*arccosh(d*x+c)/d+3/32/d*e^3*a*b^2*c^4+9/32/d*e^3*a*b^2*c^2+3/32*d^3*
e^3*a*b^2*x^4+3/2*d*x^2*a^3*c^2*e^3-9/32/d*e^3*a*b^2*arccosh(d*x+c)^2+1/4/d
*e^3*b^3*arccosh(d*x+c)^3*c^4+3/32/d*e^3*b^3*arccosh(d*x+c)*c^4+9/32/d*e^3*
b^3*arccosh(d*x+c)*c^2+3/32*d^3*e^3*b^3*arccosh(d*x+c)*x^4+9/32*d*e^3*b^3*a
```

```
rccosh(d*x+c)*x^2+1/4*d^3*e^3*b^3*arccosh(d*x+c)^3*x^4-45/256*e^3*b^3*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*x+9/16*e^3*b^3*arccosh(d*x+c)*x*c+e^3*b^3*arccos
h(d*x+c)^3*x*c^3+3/8*e^3*b^3*arccosh(d*x+c)*x*c^3+3/8*e^3*a*b^2*x*c^3+9/16*
e^3*a*b^2*x*c+9/32*d*e^3*a*b^2*x^2+d^2*x^3*a^3*c*e^3-45/256/d*e^3*b^3*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)*c+3/4/d*arccosh(d*x+c)*a^2*b*c^4*e^3-3/128/d*e^3
*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3+3*arccosh(d*x+c)*x*a^2*b*c^3*e^3-9
/32*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*a^2*b*e^3+3/2*d*e^3*b^3*arccosh(d*x+c
)^3*x^2*c^2+3/4*d^3*e^3*a*b^2*arccosh(d*x+c)^2*x^4+3/4/d*e^3*a*b^2*arccosh(
d*x+c)^2*c^4-9/32*e^3*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*
x+3/8*d^2*e^3*a*b^2*x^3*c+9/16*d*e^3*a*b^2*x^2*c^2+3/4*d^3*arccosh(d*x+c)*x
^4*a^2*b*e^3+3/8*d^2*e^3*b^3*arccosh(d*x+c)*x^3*c+9/16*d*e^3*b^3*arccosh(d*
x+c)*x^2*c^2-3/128*d^2*e^3*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3+d^2*e^3*
b^3*arccosh(d*x+c)^3*x^3*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*d^3*e^3*x^4 + a^3*c*d^2*e^3*x^3 + 3/2*a^3*c^2*d*e^3*x^2 + 9/4*(2*x^
2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*l
og(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*c^2*d*e^3 + 1/2*(6*x^3*arccosh(d*x
 + c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x
 + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x
^2 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d
^4 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^2*b*c*d^2*e^3 +
1/32*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^
2 - 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x +
2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^
2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c
^3/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^
2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a^2*b*d^3*e^3 + a^3*c^3*
e^3*x + 3*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b*c^3*e^
3/d + 1/4*(b^3*d^3*e^3*x^4 + 4*b^3*c*d^2*e^3*x^3 + 6*b^3*c^2*d*e^3*x^2 + 4*
b^3*c^3*e^3*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + integ
rate(3/4*((4*a*b^2*d^6*e^3 - b^3*d^6*e^3)*x^6 + 6*(4*a*b^2*c*d^5*e^3 - b^3*
c*d^5*e^3)*x^5 + (4*(15*c^2*d^4*e^3 - d^4*e^3)*a*b^2 - (15*c^2*d^4*e^3 - d^
4*e^3)*b^3)*x^4 + 4*(c^6*e^3 - c^4*e^3)*a*b^2 + 4*(4*(5*c^3*d^3*e^3 - c*d^3
*e^3)*a*b^2 - (5*c^3*d^3*e^3 - c*d^3*e^3)*b^3)*x^3 + 2*(6*(5*c^4*d^2*e^3 -
2*c^2*d^2*e^3)*a*b^2 - (7*c^4*d^2*e^3 - 3*c^2*d^2*e^3)*b^3)*x^2 + ((4*a*b^2
*d^5*e^3 - b^3*d^5*e^3)*x^5 + 5*(4*a*b^2*c*d^4*e^3 - b^3*c*d^4*e^3)*x^4 + 4
*(c^5*e^3 - c^3*e^3)*a*b^2 - 2*(5*b^3*c^2*d^3*e^3 - 2*(10*c^2*d^3*e^3 - d^3
*e^3)*a*b^2)*x^3 - 2*(5*b^3*c^3*d^2*e^3 - 2*(10*c^3*d^2*e^3 - 3*c*d^2*e^3)*
a*b^2)*x^2 - 4*(b^3*c^4*d*e^3 - (5*c^4*d*e^3 - 3*c^2*d*e^3)*a*b^2)*x)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + 4*(2*(3*c^5*d*e^3 - 2*c^3*d*e^3)*a*b^2 - (
c^5*d*e^3 - c^3*d*e^3)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{acosh}\,(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^3, x)
```

sympy [A] time = 9.57, size = 1828, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**3*e**3*x + 3*a**3*c**2*d*e**3*x**2/2 + a**3*c*d**2*e**3*
x**3 + a**3*d**3*e**3*x**4/4 + 3*a**2*b*c**4*e**3*acosh(c + d*x)/(4*d) + 3*
a**2*b*c**3*e**3*x*acosh(c + d*x) - 3*a**2*b*c**3*e**3*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)/(16*d) + 9*a**2*b*c**2*d*e**3*x**2*acosh(c + d*x)/2 - 9*a*
*2*b*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 + 3*a**2*b*c*d**2*
e**3*x**3*acosh(c + d*x) - 9*a**2*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**
2*x**2 - 1)/16 - 9*a**2*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(32*d
) + 3*a**2*b*d**3*e**3*x**4*acosh(c + d*x)/4 - 3*a**2*b*d**2*e**3*x**3*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/16 - 9*a**2*b*e**3*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)/32 - 9*a**2*b*e**3*acosh(c + d*x)/(32*d) + 3*a*b**2*c**4*e*
*3*acosh(c + d*x)**2/(4*d) + 3*a*b**2*c**3*e**3*x*acosh(c + d*x)**2 + 3*a*b
**2*c**3*e**3*x/8 - 3*a*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)
*acosh(c + d*x)/(8*d) + 9*a*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2/2 + 9*a
*b**2*c**2*d*e**3*x**2/16 - 9*a*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)*acosh(c + d*x)/8 + 3*a*b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 +
 3*a*b**2*c*d**2*e**3*x**3/8 - 9*a*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)/8 + 9*a*b**2*c*e**3*x/16 - 9*a*b**2*c*e**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(16*d) + 3*a*b**2*d**3*e
**3*x**4*acosh(c + d*x)**2/4 + 3*a*b**2*d**3*e**3*x**4/32 - 3*a*b**2*d**2*e
**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/8 + 9*a*b**2*d
*e**3*x**2/32 - 9*a*b**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(
c + d*x)/16 - 9*a*b**2*e**3*acosh(c + d*x)**2/(32*d) + b**3*c**4*e**3*acosh
(c + d*x)**3/(4*d) + 3*b**3*c**4*e**3*acosh(c + d*x)/(32*d) + b**3*c**3*e**
3*x*acosh(c + d*x)**3 + 3*b**3*c**3*e**3*x*acosh(c + d*x)/8 - 3*b**3*c**3*e
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(16*d) - 3*b**3*
c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(128*d) + 3*b**3*c**2*d*e**3
*x**2*acosh(c + d*x)**3/2 + 9*b**3*c**2*d*e**3*x**2*acosh(c + d*x)/16 - 9*b
**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 -
 9*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*c**2*
e**3*acosh(c + d*x)/(32*d) + b**3*c*d**2*e**3*x**3*acosh(c + d*x)**3 + 3*b*
*3*c*d**2*e**3*x**3*acosh(c + d*x)/8 - 9*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c
*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 - 9*b**3*c*d*e**3*x**2*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*c*e**3*x*acosh(c + d*x)/16 - 9*b**
3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(32*d) - 45
*b**3*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(256*d) + b**3*d**3*e**3*
x**4*acosh(c + d*x)**3/4 + 3*b**3*d**3*e**3*x**4*acosh(c + d*x)/32 - 3*b**3
*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/16 -
 3*b**3*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/128 + 9*b**3*d*
e**3*x**2*acosh(c + d*x)/32 - 9*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)**2/32 - 45*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/256 - 3*b**3*e**3*acosh(c + d*x)**3/(32*d) - 45*b**3*e**3*acosh(c + d
*x)/(256*d), Ne(d, 0)), (c**3*e**3*x*(a + b*acosh(c))**3, True))
```

3.115 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^3 dx$

Optimal. Leaf size=262

$$\frac{2b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{9d}+\frac{4}{3}ab^2e^2x-\frac{be^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\cdots$$

```
[Out] 4/3*a*b^2*e^2*x+4/3*b^3*e^2*(d*x+c)*arccosh(d*x+c)/d+2/9*b^2*e^2*(d*x+c)^3*
(a+b*arccosh(d*x+c))/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^3/d-40/27*b^3
*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-2/27*b^3*e^2*(d*x+c)^2*(d*x+c-1)^(1/
2)*(d*x+c+1)^(1/2)/d-2/3*b*e^2*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)/d-1/3*b*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)/d
```

Rubi [A] time = 0.47, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5759, 5718, 5654, 74, 100}

$$\frac{2b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{9d}+\frac{4}{3}ab^2e^2x-\frac{be^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (4*a*b^2*e^2*x)/3 - (40*b^3*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(27*d
) - (2*b^3*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(27*d) + (
4*b^3*e^2*(c + d*x)*ArcCosh[c + d*x])/(3*d) + (2*b^2*e^2*(c + d*x)^3*(a + b
*ArcCosh[c + d*x]))/(9*d) - (2*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(
a + b*ArcCosh[c + d*x])^2)/(3*d) - (b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(3*d) + (e^2*(c + d*x)^3*(a + b
*ArcCosh[c + d*x])^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^2 \text{Subst}\left(\int x^2 \left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e^2(c+dx)^3 \left(a+b\cosh^{-1}(c+dx)\right)^3}{3d} - \frac{\left(be^2\right) \text{Subst}\left(\int \frac{x^3\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\right.}{d}$$
$$= -\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3d} + \frac{e^2}{}$$
$$= \frac{2b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)}{9d} - \frac{2be^2\sqrt{-1+c+dx}\,\sqrt{1+c+}}{3d}$$
$$= \frac{4}{3}ab^2e^2x - \frac{2b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{27d} + \frac{2b^2e^2(c+dx)^3}{}$$
$$= \frac{4}{3}ab^2e^2x - \frac{2b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{27d} + \frac{4b^3e^2(c+dx)}{3}$$
$$= \frac{4}{3}ab^2e^2x - \frac{40b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{27d} - \frac{2b^3e^2\sqrt{-1+c+dx}\,(c}{27d}$$

Mathematica [**A**] time = 0.43, size = 296, normalized size = 1.13

$$\frac{e^2\left(a\left(3a^2+2b^2\right)(c+dx)^3+\frac{1}{3}b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(-\left(\left(9a^2+2b^2\right)(c+dx)^2\right)-2\left(9a^2+20b^2\right)\right)-b\cosh^{-1}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^2*(12*a*b^2*(c + d*x) + a*(3*a^2 + 2*b^2)*(c + d*x)^3 + (b*Sqrt[-1 + c +
 d*x]*Sqrt[1 + c + d*x]*(-2*(9*a^2 + 20*b^2) - (9*a^2 + 2*b^2)*(c + d*x)^2)
)/3 - b*(-12*b^2*(c + d*x) - 9*a^2*(c + d*x)^3 - 2*b^2*(c + d*x)^3 + 12*a*b
*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 6*a*b*Sqrt[-1 + c + d*x]*(c + d*x)^
2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 3*b^2*(-3*a*(c + d*x)^3 + 2*b*Sqrt[
-1 + c + d*x]*Sqrt[1 + c + d*x] + b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 +
 c + d*x])*ArcCosh[c + d*x]^2 + 3*b^3*(c + d*x)^3*ArcCosh[c + d*x]^3))/(9*d
)
```

fricas [**B**] time = 2.08, size = 607, normalized size = 2.32

$$\frac{3\left(3\,a^3+2\,ab^2\right)d^3e^2x^3+9\left(3\,a^3+2\,ab^2\right)cd^2e^2x^2+9\left(4\,ab^2+\left(3\,a^3+2\,ab^2\right)c^2\right)de^2x+9\left(b^3d^3e^2x^3+3\,b^3cd^2e^2x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(3*(3*a^3 + 2*a*b^2)*d^3*e^2*x^3 + 9*(3*a^3 + 2*a*b^2)*c*d^2*e^2*x^2 +
 9*(4*a*b^2 + (3*a^3 + 2*a*b^2)*c^2)*d*e^2*x + 9*(b^3*d^3*e^2*x^3 + 3*b^3*c
*d^2*e^2*x^2 + 3*b^3*c^2*d*e^2*x + b^3*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))^3 + 9*(3*a*b^2*d^3*e^2*x^3 + 9*a*b^2*c*d^2*e^2*x^2 +
```

```
9*a*b^2*c^2*d*e^2*x + 3*a*b^2*c^3*e^2 - (b^3*d^2*e^2*x^2 + 2*b^3*c*d*e^2*x
+ (b^3*c^2 + 2*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*((9*a^2*b + 2*b^3)*d^3*e^2*x^3 + 3*
(9*a^2*b + 2*b^3)*c*d^2*e^2*x^2 + 3*(4*b^3 + (9*a^2*b + 2*b^3)*c^2)*d*e^2*x
 + (12*b^3*c + (9*a^2*b + 2*b^3)*c^3)*e^2 - 6*(a*b^2*d^2*e^2*x^2 + 2*a*b^2*
c*d*e^2*x + (a*b^2*c^2 + 2*a*b^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - ((9*a^2*b + 2*b^3)*d^2*e^
2*x^2 + 2*(9*a^2*b + 2*b^3)*c*d*e^2*x + (18*a^2*b + 40*b^3 + (9*a^2*b + 2*b
^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^3, x)
```

maple [A] time = 0.05, size = 326, normalized size = 1.24

$$\frac{\frac{(dx+c)^3 e^2 a^3}{3} + e^2 b^3 \left(\frac{(dx+c)^3 \operatorname{arccosh}(dx+c)^3}{3} - \frac{2 \operatorname{arccosh}(dx+c)^2 \sqrt{dx+c-1}\,\sqrt{dx+c+1}}{3} - \frac{(dx+c)^2 \operatorname{arccosh}(dx+c)^2 \sqrt{dx+c-1}\,\sqrt{dx+c+1}}{3} + \frac{4(d\ldots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^3+e^2*b^3*(1/3*(d*x+c)^3*arccosh(d*x+c)^3-2/3*arcc
osh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/3*(d*x+c)^2*arccosh(d*x+c)^2
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4/3*(d*x+c)*arccosh(d*x+c)-40/27*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)+2/9*arccosh(d*x+c)*(d*x+c)^3-2/27*(d*x+c)^2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2))+3*e^2*a*b^2*(1/3*(d*x+c)^3*arccosh(d*x+c)^2-4/9*ar
ccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/9*arccosh(d*x+c)*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+4/9*d*x+4/9*c+2/27*(d*x+c)^3)+3*e^2*a^2*b*(1
/3*arccosh(d*x+c)*(d*x+c)^3-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+
2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*d^2*e^2*x^3 + a^3*c*d*e^2*x^2 + 3/2*(2*x^2*arccosh(d*x + c) - d*(3*
c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
*c/d^3))*a^2*b*c*d*e^2 + 1/6*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 +
2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9
*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d
^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*(c^2 - 1)/d^4))*a^2*b*d^2*e^2 + a^3*c^2*e^2*x + 3*((d*x + c)*a
rccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b*c^2*e^2/d + 1/3*(b^3*d^2*e^2
*x^3 + 3*b^3*c*d*e^2*x^2 + 3*b^3*c^2*e^2*x)*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^3 + integrate(((3*a*b^2*d^5*e^2 - b^3*d^5*e^2)*x^5 + 5*
```

```
(3*a*b^2*c*d^4*e^2 - b^3*c*d^4*e^2)*x^4 + 3*(c^5*e^2 - c^3*e^2)*a*b^2 + (3*
(10*c^2*d^3*e^2 - d^3*e^2)*a*b^2 - (10*c^2*d^3*e^2 - d^3*e^2)*b^3)*x^3 + 3*
((10*c^3*d^2*e^2 - 3*c*d^2*e^2)*a*b^2 - (3*c^3*d^2*e^2 - c*d^2*e^2)*b^3)*x^
2 + ((3*a*b^2*d^4*e^2 - b^3*d^4*e^2)*x^4 + 3*(c^4*e^2 - c^2*e^2)*a*b^2 + 4*
(3*a*b^2*c*d^3*e^2 - b^3*c*d^3*e^2)*x^3 - 3*(2*b^3*c^2*d^2*e^2 - (6*c^2*d^2
*e^2 - d^2*e^2)*a*b^2)*x^2 - 3*(b^3*c^3*d*e^2 - 2*(2*c^3*d*e^2 - c*d*e^2)*a
*b^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 3*((5*c^4*d*e^2 - 3*c^2*d*e^
2)*a*b^2 - (c^4*d*e^2 - c^2*d*e^2)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^
2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{acosh}\,(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^3, x)
```

sympy [A] time = 4.72, size = 1173, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c**2*e**2*x + a**3*c*d*e**2*x**2 + a**3*d**2*e**2*x**3/3 +
a**2*b*c**3*e**2*acosh(c + d*x)/d + 3*a**2*b*c**2*e**2*x*acosh(c + d*x) - a
**2*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(3*d) + 3*a**2*b*c*d*e
**2*x**2*acosh(c + d*x) - 2*a**2*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/3 + a**2*b*d**2*e**2*x**3*acosh(c + d*x) - a**2*b*d*e**2*x**2*sqrt(c*
*2 + 2*c*d*x + d**2*x**2 - 1)/3 - 2*a**2*b*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)/(3*d) + a*b**2*c**3*e**2*acosh(c + d*x)**2/d + 3*a*b**2*c**2*e**2
*x*acosh(c + d*x)**2 + 2*a*b**2*c**2*e**2*x/3 - 2*a*b**2*c**2*e**2*sqrt(c**
2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + 3*a*b**2*c*d*e**2*x**2*
acosh(c + d*x)**2 + 2*a*b**2*c*d*e**2*x**2/3 - 4*a*b**2*c*e**2*x*sqrt(c**2
+ 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/3 + a*b**2*d**2*e**2*x**3*acosh(c
 + d*x)**2 + 2*a*b**2*d**2*e**2*x**3/9 - 2*a*b**2*d*e**2*x**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/3 + 4*a*b**2*e**2*x/3 - 4*a*b**2*e**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + b**3*c**3*e**
2*acosh(c + d*x)**3/(3*d) + 2*b**3*c**3*e**2*acosh(c + d*x)/(9*d) + b**3*c*
*2*e**2*x*acosh(c + d*x)**3 + 2*b**3*c**2*e**2*x*acosh(c + d*x)/3 - b**3*c*
*2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 2*b*
*3*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + b**3*c*d*e**2*x*
*2*acosh(c + d*x)**3 + 2*b**3*c*d*e**2*x**2*acosh(c + d*x)/3 - 2*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 4*b**3*c*e**
2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27 + 4*b**3*c*e**2*acosh(c + d*x)/
(3*d) + b**3*d**2*e**2*x**3*acosh(c + d*x)**3/3 + 2*b**3*d**2*e**2*x**3*aco
sh(c + d*x)/9 - b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh
(c + d*x)**2/3 - 2*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27
 + 4*b**3*e**2*x*acosh(c + d*x)/3 - 2*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2*
x**2 - 1)*acosh(c + d*x)**2/(3*d) - 40*b**3*e**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acosh(c))**3, True))
```

3.116 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=175

$$\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{3be\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)}{}$$

```
[Out] -3/8*b^3*e*arccosh(d*x+c)/d+3/4*b^2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))/d-1/4*
e*(a+b*arccosh(d*x+c))^3/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^3/d-3/8*b^3
*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/4*b*e*(d*x+c)*(a+b*arccosh(d
*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.36, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5866, 12, 5662, 5759, 5676, 90, 52}

$$\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d}-\frac{3be\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d}+\frac{e(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (-3*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(8*d) - (3*b^3*e*
ArcCosh[c + d*x])/(8*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x]))/(4
*d) - (3*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[
c + d*x])^2)/(4*d) - (e*(a + b*ArcCosh[c + d*x])^3)/(4*d) + (e*(c + d*x)^2*
(a + b*ArcCosh[c + d*x])^3)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^3 dx, x, c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^3}{2d} - \frac{(3be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}} dx\right.}{2d}$$
$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{4d} + \frac{e(c}{}$$
$$= \frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)}{4d} - \frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+}}{4}$$
$$= -\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{8d} + \frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}\right.}{4d}$$
$$= -\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{8d} - \frac{3b^3e\cosh^{-1}(c+dx)}{8d} + \frac{3b^2}{}$$

Mathematica [A] time = 0.32, size = 244, normalized size = 1.39

$$\frac{e\left(2a\left(2a^2+3b^2\right)(c+dx)^2 - 3b\left(2a^2+b^2\right)\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1} - 3b\left(2a^2+b^2\right)\log\left(\sqrt{c+dx-1}\,\sqrt{c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e*(2*a*(2*a^2 + 3*b^2)*(c + d*x)^2 - 3*b*(2*a^2 + b^2)*Sqrt[-1 + c + d*x]*
(c + d*x)*Sqrt[1 + c + d*x] - 6*b*(c + d*x)*(-2*a^2*(c + d*x) - b^2*(c + d*
x) + 2*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 6*b^2*(
-a + 2*a*(c + d*x)^2 - b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])*Ar
cCosh[c + d*x]^2 + 2*b^3*(-1 + 2*(c + d*x)^2)*ArcCosh[c + d*x]^3 - 3*b*(2*a
^2 + b^2)*Log[c + d*x + Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(8*d)
```

fricas [B] time = 0.86, size = 395, normalized size = 2.26

$$\frac{2\left(2\,a^3+3\,ab^2\right)d^2ex^2+4\left(2\,a^3+3\,ab^2\right)cdex+2\left(2\,b^3d^2ex^2+4\,b^3cdex+\left(2\,b^3c^2-b^3\right)e\right)\log\left(dx+c+\sqrt{d^2x^2+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^3 + 3*a*b^2)*d^2*e*x^2 + 4*(2*a^3 + 3*a*b^2)*c*d*e*x + 2*(2*b^3
*d^2*e*x^2 + 4*b^3*c*d*e*x + (2*b^3*c^2 - b^3)*e)*log(d*x + c + sqrt(d^2*x^
2 + 2*c*d*x + c^2 - 1))^3 + 6*(2*a*b^2*d^2*e*x^2 + 4*a*b^2*c*d*e*x + (2*a*b
^2*c^2 - a*b^2)*e - (b^3*d*e*x + b^3*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)
)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 3*(2*(2*a^2*b + b^3)
*d^2*e*x^2 + 4*(2*a^2*b + b^3)*c*d*e*x - (2*a^2*b + b^3 - 2*(2*a^2*b + b^3)
*c^2)*e - 4*(a*b^2*d*e*x + a*b^2*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*lo
g(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*((2*a^2*b + b^3)*d*e*x +
 (2*a^2*b + b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b\,\mathrm{arcosh}\,(dx + c) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3, x)
```

maple [B] time = 0.04, size = 605, normalized size = 3.46

$$\frac{3\,\mathrm{arccosh}\,(dx+c)\,a^2b\,c^2e}{2d}+\frac{3ea\,b^2\mathrm{arccosh}\,(dx+c)^2\,c^2}{2d}-\frac{3e\,b^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}\,c}{8d}+\frac{3d\,\mathrm{arccosh}\,(dx+c)\cdots}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 3/2/d*arccosh(d*x+c)*a^2*b*c^2*e+3/2/d*e*a*b^2*arccosh(d*x+c)^2*c^2-3/8/d*e
*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c+3/2*d*arccosh(d*x+c)*x^2*a^2*b*e+3*e
*a*b^2*arccosh(d*x+c)^2*x*c+3*arccosh(d*x+c)*x*a^2*b*c*e-3/4*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)*x*a^2*b*e+3/2*d*e*a*b^2*arccosh(d*x+c)^2*x^2-3/4*e*b^3*ar
ccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-3/2*e*a*b^2*arccosh(d*x+c)
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-3/4/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a^
2*b*c*e-3/4/d*e*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c+1/2/
d*a^3*c^2*e+1/2*d*x^2*a^3*e-1/4/d*e*b^3*arccosh(d*x+c)^3+x*a^3*c*e-3/2/d*e*
a*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3/4/d*e*a^2*b*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))
-3/8*b^3*e*arccosh(d*x+c)/d+1/2/d*e*b^3*arccosh(d*x+c)^3*c^2+3/4/d*e*b^3*ar
ccosh(d*x+c)*c^2+3/4*d*e*a*b^2*x^2+3/2*e*b^3*arccosh(d*x+c)*x*c+3/4/d*e*a*b
^2*c^2-3/8*e*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+e*b^3*arccosh(d*x+c)^3*x
```

```
*c+1/2*d*e*b^3*arccosh(d*x+c)^3*x^2+3/4*d*e*b^3*arccosh(d*x+c)*x^2-3/4/d*e*
a*b^2*arccosh(d*x+c)^2+3/2*e*a*b^2*x*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^3dex^2+\frac{3}{4}\left(2x^2\operatorname{arcosh}(dx+c)-d\left(\frac{3c^2\log\left(2d^2x+2cd+2\sqrt{d^2x^2+2cdx+c^2-1}\,d\right)}{d^3}+\frac{\sqrt{d^2x^2+2cdx+c^2}}{d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*e*x^2 + 3/4*(2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*
d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c
^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x +
c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^2*b*d*e + a
^3*c*e*x + 3*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b*c*e
/d + 1/2*(b^3*d*e*x^2 + 2*b^3*c*e*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + c)^3 + integrate(3/2*((2*a*b^2*d^4*e - b^3*d^4*e)*x^4 + 2*(c^4*e
- c^2*e)*a*b^2 + 4*(2*a*b^2*c*d^3*e - b^3*c*d^3*e)*x^3 + (2*(6*c^2*d^2*e -
d^2*e)*a*b^2 - (5*c^2*d^2*e - d^2*e)*b^3)*x^2 + (2*(c^3*e - c*e)*a*b^2 + (2
*a*b^2*d^3*e - b^3*d^3*e)*x^3 + 3*(2*a*b^2*c*d^2*e - b^3*c*d^2*e)*x^2 - 2*(
b^3*c^2*d*e - (3*c^2*d*e - d*e)*a*b^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + 2*(2*(2*c^3*d*e - c*d*e)*a*b^2 - (c^3*d*e - c*d*e)*b^3)*x)*log(d*x + s
qrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d
^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d
- d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)\,\left(a+b\operatorname{acosh}(c+d\,x)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^3, x)
```

sympy [A] time = 2.00, size = 685, normalized size = 3.91

$$\begin{cases} a^3cex+\frac{a^3dex^2}{2}+\frac{3a^2bc^2e\operatorname{acosh}(c+dx)}{2d}+3a^2bcex\operatorname{acosh}(c+dx)-\frac{3a^2bce\sqrt{c^2+2cdx+d^2x^2-1}}{4d}+\frac{3a^2bdex^2\operatorname{acosh}(c+dx)}{2}-\frac{3a^2bex\sqrt{c}}{} \\ cex\left(a+b\operatorname{acosh}(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*c*e*x + a**3*d*e*x**2/2 + 3*a**2*b*c**2*e*acosh(c + d*x)/(2
*d) + 3*a**2*b*c*e*x*acosh(c + d*x) - 3*a**2*b*c*e*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)/(4*d) + 3*a**2*b*d*e*x**2*acosh(c + d*x)/2 - 3*a**2*b*e*x*sqrt
(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 - 3*a**2*b*e*acosh(c + d*x)/(4*d) + 3*a*
b**2*c**2*e*acosh(c + d*x)**2/(2*d) + 3*a*b**2*c*e*x*acosh(c + d*x)**2 + 3*
a*b**2*c*e*x/2 - 3*a*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)/(2*d) + 3*a*b**2*d*e*x**2*acosh(c + d*x)**2/2 + 3*a*b**2*d*e*x**2/4
- 3*a*b**2*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/2 - 3*a*
b**2*e*acosh(c + d*x)**2/(4*d) + b**3*c**2*e*acosh(c + d*x)**3/(2*d) + 3*b*
*3*c**2*e*acosh(c + d*x)/(4*d) + b**3*c*e*x*acosh(c + d*x)**3 + 3*b**3*c*e*
```

```
x*acosh(c + d*x)/2 - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(
c + d*x)**2/(4*d) - 3*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(8*d) +
 b**3*d*e*x**2*acosh(c + d*x)**3/2 + 3*b**3*d*e*x**2*acosh(c + d*x)/4 - 3*b
**3*e*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/4 - 3*b**3*e
*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - b**3*e*acosh(c + d*x)**3/(4*d)
- 3*b**3*e*acosh(c + d*x)/(8*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))**3, Tru
e))
```

3.117 $\int \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=114

$$6ab^2x-\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}-\frac{6b^3\sqrt{c+dx-1}\sqrt{}}{d}$$

```
[Out] 6*a*b^2*x+6*b^3*(d*x+c)*arccosh(d*x+c)/d+(d*x+c)*(a+b*arccosh(d*x+c))^3/d-6
*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3*b*(a+b*arccosh(d*x+c))^2*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5864, 5654, 5718, 74}

$$6ab^2x-\frac{3b\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}+\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}-\frac{6b^3\sqrt{c+dx-1}\sqrt{}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] 6*a*b^2*x - (6*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/d + (6*b^3*(c + d*
x)*ArcCosh[c + d*x])/d - (3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*A
rcCosh[c + d*x])^2)/d + ((c + d*x)*(a + b*ArcCosh[c + d*x])^3)/d
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^3\,dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3}{d} - \frac{(3b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+b\cosh\right.}{d}$$
$$= 6ab^2x - \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^2}{d} + \frac{(c+dx)\left(a+\right.}{}$$
$$= 6ab^2x + \frac{6b^3(c+dx)\cosh^{-1}(c+dx)}{d} - \frac{3b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\,\text{co}\right.}{d}$$
$$= 6ab^2x - \frac{6b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}}{d} + \frac{6b^3(c+dx)\cosh^{-1}(c+dx)}{d} - \frac{3b\sqrt{-}}{}$$

Mathematica [A] time = 0.19, size = 168, normalized size = 1.47

$$\frac{a\left(a^2+6b^2\right)(c+dx) - 3b\left(a^2+2b^2\right)\sqrt{c+dx-1}\sqrt{c+dx+1} - 3b\cosh^{-1}(c+dx)\left(-\left(a^2(c+dx)\right)+2ab\sqrt{c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*(c + d*x) - 3*b*(a^2 + 2*b^2)*Sqrt[-1 + c + d*x]*Sqrt[1 +
c + d*x] - 3*b*(-(a^2*(c + d*x)) - 2*b^2*(c + d*x) + 2*a*b*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] - 3*b^2*(-(a*(c + d*x)) + b*Sqrt[-1
+ c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + b^3*(c + d*x)*ArcCosh[c
+ d*x]^3)/d
```

fricas [B] time = 0.57, size = 239, normalized size = 2.10

$$\frac{\left(b^3dx+b^3c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^3 + \left(a^3+6\,ab^2\right)dx + 3\left(ab^2dx+ab^2c-\sqrt{d^2x^2+2\,cdx+c^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*d*x + b^3*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + (a^
3 + 6*a*b^2)*d*x + 3*(a*b^2*d*x + a*b^2*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1)*b^3)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 - 3*(2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*a*b^2 - (a^2*b + 2*b^3)*d*x - (a^2*b + 2*b^3)*c)*l
og(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 3*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*(a^2*b + 2*b^3))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arcosh}\,(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3, x)
```

maple [A] time = 0.06, size = 180, normalized size = 1.58

$$\frac{(dx+c)\,a^3+b^3\left((dx+c)\operatorname{arccosh}(dx+c)^3-3\operatorname{arccosh}(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}+6\,(dx+c)\operatorname{arccosh}(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*((d*x+c)*a^3+b^3*((d*x+c)*arccosh(d*x+c)^3-3*arccosh(d*x+c)^2*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)+6*(d*x+c)*arccosh(d*x+c)-6*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))+3*a*b^2*((d*x+c)*arccosh(d*x+c)^2-2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)+2*d*x+2*c)+3*a^2*b*((d*x+c)*arccosh(d*x+c)-(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3x\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^3+a^3x+\frac{3\left((dx+c)\operatorname{arcosh}(dx+c)-\sqrt{(dx+c)^2-1}\right)a^2b}{d}+\int\frac{3\left(\left(c^3-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + a^3*x + 3*((d*
x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^2*b/d + integrate(3*((c^
3 - c)*a*b^2 + (a*b^2*d^3 - b^3*d^3)*x^3 + (3*a*b^2*c*d^2 - 2*b^3*c*d^2)*x^
2 + ((c^2 - 1)*a*b^2 + (a*b^2*d^2 - b^3*d^2)*x^2 + (2*a*b^2*c*d - b^3*c*d)*
x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d - d)*a*b^2 - (c^2*d - d)
*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^3*x^3 + 3*
c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))^3, x)
```

sympy [A] time = 0.77, size = 282, normalized size = 2.47

$$\begin{cases} a^3x+\frac{3a^2bc\operatorname{acosh}(c+dx)}{d}+3a^2bx\operatorname{acosh}(c+dx)-\frac{3a^2b\sqrt{c^2+2cdx+d^2x^2-1}}{d}+\frac{3ab^2c\operatorname{acosh}^2(c+dx)}{d}+3ab^2x\operatorname{acosh}^2(c+dx)+ \\ x\left(a+b\operatorname{acosh}(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*c*acosh(c + d*x)/d + 3*a**2*b*x*acosh(c + d*x)
 - 3*a**2*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + 3*a*b**2*c*acosh(c + d
*x)**2/d + 3*a*b**2*x*acosh(c + d*x)**2 + 6*a*b**2*x - 6*a*b**2*sqrt(c**2 +
 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + b**3*c*acosh(c + d*x)**3/d + 6
*b**3*c*acosh(c + d*x)/d + b**3*x*acosh(c + d*x)**3 + 6*b**3*x*acosh(c + d*
x) - 3*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/d - 6*b*
*3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d, Ne(d, 0)), (x*(a + b*acosh(c))**
3, True))
```

3.118 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{ce+dex}\,dx$

Optimal. Leaf size=159

$$-\frac{3b^2\text{Li}_3\left(-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{2de}-\frac{3b\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de}+\frac{\left(a+b\,\text{co}\right.}{}$$

```
[Out] 1/4*(a+b*arccosh(d*x+c))^4/b/d/e+(a+b*arccosh(d*x+c))^3*ln(1+1/(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/2*b*(a+b*arccosh(d*x+c))^2*polylog(2,-
1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/2*b^2*(a+b*arccosh(d*x+c
))*polylog(3,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/4*b^3*poly
log(4,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Rubi [A] time = 0.22, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5660, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{2de}+\frac{3b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx\right.}{2de}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcCosh[c + d*x])^4/(4*b*d*e) + ((a + b*ArcCosh[c + d*x])^3*Log[1 +
 E^(2*ArcCosh[c + d*x])])/(d*e) + (3*b*(a + b*ArcCosh[c + d*x])^2*PolyLog[2
, -E^(2*ArcCosh[c + d*x])])/(2*d*e) - (3*b^2*(a + b*ArcCosh[c + d*x])*PolyL
og[3, -E^(2*ArcCosh[c + d*x])])/(2*d*e) + (3*b^3*PolyLog[4, -E^(2*ArcCosh[c
 + d*x])])/(4*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{ex}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x}\,dx,x,c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)^3\tanh(x)\,dx,x,\cosh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1+e^{2x}}\,dx,x,\cosh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} -$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{4bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$

Mathematica [A] time = 0.57, size = 217, normalized size = 1.36

$$\frac{4a^3\log(c+dx)+6a^2b\cosh^{-1}(c+dx)^2+12a^2b\cosh^{-1}(c+dx)\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)-6b^2\text{Li}_3\left(-e^{-2\cosh^{-1}(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x),x]
```

```
[Out] (6*a^2*b*ArcCosh[c + d*x]^2 + 4*a*b^2*ArcCosh[c + d*x]^3 + b^3*ArcCosh[c +
d*x]^4 + 12*a^2*b*ArcCosh[c + d*x]*Log[1 + E^(-2*ArcCosh[c + d*x])] + 12*a*
b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*ArcCosh[c + d*x])] + 4*b^3*ArcCosh[c +
 d*x]^3*Log[1 + E^(-2*ArcCosh[c + d*x])] + 4*a^3*Log[c + d*x] - 6*b*(a + b*
ArcCosh[c + d*x])^2*PolyLog[2, -E^(-2*ArcCosh[c + d*x])] - 6*b^2*(a + b*Arc
Cosh[c + d*x])*PolyLog[3, -E^(-2*ArcCosh[c + d*x])] - 3*b^3*PolyLog[4, -E^(
-2*ArcCosh[c + d*x])])/(4*d*e)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{arcosh}\,(dx+c)^3+3\,ab^2\,\text{arcosh}\,(dx+c)^2+3\,a^2b\,\text{arcosh}\,(dx+c)+a^3}{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/(d*e*x + c*e), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(dx+c)+a)^3}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e), x)
```

maple [**B**] time = 0.07, size = 471, normalized size = 2.96

$$\frac{a^3\ln(dx+c)}{de}-\frac{b^3\operatorname{arccosh}(dx+c)^4}{4de}+\frac{b^3\operatorname{arccosh}(dx+c)^3\ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{de}+\frac{3b^3\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^3/e*ln(d*x+c)-1/4/d*b^3/e*arccosh(d*x+c)^4+1/d*b^3/e*arccosh(d*x+c)^3
*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/2/d*b^3/e*arccosh(d*x+c)
^2*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2/d*b^3/e*arccos
h(d*x+c)*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/4/d*b^3/e*
polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/d*a*b^2/e*arccosh(d
*x+c)^3+3/d*a*b^2/e*arccosh(d*x+c)^2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2))^2)+3/d*a*b^2/e*arccosh(d*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2))^2)-3/2/d*a*b^2/e*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2))^2)-3/2/d*a^2*b/e*arccosh(d*x+c)^2+3/d*a^2*b/e*arccosh(d*x+c)*ln(1+(d*
x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/2/d*a^2*b/e*polylog(2,-(d*x+c+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\log(dex+ce)}{de}+\int\frac{b^3\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^3}{dex+ce}+\frac{3\,ab^2\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^2}{dex+ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^3*log(d*e*x + c*e)/(d*e) + integrate(b^3*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^3/(d*e*x + c*e) + 3*a*b^2*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)^2/(d*e*x + c*e) + 3*a^2*b*log(d*x + sqrt(d*x + c + 1)*
sqrt(d*x + c - 1) + c)/(d*e*x + c*e), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c+d\,x))^3}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c+dx}\,dx+\int \frac{b^3\operatorname{acosh}^3(c+dx)}{c+dx}\,dx+\int \frac{3ab^2\operatorname{acosh}^2(c+dx)}{c+dx}\,dx+\int \frac{3a^2b\operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**3/(c + d*x), x) + Integral(b**3*acosh(c + d*x)**3/(c + d*x), x
) + Integral(3*a*b**2*acosh(c + d*x)**2/(c + d*x), x) + Integral(3*a**2*b*a
cosh(c + d*x)/(c + d*x), x))/e
```

3.119 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{6ib^2\text{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}+\frac{6ib^2\text{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{\left(a+b\cosh^{-1}(c\right.}{de^2(c+d}$$

```
[Out] -(a+b*arccosh(d*x+c))^3/d/e^2/(d*x+c)+6*b*(a+b*arccosh(d*x+c))^2*arctan(d*x
+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-6*I*b^2*(a+b*arccosh(d*x+c))*poly
log(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+6*I*b^2*(a+b*arccos
h(d*x+c))*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+6*I*b^
3*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2-6*I*b^3*polyl
og(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2
```

Rubi [A] time = 0.36, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5761, 4180, 2531, 2282, 6589}

$$-\frac{6ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}+\frac{6ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])^3/(d*e^2*(c + d*x))) + (6*b*(a + b*ArcCosh[c + d
*x])^2*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((6*I)*b^2*(a + b*ArcCosh[c +
d*x])*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((6*I)*b^2*(a + b*ArcC
osh[c + d*x])*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^2) + ((6*I)*b^3*PolyLo
g[3, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) - ((6*I)*b^3*PolyLog[3, I*E^ArcCosh[
c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

```
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^2x^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^2}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx, x, c+dx\right)}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{(3b)\,\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \cosh^{-1}(c+dx\right.}{de^2}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{(6}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{6}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{6}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^2(c+dx)} + \frac{6b\left(a+b\cosh^{-1}(c+dx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - \frac{6}{}$$

Mathematica [A] time = 1.18, size = 327, normalized size = 1.76

$$-\frac{\frac{a^3}{c+dx} + 3a^2b\tan^{-1}\left(\frac{1}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right) + \frac{3a^2b\cosh^{-1}(c+dx)}{c+dx} + 3iab^2\left(2\text{Li}_2\left(-ie^{-\cosh^{-1}(c+dx)}\right) - 2\text{Li}_2\left(ie^{-\cosh^{-1}(c+dx)}\right) + \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^2,x]
```

```
[Out] -((a^3/(c + d*x) + (3*a^2*b*ArcCosh[c + d*x])/(c + d*x) + 3*a^2*b*ArcTan[1/
(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])] + (3*I)*a*b^2*(ArcCosh[c + d*x]*(((
-I)*ArcCosh[c + d*x])/(c + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1
 + I/E^ArcCosh[c + d*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyL
og[2, I/E^ArcCosh[c + d*x]]) + b^3*(ArcCosh[c + d*x]^3/(c + d*x) - (3*I)*(-
(ArcCosh[c + d*x]^2*(Log[1 - I/E^ArcCosh[c + d*x]] - Log[1 + I/E^ArcCosh[c
+ d*x]])) - 2*ArcCosh[c + d*x]*(PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - PolyL
og[2, I/E^ArcCosh[c + d*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + 2*Po
lyLog[3, I/E^ArcCosh[c + d*x]])))/(d*e^2))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{arcosh}\left(dx+c\right)^3 + 3\,ab^2\,\text{arcosh}\left(dx+c\right)^2 + 3\,a^2b\,\text{arcosh}\left(dx+c\right) + a^3}{d^2e^2x^2 + 2\,cde^2x + c^2e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.7sym2poly/r2sym(const gen & e,const index_m & i,c
onst vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccosh}(dx+c))^3}{(dex+ce)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3\log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^3}{d^2e^2x+cde^2}-3\,a^2b\left(\frac{\operatorname{arcosh}(dx+c)}{d^2e^2x+cde^2}+\frac{\arcsin\left(\frac{de^2}{|d^2e^2x+cde^2|}\right)}{de^2}\right)-\frac{a^3}{d^2e^2x+cde^2}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^3*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d^2*e^2*x + c*d*
e^2) - 3*a^2*b*(arccosh(d*x + c)/(d^2*e^2*x + c*d*e^2) + arcsin(d*e^2/abs(d
^2*e^2*x + c*d*e^2))/(d*e^2)) - a^3/(d^2*e^2*x + c*d*e^2) + integrate(3*((c
^3 - c)*a*b^2 + (c^3 - c)*b^3 + (a*b^2*d^3 + b^3*d^3)*x^3 + 3*(a*b^2*c*d^2
+ b^3*c*d^2)*x^2 + (b^3*c^2 + (c^2 - 1)*a*b^2 + (a*b^2*d^2 + b^3*d^2)*x^2 +
 2*(a*b^2*c*d + b^3*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d
 - d)*a*b^2 + (3*c^2*d - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
c - 1) + c)^2/(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 - c^3*e^2 + (10*c^2*
d^3*e^2 - d^3*e^2)*x^3 + (10*c^3*d^2*e^2 - 3*c*d^2*e^2)*x^2 + (d^4*e^2*x^4
+ 4*c*d^3*e^2*x^3 + c^4*e^2 - c^2*e^2 + (6*c^2*d^2*e^2 - d^2*e^2)*x^2 + 2*(
2*c^3*d*e^2 - c*d*e^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (5*c^4*d*e^
2 - 3*c^2*d*e^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c+d\,x))^3}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^2+2cdx+d^2x^2}\,dx+\int \frac{b^3\operatorname{acosh}^3(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3ab^2\operatorname{acosh}^2(c+dx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{3a^2b\operatorname{acosh}(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**3*acosh(c + d
*x)**3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a*b**2*acosh(c + d*x)*
*2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(3*a**2*b*acosh(c + d*x)/(c**
2 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.120 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{3b^2\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}+\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)}-\frac{3b}{}\;-$$

```
[Out] -3/2*b*(a+b*arccosh(d*x+c))^2/d/e^3-1/2*(a+b*arccosh(d*x+c))^3/d/e^3/(d*x+c
)^2-3*b^2*(a+b*arccosh(d*x+c))*ln(1+1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))^2)/d/e^3+3/2*b^3*polylog(2,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
/d/e^3+3/2*b*(a+b*arccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^3/(
d*x+c)
```

Rubi [A] time = 0.37, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5662, 5724, 5660, 3718, 2190, 2279, 2391}

$$-\frac{3b^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)}{2de^3}-\frac{3b^2\log\left(e^{2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}+\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx}}{2de}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (3*b*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^3) + (3*b*Sqrt[-1 + c + d*x]*Sqrt[1
 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^3*(c + d*x)) - (a + b*ArcCos
h[c + d*x])^3/(2*d*e^3*(c + d*x)^2) - (3*b^2*(a + b*ArcCosh[c + d*x])*Log[1
 + E^(2*ArcCosh[c + d*x])])/(d*e^3) - (3*b^3*PolyLog[2, -E^(2*ArcCosh[c + d
*x])])/(2*d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^3x^3}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^3}\,dx,x,c+dx\right)}{de^3}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{2de^3(c+dx)^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx,x,c+dx\right)}{2de^3}$$
$$= \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx\right.}{2de^3(c+dx)^2}$$
$$= \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx\right.}{2de^3(c+dx)^2}$$
$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^3(c+dx)}$$
$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^3(c+dx)}$$
$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^3(c+dx)}$$
$$= \frac{3b\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^3} + \frac{3b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{2de^3(c+dx)}$$

Mathematica [A] time = 1.11, size = 266, normalized size = 1.62

$$\frac{-\frac{a^3}{(c+dx)^2} + \frac{3a^2b\left(\sqrt{\frac{c+dx-1}{c+dx+1}}\left(c^2+2cdx+c+dx(dx+1)\right)-\cosh^{-1}(c+dx)\right)}{(c+dx)^2} + 6ab^2\left(-\log(c+dx) - \frac{\cosh^{-1}(c+dx)^2}{2(c+dx)^2} + \frac{\sqrt{\frac{c+dx-1}{c+dx+1}}(c+dx+1)\cos}{c+dx}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^3,x]
```

```
[Out] (-(a^3/(c + d*x)^2) + (3*a^2*b*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(c + c^2
 + 2*c*d*x + d*x*(1 + d*x)) - ArcCosh[c + d*x]))/(c + d*x)^2 - (b^3*ArcCosh
[c + d*x]^3)/(c + d*x)^2 + 6*a*b^2*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
+ c + d*x)*ArcCosh[c + d*x])/(c + d*x) - ArcCosh[c + d*x]^2/(2*(c + d*x)^2)
 - Log[c + d*x]) + 3*b^3*(ArcCosh[c + d*x]*(-ArcCosh[c + d*x] + (Sqrt[(-1 +
 c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) - 2*Log[
1 + E^(-2*ArcCosh[c + d*x])]) + PolyLog[2, -E^(-2*ArcCosh[c + d*x])]))/(2*d
*e^3)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\,\text{arcosh}\,(dx+c)^3 + 3\,ab^2\,\text{arcosh}\,(dx+c)^2 + 3\,a^2b\,\text{arcosh}\,(dx+c) + a^3}{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^3}{(dex+ce)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^3, x)
```

maple [B] time = 0.29, size = 375, normalized size = 2.29

$$-\frac{a^3}{2d\,e^3\,(dx+c)^2}+\frac{3b^3 \operatorname{arccosh}(dx+c)^2\,\sqrt{dx+c+1}\,\sqrt{dx+c-1}}{2d\,e^3\,(dx+c)}+\frac{3b^3 \operatorname{arccosh}(dx+c)^2}{2d\,e^3}-\frac{b^3 \operatorname{arccosh}(dx+c)^3}{2d\,e^3\,(dx+c)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^3/e^3/(d*x+c)^2+3/2/d*b^3/e^3*arccosh(d*x+c)^2/(d*x+c)*(d*x+c+1)^(
1/2)*(d*x+c-1)^(1/2)+3/2/d*b^3/e^3*arccosh(d*x+c)^2-1/2/d*b^3/e^3*arccosh(d
*x+c)^3/(d*x+c)^2-3/d*b^3/e^3*arccosh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2))^2)-3/2/d*b^3/e^3*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))^2)+3/d*a*b^2/e^3*arccosh(d*x+c)+3/d*a*b^2/e^3*arccosh(d*x+c)/(d*x+c
)*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)-3/2/d*a*b^2/e^3*arccosh(d*x+c)^2/(d*x+c)^
2-3/d*a*b^2/e^3*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2/d*a^2*b
/e^3/(d*x+c)^2*arccosh(d*x+c)+3/2/d*a^2*b/e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3\left(\frac{\sqrt{d^2x^2+2\,cdx+c^2-1}\,d \operatorname{arcosh}(dx+c)}{d^3e^3x+cd^2e^3}-\frac{\log(dx+c)}{de^3}\right)ab^2-\frac{1}{2}\left(\frac{\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^3}{d^3e^3x^2+2\,cd^2e^3x+c^2de^3}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] 3*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arccosh(d*x + c)/(d^3*e^3*x + c*d^2*
e^3) - log(d*x + c)/(d*e^3))*a*b^2 - 1/2*(log(d*x + sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + c)^3/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2*integrate
(3/2*(d^2*x^2 + 2*c*d*x + sqrt(d*x + c + 1)*(d*x + c)*sqrt(d*x + c - 1) + c
^2 - 1)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d^5*e^3*x^5 +
 5*c*d^4*e^3*x^4 + c^5*e^3 - c^3*e^3 + (10*c^2*d^3*e^3 - d^3*e^3)*x^3 + (10
*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (d^4*e^3*x^4 + 4*c*d^3*e^3*x^3 + c^4*e^3
- c^2*e^3 + (6*c^2*d^2*e^3 - d^2*e^3)*x^2 + 2*(2*c^3*d*e^3 - c*d*e^3)*x)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1) + (5*c^4*d*e^3 - 3*c^2*d*e^3)*x), x))*b^3
 + 3/2*a^2*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*e^3*x + c*d^2*e^3) -
 arccosh(d*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3/2*a*b^2*ar
ccosh(d*x + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^3/(d^3*e
^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}{(c\,e+d\,e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^3\,\mathrm{acosh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3ab^2\,\mathrm{acosh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{3a^2b\,\mathrm{acosh}\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**3*acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(3*a*b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(3*a**2*b*acosh(c + d*x)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.121 $\int \frac{(a+b\cosh^{-1}(c+dx))^3}{(ce+dex)^4}\,dx$

Optimal. Leaf size=297

$$-\frac{ib^2\text{Li}_2\left(-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{ib^2\text{Li}_2\left(ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{b^2\left(a+b\cosh^{-1}(c\right.}{de^4(c+dx}$$

```
[Out] b^2*(a+b*arccosh(d*x+c))/d/e^4/(d*x+c)-1/3*(a+b*arccosh(d*x+c))^3/d/e^4/(d*
x+c)^3+b*(a+b*arccosh(d*x+c))^2*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))/d/e^4-b^3*arctan((d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^4-I*b^2*(a+b*arcco
sh(d*x+c))*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+I*b^
2*(a+b*arccosh(d*x+c))*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
/d/e^4+I*b^3*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4-I*
b^3*polylog(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+1/2*b*(a+b*a
rccosh(d*x+c))^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A] time = 0.59, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5662, 5748, 5761, 4180, 2531, 2282, 6589, 92, 203}

$$-\frac{ib^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\frac{ib^2\text{PolyLog}\left(2,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] (b^2*(a + b*ArcCosh[c + d*x]))/(d*e^4*(c + d*x)) + (b*Sqrt[-1 + c + d*x]*Sq
rt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^2)/(2*d*e^4*(c + d*x)^2) - (a + b*
ArcCosh[c + d*x])^3/(3*d*e^4*(c + d*x)^3) + (b*(a + b*ArcCosh[c + d*x])^2*A
rcTan[E^ArcCosh[c + d*x]])/(d*e^4) - (b^3*ArcTan[Sqrt[-1 + c + d*x]*Sqrt[1
+ c + d*x]])/(d*e^4) - (I*b^2*(a + b*ArcCosh[c + d*x])*PolyLog[2, (-I)*E^Ar
cCosh[c + d*x]])/(d*e^4) + (I*b^2*(a + b*ArcCosh[c + d*x])*PolyLog[2, I*E^A
rcCosh[c + d*x]])/(d*e^4) + (I*b^3*PolyLog[3, (-I)*E^ArcCosh[c + d*x]])/(d*
e^4) - (I*b^3*PolyLog[3, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{e^4x^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x^4}\,dx, x, c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3} + \frac{b\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\,dx, x, c+dx\right)}{de^4}$$

$$= \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^3}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

$$= \frac{b^2\left(a+b\cosh^{-1}(c+dx)\right)}{de^4(c+dx)} + \frac{b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^2}{2de^4(c+dx)^2}$$

Mathematica [A] time = 2.38, size = 504, normalized size = 1.70

$$-\frac{2a^3}{(c+dx)^3} + \frac{3a^2b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{(c+dx)^2} - 3a^2b\tan^{-1}\left(\frac{1}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}\right) - \frac{6a^2b\cosh^{-1}(c+dx)}{(c+dx)^3} + 6ab^2\left(-i\text{Li}_2\left(-ie^{-\cosh^{-1}(c+dx)}\right) + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^4,x]
```

```
[Out] ((-2*a^3)/(c + d*x)^3 + (3*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c +
 d*x)^2 - (6*a^2*b*ArcCosh[c + d*x])/(c + d*x)^3 - 3*a^2*b*ArcTan[1/(Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])] + 6*a*b^2*((c + d*x)^(-1) + (Sqrt[(-1 + c
+ d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x)^2 - ArcCosh
[c + d*x]^2/(c + d*x)^3 - I*ArcCosh[c + d*x]*Log[1 - I/E^ArcCosh[c + d*x]]
+ I*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - I*PolyLog[2, (-I)/E^Ar
cCosh[c + d*x]] + I*PolyLog[2, I/E^ArcCosh[c + d*x]]) + b^3*((6*ArcCosh[c +
 d*x])/(c + d*x) + (3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcC
osh[c + d*x]^2)/(c + d*x)^2 - (2*ArcCosh[c + d*x]^3)/(c + d*x)^3 - (3*I)*((
```

```
-4*I)*ArcTan[Tanh[ArcCosh[c + d*x]/2]] + ArcCosh[c + d*x]^2*Log[1 - I/E^Arc
Cosh[c + d*x]] - ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c + d*x]] + 2*ArcCo
sh[c + d*x]*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*ArcCosh[c + d*x]*PolyLo
g[2, I/E^ArcCosh[c + d*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] - 2*Poly
Log[3, I/E^ArcCosh[c + d*x]])))/(6*d*e^4)
```

fricas [F] time = 2.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3 \operatorname{arcosh}(dx+c)^3 + 3\,ab^2 \operatorname{arcosh}(dx+c)^2 + 3\,a^2b \operatorname{arcosh}(dx+c) + a^3}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4
*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx+c) + a)^3}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^4, x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(dx+c))^3}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^3 \log\left(dx + \sqrt{dx+c+1}\,\sqrt{dx+c-1} + c\right)^3}{3\left(d^4e^4x^3 + 3\,cd^3e^4x^2 + 3\,c^2d^2e^4x + c^3de^4\right)} - \frac{a^3}{3\left(d^4e^4x^3 + 3\,cd^3e^4x^2 + 3\,c^2d^2e^4x + c^3de^4\right)} + \int \frac{\left(3\left(c^3 - c\right)ab^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d^4*e^4*x^3
+ 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^3/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(((3*(c^3 - c)*a*b
^2 + (c^3 - c)*b^3 + (3*a*b^2*d^3 + b^3*d^3)*x^3 + 3*(3*a*b^2*c*d^2 + b^3*c
*d^2)*x^2 + (b^3*c^2 + 3*(c^2 - 1)*a*b^2 + (3*a*b^2*d^2 + b^3*d^2)*x^2 + 2*
(3*a*b^2*c*d + b^3*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(3*c^2*
d - d)*a*b^2 + (3*c^2*d - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + c)^2 + 3*(a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d - d)*a^2*b
*x + (c^3 - c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 - 1)*a^2*b)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
 - 1) + c))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^
```

```
5*e^4 - d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4
 - 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^
6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 +
4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 +
 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*
c^6*d*e^4 - 5*c^4*d*e^4)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}{(c\,e+d\,e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^3}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{b^3\,\mathrm{acosh}^3\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{3ab^2\,\mathrm{acosh}^2\,(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\,dx+\int \frac{3a^2b}{c^4+4c^3dx+6c}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**3*acosh(c + d*x)**3/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(3*a*b**2*acosh(c + d*x)
**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(3*a**2*b*acosh(c + d*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.122 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=377

$$-\frac{3b^3e^3\sqrt{c+dx-1}\,(c+dx)^3\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{32d}-\frac{45b^3e^3\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+\right.}{64d}$$

```
[Out] 45/128*b^4*e^3*(d*x+c)^2/d+3/128*b^4*e^3*(d*x+c)^4/d-45/128*b^2*e^3*(a+b*ar
ccosh(d*x+c))^2/d+9/16*b^2*e^3*(d*x+c)^2*(a+b*arccosh(d*x+c))^2/d+3/16*b^2*
e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^2/d-3/32*e^3*(a+b*arccosh(d*x+c))^4/d+1/
4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^4/d-45/64*b^3*e^3*(d*x+c)*(a+b*arccosh
(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/32*b^3*e^3*(d*x+c)^3*(a+b*arcc
osh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-3/8*b*e^3*(d*x+c)*(a+b*arccos
h(d*x+c))^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-1/4*b*e^3*(d*x+c)^3*(a+b*arcc
osh(d*x+c))^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 1.17, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 12, 5662, 5759, 5676, 30}

$$-\frac{3b^3e^3\sqrt{c+dx-1}\,(c+dx)^3\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{32d}-\frac{45b^3e^3\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (45*b^4*e^3*(c + d*x)^2)/(128*d) + (3*b^4*e^3*(c + d*x)^4)/(128*d) - (45*b^
3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x
]))/(64*d) - (3*b^3*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a
 + b*ArcCosh[c + d*x]))/(32*d) - (45*b^2*e^3*(a + b*ArcCosh[c + d*x])^2)/(1
28*d) + (9*b^2*e^3*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(16*d) + (3*b^2*
e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^2)/(16*d) - (3*b*e^3*Sqrt[-1 + c +
 d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(8*d) - (b*e^
3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])
^3)/(4*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^4)/(32*d) + (e^3*(c + d*x)^4*(a
 + b*ArcCosh[c + d*x])^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
```

```
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx = \frac{\text{Subst}\left(\int e^3x^3\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^3\,\text{Subst}\left(\int x^3\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$
$$= \frac{e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^4}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,\sqrt{1+x}}\right.}{d}$$
$$= -\frac{be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{4d} + \ldots$$
$$= \frac{3b^2e^3(c+dx)^4\left(a+b\cosh^{-1}(c+dx)\right)^2}{16d} - \frac{3be^3\sqrt{-1+c+dx}\,(c+dx)\ldots}{\ldots}$$
$$= -\frac{3b^3e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{32d} + \ldots$$
$$= \frac{3b^4e^3(c+dx)^4}{128d} - \frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cos\ldots\right.}{64d}$$
$$= \frac{45b^4e^3(c+dx)^2}{128d} + \frac{3b^4e^3(c+dx)^4}{128d} - \frac{45b^3e^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1\ldots}}{64\ldots}$$

Mathematica [A] time = 0.89, size = 562, normalized size = 1.49

$$\frac{e^3\left(9b^2\left(8a^2+5b^2\right)(c+dx)^2+2ab\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)\left(-2\left(8a^2+3b^2\right)(c+dx)^2-3\left(8a^2+15b^2\right)\right)-\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^3*(9*b^2*(8*a^2 + 5*b^2)*(c + d*x)^2 + (32*a^4 + 24*a^2*b^2 + 3*b^4)*(c
+ d*x)^4 + 2*a*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(-3*(8*a^2
+ 15*b^2) - 2*(8*a^2 + 3*b^2)*(c + d*x)^2) + 2*b*(c + d*x)*(72*a*b^2*(c + d
*x) + 64*a^3*(c + d*x)^3 + 24*a*b^2*(c + d*x)^3 - 72*a^2*b*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x] - 45*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 48*a^2
*b*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x] - 6*b^3*Sqrt[-1 + c + d
*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 3*b^2*(-24*a^2 - 15*b
^2 + 24*b^2*(c + d*x)^2 + 64*a^2*(c + d*x)^4 + 8*b^2*(c + d*x)^4 - 48*a*b*S
qrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 32*a*b*Sqrt[-1 + c + d*x]*(
c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 16*b^3*(-3*a + 8*a*(c +
d*x)^4 - 3*b*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*Sqrt[-1 +
 c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + 4*b^4*(-3 + 8
*(c + d*x)^4)*ArcCosh[c + d*x]^4 - 6*a*b*(8*a^2 + 15*b^2)*Log[c + d*x + Sqr
t[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(128*d)
```

fricas [B] time = 0.75, size = 1236, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/128*((32*a^4 + 24*a^2*b^2 + 3*b^4)*d^4*e^3*x^4 + 4*(32*a^4 + 24*a^2*b^2 +
 3*b^4)*c*d^3*e^3*x^3 + 3*(24*a^2*b^2 + 15*b^4 + 2*(32*a^4 + 24*a^2*b^2 + 3
*b^4)*c^2)*d^2*e^3*x^2 + 2*(2*(32*a^4 + 24*a^2*b^2 + 3*b^4)*c^3 + 9*(8*a^2*
b^2 + 5*b^4)*c)*d*e^3*x + 4*(8*b^4*d^4*e^3*x^4 + 32*b^4*c*d^3*e^3*x^3 + 48*
b^4*c^2*d^2*e^3*x^2 + 32*b^4*c^3*d*e^3*x + (8*b^4*c^4 - 3*b^4)*e^3)*log(d*x
 + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 16*(8*a*b^3*d^4*e^3*x^4 + 32*
a*b^3*c*d^3*e^3*x^3 + 48*a*b^3*c^2*d^2*e^3*x^2 + 32*a*b^3*c^3*d*e^3*x + (8*
a*b^3*c^4 - 3*a*b^3)*e^3 - (2*b^4*d^3*e^3*x^3 + 6*b^4*c*d^2*e^3*x^2 + 3*(2*
b^4*c^2 + b^4)*d*e^3*x + (2*b^4*c^3 + 3*b^4*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 3*(8*(8*a^
2*b^2 + b^4)*d^4*e^3*x^4 + 32*(8*a^2*b^2 + b^4)*c*d^3*e^3*x^3 + 24*(b^4 + 2
*(8*a^2*b^2 + b^4)*c^2)*d^2*e^3*x^2 + 16*(3*b^4*c + 2*(8*a^2*b^2 + b^4)*c^3
)*d*e^3*x + (24*b^4*c^2 + 8*(8*a^2*b^2 + b^4)*c^4 - 24*a^2*b^2 - 15*b^4)*e^
3 - 16*(2*a*b^3*d^3*e^3*x^3 + 6*a*b^3*c*d^2*e^3*x^2 + 3*(2*a*b^3*c^2 + a*b^
3)*d*e^3*x + (2*a*b^3*c^3 + 3*a*b^3*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 -
1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 2*(8*(8*a^3*b + 3*
a*b^3)*d^4*e^3*x^4 + 32*(8*a^3*b + 3*a*b^3)*c*d^3*e^3*x^3 + 24*(3*a*b^3 + 2
*(8*a^3*b + 3*a*b^3)*c^2)*d^2*e^3*x^2 + 16*(9*a*b^3*c + 2*(8*a^3*b + 3*a*b^
3)*c^3)*d*e^3*x + (72*a*b^3*c^2 + 8*(8*a^3*b + 3*a*b^3)*c^4 - 24*a^3*b - 45
*a*b^3)*e^3 - 3*(2*(8*a^2*b^2 + b^4)*d^3*e^3*x^3 + 6*(8*a^2*b^2 + b^4)*c*d^
2*e^3*x^2 + 3*(8*a^2*b^2 + 5*b^4 + 2*(8*a^2*b^2 + b^4)*c^2)*d*e^3*x + (2*(8
*a^2*b^2 + b^4)*c^3 + 3*(8*a^2*b^2 + 5*b^4)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*(2*(8*a^3*
b + 3*a*b^3)*d^3*e^3*x^3 + 6*(8*a^3*b + 3*a*b^3)*c*d^2*e^3*x^2 + 3*(8*a^3*b
 + 15*a*b^3 + 2*(8*a^3*b + 3*a*b^3)*c^2)*d*e^3*x + (2*(8*a^3*b + 3*a*b^3)*c
^3 + 3*(8*a^3*b + 15*a*b^3)*c)*e^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (b\,\mathrm{arcosh}\,(dx + c) + a)^4 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^4, x)
```

maple [B] time = 0.05, size = 2465, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] -9/4*d*e^3*a*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-9/4
*d*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c+3/2/d*e
^3*a^2*b^2*arccosh(d*x+c)^2*c^4+9/8*d*e^3*a*b^3*arccosh(d*x+c)*x^2+d^3*e^3*
a*b^3*arccosh(d*x+c)^3*x^4+3/8*d^3*e^3*a*b^3*arccosh(d*x+c)*x^4+3/4*d^2*e^3
*a^2*b^2*x^3*c+9/8*d*e^3*a^2*b^2*x^2*c^2+1/d*arccosh(d*x+c)*a^3*b*c^4*e^3+1
/d*e^3*a*b^3*arccosh(d*x+c)^3*c^4+3/8/d*e^3*a*b^3*arccosh(d*x+c)*c^4+3/2*d^
3*e^3*a^2*b^2*arccosh(d*x+c)^2*x^4+d^3*arccosh(d*x+c)*x^4*a^3*b*e^3-3/8*e^3
*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+9/8/d*e^3*a*b^3*arc
cosh(d*x+c)*c^2-45/64*e^3*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+9/4*e^3*a
*b^3*arccosh(d*x+c)*x*c+4*e^3*a*b^3*arccosh(d*x+c)^3*x*c^3+6*e^3*a^2*b^2*ar
ccosh(d*x+c)^2*x*c^3+3/2*d*e^3*b^4*arccosh(d*x+c)^4*x^2*c^2+3/4*d^2*e^3*b^4
*arccosh(d*x+c)^2*x^3*c+9/8*d*e^3*b^4*arccosh(d*x+c)^2*x^2*c^2+d^2*e^3*b^4*
arccosh(d*x+c)^4*x^3*c-45/64*e^3*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2)*x+4*arccosh(d*x+c)*x*a^3*b*c^3*e^3-3/8*(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)*x*a^3*b*e^3+3/2*e^3*a*b^3*arccosh(d*x+c)*x*c^3+1/4/d*a^4*c^4*e^3-3/32/
d*e^3*b^4*arccosh(d*x+c)^4-45/128/d*e^3*b^4*arccosh(d*x+c)^2+45/128*d*e^3*b
^4*x^2+3/128*d^3*e^3*b^4*x^4+1/4*d^3*x^4*a^4*e^3+x*a^4*c^3*e^3+45/64*e^3*b^
4*x*c+3/32*e^3*b^4*x*c^3+45/128/d*e^3*b^4*c^2+3/128/d*e^3*b^4*c^4+9/8*e^3*a
^2*b^2*x*c-9/4*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x
*c^2+3/16/d*e^3*a^2*b^2*c^4+9/16/d*e^3*a^2*b^2*c^2-3/8/d*e^3*a^3*b*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2))-3
/4*d^2*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-9/32*
d*e^3*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*c-9/4*e^3*a*b^3*arccosh(d*x
+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-3/4/d*e^3*a*b^3*arccosh(d*x+c)^
2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-3/4*d^2*e^3*a*b^3*arccosh(d*x+c)^2*(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-9/32*d*e^3*b^4*arccosh(d*x+c)*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)*x^2*c-3/4*d*e^3*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)*x^2*c-3/4/d*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+
c+1)^(1/2)*c^3-3/4*d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^2*a^3*b*c*e^3-9/8/d*
e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-9/8/d*e^3*a*b^
3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c+3/4*e^3*a^2*b^2*x*c^3+
d^2*x^3*a^4*c*e^3+3/2*d*x^2*a^4*c^2*e^3+3/16*d^3*e^3*a^2*b^2*x^4+9/16*d*e^3
*a^2*b^2*x^2+9/64*d*e^3*b^4*x^2*c^2+3/32*d^2*e^3*b^4*x^3*c+e^3*b^4*arccosh(
d*x+c)^4*x*c^3+3/4*e^3*b^4*arccosh(d*x+c)^2*x*c^3+9/8*e^3*b^4*arccosh(d*x+c
)^2*x*c-45/64/d*e^3*a*b^3*arccosh(d*x+c)-3/8/d*e^3*a*b^3*arccosh(d*x+c)^3+1
/4/d*e^3*b^4*arccosh(d*x+c)^4*c^4+3/16/d*e^3*b^4*arccosh(d*x+c)^2*c^4+9/16/
d*e^3*b^4*arccosh(d*x+c)^2*c^2+9/16*d*e^3*b^4*arccosh(d*x+c)^2*x^2+1/4*d^3*
e^3*b^4*arccosh(d*x+c)^4*x^4+3/16*d^3*e^3*b^4*arccosh(d*x+c)^2*x^4-9/16/d*e
^3*a^2*b^2*arccosh(d*x+c)^2-9/8*e^3*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)*x-9/32*e^3*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-9/32
*e^3*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^2-3/8/d*e^3*b^4
*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3/32/d*e^3*b^4*arccosh(
d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c^3-45/64/d*e^3*b^4*arccosh(d*x+c)*(
d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c+4*d^2*arccosh(d*x+c)*x^3*a^3*b*c*e^3+6*d*a
rccosh(d*x+c)*x^2*a^3*b*c^2*e^3-1/4*d^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3
*a^3*b*e^3+9/4*d*e^3*a*b^3*arccosh(d*x+c)*x^2*c^2-3/32/d*e^3*a*b^3*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)*c^3-1/4/d*e^3*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)*c^3-9/8*e^3*a*b^3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)*x-3/4*e^3*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*c^
2-1/4*d^2*e^3*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-3/32
*d^2*e^3*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x^3-45/64/d*e^3
*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3/32*d^2*e^3*a*b^3*(d*x+c-1)^(1/2)
```

```
*(d*x+c+1)^(1/2)*x^3+4*d^2*e^3*a*b^3*arccosh(d*x+c)^3*x^3*c+6*d*e^3*a*b^3*a
rccosh(d*x+c)^3*x^2*c^2+6*d^2*e^3*a^2*b^2*arccosh(d*x+c)^2*x^3*c+9*d*e^3*a^
2*b^2*arccosh(d*x+c)^2*x^2*c^2-3/8/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a^3*b*
c*e^3-1/4/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a^3*b*c^3*e^3+3/2*d^2*e^3*a*b^3
*arccosh(d*x+c)*x^3*c-3/4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x*a^3*b*c^2*e^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*d^3*e^3*x^4 + a^4*c*d^2*e^3*x^3 + 3/2*a^4*c^2*d*e^3*x^2 + 3*(2*x^2*
arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log
(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^3*b*c^2*d*e^3 + 2/3*(6*x^3*arccosh(d*x +
 c) - d*(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x +
 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*
d*x + c^2 - 1)*c*x/d^3 + 9*(c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2
 + 2*c*d*x + c^2 - 1)*d)/d^4 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4
 - 4*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)/d^4))*a^3*b*c*d^2*e^3 + 1/
24*(24*x^4*arccosh(d*x + c) - (6*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*x^3/d^2
- 14*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x^2/d^3 + 105*c^4*log(2*d^2*x + 2*
c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 35*sqrt(d^2*x^2 + 2*c*d*
x + c^2 - 1)*c^2*x/d^4 - 90*(c^2 - 1)*c^2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*
x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 - 105*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^3
/d^5 - 9*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*x/d^4 + 9*(c^2 - 1)^2*
log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^5 + 55*sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)*(c^2 - 1)*c/d^5)*d)*a^3*b*d^3*e^3 + a^4*c^3*e^
3*x + 4*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b*c^3*e^3/
d + 1/4*(b^4*d^3*e^3*x^4 + 4*b^4*c*d^2*e^3*x^3 + 6*b^4*c^2*d*e^3*x^2 + 4*b^
4*c^3*e^3*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4 + integra
te((((4*a*b^3*d^6*e^3 - b^4*d^6*e^3)*x^6 + 6*(4*a*b^3*c*d^5*e^3 - b^4*c*d^5
*e^3)*x^5 + 4*(c^6*e^3 - c^4*e^3)*a*b^3 + (4*(15*c^2*d^4*e^3 - d^4*e^3)*a*b
^3 - (15*c^2*d^4*e^3 - d^4*e^3)*b^4)*x^4 + 4*(4*(5*c^3*d^3*e^3 - c*d^3*e^3)
*a*b^3 - (5*c^3*d^3*e^3 - c*d^3*e^3)*b^4)*x^3 + 2*(6*(5*c^4*d^2*e^3 - 2*c^2
*d^2*e^3)*a*b^3 - (7*c^4*d^2*e^3 - 3*c^2*d^2*e^3)*b^4)*x^2 + ((4*a*b^3*d^5*
e^3 - b^4*d^5*e^3)*x^5 + 4*(c^5*e^3 - c^3*e^3)*a*b^3 + 5*(4*a*b^3*c*d^4*e^3
 - b^4*c*d^4*e^3)*x^4 - 2*(5*b^4*c^2*d^3*e^3 - 2*(10*c^2*d^3*e^3 - d^3*e^3)
*a*b^3)*x^3 - 2*(5*b^4*c^3*d^2*e^3 - 2*(10*c^3*d^2*e^3 - 3*c*d^2*e^3)*a*b^3
)*x^2 - 4*(b^4*c^4*d*e^3 - (5*c^4*d*e^3 - 3*c^2*d*e^3)*a*b^3)*x)*sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + 4*(2*(3*c^5*d*e^3 - 2*c^3*d*e^3)*a*b^3 - (c^5*d
*e^3 - c^3*d*e^3)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c
)^3 + 6*(a^2*b^2*d^6*e^3*x^6 + 6*a^2*b^2*c*d^5*e^3*x^5 + (15*c^2*d^4*e^3 -
d^4*e^3)*a^2*b^2*x^4 + 4*(5*c^3*d^3*e^3 - c*d^3*e^3)*a^2*b^2*x^3 + 3*(5*c^4
*d^2*e^3 - 2*c^2*d^2*e^3)*a^2*b^2*x^2 + 2*(3*c^5*d*e^3 - 2*c^3*d*e^3)*a^2*b
^2*x + (c^6*e^3 - c^4*e^3)*a^2*b^2 + (a^2*b^2*d^5*e^3*x^5 + 5*a^2*b^2*c*d^4
*e^3*x^4 + (10*c^2*d^3*e^3 - d^3*e^3)*a^2*b^2*x^3 + (10*c^3*d^2*e^3 - 3*c*d
^2*e^3)*a^2*b^2*x^2 + (5*c^4*d*e^3 - 3*c^2*d*e^3)*a^2*b^2*x + (c^5*e^3 - c^
3*e^3)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + c)^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2
*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,(a+b\,\mathrm{acosh}\,(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^4, x)
```

sympy [A] time = 19.27, size = 2876, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**3*e**3*x + 3*a**4*c**2*d*e**3*x**2/2 + a**4*c*d**2*e**3*
x**3 + a**4*d**3*e**3*x**4/4 + a**3*b*c**4*e**3*acosh(c + d*x)/d + 4*a**3*b
*c**3*e**3*x*acosh(c + d*x) - a**3*b*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)/(4*d) + 6*a**3*b*c**2*d*e**3*x**2*acosh(c + d*x) - 3*a**3*b*c**2*e
**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/4 + 4*a**3*b*c*d**2*e**3*x**3*ac
osh(c + d*x) - 3*a**3*b*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/
4 - 3*a**3*b*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(8*d) + a**3*b*d**
3*e**3*x**4*acosh(c + d*x) - a**3*b*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)/4 - 3*a**3*b*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/8 - 3
*a**3*b*e**3*acosh(c + d*x)/(8*d) + 3*a**2*b**2*c**4*e**3*acosh(c + d*x)**2
/(2*d) + 6*a**2*b**2*c**3*e**3*x*acosh(c + d*x)**2 + 3*a**2*b**2*c**3*e**3*
x/4 - 3*a**2*b**2*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
d*x)/(4*d) + 9*a**2*b**2*c**2*d*e**3*x**2*acosh(c + d*x)**2 + 9*a**2*b**2*c
**2*d*e**3*x**2/8 - 9*a**2*b**2*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)/4 + 6*a**2*b**2*c*d**2*e**3*x**3*acosh(c + d*x)**2 + 3
*a**2*b**2*c*d**2*e**3*x**3/4 - 9*a**2*b**2*c*d*e**3*x**2*sqrt(c**2 + 2*c*d
*x + d**2*x**2 - 1)*acosh(c + d*x)/4 + 9*a**2*b**2*c*e**3*x/8 - 9*a**2*b**2
*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(8*d) + 3*a**2*
b**2*d**3*e**3*x**4*acosh(c + d*x)**2/2 + 3*a**2*b**2*d**3*e**3*x**4/16 - 3
*a**2*b**2*d**2*e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*
x)/4 + 9*a**2*b**2*d*e**3*x**2/16 - 9*a**2*b**2*e**3*x*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*acosh(c + d*x)/8 - 9*a**2*b**2*e**3*acosh(c + d*x)**2/(16*
d) + a*b**3*c**4*e**3*acosh(c + d*x)**3/d + 3*a*b**3*c**4*e**3*acosh(c + d*
x)/(8*d) + 4*a*b**3*c**3*e**3*x*acosh(c + d*x)**3 + 3*a*b**3*c**3*e**3*x*ac
osh(c + d*x)/2 - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*ac
osh(c + d*x)**2/(4*d) - 3*a*b**3*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)/(32*d) + 6*a*b**3*c**2*d*e**3*x**2*acosh(c + d*x)**3 + 9*a*b**3*c**2*d
*e**3*x**2*acosh(c + d*x)/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)*acosh(c + d*x)**2/4 - 9*a*b**3*c**2*e**3*x*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)/32 + 9*a*b**3*c**2*e**3*acosh(c + d*x)/(8*d) + 4*a*b**3*c
*d**2*e**3*x**3*acosh(c + d*x)**3 + 3*a*b**3*c*d**2*e**3*x**3*acosh(c + d*x
)/2 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)**2/4 - 9*a*b**3*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/32
 + 9*a*b**3*c*e**3*x*acosh(c + d*x)/4 - 9*a*b**3*c*e**3*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*acosh(c + d*x)**2/(8*d) - 45*a*b**3*c*e**3*sqrt(c**2 + 2*
c*d*x + d**2*x**2 - 1)/(64*d) + a*b**3*d**3*e**3*x**4*acosh(c + d*x)**3 + 3
*a*b**3*d**3*e**3*x**4*acosh(c + d*x)/8 - 3*a*b**3*d**2*e**3*x**3*sqrt(c**2
 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/4 - 3*a*b**3*d**2*e**3*x**3*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/32 + 9*a*b**3*d*e**3*x**2*acosh(c + d*x
)/8 - 9*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**
2/8 - 45*a*b**3*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/64 - 3*a*b**3*e
**3*acosh(c + d*x)**3/(8*d) - 45*a*b**3*e**3*acosh(c + d*x)/(64*d) + b**4*c
**4*e**3*acosh(c + d*x)**4/(4*d) + 3*b**4*c**4*e**3*acosh(c + d*x)**2/(16*d
) + b**4*c**3*e**3*x*acosh(c + d*x)**4 + 3*b**4*c**3*e**3*x*acosh(c + d*x)*
*2/4 + 3*b**4*c**3*e**3*x/32 - b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)**3/(4*d) - 3*b**4*c**3*e**3*sqrt(c**2 + 2*c*d*x + d*
*2*x**2 - 1)*acosh(c + d*x)/(32*d) + 3*b**4*c**2*d*e**3*x**2*acosh(c + d*x)
```

```
**4/2 + 9*b**4*c**2*d*e**3*x**2*acosh(c + d*x)**2/8 + 9*b**4*c**2*d*e**3*x*
*2/64 - 3*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d
*x)**3/4 - 9*b**4*c**2*e**3*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)/32 + 9*b**4*c**2*e**3*acosh(c + d*x)**2/(16*d) + b**4*c*d**2*e**3*x*
*3*acosh(c + d*x)**4 + 3*b**4*c*d**2*e**3*x**3*acosh(c + d*x)**2/4 + 3*b**4
*c*d**2*e**3*x**3/32 - 3*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2
 - 1)*acosh(c + d*x)**3/4 - 9*b**4*c*d*e**3*x**2*sqrt(c**2 + 2*c*d*x + d**2
*x**2 - 1)*acosh(c + d*x)/32 + 9*b**4*c*e**3*x*acosh(c + d*x)**2/8 + 45*b**
4*c*e**3*x/64 - 3*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c
+ d*x)**3/(8*d) - 45*b**4*c*e**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh
(c + d*x)/(64*d) + b**4*d**3*e**3*x**4*acosh(c + d*x)**4/4 + 3*b**4*d**3*e*
*3*x**4*acosh(c + d*x)**2/16 + 3*b**4*d**3*e**3*x**4/128 - b**4*d**2*e**3*x
**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/4 - 3*b**4*d**2*
e**3*x**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/32 + 9*b**4*d
*e**3*x**2*acosh(c + d*x)**2/16 + 45*b**4*d*e**3*x**2/128 - 3*b**4*e**3*x*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/8 - 45*b**4*e**3*x*sq
rt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/64 - 3*b**4*e**3*acosh(c
+ d*x)**4/(32*d) - 45*b**4*e**3*acosh(c + d*x)**2/(128*d), Ne(d, 0)), (c**3
*e**3*x*(a + b*acosh(c))**4, True))
```

3.123 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=309

$$-\frac{8b^3e^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{27d}-\frac{160b^3e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(\right.}{27d}$$

```
[Out] 160/27*b^4*e^2*x+8/81*b^4*e^2*(d*x+c)^3/d+8/3*b^2*e^2*(d*x+c)*(a+b*arccosh(
d*x+c))^2/d+4/9*b^2*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^2/d+1/3*e^2*(d*x+c)^
3*(a+b*arccosh(d*x+c))^4/d-160/27*b^3*e^2*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)/d-8/27*b^3*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/d-8/9*b*e^2*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/d-4/9*b*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.84, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5662, 5759, 5718, 5654, 8, 30}

$$-\frac{8b^3e^2\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{27d}-\frac{160b^3e^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(\right.}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (160*b^4*e^2*x)/27 + (8*b^4*e^2*(c + d*x)^3)/(81*d) - (160*b^3*e^2*Sqrt[-1
+ c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/(27*d) - (8*b^3*e^2*
Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x]))/
(27*d) + (8*b^2*e^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^2)/(3*d) + (4*b^2*e^
2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^2)/(9*d) - (8*b*e^2*Sqrt[-1 + c + d*
x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(9*d) - (4*b*e^2*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(9*d) +
 (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^4)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
```

```
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\cosh^{-1}(c+dx)\right)^4}{3d} - \frac{\left(4be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\right.}{3d}$$

$$= -\frac{4be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{9d} +$$

$$= \frac{4b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^2}{9d} - \frac{8be^2\sqrt{-1+c+dx}\,\sqrt{1+c+}}{9}$$

$$= -\frac{8b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{27d} +$$

$$= \frac{8b^4e^2(c+dx)^3}{81d} - \frac{160b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c\right.}{27d}$$

$$= \frac{160}{27}b^4e^2x + \frac{8b^4e^2(c+dx)^3}{81d} - \frac{160b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+\right.}{27d}$$

Mathematica [A] time = 0.64, size = 475, normalized size = 1.54

$$\frac{e^2\left(24b^2\left(9a^2+20b^2\right)(c+dx)+12ab\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\left(-\left(3a^2+2b^2\right)(c+dx)^2-6a^2-40b^2\right)+18b^2\cosh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^2*(24*b^2*(9*a^2 + 20*b^2)*(c + d*x) + (27*a^4 + 36*a^2*b^2 + 8*b^4)*(c
+ d*x)^3 + 12*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-6*a^2 - 40*b^2 - (
3*a^2 + 2*b^2)*(c + d*x)^2) + 12*b*(36*a*b^2*(c + d*x) + 9*a^3*(c + d*x)^3
+ 6*a*b^2*(c + d*x)^3 - 18*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 40*
b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 9*a^2*b*Sqrt[-1 + c + d*x]*(c +
d*x)^2*Sqrt[1 + c + d*x] - 2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c
+ d*x])*ArcCosh[c + d*x] + 18*b^2*(12*b^2*(c + d*x) + 9*a^2*(c + d*x)^3 + 2
*b^2*(c + d*x)^3 - 12*a*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] - 6*a*b*Sqrt
[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 - 36*b^3*(
-3*a*(c + d*x)^3 + 2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*Sqrt[-1 + c
 + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + 27*b^4*(c + d*x
)^3*ArcCosh[c + d*x]^4))/(81*d)
```

fricas [B] time = 0.54, size = 890, normalized size = 2.88

$$\frac{\left(27\,a^4+36\,a^2b^2+8\,b^4\right)d^3e^2x^3+3\left(27\,a^4+36\,a^2b^2+8\,b^4\right)cd^2e^2x^2+3\left(72\,a^2b^2+160\,b^4+\left(27\,a^4+36\,a^2b^2+8\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/81*((27*a^4 + 36*a^2*b^2 + 8*b^4)*d^3*e^2*x^3 + 3*(27*a^4 + 36*a^2*b^2 +
8*b^4)*c*d^2*e^2*x^2 + 3*(72*a^2*b^2 + 160*b^4 + (27*a^4 + 36*a^2*b^2 + 8*b
^4)*c^2)*d*e^2*x + 27*(b^4*d^3*e^2*x^3 + 3*b^4*c*d^2*e^2*x^2 + 3*b^4*c^2*d*
e^2*x + b^4*c^3*e^2)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 3
6*(3*a*b^3*d^3*e^2*x^3 + 9*a*b^3*c*d^2*e^2*x^2 + 9*a*b^3*c^2*d*e^2*x + 3*a*
b^3*c^3*e^2 - (b^4*d^2*e^2*x^2 + 2*b^4*c*d*e^2*x + (b^4*c^2 + 2*b^4)*e^2)*s
qrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^
2 - 1))^3 + 18*((9*a^2*b^2 + 2*b^4)*d^3*e^2*x^3 + 3*(9*a^2*b^2 + 2*b^4)*c*d
^2*e^2*x^2 + 3*(4*b^4 + (9*a^2*b^2 + 2*b^4)*c^2)*d*e^2*x + (12*b^4*c + (9*a
^2*b^2 + 2*b^4)*c^3)*e^2 - 6*(a*b^3*d^2*e^2*x^2 + 2*a*b^3*c*d*e^2*x + (a*b^
3*c^2 + 2*a*b^3)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt
(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 12*(3*(3*a^3*b + 2*a*b^3)*d^3*e^2*x^3 +
9*(3*a^3*b + 2*a*b^3)*c*d^2*e^2*x^2 + 9*(4*a*b^3 + (3*a^3*b + 2*a*b^3)*c^2)
*d*e^2*x + 3*(12*a*b^3*c + (3*a^3*b + 2*a*b^3)*c^3)*e^2 - ((9*a^2*b^2 + 2*b
^4)*d^2*e^2*x^2 + 2*(9*a^2*b^2 + 2*b^4)*c*d*e^2*x + (18*a^2*b^2 + 40*b^4 +
(9*a^2*b^2 + 2*b^4)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x +
c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 12*((3*a^3*b + 2*a*b^3)*d^2*e^2*x^
2 + 2*(3*a^3*b + 2*a*b^3)*c*d*e^2*x + (6*a^3*b + 40*a*b^3 + (3*a^3*b + 2*a*
b^3)*c^2)*e^2)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^4, x)
```

maple [A] time = 0.07, size = 517, normalized size = 1.67

$$\frac{\frac{(dx+c)^3 e^2 a^4}{3} + e^2 b^4 \left(\frac{(dx+c)^3 \operatorname{arccosh}(dx+c)^4}{3} - \frac{8 \operatorname{arccosh}(dx+c)^3 \sqrt{dx+c-1}\,\sqrt{dx+c+1}}{9} - \frac{4 (dx+c)^2 \operatorname{arccosh}(dx+c)^3 \sqrt{dx+c-1}\,\sqrt{dx+c+1}}{9} + \frac{8(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/3*(d*x+c)^3*e^2*a^4+e^2*b^4*(1/3*(d*x+c)^3*arccosh(d*x+c)^4-8/9*arcc
osh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4/9*(d*x+c)^2*arccosh(d*x+c)^3
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+8/3*(d*x+c)*arccosh(d*x+c)^2-160/27*arccos
h(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+160/27*d*x+160/27*c+4/9*(d*x+c)^3*
arccosh(d*x+c)^2-8/27*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(d*x+c
)^2+8/81*(d*x+c)^3)+4*e^2*a*b^3*(1/3*(d*x+c)^3*arccosh(d*x+c)^3-2/3*arccosh
(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-1/3*(d*x+c)^2*arccosh(d*x+c)^2*(d
*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4/3*(d*x+c)*arccosh(d*x+c)-40/27*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)+2/9*arccosh(d*x+c)*(d*x+c)^3-2/27*(d*x+c)^2*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2))+6*e^2*a^2*b^2*(1/3*(d*x+c)^3*arccosh(d*x+c)^2-4/9*arc
cosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-2/9*arccosh(d*x+c)*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)*(d*x+c)^2+4/9*d*x+4/9*c+2/27*(d*x+c)^3)+4*e^2*a^3*b*(1/
3*arccosh(d*x+c)*(d*x+c)^3-1/9*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*((d*x+c)^2+2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^4*d^2*e^2*x^3 + a^4*c*d*e^2*x^2 + 2*(2*x^2*arccosh(d*x + c) - d*(3*c^
2*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d
^2*x^2 + 2*c*d*x + c^2 - 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(
d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c
/d^3))*a^3*b*c*d*e^2 + 2/9*(6*x^3*arccosh(d*x + c) - d*(2*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*x^2/d^2 - 15*c^3*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1)*d)/d^4 - 5*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c*x/d^3 + 9*(
c^2 - 1)*c*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^4
 + 15*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c^2/d^4 - 4*sqrt(d^2*x^2 + 2*c*d*x
+ c^2 - 1)*(c^2 - 1)/d^4))*a^3*b*d^2*e^2 + a^4*c^2*e^2*x + 4*((d*x + c)*arc
cosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b*c^2*e^2/d + 1/3*(b^4*d^2*e^2*x
^3 + 3*b^4*c*d*e^2*x^2 + 3*b^4*c^2*e^2*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + c)^4 + integrate(2/3*(2*((3*a*b^3*d^5*e^2 - b^4*d^5*e^2)*x^5
 + 3*(c^5*e^2 - c^3*e^2)*a*b^3 + 5*(3*a*b^3*c*d^4*e^2 - b^4*c*d^4*e^2)*x^4
+ (3*(10*c^2*d^3*e^2 - d^3*e^2)*a*b^3 - (10*c^2*d^3*e^2 - d^3*e^2)*b^4)*x^3
 + 3*((10*c^3*d^2*e^2 - 3*c*d^2*e^2)*a*b^3 - (3*c^3*d^2*e^2 - c*d^2*e^2)*b^
4)*x^2 + (3*(c^4*e^2 - c^2*e^2)*a*b^3 + (3*a*b^3*d^4*e^2 - b^4*d^4*e^2)*x^4
 + 4*(3*a*b^3*c*d^3*e^2 - b^4*c*d^3*e^2)*x^3 - 3*(2*b^4*c^2*d^2*e^2 - (6*c^
2*d^2*e^2 - d^2*e^2)*a*b^3)*x^2 - 3*(b^4*c^3*d*e^2 - 2*(2*c^3*d*e^2 - c*d*e
^2)*a*b^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 3*((5*c^4*d*e^2 - 3*c^2
*d*e^2)*a*b^3 - (c^4*d*e^2 - c^2*d*e^2)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)
*sqrt(d*x + c - 1) + c)^3 + 9*(a^2*b^2*d^5*e^2*x^5 + 5*a^2*b^2*c*d^4*e^2*x^
4 + (10*c^2*d^3*e^2 - d^3*e^2)*a^2*b^2*x^3 + (10*c^3*d^2*e^2 - 3*c*d^2*e^2)
*a^2*b^2*x^2 + (5*c^4*d*e^2 - 3*c^2*d*e^2)*a^2*b^2*x + (c^5*e^2 - c^3*e^2)*
a^2*b^2 + (a^2*b^2*d^4*e^2*x^4 + 4*a^2*b^2*c*d^3*e^2*x^3 + (6*c^2*d^2*e^2 -
 d^2*e^2)*a^2*b^2*x^2 + 2*(2*c^3*d*e^2 - c*d*e^2)*a^2*b^2*x + (c^4*e^2 - c^
2*e^2)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + c)^2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2
*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,(a+b\,\mathrm{acosh}(c+d\,x))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^4, x)
```

sympy [A] time = 9.09, size = 1889, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c**2*e**2*x + a**4*c*d*e**2*x**2 + a**4*d**2*e**2*x**3/3 +
4*a**3*b*c**3*e**2*acosh(c + d*x)/(3*d) + 4*a**3*b*c**2*e**2*x*acosh(c + d*
x) - 4*a**3*b*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(9*d) + 4*a**3
*b*c*d*e**2*x**2*acosh(c + d*x) - 8*a**3*b*c*e**2*x*sqrt(c**2 + 2*c*d*x + d
**2*x**2 - 1)/9 + 4*a**3*b*d**2*e**2*x**3*acosh(c + d*x)/3 - 4*a**3*b*d*e**
2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/9 - 8*a**3*b*e**2*sqrt(c**2 + 2
*c*d*x + d**2*x**2 - 1)/(9*d) + 2*a**2*b**2*c**3*e**2*acosh(c + d*x)**2/d +
 6*a**2*b**2*c**2*e**2*x*acosh(c + d*x)**2 + 4*a**2*b**2*c**2*e**2*x/3 - 4*
a**2*b**2*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/(3*
d) + 6*a**2*b**2*c*d*e**2*x**2*acosh(c + d*x)**2 + 4*a**2*b**2*c*d*e**2*x**
```

```
2/3 - 8*a**2*b**2*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d
*x)/3 + 2*a**2*b**2*d**2*e**2*x**3*acosh(c + d*x)**2 + 4*a**2*b**2*d**2*e**
2*x**3/9 - 4*a**2*b**2*d*e**2*x**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*aco
sh(c + d*x)/3 + 8*a**2*b**2*e**2*x/3 - 8*a**2*b**2*e**2*sqrt(c**2 + 2*c*d*x
 + d**2*x**2 - 1)*acosh(c + d*x)/(3*d) + 4*a*b**3*c**3*e**2*acosh(c + d*x)*
*3/(3*d) + 8*a*b**3*c**3*e**2*acosh(c + d*x)/(9*d) + 4*a*b**3*c**2*e**2*x*a
cosh(c + d*x)**3 + 8*a*b**3*c**2*e**2*x*acosh(c + d*x)/3 - 4*a*b**3*c**2*e*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 8*a*b**3*
c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + 4*a*b**3*c*d*e**2*x
**2*acosh(c + d*x)**3 + 8*a*b**3*c*d*e**2*x**2*acosh(c + d*x)/3 - 8*a*b**3*
c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 16*a*b*
*3*c*e**2*x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/27 + 16*a*b**3*c*e**2*acos
h(c + d*x)/(3*d) + 4*a*b**3*d**2*e**2*x**3*acosh(c + d*x)**3/3 + 8*a*b**3*d
**2*e**2*x**3*acosh(c + d*x)/9 - 4*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)**2/3 - 8*a*b**3*d*e**2*x**2*sqrt(c**2 + 2*c*
d*x + d**2*x**2 - 1)/27 + 16*a*b**3*e**2*x*acosh(c + d*x)/3 - 8*a*b**3*e**2
*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2/(3*d) - 160*a*b**3*
e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(27*d) + b**4*c**3*e**2*acosh(c +
 d*x)**4/(3*d) + 4*b**4*c**3*e**2*acosh(c + d*x)**2/(9*d) + b**4*c**2*e**2*
x*acosh(c + d*x)**4 + 4*b**4*c**2*e**2*x*acosh(c + d*x)**2/3 + 8*b**4*c**2*
e**2*x/27 - 4*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c +
 d*x)**3/(9*d) - 8*b**4*c**2*e**2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acos
h(c + d*x)/(27*d) + b**4*c*d*e**2*x**2*acosh(c + d*x)**4 + 4*b**4*c*d*e**2*
x**2*acosh(c + d*x)**2/3 + 8*b**4*c*d*e**2*x**2/27 - 8*b**4*c*e**2*x*sqrt(c
**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/9 - 16*b**4*c*e**2*x*sqrt(
c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/27 + 8*b**4*c*e**2*acosh(c +
 d*x)**2/(3*d) + b**4*d**2*e**2*x**3*acosh(c + d*x)**4/3 + 4*b**4*d**2*e**2
*x**3*acosh(c + d*x)**2/9 + 8*b**4*d**2*e**2*x**3/81 - 4*b**4*d*e**2*x**2*s
qrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/9 - 8*b**4*d*e**2*x**
2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/27 + 8*b**4*e**2*x*ac
osh(c + d*x)**2/3 + 160*b**4*e**2*x/27 - 8*b**4*e**2*sqrt(c**2 + 2*c*d*x +
d**2*x**2 - 1)*acosh(c + d*x)**3/(9*d) - 160*b**4*e**2*sqrt(c**2 + 2*c*d*x
+ d**2*x**2 - 1)*acosh(c + d*x)/(27*d), Ne(d, 0)), (c**2*e**2*x*(a + b*acos
h(c))**4, True))
```

3.124 $\int (ce + dex) \left(a + b\cosh^{-1}(c + dx)\right)^4 dx$

Optimal. Leaf size=209

$$-\frac{3b^3e\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{3b^2e\left(a+\right.}{}$$

```
[Out] 3/4*b^4*e*(d*x+c)^2/d-3/4*b^2*e*(a+b*arccosh(d*x+c))^2/d+3/2*b^2*e*(d*x+c)^
2*(a+b*arccosh(d*x+c))^2/d-1/4*e*(a+b*arccosh(d*x+c))^4/d+1/2*e*(d*x+c)^2*(
a+b*arccosh(d*x+c))^4/d-3/2*b^3*e*(d*x+c)*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)/d-b*e*(d*x+c)*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.56, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5866, 12, 5662, 5759, 5676, 30}

$$-\frac{3b^3e\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,\left(a+b\cosh^{-1}(c+dx)\right)}{2d}+\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{3b^2e\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (3*b^4*e*(c + d*x)^2)/(4*d) - (3*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1
+ c + d*x]*(a + b*ArcCosh[c + d*x]))/(2*d) - (3*b^2*e*(a + b*ArcCosh[c + d*
x])^2)/(4*d) + (3*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^2)/(2*d) - (b*
e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3
)/d - (e*(a + b*ArcCosh[c + d*x])^4)/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[
c + d*x])^4)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$

$$=\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^4}{2d}-\frac{(2be)\,\text{Subst}\left(\int\frac{x^2\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,\sqrt{1+x}}\right.}{d}$$

$$=-\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{d}+\frac{e(}{}$$

$$=\frac{3b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{2d}-\frac{be\sqrt{-1+c+dx}\,(c+dx)\sqrt{}}{}$$

$$=-\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{2d}+$$

$$=\frac{3b^4e(c+dx)^2}{4d}-\frac{3b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh\right.}{2d}$$

Mathematica [A] time = 0.49, size = 360, normalized size = 1.72

$$\frac{e\left(-2ab\left(2a^2+3b^2\right)\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}\,-2ab\left(2a^2+3b^2\right)\log\left(\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,+c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e*((2*a^4 + 6*a^2*b^2 + 3*b^4)*(c + d*x)^2 - 2*a*b*(2*a^2 + 3*b^2)*Sqrt[-1
 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x] - 2*b*(c + d*x)*(-4*a^3*(c + d*x) -
 6*a*b^2*(c + d*x) + 6*a^2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 3*b^3*S
qrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x] + 3*b^2*(-2*a^2 - b^2
 + 4*a^2*(c + d*x)^2 + 2*b^2*(c + d*x)^2 - 4*a*b*Sqrt[-1 + c + d*x]*(c + d*
x)*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 + 4*b^3*(-a + 2*a*(c + d*x)^2 - b*
Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + b^4*(-
```

```
1 + 2*(c + d*x)^2)*ArcCosh[c + d*x]^4 - 2*a*b*(2*a^2 + 3*b^2)*Log[c + d*x +
 Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]]))/(4*d)
```

fricas [B] time = 0.80, size = 579, normalized size = 2.77

$$\frac{\left(2\,a^4 + 6\,a^2b^2 + 3\,b^4\right)d^2ex^2 + 2\left(2\,a^4 + 6\,a^2b^2 + 3\,b^4\right)cdex + \left(2\,b^4d^2ex^2 + 4\,b^4cdex + \left(2\,b^4c^2 - b^4\right)e\right)\log\left(dx + c + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^4 + 6*a^2*b^2 + 3*b^4)*d^2*e*x^2 + 2*(2*a^4 + 6*a^2*b^2 + 3*b^4)*
c*d*e*x + (2*b^4*d^2*e*x^2 + 4*b^4*c*d*e*x + (2*b^4*c^2 - b^4)*e)*log(d*x +
 c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 4*(2*a*b^3*d^2*e*x^2 + 4*a*b^3*
c*d*e*x + (2*a*b^3*c^2 - a*b^3)*e - (b^4*d*e*x + b^4*c*e)*sqrt(d^2*x^2 + 2*
c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + 3*(2
*(2*a^2*b^2 + b^4)*d^2*e*x^2 + 4*(2*a^2*b^2 + b^4)*c*d*e*x - (2*a^2*b^2 + b
^4 - 2*(2*a^2*b^2 + b^4)*c^2)*e - 4*(a*b^3*d*e*x + a*b^3*c*e)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 +
2*(2*(2*a^3*b + 3*a*b^3)*d^2*e*x^2 + 4*(2*a^3*b + 3*a*b^3)*c*d*e*x - (2*a^3
*b + 3*a*b^3 - 2*(2*a^3*b + 3*a*b^3)*c^2)*e - 3*((2*a^2*b^2 + b^4)*d*e*x +
(2*a^2*b^2 + b^4)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))*log(d*x + c + sqr
t(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 2*((2*a^3*b + 3*a*b^3)*d*e*x + (2*a^3*b +
 3*a*b^3)*c*e)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arcosh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4, x)
```

maple [B] time = 0.04, size = 933, normalized size = 4.46

$$-\frac{3ea\,b^3\sqrt{dx + c - 1}\,\sqrt{dx + c + 1}\,x}{2} + 4\operatorname{arccosh}(dx + c)\,x\,a^3bce + 3dea\,b^3\operatorname{arccosh}(dx + c)\,x^2 + 6e\,a^2b^2\operatorname{arccosh}(dx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] -3/2*e*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x+4*arccosh(d*x+c)*x*a^3*b*c*e
+3*d*e*a*b^3*arccosh(d*x+c)*x^2+6*e*a^2*b^2*arccosh(d*x+c)^2*x*c+2*d*e*a*b^
3*arccosh(d*x+c)^3*x^2+4*e*a*b^3*arccosh(d*x+c)^3*x*c+6*e*a*b^3*arccosh(d*x
+c)*x*c-e*b^4*arccosh(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)*x*a^3*b*e+2/d*arccosh(d*x+c)*a^3*b*c^2*e+3/d*e*a^2*b^
2*arccosh(d*x+c)^2*c^2+1/2/d*a^4*c^2*e-1/4/d*e*b^4*arccosh(d*x+c)^4-3/4/d*e
*b^4*arccosh(d*x+c)^2+3/4*d*e*b^4*x^2+1/2*d*x^2*a^4*e+3/2*e*b^4*x*c+x*a^4*c
*e+3/4/d*e*b^4*c^2+2*d*arccosh(d*x+c)*x^2*a^3*b*e+3*d*e*a^2*b^2*arccosh(d*x
+c)^2*x^2+2/d*e*a*b^3*arccosh(d*x+c)^3*c^2+3/d*e*a*b^3*arccosh(d*x+c)*c^2-3
/2*e*b^4*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-1/d*e*a^3*b*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)/((d*x+c)^2-1)^(1/2)*ln(d*x+c+((d*x+c)^2-1)^(1/2)
)-3/d*e*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3/d*e*a*b^
3*arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3*e*a*b^3*arccosh(d*x+
c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*x-3/2/d*e*b^4*arccosh(d*x+c)*(d*x+c-1)
```

```
^(1/2)*(d*x+c+1)^(1/2)*c-1/d*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*a^3*b*c*e-3/2/
d*e*a*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-1/d*e*b^4*arccosh(d*x+c)^3*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)*c-3*e*a^2*b^2*arccosh(d*x+c)*(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)*x+3/2*d*e*a^2*b^2*x^2+3*e*b^4*arccosh(d*x+c)^2*x*c+e*b^4*arcc
osh(d*x+c)^4*x*c+1/2*d*e*b^4*arccosh(d*x+c)^4*x^2+3/2*d*e*b^4*arccosh(d*x+c
)^2*x^2-3/2/d*e*a^2*b^2*arccosh(d*x+c)^2-1/d*e*a*b^3*arccosh(d*x+c)^3-3/2/d
*e*a*b^3*arccosh(d*x+c)+1/2/d*e*b^4*arccosh(d*x+c)^4*c^2+3/2/d*e*b^4*arccos
h(d*x+c)^2*c^2+3*e*a^2*b^2*x*c+3/2/d*e*a^2*b^2*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a^4dex^2+\left(2\,x^2\,\mathrm{arcosh}\,(dx+c)-d\left(\frac{3\,c^2\log\left(2\,d^2x+2\,cd+2\,\sqrt{d^2x^2+2\,cdx+c^2-1}\,d\right)}{d^3}+\frac{\sqrt{d^2x^2+2\,cdx+c^2}}{d^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*a^4*d*e*x^2 + (2*x^2*arccosh(d*x + c) - d*(3*c^2*log(2*d^2*x + 2*c*d +
2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)/d^3 + sqrt(d^2*x^2 + 2*c*d*x + c^2 -
 1)*x/d^2 - (c^2 - 1)*log(2*d^2*x + 2*c*d + 2*sqrt(d^2*x^2 + 2*c*d*x + c^2
- 1)*d)/d^3 - 3*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*c/d^3))*a^3*b*d*e + a^4*c
*e*x + 4*((d*x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b*c*e/d +
 1/2*(b^4*d*e*x^2 + 2*b^4*c*e*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)^4 + integrate(2*((2*(c^4*e - c^2*e)*a*b^3 + (2*a*b^3*d^4*e - b^4*d
^4*e)*x^4 + 4*(2*a*b^3*c*d^3*e - b^4*c*d^3*e)*x^3 + (2*(6*c^2*d^2*e - d^2*e
)*a*b^3 - (5*c^2*d^2*e - d^2*e)*b^4)*x^2 + (2*(c^3*e - c*e)*a*b^3 + (2*a*b^
3*d^3*e - b^4*d^3*e)*x^3 + 3*(2*a*b^3*c*d^2*e - b^4*c*d^2*e)*x^2 - 2*(b^4*c
^2*d*e - (3*c^2*d*e - d*e)*a*b^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
2*(2*(2*c^3*d*e - c*d*e)*a*b^3 - (c^3*d*e - c*d*e)*b^4)*x)*log(d*x + sqrt(d
*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 3*(a^2*b^2*d^4*e*x^4 + 4*a^2*b^2*c*d
^3*e*x^3 + (6*c^2*d^2*e - d^2*e)*a^2*b^2*x^2 + 2*(2*c^3*d*e - c*d*e)*a^2*b^
2*x + (c^4*e - c^2*e)*a^2*b^2 + (a^2*b^2*d^3*e*x^3 + 3*a^2*b^2*c*d^2*e*x^2
+ (3*c^2*d*e - d*e)*a^2*b^2*x + (c^3*e - c*e)*a^2*b^2)*sqrt(d*x + c + 1)*sq
rt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2)/(d^3
*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*
sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\,(c+d\,x)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^4, x)
```

sympy [A] time = 4.52, size = 1027, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*c*e*x + a**4*d*e*x**2/2 + 2*a**3*b*c**2*e*acosh(c + d*x)/d
+ 4*a**3*b*c*e*x*acosh(c + d*x) - a**3*b*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**
2 - 1)/d + 2*a**3*b*d*e*x**2*acosh(c + d*x) - a**3*b*e*x*sqrt(c**2 + 2*c*d*
x + d**2*x**2 - 1) - a**3*b*e*acosh(c + d*x)/d + 3*a**2*b**2*c**2*e*acosh(c
```

```
 + d*x)**2/d + 6*a**2*b**2*c*e*x*acosh(c + d*x)**2 + 3*a**2*b**2*c*e*x - 3*
a**2*b**2*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + 3*a**
2*b**2*d*e*x**2*acosh(c + d*x)**2 + 3*a**2*b**2*d*e*x**2/2 - 3*a**2*b**2*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x) - 3*a**2*b**2*e*acosh
(c + d*x)**2/(2*d) + 2*a*b**3*c**2*e*acosh(c + d*x)**3/d + 3*a*b**3*c**2*e*
acosh(c + d*x)/d + 4*a*b**3*c*e*x*acosh(c + d*x)**3 + 6*a*b**3*c*e*x*acosh(
c + d*x) - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)
**2/d - 3*a*b**3*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/(2*d) + 2*a*b**3*
d*e*x**2*acosh(c + d*x)**3 + 3*a*b**3*d*e*x**2*acosh(c + d*x) - 3*a*b**3*e*
x*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**2 - 3*a*b**3*e*x*sqr
t(c**2 + 2*c*d*x + d**2*x**2 - 1)/2 - a*b**3*e*acosh(c + d*x)**3/d - 3*a*b*
*3*e*acosh(c + d*x)/(2*d) + b**4*c**2*e*acosh(c + d*x)**4/(2*d) + 3*b**4*c*
*2*e*acosh(c + d*x)**2/(2*d) + b**4*c*e*x*acosh(c + d*x)**4 + 3*b**4*c*e*x*
acosh(c + d*x)**2 + 3*b**4*c*e*x/2 - b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x*
*2 - 1)*acosh(c + d*x)**3/d - 3*b**4*c*e*sqrt(c**2 + 2*c*d*x + d**2*x**2 -
1)*acosh(c + d*x)/(2*d) + b**4*d*e*x**2*acosh(c + d*x)**4/2 + 3*b**4*d*e*x*
*2*acosh(c + d*x)**2/2 + 3*b**4*d*e*x**2/4 - b**4*e*x*sqrt(c**2 + 2*c*d*x +
 d**2*x**2 - 1)*acosh(c + d*x)**3 - 3*b**4*e*x*sqrt(c**2 + 2*c*d*x + d**2*x
**2 - 1)*acosh(c + d*x)/2 - b**4*e*acosh(c + d*x)**4/(4*d) - 3*b**4*e*acosh
(c + d*x)**2/(4*d), Ne(d, 0)), (c*e*x*(a + b*acosh(c))**4, True))
```

3.125 $\int \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=129

$$-\frac{24b^3\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{c+dx-1}}{}$$

```
[Out] 24*b^4*x+12*b^2*(d*x+c)*(a+b*arccosh(d*x+c))^2/d+(d*x+c)*(a+b*arccosh(d*x+c
))^4/d-24*b^3*(a+b*arccosh(d*x+c))*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-4*b*(a
+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.28, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5864, 5654, 5718, 8}

$$-\frac{24b^3\sqrt{c+dx-1}\sqrt{c+dx+1}\left(a+b\cosh^{-1}(c+dx)\right)}{d}+\frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d}-\frac{4b\sqrt{c+dx-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] 24*b^4*x - (24*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c +
d*x]))/d + (12*b^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^2)/d - (4*b*Sqrt[-1 +
 c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/d + ((c + d*x)*(a +
 b*ArcCosh[c + d*x])^4)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^4 dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^4 dx, x, c+dx\right)}{d}$$
$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^4}{d} - \frac{(4b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c+dx\right)}{d}$$
$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^3}{d} + \frac{(c+dx)\left(a+b\cosh^{-1}\right.}{d}$$
$$= \frac{12b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2}{d} - \frac{4b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh\right.}{d}$$
$$= -\frac{24b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(a+b\,\text{c}\right.}{d}$$
$$= 24b^4x - \frac{24b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)}{d} + \frac{12b^2(c+dx)\left(\right.}{}$$

Mathematica [B] time = 0.27, size = 261, normalized size = 2.02

$$\frac{-4ab\left(a^2+6b^2\right)\sqrt{c+dx-1}\sqrt{c+dx+1} + 6b^2\cosh^{-1}(c+dx)^2\left(a^2(c+dx)-2ab\sqrt{c+dx-1}\sqrt{c+dx+1} + 2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] ((a^4 + 12*a^2*b^2 + 24*b^4)*(c + d*x) - 4*a*b*(a^2 + 6*b^2)*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x] - 4*b*(-(a^3*(c + d*x)) - 6*a*b^2*(c + d*x) + 3*a^2*
b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + 6*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 +
c + d*x])*ArcCosh[c + d*x] + 6*b^2*(a^2*(c + d*x) + 2*b^2*(c + d*x) - 2*a*b
*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^2 - 4*b^3*(-(a*(c +
 d*x)) + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])*ArcCosh[c + d*x]^3 + b^4*(
c + d*x)*ArcCosh[c + d*x]^4)/d
```

fricas [B] time = 0.69, size = 344, normalized size = 2.67

$$\frac{\left(b^4dx+b^4c\right)\log\left(dx+c+\sqrt{d^2x^2+2\,cdx+c^2-1}\right)^4 + 4\left(ab^3dx+ab^3c-\sqrt{d^2x^2+2\,cdx+c^2-1}\,b^4\right)\log\left(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] ((b^4*d*x + b^4*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^4 + 4*(
a*b^3*d*x + a*b^3*c - sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*b^4)*log(d*x + c +
sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^3 + (a^4 + 12*a^2*b^2 + 24*b^4)*d*x - 6*
(2*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*a*b^3 - (a^2*b^2 + 2*b^4)*d*x - (a^2*b
^2 + 2*b^4)*c)*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))^2 + 4*((a^3
*b + 6*a*b^3)*d*x + (a^3*b + 6*a*b^3)*c - 3*(a^2*b^2 + 2*b^4)*sqrt(d^2*x^2
+ 2*c*d*x + c^2 - 1))*log(d*x + c + sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)) - 4*
(a^3*b + 6*a*b^3)*sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arcosh}\,(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4, x)
```

maple [B] time = 0.06, size = 275, normalized size = 2.13

$$\frac{(dx+c)\,a^4+b^4\left((dx+c)\operatorname{arccosh}(dx+c)^4-4\operatorname{arccosh}(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}+12\,(dx+c)\operatorname{arccc}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*((d*x+c)*a^4+b^4*((d*x+c)*arccosh(d*x+c)^4-4*arccosh(d*x+c)^3*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)+12*(d*x+c)*arccosh(d*x+c)^2-24*arccosh(d*x+c)*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)+24*d*x+24*c)+4*a*b^3*((d*x+c)*arccosh(d*x+c)^3-3*
arccosh(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+6*(d*x+c)*arccosh(d*x+c)-6
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))+6*a^2*b^2*((d*x+c)*arccosh(d*x+c)^2-2*arc
cosh(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+2*d*x+2*c)+4*a^3*b*((d*x+c)*arc
cosh(d*x+c)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^4x\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx-c-1}+c\right)^4+a^4x+\frac{4\left((dx+c)\operatorname{arcosh}(dx+c)-\sqrt{(dx+c)^2-1}\right)a^3b}{d}+\int\frac{2\,(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4 + a^4*x + 4*((d*
x + c)*arccosh(d*x + c) - sqrt((d*x + c)^2 - 1))*a^3*b/d + integrate(2*(2*(
(c^3 - c)*a*b^3 + (a*b^3*d^3 - b^4*d^3)*x^3 + (3*a*b^3*c*d^2 - 2*b^4*c*d^2)
*x^2 + ((c^2 - 1)*a*b^3 + (a*b^3*d^2 - b^4*d^2)*x^2 + (2*a*b^3*c*d - b^4*c*
d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d - d)*a*b^3 - (c^2*d -
 d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 3*(a^2*b
^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d - d)*a^2*b^2*x + (c^3 - c)*a^2*
b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2 - 1)*a^2*b^2)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^
2)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}\left(c+d\,x\right))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((a + b*acosh(c + d*x))^4, x)
```

sympy [A] time = 1.60, size = 444, normalized size = 3.44

$$\begin{cases} a^4x+\frac{4a^3bc\operatorname{acosh}(c+dx)}{d}+4a^3bx\operatorname{acosh}(c+dx)-\frac{4a^3b\sqrt{c^2+2cdx+d^2x^2-1}}{d}+\frac{6a^2b^2c\operatorname{acosh}^2(c+dx)}{d}+6a^2b^2x\operatorname{acosh}^2(c+ \\ x\,(a+b\operatorname{acosh}(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*c*acosh(c + d*x)/d + 4*a**3*b*x*acosh(c + d*x)
 - 4*a**3*b*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d + 6*a**2*b**2*c*acosh(c
+ d*x)**2/d + 6*a**2*b**2*x*acosh(c + d*x)**2 + 12*a**2*b**2*x - 12*a**2*b*
*2*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)/d + 4*a*b**3*c*acosh
(c + d*x)**3/d + 24*a*b**3*c*acosh(c + d*x)/d + 4*a*b**3*x*acosh(c + d*x)**
3 + 24*a*b**3*x*acosh(c + d*x) - 12*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2
- 1)*acosh(c + d*x)**2/d - 24*a*b**3*sqrt(c**2 + 2*c*d*x + d**2*x**2 - 1)/d
 + b**4*c*acosh(c + d*x)**4/d + 12*b**4*c*acosh(c + d*x)**2/d + b**4*x*acos
h(c + d*x)**4 + 12*b**4*x*acosh(c + d*x)**2 + 24*b**4*x - 4*b**4*sqrt(c**2
+ 2*c*d*x + d**2*x**2 - 1)*acosh(c + d*x)**3/d - 24*b**4*sqrt(c**2 + 2*c*d*
x + d**2*x**2 - 1)*acosh(c + d*x)/d, Ne(d, 0)), (x*(a + b*acosh(c))**4, Tru
e))
```

3.126 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{ce+dex}\,dx$

Optimal. Leaf size=192

$$-\frac{3b^3\text{Li}_4\left(-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{3b^2\text{Li}_3\left(-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de}-\frac{2b\text{Li}_2\left(-\right.}{}$$

```
[Out] 1/5*(a+b*arccosh(d*x+c))^5/b/d/e+(a+b*arccosh(d*x+c))^4*ln(1+1/(d*x+c+(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-2*b*(a+b*arccosh(d*x+c))^3*polylog(2,-1/
(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3*b^2*(a+b*arccosh(d*x+c))^2
*polylog(3,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3*b^3*(a+b*arc
cosh(d*x+c))*polylog(4,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e-3/
2*b^4*polylog(5,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)/d/e
```

Rubi [A] time = 0.27, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5660, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de}+\frac{3b^3\text{PolyLog}\left(4,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+\right.}{de}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] -(a + b*ArcCosh[c + d*x])^5/(5*b*d*e) + ((a + b*ArcCosh[c + d*x])^4*Log[1 +
 E^(2*ArcCosh[c + d*x])])/(d*e) + (2*b*(a + b*ArcCosh[c + d*x])^3*PolyLog[2
, -E^(2*ArcCosh[c + d*x])])/(d*e) - (3*b^2*(a + b*ArcCosh[c + d*x])^2*PolyL
og[3, -E^(2*ArcCosh[c + d*x])])/(d*e) + (3*b^3*(a + b*ArcCosh[c + d*x])*Pol
yLog[4, -E^(2*ArcCosh[c + d*x])])/(d*e) - (3*b^4*PolyLog[5, -E^(2*ArcCosh[c
 + d*x])])/(2*d*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{ex}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x}\,dx,x,c+dx\right)}{de}$$
$$= \frac{\text{Subst}\left(\int (a+bx)^4\tanh(x)\,dx,x,\cosh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^4}{1+e^{2x}}\,dx,x,\cosh^{-1}(c+dx)\right)}{de}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} -$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^5}{5bde} + \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4\log\left(1+e^{2\cosh^{-1}(c+dx)}\right)}{de} +$$

Mathematica [A] time = 0.80, size = 308, normalized size = 1.60

$$\frac{a^4\log(c+dx)+2a^3b\cosh^{-1}(c+dx)^2+4a^3b\cosh^{-1}(c+dx)\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)+2a^2b^2\cosh^{-1}(c+dx)^3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x),x]
```

```
[Out] (2*a^3*b*ArcCosh[c + d*x]^2 + 2*a^2*b^2*ArcCosh[c + d*x]^3 + a*b^3*ArcCosh[
c + d*x]^4 + (b^4*ArcCosh[c + d*x]^5)/5 + 4*a^3*b*ArcCosh[c + d*x]*Log[1 +
E^(-2*ArcCosh[c + d*x])] + 6*a^2*b^2*ArcCosh[c + d*x]^2*Log[1 + E^(-2*ArcCo
sh[c + d*x])] + 4*a*b^3*ArcCosh[c + d*x]^3*Log[1 + E^(-2*ArcCosh[c + d*x])]
 + b^4*ArcCosh[c + d*x]^4*Log[1 + E^(-2*ArcCosh[c + d*x])] + a^4*Log[c + d*
x] - 2*b*(a + b*ArcCosh[c + d*x])^3*PolyLog[2, -E^(-2*ArcCosh[c + d*x])] -
3*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[3, -E^(-2*ArcCosh[c + d*x])] - 3*a
*b^3*PolyLog[4, -E^(-2*ArcCosh[c + d*x])] - 3*b^4*ArcCosh[c + d*x]*PolyLog[
4, -E^(-2*ArcCosh[c + d*x])] - (3*b^4*PolyLog[5, -E^(-2*ArcCosh[c + d*x])])
/2)/(d*e)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4\,\text{arcosh}\,(dx+c)^4+4\,ab^3\,\text{arcosh}\,(dx+c)^3+6\,a^2b^2\,\text{arcosh}\,(dx+c)^2+4\,a^3b\,\text{arcosh}\,(dx+c)+a^4}{dex+ce},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^4}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e), x)
```

maple [B] time = 0.07, size = 727, normalized size = 3.79

$$\frac{a^4 \ln(dx+c)}{de} - \frac{b^4 \operatorname{arccosh}(dx+c)^5}{5de} + \frac{b^4 \operatorname{arccosh}(dx+c)^4 \ln\left(1+\left(dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}\right)^2\right)}{de} + \frac{2b^4 \operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x)
```

```
[Out] 1/d*a^4/e*ln(d*x+c)-1/5/d*b^4/e*arccosh(d*x+c)^5+1/d*b^4/e*arccosh(d*x+c)^4
*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+2/d*b^4/e*arccosh(d*x+c)^3
*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/d*b^4/e*arccosh(d*
x+c)^2*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+3/d*b^4/e*arcc
osh(d*x+c)*polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/2/d*b^4/
e*polylog(5,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-1/d*a*b^3/e*arccosh
(d*x+c)^4+4/d*a*b^3/e*arccosh(d*x+c)^3*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2))^2)+6/d*a*b^3/e*arccosh(d*x+c)^2*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2))^2)-6/d*a*b^3/e*arccosh(d*x+c)*polylog(3,-(d*x+c+(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2))^2)+3/d*a*b^3/e*polylog(4,-(d*x+c+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2))^2)-2/d*a^2*b^2/e*arccosh(d*x+c)^3+6/d*a^2*b^2/e*arccosh(d*x+c
)^2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)+6/d*a^2*b^2/e*arccosh(d
*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-3/d*a^2*b^2/e*p
olylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-2/d*a^3*b/e*arccosh(d*
x+c)^2+4/d*a^3*b/e*arccosh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2))^2)+2/d*a^3*b/e*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^4 \log(dex+ce)}{de} + \int \frac{b^4 \log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^4}{dex+ce} + \frac{4\,ab^3 \log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^3}{dex+ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] a^4*log(d*e*x + c*e)/(d*e) + integrate(b^4*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^4/(d*e*x + c*e) + 4*a*b^3*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)^3/(d*e*x + c*e) + 6*a^2*b^2*log(d*x + sqrt(d*x + c + 1
)*sqrt(d*x + c - 1) + c)^2/(d*e*x + c*e) + 4*a^3*b*log(d*x + sqrt(d*x + c +
 1)*sqrt(d*x + c - 1) + c)/(d*e*x + c*e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^4}{c\,e+d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c+dx}\,dx + \int \frac{b^4 \operatorname{acosh}^4(c+dx)}{c+dx}\,dx + \int \frac{4ab^3 \operatorname{acosh}^3(c+dx)}{c+dx}\,dx + \int \frac{6a^2b^2 \operatorname{acosh}^2(c+dx)}{c+dx}\,dx + \int \frac{4a^3b \operatorname{acosh}(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**4/(c + d*x), x) + Integral(b**4*acosh(c + d*x)**4/(c + d*x), x
) + Integral(4*a*b**3*acosh(c + d*x)**3/(c + d*x), x) + Integral(6*a**2*b**
2*acosh(c + d*x)**2/(c + d*x), x) + Integral(4*a**3*b*acosh(c + d*x)/(c + d
*x), x))/e
```

3.127 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^2} dx$

Optimal. Leaf size=264

$$\frac{24ib^3\text{Li}_3\left(-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{24ib^3\text{Li}_3\left(ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{12ib^2\text{Li}_2\left(-ie^{\text{cc}}\right.}{}$$

```
[Out] -(a+b*arccosh(d*x+c))^4/d/e^2/(d*x+c)+8*b*(a+b*arccosh(d*x+c))^3*arctan(d*x
+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/d/e^2-12*I*b^2*(a+b*arccosh(d*x+c))^2*p
olylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+12*I*b^2*(a+b*ar
ccosh(d*x+c))^2*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^2+
24*I*b^3*(a+b*arccosh(d*x+c))*polylog(3,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)))/d/e^2-24*I*b^3*(a+b*arccosh(d*x+c))*polylog(3,I*(d*x+c+(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)))/d/e^2-24*I*b^4*polylog(4,-I*(d*x+c+(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)))/d/e^2+24*I*b^4*polylog(4,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)))/d/e^2
```

Rubi [A] time = 0.41, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5662, 5761, 4180, 2531, 6609, 2282, 6589}

$$\frac{24ib^3\text{PolyLog}\left(3,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}-\frac{24ib^3\text{PolyLog}\left(3,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] -((a + b*ArcCosh[c + d*x])^4/(d*e^2*(c + d*x))) + (8*b*(a + b*ArcCosh[c + d
*x])^3*ArcTan[E^ArcCosh[c + d*x]])/(d*e^2) - ((12*I)*b^2*(a + b*ArcCosh[c +
 d*x])^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((12*I)*b^2*(a + b*
ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh[c + d*x]])/(d*e^2) + ((24*I)*b^3
*(a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) - ((
24*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3, I*E^ArcCosh[c + d*x]])/(d*e^2
) - ((24*I)*b^4*PolyLog[4, (-I)*E^ArcCosh[c + d*x]])/(d*e^2) + ((24*I)*b^4*
PolyLog[4, I*E^ArcCosh[c + d*x]])/(d*e^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^2x^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^2}\,dx, x, c+dx\right)}{de^2}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx, x, c+dx\right)}{de^2}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{(4b)\,\text{Subst}\left(\int (a+bx)^3\text{sech}(x)\,dx, x, \cosh^{-1}(c+dx\right)}{de^2}$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - (1$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - 12$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - 12$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - 12$$
$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{de^2(c+dx)} + \frac{8b\left(a+b\cosh^{-1}(c+dx)\right)^3\tan^{-1}\left(e^{\cosh^{-1}(c+dx)}\right)}{de^2} - 12$$

Mathematica [B] time = 2.52, size = 872, normalized size = 3.30

$$-\frac{a^4}{c+dx} + 4b\left(2\tan^{-1}\left(\tanh\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\right) - \frac{\cosh^{-1}(c+dx)}{c+dx}\right)a^3 - 6ib^2\left(\cosh^{-1}(c+dx)\left(-\frac{i\cosh^{-1}(c+dx)}{c+dx} + 2\log\left(1\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^2,x]
```

```
[Out] (-(a^4/(c + d*x)) + 4*a^3*b*(-(ArcCosh[c + d*x]/(c + d*x)) + 2*ArcTan[Tanh[
ArcCosh[c + d*x]/2]]) - (6*I)*a^2*b^2*(ArcCosh[c + d*x]*(((-I)*ArcCosh[c +
d*x])/(c + d*x) + 2*Log[1 - I/E^ArcCosh[c + d*x]] - 2*Log[1 + I/E^ArcCosh[c
 + d*x]]) + 2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[2, I/E^ArcCos
h[c + d*x]]) + 4*a*b^3*(-(ArcCosh[c + d*x]^3/(c + d*x)) + (3*I)*(-(ArcCosh[
c + d*x]^2*(Log[1 - I/E^ArcCosh[c + d*x]] - Log[1 + I/E^ArcCosh[c + d*x]]))
 - 2*ArcCosh[c + d*x]*(PolyLog[2, (-I)/E^ArcCosh[c + d*x]] - PolyLog[2, I/E
^ArcCosh[c + d*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + 2*PolyLog[3,
I/E^ArcCosh[c + d*x]])) + b^4*(((-7*I)/16)*Pi^4 + (Pi^3*ArcCosh[c + d*x])/2
 - ((3*I)/2)*Pi^2*ArcCosh[c + d*x]^2 - 2*Pi*ArcCosh[c + d*x]^3 + I*ArcCosh[
c + d*x]^4 - ArcCosh[c + d*x]^4/(c + d*x) + (Pi^3*Log[1 + I/E^ArcCosh[c + d
*x]])/2 - (3*I)*Pi^2*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] - 6*Pi*
ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c + d*x]] + (4*I)*ArcCosh[c + d*x]^3
*Log[1 + I/E^ArcCosh[c + d*x]] + (3*I)*Pi^2*ArcCosh[c + d*x]*Log[1 - I*E^Ar
cCosh[c + d*x]] + 6*Pi*ArcCosh[c + d*x]^2*Log[1 - I*E^ArcCosh[c + d*x]] - (
Pi^3*Log[1 + I*E^ArcCosh[c + d*x]])/2 - (4*I)*ArcCosh[c + d*x]^3*Log[1 + I*
E^ArcCosh[c + d*x]] + (Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[c + d*x])/4]])/2 +
```

```
(3*I)*(Pi - (2*I)*ArcCosh[c + d*x])^2*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] -
 (12*I)*ArcCosh[c + d*x]^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]] + (3*I)*Pi^2
*PolyLog[2, I*E^ArcCosh[c + d*x]] + 12*Pi*ArcCosh[c + d*x]*PolyLog[2, I*E^A
rcCosh[c + d*x]] + 12*Pi*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] - (24*I)*ArcCo
sh[c + d*x]*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + (24*I)*ArcCosh[c + d*x]*P
olyLog[3, (-I)*E^ArcCosh[c + d*x]] - 12*Pi*PolyLog[3, I*E^ArcCosh[c + d*x]]
 - (24*I)*PolyLog[4, (-I)/E^ArcCosh[c + d*x]] - (24*I)*PolyLog[4, (-I)*E^Ar
cCosh[c + d*x]]))/(d*e^2)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4\,\mathrm{arcosh}\,(dx+c)^4+4\,ab^3\,\mathrm{arcosh}\,(dx+c)^3+6\,a^2b^2\,\mathrm{arcosh}\,(dx+c)^2+4\,a^3b\,\mathrm{arcosh}\,(dx+c)+a^4}{d^2e^2x^2+2\,cde^2x+c^2e^2},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 1.52sym2poly/r2sym(const gen & e,const index_m & i,
const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(dx+c))^4}{(dex+ce)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^4\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^4}{d^2e^2x+cde^2}-4\,a^3b\left(\frac{\mathrm{arcosh}\,(dx+c)}{d^2e^2x+cde^2}+\frac{\arcsin\left(\frac{de^2}{|d^2e^2x+cde^2|}\right)}{de^2}\right)-\frac{a^4}{d^2e^2x+cde^2}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^2,x, algorithm="maxima")
```

```
[Out] -b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^2*e^2*x + c*d*
e^2) - 4*a^3*b*(arccosh(d*x + c)/(d^2*e^2*x + c*d*e^2) + arcsin(d*e^2/abs(d
^2*e^2*x + c*d*e^2))/(d*e^2)) - a^4/(d^2*e^2*x + c*d*e^2) + integrate(2*(2*
((c^3 - c)*a*b^3 + (c^3 - c)*b^4 + (a*b^3*d^3 + b^4*d^3)*x^3 + 3*(a*b^3*c*d
^2 + b^4*c*d^2)*x^2 + (b^4*c^2 + (c^2 - 1)*a*b^3 + (a*b^3*d^2 + b^4*d^2)*x^
```

```
2 + 2*(a*b^3*c*d + b^4*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^
2*d - d)*a*b^3 + (3*c^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + c)^3 + 3*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d - d)
*a^2*b^2*x + (c^3 - c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (c^2
- 1)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c +
 1)*sqrt(d*x + c - 1) + c)^2)/(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 - c^
3*e^2 + (10*c^2*d^3*e^2 - d^3*e^2)*x^3 + (10*c^3*d^2*e^2 - 3*c*d^2*e^2)*x^2
 + (d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 - c^2*e^2 + (6*c^2*d^2*e^2 - d^
2*e^2)*x^2 + 2*(2*c^3*d*e^2 - c*d*e^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + (5*c^4*d*e^2 - 3*c^2*d*e^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^4}{(c\,e+d\,e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^2,x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^2+2cdx+d^2x^2}\,dx + \int \frac{b^4\,\mathrm{acosh}^4\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4ab^3\,\mathrm{acosh}^3\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{6a^2b^2\,\mathrm{acosh}^2\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{4a^3b\,\mathrm{acosh}\,(c+dx)}{c^2+2cdx+d^2x^2}\,dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**2,x)
```

```
[Out] (Integral(a**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(b**4*acosh(c + d
*x)**4/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a*b**3*acosh(c + d*x)*
*3/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(6*a**2*b**2*acosh(c + d*x)**
2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(4*a**3*b*acosh(c + d*x)/(c**2
 + 2*c*d*x + d**2*x**2), x))/e**2
```

3.128 $\int \frac{(a+b\cosh^{-1}(c+dx))^4}{(ce+dex)^3}\,dx$

Optimal. Leaf size=195

$$\frac{6b^3\text{Li}_2\left(-e^{-2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}-\frac{6b^2\log\left(e^{-2\cosh^{-1}(c+dx)}+1\right)\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^3}+\frac{2b\sqrt{c-}}{}$$

```
[Out] -2*b*(a+b*arccosh(d*x+c))^3/d/e^3-1/2*(a+b*arccosh(d*x+c))^4/d/e^3/(d*x+c)^
2-6*b^2*(a+b*arccosh(d*x+c))^2*ln(1+1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
))^2)/d/e^3+6*b^3*(a+b*arccosh(d*x+c))*polylog(2,-1/(d*x+c+(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2))^2)/d/e^3+3*b^4*polylog(3,-1/(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))^2)/d/e^3+2*b*(a+b*arccosh(d*x+c))^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/d/e^3/(d*x+c)
```

Rubi [A] time = 0.43, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5866, 12, 5662, 5724, 5660, 3718, 2190, 2531, 2282, 6589}

$$-\frac{6b^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^3}+\frac{3b^4\text{PolyLog}\left(3,-e^{2\cosh^{-1}(c+dx)}\right)}{de^3}-\frac{6b^2\log\left(e^{2\cosh^{-1}(c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (2*b*(a + b*ArcCosh[c + d*x])^3)/(d*e^3) + (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 +
 c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(d*e^3*(c + d*x)) - (a + b*ArcCosh[c
+ d*x])^4/(2*d*e^3*(c + d*x)^2) - (6*b^2*(a + b*ArcCosh[c + d*x])^2*Log[1 +
 E^(2*ArcCosh[c + d*x])])/(d*e^3) - (6*b^3*(a + b*ArcCosh[c + d*x])*PolyLog
[2, -E^(2*ArcCosh[c + d*x])])/(d*e^3) + (3*b^4*PolyLog[3, -E^(2*ArcCosh[c +
 d*x])])/(d*e^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^3x^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^3}\,dx,x,c+dx\right)}{de^3}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{2de^3(c+dx)^2} + \frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx,x,c+dx\right)}{de^3}$$

$$= \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx\right.}{2de^3(c+dx)^2}$$

$$= \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3(c+dx)} - \frac{\left(a+b\cosh^{-1}(c+dx\right.}{2de^3(c+dx)^2}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{de^3(c+dx)}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{de^3(c+dx)}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{de^3(c+dx)}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{de^3(c+dx)}$$

$$= \frac{2b\left(a+b\cosh^{-1}(c+dx)\right)^3}{de^3} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+\right.}{de^3(c+dx)}$$

Mathematica [B] time = 2.31, size = 398, normalized size = 2.04

$$-\frac{a^4}{(c+dx)^2} + \frac{4a^3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{c+dx} - \frac{4a^3b\cosh^{-1}(c+dx)}{(c+dx)^2} + 12a^2b^2\left(-\log(c+dx) - \frac{\cosh^{-1}(c+dx)^2}{2(c+dx)^2} + \frac{\sqrt{\frac{c+dx-1}{c+dx+1}}\,(c+dx+1)\cosh^{-1}(\right.}{c+dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^3,x]
```

```
[Out] (-(a^4/(c + d*x)^2) + (4*a^3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(c + d
*x) - (4*a^3*b*ArcCosh[c + d*x])/(c + d*x)^2 - (b^4*ArcCosh[c + d*x]^4)/(c
+ d*x)^2 + 12*a^2*b^2*((Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Ar
cCosh[c + d*x])/(c + d*x) - ArcCosh[c + d*x]^2/(2*(c + d*x)^2) - Log[c + d*
x]) + 4*a*b^3*(-(ArcCosh[c + d*x]*(3*ArcCosh[c + d*x] - (3*Sqrt[(-1 + c + d
*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d*x) + ArcCosh[c +
d*x]^2/(c + d*x)^2 + 6*Log[1 + E^(-2*ArcCosh[c + d*x])])) + 3*PolyLog[2, -E
^(-2*ArcCosh[c + d*x])]) + 2*b^4*(2*ArcCosh[c + d*x]^2*(-ArcCosh[c + d*x] +
 (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x])/(c + d
*x) - 3*Log[1 + E^(-2*ArcCosh[c + d*x])]) + 6*ArcCosh[c + d*x]*PolyLog[2, -
E^(-2*ArcCosh[c + d*x])] + 3*PolyLog[3, -E^(-2*ArcCosh[c + d*x])]))/(2*d*e^
3)
```

fricas [F] time = 1.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4 \operatorname{arcosh}(dx+c)^4 + 4ab^3 \operatorname{arcosh}(dx+c)^3 + 6a^2b^2 \operatorname{arcosh}(dx+c)^2 + 4a^3b \operatorname{arcosh}(dx+c) + a^4}{d^3e^3x^3 + 3cd^2e^3x^2 + 3c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx+c) + a)^4}{(dex+ce)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^3, x)
```

maple [B] time = 0.29, size = 605, normalized size = 3.10

$$-\frac{a^4}{2d\,e^3\,(dx+c)^2} + \frac{2b^4 \operatorname{arccosh}(dx+c)^3 \sqrt{dx+c+1}\,\sqrt{dx+c-1}}{d\,e^3\,(dx+c)} + \frac{2b^4 \operatorname{arccosh}(dx+c)^3}{d\,e^3} - \frac{b^4 \operatorname{arccosh}(dx+c)^4}{2d\,e^3\,(dx+c)^2} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x)
```

```
[Out] -1/2/d*a^4/e^3/(d*x+c)^2+2/d*b^4/e^3*arccosh(d*x+c)^3/(d*x+c)*(d*x+c+1)^(1/
2)*(d*x+c-1)^(1/2)+2/d*b^4/e^3*arccosh(d*x+c)^3-1/2/d*b^4/e^3*arccosh(d*x+c
)^4/(d*x+c)^2-6/d*b^4/e^3*arccosh(d*x+c)^2*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2))^2)-6/d*b^4/e^3*arccosh(d*x+c)*polylog(2,-(d*x+c+(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2))^2)+3/d*b^4/e^3*polylog(3,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))^2)+6/d*a*b^3/e^3*arccosh(d*x+c)^2/(d*x+c)*(d*x+c+1)^(1/2)*(d*x+c-
1)^(1/2)+6/d*a*b^3/e^3*arccosh(d*x+c)^2-2/d*a*b^3/e^3*arccosh(d*x+c)^3/(d*x
+c)^2-12/d*a*b^3/e^3*arccosh(d*x+c)*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2))^2)-6/d*a*b^3/e^3*polylog(2,-(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2
)+6/d*a^2*b^2/e^3*arccosh(d*x+c)+6/d*a^2*b^2/e^3*arccosh(d*x+c)/(d*x+c)*(d*
x+c+1)^(1/2)*(d*x+c-1)^(1/2)-3/d*a^2*b^2/e^3*arccosh(d*x+c)^2/(d*x+c)^2-6/d
*a^2*b^2/e^3*ln(1+(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))^2)-2/d*a^3*b/e^3/
(d*x+c)^2*arccosh(d*x+c)+2/d*a^3*b/e^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/(d*x
+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^4 \log\left(dx + \sqrt{dx+c+1}\,\sqrt{dx+c-1} + c\right)^4}{2\left(d^3e^3x^2 + 2cd^2e^3x + c^2de^3\right)} + 6\left(\frac{\sqrt{d^2x^2 + 2cdx + c^2 - 1}\,d \operatorname{arcosh}(dx+c)}{d^3e^3x + cd^2e^3} - \frac{\log(dx+c)}{de^3}\right)a^2b^2 + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^3*e^3*x^2
+ 2*c*d^2*e^3*x + c^2*d*e^3) + 6*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d*arcco
sh(d*x + c)/(d^3*e^3*x + c*d^2*e^3) - log(d*x + c)/(d*e^3))*a^2*b^2 + 2*a^3
```

```
*b*(sqrt(d^2*x^2 + 2*c*d*x + c^2 - 1)*d/(d^3*e^3*x + c*d^2*e^3) - arccosh(d
*x + c)/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3)) - 3*a^2*b^2*arccosh(d*x
+ c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 1/2*a^4/(d^3*e^3*x^2 + 2
*c*d^2*e^3*x + c^2*d*e^3) + integrate(2*(2*(c^3 - c)*a*b^3 + (c^3 - c)*b^4
+ (2*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(2*a*b^3*c*d^2 + b^4*c*d^2)*x^2 + (b^4*c^
2 + 2*(c^2 - 1)*a*b^3 + (2*a*b^3*d^2 + b^4*d^2)*x^2 + 2*(2*a*b^3*c*d + b^4*
c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (2*(3*c^2*d - d)*a*b^3 + (3*c
^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(d^6
*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 - c^4*e^3 + (15*c^2*d^4*e^3 - d^4*e^3)
*x^4 + 4*(5*c^3*d^3*e^3 - c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 - 2*c^2*d^2*e^3
)*x^2 + (d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^3 - c^3*e^3 + (10*c^2*d^3*e^
3 - d^3*e^3)*x^3 + (10*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 - 3*c^
2*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(3*c^5*d*e^3 - 2*c^3*d*
e^3)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + d\,x))^4}{(c\,e + d\,e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^3,x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{b^4\,\mathrm{acosh}^4\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{4ab^3\,\mathrm{acosh}^3\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{6a^2b^2\,\mathrm{acosh}^2\,(c+dx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{}{c^3}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**3,x)
```

```
[Out] (Integral(a**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(b**4*acosh(c + d*x)**4/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(4*a*b**3*acosh(c + d*x)**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3), x) + Integral(6*a**2*b**2*acosh(c + d*x)**2/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(4*a**3*b*acosh(c + d*x)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/e**3
```

3.129 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx$

Optimal. Leaf size=432

$$\frac{4ib^3\text{Li}_3\left(-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{4ib^3\text{Li}_3\left(ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{8b^3\tan^{-1}\left(e^{\cosh^{-1}\ldots}\right.}{}$$

```
[Out] 2*b^2*(a+b*arccosh(d*x+c))^2/d/e^4/(d*x+c)-1/3*(a+b*arccosh(d*x+c))^4/d/e^4
/(d*x+c)^3-8*b^3*(a+b*arccosh(d*x+c))*arctan(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2))/d/e^4+4/3*b*(a+b*arccosh(d*x+c))^3*arctan(d*x+c+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2))/d/e^4+4*I*b^4*polylog(2,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)))/d/e^4-2*I*b^2*(a+b*arccosh(d*x+c))^2*polylog(2,-I*(d*x+c+(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)))/d/e^4-4*I*b^4*polylog(2,I*(d*x+c+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)))/d/e^4+2*I*b^2*(a+b*arccosh(d*x+c))^2*polylog(2,I*(d*x+c+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+4*I*b^3*(a+b*arccosh(d*x+c))*polylog(3
,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4-4*I*b^3*(a+b*arccosh(d*x
+c))*polylog(3,I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4-4*I*b^4*pol
ylog(4,-I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+4*I*b^4*polylog(4,
I*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)))/d/e^4+2/3*b*(a+b*arccosh(d*x+c))
^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^4/(d*x+c)^2
```

Rubi [A] time = 0.84, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5866, 12, 5662, 5748, 5761, 4180, 2531, 6609, 2282, 6589, 2279, 2391}

$$\frac{4ib^3\text{PolyLog}\left(3,-ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\frac{4ib^3\text{PolyLog}\left(3,ie^{\cosh^{-1}(c+dx)}\right)\left(a+b\cosh^{-1}(c+dx)\right)}{de^4}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] (2*b^2*(a + b*ArcCosh[c + d*x])^2)/(d*e^4*(c + d*x)) + (2*b*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^3)/(3*d*e^4*(c + d*x)^2) - (
a + b*ArcCosh[c + d*x])^4/(3*d*e^4*(c + d*x)^3) - (8*b^3*(a + b*ArcCosh[c +
 d*x])*ArcTan[E^ArcCosh[c + d*x]])/(d*e^4) + (4*b*(a + b*ArcCosh[c + d*x])^
3*ArcTan[E^ArcCosh[c + d*x]])/(3*d*e^4) + ((4*I)*b^4*PolyLog[2, (-I)*E^ArcC
osh[c + d*x]])/(d*e^4) - ((2*I)*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[2, (
-I)*E^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^4*PolyLog[2, I*E^ArcCosh[c + d*
x]])/(d*e^4) + ((2*I)*b^2*(a + b*ArcCosh[c + d*x])^2*PolyLog[2, I*E^ArcCosh
[c + d*x]])/(d*e^4) + ((4*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3, (-I)*E
^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^3*(a + b*ArcCosh[c + d*x])*PolyLog[3
, I*E^ArcCosh[c + d*x]])/(d*e^4) - ((4*I)*b^4*PolyLog[4, (-I)*E^ArcCosh[c +
 d*x]])/(d*e^4) + ((4*I)*b^4*PolyLog[4, I*E^ArcCosh[c + d*x]])/(d*e^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{e^4x^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{x^4}\,dx,x,c+dx\right)}{de^4}$$

$$= -\frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3} + \frac{(4b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,x^3\sqrt{1+x}}\,dx,x,c+dx\right)}{3de^4}$$

$$= \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de^4(c+dx)^2} - \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de^4(c+dx)^3}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^2}$$

$$= \frac{2b^2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de^4(c+dx)} + \frac{2b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+d\right.}{3de^4(c+dx)^2}$$

Mathematica [B] time = 9.36, size = 1374, normalized size = 3.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^4,x]
```

```
[Out] -1/3*a^4/(d*e^4*(c + d*x)^3) + (4*a^3*b*Sqrt[-1 + c + d*x]*((Sqrt[(-1 + c +
 d*x)/(1 + c + d*x)]*(1 + c + d*x))/(6*(c + d*x)^2) - ArcCosh[c + d*x]/(3*(
c + d*x)^3) + ArcTan[Tanh[ArcCosh[c + d*x]/2]]/3))/(d*e^4*Sqrt[(-1 + c + d*
x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (2*a^2*b^2*Sqrt[-1 + c + d*x]*((c +
d*x)^(-1) + (Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d
*x])/(c + d*x)^2 - ArcCosh[c + d*x]^2/(c + d*x)^3 - I*ArcCosh[c + d*x]*Log[
1 - I/E^ArcCosh[c + d*x]] + I*ArcCosh[c + d*x]*Log[1 + I/E^ArcCosh[c + d*x]
] - I*PolyLog[2, (-I)/E^ArcCosh[c + d*x]] + I*PolyLog[2, I/E^ArcCosh[c + d*
x]]))/(d*e^4*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (4*a*b
^3*Sqrt[-1 + c + d*x]*(ArcCosh[c + d*x]/(c + d*x) + (Sqrt[(-1 + c + d*x)/(1
 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x]^2)/(2*(c + d*x)^2) - ArcCosh[c
+ d*x]^3/(3*(c + d*x)^3) - (I/2)*((-4*I)*ArcTan[Tanh[ArcCosh[c + d*x]/2]] +
 ArcCosh[c + d*x]^2*Log[1 - I/E^ArcCosh[c + d*x]] - ArcCosh[c + d*x]^2*Log[
1 + I/E^ArcCosh[c + d*x]] + 2*ArcCosh[c + d*x]*PolyLog[2, (-I)/E^ArcCosh[c
+ d*x]] - 2*ArcCosh[c + d*x]*PolyLog[2, I/E^ArcCosh[c + d*x]] + 2*PolyLog[3
, (-I)/E^ArcCosh[c + d*x]] - 2*PolyLog[3, I/E^ArcCosh[c + d*x]])))/(d*e^4*S
qrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (b^4*Sqrt[-1 + c + d
*x]*((I/2)*(8 + Pi^2 - (4*I)*Pi*ArcCosh[c + d*x] - 4*ArcCosh[c + d*x]^2)*Po
lyLog[2, (-I)/E^ArcCosh[c + d*x]] - (I/96)*(7*Pi^4 + (8*I)*Pi^3*ArcCosh[c +
 d*x] + 24*Pi^2*ArcCosh[c + d*x]^2 + ((192*I)*ArcCosh[c + d*x]^2)/(c + d*x)
 - (32*I)*Pi*ArcCosh[c + d*x]^3 + ((64*I)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)
]*(1 + c + d*x)*ArcCosh[c + d*x]^3)/(c + d*x)^2 - 16*ArcCosh[c + d*x]^4 - (
(32*I)*ArcCosh[c + d*x]^4)/(c + d*x)^3 - 384*ArcCosh[c + d*x]*Log[1 - I/E^A
rcCosh[c + d*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcCosh[c + d*x]] + 384*ArcCosh[c
 + d*x]*Log[1 + I/E^ArcCosh[c + d*x]] + 48*Pi^2*ArcCosh[c + d*x]*Log[1 + I/
E^ArcCosh[c + d*x]] - (96*I)*Pi*ArcCosh[c + d*x]^2*Log[1 + I/E^ArcCosh[c +
d*x]] - 64*ArcCosh[c + d*x]^3*Log[1 + I/E^ArcCosh[c + d*x]] - 48*Pi^2*ArcCo
sh[c + d*x]*Log[1 - I*E^ArcCosh[c + d*x]] + (96*I)*Pi*ArcCosh[c + d*x]^2*Lo
g[1 - I*E^ArcCosh[c + d*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcCosh[c + d*x]] + 64
*ArcCosh[c + d*x]^3*Log[1 + I*E^ArcCosh[c + d*x]] + (8*I)*Pi^3*Log[Tan[(Pi
+ (2*I)*ArcCosh[c + d*x])/4]] + 384*PolyLog[2, I/E^ArcCosh[c + d*x]] + 192*
ArcCosh[c + d*x]^2*PolyLog[2, (-I)*E^ArcCosh[c + d*x]] - 48*Pi^2*PolyLog[2,
 I*E^ArcCosh[c + d*x]] + (192*I)*Pi*ArcCosh[c + d*x]*PolyLog[2, I*E^ArcCosh
[c + d*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] + 384*ArcCosh[c
 + d*x]*PolyLog[3, (-I)/E^ArcCosh[c + d*x]] - 384*ArcCosh[c + d*x]*PolyLog[
3, (-I)*E^ArcCosh[c + d*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcCosh[c + d*x]] +
 384*PolyLog[4, (-I)/E^ArcCosh[c + d*x]] + 384*PolyLog[4, (-I)*E^ArcCosh[c
+ d*x]])))/(d*e^4*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4 \operatorname{arcosh}(dx+c)^4 + 4\,ab^3 \operatorname{arcosh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arcosh}(dx+c)^2 + 4\,a^3b \operatorname{arcosh}(dx+c) + a^4}{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^4}{(dex+ce)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^4, x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^4}{(dex + ce)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^4 \log\left(dx + \sqrt{dx + c + 1}\sqrt{dx + c - 1} + c\right)^4}{3\left(d^4e^4x^3 + 3\,cd^3e^4x^2 + 3\,c^2d^2e^4x + c^3de^4\right)} - \frac{a^4}{3\left(d^4e^4x^3 + 3\,cd^3e^4x^2 + 3\,c^2d^2e^4x + c^3de^4\right)} + \int \frac{2\left(2\left(3\left(c^3 - c\right)ab^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^4*e^4*x^3
+ 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 1/3*a^4/(d^4*e^4*x^3 + 3
*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) + integrate(2/3*(2*(3*(c^3 -
c)*a*b^3 + (c^3 - c)*b^4 + (3*a*b^3*d^3 + b^4*d^3)*x^3 + 3*(3*a*b^3*c*d^2 +
 b^4*c*d^2)*x^2 + (b^4*c^2 + 3*(c^2 - 1)*a*b^3 + (3*a*b^3*d^2 + b^4*d^2)*x^
2 + 2*(3*a*b^3*c*d + b^4*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(
3*c^2*d - d)*a*b^3 + (3*c^2*d - d)*b^4)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)^3 + 9*(a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d
- d)*a^2*b^2*x + (c^3 - c)*a^2*b^2 + (a^2*b^2*d^2*x^2 + 2*a^2*b^2*c*d*x + (
c^2 - 1)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + c)^2 + 6*(a^3*b*d^3*x^3 + 3*a^3*b*c*d^2*x^2 + (
3*c^2*d - d)*a^3*b*x + (c^3 - c)*a^3*b + (a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (
c^2 - 1)*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + c))/(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^
5*e^4 + (21*c^2*d^5*e^4 - d^5*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4
+ 5*(7*c^4*d^3*e^4 - 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)
*x^2 + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4
 - d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*
c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(
d*x + c - 1) + (7*c^6*d*e^4 - 5*c^4*d*e^4)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c + d\,x))^4}{(c\,e + d\,e\,x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^4,x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^4}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\, dx + \int \frac{b^4 \operatorname{acosh}^4(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\, dx + \int \frac{4ab^3 \operatorname{acosh}^3(c+dx)}{c^4+4c^3dx+6c^2d^2x^2+4cd^3x^3+d^4x^4}\, dx + \int \frac{6a^2b^2}{c^4+4c^3dx+6}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**4,x)
```

```
[Out] (Integral(a**4/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4
*x**4), x) + Integral(b**4*acosh(c + d*x)**4/(c**4 + 4*c**3*d*x + 6*c**2*d*
*2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a*b**3*acosh(c + d*x)
**3/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x)
+ Integral(6*a**2*b**2*acosh(c + d*x)**2/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x
**2 + 4*c*d**3*x**3 + d**4*x**4), x) + Integral(4*a**3*b*acosh(c + d*x)/(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4), x))/e**4
```

3.130 $\int \frac{(ce+dex)^4}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=213

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8bd}-\frac{3e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{16bd}-\frac{e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{16bd}+\ldots$$

```
[Out] 1/8*e^4*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b/d+3/16*e^4*cosh(3*a/b)*Shi(
3*(a+b*arccosh(d*x+c))/b)/b/d+1/16*e^4*cosh(5*a/b)*Shi(5*(a+b*arccosh(d*x+c
))/b)/b/d-1/8*e^4*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b/d-3/16*e^4*Chi(3*
(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b/d-1/16*e^4*Chi(5*(a+b*arccosh(d*x+c))
/b)*sinh(5*a/b)/b/d
```

Rubi [A] time = 0.43, antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{8bd}-\frac{3e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{16bd}-\frac{e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+5\cosh^{-1}(c+dx)\right)}{16bd}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e^4*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b])/(8*b*d) - (3*e^4*Cosh
Integral[(3*a)/b + 3*ArcCosh[c + d*x]]*Sinh[(3*a)/b])/(16*b*d) - (e^4*CoshI
ntegral[(5*a)/b + 5*ArcCosh[c + d*x]]*Sinh[(5*a)/b])/(16*b*d) + (e^4*Cosh[a
/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(8*b*d) + (3*e^4*Cosh[(3*a)/b]*Si
nhIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(16*b*d) + (e^4*Cosh[(5*a)/b]*Sin
hIntegral[(5*a)/b + 5*ArcCosh[c + d*x]])/(16*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ce+dex)^4}{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^4x^4}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^4\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)}+\frac{3\sinh(3x)}{16(a+bx)}+\frac{\sinh(5x)}{16(a+bx)}\right)dx,x,\cosh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{16d}+\frac{e^4\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+\right.}{8d}\\
&= \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{8d}+\frac{\left(3e^4\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}}{}\\
&= -\frac{e^4\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{8bd}-\frac{3e^4\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)\sinh\left(\right.}{16bd}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 151, normalized size = 0.71

$$\frac{e^4\left(-2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^4*(-2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*
(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcCosh[c +
 d*x])]*Sinh[(5*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*Si
nhIntegral[5*(a/b + ArcCosh[c + d*x])]))/(16*b*d)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4}{b\,\text{arcosh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b*arccosh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{b\operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a), x)
```

maple [A] time = 0.40, size = 194, normalized size = 0.91

$$\frac{\frac{e^4 e^{\frac{5a}{b}} \operatorname{Ei}\left(1,5\operatorname{arccosh}(dx+c)+\frac{5a}{b}\right)}{32b}+\frac{3e^4 e^{\frac{3a}{b}} \operatorname{Ei}\left(1,3\operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{32b}+\frac{e^4 e^{\frac{a}{b}} \operatorname{Ei}\left(1,\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{16b}-\frac{e^4 e^{-\frac{a}{b}} \operatorname{Ei}\left(1,-\operatorname{arccosh}(dx+c)-\frac{a}{b}\right)}{16b}-\frac{3e^4 e}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/32*e^4/b*exp(5*a/b)*Ei(1,5*arccosh(d*x+c)+5*a/b)+3/32*e^4/b*exp(3*a/
b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/16*e^4/b*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b
)-1/16*e^4/b*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-3/32*e^4/b*exp(-3*a/b)*Ei(
1,-3*arccosh(d*x+c)-3*a/b)-1/32*e^4/b*exp(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*
a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{b\operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{a+b\operatorname{acosh}(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{d^4x^4}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{4cd^3x^3}{a+b\operatorname{acosh}(c+dx)}\,dx+\int \frac{6c^2d^2x^2}{a+b\operatorname{acosh}(c+d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c)),x)
```

```
[Out] e**4*(Integral(c**4/(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4/(a + b*
acosh(c + d*x)), x) + Integral(4*c*d**3*x**3/(a + b*acosh(c + d*x)), x) + I
ntegral(6*c**2*d**2*x**2/(a + b*acosh(c + d*x)), x) + Integral(4*c**3*d*x/(
a + b*acosh(c + d*x)), x))
```

3.131 $\int \frac{(ce+dex)^3}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=145

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8bd}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4bd}$$

```
[Out] 1/4*e^3*cosh(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b/d+1/8*e^3*cosh(4*a/b)*S
hi(4*(a+b*arccosh(d*x+c))/b)/b/d-1/4*e^3*Chi(2*(a+b*arccosh(d*x+c))/b)*sinh
(2*a/b)/b/d-1/8*e^3*Chi(4*(a+b*arccosh(d*x+c))/b)*sinh(4*a/b)/b/d
```

Rubi [A] time = 0.33, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{4bd}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)}{8bd}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e^3*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b])/(4*b*d) - (
e^3*CoshIntegral[(4*a)/b + 4*ArcCosh[c + d*x]]*Sinh[(4*a)/b])/(8*b*d) + (e^
3*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(4*b*d) + (e^3*
Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c + d*x]])/(8*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ce+dex)^3}{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^3x^3}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\
&= \frac{e^3\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^3\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)dx,x,\cosh^{-1}(c+dx)\right)}{d}\\
&= \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{8d}+\frac{e^3\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx,x,\cosh^{-1}(c
\end{aligned}$$

$$\begin{aligned}
&= \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^3\cosh\left(\frac{4a}{b}\right)\right)\text{Sub}}{}\\
&= -\frac{e^3\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{4bd}-\frac{e^3\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)\text{sinh}}{8bd}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 109, normalized size = 0.75

$$\frac{e^3\left(-2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\right.\right.}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^3*(-2*CoshIntegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] - CoshInteg
ral[4*(a/b + ArcCosh[c + d*x])]*Sinh[(4*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegra
l[2*(a/b + ArcCosh[c + d*x])] + Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh
[c + d*x])]))/(8*b*d)
```

fricas [F] time = 1.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}{b\,\text{arcosh}\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b*arcco
sh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{b\,\operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a), x)
```

maple [A] time = 0.34, size = 134, normalized size = 0.92

$$\frac{\frac{e^3 e^{\frac{4a}{b}}\operatorname{Ei}\left(1, 4\operatorname{arccosh}(dx+c)+\frac{4a}{b}\right)}{16b}+\frac{e^3 e^{\frac{2a}{b}}\operatorname{Ei}\left(1, 2\operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{8b}-\frac{e^3 e^{-\frac{2a}{b}}\operatorname{Ei}\left(1, -2\operatorname{arccosh}(dx+c)-\frac{2a}{b}\right)}{8b}-\frac{e^3 e^{-\frac{4a}{b}}\operatorname{Ei}\left(1, -4\operatorname{arccosh}(dx+c)-\frac{4a}{b}\right)}{16b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/16*e^3/b*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/8*e^3/b*exp(2*a/b
)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8*e^3/b*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)
-2*a/b)-1/16*e^3/b*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+c)-4*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{b\,\operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^3}{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{d^3x^3}{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{3cd^2x^2}{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{3c^2dx}{a+b\,\mathrm{acosh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c)),x)
```

```
[Out] e**3*(Integral(c**3/(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3/(a + b*
acosh(c + d*x)), x) + Integral(3*c*d**2*x**2/(a + b*acosh(c + d*x)), x) + I
ntegral(3*c**2*d*x/(a + b*acosh(c + d*x)), x))
```

3.132 $\int \frac{(ce+dex)^2}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4bd}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4bd}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4bd}+\frac{e^2 c}{}$$

```
[Out] 1/4*e^2*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b/d+1/4*e^2*cosh(3*a/b)*Shi(3
*(a+b*arccosh(d*x+c))/b)/b/d-1/4*e^2*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/
b/d-1/4*e^2*Chi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b/d
```

Rubi [A] time = 0.30, antiderivative size = 137, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{4bd}-\frac{e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{4bd}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e^2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b])/(4*b*d) - (e^2*CoshIn
tegral[(3*a)/b + 3*ArcCosh[c + d*x]]*Sinh[(3*a)/b])/(4*b*d) + (e^2*Cosh[a/b
]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(4*b*d) + (e^2*Cosh[(3*a)/b]*SinhIn
tegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(ce+dex)^2}{a+b\cosh^{-1}(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{e^2x^2}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\ &= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}\\ &= \frac{e^2\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{d}\\ &= \frac{e^2\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx,x,\cosh^{-1}(c+dx)\right)}{d}\\ &= \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}+\frac{e^2\,\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right.}{4d}\\ &= \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}+\frac{\left(e^2\cosh\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \right.}{}\\ &= -\frac{e^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\sinh\left(\frac{a}{b}\right)}{4bd}-\frac{e^2\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)\sinh\left(\frac{3a}{b}\right)}{4bd}-\end{aligned}$$

Mathematica [A] time = 0.16, size = 102, normalized size = 0.72

$$\frac{e^2\left(\sinh\left(\frac{a}{b}\right)\left(-\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c\right.\right.}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (e^2*(-(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b]) - CoshIntegral[3*(a
/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcCos
h[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])]))/(4*b
*d)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2e^2x^2+2\,cde^2x+c^2e^2}{b\,\text{arcosh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b*arccosh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{b \operatorname{arcosh}(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a), x)
```

maple [A] time = 0.19, size = 130, normalized size = 0.92

$$\frac{\frac{e^2 e^{\frac{3a}{b}} \operatorname{Ei}\left(1, 3 \operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{8b} + \frac{e^2 e^{\frac{a}{b}} \operatorname{Ei}\left(1, \operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{8b} - \frac{e^2 e^{-\frac{a}{b}} \operatorname{Ei}\left(1, -\operatorname{arccosh}(dx+c)-\frac{a}{b}\right)}{8b} - \frac{e^2 e^{-\frac{3a}{b}} \operatorname{Ei}\left(1, -3 \operatorname{arccosh}(dx+c)-\frac{3a}{b}\right)}{8b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x)
[Out] 1/d*(1/8*e^2/b*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/8*e^2/b*exp(a/b)*E
i(1,arccosh(d*x+c)+a/b)-1/8*e^2/b*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/8*e
^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{b \operatorname{arcosh}(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e + d\,e\,x)^2}{a + b\,\operatorname{acosh}(c + d\,x)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x)),x)
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int \frac{c^2}{a + b \operatorname{acosh}(c + dx)} dx + \int \frac{d^2 x^2}{a + b \operatorname{acosh}(c + dx)} dx + \int \frac{2cdx}{a + b \operatorname{acosh}(c + dx)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c)),x)
[Out] e**2*(Integral(c**2/(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2/(a + b*
acosh(c + d*x)), x) + Integral(2*c*d*x/(a + b*acosh(c + d*x)), x))
```

3.133 $\int \frac{ce+dex}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2bd}$$

```
[Out] 1/2*e*cosh(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b/d-1/2*e*Chi(2*(a+b*arccos
h(d*x+c))/b)*sinh(2*a/b)/b/d
```

Rubi [A] time = 0.16, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5866, 12, 5670, 5448, 3303, 3298, 3301}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2bd}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] -(e*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b])/(2*b*d) + (e*
Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
```

```
x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int \frac{x}{a+b\cosh^{-1}(x)}\,dx, x, c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d}$$
$$= \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d} - \frac{\left(e\sinh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\right.}{}$$
$$= -\frac{e\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\sinh\left(\frac{2a}{b}\right)}{2bd} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+\right.}{2bd}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 0.88

$$-\frac{e\left(\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)\right)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x]),x]
[Out] -1/2*(e*(CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b] - Cosh[(2
*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]))/(b*d)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dex+ce}{b\,\text{arcosh}\,(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex+ce}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

maple [A] time = 0.03, size = 66, normalized size = 0.96

$$\frac{\frac{e\,\mathrm{e}^{\frac{2a}{b}}\,\mathrm{Ei}\left(1,2\,\mathrm{arccosh}(dx+c)+\frac{2a}{b}\right)}{4b}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\,\mathrm{Ei}\left(1,-2\,\mathrm{arccosh}(dx+c)-\frac{2a}{b}\right)}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

```
[Out] 1/d*(1/4*e/b*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/4*e/b*exp(-2*a/b)*Ei
(1,-2*arccosh(d*x+c)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{b\,\mathrm{arcosh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{a+b\,\mathrm{acosh}\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a + b\,\mathrm{acosh}\,(c + dx)}\,dx + \int \frac{dx}{a + b\,\mathrm{acosh}\,(c + dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
```

```
[Out] e*(Integral(c/(a + b*acosh(c + d*x)), x) + Integral(d*x/(a + b*acosh(c + d*
x)), x))
```

3.134 $\int \frac{1}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

```
[Out] cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b/d-Chi((a+b*arccosh(d*x+c))/b)*sinh(
a/b)/b/d
```

Rubi [A] time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5864, 5658, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-1),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcCosh[c + d*x])/b]*Sinh[a/b])/(b*d)) + (Cosh[a/b]*
SinhIntegral[(a + b*ArcCosh[c + d*x])/b])/(b*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\cosh^{-1}(c+dx)\right)}{bd}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\cosh^{-1}(c+dx)\right)}{bd} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\right.}{b}$$
$$= -\frac{\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bd} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{bd}$$

Mathematica [A] time = 0.12, size = 49, normalized size = 0.84

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-1),x]
[Out] (-(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b]) + Cosh[a/b]*SinhIntegral
[a/b + ArcCosh[c + d*x]])/(b*d)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b\,\text{arcosh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
[Out] integral(1/(b*arccosh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="giac")
[Out] integrate(1/(b*arccosh(d*x + c) + a), x)
```

maple [A] time = 0.03, size = 60, normalized size = 1.03

$$\frac{\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arccosh}(dx+c)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arccosh}(dx+c)-\frac{a}{b}\right)}{2b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c)),x)
[Out] 1/d*(1/2/b*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arccosh(
d*x+c)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a+b\operatorname{acosh}(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x)),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\operatorname{acosh}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*acosh(c + d*x)), x)
```

3.135 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c)),x)/e

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)}\,dx,x,c+dx\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)}\,dx,x,c+dx\right)}{de}
\end{aligned}$$

Mathematica [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])), x]

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{adex+ace+(bdex+bce)\operatorname{arcosh}(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="fricas")

[Out] integral(1/(a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arccosh(d*x + c)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\operatorname{arcosh}(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(a + b \operatorname{arccosh}(dx + c))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(b \operatorname{arcosh}(dx + c) + a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e + d\,e\,x)\,\left(a + b\operatorname{acosh}(c + d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ac + adx + bc \operatorname{acosh}(c + dx) + bdx \operatorname{acosh}(c + dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c)),x)
[Out] Integral(1/(a*c + a*d*x + b*c*acosh(c + d*x) + b*d*x*acosh(c + d*x)), x)/e
```

3.136 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=263

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^2d}+\frac{9e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}+\frac{5e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{16b^2d}-$$

```
[Out] 1/8*e^4*Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^2/d+9/16*e^4*Chi(3*(a+b*arc
cosh(d*x+c))/b)*cosh(3*a/b)/b^2/d+5/16*e^4*Chi(5*(a+b*arccosh(d*x+c))/b)*co
sh(5*a/b)/b^2/d-1/8*e^4*Shi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^2/d-9/16*e^
4*Shi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^2/d-5/16*e^4*Shi(5*(a+b*arcco
sh(d*x+c))/b)*sinh(5*a/b)/b^2/d-e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/
2)/b/d/(a+b*arccosh(d*x+c))
```

Rubi [A] time = 0.39, antiderivative size = 259, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 12, 5666, 3303, 3298, 3301}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{8b^2d}+\frac{9e^4\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{16b^2d}+\frac{5e^4\cosh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5a}{b}+5\cos\right.}{16b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^4*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])/(8*b^2
*d) + (9*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(16*
b^2*d) + (5*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c + d*x]])/(
16*b^2*d) - (e^4*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(8*b^2*d)
- (9*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(16*b^2*
d) - (5*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c + d*x]])/(16*b
^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
 NeQ[d*e - c*f, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^4\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{8(a+bx)} - \frac{9\cosh(3x)}{16(a+bx)} - \right.\right.}{bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{8bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^4\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx\right.}{8bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx\right.}{8b^2d}$$

Mathematica [A] time = 1.97, size = 293, normalized size = 1.11

$$\frac{e^4\left(-16\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^4*((-16*b*(c + d*x)^4*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/
(a + b*ArcCosh[c + d*x]) - 16*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d
*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - 3*Sinh[a/b]
*SinhIntegral[a/b + ArcCosh[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcCosh[c + d*x])]) + 5*(10*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]]
```

```
 + 5*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]
*CoshIntegral[5*(a/b + ArcCosh[c + d*x])] - 10*Sinh[a/b]*SinhIntegral[a/b +
 ArcCosh[c + d*x]] - 5*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x]
)] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c + d*x])])))/(16*b^2*d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^2\,\text{arcosh}\,(dx+c)^2 + 2\,ab\,\text{arcosh}\,(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^2, x)
```

maple [B] time = 0.41, size = 665, normalized size = 2.53

$$\frac{\left(-16(dx+c)^4\sqrt{dx+c-1}\,\sqrt{dx+c+1}+12(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+16(dx+c)^5-20(dx+c)^3+5dx+5c\right)e^4}{32b(a+b\,\text{arccosh}(dx+c))} - \frac{5e^4\mathrm{e}^{\frac{5a}{b}}\,\mathrm{Ei}\left(1,5\,\text{arccosh}\right.}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/32*(-16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+12*(d*x+c)^2*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20*
(d*x+c)^3+5*d*x+5*c)*e^4/b/(a+b*arccosh(d*x+c))-5/32*e^4/b^2*exp(5*a/b)*Ei(
1,5*arccosh(d*x+c)+5*a/b)+3/32*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4/b/(a+b*arccosh
(d*x+c))-9/32*e^4/b^2*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/16*(-(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^4/b/(a+b*arccosh(d*x+c))-1/16*e^4/b^2*ex
p(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16/b*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2))/(a+b*arccosh(d*x+c))-1/16/b^2*e^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/
b)-3/32/b*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/32/b^2*e^4*ex
p(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/32/b*e^4*(16*(d*x+c)^5-20*(d*x+c)
^3+16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d
*x+c))-5/32/b^2*e^4*exp(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^7*e^4*x^7 + 7*c*d^6*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^5*e^4 - d^5
*e^4)*x^5 + 5*(7*c^3*d^4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 - 2*c^2*d^
3*e^4)*x^3 + (21*c^5*d^2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^6 + 6*c*d^5
*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(5*c^3*d^
3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d
*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*c^6*d*e^4 -
 5*c^4*d*e^4)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*
x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d
^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate((5
*d^8*e^4*x^8 + 40*c*d^7*e^4*x^7 + 5*c^8*e^4 - 10*c^6*e^4 + 5*c^4*e^4 + 10*(
14*c^2*d^6*e^4 - d^6*e^4)*x^6 + 20*(14*c^3*d^5*e^4 - 3*c*d^5*e^4)*x^5 + 5*(
70*c^4*d^4*e^4 - 30*c^2*d^4*e^4 + d^4*e^4)*x^4 + 20*(14*c^5*d^3*e^4 - 10*c^
3*d^3*e^4 + c*d^3*e^4)*x^3 + (5*d^6*e^4*x^6 + 30*c*d^5*e^4*x^5 + 5*c^6*e^4
- 3*c^4*e^4 + 3*(25*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(25*c^3*d^3*e^4 - 3*c*d^
3*e^4)*x^3 + 3*(25*c^4*d^2*e^4 - 6*c^2*d^2*e^4)*x^2 + 6*(5*c^5*d*e^4 - 2*c^
3*d*e^4)*x)*(d*x + c + 1)*(d*x + c - 1) + 10*(14*c^6*d^2*e^4 - 15*c^4*d^2*e
^4 + 3*c^2*d^2*e^4)*x^2 + (10*d^7*e^4*x^7 + 70*c*d^6*e^4*x^6 + 10*c^7*e^4 -
 13*c^5*e^4 + 4*c^3*e^4 + (210*c^2*d^5*e^4 - 13*d^5*e^4)*x^5 + 5*(70*c^3*d^
4*e^4 - 13*c*d^4*e^4)*x^4 + 2*(175*c^4*d^3*e^4 - 65*c^2*d^3*e^4 + 2*d^3*e^4
)*x^3 + 2*(105*c^5*d^2*e^4 - 65*c^3*d^2*e^4 + 6*c*d^2*e^4)*x^2 + (70*c^6*d*
e^4 - 65*c^4*d*e^4 + 12*c^2*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) +
 20*(2*c^7*d*e^4 - 3*c^5*d*e^4 + c^3*d*e^4)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x
^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2
*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b +
 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c
^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))
, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^2+2ab\,\mathrm{acosh}\,(c+dx)+b^2\,\mathrm{acosh}^2\,(c+dx)}\,dx+\int \frac{d^4x^4}{a^2+2ab\,\mathrm{acosh}\,(c+dx)+b^2\,\mathrm{acosh}^2\,(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**4*(Integral(c**4/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**4*x**4/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(4*c*d**3*x**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh
(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x)
+ b**2*acosh(c + d*x)**2), x) + Integral(4*c**3*d*x/(a**2 + 2*a*b*acosh(c +
 d*x) + b**2*acosh(c + d*x)**2), x))
```

3.137 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^2d}-$$

```
[Out] 1/2*e^3*Chi(2*(a+b*arccosh(d*x+c))/b)*cosh(2*a/b)/b^2/d+1/2*e^3*Chi(4*(a+b*
arccosh(d*x+c))/b)*cosh(4*a/b)/b^2/d-1/2*e^3*Shi(2*(a+b*arccosh(d*x+c))/b)*
sinh(2*a/b)/b^2/d-1/2*e^3*Shi(4*(a+b*arccosh(d*x+c))/b)*sinh(4*a/b)/b^2/d-e
^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))
```

Rubi [A] time = 0.30, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5866, 12, 5666, 3303, 3298, 3301}

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2b^2d}+\frac{e^3\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)}{2b^2d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\cos\right.}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x
]])/(2*b^2*d) + (e^3*Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcCosh[c + d*x
]])/(2*b^2*d) - (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x
]])/(2*b^2*d) - (e^3*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c + d*x
]])/(2*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^3\,\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2(a+bx)} - \frac{\cosh(4x)}{2(a+bx)}\right) d\right.}{bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{2bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^3\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right.}{a+bx}\right.}{2bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c\right.}{2b^2d}$$

Mathematica [A] time = 2.58, size = 230, normalized size = 1.18

$$\frac{e^3\left(-3\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)+\log\left(a+b\cosh^{-1}(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^3*((-2*b*(c + d*x)^3*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/(
a + b*ArcCosh[c + d*x]) + 4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c +
 d*x])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c + d*x])] + 3*Log[a
+ b*ArcCosh[c + d*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d
*x])] - 3*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Log[a +
 b*ArcCosh[c + d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x]
)]) - Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c + d*x])]))/(2*b^2*d)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^2\operatorname{arcosh}\left(dx+c\right)^2 + 2\,ab\operatorname{arcosh}\left(dx+c\right) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^2*arc
cosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(b\operatorname{arcosh}(dx+c) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^2, x)
```

maple [B] time = 0.37, size = 418, normalized size = 2.14

$$\frac{\left(-8(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3}{16(a+b\operatorname{arccosh}(dx+c))b} - \frac{e^3\mathrm{e}^{\frac{4a}{b}}\operatorname{Ei}\left(1,4\operatorname{arccosh}(dx+c)+\frac{4a}{b}\right)}{4b^2} + \frac{\left(-2\sqrt{dx+c+1}\,\sqrt{dx\right.}{8(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/16*(-8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c+1)^(1/2)*(
d*x+c-1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3/(a+b*arccosh(d*x+c))/
b-1/4*e^3/b^2*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/8*(-2*(d*x+c+1)^(1/
2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e^3/(a+b*arccosh(d*x+c))/b-1/4*e^
3/b^2*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8/b*e^3*(2*(d*x+c)^2-1+2*(d
*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-1/4/b^2*e^3*exp
(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/16/b*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+
8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/
2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))-1/4/b^2*e^3*exp(-4*a/b)*Ei(1,-4*arccosh(
d*x+c)-4*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 - c^4*e^3 + (15*c^2*d^4*e^3 - d^4
*e^3)*x^4 + 4*(5*c^3*d^3*e^3 - c*d^3*e^3)*x^3 + 3*(5*c^4*d^2*e^3 - 2*c^2*d^
2*e^3)*x^2 + (d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^3 - c^3*e^3 + (10*c^2*d
^3*e^3 - d^3*e^3)*x^3 + (10*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 -
 3*c^2*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(3*c^5*d*e^3 - 2*c
^3*d*e^3)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x +
a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c -
 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate((4*d^7
*e^3*x^7 + 28*c*d^6*e^3*x^6 + 4*c^7*e^3 - 8*c^5*e^3 + 4*c^3*e^3 + 4*(21*c^2
```

```
*d^5*e^3 - 2*d^5*e^3)*x^5 + 20*(7*c^3*d^4*e^3 - 2*c*d^4*e^3)*x^4 + 4*(35*c^
4*d^3*e^3 - 20*c^2*d^3*e^3 + d^3*e^3)*x^3 + 2*(2*d^5*e^3*x^5 + 10*c*d^4*e^3
*x^4 + 2*c^5*e^3 - c^3*e^3 + (20*c^2*d^3*e^3 - d^3*e^3)*x^3 + (20*c^3*d^2*e
^3 - 3*c*d^2*e^3)*x^2 + (10*c^4*d*e^3 - 3*c^2*d*e^3)*x)*(d*x + c + 1)*(d*x
+ c - 1) + 4*(21*c^5*d^2*e^3 - 20*c^3*d^2*e^3 + 3*c*d^2*e^3)*x^2 + (8*d^6*e
^3*x^6 + 48*c*d^5*e^3*x^5 + 8*c^6*e^3 - 10*c^4*e^3 + 3*c^2*e^3 + 10*(12*c^2
*d^4*e^3 - d^4*e^3)*x^4 + 40*(4*c^3*d^3*e^3 - c*d^3*e^3)*x^3 + 3*(40*c^4*d^
2*e^3 - 20*c^2*d^2*e^3 + d^2*e^3)*x^2 + 2*(24*c^5*d*e^3 - 20*c^3*d*e^3 + 3*
c*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 4*(7*c^6*d*e^3 - 10*c^4*d
*e^3 + 3*c^2*d*e^3)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)
*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d
*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*b*
c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x
 + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2 +
4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d*x + c + 1)
*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*
log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^2+2ab\,\mathrm{acosh}\,(c+dx)+b^2\,\mathrm{acosh}^2\,(c+dx)}\,dx+\int \frac{d^3x^3}{a^2+2ab\,\mathrm{acosh}\,(c+dx)+b^2\,\mathrm{acosh}^2\,(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**3*(Integral(c**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**3*x**3/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(3*c*d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh
(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2
*acosh(c + d*x)**2), x))
```

3.138 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=191

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4b^2d}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{4b^2d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{4b^2d}-\frac{3e^2\sin}{}$$

```
[Out] 1/4*e^2*Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^2/d+3/4*e^2*Chi(3*(a+b*arcc
osh(d*x+c))/b)*cosh(3*a/b)/b^2/d-1/4*e^2*Shi((a+b*arccosh(d*x+c))/b)*sinh(a
/b)/b^2/d-3/4*e^2*Shi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^2/d-e^2*(d*x+
c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 12, 5666, 3303, 3298, 3301}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{4b^2d}+\frac{3e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{4b^2d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCos
h[c + d*x]))) + (e^2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])/(4*b^2
*d) + (3*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(4*b
^2*d) - (e^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(4*b^2*d) - (3
*e^2*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(4*b^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
```

```
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e^2\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)} - \frac{3\cosh(3x)}{4(a+bx)}\right) \ldots\right.}{bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(c\ldots\right.}{4bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e^2\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,d\ldots\right.}{4bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+d\ldots\right.}{4b^2d}$$

Mathematica [A] time = 1.96, size = 150, normalized size = 0.79

$$\frac{e^2\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}\ldots\right.}{4b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e^2*((-4*b*(c + d*x)^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x))/(
a + b*ArcCosh[c + d*x]) + Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c + d*x])] - Sinh[a/b]*SinhIn
tegral[a/b + ArcCosh[c + d*x]] - 3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcC
osh[c + d*x])]))/(4*b^2*d)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2e^2x^2+2\,cde^2x+c^2e^2}{b^2\,\text{arcosh}\,(dx+c)^2+2\,ab\,\text{arcosh}\,(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^2*arccosh(d*x + c)^2 + 2*
a*b*arccosh(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{(b\operatorname{arcosh}(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^2, x)
```

maple [B] time = 0.22, size = 374, normalized size = 1.96

$$\frac{\left(-4(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2}{8b(a+b\operatorname{arccosh}(dx+c))}-\frac{3e^2\mathrm{e}^{\frac{3a}{b}}\operatorname{Ei}\left(1,3\operatorname{arccosh}(dx+c)+\frac{3a}{b}\right)}{8b^2}+\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right.}{8b(a+b\operatorname{arccosh}(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/8*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2/b/(a+b*arccosh(d*x+c))-3/8*e^2/b^2*e
xp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/8*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2
)+d*x+c)*e^2/b/(a+b*arccosh(d*x+c))-1/8*e^2/b^2*exp(a/b)*Ei(1,arccosh(d*x+c
)+a/b)-1/8/b*e^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))-1/8/b^2*e^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/8/b*e^2*(4*(d*x+c)^3-3
*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2))/(a+b*arccosh(d*x+c))-3/8/b^2*e^2*exp(-3*a/b)*Ei(1,-3*arccosh(d*x
+c)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 - c^3*e^2 + (10*c^2*d^3*e^2 - d^3
*e^2)*x^3 + (10*c^3*d^2*e^2 - 3*c*d^2*e^2)*x^2 + (d^4*e^2*x^4 + 4*c*d^3*e^2
*x^3 + c^4*e^2 - c^2*e^2 + (6*c^2*d^2*e^2 - d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 -
 c*d*e^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (5*c^4*d*e^2 - 3*c^2*d*e
^2)*x)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*
d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^
2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*l
og(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate((3*d^6*e^2*x
^6 + 18*c*d^5*e^2*x^5 + 3*c^6*e^2 - 6*c^4*e^2 + 3*(15*c^2*d^4*e^2 - 2*d^4*e
^2)*x^4 + 3*c^2*e^2 + 12*(5*c^3*d^3*e^2 - 2*c*d^3*e^2)*x^3 + (3*d^4*e^2*x^4
 + 12*c*d^3*e^2*x^3 + 3*c^4*e^2 - c^2*e^2 + (18*c^2*d^2*e^2 - d^2*e^2)*x^2
+ 2*(6*c^3*d*e^2 - c*d*e^2)*x)*(d*x + c + 1)*(d*x + c - 1) + 3*(15*c^4*d^2*
e^2 - 12*c^2*d^2*e^2 + d^2*e^2)*x^2 + (6*d^5*e^2*x^5 + 30*c*d^4*e^2*x^4 + 6
*c^5*e^2 - 7*c^3*e^2 + (60*c^2*d^3*e^2 - 7*d^3*e^2)*x^3 + 2*c*e^2 + 3*(20*c
^3*d^2*e^2 - 7*c*d^2*e^2)*x^2 + (30*c^4*d*e^2 - 21*c^2*d*e^2 + 2*d*e^2)*x)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 6*(3*c^5*d*e^2 - 4*c^3*d*e^2 + c*d*e^
2)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3
*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x
```

```
+ c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^
2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b
^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) +
(c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^
2*x + (c^3 - c)*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*
x + c + 1)*sqrt(d*x + c - 1) + c)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^2+2ab\,\mathrm{acosh}\,(c+dx)+b^2\,\mathrm{acosh}^2\,(c+dx)}\,dx+\int \frac{d^2x^2}{a^2+2ab\,\mathrm{acosh}\,(c+dx)+b^2\,\mathrm{acosh}^2\,(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e**2*(Integral(c**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2),
 x) + Integral(d**2*x**2/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)
**2), x) + Integral(2*c*d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d
*x)**2), x))
```

3.139 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^2d}-\frac{e\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

```
[Out] e*Chi(2*(a+b*arccosh(d*x+c))/b)*cosh(2*a/b)/b^2/d-e*Shi(2*(a+b*arccosh(d*x+
c))/b)*sinh(2*a/b)/b^2/d-e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b
*arccosh(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5866, 12, 5666, 3303, 3298, 3301}

$$\frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{e\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] -((e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c
+ d*x]))) + (e*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(b
^2*d) - (e*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(b^2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
```

```
x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(c+$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\left(e\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx\right.}{bd}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+\right.}{b^2d}$$

Mathematica [A] time = 0.49, size = 108, normalized size = 0.98

$$\frac{e\left(-\frac{b\sqrt{\frac{c+dx-1}{c+dx+1}}\left(c^2+2cdx+c+dx(dx+1)\right)}{a+b\cosh^{-1}(c+dx)} + \cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+\right.\right.\right.}{b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (e*(-((b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(c + c^2 + 2*c*d*x + d*x*(1 + d
*x)))/(a + b*ArcCosh[c + d*x])) + Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCo
sh[c + d*x])] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])]))/(b
^2*d)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dex+ce}{b^2\,\text{arcosh}\,(dx+c)^2 + 2\,ab\,\text{arcosh}\,(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex+ce}{(b\operatorname{arcosh}(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^2, x)
```

maple [A] time = 0.05, size = 170, normalized size = 1.55

$$\frac{\frac{\left(-2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+2(dx+c)^2-1\right)e}{4(a+b\operatorname{arccosh}(dx+c))b}-\frac{e\,\mathrm{e}^{\frac{2a}{b}}\operatorname{Ei}\left(1,2\operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{2b^2}-\frac{e\left(2(dx+c)^2-1+2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)\right)}{4b(a+b\operatorname{arccosh}(dx+c))}-\frac{e\,\mathrm{e}^{-\frac{2a}{b}}\operatorname{Ei}\left(1,-\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(1/4*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e/(a+b*
arccosh(d*x+c))/b-1/2*e/b^2*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/4/b*e
*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x
+c))-1/2/b^2*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^4ex^4+4\,cd^3ex^3+c^4e-c^2e+\left(6\,c^2d^2e-d^2e\right)x^2+\left(d^3ex^3+3\,cd^2ex^2+c^3e-ce+\left(3\right.\right.}{abd^3x^2+2\,abcd^2x+\left(c^2d-d\right)ab+\left(abd^2x+abcd\right)\sqrt{dx+c+1}\,\sqrt{dx+c-1}+\left(b^2d^3x^2+2\,b^2cd^2x+\left(c^2d-d\right)b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e - c^2*e + (6*c^2*d^2*e - d^2*e)*x^2 + (
d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e - c*e + (3*c^2*d*e - d*e)*x)*sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + 2*(2*c^3*d*e - c*d*e)*x)/(a*b*d^3*x^2 + 2*a*b*c*d
^2*x + (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2
*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt
(d*x + c - 1) + c)) + integrate((2*d^5*e*x^5 + 10*c*d^4*e*x^4 + 2*c^5*e - 4
*c^3*e + 4*(5*c^2*d^3*e - d^3*e)*x^3 + 2*(d^3*e*x^3 + 3*c*d^2*e*x^2 + 3*c^2
*d*e*x + c^3*e)*(d*x + c + 1)*(d*x + c - 1) + 4*(5*c^3*d^2*e - 3*c*d^2*e)*x
^2 + (4*d^4*e*x^4 + 16*c*d^3*e*x^3 + 4*c^4*e - 4*c^2*e + 4*(6*c^2*d^2*e - d
^2*e)*x^2 + 8*(2*c^3*d*e - c*d*e)*x + e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + 2*c*e + 2*(5*c^4*d*e - 6*c^2*d*e + d*e)*x)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x
^3 + 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2
*a*b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b +
 2*(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sq
rt(d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c
^2*d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^2 + 2ab\operatorname{acosh}(c + dx) + b^2\operatorname{acosh}^2(c + dx)}\,dx + \int \frac{dx}{a^2 + 2ab\operatorname{acosh}(c + dx) + b^2\operatorname{acosh}^2(c + dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] e*(Integral(c/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2), x) +
Integral(d*x/(a**2 + 2*a*b*acosh(c + d*x) + b**2*acosh(c + d*x)**2), x))
```

3.140 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^2d}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

```
[Out] Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^2/d-Shi((a+b*arccosh(d*x+c))/b)*sin
h(a/b)/b^2/d-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))
```

Rubi [A] time = 0.25, antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5864, 5656, 5781, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-2),x]
```

```
[Out] -((Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*(a + b*ArcCosh[c + d*x]))) +
(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])/(b^2*d) - (Sinh[a/b]*SinhI
ntegral[a/b + ArcCosh[c + d*x]])/(b^2*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
```

```
, 0] && LtQ[d2, 0])
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx=\frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)}\,dx,x,c+dx\right)}{bd}$$

$$=-\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\cosh^{-1}(c+dx)\right)}{bd}$$

$$=-\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(c+\right.}{bd}$$

$$=-\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{b^2d}-\frac{\sinh\left(\frac{a}{b}\right.}{}$$

Mathematica [A] time = 1.03, size = 143, normalized size = 1.46

$$\frac{\sqrt{\frac{c+dx-1}{c+dx+1}}\coth\left(\frac{1}{2}\cosh^{-1}(c+dx)\right)\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)-\text{l}\right.}{b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x])) + Log
[1 + (b*ArcCosh[c + d*x])/a] + Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Coth[ArcC
osh[c + d*x]/2]*(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] - Log[a + b
*ArcCosh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]]))/(b^2*
d)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2\,\text{arcosh}\,(dx+c)^2+2\,ab\,\text{arcosh}\,(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\text{arcosh}\,(dx+c)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-2), x)
```

maple [A] time = 0.04, size = 139, normalized size = 1.42

$$\frac{-\frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{2b(a+b\,\mathrm{arccosh}(dx+c))}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}\left(1,-\mathrm{arccosh}(dx+c)-\frac{a}{b}\right)}{2b^{2}}+\frac{-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c}{2b(a+b\,\mathrm{arccosh}(dx+c))}-\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\mathrm{arccosh}(dx+c)+\frac{a}{b}\right)}{2b^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/2/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/
2/b^2*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)+1/2*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)+d*x+c)/b/(a+b*arccosh(d*x+c))-1/2/b^2*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^{3}x^{3}+3\,cd^{2}x^{2}+c^{3}+\left(d^{2}x^{2}+2\,cdx+c^{2}-1\right)\sqrt{dx+c+1}}{abd^{3}x^{2}+2\,abcd^{2}x+\left(c^{2}d-d\right)ab+\left(abd^{2}x+abcd\right)\sqrt{dx+c+1}\,\sqrt{dx+c-1}+\left(b^{2}d^{3}x^{2}+2\,b^{2}cd^{2}x+\left(c^{2}d-d\right)b^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x
+ (c^2*d - d)*a*b + (a*b*d^2*x + a*b*c*d)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + (c^2*d - d)*b^2 + (b^2*d^2*x + b^2*c*d)
*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c)) + integrate((d^4*x^4 + 4*c*d^3*x^3 + c^4 + (d^2*x^2 + 2*c*d*
x + c^2 + 1)*(d*x + c + 1)*(d*x + c - 1) + 2*(3*c^2*d^2 - d^2)*x^2 + (2*d^3
*x^3 + 6*c*d^2*x^2 + 2*c^3 + (6*c^2*d - d)*x - c)*sqrt(d*x + c + 1)*sqrt(d*
x + c - 1) - 2*c^2 + 4*(c^3*d - c*d)*x + 1)/(a*b*d^4*x^4 + 4*a*b*c*d^3*x^3
+ 2*(3*c^2*d^2 - d^2)*a*b*x^2 + 4*(c^3*d - c*d)*a*b*x + (a*b*d^2*x^2 + 2*a*
b*c*d*x + a*b*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*a*b + 2*
(a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 + (3*c^2*d - d)*a*b*x + (c^3 - c)*a*b)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 2*(3*c^2*
d^2 - d^2)*b^2*x^2 + 4*(c^3*d - c*d)*b^2*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*(d*x + c + 1)*(d*x + c - 1) + (c^4 - 2*c^2 + 1)*b^2 + 2*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + (3*c^2*d - d)*b^2*x + (c^3 - c)*b^2)*sqrt(d*x + c + 1
)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-2), x)
```

3.141 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^2},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^2,x)/e
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^2), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^2}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 7.77, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^2), x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2dex + a^2ce + \left(b^2dex + b^2ce\right)\text{arcosh}\left(dx+c\right)^2 + 2\left(abdex + abce\right)\text{arcosh}\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arccosh(d*x + c)^2
+ 2*(a*b*d*e*x + a*b*c*e)*arccosh(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\operatorname{arcosh}(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2), x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,(a+b\operatorname{arccosh}(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^3x^3+3\,cd^2}{abd^4ex^3+3\,abcd^3ex^2+\left(3\,c^2d^2e-d^2e\right)abx+\left(c^3de-cde\right)ab+\left(abd^3ex^2+2\,abcd^2ex+abc^2de\right)\sqrt{dx+c+1}\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c)/(a*b*d^4*e*x^3 + 3*a*b*c*d^3*
e*x^2 + (3*c^2*d^2*e - d^2*e)*a*b*x + (c^3*d*e - c*d*e)*a*b + (a*b*d^3*e*x^
2 + 2*a*b*c*d^2*e*x + a*b*c^2*d*e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (b
^2*d^4*e*x^3 + 3*b^2*c*d^3*e*x^2 + (3*c^2*d^2*e - d^2*e)*b^2*x + (c^3*d*e -
 c*d*e)*b^2 + (b^2*d^3*e*x^2 + 2*b^2*c*d^2*e*x + b^2*c^2*d*e)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))
 + integrate((2*(d*x + c + 1)*(d*x + c)*(d*x + c - 1) + (2*d^2*x^2 + 4*c*d*
x + 2*c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))/(a*b*d^6*e*x^6 + 6*a*b*
c*d^5*e*x^5 + (15*c^2*d^4*e - 2*d^4*e)*a*b*x^4 + 4*(5*c^3*d^3*e - 2*c*d^3*e
)*a*b*x^3 + (15*c^4*d^2*e - 12*c^2*d^2*e + d^2*e)*a*b*x^2 + 2*(3*c^5*d*e -
4*c^3*d*e + c*d*e)*a*b*x + (a*b*d^4*e*x^4 + 4*a*b*c*d^3*e*x^3 + 6*a*b*c^2*d
^2*e*x^2 + 4*a*b*c^3*d*e*x + a*b*c^4*e)*(d*x + c + 1)*(d*x + c - 1) + (c^6*
e - 2*c^4*e + c^2*e)*a*b + 2*(a*b*d^5*e*x^5 + 5*a*b*c*d^4*e*x^4 + (10*c^2*d
^3*e - d^3*e)*a*b*x^3 + (10*c^3*d^2*e - 3*c*d^2*e)*a*b*x^2 + (5*c^4*d*e - 3
*c^2*d*e)*a*b*x + (c^5*e - c^3*e)*a*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ (b^2*d^6*e*x^6 + 6*b^2*c*d^5*e*x^5 + (15*c^2*d^4*e - 2*d^4*e)*b^2*x^4 + 4
*(5*c^3*d^3*e - 2*c*d^3*e)*b^2*x^3 + (15*c^4*d^2*e - 12*c^2*d^2*e + d^2*e)*
b^2*x^2 + 2*(3*c^5*d*e - 4*c^3*d*e + c*d*e)*b^2*x + (b^2*d^4*e*x^4 + 4*b^2*
c*d^3*e*x^3 + 6*b^2*c^2*d^2*e*x^2 + 4*b^2*c^3*d*e*x + b^2*c^4*e)*(d*x + c +
 1)*(d*x + c - 1) + (c^6*e - 2*c^4*e + c^2*e)*b^2 + 2*(b^2*d^5*e*x^5 + 5*b^
2*c*d^4*e*x^4 + (10*c^2*d^3*e - d^3*e)*b^2*x^3 + (10*c^3*d^2*e - 3*c*d^2*e)
*b^2*x^2 + (5*c^4*d*e - 3*c^2*d*e)*b^2*x + (c^5*e - c^3*e)*b^2)*sqrt(d*x +
c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c
)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e+d\,e\,x)\,(a+b\operatorname{acosh}(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^2),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2c+a^2dx+2abc\operatorname{acosh}(c+dx)+2abdx\operatorname{acosh}(c+dx)+b^2c\operatorname{acosh}^2(c+dx)+b^2dx\operatorname{acosh}^2(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral(1/(a**2*c + a**2*d*x + 2*a*b*c*acosh(c + d*x) + 2*a*b*d*x*acosh(c
+ d*x) + b**2*c*acosh(c + d*x)**2 + b**2*d*x*acosh(c + d*x)**2), x)/e
```

3.142 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=327

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{16b^3d}-\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}-\frac{25e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^3d}$$

```
[Out] 2*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))-5/2*e^4*(d*x+c)^5/b^2/d/(a+b*arc
cosh(d*x+c))+1/16*e^4*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b^3/d+27/32*e^4
*cosh(3*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b^3/d+25/32*e^4*cosh(5*a/b)*Shi(
5*(a+b*arccosh(d*x+c))/b)/b^3/d-1/16*e^4*Chi((a+b*arccosh(d*x+c))/b)*sinh(a
/b)/b^3/d-27/32*e^4*Chi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^3/d-25/32*e
^4*Chi(5*(a+b*arccosh(d*x+c))/b)*sinh(5*a/b)/b^3/d-1/2*e^4*(d*x+c)^4*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^2
```

Rubi [A] time = 1.11, antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 26, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^4\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{16b^3d}-\frac{27e^4\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{32b^3d}-\frac{25e^4\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}+\ldots\right)}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -(e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (2*e^4*(c + d*x)^3)/(b^2*d*(a + b*ArcCosh[c + d*x])) - (5
*e^4*(c + d*x)^5)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e^4*CoshIntegral[a/
b + ArcCosh[c + d*x]]*Sinh[a/b])/(16*b^3*d) - (27*e^4*CoshIntegral[(3*a)/b
+ 3*ArcCosh[c + d*x]]*Sinh[(3*a)/b])/(32*b^3*d) - (25*e^4*CoshIntegral[(5*a
)/b + 5*ArcCosh[c + d*x]]*Sinh[(5*a)/b])/(32*b^3*d) + (e^4*Cosh[a/b]*SinhIn
tegral[a/b + ArcCosh[c + d*x]])/(16*b^3*d) + (27*e^4*Cosh[(3*a)/b]*SinhInte
gral[(3*a)/b + 3*ArcCosh[c + d*x]])/(32*b^3*d) + (25*e^4*Cosh[(5*a)/b]*Sinh
Integral[(5*a)/b + 5*ArcCosh[c + d*x]])/(32*b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{(2e^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}\right.}\right.}{bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{2e^4(c+dx)^3}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

Mathematica [A] time = 1.28, size = 323, normalized size = 0.99

$$e^4\left(-\frac{16b^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + 48\left(\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+d\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^4*((-16*b^2*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^2 + (16*b*(4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcCosh[c
+ d*x]) + 48*(CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] + CoshIntegral
[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c + d*x]] - Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])])
+ 25*(-2*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 3*CoshIntegral[3*
(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] - CoshIntegral[5*(a/b + ArcCosh[c +
 d*x])]*Sinh[(5*a)/b] + 2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] +
3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*Si
nhIntegral[5*(a/b + ArcCosh[c + d*x])])))/(32*b^3*d)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^3\,\text{arcosh}\,(dx + c)^3 + 3\,ab^2\,\text{arcosh}\,(dx + c)^2 + 3\,a^2b\,\text{arcosh}\,(dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*
arccosh(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(b\,\text{arcosh}\,(dx + c) + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^3, x)
```

maple [B] time = 0.45, size = 993, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/64*(-16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+12*(d*x+c)^2*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20
*(d*x+c)^3+5*d*x+5*c)*e^4*(5*b*arccosh(d*x+c)+5*a-b)/b^2/(b^2*arccosh(d*x+c
)^2+2*a*b*arccosh(d*x+c)+a^2)+25/64*e^4/b^3*exp(5*a/b)*Ei(1,5*arccosh(d*x+c
)+5*a/b)-3/64*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4*(3*b*arccosh(d*x+c)+3*a-b)/b^2/
(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+27/64*e^4/b^3*exp(3*a/b)*Ei
(1,3*arccosh(d*x+c)+3*a/b)-1/32*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^
4*(b*arccosh(d*x+c)+a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2
)+1/32*e^4/b^3*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/32/b*e^4*(d*x+c+(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/32/b^2*e^4*(d*x+c+(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/32/b^3*e^4*exp(-a/b)*Ei(1,
-arccosh(d*x+c)-a/b)-3/64/b*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^2-9/64/b^2*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c
+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-27/64/b^3*e
^4*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/64/b*e^4*(16*(d*x+c)^5-20*(d
*x+c)^3+16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2
*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arcc
osh(d*x+c))^2-5/64/b^2*e^4*(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c)^4*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-25/64/b^3*e^4*exp
(-5*a/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^3+3a^2b\,\mathrm{acosh}\,(c+dx)+3ab^2\,\mathrm{acosh}^2\,(c+dx)+b^3\,\mathrm{acosh}^3\,(c+dx)}\,dx+\int \frac{}{a^3+3a^2b\,\mathrm{acosh}\,(c+dx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e**4*(Integral(c**4/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*
x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**4*x**4/(a**3 + 3*a**2*b*a
cosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) +
Integral(4*c*d**3*x**3/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c +
 d*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(6*c**2*d**2*x**2/(a**3 +
3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)*
*3), x) + Integral(4*c**3*d*x/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*ac
osh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x))
```

3.143 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=254

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^3d}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{2b^3d}-$$

```
[Out] 3/2*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))-2*e^3*(d*x+c)^4/b^2/d/(a+b*arc
cosh(d*x+c))+1/2*e^3*cosh(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b^3/d+e^3*co
sh(4*a/b)*Shi(4*(a+b*arccosh(d*x+c))/b)/b^3/d-1/2*e^3*Chi(2*(a+b*arccosh(d*
x+c))/b)*sinh(2*a/b)/b^3/d-e^3*Chi(4*(a+b*arccosh(d*x+c))/b)*sinh(4*a/b)/b^
3/d-1/2*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+
c))^2
```

Rubi [A] time = 0.90, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2b^3d}-\frac{e^3\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)}{b^3d}+\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -(e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (3*e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) -
(2*e^3*(c + d*x)^4)/(b^2*d*(a + b*ArcCosh[c + d*x])) - (e^3*CoshIntegral[(2
*a)/b + 2*ArcCosh[c + d*x]]*Sinh[(2*a)/b])/(2*b^3*d) - (e^3*CoshIntegral[(4
*a)/b + 4*ArcCosh[c + d*x]]*Sinh[(4*a)/b])/(b^3*d) + (e^3*Cosh[(2*a)/b]*Sin
hIntegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(2*b^3*d) + (e^3*Cosh[(4*a)/b]*Sin
hIntegral[(4*a)/b + 4*ArcCosh[c + d*x]])/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\left(3e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right.}\right.}{2bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{b^2d\left(a\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{b^2d\left(a\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{b^2d\left(a\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{b^2d\left(a\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{b^2d\left(a\right.}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{3e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{b^2d\left(a\right.}$$

Mathematica [A] time = 0.56, size = 186, normalized size = 0.73

$$\frac{e^3\left(-\frac{b^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^2} - \sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - 2\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right.\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^3*(-((b^2*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(a + b*ArcCo
sh[c + d*x])^2) + (b*(3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcCosh[c + d*
x]) - CoshIntegral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] - 2*CoshIntegr
al[4*(a/b + ArcCosh[c + d*x])]*Sinh[(4*a)/b] + Cosh[(2*a)/b]*SinhIntegral[2
*(a/b + ArcCosh[c + d*x])] + 2*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[
c + d*x])]))/(2*b^3*d)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^3\,\text{arcosh}\,(dx+c)^3 + 3\,ab^2\,\text{arcosh}\,(dx+c)^2 + 3\,a^2b\,\text{arcosh}\,(dx+c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^3*arc
cosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c) + a
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(b \operatorname{arcosh}(dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^3, x)
```

maple [B] time = 0.40, size = 624, normalized size = 2.46

$$-\frac{\frac{\left(-8(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3(4b\operatorname{arccosh}(dx+c)+4a-b)}{32b^2\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)+a^2\right)}+\frac{e^3\mathrm{e}^{\frac{4a}{b}}\operatorname{Ei}\left(1,4\operatorname{arccosh}(dx\right.}{2b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/32*(-8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c+1)^(1/2)*
(d*x+c-1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3*(4*b*arccosh(d*x+c)+
4*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/2*e^3/b^3*exp(
4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)-1/16*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2
)*(d*x+c)+2*(d*x+c)^2-1)*e^3*(2*b*arccosh(d*x+c)+2*a-b)/b^2/(b^2*arccosh(d*
x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/4*e^3/b^3*exp(2*a/b)*Ei(1,2*arccosh(d*x+
c)+2*a/b)-1/16/b*e^3*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+
c))/(a+b*arccosh(d*x+c))^2-1/8/b^2*e^3*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*
x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-1/4/b^3*e^3*exp(-2*a/b)*Ei(1,-2*
arccosh(d*x+c)-2*a/b)-1/32/b*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+
b*arccosh(d*x+c))^2-1/8/b^2*e^3*(8*(d*x+c)^4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b
*arccosh(d*x+c))-1/2/b^3*e^3*exp(-4*a/b)*Ei(1,-4*arccosh(d*x+c)-4*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^3}{(a + b\,\mathrm{acosh}\,(c + d\,x))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^3 + 3a^2b\operatorname{acosh}(c+dx) + 3ab^2\operatorname{acosh}^2(c+dx) + b^3\operatorname{acosh}^3(c+dx)}\,dx + \int \frac{}{a^3 + 3a^2b\operatorname{acosh}(c+dx) + 3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e**3*(Integral(c**3/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*
x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**3*x**3/(a**3 + 3*a**2*b*a
cosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) +
Integral(3*c*d**2*x**2/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c +
 d*x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(3*c**2*d*x/(a**3 + 3*a**2
*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x
))
```

3.144 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^3} dx$

Optimal. Leaf size=252

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^3d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8b^3d}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{8b^3d}+\frac{9e}{\ldots}$$

```
[Out] e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))-3/2*e^2*(d*x+c)^3/b^2/d/(a+b*arccosh
(d*x+c))+1/8*e^2*cosh(a/b)*Shi((a+b*arccosh(d*x+c))/b)/b^3/d+9/8*e^2*cosh(3
*a/b)*Shi(3*(a+b*arccosh(d*x+c))/b)/b^3/d-1/8*e^2*Chi((a+b*arccosh(d*x+c))/
b)*sinh(a/b)/b^3/d-9/8*e^2*Chi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^3/d-
1/2*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^
2
```

Rubi [A] time = 0.80, antiderivative size = 311, normalized size of antiderivative = 1.23, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3303, 3298, 3301, 5658}

$$-\frac{9e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{8b^3d}+\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{b^3d}-\frac{9e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(\ldots\right)}{8b^3d}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -(e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCo
sh[c + d*x])^2) + (e^2*(c + d*x))/(b^2*d*(a + b*ArcCosh[c + d*x])) - (3*e^2
*(c + d*x)^3)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (9*e^2*CoshIntegral[a/b
+ ArcCosh[c + d*x]]*Sinh[a/b])/(8*b^3*d) + (e^2*CoshIntegral[(a + b*ArcCosh
[c + d*x])/b]*Sinh[a/b])/(b^3*d) - (9*e^2*CoshIntegral[(3*a)/b + 3*ArcCosh[
c + d*x]]*Sinh[(3*a)/b])/(8*b^3*d) + (9*e^2*Cosh[a/b]*SinhIntegral[a/b + Ar
cCosh[c + d*x]])/(8*b^3*d) + (9*e^2*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcCosh[c + d*x]])/(8*b^3*d) - (e^2*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c
 + d*x])/b])/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e^2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right.}\right.}{bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e^2(c+dx)}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{}{2b^2d}$$

Mathematica [A] time = 0.69, size = 223, normalized size = 0.88

$$\frac{e^2\left(-\frac{4b^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + 9\left(\sinh\left(\frac{a}{b}\right)\left(-\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(c+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e^2*((-4*b^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^2 + (4*b*(2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcCosh[c + d*
x]) + 8*CoshIntegral[a/b + ArcCosh[c + d*x]]*Sinh[a/b] - 8*Cosh[a/b]*SinhIn
tegral[a/b + ArcCosh[c + d*x]] + 9*(-(CoshIntegral[a/b + ArcCosh[c + d*x]]*
Sinh[a/b]) - CoshIntegral[3*(a/b + ArcCosh[c + d*x])]*Sinh[(3*a)/b] + Cosh[
a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a
/b + ArcCosh[c + d*x])])))/(8*b^3*d)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2e^2x^2 + 2\,cde^2x + c^2e^2}{b^3\,\text{arcosh}\left(dx+c\right)^3 + 3\,ab^2\,\text{arcosh}\left(dx+c\right)^2 + 3\,a^2b\,\text{arcosh}\left(dx+c\right) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^3*arccosh(d*x + c)^3 + 3*
a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{(b\,\mathrm{arcosh}\,(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^3, x)
```

maple [B] time = 0.24, size = 557, normalized size = 2.21

$$-\frac{\left(-4(dx+c)^2\sqrt{dx+c-1}\,\sqrt{dx+c+1}+\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2(3b\,\mathrm{arccosh}(dx+c)+3a-b)}{16b^2\left(b^2\mathrm{arccosh}(dx+c)^2+2ab\,\mathrm{arccosh}(dx+c)+a^2\right)}+\frac{9e^2\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}\left(1,3\,\mathrm{arccosh}(dx+c)+\frac{3a}{b}\right)}{16b^3}-\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/16*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2*(3*b*arccosh(d*x+c)+3*a-b)/b^2/(b^
2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+9/16*e^2/b^3*exp(3*a/b)*Ei(1,3
*arccosh(d*x+c)+3*a/b)-1/16*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*e^2*(b
*arccosh(d*x+c)+a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/
16*e^2/b^3*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/16/b*e^2*(d*x+c+(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/16/b^2*e^2*(d*x+c+(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/16/b^3*e^2*exp(-a/b)*Ei(1,-arc
cosh(d*x+c)-a/b)-1/16/b*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1
/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2
-3/16/b^2*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^
(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/16/b^3*e^2*ex
p(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^3 + 3a^2b\operatorname{acosh}(c+dx) + 3ab^2\operatorname{acosh}^2(c+dx) + b^3\operatorname{acosh}^3(c+dx)}\,dx + \int \frac{}{a^3 + 3a^2b\operatorname{acosh}(c+dx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e**2*(Integral(c**2/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*
x)**2 + b**3*acosh(c + d*x)**3), x) + Integral(d**2*x**2/(a**3 + 3*a**2*b*a
cosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x) +
Integral(2*c*d*x/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)*
*2 + b**3*acosh(c + d*x)**3), x))
```

3.145 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}+\frac{}{2b^2d\left(a+\right.}$$

```
[Out] 1/2*e/b^2/d/(a+b*arccosh(d*x+c))-e*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))+e*c
osh(2*a/b)*Shi(2*(a+b*arccosh(d*x+c))/b)/b^3/d-e*Chi(2*(a+b*arccosh(d*x+c))
/b)*sinh(2*a/b)/b^3/d-1/2*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+
b*arccosh(d*x+c))^2
```

Rubi [A] time = 0.51, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3303, 3298, 3301, 5676}

$$-\frac{e\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^3d}+\frac{e\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{b^3d}-\frac{e(c+dx)^2}{b^2d\left(a+b\cosh^{-1}(c+dx)\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] -(e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCosh[c
 + d*x])^2) + e/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (e*(c + d*x)^2)/(b^2*d
*(a + b*ArcCosh[c + d*x])) - (e*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x]]*
Sinh[(2*a)/b])/(b^3*d) + (e*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[
c + d*x]])/(b^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^2}\,dx,\right.}{2bd}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e}{b^2d\left(a+\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e}{b^2d\left(a+\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e}{b^2d\left(a+\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e}{b^2d\left(a+\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e}{b^2d\left(a+\right.}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{e}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{e}{b^2d\left(a+\right.}$$

Mathematica [A] time = 0.33, size = 127, normalized size = 0.78

$$\frac{e\left(-\frac{b^2\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}}{\left(a+b\cosh^{-1}(c+dx)\right)^2} - 2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) + 2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right)\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (e*(-((b^2*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c
 + d*x])^2) + (b*(1 - 2*(c + d*x)^2))/(a + b*ArcCosh[c + d*x]) - 2*CoshInte
gral[2*(a/b + ArcCosh[c + d*x])]*Sinh[(2*a)/b] + 2*Cosh[(2*a)/b]*SinhIntegr
al[2*(a/b + ArcCosh[c + d*x])]))/(2*b^3*d)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dex+ce}{b^3\,\text{arcosh}\,(dx+c)^3 + 3\,ab^2\,\text{arcosh}\,(dx+c)^2 + 3\,a^2b\,\text{arcosh}\,(dx+c) + a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2
 + 3*a^2*b*arccosh(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^3, x)
```

maple [A] time = 0.06, size = 254, normalized size = 1.56

$$\frac{-\frac{\left(-2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+2(dx+c)^2-1\right)e(2b\operatorname{arccosh}(dx+c)+2a-b)}{8b^2\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)+a^2\right)}+\frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}\left(1,2\operatorname{arccosh}(dx+c)+\frac{2a}{b}\right)}{2b^3}-\frac{e\left(2(dx+c)^2-1+2\sqrt{dx+c+1}\,\sqrt{d}\right.}{8b(a+b\operatorname{arccosh}(dx+}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/8*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e*(2*b
*arccosh(d*x+c)+2*a-b)/b^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+
1/2*e/b^3*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/8/b*e*(2*(d*x+c)^2-1+2*
(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/4/b^2*e*(
2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c
))-1/2/b^3*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e + d\,e\,x}{(a + b \operatorname{acosh}(c + d\,x))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^3 + 3a^2b \operatorname{acosh}(c + dx) + 3ab^2 \operatorname{acosh}^2(c + dx) + b^3 \operatorname{acosh}^3(c + dx)} \, dx + \int \frac{}{a^3 + 3a^2b \operatorname{acosh}(c + dx) +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] e*(Integral(c/(a**3 + 3*a**2*b*acosh(c + d*x) + 3*a*b**2*acosh(c + d*x)**2
+ b**3*acosh(c + d*x)**3), x) + Integral(d*x/(a**3 + 3*a**2*b*acosh(c + d*x
) + 3*a*b**2*acosh(c + d*x)**2 + b**3*acosh(c + d*x)**3), x))
```

3.146 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx$

Optimal. Leaf size=132

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{\sqrt{c+dx-1}\sqrt{c+}}{2bd\left(a+b\cosh^{-1}(\right.}$$

```
[Out] 1/2*(-d*x-c)/b^2/d/(a+b*arccosh(d*x+c))+1/2*cosh(a/b)*Shi((a+b*arccosh(d*x+
c))/b)/b^3/d-1/2*Chi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^3/d-1/2*(d*x+c-1)^
(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^2
```

Rubi [A] time = 0.26, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5864, 5656, 5775, 5658, 3303, 3298, 3301}

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}+\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{2b^3d}-\frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{\sqrt{c+dx-1}\sqrt{c+}}{2bd\left(a+b\cosh^{-1}(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-3),x]
```

```
[Out] -(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(2*b*d*(a + b*ArcCosh[c + d*x])^2)
- (c + d*x)/(2*b^2*d*(a + b*ArcCosh[c + d*x])) - (CoshIntegral[(a + b*ArcCo
sh[c + d*x])/b]*Sinh[a/b])/(2*b^3*d) + (Cosh[a/b]*SinhIntegral[(a + b*ArcCo
sh[c + d*x])/b])/(2*b^3*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
```

```
, b, c, n}, x]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^2}\,dx, x, c+ \ldots\right.}{2bd}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\ldots}{a+b\ldots}\right.}{\ldots}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\sinh\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} + \frac{\cosh\left(\frac{a}{b}\right)\text{Sub}\ldots}{\ldots}$$

$$= -\frac{\sqrt{-1+c+dx}\sqrt{1+c+dx}}{2bd\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{c+dx}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)} - \frac{\text{Chi}\left(\frac{a+b\cosh^{-}\ldots}{b}\right.}{2\ldots}$$

Mathematica [A] time = 0.42, size = 109, normalized size = 0.83

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) - \cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \frac{b\left(ac+adx+b\sqrt{c+dx-1}\sqrt{c+dx+1}+b(c+dx)\,\text{co}\ldots\right.}{\left(a+b\cosh^{-1}(c+dx)\right)^2}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-3),x]
```

```
[Out] -1/2*((b*(a*c + a*d*x + b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x] + b*(c + d*x
)*ArcCosh[c + d*x]))/(a + b*ArcCosh[c + d*x])^2 + CoshIntegral[a/b + ArcCos
```

```
h[c + d*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(b^
3*d)
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^3 \operatorname{arcosh}\left(dx+c\right)^3 + 3\,ab^2 \operatorname{arcosh}\left(dx+c\right)^2 + 3\,a^2 b \operatorname{arcosh}\left(dx+c\right) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*a
rccosh(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arcosh}\left(dx+c\right) + a\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-3), x)
```

maple [A] time = 0.06, size = 207, normalized size = 1.57

$$\frac{-\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right)\left(b \operatorname{arccosh}(dx+c)+a-b\right)}{4b^2\left(b^2 \operatorname{arccosh}(dx+c)^2+2ab \operatorname{arccosh}(dx+c)+a^2\right)} + \frac{e^{\frac{a}{b}} \operatorname{Ei}\left(1, \operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{4b^3} - \frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4b\left(a+b \operatorname{arccosh}(dx+c)\right)^2} - \frac{dx+c+\sqrt{dx+c-1}\,\sqrt{dx+c+1}}{4b^2\left(a+b \operatorname{arccosh}(dx+c)\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/4*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*(b*arccosh(d*x+c)+a-b)/b
^2/(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)+a^2)+1/4/b^3*exp(a/b)*Ei(1,ar
ccosh(d*x+c)+a/b)-1/4/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccos
h(d*x+c))^2-1/4/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*
x+c))-1/4/b^3*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*d^7 + b*d^7)*x^7 + 7*(a*c*d^6 + b*c*d^6)*x^6 + 3*((7*c^2*d^5 - d^5
)*a + (7*c^2*d^5 - d^5)*b)*x^5 + 5*((7*c^3*d^4 - 3*c*d^4)*a + (7*c^3*d^4 -
3*c*d^4)*b)*x^4 + ((a*d^4 + b*d^4)*x^4 + 4*(a*c*d^3 + b*c*d^3)*x^3 + (6*a*c
^2*d^2 + (6*c^2*d^2 - d^2)*b)*x^2 + (c^4 - 1)*a + (c^4 - c^2)*b + 2*(2*a*c^
3*d + (2*c^3*d - c*d)*b)*x)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + ((35*
c^4*d^3 - 30*c^2*d^3 + 3*d^3)*a + (35*c^4*d^3 - 30*c^2*d^3 + 3*d^3)*b)*x^3
+ (3*(a*d^5 + b*d^5)*x^5 + 15*(a*c*d^4 + b*c*d^4)*x^4 + (3*(10*c^2*d^3 - d^
3)*a + 5*(6*c^2*d^3 - d^3)*b)*x^3 + 3*((10*c^3*d^2 - 3*c*d^2)*a + 5*(2*c^3*
d^2 - c*d^2)*b)*x^2 + 3*(c^5 - c^3)*a + (3*c^5 - 5*c^3 + 2*c)*b + (3*(5*c^4
*d - 3*c^2*d)*a + (15*c^4*d - 15*c^2*d + 2*d)*b)*x)*(d*x + c + 1)*(d*x + c
- 1) + 3*((7*c^5*d^2 - 10*c^3*d^2 + 3*c*d^2)*a + (7*c^5*d^2 - 10*c^3*d^2 +
3*c*d^2)*b)*x^2 + (3*(a*d^6 + b*d^6)*x^6 + 18*(a*c*d^5 + b*c*d^5)*x^5 + (3*
```

```
(15*c^2*d^4 - 2*d^4)*a + (45*c^2*d^4 - 7*d^4)*b)*x^4 + 4*(3*(5*c^3*d^3 - 2*
c*d^3)*a + (15*c^3*d^3 - 7*c*d^3)*b)*x^3 + ((45*c^4*d^2 - 36*c^2*d^2 + 4*d^
2)*a + (45*c^4*d^2 - 42*c^2*d^2 + 5*d^2)*b)*x^2 + (3*c^6 - 6*c^4 + 4*c^2 -
1)*a + (3*c^6 - 7*c^4 + 5*c^2 - 1)*b + 2*((9*c^5*d - 12*c^3*d + 4*c*d)*a +
(9*c^5*d - 14*c^3*d + 5*c*d)*b)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c
^7 - 3*c^5 + 3*c^3 - c)*a + (c^7 - 3*c^5 + 3*c^3 - c)*b + ((7*c^6*d - 15*c^
4*d + 9*c^2*d - d)*a + (7*c^6*d - 15*c^4*d + 9*c^2*d - d)*b)*x + (b*d^7*x^7
 + 7*b*c*d^6*x^6 + 3*(7*c^2*d^5 - d^5)*b*x^5 + 5*(7*c^3*d^4 - 3*c*d^4)*b*x^
4 + (35*c^4*d^3 - 30*c^2*d^3 + 3*d^3)*b*x^3 + (b*d^4*x^4 + 4*b*c*d^3*x^3 +
6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + (c^4 - 1)*b)*(d*x + c + 1)^(3/2)*(d*x + c -
 1)^(3/2) + 3*(7*c^5*d^2 - 10*c^3*d^2 + 3*c*d^2)*b*x^2 + 3*(b*d^5*x^5 + 5*b
*c*d^4*x^4 + (10*c^2*d^3 - d^3)*b*x^3 + (10*c^3*d^2 - 3*c*d^2)*b*x^2 + (5*c
^4*d - 3*c^2*d)*b*x + (c^5 - c^3)*b)*(d*x + c + 1)*(d*x + c - 1) + (7*c^6*d
 - 15*c^4*d + 9*c^2*d - d)*b*x + (3*b*d^6*x^6 + 18*b*c*d^5*x^5 + 3*(15*c^2*
d^4 - 2*d^4)*b*x^4 + 12*(5*c^3*d^3 - 2*c*d^3)*b*x^3 + (45*c^4*d^2 - 36*c^2*
d^2 + 4*d^2)*b*x^2 + 2*(9*c^5*d - 12*c^3*d + 4*c*d)*b*x + (3*c^6 - 6*c^4 +
4*c^2 - 1)*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (c^7 - 3*c^5 + 3*c^3 -
c)*b)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(a^2*b^2*d^7*x^6
+ 6*a^2*b^2*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*a^2*b^2*x^4 + 4*(5*c^3*d^4 - 3*
c*d^4)*a^2*b^2*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 + d^3)*a^2*b^2*x^2 + 6*(c^5*d
^2 - 2*c^3*d^2 + c*d^2)*a^2*b^2*x + (c^6*d - 3*c^4*d + 3*c^2*d - d)*a^2*b^2
 + (a^2*b^2*d^4*x^3 + 3*a^2*b^2*c*d^3*x^2 + 3*a^2*b^2*c^2*d^2*x + a^2*b^2*c
^3*d)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^(3/2) + 3*(a^2*b^2*d^5*x^4 + 4*a^2*
b^2*c*d^4*x^3 + (6*c^2*d^3 - d^3)*a^2*b^2*x^2 + 2*(2*c^3*d^2 - c*d^2)*a^2*b
^2*x + (c^4*d - c^2*d)*a^2*b^2)*(d*x + c + 1)*(d*x + c - 1) + (b^4*d^7*x^6
+ 6*b^4*c*d^6*x^5 + 3*(5*c^2*d^5 - d^5)*b^4*x^4 + 4*(5*c^3*d^4 - 3*c*d^4)*b
^4*x^3 + 3*(5*c^4*d^3 - 6*c^2*d^3 + d^3)*b^4*x^2 + 6*(c^5*d^2 - 2*c^3*d^2 +
 c*d^2)*b^4*x + (c^6*d - 3*c^4*d + 3*c^2*d - d)*b^4 + (b^4*d^4*x^3 + 3*b^4*
c*d^3*x^2 + 3*b^4*c^2*d^2*x + b^4*c^3*d)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^
(3/2) + 3*(b^4*d^5*x^4 + 4*b^4*c*d^4*x^3 + (6*c^2*d^3 - d^3)*b^4*x^2 + 2*(2
*c^3*d^2 - c*d^2)*b^4*x + (c^4*d - c^2*d)*b^4)*(d*x + c + 1)*(d*x + c - 1)
+ 3*(b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 2*(5*c^2*d^4 - d^4)*b^4*x^3 + 2*(5*c^3
*d^3 - 3*c*d^3)*b^4*x^2 + (5*c^4*d^2 - 6*c^2*d^2 + d^2)*b^4*x + (c^5*d - 2*
c^3*d + c*d)*b^4)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x +
 c + 1)*sqrt(d*x + c - 1) + c)^2 + 3*(a^2*b^2*d^6*x^5 + 5*a^2*b^2*c*d^5*x^4
 + 2*(5*c^2*d^4 - d^4)*a^2*b^2*x^3 + 2*(5*c^3*d^3 - 3*c*d^3)*a^2*b^2*x^2 +
(5*c^4*d^2 - 6*c^2*d^2 + d^2)*a^2*b^2*x + (c^5*d - 2*c^3*d + c*d)*a^2*b^2)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(a*b^3*d^7*x^6 + 6*a*b^3*c*d^6*x^5
+ 3*(5*c^2*d^5 - d^5)*a*b^3*x^4 + 4*(5*c^3*d^4 - 3*c*d^4)*a*b^3*x^3 + 3*(5*
c^4*d^3 - 6*c^2*d^3 + d^3)*a*b^3*x^2 + 6*(c^5*d^2 - 2*c^3*d^2 + c*d^2)*a*b^
3*x + (c^6*d - 3*c^4*d + 3*c^2*d - d)*a*b^3 + (a*b^3*d^4*x^3 + 3*a*b^3*c*d^
3*x^2 + 3*a*b^3*c^2*d^2*x + a*b^3*c^3*d)*(d*x + c + 1)^(3/2)*(d*x + c - 1)^
(3/2) + 3*(a*b^3*d^5*x^4 + 4*a*b^3*c*d^4*x^3 + (6*c^2*d^3 - d^3)*a*b^3*x^2
+ 2*(2*c^3*d^2 - c*d^2)*a*b^3*x + (c^4*d - c^2*d)*a*b^3)*(d*x + c + 1)*(d*x
 + c - 1) + 3*(a*b^3*d^6*x^5 + 5*a*b^3*c*d^5*x^4 + 2*(5*c^2*d^4 - d^4)*a*b^
3*x^3 + 2*(5*c^3*d^3 - 3*c*d^3)*a*b^3*x^2 + (5*c^4*d^2 - 6*c^2*d^2 + d^2)*a
*b^3*x + (c^5*d - 2*c^3*d + c*d)*a*b^3)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)) + integrate(1/2*(d^8*
x^8 + 8*c*d^7*x^7 + c^8 + 4*(7*c^2*d^6 - d^6)*x^6 - 4*c^6 + 8*(7*c^3*d^5 -
3*c*d^5)*x^5 + (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4 + 3
)*(d*x + c + 1)^2*(d*x + c - 1)^2 + 2*(35*c^4*d^4 - 30*c^2*d^4 + 3*d^4)*x^4
 + (4*d^5*x^5 + 20*c*d^4*x^4 + 4*c^5 + 4*(10*c^2*d^3 - d^3)*x^3 - 4*c^3 + 4
*(10*c^3*d^2 - 3*c*d^2)*x^2 + (20*c^4*d - 12*c^2*d + 3*d)*x + 3*c)*(d*x + c
 + 1)^(3/2)*(d*x + c - 1)^(3/2) + 6*c^4 + 8*(7*c^5*d^3 - 10*c^3*d^3 + 3*c*d
^3)*x^3 + 3*(2*d^6*x^6 + 12*c*d^5*x^5 + 2*c^6 + 2*(15*c^2*d^4 - 2*d^4)*x^4
- 4*c^4 + 8*(5*c^3*d^3 - 2*c*d^3)*x^3 + (30*c^4*d^2 - 24*c^2*d^2 + d^2)*x^2
 + c^2 + 2*(6*c^5*d - 8*c^3*d + c*d)*x + 1)*(d*x + c + 1)*(d*x + c - 1) + 4
*(7*c^6*d^2 - 15*c^4*d^2 + 9*c^2*d^2 - d^2)*x^2 + (4*d^7*x^7 + 28*c*d^6*x^6
 + 4*c^7 + 12*(7*c^2*d^5 - d^5)*x^5 - 12*c^5 + 20*(7*c^3*d^4 - 3*c*d^4)*x^4
```

```
 + (140*c^4*d^3 - 120*c^2*d^3 + 9*d^3)*x^3 + 9*c^3 + 3*(28*c^5*d^2 - 40*c^3
*d^2 + 9*c*d^2)*x^2 + (28*c^6*d - 60*c^4*d + 27*c^2*d - d)*x - c)*sqrt(d*x
+ c + 1)*sqrt(d*x + c - 1) - 4*c^2 + 8*(c^7*d - 3*c^5*d + 3*c^3*d - c*d)*x
+ 1)/(a*b^2*d^8*x^8 + 8*a*b^2*c*d^7*x^7 + 4*(7*c^2*d^6 - d^6)*a*b^2*x^6 + 8
*(7*c^3*d^5 - 3*c*d^5)*a*b^2*x^5 + 2*(35*c^4*d^4 - 30*c^2*d^4 + 3*d^4)*a*b^
2*x^4 + 8*(7*c^5*d^3 - 10*c^3*d^3 + 3*c*d^3)*a*b^2*x^3 + 4*(7*c^6*d^2 - 15*
c^4*d^2 + 9*c^2*d^2 - d^2)*a*b^2*x^2 + (a*b^2*d^4*x^4 + 4*a*b^2*c*d^3*x^3 +
 6*a*b^2*c^2*d^2*x^2 + 4*a*b^2*c^3*d*x + a*b^2*c^4)*(d*x + c + 1)^2*(d*x +
c - 1)^2 + 8*(c^7*d - 3*c^5*d + 3*c^3*d - c*d)*a*b^2*x + 4*(a*b^2*d^5*x^5 +
 5*a*b^2*c*d^4*x^4 + (10*c^2*d^3 - d^3)*a*b^2*x^3 + (10*c^3*d^2 - 3*c*d^2)*
a*b^2*x^2 + (5*c^4*d - 3*c^2*d)*a*b^2*x + (c^5 - c^3)*a*b^2)*(d*x + c + 1)^
(3/2)*(d*x + c - 1)^(3/2) + (c^8 - 4*c^6 + 6*c^4 - 4*c^2 + 1)*a*b^2 + 6*(a*
b^2*d^6*x^6 + 6*a*b^2*c*d^5*x^5 + (15*c^2*d^4 - 2*d^4)*a*b^2*x^4 + 4*(5*c^3
*d^3 - 2*c*d^3)*a*b^2*x^3 + (15*c^4*d^2 - 12*c^2*d^2 + d^2)*a*b^2*x^2 + 2*(
3*c^5*d - 4*c^3*d + c*d)*a*b^2*x + (c^6 - 2*c^4 + c^2)*a*b^2)*(d*x + c + 1)
*(d*x + c - 1) + 4*(a*b^2*d^7*x^7 + 7*a*b^2*c*d^6*x^6 + 3*(7*c^2*d^5 - d^5)
*a*b^2*x^5 + 5*(7*c^3*d^4 - 3*c*d^4)*a*b^2*x^4 + (35*c^4*d^3 - 30*c^2*d^3 +
 3*d^3)*a*b^2*x^3 + 3*(7*c^5*d^2 - 10*c^3*d^2 + 3*c*d^2)*a*b^2*x^2 + (7*c^6
*d - 15*c^4*d + 9*c^2*d - d)*a*b^2*x + (c^7 - 3*c^5 + 3*c^3 - c)*a*b^2)*sqr
t(d*x + c + 1)*sqrt(d*x + c - 1) + (b^3*d^8*x^8 + 8*b^3*c*d^7*x^7 + 4*(7*c^
2*d^6 - d^6)*b^3*x^6 + 8*(7*c^3*d^5 - 3*c*d^5)*b^3*x^5 + 2*(35*c^4*d^4 - 30
*c^2*d^4 + 3*d^4)*b^3*x^4 + 8*(7*c^5*d^3 - 10*c^3*d^3 + 3*c*d^3)*b^3*x^3 +
4*(7*c^6*d^2 - 15*c^4*d^2 + 9*c^2*d^2 - d^2)*b^3*x^2 + (b^3*d^4*x^4 + 4*b^3
*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*(d*x + c + 1)^2*(
d*x + c - 1)^2 + 8*(c^7*d - 3*c^5*d + 3*c^3*d - c*d)*b^3*x + 4*(b^3*d^5*x^5
 + 5*b^3*c*d^4*x^4 + (10*c^2*d^3 - d^3)*b^3*x^3 + (10*c^3*d^2 - 3*c*d^2)*b^
3*x^2 + (5*c^4*d - 3*c^2*d)*b^3*x + (c^5 - c^3)*b^3)*(d*x + c + 1)^(3/2)*(d
*x + c - 1)^(3/2) + (c^8 - 4*c^6 + 6*c^4 - 4*c^2 + 1)*b^3 + 6*(b^3*d^6*x^6
+ 6*b^3*c*d^5*x^5 + (15*c^2*d^4 - 2*d^4)*b^3*x^4 + 4*(5*c^3*d^3 - 2*c*d^3)*
b^3*x^3 + (15*c^4*d^2 - 12*c^2*d^2 + d^2)*b^3*x^2 + 2*(3*c^5*d - 4*c^3*d +
c*d)*b^3*x + (c^6 - 2*c^4 + c^2)*b^3)*(d*x + c + 1)*(d*x + c - 1) + 4*(b^3*
d^7*x^7 + 7*b^3*c*d^6*x^6 + 3*(7*c^2*d^5 - d^5)*b^3*x^5 + 5*(7*c^3*d^4 - 3*
c*d^4)*b^3*x^4 + (35*c^4*d^3 - 30*c^2*d^3 + 3*d^3)*b^3*x^3 + 3*(7*c^5*d^2 -
 10*c^3*d^2 + 3*c*d^2)*b^3*x^2 + (7*c^6*d - 15*c^4*d + 9*c^2*d - d)*b^3*x +
 (c^7 - 3*c^5 + 3*c^3 - c)*b^3)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*
x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(c + d x \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-3), x)
```

3.147 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3} dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^3},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^3,x)/e
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3),x]
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^3), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^3} dx, x, c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^3} dx, x, c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.74, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3),x]
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^3), x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^3dex + a^3ce + \left(b^3dex + b^3ce\right)\text{arcosh}\left(dx+c\right)^3 + 3\left(ab^2dex + ab^2ce\right)\text{arcosh}\left(dx+c\right)^2 + 3\left(a^2bdex + \right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
[Out] integral(1/(a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arccosh(d*x + c)^3
+ 3*(a*b^2*d*e*x + a*b^2*c*e)*arccosh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c
*e)*arccosh(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(b \operatorname{arcosh}(dx + c) + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\,(a + b \operatorname{arccosh}(dx + c))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e + d\,e\,x)\,\left(a + b \operatorname{acosh}(c + d\,x)\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^3),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^3c + a^3dx + 3a^2bc \operatorname{acosh}(c+dx) + 3a^2bdx \operatorname{acosh}(c+dx) + 3ab^2c \operatorname{acosh}^2(c+dx) + 3ab^2dx \operatorname{acosh}^2(c+dx) + b^3c \operatorname{acosh}^3(c+dx) + b^3dx \operatorname{acosh}^3(c+dx)}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**3,x)
[Out] Integral(1/(a**3*c + a**3*d*x + 3*a**2*b*c*acosh(c + d*x) + 3*a**2*b*d*x*ac
osh(c + d*x) + 3*a*b**2*c*acosh(c + d*x)**2 + 3*a*b**2*d*x*acosh(c + d*x)**
2 + b**3*c*acosh(c + d*x)**3 + b**3*d*x*acosh(c + d*x)**3), x)/e
```

3.148 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=431

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{48b^4d}+\frac{27e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{32b^4d}+\frac{125e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(c+\right.}{b}\right.}{96b^4d}$$

```
[Out] 2/3*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^2-5/6*e^4*(d*x+c)^5/b^2/d/(a+b
*arccosh(d*x+c))^2+1/48*e^4*Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^4/d+27/
32*e^4*Chi(3*(a+b*arccosh(d*x+c))/b)*cosh(3*a/b)/b^4/d+125/96*e^4*Chi(5*(a+
b*arccosh(d*x+c))/b)*cosh(5*a/b)/b^4/d-1/48*e^4*Shi((a+b*arccosh(d*x+c))/b)
*sinh(a/b)/b^4/d-27/32*e^4*Shi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a/b)/b^4/d-
125/96*e^4*Shi(5*(a+b*arccosh(d*x+c))/b)*sinh(5*a/b)/b^4/d-1/3*e^4*(d*x+c)^
4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^3+2*e^4*(d*x+c)^
2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))-25/6*e^4*(d*x+
c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))
```

Rubi [A] time = 1.10, antiderivative size = 427, normalized size of antiderivative = 0.99, number of steps used = 24, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5668, 5775, 5666, 3303, 3298, 3301}

$$\frac{e^4\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{48b^4d}+\frac{27e^4\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{32b^4d}+\frac{125e^4\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5a}{b}\right.}{96b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -(e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (2*e^4*(c + d*x)^3)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2)
- (5*e^4*(c + d*x)^5)/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (2*e^4*Sqrt[-1
 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b^3*d*(a + b*ArcCosh[c + d*x]))
 - (25*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(6*b^3*d*(a +
b*ArcCosh[c + d*x])) + (e^4*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])
/(48*b^4*d) + (27*e^4*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c + d*
x]])/(32*b^4*d) + (125*e^4*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c
 + d*x]])/(96*b^4*d) - (e^4*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])
/(48*b^4*d) - (27*e^4*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c + d*
x]])/(32*b^4*d) - (125*e^4*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c
 + d*x]])/(96*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{\left(4e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}\right.}\right.}{3bd}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{6b}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{6b}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{6b}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{6b}$$

$$= -\frac{e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{2e^4(c+dx)^3}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{6b}$$

Mathematica [A] time = 2.01, size = 424, normalized size = 0.98

$$e^4\left(-\frac{32b^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{16b^2\left(4(c+dx)^3-5(c+dx)^5\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + 384\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) - \sinh\left(\frac{a}{b}\right)\text{Sh}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^4*((-32*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(a + b*Arc
Cosh[c + d*x])^3 + (16*b^2*(4*(c + d*x)^3 - 5*(c + d*x)^5))/(a + b*ArcCosh[
c + d*x])^2 - (16*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-12*(c + d*x)^2 +
 25*(c + d*x)^4))/(a + b*ArcCosh[c + d*x]) + 384*(Cosh[a/b]*CoshIntegral[a/
b + ArcCosh[c + d*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]]) - 5
44*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*CoshIn
tegral[3*(a/b + ArcCosh[c + d*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh
[c + d*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])]) + 125*
(10*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]] + 5*Cosh[(3*a)/b]*CoshIn
tegral[3*(a/b + ArcCosh[c + d*x])] + Cosh[(5*a)/b]*CoshIntegral[5*(a/b + Ar
cCosh[c + d*x])] - 10*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] - 5*Si
nh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])] - Sinh[(5*a)/b]*SinhIn
tegral[5*(a/b + ArcCosh[c + d*x])])))/(96*b^4*d)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4e^4x^4 + 4\,cd^3e^4x^3 + 6\,c^2d^2e^4x^2 + 4\,c^3de^4x + c^4e^4}{b^4\,\text{arcosh}\,(dx+c)^4 + 4\,ab^3\,\text{arcosh}\,(dx+c)^3 + 6\,a^2b^2\,\text{arcosh}\,(dx+c)^2 + 4\,a^3b\,\text{arcosh}\,(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
 + c^4*e^4)/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^
2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\,\text{arcosh}\,(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^4, x)
```

maple [B] time = 0.47, size = 1375, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/192*(-16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+12*(d*x+c)^2*(d*x
+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+16*(d*x+c)^5-20
*(d*x+c)^3+5*d*x+5*c)*e^4*(25*b^2*arccosh(d*x+c)^2+50*a*b*arccosh(d*x+c)-5*
arccosh(d*x+c)*b^2+25*a^2-5*a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*ar
ccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-125/192*e^4/b^4*exp(5*a/b)*Ei(1,
5*arccosh(d*x+c)+5*a/b)+1/64*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^4*(9*b^2*arccosh(d
*x+c)^2+18*a*b*arccosh(d*x+c)-3*arccosh(d*x+c)*b^2+9*a^2-3*a*b+2*b^2)/b^3/(
b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-2
7/64*e^4/b^4*exp(3*a/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/96*(-(d*x+c-1)^(1/2)
*(d*x+c+1)^(1/2)+d*x+c)*e^4*(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)-arcc
osh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x
+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/96*e^4/b^4*exp(a/b)*Ei(1,arccosh(d*x+c)
+a/b)-1/48/b*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^3-1/96/b^2*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+
c))^2-1/96/b^3*e^4*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x
+c))-1/96/b^4*e^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/32/b*e^4*(4*(d*x+c)
^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d
*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^3-3/64/b^2*e^4*(4*(d*x+c)^3-3*d*x-3*c+4
*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))
/(a+b*arccosh(d*x+c))^2-9/64/b^3*e^4*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(
d*x+c))-27/64/b^4*e^4*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b)-1/96/b*e^4*
(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+5*d
*x+5*c-12*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2))/(a+b*arccosh(d*x+c))^3-5/192/b^2*e^4*(16*(d*x+c)^5-20*(d*x+c)^3+1
6*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c
))^2-25/192/b^3*e^4*(16*(d*x+c)^5-20*(d*x+c)^3+16*(d*x+c)^4*(d*x+c-1)^(1/2)
```

```
*(d*x+c+1)^(1/2)+5*d*x+5*c-12*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*
x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-125/192/b^4*e^4*exp(-5*a
/b)*Ei(1,-5*arccosh(d*x+c)-5*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{acosh}\,(c+d\,x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^4+4a^3b\,\mathrm{acosh}\,(c+dx)+6a^2b^2\,\mathrm{acosh}^2\,(c+dx)+4ab^3\,\mathrm{acosh}^3\,(c+dx)+b^4\,\mathrm{acosh}^4\,(c+dx)}\,dx+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] e**4*(Integral(c**4/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c +
 d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integ
ral(d**4*x**4/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(4*
c*d**3*x**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2
 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(6*c*
*2*d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**
2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(4*c
**3*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4
*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

3.149 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=360

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-$$

```
[Out] 1/2*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^2-2/3*e^3*(d*x+c)^4/b^2/d/(a+b
*arccosh(d*x+c))^2+1/3*e^3*Chi(2*(a+b*arccosh(d*x+c))/b)*cosh(2*a/b)/b^4/d+
4/3*e^3*Chi(4*(a+b*arccosh(d*x+c))/b)*cosh(4*a/b)/b^4/d-1/3*e^3*Shi(2*(a+b*
arccosh(d*x+c))/b)*sinh(2*a/b)/b^4/d-4/3*e^3*Shi(4*(a+b*arccosh(d*x+c))/b)*
sinh(4*a/b)/b^4/d-1/3*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+
b*arccosh(d*x+c))^3+e^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*
arccosh(d*x+c))-8/3*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+
b*arccosh(d*x+c))
```

Rubi [A] time = 0.89, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5668, 5775, 5666, 3303, 3298, 3301}

$$\frac{e^3\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{3b^4d}+\frac{4e^3\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(c+dx)\right)}{3b^4d}-\frac{e^3\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{3b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -(e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (e^3*(c + d*x)^2)/(2*b^2*d*(a + b*ArcCosh[c + d*x])^2) -
(2*e^3*(c + d*x)^4)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (e^3*Sqrt[-1 + c
 + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b^3*d*(a + b*ArcCosh[c + d*x])) - (8*
e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b^3*d*(a + b*ArcCo
sh[c + d*x])) + (e^3*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c + d*x
]])/(3*b^4*d) + (4*e^3*Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcCosh[c + d
*x]])/(3*b^4*d) - (e^3*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c + d
*x]])/(3*b^4*d) - (4*e^3*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c +
 d*x]])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{e^3\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^3}\,d\right.}{bd}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(}$$

$$= -\frac{e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^3(c+dx)^2}{2b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(}$$

Mathematica [A] time = 1.30, size = 330, normalized size = 0.92

$$e^3\left(-\frac{2b^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{b^2\left(3(c+dx)^2-4(c+dx)^4\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} - 30\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^3*((-2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^3 + (b^2*(3*(c + d*x)^2 - 4*(c + d*x)^4))/(a + b*ArcCosh[c +
d*x])^2 - (2*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-3*(c + d*x) + 8*(c +
d*x)^3))/(a + b*ArcCosh[c + d*x]) + 6*Log[a + b*ArcCosh[c + d*x]] - 30*(Cos
h[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Log[a + b*ArcCosh[c +
 d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])]) + 8*(4*Cos
h[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c + d*x])] + Cosh[(4*a)/b]*CoshInt
egral[4*(a/b + ArcCosh[c + d*x])] + 3*Log[a + b*ArcCosh[c + d*x]] - 4*Sinh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c + d*x])] - Sinh[(4*a)/b]*SinhInteg
ral[4*(a/b + ArcCosh[c + d*x])])))/(6*b^4*d)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}{b^4\,\text{arcosh}\,(dx+c)^4 + 4\,ab^3\,\text{arcosh}\,(dx+c)^3 + 6\,a^2b^2\,\text{arcosh}\,(dx+c)^2 + 4\,a^3b\,\text{arcosh}\,(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3)/(b^4*arc
cosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)^2
 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(b \operatorname{arcosh}(dx + c) + a)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^4, x)
```

maple [B] time = 0.42, size = 860, normalized size = 2.39

$$\frac{\left(-8(dx+c)^3\sqrt{dx+c-1}\,\sqrt{dx+c+1}+4\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+8(dx+c)^4-8(dx+c)^2+1\right)e^3\left(8b^2\operatorname{arccosh}(dx+c)^2+16ab\operatorname{arccosh}(dx+c)-2\operatorname{arccosh}(dx\right.}{48b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/48*(-8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+4*(d*x+c+1)^(1/2)*(
d*x+c-1)^(1/2)*(d*x+c)+8*(d*x+c)^4-8*(d*x+c)^2+1)*e^3*(8*b^2*arccosh(d*x+c)
^2+16*a*b*arccosh(d*x+c)-2*arccosh(d*x+c)*b^2+8*a^2-2*a*b+b^2)/b^3/(b^3*arc
cosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-2/3*e^3/
b^4*exp(4*a/b)*Ei(1,4*arccosh(d*x+c)+4*a/b)+1/24*(-2*(d*x+c+1)^(1/2)*(d*x+c
-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e^3*(2*b^2*arccosh(d*x+c)^2+4*a*b*arccosh(
d*x+c)-arccosh(d*x+c)*b^2+2*a^2-a*b+b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*
arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/6*e^3/b^4*exp(2*a/b)*Ei(1,2*
arccosh(d*x+c)+2*a/b)-1/24/b*e^3*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)
^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^3-1/24/b^2*e^3*(2*(d*x+c)^2-1+2*(d*x+c
+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^2-1/12/b^3*e^3*(2*(
d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))-
1/6/b^4*e^3*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b)-1/48/b*e^3*(8*(d*x+c)
^4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))^3-1/24/b^2*e^3*(8*(d*x+c)
^4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2
)*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))^2-1/6/b^3*e^3*(8*(d*x+c)^
4-8*(d*x+c)^2+8*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-4*(d*x+c+1)^(1/2)
*(d*x+c-1)^(1/2)*(d*x+c)+1)/(a+b*arccosh(d*x+c))-2/3/b^4*e^3*exp(-4*a/b)*Ei
(1,-4*arccosh(d*x+c)-4*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^3}{(a + b\operatorname{acosh}(c + d\,x))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{}{a^4 +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] e**3*(Integral(c**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c +
 d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integ
ral(d**3*x**3/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(3*
c*d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2
 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(3*c*
*2*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4*
a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

3.150 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=352

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{24b^4d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{8b^4d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{24b^4d}-\frac{9e^2}{}$$

```
[Out] 1/3*e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^2-1/2*e^2*(d*x+c)^3/b^2/d/(a+b*a
rccosh(d*x+c))^2+1/24*e^2*Chi((a+b*arccosh(d*x+c))/b)*cosh(a/b)/b^4/d+9/8*e
^2*Chi(3*(a+b*arccosh(d*x+c))/b)*cosh(3*a/b)/b^4/d-1/24*e^2*Shi((a+b*arccos
h(d*x+c))/b)*sinh(a/b)/b^4/d-9/8*e^2*Shi(3*(a+b*arccosh(d*x+c))/b)*sinh(3*a
/b)/b^4/d-1/3*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccos
h(d*x+c))^3+1/3*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+
c))-3/2*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*
x+c))
```

Rubi [A] time = 0.96, antiderivative size = 348, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5866, 12, 5668, 5775, 5666, 3303, 3298, 3301, 5656, 5781}

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{24b^4d}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(c+dx)\right)}{8b^4d}-\frac{e^2\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}\right)}{24b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -(e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCo
sh[c + d*x])^3) + (e^2*(c + d*x))/(3*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (e
^2*(c + d*x)^3)/(2*b^2*d*(a + b*ArcCosh[c + d*x])^2) + (e^2*Sqrt[-1 + c + d
*x]*Sqrt[1 + c + d*x])/(3*b^3*d*(a + b*ArcCosh[c + d*x])) - (3*e^2*Sqrt[-1
+ c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(2*b^3*d*(a + b*ArcCosh[c + d*x])
) + (e^2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]])/(24*b^4*d) + (9*e^
2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(8*b^4*d) - (e^
2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(24*b^4*d) - (9*e^2*Sinh[
(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c + d*x]])/(8*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{\left(2e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh\right.}\right.}{3bd}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{2b}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{2b}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{2b}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{2b}$$

$$= -\frac{e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e^2(c+dx)}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{2b}$$

Mathematica [A] time = 1.33, size = 272, normalized size = 0.77

$$\frac{e^2\left(-\frac{8b^3\sqrt{c+dx-1}\,(c+dx)^2\sqrt{c+dx+1}}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{4b^2\left(2(c+dx)-3(c+dx)^3\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + 27\left(3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Ch}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e^2*((-8*b^3*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(a + b*ArcC
osh[c + d*x])^3 + (4*b^2*(2*(c + d*x) - 3*(c + d*x)^3))/(a + b*ArcCosh[c +
d*x])^2 - (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(-2 + 9*(c + d*x)^2))/(
a + b*ArcCosh[c + d*x]) - 80*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c + d*x]]
 + 80*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] + 27*(3*Cosh[a/b]*Cosh
Integral[a/b + ArcCosh[c + d*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcC
osh[c + d*x])] - 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]] - Sinh[(3
*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c + d*x])])))/(24*b^4*d)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2e^2x^2+2\,cde^2x+c^2e^2}{b^4\,\text{arcosh}\,(dx+c)^4+4\,ab^3\,\text{arcosh}\,(dx+c)^3+6\,a^2b^2\,\text{arcosh}\,(dx+c)^2+4\,a^3b\,\text{arcosh}\,(dx+c)+a^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)/(b^4*arccosh(d*x + c)^4 + 4*
a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d
*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^2}{(b\operatorname{arcosh}(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^4, x)
```

maple [B] time = 0.26, size = 777, normalized size = 2.21

$$\frac{\left(-4(dx+c)^2\sqrt{dx+c-1}\ \sqrt{dx+c+1}+\sqrt{dx+c-1}\ \sqrt{dx+c+1}+4(dx+c)^3-3dx-3c\right)e^2\left(9b^2\operatorname{arccosh}(dx+c)^2+18ab\operatorname{arccosh}(dx+c)-3\operatorname{arccosh}(dx+c)b^2+9a^2-3ab+\right.}{48b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/48*(-4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)+4*(d*x+c)^3-3*d*x-3*c)*e^2*(9*b^2*arccosh(d*x+c)^2+18*a*b*arcc
osh(d*x+c)-3*arccosh(d*x+c)*b^2+9*a^2-3*a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^
3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-9/16*e^2/b^4*exp(3*a
/b)*Ei(1,3*arccosh(d*x+c)+3*a/b)+1/48*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x
+c)*e^2*(b^2*arccosh(d*x+c)^2+2*a*b*arccosh(d*x+c)-arccosh(d*x+c)*b^2+a^2-a
*b+2*b^2)/b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccos
h(d*x+c)+a^3)-1/48*e^2/b^4*exp(a/b)*Ei(1,arccosh(d*x+c)+a/b)-1/24/b*e^2*(d*
x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^3-1/48/b^2*e^2*(d
*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))^2-1/48/b^3*e^2*(
d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-1/48/b^4*e^2*ex
p(-a/b)*Ei(1,-arccosh(d*x+c)-a/b)-1/24/b*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+
c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*
arccosh(d*x+c))^3-1/16/b^2*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c)
)^2-3/16/b^3*e^2*(4*(d*x+c)^3-3*d*x-3*c+4*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2)-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arccosh(d*x+c))-9/16/b^4*e^2
*exp(-3*a/b)*Ei(1,-3*arccosh(d*x+c)-3*a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^4 + 4a^3b\operatorname{acosh}(c+dx) + 6a^2b^2\operatorname{acosh}^2(c+dx) + 4ab^3\operatorname{acosh}^3(c+dx) + b^4\operatorname{acosh}^4(c+dx)}\,dx + \int \frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] e**2*(Integral(c**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c +
 d*x)**2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integ
ral(d**2*x**2/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(2*
c*d*x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4*a
*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

3.151 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=218

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)}{3b^4d}-\frac{2e\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)}{3b^3d\left(a+b\cosh^{-1}(c+dx)\right)}-$$

```
[Out] 1/6*e/b^2/d/(a+b*arccosh(d*x+c))^2-1/3*e*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c
))^2+2/3*e*Chi(2*(a+b*arccosh(d*x+c))/b)*cosh(2*a/b)/b^4/d-2/3*e*Shi(2*(a+b
*arccosh(d*x+c))/b)*sinh(2*a/b)/b^4/d-1/3*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+
1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^3-2/3*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)/b^3/d/(a+b*arccosh(d*x+c))
```

Rubi [A] time = 0.51, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5866, 12, 5668, 5775, 5666, 3303, 3298, 3301, 5676}

$$\frac{2e\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{3b^4d}-\frac{2e\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(c+dx)\right)}{3b^4d}-\frac{e(c+dx)^2}{3b^2d\left(a+b\cosh^{-1}(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] -(e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c
 + d*x])^3) + e/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (e*(c + d*x)^2)/(3*b
^2*d*(a + b*ArcCosh[c + d*x])^2) - (2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1
 + c + d*x])/(3*b^3*d*(a + b*ArcCosh[c + d*x])) + (2*e*Cosh[(2*a)/b]*CoshIn
tegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(3*b^4*d) - (2*e*Sinh[(2*a)/b]*SinhIn
tegral[(2*a)/b + 2*ArcCosh[c + d*x]])/(3*b^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{e\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^3}\,dx,\right.}{3bd}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(a}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(a}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(a}$$

$$= -\frac{e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{e}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{}{3b^2d\,(a}$$

Mathematica [A] time = 1.03, size = 195, normalized size = 0.89

$$\frac{e\left(-\frac{2b^3\sqrt{c+dx-1}\,(c+dx)\sqrt{c+dx+1}}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{b^2\left(1-2(c+dx)^2\right)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} + 4\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)\right) - \sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}\right.\right.\right.\right.}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (e*((-2*b^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[
c + d*x])^3 + (b^2*(1 - 2*(c + d*x)^2))/(a + b*ArcCosh[c + d*x])^2 - (4*b*S
qrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x]) - 4
*Log[a + b*ArcCosh[c + d*x]] + 4*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCo
sh[c + d*x])] + Log[a + b*ArcCosh[c + d*x]] - Sinh[(2*a)/b]*SinhIntegral[2*
(a/b + ArcCosh[c + d*x])]))))/(6*b^4*d)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dex+ce}{b^4\operatorname{arcosh}(dx+c)^4 + 4\,ab^3\operatorname{arcosh}(dx+c)^3 + 6\,a^2b^2\operatorname{arcosh}(dx+c)^2 + 4\,a^3b\operatorname{arcosh}(dx+c) + a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((d*e*x + c*e)/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3
 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex+ce}{\left(b \operatorname{arcosh}\left(dx+c\right)+a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^4, x)
```

maple [A] time = 0.07, size = 353, normalized size = 1.62

$$\frac{\frac{\left(-2\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,(dx+c)+2(dx+c)^{2}-1\right)e\left(2b^{2}\operatorname{arccosh}(dx+c)^{2}+4ab\operatorname{arccosh}(dx+c)-\operatorname{arccosh}(dx+c)b^{2}+2a^{2}-ab+b^{2}\right)}{12b^{3}\left(b^{3}\operatorname{arccosh}(dx+c)^{3}+3a\,b^{2}\operatorname{arccosh}(dx+c)^{2}+3a^{2}b\operatorname{arccosh}(dx+c)+a^{3}\right)}-\frac{e\,e^{\frac{2a}{b}}\operatorname{Ei}\left(1,2\operatorname{arccosh}(d\right.}{3b^{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/12*(-2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c)+2*(d*x+c)^2-1)*e*(2*b
^2*arccosh(d*x+c)^2+4*a*b*arccosh(d*x+c)-arccosh(d*x+c)*b^2+2*a^2-a*b+b^2)/
b^3/(b^3*arccosh(d*x+c)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a
^3)-1/3*e/b^4*exp(2*a/b)*Ei(1,2*arccosh(d*x+c)+2*a/b)-1/12/b*e*(2*(d*x+c)^2
-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh(d*x+c))^3-1/12/b
^2*e*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+c))/(a+b*arccosh
(d*x+c))^2-1/6/b^3*e*(2*(d*x+c)^2-1+2*(d*x+c+1)^(1/2)*(d*x+c-1)^(1/2)*(d*x+
c))/(a+b*arccosh(d*x+c))-1/3/b^4*e*exp(-2*a/b)*Ei(1,-2*arccosh(d*x+c)-2*a/b
))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^{4}+4a^{3}b\operatorname{acosh}\left(c+dx\right)+6a^{2}b^{2}\operatorname{acosh}^{2}\left(c+dx\right)+4ab^{3}\operatorname{acosh}^{3}\left(c+dx\right)+b^{4}\operatorname{acosh}^{4}\left(c+dx\right)}\,dx+\int \frac{}{a^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] e*(Integral(c/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)*
*2 + 4*a*b**3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x) + Integral(d*
x/(a**4 + 4*a**3*b*acosh(c + d*x) + 6*a**2*b**2*acosh(c + d*x)**2 + 4*a*b**
3*acosh(c + d*x)**3 + b**4*acosh(c + d*x)**4), x))
```

3.152 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=174

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{6b^4d}-\frac{\sqrt{c+dx-1}\sqrt{c+dx+1}}{6b^3d\left(a+b\cosh^{-1}(c+dx)\right)}-\frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c\right.}$$

```
[Out] 1/6*(-d*x-c)/b^2/d/(a+b*arccosh(d*x+c))^2+1/6*Chi((a+b*arccosh(d*x+c))/b)*c
osh(a/b)/b^4/d-1/6*Shi((a+b*arccosh(d*x+c))/b)*sinh(a/b)/b^4/d-1/3*(d*x+c-1
)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^3-1/6*(d*x+c-1)^(1/2)*(d*x
+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))
```

Rubi [A] time = 0.45, antiderivative size = 170, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5864, 5656, 5775, 5781, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{6b^4d}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{6b^4d}-\frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2}-\frac{\sqrt{c+dx}}{6b^3d\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-4),x]
```

```
[Out] -(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c + d*x])^3)
- (c + d*x)/(6*b^2*d*(a + b*ArcCosh[c + d*x])^2) - (Sqrt[-1 + c + d*x]*Sqrt
[1 + c + d*x])/(6*b^3*d*(a + b*ArcCosh[c + d*x])) + (Cosh[a/b]*CoshIntegral
[a/b + ArcCosh[c + d*x]])/(6*b^4*d) - (Sinh[a/b]*SinhIntegral[a/b + ArcCosh
[c + d*x]])/(6*b^4*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^4}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^3}\,dx, x, c+\right.}{3bd}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+c+}}{6b^3d\left(a+b\right.}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+c+}}{6b^3d\left(a+b\right.}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+c+}}{6b^3d\left(a+b\right.}$$

$$= -\frac{\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^3} - \frac{c+dx}{6b^2d\left(a+b\cosh^{-1}(c+dx)\right)^2} - \frac{\sqrt{-1+c+}}{6b^3d\left(a+b\right.}$$

Mathematica [A] time = 0.67, size = 144, normalized size = 0.83

$$-\frac{\frac{2b^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{\left(a+b\cosh^{-1}(c+dx)\right)^3} + \frac{b^2(c+dx)}{\left(a+b\cosh^{-1}(c+dx)\right)^2} - \cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right) + \sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(c+\right.}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-4),x]
```

```
[Out] -1/6*((2*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(a + b*ArcCosh[c + d*x])
^3 + (b^2*(c + d*x))/(a + b*ArcCosh[c + d*x])^2 + (b*Sqrt[-1 + c + d*x]*Sqr
t[1 + c + d*x])/(a + b*ArcCosh[c + d*x]) - Cosh[a/b]*CoshIntegral[a/b + Arc
Cosh[c + d*x]] + Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c + d*x]])/(b^4*d)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^4 \operatorname{arcosh}(dx+c)^4 + 4\,ab^3 \operatorname{arcosh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arcosh}(dx+c)^2 + 4\,a^3b \operatorname{arcosh}(dx+c) + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral(1/(b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2
*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcosh}(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-4), x)
```

maple [A] time = 0.07, size = 295, normalized size = 1.70

$$\frac{\frac{\left(-\sqrt{dx+c-1}\,\sqrt{dx+c+1}+dx+c\right)\left(b^2\operatorname{arccosh}(dx+c)^2+2ab\operatorname{arccosh}(dx+c)-\operatorname{arccosh}(dx+c)b^2+a^2-ab+2b^2\right)}{12b^3\left(b^3\operatorname{arccosh}(dx+c)^3+3a\,b^2\operatorname{arccosh}(dx+c)^2+3a^2b\operatorname{arccosh}(dx+c)+a^3\right)}-\frac{e^{\frac{a}{b}}\operatorname{Ei}\left(1,\operatorname{arccosh}(dx+c)+\frac{a}{b}\right)}{12b^4}-\frac{dx+c+\sqrt{dx+\cdots}}{6b(a+b\operatorname{arc}\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] 1/d*(1/12*(-(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)+d*x+c)*(b^2*arccosh(d*x+c)^2+2*
a*b*arccosh(d*x+c)-arccosh(d*x+c)*b^2+a^2-a*b+2*b^2)/b^3/(b^3*arccosh(d*x+c
)^3+3*a*b^2*arccosh(d*x+c)^2+3*a^2*b*arccosh(d*x+c)+a^3)-1/12/b^4*exp(a/b)*
Ei(1,arccosh(d*x+c)+a/b)-1/6/b*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b
*arccosh(d*x+c))^3-1/12/b^2*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*ar
ccosh(d*x+c))^2-1/12/b^3*(d*x+c+(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2))/(a+b*arcco
sh(d*x+c))-1/12/b^4*exp(-a/b)*Ei(1,-arccosh(d*x+c)-a/b))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-4), x)
```

3.153 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$

Optimal. Leaf size=27

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^4},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^4,x)/e

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^4), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^4}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 14.55, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^4}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^4), x]

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^4dex + a^4ce + \left(b^4dex + b^4ce\right)\text{arcosh}\,(dx+c)^4 + 4\left(ab^3dex + ab^3ce\right)\text{arcosh}\,(dx+c)^3 + 6\left(a^2b^2dex + a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")

[Out] integral(1/(a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arccosh(d*x + c)^4 + 4*(a*b^3*d*e*x + a*b^3*c*e)*arccosh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b

```
^2*c*e)*arccosh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arccosh(d*x + c)),
 x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(b \operatorname{arcosh}(dx + c) + a)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\,(a + b \operatorname{arccosh}(dx + c))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(c\,e + d\,e\,x)\,(a + b \operatorname{acosh}(c + d\,x))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^4),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^4), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4c + a^4dx + 4a^3bc \operatorname{acosh}(c+dx) + 4a^3bdx \operatorname{acosh}(c+dx) + 6a^2b^2c \operatorname{acosh}^2(c+dx) + 6a^2b^2dx \operatorname{acosh}^2(c+dx) + 4ab^3c \operatorname{acosh}^3(c+dx) + 4ab^3dx \operatorname{acosh}^3(c+d}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**4,x)
[Out] Integral(1/(a**4*c + a**4*d*x + 4*a**3*b*c*acosh(c + d*x) + 4*a**3*b*d*x*ac
osh(c + d*x) + 6*a**2*b**2*c*acosh(c + d*x)**2 + 6*a**2*b**2*d*x*acosh(c +
d*x)**2 + 4*a*b**3*c*acosh(c + d*x)**3 + 4*a*b**3*d*x*acosh(c + d*x)**3 + b
**4*c*acosh(c + d*x)**4 + b**4*d*x*acosh(c + d*x)**4), x)/e
```

3.154 $\int (ce+dex)^4 \sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=361

$$-\frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}$$

```
[Out] -1/1600*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1
/2)*5^(1/2)*Pi^(1/2)/d-1/1600*e^4*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b
^(1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)-1/192*e^4*exp(3*a/b)*erf(3^(1
/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d-1/192*e^
4*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)
/d/exp(3*a/b)-1/32*e^4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(
1/2)*Pi^(1/2)/d-1/32*e^4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*P
i^(1/2)/d/exp(a/b)+1/5*e^4*(d*x+c)^5*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.95, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5866, 12, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^4e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^4e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^4e^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{320d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^4*(c + d*x)^5*Sqrt[a + b*ArcCosh[c + d*x]])/(5*d) - (Sqrt[b]*e^4*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d) - (Sqrt[b]*e^4*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(64*d) - (Sqrt[b]*e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCo
sh[c + d*x]])/Sqrt[b]])/(320*d) - (Sqrt[b]*e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*Arc
Cosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) - (Sqrt[b]*e^4*Sqrt[Pi/3]*Erfi[(Sqr
t[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d*E^((3*a)/b)) - (Sqrt[b]*
e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(320*d
*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^4\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^4x^4\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int x^4\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\,\text{co}}}\right.}{10d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\cosh^5(x)}{\sqrt{a+bx}}\,dx,x,\text{cosh}\right.}{10d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8\sqrt{a+bx}}+\frac{5\cosh(3}{16\sqrt{a+b}}\right.\right.}{}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx,x,\text{cosh}\right.}{160d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{(be^4)\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx,x,\text{cosh}\right.}{320d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{e^4\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b}\right.}{160d}$$

$$= \frac{e^4(c+dx)^5\sqrt{a+b\cosh^{-1}(c+dx)}}{5d} - \frac{\sqrt{b}\,e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right.}{32d}$$

Mathematica [A] time = 0.74, size = 342, normalized size = 0.95

$$\frac{e^4e^{-\frac{5a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(150e^{\frac{6a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+3\sqrt{5}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^4*Sqrt[a + b*ArcCosh[c + d*x]]*(150*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c
+ d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + 3*Sqrt[5]*Sqrt[a/b + ArcCo
sh[c + d*x]]*Gamma[3/2, (-5*(a + b*ArcCosh[c + d*x]))/b] + 25*Sqrt[3]*E^((2
*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x])
)/b] + 150*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*Arc
Cosh[c + d*x])/b)] + 25*Sqrt[3]*E^((8*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])
/b)]*Gamma[3/2, (3*(a + b*ArcCosh[c + d*x]))/b] + 3*Sqrt[5]*E^((10*a)/b)*Sq
rt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (5*(a + b*ArcCosh[c + d*x]))/b
]))/(2400*d*E^((5*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^4 \sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^4\sqrt{b\,\mathrm{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4*sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^4\,\sqrt{a+b\,\mathrm{acosh}\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^4*(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int c^4\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int d^4x^4\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int 4cd^3x^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4*sqr
t(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3*sqrt(a + b*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2*sqrt(a + b*acosh(c + d*x)), x) + Integ
ral(4*c**3*d*x*sqrt(a + b*acosh(c + d*x)), x))
```

3.155 $\int (ce+dex)^3 \sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=272

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^3 e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^3 e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}\,e^3 e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}$$

```
[Out] -1/64*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2
)*2^(1/2)*Pi^(1/2)/d-1/64*e^3*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-1/256*e^3*exp(4*a/b)*erf(2*(a+b*a
rccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/256*e^3*erfi(2*(a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(4*a/b)-3/32*e^3*(a+b*arcc
osh(d*x+c))^(1/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.79, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5866, 12, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}\,e^3 e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\,e^3 e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\pi}\sqrt{b}\,e^3 e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (-3*e^3*Sqrt[a + b*ArcCosh[c + d*x]])/(32*d) + (e^3*(c + d*x)^4*Sqrt[a + b*
ArcCosh[c + d*x]])/(4*d) - (Sqrt[b]*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d) - (Sqrt[b]*e^3*E^((2*a)/b)*Sqrt[Pi
/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*d) - (Sqrt[b]*
e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d*E^((4*a
)/b)) - (Sqrt[b]*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])
/Sqrt[b]])/(32*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^3x^3\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh^{-1}(x)}} \cdots\right.}{8d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{\sqrt{a+bx}}\,dx,x,\cosh \cdots\right.}{8d}$$

$$= \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{\left(be^3\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}} + \frac{\cosh(2x)}{2\sqrt{a+bx}} \cdots\right.\right.}{8 \cdots}$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \cdots$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \cdots$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \cdots$$

$$= -\frac{3e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} + \frac{e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \cdots$$

Mathematica [A] time = 0.54, size = 223, normalized size = 0.82

$$\frac{e^3e^{-\frac{4a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+4\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\cdots\right.\right.}{128d\sqrt{-\frac{\left(a+b\cosh \cdots\right.}{\cdots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^3*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2,
(-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcCosh
[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[-
((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[c + d
*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(128*d
*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 \sqrt{a + b\,\mathrm{arccosh}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x)
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 \sqrt{b\,\mathrm{arcosh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^3*sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^3 \sqrt{a + b\,\mathrm{acosh}\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(1/2),x)
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3 \left(\int c^3 \sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx + \int d^3x^3 \sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx + \int 3cd^2x^2 \sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\,dx + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(1/2),x)
[Out] e**3*(Integral(c**3*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3*sqr
t(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2*sqrt(a + b*acosh(c + d
*x)), x) + Integral(3*c**2*d*x*sqrt(a + b*acosh(c + d*x)), x))
```

3.156 $\int (ce+dex)^2\sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}-\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

```
[Out] -1/144*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/
2)*3^(1/2)*Pi^(1/2)/d-1/144*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-1/16*e^2*exp(a/b)*erf((a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/16*e^2*erfi((a+b*arccosh(d*
x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+1/3*e^2*(d*x+c)^3*(a+b*arc
cosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.76, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5866, 12, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{48d}-\frac{\sqrt{\pi}\sqrt{b}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^2*(c + d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/(3*d) - (Sqrt[b]*e^2*E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d) - (Sqrt[b]*e^2*
E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(48*d) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])
/(16*d*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(48*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int e^2x^2\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh}}\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{\cosh^3(x)}{\sqrt{a+bx}}\,dx,x,\cosh\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4\sqrt{a+bx}}+\frac{\cosh(3x}{4\sqrt{a+bx}}\right.\right.}{6d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx,x,\cosh\right.}{24d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{(be^2)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx,x,\cosh\right.}{48d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{e^2\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b}\right.}{24d}$$

$$= \frac{e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{\sqrt{b}\,e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right.}{16d}$$

Mathematica [A] time = 0.51, size = 237, normalized size = 0.97

$$\frac{e^2e^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},\right.\right.}{72d\sqrt{-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^2*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c +
d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcCosh[c
 + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*Sqrt[a
/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[3]*
E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*ArcCos
h[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2 \sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2 \sqrt{b\,\mathrm{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^2\,\sqrt{a+b\,\mathrm{acosh}\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int c^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int d^2x^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int 2cdx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2*sqr
t(a + b*acosh(c + d*x)), x) + Integral(2*c*d*x*sqrt(a + b*acosh(c + d*x)),
x))
```

3.157 $\int (ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=164

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}$$

```
[Out] -1/32*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*
2^(1/2)*Pi^(1/2)/d-1/32*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*
b^(1/2)*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-1/4*e*(a+b*arccosh(d*x+c))^(1/2)/d+1/
2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.58, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5866, 12, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(e*Sqrt[a + b*ArcCosh[c + d*x]])/(4*d) + (e*(c + d*x)^2*Sqrt[a + b*ArcCosh
[c + d*x]])/(2*d) - (Sqrt[b]*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*d) - (Sqrt[b]*e*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int ex\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e\,\text{Subst}\left(\int x\sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$
$$= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\sqrt{1+x}\sqrt{a+b\cosh^{-1}}}\right.}{4d}$$
$$= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \frac{\cosh^2(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(\right.}{4d}$$
$$= \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}}+\frac{\cosh(2x)}{2\sqrt{a+bx}}\right) d\right.}{4d}$$
$$= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{S}}{}$$
$$= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{(be)\,\text{S}}{}$$
$$= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{e\,\text{Subs}}{}$$
$$= -\frac{e\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} + \frac{e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{2d} - \frac{\sqrt{b}\,ee}{}$$

Mathematica [B] time = 2.52, size = 437, normalized size = 2.66

$$\frac{e\left(8\sqrt{\pi}\sqrt{b}\,c\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c}}{\sqrt{b}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e*(-32*c*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[a + b*ArcCosh[c +
 d*x]]*Cosh[2*ArcCosh[c + d*x]] + 8*Sqrt[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(S
qrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] + (16*c*Sqrt[a + b*ArcCosh[c
+ d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCos
h[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCo
sh[c + d*x])/b)]))/E^(a/b) - 8*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c
 + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[b]*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c
 + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - Sqrt[b]*Sqrt[2*Pi]
*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[
(2*a)/b])))/(32*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (dex + ce) \sqrt{a + b \,\mathrm{arccosh}\,(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)\sqrt{b\,\mathrm{arcosh}\,(dx + c) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)\,\sqrt{a + b\,\mathrm{acosh}\,(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int c\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\, dx + \int dx\sqrt{a + b\,\mathrm{acosh}\,(c + dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c*sqrt(a + b*acosh(c + d*x)), x) + Integral(d*x*sqrt(a + b*acos
h(c + d*x)), x))
```

3.158 $\int \sqrt{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}$$

```
[Out] -1/4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d-1/
4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/d/exp(a/b)+(d*x
+c)*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.35, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5864, 5654, 5781, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}+\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] ((c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/d - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*d) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}(c+dx)}\,dx=\frac{\text{Subst}\left(\int \sqrt{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}-\frac{b\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c\right.}{2d}$$

$$=\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}-\frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{2d}$$

$$=\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}-\frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c+dx)\right)}{4d}-$$

$$=\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}-\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(c+dx}\right.}{2d}$$

$$=\frac{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{d}-\frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4d}-\frac{\sqrt{b}\,e^{-\frac{a}{b}}}{}$$

Mathematica [A] time = 0.23, size = 110, normalized size = 0.96

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c + d*
x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b
)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(a/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(d*x + c) + a), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \sqrt{a+b \,\mathrm{arccosh}(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arcosh}(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b \,\mathrm{acosh}(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b \,\mathrm{acosh}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c + d*x)), x)
```

3.159 $\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{c+dx},x\right)}{e}$$

[Out] Unintegrable((a+b*arccosh(d*x+c))^(1/2)/(d*x+c),x)/e

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x),x]

[Out] Defer[Subst][Defer[Int][Sqrt[a + b*ArcCosh[x]]/x, x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\cosh^{-1}(x)}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+b\cosh^{-1}(x)}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 2.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x),x]

[Out] Integrate[Sqrt[a + b*ArcCosh[c + d*x]]/(c*e + d*e*x), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\text{arcosh}\left(dx+c\right)+a}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate(sqrt(b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \operatorname{arccosh}(dx + c)}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
[Out] int((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \operatorname{arcosh}(dx + c) + a}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(1/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate(sqrt(b*arccosh(d*x + c) + a)/(d*e*x + c*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a + b \operatorname{acosh}(c + d\,x)}}{c\,e + d\,e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(1/2)/(c*e + d*e*x),x)
[Out] int((a + b*acosh(c + d*x))^(1/2)/(c*e + d*e*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a+b \operatorname{acosh}(c+dx)}}{c+dx}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(1/2)/(d*e*x+c*e),x)
[Out] Integral(sqrt(a + b*acosh(c + d*x))/(c + d*x), x)/e
```

3.160 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=374

$$-\frac{3\sqrt{\pi}\, b^{3/2}e^3 e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}-\frac{3\sqrt{\frac{\pi}{2}}\, b^{3/2}e^3 e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}\, b^{3/2}e^3 e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}$$

```
[Out] -3/32*e^3*(a+b*arccosh(d*x+c))^(3/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c
))^(3/2)/d-3/256*b^(3/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+3/256*b^(3/2)*e^3*erfi(2^(1/2)*(a+b*arccosh
(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/2048*b^(3/2)*e^3*ex
p(4*a/b)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/2048*b^(3/2
)*e^3*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)-9/64
*b*e^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d
-3/32*b*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^
(1/2)/d
```

Rubi [A] time = 1.41, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5866, 12, 5664, 5759, 5676, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\, b^{3/2}e^3 e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}-\frac{3\sqrt{\frac{\pi}{2}}\, b^{3/2}e^3 e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}+\frac{3\sqrt{\pi}\, b^{3/2}e^3 e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2048d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-9*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh
[c + d*x]])/(64*d) - (3*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d
*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(32*d) - (3*e^3*(a + b*ArcCosh[c + d*x])^
(3/2))/(32*d) + (e^3*(c + d*x)^4*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) - (3
*b^(3/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(2048*d) - (3*b^(3/2)*e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]])/(128*d) + (3*b^(3/2)*e^3*Sqrt[Pi]*Erfi[(2*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2048*d*E^((4*a)/b)) + (3*b^(3/2)*e
^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(128*d*
E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3 \left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{\left(3be^3\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+b\cosh}}{\sqrt{-1+x}}\right.}{8d}$$

$$= -\frac{3be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{32d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}$$

$$= -\frac{9be^3\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{64d}$$

Mathematica [A] time = 3.95, size = 558, normalized size = 1.49

$$e^3\left(\frac{ae^{-\frac{4a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{3}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+4\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+d}\right.}{128d\sqrt{-\frac{(}{}}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] e^3*((a*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[3/
2, (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqr
t[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[c
+ d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(12
8*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]) + (Sqrt[b]*((8*a +
 3*b)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b
```

```
] - Sinh[(4*a)/b]) + (8*a - 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) + 8*((4*a + 3*b)*Sqrt[2*Pi]*Er
fi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2
*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])
/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c
+ d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh[c +
 d*x]])) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(8*ArcCosh[c + d*x]*Cosh[
4*ArcCosh[c + d*x]] - 3*Sinh[4*ArcCosh[c + d*x]])))/(2048*d))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3 \left(a+b \operatorname{arccosh}(dx+c)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^3 (b \operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3 \left(\int ac^3 \sqrt{a+b \operatorname{acosh}(c+dx)}\, dx + \int ad^3x^3 \sqrt{a+b \operatorname{acosh}(c+dx)}\, dx + \int bc^3 \sqrt{a+b \operatorname{acosh}(c+dx)} \operatorname{acosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(a*c**3*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d**3*x**3
*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*c**3*sqrt(a + b*acosh(c + d*x)
)*acosh(c + d*x), x) + Integral(3*a*c*d**2*x**2*sqrt(a + b*acosh(c + d*x)),
 x) + Integral(3*a*c**2*d*x*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*d**
3*x**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integral(3*b*c*d**2*
x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integral(3*b*c**2*d*x*
sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

3.161 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx$

Optimal. Leaf size=342

$$-\frac{3\sqrt{\pi}b^{3/2}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}+\frac{3\sqrt{\pi}b^{3/2}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(3/2)/d-1/288*b^(3/2)*e^2*exp(3*a/b)
*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d+1/288*b
^(3/2)*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2
)/d/exp(3*a/b)-3/32*b^(3/2)*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*Pi^(1/2)/d+3/32*b^(3/2)*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*
Pi^(1/2)/d/exp(a/b)-1/3*b*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(
d*x+c))^(1/2)/d-1/6*b*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*ar
ccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.11, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {5866, 12, 5664, 5759, 5718, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{3\sqrt{\pi}b^{3/2}e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{96d}+\frac{3\sqrt{\pi}b^{3/2}e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] -(b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/
(3*d) - (b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*Sqrt[a + b*
ArcCosh[c + d*x]])/(6*d) + (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^(3/2))
/(3*d) - (3*b^(3/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/S
qrt[b]])/(32*d) - (b^(3/2)*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(96*d) + (3*b^(3/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b)) + (b^(3/2)*e^2*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(96*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{3/2} dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\cosh^{-1}(x)\right)^{3/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{3d} - \frac{\left(be^2\right)\text{Subst}\left(\int \frac{x^3\sqrt{a+b\cosh^{-1}(}}{\sqrt{-1+x}\,\sqrt{1+x}}\right.}{2d}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{e}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+}}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+}}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+}}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+}}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+}}{}$$

$$= -\frac{be^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{3d} - \frac{be^2\sqrt{-1+}}{}$$

Mathematica [A] time = 2.51, size = 592, normalized size = 1.73

$$e^2\left(\frac{ae^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\right.\right.}{72d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] e^2*((a*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c
 + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcCos
h[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*Sqr
```

```
t[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[
3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*Arc
Cosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^
2)]) + (Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c +
 d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c + d*x]*S
qrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[
a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[
3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] -
 Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*Arc
Cosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*ArcCos
h[c + d*x]])))/(288*d))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2 (a+b \operatorname{arccosh}(dx+c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^2 (b \operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,e+d\,e\,x\right)}^2\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int ac^2 \sqrt{a + b \operatorname{acosh}(c + dx)}\, dx + \int ad^2x^2 \sqrt{a + b \operatorname{acosh}(c + dx)}\, dx + \int bc^2 \sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(a*c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d**2*x**2
*sqrt(a + b*acosh(c + d*x)), x) + Integral(b*c**2*sqrt(a + b*acosh(c + d*x)
)*acosh(c + d*x), x) + Integral(2*a*c*d*x*sqrt(a + b*acosh(c + d*x)), x) +
Integral(b*d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x) + Integr
al(2*b*c*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

3.162 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=212

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c-\right.}{2d}$$

```
[Out] -1/4*e*(a+b*arccosh(d*x+c))^(3/2)/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(3
/2)/d-3/128*b^(3/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*2^(1/2)*Pi^(1/2)/d+3/128*b^(3/2)*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-3/8*b*e*(d*x+c)*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.69, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5664, 5759, 5676, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}+\frac{e(c+dx)^2\left(a+b\cosh^{-1}(c\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c
 + d*x]])/(8*d) - (e*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) + (e*(c + d*x)^2
*(a + b*ArcCosh[c + d*x])^(3/2))/(2*d) - (3*b^(3/2)*e*E^((2*a)/b)*Sqrt[Pi/2
]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d) + (3*b^(3/2)*
e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(64*d*E^
((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d} - \frac{(3be)\,\text{Subst}\left(\int \frac{x^2\sqrt{a+b\cosh^{-1}}}{\sqrt{-1+x}\sqrt{1+}}\right.}{4d}$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} +$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} -$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} -$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} -$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} -$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} -$$

$$= -\frac{3be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} -$$

Mathematica [B] time = 7.94, size = 1144, normalized size = 5.40

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] e*((a*c*Sqrt[-1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma
[3/2, a/b + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -(
(a + b*ArcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(
a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (b*c*Sqrt[-1 +
 c + d*x]*(-12*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*
ArcCosh[c + d*x]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x
]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh
[a/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*d*Sqrt[(-1 + c + d
*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (a*Sqrt[-1 + c + d*x]*(-32*c*(c + d
*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[a + b*ArcCosh[c + d*x]]*Cosh[2*Ar
cCosh[c + d*x]] + 8*Sqrt[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi[Sqrt[a + b*ArcCosh[c
+ d*x]]/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]] - 8*Sqrt[b]*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCos
h[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[b]*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCos
h[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + Sqrt[b]*Sqrt[2*Pi]*Erfi[(Sqr
```

```
t[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - Sqrt[b]*Sqrt[2*
Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Si
nh[(2*a)/b])))/(32*d*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x])
+ (Sqrt[-1 + c + d*x]*(-16*c*(-12*b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
 c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*b*(c + d*x)*ArcCosh[c + d*x]*Sqr
t[a + b*ArcCosh[c + d*x]] + Sqrt[b]*(2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[b]*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + S
qrt[b]*(4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + (4*a - 3*b)*Sqrt[b]*Sqrt[2*Pi]*Er
f[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*
a)/b]) + 8*b*Sqrt[a + b*ArcCosh[c + d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCos
h[c + d*x]] - 3*Sinh[2*ArcCosh[c + d*x]])))/(128*d*Sqrt[(-1 + c + d*x)/(1 +
 c + d*x)]*Sqrt[1 + c + d*x]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (dex+ce)\,(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)(b\,\mathrm{arcosh}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int ac\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int adx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,dx+\int bc\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(a*c*sqrt(a + b*acosh(c + d*x)), x) + Integral(a*d*x*sqrt(a + b*
acosh(c + d*x)), x) + Integral(b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
), x) + Integral(b*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

3.163 $\int \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=157

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{a+b\cos}}{2d}$$

```
[Out] (d*x+c)*(a+b*arccosh(d*x+c))^(3/2)/d-3/8*b^(3/2)*exp(a/b)*erf((a+b*arccosh(
d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+3/8*b^(3/2)*erfi((a+b*arccosh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-3/2*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*
arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.36, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5864, 5654, 5718, 5658, 3308, 2180, 2205, 2204}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8d}-\frac{3b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{a+b\cos}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]])/(2
*d) + ((c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2))/d - (3*b^(3/2)*E^(a/b)*Sqr
t[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*d) + (3*b^(3/2)*Sqrt[Pi
]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{3/2}\,dx,x,c+dx\right)}{d}$$

$$=\frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{d}-\frac{(3b)\,\text{Subst}\left(\int \frac{x\sqrt{a+b\cosh^{-1}(x)}}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,c+

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-12*b*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[
c + d*x]] + 8*b*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (
4*a*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c +
 d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x]
)/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) + Sqrt[b]*(2*a + 3*b)*S
qrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b])
+ (2*a - 3*b)*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(C
osh[a/b] + Sinh[a/b]))/(8*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acosh}(c + d\,x))^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acosh}\,(c + dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(3/2),x)
[Out] Integral((a + b*acosh(c + d*x))**(3/2), x)
```

3.164 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arccosh(d*x+c))^(3/2)/(d*x+c),x)/e
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(3/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{3/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{3/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(3/2)/(c*e + d*e*x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\left(dx+c\right)+a\right)^{\frac{3}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arccosh}\,(dx + c))^{\frac{3}{2}}}{dex + ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
[Out] int((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(dx + c) + a)^{\frac{3}{2}}}{dex + ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(3/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x + c) + a)^(3/2)/(d*e*x + c*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + d\,x))^{3/2}}{c\,e + d\,e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(3/2)/(c*e + d*e*x),x)
[Out] int((a + b*acosh(c + d*x))^(3/2)/(c*e + d*e*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}{c+dx}\,dx + \int \frac{b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}{c+dx}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(3/2)/(d*e*x+c*e),x)
[Out] (Integral(a*sqrt(a + b*acosh(c + d*x))/(c + d*x), x) + Integral(b*sqrt(a +
b*acosh(c + d*x))*acosh(c + d*x)/(c + d*x), x))/e
```

3.165 $\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=469

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}\,b^{5/2}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}$$

```
[Out] -3/32*e^3*(a+b*arccosh(d*x+c))^(5/2)/d+1/4*e^3*(d*x+c)^4*(a+b*arccosh(d*x+c
))^(5/2)/d-15/1024*b^(5/2)*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^
(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d-15/1024*b^(5/2)*e^3*erfi(2^(1/2)*(a+b*arc
cosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-15/16384*b^(5/2)*
e^3*exp(4*a/b)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/1638
4*b^(5/2)*e^3*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a
/b)-15/64*b*e^3*(d*x+c)*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(d*x+c+1
)^(1/2)/d-5/32*b*e^3*(d*x+c)^3*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/d-225/2048*b^2*e^3*(a+b*arccosh(d*x+c))^(1/2)/d+45/256*b^2*e
^3*(d*x+c)^2*(a+b*arccosh(d*x+c))^(1/2)/d+15/256*b^2*e^3*(d*x+c)^4*(a+b*arc
cosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 2.24, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5866, 12, 5664, 5759, 5676, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{512d}-\frac{15\sqrt{\pi}\,b^{5/2}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16384d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-225*b^2*e^3*Sqrt[a + b*ArcCosh[c + d*x]])/(2048*d) + (45*b^2*e^3*(c + d*x
)^2*Sqrt[a + b*ArcCosh[c + d*x]])/(256*d) + (15*b^2*e^3*(c + d*x)^4*Sqrt[a
+ b*ArcCosh[c + d*x]])/(256*d) - (15*b*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqr
t[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(64*d) - (5*b*e^3*Sqrt[-1 +
c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(32*
d) - (3*e^3*(a + b*ArcCosh[c + d*x])^(5/2))/(32*d) + (e^3*(c + d*x)^4*(a +
b*ArcCosh[c + d*x])^(5/2))/(4*d) - (15*b^(5/2)*e^3*E^((4*a)/b)*Sqrt[Pi]*Erf
[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16384*d) - (15*b^(5/2)*e^3*E^(
(2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5
12*d) - (15*b^(5/2)*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(16384*d*E^((4*a)/b)) - (15*b^(5/2)*e^3*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(512*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^3x^3 \left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int x^3 \left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^3(c+dx)^4 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{4d} - \frac{\left(5be^3\right)\text{Subst}\left(\int \frac{x^4\left(a+b\cosh^{-1}\right.}{\sqrt{-1+x}\,\sqrt{1\cdot}}\right.}{8d}$$

$$= -\frac{5be^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{32d}$$

$$= \frac{15b^2e^3(c+dx)^4\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} - \frac{15be^3\sqrt{-1+c+dx}\,(c+d}{}$$

$$= \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\,\mathrm{cc}}}{256d}$$

$$= \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{256d} + \frac{15b^2e^3(c+dx)^4\sqrt{a+b\,\mathrm{cc}}}{256d}$$

$$= -\frac{45b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(c}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(}}{256d}$$

$$= -\frac{225b^2e^3\sqrt{a+b\cosh^{-1}(c+dx)}}{2048d} + \frac{45b^2e^3(c+dx)^2\sqrt{a+b\cosh^{-1}(}}{256d}$$

Mathematica [B] time = 11.22, size = 968, normalized size = 2.06

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] e^3*((a^2*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[
3/2, (-4*(a + b*ArcCosh[c + d*x]))/b] + 4*Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + Ar
cCosh[c + d*x]]*Gamma[3/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*S
qrt[-((a + b*ArcCosh[c + d*x])/b)]*(4*Sqrt[2]*Gamma[3/2, (2*(a + b*ArcCosh[
c + d*x]))/b] + E^((2*a)/b)*Gamma[3/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(
128*d*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/b^2)]) + (a*Sqrt[b]*((8
*a + 3*b)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*
a)/b] - Sinh[(4*a)/b]) + (8*a - 3*b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]]*(Cosh[(4*a)/b] + Sinh[(4*a)/b]) + 8*((4*a + 3*b)*Sqrt[2*Pi
]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sin
h[(2*a)/b]) + (4*a - 3*b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*Sqrt[b]*Sqrt[a + b*ArcCos
h[c + d*x]]*(4*ArcCosh[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh
[c + d*x]])) + 8*Sqrt[b]*Sqrt[a + b*ArcCosh[c + d*x]]*(8*ArcCosh[c + d*x]*C
osh[4*ArcCosh[c + d*x]] - 3*Sinh[4*ArcCosh[c + d*x]])))/(1024*d) + (-(Sqrt[
b]*(64*a^2 + 48*a*b + 15*b^2)*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]]*(Cosh[(4*a)/b] - Sinh[(4*a)/b])) - Sqrt[b]*(64*a^2 - 48*a*b + 15
*b^2)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(4*a)/b]
 + Sinh[(4*a)/b]) - 16*(Sqrt[b]*(16*a^2 + 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)
/b]) + Sqrt[b]*(16*a^2 - 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 8*b*Sqrt[a
+ b*ArcCosh[c + d*x]]*(b*(15 + 16*ArcCosh[c + d*x]^2)*Cosh[2*ArcCosh[c + d*
x]] + 4*(a - 5*b*ArcCosh[c + d*x])*Sinh[2*ArcCosh[c + d*x]])) + 8*b*Sqrt[a
+ b*ArcCosh[c + d*x]]*(b*(15 + 64*ArcCosh[c + d*x]^2)*Cosh[4*ArcCosh[c + d*
x]] + 8*(a - 5*b*ArcCosh[c + d*x])*Sinh[4*ArcCosh[c + d*x]]))/(16384*d))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^3 (b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3*(b*arccosh(d*x + c) + a)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^3\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^3*(a + b*acosh(c + d*x))^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3*(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.166 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=408

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{5\sqrt{\frac{\pi}{3}}\,b^{5/2}e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}-\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\ldots\right)}{64d}$$

```
[Out] 1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(5/2)/d-5/1728*b^(5/2)*e^2*exp(3*a/b
)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d-5/1728
*b^(5/2)*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/d/exp(3*a/b)-15/64*b^(5/2)*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/d-15/64*b^(5/2)*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*Pi^(1/2)/d/exp(a/b)-5/9*b*e^2*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)/d-5/18*b*e^2*(d*x+c)^2*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c-
1)^(1/2)*(d*x+c+1)^(1/2)/d+5/6*b^2*e^2*(d*x+c)*(a+b*arccosh(d*x+c))^(1/2)/d
+5/36*b^2*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.75, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {5866, 12, 5664, 5759, 5718, 5654, 5781, 3307, 2180, 2204, 2205, 3312}

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{64d}-\frac{5\sqrt{\frac{\pi}{3}}\,b^{5/2}e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{576d}-\frac{15\sqrt{\pi}\,b^{5/2}e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\ldots\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (5*b^2*e^2*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/(6*d) + (5*b^2*e^2*(c +
d*x)^3*Sqrt[a + b*ArcCosh[c + d*x]])/(36*d) - (5*b*e^2*Sqrt[-1 + c + d*x]*S
qrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(9*d) - (5*b*e^2*Sqrt[-1 +
 c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(18
*d) + (e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x])^(5/2))/(3*d) - (15*b^(5/2)*
e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(64*d) - (5
*b^(5/2)*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(576*d) - (15*b^(5/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c
+ d*x]]/Sqrt[b]])/(64*d*E^(a/b)) - (5*b^(5/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(576*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
```

```
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2} dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\cosh^{-1}(x)\right)^{5/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{3d} - \frac{\left(5be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\cos\right.}{\sqrt{-1+x}}\right.}{6d}$$

$$= -\frac{5be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)}{18d}$$

$$= \frac{5b^2e^2(c+dx)^3\sqrt{a+b\cosh^{-1}(c+dx)}}{36d} - \frac{5be^2\sqrt{-1+c+dx}\,\sqrt{1+}}{}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{co}}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{co}}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{co}}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{co}}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{co}}}{36d}$$

$$= \frac{5b^2e^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{6d} + \frac{5b^2e^2(c+dx)^3\sqrt{a+b\,\text{co}}}{36d}$$

Mathematica [B] time = 9.31, size = 1008, normalized size = 2.47

$$e^2\left(\frac{e^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\right.}{7}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] e^2*((a^2*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh
[c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*S
qrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqr
t[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/
b^2)]) + (a*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 +
 c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c + d*
x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcC
osh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*S
qrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/
b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b
*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*Ar
cCosh[c + d*x]])))/(144*d) + (-27*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqr
t[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]) +
b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2
)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b
]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c +
d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) - Sqrt[b]*(12*a^2 + 12*a*b + 5*b^2)
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a
)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(12*a^2 - 12*a*b + 5*b^2)*Sqrt[3*Pi]*Erf[(S
qrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b
]) + 12*b*Sqrt[a + b*ArcCosh[c + d*x]]*(b*(5 + 12*ArcCosh[c + d*x]^2)*Cosh[
3*ArcCosh[c + d*x]] + 2*(a - 5*b*ArcCosh[c + d*x])*Sinh[3*ArcCosh[c + d*x]]
))/(1728*d))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^2\,(a + b\operatorname{acosh}(c + d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(5/2),x)
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int a^2 c^2 \sqrt{a + b \operatorname{acosh}(c + dx)}\, dx + \int a^2 d^2 x^2 \sqrt{a + b \operatorname{acosh}(c + dx)}\, dx + \int b^2 c^2 \sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(5/2),x)
[Out] e**2*(Integral(a**2*c**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(a**2*d**
2*x**2*sqrt(a + b*acosh(c + d*x)), x) + Integral(b**2*c**2*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x)**2, x) + Integral(2*a*b*c**2*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x), x) + Integral(2*a**2*c*d*x*sqrt(a + b*acosh(c + d*x)
), x) + Integral(b**2*d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**
2, x) + Integral(2*a*b*d**2*x**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x),
 x) + Integral(2*b**2*c*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2, x
) + Integral(4*a*b*c*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

3.167 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=269

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-}}}{32d}$$

```
[Out] -1/4*e*(a+b*arccosh(d*x+c))^(5/2)/d+1/2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(5
/2)/d-15/512*b^(5/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^
(1/2))*2^(1/2)*Pi^(1/2)/d-15/512*b^(5/2)*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-5/8*b*e*(d*x+c)*(a+b*arccosh
(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-15/64*b^2*e*(a+b*arccosh(d
*x+c))^(1/2)/d+15/32*b^2*e*(d*x+c)^2*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 1.11, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5866, 12, 5664, 5759, 5676, 5781, 3312, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}-\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}ee^{-\frac{2a}{b}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{256d}+\frac{15b^2e(c+dx)^2\sqrt{a+b\cosh}}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-15*b^2*e*Sqrt[a + b*ArcCosh[c + d*x]])/(64*d) + (15*b^2*e*(c + d*x)^2*Sqr
t[a + b*ArcCosh[c + d*x]])/(32*d) - (5*b*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqr
t[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(8*d) - (e*(a + b*ArcCosh[c
+ d*x])^(5/2))/(4*d) + (e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(5/2))/(2*d)
 - (15*b^(5/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]])/(256*d) - (15*b^(5/2)*e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(256*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{5/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{5/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{2d} - \frac{(5be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}(x)\right)^{3}}{\sqrt{-1+x}\sqrt{1+x}}\right.}{4d}$$

$$= -\frac{5be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{8d} +$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} - \frac{5be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1}}{}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} - \frac{5be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1}}{}$$

$$= \frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+dx)}}{32d} - \frac{5be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1}}{}$$

$$= -\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+d}}{32d}$$

$$= -\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+d}}{32d}$$

$$= -\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+d}}{32d}$$

$$= -\frac{15b^2e\sqrt{a+b\cosh^{-1}(c+dx)}}{64d} + \frac{15b^2e(c+dx)^2\sqrt{a+b\cosh^{-1}(c+d}}{32d}$$

Mathematica [B] time = 9.26, size = 1846, normalized size = 6.86

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] e*((a^2*c*Sqrt[-1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gam
ma[3/2, a/b + ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2,
-((a + b*ArcCosh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E
^(a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (a*b*c*Sqrt[
-1 + c + d*x]*(-12*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a
+ b*ArcCosh[c + d*x]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c +
 d*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(
Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcC
osh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(4*d*Sqrt[(-1 + c
 + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) - (c*Sqrt[-1 + c + d*x]*(-4*b*Sqr
t[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*
x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) +
```

```
Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]
]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt
[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(1
6*d*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) + (a^2*Sqrt[-1 +
c + d*x]*(-32*c*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[a + b*ArcCo
sh[c + d*x]]*Cosh[2*ArcCosh[c + d*x]] + 8*Sqrt[b]*c*Sqrt[Pi]*Cosh[a/b]*Erfi
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*Cosh[(2*a)/b]*E
rfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - 8*Sqrt[b]*c*Sqrt[Pi]*
Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 8*Sqrt[b]*c*Sqrt[Pi]
*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + Sqrt[b
]*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a
)/b] - Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b])))/(32*d*Sqrt[(-1 + c + d*x)/(1 + c + d*x
)]*Sqrt[1 + c + d*x]) + (a*Sqrt[-1 + c + d*x]*(-16*c*(-12*b*Sqrt[(-1 + c +
d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*b*(c + d
*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + Sqrt[b]*(2*a + 3*b)*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) +
(2*a - 3*b)*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cos
h[a/b] + Sinh[a/b])) + Sqrt[b]*(4*a + 3*b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
+ b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + (4*a - 3*
b)*Sqrt[b]*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(
Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*b*Sqrt[a + b*ArcCosh[c + d*x]]*(4*ArcCos
h[c + d*x]*Cosh[2*ArcCosh[c + d*x]] - 3*Sinh[2*ArcCosh[c + d*x]])))/(64*d*S
qrt[(-1 + c + d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]) - (Sqrt[-1 + c + d*x]*
(-32*c*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d
*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh
[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*
ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + Sqrt[b]*(4*a^2 - 12*a*
b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] +
 Sinh[a/b])) + Sqrt[b]*(16*a^2 + 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) + Sq
rt[b]*(16*a^2 - 24*a*b + 15*b^2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) - 8*b*Sqrt[a + b*ArcCo
sh[c + d*x]]*(b*(15 + 16*ArcCosh[c + d*x]^2)*Cosh[2*ArcCosh[c + d*x]] + 4*(
a - 5*b*ArcCosh[c + d*x])*Sinh[2*ArcCosh[c + d*x]])))/(512*d*Sqrt[(-1 + c +
 d*x)/(1 + c + d*x)]*Sqrt[1 + c + d*x]))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (dex + ce)\,(a + b\,\mathrm{arccosh}\,(dx + c))^{\frac{5}{2}}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)\,\left(a + b\,\mathrm{acosh}\left(c + d\,x\right)\right)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int a^2 c\sqrt{a + b \operatorname{acosh}(c + dx)} \, dx + \int a^2 dx\sqrt{a + b \operatorname{acosh}(c + dx)} \, dx + \int b^2 c\sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}^2 (c +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(a**2*c*sqrt(a + b*acosh(c + d*x)), x) + Integral(a**2*d*x*sqrt(
a + b*acosh(c + d*x)), x) + Integral(b**2*c*sqrt(a + b*acosh(c + d*x))*acos
h(c + d*x)**2, x) + Integral(2*a*b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x), x) + Integral(b**2*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2, x
) + Integral(2*a*b*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x), x))
```

3.168 $\int \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=186

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+d}}{4d}$$

```
[Out] (d*x+c)*(a+b*arccosh(d*x+c))^(5/2)/d-15/16*b^(5/2)*exp(a/b)*erf((a+b*arccos
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d-15/16*b^(5/2)*erfi((a+b*arccosh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-5/2*b*(a+b*arccosh(d*x+c))^(3/2)*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/d+15/4*b^2*(d*x+c)*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.61, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5864, 5654, 5718, 5781, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}\,b^{5/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}-\frac{15\sqrt{\pi}\,b^{5/2}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16d}+\frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+d}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (15*b^2*(c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]])/(4*d) - (5*b*Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(3/2))/(2*d) + ((c + d*x)*(
a + b*ArcCosh[c + d*x])^(5/2))/d - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a
+ b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d) - (15*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
```

```
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{5/2}\,dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{5/2}\,dx,x,c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{d} - \frac{(5b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^{3/2}}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,c+dx\right.}{2d}$$

$$= -\frac{5b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{2d} + \frac{(c+dx)\left(a+b\cosh\right.}{d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cos\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cos\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cos\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cos\right.}{2d}$$

$$= \frac{15b^2(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}}{4d} - \frac{5b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\left(a+b\cos\right.}{2d}$$

Mathematica [B] time = 3.56, size = 494, normalized size = 2.66

$$-\sqrt{\pi}\sqrt{b}\left(4a^2-12ab+15b^2\right)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)-\sqrt{\pi}\sqrt{b}\left(4a^2+12ab+15b^2\right)\left(\cos\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
+ c + d*x)*(a - 5*b*ArcCosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x
]^2)) + (8*a^2*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b +
ArcCosh[c + d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCo
sh[c + d*x])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b) - Sqrt[b]*(4
*a^2 + 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]
*(Cosh[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]) + 4*a*b*(-12*S
qrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]
] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + ((2*a + 3*b
)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b
]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b
]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(16*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arccosh}(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\text{acosh}\,(c + d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\text{acosh}\,(c + dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(5/2), x)
```

3.169 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arccosh(d*x+c))^(5/2)/(d*x+c),x)/e
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(5/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{5/2}}{ex}\,dx,x,c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{5/2}}{x}\,dx,x,c+dx\right)}{de}$$

Mathematica [A] time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(5/2)/(c*e + d*e*x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\left(dx+c\right)+a\right)^{\frac{5}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

maple [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}}{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(5/2)/(d*e*x+c*e),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(5/2)/(d*e*x + c*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b \operatorname{acosh}(c + d\, x))^{5/2}}{c\, e + d\, e\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(5/2)/(c*e + d*e*x),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(5/2)/(c*e + d*e*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a^2 \sqrt{a+b \operatorname{acosh}(c+dx)}}{c+dx}\, dx + \int \frac{b^2 \sqrt{a+b \operatorname{acosh}(c+dx)} \operatorname{acosh}^2(c+dx)}{c+dx}\, dx + \int \frac{2ab \sqrt{a+b \operatorname{acosh}(c+dx)} \operatorname{acosh}(c+dx)}{c+dx}\, dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(5/2)/(d*e*x+c*e),x)
```

```
[Out] (Integral(a**2*sqrt(a + b*acosh(c + d*x))/(c + d*x), x) + Integral(b**2*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x)**2/(c + d*x), x) + Integral(2*a*b*sq
rt(a + b*acosh(c + d*x))*acosh(c + d*x)/(c + d*x), x))/e
```

3.170 $\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{7/2} dx$

Optimal. Leaf size=509

$$-\frac{105\sqrt{\pi}\, b^{7/2}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{35\sqrt{\frac{\pi}{3}}\, b^{7/2}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}+\frac{105\sqrt{\pi}\, b^{7/2}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b}}{}\right.}{128d}$$

```
[Out] 35/18*b^2*e^2*(d*x+c)*(a+b*arccosh(d*x+c))^(3/2)/d+35/108*b^2*e^2*(d*x+c)^3
*(a+b*arccosh(d*x+c))^(3/2)/d+1/3*e^2*(d*x+c)^3*(a+b*arccosh(d*x+c))^(7/2)/
d-35/10368*b^(7/2)*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/d+35/10368*b^(7/2)*e^2*erfi(3^(1/2)*(a+b*arccosh(d*
x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)-105/128*b^(7/2)*e^2*exp(
a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d+105/128*b^(7/2)*e^2
*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)-7/9*b*e^2*(a+
b*arccosh(d*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-7/18*b*e^2*(d*x+c
)^2*(a+b*arccosh(d*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-175/54*b^3
*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d-35/216*b^
3*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/
d
```

Rubi [A] time = 2.14, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {5866, 12, 5664, 5759, 5718, 5654, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{105\sqrt{\pi}\, b^{7/2}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{128d}-\frac{35\sqrt{\frac{\pi}{3}}\, b^{7/2}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3456d}+\frac{105\sqrt{\pi}\, b^{7/2}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+}}{}\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-175*b^3*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d
*x]])/(54*d) - (35*b^3*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x]
*Sqrt[a + b*ArcCosh[c + d*x]])/(216*d) + (35*b^2*e^2*(c + d*x)*(a + b*ArcCo
sh[c + d*x])^(3/2))/(18*d) + (35*b^2*e^2*(c + d*x)^3*(a + b*ArcCosh[c + d*x
])^(3/2))/(108*d) - (7*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*Ar
cCosh[c + d*x])^(5/2))/(9*d) - (7*b*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt
[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(18*d) + (e^2*(c + d*x)^3*(a
+ b*ArcCosh[c + d*x])^(7/2))/(3*d) - (105*b^(7/2)*e^2*E^(a/b)*Sqrt[Pi]*Erf[
Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(128*d) - (35*b^(7/2)*e^2*E^((3*a)/b
)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3456*d)
+ (105*b^(7/2)*e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12
8*d*E^(a/b)) + (35*b^(7/2)*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(3456*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
```

```
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^2 \left(a+b\cosh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int e^2x^2 \left(a+b\cosh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int x^2 \left(a+b\cosh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{e^2(c+dx)^3 \left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{3d} - \frac{\left(7be^2\right)\text{Subst}\left(\int \frac{x^3\left(a+b\cosh^{-1}\right.}{\sqrt{-1+x}\,\sqrt{1}}\right.}{6d}$$

$$= -\frac{7be^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{18d}$$

$$= \frac{35b^2e^2(c+dx)^3\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{108d} - \frac{7be^2\sqrt{-1+c+dx}\,\sqrt{1+}}{}$$

$$= -\frac{35b^3e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{216d}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e}{}$$

$$= -\frac{175b^3e^2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{54d} - \frac{35b^3e}{}$$

Mathematica [B] time = 14.07, size = 1523, normalized size = 2.99

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] e^2*((a^3*Sqrt[a + b*ArcCosh[c + d*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh
[c + d*x])/b)]*Gamma[3/2, a/b + ArcCosh[c + d*x]] + Sqrt[3]*Sqrt[a/b + ArcC
osh[c + d*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 9*E^((2*a)/b)*S
qrt[a/b + ArcCosh[c + d*x]]*Gamma[3/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqr
t[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c + d*x]))/b]))/(72*d*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])^2/
```

```
b^2)]) + (a^2*Sqrt[b]*(9*(-12*Sqrt[b]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1
 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x]] + 8*Sqrt[b]*(c + d*x)*ArcCosh[c +
d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)
*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a
)/b] - Sinh[(3*a)/b]) + (2*a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCos
h[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a +
 b*ArcCosh[c + d*x]]*(2*ArcCosh[c + d*x]*Cosh[3*ArcCosh[c + d*x]] - Sinh[3*
ArcCosh[c + d*x]])))/(96*d) + (a*(-27*(-4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(2
*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*ArcCosh[c + d*x]
) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) + Sqrt[b]*(4*a^2 + 12*a*b + 15
*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh
[a/b]) + Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[
c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) - Sqrt[b]*(12*a^2 + 12*a*b + 5*
b^2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[
(3*a)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(12*a^2 - 12*a*b + 5*b^2)*Sqrt[3*Pi]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*
a)/b]) + 12*b*Sqrt[a + b*ArcCosh[c + d*x]]*(b*(5 + 12*ArcCosh[c + d*x]^2)*C
osh[3*ArcCosh[c + d*x]] + 2*(a - 5*b*ArcCosh[c + d*x])*Sinh[3*ArcCosh[c + d
*x]])))/(576*d) + (-81*(4*b*Sqrt[a + b*ArcCosh[c + d*x]]*(Sqrt[(-1 + c + d*
x)/(1 + c + d*x)]*(1 + c + d*x)*(4*a^2 - 4*a*b*ArcCosh[c + d*x] + 7*b^2*(15
 + 4*ArcCosh[c + d*x]^2)) - 2*b*(c + d*x)*(-10*a + b*ArcCosh[c + d*x]*(35 +
 4*ArcCosh[c + d*x]^2))) + Sqrt[b]*(8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]
) + Sqrt[b]*(-8*a^3 + 36*a^2*b - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a +
b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + Sqrt[b]*(72*a^3 + 1
08*a^2*b + 90*a*b^2 + 35*b^3)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) - Sqrt[b]*(-72*a^3 + 108
*a^2*b - 90*a*b^2 + 35*b^3)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]]*(Cosh[(3*a)/b] + Sinh[(3*a)/b]) - 12*b*Sqrt[a + b*ArcCosh[c
 + d*x]]*(-2*b*(-10*a + b*ArcCosh[c + d*x]*(35 + 36*ArcCosh[c + d*x]^2))*Co
sh[3*ArcCosh[c + d*x]] + (12*a^2 - 12*a*b*ArcCosh[c + d*x] + 7*b^2*(5 + 12*
ArcCosh[c + d*x]^2))*Sinh[3*ArcCosh[c + d*x]]))/(10368*d))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x)
[Out] int((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^2 (b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^2*(b*arccosh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)^2 \,(a + b\,\mathrm{acosh}\,(c + d\,x))^{7/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(7/2),x)
[Out] int((c*e + d*e*x)^2*(a + b*acosh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2*(a+b*acosh(d*x+c))**(7/2),x)
[Out] Timed out
```

3.171 $\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}\,dx$

Optimal. Leaf size=319

$$-\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e\sqrt{c+dx-1}\,(c+}{}$$

```
[Out] -35/64*b^2*e*(a+b*arccosh(d*x+c))^(3/2)/d+35/32*b^2*e*(d*x+c)^2*(a+b*arccos
h(d*x+c))^(3/2)/d-1/4*e*(a+b*arccosh(d*x+c))^(7/2)/d+1/2*e*(d*x+c)^2*(a+b*a
rccosh(d*x+c))^(7/2)/d-105/2048*b^(7/2)*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arcco
sh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d+105/2048*b^(7/2)*e*erfi(2^(1/2
)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)-7/8*b*e
*(d*x+c)*(a+b*arccosh(d*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-105/1
28*b^3*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)
/d
```

Rubi [A] time = 1.28, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5664, 5759, 5676, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}+\frac{105\sqrt{\frac{\pi}{2}}\,b^{7/2}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{1024d}-\frac{105b^3e\sqrt{c+dx-1}\,(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCo
sh[c + d*x]])/(128*d) - (35*b^2*e*(a + b*ArcCosh[c + d*x])^(3/2))/(64*d) +
(35*b^2*e*(c + d*x)^2*(a + b*ArcCosh[c + d*x])^(3/2))/(32*d) - (7*b*e*Sqrt[
-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(
8*d) - (e*(a + b*ArcCosh[c + d*x])^(7/2))/(4*d) + (e*(c + d*x)^2*(a + b*Arc
Cosh[c + d*x])^(7/2))/(2*d) - (105*b^(7/2)*e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(1024*d) + (105*b^(7/2)*e*Sqr
t[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(1024*d*E^((2
*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}dx = \frac{\text{Subst}\left(\int ex\left(a+b\cosh^{-1}(x)\right)^{7/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int x\left(a+b\cosh^{-1}(x)\right)^{7/2}dx,x,c+dx\right)}{d}$$

$$= \frac{e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{2d} - \frac{(7be)\,\text{Subst}\left(\int \frac{x^2\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+x}\sqrt{1+}}\right.}{4d}$$

$$= -\frac{7be\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{8d}$$

$$= \frac{35b^2e(c+dx)^2\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{32d} - \frac{7be\sqrt{-1+c+dx}\,(c+}{}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}$$

$$= -\frac{105b^3e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{128d}$$

Mathematica [A] time = 5.75, size = 288, normalized size = 0.90

$$\frac{e\left(8\sqrt{a+b\cosh^{-1}(c+dx)}\,\left(4a\left(16a^2+35b^2\right)\cosh\left(2\cosh^{-1}(c+dx)\right)+4b\cosh^{-1}(c+dx)\left(\left(48a^2+35b^2\right)\cos\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e*(105*b^(7/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]]*(Cosh[(2*a)/b] - Sinh[(2*a)/b]) - 105*b^(7/2)*Sqrt[2*Pi]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Cosh[(2*a)/b] + Sinh[(2*a)/b]) + 8*
Sqrt[a + b*ArcCosh[c + d*x]]*(4*a*(16*a^2 + 35*b^2)*Cosh[2*ArcCosh[c + d*x]
] + 64*b^3*ArcCosh[c + d*x]^3*Cosh[2*ArcCosh[c + d*x]] - 7*b*(16*a^2 + 15*b
^2)*Sinh[2*ArcCosh[c + d*x]] + 16*b^2*ArcCosh[c + d*x]^2*(12*a*Cosh[2*ArcCo
```

```
sh[c + d*x]] - 7*b*Sinh[2*ArcCosh[c + d*x]]) + 4*b*ArcCosh[c + d*x]*((48*a^
2 + 35*b^2)*Cosh[2*ArcCosh[c + d*x]] - 56*a*b*Sinh[2*ArcCosh[c + d*x]])))))/
(2048*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int (dex + ce)(a + b\,\mathrm{arccosh}(dx + c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)(b\,\mathrm{arcosh}(dx + c) + a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e + d\,e\,x)\,\left(a + b\,\mathrm{acosh}\left(c + d\,x\right)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)*(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.172 $\int \left(a + b\cosh^{-1}(c + dx)\right)^{7/2} dx$

Optimal. Leaf size=230

$$-\frac{105\sqrt{\pi}\, b^{7/2} e^{a/b} \operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}\, b^{7/2} e^{-\frac{a}{b}} \operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{8d}$$

```
[Out] 35/4*b^2*(d*x+c)*(a+b*arccosh(d*x+c))^(3/2)/d+(d*x+c)*(a+b*arccosh(d*x+c))^
(7/2)/d-105/32*b^(7/2)*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^
(1/2)/d+105/32*b^(7/2)*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/
exp(a/b)-7/2*b*(a+b*arccosh(d*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
-105/8*b^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.64, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5864, 5654, 5718, 5658, 3308, 2180, 2205, 2204}

$$-\frac{105\sqrt{\pi}\, b^{7/2} e^{a/b} \text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}+\frac{105\sqrt{\pi}\, b^{7/2} e^{-\frac{a}{b}} \text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32d}-\frac{105b^3\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-105*b^3*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]]
)/(8*d) + (35*b^2*(c + d*x)*(a + b*ArcCosh[c + d*x])^(3/2))/(4*d) - (7*b*Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x]*(a + b*ArcCosh[c + d*x])^(5/2))/(2*d) +
((c + d*x)*(a + b*ArcCosh[c + d*x])^(7/2))/d - (105*b^(7/2)*E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d) + (105*b^(7/2)*Sqrt[Pi]
*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(32*d*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(c+dx)\right)^{7/2} dx = \frac{\text{Subst}\left(\int \left(a+b\cosh^{-1}(x)\right)^{7/2} dx, x, c+dx\right)}{d}$$

$$= \frac{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{d} - \frac{(7b)\,\text{Subst}\left(\int \frac{x\left(a+b\cosh^{-1}(x)\right)^{5/2}}{\sqrt{-1+x}\sqrt{1+x}} dx, x, c+\right.}{2d}$$

$$= -\frac{7b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}{2d} + \frac{(c+dx)\left(a+b\,\mathrm{c}\right.}{}$$

$$= \frac{35b^2(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}{4d} - \frac{7b\sqrt{-1+c+dx}\sqrt{1+c+dx}\left(a+\right.}{2d}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a\right.}{}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a\right.}{}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a\right.}{}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a\right.}{}$$

$$= -\frac{105b^3\sqrt{-1+c+dx}\sqrt{1+c+dx}\sqrt{a+b\cosh^{-1}(c+dx)}}{8d} + \frac{35b^2(c+dx)\left(a\right.}{}$$

Mathematica [B] time = 7.51, size = 765, normalized size = 3.33

$$\frac{a^3 e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(c+dx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}}\right)}{2d} + \frac{3a\left(-\sqrt{\pi}\sqrt{b}\left(4a^2-12ab+15b^2\right)\left(\mathrm{s}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (a^3*Sqrt[a + b*ArcCosh[c + d*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c
+ d*x]])/Sqrt[a/b + ArcCosh[c + d*x]] + Gamma[3/2, -((a + b*ArcCosh[c + d*x
])/b)]/Sqrt[-((a + b*ArcCosh[c + d*x])/b)]))/(2*d*E^(a/b)) + (3*a^2*b*(-12*
Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*Sqrt[a + b*ArcCosh[c + d*x
]] + 8*(c + d*x)*ArcCosh[c + d*x]*Sqrt[a + b*ArcCosh[c + d*x]] + ((2*a + 3*
b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/
b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[
b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*d) + (3*a*(4*b*Sqrt[a + b*ArcCosh
[c + d*x]]*(2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*(a - 5*b*Arc
Cosh[c + d*x]) + b*(c + d*x)*(15 + 4*ArcCosh[c + d*x]^2)) - Sqrt[b]*(4*a^2
+ 12*a*b + 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cos
h[a/b] - Sinh[a/b]) - Sqrt[b]*(4*a^2 - 12*a*b + 15*b^2)*Sqrt[Pi]*Erf[Sqrt[a
 + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])))/(16*d) + (-4*b*Sq
```

```
rt[a + b*ArcCosh[c + d*x]]*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x
)*(4*a^2 - 4*a*b*ArcCosh[c + d*x] + 7*b^2*(15 + 4*ArcCosh[c + d*x]^2)) - 2*
b*(c + d*x)*(-10*a + b*ArcCosh[c + d*x]*(35 + 4*ArcCosh[c + d*x]^2))) - Sqr
t[b]*(8*a^3 + 36*a^2*b + 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCo
sh[c + d*x]]/Sqrt[b]]*(-Cosh[a/b] + Sinh[a/b]) - Sqrt[b]*(-8*a^3 + 36*a^2*b
 - 90*a*b^2 + 105*b^3)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(
Cosh[a/b] + Sinh[a/b]))/(32*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{acosh}(c + d\, x))^{7/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.173 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{c+dx},x\right)}{e}$$

```
[Out] Unintegrable((a+b*arccosh(d*x+c))^(7/2)/(d*x+c),x)/e
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x),x]
[Out] Defer[Subst][Defer[Int][(a + b*ArcCosh[x])^(7/2)/x, x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{7/2}}{ex}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^{7/2}}{x}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}{ce+dex}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x),x]
[Out] Integrate[(a + b*ArcCosh[c + d*x])^(7/2)/(c*e + d*e*x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\left(dx+c\right)+a\right)^{\frac{7}{2}}}{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}}{dex + ce} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
[Out] int((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}{dex + ce} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^(7/2)/(d*e*x+c*e),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x + c) + a)^(7/2)/(d*e*x + c*e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b \operatorname{acosh}(c + d x))^{7/2}}{c e + d e x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^(7/2)/(c*e + d*e*x),x)
[Out] int((a + b*acosh(c + d*x))^(7/2)/(c*e + d*e*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**(7/2)/(d*e*x+c*e),x)
[Out] Timed out
```

3.174 $\int \frac{(ce+dex)^4}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=326

$$-\frac{\sqrt{\pi}e^4e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d}-\frac{\sqrt{3\pi}e^4e^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{5}}e^4e^{\frac{5a}{b}}\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\pi}e^4}{}$$

```
[Out] -1/160*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/
2)*Pi^(1/2)/d/b^(1/2)+1/160*e^4*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*5^(1/2)*Pi^(1/2)/d/exp(5*a/b)/b^(1/2)-1/16*e^4*exp(a/b)*erf((a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/16*e^4*erfi((a+b*arccosh(d*
x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)-1/32*e^4*exp(3*a/b)*erf(3^
(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/b^(1/2)+1/32*e
^4*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/d/exp(
3*a/b)/b^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5866, 12, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}d}-\frac{\sqrt{3\pi}e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{5}}e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}d}+\frac{\sqrt{\pi}e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*Sqrt[
b]*d) - (e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(32*Sqrt[b]*d) - (e^4*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d) + (e^4*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(16*Sqrt[b]*d*E^(a/b)) + (e^4*Sqrt[3*P
i]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((
3*a)/b)) + (e^4*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/(32*Sqrt[b]*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8\sqrt{a+bx}} + \frac{3\sinh(3x)}{16\sqrt{a+bx}} + \frac{\sinh(5x)}{16\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{16d} + \frac{e^4\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+\right.}{8d}$$

$$= -\frac{e^4\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{32d} + \frac{e^4\,\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c\right.}{32d}$$

$$= -\frac{e^4\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{16bd} + \frac{e^4\,\text{Subst}\left(\int e^{-\frac{5a}{b}+\frac{5x^2}{b}}\,dx, x\right.}{16b}$$

$$= -\frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,d} - \frac{e^4e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d} - \frac{e^4e^{\frac{5a}{b}}\sqrt{}}{}$$

Mathematica [A] time = 0.64, size = 319, normalized size = 0.98

$$\frac{e^4e^{-\frac{5a}{b}}\left(10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{5}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^4*(10*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[
c + d*x]] + Sqrt[5]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-5*(a +
 b*ArcCosh[c + d*x]))/b] + 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c +
d*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 10*E^((4*a)/b)*Sqrt
[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] +
 5*Sqrt[3]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*Ar
cCosh[c + d*x]))/b] + Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gam
ma[1/2, (5*(a + b*ArcCosh[c + d*x]))/b]))/(160*d*E^((5*a)/b)*Sqrt[a + b*Arc
Cosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{\sqrt{b \operatorname{arcosh}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arccosh(d*x + c) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{\sqrt{a + b \operatorname{arccosh}(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{\sqrt{b \operatorname{arcosh}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^4}{\sqrt{a + b \operatorname{acosh}(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4 \left(\int \frac{c^4}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{d^4 x^4}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{4 c d^3 x^3}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\, dx + \int \frac{6}{\sqrt{a + b\, \mathrm{a}}} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**4*(Integral(c**4/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**4*x**4/sqr
t(a + b*acosh(c + d*x)), x) + Integral(4*c*d**3*x**3/sqrt(a + b*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/sqrt(a + b*acosh(c + d*x)), x) + Integ
ral(4*c**3*d*x/sqrt(a + b*acosh(c + d*x)), x))
```

3.175 $\int \frac{(ce+dex)^3}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=217

$$-\frac{\sqrt{\pi}\,e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\sqrt{\frac{\pi}{2}}\,e^3$$

```
[Out] -1/16*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2
)*Pi^(1/2)/d/b^(1/2)+1/16*e^3*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*2^(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)-1/32*e^3*exp(4*a/b)*erf(2*(a+b*ar
ccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/32*e^3*erfi(2*(a+b*arccos
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(4*a/b)/b^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5866, 12, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{2}}\,e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d}+\sqrt{\frac{\pi}{2}}\,e$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(
32*Sqrt[b]*d) - (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcC
osh[c + d*x]])/Sqrt[b]])/(32*Sqrt[b]*d*E^((4*a)/b)) + (e^3*Sqrt[Pi/2]*Erfi[
(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4\sqrt{a+bx}}+\frac{\sinh(4x)}{8\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{8d} + \frac{e^3\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\right.}{4d}$$

$$= -\frac{e^3\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{16d} + \frac{e^3\,\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\right.}{16d}$$

$$= -\frac{e^3\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{8bd} + \frac{e^3\,\text{Subst}\left(\int e^{-\frac{4a}{b}+\frac{4x^2}{b}}\,dx\right.}{}$$

$$= -\frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,d} - \frac{e^3e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d} + \frac{e^3e^{-}}{}$$

Mathematica [A] time = 0.41, size = 205, normalized size = 0.94

$$\frac{e^3 e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+2\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+e^{\frac{6a}{b}}\sqrt{\cdots}\right.}{32d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c +
 d*x]))/b] + 2*Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamm
a[1/2, (-2*(a + b*ArcCosh[c + d*x]))/b] + E^((6*a)/b)*Sqrt[a/b + ArcCosh[c
+ d*x]]*(2*Sqrt[2]*Gamma[1/2, (2*(a + b*ArcCosh[c + d*x]))/b] + E^((2*a)/b)
*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(32*d*E^((4*a)/b)*Sqrt[a + b
*ArcCosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{\sqrt{b\,\mathrm{arcosh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arccosh(d*x + c) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{\sqrt{a+b\,\mathrm{arccosh}\,(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{\sqrt{b\,\mathrm{arcosh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx+\int \frac{d^3x^3}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx+\int \frac{3cd^2x^2}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx+\int \frac{3}{\sqrt{a+b\,\mathrm{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**3*(Integral(c**3/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**3*x**3/sqr
t(a + b*acosh(c + d*x)), x) + Integral(3*c*d**2*x**2/sqrt(a + b*acosh(c + d
*x)), x) + Integral(3*c**2*d*x/sqrt(a + b*acosh(c + d*x)), x))
```

3.176 $\int \frac{(ce+dex)^2}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=214

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{3}}\,e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{3}}\,e^2e^{-}}{}$$

```
[Out] -1/24*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2
)*Pi^(1/2)/d/b^(1/2)+1/24*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*3^(1/2)*Pi^(1/2)/d/exp(3*a/b)/b^(1/2)-1/8*e^2*exp(a/b)*erf((a+b*arccosh
(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/8*e^2*erfi((a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5866, 12, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}-\frac{\sqrt{\frac{\pi}{3}}\,e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\,\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d}+\frac{\sqrt{\frac{\pi}{3}}\,e^2e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*Sqrt[b
]*d) - (e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]])/(8*Sqrt[b]*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]
]/Sqrt[b]])/(8*Sqrt[b]*d*E^(a/b)) + (e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a +
b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*Sqrt[b]*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4\sqrt{a+bx}}+\frac{\sinh(3x)}{4\sqrt{a+bx}}\right)dx, x, \cosh^{-1}(c+dx)\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d} + \frac{e^2\,\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(\cdots\right)}{4d}$$

$$= -\frac{e^2\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{8d} - \frac{e^2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(\cdots\right)}{8d}$$

$$= -\frac{e^2\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{4bd} - \frac{e^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x \cdots\right)}{4\cdots}$$

$$= -\frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d} - \frac{e^2e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,d} + \frac{e^2e^{-\cdots}}{\cdots}$$

Mathematica [A] time = 0.38, size = 216, normalized size = 1.01

$$\frac{e^2 e^{-\frac{3a}{b}} \left(3e^{\frac{4a}{b}} \sqrt{\frac{a}{b} + \cosh^{-1}(c + dx)}\, \Gamma\left(\frac{1}{2}, \frac{a}{b} + \cosh^{-1}(c + dx)\right) + \sqrt{3} \sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\, \Gamma\left(\frac{1}{2}, -\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right) + 3\right.}{24d\sqrt{a + b\cosh^{-1}(c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e^2*(3*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c
 + d*x]] + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcCosh[c + d*x]))/b] + 3*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]
*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
 ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]))/(24*d*E^((3
*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{\sqrt{b \operatorname{arcosh}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arccosh(d*x + c) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{\sqrt{a + b \operatorname{arccosh}(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{\sqrt{b \operatorname{arcosh}(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^2}{\sqrt{a+b\,\mathrm{acosh}\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx+\int \frac{d^2x^2}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx+\int \frac{2cdx}{\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e**2*(Integral(c**2/sqrt(a + b*acosh(c + d*x)), x) + Integral(d**2*x**2/sqr
t(a + b*acosh(c + d*x)), x) + Integral(2*c*d*x/sqrt(a + b*acosh(c + d*x)),
x))
```

3.177 $\int \frac{ce+dex}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

```
[Out] -1/8*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/d/b^(1/2)+1/8*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/d/exp(2*a/b)/b^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5866, 12, 5670, 5448, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}-\frac{\sqrt{\frac{\pi}{2}}ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[
b]])/(4*Sqrt[b]*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x
]])/Sqrt[b]])/(4*Sqrt[b]*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ce+dex}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{ex}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\,\text{Subst}\left(\int \frac{x}{\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+dx\right)}{d} \\
&= \frac{e\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d} \\
&= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{d} \\
&= \frac{e\,\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{2d} \\
&= -\frac{e\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{4d} + \frac{e\,\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c\right.}{4d} \\
&= -\frac{e\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right)}{2bd} + \frac{e\,\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, \right.}{2} \\
&= -\frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d} + \frac{ee^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,d}
\end{aligned}
$$

Mathematica [B] time = 1.43, size = 306, normalized size = 2.71

$$
\frac{e\left(\frac{4\sqrt{\pi}\,ce^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{\sqrt{2\pi}\,e^{\frac{2a}{b}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{4\sqrt{\pi}\,ce^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}} + \frac{\sqrt{2\pi}\,e^{-\frac{2a}{b}}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh\right.}{\sqrt{b}}\right.}{\sqrt{b}}\right.}{8d}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (e*((4*c*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/Sqrt[b
] - (E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt
[b]])/Sqrt[b] - (4*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(
Sqrt[b]*E^(a/b)) + (Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/
Sqrt[b]])/(Sqrt[b]*E^((2*a)/b)) + (4*c*E^(a/b)*Sqrt[a/b + ArcCosh[c + d*x]]
*Gamma[1/2, a/b + ArcCosh[c + d*x]])/Sqrt[a + b*ArcCosh[c + d*x]] + (4*c*Sq
rt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]
)/(E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])))/(8*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\sqrt{b \operatorname{arcosh}\left(dx + c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arccosh(d*x + c) + a), x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\sqrt{a + b \,\mathrm{arccosh}\left(dx + c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{\sqrt{b \operatorname{arcosh}\left(dx + c\right) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e + d\,e\,x}{\sqrt{a + b\,\mathrm{acosh}\left(c + d\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{\sqrt{a + b\operatorname{acosh}(c + dx)}}\,dx + \int \frac{dx}{\sqrt{a + b\operatorname{acosh}(c + dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] e*(Integral(c/sqrt(a + b*acosh(c + d*x)), x) + Integral(d*x/sqrt(a + b*acos
h(c + d*x)), x))
```

3.178 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

```
[Out] -1/2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/b^(1/2)+1/
2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/d/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5864, 5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d)
 + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(2*Sqrt[b]*d*E^(a/
b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(c+dx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\cosh^{-1}(c+dx)\right)}{2bd} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+\right.}{2bd}$$
$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(c+dx)}\right)}{bd} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a}\right.}{bd}$$
$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,d}$$

Mathematica [A] time = 0.11, size = 110, normalized size = 1.20

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)\right)}{2d\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c + d*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]
] + Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x
])/b)])/(2*d*E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\text{arcosh}\,(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{arccosh}(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \operatorname{arcosh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(c + d*x)), x)
```

3.179 $\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\sqrt{a+b\cosh^{-1}(c+dx)}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^(1/2),x)/e
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]),x]
[Out] Defer[Subst][Defer[Int][1/(x*Sqrt[a + b*ArcCosh[x]]), x], x, c + d*x]/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\cosh^{-1}(x)}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\sqrt{a+b\cosh^{-1}(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]),x]
[Out] Integrate[1/((c*e + d*e*x)*Sqrt[a + b*ArcCosh[c + d*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\sqrt{b\operatorname{arcosh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a)), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\sqrt{a + b\operatorname{arccosh}(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\sqrt{b\operatorname{arcosh}(dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*e*x + c*e)*sqrt(b*arccosh(d*x + c) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e + d\,e\,x)\,\sqrt{a + b\operatorname{acosh}(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\sqrt{a+b\operatorname{acosh}(c+dx)}+dx\sqrt{a+b\operatorname{acosh}(c+dx)}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(c*sqrt(a + b*acosh(c + d*x)) + d*x*sqrt(a + b*acosh(c + d*x))),
 x)/e
```

3.180 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=374

$$\frac{\sqrt{\pi}\,e^4 e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}\,e^4 e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{5\pi}\,e^4 e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\cdots$$

```
[Out] 1/8*e^4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d
+1/8*e^4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/
b)+3/16*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/d+3/16*e^4*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)+1/16*e^4*exp(5*a/b)*erf(5^(1/2)
*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d+1/16*e^4*er
fi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/d/e
xp(5*a/b)-2*e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(
d*x+c))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5866, 12, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^4 e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d}+\frac{3\sqrt{3\pi}\,e^4 e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\frac{\sqrt{5\pi}\,e^4 e^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{16b^{3/2}d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(8*b^(3/2)*d) + (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (e^4*E^((5*a)/b)*Sqrt[5*
Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d) + (
e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(8*b^(3/2)*d*E^(a/
b)) + (3*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]
])/(16*b^(3/2)*d*E^((3*a)/b)) + (e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(16*b^(3/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{\left(2e^4\right)\text{Subst}\left(\int\left(-\frac{\cosh(x)}{8\sqrt{a+bx}} - \frac{9\cosh(3x}{16\sqrt{a+bx}}\right.\right.}{b}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c\right.}{4bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c\right.}{8bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cos}\right.}{4b^2d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^4e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{3/2}d} + \cdots$$

Mathematica [A] time = 1.54, size = 396, normalized size = 1.06

$$e^4e^{-\frac{5a}{b}}\left(-4e^{\frac{5a}{b}}\sqrt{\frac{c+dx-1}{c+dx+1}}(c+dx+1)-6e^{\frac{5a}{b}}\sinh\left(3\cosh^{-1}(c+dx)\right)-2e^{\frac{5a}{b}}\sinh\left(5\cosh^{-1}(c+dx)\right)-2e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^4*(-4*E^((5*a)/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - 2*E
^((6*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]]
+ Sqrt[5]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-5*(a + b*ArcCosh
[c + d*x]))/b] + 3*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*
Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b] + 2*E^((4*a)/b)*Sqrt[-((a + b*A
rcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] - 3*Sqrt[3]*
E^((8*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d
*x]))/b] - Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (5*
(a + b*ArcCosh[c + d*x]))/b] - 6*E^((5*a)/b)*Sinh[3*ArcCosh[c + d*x]] - 2*E
^((5*a)/b)*Sinh[5*ArcCosh[c + d*x]]))/(16*b*d*E^((5*a)/b)*Sqrt[a + b*ArcCos
h[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arccosh}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^4}{(a+b\,\mathrm{acosh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{d^4}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**4*(Integral(c**4/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**4*x**4/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(4*c*d**3*x*
*3/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)), x) + Integral(6*c**2*d**2*x**2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt
(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(4*c**3*d*x/(a*sqrt(a
+ b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x))
```

3.181 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\sqrt{\frac{\pi}{2}}\ldots$$

```
[Out] 1/4*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(3/2)/d+1/4*e^3*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))
*2^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)+1/4*e^3*exp(4*a/b)*erf(2*(a+b*arccos
h(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+1/4*e^3*erfi(2*(a+b*arccosh(d*x
+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(4*a/b)-2*e^3*(d*x+c)^3*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5866, 12, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{3/2}d}+\frac{\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\sqrt{\frac{\pi}{2}}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c +
 d*x]])/Sqrt[b]])/(4*b^(3/2)*d) + (e^3*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*
Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*b^(3/2)*d) + (e^3*Sqrt[Pi]*Erfi[
(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((4*a)/b)) + (e^3
*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(2*b^(3/2
)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{\left(2e^3\right)\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2\sqrt{a+bx}} - \frac{\cosh(4x)}{2\sqrt{a+bx}}\right.\right.}{bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c\right.}{bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c\right.}{2bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\,\text{co}}\right.}{b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^3e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} +$$

Mathematica [A] time = 1.02, size = 265, normalized size = 0.99

$$e^3 e^{-\frac{4a}{b}}\left(\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)+\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)-e^{\frac{4a}{b}}\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^3*(Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c +
 d*x]))/b] + Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcCosh[c + d*x]))/b] - E^((4*a)/b)*(8*(c + d*x)^3*Sqrt[(-1
 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + A
rcCosh[c + d*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c + d*x]))/b] + E^((4*a)/b)*S
qrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(
4*b*d*E^((4*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\,\mathrm{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\,\mathrm{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e+d\,e\,x)^3}{(a+b\,\mathrm{acosh}\,(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx+\int \frac{d^3}{a\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+b\sqrt{a+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**3*(Integral(c**3/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**3*x**3/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(3*c*d**2*x*
*2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)), x) + Integral(3*c**2*d*x/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b
*acosh(c + d*x))*acosh(c + d*x)), x))
```

3.182 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{\sqrt{\pi}\,e^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\sqrt{3\pi}\,e$$

```
[Out] 1/4*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d
+1/4*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/
b)+1/4*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/
2)*Pi^(1/2)/b^(3/2)/d+1/4*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*3^(1/2)*Pi^(1/2)/b^(3/2)/d/exp(3*a/b)-2*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(
d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5866, 12, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d}+\sqrt{3\pi}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*A
rcCosh[c + d*x]]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/
Sqrt[b]])/(4*b^(3/2)*d) + (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a +
 b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcCosh[c + d*x]]/Sqrt[b]])/(4*b^(3/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi
[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(4*b^(3/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{\left(2e^2\right)\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}} - \frac{3\cosh(3x}{4\sqrt{a+bx}}\right.\right.}{bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c\right.}{2bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(c\right.}{4bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\cos\right.}{2b^2d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{4b^{3/2}d} + \cdots$$

Mathematica [A] time = 1.63, size = 265, normalized size = 1.01

$$e^2e^{-\frac{3a}{b}}\left(-2e^{\frac{3a}{b}}\left(\sqrt{\frac{c+dx-1}{c+dx+1}}\,(c+dx+1)+\sinh\left(3\cosh^{-1}(c+dx)\right)\right)-e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e^2*(-(E^((4*a)/b)*Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c
 + d*x]]) + Sqrt[3]*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, (-3*(a +
 b*ArcCosh[c + d*x]))/b] + E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c + d*x])/b)]*
Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b +
ArcCosh[c + d*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b] - 2*E^((3*a)/b
)*(Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + Sinh[3*ArcCosh[c + d*
x]])))/(4*b*d*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^2}{(a + b\,\mathrm{acosh}(c + d\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)}\,dx+\int \frac{d^2}{a\sqrt{a+b\operatorname{acosh}(c+dx)}+b\sqrt{a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e**2*(Integral(c**2/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x)), x) + Integral(d**2*x**2/(a*sqrt(a + b*acosh(c + d*x)
) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)), x) + Integral(2*c*d*x/(a*
sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)),
x))
```

3.183 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

```
[Out] 1/2*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi
^(1/2)/b^(3/2)/d+1/2*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/d/exp(2*a/b)-2*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 12, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\frac{\pi}{2}}\,ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2e\sqrt{c+dx-1}\sqrt{c+dx+1}\,(c+dx)}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*ArcCo
sh[c + d*x]]) + (e*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c
 + d*x]])/Sqrt[b]])/(b^(3/2)*d) + (e*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*Ar
cCosh[c + d*x]])/Sqrt[b]])/(b^(3/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{(2e)\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c\right.}{bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+d\right.}{bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{(2e)\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\,\text{co}}\right.}{b^2d}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{ee^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \cdots e$$

Mathematica [B] time = 6.71, size = 314, normalized size = 2.03

$$\frac{e\left(-2\sqrt{\pi}\,c\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)+\sqrt{2\pi}\left(\sinh\left(\frac{2a}{b}\right)+\cosh\left(\frac{2a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(3/2),x]
```

```
[Out] (e*((-2*c*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/E^(a/b) + (S
qrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/E^((2*a)/b)
 - 2*c*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh
[a/b]) + Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*(Co
sh[(2*a)/b] + Sinh[(2*a)/b]) - (2*Sqrt[b]*(c*E^((2*a)/b)*Sqrt[a/b + ArcCosh
[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]] - c*Sqrt[-((a + b*ArcCosh[c +
 d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + E^(a/b)*Sinh[2*ArcCo
sh[c + d*x]]))/(E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])))/(2*b^(3/2)*d)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(3/2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b \operatorname{arccosh}(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,e + d\,e\,x}{{\left(a + b\,\mathrm{acosh}\left(c + d\,x\right)\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a\sqrt{a + b\operatorname{acosh}(c + dx)} + b\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx)}\,dx + \int \frac{dx}{a\sqrt{a + b\operatorname{acosh}(c + dx)} + b\sqrt{a +}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(3/2),x)
```

```
[Out] e*(Integral(c/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a + b*acosh(c + d*x))*
acosh(c + d*x)), x) + Integral(d*x/(a*sqrt(a + b*acosh(c + d*x)) + b*sqrt(a
 + b*acosh(c + d*x))*acosh(c + d*x)), x))
```

3.184 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{\sqrt{\pi}\, e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}\, e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

```
[Out] exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d+erfi((a
+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/d/exp(a/b)-2*(d*x+c-1)^(
1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5864, 5656, 5781, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\, e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}+\frac{\sqrt{\pi}\, e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d}-\frac{2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(b*d*Sqrt[a + b*ArcCosh[c + d*x]]
) + (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d
) + (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(b^(3/2)*d*E^(a/b
))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,\sqrt{a+b\cosh^{-1}(x)}}\,dx, x, c+\right.}{bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(c+dx)\right)}{bd} + \frac{\text{Sub}}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c+dx)}\right.}{b^2d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{bd\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\right.}{b}$$

Mathematica [A] time = 0.18, size = 145, normalized size = 1.13

$$\frac{e^{-\frac{a}{b}}\left(-2e^{a/b}\sqrt{\frac{c+dx-1}{c+dx+1}}\,(c+dx+1)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\,\Gamma\left(\frac{1}{2},-\right.\right.}{bd\sqrt{a+b\cosh^{-1}(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*E^(a/b)*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - E^((2*a)/b)*
Sqrt[a/b + ArcCosh[c + d*x]]*Gamma[1/2, a/b + ArcCosh[c + d*x]] + Sqrt[-((a
 + b*ArcCosh[c + d*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])/(b*d*
E^(a/b)*Sqrt[a + b*ArcCosh[c + d*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-3/2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{arccosh}(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\operatorname{acosh}(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(3/2),x)
[Out] Integral((a + b*acosh(c + d*x))**(-3/2), x)
```

3.185 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^(3/2),x)/e

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(3/2)), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(3/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\,\mathrm{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2)), x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\,(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\,\mathrm{arcosh}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\,(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ac\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+adx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+bc\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+bdx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(3/2),x)
[Out] Integral(1/(a*c*sqrt(a + b*acosh(c + d*x)) + a*d*x*sqrt(a + b*acosh(c + d*x
)) + b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b*d*x*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x)), x)/e
```

3.186 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{\sqrt{\pi}\,e^4e^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}-\frac{5\sqrt{5\pi}\,e^4e^{\frac{5a}{b}}\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}+$$

```
[Out] -1/12*e^4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)
/d+1/12*e^4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp
(a/b)-3/8*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^
(1/2)*Pi^(1/2)/b^(5/2)/d+3/8*e^4*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^
(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)-5/24*e^4*exp(5*a/b)*erf(5^(1/2
)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d+5/24*e^4*e
rfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(5/2)/d/
exp(5*a/b)-2/3*e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arcco
sh(d*x+c))^(3/2)+16/3*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^(1/2)-20/3*e
^4*(d*x+c)^5/b^2/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 1.77, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^4e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{5/2}d}-\frac{3\sqrt{3\pi}\,e^4e^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{8b^{5/2}d}-\frac{5\sqrt{5\pi}\,e^4e^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{24b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (16*e^4*(c + d*x)^3)/(3*b^2*d*Sqrt[a + b*ArcCosh[c
+ d*x]]) - (20*e^4*(c + d*x)^5)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (e
^4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*
d) - (3*e^4*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]
])/Sqrt[b]])/(8*b^(5/2)*d) - (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(24*b^(5/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(12*b^(5/2)*d*E^(a/b)) + (3*e^4*Sqrt[3
*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(8*b^(5/2)*d*E^(
(3*a)/b)) + (5*e^4*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(24*b^(5/2)*d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{\left(8e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\,\sqrt{1+x}\,\left(a+b\operatorname{co}\right.}\right.}{3bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - $$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - $$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - $$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - $$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - $$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - $$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{16e^4(c+dx)^3}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - $$

Mathematica [A] time = 3.66, size = 615, normalized size = 1.39

$$e^4e^{-5\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\left(-10\sqrt{5}\,be^{5\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{5\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)-18\sqrt{3}\,be^{\frac{2a}{b}+5\cosh^{-1}(c\cdot}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^4*(-10*Sqrt[5]*b*E^(5*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^
(3/2)*Gamma[1/2, (-5*(a + b*ArcCosh[c + d*x]))/b] - 18*Sqrt[3]*b*E^((2*a)/b
 + 5*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3
*(a + b*ArcCosh[c + d*x]))/b] + 2*E^(4*(a/b + ArcCosh[c + d*x]))*(2*E^((2*a
```

```
)/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x
])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c + d*x]*Sq
rt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + (1 + E^(2*ArcCosh[c + d*x]
))*(a + b*ArcCosh[c + d*x])) + b*E^ArcCosh[c + d*x]*(-((a + b*ArcCosh[c + d
*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])) + 3*E^((5*a)/b +
 2*ArcCosh[c + d*x])*(b - 6*a*(1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c
+ d*x] - b*E^(6*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3
*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c +
d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b]) + 2*E^((5*a)/b)*(-1/2*(b*
(-1 + E^(10*ArcCosh[c + d*x]))) - 5*(1 + E^(10*ArcCosh[c + d*x]))*(a + b*Ar
cCosh[c + d*x]) + 5*Sqrt[5]*E^(5*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCo
sh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (5*(a + b*ArcCosh[c + d*x]
))/b]))))/(48*b^2*d*E^(5*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^
(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\,\mathrm{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^4}{(b\,\mathrm{arcosh}\,(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^2\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)**2), x) + Integral(d**4*x**4/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*ac
osh(c + d*x)**2), x) + Integral(4*c*d**3*x**3/(a**2*sqrt(a + b*acosh(c + d*
x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x)**2), x) + Integral(6*c**2*d**2*x**2/(a**2*sqrt(
a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b
**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x) + Integral(4*c**3*d*x
/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(
c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x))
```

3.187 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{2\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{2}}{}$$

```
[Out] -2/3*e^3*exp(4*a/b)*erf(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5
/2)/d+2/3*e^3*erfi(2*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d
/exp(4*a/b)-1/3*e^3*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/
2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d+1/3*e^3*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e^3*(d*x+c)^3*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)+4*e^3*(d*x+c)^2/b^
2/d/(a+b*arccosh(d*x+c))^(1/2)-16/3*e^3*(d*x+c)^4/b^2/d/(a+b*arccosh(d*x+c)
)^(1/2)
```

Rubi [A] time = 1.44, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}e^3e^{\frac{4a}{b}}\text{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{\sqrt{2\pi}e^3e^{\frac{2a}{b}}\text{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^3e^{-\frac{4a}{b}}\text{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (4*e^3*(c + d*x)^2)/(b^2*d*Sqrt[a + b*ArcCosh[c + d
*x]]) - (16*e^3*(c + d*x)^4)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*e^
3*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^
(5/2)*d) - (e^3*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c +
d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((4*a)/b)) + (e^3*Sqrt[2*Pi]*Erfi[(Sqr
t[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{\left(2e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-}\right.}\right.}{bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e^3(c+dx)^2}{b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d}$$

Mathematica [A] time = 2.47, size = 391, normalized size = 1.17

$$e^3e^{-4\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\left(-16be^{4\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)-8\sqrt{2}\,be^{\frac{2a}{b}+4\cosh^{-1}(c+dx)}\left(-\frac{a}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^3*(-16*b*E^(4*ArcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Ga
mma[1/2, (-4*(a + b*ArcCosh[c + d*x]))/b] - 8*Sqrt[2]*b*E^((2*a)/b + 4*ArcC
osh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-2*(a + b*A
rcCosh[c + d*x]))/b] + E^((4*a)/b)*(-((1 + E^(2*ArcCosh[c + d*x]))^2*(b*(-1
```

```
 + E^(4*ArcCosh[c + d*x])) + 8*a*(1 - E^(2*ArcCosh[c + d*x]) + E^(4*ArcCosh
[c + d*x])) + 8*b*(1 - E^(2*ArcCosh[c + d*x]) + E^(4*ArcCosh[c + d*x]))*Arc
Cosh[c + d*x])) + 8*Sqrt[2]*E^((2*a)/b + 4*ArcCosh[c + d*x])*Sqrt[a/b + Arc
Cosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (2*(a + b*ArcCosh[c + d*
x]))/b] + 16*E^(4*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a
 + b*ArcCosh[c + d*x])*Gamma[1/2, (4*(a + b*ArcCosh[c + d*x]))/b])))/(24*b^
2*d*E^(4*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3/2))
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(5/2), x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(a+b\,\mathrm{arccosh}\,(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^3}{(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3\left(\int \frac{c^3}{a^2\sqrt{a+b\operatorname{acosh}(c+dx)}+2ab\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+b^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}^2(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)**2), x) + Integral(d**3*x**3/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*ac
osh(c + d*x)**2), x) + Integral(3*c*d**2*x**2/(a**2*sqrt(a + b*acosh(c + d*
x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x)**2), x) + Integral(3*c**2*d*x/(a**2*sqrt(a + b*
acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sq
rt(a + b*acosh(c + d*x))*acosh(c + d*x)**2), x))
```

3.188 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=328

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}}{}$$

```
[Out] -1/6*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/
d+1/6*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a
/b)-1/2*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(5/2)/d+1/2*e^2*erfi(3^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1
/2))*3^(1/2)*Pi^(1/2)/b^(5/2)/d/exp(3*a/b)-2/3*e^2*(d*x+c)^2*(d*x+c-1)^(1/2
)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)+8/3*e^2*(d*x+c)/b^2/d/(a+b
*arccosh(d*x+c))^(1/2)-4*e^2*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205, 5658}

$$-\frac{\sqrt{\pi}e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}-\frac{\sqrt{3\pi}e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{2b^{5/2}d}+\frac{\sqrt{\pi}e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{6b^{5/2}d}+\frac{\sqrt{3\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(3*b*d*(a + b*Arc
Cosh[c + d*x])^(3/2)) + (8*e^2*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d
*x]]) - (4*e^2*(c + d*x)^3)/(b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (e^2*E^(
a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(6*b^(5/2)*d) - (e
^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b
]])/(2*b^(5/2)*d) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]
])/(6*b^(5/2)*d*E^(a/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]])/(2*b^(5/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,(a+b\cosh\cdots}\right.}{3bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \cdots$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \cdots$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \cdots$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \cdots$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \cdots$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \cdots$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{8e^2(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \cdots$$

Mathematica [A] time = 3.04, size = 391, normalized size = 1.19

$$e^2e^{-3\left(\frac{a}{b}+\cosh^{-1}(c+dx)\right)}\left(-6\sqrt{3}\,be^{3\cosh^{-1}(c+dx)}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(c+dx)\right)}{b}\right)-2be^{\frac{2a}{b}+3\cosh^{-1}(c+dx)}\left(-\cdots\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(2*E^((4*a)/b + 3*ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a +
b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 6*Sqrt[3]*b*E^(3*A
rcCosh[c + d*x])*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-3*(a +
b*ArcCosh[c + d*x]))/b] - 2*b*E^((2*a)/b + 3*ArcCosh[c + d*x])*(-((a + b*Ar
cCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)] + E^((3
*a)/b)*(-((1 + E^(2*ArcCosh[c + d*x]))*(a*(6 - 4*E^(2*ArcCosh[c + d*x]) + 6
*E^(4*ArcCosh[c + d*x])) + b*(-1 + 6*ArcCosh[c + d*x] - 4*E^(2*ArcCosh[c +
d*x])*ArcCosh[c + d*x] + E^(4*ArcCosh[c + d*x])*(1 + 6*ArcCosh[c + d*x]))))
 + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a
 + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a + b*ArcCosh[c + d*x]))/b])))/(12*b^
2*d*E^(3*(a/b + ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(5/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arccosh}(dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^2}{(a + b \operatorname{acosh}(c + d\,x))^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2 \left(\int \frac{c^2}{a^2 \sqrt{a + b \operatorname{acosh}(c + dx)} + 2ab \sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}(c + dx) + b^2 \sqrt{a + b \operatorname{acosh}(c + dx)} \operatorname{acosh}^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*aco
sh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x
)**2), x) + Integral(d**2*x**2/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqr
t(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*ac
osh(c + d*x)**2), x) + Integral(2*c*d*x/(a**2*sqrt(a + b*acosh(c + d*x)) +
2*a*b*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x)**2), x))
```

3.189 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{2\sqrt{2\pi}\,ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{}{3b^2d}$$

```
[Out] -2/3*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(5/2)/d+2/3*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)/d/exp(2*a/b)-2/3*e*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)
^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)+4/3*e/b^2/d/(a+b*arccosh(d*x+c))^(1/2
)-8/3*e*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5866, 12, 5668, 5775, 5670, 5448, 3308, 2180, 2204, 2205, 5676}

$$-\frac{2\sqrt{2\pi}\,ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{8e(c+dx)^2}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}+\frac{}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh
[c + d*x])^(3/2)) + (4*e)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (8*e*(c
+ d*x)^2)/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*e*E^((2*a)/b)*Sqrt[2*
Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5/2)*d) + (2
*e*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(3*b^(5
/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)}\right.}{3bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4e}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{}{3b^2d\sqrt{}}$$

Mathematica [B] time = 5.48, size = 687, normalized size = 3.18

$$\frac{e\left(-2b^{3/2}ce^{-\frac{a}{b}}\left(-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(c+dx)}{b}\right)+2\sqrt{\pi}\,c\cosh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}\text{erf}\left(\frac{\sqrt{a+b\cos}}{\sqrt{}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(5/2),x]
```

```
[Out] (e*(4*a*Sqrt[b]*c*(c + d*x) + 4*b^(3/2)*c*(c + d*x)*ArcCosh[c + d*x] - (2*S
qrt[b]*c*(1 + E^(2*ArcCosh[c + d*x]))*(a + b*ArcCosh[c + d*x]))/E^ArcCosh[c
 + d*x] - 4*a*Sqrt[b]*Cosh[2*ArcCosh[c + d*x]] - 4*b^(3/2)*ArcCosh[c + d*x]
```

```
*Cosh[2*ArcCosh[c + d*x]] + 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Cos
h[a/b]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] - 2*Sqrt[2*Pi]*(a + b*ArcC
osh[c + d*x])^(3/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]]
)/Sqrt[b]] - 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Cosh[a/b]*Erfi[Sqr
t[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 2*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^
(3/2)*Cosh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] +
2*Sqrt[b]*c*E^(a/b)*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*G
amma[1/2, a/b + ArcCosh[c + d*x]] - (2*b^(3/2)*c*(-((a + b*ArcCosh[c + d*x]
)/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)])/E^(a/b) + 2*c*Sqrt[P
i]*(a + b*ArcCosh[c + d*x])^(3/2)*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]
*Sinh[a/b] + 2*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] - 2*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^
(3/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - 2
*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(3/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh
[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - b^(3/2)*Sinh[2*ArcCosh[c + d*x]]))/(3*
b^(5/2)*d*(a + b*ArcCosh[c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(5/2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b \,\mathrm{arccosh}(dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e+d\,e\,x}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(5/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+2ab\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+b^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}^2\,(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(5/2),x)
```

```
[Out] e*(Integral(c/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acosh(c +
 d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2),
 x) + Integral(d*x/(a**2*sqrt(a + b*acosh(c + d*x)) + 2*a*b*sqrt(a + b*acos
h(c + d*x))*acosh(c + d*x) + b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)
**2), x))
```

3.190 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2\sqrt{c+dx-}}{3bd\left(a+b\cosh\right.}$$

[Out] -2/3*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d+2/3*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/d/exp(a/b)-2/3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(3/2)-4/3*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(1/2)

Rubi [A] time = 0.41, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5864, 5656, 5775, 5658, 3308, 2180, 2205, 2204}

$$-\frac{2\sqrt{\pi}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}+\frac{2\sqrt{\pi}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{3b^{5/2}d}-\frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{2\sqrt{c+dx-}}{3bd\left(a+b\cosh\right.}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c + d*x])^(-5/2),x]

[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*b*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (4*(c + d*x))/(3*b^2*d*Sqrt[a + b*ArcCosh[c + d*x]]) - (2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d) + (2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(3*b^(5/2)*d*E^(a/b))

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{3/2}}\,dx, x\right.}{3bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} + \frac{4\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{4\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\sqrt{1+c+dx}}{3bd\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{4(c+dx)}{3b^2d\sqrt{a+b\cosh^{-1}(c+dx)}} - \frac{2e^{a/b}\sqrt{\pi}\,\text{er}}{}$$

Mathematica [A] time = 1.21, size = 219, normalized size = 1.33

$$\frac{e^{-\frac{a+b\cosh^{-1}(c+dx)}{b}}\left(2e^{\frac{2a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-2\left(e^{a}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-5/2),x]
```

```
[Out] (2*E^((2*a)/b + ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCo
sh[c + d*x])*Gamma[1/2, a/b + ArcCosh[c + d*x]] - 2*(E^(a/b)*(b*E^ArcCosh[c
 + d*x]*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + (1 + E^(2*ArcCos
h[c + d*x]))*(a + b*ArcCosh[c + d*x])) + b*E^ArcCosh[c + d*x]*(-((a + b*Arc
Cosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/(3*b^2
*d*E^((a + b*ArcCosh[c + d*x])/b)*(a + b*ArcCosh[c + d*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x + c) + a)^(-5/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b \operatorname{arccosh}(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(5/2),x)
[Out] int(1/(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x + c) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(5/2),x)
[Out] int(1/(a + b*acosh(c + d*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b \operatorname{acosh}(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(5/2),x)
[Out] Integral((a + b*acosh(c + d*x))**(-5/2), x)
```

3.191 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\operatorname{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}},x\right)}{e}$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^(5/2),x)/e
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(5/2)), x], x, c + d*x]/(d*
e)
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(5/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2)), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)\left(a+b\operatorname{arccosh}(dx+c)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex+ce)(b\operatorname{arcosh}(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(5/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e+d\,e\,x)\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^2c\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+a^2dx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}+2abc\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+2abdx\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\,\mathrm{acosh}\,(c+dx)+b^2c\sqrt{a+b\,\mathrm{acosh}}}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(5/2),x)
[Out] Integral(1/(a**2*c*sqrt(a + b*acosh(c + d*x)) + a**2*d*x*sqrt(a + b*acosh(c
 + d*x)) + 2*a*b*c*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x) + 2*a*b*d*x*sq
rt(a + b*acosh(c + d*x))*acosh(c + d*x) + b**2*c*sqrt(a + b*acosh(c + d*x))
*acosh(c + d*x)**2 + b**2*d*x*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2)
, x)/e
```

3.192 $\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=552

$$\frac{\sqrt{\pi}\,e^4 e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}\,e^4 e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}+\frac{5\sqrt{5\pi}\,e^4 e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\cdots$$

```
[Out] 16/15*e^4*(d*x+c)^3/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-4/3*e^4*(d*x+c)^5/b^2/
d/(a+b*arccosh(d*x+c))^(3/2)+1/30*e^4*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/
2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+1/30*e^4*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)+9/20*e^4*exp(3*a/b)*erf(3^(1/2)*(a+b*arcc
osh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d+9/20*e^4*erfi(3^(1/2)
*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)+
5/12*e^4*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*5^(1/2)
*Pi^(1/2)/b^(7/2)/d+5/12*e^4*erfi(5^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2
))*5^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(5*a/b)-2/5*e^4*(d*x+c)^4*(d*x+c-1)^(1/2)*
(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)+32/5*e^4*(d*x+c)^2*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)-40/3*e^4*(d*x+c)^4*
(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 1.78, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5866, 12, 5668, 5775, 5666, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^4 e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{30b^{7/2}d}+\frac{9\sqrt{3\pi}\,e^4 e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{20b^{7/2}d}+\frac{5\sqrt{5\pi}\,e^4 e^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{12b^{7/2}d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (16*e^4*(c + d*x)^3)/(15*b^2*d*(a + b*ArcCosh[c + d
*x])^(3/2)) - (4*e^4*(c + d*x)^5)/(3*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2))
+ (32*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b^3*d*Sqrt[a
 + b*ArcCosh[c + d*x]]) - (40*e^4*Sqrt[-1 + c + d*x]*(c + d*x)^4*Sqrt[1 + c
 + d*x])/(3*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (e^4*E^(a/b)*Sqrt[Pi]*Erf
[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(30*b^(7/2)*d) + (9*e^4*E^((3*a)/b)
*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(20*b^(7/2
)*d) + (5*e^4*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*
x]])/Sqrt[b]])/(12*b^(7/2)*d) + (e^4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d
*x]]/Sqrt[b]])/(30*b^(7/2)*d*E^(a/b)) + (9*e^4*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqr
t[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(20*b^(7/2)*d*E^((3*a)/b)) + (5*e^4*Sq
rt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(12*b^(7/2)*
d*E^((5*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^4}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^4x^4}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^4\,\text{Subst}\left(\int \frac{x^4}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{\left(8e^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh\right.}\right.}{5bd}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^4\sqrt{-1+c+dx}\,(c+dx)^4\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{16e^4(c+dx)^3}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Mathematica [A] time = 5.07, size = 654, normalized size = 1.18

$$\frac{e^4\left(-4\left(e^{-\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\left(2e^{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\left(a+b\cosh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^4/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e^4*(-4*(3*b^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) + ((a + b*
ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c +
 d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b
 + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] + ((a + b*ArcCosh[c + d*x])*(E^(a
/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*Arc
Cosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/E^(a/b
)) - 9*(a + b*ArcCosh[c + d*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c + d*x])/
b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b])/E^((3*a)/b) + (2*(b
```

```
+ 6*a*(-1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c + d*x] + b*E^(6*ArcCosh
[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x
]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a
+ b*ArcCosh[c + d*x]))/b]))/E^(3*ArcCosh[c + d*x])) - 5*(a + b*ArcCosh[c +
d*x])*((2*(b + 10*a*(-1 + E^(10*ArcCosh[c + d*x])) - 10*b*ArcCosh[c + d*x]
+ b*E^(10*ArcCosh[c + d*x])*(1 + 10*ArcCosh[c + d*x])))/E^(5*ArcCosh[c + d*
x]) + (20*Sqrt[5]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-5*(a
 + b*ArcCosh[c + d*x]))/b])/E^((5*a)/b) + 20*Sqrt[5]*E^((5*a)/b)*Sqrt[a/b +
 ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (5*(a + b*ArcCosh[c
+ d*x]))/b]) - 18*b^2*Sinh[3*ArcCosh[c + d*x]] - 6*b^2*Sinh[5*ArcCosh[c + d
*x]]))/(240*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^4}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^4/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^4/(b*arccosh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^4}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^4/(a + b*acosh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^4\left(\int \frac{c^4}{a^3\sqrt{a+b\operatorname{acosh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+3ab^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**4/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] e**4*(Integral(c**4/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*
acosh(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(
c + d*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Inte
gral(d**4*x**4/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(
4*c*d**3*x**3/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(
c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*
x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(6
*c**2*d**2*x**2/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acos
h(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c +
d*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral
(4*c**3*d*x/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(c
+ d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)
**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x))
```

3.193 $\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=441

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\cdots$$

```
[Out] 4/5*e^3*(d*x+c)^2/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-16/15*e^3*(d*x+c)^4/b^2/
d/(a+b*arccosh(d*x+c))^(3/2)+16/15*e^3*exp(4*a/b)*erf(2*(a+b*arccosh(d*x+c)
)^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+16/15*e^3*erfi(2*(a+b*arccosh(d*x+c))^(
1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(4*a/b)+4/15*e^3*exp(2*a/b)*erf(2^(1/2)
*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d+4/15*e^3*er
fi(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/e
xp(2*a/b)-2/5*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccos
h(d*x+c))^(5/2)+16/5*e^3*(d*x+c)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b
*arccosh(d*x+c))^(1/2)-128/15*e^3*(d*x+c)^3*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)
/b^3/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 1.41, antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5866, 12, 5668, 5775, 5666, 3307, 2180, 2204, 2205}

$$\frac{16\sqrt{\pi}e^3e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{2\pi}e^3e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{16\sqrt{\pi}e^3e^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (4*e^3*(c + d*x)^2)/(5*b^2*d*(a + b*ArcCosh[c + d*x
])^(3/2)) - (16*e^3*(c + d*x)^4)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2))
+ (16*e^3*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b^3*d*Sqrt[a +
 b*ArcCosh[c + d*x]]) - (128*e^3*Sqrt[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c
+ d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (16*e^3*E^((4*a)/b)*Sqrt[
Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d) + (4*e^3*
E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])
/(15*b^(7/2)*d) + (16*e^3*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c + d*x]])/Sq
rt[b]])/(15*b^(7/2)*d*E^((4*a)/b)) + (4*e^3*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a
 + b*ArcCosh[c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^3}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^3x^3}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^3\,\text{Subst}\left(\int \frac{x^3}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{\left(6e^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-}\right.}\right.}{5bd}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e^3\sqrt{-1+c+dx}\,(c+dx)^3\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e^3(c+dx)^2}{5b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

Mathematica [A] time = 3.10, size = 445, normalized size = 1.01

$$\frac{e^3\left(-2\left(\left(a+b\cosh^{-1}(c+dx)\right)\left(2e^{-2\cosh^{-1}(c+dx)}\left(4a\left(e^{4\cosh^{-1}(c+dx)}-1\right)-4b\cosh^{-1}(c+dx)+be^{4\cosh^{-1}(c+dx)}\left(4\cos\right.\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^3/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e^3*((-4*(a + b*ArcCosh[c + d*x])*(16*b*E^(4*ArcCosh[c + d*x])*(-((a + b*A
rcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, (-4*(a + b*ArcCosh[c + d*x]))/b] + E^
((4*a)/b)*(b + 8*a*(-1 + E^(8*ArcCosh[c + d*x])) - 8*b*ArcCosh[c + d*x] + b
*E^(8*ArcCosh[c + d*x])*(1 + 8*ArcCosh[c + d*x]) + 16*E^(4*(a/b + ArcCosh[c
 + d*x]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2,
(4*(a + b*ArcCosh[c + d*x]))/b])))/E^(4*(a/b + ArcCosh[c + d*x])) - 2*((a +
 b*ArcCosh[c + d*x])*((2*(b + 4*a*(-1 + E^(4*ArcCosh[c + d*x])) - 4*b*ArcCo
```

```
sh[c + d*x] + b*E^(4*ArcCosh[c + d*x])*(1 + 4*ArcCosh[c + d*x])))/E^(2*ArcC
osh[c + d*x]) + (8*Sqrt[2]*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/
2, (-2*(a + b*ArcCosh[c + d*x]))/b])/E^((2*a)/b) + 8*Sqrt[2]*E^((2*a)/b)*Sq
rt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (2*(a + b*Ar
cCosh[c + d*x]))/b]) + 3*b^2*Sinh[2*ArcCosh[c + d*x]] - 3*b^2*Sinh[4*ArcCo
sh[c + d*x]]))/(60*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^3}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^3/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^3/(b*arccosh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,e+d\,e\,x\right)}^3}{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^3/(a + b*acosh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^3 \left(\int \frac{c^3}{a^3\sqrt{a+b\,\mathrm{acosh}\,(c+dx)} + 3a^2b\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\ \mathrm{acosh}\,(c+dx) + 3ab^2\sqrt{a+b\,\mathrm{acosh}\,(c+dx)}\ \mathrm{acosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**3/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] e**3*(Integral(c**3/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*
acosh(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(
c + d*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Inte
gral(d**3*x**3/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(
3*c*d**2*x**2/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(
c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*
x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(3
*c**2*d*x/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(c +
d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**
2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x))
```

3.194 $\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=431

$$\frac{\sqrt{\pi}\,e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3}}{}$$

```
[Out] 8/15*e^2*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-4/5*e^2*(d*x+c)^3/b^2/d/(
a+b*arccosh(d*x+c))^(3/2)+1/15*e^2*exp(a/b)*erf((a+b*arccosh(d*x+c))^(1/2)/
b^(1/2))*Pi^(1/2)/b^(7/2)/d+1/15*e^2*erfi((a+b*arccosh(d*x+c))^(1/2)/b^(1/2
))*Pi^(1/2)/b^(7/2)/d/exp(a/b)+3/5*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(
d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d+3/5*e^2*erfi(3^(1/2)*(a+b
*arccosh(d*x+c))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(3*a/b)-2/5*e
^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)
+16/15*e^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)
-24/5*e^2*(d*x+c)^2*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+
c))^(1/2)
```

Rubi [A] time = 1.47, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5866, 12, 5668, 5775, 5666, 3307, 2180, 2204, 2205, 5656, 5781}

$$\frac{\sqrt{\pi}\,e^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{5b^{7/2}d}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{3\sqrt{3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b*d*(a + b*Arc
Cosh[c + d*x])^(5/2)) + (8*e^2*(c + d*x))/(15*b^2*d*(a + b*ArcCosh[c + d*x]
)^(3/2)) - (4*e^2*(c + d*x)^3)/(5*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) + (
16*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c
 + d*x]]) - (24*e^2*Sqrt[-1 + c + d*x]*(c + d*x)^2*Sqrt[1 + c + d*x])/(5*b^
3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*Ar
cCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d) + (3*e^2*E^((3*a)/b)*Sqrt[3*Pi]*Er
f[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5*b^(7/2)*d) + (e^2*Sqr
t[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E^(a/b)) +
(3*e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]])/(5*
b^(7/2)*d*E^((3*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(ce+dex)^2}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{e^2x^2}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e^2\,\text{Subst}\left(\int \frac{x^2}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{\left(4e^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh\right.}\right.}{5bd}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

$$= -\frac{2e^2\sqrt{-1+c+dx}\,(c+dx)^2\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{8e^2(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}$$

Mathematica [A] time = 2.92, size = 452, normalized size = 1.05

$$e^2\left(-2e^{-\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\left(2e^{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\,\left(a+b\cosh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)^2/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e^2*(-6*b^2*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - (2*(a + b*A
rcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c +
d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b
+ ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] - (2*(a + b*ArcCosh[c + d*x])*(E^(
a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*Ar
cCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/E^(a/
b) - 3*(a + b*ArcCosh[c + d*x])*((12*Sqrt[3]*b*(-((a + b*ArcCosh[c + d*x])/
b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcCosh[c + d*x]))/b])/E^((3*a)/b) + (2*(b
+ 6*a*(-1 + E^(6*ArcCosh[c + d*x])) - 6*b*ArcCosh[c + d*x] + b*E^(6*ArcCosh
[c + d*x])*(1 + 6*ArcCosh[c + d*x]) + 6*Sqrt[3]*E^(3*(a/b + ArcCosh[c + d*x
]))*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, (3*(a
+ b*ArcCosh[c + d*x]))/b]))/E^(3*ArcCosh[c + d*x])) - 6*b^2*Sinh[3*ArcCosh[
c + d*x]]))/(60*b^3*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^2}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^2/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)^2/(b*arccosh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,e + d\,e\,x)^2}{(a + b\,\mathrm{acosh}(c + d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)^2/(a + b*acosh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e^2\left(\int \frac{c^2}{a^3\sqrt{a+b\operatorname{acosh}(c+dx)}+3a^2b\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acosh}(c+dx)+3ab^2\sqrt{a+b\operatorname{acosh}(c+dx)}\operatorname{acc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**2/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] e**2*(Integral(c**2/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*
acosh(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(
c + d*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Inte
gral(d**2*x**2/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh
(c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d
*x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(
2*c*d*x/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(c + d*
x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2
+ b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x))
```

3.195 $\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=266

$$\frac{8\sqrt{2\pi}\,ee^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{32e\sqrt{c+dx-1}\sqrt{c+dx+1}(c+dx)}{15b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}-$$

```
[Out] 4/15*e/b^2/d/(a+b*arccosh(d*x+c))^(3/2)-8/15*e*(d*x+c)^2/b^2/d/(a+b*arccosh
(d*x+c))^(3/2)+8/15*e*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(d*x+c))^(1/2)/b^(
1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d+8/15*e*erfi(2^(1/2)*(a+b*arccosh(d*x+c))^(
1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/exp(2*a/b)-2/5*e*(d*x+c)*(d*x+c-1)
^(1/2)*(d*x+c+1)^(1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)-32/15*e*(d*x+c)*(d*x+
c-1)^(1/2)*(d*x+c+1)^(1/2)/b^3/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5866, 12, 5668, 5775, 5666, 3307, 2180, 2204, 2205, 5676}

$$\frac{8\sqrt{2\pi}\,ee^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{8\sqrt{2\pi}\,ee^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8e(c+dx)^2}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{32e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (-2*e*Sqrt[-1 + c + d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(5*b*d*(a + b*ArcCosh
[c + d*x])^(5/2)) + (4*e)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (8*e*
(c + d*x)^2)/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (32*e*Sqrt[-1 + c
+ d*x]*(c + d*x)*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]])
 + (8*e*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/S
qrt[b]])/(15*b^(7/2)*d) + (8*e*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[
c + d*x]])/Sqrt[b]])/(15*b^(7/2)*d*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5668

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + (-Dist[(c*(m + 1))/(b*(n + 1)), Int[(x^(m + 1)*(a + b*ArcCosh[c*x]
)^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] + Dist[m/(b*c*(n + 1)), I
nt[(x^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),
 x], x]) /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && LtQ[n, -2]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{ce+dex}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{ex}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{e\,\text{Subst}\left(\int \frac{x}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{(2e)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)}\right.}{5bd}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

$$= -\frac{2e\sqrt{-1+c+dx}\,(c+dx)\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{4e}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{}{15}$$

Mathematica [B] time = 4.77, size = 916, normalized size = 3.44

$$\frac{e\left(8c\sqrt{\frac{c+dx-1}{c+dx+1}}\,(c+dx+1)\cosh^{-1}(c+dx)^2b^{5/2} + 4c(c+dx)\cosh^{-1}(c+dx)b^{5/2} - 4\cosh^{-1}(c+dx)\cosh\left(2\cosh^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*e + d*e*x)/(a + b*ArcCosh[c + d*x])^(7/2),x]
```

```
[Out] (e*(4*a*b^(3/2)*c*(c + d*x) + 8*a^2*Sqrt[b]*c*Sqrt[(-1 + c + d*x)/(1 + c +
d*x)]*(1 + c + d*x) + 4*b^(5/2)*c*(c + d*x)*ArcCosh[c + d*x] + 16*a*b^(3/2)
*c*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x] + 8*b^
(5/2)*c*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x)*ArcCosh[c + d*x]^2
 - 4*a*b^(3/2)*Cosh[2*ArcCosh[c + d*x]] - 4*b^(5/2)*ArcCosh[c + d*x]*Cosh[2
*ArcCosh[c + d*x]] - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Cosh[a/b]*
Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a + b*ArcCosh[c +
 d*x])^(5/2)*Cosh[(2*a)/b]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[
b]] - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Cosh[a/b]*Erfi[Sqrt[a + b
*ArcCosh[c + d*x]]/Sqrt[b]] + 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*C
osh[(2*a)/b]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]] - (2*Sqrt
```

```
[b]*c*(a + b*ArcCosh[c + d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b
+ ArcCosh[c + d*x])*Sqrt[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*G
amma[1/2, a/b + ArcCosh[c + d*x]]))/E^ArcCosh[c + d*x] - (2*Sqrt[b]*c*(a +
b*ArcCosh[c + d*x])*(E^(a/b + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c +
d*x]) + 2*b*(-((a + b*ArcCosh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCo
sh[c + d*x])/b)]))/E^(a/b) - 4*c*Sqrt[Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Er
f[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] + 4*c*Sqrt[Pi]*(a + b*Arc
Cosh[c + d*x])^(5/2)*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]]*Sinh[a/b] +
 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])^(5/2)*Erf[(Sqrt[2]*Sqrt[a + b*ArcCos
h[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] - 8*Sqrt[2*Pi]*(a + b*ArcCosh[c + d*x])
^(5/2)*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c + d*x]])/Sqrt[b]]*Sinh[(2*a)/b] -
 16*a^2*Sqrt[b]*Sinh[2*ArcCosh[c + d*x]] - 3*b^(5/2)*Sinh[2*ArcCosh[c + d*x
]] - 32*a*b^(3/2)*ArcCosh[c + d*x]*Sinh[2*ArcCosh[c + d*x]] - 16*b^(5/2)*Ar
cCosh[c + d*x]^2*Sinh[2*ArcCosh[c + d*x]]))/(15*b^(7/2)*d*(a + b*ArcCosh[c
+ d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(7/2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dex + ce}{(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*e*x + c*e)/(b*arccosh(d*x + c) + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,e + d\,e\,x}{(a + b \operatorname{acosh}(c + d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int((c*e + d*e*x)/(a + b*acosh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$e\left(\int \frac{c}{a^{3}\sqrt{a + b\operatorname{acosh}(c + dx)} + 3a^{2}b\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}(c + dx) + 3ab^{2}\sqrt{a + b\operatorname{acosh}(c + dx)}\operatorname{acosh}^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] e*(Integral(c/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(
c + d*x))*acosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*
x)**2 + b**3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x) + Integral(d
*x/(a**3*sqrt(a + b*acosh(c + d*x)) + 3*a**2*b*sqrt(a + b*acosh(c + d*x))*a
cosh(c + d*x) + 3*a*b**2*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**2 + b**
3*sqrt(a + b*acosh(c + d*x))*acosh(c + d*x)**3), x))
```

3.196 $\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=209

$$\frac{4\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{8\sqrt{c+dx-1}\sqrt{c+dx+1}}{15b^3d\sqrt{a+b\cosh^{-1}(c+dx)}}-\frac{4(\ldots}{15b^2d\left(a+b\,c\ldots\right.}$$

```
[Out] -4/15*(d*x+c)/b^2/d/(a+b*arccosh(d*x+c))^(3/2)+4/15*exp(a/b)*erf((a+b*arcco
sh(d*x+c))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d+4/15*erfi((a+b*arccosh(d*x+c))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/d/exp(a/b)-2/5*(d*x+c-1)^(1/2)*(d*x+c+1)^(
1/2)/b/d/(a+b*arccosh(d*x+c))^(5/2)-8/15*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/b^
3/d/(a+b*arccosh(d*x+c))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5864, 5656, 5775, 5781, 3307, 2180, 2204, 2205}

$$\frac{4\sqrt{\pi}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}+\frac{4\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(c+dx)}}{\sqrt{b}}\right)}{15b^{7/2}d}-\frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}}-\frac{8\sqrt{c+dx\ldots}}{15b^3d\sqrt{a+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(5*b*d*(a + b*ArcCosh[c + d*x])^(
5/2)) - (4*(c + d*x))/(15*b^2*d*(a + b*ArcCosh[c + d*x])^(3/2)) - (8*Sqrt[-
1 + c + d*x]*Sqrt[1 + c + d*x])/(15*b^3*d*Sqrt[a + b*ArcCosh[c + d*x]]) + (
4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d
) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c + d*x]]/Sqrt[b]])/(15*b^(7/2)*d*E
^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\left(a+b\cosh^{-1}(x)\right)^{5/2}}\,dx, x\right.}{5bd}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} + \frac{4\,\text{Subs}}{}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1}}{15b^3d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1}}{15b^3d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1}}{15b^3d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1}}{15b^3d}$$

$$= -\frac{2\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{5bd\left(a+b\cosh^{-1}(c+dx)\right)^{5/2}} - \frac{4(c+dx)}{15b^2d\left(a+b\cosh^{-1}(c+dx)\right)^{3/2}} - \frac{8\sqrt{-1}}{15b^3d}$$

Mathematica [A] time = 0.81, size = 243, normalized size = 1.16

$$\frac{-\frac{2e^{-\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\left(2e^{\frac{a}{b}+\cosh^{-1}(c+dx)}\sqrt{\frac{a}{b}+\cosh^{-1}(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(c+dx)\right)-2a-2b\cosh^{-1}(c+dx)+b\right)}{b^2} -}{15bd\left(a+b\,c\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[(-1 + c + d*x)/(1 + c + d*x)]*(1 + c + d*x) - (2*(a + b*ArcCosh[c
+ d*x])*(-2*a + b - 2*b*ArcCosh[c + d*x] + 2*E^(a/b + ArcCosh[c + d*x])*Sqr
t[a/b + ArcCosh[c + d*x]]*(a + b*ArcCosh[c + d*x])*Gamma[1/2, a/b + ArcCosh
[c + d*x]]))/(b^2*E^ArcCosh[c + d*x]) - (2*(a + b*ArcCosh[c + d*x])*(E^(a/b
 + ArcCosh[c + d*x])*(2*a + b + 2*b*ArcCosh[c + d*x]) + 2*b*(-((a + b*ArcCo
sh[c + d*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcCosh[c + d*x])/b)]))/(b^2*E^(
a/b)))/(15*b*d*(a + b*ArcCosh[c + d*x])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcosh}(dx+c)+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-7/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b \operatorname{arccosh}(dx+c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x+c))^(7/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcosh}(dx+c)+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b \operatorname{acosh}(c+d\,x))^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a + b*acosh(c + d*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b \operatorname{acosh}(c+dx))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x+c))**(7/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**(-7/2), x)
```

3.197 $\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Int}\left(\frac{1}{(c+dx)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}},x\right)}{e}$$

[Out] Unintegrable(1/(d*x+c)/(a+b*arccosh(d*x+c))^(7/2),x)/e

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)),x]

[Out] Defer[Subst][Defer[Int][1/(x*(a + b*ArcCosh[x])^(7/2)), x], x, c + d*x]/(d*e)

Rubi steps

$$\begin{aligned}\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ex\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\left(a+b\cosh^{-1}(x)\right)^{7/2}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(ce+dex)\left(a+b\cosh^{-1}(c+dx)\right)^{7/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)),x]

[Out] Integrate[1/((c*e + d*e*x)*(a + b*ArcCosh[c + d*x])^(7/2)), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2)), x)
```

maple [**A**] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)\,(a + b \operatorname{arccosh}(dx + c))^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
[Out] int(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dex + ce)(b \operatorname{arcosh}(dx + c) + a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*arccosh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate(1/((d*e*x + c*e)*(b*arccosh(d*x + c) + a)^(7/2)), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(c\,e + d\,e\,x)\,\left(a + b \operatorname{acosh}(c + d\,x)\right)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2)),x)
[Out] int(1/((c*e + d*e*x)*(a + b*acosh(c + d*x))^(7/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*e*x+c*e)/(a+b*acosh(d*x+c))**(7/2),x)
[Out] Timed out
```

3.198 $\int (ce+dex)^{7/2} \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=189

$$\frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)}{9de}-\frac{28be^3\sqrt{-c-dx+1}\sqrt{e(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{135d\sqrt{-c-dx}\sqrt{c+dx-1}}-\frac{28be^2\sqrt{c+dx}}{}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arccosh(d*x+c))/d/e-28/135*b*e^3*EllipticE(1/2*(
d*x+c+1)^(1/2)*2^(1/2),2^(1/2))*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d/(-d*x-
c)^(1/2)/(d*x+c-1)^(1/2)-28/405*b*e^2*(e*(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(d*
x+c+1)^(1/2)/d-4/81*b*(e*(d*x+c))^(7/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5866, 5662, 102, 12, 114, 113}

$$\frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)}{9de}-\frac{28be^2\sqrt{c+dx-1}\sqrt{c+dx+1}\,(e(c+dx))^{3/2}}{405d}-\frac{28be^3\sqrt{-c-dx+1}\sqrt{e(c}{135d\sqrt{-c-}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-28*b*e^2*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/(405*d
) - (4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(7/2)*Sqrt[1 + c + d*x])/(81*d) +
 (2*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x]))/(9*d*e) - (28*b*e^3*Sqrt[
1 - c - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]],
 2])/(135*d*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
```

```
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\cosh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)}{9de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right.}{9de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{7/2}\sqrt{1+c+dx}}{81d} + \frac{2(e(c+dx))^{9/2}\left(a+b\,c\right.}{9de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{7/2}\sqrt{1+c+dx}}{81d} + \frac{2(e(c+dx))^{9/2}\left(a+b\,c\right.}{9de}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+c+dx}\,(e(}{}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+c+dx}\,(e(}{}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+c+dx}\,(e(}{}$$

$$= -\frac{28be^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{405d} - \frac{4b\sqrt{-1+c+dx}\,(e(}{}$$

Mathematica [C] time = 0.37, size = 150, normalized size = 0.79

$$\frac{2(e(c+dx))^{7/2}\left((c+dx)^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)+\frac{2b(c+dx)^{3/2}\left(-7\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)+5\left(1-(c+dx)^2\right)(c+dx)^2+7\left(1-(c\right.\right.}{45\sqrt{c+dx-1}\sqrt{c+dx+1}}\right.}{9d(c+dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*((c + d*x)^(9/2)*(a + b*ArcCosh[c + d*x]) + (2*b*(c
+ d*x)^(3/2)*(7*(1 - (c + d*x)^2) + 5*(c + d*x)^2*(1 - (c + d*x)^2) - 7*Sqr
```

```
t[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2]))/(45*Sqrt
[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(9*d*(c + d*x)^(7/2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ad^3e^3x^3 + 3\,acd^2e^3x^2 + 3\,ac^2de^3x + ac^3e^3 + \left(bd^3e^3x^3 + 3\,bcd^2e^3x^2 + 3\,bc^2de^3x + bc^3e^3\right)\text{arcosh}\,(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d^3*e^3*x^3 + 3*a*c*d^2*e^3*x^2 + 3*a*c^2*d*e^3*x + a*c^3*e^3 +
 (b*d^3*e^3*x^3 + 3*b*c*d^2*e^3*x^2 + 3*b*c^2*d*e^3*x + b*c^3*e^3)*arccosh(
d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{7}{2}}(b\,\text{arcosh}\,(dx + c) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arccosh(d*x + c) + a), x)
```

maple [C] time = 0.05, size = 277, normalized size = 1.47

$$\frac{\frac{2(dex+ce)^{\frac{9}{2}}a}{9} + 2b\left(\frac{(dex+ce)^{\frac{9}{2}}\text{arccosh}\left(\frac{dex+ce}{e}\right)}{9} - \frac{2\left(5\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{11}{2}}+2\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{7}{2}}e^2+21e^5\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\,\text{EllipticF}\left(\sqrt{dex+ce}\right.\right.}{405e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+c}{e}}}\right.}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/9*(d*e*x+c*e)^(9/2)*a+b*(1/9*(d*e*x+c*e)^(9/2)*arccosh((d*e*x+c*e)
/e)-2/405/e*(5*(-1/e)^(1/2)*(d*e*x+c*e)^(11/2)+2*(-1/e)^(1/2)*(d*e*x+c*e)^(
7/2)*e^2+21*e^5*((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*EllipticF(
(d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-21*e^5*((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c
*e-e)/e)^(1/2)*EllipticE((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-7*(-1/e)^(1/2)*(
d*e*x+c*e)^(3/2)*e^4)/(-1/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/((d*e*x+c*e-e)/e
)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex + ce)^{\frac{9}{2}}a}{9\,de} + \frac{1}{405}\left(\frac{90\left(d^4e^{\frac{7}{2}}x^4 + 4\,cd^3e^{\frac{7}{2}}x^3 + 6\,c^2d^2e^{\frac{7}{2}}x^2 + 4\,c^3de^{\frac{7}{2}}x + c^4e^{\frac{7}{2}}\right)\sqrt{dx + c}\,\log\left(dx + \sqrt{dx + c + 1}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/9*(d*e*x + c*e)^(9/2)*a/(d*e) + 1/405*(90*(d^4*e^(7/2)*x^4 + 4*c*d^3*e^(7
/2)*x^3 + 6*c^2*d^2*e^(7/2)*x^2 + 4*c^3*d*e^(7/2)*x + c^4*e^(7/2))*sqrt(d*x
 + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/d - (20*(d*x + c)^
(9/2)*e^(7/2) + 36*(d*x + c)^(5/2)*e^(7/2) + 45*I*e^(7/2)*(log(I*sqrt(d*x +
 c) + 1) - log(-I*sqrt(d*x + c) + 1)) - 45*e^(7/2)*log(sqrt(d*x + c) + 1) +
 45*e^(7/2)*log(sqrt(d*x + c) - 1) + 180*sqrt(d*x + c)*e^(7/2))/d + 405*int
egrate(2/9*(d^4*e^(7/2)*x^4 + 4*c*d^3*e^(7/2)*x^3 + 6*c^2*d^2*e^(7/2)*x^2 +
```

```
 4*c^3*d*e^(7/2)*x + c^4*e^(7/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^
3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*
c^2*d - d)*x - c), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{7/2}\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(7/2)*(a + b*acosh(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Timed out
```

3.199 $\int (ce+dex)^{5/2} \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=169

$$\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)}{7de}-\frac{20be^{5/2}\sqrt{-c-dx+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{147d\sqrt{c+dx-1}}-\frac{20be^2\sqrt{c+dx-1}\,\sqrt{c+}}{147d}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arccosh(d*x+c))/d/e-20/147*b*e^(5/2)*EllipticF((
e*(d*x+c))^(1/2)/e^(1/2),I)*(-d*x-c+1)^(1/2)/d/(d*x+c-1)^(1/2)-4/49*b*(e*(d
*x+c))^(5/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d-20/147*b*e^2*(d*x+c-1)^(1/2)
*(e*(d*x+c))^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 5662, 102, 12, 117, 116}

$$\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)}{7de}-\frac{20be^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{e(c+dx)}}{147d}-\frac{20be^{5/2}\sqrt{-c-dx+1}\,F\left(\sin\right.}{147d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-20*b*e^2*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/(147*d)
- (4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(5/2)*Sqrt[1 + c + d*x])/(49*d) + (
2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x]))/(7*d*e) - (20*b*e^(5/2)*Sqr
t[1 - c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(147*d*Sqr
t[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] && !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)dx=\frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)}{7de}-\frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right.}{7de}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{5/2}\sqrt{1+c+dx}}{49d}+\frac{2(e(c+dx))^{7/2}\left(a+b\right.}{7de}$$

$$=-\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{5/2}\sqrt{1+c+dx}}{49d}+\frac{2(e(c+dx))^{7/2}\left(a+b\right.}{7de}$$

$$=-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c+dx}\,(e(c}{4}$$

$$=-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c+dx}\,(e(c}{4}$$

$$=-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c+dx}\,(e(c}{4}$$

$$=-\frac{20be^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+c+dx}}{147d}-\frac{4b\sqrt{-1+c+dx}\,(e(c}{4}$$

Mathematica [C] time = 0.28, size = 180, normalized size = 1.07

$$\frac{2(e(c+dx))^{5/2}\left(21a\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(c+dx)^3-10b\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)-6b(c+dx)^4-\right.}{147d\sqrt{\frac{c+dx-1}{c+dx}}\,(c+dx)^{5/2}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(10*b - 4*b*(c + d*x)^2 - 6*b*(c + d*x)^4 + 21*a*Sqr
t[-1 + c + d*x]*(c + d*x)^3*Sqrt[1 + c + d*x] + 21*b*Sqrt[-1 + c + d*x]*(c
+ d*x)^3*Sqrt[1 + c + d*x]*ArcCosh[c + d*x] - 10*b*Sqrt[1 - (c + d*x)^2]*Hy
pergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2]))/(147*d*Sqrt[(-1 + c + d*x)/(c
 + d*x)]*(c + d*x)^(5/2)*Sqrt[1 + c + d*x])
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ad^2e^2x^2 + 2\,acde^2x + ac^2e^2 + \left(bd^2e^2x^2 + 2\,bcde^2x + bc^2e^2\right)\text{arcosh}\left(dx + c\right)\right)\sqrt{dex + ce}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d^2*e^2*x^2 + 2*a*c*d*e^2*x + a*c^2*e^2 + (b*d^2*e^2*x^2 + 2*b*
c*d*e^2*x + b*c^2*e^2)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{5}{2}}(b\,\text{arcosh}\left(dx + c\right) + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arccosh(d*x + c) + a), x)
```

maple [A] time = 0.03, size = 218, normalized size = 1.29

$$\frac{\frac{2(dex+ce)^{\frac{7}{2}}a}{7} + 2b\left(\frac{(dex+ce)^{\frac{7}{2}}\text{arccosh}\left(\frac{dex+ce}{e}\right)}{7} - \frac{2\left(3\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{9}{2}}+2\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{5}{2}}e^2+5e^4\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\,\text{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{\right.\right.}{147e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{dex+ce-e}{e}}}\right.}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/7*(d*e*x+c*e)^(7/2)*a+b*(1/7*(d*e*x+c*e)^(7/2)*arccosh((d*e*x+c*e)
/e)-2/147/e*(3*(-1/e)^(1/2)*(d*e*x+c*e)^(9/2)+2*(-1/e)^(1/2)*(d*e*x+c*e)^(5
/2)*e^2+5*e^4*((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*EllipticF((d
*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-5*(-1/e)^(1/2)*(d*e*x+c*e)^(1/2)*e^4)/(-1/e
)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex + ce)^{\frac{7}{2}}a}{7\,de} + \frac{1}{147}\left(\frac{42\left(d^3e^{\frac{5}{2}}x^3 + 3\,cd^2e^{\frac{5}{2}}x^2 + 3\,c^2de^{\frac{5}{2}}x + c^3e^{\frac{5}{2}}\right)\sqrt{dx + c}\,\log\left(dx + \sqrt{dx + c + 1}\,\sqrt{dx + c - 1}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/7*(d*e*x + c*e)^(7/2)*a/(d*e) + 1/147*(42*(d^3*e^(5/2)*x^3 + 3*c*d^2*e^(5
/2)*x^2 + 3*c^2*d*e^(5/2)*x + c^3*e^(5/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x
 + c + 1)*sqrt(d*x + c - 1) + c)/d - (12*(d*x + c)^(7/2)*e^(5/2) + 28*(d*x
+ c)^(3/2)*e^(5/2) - 21*I*e^(5/2)*(log(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d
*x + c) + 1)) - 21*e^(5/2)*log(sqrt(d*x + c) + 1) + 21*e^(5/2)*log(sqrt(d*x
 + c) - 1))/d + 147*integrate(2/7*(d^3*e^(5/2)*x^3 + 3*c*d^2*e^(5/2)*x^2 +
3*c^2*d*e^(5/2)*x + c^3*e^(5/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3
 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c
^2*d - d)*x - c), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{5/2}\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*acosh(d*x+c)),x)
```

```
[Out] Timed out
```

3.200 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=145

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(e(c+dx))^{3/2}}{25d}-\frac{12be\sqrt{-c-dx+1}\,\sqrt{e(c+dx}}{25d\sqrt{-c-dx}}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))/d/e-12/25*b*e*EllipticE(1/2*(d*x
+c+1)^(1/2)*2^(1/2),2^(1/2))*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d/(-d*x-c)^
(1/2)/(d*x+c-1)^(1/2)-4/25*b*(e*(d*x+c))^(3/2)*(d*x+c-1)^(1/2)*(d*x+c+1)^(1
/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 5662, 102, 12, 114, 113}

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(e(c+dx))^{3/2}}{25d}-\frac{12be\sqrt{-c-dx+1}\,\sqrt{e(c+dx}}{25d\sqrt{-c-dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d*x])/(25*d) + (2
*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x]))/(5*d*e) - (12*b*e*Sqrt[1 - c
 - d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/
(25*d*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
```

```
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)}{5de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx\right.}{5de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\right.}{5de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\right.}{5de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\right.}{5de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c+dx}}{25d} + \frac{2(e(c+dx))^{5/2}\left(a+b\right.}{5de}$$

Mathematica [C] time = 0.49, size = 109, normalized size = 0.75

$$\frac{2(e(c+dx))^{3/2}\left(5(c+dx)\left(a+b\cosh^{-1}(c+dx)\right) - \frac{2b\left(c^2+\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)+2cdx+d^2x^2-1\right)}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}\right)}{25d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(5*(c + d*x)*(a + b*ArcCosh[c + d*x]) - (2*b*(-1 + c
^2 + 2*c*d*x + d^2*x^2 + Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, 3/4,
7/4, (c + d*x)^2]))/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(25*d)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((adex+ace+(bdex+bce)\,\text{arcosh}(dx+c))\sqrt{dex+ce}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*d*e*x + a*c*e + (b*d*e*x + b*c*e)*arccosh(d*x + c))*sqrt(d*e*x
+ c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}(b \operatorname{arcosh}(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a), x)
```

maple [C] time = 0.02, size = 254, normalized size = 1.75

$$\frac{\frac{2(dex+ce)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dex+ce)^{\frac{5}{2}}\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{7}{2}}+3\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\,e^{3}\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)-3e^{3}\sqrt{\frac{dex+ce+e}{e}}\cdots\right)}{25e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{dex+ce-e}{e}}\cdots}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x)
```

```
[Out] 2/d/e*(1/5*(d*e*x+c*e)^(5/2)*a+b*(1/5*(d*e*x+c*e)^(5/2)*arccosh((d*e*x+c*e)
/e)-2/25/e*((-1/e)^(1/2)*(d*e*x+c*e)^(7/2)+3*((d*e*x+c*e+e)/e)^(1/2)*(-(d*e
*x+c*e-e)/e)^(1/2)*e^3*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-3*e^3*((
d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*EllipticE((d*e*x+c*e)^(1/2)*
(-1/e)^(1/2),I)-(-1/e)^(1/2)*(d*e*x+c*e)^(3/2)*e^2)/(-1/e)^(1/2)/((d*e*x+c*
e+e)/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{25}\left(\frac{10\left(d^{2}e^{\frac{3}{2}}x^{2}+2\,cde^{\frac{3}{2}}x+c^{2}e^{\frac{3}{2}}\right)\sqrt{dx+c}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)}{d}-\frac{4\,(dx+c)^{\frac{5}{2}}e^{\frac{3}{2}}+5i\,e^{\frac{3}{2}}\left(\log\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/25*(10*(d^2*e^(3/2)*x^2 + 2*c*d*e^(3/2)*x + c^2*e^(3/2))*sqrt(d*x + c)*lo
g(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/d - (4*(d*x + c)^(5/2)*e^(
3/2) + 5*I*e^(3/2)*(log(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1)) -
 5*e^(3/2)*log(sqrt(d*x + c) + 1) + 5*e^(3/2)*log(sqrt(d*x + c) - 1) + 20*s
qrt(d*x + c)*e^(3/2))/d + 25*integrate(2/5*(d^2*e^(3/2)*x^2 + 2*c*d*e^(3/2)
*x + c^2*e^(3/2))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2
*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c
), x))*b + 2/5*(d*e*x + c*e)^(5/2)*a/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{3/2}\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}}(a+b\operatorname{acosh}(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x)), x)
```

3.201 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=127

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{e(c+dx)}}{9d}-\frac{4b\sqrt{e}\,\sqrt{-c-dx+1}\,F\left(\sin^{-1}\left(\right.\right.}{9d\sqrt{c+dx-1}}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))/d/e-4/9*b*EllipticF((e*(d*x+c))^
(1/2)/e^(1/2),I)*e^(1/2)*(-d*x-c+1)^(1/2)/d/(d*x+c-1)^(1/2)-4/9*b*(d*x+c-1)
^(1/2)*(e*(d*x+c))^(1/2)*(d*x+c+1)^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5866, 5662, 102, 12, 117, 116}

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de}-\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,\sqrt{e(c+dx)}}{9d}-\frac{4b\sqrt{e}\,\sqrt{-c-dx+1}\,F\left(\sin^{-1}\left(\right.\right.}{9d\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (-4*b*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x])/(9*d) + (2*(e
*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x]))/(3*d*e) - (4*b*Sqrt[e]*Sqrt[1 -
 c - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(9*d*Sqrt[-1 +
c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)dx = \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)dx,x,c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)}{3de} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}}{\sqrt{-1+x}\sqrt{1+x}}dx,\right.}{3de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\cos\right.}{3de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\cos\right.}{3de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\cos\right.}{3de}$$

$$= -\frac{4b\sqrt{-1+c+dx}\sqrt{e(c+dx)}\sqrt{1+c+dx}}{9d} + \frac{2(e(c+dx))^{3/2}\left(a+b\cos\right.}{3de}$$

Mathematica [C] time = 0.46, size = 131, normalized size = 1.03

$$\frac{\sqrt{e(c+dx)}\left(\frac{2}{3}(c+dx)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)-\frac{4b\left(c^2+\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)+2cdx+d^2x^2-1\right)}{9\sqrt{\frac{c+dx-1}{c+dx}}\sqrt{c+dx+1}}\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] (Sqrt[e*(c + d*x)]*((2*(c + d*x)^(3/2)*(a + b*ArcCosh[c + d*x]))/3 - (4*b*(
-1 + c^2 + 2*c*d*x + d^2*x^2 + Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/4,
 1/2, 5/4, (c + d*x)^2]))/(9*Sqrt[(-1 + c + d*x)/(c + d*x)]*Sqrt[1 + c + d*
x])))/(d*Sqrt[c + d*x])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{dex+ce}\,(b\,\text{arcosh}\,(dx+c)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dex+ce}\,(b\,\mathrm{arcosh}\,(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a), x)
```

maple [A] time = 0.02, size = 194, normalized size = 1.53

$$\frac{\frac{2(dex+ce)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dex+ce)^{\frac{3}{2}}\mathrm{arccosh}\left(\frac{dex+ce}{e}\right)}{3}-\frac{2\left(\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{5}{2}}+\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\,\mathrm{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)e^{2}-\sqrt{-\frac{1}{e}}\,\sqrt{dex+ce}\right.}{9e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{dex+ce-e}{e}}}\right.}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x)
[Out] 2/d/e*(1/3*(d*e*x+c*e)^(3/2)*a+b*(1/3*(d*e*x+c*e)^(3/2)*arccosh((d*e*x+c*e)
/e)-2/9/e*((-1/e)^(1/2)*(d*e*x+c*e)^(5/2)+((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+
c*e-e)/e)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*e^2-(-1/e)^(1/2
)*(d*e*x+c*e)^(1/2)*e^2)/(-1/e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/((d*e*x+c*e-e
)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{9}\left(\frac{6\left(d\sqrt{e}\,x+c\sqrt{e}\right)\sqrt{dx+c}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)}{d}-\frac{4\,(dx+c)^{\frac{3}{2}}\sqrt{e}\,-3i\,\sqrt{e}\left(\log\left(i\,\sqrt{dx+c}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
[Out] 1/9*(6*(d*sqrt(e)*x + c*sqrt(e))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*
sqrt(d*x + c - 1) + c)/d - (4*(d*x + c)^(3/2)*sqrt(e) - 3*I*sqrt(e)*(log(I*
sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1)) - 3*sqrt(e)*log(sqrt(d*x +
c) + 1) + 3*sqrt(e)*log(sqrt(d*x + c) - 1))/d + 9*integrate(2/3*(d*sqrt(e)*
x + c*sqrt(e))*sqrt(d*x + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*
d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c),
x))*b + 2/3*(d*e*x + c*e)^(3/2)*a/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x)),x)
[Out] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\,(c+dx)}\,\left(a+b\,\mathrm{acosh}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))*(d*e*x+c*e)**(1/2),x)
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x)), x)
```

3.202 $\int \frac{a+b\cosh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{4b\sqrt{-c-dx+1}\sqrt{e(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{de\sqrt{-c-dx}\sqrt{c+dx-1}}$$

```
[Out] 2*(a+b*arccosh(d*x+c))*(e*(d*x+c))^(1/2)/d/e-4*b*EllipticE(1/2*(d*x+c+1)^(1
/2)*2^(1/2),2^(1/2))*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d/e/(-d*x-c)^(1/2)/
(d*x+c-1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5866, 5662, 114, 113}

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de}-\frac{4b\sqrt{-c-dx+1}\sqrt{e(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{de\sqrt{-c-dx}\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x]))/(d*e) - (4*b*Sqrt[1 - c - d*
x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(d*e*
Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[(((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
```

```
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{ex}}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{\left(\sqrt{2}\,b\sqrt{1-c-dx}\sqrt{e(c+dx)}\right)\text{Subst}\left(\int -\right.}{de\sqrt{-c-dx}\sqrt{-1+c-}}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)}{de} - \frac{4b\sqrt{1-c-dx}\sqrt{e(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{1+c-}}{\sqrt{2}}\right.\right.}{de\sqrt{-c-dx}\sqrt{-1+c+dx}}$$

Mathematica [C] time = 0.19, size = 94, normalized size = 0.90

$$\frac{2\sqrt{e(c+dx)}\left(3\left(a+b\cosh^{-1}(c+dx)\right)-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(3*(a + b*ArcCosh[c + d*x]) - (2*b*(c + d*x)*Sqrt[1 -
(c + d*x)^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (c + d*x)^2])/(Sqrt[-1 + c +
d*x]*Sqrt[1 + c + d*x])))/(3*d*e)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(dx+c)+a}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(dx+c)+a}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/sqrt(d*e*x + c*e), x)
```

maple [C] time = 0.02, size = 138, normalized size = 1.33

$$\frac{2a\sqrt{dex+ce}+2b\left(\sqrt{dex+ce}\,\text{arccosh}\left(\frac{dex+ce}{e}\right)-\frac{2\left(\text{EllipticF}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)-\text{EllipticE}\left(\sqrt{dex+ce}\sqrt{-\frac{1}{e}},i\right)\right)\sqrt{-\frac{dex+ce-e}{e}}}{\sqrt{-\frac{1}{e}}\sqrt{\frac{dex+ce-e}{e}}}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x)
```

```
[Out] 2/d/e*(a*(d*e*x+c*e)^(1/2)+b*((d*e*x+c*e)^(1/2)*arccosh((d*e*x+c*e)/e)-2*(E
llipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)-EllipticE((d*e*x+c*e)^(1/2)*(-1/
e)^(1/2),I))*(-(d*e*x+c*e-e)/e)^(1/2)/(-1/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-b\left(\frac{\frac{i\left(\log\left(i\sqrt{dx+c}+1\right)-\log\left(-i\sqrt{dx+c}+1\right)\right)}{\sqrt{e}}-\frac{\log\left(\sqrt{dx+c}+1\right)}{\sqrt{e}}+\frac{\log\left(\sqrt{dx+c}-1\right)}{\sqrt{e}}+\frac{4\sqrt{dx+c}}{\sqrt{e}}}{d}-\frac{2\left(d\sqrt{e}x+c\sqrt{e}\right)\log\left(dx+\sqrt{dx+c}\right.}{\sqrt{dx+c}\,de}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*((I*(log(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))/sqrt(e) - log
(sqrt(d*x + c) + 1)/sqrt(e) + log(sqrt(d*x + c) - 1)/sqrt(e) + 4*sqrt(d*x +
 c)/sqrt(e))/d - 2*(d*sqrt(e)*x + c*sqrt(e))*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)/(sqrt(d*x + c)*d*e) - integrate(2*(d*sqrt(e)*x + c*sqr
t(e))/((d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e + (d^2*e*x^2 + 2*c*d*e*x + c^2*e
- e)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*e + (3*c^2*d*e - d*e)*x)*sqrt(
d*x + c)), x)) + 2*sqrt(d*e*x + c*e)*a/(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c+d\,x\right)}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{acosh}\left(c+dx\right)}{\sqrt{e\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/sqrt(e*(c + d*x)), x)
```

3.203 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{4b\sqrt{-c-dx+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}\sqrt{c+dx-1}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

```
[Out] 4*b*EllipticF((e*(d*x+c))^(1/2)/e^(1/2),I)*(-d*x-c+1)^(1/2)/d/e^(3/2)/(d*x+
c-1)^(1/2)-2*(a+b*arccosh(d*x+c))/d/e/(e*(d*x+c))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5866, 5662, 117, 116}

$$\frac{4b\sqrt{-c-dx+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}\sqrt{c+dx-1}}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x]))/(d*e*Sqrt[e*(c + d*x)]) + (4*b*Sqrt[1 - c - d
*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(d*e^(3/2)*Sqrt[-1 +
c + d*x])
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}$$
$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{ex}\,\sqrt{1+x}}\,dx,x,c+dx\right)}{de}$$
$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{\left(2b\sqrt{1-c-dx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{ex}\,\sqrt{1+x}}\,dx,x,c+dx\right)}{de\sqrt{-1+c+dx}}$$
$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}} + \frac{4b\sqrt{1-c-dx}\,F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{de^{3/2}\sqrt{-1+c+dx}}$$

Mathematica [C] time = 0.17, size = 92, normalized size = 1.10

$$\frac{2\left(-a+\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}-b\cosh^{-1}(c+dx)\right)}{de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-a - b*ArcCosh[c + d*x] + (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hypergeo
metric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x
])))/(d*e*Sqrt[e*(c + d*x)])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dex+ce}\,(b\,\text{arcosh}\,(dx+c)+a)}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)/(d^2*e^2*x^2 + 2*c*d*e^
2*x + c^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(dx+c)+a}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^(3/2), x)
```

maple [A] time = 0.02, size = 119, normalized size = 1.42

$$\frac{-\frac{2a}{\sqrt{dex+ce}}+2b\left(-\frac{\text{arccosh}\left(\frac{dex+ce}{e}\right)}{\sqrt{dex+ce}}+\frac{2\,\text{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)\sqrt{-\frac{dex+ce-e}{e}}}{e\sqrt{-\frac{1}{e}}\,\sqrt{\frac{dex+ce-e}{e}}}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x)
```

```
[Out] 2/d/e*(-a/(d*e*x+c*e)^(1/2)+b*(-1/(d*e*x+c*e)^(1/2)*arccosh((d*e*x+c*e)/e)+
2/e*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*(-(d*e*x+c*e-e)/e)^(1/2)/(-
1/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{-\frac{i\left(\log\left(i\sqrt{dx+c}+1\right)-\log\left(-i\sqrt{dx+c}+1\right)\right)}{e^{\frac{3}{2}}}-\frac{\log\left(\sqrt{dx+c}+1\right)}{e^{\frac{3}{2}}}+\frac{\log\left(\sqrt{dx+c}-1\right)}{e^{\frac{3}{2}}}}{d}-\frac{2\log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)}{\sqrt{dx+c}\,de^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
```

```
[Out] b*((-I*(log(I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))/e^(3/2) - log
(sqrt(d*x + c) + 1)/e^(3/2) + log(sqrt(d*x + c) - 1)/e^(3/2))/d - 2*log(d*x
 + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(sqrt(d*x + c)*d*e^(3/2)) - 2*i
ntegrate(1/((d^2*e^(3/2)*x^2 + 2*c*d*e^(3/2)*x + c^2*e^(3/2) - e^(3/2))*sqr
t(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (d^3*e^(3/2)*x^3 + 3*c*d^2
*e^(3/2)*x^2 + c^3*e^(3/2) - c*e^(3/2) + (3*c^2*d*e^(3/2) - d*e^(3/2))*x)*s
qrt(d*x + c)), x)) - 2*a/(sqrt(d*e*x + c*e)*d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c+d\,x\right)}{{\left(c\,e+d\,e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{acosh}\left(c+dx\right)}{\left(e\left(c+dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/(e*(c + d*x))**(3/2), x)
```

3.204 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}-\frac{4b\sqrt{-c-dx+1}\,\sqrt{e(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{3de^3\sqrt{-c-dx}\,\sqrt{c+dx-1}}+\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{3de^2\sqrt{e(c+dx)}}$$

```
[Out] -2/3*(a+b*arccosh(d*x+c))/d/e/(e*(d*x+c))^(3/2)-4/3*b*EllipticE(1/2*(d*x+c+
1)^(1/2)*2^(1/2),2^(1/2))*(-d*x-c+1)^(1/2)*(e*(d*x+c))^(1/2)/d/e^3/(-d*x-c)
^(1/2)/(d*x+c-1)^(1/2)+4/3*b*(d*x+c-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^2/(e*(d*x+
c))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 5662, 104, 12, 16, 114, 113}

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}+\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{3de^2\sqrt{e(c+dx)}}-\frac{4b\sqrt{-c-dx+1}\,\sqrt{e(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{c+dx+1}}{\sqrt{2}}\right)\middle|2\right)}{3de^3\sqrt{-c-dx}\,\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(3*d*e^2*Sqrt[e*(c + d*x)]) - (2
*(a + b*ArcCosh[c + d*x]))/(3*d*e*(e*(c + d*x))^(3/2)) - (4*b*Sqrt[1 - c -
d*x]*Sqrt[e*(c + d*x)]*EllipticE[ArcSin[Sqrt[1 + c + d*x]/Sqrt[2]], 2])/(3*
d*e^3*Sqrt[-c - d*x]*Sqrt[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(ex)^{3/2}\sqrt{1+x}}\,dx, x, c+dx\right)}{3de}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int -\frac{\cdots}{2\sqrt{-1\cdots}}\right.}{\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{\cdots}{\sqrt{-1+x}\cdots}\right.}{3\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{(2b)\,\text{Subst}\left(\int \frac{\sqrt{e\cdots}}{\sqrt{-1+x}\cdots}\right.}{3de\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{\left(\sqrt{2}\,b\sqrt{1-c-dx}\,\sqrt{e\cdots}\right.}{3d\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{3de^2\sqrt{e(c+dx)}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}} - \frac{4b\sqrt{1-c-dx}\,\sqrt{e(c\cdots}}{3de^3\sqrt{-c\cdots}}$$

Mathematica [C] time = 0.15, size = 94, normalized size = 0.63

$$\frac{2\left(-a-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}-b\cosh^{-1}(c+dx)\right)}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (2*(-a - b*ArcCosh[c + d*x] - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hypergeo
metric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x])))/(3*d*e*(e*(c + d*x))^(3/2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{dex+ce}\,(b\operatorname{arcosh}(dx+c)+a)}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)/(d^3*e^3*x^3 + 3*c*d^2*
e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(t_nostep)]sym2poly/r2sym(co
nst gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.03, size = 269, normalized size = 1.79

$$\frac{-\frac{2a}{3(dex+ce)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{3(dex+ce)^{\frac{3}{2}}}+\frac{-\frac{2\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\,\sqrt{dex+ce}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)e}{3}+\frac{2\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\,\sqrt{dex+ce}\,\operatorname{EllipticE}\left(\cdots\right.}{3}}{e^3\sqrt{-\frac{1}{e}}\,\sqrt{dex+ce}\,\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{dex+ce-e}{e}}}\right.}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x)
```

```
[Out] 2/d/e*(-1/3*a/(d*e*x+c*e)^(3/2)+b*(-1/3/(d*e*x+c*e)^(3/2)*arccosh((d*e*x+c*
e)/e)+2/3/e^3*(-((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*(d*e*x+c*e
)^(1/2)*EllipticF((d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*e+((d*e*x+c*e+e)/e)^(1/
2)*(-(d*e*x+c*e-e)/e)^(1/2)*(d*e*x+c*e)^(1/2)*EllipticE((d*e*x+c*e)^(1/2)*(
-1/e)^(1/2),I)*e+(-1/e)^(1/2)*(d*e*x+c*e)^2-(-1/e)^(1/2)*e^2)/(-1/e)^(1/2)/
(d*e*x+c*e)^(1/2)/((d*e*x+c*e+e)/e)^(1/2)/((d*e*x+c*e-e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(6\sqrt{e}\int\frac{1}{3\left(d^4e^3x^4+4\,cd^3e^3x^3+c^4e^3-c^2e^3+\left(6\,c^2d^2e^3-d^2e^3\right)x^2+\left(d^3e^3x^3+3\,cd^2e^3x^2+c^3e^3-ce^3+\left(3\,c^2d\cdots\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(6*sqrt(e)*integrate(1/3/((d^4*e^3*x^4 + 4*c*d^3*e^3*x^3 + c^4*e^3 - c
^2*e^3 + (6*c^2*d^2*e^3 - d^2*e^3)*x^2 + (d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + c
^3*e^3 - c*e^3 + (3*c^2*d*e^3 - d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + 2*(2*c^3*d*e^3 - c*d*e^3)*x)*sqrt(d*x + c)), x) + sqrt(e)*(-I*(log(I*s
qrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))/e^3 + log(sqrt(d*x + c) + 1)
/e^3 - log(sqrt(d*x + c) - 1)/e^3)/d + 2*sqrt(e)*log(d*x + sqrt(d*x + c + 1
)*sqrt(d*x + c - 1) + c)/((d^2*e^3*x + c*d*e^3)*sqrt(d*x + c)))*b - 2/3*a/(
(d*e*x + c*e)^(3/2)*d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\,(c+d\,x)}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acosh}\,(c+dx)}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))/(e*(c + d*x))**(5/2), x)
```

3.205 $\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}+\frac{4b\sqrt{-c-dx+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{15de^{7/2}\sqrt{c+dx-1}}+\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{15de^{2}(e(c+dx))^{3/2}}$$

```
[Out] -2/5*(a+b*arccosh(d*x+c))/d/e/(e*(d*x+c))^(5/2)+4/15*b*EllipticF((e*(d*x+c)
)^(1/2)/e^(1/2),I)*(-d*x-c+1)^(1/2)/d/e^(7/2)/(d*x+c-1)^(1/2)+4/15*b*(d*x+c
-1)^(1/2)*(d*x+c+1)^(1/2)/d/e^2/(e*(d*x+c))^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {5866, 5662, 104, 12, 16, 117, 116}

$$-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}}+\frac{4b\sqrt{c+dx-1}\,\sqrt{c+dx+1}}{15de^{2}(e(c+dx))^{3/2}}+\frac{4b\sqrt{-c-dx+1}\,F\left(\sin^{-1}\left(\frac{\sqrt{e(c+dx)}}{\sqrt{e}}\right)\middle|-1\right)}{15de^{7/2}\sqrt{c+dx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (4*b*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])/(15*d*e^2*(e*(c + d*x))^(3/2)) -
 (2*(a + b*ArcCosh[c + d*x]))/(5*d*e*(e*(c + d*x))^(5/2)) + (4*b*Sqrt[1 - c
 - d*x]*EllipticF[ArcSin[Sqrt[e*(c + d*x)]/Sqrt[e]], -1])/(15*d*e^(7/2)*Sqr
t[-1 + c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 116

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-(
b/d), 2])], (c*f)/(d*e)])/(b*Sqrt[e]), x] /; FreeQ[{b, c, d, e, f}, x] && G
tQ[c, 0] && GtQ[e, 0] && (PosQ[-(b/d)] || NegQ[-(b/f)])
```

Rule 117

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_Symbol] :> Dist[(Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e])/(Sqrt[c + d*x]*Sqrt[
```

```
e + f*x]), Int[1/(Sqrt[b*x]*Sqrt[1 + (d*x)/c]*Sqrt[1 + (f*x)/e]), x], x] /;
 FreeQ[{b, c, d, e, f}, x] && !(GtQ[c, 0] && GtQ[e, 0])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(c+dx)}{(ce+dex)^{7/2}}\,dx = \frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}$$

$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(ex)^{5/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{5de}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{}{2\sqrt{-1+}\cdots}\right.}{1\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\cdots}\right.}{1\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{(2b)\,\text{Subst}\left(\int \frac{}{\sqrt{-1+x}\cdots}\right.}{15\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{\left(2b\sqrt{1-c-dx}\right)\text{Sul}\cdots}{15d\cdots}$$

$$= \frac{4b\sqrt{-1+c+dx}\,\sqrt{1+c+dx}}{15de^2(e(c+dx))^{3/2}} - \frac{2\left(a+b\cosh^{-1}(c+dx)\right)}{5de(e(c+dx))^{5/2}} + \frac{4b\sqrt{1-c-dx}\,F\left(\text{sin}\cdots\right.}{15de^{7/2}\sqrt{-1}\cdots}$$

Mathematica [C] time = 0.16, size = 94, normalized size = 0.72

$$\frac{2\left(-3\left(a+b\cosh^{-1}(c+dx)\right)-\frac{2b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}\right)}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (2*(-3*(a + b*ArcCosh[c + d*x]) - (2*b*(c + d*x)*Sqrt[1 - (c + d*x)^2]*Hype
rgeometric2F1[-3/4, 1/2, 1/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c
+ d*x])))/(15*d*e*(e*(c + d*x))^(5/2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dex+ce}\,(b\operatorname{arcosh}(dx+c)+a)}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)/(d^4*e^4*x^4 + 4*c*d^3*
e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcosh}(dx+c)+a}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)/(d*e*x + c*e)^(7/2), x)
```

maple [A] time = 0.03, size = 201, normalized size = 1.55

$$\frac{-\frac{2a}{5(dex+ce)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arccosh}\left(\frac{dex+ce}{e}\right)}{5(dex+ce)^{\frac{5}{2}}}+\frac{\frac{2\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{-\frac{dex+ce-e}{e}}\,\operatorname{EllipticF}\left(\sqrt{dex+ce}\,\sqrt{-\frac{1}{e}},i\right)(dex+ce)^{\frac{3}{2}}}{15}+\frac{2\sqrt{-\frac{1}{e}}\,(dex+ce)^2}{15}-\frac{2\sqrt{-\frac{1}{e}}\,e^2}{15}}{e^3\sqrt{-\frac{1}{e}}\,(dex+ce)^{\frac{3}{2}}\sqrt{\frac{dex+ce+e}{e}}\,\sqrt{\frac{dex+ce-e}{e}}}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x)
```

```
[Out] 2/d/e*(-1/5*a/(d*e*x+c*e)^(5/2)+b*(-1/5/(d*e*x+c*e)^(5/2)*arccosh((d*e*x+c*
e)/e)+2/15/e^3*(((d*e*x+c*e+e)/e)^(1/2)*(-(d*e*x+c*e-e)/e)^(1/2)*EllipticF(
(d*e*x+c*e)^(1/2)*(-1/e)^(1/2),I)*(d*e*x+c*e)^(3/2)+(-1/e)^(1/2)*(d*e*x+c*e
)^2-(-1/e)^(1/2)*e^2)/(-1/e)^(1/2)/(d*e*x+c*e)^(3/2)/((d*e*x+c*e+e)/e)^(1/2
)/((d*e*x+c*e-e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{5}\left(10\sqrt{e}\int \frac{}{5\left(d^5e^4x^5+5\,cd^4e^4x^4+c^5e^4-c^3e^4+\left(10\,c^2d^3e^4-d^3e^4\right)x^3+\left(10\,c^3d^2e^4-3\,cd^2e^4\right)x^2+\left(d^4e^4x^4+4\,c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(10*sqrt(e)*integrate(1/5/((d^5*e^4*x^5 + 5*c*d^4*e^4*x^4 + c^5*e^4 -
c^3*e^4 + (10*c^2*d^3*e^4 - d^3*e^4)*x^3 + (10*c^3*d^2*e^4 - 3*c*d^2*e^4)*x
^2 + (d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + c^4*e^4 - c^2*e^4 + (6*c^2*d^2*e^4 -
d^2*e^4)*x^2 + 2*(2*c^3*d*e^4 - c*d*e^4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c
- 1) + (5*c^4*d*e^4 - 3*c^2*d*e^4)*x)*sqrt(d*x + c)), x) + sqrt(e)*(I*(log(
I*sqrt(d*x + c) + 1) - log(-I*sqrt(d*x + c) + 1))/e^4 + log(sqrt(d*x + c) +
 1)/e^4 - log(sqrt(d*x + c) - 1)/e^4 - 4/(sqrt(d*x + c)*e^4))/d + 2*sqrt(e)
*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/((d^3*e^4*x^2 + 2*c*d^2
*e^4*x + c^2*d*e^4)*sqrt(d*x + c)))*b - 2/5*a/((d*e*x + c*e)^(5/2)*d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{acosh}(c + d\,x)}{(c\,e + d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(7/2),x)
[Out] int((a + b*acosh(c + d*x))/(c*e + d*e*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))/(d*e*x+c*e)**(7/2),x)
[Out] Timed out
```

3.206 $\int (ce+dex)^{7/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=153

$$-\frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};(c+dx)^2\right)}{1287de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};(c+dx)^2\right)\left(a+\right.}{99de^2\sqrt{c+dx-1}}$$

```
[Out] 2/9*(e*(d*x+c))^(9/2)*(a+b*arccosh(d*x+c))^2/d/e-16/1287*b^2*(e*(d*x+c))^(1
3/2)*HypergeometricPFQ([1, 13/4, 13/4],[15/4, 17/4],(d*x+c)^2)/d/e^3-8/99*b
*(e*(d*x+c))^(11/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 11/4],[15/4],(d*x+
c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{13/2}\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};(c+dx)^2\right)}{1287de^3}-\frac{8b\sqrt{1-(c+dx)^2}\,(e(c+dx))^{11/2}\,{}_2F_1\left(\frac{1}{2},\frac{11}{4};\frac{15}{4};(c+dx)^2\right)\left(a-\right.}{99de^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(a + b*ArcCosh[c + d*x])^2)/(9*d*e) - (8*b*(e*(c + d
*x))^(11/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F
1[1/2, 11/4, 15/4, (c + d*x)^2])/(99*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c +
d*x]) - (16*b^2*(e*(c + d*x))^(13/2)*HypergeometricPFQ[{1, 13/4, 13/4}, {15
/4, 17/4}, (c + d*x)^2])/(1287*d*e^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int (ex)^{7/2}\left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{9de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{9/2}(a+b}{\sqrt{-1+x}}\right.}{9a}$$

$$= \frac{2(e(c+dx))^{9/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{9de} - \frac{8b(e(c+dx))^{11/2}\sqrt{1-(}}{}$$

Mathematica [A] time = 0.54, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{9/2}\left(143\left(a+b\cosh^{-1}(c+dx)\right)^2 - 4b(c+dx)\left(2b(c+dx)\,{}_3F_2\left(1,\frac{13}{4},\frac{13}{4};\frac{15}{4},\frac{17}{4};(c+dx)^2\right) + \frac{13\sqrt{1-(c+}}{}\right.\right.}{1287de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(7/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(9/2)*(143*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((13
*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 11/4
, 15/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x
)*HypergeometricPFQ[{1, 13/4, 13/4}, {15/4, 17/4}, (c + d*x)^2])))/(1287*d*
e)
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2d^3e^3x^3 + 3\,a^2cd^2e^3x^2 + 3\,a^2c^2de^3x + a^2c^3e^3 + \left(b^2d^3e^3x^3 + 3\,b^2cd^2e^3x^2 + 3\,b^2c^2de^3x + b^2c^3e^3\right)\text{arc}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^3*e^3*x^3 + 3*a^2*c*d^2*e^3*x^2 + 3*a^2*c^2*d*e^3*x + a^2*c
^3*e^3 + (b^2*d^3*e^3*x^3 + 3*b^2*c*d^2*e^3*x^2 + 3*b^2*c^2*d*e^3*x + b^2*c
^3*e^3)*arccosh(d*x + c)^2 + 2*(a*b*d^3*e^3*x^3 + 3*a*b*c*d^2*e^3*x^2 + 3*a
*b*c^2*d*e^3*x + a*b*c^3*e^3)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}}(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(7/2)*(b*arccosh(d*x + c) + a)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{7}{2}}\left(a+b\,\text{arccosh}\,(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex+ce)^{\frac{9}{2}}a^2}{9\,de}+\frac{2\left(b^2d^4e^{\frac{7}{2}}x^4+4\,b^2cd^3e^{\frac{7}{2}}x^3+6\,b^2c^2d^2e^{\frac{7}{2}}x^2+4\,b^2c^3de^{\frac{7}{2}}x+b^2c^4e^{\frac{7}{2}}\right)\sqrt{dx+c}\,\log\left(dx+\sqrt{dx+c+\right.}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(7/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/9*(d*e*x + c*e)^(9/2)*a^2/(d*e) + 2/9*(b^2*d^4*e^(7/2)*x^4 + 4*b^2*c*d^3*
e^(7/2)*x^3 + 6*b^2*c^2*d^2*e^(7/2)*x^2 + 4*b^2*c^3*d*e^(7/2)*x + b^2*c^4*e
^(7/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/
d + integrate(-2/9*((2*b^2*c^5*e^(7/2) - (9*a*b*d^5*e^(7/2) - 2*b^2*d^5*e^(
7/2))*x^5 - 5*(9*a*b*c*d^4*e^(7/2) - 2*b^2*c*d^4*e^(7/2))*x^4 + (20*b^2*c^2
*d^3*e^(7/2) - 9*(10*c^2*d^3*e^(7/2) - d^3*e^(7/2))*a*b)*x^3 - 9*(c^5*e^(7/
2) - c^3*e^(7/2))*a*b + (20*b^2*c^3*d^2*e^(7/2) - 9*(10*c^3*d^2*e^(7/2) - 3
*c*d^2*e^(7/2))*a*b)*x^2 + (10*b^2*c^4*d*e^(7/2) - 9*(5*c^4*d*e^(7/2) - 3*c
^2*d*e^(7/2))*a*b)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) - (
(9*a*b*d^6*e^(7/2) - 2*b^2*d^6*e^(7/2))*x^6 + 6*(9*a*b*c*d^5*e^(7/2) - 2*b^
2*c*d^5*e^(7/2))*x^5 + (9*(15*c^2*d^4*e^(7/2) - d^4*e^(7/2))*a*b - 2*(15*c^
2*d^4*e^(7/2) - d^4*e^(7/2))*b^2)*x^4 + 4*(9*(5*c^3*d^3*e^(7/2) - c*d^3*e^(
7/2))*a*b - 2*(5*c^3*d^3*e^(7/2) - c*d^3*e^(7/2))*b^2)*x^3 + 9*(c^6*e^(7/2)
 - c^4*e^(7/2))*a*b - 2*(c^6*e^(7/2) - c^4*e^(7/2))*b^2 + 3*(9*(5*c^4*d^2*e
^(7/2) - 2*c^2*d^2*e^(7/2))*a*b - 2*(5*c^4*d^2*e^(7/2) - 2*c^2*d^2*e^(7/2))
*b^2)*x^2 + 2*(9*(3*c^5*d*e^(7/2) - 2*c^3*d*e^(7/2))*a*b - 2*(3*c^5*d*e^(7/
2) - 2*c^3*d*e^(7/2))*b^2)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^
2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^{7/2}\,{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(7/2)*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(7/2)*(a + b*acosh(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(7/2)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.207 $\int (ce+dex)^{5/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=153

$$-\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};(c+dx)^2\right)}{693de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};(c+dx)^2\right)\left(a-\right.}{63de^2\sqrt{c+dx-1}}$$

```
[Out] 2/7*(e*(d*x+c))^(7/2)*(a+b*arccosh(d*x+c))^2/d/e-16/693*b^2*(e*(d*x+c))^(11
/2)*HypergeometricPFQ([1, 11/4, 11/4],[13/4, 15/4],(d*x+c)^2)/d/e^3-8/63*b*
(e*(d*x+c))^(9/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 9/4],[13/4],(d*x+c)^
2)*(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};(c+dx)^2\right)}{693de^3}-\frac{8b\sqrt{1-(c+dx)^2}\,(e(c+dx))^{9/2}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{13}{4};(c+dx)^2\right)\left(a\right.}{63de^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(a + b*ArcCosh[c + d*x])^2)/(7*d*e) - (8*b*(e*(c + d
*x))^(9/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1
[1/2, 9/4, 13/4, (c + d*x)^2])/(63*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x]) - (16*b^2*(e*(c + d*x))^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4
, 15/4}, (c + d*x)^2])/(693*d*e^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int (ex)^{5/2}\left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$
$$= \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{7de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{7/2}(a+b\cos}{\sqrt{-1+x}\sqrt{1}}\right.}{7de}$$
$$= \frac{2(e(c+dx))^{7/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{7de} - \frac{8b(e(c+dx))^{9/2}\sqrt{1-(c+}}{63d}$$

Mathematica [A] time = 0.48, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{7/2}\left(99\left(a+b\cosh^{-1}(c+dx)\right)^2 - 4b(c+dx)\left(2b(c+dx)\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};(c+dx)^2\right) + \frac{11\sqrt{1-(c+dx)^2}}{}\right.\right.}{693de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(5/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(7/2)*(99*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((11*
Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 9/4,
13/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*
HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, (c + d*x)^2])))/(693*d*e)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2d^2e^2x^2 + 2\,a^2cde^2x + a^2c^2e^2 + \left(b^2d^2e^2x^2 + 2\,b^2cde^2x + b^2c^2e^2\right)\text{arcosh}\,(dx+c)^2 + 2\left(abd^2e^2x^2 + 2\,ab\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d^2*e^2*x^2 + 2*a^2*c*d*e^2*x + a^2*c^2*e^2 + (b^2*d^2*e^2*x^
2 + 2*b^2*c*d*e^2*x + b^2*c^2*e^2)*arccosh(d*x + c)^2 + 2*(a*b*d^2*e^2*x^2
+ 2*a*b*c*d*e^2*x + a*b*c^2*e^2)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(5/2)*(b*arccosh(d*x + c) + a)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{5}{2}}(a+b\,\text{arccosh}\,(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex+ce)^{\frac{7}{2}}a^2}{7\,de}+\frac{2\left(b^2d^3e^{\frac{5}{2}}x^3+3\,b^2cd^2e^{\frac{5}{2}}x^2+3\,b^2c^2de^{\frac{5}{2}}x+b^2c^3e^{\frac{5}{2}}\right)\sqrt{dx+c}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-}\right.}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(5/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/7*(d*e*x + c*e)^(7/2)*a^2/(d*e) + 2/7*(b^2*d^3*e^(5/2)*x^3 + 3*b^2*c*d^2*
e^(5/2)*x^2 + 3*b^2*c^2*d*e^(5/2)*x + b^2*c^3*e^(5/2))*sqrt(d*x + c)*log(d*
x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/d + integrate(-2/7*((2*b^2*c
^4*e^(5/2) - (7*a*b*d^4*e^(5/2) - 2*b^2*d^4*e^(5/2))*x^4 - 4*(7*a*b*c*d^3*e
^(5/2) - 2*b^2*c*d^3*e^(5/2))*x^3 - 7*(c^4*e^(5/2) - c^2*e^(5/2))*a*b + (12
*b^2*c^2*d^2*e^(5/2) - 7*(6*c^2*d^2*e^(5/2) - d^2*e^(5/2))*a*b)*x^2 + 2*(4*
b^2*c^3*d*e^(5/2) - 7*(2*c^3*d*e^(5/2) - c*d*e^(5/2))*a*b)*x)*sqrt(d*x + c
+ 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) - ((7*a*b*d^5*e^(5/2) - 2*b^2*d^5*e^(5
/2))*x^5 + 5*(7*a*b*c*d^4*e^(5/2) - 2*b^2*c*d^4*e^(5/2))*x^4 + (7*(10*c^2*d
^3*e^(5/2) - d^3*e^(5/2))*a*b - 2*(10*c^2*d^3*e^(5/2) - d^3*e^(5/2))*b^2)*x
^3 + 7*(c^5*e^(5/2) - c^3*e^(5/2))*a*b - 2*(c^5*e^(5/2) - c^3*e^(5/2))*b^2
+ (7*(10*c^3*d^2*e^(5/2) - 3*c*d^2*e^(5/2))*a*b - 2*(10*c^3*d^2*e^(5/2) - 3
*c*d^2*e^(5/2))*b^2)*x^2 + (7*(5*c^4*d*e^(5/2) - 3*c^2*d*e^(5/2))*a*b - 2*(
5*c^4*d*e^(5/2) - 3*c^2*d*e^(5/2))*b^2)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*
x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 +
 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x -
 c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{5/2}\,(a+b\,\mathrm{acosh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(5/2)*(a + b*acosh(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(5/2)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.208 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^2 dx$

Optimal. Leaf size=153

$$-\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};(c+dx)^2\right)}{315de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};(c+dx)^2\right)\left(a+b\cos\right.}{35de^2\sqrt{c+dx-1}}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))^2/d/e-16/315*b^2*(e*(d*x+c))^(9/
2)*HypergeometricPFQ([1, 9/4, 9/4],[11/4, 13/4],(d*x+c)^2)/d/e^3-8/35*b*(e*
(d*x+c))^(7/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 7/4],[11/4],(d*x+c)^2)*
(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};(c+dx)^2\right)}{315de^3}-\frac{8b\sqrt{1-(c+dx)^2}\,(e(c+dx))^{7/2}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{11}{4};(c+dx)^2\right)\left(a+b\cosh\right.}{35de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^2)/(5*d*e) - (8*b*(e*(c + d
*x))^(7/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1
[1/2, 7/4, 11/4, (c + d*x)^2])/(35*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*
x]) - (16*b^2*(e*(c + d*x))^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 1
3/4}, (c + d*x)^2])/(315*d*e^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^2\,dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de}-\frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}(a+b}{\sqrt{-1+x}}\right.}{5a}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de}-\frac{8b(e(c+dx))^{7/2}\sqrt{1-(c}}{}$$

Mathematica [A] time = 0.49, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{5/2}\left(63\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(2b(c+dx)\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};(c+dx)^2\right)+\frac{9\sqrt{1-(c+dx)^2}}{}\right.\right.}{315de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(63*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((9*S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 7/4, 1
1/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*H
ypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, (c + d*x)^2])))/(315*d*e)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2dex+a^2ce+\left(b^2dex+b^2ce\right)\text{arcosh}\,(dx+c)^2+2\,(abdex+abce)\,\text{arcosh}\,(dx+c)\right)\sqrt{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*d*e*x + a^2*c*e + (b^2*d*e*x + b^2*c*e)*arccosh(d*x + c)^2 +
2*(a*b*d*e*x + a*b*c*e)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^{\frac{3}{2}}\,(a+b\,\text{arccosh}\,(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,(dex+ce)^{\frac{5}{2}}a^2}{5\,de}+\frac{2\left(b^2d^2e^{\frac{3}{2}}x^2+2\,b^2cde^{\frac{3}{2}}x+b^2c^2e^{\frac{3}{2}}\right)\sqrt{dx+c}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^2}{5\,d}+\int-\frac{2\left(\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/5*(d*e*x + c*e)^(5/2)*a^2/(d*e) + 2/5*(b^2*d^2*e^(3/2)*x^2 + 2*b^2*c*d*e^
(3/2)*x + b^2*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d
*x + c - 1) + c)^2/d + integrate(-2/5*((2*b^2*c^3*e^(3/2) - (5*a*b*d^3*e^(3
/2) - 2*b^2*d^3*e^(3/2))*x^3 - 5*(c^3*e^(3/2) - c*e^(3/2))*a*b - 3*(5*a*b*c
*d^2*e^(3/2) - 2*b^2*c*d^2*e^(3/2))*x^2 + (6*b^2*c^2*d*e^(3/2) - 5*(3*c^2*d
*e^(3/2) - d*e^(3/2))*a*b)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c
- 1) - ((5*a*b*d^4*e^(3/2) - 2*b^2*d^4*e^(3/2))*x^4 + 4*(5*a*b*c*d^3*e^(3/2
) - 2*b^2*c*d^3*e^(3/2))*x^3 + 5*(c^4*e^(3/2) - c^2*e^(3/2))*a*b - 2*(c^4*e
^(3/2) - c^2*e^(3/2))*b^2 + (5*(6*c^2*d^2*e^(3/2) - d^2*e^(3/2))*a*b - 2*(6
*c^2*d^2*e^(3/2) - d^2*e^(3/2))*b^2)*x^2 + 2*(5*(2*c^3*d*e^(3/2) - c*d*e^(3
/2))*a*b - 2*(2*c^3*d*e^(3/2) - c*d*e^(3/2))*b^2)*x)*sqrt(d*x + c))*log(d*x
 + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x^3 + 3*c*d^2*x^2 + c^3 +
(d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*
d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^{3/2}\,(a+b\,\mathrm{acosh}\,(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\,(c+dx))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(c+dx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**2, x)
```

3.209 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx$

Optimal. Leaf size=153

$$-\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};(c+dx)^2\right)}{105de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};(c+dx)^2\right)\left(a+b\,\mathrm{c}\right.}{15de^2\sqrt{c+dx-1}}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))^2/d/e-16/105*b^2*(e*(d*x+c))^(7/
2)*HypergeometricPFQ([1, 7/4, 7/4],[9/4, 11/4],(d*x+c)^2)/d/e^3-8/15*b*(e*(
d*x+c))^(5/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2, 5/4],[9/4],(d*x+c)^2)*(-
d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};(c+dx)^2\right)}{105de^3}-\frac{8b\sqrt{1-(c+dx)^2}\,(e(c+dx))^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};(c+dx)^2\right)\left(a+b\right.}{15de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^2)/(3*d*e) - (8*b*(e*(c + d
*x))^(5/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1
[1/2, 5/4, 9/4, (c + d*x)^2])/(15*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x
]) - (16*b^2*(e*(c + d*x))^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/
4}, (c + d*x)^2])/(105*d*e^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{ce+dex}\,\left(a+b\cosh^{-1}(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \sqrt{ex}\,\left(a+b\cosh^{-1}(x)\right)^2\,dx, x, c+dx\right)}{d}$$
$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}(a+b\cosh}{\sqrt{-1+x}\,\sqrt{1+}}\right.}{3de}$$
$$= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de} - \frac{8b(e(c+dx))^{5/2}\sqrt{1-(c+d}}{15de^2}$$

Mathematica [A] time = 0.43, size = 140, normalized size = 0.92

$$\frac{2(e(c+dx))^{3/2}\left(35\left(a+b\cosh^{-1}(c+dx)\right)^2 - 4b(c+dx)\left(2b(c+dx)\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};(c+dx)^2\right) + \frac{7\sqrt{1-(c+dx)^2}\,{}_2F_1(}{\sqrt{}}\right.\right.}{105de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(35*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((7*S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 5/4, 9
/4, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*Hy
pergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, (c + d*x)^2])))/(105*d*e)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\,\text{arcosh}\,(dx+c)^2 + 2\,ab\,\text{arcosh}\,(dx+c) + a^2\right)\sqrt{dex+ce}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dex+ce}\,(b\,\text{arcosh}\,(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^2, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\,\text{arccosh}\,(dx+c))^2\,\sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(b^2d\sqrt{e}\,x+b^2c\sqrt{e}\right)\sqrt{dx+c}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^2}{3\,d}+\frac{2\,(dex+ce)^{\frac{3}{2}}a^2}{3\,de}+\int-\frac{2\left(\left(2\,b^2c^2\sqrt{e}\,-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*d*sqrt(e)*x + b^2*c*sqrt(e))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + c)^2/d + 2/3*(d*e*x + c*e)^(3/2)*a^2/(d*e) + integ
rate(-2/3*((2*b^2*c^2*sqrt(e) - 3*(c^2*sqrt(e) - sqrt(e))*a*b - (3*a*b*d^2*
sqrt(e) - 2*b^2*d^2*sqrt(e))*x^2 - 2*(3*a*b*c*d*sqrt(e) - 2*b^2*c*d*sqrt(e)
)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) - ((3*a*b*d^3*sqrt(e
) - 2*b^2*d^3*sqrt(e))*x^3 + 3*(c^3*sqrt(e) - c*sqrt(e))*a*b - 2*(c^3*sqrt(
e) - c*sqrt(e))*b^2 + 3*(3*a*b*c*d^2*sqrt(e) - 2*b^2*c*d^2*sqrt(e))*x^2 + (
3*(3*c^2*d*sqrt(e) - d*sqrt(e))*a*b - 2*(3*c^2*d*sqrt(e) - d*sqrt(e))*b^2)*
x)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*x
^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sq
rt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\,(c+dx)}\,\left(a+b\,\mathrm{acosh}\,(c+dx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**2, x)
```

3.210 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=151

$$-\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};(c+dx)^2\right)}{15de^3}-\frac{8b\sqrt{-c-dx+1}\,(e(c+dx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)\left(a+b\cosh\right.}{3de^2\sqrt{c+dx-1}}$$

```
[Out] -16/15*b^2*(e*(d*x+c))^(5/2)*HypergeometricPFQ([1, 5/4, 5/4],[7/4, 9/4],(d*
x+c)^2)/d/e^3-8/3*b*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))*hypergeom([1/2,
3/4],[7/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(1/2)+2*(a+b*arccosh
(d*x+c))^2*(e*(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.29, antiderivative size = 163, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2(e(c+dx))^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};(c+dx)^2\right)}{15de^3}-\frac{8b\sqrt{1-(c+dx)^2}\,(e(c+dx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(c+dx)^2\right)\left(a+b\cosh\right.}{3de^2\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^2)/(d*e) - (8*b*(e*(c + d*x))
^(3/2)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2
, 3/4, 7/4, (c + d*x)^2])/(3*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) -
(16*b^2*(e*(c + d*x))^(5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c
 + d*x)^2])/(15*d*e^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{ce+dex}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de} - \frac{(4b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\cosh^{-1}(x)\right)}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,c\right.}{de}$$

$$= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de} - \frac{8b(e(c+dx))^{3/2}\sqrt{1-(c+dx)^2}\left(a+b\right.}{3de^2\sqrt{-1+c+}}$$

Mathematica [A] time = 0.30, size = 140, normalized size = 0.93

$$\frac{2\sqrt{e(c+dx)}\left(15\left(a+b\cosh^{-1}(c+dx)\right)^2-4b(c+dx)\left(2b(c+dx)\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};(c+dx)^2\right)+\frac{5\sqrt{1-(c+dx)^2}\,{}_2F_1(}{\sqrt{}}\right.\right.}{15de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(15*(a + b*ArcCosh[c + d*x])^2 - 4*b*(c + d*x)*((5*Sqr
t[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4
, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*Hype
rgeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, (c + d*x)^2])))/(15*d*e)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arcosh}\,(dx+c)^2+2\,ab\,\text{arcosh}\,(dx+c)+a^2}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)/sqrt(d*e*x
 + c*e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^2}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/sqrt(d*e*x + c*e), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccosh}\,(dx+c))^2}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)
```

[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{dx+c}\,b^2\log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^2}{d\sqrt{e}}+\frac{2\sqrt{dex+ce}\,a^2}{de}+\int-\frac{2\left(\left(2\,b^2c^2\sqrt{e}-\left(c^2\sqrt{e}-\sqrt{e}\right)ab-\left(a\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)*b^2*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2/(d
*sqrt(e)) + 2*sqrt(d*e*x + c*e)*a^2/(d*e) + integrate(-2*((2*b^2*c^2*sqrt(e
) - (c^2*sqrt(e) - sqrt(e))*a*b - (a*b*d^2*sqrt(e) - 2*b^2*d^2*sqrt(e))*x^2
 - 2*(a*b*c*d*sqrt(e) - 2*b^2*c*d*sqrt(e))*x)*sqrt(d*x + c + 1)*sqrt(d*x +
c)*sqrt(d*x + c - 1) - ((a*b*d^3*sqrt(e) - 2*b^2*d^3*sqrt(e))*x^3 + (c^3*sq
rt(e) - c*sqrt(e))*a*b - 2*(c^3*sqrt(e) - c*sqrt(e))*b^2 + 3*(a*b*c*d^2*sqr
t(e) - 2*b^2*c*d^2*sqrt(e))*x^2 + ((3*c^2*d*sqrt(e) - d*sqrt(e))*a*b - 2*(3
*c^2*d*sqrt(e) - d*sqrt(e))*b^2)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c +
 1)*sqrt(d*x + c - 1) + c)/(d^4*e*x^4 + 4*c*d^3*e*x^3 + c^4*e - c^2*e + (6*
c^2*d^2*e - d^2*e)*x^2 + (d^3*e*x^3 + 3*c*d^2*e*x^2 + c^3*e - c*e + (3*c^2*
d*e - d*e)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(2*c^3*d*e - c*d*e)*x
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^2}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\operatorname{acosh}\left(c+dx\right)\right)^2}{\sqrt{e\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/sqrt(e*(c + d*x)), x)
```

3.211 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=149

$$\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{-c-dx+1}\,\sqrt{e(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}\right.}{de^2\sqrt{c+dx-1}}$$

```
[Out] 16/3*b^2*(e*(d*x+c))^(3/2)*HypergeometricPFQ([3/4, 3/4, 1],[5/4, 7/4],(d*x+
c)^2)/d/e^3-2*(a+b*arccosh(d*x+c))^2/d/e/(e*(d*x+c))^(1/2)+8*b*(a+b*arccosh
(d*x+c))*hypergeom([1/4, 1/2],[5/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)*(e*(d*x+c))
^(1/2)/d/e^2/(d*x+c-1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 161, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5866, 5662, 5763}

$$\frac{16b^2(e(c+dx))^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};(c+dx)^2\right)}{3de^3}+\frac{8b\sqrt{1-(c+dx)^2}\,\sqrt{e(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\left(a+b\cosh\right.}{de^2\sqrt{c+dx-1}\,\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Sqrt[e*(c +
d*x)]*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4,
 1/2, 5/4, (c + d*x)^2])/(d*e^2*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + (16
*b^2*(e*(c + d*x))^(3/2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, (c +
d*x)^2])/(3*d*e^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{(ex)^{3/2}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx, x, c+dx\right)}{de}$$
$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{de\sqrt{e(c+dx)}} + \frac{8b\sqrt{e(c+dx)}\sqrt{1-(c+dx)^2}\left(a+b\cosh^{-1}(c+dx\right.}{de^2\sqrt{-1+c+dx}\sqrt{1+c+}}$$

Mathematica [A] time = 0.30, size = 140, normalized size = 0.94

$$\frac{2\left(4b(c+dx)\left(2b(c+dx)\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};(c+dx)^2\right)+\frac{3\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{\sqrt{c+dx-1}\sqrt{c+dx+1}}\right)-3\left(a+b\cosh\right.}{3de\sqrt{e(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (2*(-3*(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*((3*Sqrt[1 - (c + d*x)^2]
*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/4, 1/2, 5/4, (c + d*x)^2])/(S
qrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + 2*b*(c + d*x)*HypergeometricPFQ[{3/4
, 3/4, 1}, {5/4, 7/4}, (c + d*x)^2])))/(3*d*e*Sqrt[e*(c + d*x)])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\,\text{arcosh}\,(dx+c)^2+2\,ab\,\text{arcosh}\,(dx+c)+a^2\right)\sqrt{dex+ce}}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(dx+c)+a)^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccosh}\,(dx+c))^2}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^2\sqrt{e}\log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^2}{d^2e^2x+cde^2}-\frac{2\,a^2}{\sqrt{dex+ce}\,de}+\int\frac{2\left(\left(2\,b^2c^2+\left(c^2-1\right)ab+\left(abd^2\right.\right.\right.}{d^5e^{\frac{3}{2}}x^5+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
[Out] -2*sqrt(d*x + c)*b^2*sqrt(e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ c)^2/(d^2*e^2*x + c*d*e^2) - 2*a^2/(sqrt(d*e*x + c*e)*d*e) + integrate(2*
((2*b^2*c^2 + (c^2 - 1)*a*b + (a*b*d^2 + 2*b^2*d^2)*x^2 + 2*(a*b*c*d + 2*b^
2*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((a*b*d^3 + 2
*b^2*d^3)*x^3 + (c^3 - c)*a*b + 2*(c^3 - c)*b^2 + 3*(a*b*c*d^2 + 2*b^2*c*d^
2)*x^2 + ((3*c^2*d - d)*a*b + 2*(3*c^2*d - d)*b^2)*x)*sqrt(d*x + c))*log(d*
x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^5*e^(3/2)*x^5 + 5*c*d^4*e^(
3/2)*x^4 + c^5*e^(3/2) - c^3*e^(3/2) + (10*c^2*d^3*e^(3/2) - d^3*e^(3/2))*x
^3 + (10*c^3*d^2*e^(3/2) - 3*c*d^2*e^(3/2))*x^2 + (d^4*e^(3/2)*x^4 + 4*c*d^
3*e^(3/2)*x^3 + c^4*e^(3/2) - c^2*e^(3/2) + (6*c^2*d^2*e^(3/2) - d^2*e^(3/2
))*x^2 + 2*(2*c^3*d*e^(3/2) - c*d*e^(3/2))*x)*sqrt(d*x + c + 1)*sqrt(d*x +
c - 1) + (5*c^4*d*e^(3/2) - 3*c^2*d*e^(3/2))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(3/2),x)
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\mathrm{acosh}\,(c+dx))^2}{(e\,(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(3/2),x)
[Out] Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(3/2), x)
```

3.212 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};(c+dx)^2\right)}{3de^3}-\frac{8b\sqrt{-c-dx+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{3de^2\sqrt{c+dx-1}\sqrt{e(c+dx)}}-\cdots$$

```
[Out] -2/3*(a+b*arccosh(d*x+c))^2/d/e/(e*(d*x+c))^(3/2)-8/3*b*(a+b*arccosh(d*x+c)
)*hypergeom([-1/4, 1/2],[3/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(d*x+c-1)^(
1/2)/(e*(d*x+c))^(1/2)-16/3*b^2*HypergeometricPFQ([1/4, 1/4, 1],[3/4, 5/4],
(d*x+c)^2)*(e*(d*x+c))^(1/2)/d/e^3
```

Rubi [A] time = 0.31, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5866, 5662, 5763}

$$-\frac{16b^2\sqrt{e(c+dx)}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};(c+dx)^2\right)}{3de^3}-\frac{8b\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{3de^2\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(3*d*e*(e*(c + d*x))^(3/2)) - (8*b*Sqrt[1 -
 (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, (c
 + d*x)^2])/(3*d*e^2*Sqrt[-1 + c + d*x]*Sqrt[e*(c + d*x)]*Sqrt[1 + c + d*x]
) - (16*b^2*Sqrt[e*(c + d*x)]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4},
(c + d*x)^2])/(3*d*e^3)
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{(ex)^{5/2}}\,dx, x, c+dx\right)}{d}$$
$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} + \frac{(4b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,(ex)^{3/2}\sqrt{1+x}}\,dx, x, c+dx\right)}{3de}$$
$$= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{3de(e(c+dx))^{3/2}} - \frac{8b\sqrt{1-(c+dx)^2}\,\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(\right.}{3de^2\sqrt{-1+c+dx}\,\sqrt{e(c+dx)}\,\sqrt{1+}}$$

Mathematica [A] time = 0.34, size = 140, normalized size = 0.92

$$\frac{2\left(4b(c+dx)\left(-2b(c+dx)\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};(c+dx)^2\right) - \frac{\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}\right) - \left(a+b\,c\right.}{3de(e(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (2*(-(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*(-((Sqrt[1 - (c + d*x)^2]*(
a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-1/4, 1/2, 3/4, (c + d*x)^2])/(Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x])) - 2*b*(c + d*x)*HypergeometricPFQ[{1/4
, 1/4, 1}, {3/4, 5/4}, (c + d*x)^2])))/(3*d*e*(e*(c + d*x))^(3/2))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\,\text{arcosh}\,(dx+c)^2 + 2\,ab\,\text{arcosh}\,(dx+c) + a^2\right)\sqrt{dex+ce}}{d^3e^3x^3 + 3\,cd^2e^3x^2 + 3\,c^2de^3x + c^3e^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(t_nostep)]Evaluation time:
0.82sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccosh}\,(dx+c))^2}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^2\sqrt{e}\log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^2}{3\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}-\frac{2\,a^2}{3\,(dex+ce)^{\frac{3}{2}}de}+\int\frac{2\left(\left(2\,b^2c^2+3\left(c^2-1\right)ab+\left(3\right.\right.\right.}{3\left(d^6e^{\frac{5}{2}}x^6+6\,cd^5e^{\frac{5}{2}}x^5+c^6e^{\frac{5}{2}}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(d*x + c)*b^2*sqrt(e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^2/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2/3*a^2/((d*e*x + c*e)
^(3/2)*d*e) + integrate(2/3*((2*b^2*c^2 + 3*(c^2 - 1)*a*b + (3*a*b*d^2 + 2*
b^2*d^2)*x^2 + 2*(3*a*b*c*d + 2*b^2*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)
*sqrt(d*x + c - 1) + ((3*a*b*d^3 + 2*b^2*d^3)*x^3 + 3*(c^3 - c)*a*b + 2*(c^
3 - c)*b^2 + 3*(3*a*b*c*d^2 + 2*b^2*c*d^2)*x^2 + (3*(3*c^2*d - d)*a*b + 2*(
3*c^2*d - d)*b^2)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
c - 1) + c)/(d^6*e^(5/2)*x^6 + 6*c*d^5*e^(5/2)*x^5 + c^6*e^(5/2) - c^4*e^(5
/2) + (15*c^2*d^4*e^(5/2) - d^4*e^(5/2))*x^4 + 4*(5*c^3*d^3*e^(5/2) - c*d^3
*e^(5/2))*x^3 + 3*(5*c^4*d^2*e^(5/2) - 2*c^2*d^2*e^(5/2))*x^2 + (d^5*e^(5/2
)*x^5 + 5*c*d^4*e^(5/2)*x^4 + c^5*e^(5/2) - c^3*e^(5/2) + (10*c^2*d^3*e^(5/
2) - d^3*e^(5/2))*x^3 + (10*c^3*d^2*e^(5/2) - 3*c*d^2*e^(5/2))*x^2 + (5*c^4
*d*e^(5/2) - 3*c^2*d*e^(5/2))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(3
*c^5*d*e^(5/2) - 2*c^3*d*e^(5/2))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\mathrm{acosh}\,(c+dx))^2}{(e\,(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(5/2), x)
```

3.213 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=153

$$\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\sqrt{-c-dx+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{15de^2\sqrt{c+dx-1}\,(e(c+dx))^{3/2}}-\frac{2\left(a+b\right.}{5de}$$

```
[Out] -2/5*(a+b*arccosh(d*x+c))^2/d/e/(e*(d*x+c))^(5/2)-8/15*b*(a+b*arccosh(d*x+c
))*hypergeom([-3/4, 1/2],[1/4],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(e*(d*x+c)
)^(3/2)/(d*x+c-1)^(1/2)+16/15*b^2*HypergeometricPFQ([-1/4, -1/4, 1],[1/4, 3
/4],(d*x+c)^2)/d/e^3/(e*(d*x+c))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5866, 5662, 5763}

$$\frac{16b^2\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};(c+dx)^2\right)}{15de^3\sqrt{e(c+dx)}}-\frac{8b\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{15de^2\sqrt{c+dx-1}\sqrt{c+dx+1}\,(e(c+dx))^{3/2}}-\frac{2\left(a+\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^2)/(5*d*e*(e*(c + d*x))^(5/2)) - (8*b*Sqrt[1 -
 (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, (c
 + d*x)^2])/(15*d*e^2*Sqrt[-1 + c + d*x]*(e*(c + d*x))^(3/2)*Sqrt[1 + c + d
*x]) + (16*b^2*HypergeometricPFQ[{-1/4, -1/4, 1}, {1/4, 3/4}, (c + d*x)^2])
/(15*d*e^3*Sqrt[e*(c + d*x)])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{(ce+dex)^{7/2}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}}+\frac{(4b)\,\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{\sqrt{-1+x}\,(ex)^{5/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{5de}$$

$$=-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^2}{5de(e(c+dx))^{5/2}}-\frac{8b\sqrt{1-(c+dx)^2}\,\left(a+b\cosh^{-1}(c+dx)\right)\,{}_2F_1\left(-\frac{3}{4},\right.}{15de^2\sqrt{-1+c+dx}\,(e(c+dx))^{3/2}\sqrt{1+c}}$$

Mathematica [A] time = 0.36, size = 140, normalized size = 0.92

$$\frac{2\left(4b(c+dx)\left(2b(c+dx)\,{}_3F_2\left(-\frac{1}{4},-\frac{1}{4},1;\frac{1}{4},\frac{3}{4};(c+dx)^2\right)-\frac{\sqrt{1-(c+dx)^2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};(c+dx)^2\right)\left(a+b\cosh^{-1}(c+dx)\right)}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}}\right)-3\left(a+b\,\text{c}\right.}{15de(e(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^2/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (2*(-3*(a + b*ArcCosh[c + d*x])^2 + 4*b*(c + d*x)*(-((Sqrt[1 - (c + d*x)^2]
*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[-3/4, 1/2, 1/4, (c + d*x)^2])/(
Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])) + 2*b*(c + d*x)*HypergeometricPFQ[{-
1/4, -1/4, 1}, {1/4, 3/4}, (c + d*x)^2])))/(15*d*e*(e*(c + d*x))^(5/2))
```

fricas [F] time = 1.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\operatorname{arcosh}(dx+c)^2+2\,ab\operatorname{arcosh}(dx+c)+a^2\right)\sqrt{dex+ce}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*sqrt(d*e*x
 + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x
+ c^4*e^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(dx+c)+a)^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2/(d*e*x + c*e)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccosh}(dx+c))^2}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)

[Out] int((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^2\sqrt{e}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^2}{5\left(d^4e^4x^3+3\,cd^3e^4x^2+3\,c^2d^2e^4x+c^3de^4\right)}-\frac{2\,a^2}{5\,(dex+ce)^{\frac{5}{2}}de}+\int\frac{}{5\left(d^7e^{\frac{7}{2}}x^7+7\,cd^6e^{\frac{7}{2}}x^6+c^7e^{\frac{7}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^2/(d*e*x+c*e)^(7/2),x, algorithm="maxima")

[Out] -2/5*sqrt(d*x + c)*b^2*sqrt(e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^2/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 2/
5*a^2/((d*e*x + c*e)^(5/2)*d*e) + integrate(2/5*((2*b^2*c^2 + 5*(c^2 - 1)*a
*b + (5*a*b*d^2 + 2*b^2*d^2)*x^2 + 2*(5*a*b*c*d + 2*b^2*c*d)*x)*sqrt(d*x +
c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((5*a*b*d^3 + 2*b^2*d^3)*x^3 + 5*(
c^3 - c)*a*b + 2*(c^3 - c)*b^2 + 3*(5*a*b*c*d^2 + 2*b^2*c*d^2)*x^2 + (5*(3*
c^2*d - d)*a*b + 2*(3*c^2*d - d)*b^2)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*x
+ c + 1)*sqrt(d*x + c - 1) + c)/(d^7*e^(7/2)*x^7 + 7*c*d^6*e^(7/2)*x^6 + c^
7*e^(7/2) - c^5*e^(7/2) + (21*c^2*d^5*e^(7/2) - d^5*e^(7/2))*x^5 + 5*(7*c^3
*d^4*e^(7/2) - c*d^4*e^(7/2))*x^4 + 5*(7*c^4*d^3*e^(7/2) - 2*c^2*d^3*e^(7/2
))*x^3 + (21*c^5*d^2*e^(7/2) - 10*c^3*d^2*e^(7/2))*x^2 + (d^6*e^(7/2)*x^6 +
 6*c*d^5*e^(7/2)*x^5 + c^6*e^(7/2) - c^4*e^(7/2) + (15*c^2*d^4*e^(7/2) - d^
4*e^(7/2))*x^4 + 4*(5*c^3*d^3*e^(7/2) - c*d^3*e^(7/2))*x^3 + 3*(5*c^4*d^2*e
^(7/2) - 2*c^2*d^2*e^(7/2))*x^2 + 2*(3*c^5*d*e^(7/2) - 2*c^3*d*e^(7/2))*x)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*c^6*d*e^(7/2) - 5*c^4*d*e^(7/2))*x
), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^2}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(7/2),x)

[Out] int((a + b*acosh(c + d*x))^2/(c*e + d*e*x)^(7/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+dx))^2}{(e\,(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acosh(d*x+c))**2/(d*e*x+c*e)**(7/2),x)

[Out] Integral((a + b*acosh(c + d*x))**2/(e*(c + d*x))**(7/2), x)

3.214 $\int (ce+dex)^{3/2} \left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$

Optimal. Leaf size=89

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de}-\frac{6b\text{Int}\left(\frac{(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))^3/d/e-6/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arccosh(d*x+c))^2/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^3)/(5*d*e) - (6*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^3 \, dx,x,c+dx\right)}{d}$$

$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de}-\frac{(6b)\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\cosh\right.}{\sqrt{-1+x}\sqrt{1}}\right.}{5de}$$

Mathematica [A] time = 107.34, size = 0, normalized size = 0.00

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^3, x]
```

fricas [A] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^3dex+a^3ce+\left(b^3dex+b^3ce\right)\text{arcosh}\left(dx+c\right)^3+3\left(ab^2dex+ab^2ce\right)\text{arcosh}\left(dx+c\right)^2+3\left(a^2bdex+a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*d*e*x + a^3*c*e + (b^3*d*e*x + b^3*c*e)*arccosh(d*x + c)^3 +
3*(a*b^2*d*e*x + a*b^2*c*e)*arccosh(d*x + c)^2 + 3*(a^2*b*d*e*x + a^2*b*c*e
)*arccosh(d*x + c))*sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{3}{2}} (b \operatorname{arcosh}(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^3, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{3}{2}} (a + b \operatorname{arccosh}(dx + c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 (dex + ce)^{\frac{5}{2}} a^3}{5 de} + \frac{2 \left(b^3 d^2 e^{\frac{3}{2}} x^2 + 2 b^3 c d e^{\frac{3}{2}} x + b^3 c^2 e^{\frac{3}{2}}\right) \sqrt{dx + c} \log\left(dx + \sqrt{dx + c + 1} \sqrt{dx + c - 1} + c\right)^3}{5 d} + \int -- 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/5*(d*e*x + c*e)^(5/2)*a^3/(d*e) + 2/5*(b^3*d^2*e^(3/2)*x^2 + 2*b^3*c*d*e^
(3/2)*x + b^3*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d
*x + c - 1) + c)^3/d + integrate(-3/5*(((2*b^3*c^3*e^(3/2) - 5*(c^3*e^(3/2)
 - c*e^(3/2))*a*b^2 - (5*a*b^2*d^3*e^(3/2) - 2*b^3*d^3*e^(3/2))*x^3 - 3*(5*
a*b^2*c*d^2*e^(3/2) - 2*b^3*c*d^2*e^(3/2))*x^2 + (6*b^3*c^2*d*e^(3/2) - 5*(
3*c^2*d*e^(3/2) - d*e^(3/2))*a*b^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt
(d*x + c - 1) - ((5*a*b^2*d^4*e^(3/2) - 2*b^3*d^4*e^(3/2))*x^4 + 5*(c^4*e^(
3/2) - c^2*e^(3/2))*a*b^2 - 2*(c^4*e^(3/2) - c^2*e^(3/2))*b^3 + 4*(5*a*b^2*
c*d^3*e^(3/2) - 2*b^3*c*d^3*e^(3/2))*x^3 + (5*(6*c^2*d^2*e^(3/2) - d^2*e^(3
/2))*a*b^2 - 2*(6*c^2*d^2*e^(3/2) - d^2*e^(3/2))*b^3)*x^2 + 2*(5*(2*c^3*d*e
^(3/2) - c*d*e^(3/2))*a*b^2 - 2*(2*c^3*d*e^(3/2) - c*d*e^(3/2))*b^3)*x)*sqr
t(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - 5*((a^2*
b*d^3*e^(3/2)*x^3 + 3*a^2*b*c*d^2*e^(3/2)*x^2 + (3*c^2*d*e^(3/2) - d*e^(3/2
))*a^2*b*x + (c^3*e^(3/2) - c*e^(3/2))*a^2*b)*sqrt(d*x + c + 1)*sqrt(d*x +
c)*sqrt(d*x + c - 1) + (a^2*b*d^4*e^(3/2)*x^4 + 4*a^2*b*c*d^3*e^(3/2)*x^3 +
 (6*c^2*d^2*e^(3/2) - d^2*e^(3/2))*a^2*b*x^2 + 2*(2*c^3*d*e^(3/2) - c*d*e^(
3/2))*a^2*b*x + (c^4*e^(3/2) - c^2*e^(3/2))*a^2*b)*sqrt(d*x + c))*log(d*x +
 sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (
d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d
 - d)*x - c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)^{3/2} \, (a + b \operatorname{acosh}(c + d\,x))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^3, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\,(c+dx))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(c+dx))^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**3,x)
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**3, x)
```

3.215 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de}-\frac{2b\mathrm{Int}\left(\frac{(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))^3/d/e-2*b*Unintegrable((e*(d*x+c
))^(3/2)*(a+b*arccosh(d*x+c))^2/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^3)/(3*d*e) - (2*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(d*e)
```

Rubi steps

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^3\,dx=\frac{\mathrm{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de}-\frac{(2b)\,\mathrm{Subst}\left(\int \frac{(ex)^{3/2}(a+b\,\mathrm{c}}{\sqrt{-1+x}}\right.}{de}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^3\,\mathrm{arcosh}\left(dx+c\right)^3+3\,ab^2\,\mathrm{arcosh}\left(dx+c\right)^2+3\,a^2b\,\mathrm{arcosh}\left(dx+c\right)+a^3\right)\sqrt{dex+ce},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dex+ce}\,(b\,\mathrm{arcosh}\,(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^3, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{arccosh}\,(dx+c))^3 \sqrt{dex+ce}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(b^3d\sqrt{e}\,x+b^3c\sqrt{e}\right)\sqrt{dx+c}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^3}{3\,d}+\frac{2\,(dex+ce)^{\frac{3}{2}}a^3}{3\,de}+\int-\frac{\left(\left(2\,b^3c^2\sqrt{e}\,-3\,(c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3*(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^3*d*sqrt(e)*x + b^3*c*sqrt(e))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1) + c)^3/d + 2/3*(d*e*x + c*e)^(3/2)*a^3/(d*e) + integ
rate(-(((2*b^3*c^2*sqrt(e) - 3*(c^2*sqrt(e) - sqrt(e))*a*b^2 - (3*a*b^2*d^2
*sqrt(e) - 2*b^3*d^2*sqrt(e))*x^2 - 2*(3*a*b^2*c*d*sqrt(e) - 2*b^3*c*d*sqrt
(e))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) - (3*(c^3*sqrt(e)
 - c*sqrt(e))*a*b^2 - 2*(c^3*sqrt(e) - c*sqrt(e))*b^3 + (3*a*b^2*d^3*sqrt(e
) - 2*b^3*d^3*sqrt(e))*x^3 + 3*(3*a*b^2*c*d^2*sqrt(e) - 2*b^3*c*d^2*sqrt(e)
)*x^2 + (3*(3*c^2*d*sqrt(e) - d*sqrt(e))*a*b^2 - 2*(3*c^2*d*sqrt(e) - d*sqr
t(e))*b^3)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1)
+ c)^2 - 3*((a^2*b*d^2*sqrt(e)*x^2 + 2*a^2*b*c*d*sqrt(e)*x + (c^2*sqrt(e) -
 sqrt(e))*a^2*b)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b
*d^3*sqrt(e)*x^3 + 3*a^2*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e)
)*a^2*b*x + (c^3*sqrt(e) - c*sqrt(e))*a^2*b)*sqrt(d*x + c))*log(d*x + sqrt(
d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^
2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*
x - c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e+d\,e\,x}\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\,(c+dx)}\,(a+b\,\mathrm{acosh}\,(c+dx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**3, x)
```

3.216 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de}-\frac{6b\text{Int}\left(\frac{\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e}$$

```
[Out] 2*(a+b*arccosh(d*x+c))^3*(e*(d*x+c))^(1/2)/d/e-6*b*Unintegrable((a+b*arccos
h(d*x+c))^2*(e*(d*x+c))^(1/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^3)/(d*e) - (6*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqrt[1 + x]), x],
 x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}\\ &= \frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de}-\frac{(6b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\right.}{de}\end{aligned}$$

Mathematica [A] time = 24.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/Sqrt[c*e + d*e*x], x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\operatorname{arcosh}(dx+c)^3+3\,ab^2\operatorname{arcosh}(dx+c)^2+3\,a^2b\operatorname{arcosh}(dx+c)+a^3}{\sqrt{dex+ce}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)/sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(dx+c)+a)^3}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/sqrt(d*e*x + c*e), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccosh}(dx+c))^3}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{dex+ce}\,a^3}{de}+\frac{2\left(b^3d\sqrt{e}\,x+b^3c\sqrt{e}\right)\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^3}{\sqrt{dx+c}\,de}+\int \frac{3\left(\left(\left(c^3\sqrt{e}-c\sqrt{e}\right)ab^2-2\left(c^3\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*e*x + c*e)*a^3/(d*e) + 2*(b^3*d*sqrt(e)*x + b^3*c*sqrt(e))*log(d*x
 + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(sqrt(d*x + c)*d*e) + integra
te(3*(((c^3*sqrt(e) - c*sqrt(e))*a*b^2 - 2*(c^3*sqrt(e) - c*sqrt(e))*b^3 +
(a*b^2*d^3*sqrt(e) - 2*b^3*d^3*sqrt(e))*x^3 + 3*(a*b^2*c*d^2*sqrt(e) - 2*b^
3*c*d^2*sqrt(e))*x^2 - (2*b^3*c^2*sqrt(e) - (c^2*sqrt(e) - sqrt(e))*a*b^2 -
 (a*b^2*d^2*sqrt(e) - 2*b^3*d^2*sqrt(e))*x^2 - 2*(a*b^2*c*d*sqrt(e) - 2*b^3
*c*d*sqrt(e))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d*sqrt(e) -
d*sqrt(e))*a*b^2 - 2*(3*c^2*d*sqrt(e) - d*sqrt(e))*b^3)*x)*log(d*x + sqrt(d
*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + (a^2*b*d^3*sqrt(e)*x^3 + 3*a^2*b*c*d
^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^2*b*x + (c^3*sqrt(e) - c*s
qrt(e))*a^2*b + (a^2*b*d^2*sqrt(e)*x^2 + 2*a^2*b*c*d*sqrt(e)*x + (c^2*sqrt(
e) - sqrt(e))*a^2*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*
x + c + 1)*sqrt(d*x + c - 1) + c))/((d^2*e*x^2 + 2*c*d*e*x + c^2*e - e)*sqr
t(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (d^3*e*x^3 + 3*c*d^2*e*x^2
 + c^3*e - c*e + (3*c^2*d*e - d*e)*x)*sqrt(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{acosh}(c+d\,x))^3}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+dx))^3}{\sqrt{e\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/sqrt(e*(c + d*x)), x)
```

3.217 $\int \frac{(a+b\cosh^{-1}(c+dx))^3}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{6b\text{Int}\left(\frac{(a+b\cosh^{-1}(c+dx))^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}},x\right)}{e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}$$

```
[Out] -2*(a+b*arccosh(d*x+c))^3/d/e/(e*(d*x+c))^(1/2)+6*b*Unintegrable((a+b*arcco
sh(d*x+c))^2/(d*x+c-1)^(1/2)/(e*(d*x+c))^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(d*e*Sqrt[e*(c + d*x)]) + (6*b*Defer[Subst]
[Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*Sqrt[e*x]*Sqrt[1 + x]), x],
x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{de\sqrt{e(c+dx)}}+\frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 32.21, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(3/2), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\,\text{arcosh}\left(dx+c\right)^3+3\,ab^2\,\text{arcosh}\left(dx+c\right)^2+3\,a^2b\,\text{arcosh}\left(dx+c\right)+a^3\right)\sqrt{dex+ce}}{d^2e^2x^2+2\,cde^2x+c^2e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2
), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^3}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arccosh}(dx+c))^3}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2 b^3 \log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}+c\right)^3}{\sqrt{dx+c}\,de^{\frac{3}{2}}}-\frac{2 a^3}{\sqrt{dex+ce}\,de}+\int \frac{3\left(\left(\left(c^3-c\right)ab^2+2\left(c^3-c\right)b^3+\left(ab^2d^3+2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(3/2),x, algorithm="maxima")
[Out] -2*b^3*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/(sqrt(d*x + c)*
d*e^(3/2)) - 2*a^3/(sqrt(d*e*x + c*e)*d*e) + integrate(3*(((c^3 - c)*a*b^2
+ 2*(c^3 - c)*b^3 + (a*b^2*d^3 + 2*b^3*d^3)*x^3 + 3*(a*b^2*c*d^2 + 2*b^3*c*
d^2)*x^2 + (2*b^3*c^2 + (c^2 - 1)*a*b^2 + (a*b^2*d^2 + 2*b^3*d^2)*x^2 + 2*(
a*b^2*c*d + 2*b^3*c*d)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + ((3*c^2*d -
 d)*a*b^2 + 2*(3*c^2*d - d)*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x +
c - 1) + c)^2 + (a^2*b*d^3*x^3 + 3*a^2*b*c*d^2*x^2 + (3*c^2*d - d)*a^2*b*x
+ (c^3 - c)*a^2*b + (a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + (c^2 - 1)*a^2*b)*sqrt(
d*x + c + 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
1) + c))/((d^3*e^(3/2)*x^3 + 3*c*d^2*e^(3/2)*x^2 + c^3*e^(3/2) - c*e^(3/2)
+ (3*c^2*d*e^(3/2) - d*e^(3/2))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x
 + c - 1) + (d^4*e^(3/2)*x^4 + 4*c*d^3*e^(3/2)*x^3 + c^4*e^(3/2) - c^2*e^(3
/2) + (6*c^2*d^2*e^(3/2) - d^2*e^(3/2))*x^2 + 2*(2*c^3*d*e^(3/2) - c*d*e^(3
/2))*x)*sqrt(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b \operatorname{acosh}(c+d\,x))^3}{(c\,e+d\,e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(3/2),x)
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + dx))^3}{(e\,(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(3/2), x)
```

3.218 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2b\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}\,(e(c+dx))^{3/2}},x\right)}{e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}$$

```
[Out] -2/3*(a+b*arccosh(d*x+c))^3/d/e/(e*(d*x+c))^(3/2)+2*b*Unintegrable((a+b*arc
cosh(d*x+c))^2/(e*(d*x+c))^(3/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(3*d*e*(e*(c + d*x))^(3/2)) + (2*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*(e*x)^(3/2)*Sqrt[1 + x])
, x], x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{3de(e(c+dx))^{3/2}}+\frac{(2b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,(ex)^{3/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 36.88, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(5/2), x]
```

fricas [A] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\operatorname{arcosh}(dx+c)^3+3\,ab^2\operatorname{arcosh}(dx+c)^2+3\,a^2b\operatorname{arcosh}(dx+c)+a^3\right)\sqrt{dex+ce}}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^3*e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c
^2*d*e^3*x + c^3*e^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(t_nostep)]Evaluation time:
2.05sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(dx+c))^3}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,b^3\sqrt{e}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^3}{3\left(d^2e^3x+cde^3\right)\sqrt{dx+c}}-\frac{2\,a^3}{3\,(dex+ce)^{\frac{3}{2}}de}+\int\frac{\left(3\left(c^3\sqrt{e}\,-c\sqrt{e}\right)ab^2+2\left(c^3\sqrt{e}\,-c\sqrt{e}\right)b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*b^3*sqrt(e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/((d^2
*e^3*x + c*d*e^3)*sqrt(d*x + c)) - 2/3*a^3/((d*e*x + c*e)^(3/2)*d*e) + inte
grate(((3*(c^3*sqrt(e) - c*sqrt(e))*a*b^2 + 2*(c^3*sqrt(e) - c*sqrt(e))*b^3
 + (3*a*b^2*d^3*sqrt(e) + 2*b^3*d^3*sqrt(e))*x^3 + 3*(3*a*b^2*c*d^2*sqrt(e)
 + 2*b^3*c*d^2*sqrt(e))*x^2 + (2*b^3*c^2*sqrt(e) + 3*(c^2*sqrt(e) - sqrt(e)
)*a*b^2 + (3*a*b^2*d^2*sqrt(e) + 2*b^3*d^2*sqrt(e))*x^2 + 2*(3*a*b^2*c*d*sq
rt(e) + 2*b^3*c*d*sqrt(e))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*(3*c
^2*d*sqrt(e) - d*sqrt(e))*a*b^2 + 2*(3*c^2*d*sqrt(e) - d*sqrt(e))*b^3)*x)*l
og(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 3*(a^2*b*d^3*sqrt(e)*
x^3 + 3*a^2*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^2*b*x + (
c^3*sqrt(e) - c*sqrt(e))*a^2*b + (a^2*b*d^2*sqrt(e)*x^2 + 2*a^2*b*c*d*sqrt(
e)*x + (c^2*sqrt(e) - sqrt(e))*a^2*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1))*
log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/((d^4*e^3*x^4 + 4*c*d^3
*e^3*x^3 + c^4*e^3 - c^2*e^3 + (6*c^2*d^2*e^3 - d^2*e^3)*x^2 + 2*(2*c^3*d*e
^3 - c*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (d^5*e
^3*x^5 + 5*c*d^4*e^3*x^4 + c^5*e^3 - c^3*e^3 + (10*c^2*d^3*e^3 - d^3*e^3)*x
^3 + (10*c^3*d^2*e^3 - 3*c*d^2*e^3)*x^2 + (5*c^4*d*e^3 - 3*c^2*d*e^3)*x)*sq
rt(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}{(c\,e+d\,e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + dx))^3}{(e\,(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(5/2), x)
```

3.219 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=89

$$\frac{6b\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{5/2}},x\right)}{5e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}$$

```
[Out] -2/5*(a+b*arccosh(d*x+c))^3/d/e/(e*(d*x+c))^(5/2)+6/5*b*Unintegrable((a+b*a
rccosh(d*x+c))^2/(e*(d*x+c))^(5/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^3)/(5*d*e*(e*(c + d*x))^(5/2)) + (6*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^2/(Sqrt[-1 + x]*(e*x)^(5/2)*Sqrt[1 + x])
, x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^3}{5de(e(c+dx))^{5/2}}+\frac{(6b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{\sqrt{-1+x}\,(ex)^{5/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A] time = 123.32, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^3/(c*e + d*e*x)^(7/2), x]
```

fricas [A] time = 1.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^3\operatorname{arcosh}(dx+c)^3+3\,ab^2\operatorname{arcosh}(dx+c)^2+3\,a^2b\operatorname{arcosh}(dx+c)+a^3\right)\sqrt{dex+ce}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*sqrt(d*e*x + c*e)/(d^4*e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c
^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(dx+c)+a)^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3/(d*e*x + c*e)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(dx+c))^3}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,b^3\sqrt{e}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^3}{5\left(d^3e^4x^2+2\,cd^2e^4x+c^2de^4\right)\sqrt{dx+c}}-\frac{2\,a^3}{5\,(dex+ce)^{\frac{5}{2}}de}+\int\frac{3\left(\left(5\left(c^3\sqrt{e}\,-c\sqrt{e}\right)ab^2+2\left(c^3\sqrt{e}\,-c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^3/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*b^3*sqrt(e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3/((d^3
*e^4*x^2 + 2*c*d^2*e^4*x + c^2*d*e^4)*sqrt(d*x + c)) - 2/5*a^3/((d*e*x + c*
e)^(5/2)*d*e) + integrate(3/5*((5*(c^3*sqrt(e) - c*sqrt(e))*a*b^2 + 2*(c^3*
sqrt(e) - c*sqrt(e))*b^3 + (5*a*b^2*d^3*sqrt(e) + 2*b^3*d^3*sqrt(e))*x^3 +
3*(5*a*b^2*c*d^2*sqrt(e) + 2*b^3*c*d^2*sqrt(e))*x^2 + (2*b^3*c^2*sqrt(e) +
5*(c^2*sqrt(e) - sqrt(e))*a*b^2 + (5*a*b^2*d^2*sqrt(e) + 2*b^3*d^2*sqrt(e))
*x^2 + 2*(5*a*b^2*c*d*sqrt(e) + 2*b^3*c*d*sqrt(e))*x)*sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + (5*(3*c^2*d*sqrt(e) - d*sqrt(e))*a*b^2 + 2*(3*c^2*d*sqrt(e
) - d*sqrt(e))*b^3)*x)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2
 + 5*(a^2*b*d^3*sqrt(e)*x^3 + 3*a^2*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e)
- d*sqrt(e))*a^2*b*x + (c^3*sqrt(e) - c*sqrt(e))*a^2*b + (a^2*b*d^2*sqrt(e)
*x^2 + 2*a^2*b*c*d*sqrt(e)*x + (c^2*sqrt(e) - sqrt(e))*a^2*b)*sqrt(d*x + c
+ 1)*sqrt(d*x + c - 1))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))
/((d^5*e^4*x^5 + 5*c*d^4*e^4*x^4 + c^5*e^4 - c^3*e^4 + (10*c^2*d^3*e^4 - d^
3*e^4)*x^3 + (10*c^3*d^2*e^4 - 3*c*d^2*e^4)*x^2 + (5*c^4*d*e^4 - 3*c^2*d*e^
4)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (d^6*e^4*x^6 + 6*
c*d^5*e^4*x^5 + c^6*e^4 - c^4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(5*c
^3*d^3*e^4 - c*d^3*e^4)*x^3 + 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 + 2*(3*
c^5*d*e^4 - 2*c^3*d*e^4)*x)*sqrt(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c+d\,x))^3}{(c\,e+d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(7/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^3/(c*e + d*e*x)^(7/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^3}{(e(c + dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**3/(d*e*x+c*e)**(7/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**3/(e*(c + d*x))**(7/2), x)
```

3.220 $\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx$

Optimal. Leaf size=89

$$\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\,\sqrt{c+dx+1}},x\right)}{5e}$$

```
[Out] 2/5*(e*(d*x+c))^(5/2)*(a+b*arccosh(d*x+c))^4/d/e-8/5*b*Unintegrable((e*(d*x
+c))^(5/2)*(a+b*arccosh(d*x+c))^3/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(5/2)*(a + b*ArcCosh[c + d*x])^4)/(5*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(5/2)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx=\frac{\text{Subst}\left(\int (ex)^{3/2}\left(a+b\cosh^{-1}(x)\right)^4\,dx,x,c+dx\right)}{d}$$
$$=\frac{2(e(c+dx))^{5/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{5/2}\left(a+b\right.}{\sqrt{-1+}}\right.}{5}$$

Mathematica [A] time = 75.45, size = 0, normalized size = 0.00

$$\int (ce+dex)^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^(3/2)*(a + b*ArcCosh[c + d*x])^4, x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^4dex+a^4ce+\left(b^4dex+b^4ce\right)\text{arcosh}\,(dx+c)^4+4\left(ab^3dex+ab^3ce\right)\text{arcosh}\,(dx+c)^3+6\left(a^2b^2dex\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*d*e*x + a^4*c*e + (b^4*d*e*x + b^4*c*e)*arccosh(d*x + c)^4 +
4*(a*b^3*d*e*x + a*b^3*c*e)*arccosh(d*x + c)^3 + 6*(a^2*b^2*d*e*x + a^2*b^2
*c*e)*arccosh(d*x + c)^2 + 4*(a^3*b*d*e*x + a^3*b*c*e)*arccosh(d*x + c))*sq
rt(d*e*x + c*e), x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{3}{2}} (b \operatorname{arcosh}(dx + c) + a)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((d*e*x + c*e)^(3/2)*(b*arccosh(d*x + c) + a)^4, x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int (dex + ce)^{\frac{3}{2}} (a + b \operatorname{arccosh}(dx + c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^(3/2)*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 2/5*(d*e*x + c*e)^(5/2)*a^4/(d*e) + 2/5*(b^4*d^2*e^(3/2)*x^2 + 2*b^4*c*d*e^
(3/2)*x + b^4*c^2*e^(3/2))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d
*x + c - 1) + c)^4/d + integrate(-2/5*(2*((2*b^4*c^3*e^(3/2) - 5*(c^3*e^(3/
2) - c*e^(3/2))*a*b^3 - (5*a*b^3*d^3*e^(3/2) - 2*b^4*d^3*e^(3/2))*x^3 - 3*(
5*a*b^3*c*d^2*e^(3/2) - 2*b^4*c*d^2*e^(3/2))*x^2 + (6*b^4*c^2*d*e^(3/2) - 5
*(3*c^2*d*e^(3/2) - d*e^(3/2))*a*b^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sq
rt(d*x + c - 1) - (5*(c^4*e^(3/2) - c^2*e^(3/2))*a*b^3 - 2*(c^4*e^(3/2) - c
^2*e^(3/2))*b^4 + (5*a*b^3*d^4*e^(3/2) - 2*b^4*d^4*e^(3/2))*x^4 + 4*(5*a*b^
3*c*d^3*e^(3/2) - 2*b^4*c*d^3*e^(3/2))*x^3 + (5*(6*c^2*d^2*e^(3/2) - d^2*e^
(3/2))*a*b^3 - 2*(6*c^2*d^2*e^(3/2) - d^2*e^(3/2))*b^4)*x^2 + 2*(5*(2*c^3*d
*e^(3/2) - c*d*e^(3/2))*a*b^3 - 2*(2*c^3*d*e^(3/2) - c*d*e^(3/2))*b^4)*x)*s
qrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 - 15*((a
^2*b^2*d^3*e^(3/2)*x^3 + 3*a^2*b^2*c*d^2*e^(3/2)*x^2 + (3*c^2*d*e^(3/2) - d
*e^(3/2))*a^2*b^2*x + (c^3*e^(3/2) - c*e^(3/2))*a^2*b^2)*sqrt(d*x + c + 1)*
sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b^2*d^4*e^(3/2)*x^4 + 4*a^2*b^2*c*d^
3*e^(3/2)*x^3 + (6*c^2*d^2*e^(3/2) - d^2*e^(3/2))*a^2*b^2*x^2 + 2*(2*c^3*d*
e^(3/2) - c*d*e^(3/2))*a^2*b^2*x + (c^4*e^(3/2) - c^2*e^(3/2))*a^2*b^2)*sqr
t(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - 10*((a^3
*b*d^3*e^(3/2)*x^3 + 3*a^3*b*c*d^2*e^(3/2)*x^2 + (3*c^2*d*e^(3/2) - d*e^(3/
2))*a^3*b*x + (c^3*e^(3/2) - c*e^(3/2))*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x +
 c)*sqrt(d*x + c - 1) + (a^3*b*d^4*e^(3/2)*x^4 + 4*a^3*b*c*d^3*e^(3/2)*x^3
+ (6*c^2*d^2*e^(3/2) - d^2*e^(3/2))*a^3*b*x^2 + 2*(2*c^3*d*e^(3/2) - c*d*e^
(3/2))*a^3*b*x + (c^4*e^(3/2) - c^2*e^(3/2))*a^3*b)*sqrt(d*x + c))*log(d*x
+ sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 +
(d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*
d - d)*x - c), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)^{3/2}\,(a + b\,\mathrm{acosh}(c + d\,x))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^(3/2)*(a + b*acosh(c + d*x))^4, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^{\frac{3}{2}} (a+b\operatorname{acosh}(c+dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**(3/2)*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Integral((e*(c + d*x))**(3/2)*(a + b*acosh(c + d*x))**4, x)
```

3.221 $\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4 dx$

Optimal. Leaf size=89

$$\frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de}-\frac{8b\text{Int}\left(\frac{(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{3e}$$

```
[Out] 2/3*(e*(d*x+c))^(3/2)*(a+b*arccosh(d*x+c))^4/d/e-8/3*b*Unintegrable((e*(d*x
+c))^(3/2)*(a+b*arccosh(d*x+c))^3/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4 dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] (2*(e*(c + d*x))^(3/2)*(a + b*ArcCosh[c + d*x])^4)/(3*d*e) - (8*b*Defer[Sub
st][Defer[Int][((e*x)^(3/2)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]
), x], x, c + d*x])/(3*d*e)
```

Rubi steps

$$\begin{aligned}\int \sqrt{ce+dex}\left(a+b\cosh^{-1}(c+dx)\right)^4 dx &= \frac{\text{Subst}\left(\int \sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^4 dx,x,c+dx\right)}{d}\\ &= \frac{2(e(c+dx))^{3/2}\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de}-\frac{(8b)\,\text{Subst}\left(\int \frac{(ex)^{3/2}\left(a+b\cosh^{-1}\right.}{\sqrt{-1+x}\sqrt{1+}}\right.}{3de}\end{aligned}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c*e + d*e*x]*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4 \operatorname{arcosh}(dx+c)^4+4ab^3\operatorname{arcosh}(dx+c)^3+6a^2b^2\operatorname{arcosh}(dx+c)^2+4a^3b\operatorname{arcosh}(dx+c)+a^4\right)\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{dex+ce}\,(b\operatorname{arcosh}(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*e*x + c*e)*(b*arccosh(d*x + c) + a)^4, x)

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(dx + c))^4 \sqrt{dex + ce}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)

[Out] int((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(b^4 d\sqrt{e}\,x + b^4 c\sqrt{e}\right)\sqrt{dx+c}\,\log\left(dx + \sqrt{dx+c+1}\,\sqrt{dx+c-1} + c\right)^4}{3\,d} + \frac{2\,(dex+ce)^{\frac{3}{2}}a^4}{3\,de} + \int -\frac{2\left(2\left(\left(2\,b^4c^2\sqrt{e}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^4*(d*e*x+c*e)^(1/2),x, algorithm="maxima")

[Out] 2/3*(b^4*d*sqrt(e)*x + b^4*c*sqrt(e))*sqrt(d*x + c)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/d + 2/3*(d*e*x + c*e)^(3/2)*a^4/(d*e) + integrate(-2/3*(2*((2*b^4*c^2*sqrt(e) - 3*(c^2*sqrt(e) - sqrt(e))*a*b^3 - (3*a*b^3*d^2*sqrt(e) - 2*b^4*d^2*sqrt(e))*x^2 - 2*(3*a*b^3*c*d*sqrt(e) - 2*b^4*c*d*sqrt(e))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) - (3*(c^3*sqrt(e) - c*sqrt(e))*a*b^3 - 2*(c^3*sqrt(e) - c*sqrt(e))*b^4 + (3*a*b^3*d^3*sqrt(e) - 2*b^4*d^3*sqrt(e))*x^3 + 3*(3*a*b^3*c*d^2*sqrt(e) - 2*b^4*c*d^2*sqrt(e))*x^2 + (3*(3*c^2*d*sqrt(e) - d*sqrt(e))*a*b^3 - 2*(3*c^2*d*sqrt(e) - d*sqrt(e))*b^4)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 - 9*((a^2*b^2*d^2*sqrt(e)*x^2 + 2*a^2*b^2*c*d*sqrt(e)*x + (c^2*sqrt(e) - sqrt(e))*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b^2*d^3*sqrt(e)*x^3 + 3*a^2*b^2*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^2*b^2*x + (c^3*sqrt(e) - c*sqrt(e))*a^2*b^2)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - 6*((a^3*b*d^2*sqrt(e)*x^2 + 2*a^3*b*c*d*sqrt(e)*x + (c^2*sqrt(e) - sqrt(e))*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^3*b*d^3*sqrt(e)*x^3 + 3*a^3*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^3*b*x + (c^3*sqrt(e) - c*sqrt(e))*a^3*b)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*x^3 + 3*c*d^2*x^2 + c^3 + (d^2*x^2 + 2*c*d*x + c^2 - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (3*c^2*d - d)*x - c), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,e + d\,e\,x}\,(a + b\,\mathrm{acosh}(c + d\,x))^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^4,x)

[Out] int((c*e + d*e*x)^(1/2)*(a + b*acosh(c + d*x))^4, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e(c + dx)}\,(a + b\,\mathrm{acosh}(c + dx))^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4*(d*e*x+c*e)**(1/2),x)
```

```
[Out] Integral(sqrt(e*(c + d*x))*(a + b*acosh(c + d*x))**4, x)
```

3.222 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de}-\frac{8b\text{Int}\left(\frac{\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e}$$

```
[Out] 2*(a+b*arccosh(d*x+c))^4*(e*(d*x+c))^(1/2)/d/e-8*b*Unintegrable((a+b*arccos
h(d*x+c))^3*(e*(d*x+c))^(1/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] (2*Sqrt[e*(c + d*x)]*(a + b*ArcCosh[c + d*x])^4)/(d*e) - (8*b*Defer[Subst][
Defer[Int][(Sqrt[e*x]*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqrt[1 + x]), x],
 x, c + d*x])/(d*e)
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx=\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{\sqrt{ex}}\,dx,x,c+dx\right)}{d}$$

$$=\frac{2\sqrt{e(c+dx)}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de}-\frac{(8b)\,\text{Subst}\left(\int \frac{\sqrt{ex}\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\right.}{de}$$

Mathematica [A] time = 15.19, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{\sqrt{ce+dex}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x],x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/Sqrt[c*e + d*e*x], x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^4\operatorname{arcosh}(dx+c)^4+4\,ab^3\operatorname{arcosh}(dx+c)^3+6\,a^2b^2\operatorname{arcosh}(dx+c)^2+4\,a^3b\operatorname{arcosh}(dx+c)+a^4}{\sqrt{dex+ce}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)/sqrt(d*e*x + c*e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(dx+c)+a)^4}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/sqrt(d*e*x + c*e), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccosh}(dx+c))^4}{\sqrt{dex+ce}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\sqrt{dx+c}\,b^4\log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^4}{d\sqrt{e}}+\frac{2\sqrt{dex+ce}\,a^4}{de}+\int -\frac{2\left(2\left(\left(2\,b^4c^2-\left(c^2-1\right)ab^3-\left(ab^3d^2\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)*b^4*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d
*sqrt(e)) + 2*sqrt(d*e*x + c*e)*a^4/(d*e) + integrate(-2*(2*((2*b^4*c^2 - (
c^2 - 1)*a*b^3 - (a*b^3*d^2 - 2*b^4*d^2)*x^2 - 2*(a*b^3*c*d - 2*b^4*c*d)*x)
*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) - ((c^3 - c)*a*b^3 - 2*(
c^3 - c)*b^4 + (a*b^3*d^3 - 2*b^4*d^3)*x^3 + 3*(a*b^3*c*d^2 - 2*b^4*c*d^2)*
x^2 + ((3*c^2*d - d)*a*b^3 - 2*(3*c^2*d - d)*b^4)*x)*sqrt(d*x + c))*log(d*x
 + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 - 3*((a^2*b^2*d^2*x^2 + 2*a^2
*b^2*c*d*x + (c^2 - 1)*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x +
c - 1) + (a^2*b^2*d^3*x^3 + 3*a^2*b^2*c*d^2*x^2 + (3*c^2*d - d)*a^2*b^2*x +
 (c^3 - c)*a^2*b^2)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c
 - 1) + c)^2 - 2*((a^3*b*d^2*x^2 + 2*a^3*b*c*d*x + (c^2 - 1)*a^3*b)*sqrt(d*
x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^3*b*d^3*x^3 + 3*a^3*b*c*d^2
*x^2 + (3*c^2*d - d)*a^3*b*x + (c^3 - c)*a^3*b)*sqrt(d*x + c))*log(d*x + sq
rt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^4*sqrt(e)*x^4 + 4*c*d^3*sqrt(e)*
x^3 + c^4*sqrt(e) + (6*c^2*d^2*sqrt(e) - d^2*sqrt(e))*x^2 - c^2*sqrt(e) + (
d^3*sqrt(e)*x^3 + 3*c*d^2*sqrt(e)*x^2 + c^3*sqrt(e) + (3*c^2*d*sqrt(e) - d*
sqrt(e))*x - c*sqrt(e))*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + 2*(2*c^3*d*sq
rt(e) - c*d*sqrt(e))*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c+d\,x))^4}{\sqrt{c\,e+d\,e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(c + dx))^4}{\sqrt{e(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(1/2),x)
[Out] Integral((a + b*acosh(c + d*x))**4/sqrt(e*(c + d*x)), x)
```

3.223 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{8b\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}\sqrt{e(c+dx)}},x\right)}{e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}$$

```
[Out] -2*(a+b*arccosh(d*x+c))^4/d/e/(e*(d*x+c))^(1/2)+8*b*Unintegrable((a+b*arcco
sh(d*x+c))^3/(d*x+c-1)^(1/2)/(e*(d*x+c))^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(d*e*Sqrt[e*(c + d*x)]) + (8*b*Defer[Subst]
[Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*Sqrt[e*x]*Sqrt[1 + x]), x],
x, c + d*x])/(d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{(ex)^{3/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{de\sqrt{e(c+dx)}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\sqrt{ex}\sqrt{1+x}}\,dx,x,c+dx\right)}{de}\end{aligned}$$

Mathematica [A] time = 39.13, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(3/2), x]
```

fricas [A] time = 1.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\operatorname{arcosh}(dx+c)^4+4ab^3\operatorname{arcosh}(dx+c)^3+6a^2b^2\operatorname{arcosh}(dx+c)^2+4a^3b\operatorname{arcosh}(dx+c)+a^4\right)\sqrt{}}{d^2e^2x^2+2cde^2x+c^2e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="fricas")
```

[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*arccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(dx+c)+a)^4}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="giac")

[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^(3/2), x)

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b \operatorname{arccosh}(dx+c))^4}{(dex+ce)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)

[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\sqrt{dx+c}\,b^4\sqrt{e}\log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^4}{d^2e^2x+cde^2}-\frac{2a^4}{\sqrt{dex+ce}\,de}+\int \frac{2\left(2\left(\left(2b^4c^2\sqrt{e}+\left(c^2\sqrt{e}-\sqrt{e}\right)\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(3/2),x, algorithm="maxima")

[Out] -2*sqrt(d*x + c)*b^4*sqrt(e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^4/(d^2*e^2*x + c*d*e^2) - 2*a^4/(sqrt(d*e*x + c*e)*d*e) + integrate(2*(2*((2*b^4*c^2*sqrt(e) + (c^2*sqrt(e) - sqrt(e))*a*b^3 + (a*b^3*d^2*sqrt(e) + 2*b^4*d^2*sqrt(e))*x^2 + 2*(a*b^3*c*d*sqrt(e) + 2*b^4*c*d*sqrt(e))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + ((c^3*sqrt(e) - c*sqrt(e))*a*b^3 + 2*(c^3*sqrt(e) - c*sqrt(e))*b^4 + (a*b^3*d^3*sqrt(e) + 2*b^4*d^3*sqrt(e))*x^3 + 3*(a*b^3*c*d^2*sqrt(e) + 2*b^4*c*d^2*sqrt(e))*x^2 + ((3*c^2*d*sqrt(e) - d*sqrt(e))*a*b^3 + 2*(3*c^2*d*sqrt(e) - d*sqrt(e))*b^4)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 3*((a^2*b^2*d^2*sqrt(e)*x^2 + 2*a^2*b^2*c*d*sqrt(e)*x + (c^2*sqrt(e) - sqrt(e))*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b^2*d^3*sqrt(e)*x^3 + 3*a^2*b^2*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^2*b^2*x + (c^3*sqrt(e) - c*sqrt(e))*a^2*b^2)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 + 2*((a^3*b*d^2*sqrt(e)*x^2 + 2*a^3*b*c*d*sqrt(e)*x + (c^2*sqrt(e) - sqrt(e))*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^3*b*d^3*sqrt(e)*x^3 + 3*a^3*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^3*b*x + (c^3*sqrt(e) - c*sqrt(e))*a^3*b)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^5*e^2*x^5 + 5*c*d^4*e^2*x^4 + c^5*e^2 - c^3*e^2 + (10*c^2*d^3*e^2 - d^3*e^2)*x^3 + (10*c^3*d^2*e^2 - 3*c*d^2*e^2)*x^2 + (d^4*e^2*x^4 + 4*c*d^3*e^2*x^3 + c^4*e^2 - c^2*e^2 + (6*c^2*d^2*e^2 - d^2*e^2)*x^2 + 2*(2*c^3*d*e^2 - c*d*e^2)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (5*c^4*d*e^2 - 3*c^2*d*e^2)*x), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)}^4}{{\left(c\,e+d\,e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(3/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + dx))^4}{(e\,(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/(e*(c + d*x))**(3/2), x)
```

3.224 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{8b\text{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{3/2}},x\right)}{3e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}$$

```
[Out] -2/3*(a+b*arccosh(d*x+c))^4/d/e/(e*(d*x+c))^(3/2)+8/3*b*Unintegrable((a+b*a
rccosh(d*x+c))^3/(e*(d*x+c))^(3/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(3*d*e*(e*(c + d*x))^(3/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*(e*x)^(3/2)*Sqrt[1 + x])
, x], x, c + d*x])/(3*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{(ex)^{5/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{3de(e(c+dx))^{3/2}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,(ex)^{3/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{3de}\end{aligned}$$

Mathematica [A] time = 39.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(5/2), x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\operatorname{arcosh}(dx+c)^4+4\,ab^3\operatorname{arcosh}(dx+c)^3+6\,a^2b^2\operatorname{arcosh}(dx+c)^2+4\,a^3b\operatorname{arcosh}(dx+c)+a^4\right.}{d^3e^3x^3+3\,cd^2e^3x^2+3\,c^2de^3x+c^3e^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^3*
e^3*x^3 + 3*c*d^2*e^3*x^2 + 3*c^2*d*e^3*x + c^3*e^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [abs(t_nostep)]Evaluation time:
4.49sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error
: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(dx+c))^4}{(dex+ce)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,\sqrt{dx+c}\,b^4\sqrt{e}\,\log\left(dx+\sqrt{dx+c+1}\,\sqrt{dx+c-1}\,+c\right)^4}{3\left(d^3e^3x^2+2\,cd^2e^3x+c^2de^3\right)}-\frac{2\,a^4}{3\,(dex+ce)^{\frac{3}{2}}de}+\int \frac{2\left(2\left(\left(2\,b^4c^2\sqrt{e}\,+3\left(c^2\sqrt{e}\,-\sqrt{e}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(d*x + c)*b^4*sqrt(e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^4/(d^3*e^3*x^2 + 2*c*d^2*e^3*x + c^2*d*e^3) - 2/3*a^4/((d*e*x + c*e)
^(3/2)*d*e) + integrate(2/3*(2*((2*b^4*c^2*sqrt(e) + 3*(c^2*sqrt(e) - sqrt(
e))*a*b^3 + (3*a*b^3*d^2*sqrt(e) + 2*b^4*d^2*sqrt(e))*x^2 + 2*(3*a*b^3*c*d*
sqrt(e) + 2*b^4*c*d*sqrt(e))*x)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x +
c - 1) + (3*(c^3*sqrt(e) - c*sqrt(e))*a*b^3 + 2*(c^3*sqrt(e) - c*sqrt(e))*b
^4 + (3*a*b^3*d^3*sqrt(e) + 2*b^4*d^3*sqrt(e))*x^3 + 3*(3*a*b^3*c*d^2*sqrt(
e) + 2*b^4*c*d^2*sqrt(e))*x^2 + (3*(3*c^2*d*sqrt(e) - d*sqrt(e))*a*b^3 + 2*
(3*c^2*d*sqrt(e) - d*sqrt(e))*b^4)*x)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c
 + 1)*sqrt(d*x + c - 1) + c)^3 + 9*((a^2*b^2*d^2*sqrt(e)*x^2 + 2*a^2*b^2*c*
d*sqrt(e)*x + (c^2*sqrt(e) - sqrt(e))*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x +
 c)*sqrt(d*x + c - 1) + (a^2*b^2*d^3*sqrt(e)*x^3 + 3*a^2*b^2*c*d^2*sqrt(e)*
x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^2*b^2*x + (c^3*sqrt(e) - c*sqrt(e))*a
^2*b^2)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2
 + 6*((a^3*b*d^2*sqrt(e)*x^2 + 2*a^3*b*c*d*sqrt(e)*x + (c^2*sqrt(e) - sqrt(
e))*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^3*b*d^3*s
qrt(e)*x^3 + 3*a^3*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^3*
b*x + (c^3*sqrt(e) - c*sqrt(e))*a^3*b)*sqrt(d*x + c))*log(d*x + sqrt(d*x +
c + 1)*sqrt(d*x + c - 1) + c))/(d^6*e^3*x^6 + 6*c*d^5*e^3*x^5 + c^6*e^3 - c
^4*e^3 + (15*c^2*d^4*e^3 - d^4*e^3)*x^4 + 4*(5*c^3*d^3*e^3 - c*d^3*e^3)*x^3
```

```
 + 3*(5*c^4*d^2*e^3 - 2*c^2*d^2*e^3)*x^2 + (d^5*e^3*x^5 + 5*c*d^4*e^3*x^4 +
 c^5*e^3 - c^3*e^3 + (10*c^2*d^3*e^3 - d^3*e^3)*x^3 + (10*c^3*d^2*e^3 - 3*c
*d^2*e^3)*x^2 + (5*c^4*d*e^3 - 3*c^2*d*e^3)*x)*sqrt(d*x + c + 1)*sqrt(d*x +
 c - 1) + 2*(3*c^5*d*e^3 - 2*c^3*d*e^3)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^4}{\left(c\,e+d\,e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c+dx\right)\right)^4}{\left(e\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c + d*x))**4/(e*(c + d*x))**(5/2), x)
```

3.225 $\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$

Optimal. Leaf size=89

$$\frac{8b\mathrm{Int}\left(\frac{\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}(e(c+dx))^{5/2}},x\right)}{5e}-\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}$$

```
[Out] -2/5*(a+b*arccosh(d*x+c))^4/d/e/(e*(d*x+c))^(5/2)+8/5*b*Unintegrable((a+b*a
rccosh(d*x+c))^3/(e*(d*x+c))^(5/2)/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e
```

Rubi [A] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] (-2*(a + b*ArcCosh[c + d*x])^4)/(5*d*e*(e*(c + d*x))^(5/2)) + (8*b*Defer[Su
bst][Defer[Int][(a + b*ArcCosh[x])^3/(Sqrt[-1 + x]*(e*x)^(5/2)*Sqrt[1 + x])
, x], x, c + d*x])/(5*d*e)
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^4}{(ex)^{7/2}}\,dx,x,c+dx\right)}{d}\\ &= -\frac{2\left(a+b\cosh^{-1}(c+dx)\right)^4}{5de(e(c+dx))^{5/2}}+\frac{(8b)\,\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{\sqrt{-1+x}\,(ex)^{5/2}\sqrt{1+x}}\,dx,x,c+dx\right)}{5de}\end{aligned}$$

Mathematica [A] time = 175.70, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(c+dx)\right)^4}{(ce+dex)^{7/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c + d*x])^4/(c*e + d*e*x)^(7/2), x]
```

fricas [A] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^4\,\text{arcosh}\,(dx+c)^4+4\,ab^3\,\text{arcosh}\,(dx+c)^3+6\,a^2b^2\,\text{arcosh}\,(dx+c)^2+4\,a^3b\,\text{arcosh}\,(dx+c)+a^4\right)\sqrt{}}{d^4e^4x^4+4\,cd^3e^4x^3+6\,c^2d^2e^4x^2+4\,c^3de^4x+c^4e^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*sqrt(d*e*x + c*e)/(d^4*
e^4*x^4 + 4*c*d^3*e^4*x^3 + 6*c^2*d^2*e^4*x^2 + 4*c^3*d*e^4*x + c^4*e^4), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(dx+c)+a)^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4/(d*e*x + c*e)^(7/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(dx+c))^4}{(dex+ce)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

```
[Out] int((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x+c))^4/(d*e*x+c*e)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(d*x + c)*b^4*sqrt(e)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1
) + c)^4/(d^4*e^4*x^3 + 3*c*d^3*e^4*x^2 + 3*c^2*d^2*e^4*x + c^3*d*e^4) - 2/
5*a^4/((d*e*x + c*e)^(5/2)*d*e) + integrate(2/5*(2*((2*b^4*c^2*sqrt(e) + 5*
(c^2*sqrt(e) - sqrt(e))*a*b^3 + (5*a*b^3*d^2*sqrt(e) + 2*b^4*d^2*sqrt(e))*x
^2 + 2*(5*a*b^3*c*d*sqrt(e) + 2*b^4*c*d*sqrt(e))*x)*sqrt(d*x + c + 1)*sqrt(
d*x + c)*sqrt(d*x + c - 1) + (5*(c^3*sqrt(e) - c*sqrt(e))*a*b^3 + 2*(c^3*sq
rt(e) - c*sqrt(e))*b^4 + (5*a*b^3*d^3*sqrt(e) + 2*b^4*d^3*sqrt(e))*x^3 + 3*
(5*a*b^3*c*d^2*sqrt(e) + 2*b^4*c*d^2*sqrt(e))*x^2 + (5*(3*c^2*d*sqrt(e) - d
*sqrt(e))*a*b^3 + 2*(3*c^2*d*sqrt(e) - d*sqrt(e))*b^4)*x)*sqrt(d*x + c))*lo
g(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 + 15*((a^2*b^2*d^2*sqrt(
e)*x^2 + 2*a^2*b^2*c*d*sqrt(e)*x + (c^2*sqrt(e) - sqrt(e))*a^2*b^2)*sqrt(d*
x + c + 1)*sqrt(d*x + c)*sqrt(d*x + c - 1) + (a^2*b^2*d^3*sqrt(e)*x^3 + 3*a
^2*b^2*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqrt(e) - d*sqrt(e))*a^2*b^2*x + (c^3*s
qrt(e) - c*sqrt(e))*a^2*b^2)*sqrt(d*x + c))*log(d*x + sqrt(d*x + c + 1)*sqr
t(d*x + c - 1) + c)^2 + 10*((a^3*b*d^2*sqrt(e)*x^2 + 2*a^3*b*c*d*sqrt(e)*x
+ (c^2*sqrt(e) - sqrt(e))*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c)*sqrt(d*x +
 c - 1) + (a^3*b*d^3*sqrt(e)*x^3 + 3*a^3*b*c*d^2*sqrt(e)*x^2 + (3*c^2*d*sqr
t(e) - d*sqrt(e))*a^3*b*x + (c^3*sqrt(e) - c*sqrt(e))*a^3*b)*sqrt(d*x + c))
*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^7*e^4*x^7 + 7*c*d^6
*e^4*x^6 + c^7*e^4 - c^5*e^4 + (21*c^2*d^5*e^4 - d^5*e^4)*x^5 + 5*(7*c^3*d^
4*e^4 - c*d^4*e^4)*x^4 + 5*(7*c^4*d^3*e^4 - 2*c^2*d^3*e^4)*x^3 + (21*c^5*d^
2*e^4 - 10*c^3*d^2*e^4)*x^2 + (d^6*e^4*x^6 + 6*c*d^5*e^4*x^5 + c^6*e^4 - c^
4*e^4 + (15*c^2*d^4*e^4 - d^4*e^4)*x^4 + 4*(5*c^3*d^3*e^4 - c*d^3*e^4)*x^3
+ 3*(5*c^4*d^2*e^4 - 2*c^2*d^2*e^4)*x^2 + 2*(3*c^5*d*e^4 - 2*c^3*d*e^4)*x)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + (7*c^6*d*e^4 - 5*c^4*d*e^4)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + d\,x))^4}{(c\,e + d\,e\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(7/2),x)
[Out] int((a + b*acosh(c + d*x))^4/(c*e + d*e*x)^(7/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c + dx))^4}{(e\,(c + dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x+c))**4/(d*e*x+c*e)**(7/2),x)
[Out] Integral((a + b*acosh(c + d*x))**4/(e*(c + d*x))**(7/2), x)
```

3.226 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$

Optimal. Leaf size=94

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de(m+1)}-\frac{4b\text{Int}\left(\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^3}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e(m+1)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))^4/d/e/(1+m)-4*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arccosh(d*x+c))^3/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e/(1+m
)
```

Rubi [A] time = 0.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^4)/(d*e*(1 + m)) - (4*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcCosh[x])^3)/(Sqrt[-1 + x]*Sqr
t[1 + x]), x], x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^4 \, dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^4}{de(1+m)} - \frac{(4b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\right.}{\sqrt{-1+x}}\right.}{de(1}$$

Mathematica [A] time = 4.12, size = 0, normalized size = 0.00

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^4 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^4, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^4 \operatorname{arcosh}(dx+c)^4 + 4\,ab^3 \operatorname{arcosh}(dx+c)^3 + 6\,a^2b^2 \operatorname{arcosh}(dx+c)^2 + 4\,a^3b \operatorname{arcosh}(dx+c) + a^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*arccosh(d*x + c)^4 + 4*a*b^3*arccosh(d*x + c)^3 + 6*a^2*b^2*a
rccosh(d*x + c)^2 + 4*a^3*b*arccosh(d*x + c) + a^4)*(d*e*x + c*e)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(dx + c) + a)^4 (dex + ce)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^4*(d*e*x + c*e)^m, x)
```

maple [A] time = 2.65, size = 0, normalized size = 0.00

$$\int (dex + ce)^m (a + b \operatorname{arccosh}(dx + c))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] (b^4*d*e^m*x + b^4*c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c)^4/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^4/(d*e*(m + 1)) + int
egrate(-2*(2*((b^4*c^2*e^m - (c^2*e^m*(m + 1) - e^m*(m + 1))*a*b^3 - (a*b^3
*d^2*e^m*(m + 1) - b^4*d^2*e^m)*x^2 - 2*(a*b^3*c*d*e^m*(m + 1) - b^4*c*d*e^
m)*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - ((c^3*e^m*(m + 1) -
 c*e^m*(m + 1))*a*b^3 - (c^3*e^m - c*e^m)*b^4 + (a*b^3*d^3*e^m*(m + 1) - b^
4*d^3*e^m)*x^3 + 3*(a*b^3*c*d^2*e^m*(m + 1) - b^4*c*d^2*e^m)*x^2 + ((3*c^2*
d*e^m*(m + 1) - d*e^m*(m + 1))*a*b^3 - (3*c^2*d*e^m - d*e^m)*b^4)*x)*(d*x +
 c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^3 - 3*((a^2*b^2*d
^2*e^m*(m + 1)*x^2 + 2*a^2*b^2*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) - e^m*(
m + 1))*a^2*b^2)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m + (a^2*b^2
*d^3*e^m*(m + 1)*x^3 + 3*a^2*b^2*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m +
1) - d*e^m*(m + 1))*a^2*b^2*x + (c^3*e^m*(m + 1) - c*e^m*(m + 1))*a^2*b^2)*
(d*x + c)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - 2*((a^3
*b*d^2*e^m*(m + 1)*x^2 + 2*a^3*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) - e^m
*(m + 1))*a^3*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m + (a^3*b*d
^3*e^m*(m + 1)*x^3 + 3*a^3*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) -
 d*e^m*(m + 1))*a^3*b*x + (c^3*e^m*(m + 1) - c*e^m*(m + 1))*a^3*b)*(d*x + c
)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*(m + 1)*x^3 +
 3*c*d^2*(m + 1)*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c
^2*(m + 1) - m - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) + (3*c^
2*d*(m + 1) - d*(m + 1))*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e + d\,e\,x)^m \,(a + b \operatorname{acosh}(c + d\,x))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^4,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^4, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\,(c+dx))^m\,(a+b\,\mathrm{acosh}\,(c+dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**4,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**4, x)
```

3.227 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$

Optimal. Leaf size=94

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de(m+1)}-\frac{3b\text{Int}\left(\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)^2}{\sqrt{c+dx-1}\sqrt{c+dx+1}},x\right)}{e(m+1)}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))^3/d/e/(1+m)-3*b*Unintegrable((e*(d*x
+c))^(1+m)*(a+b*arccosh(d*x+c))^2/(d*x+c-1)^(1/2)/(d*x+c+1)^(1/2),x)/e/(1+m
)
```

Rubi [A] time = 0.29, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^3)/(d*e*(1 + m)) - (3*b*Def
er[Subst][Defer[Int][((e*x)^(1 + m)*(a + b*ArcCosh[x])^2)/(Sqrt[-1 + x]*Sqr
t[1 + x]), x], x, c + d*x])/(d*e*(1 + m))
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^3\,dx,x,c+dx\right)}{d}$$
$$=\frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^3}{de(1+m)}-\frac{(3b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}\left(a+b\cosh\right.}{\sqrt{-1+x}\sqrt{1}\ldots}\right.}{de(1+m\ldots}$$

Mathematica [A] time = 1.98, size = 0, normalized size = 0.00

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3,x]
```

```
[Out] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^3, x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\operatorname{arcosh}(dx+c)^3+3\,ab^2\operatorname{arcosh}(dx+c)^2+3\,a^2b\operatorname{arcosh}(dx+c)+a^3\right)(dex+ce)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccosh(d*x + c)^3 + 3*a*b^2*arccosh(d*x + c)^2 + 3*a^2*b*arc
cosh(d*x + c) + a^3)*(d*e*x + c*e)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(dx+c)+a)^3 (dex+ce)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^3*(d*e*x + c*e)^m, x)
```

maple [A] time = 2.54, size = 0, normalized size = 0.00

$$\int (dex+ce)^m (a+b \operatorname{arccosh}(dx+c))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^3 de^m x + b^3 ce^m\right)(dx+c)^m \log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^3}{d(m+1)} + \frac{(dex+ce)^{m+1} a^3}{de(m+1)} + \int -\frac{3\left(\left(\left(b^3c^2e^m - \left(c^2\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (b^3*d*e^m*x + b^3*c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c)^3/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^3/(d*e*(m + 1)) + int
egrate(-3*(((b^3*c^2*e^m - (c^2*e^m*(m + 1) - e^m*(m + 1))*a*b^2 - (a*b^2*d
^2*e^m*(m + 1) - b^3*d^2*e^m)*x^2 - 2*(a*b^2*c*d*e^m*(m + 1) - b^3*c*d*e^m)
*x)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - ((c^3*e^m*(m + 1) - c
*e^m*(m + 1))*a*b^2 - (c^3*e^m - c*e^m)*b^3 + (a*b^2*d^3*e^m*(m + 1) - b^3*
d^3*e^m)*x^3 + 3*(a*b^2*c*d^2*e^m*(m + 1) - b^3*c*d^2*e^m)*x^2 + ((3*c^2*d*
e^m*(m + 1) - d*e^m*(m + 1))*a*b^2 - (3*c^2*d*e^m - d*e^m)*b^3)*x)*(d*x + c
)^m)*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)^2 - ((a^2*b*d^2*e^m
*(m + 1)*x^2 + 2*a^2*b*c*d*e^m*(m + 1)*x + (c^2*e^m*(m + 1) - e^m*(m + 1))*
a^2*b)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m + (a^2*b*d^3*e^m*(m
+ 1)*x^3 + 3*a^2*b*c*d^2*e^m*(m + 1)*x^2 + (3*c^2*d*e^m*(m + 1) - d*e^m*(m
+ 1))*a^2*b*x + (c^3*e^m*(m + 1) - c*e^m*(m + 1))*a^2*b)*(d*x + c)^m)*log(d
*x + sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c))/(d^3*(m + 1)*x^3 + 3*c*d^2*(
m + 1)*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1)
 - m - 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) + (3*c^2*d*(m + 1
) - d*(m + 1))*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,e+d\,e\,x)^m\,(a+b\,\mathrm{acosh}(c+d\,x))^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^3,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e(c+dx))^m (a+b \operatorname{acosh}(c+dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**3,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**3, x)
```

3.228 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^2 \, dx$

Optimal. Leaf size=206

$$-\frac{2b^2(e(c+dx))^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}-\frac{2b\sqrt{-c-dx+1}\,(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2}\right.}{de^2(m+1)(m+2}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))^2/d/e/(1+m)-2*b^2*(e*(d*x+c))^(3+m)*
HypergeometricPFQ([1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],(d*x+c)^2)
/d/e^3/(3+m)/(m^2+3*m+2)-2*b*(e*(d*x+c))^(2+m)*(a+b*arccosh(d*x+c))*hyperge
om([1/2, 1+1/2*m],[2+1/2*m],(d*x+c)^2)*(-d*x-c+1)^(1/2)/d/e^2/(1+m)/(2+m)/(
d*x+c-1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 218, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5866, 5662, 5763}

$$-\frac{2b^2(e(c+dx))^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};(c+dx)^2\right)}{de^3(m+1)(m+2)(m+3)}-\frac{2b\sqrt{1-(c+dx)^2}\,(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+}{2}\right.}{de^2(m+1)(m+2)\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x])^2)/(d*e*(1 + m)) - (2*b*(e*
(c + d*x))^(2 + m)*Sqrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeom
etric2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)*Sq
rt[-1 + c + d*x]*Sqrt[1 + c + d*x]) - (2*b^2*(e*(c + d*x))^(3 + m)*Hypergeo
metricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, (c + d*x)^2])/(d
*e^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right)^2 dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{(2b)\,\text{Subst}\left(\int \frac{(ex)^{1+m}(a+b\cos}{\sqrt{-1+x}\sqrt{1-}}\right.}{de(1+m}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)^2}{de(1+m)} - \frac{2b(e(c+dx))^{2+m}\sqrt{1-(c+}}{de^2(1+}$$

Mathematica [A] time = 0.43, size = 178, normalized size = 0.86

$$\frac{(c+dx)(e(c+dx))^m\left(\left(a+b\cosh^{-1}(c+dx)\right)^2 - \frac{2b(c+dx)\left(\frac{b(c+dx)\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};(c+dx)^2\right)}{m+3} + \frac{\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};(c+d\right.}{\sqrt{c+dx-1}\sqrt{c+dx}}\right.}{m+2}\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*((a + b*ArcCosh[c + d*x])^2 - (2*b*(c + d*x)*((S
qrt[1 - (c + d*x)^2]*(a + b*ArcCosh[c + d*x])*Hypergeometric2F1[1/2, (2 + m
)/2, (4 + m)/2, (c + d*x)^2])/(Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x]) + (b*(
c + d*x)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
 (c + d*x)^2])/(3 + m)))/(2 + m)))/(d*(1 + m))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \operatorname{arcosh}(dx+c)^2 + 2\,ab\operatorname{arcosh}(dx+c) + a^2\right)(dex+ce)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccosh(d*x + c)^2 + 2*a*b*arccosh(d*x + c) + a^2)*(d*e*x + c
*e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\operatorname{arcosh}(dx+c)+a)^2(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)^2*(d*e*x + c*e)^m, x)
```

maple [F] time = 3.00, size = 0, normalized size = 0.00

$$\int (dex+ce)^m\,(a+b\operatorname{arccosh}(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(b^2de^mx + b^2ce^m\right)(dx+c)^m \log\left(dx+\sqrt{dx+c+1}\sqrt{dx+c-1}+c\right)^2}{d(m+1)} + \frac{(dex+ce)^{m+1}a^2}{de(m+1)} + \int -\frac{2\left(\left(b^2c^2e^m - \left(c^2e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2*d*e^m*x + b^2*c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x
+ c - 1) + c)^2/(d*(m + 1)) + (d*e*x + c*e)^(m + 1)*a^2/(d*e*(m + 1)) + int
egrate(-2*((b^2*c^2*e^m - (c^2*e^m*(m + 1) - e^m*(m + 1))*a*b - (a*b*d^2*e^
m*(m + 1) - b^2*d^2*e^m)*x^2 - 2*(a*b*c*d*e^m*(m + 1) - b^2*c*d*e^m)*x)*sqr
t(d*x + c + 1)*sqrt(d*x + c - 1)*(d*x + c)^m - ((a*b*d^3*e^m*(m + 1) - b^2*
d^3*e^m)*x^3 + (c^3*e^m*(m + 1) - c*e^m*(m + 1))*a*b - (c^3*e^m - c*e^m)*b^
2 + 3*(a*b*c*d^2*e^m*(m + 1) - b^2*c*d^2*e^m)*x^2 + ((3*c^2*d*e^m*(m + 1) -
 d*e^m*(m + 1))*a*b - (3*c^2*d*e^m - d*e^m)*b^2)*x)*(d*x + c)^m)*log(d*x +
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) + c)/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)
*x^2 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m -
 1)*sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) + (3*c^2*d*(m + 1) - d*
(m + 1))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\,e+d\,e\,x)^m\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^2,x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\left(c+dx\right)\right)^m \left(a+b\operatorname{acosh}\left(c+dx\right)\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c))**2,x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x))**2, x)
```

3.229 $\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right) dx$

Optimal. Leaf size=118

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b\left(1-(c+dx)^2\right)(e(c+dx))^{m+2}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+4}{2};(c+dx)^2\right)}{de^2(m+1)(m+2)\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

```
[Out] (e*(d*x+c))^(1+m)*(a+b*arccosh(d*x+c))/d/e/(1+m)-b*(e*(d*x+c))^(2+m)*(1-(d*
x+c)^2)*hypergeom([1, 3/2+1/2*m],[2+1/2*m],(d*x+c)^2)/d/e^2/(1+m)/(2+m)/(d*
x+c-1)^(1/2)/(d*x+c+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5866, 5662, 126, 365, 364}

$$\frac{(e(c+dx))^{m+1}\left(a+b\cosh^{-1}(c+dx)\right)}{de(m+1)}-\frac{b\sqrt{1-(c+dx)^2}(e(c+dx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};(c+dx)^2\right)}{de^2(m+1)(m+2)\sqrt{c+dx-1}\sqrt{c+dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] ((e*(c + d*x))^(1 + m)*(a + b*ArcCosh[c + d*x]))/(d*e*(1 + m)) - (b*(e*(c +
 d*x))^(2 + m)*Sqrt[1 - (c + d*x)^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 +
 m)/2, (c + d*x)^2])/(d*e^2*(1 + m)*(2 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c +
 d*x])
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int (ce+dex)^m \left(a+b\cosh^{-1}(c+dx)\right) dx = \frac{\text{Subst}\left(\int (ex)^m \left(a+b\cosh^{-1}(x)\right) dx, x, c+dx\right)}{d}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b\,\text{Subst}\left(\int \frac{(ex)^{1+m}}{\sqrt{-1+x}\sqrt{1+x}} dx,\right.}{de(1+m)}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{\left(b\sqrt{-1+(c+dx)^2}\right)\text{Subst}}{de(1+m)\sqrt{-1+c}}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{\left(b\sqrt{1-(c+dx)^2}\right)\text{Subst}\Big(}{de(1+m)\sqrt{-1+c+}}$$

$$= \frac{(e(c+dx))^{1+m}\left(a+b\cosh^{-1}(c+dx)\right)}{de(1+m)} - \frac{b(e(c+dx))^{2+m}\sqrt{1-(c+}}{de^2(1+m)(2+m)}$$

Mathematica [A] time = 0.20, size = 106, normalized size = 0.90

$$\frac{(c+dx)(e(c+dx))^m \left(a - \frac{b(c+dx)\sqrt{1-(c+dx)^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};(c+dx)^2\right)}{(m+2)\sqrt{c+dx-1}\sqrt{c+dx+1}} + b\cosh^{-1}(c+dx)\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*e + d*e*x)^m*(a + b*ArcCosh[c + d*x]),x]
```

```
[Out] ((c + d*x)*(e*(c + d*x))^m*(a + b*ArcCosh[c + d*x] - (b*(c + d*x)*Sqrt[1 -
(c + d*x)^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, (c + d*x)^2])/((2
 + m)*Sqrt[-1 + c + d*x]*Sqrt[1 + c + d*x])))/(d*(1 + m))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left((b\,\text{arcosh}\,(dx+c)+a)(dex+ce)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\,\text{arcosh}\,(dx+c)+a)(dex+ce)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x + c) + a)*(d*e*x + c*e)^m, x)
```

maple [F] time = 2.69, size = 0, normalized size = 0.00

$$\int (dex+ce)^m \left(a+b\,\text{arccosh}\,(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x)
```

```
[Out] int((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{(de^m x + ce^m)(dx+c)^m \log\left(dx + \sqrt{dx+c+1}\,\sqrt{dx+c-1} + c\right)}{d(m+1)} - \int \frac{\left(d^2e^m x^2 + 2\,cde^m x + c^2e^m\right)(dx+c)^m}{d^2(m+1)x^2 + 2\,cd(m+1)x + c^2(m+1) -}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m*(a+b*arccosh(d*x+c)),x, algorithm="maxima")
```

```
[Out] b*((d*e^m*x + c*e^m)*(d*x + c)^m*log(d*x + sqrt(d*x + c + 1)*sqrt(d*x + c -
 1) + c)/(d*(m + 1)) - integrate((d^2*e^m*x^2 + 2*c*d*e^m*x + c^2*e^m)*(d*x
 + c)^m/(d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m - 1), x) + int
egrate((d*e^m*x + c*e^m)*(d*x + c)^m/(d^3*(m + 1)*x^3 + 3*c*d^2*(m + 1)*x^2
 + c^3*(m + 1) + (d^2*(m + 1)*x^2 + 2*c*d*(m + 1)*x + c^2*(m + 1) - m - 1)*
sqrt(d*x + c + 1)*sqrt(d*x + c - 1) - c*(m + 1) + (3*c^2*d*(m + 1) - d*(m +
 1))*x), x)) + (d*e*x + c*e)^(m + 1)*a/(d*e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,e+d\,e\,x\right)}^m\,\left(a+b\,\mathrm{acosh}\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x)),x)
```

```
[Out] int((c*e + d*e*x)^m*(a + b*acosh(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e \left(c + dx\right)\right)^{m} \left(a + b \operatorname{acosh}{\left(c + dx \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m*(a+b*acosh(d*x+c)),x)
```

```
[Out] Integral((e*(c + d*x))**m*(a + b*acosh(c + d*x)), x)
```

3.230 $\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{(e(c+dx))^m}{a+b\cosh^{-1}(c+dx)},x\right)$$

```
[Out] Unintegrable((e*(d*x+c))^m/(a+b*arccosh(d*x+c)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]),x]
[Out] Defer[Subst][Defer[Int][(e*x)^m/(a + b*ArcCosh[x]), x], x, c + d*x]/d
```

Rubi steps

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(ex)^m}{a+b\cosh^{-1}(x)}\,dx,x,c+dx\right)}{d}$$

Mathematica [A] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{(ce+dex)^m}{a+b\cosh^{-1}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]),x]
[Out] Integrate[(c*e + d*e*x)^m/(a + b*ArcCosh[c + d*x]), x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dex+ce)^m}{b\,\text{arcosh}\,(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="fricas")
[Out] integral((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex+ce)^m}{b\,\text{arcosh}\,(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="giac")
[Out] integrate((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

maple [A]   time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^m}{a + b\,\mathrm{arccosh}\,(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x)
[Out] int((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x)
```

maxima [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dex + ce)^m}{b\,\mathrm{arcosh}\,(dx + c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)^m/(a+b*arccosh(d*x+c)),x, algorithm="maxima")
[Out] integrate((d*e*x + c*e)^m/(b*arccosh(d*x + c) + a), x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,e + d\,e\,x)^m}{a + b\,\mathrm{acosh}\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*e + d*e*x)^m/(a + b*acosh(c + d*x)),x)
[Out] int((c*e + d*e*x)^m/(a + b*acosh(c + d*x)), x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\,(c + dx))^m}{a + b\,\mathrm{acosh}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*e*x+c*e)**m/(a+b*acosh(d*x+c)),x)
[Out] Integral((e*(c + d*x))**m/(a + b*acosh(c + d*x)), x)
```

3.231 $\int \frac{\cosh^{-1}(ax^5)}{x} dx$

Optimal. Leaf size=54

$$\frac{1}{10}\mathrm{Li}_2\left(-e^{2\cosh^{-1}(ax^5)}\right)-\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(e^{2\cosh^{-1}(ax^5)}+1\right)$$

```
[Out] -1/10*arccosh(a*x^5)^2+1/5*arccosh(a*x^5)*ln(1+(a*x^5+(a*x^5-1)^(1/2)*(a*x^
5+1)^(1/2))^2)+1/10*polylog(2,-(a*x^5+(a*x^5-1)^(1/2)*(a*x^5+1)^(1/2))^2)
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5891, 3718, 2190, 2279, 2391}

$$\frac{1}{10}\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax^5)}\right)-\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2+\frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(e^{2\cosh^{-1}(ax^5)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x^5]/x,x]
```

```
[Out] -ArcCosh[a*x^5]^2/10 + (ArcCosh[a*x^5]*Log[1 + E^(2*ArcCosh[a*x^5])])/5 + P
olyLog[2, -E^(2*ArcCosh[a*x^5])]/10
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5891

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}\left(ax^5\right)}{x}\,dx &= \frac{1}{5}\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(ax^5\right)\right)\\
&= -\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2 + \frac{2}{5}\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(ax^5\right)\right)\\
&= -\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2 + \frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(1+e^{2\cosh^{-1}\left(ax^5\right)}\right) - \frac{1}{5}\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx,\right.\\
&= -\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2 + \frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(1+e^{2\cosh^{-1}\left(ax^5\right)}\right) - \frac{1}{10}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x\right.\\
&= -\frac{1}{10}\cosh^{-1}\left(ax^5\right)^2 + \frac{1}{5}\cosh^{-1}\left(ax^5\right)\log\left(1+e^{2\cosh^{-1}\left(ax^5\right)}\right) + \frac{1}{10}\text{Li}_2\left(-e^{2\cosh^{-1}\left(ax^5\right)}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.93

$$\frac{1}{10}\left(\cosh^{-1}\left(ax^5\right)\left(\cosh^{-1}\left(ax^5\right)+2\log\left(e^{-2\cosh^{-1}\left(ax^5\right)}+1\right)\right)-\text{Li}_2\left(-e^{-2\cosh^{-1}\left(ax^5\right)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x^5]/x,x]
```

```
[Out] (ArcCosh[a*x^5]*(ArcCosh[a*x^5] + 2*Log[1 + E^(-2*ArcCosh[a*x^5])]) - PolyL
og[2, -E^(-2*ArcCosh[a*x^5])])/10
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcosh}\left(ax^5\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x^5)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x^5)/x, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\text{arccosh}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x^5)/x,x)
```

```
[Out] int(arccosh(a*x^5)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}\left(ax^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x^5)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(a\,x^5\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x^5)/x,x)
```

```
[Out] int(acosh(a*x^5)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}\left(ax^{5}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x**5)/x,x)
```

```
[Out] Integral(acosh(a*x**5)/x, x)
```

3.232 $\int x^2 \cosh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=117

$$-\frac{1}{18}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{5}{72}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{5}{48}\cosh$$

```
[Out] -5/48*arccosh(x^(1/2))+1/3*x^3*arccosh(x^(1/2))-5/72*x^(3/2)*(-1+x^(1/2))^(
1/2)*(1+x^(1/2))^(1/2)-1/18*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-5/
48*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5903, 12, 323, 330, 52}

$$-\frac{1}{18}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{5}{72}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right)-\frac{5}{48}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{5}{48}\cosh$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/48 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/72 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/18 - (5*ArcCosh[Sqrt[x]])/48 + (x^3*ArcCosh[Sqrt[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 5903

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCosh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(Sqrt[-1 + u]*Sqrt[1
```

```
 + u]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rubi steps

$$\int x^2\cosh^{-1}\left(\sqrt{x}\right)dx = \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{3}\int\frac{x^{5/2}}{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{6}\int\frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{18}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right) - \frac{5}{36}\int\frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{5}{72}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{18}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{1}{3}x^3\cosh^{-1}\left(\sqrt{x}\right) - \frac{5}{4}$$

$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{72}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{18}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{}}$$

$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{72}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{18}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{}}$$

$$= -\frac{5}{48}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{72}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{18}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{}}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.68

$$\frac{1}{144}\left(48x^3\cosh^{-1}\left(\sqrt{x}\right) - \sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8x^2+10x+15\right)\sqrt{x} - 30\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 + 10*x + 8*x^2)) + 48*x
^3*ArcCosh[Sqrt[x]] - 30*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/144
```

fricas [A] time = 0.59, size = 40, normalized size = 0.34

$$-\frac{1}{144}\left(8\,x^2+10\,x+15\right)\sqrt{x-1}\,\sqrt{x} + \frac{1}{48}\left(16\,x^3-5\right)\log\left(\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/144*(8*x^2 + 10*x + 15)*sqrt(x - 1)*sqrt(x) + 1/48*(16*x^3 - 5)*log(sqrt
(x - 1) + sqrt(x))
```

giac [A] time = 1.21, size = 60, normalized size = 0.51

$$\frac{1}{3}\,x^3\log\left(\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\sqrt{x}\right) - \frac{1}{144}\left(2\left(4\,x+5\right)x+15\right)\sqrt{x-1}\,\sqrt{x} + \frac{5}{48}\log\left(-\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/144*(2*(4*x
+ 5)*x + 15)*sqrt(x - 1)*sqrt(x) + 5/48*log(-sqrt(x - 1) + sqrt(x))
```

maple [A] time = 0.03, size = 75, normalized size = 0.64

$$\frac{x^3\text{arccosh}\left(\sqrt{x}\right)}{3}-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+\right.}{144\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(x^(1/2)),x)
```

```
[Out] 1/3*x^3*arccosh(x^(1/2))-1/144*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(8*x^(5
/2)*(-1+x)^(1/2)+10*x^(3/2)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^(1
/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

maxima [A] time = 0.50, size = 56, normalized size = 0.48

$$\frac{1}{3}\,x^3\,\text{arcosh}\left(\sqrt{x}\right)-\frac{1}{18}\,\sqrt{x-1}\,x^{\frac{5}{2}}-\frac{5}{72}\,\sqrt{x-1}\,x^{\frac{3}{2}}-\frac{5}{48}\,\sqrt{x-1}\,\sqrt{x}-\frac{5}{48}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccosh(sqrt(x)) - 1/18*sqrt(x - 1)*x^(5/2) - 5/72*sqrt(x - 1)*x^(3
/2) - 5/48*sqrt(x - 1)*sqrt(x) - 5/48*log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\text{acosh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acosh(x^(1/2)),x)
```

```
[Out] int(x^2*acosh(x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,\text{acosh}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(x**(1/2)),x)
```

```
[Out] Integral(x**2*acosh(sqrt(x)), x)
```

3.233 $\int x \cosh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=86

$$-\frac{1}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{3}{16}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -3/16*arccosh(x^(1/2))+1/2*x^2*arccosh(x^(1/2))-1/8*x^(3/2)*(-1+x^(1/2))^(1
/2)*(1+x^(1/2))^(1/2)-3/16*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5903, 12, 323, 330, 52}

$$-\frac{1}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{3}{16}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/16 - (Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/8 - (3*ArcCosh[Sqrt[x]])/16 + (x^2*ArcCosh[Sqrt[
x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 5903

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCosh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(Sqrt[-1 + u]*Sqrt[1
 + u]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
```

```
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rubi steps

$$\int x\cosh^{-1}\left(\sqrt{x}\right)\,dx = \frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{x^{3/2}}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$
$$=\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$
$$=-\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\int\frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$
$$=-\frac{3}{16}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{32}\int$$
$$=-\frac{3}{16}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}+\frac{1}{2}x^2\cosh^{-1}\left(\sqrt{x}\right)-\frac{3}{16}\text{Sub}$$
$$=-\frac{3}{16}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}-\frac{3}{16}\cosh^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\cos$$

Mathematica [A] time = 0.04, size = 74, normalized size = 0.86

$$\frac{1}{16}\left(8x^2\cosh^{-1}\left(\sqrt{x}\right)-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,(2x+3)\sqrt{x}-6\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*ArcCosh[Sqrt[x]],x]
```

```
[Out] (-(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 + 2*x)) + 8*x^2*ArcCosh[
Sqrt[x]] - 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/16
```

fricas [A] time = 0.62, size = 35, normalized size = 0.41

$$-\frac{1}{16}(2x+3)\sqrt{x-1}\sqrt{x}+\frac{1}{16}\left(8x^2-3\right)\log\left(\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/16*(2*x + 3)*sqrt(x - 1)*sqrt(x) + 1/16*(8*x^2 - 3)*log(sqrt(x - 1) + sq
rt(x))
```

giac [A] time = 0.64, size = 55, normalized size = 0.64

$$\frac{1}{2}x^2\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)-\frac{1}{16}(2x+3)\sqrt{x-1}\sqrt{x}+\frac{3}{16}\log\left(-\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/16*(2*x + 3)
*sqrt(x - 1)*sqrt(x) + 3/16*log(-sqrt(x - 1) + sqrt(x))
```

maple [A] time = 0.00, size = 65, normalized size = 0.76

$$\frac{x^{2}\operatorname{arccosh}\left(\sqrt{x}\right)}{2}-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{16\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(x^(1/2)),x)
```

```
[Out] 1/2*x^2*arccosh(x^(1/2))-1/16*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x^(3/
2)*(-1+x)^(1/2)+3*x^(1/2)*(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(
1/2)
```

maxima [A] time = 0.38, size = 46, normalized size = 0.53

$$\frac{1}{2}\,x^{2}\operatorname{arcosh}\left(\sqrt{x}\right)-\frac{1}{8}\,\sqrt{x-1}\,x^{\frac{3}{2}}-\frac{3}{16}\,\sqrt{x-1}\,\sqrt{x}-\frac{3}{16}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccosh(sqrt(x)) - 1/8*sqrt(x - 1)*x^(3/2) - 3/16*sqrt(x - 1)*sqrt(
x) - 3/16*log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\operatorname{acosh}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acosh(x^(1/2)),x)
```

```
[Out] int(x*acosh(x^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\operatorname{acosh}\left(\sqrt{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(x**(1/2)),x)
```

```
[Out] Integral(x*acosh(sqrt(x)), x)
```

3.234 $\int \cosh^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=50

$$-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+x\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/2*arccosh(x^(1/2))+x*arccosh(x^(1/2))-1/2*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5901, 12, 323, 330, 52}

$$-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+x\cosh^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]],x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/2 - ArcCosh[Sqrt[x]]/2 + x*
ArcCosh[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^{-1}\left(\sqrt{x}\right)dx &= x\cosh^{-1}\left(\sqrt{x}\right) - \int \frac{\sqrt{x}}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx \\ &= x\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx \\ &= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + x\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{4}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}dx \\ &= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + x\cosh^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}dx, x, \sqrt{x} \\ &= -\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{2}\cosh^{-1}\left(\sqrt{x}\right) + x\cosh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 1.28

$$-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x} + x\cosh^{-1}\left(\sqrt{x}\right) - \tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[Sqrt[x]],x]
[Out] -1/2*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]) + x*ArcCosh[Sqrt[x]] -
ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]]
```

fricas [A] time = 0.83, size = 28, normalized size = 0.56

$$\frac{1}{2}(2x-1)\log\left(\sqrt{x-1}+\sqrt{x}\right) - \frac{1}{2}\sqrt{x-1}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(2*x - 1)*log(sqrt(x - 1) + sqrt(x)) - 1/2*sqrt(x - 1)*sqrt(x)
```

giac [A] time = 1.46, size = 47, normalized size = 0.94

$$x\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right) - \frac{1}{2}\sqrt{x-1}\sqrt{x} + \frac{1}{2}\log\left(-\sqrt{x-1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="giac")
[Out] x*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)) - 1/2*sqrt(x - 1)*sqrt
(x) + 1/2*log(-sqrt(x - 1) + sqrt(x))
```

maple [A] time = 0.00, size = 49, normalized size = 0.98

$$x\,\text{arccosh}\left(\sqrt{x}\right) - \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(\sqrt{x}\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{2\sqrt{-1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2)),x)
```

```
[Out] x*arccosh(x^(1/2))-1/2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x^(1/2)*(-1+x)
^(1/2)+ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

maxima [A] time = 0.62, size = 33, normalized size = 0.66

$$x \operatorname{arcosh}\left(\sqrt{x}\right) - \frac{1}{2}\sqrt{x-1}\sqrt{x} - \frac{1}{2}\log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arccosh(sqrt(x)) - 1/2*sqrt(x - 1)*sqrt(x) - 1/2*log(2*sqrt(x - 1) + 2*sq
rt(x))
```

mupad [B] time = 1.43, size = 40, normalized size = 0.80

$$-2\sqrt{x}\operatorname{acosh}\left(\sqrt{x}\right)\left(\frac{1}{4\sqrt{x}} - \frac{\sqrt{x}}{2}\right) - \frac{\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x^(1/2)),x)
```

```
[Out] - 2*x^(1/2)*acosh(x^(1/2))*(1/(4*x^(1/2)) - x^(1/2)/2) - (x^(1/2)*(x^(1/2)
- 1)^(1/2)*(x^(1/2) + 1)^(1/2))/2
```

sympy [A] time = 0.26, size = 29, normalized size = 0.58

$$-\frac{\sqrt{x}\sqrt{x-1}}{2} + x\operatorname{acosh}\left(\sqrt{x}\right) - \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2)),x)
```

```
[Out] -sqrt(x)*sqrt(x - 1)/2 + x*acosh(sqrt(x)) - acosh(sqrt(x))/2
```

3.235 $\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}\,dx$

Optimal. Leaf size=46

$$\text{Li}_2\left(-e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)-\cosh^{-1}\left(\sqrt{x}\right)^2+2\cosh^{-1}\left(\sqrt{x}\right)\log\left(e^{2\cosh^{-1}\left(\sqrt{x}\right)}+1\right)$$

```
[Out] -arccosh(x^(1/2))^2+2*arccosh(x^(1/2))*ln(1+(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2))^2)+polylog(2,-(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
)^2)
```

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5891, 3718, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)-\cosh^{-1}\left(\sqrt{x}\right)^2+2\cosh^{-1}\left(\sqrt{x}\right)\log\left(e^{2\cosh^{-1}\left(\sqrt{x}\right)}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x,x]
```

```
[Out] -ArcCosh[Sqrt[x]]^2 + 2*ArcCosh[Sqrt[x]]*Log[1 + E^(2*ArcCosh[Sqrt[x]])] +
PolyLog[2, -E^(2*ArcCosh[Sqrt[x]])]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5891

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}\,dx &= 2\,\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(\sqrt{x}\right)\right) \\
&= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 4\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\sqrt{x}\right)\right) \\
&= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) - 2\,\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, c \right. \\
&= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) - \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cos} \right. \\
&= -\cosh^{-1}\left(\sqrt{x}\right)^2 + 2\cosh^{-1}\left(\sqrt{x}\right)\log\left(1+e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right) + \text{Li}_2\left(-e^{2\cosh^{-1}\left(\sqrt{x}\right)}\right)
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$\cosh^{-1}\left(\sqrt{x}\right)\left(\cosh^{-1}\left(\sqrt{x}\right) + 2\log\left(e^{-2\cosh^{-1}\left(\sqrt{x}\right)}+1\right)\right) - \text{Li}_2\left(-e^{-2\cosh^{-1}\left(\sqrt{x}\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x,x]
[Out] ArcCosh[Sqrt[x]]*(ArcCosh[Sqrt[x]] + 2*Log[1 + E^(-2*ArcCosh[Sqrt[x]])]) -
PolyLog[2, -E^(-2*ArcCosh[Sqrt[x]])]
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcosh}\left(\sqrt{x}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="fricas")
[Out] integral(arccosh(sqrt(x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="giac")
[Out] integrate(arccosh(sqrt(x))/x, x)
```

maple [A] time = 0.09, size = 65, normalized size = 1.41

$$-\text{arccosh}\left(\sqrt{x}\right)^2 + 2\,\text{arccosh}\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\right)^2\right) + \text{polylog}\left(2, -\left(\sqrt{x}+\sqrt{-1+\sqrt{x}}\,\sqrt{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x,x)
[Out] -arccosh(x^(1/2))^2+2*arccosh(x^(1/2))*ln(1+(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2))^2)+polylog(2,-(x^(1/2)+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
)^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(arccosh(sqrt(x))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x^(1/2))/x,x)
```

```
[Out] int(acosh(x^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x,x)
```

```
[Out] Integral(acosh(sqrt(x))/x, x)
```

3.236 $\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^2} dx$

Optimal. Leaf size=40

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}$$

```
[Out] -arccosh(x^(1/2))/x+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5903, 12, 265}

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x^2,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 5903

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCosh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(Sqrt[-1 + u]*Sqrt[1
 + u]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^2} dx &= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}+\int \frac{1}{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}} dx\\ &= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}} dx\\ &= \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x^2,x]
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x] - ArcCosh[Sqrt[x]]/x
```

fricas [A] time = 0.58, size = 26, normalized size = 0.65

$$\frac{\sqrt{x-1}\sqrt{x}-\log\left(\sqrt{x-1}+\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="fricas")
[Out] (sqrt(x - 1)*sqrt(x) - log(sqrt(x - 1) + sqrt(x)))/x
```

giac [A] time = 0.69, size = 45, normalized size = 1.12

$$-\frac{\log\left(\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\sqrt{x}\right)}{x}+\frac{2}{\left(\sqrt{x-1}-\sqrt{x}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="giac")
[Out] -log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x))/x + 2/((sqrt(x - 1) - s
qrt(x))^2 + 1)
```

maple [A] time = 0.00, size = 29, normalized size = 0.72

$$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{x}+\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x^2,x)
[Out] -arccosh(x^(1/2))/x+(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

maxima [A] time = 1.30, size = 19, normalized size = 0.48

$$\frac{\sqrt{x-1}}{\sqrt{x}}-\frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^2,x, algorithm="maxima")
[Out] sqrt(x - 1)/sqrt(x) - arccosh(sqrt(x))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x^(1/2))/x^2,x)
```

```
[Out] int(acosh(x^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x**2,x)
```

```
[Out] Integral(acosh(sqrt(x))/x**2, x)
```

3.237 $\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^3} dx$

Optimal. Leaf size=76

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{6x^{3/2}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

```
[Out] -1/2*arccosh(x^(1/2))/x^2+1/6*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2)+
1/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5903, 12, 272, 265}

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{6x^{3/2}}-\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[x]]/x^3,x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(6*x^(3/2)) + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]])/(3*Sqrt[x]) - ArcCosh[Sqrt[x]]/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rule 5903

```
Int[((a_.) + ArcCosh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcCosh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(Sqrt[-1 + u]*Sqrt[1
 + u]), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFun
ctionFreeQ[u, x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfE
xponentialQ[u, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{2}\int \frac{1}{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx$$
$$= -\frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{4}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx$$
$$= \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{6x^{3/2}} - \frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2} + \frac{1}{6}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx$$
$$= \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{6x^{3/2}} + \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3\sqrt{x}} - \frac{\cosh^{-1}\left(\sqrt{x}\right)}{2x^2}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.64

$$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}\,(2x+1) - 3\cosh^{-1}\left(\sqrt{x}\right)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[x]]/x^3,x]
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(1 + 2*x) - 3*ArcCosh[Sqrt[x]
])/(6*x^2)
```

fricas [A] time = 0.62, size = 32, normalized size = 0.42

$$\frac{(2\,x+1)\sqrt{x-1}\,\sqrt{x} - 3\,\log\left(\sqrt{x-1}+\sqrt{x}\right)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/6*((2*x + 1)*sqrt(x - 1)*sqrt(x) - 3*log(sqrt(x - 1) + sqrt(x)))/x^2
```

giac [A] time = 1.53, size = 62, normalized size = 0.82

$$-\frac{\log\left(\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\sqrt{x}\right)}{2\,x^2} + \frac{2\left(3\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)}{3\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="giac")
[Out] -1/2*log(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x))/x^2 + 2/3*(3*(sqrt(
x - 1) - sqrt(x))^2 + 1)/((sqrt(x - 1) - sqrt(x))^2 + 1)^3
```

maple [A] time = 0.00, size = 35, normalized size = 0.46

$$-\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{2x^2} + \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(1+2x)}{6x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x^(1/2))/x^3,x)
```

```
[Out] -1/2*arccosh(x^(1/2))/x^2+1/6*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(1+2*x)/
x^(3/2)
```

maxima [A] time = 0.81, size = 30, normalized size = 0.39

$$\frac{\sqrt{x-1}}{3\sqrt{x}}+\frac{\sqrt{x-1}}{6\,x^{\frac{3}{2}}}-\frac{\operatorname{arcosh}\left(\sqrt{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x - 1)/sqrt(x) + 1/6*sqrt(x - 1)/x^(3/2) - 1/2*arccosh(sqrt(x))/x^
2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x^(1/2))/x^3,x)
```

```
[Out] int(acosh(x^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}\left(\sqrt{x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x**(1/2))/x**3,x)
```

```
[Out] Integral(acosh(sqrt(x))/x**3, x)
```

3.238 $\int \cosh^{-1}\left(\frac{1}{x}\right) dx$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x+1}}\sqrt{x+1}\,\sin^{-1}(x)+x\text{sech}^{-1}(x)$$

[Out] x*arcsech(x)+arcsin(x)*(1/(1+x))^(1/2)*(1+x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5893, 6277, 216}

$$\sqrt{\frac{1}{x+1}}\sqrt{x+1}\,\sin^{-1}(x)+x\text{sech}^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[x^(-1)],x]

[Out] x*ArcSech[x] + Sqrt[(1 + x)^(-1)]*Sqrt[1 + x]*ArcSin[x]

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5893

```
Int[ArcCosh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSech[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6277

```
Int[ArcSech[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1
 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}\left(\frac{1}{x}\right) dx &= \int \text{sech}^{-1}(x)\,dx\\
&= x\text{sech}^{-1}(x)+\left(\sqrt{\frac{1}{1+x}}\sqrt{1+x}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= x\text{sech}^{-1}(x)+\sqrt{\frac{1}{1+x}}\sqrt{1+x}\,\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 1.92

$$x\cosh^{-1}\left(\frac{1}{x}\right)-\frac{\sqrt{\frac{1}{x^2}-1}\,\tan^{-1}\left(\sqrt{\frac{1}{x^2}-1}\right)}{\sqrt{\frac{1}{x}-1}\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[x^(-1)],x]

```
[Out] x*ArcCosh[x^(-1)] - (Sqrt[-1 + x^(-2)]*ArcTan[Sqrt[-1 + x^(-2)]])/(Sqrt[-1
+ x^(-1)]*Sqrt[1 + x^(-1)])
```

fricas [B] time = 1.02, size = 72, normalized size = 3.00

$$(x-2)\log\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}+1}{x}\right)-2\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)-2\log\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="fricas")
```

```
[Out] (x - 2)*log((x*sqrt(-(x^2 - 1)/x^2) + 1)/x) - 2*arctan((x*sqrt(-(x^2 - 1)/x
^2) - 1)/x) - 2*log((x*sqrt(-(x^2 - 1)/x^2) - 1)/x)
```

giac [B] time = 0.23, size = 22, normalized size = 0.92

$$x\log\left(\sqrt{\frac{1}{x^2}-1}+\frac{1}{x}\right)+\frac{\arcsin(x)}{\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="giac")
```

```
[Out] x*log(sqrt(1/x^2 - 1) + 1/x) + arcsin(x)/sgn(x)
```

maple [A] time = 0.02, size = 38, normalized size = 1.58

$$\operatorname{arccosh}\left(\frac{1}{x}\right)x+\frac{\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}\right)}{\sqrt{\frac{1}{x^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(1/x),x)
```

```
[Out] arccosh(1/x)*x+(1/x-1)^(1/2)*(1/x+1)^(1/2)/(1/x^2-1)^(1/2)*arctan(1/(1/x^2-
1)^(1/2))
```

maxima [B] time = 0.80, size = 17, normalized size = 0.71

$$x\operatorname{arcosh}\left(\frac{1}{x}\right)-\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(1/x),x, algorithm="maxima")
```

```
[Out] x*arccosh(1/x) - arctan(sqrt(1/x^2 - 1))
```

mupad [B] time = 0.37, size = 23, normalized size = 0.96

$$\operatorname{atan}\left(\frac{1}{\sqrt{\frac{1}{x}-1}\,\sqrt{\frac{1}{x}+1}}\right)+x\operatorname{acosh}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(1/x),x)
```

```
[Out] atan(1/((1/x - 1)^(1/2)*(1/x + 1)^(1/2))) + x*acosh(1/x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}\left(\frac{1}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(1/x),x)
```

```
[Out] Integral(acosh(1/x), x)
```

3.239 $\int \frac{\cosh^{-1}(ax^n)}{x} dx$

Optimal. Leaf size=60

$$\frac{\text{Li}_2\left(-e^{2\cosh^{-1}(ax^n)}\right)}{2n} - \frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(e^{2\cosh^{-1}(ax^n)}+1\right)}{n}$$

```
[Out] -1/2*arccosh(a*x^n)^2/n+arccosh(a*x^n)*ln(1+(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1
)^(1/2))^2)/n+1/2*polylog(2,-(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1)^(1/2))^2)/n
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5891, 3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax^n)}\right)}{2n} - \frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(e^{2\cosh^{-1}(ax^n)}+1\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x^n]/x,x]
```

```
[Out] -ArcCosh[a*x^n]^2/(2*n) + (ArcCosh[a*x^n]*Log[1 + E^(2*ArcCosh[a*x^n])])/n
+ PolyLog[2, -E^(2*ArcCosh[a*x^n])]/(2*n)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5891

```
Int[ArcCosh[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Dist[1/p, Subst[Int[x
^n*Tanh[x], x], x, ArcCosh[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax^n)}{x}\,dx = \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(ax^n)\right)}{n}$$
$$= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}(ax^n)\right)}{n}$$
$$= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx, x, \cosh$$
$$= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}(ax^n}}{2n}$$
$$= -\frac{\cosh^{-1}(ax^n)^2}{2n} + \frac{\cosh^{-1}(ax^n)\log\left(1+e^{2\cosh^{-1}(ax^n)}\right)}{n} + \frac{\text{Li}_2\left(-e^{2\cosh^{-1}(ax^n)}\right)}{2n}$$

Mathematica [B] time = 0.51, size = 179, normalized size = 2.98

$$\frac{a\sqrt{1-a^2x^{2n}}\left(-\text{Li}_2\left(e^{-2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)}\right)+\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)^2+2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)\log\left(1-e^{-2\sinh^{-1}\left(\sqrt{-a^2}\,x^n\right)}\right)-2}{2\sqrt{-a^2}\,n\sqrt{ax^n-1}\,\sqrt{ax^n+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x^n]/x,x]
```

```
[Out] ArcCosh[a*x^n]*Log[x] + (a*Sqrt[1 - a^2*x^(2*n)]*(ArcSinh[Sqrt[-a^2]*x^n]^2
 + 2*ArcSinh[Sqrt[-a^2]*x^n]*Log[1 - E^(-2*ArcSinh[Sqrt[-a^2]*x^n])] - 2*n*
Log[x]*Log[Sqrt[-a^2]*x^n + Sqrt[1 - a^2*x^(2*n)]] - PolyLog[2, E^(-2*ArcSi
nh[Sqrt[-a^2]*x^n])]))/(2*Sqrt[-a^2]*n*Sqrt[-1 + a*x^n]*Sqrt[1 + a*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x^n)/x, x)
```

maple [A] time = 0.02, size = 91, normalized size = 1.52

$$-\frac{\operatorname{arccosh}(a\,x^n)^2}{2n}+\frac{\operatorname{arccosh}(a\,x^n)\ln\left(1+\left(a\,x^n+\sqrt{a\,x^n-1}\,\sqrt{a\,x^n+1}\right)^2\right)}{n}+\frac{\operatorname{polylog}\left(2,-\left(a\,x^n+\sqrt{a\,x^n-1}\,\sqrt{a\,x^n}\right.\right.}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x^n)/x,x)
```

```
[Out] -1/2*arccosh(a*x^n)^2/n+arccosh(a*x^n)*ln(1+(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1
)^(1/2))^2)/n+1/2*polylog(2,-(a*x^n+(a*x^n-1)^(1/2)*(a*x^n+1)^(1/2))^2)/n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$an\int\frac{x^n\log(x)}{a^3xx^{3n}-axx^n+\left(a^2xx^{2n}-x\right)\sqrt{ax^n+1}\sqrt{ax^n-1}}\,dx-\frac{1}{2}n\log(x)^2+n\int\frac{\log(x)}{2\,(axx^n+x)}\,dx-n\int\frac{\log(x)}{2\,(axx^n-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(x^n*log(x)/(a^3*x*x^(3*n) - a*x*x^n + (a^2*x*x^(2*n) - x)*sqr
t(a*x^n + 1)*sqrt(a*x^n - 1)), x) - 1/2*n*log(x)^2 + n*integrate(1/2*log(x)
/(a*x*x^n + x), x) - n*integrate(1/2*log(x)/(a*x*x^n - x), x) + log(a*x^n +
 sqrt(a*x^n + 1)*sqrt(a*x^n - 1))*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\mathrm{acosh}\left(a\,x^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x^n)/x,x)
```

```
[Out] int(acosh(a*x^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acosh}\left(ax^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x**n)/x,x)
```

```
[Out] Integral(acosh(a*x**n)/x, x)
```

3.240 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4\,dx$

Optimal. Leaf size=145

$$-\frac{192b^3\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^4-\frac{8b\left(\right.}{}$$

```
[Out] 384*b^4*x+48*b^2*x*(a+b*arccosh(d*x^2+1))^2+x*(a+b*arccosh(d*x^2+1))^4-192*
b^3*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-8*
b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^3/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5880, 8}

$$-\frac{192b^3\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2-\frac{8b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2\right.\right.}{x\sqrt{dx^2}\sqrt{dx^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x - (192*b^3*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*
x^2]*Sqrt[2 + d*x^2]) + 48*b^2*x*(a + b*ArcCosh[1 + d*x^2])^2 - (8*b*(2*x^2
 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x
*(a + b*ArcCosh[1 + d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4\,dx = -\frac{8b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3}{x\sqrt{dx^2}\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^4+(48$$

$$= -\frac{192b^3\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}}+48b^2x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)$$

$$= 384b^4x-\frac{192b^3\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}}+48b^2x\left(a+b\cosh^{-1}\left(1\right.\right.$$

Mathematica [A] time = 0.23, size = 264, normalized size = 1.82

$$-8ab\left(a^2+24b^2\right)\sqrt{dx^2}\sqrt{dx^2+2}+6b^2\cosh^{-1}\left(dx^2+1\right)^2\left(a^2dx^2-4ab\sqrt{dx^2}\sqrt{dx^2+2}+8b^2dx^2\right)+dx^2\left(a^4+48\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^4,x]
```

```
[Out] ((a^4 + 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 + 24*b^2)*Sqrt[d*x^2]*Sqrt
[2 + d*x^2] + 4*b*(a^3*d*x^2 + 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[d*x^2]*Sqrt[2
+ d*x^2] - 48*b^3*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^2] + 6*b^2*(
a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*
x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^
2]^3 + b^4*d*x^2*ArcCosh[1 + d*x^2]^4)/(d*x)
```

fricas [B] time = 0.59, size = 298, normalized size = 2.06

$$\frac{b^4dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^4+\left(a^4+48\,a^2b^2+384\,b^4\right)dx^2+4\left(ab^3dx^2-2\sqrt{d^2x^4+2\,dx^2}\,b^4\right)\log\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^4 + (a^4 + 48*a^2*b^2 +
 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)*b^4)*log(d*x^2
 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^3 - 6*(4*sqrt(d^2*x^4 + 2*d*x^2)*a*b^3 - (a
^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^2 + 4*((a^3
*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 + 2*d*x^2)*(a^2*b^2 + 8*b^4))*log(d*x
^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) - 8*sqrt(d^2*x^4 + 2*d*x^2)*(a^3*b + 24*a
*b^3))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(x)]index.cc ind
ex_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m operator +
Error: Bad Argument Value
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^4,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^4x\log\left(dx^2+\sqrt{dx^2+2}\,\sqrt{d}\,x+1\right)^4+6\,a^2b^2x\,\mathrm{arcosh}\left(dx^2+1\right)^2+24\,a^2b^2d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^2+2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{d}\right.}{\sqrt{dx^2+2}\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^4 + 6*a^2*b^2*x*arccosh(d*
x^2 + 1)^2 + 24*a^2*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(d))*log(d*x^2 + sq
rt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 4*(x*arccosh(d*x^2
+ 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a^3*b + a^4*x + int
egrate(4*((a*b^3*d^2 - 2*b^4*d^2)*x^4 + 2*a*b^3 + (3*a*b^3*d - 4*b^4*d)*x^2
 + ((a*b^3*d - 2*b^4*d)*sqrt(d)*x^3 + 2*(a*b^3 - b^4)*sqrt(d)*x)*sqrt(d*x^2
 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^3/(d^2*x^4 + 3*d*x^2 + (d
^(3/2)*x^3 + 2*sqrt(d)*x)*sqrt(d*x^2 + 2) + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\,\mathrm{acosh}\left(d\,x^2 + 1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^4,x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{acosh}\left(dx^2 + 1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**4,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**4, x)
```

3.241 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3\,dx$

Optimal. Leaf size=125

$$24ab^2x+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^3-\frac{6b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2+2}}-\frac{48b^3\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}+24b^3x$$

```
[Out] 24*a*b^2*x+24*b^3*x*arccosh(d*x^2+1)+x*(a+b*arccosh(d*x^2+1))^3-48*b^3*(d*x
^2+2)*(d*x^2/(d*x^2+2))^(1/2)/d/x-6*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^
2/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5880, 5901, 12, 6719, 261}

$$24ab^2x-\frac{6b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2+2}}+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^3-\frac{48b^3\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}+24b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - (48*b^3*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + 24*b^3*
x*ArcCosh[1 + d*x^2] - (6*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^2)/(
x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3\,dx = -\frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3 + (24$$

$$= 24ab^2x - \frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right.$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(1+dx^2\right) - \frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}}$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(1+dx^2\right) - \frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}}$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(1+dx^2\right) - \frac{6b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{2+dx^2}}$$

$$= 24ab^2x - \frac{48b^3\sqrt{\frac{dx^2}{2+dx^2}}\left(2+dx^2\right)}{dx} + 24b^3x\cosh^{-1}\left(1+dx^2\right) - \frac{6b\left(2x^2+dx^4\right)(}{x\sqrt{d}}$$

Mathematica [A] time = 0.13, size = 171, normalized size = 1.37

$$\frac{adx^2\left(a^2+24b^2\right)-6b\left(a^2+8b^2\right)\sqrt{dx^2}\sqrt{dx^2+2}+3b\cosh^{-1}\left(dx^2+1\right)\left(a^2dx^2-4ab\sqrt{dx^2}\sqrt{dx^2+2}+8b^2dx^2\right)+}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2]*Sqrt[2 + d*x^2] + 3
*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1
+ d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*
x^2]^2 + b^3*d*x^2*ArcCosh[1 + d*x^2]^3)/(d*x)
```

fricas [A] time = 0.72, size = 210, normalized size = 1.68

$$b^3dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^3+\left(a^3+24\,ab^2\right)dx^2+3\left(ab^2dx^2-2\sqrt{d^2x^4+2\,dx^2}\,b^3\right)\log\left(dx^2+\sqrt{d^2x^4+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^3 + (a^3 + 24*a*b^2)*d*
x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)*b^3)*log(d*x^2 + sqrt(d^2*
x^4 + 2*d*x^2) + 1)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 + 2*d*x^2
)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) - 6*sqrt(d^2*x^4 + 2*d*x^
2)*(a^2*b + 8*b^3))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(x)]index.cc ind
ex_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m operator +
Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^{2}+1\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^3,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3\,ab^{2}x\,\mathrm{arcosh}\left(dx^{2}+1\right)^{2}+12\,ab^{2}d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^{2}+2\,\sqrt{d}\right)\log\left(dx^{2}+\sqrt{dx^{2}+2}\,\sqrt{dx^{2}}+1\right)}{\sqrt{dx^{2}+2}\,d^{2}}\right)+3\left(x\,\mathrm{arcosh}\left(dx^{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*x*arccosh(d*x^2 + 1)^2 + 12*a*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(
d))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 3
*(x*arccosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a
^2*b + (x*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1)^3 - integrate(6*(d^2*x
^4 + 2*d*x^2 + (d^(3/2)*x^3 + sqrt(d)*x)*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(
d*x^2 + 2)*sqrt(d)*x + 1)^2/(d^2*x^4 + 3*d*x^2 + (d^(3/2)*x^3 + 2*sqrt(d)*x
)*sqrt(d*x^2 + 2) + 2), x))*b^3 + a^3*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^{2}+1\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^3,x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{acosh}\left(dx^{2}+1\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**3, x)
```

3.242 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2\,dx$

Optimal. Leaf size=72

$$x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2-\frac{4b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+8b^2x$$

```
[Out] 8*b^2*x+x*(a+b*arccosh(d*x^2+1))^2-4*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))
/x/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5880, 8}

$$-\frac{4b\left(dx^4+2x^2\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2+2}}+x\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^2,x]
```

```
[Out] 8*b^2*x - (4*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2]))/(x*Sqrt[d*x^2]*S
qrt[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2\,dx &= -\frac{4b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2+\left(8b^2\right.\\ &= 8b^2x-\frac{4b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2\end{aligned}$$

Mathematica [A] time = 0.07, size = 104, normalized size = 1.44

$$x\left(a^2+8b^2\right)-\frac{4ab\sqrt{dx^2}\sqrt{dx^2+2}}{dx}+\frac{2b\cosh^{-1}\left(dx^2+1\right)\left(adx^2-2b\sqrt{dx^2}\sqrt{dx^2+2}\right)}{dx}+b^2x\cosh^{-1}\left(dx^2+1\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^2,x]
```

```
[Out] (a^2 + 8*b^2)*x - (4*a*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(d*x) + (2*b*(a*d*x^2
 - 2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])*ArcCosh[1 + d*x^2])/(d*x) + b^2*x*ArcCo
sh[1 + d*x^2]^2
```

fricas [A] time = 0.69, size = 131, normalized size = 1.82

$$\frac{b^2dx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)^2+\left(a^2+8\,b^2\right)dx^2-4\,\sqrt{d^2x^4+2\,dx^2}\,ab+2\left(abdx^2-2\,\sqrt{d^2x^4+2\,dx^2}\,b^2\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1)^2 + (a^2 + 8*b^2)*d*x^2
 - 4*sqrt(d^2*x^4 + 2*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 + 2*d*x^2)
*b^2)*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(x)]index.cc ind
ex_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m operator +
Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^2,x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^2,x)
```

maxima [A] time = 0.74, size = 128, normalized size = 1.78

$$b^2x\,\mathrm{arcosh}\left(dx^2+1\right)^2+4\,b^2d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^2+2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2+2}\,\sqrt{dx^2}+1\right)}{\sqrt{dx^2+2}\,d^2}\right)+2\left(x\,\mathrm{arcosh}\left(dx^2+1\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccosh(d*x^2 + 1)^2 + 4*b^2*d*(2*x/d - (d^(3/2)*x^2 + 2*sqrt(d))*log
(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d*x^2) + 1)/(sqrt(d*x^2 + 2)*d^2)) + 2*(x*arc
cosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*a*b + a^
2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^2,x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\,\mathrm{acosh}\left(dx^2 + 1\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**2, x)
```

3.243 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)\,dx$

Optimal. Leaf size=49

$$ax-\frac{2b\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}+bx\cosh^{-1}\left(dx^2+1\right)$$

```
[Out] a*x+b*x*arccosh(d*x^2+1)-2*b*(d*x^2+2)*(d*x^2/(d*x^2+2))^(1/2)/d/x
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5901, 12, 6719, 261}

$$ax-\frac{2b\sqrt{\frac{dx^2}{dx^2+2}}\left(dx^2+2\right)}{dx}+bx\cosh^{-1}\left(dx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[1 + d*x^2],x]
```

```
[Out] a*x - (2*b*Sqrt[(d*x^2)/(2 + d*x^2)]*(2 + d*x^2))/(d*x) + b*x*ArcCosh[1 + d
*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)dx = ax+b\int \cosh^{-1}\left(1+dx^2\right)dx$$
$$= ax+bx\cosh^{-1}\left(1+dx^2\right)-b\int 2\sqrt{\frac{dx^2}{2+dx^2}}\,dx$$
$$= ax+bx\cosh^{-1}\left(1+dx^2\right)-(2b)\int \sqrt{\frac{dx^2}{2+dx^2}}\,dx$$
$$= ax+bx\cosh^{-1}\left(1+dx^2\right)-\frac{\left(2b\sqrt{\frac{dx^2}{2+dx^2}}\sqrt{2+dx^2}\right)\int \frac{x}{\sqrt{2+dx^2}}\,dx}{x}$$
$$= ax-\frac{2b\sqrt{\frac{dx^2}{2+dx^2}}\left(2+dx^2\right)}{dx}+bx\cosh^{-1}\left(1+dx^2\right)$$

Mathematica [**A**] time = 0.07, size = 37, normalized size = 0.76

$$ax-\frac{2bx}{\sqrt{\frac{dx^2}{dx^2+2}}}+bx\cosh^{-1}\left(dx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[1 + d*x^2],x]
```

```
[Out] a*x - (2*b*x)/Sqrt[(d*x^2)/(2 + d*x^2)] + b*x*ArcCosh[1 + d*x^2]
```

fricas [**A**] time = 0.58, size = 63, normalized size = 1.29

$$\frac{bdx^2\log\left(dx^2+\sqrt{d^2x^4+2\,dx^2}+1\right)+adx^2-2\,\sqrt{d^2x^4+2\,dx^2}\,b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 + 2*d*x^2) + 1) + a*d*x^2 - 2*sqrt(d^2*x^
4 + 2*d*x^2)*b)/(d*x)
```

giac [**A**] time = 0.20, size = 62, normalized size = 1.27

$$\left(x\log\left(dx^2+\sqrt{\left(dx^2+1\right)^2-1}+1\right)+\frac{2\sqrt{2}\,\mathrm{sgn}(x)}{\sqrt{d}}-\frac{2\sqrt{d^2x^2+2\,d}}{d\mathrm{sgn}(x)}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="giac")
```

```
[Out] (x*log(d*x^2 + sqrt((d*x^2 + 1)^2 - 1) + 1) + 2*sqrt(2)*sgn(x)/sqrt(d) - 2*
sqrt(d^2*x^2 + 2*d)/(d*sgn(x)))*b + a*x
```

maple [**A**] time = 0.01, size = 37, normalized size = 0.76

$$ax+b\left(x\,\mathrm{arccosh}\left(d\,x^2+1\right)-\frac{2x\sqrt{d\,x^2+2}}{\sqrt{d\,x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x^2+1),x)
```

```
[Out] a*x+b*(x*arccosh(d*x^2+1)-2/(d*x^2)^(1/2)*x*(d*x^2+2)^(1/2))
```

maxima [A] time = 0.95, size = 44, normalized size = 0.90

$$\left(x \operatorname{arcosh}\left(dx^2+1\right)-\frac{2\left(d^{\frac{3}{2}}x^2+2\sqrt{d}\right)}{\sqrt{dx^2+2}\,d}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2+1),x, algorithm="maxima")
```

```
[Out] (x*arccosh(d*x^2 + 1) - 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(d*x^2 + 2)*d))*b
+ a*x
```

mupad [B] time = 0.59, size = 32, normalized size = 0.65

$$a\,x+b\,x\,\operatorname{acosh}\left(d\,x^2+1\right)-\frac{2\,b\,\operatorname{sign}(x)\,\sqrt{d\,x^2+2}}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acosh(d*x^2 + 1),x)
```

```
[Out] a*x + b*x*acosh(d*x^2 + 1) - (2*b*sign(x)*(d*x^2 + 2)^(1/2))/d^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x**2+1),x)
```

```
[Out] Integral(a + b*acosh(d*x**2 + 1), x)
```

3.244 $\int \frac{1}{a+b\cosh^{-1}(1+dx^2)}\,dx$

Optimal. Leaf size=98

$$\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

```
[Out] 1/2*x*Chi(1/2*(a+b*arccosh(d*x^2+1))/b)*cosh(1/2*a/b)/b*2^(1/2)/(d*x^2)^(1/
2)-1/2*x*Shi(1/2*(a+b*arccosh(d*x^2+1))/b)*sinh(1/2*a/b)/b*2^(1/2)/(d*x^2)^
(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5881}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-1),x]
```

```
[Out] (x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b
*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2
*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rule 5881

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(x*Co
sh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt
[d*x^2]), x] - Simp[(x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}\left(1+dx^2\right)}\,dx=\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Mathematica [A] time = 0.14, size = 118, normalized size = 1.20

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)\right)}{b\sqrt{dx^2}\sqrt{\frac{dx^2}{dx^2+2}}\sqrt{dx^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-1),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1
+ d*x^2])/(2*b)] - Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2
*b)]))/(b*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b\,\text{arcosh}\left(dx^2+1\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="fricas")
[Out] integral(1/(b*arccosh(d*x^2 + 1) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arcosh}\left(dx^2+1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="giac")
[Out] integrate(1/(b*arccosh(d*x^2 + 1) + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\text{arccosh}\left(d\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1)),x)
[Out] int(1/(a+b*arccosh(d*x^2+1)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arcosh}\left(dx^2+1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1)),x, algorithm="maxima")
[Out] integrate(1/(b*arccosh(d*x^2 + 1) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b\,\text{acosh}\left(d\,x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1)),x)
[Out] int(1/(a + b*acosh(d*x^2 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\text{acosh}\left(dx^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1)),x)
[Out] Integral(1/(a + b*acosh(d*x**2 + 1)), x)
```

3.245 $\int \frac{1}{\left(a+b\cosh^{-1}(1+dx^2)\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2+2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}$$

```
[Out] 1/4*x*cosh(1/2*a/b)*Shi(1/2*(a+b*arccosh(d*x^2+1))/b)/b^2*2^(1/2)/(d*x^2)^(
1/2)-1/4*x*Chi(1/2*(a+b*arccosh(d*x^2+1))/b)*sinh(1/2*a/b)/b^2*2^(1/2)/(d*x
^2)^(1/2)-1/2*(d*x^2)^(1/2)*(d*x^2+2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2+1))
```

Rubi [A] time = 0.02, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5887}

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2+2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-2),x]
```

```
[Out] -(Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[1 + d*x^2])) - (x*Co
shIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(2*Sqrt[2]*b^2*
Sqrt[d*x^2]) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*
b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Rule 5887

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> -Simp[(Sqr
t[d*x^2]*Sqrt[2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[1 + d*x^2])), x] + (-Simp
[(x*Sinh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(2*Sqrt[2
]*b^2*Sqrt[d*x^2]), x] + Simp[(x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[
1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2]), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}\,dx=-\frac{\sqrt{dx^2}\sqrt{2+dx^2}}{2bdx\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)}-\frac{x\text{Chi}\left(\frac{a+b\cosh^{-1}(1+dx^2)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}+\frac{x\cosh\cdots}{\cdots}$$

Mathematica [A] time = 0.93, size = 130, normalized size = 0.87

$$-\frac{x^2\text{csch}\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(dx^2+1)}{2b}\right)\right)+\frac{2b\sqrt{dx^2}\sqrt{dx\cdots}}{ad+bd\cosh^{-1}(\cdots}}{4b^2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-2),x]
```

```
[Out] -1/4*((2*b*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(a*d + b*d*ArcCosh[1 + d*x^2]) + x^
2*Csch[ArcCosh[1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)
```

```
]*Sinh[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*
b)]))/(b^2*x)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2 \operatorname{arcosh}\left(dx^2+1\right)^2+2\,ab \operatorname{arcosh}\left(dx^2+1\right)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(d*x^2 + 1)^2 + 2*a*b*arccosh(d*x^2 + 1) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arcosh}\left(dx^2+1\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b \operatorname{arccosh}\left(d\,x^2+1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^2,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^2x^4+3\,dx^2+\left(d^{\frac{3}{2}}x^3+2\,\sqrt{d}\,x\right)\sqrt{dx^2+2}+2}{2\left(abd^2x^3+2\,abdx+\left(b^2d^2x^3+2\,b^2dx+\left(b^2d^{\frac{3}{2}}x^2+b^2\sqrt{d}\right)\sqrt{dx^2+2}\right)\log\left(dx^2+\sqrt{dx^2+2}\,\sqrt{d}\,x+1\right)+\left(abd\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 + 3*d*x^2 + (d^(3/2)*x^3 + 2*sqrt(d)*x)*sqrt(d*x^2 + 2) + 2)/
(a*b*d^2*x^3 + 2*a*b*d*x + (b^2*d^2*x^3 + 2*b^2*d*x + (b^2*d^(3/2)*x^2 + b^
2*sqrt(d))*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + (a
*b*d^(3/2)*x^2 + a*b*sqrt(d))*sqrt(d*x^2 + 2)) + integrate(1/2*(d^3*x^6 + 3
*d^2*x^4 + (d^2*x^4 + d*x^2 + 2)*(d*x^2 + 2) + (2*d^(5/2)*x^5 + 4*d^(3/2)*x
^3 + sqrt(d)*x)*sqrt(d*x^2 + 2) - 4)/(a*b*d^3*x^6 + 4*a*b*d^2*x^4 + 4*a*b*d
*x^2 + (a*b*d^2*x^4 + 2*a*b*d*x^2 + a*b)*(d*x^2 + 2) + (b^2*d^3*x^6 + 4*b^2
*d^2*x^4 + 4*b^2*d*x^2 + (b^2*d^2*x^4 + 2*b^2*d*x^2 + b^2)*(d*x^2 + 2) + 2*
(b^2*d^(5/2)*x^5 + 3*b^2*d^(3/2)*x^3 + 2*b^2*sqrt(d)*x)*sqrt(d*x^2 + 2))*lo
g(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + 2*(a*b*d^(5/2)*x^5 + 3*a*b*d^(3/
2)*x^3 + 2*a*b*sqrt(d)*x)*sqrt(d*x^2 + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^2,x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\,\mathrm{acosh}\left(dx^2 + 1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-2), x)
```

3.246 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3}\,dx$

Optimal. Leaf size=180

$$\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}-\frac{\ldots}{4bx\sqrt{dx^2\ldots}}$$

```
[Out] -1/8*x/b^2/(a+b*arccosh(d*x^2+1))+1/16*x*Chi(1/2*(a+b*arccosh(d*x^2+1))/b)*
cosh(1/2*a/b)/b^3*2^(1/2)/(d*x^2)^(1/2)-1/16*x*Shi(1/2*(a+b*arccosh(d*x^2+1
))/b)*sinh(1/2*a/b)/b^3*2^(1/2)/(d*x^2)^(1/2)+1/4*(-d*x^4-2*x^2)/b/x/(a+b*a
rccosh(d*x^2+1))^2/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5889, 5881}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)}-\frac{\ldots}{4bx\sqrt{dx^2\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-3),x]
```

```
[Out] -(2*x^2 + d*x^4)/(4*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^2) - x/(8*b^2*(a + b*ArcCosh[1 + d*x^2])) + (x*Cosh[a/(2*b)]*CoshIntegr
al[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) - (x*Sinh
[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^3*Sq
rt[d*x^2])
```

Rule 5881

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[(x*Co
sh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt
[d*x^2]), x] - Simp[(x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[1 + d*x^2]
)/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^3}\,dx &= -\frac{2x^2+dx^4}{4bx\sqrt{dx^2}\,\sqrt{2+dx^2}\,\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(1+dx\ldots\right.\right.}\\ &= -\frac{2x^2+dx^4}{4bx\sqrt{dx^2}\,\sqrt{2+dx^2}\,\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^2}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(1+dx\ldots\right.\right.}\end{aligned}$$

Mathematica [A] time = 0.48, size = 152, normalized size = 0.84

$$\frac{-\frac{2b^2\sqrt{dx^2}\sqrt{dx^2+2}}{d\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2}+\frac{\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2+1\right)}{2b}\right)\right)}{d}-\frac{bx^2}{a+b\cosh^{-1}\left(dx^2+1\right)}}{8b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-3),x]
```

```
[Out] ((-2*b^2*Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(d*(a + b*ArcCosh[1 + d*x^2])^2) - (b
*x^2)/(a + b*ArcCosh[1 + d*x^2]) + (Sinh[ArcCosh[1 + d*x^2]/2]*(Cosh[a/(2*b
)]*CoshIntegral[(a + b*ArcCosh[1 + d*x^2])/(2*b)] - Sinh[a/(2*b)]*SinhInteg
ral[(a + b*ArcCosh[1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{1}{b^3\operatorname{arcosh}\left(dx^2+1\right)^3+3\,ab^2\operatorname{arcosh}\left(dx^2+1\right)^2+3\,a^2b\operatorname{arcosh}\left(dx^2+1\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccosh(d*x^2 + 1)^3 + 3*a*b^2*arccosh(d*x^2 + 1)^2 + 3*a^2
*b*arccosh(d*x^2 + 1) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\operatorname{arcosh}\left(dx^2+1\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{arccosh}\left(d\,x^2+1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^3,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^5 + 2*b*d^5)*sqrt(d)*x^10 + 2*(3*a*d^4 + 7*b*d^4)*sqrt(d)*x^8 +
(11*a*d^3 + 36*b*d^3)*sqrt(d)*x^6 + 2*(a*d^2 + 20*b*d^2)*sqrt(d)*x^4 - 4*(3
*a*d - 4*b*d)*sqrt(d)*x^2 + ((a*d^4 + 2*b*d^4)*x^7 + (3*a*d^3 + 8*b*d^3)*x^
```

```
5 + 2*(2*a*d^2 + 5*b*d^2)*x^3 + 4*(a*d + b*d)*x)*(d*x^2 + 2)^(3/2) + (3*(a*
d^4 + 2*b*d^4)*sqrt(d)*x^8 + 6*(2*a*d^3 + 5*b*d^3)*sqrt(d)*x^6 + 2*(8*a*d^2
 + 25*b*d^2)*sqrt(d)*x^4 + 10*(a*d + 3*b*d)*sqrt(d)*x^2 + 4*(a + b)*sqrt(d)
)*(d*x^2 + 2) + (b*d^(11/2)*x^10 + 6*b*d^(9/2)*x^8 + 11*b*d^(7/2)*x^6 + 2*b
*d^(5/2)*x^4 - 12*b*d^(3/2)*x^2 + (b*d^4*x^7 + 3*b*d^3*x^5 + 4*b*d^2*x^3 +
4*b*d*x)*(d*x^2 + 2)^(3/2) + (3*b*d^(9/2)*x^8 + 12*b*d^(7/2)*x^6 + 16*b*d^(
5/2)*x^4 + 10*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 + 2) + (3*b*d^5*x^9 + 15*
b*d^4*x^7 + 23*b*d^3*x^5 + 7*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 + 2) - 8*b*sqr
t(d))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + (3*(a*d^5 + 2*b*d^5)*x^9
 + 3*(5*a*d^4 + 12*b*d^4)*x^7 + (23*a*d^3 + 76*b*d^3)*x^5 + (7*a*d^2 + 64*b
*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 + 2) - 8*a*sqrt(d))/(a^2*b^2*d^
(11/2)*x^9 + 6*a^2*b^2*d^(9/2)*x^7 + 12*a^2*b^2*d^(7/2)*x^5 + 8*a^2*b^2*d^(
5/2)*x^3 + (b^4*d^(11/2)*x^9 + 6*b^4*d^(9/2)*x^7 + 12*b^4*d^(7/2)*x^5 + 8*b
^4*d^(5/2)*x^3 + (b^4*d^4*x^6 + 3*b^4*d^3*x^4 + 3*b^4*d^2*x^2 + b^4*d)*(d*x
^2 + 2)^(3/2) + 3*(b^4*d^(9/2)*x^7 + 4*b^4*d^(7/2)*x^5 + 5*b^4*d^(5/2)*x^3
+ 2*b^4*d^(3/2)*x)*(d*x^2 + 2) + 3*(b^4*d^5*x^8 + 5*b^4*d^4*x^6 + 8*b^4*d^3
*x^4 + 4*b^4*d^2*x^2)*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*
x + 1)^2 + (a^2*b^2*d^4*x^6 + 3*a^2*b^2*d^3*x^4 + 3*a^2*b^2*d^2*x^2 + a^2*b
^2*d)*(d*x^2 + 2)^(3/2) + 3*(a^2*b^2*d^(9/2)*x^7 + 4*a^2*b^2*d^(7/2)*x^5 +
5*a^2*b^2*d^(5/2)*x^3 + 2*a^2*b^2*d^(3/2)*x)*(d*x^2 + 2) + 2*(a*b^3*d^(11/2
)*x^9 + 6*a*b^3*d^(9/2)*x^7 + 12*a*b^3*d^(7/2)*x^5 + 8*a*b^3*d^(5/2)*x^3 +
(a*b^3*d^4*x^6 + 3*a*b^3*d^3*x^4 + 3*a*b^3*d^2*x^2 + a*b^3*d)*(d*x^2 + 2)^(
3/2) + 3*(a*b^3*d^(9/2)*x^7 + 4*a*b^3*d^(7/2)*x^5 + 5*a*b^3*d^(5/2)*x^3 + 2
*a*b^3*d^(3/2)*x)*(d*x^2 + 2) + 3*(a*b^3*d^5*x^8 + 5*a*b^3*d^4*x^6 + 8*a*b^
3*d^3*x^4 + 4*a*b^3*d^2*x^2)*sqrt(d*x^2 + 2))*log(d*x^2 + sqrt(d*x^2 + 2)*s
qrt(d)*x + 1) + 3*(a^2*b^2*d^5*x^8 + 5*a^2*b^2*d^4*x^6 + 8*a^2*b^2*d^3*x^4
+ 4*a^2*b^2*d^2*x^2)*sqrt(d*x^2 + 2)) + integrate(1/8*(d^6*x^12 + 8*d^5*x^1
0 + 27*d^4*x^8 + 56*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 + 4*d^3*x^6 + 3*d^2*x^4
 - 8*d*x^2 + 4)*(d*x^2 + 2)^2 + 96*d*x^2 + 2*(2*d^(9/2)*x^9 + 10*d^(7/2)*x^
7 + 15*d^(5/2)*x^5 - d^(3/2)*x^3 - 11*sqrt(d)*x)*(d*x^2 + 2)^(3/2) + 3*(2*d
^5*x^10 + 12*d^4*x^8 + 26*d^3*x^6 + 24*d^2*x^4 + 3*d*x^2 - 10)*(d*x^2 + 2)
+ 2*(2*d^(11/2)*x^11 + 14*d^(9/2)*x^9 + 39*d^(7/2)*x^7 + 61*d^(5/2)*x^5 + 6
1*d^(3/2)*x^3 + 30*sqrt(d)*x)*sqrt(d*x^2 + 2) + 48)/(a*b^2*d^6*x^12 + 8*a*b
^2*d^5*x^10 + 24*a*b^2*d^4*x^8 + 32*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b
^2*d^4*x^8 + 4*a*b^2*d^3*x^6 + 6*a*b^2*d^2*x^4 + 4*a*b^2*d*x^2 + a*b^2)*(d*
x^2 + 2)^2 + 4*(a*b^2*d^(9/2)*x^9 + 5*a*b^2*d^(7/2)*x^7 + 9*a*b^2*d^(5/2)*x
^5 + 7*a*b^2*d^(3/2)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 + 2)^(3/2) + 6*(a*b^2*
d^5*x^10 + 6*a*b^2*d^4*x^8 + 13*a*b^2*d^3*x^6 + 12*a*b^2*d^2*x^4 + 4*a*b^2*
d*x^2)*(d*x^2 + 2) + (b^3*d^6*x^12 + 8*b^3*d^5*x^10 + 24*b^3*d^4*x^8 + 32*b
^3*d^3*x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 + 4*b^3*d^3*x^6 + 6*b^3*d^2*x^4
+ 4*b^3*d*x^2 + b^3)*(d*x^2 + 2)^2 + 4*(b^3*d^(9/2)*x^9 + 5*b^3*d^(7/2)*x^7
 + 9*b^3*d^(5/2)*x^5 + 7*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 + 2)^(3/
2) + 6*(b^3*d^5*x^10 + 6*b^3*d^4*x^8 + 13*b^3*d^3*x^6 + 12*b^3*d^2*x^4 + 4*
b^3*d*x^2)*(d*x^2 + 2) + 4*(b^3*d^(11/2)*x^11 + 7*b^3*d^(9/2)*x^9 + 18*b^3*
d^(7/2)*x^7 + 20*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d*x^2 + 2))*log(
d*x^2 + sqrt(d*x^2 + 2)*sqrt(d)*x + 1) + 4*(a*b^2*d^(11/2)*x^11 + 7*a*b^2*d
^(9/2)*x^9 + 18*a*b^2*d^(7/2)*x^7 + 20*a*b^2*d^(5/2)*x^5 + 8*a*b^2*d^(3/2)*
x^3)*sqrt(d*x^2 + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^3,x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**3,x)
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-3), x)
```

3.247 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^4\,dx$

Optimal. Leaf size=147

$$\frac{192b^3\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^4+\frac{8b}{}$$

```
[Out] 384*b^4*x+48*b^2*x*(a+b*arccosh(d*x^2-1))^2+x*(a+b*arccosh(d*x^2-1))^4+192*
b^3*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)+8
*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^3/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5880, 8}

$$\frac{192b^3\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+48b^2x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+\frac{8b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(d\right.\right.}{x\sqrt{dx^2}\sqrt{dx^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^4,x]
```

```
[Out] 384*b^4*x + (192*b^3*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d
*x^2]*Sqrt[-2 + d*x^2]) + 48*b^2*x*(a + b*ArcCosh[-1 + d*x^2])^2 + (8*b*(2*
x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^3)/(x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]
) + x*(a + b*ArcCosh[-1 + d*x^2])^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^4\,dx &= \frac{8b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^4\\ &= \frac{192b^3\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+48b^2x\left(a+b\cosh^{-1}\left(-1+\right.\right.\\ &= 384b^4x+\frac{192b^3\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+48b^2x\left(a+b\cos\right.\end{aligned}$$

Mathematica [A] time = 0.24, size = 264, normalized size = 1.80

$$\frac{-8ab\left(a^2+24b^2\right)\sqrt{dx^2}\sqrt{dx^2-2}+6b^2\cosh^{-1}\left(dx^2-1\right)^2\left(a^2dx^2-4ab\sqrt{dx^2}\sqrt{dx^2-2}+8b^2dx^2\right)+dx^2\left(a^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^4,x]
```

```
[Out] ((a^4 + 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 + 24*b^2)*Sqrt[d*x^2]*Sqrt
[-2 + d*x^2] + 4*b*(a^3*d*x^2 + 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[d*x^2]*Sqrt[-
2 + d*x^2] - 48*b^3*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1 + d*x^2] + 6*b
^2*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-
1 + d*x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-
1 + d*x^2]^3 + b^4*d*x^2*ArcCosh[-1 + d*x^2]^4)/(d*x)
```

fricas [B] time = 0.61, size = 298, normalized size = 2.03

$$b^4dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}\,-1\right)^4+\left(a^4+48\,a^2b^2+384\,b^4\right)dx^2+4\left(ab^3dx^2-2\,\sqrt{d^2x^4-2\,dx^2}\,b^4\right)\log\left(dx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="fricas")
```

```
[Out] (b^4*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^4 + (a^4 + 48*a^2*b^2 +
 384*b^4)*d*x^2 + 4*(a*b^3*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)*b^4)*log(d*x^2
 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^3 - 6*(4*sqrt(d^2*x^4 - 2*d*x^2)*a*b^3 - (a
^2*b^2 + 8*b^4)*d*x^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^2 + 4*((a^3
*b + 24*a*b^3)*d*x^2 - 6*sqrt(d^2*x^4 - 2*d*x^2)*(a^2*b^2 + 8*b^4))*log(d*x
^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) - 8*sqrt(d^2*x^4 - 2*d*x^2)*(a^3*b + 24*a
*b^3))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(x)]index.cc ind
ex_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m operator +
Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^4,x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b^4x\log\left(dx^2+\sqrt{dx^2-2}\,\sqrt{d}\,x-1\right)^4+6\,a^2b^2x\,\mathrm{arcosh}\left(dx^2-1\right)^2+24\,a^2b^2d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^2-2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2\right.}{\sqrt{dx^2-2}\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^4,x, algorithm="maxima")
```

```
[Out] b^4*x*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^4 + 6*a^2*b^2*x*arccosh(d*
x^2 - 1)^2 + 24*a^2*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(d))*log(d*x^2 + sq
rt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 4*(x*arccosh(d*x^2
- 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a^3*b + a^4*x + int
egrate(4*((a*b^3*d^2 - 2*b^4*d^2)*x^4 + 2*a*b^3 - (3*a*b^3*d - 4*b^4*d)*x^2
 + ((a*b^3*d - 2*b^4*d)*sqrt(d)*x^3 - 2*(a*b^3 - b^4)*sqrt(d)*x)*sqrt(d*x^2
 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^3/(d^2*x^4 - 3*d*x^2 + (d
^(3/2)*x^3 - 2*sqrt(d)*x)*sqrt(d*x^2 - 2) + 2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^4,x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^4, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{acosh}\left(dx^2-1\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**4,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**4, x)
```

3.248 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3\,dx$

Optimal. Leaf size=110

$$24ab^2x+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^3+\frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2-2}}-48b^3x\sqrt{1-\frac{2}{dx^2}}+24b^3x\cosh^{-1}\left(\right.$$

```
[Out] 24*a*b^2*x+24*b^3*x*arccosh(d*x^2-1)+x*(a+b*arccosh(d*x^2-1))^3-48*b^3*x*(1
-2/d/x^2)^(1/2)+6*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^2/x/(d*x^2)^(1/2)
/(d*x^2-2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5880, 5901, 12, 191}

$$24ab^2x+\frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2}{x\sqrt{dx^2}\sqrt{dx^2-2}}+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^3-48b^3x\sqrt{1-\frac{2}{dx^2}}+24b^3x\cosh^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^3,x]
```

```
[Out] 24*a*b^2*x - 48*b^3*Sqrt[1 - 2/(d*x^2)]*x + 24*b^3*x*ArcCosh[-1 + d*x^2] +
(6*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^2)/(x*Sqrt[d*x^2]*Sqrt[-2
+ d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3 dx = \frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3$$

$$= 24ab^2x + \frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-\right.\right.$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(-1+dx^2\right) + \frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+\right.\right.}{x\sqrt{dx^2}\sqrt{-2+dx^2}}$$

$$= 24ab^2x + 24b^3x\cosh^{-1}\left(-1+dx^2\right) + \frac{6b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+\right.\right.}{x\sqrt{dx^2}\sqrt{-2+dx^2}}$$

$$= 24ab^2x - 48b^3\sqrt{1-\frac{2}{dx^2}}\,x + 24b^3x\cosh^{-1}\left(-1+dx^2\right) + \frac{6b\left(2x^2-dx^4\right)\left(a\right.}{x\sqrt{dx}}$$

Mathematica [A] time = 0.13, size = 171, normalized size = 1.55

$$\frac{adx^2\left(a^2+24b^2\right)-6b\left(a^2+8b^2\right)\sqrt{dx^2}\sqrt{dx^2-2}+3b\cosh^{-1}\left(dx^2-1\right)\left(a^2dx^2-4ab\sqrt{dx^2}\sqrt{dx^2-2}+8b^2dx^2\right.}{dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^3,x]
```

```
[Out] (a*(a^2 + 24*b^2)*d*x^2 - 6*b*(a^2 + 8*b^2)*Sqrt[d*x^2]*Sqrt[-2 + d*x^2] +
3*b*(a^2*d*x^2 + 8*b^2*d*x^2 - 4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[
-1 + d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1
 + d*x^2]^2 + b^3*d*x^2*ArcCosh[-1 + d*x^2]^3)/(d*x)
```

fricas [B] time = 0.68, size = 210, normalized size = 1.91

$$b^3dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^3+\left(a^3+24\,ab^2\right)dx^2+3\left(ab^2dx^2-2\,\sqrt{d^2x^4-2\,dx^2}\,b^3\right)\log\left(dx^2+\sqrt{d^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="fricas")
```

```
[Out] (b^3*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^3 + (a^3 + 24*a*b^2)*d*
x^2 + 3*(a*b^2*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)*b^3)*log(d*x^2 + sqrt(d^2*
x^4 - 2*d*x^2) - 1)^2 + 3*((a^2*b + 8*b^3)*d*x^2 - 4*sqrt(d^2*x^4 - 2*d*x^2
)*a*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) - 6*sqrt(d^2*x^4 - 2*d*x^
2)*(a^2*b + 8*b^3))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(x)]index.cc ind
```

```
ex_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m operator +
Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a + b\operatorname{arccosh}\left(d\,x^2 - 1\right)\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^3,x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$3\,ab^2x\operatorname{arccosh}\left(dx^2-1\right)^2+12\,ab^2d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^2-2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2-2}\,\sqrt{dx^2}\,-1\right)}{\sqrt{dx^2-2}\,d^2}\right)+3\left(x\operatorname{arccosh}\left(dx^2-1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*x*arccosh(d*x^2 - 1)^2 + 12*a*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(
d))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 3
*(x*arccosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a
^2*b + (x*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1)^3 - integrate(6*(d^2*x
^4 - 2*d*x^2 + (d^(3/2)*x^3 - sqrt(d)*x)*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(
d*x^2 - 2)*sqrt(d)*x - 1)^2/(d^2*x^4 - 3*d*x^2 + (d^(3/2)*x^3 - 2*sqrt(d)*x
)*sqrt(d*x^2 - 2) + 2), x))*b^3 + a^3*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)}^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^3,x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acosh}{\left(dx^{2} - 1 \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**3, x)
```

3.249 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2\,dx$

Optimal. Leaf size=73

$$x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+\frac{4b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+8b^2x$$

```
[Out] 8*b^2*x+x*(a+b*arccosh(d*x^2-1))^2+4*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1)
)/x/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5880, 8}

$$\frac{4b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}{x\sqrt{dx^2}\sqrt{dx^2-2}}+x\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+8b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^2,x]
```

```
[Out] 8*b^2*x + (4*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2]))/(x*Sqrt[d*x^2]*
Sqrt[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2\,dx=\frac{4b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2$$
$$=8b^2x+\frac{4b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}{x\sqrt{dx^2}\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+\right.\right.$$

Mathematica [A] time = 0.07, size = 104, normalized size = 1.42

$$x\left(a^2+8b^2\right)-\frac{4ab\sqrt{dx^2}\sqrt{dx^2-2}}{dx}+\frac{2b\cosh^{-1}\left(dx^2-1\right)\left(adx^2-2b\sqrt{dx^2}\sqrt{dx^2-2}\right)}{dx}+b^2x\cosh^{-1}\left(dx^2-1\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^2,x]
```

```
[Out] (a^2 + 8*b^2)*x - (4*a*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(d*x) + (2*b*(a*d*x^
2 - 2*b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])*ArcCosh[-1 + d*x^2])/(d*x) + b^2*x*Ar
cCosh[-1 + d*x^2]^2
```

fricas [A] time = 0.48, size = 131, normalized size = 1.79

$$\frac{b^2dx^2\log\left(dx^2+\sqrt{d^2x^4-2\,dx^2}-1\right)^2+\left(a^2+8\,b^2\right)dx^2-4\,\sqrt{d^2x^4-2\,dx^2}\,ab+2\left(abdx^2-2\,\sqrt{d^2x^4-2\,dx^2}\,b^2\right)\log}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1)^2 + (a^2 + 8*b^2)*d*x^2
 - 4*sqrt(d^2*x^4 - 2*d*x^2)*a*b + 2*(a*b*d*x^2 - 2*sqrt(d^2*x^4 - 2*d*x^2)
*b^2)*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(x)]index.cc ind
ex_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m operator +
Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^2,x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^2,x)
```

maxima [A] time = 0.59, size = 128, normalized size = 1.75

$$b^2x\,\mathrm{arcosh}\left(dx^2-1\right)^2+4\,b^2d\left(\frac{2\,x}{d}-\frac{\left(d^{\frac{3}{2}}x^2-2\,\sqrt{d}\right)\log\left(dx^2+\sqrt{dx^2-2}\,\sqrt{dx^2}-1\right)}{\sqrt{dx^2-2}\,d^2}\right)+2\left(x\,\mathrm{arcosh}\left(dx^2-1\right)-\frac{2\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccosh(d*x^2 - 1)^2 + 4*b^2*d*(2*x/d - (d^(3/2)*x^2 - 2*sqrt(d))*log
(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d*x^2) - 1)/(sqrt(d*x^2 - 2)*d^2)) + 2*(x*arc
cosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*a*b + a^
2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^2,x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acosh}\left(dx^2 - 1\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**2, x)
```

3.250 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)\,dx$

Optimal. Leaf size=33

$$ax-2bx\sqrt{1-\frac{2}{dx^2}}+bx\cosh^{-1}\left(dx^2-1\right)$$

```
[Out] a*x+b*x*arccosh(d*x^2-1)-2*b*x*(1-2/d/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5901, 12, 191}

$$ax-2bx\sqrt{1-\frac{2}{dx^2}}+bx\cosh^{-1}\left(dx^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCosh[-1 + d*x^2],x]
```

```
[Out] a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5901

```
Int[ArcCosh[u_], x_Symbol] :> Simp[x*ArcCosh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(Sqrt[-1 + u]*Sqrt[1 + u]), x], x] /; InverseFunctionFreeQ[u,
x] &&  !FunctionOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)\,dx &= ax+b\int \cosh^{-1}\left(-1+dx^2\right)\,dx\\ &= ax+bx\cosh^{-1}\left(-1+dx^2\right)-b\int \frac{2}{\sqrt{1-\frac{2}{dx^2}}}\,dx\\ &= ax+bx\cosh^{-1}\left(-1+dx^2\right)-(2b)\int \frac{1}{\sqrt{1-\frac{2}{dx^2}}}\,dx\\ &= ax-2b\sqrt{1-\frac{2}{dx^2}}\,x+bx\cosh^{-1}\left(-1+dx^2\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.00

$$ax-2bx\sqrt{1-\frac{2}{dx^2}}+bx\cosh^{-1}\left(dx^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCosh[-1 + d*x^2],x]
```

```
[Out] a*x - 2*b*Sqrt[1 - 2/(d*x^2)]*x + b*x*ArcCosh[-1 + d*x^2]
```

fricas [B] time = 1.22, size = 63, normalized size = 1.91

$$\frac{bdx^2\log\left(dx^2 + \sqrt{d^2x^4 - 2\,dx^2} - 1\right) + adx^2 - 2\sqrt{d^2x^4 - 2\,dx^2}\,b}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="fricas")
```

```
[Out] (b*d*x^2*log(d*x^2 + sqrt(d^2*x^4 - 2*d*x^2) - 1) + a*d*x^2 - 2*sqrt(d^2*x^
4 - 2*d*x^2)*b)/(d*x)
```

giac [B] time = 0.24, size = 67, normalized size = 2.03

$$\left(x\log\left(dx^2 + \sqrt{\left(dx^2 - 1\right)^2 - 1} - 1\right) + \frac{2\sqrt{2}\sqrt{-d}\,\mathrm{sgn}(x)}{d} - \frac{2\sqrt{d^2x^2 - 2\,d}}{d\mathrm{sgn}(x)}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="giac")
```

```
[Out] (x*log(d*x^2 + sqrt((d*x^2 - 1)^2 - 1) - 1) + 2*sqrt(2)*sqrt(-d)*sgn(x)/d -
 2*sqrt(d^2*x^2 - 2*d)/(d*sgn(x)))*b + a*x
```

maple [A] time = 0.00, size = 37, normalized size = 1.12

$$ax + b\left(x\,\mathrm{arccosh}\left(d\,x^2 - 1\right) - \frac{2x\sqrt{d\,x^2 - 2}}{\sqrt{d\,x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccosh(d*x^2-1),x)
```

```
[Out] a*x+b*(x*arccosh(d*x^2-1)-2/(d*x^2)^(1/2)*x*(d*x^2-2)^(1/2))
```

maxima [A] time = 0.65, size = 44, normalized size = 1.33

$$\left(x\,\mathrm{arcosh}\left(dx^2 - 1\right) - \frac{2\left(d^{\frac{3}{2}}x^2 - 2\sqrt{d}\right)}{\sqrt{dx^2 - 2}\,d}\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccosh(d*x^2-1),x, algorithm="maxima")
```

```
[Out] (x*arccosh(d*x^2 - 1) - 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(d*x^2 - 2)*d))*b
+ a*x
```

mupad [B] time = 1.53, size = 32, normalized size = 0.97

$$a\,x + b\,x\,\mathrm{acosh}\left(d\,x^2 - 1\right) - \frac{2\,b\,\mathrm{sign}(x)\,\sqrt{d\,x^2 - 2}}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*acosh(d*x^2 - 1),x)
```

```
[Out] a*x + b*x*acosh(d*x^2 - 1) - (2*b*sign(x)*(d*x^2 - 2)^(1/2))/d^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{acosh}\left(dx^2-1\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acosh(d*x**2-1),x)
```

```
[Out] Integral(a + b*acosh(d*x**2 - 1), x)
```

3.251 $\int \frac{1}{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx$

Optimal. Leaf size=98

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

```
[Out] 1/2*x*cosh(1/2*a/b)*Shi(1/2*(a+b*arccosh(d*x^2-1))/b)/b*2^(1/2)/(d*x^2)^(1/
2)-1/2*x*Chi(1/2*(a+b*arccosh(d*x^2-1))/b)*sinh(1/2*a/b)/b*2^(1/2)/(d*x^2)^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5882}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-1),x]
```

```
[Out] -((x*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(Sqrt[2
]*b*Sqrt[d*x^2])) + (x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2
])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

Rule 5882

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x*
Sinh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(Sqrt[2]*b*S
qrt[d*x^2]), x] + Simp[(x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*
x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx=-\frac{x\text{Chi}\left(\frac{a+b\cosh^{-1}\left(-1+dx^2\right)}{2b}\right)\sinh\left(\frac{a}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(-1+dx^2\right)}{2b}\right)}{\sqrt{2}\,b\sqrt{dx^2}}$$

Mathematica [A] time = 0.14, size = 86, normalized size = 0.88

$$-\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)\right)}{bdx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-1),x]
```

```
[Out] -((Cosh[ArcCosh[-1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2
*b)]*Sinh[a/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])
/(2*b)]))/(b*d*x))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b \operatorname{arcosh}\left(dx^2-1\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="fricas")
[Out] integral(1/(b*arccosh(d*x^2 - 1) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \operatorname{arcosh}\left(dx^2-1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="giac")
[Out] integrate(1/(b*arccosh(d*x^2 - 1) + a), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b \operatorname{arccosh}\left(d\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1)),x)
[Out] int(1/(a+b*arccosh(d*x^2-1)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \operatorname{arcosh}\left(dx^2-1\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1)),x, algorithm="maxima")
[Out] integrate(1/(b*arccosh(d*x^2 - 1) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a+b \operatorname{acosh}\left(d\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1)),x)
[Out] int(1/(a + b*acosh(d*x^2 - 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b \operatorname{acosh}\left(dx^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1)),x)
[Out] Integral(1/(a + b*acosh(d*x**2 - 1)), x)
```

3.252 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}\,dx$

Optimal. Leaf size=150

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2-2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}$$

```
[Out] 1/4*x*Chi(1/2*(a+b*arccosh(d*x^2-1))/b)*cosh(1/2*a/b)/b^2*2^(1/2)/(d*x^2)^(
1/2)-1/4*x*Shi(1/2*(a+b*arccosh(d*x^2-1))/b)*sinh(1/2*a/b)/b^2*2^(1/2)/(d*x
^2)^(1/2)-1/2*(d*x^2)^(1/2)*(d*x^2-2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2-1))
```

Rubi [A] time = 0.02, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5888}

$$\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-\frac{\sqrt{dx^2}\sqrt{dx^2-2}}{2bdx\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-2),x]
```

```
[Out] -(Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[-1 + d*x^2])) + (x*
Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b
^2*Sqrt[d*x^2]) - (x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])
/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2])
```

Rule 5888

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> -Simp[(Sq
rt[d*x^2]*Sqrt[-2 + d*x^2])/(2*b*d*x*(a + b*ArcCosh[-1 + d*x^2])), x] + (Si
mp[(x*Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(2*Sqr
t[2]*b^2*Sqrt[d*x^2]), x] - Simp[(x*Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCo
sh[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*x^2]), x]) /; FreeQ[{a, b, d}
, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}\,dx=-\frac{\sqrt{dx^2}\sqrt{-2+dx^2}}{2bdx\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(-1+dx^2\right)}{2b}\right)}{2\sqrt{2}\,b^2\sqrt{dx^2}}-$$

Mathematica [A] time = 0.75, size = 141, normalized size = 0.94

$$\frac{\frac{\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)\right)}{\sqrt{1-\frac{2}{dx^2}}}-\frac{b\sqrt{dx^2}\sqrt{dx^2-2}}{a+b\cosh^{-1}\left(dx^2-1\right)}}{2b^2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-2),x]
```

```
[Out] (-((b*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(a + b*ArcCosh[-1 + d*x^2])) + (Sinh[Ar
cCosh[-1 + d*x^2]/2]*(Cosh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2]
)/(2*b)] - Sinh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]))/
Sqrt[1 - 2/(d*x^2)])/(2*b^2*d*x)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2\operatorname{arcosh}\left(dx^2-1\right)^2+2\,ab\operatorname{arcosh}\left(dx^2-1\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(d*x^2 - 1)^2 + 2*a*b*arccosh(d*x^2 - 1) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\operatorname{arcosh}\left(dx^2-1\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-2), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{arccosh}\left(d\,x^2-1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^2,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^2x^4-3\,dx^2+\left(d^{\frac{3}{2}}x^3-2\,\sqrt{d}\,x\right)\sqrt{dx^2-2}+2}{2\left(abd^2x^3-2\,abdx+\left(b^2d^2x^3-2\,b^2dx+\left(b^2d^{\frac{3}{2}}x^2-b^2\sqrt{d}\right)\sqrt{dx^2-2}\right)\log\left(dx^2+\sqrt{dx^2-2}\,\sqrt{d}\,x-1\right)+\left(abd^{\frac{3}{2}}x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d^2*x^4 - 3*d*x^2 + (d^(3/2)*x^3 - 2*sqrt(d)*x)*sqrt(d*x^2 - 2) + 2)/
(a*b*d^2*x^3 - 2*a*b*d*x + (b^2*d^2*x^3 - 2*b^2*d*x + (b^2*d^(3/2)*x^2 - b^
2*sqrt(d))*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + (a
*b*d^(3/2)*x^2 - a*b*sqrt(d))*sqrt(d*x^2 - 2)) + integrate(1/2*(d^3*x^6 - 3
*d^2*x^4 + (d^2*x^4 - d*x^2 + 2)*(d*x^2 - 2) + (2*d^(5/2)*x^5 - 4*d^(3/2)*x
^3 + sqrt(d)*x)*sqrt(d*x^2 - 2) + 4)/(a*b*d^3*x^6 - 4*a*b*d^2*x^4 + 4*a*b*d
*x^2 + (a*b*d^2*x^4 - 2*a*b*d*x^2 + a*b)*(d*x^2 - 2) + (b^2*d^3*x^6 - 4*b^2
*d^2*x^4 + 4*b^2*d*x^2 + (b^2*d^2*x^4 - 2*b^2*d*x^2 + b^2)*(d*x^2 - 2) + 2*
(b^2*d^(5/2)*x^5 - 3*b^2*d^(3/2)*x^3 + 2*b^2*sqrt(d)*x)*sqrt(d*x^2 - 2))*lo
g(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + 2*(a*b*d^(5/2)*x^5 - 3*a*b*d^(3/
2)*x^3 + 2*a*b*sqrt(d)*x)*sqrt(d*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^2,x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(dx^2-1\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**2,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-2), x)
```

3.253 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}\,dx$

Optimal. Leaf size=181

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}+\frac{}{4bx\sqrt{dx^2}\,\cdots}$$

```
[Out] -1/8*x/b^2/(a+b*arccosh(d*x^2-1))+1/16*x*cosh(1/2*a/b)*Shi(1/2*(a+b*arccosh
(d*x^2-1))/b)/b^3*2^(1/2)/(d*x^2)^(1/2)-1/16*x*Chi(1/2*(a+b*arccosh(d*x^2-1
))/b)*sinh(1/2*a/b)/b^3*2^(1/2)/(d*x^2)^(1/2)+1/4*(-d*x^4+2*x^2)/b/x/(a+b*a
rccosh(d*x^2-1))^2/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5889, 5882}

$$-\frac{x\sinh\left(\frac{a}{2b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}+\frac{x\cosh\left(\frac{a}{2b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}\left(dx^2-1\right)}{2b}\right)}{8\sqrt{2}\,b^3\sqrt{dx^2}}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)}+\frac{}{4bx\sqrt{dx^2}\,\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-3),x]
```

```
[Out] (2*x^2 - d*x^4)/(4*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^2) - x/(8*b^2*(a + b*ArcCosh[-1 + d*x^2])) - (x*CoshIntegral[(a + b*Ar
cCosh[-1 + d*x^2])/(2*b)]*Sinh[a/(2*b)])/(8*Sqrt[2]*b^3*Sqrt[d*x^2]) + (x*C
osh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(8*Sqrt[2]*b^
3*Sqrt[d*x^2])
```

Rule 5882

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> -Simp[(x*
Sinh[a/(2*b)]*CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)])/(Sqrt[2]*b*S
qrt[d*x^2]), x] + Simp[(x*Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*
x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]), x] /; FreeQ[{a, b, d}, x]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^3}\,dx=\frac{2x^2-dx^4}{4bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(-1+\cdots\right.\right.}$$

$$=\frac{2x^2-dx^4}{4bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^2}-\frac{x}{8b^2\left(a+b\cosh^{-1}\left(-1+\cdots\right.\right.}$$

Mathematica [A] time = 0.69, size = 168, normalized size = 0.93

$$-\frac{\frac{2b^2\sqrt{dx^2}\sqrt{dx^2-2}}{d\left(a+b\cosh^{-1}(dx^2-1)\right)^2}+\frac{1}{2}x^2\sqrt{1-\frac{2}{dx^2}}\operatorname{csch}\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(dx^2-1)}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\operatorname{Sh}}{8b^3x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-3),x]
```

```
[Out] -1/8*((2*b^2*Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(d*(a + b*ArcCosh[-1 + d*x^2])^2
) + (b*x^2)/(a + b*ArcCosh[-1 + d*x^2]) + (Sqrt[1 - 2/(d*x^2)]*x^2*Csch[Arc
Cosh[-1 + d*x^2]/2]*(CoshIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]*Sinh[a
/(2*b)] - Cosh[a/(2*b)]*SinhIntegral[(a + b*ArcCosh[-1 + d*x^2])/(2*b)]))/2
)/(b^3*x)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{b^3\operatorname{arcosh}\left(dx^2-1\right)^3+3\,ab^2\operatorname{arcosh}\left(dx^2-1\right)^2+3\,a^2b\operatorname{arcosh}\left(dx^2-1\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccosh(d*x^2 - 1)^3 + 3*a*b^2*arccosh(d*x^2 - 1)^2 + 3*a^2
*b*arccosh(d*x^2 - 1) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\operatorname{arcosh}\left(dx^2-1\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a+b\operatorname{arccosh}\left(d\,x^2-1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^3,x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((a*d^5 + 2*b*d^5)*sqrt(d)*x^10 - 2*(3*a*d^4 + 7*b*d^4)*sqrt(d)*x^8 +
(11*a*d^3 + 36*b*d^3)*sqrt(d)*x^6 - 2*(a*d^2 + 20*b*d^2)*sqrt(d)*x^4 - 4*(3
*a*d - 4*b*d)*sqrt(d)*x^2 + ((a*d^4 + 2*b*d^4)*x^7 - (3*a*d^3 + 8*b*d^3)*x^
```

```
5 + 2*(2*a*d^2 + 5*b*d^2)*x^3 - 4*(a*d + b*d)*x)*(d*x^2 - 2)^(3/2) + (3*(a*
d^4 + 2*b*d^4)*sqrt(d)*x^8 - 6*(2*a*d^3 + 5*b*d^3)*sqrt(d)*x^6 + 2*(8*a*d^2
 + 25*b*d^2)*sqrt(d)*x^4 - 10*(a*d + 3*b*d)*sqrt(d)*x^2 + 4*(a + b)*sqrt(d)
)*(d*x^2 - 2) + (b*d^(11/2)*x^10 - 6*b*d^(9/2)*x^8 + 11*b*d^(7/2)*x^6 - 2*b
*d^(5/2)*x^4 - 12*b*d^(3/2)*x^2 + (b*d^4*x^7 - 3*b*d^3*x^5 + 4*b*d^2*x^3 -
4*b*d*x)*(d*x^2 - 2)^(3/2) + (3*b*d^(9/2)*x^8 - 12*b*d^(7/2)*x^6 + 16*b*d^(
5/2)*x^4 - 10*b*d^(3/2)*x^2 + 4*b*sqrt(d))*(d*x^2 - 2) + (3*b*d^5*x^9 - 15*
b*d^4*x^7 + 23*b*d^3*x^5 - 7*b*d^2*x^3 - 6*b*d*x)*sqrt(d*x^2 - 2) + 8*b*sqr
t(d))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + (3*(a*d^5 + 2*b*d^5)*x^9
 - 3*(5*a*d^4 + 12*b*d^4)*x^7 + (23*a*d^3 + 76*b*d^3)*x^5 - (7*a*d^2 + 64*b
*d^2)*x^3 - 2*(3*a*d - 8*b*d)*x)*sqrt(d*x^2 - 2) + 8*a*sqrt(d))/(a^2*b^2*d^
(11/2)*x^9 - 6*a^2*b^2*d^(9/2)*x^7 + 12*a^2*b^2*d^(7/2)*x^5 - 8*a^2*b^2*d^(
5/2)*x^3 + (b^4*d^(11/2)*x^9 - 6*b^4*d^(9/2)*x^7 + 12*b^4*d^(7/2)*x^5 - 8*b
^4*d^(5/2)*x^3 + (b^4*d^4*x^6 - 3*b^4*d^3*x^4 + 3*b^4*d^2*x^2 - b^4*d)*(d*x
^2 - 2)^(3/2) + 3*(b^4*d^(9/2)*x^7 - 4*b^4*d^(7/2)*x^5 + 5*b^4*d^(5/2)*x^3
- 2*b^4*d^(3/2)*x)*(d*x^2 - 2) + 3*(b^4*d^5*x^8 - 5*b^4*d^4*x^6 + 8*b^4*d^3
*x^4 - 4*b^4*d^2*x^2)*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*
x - 1)^2 + (a^2*b^2*d^4*x^6 - 3*a^2*b^2*d^3*x^4 + 3*a^2*b^2*d^2*x^2 - a^2*b
^2*d)*(d*x^2 - 2)^(3/2) + 3*(a^2*b^2*d^(9/2)*x^7 - 4*a^2*b^2*d^(7/2)*x^5 +
5*a^2*b^2*d^(5/2)*x^3 - 2*a^2*b^2*d^(3/2)*x)*(d*x^2 - 2) + 2*(a*b^3*d^(11/2
)*x^9 - 6*a*b^3*d^(9/2)*x^7 + 12*a*b^3*d^(7/2)*x^5 - 8*a*b^3*d^(5/2)*x^3 +
(a*b^3*d^4*x^6 - 3*a*b^3*d^3*x^4 + 3*a*b^3*d^2*x^2 - a*b^3*d)*(d*x^2 - 2)^(
3/2) + 3*(a*b^3*d^(9/2)*x^7 - 4*a*b^3*d^(7/2)*x^5 + 5*a*b^3*d^(5/2)*x^3 - 2
*a*b^3*d^(3/2)*x)*(d*x^2 - 2) + 3*(a*b^3*d^5*x^8 - 5*a*b^3*d^4*x^6 + 8*a*b^
3*d^3*x^4 - 4*a*b^3*d^2*x^2)*sqrt(d*x^2 - 2))*log(d*x^2 + sqrt(d*x^2 - 2)*s
qrt(d)*x - 1) + 3*(a^2*b^2*d^5*x^8 - 5*a^2*b^2*d^4*x^6 + 8*a^2*b^2*d^3*x^4
- 4*a^2*b^2*d^2*x^2)*sqrt(d*x^2 - 2)) + integrate(1/8*(d^6*x^12 - 8*d^5*x^1
0 + 27*d^4*x^8 - 56*d^3*x^6 + 88*d^2*x^4 + (d^4*x^8 - 4*d^3*x^6 + 3*d^2*x^4
 + 8*d*x^2 + 4)*(d*x^2 - 2)^2 - 96*d*x^2 + 2*(2*d^(9/2)*x^9 - 10*d^(7/2)*x^
7 + 15*d^(5/2)*x^5 + d^(3/2)*x^3 - 11*sqrt(d)*x)*(d*x^2 - 2)^(3/2) + 3*(2*d
^5*x^10 - 12*d^4*x^8 + 26*d^3*x^6 - 24*d^2*x^4 + 3*d*x^2 + 10)*(d*x^2 - 2)
+ 2*(2*d^(11/2)*x^11 - 14*d^(9/2)*x^9 + 39*d^(7/2)*x^7 - 61*d^(5/2)*x^5 + 6
1*d^(3/2)*x^3 - 30*sqrt(d)*x)*sqrt(d*x^2 - 2) + 48)/(a*b^2*d^6*x^12 - 8*a*b
^2*d^5*x^10 + 24*a*b^2*d^4*x^8 - 32*a*b^2*d^3*x^6 + 16*a*b^2*d^2*x^4 + (a*b
^2*d^4*x^8 - 4*a*b^2*d^3*x^6 + 6*a*b^2*d^2*x^4 - 4*a*b^2*d*x^2 + a*b^2)*(d*
x^2 - 2)^2 + 4*(a*b^2*d^(9/2)*x^9 - 5*a*b^2*d^(7/2)*x^7 + 9*a*b^2*d^(5/2)*x
^5 - 7*a*b^2*d^(3/2)*x^3 + 2*a*b^2*sqrt(d)*x)*(d*x^2 - 2)^(3/2) + 6*(a*b^2*
d^5*x^10 - 6*a*b^2*d^4*x^8 + 13*a*b^2*d^3*x^6 - 12*a*b^2*d^2*x^4 + 4*a*b^2*
d*x^2)*(d*x^2 - 2) + (b^3*d^6*x^12 - 8*b^3*d^5*x^10 + 24*b^3*d^4*x^8 - 32*b
^3*d^3*x^6 + 16*b^3*d^2*x^4 + (b^3*d^4*x^8 - 4*b^3*d^3*x^6 + 6*b^3*d^2*x^4
- 4*b^3*d*x^2 + b^3)*(d*x^2 - 2)^2 + 4*(b^3*d^(9/2)*x^9 - 5*b^3*d^(7/2)*x^7
 + 9*b^3*d^(5/2)*x^5 - 7*b^3*d^(3/2)*x^3 + 2*b^3*sqrt(d)*x)*(d*x^2 - 2)^(3/
2) + 6*(b^3*d^5*x^10 - 6*b^3*d^4*x^8 + 13*b^3*d^3*x^6 - 12*b^3*d^2*x^4 + 4*
b^3*d*x^2)*(d*x^2 - 2) + 4*(b^3*d^(11/2)*x^11 - 7*b^3*d^(9/2)*x^9 + 18*b^3*
d^(7/2)*x^7 - 20*b^3*d^(5/2)*x^5 + 8*b^3*d^(3/2)*x^3)*sqrt(d*x^2 - 2))*log(
d*x^2 + sqrt(d*x^2 - 2)*sqrt(d)*x - 1) + 4*(a*b^2*d^(11/2)*x^11 - 7*a*b^2*d
^(9/2)*x^9 + 18*a*b^2*d^(7/2)*x^7 - 20*a*b^2*d^(5/2)*x^5 + 8*a*b^2*d^(3/2)*
x^3)*sqrt(d*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^3,x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**3,x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-3), x)
```

3.254 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}\,dx$

Optimal. Leaf size=280

$$\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\left(\cosh\left(\frac{a}{2b}\right)-\sin\ldots\right.}{}$$

```
[Out] x*(a+b*arccosh(d*x^2+1))^(5/2)-15/2*b^(5/2)*erfi(1/2*(a+b*arccosh(d*x^2+1))
^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^
2+1))*2^(1/2)*Pi^(1/2)/d/x+15/2*b^(5/2)*erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2
)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))
*2^(1/2)*Pi^(1/2)/d/x-5*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^(3/2)/x/(d*x
^2)^(1/2)/(d*x^2+2)^(1/2)+30*b^2*sinh(1/2*arccosh(d*x^2+1))^2*(a+b*arccosh(
d*x^2+1))^(1/2)/d/x
```

Rubi [A] time = 0.11, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5880, 5878}

$$\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\left(\cosh\left(\frac{a}{2b}\right)-\sin\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(5/2),x]
```

```
[Out] (-5*b*(2*x^2 + d*x^4)*(a + b*ArcCosh[1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqrt
[2 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(5/2) - (15*b^(5/2)*Sqrt[Pi/2]*
Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sin
h[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (15*b^(5/2)*Sqrt[Pi/2]*Erf[
Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(
2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (30*b^2*Sqrt[a + b*ArcCosh[1 + d
*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```

Rule 5878

```
Int[Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sqr
t[a + b*ArcCosh[1 + d*x^2]]*Sinh[(1/2)*ArcCosh[1 + d*x^2]]^2)/(d*x), x] + (
Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh
[1 + d*x^2]]*Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]])/(d*x), x] -
 Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[(1/2)*ArcCos
h[1 + d*x^2]]*Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]])/(d*x), x]
) /; FreeQ[{a, b, d}, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}\,dx = -\frac{5b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^5$$

$$= -\frac{5b\left(2x^2+dx^4\right)\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{2+dx^2}} + x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^5$$

Mathematica [A] time = 3.61, size = 311, normalized size = 1.11

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(4\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}\left(\left(a^2+15b^2\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)-5ab\cosh\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(5/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCo
sh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) + 15*b^(5
/2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[
a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*(-5*a*b*Cosh[A
rcCosh[1 + d*x^2]/2] + (a^2 + 15*b^2)*Sinh[ArcCosh[1 + d*x^2]/2] + b^2*ArcC
osh[1 + d*x^2]^2*Sinh[ArcCosh[1 + d*x^2]/2] - b*ArcCosh[1 + d*x^2]*(5*b*Cos
h[ArcCosh[1 + d*x^2]/2] - 2*a*Sinh[ArcCosh[1 + d*x^2]/2]))))/(2*Sqrt[d*x^2]
*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]index.cc index_m i_lex_i
s_greater Error: Bad Argument Valueindex.cc index_m operator + Error: Bad A
rgument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(5/2),x)
[Out] int((a+b*arccosh(d*x^2+1))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{arcosh}\left(dx^2+1\right)+a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b \operatorname{acosh}\left(d x^2+1\right)\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^(5/2),x)
[Out] int((a + b*acosh(d*x^2 + 1))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b \operatorname{acosh}\left(dx^2+1\right)\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(5/2),x)
[Out] Integral((a + b*acosh(d*x**2 + 1))**(5/2), x)
```

3.255 $\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2} dx$

Optimal. Leaf size=238

$$\frac{3\sqrt{\frac{\pi}{2}}\, b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}\, b^{3/2}\left(\cosh\left(\frac{a}{2b}\right)-\text{si}\right.}{}$$

```
[Out] x*(a+b*arccosh(d*x^2+1))^(3/2)+3/2*b^(3/2)*erfi(1/2*(a+b*arccosh(d*x^2+1))^
(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2
+1))*2^(1/2)*Pi^(1/2)/d/x+3/2*b^(3/2)*erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*
2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2
^(1/2)*Pi^(1/2)/d/x-3*b*(d*x^4+2*x^2)*(a+b*arccosh(d*x^2+1))^(1/2)/x/(d*x^2
)^(1/2)/(d*x^2+2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5880, 5883}

$$\frac{3\sqrt{\frac{\pi}{2}}\, b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}\, b^{3/2}\left(\cosh\left(\frac{a}{2b}\right)-\text{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(3/2),x]
```

```
[Out] (-3*b*(2*x^2 + d*x^4)*Sqrt[a + b*ArcCosh[1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[2
 + d*x^2]) + x*(a + b*ArcCosh[1 + d*x^2])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*Erf
i[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a
/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (3*b^(3/2)*Sqrt[Pi/2]*Erf[Sqrt
[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)
])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5883

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] + Simp[(Sqrt[Pi/2
]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}\,dx = -\frac{3b\left(2x^2+dx^4\right)\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}+($$

$$= -\frac{3b\left(2x^2+dx^4\right)\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{x\sqrt{dx^2}\,\sqrt{2+dx^2}}+x\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}+$$

Mathematica [A] time = 0.72, size = 254, normalized size = 1.07

$$x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(3\sqrt{2\pi}\,b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)+3\sqrt{2\pi}\,b^{3/2}\left(\cosh\left(\frac{a}{2b}\right)-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(3/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh
[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) + 3*b^(3/2)
*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(
2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*(-3*b*Cosh[ArcCos
h[1 + d*x^2]/2] + a*Sinh[ArcCosh[1 + d*x^2]/2] + b*ArcCosh[1 + d*x^2]*Sinh[
ArcCosh[1 + d*x^2]/2])))/(2*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 +
d*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]index.cc index_m i_lex_i
s_greater Error: Bad Argument Valueindex.cc index_m operator + Error: Bad A
rgument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \operatorname{arcosh}\left(dx^2+1\right)+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b \operatorname{acosh}\left(d x^2+1\right)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^(3/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b \operatorname{acosh}\left(dx^2+1\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(3/2), x)
```

3.256 $\int \sqrt{a+b\cosh^{-1}(1+dx^2)}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right.}{}$$

```
[Out] -1/2*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-
sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/x+1/2*
erf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1
/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/x+2*sinh(1/2
*arccosh(d*x^2+1))^2*(a+b*arccosh(d*x^2+1))^(1/2)/d/x
```

Rubi [A] time = 0.03, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5878}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] -((Sqrt[b]*Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])
]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x)) + (Sqr
t[b]*Sqrt[Pi/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(d*x) + (2*Sqrt[a +
b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x^2]/2]^2)/(d*x)
```

Rule 5878

```
Int[Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sqr
t[a + b*ArcCosh[1 + d*x^2]]*Sinh[(1/2)*ArcCosh[1 + d*x^2]]^2)/(d*x), x] + (
Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[(1/2)*ArcCosh
[1 + d*x^2]]*Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]])/(d*x), x] -
 Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[(1/2)*ArcCos
h[1 + d*x^2]]*Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[1 + d*x^2]]])/(d*x), x]
) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}\,dx=-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+d\right.\right.}{dx}$$

Mathematica [A] time = 0.31, size = 210, normalized size = 1.02

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)-\text{co}\right.\right.}{2\sqrt{dx^2}\sqrt{\frac{dx^2}{dx^2+2}}\sqrt{dx^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[b]*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[1
 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) + Sqrt[b]*Sq
rt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b
)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[1 + d*x^2]]*Sinh[ArcCosh[1 + d*x
^2]/2]))/(2*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m
 operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,\mathrm{arccosh}\left(d\,x^{2}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2+1))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\mathrm{arcosh}\left(dx^{2}+1\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x^2 + 1) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{acosh}\left(d\,x^{2}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 + 1))^(1/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acosh}\left(dx^2 + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2+1))**(1/2),x)
[Out] Integral(sqrt(a + b*acosh(d*x**2 + 1)), x)
```

3.257 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(1+dx^2)}}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}\,dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)s}{}$$

```
[Out] 1/2*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-s
inh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/d/x/b^(1/2)+1/2*e
rf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/
2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/d/x/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5883}

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}\,dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)s}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/
(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x) + (Sqrt[P
i/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] +
 Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[b]*d*x)
```

Rule 5883

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] + Simp[(Sqrt[Pi/2
]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}\,dx=\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(1+dx^2)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(1+dx\right.\right.}{\sqrt{b}\,dx}$$

Mathematica [A] time = 0.31, size = 166, normalized size = 1.01

$$\frac{\sqrt{\frac{\pi}{2}}\,x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2+1)}}{\sqrt{2}\sqrt{b}}\right)+\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\operatorname{er}\right.}{\sqrt{b}\sqrt{dx^2}\sqrt{\frac{dx^2}{dx^2+2}}\sqrt{dx^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*x*(Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] - Sinh[a/(2*b)]) + Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))*Sinh[ArcCosh[1 + d*x^2]/2])/(Sqrt[
b]*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m
 operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{arccosh}\left(d\,x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\mathrm{arcosh}\left(dx^{2}+1\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x^2 + 1) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(d\,x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \operatorname{acosh}\left(dx^2 + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(d*x**2 + 1)), x)
```

3.258 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{si}}{}$$

```
[Out] 1/2*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-s
inh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/x-1/2*e
rf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/
2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/x-(d*x^2)^(1/
2)*(d*x^2+2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2+1))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5885}

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-3/2),x]
```

```
[Out] -((Sqrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]])) + (
Sqrt[Pi/2]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(
2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x) - (Sqrt[Pi
/2]*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] +
Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*d*x)
```

Rule 5885

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[(S
qrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]]), x] + (-
Simp[(Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]
*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x] + Simp[(S
qrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x]) /; FreeQ[{a, b
, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}}\,dx=-\frac{\sqrt{dx^2}\sqrt{2+dx^2}}{bdx\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\text{si}\right.}{b^{3/2}dx}$$

Mathematica [A] time = 1.11, size = 242, normalized size = 1.14

$$-\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\sqrt{2\pi}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{}\right.}{2b^{3/2}\sqrt{dx^2}\sqrt{\frac{dx^2}{dx^2+2}}\sqrt{dx^2+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-3/2),x]
```

```
[Out] -1/2*(x*(4*Sqrt[b]*Cosh[ArcCosh[1 + d*x^2]/2] + Sqrt[2*Pi]*Sqrt[a + b*ArcCo
sh[1 + d*x^2]]*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cos
h[a/(2*b)] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[1 + d*x^2]]*Erf
[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/
(2*b)]))*Sinh[ArcCosh[1 + d*x^2]/2])/(b^(3/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 +
 d*x^2)]*Sqrt[2 + d*x^2]*Sqrt[a + b*ArcCosh[1 + d*x^2]])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m i_lex_is_greater Error: Bad Argument ValueEvaluatio
n time: 0.6index.cc index_m operator + Error: Bad Argument Value
```

maple [**F**] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b \,\mathrm{arccosh}\left(d \,x^{2}+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(b \operatorname{arcosh}\left(dx^{2} + 1\right) + a\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^(3/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-3/2), x)
```

3.259 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=252

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{s}}{}$$

```
[Out] 1/6*erfi(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-s
inh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/x+1/6*e
rf(1/2*(a+b*arccosh(d*x^2+1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/
2*a/b))*sinh(1/2*arccosh(d*x^2+1))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/x+1/3*(-d*x^4
-2*x^2)/b/x/(a+b*arccosh(d*x^2+1))^(3/2)/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-1/3*
x/b^2/(a+b*arccosh(d*x^2+1))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5889, 5883}

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-5/2),x]
```

```
[Out] -(2*x^2 + d*x^4)/(3*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + (Sqrt[Pi/2]*Erfi[Sq
rt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*
b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(3*b^(5/2)*d*x) + (Sqrt[Pi/2]*Erf[Sqrt[a +
 b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*S
inh[ArcCosh[1 + d*x^2]/2])/(3*b^(5/2)*d*x)
```

Rule 5883

```
Int[1/Sqrt[(a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sqr
t[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] + Simp[(Sqrt[Pi/2
]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erf[Sqrt[a + b
*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d}, x]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}}\,dx = -\frac{2x^2+dx^4}{3bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(1+d\right.}}$$

$$= -\frac{2x^2+dx^4}{3bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(1+d\right.}}$$

Mathematica [A] time = 1.04, size = 273, normalized size = 1.08

$$\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\sqrt{2\pi}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^{3/2}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-5/2),x]
```

```
[Out] (x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(3/2)*
Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sin
h[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(3/2)*Erf[Sqrt[a + b*Ar
cCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sq
rt[b]*(-(b*Cosh[ArcCosh[1 + d*x^2]/2]) - (a + b*ArcCosh[1 + d*x^2])*Sinh[Ar
cCosh[1 + d*x^2]/2])))/(6*b^(5/2)*Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqr
t[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^2])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m i_lex_is_greater Error: Bad Argument ValueEvaluatio
n time: 0.78index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arcosh}\left(dx^2+1\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(d\,x^2+1\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^(5/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\operatorname{acosh}\left(dx^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-5/2), x)
```

3.260 $\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\sin}{}$$

```
[Out] -1/15*x/b^2/(a+b*arccosh(d*x^2+1))^(3/2)+1/30*erfi(1/2*(a+b*arccosh(d*x^2+1
))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*sinh(1/2*arccosh(d*
x^2+1))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/x-1/30*erf(1/2*(a+b*arccosh(d*x^2+1))^(1
/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*sinh(1/2*arccosh(d*x^2+1
))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/x+1/5*(-d*x^4-2*x^2)/b/x/(a+b*arccosh(d*x^2+1
))^(5/2)/(d*x^2)^(1/2)/(d*x^2+2)^(1/2)-1/15*(d*x^2)^(1/2)*(d*x^2+2)^(1/2)/b
^3/d/x/(a+b*arccosh(d*x^2+1))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5889, 5885}

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2+1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[1 + d*x^2])^(-7/2),x]
```

```
[Out] -(2*x^2 + d*x^4)/(5*b*x*Sqrt[d*x^2]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*x^
2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[1 + d*x^2])^(3/2)) - (Sqrt[d*x^2]*Sqr
t[2 + d*x^2])/(15*b^3*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]]) + (Sqrt[Pi/2]*Erf
i[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a
/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2])/(15*b^(7/2)*d*x) - (Sqrt[Pi/2]*Erf[Sqr
t[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b
)])*Sinh[ArcCosh[1 + d*x^2]/2])/(15*b^(7/2)*d*x)
```

Rule 5885

```
Int[((a_.) + ArcCosh[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[(S
qrt[d*x^2]*Sqrt[2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[1 + d*x^2]]), x] + (-
Simp[(Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]
*Erf[Sqrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x] + Simp[(S
qrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Sinh[ArcCosh[1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x]) /; FreeQ[{a, b
, d}, x]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{7/2}}\,dx = -\frac{2x^2+dx^4}{5bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+b\cosh^{-1}\left(1\right.\right.}$$

$$= -\frac{2x^2+dx^4}{5bx\sqrt{dx^2}\sqrt{2+dx^2}\left(a+b\cosh^{-1}\left(1+dx^2\right)\right)^{5/2}} - \frac{x}{15b^2\left(a+b\cosh^{-1}\left(1\right.\right.}$$

Mathematica [A] time = 1.34, size = 291, normalized size = 0.97

$$-\frac{x\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(4\sqrt{b}\left(\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2+1\right)\right)\left(\left(a+b\cosh^{-1}\left(dx^2+1\right)\right)^2+3b^2\right)+b\sinh\left(\frac{1}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[1 + d*x^2])^(-7/2),x]
```

```
[Out] -1/30*(x*Sinh[ArcCosh[1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^
(5/2)*Erfi[Sqrt[a + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)
] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[1 + d*x^2])^(5/2)*Erf[Sqrt[a
 + b*ArcCosh[1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])
 + 4*Sqrt[b]*((3*b^2 + (a + b*ArcCosh[1 + d*x^2])^2)*Cosh[ArcCosh[1 + d*x^2
]/2] + b*(a + b*ArcCosh[1 + d*x^2])*Sinh[ArcCosh[1 + d*x^2]/2])))/(b^(7/2)*
Sqrt[d*x^2]*Sqrt[(d*x^2)/(2 + d*x^2)]*Sqrt[2 + d*x^2]*(a + b*ArcCosh[1 + d*
x^2])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m i_lex_is_greater Error: Bad Argument ValueEvaluatio
n time: 0.88index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2+1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2+1))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{arcosh}\left(dx^2+1\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2+1))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 + 1) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^2+1\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 + 1))^(7/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 + 1))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(dx^2+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2+1))**(7/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 + 1))**(-7/2), x)
```

3.261 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2} dx$

Optimal. Leaf size=281

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\cosh\left(\frac{1}{2}\cos\ldots\right.}{}$$

```
[Out] x*(a+b*arccosh(d*x^2-1))^(5/2)-15/2*b^(5/2)*cosh(1/2*arccosh(d*x^2-1))*erfi
(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*
a/b))*2^(1/2)*Pi^(1/2)/d/x-15/2*b^(5/2)*cosh(1/2*arccosh(d*x^2-1))*erf(1/2*
(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))
*2^(1/2)*Pi^(1/2)/d/x+5*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^(3/2)/x/(d*
x^2)^(1/2)/(d*x^2-2)^(1/2)+30*b^2*cosh(1/2*arccosh(d*x^2-1))^2*(a+b*arccosh
(d*x^2-1))^(1/2)/d/x
```

Rubi [A] time = 0.06, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5880, 5879}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{15\sqrt{\frac{\pi}{2}}\,b^{5/2}\cosh\left(\frac{1}{2}\cos\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(5/2),x]
```

```
[Out] (5*b*(2*x^2 - d*x^4)*(a + b*ArcCosh[-1 + d*x^2])^(3/2))/(x*Sqrt[d*x^2]*Sqrt
[-2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(5/2) + (30*b^2*Sqrt[a + b*Ar
cCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (15*b^(5/2)*Sqrt[
Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (15*b^(5/2)*Sqrt[Pi
/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2
]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rule 5879

```
Int[Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sq
rt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[(1/2)*ArcCosh[-1 + d*x^2]]^2)/(d*x), x]
+ (-Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[(1/2)*Arc
Cosh[-1 + d*x^2]]*Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]])/(d*x)
, x] - Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[(1/2)*
ArcCosh[-1 + d*x^2]]*Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]])/(
d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}\,dx = \frac{5b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5}$$

$$= \frac{5b\left(2x^2-dx^4\right)\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}{x\sqrt{dx^2}\sqrt{-2+dx^2}} + x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5}$$

Mathematica [A] time = 1.68, size = 277, normalized size = 0.99

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(4\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}\left(\left(a^2+15b^2\right)\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)+b\cosh^{-1}\left(dx^2-\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(5/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(-15*b^(5/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCos
h[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) - 15*b^(5
/2)*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh
[a/(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*((a^2 + 15*b
^2)*Cosh[ArcCosh[-1 + d*x^2]/2] + b^2*ArcCosh[-1 + d*x^2]^2*Cosh[ArcCosh[-1
 + d*x^2]/2] - 5*a*b*Sinh[ArcCosh[-1 + d*x^2]/2] + b*ArcCosh[-1 + d*x^2]*(2
*a*Cosh[ArcCosh[-1 + d*x^2]/2] - 5*b*Sinh[ArcCosh[-1 + d*x^2]/2]))))/(2*d*x
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]index.cc index_m i_lex_i
s_greater Error: Bad Argument ValuePolynomial exponent overflow. Error: Bad
 Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(5/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{arcosh}\left(dx^2-1\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^(5/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(5/2), x)
```

3.262 $\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2} dx$

Optimal. Leaf size=239

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(\right.\right.}{}$$

```
[Out] x*(a+b*arccosh(d*x^2-1))^(3/2)+3/2*b^(3/2)*cosh(1/2*arccosh(d*x^2-1))*erfi(
1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a
/b))*2^(1/2)*Pi^(1/2)/d/x-3/2*b^(3/2)*cosh(1/2*arccosh(d*x^2-1))*erf(1/2*(a
+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*2
^(1/2)*Pi^(1/2)/d/x+3*b*(-d*x^4+2*x^2)*(a+b*arccosh(d*x^2-1))^(1/2)/x/(d*x^
2)^(1/2)/(d*x^2-2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5880, 5884}

$$-\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\sqrt{b}}\right)}{dx}+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cosh\left(\frac{1}{2}\cosh^{-1}\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(3/2),x]
```

```
[Out] (3*b*(2*x^2 - d*x^4)*Sqrt[a + b*ArcCosh[-1 + d*x^2]])/(x*Sqrt[d*x^2]*Sqrt[-
2 + d*x^2]) + x*(a + b*ArcCosh[-1 + d*x^2])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*C
osh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sq
rt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (3*b^(3/2)*Sqrt[Pi/2]*Cosh
[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rule 5880

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*
(a + b*ArcCosh[c + d*x^2])^n, x] + (Dist[4*b^2*n*(n - 1), Int[(a + b*ArcCos
h[c + d*x^2])^(n - 2), x], x] - Simp[(2*b*n*(2*c*d*x^2 + d^2*x^4)*(a + b*Ar
cCosh[c + d*x^2])^(n - 1))/(d*x*Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]),
x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && GtQ[n, 1]
```

Rule 5884

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sq
rt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] - Simp[(Sqrt[P
i/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d},
 x]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}\,dx=\frac{3b\left(2x^2-dx^4\right)\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)$$

$$=\frac{3b\left(2x^2-dx^4\right)\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{x\sqrt{dx^2}\,\sqrt{-2+dx^2}}+x\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)$$

Mathematica [A] time = 0.65, size = 221, normalized size = 0.92

$$\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(-3\sqrt{2\pi}\,b^{3/2}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(dx^2-1)}}{\sqrt{2}\,\sqrt{b}}\right)+3\sqrt{2\pi}\,b^{3/2}\left(\cosh\left(\frac{a}{2b}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(3/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[
-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]) - 3*b^(3/2)
*Sqrt[2*Pi]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/
(2*b)] + Sinh[a/(2*b)]) + 4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*(a*Cosh[ArcCosh
[-1 + d*x^2]/2] + b*ArcCosh[-1 + d*x^2]*Cosh[ArcCosh[-1 + d*x^2]/2] - 3*b*S
inh[ArcCosh[-1 + d*x^2]/2])))/(2*d*x)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]index.cc index_m i_lex_i
s_greater Error: Bad Argument Valueindex.cc index_m operator + Error: Bad A
rgument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(3/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\operatorname{arcosh}\left(dx^2-1\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(d\,x^2-1\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^(3/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{acosh}\left(dx^2-1\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(3/2), x)
```

3.263 $\int \sqrt{a+b\cosh^{-1}(-1+dx^2)}\,dx$

Optimal. Leaf size=206

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(\right.\right.}{}$$

```
[Out] -1/2*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/
2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/x-1/2*
cosh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(
1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*b^(1/2)*2^(1/2)*Pi^(1/2)/d/x+2*cosh(1/2
*arccosh(d*x^2-1))^2*(a+b*arccosh(d*x^2-1))^(1/2)/d/x
```

Rubi [A] time = 0.03, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5879}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{dx}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cosh\left(\frac{1}{2}\cosh^{-1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] (2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCosh[-1 + d*x^2]/2]^2)/(d*x) - (
Sqrt[b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 +
 d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(d*x) - (Sqrt[
b]*Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(d*x)
```

Rule 5879

```
Int[Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(2*Sq
rt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[(1/2)*ArcCosh[-1 + d*x^2]]^2)/(d*x), x]
+ (-Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[(1/2)*Arc
Cosh[-1 + d*x^2]]*Erf[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]])/(d*x)
, x] - Simp[(Sqrt[b]*Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[(1/2)*
ArcCosh[-1 + d*x^2]]*Erfi[(1/Sqrt[2*b])*Sqrt[a + b*ArcCosh[-1 + d*x^2]]])/(
d*x), x]) /; FreeQ[{a, b, d}, x]
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\,dx = \frac{2\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}\cosh^2\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)}{dx}-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cosh\left(\right.}{}$$

Mathematica [A] time = 0.31, size = 178, normalized size = 0.86

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(-\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{2\pi}\sqrt{b}\left(\sinh\left(\frac{a}{2b}\right)-\right.\right.}{2dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(4*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Cosh[ArcCos
h[-1 + d*x^2]/2] + Sqrt[b]*Sqrt[2*Pi]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/
(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) - Sqrt[b]*Sqrt[2*Pi]*Er
f[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[
a/(2*b)])))/(2*d*x)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m
 operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \,\mathrm{arccosh}\left(d\,x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(d*x^2-1))^(1/2),x)
```

```
[Out] int((a+b*arccosh(d*x^2-1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \,\mathrm{arcosh}\left(dx^2 - 1\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(d*x^2 - 1) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b \,\mathrm{acosh}\left(d\,x^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(d*x^2 - 1))^(1/2),x)
```

```
[Out] int((a + b*acosh(d*x^2 - 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acosh}\left(dx^2 - 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(d*x**2 - 1)), x)
```

3.264 $\int \frac{1}{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}\,dx}-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\right.}{}$$

```
[Out] 1/2*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2
)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/d/x/b^(1/2)-1/2*c
osh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1
/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/d/x/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5884}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\frac{a}{2b}\right)\right)\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{\sqrt{b}\,dx}-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]
]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(Sqrt[b]*d*x) - (Sqrt
[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqr
t[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(Sqrt[b]*d*x)
```

Rule 5884

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sq
rt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] - Simp[(Sqrt[P
i/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d},
 x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}\,dx=\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}{\sqrt{2}\sqrt{b}}\right)\left(\cosh\left(\frac{a}{2b}\right)-\sinh\left(\right.\right.}{\sqrt{b}\,dx}$$

Mathematica [A] time = 0.28, size = 134, normalized size = 0.81

$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)+\left(\sinh\left(\frac{a}{2b}\right)-\cosh\left(\frac{a}{2b}\right)\right)\text{erfi}\left(\right.\right.}{\sqrt{b}\,dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[-1 + d*x^2]],x]
```

```
[Out] -((Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*(Erfi[Sqrt[a + b*ArcCosh[-1 + d*x
^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)] + Sinh[a/(2*b)]) + Erf[Sqrt[a + b*A
rcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])))/(S
qrt[b]*d*x))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
index.cc index_m i_lex_is_greater Error: Bad Argument Valueindex.cc index_m
 operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\,\mathrm{arccosh}\left(d\,x^2 - 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\mathrm{arcosh}\left(dx^2 - 1\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*arccosh(d*x^2 - 1) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b\,\mathrm{acosh}\left(d\,x^2 - 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^(1/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\operatorname{acosh}\left(dx^2 - 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*acosh(d*x**2 - 1)), x)
```

3.265 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right.}{}$$

```
[Out] 1/2*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2
)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/x+1/2*c
osh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1
/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/b^(3/2)/d/x-(d*x^2)^(1/
2)*(d*x^2-2)^(1/2)/b/d/x/(a+b*arccosh(d*x^2-1))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5886}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{b^{3/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-3/2),x]
```

```
[Out] -((Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]])) +
 (Sqrt[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2
]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(b^(3/2)*d*x) + (Sqr
t[Pi/2]*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sq
rt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(b^(3/2)*d*x)
```

Rule 5886

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[(
Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]), x] +
 (Simp[(Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]
/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x] + Sim
p[(Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*E
rfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x]) /; FreeQ
[{a, b, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}}\,dx=-\frac{\sqrt{dx^2}\sqrt{-2+dx^2}}{bdx\sqrt{a+b\cosh^{-1}\left(-1+dx^2\right)}}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(-1+dx^2\right)\right)\text{erfi}}{}$$

Mathematica [A] time = 1.05, size = 209, normalized size = 0.99

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sqrt{2\pi}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)+\sqrt{}\right.}{2b^{3/2}dx\sqrt{a+b\,c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-3/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Er
fi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Sinh
[a/(2*b)]) + Sqrt[2*Pi]*Sqrt[a + b*ArcCosh[-1 + d*x^2]]*Erf[Sqrt[a + b*ArcC
osh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) - 4*Sqr
t[b]*Sinh[ArcCosh[-1 + d*x^2]/2]))/(2*b^(3/2)*d*x*Sqrt[a + b*ArcCosh[-1 + d
*x^2]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m i_lex_is_greater Error: Bad Argument ValueEvaluatio
n time: 0.71index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^{2}-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,\mathrm{arcosh}\left(dx^{2}-1\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(d\,x^{2}-1\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^(3/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^2 - 1\right)\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-3/2), x)
```

3.266 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)}{}$$

```
[Out] 1/6*cosh(1/2*arccosh(d*x^2-1))*erfi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2
)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/x-1/6*c
osh(1/2*arccosh(d*x^2-1))*erf(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1
/2))*(cosh(1/2*a/b)+sinh(1/2*a/b))*2^(1/2)*Pi^(1/2)/b^(5/2)/d/x+1/3*(-d*x^4
+2*x^2)/b/x/(a+b*arccosh(d*x^2-1))^(3/2)/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)-1/3*
x/b^2/(a+b*arccosh(d*x^2-1))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5889, 5884}

$$-\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{3b^{5/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-5/2),x]
```

```
[Out] (2*x^2 - d*x^4)/(3*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^(3/2)) - x/(3*b^2*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + (Sqrt[Pi/2]*Cosh[
ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b
])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(3*b^(5/2)*d*x) - (Sqrt[Pi/2]*Cosh[Arc
Cosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*
(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(3*b^(5/2)*d*x)
```

Rule 5884

```
Int[1/Sqrt[(a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[(Sq
rt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[S
qrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] - Simp[(Sqrt[P
i/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a
 + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(Sqrt[b]*d*x), x] /; FreeQ[{a, b, d},
 x]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}\,dx = \frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(\right.}}$$

$$= \frac{2x^2-dx^4}{3bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{3/2}} - \frac{x}{3b^2\sqrt{a+b\cosh^{-1}\left(\right.}}$$

Mathematica [A] time = 0.90, size = 238, normalized size = 0.94

$$-\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sqrt{2\pi}\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^{3/2}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-5/2),x]
```

```
[Out] -1/6*(Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(
3/2)*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(-Cosh[a/(2*b)
] + Sinh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(3/2)*Erf[Sqrt[
a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)
]) + 4*Sqrt[b]*((a + b*ArcCosh[-1 + d*x^2])*Cosh[ArcCosh[-1 + d*x^2]/2] + b
*Sinh[ArcCosh[-1 + d*x^2]/2])))/(b^(5/2)*d*x*(a + b*ArcCosh[-1 + d*x^2])^(3
/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m i_lex_is_greater Error: Bad Argument ValueEvaluatio
n time: 0.88index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(5/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arcosh}\left(dx^2 - 1\right) + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + b\,\mathrm{acosh}\left(d\,x^2 - 1\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^(5/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^2 - 1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(5/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-5/2), x)
```

3.267 $\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx$

Optimal. Leaf size=302

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right.}{}$$

```
[Out] -1/15*x/b^2/(a+b*arccosh(d*x^2-1))^(3/2)+1/30*cosh(1/2*arccosh(d*x^2-1))*er
fi(1/2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)-sinh(1/
2*a/b))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/x+1/30*cosh(1/2*arccosh(d*x^2-1))*erf(1/
2*(a+b*arccosh(d*x^2-1))^(1/2)*2^(1/2)/b^(1/2))*(cosh(1/2*a/b)+sinh(1/2*a/b
))*2^(1/2)*Pi^(1/2)/b^(7/2)/d/x+1/5*(-d*x^4+2*x^2)/b/x/(a+b*arccosh(d*x^2-1
))^(5/2)/(d*x^2)^(1/2)/(d*x^2-2)^(1/2)-1/15*(d*x^2)^(1/2)*(d*x^2-2)^(1/2)/b
^3/d/x/(a+b*arccosh(d*x^2-1))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5889, 5886}

$$\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\sinh\left(\frac{a}{2b}\right)+\cosh\left(\frac{a}{2b}\right)\right)\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}\left(dx^2-1\right)}}{\sqrt{2}\sqrt{b}}\right)}{15b^{7/2}dx}+\frac{\sqrt{\frac{\pi}{2}}\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[-1 + d*x^2])^(-7/2),x]
```

```
[Out] (2*x^2 - d*x^4)/(5*b*x*Sqrt[d*x^2]*Sqrt[-2 + d*x^2]*(a + b*ArcCosh[-1 + d*x
^2])^(5/2)) - x/(15*b^2*(a + b*ArcCosh[-1 + d*x^2])^(3/2)) - (Sqrt[d*x^2]*S
qrt[-2 + d*x^2])/(15*b^3*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]) + (Sqrt[Pi/2]
*Cosh[ArcCosh[-1 + d*x^2]/2]*Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*
Sqrt[b])]*(Cosh[a/(2*b)] - Sinh[a/(2*b)]))/(15*b^(7/2)*d*x) + (Sqrt[Pi/2]*C
osh[ArcCosh[-1 + d*x^2]/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqr
t[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]))/(15*b^(7/2)*d*x)
```

Rule 5886

```
Int[((a_.) + ArcCosh[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> -Simp[(
Sqrt[d*x^2]*Sqrt[-2 + d*x^2])/(b*d*x*Sqrt[a + b*ArcCosh[-1 + d*x^2]]), x] +
 (Simp[(Sqrt[Pi/2]*(Cosh[a/(2*b)] + Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]
/2]*Erf[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x] + Sim
p[(Sqrt[Pi/2]*(Cosh[a/(2*b)] - Sinh[a/(2*b)])*Cosh[ArcCosh[-1 + d*x^2]/2]*E
rfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/Sqrt[2*b]])/(b^(3/2)*d*x), x]) /; FreeQ
[{a, b, d}, x]
```

Rule 5889

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> -Simp[(
x*(a + b*ArcCosh[c + d*x^2])^(n + 2))/(4*b^2*(n + 1)*(n + 2)), x] + (Dist[1
/(4*b^2*(n + 1)*(n + 2)), Int[(a + b*ArcCosh[c + d*x^2])^(n + 2), x], x] +
Simp[((2*c*x^2 + d*x^4)*(a + b*ArcCosh[c + d*x^2])^(n + 1))/(2*b*(n + 1)*x*
Sqrt[-1 + c + d*x^2]*Sqrt[1 + c + d*x^2]), x]) /; FreeQ[{a, b, c, d}, x] &&
 EqQ[c^2, 1] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{7/2}}\,dx=\frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(-1\right.\right.}$$

$$=\frac{2x^2-dx^4}{5bx\sqrt{dx^2}\sqrt{-2+dx^2}\left(a+b\cosh^{-1}\left(-1+dx^2\right)\right)^{5/2}}-\frac{x}{15b^2\left(a+b\cosh^{-1}\left(-1\right.\right.}$$

Mathematica [A] time = 1.08, size = 260, normalized size = 0.86

$$\frac{\cosh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(4\sqrt{b}\left(-\sinh\left(\frac{1}{2}\cosh^{-1}\left(dx^2-1\right)\right)\left(\left(a+b\cosh^{-1}\left(dx^2-1\right)\right)^2+3b^2\right)-b\cosh\left(\frac{1}{2}\cosh\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[-1 + d*x^2])^(-7/2),x]
```

```
[Out] (Cosh[ArcCosh[-1 + d*x^2]/2]*(Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(5/2)*
Erfi[Sqrt[a + b*ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] - Si
nh[a/(2*b)]) + Sqrt[2*Pi]*(a + b*ArcCosh[-1 + d*x^2])^(5/2)*Erf[Sqrt[a + b*
ArcCosh[-1 + d*x^2]]/(Sqrt[2]*Sqrt[b])]*(Cosh[a/(2*b)] + Sinh[a/(2*b)]) + 4
*Sqrt[b]*(-(b*(a + b*ArcCosh[-1 + d*x^2])*Cosh[ArcCosh[-1 + d*x^2]/2]) - (3
*b^2 + (a + b*ArcCosh[-1 + d*x^2])^2)*Sinh[ArcCosh[-1 + d*x^2]/2])))/(30*b^
(7/2)*d*x*(a + b*ArcCosh[-1 + d*x^2])^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m i_lex_is_greater Error: Bad Argument ValueEvaluatio
n time: 1.05index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(d\,x^2-1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(d*x^2-1))^(7/2),x)
```

```
[Out] int(1/(a+b*arccosh(d*x^2-1))^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \operatorname{arcosh}\left(dx^2 - 1\right) + a\right)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(d*x^2-1))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(d*x^2 - 1) + a)^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(d\,x^2 - 1\right)\right)^{7/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(d*x^2 - 1))^(7/2),x)
```

```
[Out] int(1/(a + b*acosh(d*x^2 - 1))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}\left(dx^2 - 1\right)\right)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(d*x**2-1))**(7/2),x)
```

```
[Out] Integral((a + b*acosh(d*x**2 - 1))**(-7/2), x)
```

3.268 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Defer[Int][(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx = \int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

```
[Out] integral(-(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,-4,0,%%%{4,[2
,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [-82,7]sym2poly/r2sym(const gen & e,co
nst index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{n}}{-c^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

```
[Out] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(b\,\mathrm{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^{n}}{c^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] -integrate((b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^{n}}{c^{2}\,x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

```
[Out] -int((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

```
[Out] Timed out
```

3.269 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

Optimal. Leaf size=265

$$\frac{3b^2\mathrm{Li}_3\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}+\frac{3b\mathrm{Li}_2\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2c}-\frac{\left(a+b\,\mathrm{co}\right.}{}$$

```
[Out] -1/4*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^3*ln(1+1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1
/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c+3/2
*b*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1/((-c*x+1)^(1/
2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*
x+1)^(1/2)+1)^(1/2))^2)/c+3/2*b^2*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)
))*polylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)
-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c+3/4*b^3*polylog(4,-1
/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*
x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c
```

Rubi [A] time = 0.22, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6681, 5660, 3718, 2190, 2531, 6609, 2282, 6589}

$$\frac{3b^2\mathrm{PolyLog}\left(3,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{2c}-\frac{3b\mathrm{PolyLog}\left(2,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right.\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4/(4*b*c) - ((a + b*ArcCosh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 + E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c - (3*b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*PolyLog[2,
-E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c) + (3*b^2*(a + b*ArcCosh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])])/(2*c) - (3*b^3*PolyLog[4, -E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 +
 c*x]])])/(4*c)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^3}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$
$$= -\frac{\text{Subst}\left(\int (a+bx)^3\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} +$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} -$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} -$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} -$$
$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} -$$

Mathematica [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b^3\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3 + 3\,ab^2\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 3\,a^2b\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^3}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")

[Out] integral(-(b^3*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccosh(sq
rt(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1
)) + a^3)/(c^2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,-4,0,%%%{4,[2
,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [-82,7]sym2poly/r2sym(const gen & e,co
nst index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.90, size = 870, normalized size = 3.28

$$\frac{a^3\ln(cx+1)}{2c}-\frac{a^3\ln(cx-1)}{2c}+\frac{b^3\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4}{4c}-\frac{b^3\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+\sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}-1}\sqrt{\right.}\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a^3/c*ln(c*x+1)-1/2*a^3/c*ln(c*x-1)+1/4*b^3/c*arccosh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^4-b^3/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)
/(c*x+1)^(1/2)+1)^(1/2))^2)-3/2*b^3/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))
^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1
)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*b^3/c*arccosh((-c*x+
1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^
(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)-3/4
*b^3/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/
2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+a*b^2/c*arccosh((-c*
x+1)^(1/2)/(c*x+1)^(1/2))^3-3*a*b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))
^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2
)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)-3*a*b^2/c*arccosh((-c*x+1)^(1/
2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/
(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*a*b^2
/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1
)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+3/2*a^2*b/c*arccosh((-c*
x+1)^(1/2)/(c*x+1)^(1/2))^2-3*a^2*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*
((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)-3/2*a^2*b/c*polylog(2,-((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)
/(c*x+1)^(1/2)+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^3*log(c*x + 1) - b^3*log
(-c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
sqrt(-c*x + 1)) + sqrt(-c*x + 1))^3/c + integrate(1/8*(((c*x + 1)*sqrt(-c*x
 + 1)*b^3 - (-c*x + 1)^(3/2)*b^3)*log(c*x + 1)^3 - 6*((c*x + 1)*sqrt(-c*x +
```

```
1)*a*b^2 - (-c*x + 1)^(3/2)*a*b^2)*log(c*x + 1)^2 - 6*((4*a*b^2 + (b^3*c*x
+ b^3)*log(c*x + 1) - (b^3*c*x + b^3)*log(-c*x + 1))*(c*x + 1)*sqrt(-c*x +
1) - (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) - (b^3*c*x + b^3)*log(-c*x +
1))*(-c*x + 1)^(3/2) + ((4*a*b^2 + (b^3*c*x - b^3)*log(c*x + 1) - (b^3*c*x
- b^3)*log(-c*x + 1))*(c*x + 1) + (4*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1) -
(b^3*c*x + b^3)*log(-c*x + 1))*(c*x - 1) - 2*((c*x + 1)*b^3 + (c*x - 1)*b^
3)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
sqrt(-c*x + 1)) - 2*((c*x + 1)*sqrt(-c*x + 1)*b^3 - (-c*x + 1)^(3/2)*b^3)*
log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1)
+ sqrt(-c*x + 1)) + sqrt(-c*x + 1))^2 + (((c*x + 1)*b^3 + (c*x - 1)*b^3)*lo
g(c*x + 1)^3 - 6*((c*x + 1)*a*b^2 + (c*x - 1)*a*b^2)*log(c*x + 1)^2 + 12*((
c*x + 1)*a^2*b + (c*x - 1)*a^2*b)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-
c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + 12*((c*x + 1)*sqrt(-c*x +
1)*a^2*b - (-c*x + 1)^(3/2)*a^2*b)*log(c*x + 1) - 6*(4*(c*x + 1)*sqrt(-c*x
+ 1)*a^2*b - 4*(-c*x + 1)^(3/2)*a^2*b + ((c*x + 1)*sqrt(-c*x + 1)*b^3 - (-
c*x + 1)^(3/2)*b^3)*log(c*x + 1)^2 + (4*(c*x + 1)*a^2*b + 4*(c*x - 1)*a^2*b
+ ((c*x + 1)*b^3 + (c*x - 1)*b^3)*log(c*x + 1)^2 - 4*((c*x + 1)*a*b^2 + (c
*x - 1)*a*b^2)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqr
t(c*x + 1) + sqrt(-c*x + 1)) - 4*((c*x + 1)*sqrt(-c*x + 1)*a*b^2 - (-c*x +
1)^(3/2)*a*b^2)*log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt
(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1)))/((c^2*x^2 - 1)*(c*x +
1)*sqrt(-c*x + 1) - (c^2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x +
1) + (c^2*x^2 - 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^3}{c^2x^2-1}\,dx-\int \frac{b^3\,\mathrm{acosh}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{3ab^2\,\mathrm{acosh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int \frac{3a^2b\,\mathrm{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*acosh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*acosh(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*acosh(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.270 $\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=196

$$\frac{b\text{Li}_2\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3bc}-\frac{\log\left(e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)\left(a+b\,c\right.}{c}$$

```
[Out] -1/3*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*ln(1+1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1
/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c+b*(
a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(2,-1/((-c*x+1)^(1/2)/(c*
x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+1)^(1/2))^2)/c+1/2*b^2*polylog(3,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c
*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^
2)/c
```

Rubi [A] time = 0.20, antiderivative size = 197, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6681, 5660, 3718, 2190, 2531, 2282, 6589}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b^2\text{PolyLog}\left(3,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{3bc}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(3*b*c) - ((a + b*ArcCosh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 + E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c
*x]])])/c - (b*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -E^(
2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/c + (b^2*PolyLog[3, -E^(2*ArcCosh
[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(a+b\cosh^{-1}(x)\right)^2}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}\\
&= -\frac{\text{Subst}\left(\int (a+bx)^2\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}\\
&= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}\\
&= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} +\\
&= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} -\\
&= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} -\\
&= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} -
\end{aligned}$$

Mathematica [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1 - c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b^2\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 + 2\,ab\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a^2}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
[Out] integral(-(b^2*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccosh(sqrt
(-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,-4,0,%%%{4,[2
,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [-82,7]sym2poly/r2sym(const gen & e,co
nst index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 491, normalized size = 2.51

$$\frac{a^2\ln(cx+1)}{2c} - \frac{a^2\ln(cx-1)}{2c} + \frac{b^2\text{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3}{3c} - \frac{b^2\text{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2 \ln\left(1 + \left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} + \sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1}\,\sqrt{\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x)
[Out] 1/2*a^2/c*ln(c*x+1)-1/2*a^2/c*ln(c*x-1)+1/3*b^2/c*arccosh((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^3-b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)
/(c*x+1)^(1/2)+1)^(1/2))^2)-b^2/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*pol
ylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2
)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)+1/2*b^2/c*polylog(3,-((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)
/(c*x+1)^(1/2)+1)^(1/2))^2)+a*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2-2
*a*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1
```

```
)^(1/2))^2)-a*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/
(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a^2\left(\frac{\log(cx+1)}{c}-\frac{\log(cx-1)}{c}\right)+\frac{\left(b^2\log(cx+1)-b^2\log(-cx+1)\right)\log\left(\sqrt{\sqrt{cx+1}+\sqrt{-cx+1}}\sqrt{-\sqrt{cx+1}+\right.}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

```
[Out] 1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*(b^2*log(c*x + 1) - b^2*log
(-c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) +
sqrt(-c*x + 1)) + sqrt(-c*x + 1))^2/c + integrate(-1/4*(((c*x + 1)*sqrt(-c*
x + 1)*b^2 - (-c*x + 1)^(3/2)*b^2)*log(c*x + 1)^2 + (((c*x + 1)*b^2 + (c*x
- 1)*b^2)*log(c*x + 1)^2 - 4*((c*x + 1)*a*b + (c*x - 1)*a*b)*log(c*x + 1))*
sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1))
- 4*((c*x + 1)*sqrt(-c*x + 1)*a*b - (-c*x + 1)^(3/2)*a*b)*log(c*x + 1) + 2*
((4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x + b^2)*log(-c*x + 1))*(c*
x + 1)*sqrt(-c*x + 1) - (4*a*b + (b^2*c*x + b^2)*log(c*x + 1) - (b^2*c*x +
b^2)*log(-c*x + 1))*(-c*x + 1)^(3/2) + ((4*a*b + (b^2*c*x - b^2)*log(c*x +
1) - (b^2*c*x - b^2)*log(-c*x + 1))*(c*x + 1) + (4*a*b + (b^2*c*x + b^2)*lo
g(c*x + 1) - (b^2*c*x + b^2)*log(-c*x + 1))*(c*x - 1) - 2*((c*x + 1)*b^2 +
(c*x - 1)*b^2)*log(c*x + 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqr
t(c*x + 1) + sqrt(-c*x + 1)) - 2*((c*x + 1)*sqrt(-c*x + 1)*b^2 - (-c*x + 1)
^(3/2)*b^2)*log(c*x + 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1)))/((c^2*x^2 - 1)*(c*x + 1)*s
qrt(-c*x + 1) - (c^2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x + 1) +
 (c^2*x^2 - 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c
*x + 1) + sqrt(-c*x + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{a^2}{c^2x^2-1}\,dx-\int\frac{b^2\,\mathrm{acosh}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx-\int\frac{2ab\,\mathrm{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*acosh(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*acosh(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.271 $\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}+\frac{b\text{Li}_2\left(-e^{-2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}$$

```
[Out] -1/2*(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arccosh((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))*ln(1+1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2
)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c+1/2*b
*polylog(2,-1/((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1
)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)/c
```

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {206, 6681, 5660, 3718, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{2c}+\frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2bc}-\frac{\log\left(e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}+1\right)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

```
[Out] (a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(2*b*c) - ((a + b*ArcCosh[Sq
rt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x
]])])/c - (b*PolyLog[2, -E^(2*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/(2*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 6681

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[(2*e*g)/(C*(e*f -
d*g)), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{a+b\cosh^{-1}(x)}{x}\,dx, x, \frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}{c}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\text{S}}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{b\,\text{S}}{}$$

$$= \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} - \frac{b\text{Li}}{}$$

Mathematica [F] time = 3.16, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
[Out] Integrate[(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2), x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b \operatorname{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right) + a}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

```
[Out] integral(-(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,-4,0,%%%{4,[2
,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [-82,7]sym2poly/r2sym(const gen & e,co
nst index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 207, normalized size = 1.56

$$\frac{a\ln(cx+1)}{2c} - \frac{a\ln(cx-1)}{2c} + \frac{b\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2}{2c} - \frac{b\operatorname{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1 + \left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} + \sqrt{\frac{\sqrt{-cx+1}}{\sqrt{cx+1}} - 1}\sqrt{\frac{\sqrt{-c}}{\sqrt{cx}}}\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x)
```

```
[Out] 1/2*a/c*ln(c*x+1)-1/2*a/c*ln(c*x-1)+1/2*b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^
(1/2))^2-b/c*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*
x+1)^(1/2)+((-c*x+1)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(
1/2)+1)^(1/2))^2)-1/2*b/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+((-c*x+1
)^(1/2)/(c*x+1)^(1/2)-1)^(1/2)*((-c*x+1)^(1/2)/(c*x+1)^(1/2)+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{2\left(\log(cx+1) - \log(-cx+1)\right)\log(cx+1) - \log(cx+1)^2 + 2\log(cx+1)\log(-cx+1) - \log(-cx+1}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

```
[Out] -1/8*b*((2*(log(c*x + 1) - log(-c*x + 1))*log(c*x + 1) - log(c*x + 1)^2 + 2
*log(c*x + 1)*log(-c*x + 1) - log(-c*x + 1)^2 - 4*(log(c*x + 1) - log(-c*x
+ 1))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-
c*x + 1)) + sqrt(-c*x + 1)))/c + 8*integrate(1/2*(c*x + 1)*sqrt(-c*x + 1)*(
log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1)*sqrt(-c*x + 1) - (c^
```

```
2*x^2 - 1)*(-c*x + 1)^(3/2) + ((c^2*x^2 - 1)*(c*x + 1) + (c^2*x^2 - 1)*(c*x
 - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x
 + 1))), x) + 8*integrate(-1/4*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))
/((c^2*x^2 - 1)*sqrt(c*x + 1) + (c^2*x^2 - 1)*sqrt(-c*x + 1)), x) - 8*integ
rate(1/4*sqrt(c*x + 1)*(log(c*x + 1) - log(-c*x + 1))/((c^2*x^2 - 1)*sqrt(c
*x + 1) - (c^2*x^2 - 1)*sqrt(-c*x + 1)), x)) + 1/2*a*(log(c*x + 1)/c - log(
c*x - 1)/c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

```
[Out] int(-(a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{c^2x^2-1}\,dx-\int \frac{b\,\mathrm{acosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a/(c**2*x**2 - 1), x) - Integral(b*acosh(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

3.272 $\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),
x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx = \int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{ac^2x^2+(bc^2x^2-b)\,\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

```
[Out] integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,-4,0,%%%{4,[2
,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [-82,7]sym2poly/r2sym(const gen & e,co
nst index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2}x^{2}+1\right)\left(a+b\,\mathrm{arccosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(c^{2}x^{2}-1\right)\left(b\,\mathrm{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

```
[Out] -integrate(1/((c^2*x^2 - 1)*(b*arccosh(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

```
[Out] Timed out
```

3.273 $\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

Optimal. Leaf size=43

$$\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

```
[Out] Defer[Int][1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)
, x]
```

Rubi steps

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Mathematica [A] time = 5.34, size = 0, normalized size = 0.00

$$\int \frac{1}{(1-c^2x^2)\left(a+b\cosh^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)*(a + b*ArcCosh[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2-a^2+2\left(abc^2x^2-ab\right)\text{arcosh}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

```
[Out] integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,-4,0,%%%{4,[2
,2]%%%}] at parameters values [86,-97]Warning, choosing root of [1,0,-4,0,%
%%{4,[2,2]%%%}] at parameters values [-82,7]sym2poly/r2sym(const gen & e,co
nst index_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2} x^{2}+1\right) \left(a +b \,\mathrm{arccosh}\left(\frac{\sqrt{-c x +1}}{\sqrt{c x +1}}\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)/(a+b*arccosh((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

```
[Out] 2*(2*c*x*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c
*x + 1)) + (c*x + 1)*sqrt(-c*x + 1) - (-c*x + 1)^(3/2))/(2*(c*x + 1)*sqrt(-
c*x + 1)*a*b*c - 2*(-c*x + 1)^(3/2)*a*b*c - ((c*x - 1)*b^2*c*log(c*x + 1) -
 2*(c*x - 1)*a*b*c)*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1
) + sqrt(-c*x + 1)) - ((c*x + 1)*sqrt(-c*x + 1)*b^2*c - (-c*x + 1)^(3/2)*b^
2*c)*log(c*x + 1) + 2*((c*x - 1)*b^2*c*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))
*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + (c*x + 1)*sqrt(-c*x + 1)*b^2*c - (
-c*x + 1)^(3/2)*b^2*c)*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(
c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1))) - integrate(-2*(2*(c*x + 1)*s
qrt(-c*x + 1)*(sqrt(c*x + 1) + sqrt(-c*x + 1))*(sqrt(c*x + 1) - sqrt(-c*x +
 1)) + ((c*x + 1)^2 + 2*(c*x + 1)*(c*x - 1))*sqrt(sqrt(c*x + 1) + sqrt(-c*x
 + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)))/(2*(a*b*c^2*x^2 - a*b)*(c*x +
 1)^2*sqrt(-c*x + 1) - 4*(a*b*c^2*x^2 - a*b)*(c*x + 1)*(-c*x + 1)^(3/2) + 2
*(a*b*c^2*x^2 - a*b)*(-c*x + 1)^(5/2) + ((b^2*c^2*x^2 - b^2)*(-c*x + 1)^(3/
2)*log(c*x + 1) - 2*(a*b*c^2*x^2 - a*b)*(-c*x + 1)^(3/2))*(sqrt(c*x + 1) +
sqrt(-c*x + 1))*(sqrt(c*x + 1) - sqrt(-c*x + 1)) + 2*(2*(a*b*c^2*x^2 - a*b)
*(c*x + 1)*(c*x - 1) + 2*(a*b*c^2*x^2 - a*b)*(c*x - 1)^2 - ((b^2*c^2*x^2 -
b^2)*(c*x + 1)*(c*x - 1) + (b^2*c^2*x^2 - b^2)*(c*x - 1)^2)*log(c*x + 1))*s
qrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sqrt(c*x + 1) + sqrt(-c*x + 1)) -
 ((b^2*c^2*x^2 - b^2)*(c*x + 1)^2*sqrt(-c*x + 1) - 2*(b^2*c^2*x^2 - b^2)*(c
```

```
*x + 1)*(-c*x + 1)^(3/2) + (b^2*c^2*x^2 - b^2)*(-c*x + 1)^(5/2))*log(c*x +
1) - 2*((b^2*c^2*x^2 - b^2)*(-c*x + 1)^(3/2)*(sqrt(c*x + 1) + sqrt(-c*x + 1
))*(sqrt(c*x + 1) - sqrt(-c*x + 1)) - (b^2*c^2*x^2 - b^2)*(c*x + 1)^2*sqrt(
-c*x + 1) + 2*(b^2*c^2*x^2 - b^2)*(c*x + 1)*(-c*x + 1)^(3/2) - (b^2*c^2*x^2
 - b^2)*(-c*x + 1)^(5/2) - 2*((b^2*c^2*x^2 - b^2)*(c*x + 1)*(c*x - 1) + (b^
2*c^2*x^2 - b^2)*(c*x - 1)^2)*sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqrt(-sq
rt(c*x + 1) + sqrt(-c*x + 1)))*log(sqrt(sqrt(c*x + 1) + sqrt(-c*x + 1))*sqr
t(-sqrt(c*x + 1) + sqrt(-c*x + 1)) + sqrt(-c*x + 1))), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,\left(c^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

```
[Out] -int(1/((a + b*acosh((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)/(a+b*acosh((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

```
[Out] Timed out
```

3.274 $\int \cosh^{-1}\left(ce^{a+bx}\right) dx$

Optimal. Leaf size=76

$$\frac{\text{Li}_2\left(-e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(e^{2\cosh^{-1}\left(ce^{a+bx}\right)}+1\right)}{b}$$

```
[Out] -1/2*arccosh(c*exp(b*x+a))^2/b+arccosh(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+(c*
exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)/b+1/2*polylog(2,-(c*exp(b*x+
a)+(c*exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)/b
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2282, 5660, 3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}-\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b}+\frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(e^{2\cosh^{-1}\left(ce^{a+bx}\right)}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c*E^(a + b*x)],x]
```

```
[Out] -ArcCosh[c*E^(a + b*x)]^2/(2*b) + (ArcCosh[c*E^(a + b*x)]*Log[1 + E^(2*ArcC
osh[c*E^(a + b*x)])])/b + PolyLog[2, -E^(2*ArcCosh[c*E^(a + b*x)])]/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rubi steps

$$\begin{aligned}
\int \cosh^{-1}\left(ce^{a+bx}\right)dx &= \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(cx)}{x}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}\left(ce^{a+bx}\right)\right)}{b} \\
&= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right.\right.}{} \\
&= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,\right.}{2b} \\
&= -\frac{\cosh^{-1}\left(ce^{a+bx}\right)^2}{2b} + \frac{\cosh^{-1}\left(ce^{a+bx}\right)\log\left(1+e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{b} + \frac{\text{Li}_2\left(-e^{2\cosh^{-1}\left(ce^{a+bx}\right)}\right)}{2b}
\end{aligned}$$

Mathematica [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \cosh^{-1}\left(ce^{a+bx}\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[c*E^(a + b*x)],x]
[Out] Integrate[ArcCosh[c*E^(a + b*x)], x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{arcosh}\left(ce^{(bx+a)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="giac")
[Out] integrate(arccosh(c*e^(b*x + a)), x)
```

maple [A] time = 0.01, size = 115, normalized size = 1.51

$$-\frac{\operatorname{arccosh}\left(c\,e^{bx+a}\right)^2}{2b}+\frac{\operatorname{arccosh}\left(c\,e^{bx+a}\right)\ln\left(1+\left(c\,e^{bx+a}+\sqrt{c\,e^{bx+a}-1}\,\sqrt{c\,e^{bx+a}+1}\right)^2\right)}{b}+\frac{\operatorname{polylog}\left(2,-\left(c\,e^{bx+a}+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c*exp(b*x+a)),x)
```

```
[Out] -1/2*arccosh(c*exp(b*x+a))^2/b+arccosh(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+(c*
exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)/b+1/2*polylog(2,-(c*exp(b*x+
a)+(c*exp(b*x+a)-1)^(1/2)*(c*exp(b*x+a)+1)^(1/2))^2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$bc\int\frac{xe^{(bx+a)}}{c^3e^{(3\,bx+3\,a)}-ce^{(bx+a)}+\left(c^2e^{(2\,bx+2\,a)}-1\right)e^{\left(\frac{1}{2}\,\log\left(ce^{(bx+a)}+1\right)+\frac{1}{2}\,\log\left(ce^{(bx+a)}-1\right)\right)}}\,dx+x\log\left(ce^{(bx+a)}+\sqrt{ce^{(bx+a)}+1}\,\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c*exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] b*c*integrate(x*e^(b*x + a)/(c^3*e^(3*b*x + 3*a) - c*e^(b*x + a) + (c^2*e^(
2*b*x + 2*a) - 1)*e^(1/2*log(c*e^(b*x + a) + 1) + 1/2*log(c*e^(b*x + a) - 1
))), x) + x*log(c*e^(b*x + a) + sqrt(c*e^(b*x + a) + 1)*sqrt(c*e^(b*x + a)
- 1)) - 1/2*(b*x*log(c*e^(b*x + a) + 1) + dilog(-c*e^(b*x + a)))/b - 1/2*(b
*x*log(-c*e^(b*x + a) + 1) + dilog(c*e^(b*x + a)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}\left(c\,e^{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c*exp(a + b*x)),x)
```

```
[Out] int(acosh(c*exp(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}\left(ce^{a+bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c*exp(b*x+a)),x)
```

```
[Out] Integral(acosh(c*exp(a + b*x)), x)
```

3.275 $\int e^{\cosh^{-1}(a+bx)}x^3\,dx$

Optimal. Leaf size=165

$$-\frac{\left(4a^2+3\right)ae^{2\cosh^{-1}(a+bx)}}{16b^4}+\frac{\left(4a^2+3\right)a\cosh^{-1}(a+bx)}{8b^4}+\frac{\left(6a^2+1\right)e^{-\cosh^{-1}(a+bx)}}{8b^4}+\frac{\left(6a^2+1\right)e^{3\cosh^{-1}(a+bx)}}{24b^4}-\frac{3ae^{-}}{}$$

```
[Out] 1/48/b^4/(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^3-3/16*a/b^4/(b*x+a+(b*x+a
-1)^(1/2)*(b*x+a+1)^(1/2))^2+1/8*(6*a^2+1)/b^4/(b*x+a+(b*x+a-1)^(1/2)*(b*x+
a+1)^(1/2))-1/16*a*(4*a^2+3)*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2/b^4+
1/24*(6*a^2+1)*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^3/b^4-3/32*a*(b*x+a+
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^4/b^4+1/80*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1
)^(1/2))^5/b^4+1/8*a*(4*a^2+3)*arccosh(b*x+a)/b^4
```

Rubi [A] time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5899, 2282, 12, 1628}

$$-\frac{\left(4a^2+3\right)ae^{2\cosh^{-1}(a+bx)}}{16b^4}+\frac{\left(4a^2+3\right)a\cosh^{-1}(a+bx)}{8b^4}+\frac{\left(6a^2+1\right)e^{-\cosh^{-1}(a+bx)}}{8b^4}+\frac{\left(6a^2+1\right)e^{3\cosh^{-1}(a+bx)}}{24b^4}-\frac{3ae^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x^3,x]
```

```
[Out] 1/(48*b^4*E^(3*ArcCosh[a + b*x])) - (3*a)/(16*b^4*E^(2*ArcCosh[a + b*x])) +
 (1 + 6*a^2)/(8*b^4*E^ArcCosh[a + b*x]) - (a*(3 + 4*a^2)*E^(2*ArcCosh[a + b
*x]))/(16*b^4) + ((1 + 6*a^2)*E^(3*ArcCosh[a + b*x]))/(24*b^4) - (3*a*E^(4*
ArcCosh[a + b*x]))/(32*b^4) + E^(5*ArcCosh[a + b*x])/(80*b^4) + (a*(3 + 4*a
^2)*ArcCosh[a + b*x])/(8*b^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)}x^3\,dx = \frac{\text{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^3\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int\frac{(-1+x^2)(1-2ax+x^2)^3}{16b^3x^4}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int\frac{(-1+x^2)(1-2ax+x^2)^3}{x^4}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{16b^4}$$
$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{x^4}+\frac{6a}{x^3}-\frac{2(1+6a^2)}{x^2}+\frac{2a(3+4a^2)}{x}-2a(3+4a^2)x+2(1+6a^2)x^2-6ax^3+x^4\right)d\right.}{16b^4}$$
$$= \frac{e^{-3\cosh^{-1}(a+bx)}}{48b^4}-\frac{3ae^{-2\cosh^{-1}(a+bx)}}{16b^4}+\frac{(1+6a^2)e^{-\cosh^{-1}(a+bx)}}{8b^4}-\frac{a(3+4a^2)e^{2\cosh^{-1}(a+bx)}}{16b^4}+$$

Mathematica [A] time = 0.10, size = 138, normalized size = 0.84

$$\frac{15a(4a^2+3)\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)+\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-6a^4-2(3a^2+4)b^2x^2+(6a\right.}{120b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]*x^3,x]
```

```
[Out] (30*a*b^4*x^4 + 24*b^5*x^5 + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-16 - 83
*a^2 - 6*a^4 + a*(29 + 6*a^2)*b*x - 2*(4 + 3*a^2)*b^2*x^2 + 6*a*b^3*x^3 + 2
4*b^4*x^4) + 15*a*(3 + 4*a^2)*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a +
 b*x]])/(120*b^4)
```

fricas [A] time = 0.78, size = 133, normalized size = 0.81

$$\frac{24\,b^5x^5+30\,ab^4x^4+\left(24\,b^4x^4+6\,ab^3x^3-2\left(3\,a^2+4\right)b^2x^2-6\,a^4+\left(6\,a^3+29\,a\right)bx-83\,a^2-16\right)\sqrt{bx+a+1}\,\sqrt{b}}{120\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="fricas"
)
```

```
[Out] 1/120*(24*b^5*x^5 + 30*a*b^4*x^4 + (24*b^4*x^4 + 6*a*b^3*x^3 - 2*(3*a^2 + 4
)*b^2*x^2 - 6*a^4 + (6*a^3 + 29*a)*b*x - 83*a^2 - 16)*sqrt(b*x + a + 1)*sqr
t(b*x + a - 1) - 15*(4*a^3 + 3*a)*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a
 - 1) - a))/b^4
```

giac [B] time = 0.25, size = 564, normalized size = 3.42

$$24\,b^2x^5+30\,abx^4+5\left((bx+a+1)\left(2\,(bx+a+1)\left(\frac{3\,(bx+a+1)}{b^3}-\frac{12\,ab^{12}+13\,b^{12}}{b^{15}}\right)+\frac{36\,a^2b^{12}+84\,ab^{12}+43\,b^{12}}{b^{15}}\right)-\frac{3\left(8\,a^3b^{12}+36\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="giac")
```

```
[Out] 1/120*(24*b^2*x^5 + 30*a*b*x^4 + 5*((b*x + a + 1)*(2*(b*x + a + 1)*(3*(b*x
+ a + 1)/b^3 - (12*a*b^12 + 13*b^12)/b^15) + (36*a^2*b^12 + 84*a*b^12 + 43*
```

```
b^12)/b^15) - 3*(8*a^3*b^12 + 36*a^2*b^12 + 36*a*b^12 + 13*b^12)/b^15)*sqrt
(b*x + a + 1)*sqrt(b*x + a - 1) + 5*(((b*x + a + 1)*(2*(b*x + a + 1)*(3*(b*
x + a + 1)/b^3 - (12*a*b^12 + 13*b^12)/b^15) + (36*a^2*b^12 + 84*a*b^12 + 4
3*b^12)/b^15) - 3*(8*a^3*b^12 + 36*a^2*b^12 + 36*a*b^12 + 13*b^12)/b^15)*sq
rt(b*x + a + 1)*sqrt(b*x + a - 1) - 6*(8*a^3 + 12*a^2 + 12*a + 3)*log(sqrt(
b*x + a + 1) - sqrt(b*x + a - 1))/b^3)*a + (((2*(b*x + a + 1)*(3*(b*x + a +
 1)*(4*(b*x + a + 1)/b^4 - (20*a*b^20 + 21*b^20)/b^24) + (120*a^2*b^20 + 26
0*a*b^20 + 133*b^20)/b^24) - 5*(48*a^3*b^20 + 168*a^2*b^20 + 172*a*b^20 + 5
9*b^20)/b^24)*(b*x + a + 1) + 15*(8*a^4*b^20 + 48*a^3*b^20 + 72*a^2*b^20 +
52*a*b^20 + 13*b^20)/b^24)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 30*(8*a^4
+ 16*a^3 + 24*a^2 + 12*a + 3)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))/b^
4)*b - 30*(8*a^3 + 12*a^2 + 12*a + 3)*log(sqrt(b*x + a + 1) - sqrt(b*x + a
- 1))/b^3)/b
```

maple [C] time = 0.04, size = 376, normalized size = 2.28

$$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(24\,\mathrm{csgn}(b)x^4b^4\sqrt{b^2x^2+2abx+a^2-1}+6\,\mathrm{csgn}(b)x^3a\,b^3\sqrt{b^2x^2+2abx+a^2-1}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x)
```

```
[Out] 1/120*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(24*csgn(b)*x^4*b^4*(b^2*x^2+2*a*b*x+
a^2-1)^(1/2)+6*csgn(b)*x^3*a*b^3*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-6*csgn(b)*x^
2*a^2*b^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csg
n(b)*x*a^3*b-8*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*x^2*b^2-6*(b^2*x^2+2*a
*b*x+a^2-1)^(1/2)*csgn(b)*a^4+29*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*x*a*
b-83*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a^2+60*ln(((b^2*x^2+2*a*b*x+a^2-
1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a^3-16*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(
b)+45*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a)*csgn(b)/
b^4/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+1/5*b*x^5+1/4*x^4*a
```

maxima [B] time = 0.33, size = 495, normalized size = 3.00

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4+\frac{\left(b^2x^2+2\,abx+a^2-1\right)^{\frac{3}{2}}x^2}{5\,b^2}-\frac{\left(a^2-1\right)a^3\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{5\,b^4}-\frac{7\left(b^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^3,x, algorithm="maxima"
)
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4 + 1/5*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)*x^2/b^2 - 1
/5*(a^2 - 1)*a^3*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*
b)/b^4 - 7/20*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)*a*x/b^3 + 1/5*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*(a^2 - 1)*a*x/b^3 + 1/5*(a^2 - 1)^2*a*log(2*b^2*x +
2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^4 + 7/12*(b^2*x^2 + 2*a*b*
x + a^2 - 1)^(3/2)*a^2/b^4 + 1/5*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*(a^2 - 1
)*a^2/b^4 + 7/40*(5*a^2*b^2 - (a^2 - 1)*b^2)*a^3*log(2*b^2*x + 2*a*b + 2*sq
rt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^6 - 2/15*(b^2*x^2 + 2*a*b*x + a^2 - 1)
^(3/2)*(a^2 - 1)/b^4 - 7/40*(5*a^2*b^2 - (a^2 - 1)*b^2)*sqrt(b^2*x^2 + 2*a*
b*x + a^2 - 1)*a*x/b^5 - 7/40*(5*a^2*b^2 - (a^2 - 1)*b^2)*(a^2 - 1)*a*log(2
*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^6 - 7/40*(5*a^2*b
^2 - (a^2 - 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a^2/b^6
```

mupad [B] time = 69.81, size = 1408, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x),x)
```

```
[Out] (a*x^4)/4 - ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*((3*a)/2 + 2*a^3))/(b^4
*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))) + (((a - 1)^(1/2) - (a + b*x - 1)^(
1/2))^19*((3*a)/2 + 2*a^3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^19)
- (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*((29*a)/2 + (58*a^3)/3))/(b^4*((
a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) - (((a - 1)^(1/2) - (a + b*x - 1)^(1
/2))^17*((29*a)/2 + (58*a^3)/3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))
^17) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*(654*a - (4552*a^3)/3 + (35
84*a^5)/5))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5) - (((a - 1)^(1/2)
 - (a + b*x - 1)^(1/2))^15*(654*a - (4552*a^3)/3 + (3584*a^5)/5))/(b^4*((a
+ 1)^(1/2) - (a + b*x + 1)^(1/2))^15) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/
2))^7*(4622*a - 16024*a^3 + 11776*a^5))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)
^(1/2))^7) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^13*(4622*a - 16024*a^3
+ 11776*a^5))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^13) - (((a - 1)^(1
/2) - (a + b*x - 1)^(1/2))^9*(11095*a - 48012*a^3 + 39936*a^5))/(b^4*((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^9) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))
^11*(11095*a - 48012*a^3 + 39936*a^5))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^
(1/2))^11) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(a - 1)^(1/2)*(a + 1)
^(1/2)*(64*a^4 - 128*a^2 + 64))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^
4) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^16*(a - 1)^(1/2)*(a + 1)^(1/2)*
(64*a^4 - 128*a^2 + 64))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^16) + (
((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*(a - 1)^(1/2)*(a + 1)^(1/2)*(3712*a
^4 - (12544*a^2)/3 + 1408/3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6)
 + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^14*(a - 1)^(1/2)*(a + 1)^(1/2)*(3
712*a^4 - (12544*a^2)/3 + 1408/3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1)^(1/2
))^14) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*(a - 1)^(1/2)*(a + 1)^(1/
2)*(25536*a^4 - (56960*a^2)/3 + 4928/3))/(b^4*((a + 1)^(1/2) - (a + b*x + 1
)^(1/2))^8) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^12*(a - 1)^(1/2)*(a +
1)^(1/2)*(25536*a^4 - (56960*a^2)/3 + 4928/3))/(b^4*((a + 1)^(1/2) - (a + b
*x + 1)^(1/2))^12) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10*(a - 1)^(1/2
)*(a + 1)^(1/2)*((231168*a^4)/5 - (160256*a^2)/5 + 11008/5))/(b^4*((a + 1)^
(1/2) - (a + b*x + 1)^(1/2))^10))/((45*((a - 1)^(1/2) - (a + b*x - 1)^(1/2)
)^4)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4 - (10*((a - 1)^(1/2) - (a + b*
x - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (120*((a - 1)^(1
/2) - (a + b*x - 1)^(1/2))^6)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6 + (21
0*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8)/((a + 1)^(1/2) - (a + b*x + 1)^(
1/2))^8 - (252*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10)/((a + 1)^(1/2) - (
a + b*x + 1)^(1/2))^10 + (210*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^12)/((a
 + 1)^(1/2) - (a + b*x + 1)^(1/2))^12 - (120*((a - 1)^(1/2) - (a + b*x - 1)
^(1/2))^14)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^14 + (45*((a - 1)^(1/2) -
 (a + b*x - 1)^(1/2))^16)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^16 - (10*((
a - 1)^(1/2) - (a + b*x - 1)^(1/2))^18)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2
))^18 + ((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^20/((a + 1)^(1/2) - (a + b*x
+ 1)^(1/2))^20 + 1) + (b*x^5)/5 + (a*atanh(((a - 1)^(1/2) - (a + b*x - 1)^(
1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)))*(4*a^2 + 3))/(2*b^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + bx + \sqrt{a + bx - 1}\,\sqrt{a + bx + 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x**3,x)
```

```
[Out] Integral(x**3*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)
```

3.276 $\int e^{\cosh^{-1}(a+bx)}x^2\,dx$

Optimal. Leaf size=115

$$\frac{(4a^2+1)\,e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{(4a^2+1)\cosh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\cosh^{-1}(a+bx)}}{32b^3}$$

```
[Out] 1/16/b^3/(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2-1/2*a/b^3/(b*x+a+(b*x+a-
1)^(1/2)*(b*x+a+1)^(1/2))+1/16*(4*a^2+1)*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(
1/2))^2/b^3-1/6*a*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^3/b^3+1/32*(b*x+a
+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^4/b^3-1/8*(4*a^2+1)*arccosh(b*x+a)/b^3
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5899, 2282, 12, 1628}

$$\frac{(4a^2+1)\,e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{(4a^2+1)\cosh^{-1}(a+bx)}{8b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}+\frac{e^{4\cosh^{-1}(a+bx)}}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x^2,x]
```

```
[Out] 1/(16*b^3*E^(2*ArcCosh[a + b*x])) - a/(2*b^3*E^ArcCosh[a + b*x]) + ((1 + 4*
a^2)*E^(2*ArcCosh[a + b*x]))/(16*b^3) - (a*E^(3*ArcCosh[a + b*x]))/(6*b^3)
+ E^(4*ArcCosh[a + b*x])/(32*b^3) - ((1 + 4*a^2)*ArcCosh[a + b*x])/(8*b^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)}x^2\,dx = \frac{\text{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int\frac{(-1+x^2)(1-2ax+x^2)^2}{8b^2x^3}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int\frac{(-1+x^2)(1-2ax+x^2)^2}{x^3}\,dx,x,e^{\cosh^{-1}(a+bx)}\right)}{8b^3}$$
$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{x^3}+\frac{4a}{x^2}+\frac{-1-4a^2}{x}+(1+4a^2)x-4ax^2+x^3\right)dx,x,e^{\cosh^{-1}(a+bx)}\right)}{8b^3}$$
$$= \frac{e^{-2\cosh^{-1}(a+bx)}}{16b^3}-\frac{ae^{-\cosh^{-1}(a+bx)}}{2b^3}+\frac{(1+4a^2)e^{2\cosh^{-1}(a+bx)}}{16b^3}-\frac{ae^{3\cosh^{-1}(a+bx)}}{6b^3}+\frac{e^{4\cosh^{-1}(a+bx)}}{32b^3}$$

Mathematica [A] time = 0.14, size = 119, normalized size = 1.03

$$\frac{-3(4a^2+1)\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)+\sqrt{a+bx-1}\sqrt{a+bx+1}\left(2a^3-2a^2bx+a(2b^2x^2+13)+\right.}{24b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]*x^2,x]
```

```
[Out] (8*a*b^3*x^3 + 6*b^4*x^4 + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(2*a^3 - 3*
b*x - 2*a^2*b*x + 6*b^3*x^3 + a*(13 + 2*b^2*x^2)) - 3*(1 + 4*a^2)*Log[a + b
*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(24*b^3)
```

fricas [A] time = 0.55, size = 112, normalized size = 0.97

$$\frac{6\,b^4x^4+8\,ab^3x^3+\left(6\,b^3x^3+2\,ab^2x^2+2\,a^3-(2\,a^2+3)bx+13\,a\right)\sqrt{bx+a+1}\sqrt{bx+a-1}+3\,(4\,a^2+1)\log\left(-b\right.}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="fricas"
)
```

```
[Out] 1/24*(6*b^4*x^4 + 8*a*b^3*x^3 + (6*b^3*x^3 + 2*a*b^2*x^2 + 2*a^3 - (2*a^2 +
 3)*b*x + 13*a)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 3*(4*a^2 + 1)*log(-b*
x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a))/b^3
```

giac [B] time = 0.25, size = 417, normalized size = 3.63

$$6\,b^2x^4+8\,abx^3+4\sqrt{bx+a+1}\sqrt{bx+a-1}\left((bx+a+1)\left(\frac{2\,(bx+a+1)}{b^2}-\frac{6\,ab^6+7\,b^6}{b^8}\right)+\frac{3\left(2\,a^2b^6+6\,ab^6+3\,b^6\right)}{b^8}\right)+4\left(\sqrt{bx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="giac")
```

```
[Out] 1/24*(6*b^2*x^4 + 8*a*b*x^3 + 4*sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*((b*x +
 a + 1)*(2*(b*x + a + 1)/b^2 - (6*a*b^6 + 7*b^6)/b^8) + 3*(2*a^2*b^6 + 6*a*
b^6 + 3*b^6)/b^8) + 4*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*((b*x + a + 1)*(
2*(b*x + a + 1)/b^2 - (6*a*b^6 + 7*b^6)/b^8) + 3*(2*a^2*b^6 + 6*a*b^6 + 3*b
^6)/b^8) + 6*(2*a^2 + 2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))/b
```

```
^2)*a + (((b*x + a + 1)*(2*(b*x + a + 1)*(3*(b*x + a + 1)/b^3 - (12*a*b^12
+ 13*b^12)/b^15) + (36*a^2*b^12 + 84*a*b^12 + 43*b^12)/b^15) - 3*(8*a^3*b^1
2 + 36*a^2*b^12 + 36*a*b^12 + 13*b^12)/b^15)*sqrt(b*x + a + 1)*sqrt(b*x + a
 - 1) - 6*(8*a^3 + 12*a^2 + 12*a + 3)*log(sqrt(b*x + a + 1) - sqrt(b*x + a
- 1))/b^3)*b + 24*(2*a^2 + 2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a -
1))/b^2)/b
```

maple [C] time = 0.01, size = 288, normalized size = 2.50

$$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(6\,\mathrm{csgn}(b)x^3b^3\sqrt{b^2x^2+2abx+a^2-1}+2\,\mathrm{csgn}(b)x^2a\,b^2\sqrt{b^2x^2+2abx+a^2-1}-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x)
```

```
[Out] 1/24*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(6*csgn(b)*x^3*b^3*(b^2*x^2+2*a*b*x+a^
2-1)^(1/2)+2*csgn(b)*x^2*a*b^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-2*csgn(b)*(b^2
*x^2+2*a*b*x+a^2-1)^(1/2)*x*a^2*b+2*csgn(b)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*a
^3-3*csgn(b)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x*b+13*csgn(b)*(b^2*x^2+2*a*b*x+
a^2-1)^(1/2)*a-12*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))
*a^2-3*ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b)))*csgn(b)/b
^3/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+1/4*b*x^4+1/3*x^3*a
```

maxima [A] time = 0.32, size = 275, normalized size = 2.39

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3+\frac{\left(b^2x^2+2\,abx+a^2-1\right)^{\frac{3}{2}}x}{4\,b^2}-\frac{5\left(b^2x^2+2\,abx+a^2-1\right)^{\frac{3}{2}}a}{12\,b^3}-\frac{\left(5\,a^2b^2-\left(a^2-1\right)b^2\right)a^2\log\left(2\,b^2x+2\right.}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x^2,x, algorithm="maxima"
)
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3 + 1/4*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)*x/b^2 - 5/1
2*(b^2*x^2 + 2*a*b*x + a^2 - 1)^(3/2)*a/b^3 - 1/8*(5*a^2*b^2 - (a^2 - 1)*b^
2)*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 1
/8*(5*a^2*b^2 - (a^2 - 1)*b^2)*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x/b^4 + 1/
8*(5*a^2*b^2 - (a^2 - 1)*b^2)*(a^2 - 1)*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*b)/b^5 + 1/8*(5*a^2*b^2 - (a^2 - 1)*b^2)*sqrt(b^2*x^
2 + 2*a*b*x + a^2 - 1)*a/b^5
```

mupad [B] time = 33.48, size = 1067, normalized size = 9.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x),x)
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(2*a^2 + 1/
2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))) + (((a - 1)^(1/2) - (a + b*
x - 1)^(1/2))^15*(2*a^2 + 1/2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^
15) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*((64*a^4)/3 - 58*a^2 + 35/2)
)/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) + (((a - 1)^(1/2) - (a + b*
x - 1)^(1/2))^13*((64*a^4)/3 - 58*a^2 + 35/2))/(b^3*((a + 1)^(1/2) - (a + b
*x + 1)^(1/2))^13) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*((2368*a^4)/3
 - 862*a^2 + 273/2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5) + (((a -
 1)^(1/2) - (a + b*x - 1)^(1/2))^11*((2368*a^4)/3 - 862*a^2 + 273/2))/(b^3*
((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^11) + (((a - 1)^(1/2) - (a + b*x - 1)
^(1/2))^7*((9856*a^4)/3 - 3178*a^2 + 715/2))/(b^3*((a + 1)^(1/2) - (a + b*x
```

```
 + 1)^(1/2))^7) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^9*((9856*a^4)/3 -
3178*a^2 + 715/2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^9) + (((a - 1
)^(1/2) - (a + b*x - 1)^(1/2))^4*(192*a - 192*a^3)*(a - 1)^(1/2)*(a + 1)^(1
/2))/(b^3*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4) + (((a - 1)^(1/2) - (a +
 b*x - 1)^(1/2))^12*(192*a - 192*a^3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^3*((a
 + 1)^(1/2) - (a + b*x + 1)^(1/2))^12) + (((a - 1)^(1/2) - (a + b*x - 1)^(1
/2))^6*((2816*a)/3 - (5888*a^3)/3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^3*((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^6) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))
^10*((2816*a)/3 - (5888*a^3)/3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^3*((a + 1)^
(1/2) - (a + b*x + 1)^(1/2))^10) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8
*((5504*a)/3 - (11648*a^3)/3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^3*((a + 1)^(1
/2) - (a + b*x + 1)^(1/2))^8))/((28*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4
)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4 - (8*((a - 1)^(1/2) - (a + b*x -
1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (56*((a - 1)^(1/2) -
 (a + b*x - 1)^(1/2))^6)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6 + (70*((a
- 1)^(1/2) - (a + b*x - 1)^(1/2))^8)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^
8 - (56*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10)/((a + 1)^(1/2) - (a + b*x
 + 1)^(1/2))^10 + (28*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^12)/((a + 1)^(1
/2) - (a + b*x + 1)^(1/2))^12 - (8*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^14
)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^14 + ((a - 1)^(1/2) - (a + b*x - 1)
^(1/2))^16/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^16 + 1) - (2*atanh(((a - 1
)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)))*(a^2
+ 1/4))/b^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx + \sqrt{a + bx - 1}\,\sqrt{a + bx + 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x**2,x)
```

```
[Out] Integral(x**2*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)
```

3.277 $\int e^{\cosh^{-1}(a+bx)} x \, dx$

Optimal. Leaf size=67

$$-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}+\frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}$$

```
[Out] 1/4/b^2/(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))-1/4*a*(b*x+a+(b*x+a-1)^(1/2
)*(b*x+a+1)^(1/2))^2/b^2+1/12*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^3/b^2
+1/2*a*arccosh(b*x+a)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5899, 2282, 12, 1628}

$$-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}+\frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]*x,x]
```

```
[Out] 1/(4*b^2*E^ArcCosh[a + b*x]) - (a*E^(2*ArcCosh[a + b*x]))/(4*b^2) + E^(3*Ar
cCosh[a + b*x])/(12*b^2) + (a*ArcCosh[a + b*x])/(2*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)}x\,dx = \frac{\text{Subst}\left(\int e^x\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)(-1+2ax-x^2)}{4bx^2}\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)(-1+2ax-x^2)}{x^2}\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{4b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{2a}{x}-2ax+x^2\right)dx, x, e^{\cosh^{-1}(a+bx)}\right)}{4b^2}$$
$$= \frac{e^{-\cosh^{-1}(a+bx)}}{4b^2}-\frac{ae^{2\cosh^{-1}(a+bx)}}{4b^2}+\frac{e^{3\cosh^{-1}(a+bx)}}{12b^2}+\frac{a\cosh^{-1}(a+bx)}{2b^2}$$

Mathematica [A] time = 0.17, size = 93, normalized size = 1.39

$$\frac{1}{6}\left(\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-a^2+abx+2b^2x^2-2\right)}{b^2}+\frac{3a\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)}{b^2}+3ax^2+2b\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]*x,x]
```

```
[Out] (3*a*x^2 + 2*b*x^3 + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-2 - a^2 + a*b*
x + 2*b^2*x^2))/b^2 + (3*a*Log[a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*
x]])/b^2)/6
```

fricas [A] time = 0.62, size = 88, normalized size = 1.31

$$\frac{2\,b^3x^3+3\,ab^2x^2+\left(2\,b^2x^2+abx-a^2-2\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1}-3\,a\log\left(-bx+\sqrt{bx+a+1}\,\sqrt{bx+a-1}-a\right)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 + 3*a*b^2*x^2 + (2*b^2*x^2 + a*b*x - a^2 - 2)*sqrt(b*x + a +
 1)*sqrt(b*x + a - 1) - 3*a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1)
- a))/b^2
```

giac [B] time = 0.51, size = 280, normalized size = 4.18

$$2\,b^2x^3+\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}\left((bx+a+1)\left(\frac{2\,(bx+a+1)}{b^2}-\frac{6\,ab^6+7\,b^6}{b^8}\right)+\frac{3\left(2\,a^2b^6+6\,ab^6+3\,b^6\right)}{b^8}\right)+\frac{6\left(2\,a^2+2\,a+1\right)\log\left(\sqrt{bx+\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*x^3 + (sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*((b*x + a + 1)*(2*(b*
x + a + 1)/b^2 - (6*a*b^6 + 7*b^6)/b^8) + 3*(2*a^2*b^6 + 6*a*b^6 + 3*b^6)/b
^8) + 6*(2*a^2 + 2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))/b^2)*b
 + 3*((b*x + a + 1)^2 - 2*(b*x + a + 1)*a - 2*b*x - 2*a - 2)*a/b + 3*(sqrt(
b*x + a + 1)*sqrt(b*x + a - 1)*(b*x - a - 2) - 2*(2*a + 1)*log(sqrt(b*x + a
 + 1) - sqrt(b*x + a - 1)))*a/b + 3*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*(b
*x - a - 2) - 2*(2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1)))/b)/b
```

maple [C] time = 0.01, size = 194, normalized size = 2.90

$$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(2\sqrt{b^2x^2+2abx+a^2-1}\,\mathrm{csgn}(b)x^2b^2+\sqrt{b^2x^2+2abx+a^2-1}\,\mathrm{csgn}(b)xab-\sqrt{b^2x}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x)
```

```
[Out] 1/6*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b
)*x^2*b^2+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*x*a*b-(b^2*x^2+2*a*b*x+a^2-
1)^(1/2)*csgn(b)*a^2-2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+3*ln(((b^2*x^2
+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*a)*csgn(b)/b^2/(b^2*x^2+2*a*b
*x+a^2-1)^(1/2)+1/3*b*x^3+1/2*a*x^2
```

maxima [A] time = 0.31, size = 177, normalized size = 2.64

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2+\frac{a^3\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+2\,abx+a^2-1}\,b\right)}{2\,b^2}-\frac{\sqrt{b^2x^2+2\,abx+a^2-1}\,ax}{2\,b}-\frac{\left(a^2-1\right)a\log\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))*x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2 + 1/2*a^3*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*
b*x + a^2 - 1)*b)/b^2 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a*x/b - 1/2*(
a^2 - 1)*a*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b^2
 - 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*a^2/b^2 + 1/3*(b^2*x^2 + 2*a*b*x +
 a^2 - 1)^(3/2)/b^2
```

mupad [B] time = 16.72, size = 852, normalized size = 12.72

$$\frac{a\,x^2}{2}-\frac{\frac{2\,a\,\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}{b^2\,\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^3\,\left(42\,a-\frac{160\,a^3}{3}\right)}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^3}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^9\,\left(42\,a-\frac{160\,a^3}{3}\right)}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^9}+\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)}^5\,\left(212\,a-2\right.}{b^2\,{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)}^5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x),x)
```

```
[Out] (a*x^2)/2 - ((2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2)))/(b^2*((a + 1)^(1/2
) - (a + b*x + 1)^(1/2))) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*(42*a
- (160*a^3)/3))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) + (((a - 1)^(
1/2) - (a + b*x - 1)^(1/2))^9*(42*a - (160*a^3)/3))/(b^2*((a + 1)^(1/2) - (
a + b*x + 1)^(1/2))^9) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*(212*a -
288*a^3))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5) + (((a - 1)^(1/2) -
 (a + b*x - 1)^(1/2))^7*(212*a - 288*a^3))/(b^2*((a + 1)^(1/2) - (a + b*x +
 1)^(1/2))^7) + (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^11)/(b^2*((a + 1
)^(1/2) - (a + b*x + 1)^(1/2))^11) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))
^2*(8*a^2 - 8)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a + 1)^(1/2) - (a + b*x
+ 1)^(1/2))^2) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10*(8*a^2 - 8)*(a -
 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^10) + (
((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(160*a^2 - 32)*(a - 1)^(1/2)*(a + 1
)^(1/2))/(b^2*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4) + (((a - 1)^(1/2) -
(a + b*x - 1)^(1/2))^8*(160*a^2 - 32)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a
 + 1)^(1/2) - (a + b*x + 1)^(1/2))^8) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/
2))^6*((1040*a^2)/3 - 272/3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(b^2*((a + 1)^(1/
```

```
2) - (a + b*x + 1)^(1/2))^6))/((15*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4)
/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4 - (6*((a - 1)^(1/2) - (a + b*x - 1
)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (20*((a - 1)^(1/2) -
(a + b*x - 1)^(1/2))^6)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6 + (15*((a -
 1)^(1/2) - (a + b*x - 1)^(1/2))^8)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8
 - (6*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10)/((a + 1)^(1/2) - (a + b*x +
 1)^(1/2))^10 + ((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^12/((a + 1)^(1/2) - (
a + b*x + 1)^(1/2))^12 + 1) + (b*x^3)/3 + (2*a*atanh(((a - 1)^(1/2) - (a +
b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))))/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx+\sqrt{a+bx-1}\,\sqrt{a+bx+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))*x,x)
```

```
[Out] Integral(x*(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1)), x)
```

3.278 $\int e^{\cosh^{-1}(a+bx)}\,dx$

Optimal. Leaf size=31

$$\frac{e^{2\cosh^{-1}(a+bx)}}{4b}-\frac{\cosh^{-1}(a+bx)}{2b}$$

```
[Out] 1/4*(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2/b-1/2*arccosh(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5897, 2282, 12, 14}

$$\frac{e^{2\cosh^{-1}(a+bx)}}{4b}-\frac{\cosh^{-1}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x],x]
```

```
[Out] E^(2*ArcCosh[a + b*x])/(4*b) - ArcCosh[a + b*x]/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5897

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Sinh[x], x], x, ArcCosh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)}\,dx = \frac{\text{Subst}\left(\int e^x\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{\cosh^{-1}(a+bx)}\right)}{2b}$$
$$= \frac{e^{2\cosh^{-1}(a+bx)}}{4b} - \frac{\cosh^{-1}(a+bx)}{2b}$$

Mathematica [**B**] time = 0.03, size = 69, normalized size = 2.23

$$\frac{(a+bx)\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)-\log\left(\sqrt{a+bx-1}\sqrt{a+bx+1}+a+bx\right)}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x],x]
```

```
[Out] ((a + b*x)*(a + b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]) - Log[a + b*x +
 Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]])/(2*b)
```

fricas [**A**] time = 0.53, size = 66, normalized size = 2.13

$$\frac{b^2x^2+2\,abx+\sqrt{bx+a+1}\,(bx+a)\sqrt{bx+a-1}+\log\left(-bx+\sqrt{bx+a+1}\,\sqrt{bx+a-1}-a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 + 2*a*b*x + sqrt(b*x + a + 1)*(b*x + a)*sqrt(b*x + a - 1) + lo
g(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a))/b
```

giac [**B**] time = 0.26, size = 151, normalized size = 4.87

$$\frac{1}{2}bx^2+ax+\frac{\sqrt{bx+a+1}\,\sqrt{bx+a-1}\,(bx-a-2)+2\left(\sqrt{bx+a+1}\,\sqrt{bx+a-1}+2\log\left(\sqrt{bx+a+1}-\sqrt{bx+a}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + 1/2*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1)*(b*x - a - 2) +
2*(sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 2*log(sqrt(b*x + a + 1) - sqrt(b*x
 + a - 1)))*a - 2*(2*a + 1)*log(sqrt(b*x + a + 1) - sqrt(b*x + a - 1)) + 2*
sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 4*log(sqrt(b*x + a + 1) - sqrt(b*x +
a - 1)))/b
```

maple [**B**] time = 0.02, size = 147, normalized size = 4.74

$$\frac{b\,x^2}{2}+ax+\frac{\sqrt{bx+a-1}\,\left(bx+a+1\right)^{\frac{3}{2}}}{2b}-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}}{2b}-\frac{\sqrt{(bx+a-1)\,(bx+a+1)}\,\ln\left(\frac{\frac{(a-1)b}{2}+\frac{b(1+a)}{2}+b^2x}{\sqrt{b^2}}\right.}{2\sqrt{bx+a+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x)
```

```
[Out] 1/2*b*x^2+a*x+1/2/b*(b*x+a-1)^(1/2)*(b*x+a+1)^(3/2)-1/2*(b*x+a-1)^(1/2)*(b*
x+a+1)^(1/2)/b-1/2*((b*x+a-1)*(b*x+a+1))^(1/2)/(b*x+a+1)^(1/2)/(b*x+a-1)^(1
/2)*ln((1/2*(a-1)*b+1/2*b*(1+a)+b^2*x)/(b^2)^(1/2)+(b^2*x^2+((a-1)*b+b*(1+a
))*x+(a-1)*(1+a))^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 0.31, size = 143, normalized size = 4.61

$$\frac{1}{2}bx^2+ax-\frac{a^2\log\left(2b^2x+2ab+2\sqrt{b^2x^2+2abx+a^2-1}\,b\right)}{2b}+\frac{1}{2}\sqrt{b^2x^2+2abx+a^2-1}\,x+\frac{\left(a^2-1\right)\log\left(2b^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x - 1/2*a^2*log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x +
a^2 - 1)*b)/b + 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*x + 1/2*(a^2 - 1)*log
(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2 - 1)*b)/b + 1/2*sqrt(b^2*
x^2 + 2*a*b*x + a^2 - 1)*a/b
```

mupad [B] time = 0.17, size = 79, normalized size = 2.55

$$a\,x+\frac{b\,x^2}{2}-\frac{\ln\left(a+\sqrt{a+b\,x-1}\,\sqrt{a+b\,x+1}+b\,x\right)}{2\,b}+\frac{x\,\sqrt{a+b\,x-1}\,\sqrt{a+b\,x+1}}{2}+\frac{a\,\sqrt{a+b\,x-1}\,\sqrt{a+b\,x+1}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x,x)
```

```
[Out] a*x + (b*x^2)/2 - log(a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/(2
*b) + (x*(a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2))/2 + (a*(a + b*x - 1)^(1/2
)*(a + b*x + 1)^(1/2))/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+bx+\sqrt{a+bx-1}\,\sqrt{a+bx+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2),x)
```

```
[Out] Integral(a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1), x)
```

3.279 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x}\,dx$

Optimal. Leaf size=100

$$2\sqrt{1-a^2}\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)+\sqrt{a+bx-1}\sqrt{a+bx+1}+2a\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)+a\log(x)+bx$$

```
[Out] b*x+2*a*arcsinh(1/2*(b*x+a-1)^(1/2)*2^(1/2))+a*ln(x)+2*arctan((1-a)^(1/2)*(
b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)^(1/2))*(-a^2+1)^(1/2)+(b*x+a-1)^(1/2)*
(b*x+a+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5909, 14, 101, 157, 63, 215, 93, 205}

$$2\sqrt{1-a^2}\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)+\sqrt{a+bx-1}\sqrt{a+bx+1}+2a\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)+a\log(x)+bx$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + 2*a*ArcSinh[Sqrt[-1 + a + b*x]
/Sqrt[2]] + 2*Sqrt[1 - a^2]*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1
+ a]*Sqrt[-1 + a + b*x])] + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\cosh^{-1}(a+bx)}}{x}\,dx &= \int \frac{a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}\,dx \\
&= \int \left(b+\frac{a}{x}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}\right)dx \\
&= bx+a\log(x)+\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}\,dx \\
&= bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+a\log(x)-\int \frac{1-a^2-abx}{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}\,dx \\
&= bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+a\log(x)-\left(1-a^2\right)\int \frac{1}{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}\,dx \\
&= bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+a\log(x)+(2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx,x,\sqrt{-1+a+bx} \right. \\
&= bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}+2a\sinh^{-1}\left(\frac{\sqrt{-1+a+bx}}{\sqrt{2}}\right)+2\sqrt{1-a^2}\,\tan^{-1}\left(\frac{\sqrt{1-a}}{\sqrt{1+a}}\right.
\end{aligned}$$

Mathematica [C] time = 0.10, size = 141, normalized size = 1.41

$$i\sqrt{1-a^2}\,\log\left(\frac{2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{\left(a^2-1\right)x}+\frac{2i\left(a^2+abx-1\right)}{\sqrt{1-a^2}\,\left(a^2-1\right)x}\right)+\sqrt{a+bx-1}\,\sqrt{a+bx+1}+a\log\left(\sqrt{a+bx-1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x,x]
```

```
[Out] b*x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x] + a*Log[x] + a*Log[a + b*x + Sqr
t[-1 + a + b*x]*Sqrt[1 + a + b*x]] + I*Sqrt[1 - a^2]*Log[(2*Sqrt[-1 + a + b
*x]*Sqrt[1 + a + b*x])/((-1 + a^2)*x) + ((2*I)*(-1 + a^2 + a*b*x))/(Sqrt[1
- a^2]*(-1 + a^2)*x)]
```

fricas [A] time = 0.98, size = 246, normalized size = 2.46

$$\left[bx - a\log\left(-bx + \sqrt{bx+a+1}\,\sqrt{bx+a-1} - a\right) + a\log(x) + \sqrt{a^2-1}\,\log\left(\frac{a^2bx + a^3 + \left(a^2 - \sqrt{a^2-1}\,a - 1\right)\sqrt{bx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] [b*x - a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + a*log(x) + s
qrt(a^2 - 1)*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a - 1)*sqrt(b*x + a
+ 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) + sqrt(b*x
 + a + 1)*sqrt(b*x + a - 1), b*x - a*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x
+ a - 1) - a) + a*log(x) + 2*sqrt(-a^2 + 1)*arctan(-(sqrt(-a^2 + 1)*b*x - s
qrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + sqrt(b*x +
a + 1)*sqrt(b*x + a - 1)]
```

giac [A] time = 0.23, size = 103, normalized size = 1.03

$$bx - a\log\left(\left(\sqrt{bx+a+1} - \sqrt{bx+a-1}\right)^2\right) + a\log\left(|bx|\right) + 2\sqrt{-a^2+1}\,\arctan\left(\frac{\left(\sqrt{bx+a+1} - \sqrt{bx+a-1}\right)^2 - 2a}{2\sqrt{-a^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="giac")
```

```
[Out] b*x - a*log((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2) + a*log(abs(b*x)) +
2*sqrt(-a^2 + 1)*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*
a)/sqrt(-a^2 + 1)) + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + a + 1
```

maple [C] time = 0.01, size = 156, normalized size = 1.56

$$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(-\mathrm{csgn}(b)\ln\left(\frac{2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}+2a^2-2}{x}\right)\sqrt{a^2-1} + \ln\left(\left(\sqrt{b^2x^2+2abx+a^2-\right.\right.\right.}{\sqrt{b^2x^2+2abx+a^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x)
```

```
[Out] (b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(-csgn(b)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^
2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*(a^2-1)^(1/2)+ln(((b^2*x^2+2*a*b*x+a^2-1)^
(1/2)*csgn(b)+b*x+a)*csgn(b))*a+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b))*csgn
(b)/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+b*x+a*ln(x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 zero or nonzero?
```

mupad [B] time = 23.41, size = 8883, normalized size = 88.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x,x)
```

```
[Out] b*x + a*log(x) - a*atan(-(a*(2*a*((32*(a*(a - 1)^(3/2)*(a + 1)^(3/2) - 2*a*
(a - 1)^(1/2)*(a + 1)^(1/2) + 22*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) - 68*a^5*(
a - 1)^(1/2)*(a + 1)^(1/2) - 4*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) + 92*a^7*(a
- 1)^(1/2)*(a + 1)^(1/2) - 22*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) - 58*a^9*(a -
 1)^(1/2)*(a + 1)^(1/2) - 5*a^3*(a - 1)^(5/2)*(a + 1)^(5/2) + 52*a^7*(a - 1
)^(3/2)*(a + 1)^(3/2) + 14*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 12*a^5*(a - 1
)^(5/2)*(a + 1)^(5/2) - 27*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + 9*a^7*(a - 1)^
(5/2)*(a + 1)^(5/2) + 4*a^5*(a - 1)^(7/2)*(a + 1)^(7/2)))/(5*a^2 - 10*a^4 +
 10*a^6 - 5*a^8 + a^10 - 1) - 2*a*((32*(2*a - 10*a^3 + 20*a^5 - 20*a^7 + 10
*a^9 - 2*a^11 - 2*a*(a - 1)*(a + 1) + 2*a^3*(a - 1)^2*(a + 1)^2 - 6*a^5*(a
- 1)^2*(a + 1)^2 + 4*a^7*(a - 1)^2*(a + 1)^2 + 8*a^3*(a - 1)*(a + 1) - 12*a
^5*(a - 1)*(a + 1) + 8*a^7*(a - 1)*(a + 1) - 2*a^9*(a - 1)*(a + 1)))/(5*a^2
 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(a*(a - 1)^(1/2)*(a + 1)^
(1/2) - 4*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 6*a^5*(a - 1)^(1/2)*(a + 1)^(1/
2) - 5*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) - 4*a^7*(a - 1)^(1/2)*(a + 1)^(1/2)
+ 10*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + a^9*(a - 1)^(1/2)*(a + 1)^(1/2) - 5*
a^7*(a - 1)^(3/2)*(a + 1)^(3/2) + 4*a^5*(a - 1)^(5/2)*(a + 1)^(5/2)))/(5*a^
2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) - (32*((a - 1)^(1/2) - (a + b*x - 1
)^(1/2))*(60*a^2 - 150*a^4 + 200*a^6 - 150*a^8 + 60*a^10 - 10*a^12 - 39*a^4
*(a - 1)^2*(a + 1)^2 + 78*a^6*(a - 1)^2*(a + 1)^2 + 16*a^6*(a - 1)^3*(a + 1
)^3 - 39*a^8*(a - 1)^2*(a + 1)^2 + 33*a^2*(a - 1)*(a + 1) - 132*a^4*(a - 1)
*(a + 1) + 198*a^6*(a - 1)*(a + 1) - 132*a^8*(a - 1)*(a + 1) + 33*a^10*(a -
 1)*(a + 1) - 10))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 -
 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1
)^(1/2))*(17*(a - 1)^(1/2)*(a + 1)^(1/2) - 106*a^2*(a - 1)^(1/2)*(a + 1)^(1
/2) + 275*a^4*(a - 1)^(1/2)*(a + 1)^(1/2) - 48*a^2*(a - 1)^(3/2)*(a + 1)^(3
/2) - 380*a^6*(a - 1)^(1/2)*(a + 1)^(1/2) + 196*a^4*(a - 1)^(3/2)*(a + 1)^(
3/2) + 295*a^8*(a - 1)^(1/2)*(a + 1)^(1/2) - 300*a^6*(a - 1)^(3/2)*(a + 1)^
(3/2) - 122*a^10*(a - 1)^(1/2)*(a + 1)^(1/2) + 47*a^4*(a - 1)^(5/2)*(a + 1)
^(5/2) + 204*a^8*(a - 1)^(3/2)*(a + 1)^(3/2) + 21*a^12*(a - 1)^(1/2)*(a + 1
)^(1/2) - 94*a^6*(a - 1)^(5/2)*(a + 1)^(5/2) - 52*a^10*(a - 1)^(3/2)*(a + 1
)^(3/2) + 47*a^8*(a - 1)^(5/2)*(a + 1)^(5/2) - 16*a^6*(a - 1)^(7/2)*(a + 1)
^(7/2)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 +
15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(
36*a^2 - 216*a^4 + 540*a^6 - 720*a^8 + 540*a^10 - 216*a^12 + 36*a^14 - 6*(a
 - 1)*(a + 1) + 15*a^2*(a - 1)^2*(a + 1)^2 - 64*a^4*(a - 1)^2*(a + 1)^2 - 9
*a^4*(a - 1)^3*(a + 1)^3 + 262*a^6*(a - 1)^2*(a + 1)^2 + 18*a^6*(a - 1)^3*(
a + 1)^3 - 392*a^8*(a - 1)^2*(a + 1)^2 - 73*a^8*(a - 1)^3*(a + 1)^3 + 179*a
^10*(a - 1)^2*(a + 1)^2 + 40*a^2*(a - 1)*(a + 1) - 242*a^4*(a - 1)*(a + 1)
+ 688*a^6*(a - 1)*(a + 1) - 922*a^8*(a - 1)*(a + 1) + 584*a^10*(a - 1)*(a +
 1) - 142*a^12*(a - 1)*(a + 1)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15
*a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) - (32*(2*a*(a - 1)^2*
(a + 1)^2 - 2*a*(a - 1)*(a + 1) - 8*a^3*(a - 1)^2*(a + 1)^2 - 2*a^3*(a - 1)
^3*(a + 1)^3 + 12*a^5*(a - 1)^2*(a + 1)^2 + 6*a^5*(a - 1)^3*(a + 1)^3 - 8*a
^7*(a - 1)^2*(a + 1)^2 - 4*a^7*(a - 1)^3*(a + 1)^3 + 2*a^9*(a - 1)^2*(a + 1
)^2 + 10*a^3*(a - 1)*(a + 1) - 20*a^5*(a - 1)*(a + 1) + 20*a^7*(a - 1)*(a +
 1) - 10*a^9*(a - 1)*(a + 1) + 2*a^11*(a - 1)*(a + 1)))/(5*a^2 - 10*a^4 + 1
0*a^6 - 5*a^8 + a^10 - 1) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*((a -
 1)^(3/2)*(a + 1)^(3/2) + 64*a^2*(a - 1)^(1/2)*(a + 1)^(1/2) - 384*a^4*(a -
 1)^(1/2)*(a + 1)^(1/2) - 10*a^2*(a - 1)^(3/2)*(a + 1)^(3/2) + 960*a^6*(a -
 1)^(1/2)*(a + 1)^(1/2) - 157*a^4*(a - 1)^(3/2)*(a + 1)^(3/2) - 1280*a^8*(a
 - 1)^(1/2)*(a + 1)^(1/2) + 708*a^6*(a - 1)^(3/2)*(a + 1)^(3/2) + 960*a^10*
(a - 1)^(1/2)*(a + 1)^(1/2) + 4*a^4*(a - 1)^(5/2)*(a + 1)^(5/2) - 1097*a^8*
(a - 1)^(3/2)*(a + 1)^(3/2) - 384*a^12*(a - 1)^(1/2)*(a + 1)^(1/2) + 180*a^
6*(a - 1)^(5/2)*(a + 1)^(5/2) + 742*a^10*(a - 1)^(3/2)*(a + 1)^(3/2) + 64*a
^14*(a - 1)^(1/2)*(a + 1)^(1/2) - a^4*(a - 1)^(7/2)*(a + 1)^(7/2) - 372*a^8
```

(a - 1)^(5/2)(a + 1)^(5/2) - 187*a^12*(a - 1)^(3/2)*(a + 1)^(3/2) + 2*a^6
(a - 1)^(7/2)(a + 1)^(7/2) + 188*a^10*(a - 1)^(5/2)*(a + 1)^(5/2) - 65*a^
8*(a - 1)^(7/2)*(a + 1)^(7/2)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*
a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1)))*2i - a*((32*(2*a*(a -
1)^2*(a + 1)^2 - 2*a*(a - 1)*(a + 1) - 8*a^3*(a - 1)^2*(a + 1)^2 - 2*a^3*(a
- 1)^3*(a + 1)^3 + 12*a^5*(a - 1)^2*(a + 1)^2 + 6*a^5*(a - 1)^3*(a + 1)^3
- 8*a^7*(a - 1)^2*(a + 1)^2 - 4*a^7*(a - 1)^3*(a + 1)^3 + 2*a^9*(a - 1)^2*(
a + 1)^2 + 10*a^3*(a - 1)*(a + 1) - 20*a^5*(a - 1)*(a + 1) + 20*a^7*(a - 1)
*(a + 1) - 10*a^9*(a - 1)*(a + 1) + 2*a^11*(a - 1)*(a + 1)))/(5*a^2 - 10*a^
4 + 10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(a*(a - 1)^(3/2)*(a + 1)^(3/2) -
2*a*(a - 1)^(1/2)*(a + 1)^(1/2) + 22*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) - 68*a
^5*(a - 1)^(1/2)*(a + 1)^(1/2) - 4*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) + 92*a^7
(a - 1)^(1/2)(a + 1)^(1/2) - 22*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) - 58*a^9*
(a - 1)^(1/2)*(a + 1)^(1/2) - 5*a^3*(a - 1)^(5/2)*(a + 1)^(5/2) + 52*a^7*(a
- 1)^(3/2)*(a + 1)^(3/2) + 14*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 12*a^5*(a
- 1)^(5/2)*(a + 1)^(5/2) - 27*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + 9*a^7*(a -
1)^(5/2)*(a + 1)^(5/2) + 4*a^5*(a - 1)^(7/2)*(a + 1)^(7/2)))/(5*a^2 - 10*a
^4 + 10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(2*a - 10*a^3 + 20*a^5 - 20*a^7
+ 10*a^9 - 2*a^11 - 2*a*(a - 1)*(a + 1) + 2*a^3*(a - 1)^2*(a + 1)^2 - 6*a^5
(a - 1)^2(a + 1)^2 + 4*a^7*(a - 1)^2*(a + 1)^2 + 8*a^3*(a - 1)*(a + 1) -
12*a^5*(a - 1)*(a + 1) + 8*a^7*(a - 1)*(a + 1) - 2*a^9*(a - 1)*(a + 1)))/(5
*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) - 2*a*((32*(a*(a - 1)^(1/2)*(a +
1)^(1/2) - 4*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 6*a^5*(a - 1)^(1/2)*(a + 1)
^(1/2) - 5*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) - 4*a^7*(a - 1)^(1/2)*(a + 1)^(1
/2) + 10*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + a^9*(a - 1)^(1/2)*(a + 1)^(1/2)
- 5*a^7*(a - 1)^(3/2)*(a + 1)^(3/2) + 4*a^5*(a - 1)^(5/2)*(a + 1)^(5/2)))/(
5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) - (32*((a - 1)^(1/2) - (a + b*x
- 1)^(1/2))*(60*a^2 - 150*a^4 + 200*a^6 - 150*a^8 + 60*a^10 - 10*a^12 - 39
a^4(a - 1)^2*(a + 1)^2 + 78*a^6*(a - 1)^2*(a + 1)^2 + 16*a^6*(a - 1)^3*(a
+ 1)^3 - 39*a^8*(a - 1)^2*(a + 1)^2 + 33*a^2*(a - 1)*(a + 1) - 132*a^4*(a
- 1)*(a + 1) + 198*a^6*(a - 1)*(a + 1) - 132*a^8*(a - 1)*(a + 1) + 33*a^10*
(a - 1)*(a + 1) - 10))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a
^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x
- 1)^(1/2))*(17*(a - 1)^(1/2)*(a + 1)^(1/2) - 106*a^2*(a - 1)^(1/2)*(a + 1
)^(1/2) + 275*a^4*(a - 1)^(1/2)*(a + 1)^(1/2) - 48*a^2*(a - 1)^(3/2)*(a + 1
)^(3/2) - 380*a^6*(a - 1)^(1/2)*(a + 1)^(1/2) + 196*a^4*(a - 1)^(3/2)*(a +
1)^(3/2) + 295*a^8*(a - 1)^(1/2)*(a + 1)^(1/2) - 300*a^6*(a - 1)^(3/2)*(a +
1)^(3/2) - 122*a^10*(a - 1)^(1/2)*(a + 1)^(1/2) + 47*a^4*(a - 1)^(5/2)*(a
+ 1)^(5/2) + 204*a^8*(a - 1)^(3/2)*(a + 1)^(3/2) + 21*a^12*(a - 1)^(1/2)*(a
+ 1)^(1/2) - 94*a^6*(a - 1)^(5/2)*(a + 1)^(5/2) - 52*a^10*(a - 1)^(3/2)*(a
+ 1)^(3/2) + 47*a^8*(a - 1)^(5/2)*(a + 1)^(5/2) - 16*a^6*(a - 1)^(7/2)*(a
+ 1)^(7/2)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^
6 + 15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2
))*(36*a^2 - 216*a^4 + 540*a^6 - 720*a^8 + 540*a^10 - 216*a^12 + 36*a^14 -
6*(a - 1)*(a + 1) + 15*a^2*(a - 1)^2*(a + 1)^2 - 64*a^4*(a - 1)^2*(a + 1)^2
- 9*a^4*(a - 1)^3*(a + 1)^3 + 262*a^6*(a - 1)^2*(a + 1)^2 + 18*a^6*(a - 1)
^3*(a + 1)^3 - 392*a^8*(a - 1)^2*(a + 1)^2 - 73*a^8*(a - 1)^3*(a + 1)^3 + 1
79*a^10*(a - 1)^2*(a + 1)^2 + 40*a^2*(a - 1)*(a + 1) - 242*a^4*(a - 1)*(a +
1) + 688*a^6*(a - 1)*(a + 1) - 922*a^8*(a - 1)*(a + 1) + 584*a^10*(a - 1)*
(a + 1) - 142*a^12*(a - 1)*(a + 1)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))
*(15*a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) - (32*((a - 1)^(1
/2) - (a + b*x - 1)^(1/2))*((a - 1)^(3/2)*(a + 1)^(3/2) + 64*a^2*(a - 1)^(1
/2)*(a + 1)^(1/2) - 384*a^4*(a - 1)^(1/2)*(a + 1)^(1/2) - 10*a^2*(a - 1)^(3
/2)*(a + 1)^(3/2) + 960*a^6*(a - 1)^(1/2)*(a + 1)^(1/2) - 157*a^4*(a - 1)^(
3/2)*(a + 1)^(3/2) - 1280*a^8*(a - 1)^(1/2)*(a + 1)^(1/2) + 708*a^6*(a - 1)
^(3/2)*(a + 1)^(3/2) + 960*a^10*(a - 1)^(1/2)*(a + 1)^(1/2) + 4*a^4*(a - 1)
^(5/2)*(a + 1)^(5/2) - 1097*a^8*(a - 1)^(3/2)*(a + 1)^(3/2) - 384*a^12*(a -
1)^(1/2)*(a + 1)^(1/2) + 180*a^6*(a - 1)^(5/2)*(a + 1)^(5/2) + 742*a^10*(a
- 1)^(3/2)*(a + 1)^(3/2) + 64*a^14*(a - 1)^(1/2)*(a + 1)^(1/2) - a^4*(a -

```
1)^(7/2)*(a + 1)^(7/2) - 372*a^8*(a - 1)^(5/2)*(a + 1)^(5/2) - 187*a^12*(a
- 1)^(3/2)*(a + 1)^(3/2) + 2*a^6*(a - 1)^(7/2)*(a + 1)^(7/2) + 188*a^10*(a
- 1)^(5/2)*(a + 1)^(5/2) - 65*a^8*(a - 1)^(7/2)*(a + 1)^(7/2)))/(((a + 1)^(
1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^
12 + 1)))*2i)/(2*a*(2*a*((32*(a*(a - 1)^(3/2)*(a + 1)^(3/2) - 2*a*(a - 1)^(
1/2)*(a + 1)^(1/2) + 22*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) - 68*a^5*(a - 1)^(1
/2)*(a + 1)^(1/2) - 4*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) + 92*a^7*(a - 1)^(1/2
)*(a + 1)^(1/2) - 22*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) - 58*a^9*(a - 1)^(1/2)
*(a + 1)^(1/2) - 5*a^3*(a - 1)^(5/2)*(a + 1)^(5/2) + 52*a^7*(a - 1)^(3/2)*(
a + 1)^(3/2) + 14*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 12*a^5*(a - 1)^(5/2)*(
a + 1)^(5/2) - 27*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + 9*a^7*(a - 1)^(5/2)*(a
+ 1)^(5/2) + 4*a^5*(a - 1)^(7/2)*(a + 1)^(7/2)))/(5*a^2 - 10*a^4 + 10*a^6 -
 5*a^8 + a^10 - 1) - 2*a*((32*(2*a - 10*a^3 + 20*a^5 - 20*a^7 + 10*a^9 - 2*
a^11 - 2*a*(a - 1)*(a + 1) + 2*a^3*(a - 1)^2*(a + 1)^2 - 6*a^5*(a - 1)^2*(a
 + 1)^2 + 4*a^7*(a - 1)^2*(a + 1)^2 + 8*a^3*(a - 1)*(a + 1) - 12*a^5*(a - 1
)*(a + 1) + 8*a^7*(a - 1)*(a + 1) - 2*a^9*(a - 1)*(a + 1)))/(5*a^2 - 10*a^4
 + 10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(a*(a - 1)^(1/2)*(a + 1)^(1/2) - 4
*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 6*a^5*(a - 1)^(1/2)*(a + 1)^(1/2) - 5*a^
3*(a - 1)^(3/2)*(a + 1)^(3/2) - 4*a^7*(a - 1)^(1/2)*(a + 1)^(1/2) + 10*a^5*
(a - 1)^(3/2)*(a + 1)^(3/2) + a^9*(a - 1)^(1/2)*(a + 1)^(1/2) - 5*a^7*(a -
1)^(3/2)*(a + 1)^(3/2) + 4*a^5*(a - 1)^(5/2)*(a + 1)^(5/2)))/(5*a^2 - 10*a^
4 + 10*a^6 - 5*a^8 + a^10 - 1) - (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*
(60*a^2 - 150*a^4 + 200*a^6 - 150*a^8 + 60*a^10 - 10*a^12 - 39*a^4*(a - 1)^
2*(a + 1)^2 + 78*a^6*(a - 1)^2*(a + 1)^2 + 16*a^6*(a - 1)^3*(a + 1)^3 - 39*
a^8*(a - 1)^2*(a + 1)^2 + 33*a^2*(a - 1)*(a + 1) - 132*a^4*(a - 1)*(a + 1)
+ 198*a^6*(a - 1)*(a + 1) - 132*a^8*(a - 1)*(a + 1) + 33*a^10*(a - 1)*(a +
1) - 10))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 +
 15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*
(17*(a - 1)^(1/2)*(a + 1)^(1/2) - 106*a^2*(a - 1)^(1/2)*(a + 1)^(1/2) + 275
*a^4*(a - 1)^(1/2)*(a + 1)^(1/2) - 48*a^2*(a - 1)^(3/2)*(a + 1)^(3/2) - 380
*a^6*(a - 1)^(1/2)*(a + 1)^(1/2) + 196*a^4*(a - 1)^(3/2)*(a + 1)^(3/2) + 29
5*a^8*(a - 1)^(1/2)*(a + 1)^(1/2) - 300*a^6*(a - 1)^(3/2)*(a + 1)^(3/2) - 1
22*a^10*(a - 1)^(1/2)*(a + 1)^(1/2) + 47*a^4*(a - 1)^(5/2)*(a + 1)^(5/2) +
204*a^8*(a - 1)^(3/2)*(a + 1)^(3/2) + 21*a^12*(a - 1)^(1/2)*(a + 1)^(1/2) -
 94*a^6*(a - 1)^(5/2)*(a + 1)^(5/2) - 52*a^10*(a - 1)^(3/2)*(a + 1)^(3/2) +
 47*a^8*(a - 1)^(5/2)*(a + 1)^(5/2) - 16*a^6*(a - 1)^(7/2)*(a + 1)^(7/2)))/
(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 + 15*a^8 -
6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(36*a^2 -
216*a^4 + 540*a^6 - 720*a^8 + 540*a^10 - 216*a^12 + 36*a^14 - 6*(a - 1)*(a
+ 1) + 15*a^2*(a - 1)^2*(a + 1)^2 - 64*a^4*(a - 1)^2*(a + 1)^2 - 9*a^4*(a -
 1)^3*(a + 1)^3 + 262*a^6*(a - 1)^2*(a + 1)^2 + 18*a^6*(a - 1)^3*(a + 1)^3
- 392*a^8*(a - 1)^2*(a + 1)^2 - 73*a^8*(a - 1)^3*(a + 1)^3 + 179*a^10*(a -
1)^2*(a + 1)^2 + 40*a^2*(a - 1)*(a + 1) - 242*a^4*(a - 1)*(a + 1) + 688*a^6
*(a - 1)*(a + 1) - 922*a^8*(a - 1)*(a + 1) + 584*a^10*(a - 1)*(a + 1) - 142
*a^12*(a - 1)*(a + 1)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*
a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) - (32*(2*a*(a - 1)^2*(a + 1)^2
 - 2*a*(a - 1)*(a + 1) - 8*a^3*(a - 1)^2*(a + 1)^2 - 2*a^3*(a - 1)^3*(a + 1
)^3 + 12*a^5*(a - 1)^2*(a + 1)^2 + 6*a^5*(a - 1)^3*(a + 1)^3 - 8*a^7*(a - 1
)^2*(a + 1)^2 - 4*a^7*(a - 1)^3*(a + 1)^3 + 2*a^9*(a - 1)^2*(a + 1)^2 + 10*
a^3*(a - 1)*(a + 1) - 20*a^5*(a - 1)*(a + 1) + 20*a^7*(a - 1)*(a + 1) - 10*
a^9*(a - 1)*(a + 1) + 2*a^11*(a - 1)*(a + 1)))/(5*a^2 - 10*a^4 + 10*a^6 - 5
*a^8 + a^10 - 1) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*((a - 1)^(3/2)
*(a + 1)^(3/2) + 64*a^2*(a - 1)^(1/2)*(a + 1)^(1/2) - 384*a^4*(a - 1)^(1/2)
*(a + 1)^(1/2) - 10*a^2*(a - 1)^(3/2)*(a + 1)^(3/2) + 960*a^6*(a - 1)^(1/2)
*(a + 1)^(1/2) - 157*a^4*(a - 1)^(3/2)*(a + 1)^(3/2) - 1280*a^8*(a - 1)^(1/
2)*(a + 1)^(1/2) + 708*a^6*(a - 1)^(3/2)*(a + 1)^(3/2) + 960*a^10*(a - 1)^(
1/2)*(a + 1)^(1/2) + 4*a^4*(a - 1)^(5/2)*(a + 1)^(5/2) - 1097*a^8*(a - 1)^(
3/2)*(a + 1)^(3/2) - 384*a^12*(a - 1)^(1/2)*(a + 1)^(1/2) + 180*a^6*(a - 1)
^(5/2)*(a + 1)^(5/2) + 742*a^10*(a - 1)^(3/2)*(a + 1)^(3/2) + 64*a^14*(a -
```

1)^(1/2)*(a + 1)^(1/2) - a^4*(a - 1)^(7/2)*(a + 1)^(7/2) - 372*a^8*(a - 1)^
(5/2)*(a + 1)^(5/2) - 187*a^12*(a - 1)^(3/2)*(a + 1)^(3/2) + 2*a^6*(a - 1)^
(7/2)*(a + 1)^(7/2) + 188*a^10*(a - 1)^(5/2)*(a + 1)^(5/2) - 65*a^8*(a - 1)
^(7/2)*(a + 1)^(7/2)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a
^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) - (64*(22*a^3*(a - 1)^(3/2)*(a
+ 1)^(3/2) - 2*a*(a - 1)^(3/2)*(a + 1)^(3/2) - 68*a^5*(a - 1)^(3/2)*(a + 1)
^(3/2) + 92*a^7*(a - 1)^(3/2)*(a + 1)^(3/2) - 28*a^5*(a - 1)^(5/2)*(a + 1)^
(5/2) - 58*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + 56*a^7*(a - 1)^(5/2)*(a + 1)^(
5/2) + 14*a^11*(a - 1)^(3/2)*(a + 1)^(3/2) + 2*a^5*(a - 1)^(7/2)*(a + 1)^(7
/2) - 28*a^9*(a - 1)^(5/2)*(a + 1)^(5/2) + 14*a^7*(a - 1)^(7/2)*(a + 1)^(7/
2)))/(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(2*a*(a - 1)^2
*(a + 1)^2 - 2*a*(a - 1)*(a + 1) - 8*a^3*(a - 1)^2*(a + 1)^2 - 2*a^3*(a - 1
)^3*(a + 1)^3 + 12*a^5*(a - 1)^2*(a + 1)^2 + 6*a^5*(a - 1)^3*(a + 1)^3 - 8*
a^7*(a - 1)^2*(a + 1)^2 - 4*a^7*(a - 1)^3*(a + 1)^3 + 2*a^9*(a - 1)^2*(a +
1)^2 + 10*a^3*(a - 1)*(a + 1) - 20*a^5*(a - 1)*(a + 1) + 20*a^7*(a - 1)*(a
+ 1) - 10*a^9*(a - 1)*(a + 1) + 2*a^11*(a - 1)*(a + 1)))/(5*a^2 - 10*a^4 +
10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(a*(a - 1)^(3/2)*(a + 1)^(3/2) - 2*a*
(a - 1)^(1/2)*(a + 1)^(1/2) + 22*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) - 68*a^5*(
a - 1)^(1/2)*(a + 1)^(1/2) - 4*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) + 92*a^7*(a
- 1)^(1/2)*(a + 1)^(1/2) - 22*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) - 58*a^9*(a -
1)^(1/2)*(a + 1)^(1/2) - 5*a^3*(a - 1)^(5/2)*(a + 1)^(5/2) + 52*a^7*(a - 1
)^(3/2)*(a + 1)^(3/2) + 14*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 12*a^5*(a - 1
)^(5/2)*(a + 1)^(5/2) - 27*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + 9*a^7*(a - 1)^
(5/2)*(a + 1)^(5/2) + 4*a^5*(a - 1)^(7/2)*(a + 1)^(7/2)))/(5*a^2 - 10*a^4 +
10*a^6 - 5*a^8 + a^10 - 1) + 2*a*((32*(2*a - 10*a^3 + 20*a^5 - 20*a^7 + 10
*a^9 - 2*a^11 - 2*a*(a - 1)*(a + 1) + 2*a^3*(a - 1)^2*(a + 1)^2 - 6*a^5*(a
- 1)^2*(a + 1)^2 + 4*a^7*(a - 1)^2*(a + 1)^2 + 8*a^3*(a - 1)*(a + 1) - 12*a
^5*(a - 1)*(a + 1) + 8*a^7*(a - 1)*(a + 1) - 2*a^9*(a - 1)*(a + 1)))/(5*a^2
- 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) - 2*a*((32*(a*(a - 1)^(1/2)*(a + 1)^
(1/2) - 4*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 6*a^5*(a - 1)^(1/2)*(a + 1)^(1/
2) - 5*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) - 4*a^7*(a - 1)^(1/2)*(a + 1)^(1/2)
+ 10*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + a^9*(a - 1)^(1/2)*(a + 1)^(1/2) - 5*
a^7*(a - 1)^(3/2)*(a + 1)^(3/2) + 4*a^5*(a - 1)^(5/2)*(a + 1)^(5/2)))/(5*a^
2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1) - (32*((a - 1)^(1/2) - (a + b*x - 1
)^(1/2))*(60*a^2 - 150*a^4 + 200*a^6 - 150*a^8 + 60*a^10 - 10*a^12 - 39*a^4
(a - 1)^2(a + 1)^2 + 78*a^6*(a - 1)^2*(a + 1)^2 + 16*a^6*(a - 1)^3*(a + 1
)^3 - 39*a^8*(a - 1)^2*(a + 1)^2 + 33*a^2*(a - 1)*(a + 1) - 132*a^4*(a - 1)
*(a + 1) + 198*a^6*(a - 1)*(a + 1) - 132*a^8*(a - 1)*(a + 1) + 33*a^10*(a -
1)*(a + 1) - 10))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 -
20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1
)^(1/2))*(17*(a - 1)^(1/2)*(a + 1)^(1/2) - 106*a^2*(a - 1)^(1/2)*(a + 1)^(1
/2) + 275*a^4*(a - 1)^(1/2)*(a + 1)^(1/2) - 48*a^2*(a - 1)^(3/2)*(a + 1)^(3
/2) - 380*a^6*(a - 1)^(1/2)*(a + 1)^(1/2) + 196*a^4*(a - 1)^(3/2)*(a + 1)^(
3/2) + 295*a^8*(a - 1)^(1/2)*(a + 1)^(1/2) - 300*a^6*(a - 1)^(3/2)*(a + 1)^
(3/2) - 122*a^10*(a - 1)^(1/2)*(a + 1)^(1/2) + 47*a^4*(a - 1)^(5/2)*(a + 1)
^(5/2) + 204*a^8*(a - 1)^(3/2)*(a + 1)^(3/2) + 21*a^12*(a - 1)^(1/2)*(a + 1
)^(1/2) - 94*a^6*(a - 1)^(5/2)*(a + 1)^(5/2) - 52*a^10*(a - 1)^(3/2)*(a + 1
)^(3/2) + 47*a^8*(a - 1)^(5/2)*(a + 1)^(5/2) - 16*a^6*(a - 1)^(7/2)*(a + 1)
^(7/2)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 +
15*a^8 - 6*a^10 + a^12 + 1))) + (32*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(
36*a^2 - 216*a^4 + 540*a^6 - 720*a^8 + 540*a^10 - 216*a^12 + 36*a^14 - 6*(a
- 1)*(a + 1) + 15*a^2*(a - 1)^2*(a + 1)^2 - 64*a^4*(a - 1)^2*(a + 1)^2 - 9
a^4(a - 1)^3*(a + 1)^3 + 262*a^6*(a - 1)^2*(a + 1)^2 + 18*a^6*(a - 1)^3*(
a + 1)^3 - 392*a^8*(a - 1)^2*(a + 1)^2 - 73*a^8*(a - 1)^3*(a + 1)^3 + 179*a
^10*(a - 1)^2*(a + 1)^2 + 40*a^2*(a - 1)*(a + 1) - 242*a^4*(a - 1)*(a + 1)
+ 688*a^6*(a - 1)*(a + 1) - 922*a^8*(a - 1)*(a + 1) + 584*a^10*(a - 1)*(a +
1) - 142*a^12*(a - 1)*(a + 1)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15
*a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1))) - (32*((a - 1)^(1/2)
- (a + b*x - 1)^(1/2))*((a - 1)^(3/2)*(a + 1)^(3/2) + 64*a^2*(a - 1)^(1/2)*

```
(a + 1)^(1/2) - 384*a^4*(a - 1)^(1/2)*(a + 1)^(1/2) - 10*a^2*(a - 1)^(3/2)*
(a + 1)^(3/2) + 960*a^6*(a - 1)^(1/2)*(a + 1)^(1/2) - 157*a^4*(a - 1)^(3/2)
*(a + 1)^(3/2) - 1280*a^8*(a - 1)^(1/2)*(a + 1)^(1/2) + 708*a^6*(a - 1)^(3/
2)*(a + 1)^(3/2) + 960*a^10*(a - 1)^(1/2)*(a + 1)^(1/2) + 4*a^4*(a - 1)^(5/
2)*(a + 1)^(5/2) - 1097*a^8*(a - 1)^(3/2)*(a + 1)^(3/2) - 384*a^12*(a - 1)^
(1/2)*(a + 1)^(1/2) + 180*a^6*(a - 1)^(5/2)*(a + 1)^(5/2) + 742*a^10*(a - 1
)^(3/2)*(a + 1)^(3/2) + 64*a^14*(a - 1)^(1/2)*(a + 1)^(1/2) - a^4*(a - 1)^(
7/2)*(a + 1)^(7/2) - 372*a^8*(a - 1)^(5/2)*(a + 1)^(5/2) - 187*a^12*(a - 1)
^(3/2)*(a + 1)^(3/2) + 2*a^6*(a - 1)^(7/2)*(a + 1)^(7/2) + 188*a^10*(a - 1)
^(5/2)*(a + 1)^(5/2) - 65*a^8*(a - 1)^(7/2)*(a + 1)^(7/2)))/(((a + 1)^(1/2)
 - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 +
 1))) + (64*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(4*a^4*(a - 1)^3*(a + 1)^
3 - 40*a^4*(a - 1)^2*(a + 1)^2 - 4*a^2*(a - 1)^2*(a + 1)^2 + 200*a^6*(a - 1
)^2*(a + 1)^2 + 20*a^6*(a - 1)^3*(a + 1)^3 - 320*a^8*(a - 1)^2*(a + 1)^2 -
52*a^8*(a - 1)^3*(a + 1)^3 + 220*a^10*(a - 1)^2*(a + 1)^2 + 28*a^10*(a - 1)
^3*(a + 1)^3 - 56*a^12*(a - 1)^2*(a + 1)^2 + 28*a^2*(a - 1)*(a + 1) - 168*a
^4*(a - 1)*(a + 1) + 420*a^6*(a - 1)*(a + 1) - 560*a^8*(a - 1)*(a + 1) + 42
0*a^10*(a - 1)*(a + 1) - 168*a^12*(a - 1)*(a + 1) + 28*a^14*(a - 1)*(a + 1)
))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(15*a^4 - 6*a^2 - 20*a^6 + 15*a^8
 - 6*a^10 + a^12 + 1))))*4i + ((4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2)))/
((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) + (4*a*((a - 1)^(1/2) - (a + b*x - 1)
^(1/2))^3)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3 - (8*((a - 1)^(1/2) - (a
 + b*x - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/((a + 1)^(1/2) - (a + b*x
 + 1)^(1/2))^2)/(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4/((a + 1)^(1/2) - (
a + b*x + 1)^(1/2))^4 - (2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a + 1
)^(1/2) - (a + b*x + 1)^(1/2))^2 + 1) + log(((a - 1)^(1/2) - (a + b*x - 1)^
(1/2))^2/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - a^2 - (a^2*((a - 1)^(1/2
) - (a + b*x - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 + (2*a*
((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(a - 1)^(1/2)*(a + 1)^(1/2))/((a + 1)
^(1/2) - (a + b*x + 1)^(1/2)) + 1)*(a - 1)^(1/2)*(a + 1)^(1/2) - log(((a -
1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)))*(a -
 1)^(1/2)*(a + 1)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a + bx - 1}\,\sqrt{a + bx + 1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x,x)
```

```
[Out] Integral((a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1))/x, x)
```

3.280 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2ab\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)-\frac{a}{x}+b\log(x)$$

```
[Out] -a/x+2*b*arcsinh(1/2*(b*x+a-1)^(1/2)*2^(1/2))+b*ln(x)-2*a*b*arctan((1-a)^(1
/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(1/2)-(b*x+a-1)^(
1/2)*(b*x+a+1)^(1/2)/x
```

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5909, 14, 97, 157, 63, 215, 93, 205}

$$-\frac{2ab\tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\sqrt{1-a^2}}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{a+bx-1}}{\sqrt{2}}\right)-\frac{a}{x}+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/x + 2*b*ArcSinh[Sqrt[-1 + a
 + b*x]/Sqrt[2]] - (2*a*b*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 +
a]*Sqrt[-1 + a + b*x])])/Sqrt[1 - a^2] + b*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\cosh^{-1}(a+bx)}}{x^2}\,dx &= \int \frac{a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^2}\,dx\\
&= \int \left(\frac{a}{x^2}+\frac{b}{x}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^2}\right)dx\\
&= -\frac{a}{x}+b\log(x)+\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^2}\,dx\\
&= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+b\log(x)+\int \frac{ab+b^2x}{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}\,dx\\
&= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+b\log(x)+(ab)\int \frac{1}{x\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}\,dx+b\\
&= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+b\log(x)+(2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx,x,\sqrt{-1+a+b}\right.\\
&= -\frac{a}{x}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x}+2b\sinh^{-1}\left(\frac{\sqrt{-1+a+bx}}{\sqrt{2}}\right)-\frac{2ab\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{1+a+b}}{\sqrt{1+a}\,\sqrt{-1+a+b}}\right.}{\sqrt{1-a^2}}\end{aligned}$$

Mathematica [C] time = 0.17, size = 140, normalized size = 1.28

$$-\frac{iab\log\left(\frac{2\left(\sqrt{a+bx-1}\,\sqrt{a+bx+1}+\frac{i\left(a^2+abx-1\right)}{\sqrt{1-a^2}}\right)}{abx}\right)}{\sqrt{1-a^2}}-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{x}+b\log\left(\sqrt{a+bx-1}\,\sqrt{a+bx+1}+a+bx\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^2,x]
```

```
[Out] -(a/x) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/x + b*Log[x] + b*Log[a + b*
x + Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]] - (I*a*b*Log[(2*(Sqrt[-1 + a + b*
```

```
x]*Sqrt[1 + a + b*x] + (I*(-1 + a^2 + a*b*x))/Sqrt[1 - a^2]))/(a*b*x)])/Sqr
t[1 - a^2]
```

fricas [A] time = 0.50, size = 334, normalized size = 3.06

$$\left[\frac{\sqrt{a^2-1}\,abx\log\left(\frac{a^2bx+a^3+\left(a^2-\sqrt{a^2-1}\,a-1\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1}-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-\left(a^2-1\right)bx\log\left(-bx+\sqrt{bx+a+1}\,\sqrt{}\right.}{\left(a^2-1\right)x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="fricas"
)
```

```
[Out] [(sqrt(a^2 - 1)*a*b*x*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a - 1)*sqrt
(b*x + a + 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a)/x) -
 (a^2 - 1)*b*x*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + (a^2 -
 1)*b*x*log(x) - a^3 - (a^2 - 1)*b*x - (a^2 - 1)*sqrt(b*x + a + 1)*sqrt(b*x
 + a - 1) + a)/((a^2 - 1)*x), (2*sqrt(-a^2 + 1)*a*b*x*arctan(-(sqrt(-a^2 +
1)*b*x - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) - (
a^2 - 1)*b*x*log(-b*x + sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a) + (a^2 - 1
)*b*x*log(x) - a^3 - (a^2 - 1)*b*x - (a^2 - 1)*sqrt(b*x + a + 1)*sqrt(b*x +
 a - 1) + a)/((a^2 - 1)*x)]
```

giac [B] time = 0.25, size = 200, normalized size = 1.83

$$-\frac{\frac{2\,ab^2\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2\,a}{2\,\sqrt{-a^2+1}}\right)}{\sqrt{-a^2+1}}+b^2\log\left(\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2\right)-b^2\log\left(|bx|\right)-\frac{4\left(ab^2\left(\sqrt{bx+a+1}-\right.\right.}{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-4\,a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -(2*a*b^2*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/sqrt
(-a^2 + 1))/sqrt(-a^2 + 1) + b^2*log((sqrt(b*x + a + 1) - sqrt(b*x + a - 1)
)^2) - b^2*log(abs(b*x)) - 4*(a*b^2*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))
^2 - 2*b^2)/((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a
+ 1) - sqrt(b*x + a - 1))^2 + 4) + ((b*x + a + 1)*b^2 - b^2)/(b*x))/b
```

maple [C] time = 0.02, size = 237, normalized size = 2.17

$$\frac{\left(-\mathrm{csgn}(b)\sqrt{a^2-1}\;\ln\left(\frac{2abx+2\sqrt{a^2-1}\;\sqrt{b^2x^2+2abx+a^2-1}+2a^2-2}{x}\right)xab+\ln\left(\left(\sqrt{b^2x^2+2abx+a^2-1}\;\mathrm{csgn}(b)+bx+a\right)\mathrm{cs}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x)
```

```
[Out] (-csgn(b)*(a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(
1/2)+a^2-1)/x)*x*a*b+ln(((b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(
b))*x*a^2*b-(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn(b)*a^2-ln(((b^2*x^2+2*a*b*x+
a^2-1)^(1/2)*csgn(b)+b*x+a)*csgn(b))*x*b+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*csgn
(b))*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*csgn(b)/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/
(a^2-1)/x+b*ln(x)-a/x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 zero or nonzero?
```

mupad [B] time = 15.82, size = 3029, normalized size = 27.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x^2,x)
```

```
[Out] ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*((5*b)/4 - (a^2*b)/4))/((a^2 - 1)
*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2) - b/4 + (a*b*((a - 1)^(1/2) - (a
+ b*x - 1)^(1/2))*(a - 1)^(1/2)*(a + 1)^(1/2))/(2*(a^2 - 1)*((a + 1)^(1/2)
- (a + b*x + 1)^(1/2))))/(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1
/2) - (a + b*x + 1)^(1/2)) + ((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3/((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^3 - (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/
2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1
)^(1/2))^2)) - a/x + b*log(x) - b*atan((((14*a^2 - 42*a^4 + 70*a^6 - 70*a^8
 + 42*a^10 - 14*a^12 + 2*a^14 + 4*a^4*(a - 1)^2*(a + 1)^2 - 12*a^6*(a - 1)^
2*(a + 1)^2 + 12*a^8*(a - 1)^2*(a + 1)^2 - 4*a^10*(a - 1)^2*(a + 1)^2 - 2*a
^2*(a - 1)*(a + 1) + 10*a^4*(a - 1)*(a + 1) - 20*a^6*(a - 1)*(a + 1) + 20*a
^8*(a - 1)*(a + 1) - 10*a^10*(a - 1)*(a + 1) + 2*a^12*(a - 1)*(a + 1) - 2)*
512i)/(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1) + ((
2*a^4*(a - 1)^2*(a + 1)^2 - 6*a^6*(a - 1)^2*(a + 1)^2 - 4*a^6*(a - 1)^3*(a
+ 1)^3 + 6*a^8*(a - 1)^2*(a + 1)^2 + 4*a^8*(a - 1)^3*(a + 1)^3 - 2*a^10*(a
- 1)^2*(a + 1)^2 + 2*a^2*(a - 1)*(a + 1) - 10*a^4*(a - 1)*(a + 1) + 20*a^6*
(a - 1)*(a + 1) - 20*a^8*(a - 1)*(a + 1) + 10*a^10*(a - 1)*(a + 1) - 2*a^12
*(a - 1)*(a + 1))*128i)/(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^1
2 + a^14 - 1) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(21*a*(a - 1)^(1/2)*
(a + 1)^(1/2) - 126*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 315*a^5*(a - 1)^(1/2)
*(a + 1)^(1/2) - 52*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) - 420*a^7*(a - 1)^(1/2)
*(a + 1)^(1/2) + 208*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + 315*a^9*(a - 1)^(1/2
)*(a + 1)^(1/2) - 312*a^7*(a - 1)^(3/2)*(a + 1)^(3/2) - 126*a^11*(a - 1)^(1
/2)*(a + 1)^(1/2) + 47*a^5*(a - 1)^(5/2)*(a + 1)^(5/2) + 208*a^9*(a - 1)^(3
/2)*(a + 1)^(3/2) + 21*a^13*(a - 1)^(1/2)*(a + 1)^(1/2) - 94*a^7*(a - 1)^(5
/2)*(a + 1)^(5/2) - 52*a^11*(a - 1)^(3/2)*(a + 1)^(3/2) + 47*a^9*(a - 1)^(5
/2)*(a + 1)^(5/2) - 16*a^7*(a - 1)^(7/2)*(a + 1)^(7/2))*512i)/(((a + 1)^(1/
2) - (a + b*x + 1)^(1/2))*(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a
^12 + a^14 - 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(64*a*(a - 1)^(1/
2)*(a + 1)^(1/2) - 384*a^3*(a - 1)^(1/2)*(a + 1)^(1/2) + 960*a^5*(a - 1)^(1
/2)*(a + 1)^(1/2) - 187*a^3*(a - 1)^(3/2)*(a + 1)^(3/2) - 1280*a^7*(a - 1)^
(1/2)*(a + 1)^(1/2) + 748*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + 960*a^9*(a - 1)
^(1/2)*(a + 1)^(1/2) - 1122*a^7*(a - 1)^(3/2)*(a + 1)^(3/2) - 384*a^11*(a -
 1)^(1/2)*(a + 1)^(1/2) + 188*a^5*(a - 1)^(5/2)*(a + 1)^(5/2) + 748*a^9*(a
- 1)^(3/2)*(a + 1)^(3/2) + 64*a^13*(a - 1)^(1/2)*(a + 1)^(1/2) - 376*a^7*(a
 - 1)^(5/2)*(a + 1)^(5/2) - 187*a^11*(a - 1)^(3/2)*(a + 1)^(3/2) + 188*a^9*
(a - 1)^(5/2)*(a + 1)^(5/2) - 65*a^7*(a - 1)^(7/2)*(a + 1)^(7/2))*128i)/(((
a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*
a^10 - 7*a^12 + a^14 - 1)))/((1024*(a*(a - 1)^(1/2)*(a + 1)^(1/2) - 6*a^3*(
a - 1)^(1/2)*(a + 1)^(1/2) + 15*a^5*(a - 1)^(1/2)*(a + 1)^(1/2) - 5*a^3*(a
- 1)^(3/2)*(a + 1)^(3/2) - 20*a^7*(a - 1)^(1/2)*(a + 1)^(1/2) + 20*a^5*(a -
 1)^(3/2)*(a + 1)^(3/2) + 15*a^9*(a - 1)^(1/2)*(a + 1)^(1/2) - 30*a^7*(a -
1)^(3/2)*(a + 1)^(3/2) - 6*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 4*a^5*(a - 1)
```

```
^(5/2)*(a + 1)^(5/2) + 20*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + a^13*(a - 1)^(1
/2)*(a + 1)^(1/2) - 8*a^7*(a - 1)^(5/2)*(a + 1)^(5/2) - 5*a^11*(a - 1)^(3/2
)*(a + 1)^(3/2) + 4*a^9*(a - 1)^(5/2)*(a + 1)^(5/2)))/(7*a^2 - 21*a^4 + 35*
a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1) + (64*(14*a^3*(a - 1)^(3/2)*(a
+ 1)^(3/2) - 56*a^5*(a - 1)^(3/2)*(a + 1)^(3/2) + 84*a^7*(a - 1)^(3/2)*(a +
 1)^(3/2) - 28*a^5*(a - 1)^(5/2)*(a + 1)^(5/2) - 56*a^9*(a - 1)^(3/2)*(a +
1)^(3/2) + 56*a^7*(a - 1)^(5/2)*(a + 1)^(5/2) + 14*a^11*(a - 1)^(3/2)*(a +
1)^(3/2) - 28*a^9*(a - 1)^(5/2)*(a + 1)^(5/2) + 14*a^7*(a - 1)^(7/2)*(a + 1
)^(7/2)))/(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1)
- (256*(14*a*(a - 1)^(1/2)*(a + 1)^(1/2) - 84*a^3*(a - 1)^(1/2)*(a + 1)^(1/
2) + 210*a^5*(a - 1)^(1/2)*(a + 1)^(1/2) - 27*a^3*(a - 1)^(3/2)*(a + 1)^(3/
2) - 280*a^7*(a - 1)^(1/2)*(a + 1)^(1/2) + 108*a^5*(a - 1)^(3/2)*(a + 1)^(3
/2) + 210*a^9*(a - 1)^(1/2)*(a + 1)^(1/2) - 162*a^7*(a - 1)^(3/2)*(a + 1)^(
3/2) - 84*a^11*(a - 1)^(1/2)*(a + 1)^(1/2) + 9*a^5*(a - 1)^(5/2)*(a + 1)^(5
/2) + 108*a^9*(a - 1)^(3/2)*(a + 1)^(3/2) + 14*a^13*(a - 1)^(1/2)*(a + 1)^(
1/2) - 18*a^7*(a - 1)^(5/2)*(a + 1)^(5/2) - 27*a^11*(a - 1)^(3/2)*(a + 1)^(
3/2) + 9*a^9*(a - 1)^(5/2)*(a + 1)^(5/2) + 4*a^7*(a - 1)^(7/2)*(a + 1)^(7/2
)))/(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1) + (102
4*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(70*a^2 - 210*a^4 + 350*a^6 - 350*a
^8 + 210*a^10 - 70*a^12 + 10*a^14 - 39*a^4*(a - 1)^2*(a + 1)^2 + 117*a^6*(a
 - 1)^2*(a + 1)^2 + 16*a^6*(a - 1)^3*(a + 1)^3 - 117*a^8*(a - 1)^2*(a + 1)^
2 - 16*a^8*(a - 1)^3*(a + 1)^3 + 39*a^10*(a - 1)^2*(a + 1)^2 + 33*a^2*(a -
1)*(a + 1) - 165*a^4*(a - 1)*(a + 1) + 330*a^6*(a - 1)*(a + 1) - 330*a^8*(a
 - 1)*(a + 1) + 165*a^10*(a - 1)*(a + 1) - 33*a^12*(a - 1)*(a + 1) - 10))/(
((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 2
1*a^10 - 7*a^12 + a^14 - 1)) - (256*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(
252*a^2 - 756*a^4 + 1260*a^6 - 1260*a^8 + 756*a^10 - 252*a^12 + 36*a^14 - 1
79*a^4*(a - 1)^2*(a + 1)^2 + 537*a^6*(a - 1)^2*(a + 1)^2 + 73*a^6*(a - 1)^3
*(a + 1)^3 - 537*a^8*(a - 1)^2*(a + 1)^2 - 73*a^8*(a - 1)^3*(a + 1)^3 + 179
*a^10*(a - 1)^2*(a + 1)^2 + 142*a^2*(a - 1)*(a + 1) - 710*a^4*(a - 1)*(a +
1) + 1420*a^6*(a - 1)*(a + 1) - 1420*a^8*(a - 1)*(a + 1) + 710*a^10*(a - 1)
*(a + 1) - 142*a^12*(a - 1)*(a + 1) - 36))/(((a + 1)^(1/2) - (a + b*x + 1)^
(1/2))*(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 + 21*a^10 - 7*a^12 + a^14 - 1)) -
(64*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(56*a^4*(a - 1)^2*(a + 1)^2 - 168
*a^6*(a - 1)^2*(a + 1)^2 - 28*a^6*(a - 1)^3*(a + 1)^3 + 168*a^8*(a - 1)^2*(
a + 1)^2 + 28*a^8*(a - 1)^3*(a + 1)^3 - 56*a^10*(a - 1)^2*(a + 1)^2 - 28*a^
2*(a - 1)*(a + 1) + 140*a^4*(a - 1)*(a + 1) - 280*a^6*(a - 1)*(a + 1) + 280
*a^8*(a - 1)*(a + 1) - 140*a^10*(a - 1)*(a + 1) + 28*a^12*(a - 1)*(a + 1)))
/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(7*a^2 - 21*a^4 + 35*a^6 - 35*a^8 +
 21*a^10 - 7*a^12 + a^14 - 1))))*4i - (b*(a^2 - 1)*((a - 1)^(1/2) - (a + b*
x - 1)^(1/2)))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(4*a^2 - 4)) + (a*b*l
og(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2/((a + 1)^(1/2) - (a + b*x + 1)^(
1/2))^2 - a^2 - (a^2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a + 1)^(1/2
) - (a + b*x + 1)^(1/2))^2 + (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(a
- 1)^(1/2)*(a + 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) + 1)*(a - 1
)^(1/2)*(a + 1)^(1/2))/(a^2 - 1) - (a*b*log(((a - 1)^(1/2) - (a + b*x - 1)^
(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)))*(a - 1)^(1/2)*(a + 1)^(1/2))/
(a^2 - 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + \sqrt{a + bx - 1}\,\sqrt{a + bx + 1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**2,x)
```

```
[Out] Integral((a + b*x + sqrt(a + b*x - 1)*sqrt(a + b*x + 1))/x**2, x)
```

3.281 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^3} dx$

Optimal. Leaf size=138

$$-\frac{b^2 \tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\sqrt{a+bx-1}\,(a+bx+1)^{3/2}}{2(a+1)x^2}-\frac{a}{2x^2}-\frac{b}{x}$$

```
[Out] -1/2*a/x^2-b/x-b^2*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)
^(1/2))/(-a^2+1)^(3/2)-1/2*(b*x+a+1)^(3/2)*(b*x+a-1)^(1/2)/(1+a)/x^2+1/2*b*
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)/x
```

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5909, 14, 94, 93, 205}

$$-\frac{b^2 \tan^{-1}\left(\frac{\sqrt{1-a}\sqrt{a+bx+1}}{\sqrt{a+1}\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{3/2}}+\frac{b\sqrt{a+bx-1}\sqrt{a+bx+1}}{2\left(1-a^2\right)x}-\frac{\sqrt{a+bx-1}\,(a+bx+1)^{3/2}}{2(a+1)x^2}-\frac{a}{2x^2}-\frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^3,x]
```

```
[Out] -a/(2*x^2) - b/x + (b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1 - a^2)*x)
 - (Sqrt[-1 + a + b*x]*(1 + a + b*x)^(3/2))/(2*(1 + a)*x^2) - (b^2*ArcTan[(
Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a + b*x])])/(1 - a^2)
^(3/2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)}}{x^3}\,dx = \int \frac{a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^3}\,dx$$
$$= \int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^3}\right)dx$$
$$= -\frac{a}{2x^2}-\frac{b}{x}+\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^3}\,dx$$
$$= -\frac{a}{2x^2}-\frac{b}{x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b\int \frac{\sqrt{1+a+bx}}{x^2\sqrt{-1+a+bx}}\,dx}{2(1+a)}$$
$$= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b^2\int \frac{1}{x\sqrt{-1+a+bx}\,\sqrt{}}}{2\left(1-a^2\right.}$$
$$= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{}{-1-a}\right.}{}$$
$$= -\frac{a}{2x^2}-\frac{b}{x}+\frac{b\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{2\left(1-a^2\right)x}-\frac{\sqrt{-1+a+bx}\,(1+a+bx)^{3/2}}{2(1+a)x^2}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{}}{\sqrt{1+a}\,\sqrt{}}\right.}{\left(1-a^2\right)^{3/}}$$

Mathematica [C] time = 0.27, size = 142, normalized size = 1.03

$$\frac{1}{2}\left(-\frac{ib^2\log\left(\frac{4i\sqrt{1-a^2}\left(-i\sqrt{1-a^2}\,\sqrt{a+bx-1}\,\sqrt{a+bx+1}+a^2+abx-1\right)}{b^2x}\right)}{\left(1-a^2\right)^{3/2}}-\frac{\sqrt{a+bx-1}\,\sqrt{a+bx+1}\,\left(a^2+abx-1\right)}{\left(a^2-1\right)x^2}-\frac{a}{x^2}-\frac{2b}{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^3,x]
```

```
[Out] (-(a/x^2) - (2*b)/x - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-1 + a^2 + a*b
*x))/((-1 + a^2)*x^2) - (I*b^2*Log[((4*I)*Sqrt[1 - a^2]*(-1 + a^2 + a*b*x -
 I*Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]))/(b^2*x)])/(1 - a^2)
^(3/2))/2
```

fricas [A] time = 0.47, size = 338, normalized size = 2.45

$$\left[\frac{\sqrt{a^2-1}\,b^2x^2\log\left(\frac{a^2bx+a^3+\left(a^2+\sqrt{a^2-1}\,a-1\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1}+\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-a^5-\left(a^3-a\right)b^2x^2+2\,a^3-2\left(a^4-2\right.}{2\left(a^4-2\,a^2+1\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="fricas"
)
```

```
[Out] [1/2*(sqrt(a^2 - 1)*b^2*x^2*log((a^2*b*x + a^3 + (a^2 + sqrt(a^2 - 1)*a - 1
)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + (a*b*x + a^2 - 1)*sqrt(a^2 - 1) - a
)/x) - a^5 - (a^3 - a)*b^2*x^2 + 2*a^3 - 2*(a^4 - 2*a^2 + 1)*b*x - (a^4 + (
a^3 - a)*b*x - 2*a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - a)/((a^4 -
2*a^2 + 1)*x^2), -1/2*(2*sqrt(-a^2 + 1)*b^2*x^2*arctan(-(sqrt(-a^2 + 1)*b*x
 - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + a^5 + (
a^3 - a)*b^2*x^2 - 2*a^3 + 2*(a^4 - 2*a^2 + 1)*b*x + (a^4 + (a^3 - a)*b*x -
 2*a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + a)/((a^4 - 2*a^2 + 1)*x^2
)]
```

giac [B] time = 0.33, size = 326, normalized size = 2.36

$$\frac{\frac{2b^3\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2a}{2\sqrt{-a^2+1}}\right)}{(a^2-1)\sqrt{-a^2+1}}+\frac{4\left(2a^2b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-4a^3b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-b^3\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-2ab^3(\cdots\right.}{\left(\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^4-4a\left(\sqrt{bx+\cdots}\right.\right.}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/sqr
t(-a^2 + 1))/((a^2 - 1)*sqrt(-a^2 + 1)) + 4*(2*a^2*b^3*(sqrt(b*x + a + 1) -
 sqrt(b*x + a - 1))^6 - 4*a^3*b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4
 - b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 2*a*b^3*(sqrt(b*x + a +
1) - sqrt(b*x + a - 1))^4 + 8*a^2*b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1
))^2 + 4*b^3*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 8*a*b^3)/(((sqrt(b
*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a + 1) - sqrt(b*x + a
- 1))^2 + 4)^2*(a^2 - 1)) - (2*(b*x + a + 1)*b^3 - a*b^3 - 2*b^3)/(b^2*x^2)
)/b
```

maple [B] time = 0.02, size = 236, normalized size = 1.71

$$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(\sqrt{a^2-1}\,\ln\left(\frac{2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}+2a^2-2}{x}\right)x^2b^2-x\,a^3b\sqrt{b^2x^2+2abx+a^2-1}\cdots\right.}{2\sqrt{b^2x^2+2abx\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x)
```

```
[Out] 1/2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*((a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2
)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*x^2*b^2-x*a^3*b*(b^2*x^2+2*a*b*x+
a^2-1)^(1/2)-a^4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2
)*x*a*b+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*a^2-(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/
(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/(a^2-1)^2/x^2-b/x-1/2*a/x^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 zero or nonzero?
```

mupad [B] time = 14.15, size = 958, normalized size = 6.94

$$\frac{b^2 \ln\left(\frac{\sqrt{a-1}-\sqrt{a+b\,x-1}}{\sqrt{a+1}-\sqrt{a+b\,x+1}}\right)\sqrt{a-1}\,\sqrt{a+1}}{2\,a^4-4\,a^2+2}-\frac{\frac{a\,b^2\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^5}{8\left(a^2-1\right)\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^5}-\frac{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^3\left(\frac{3\,a\,b^2}{8}-\frac{7\,a^3\,b^2}{8}\right)}{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^3\left(a^4-2\,a^2+1\right)}-\frac{b^2\,\sqrt{a-1}\,\sqrt{a+1}}{32\left(a^2-1\right)}+\cdots}{\frac{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^2}+\frac{\left(\sqrt{a-1}-\sqrt{a+b\,x-1}\right)^6}{\left(\sqrt{a+1}-\sqrt{a+b\,x+1}\right)^6}+\frac{\left(\sqrt{a-\cdots}\right.}{\left(a^2-\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x^3,x)
```

```
[Out] (b^2*log(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x +
1)^(1/2)))*(a - 1)^(1/2)*(a + 1)^(1/2))/(2*a^4 - 4*a^2 + 2) - ((a*b^2*((a -
 1)^(1/2) - (a + b*x - 1)^(1/2))^5)/(8*(a^2 - 1)*((a + 1)^(1/2) - (a + b*x
+ 1)^(1/2))^5) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*((3*a*b^2)/8 - (7
*a^3*b^2)/8))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3*(a^4 - 2*a^2 + 1)) -
 (b^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(32*(a^2 - 1)) + (a*b^2*((a - 1)^(1/2) -
 (a + b*x - 1)^(1/2)))/(4*(a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2)))
+ (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*(b^2/16 - (11*a^2*b^2)/16)*(a -
1)^(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2*(a^4 - 2*a
^2 + 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(a - 1)^(1/2)*(a + 1)^(
1/2)*((15*b^2)/32 + (9*a^2*b^2)/16 - (17*a^4*b^2)/32))/(((a + 1)^(1/2) - (a
 + b*x + 1)^(1/2))^4*(3*a^2 - 3*a^4 + a^6 - 1)))/(((a - 1)^(1/2) - (a + b*x
 - 1)^(1/2))^2/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 + ((a - 1)^(1/2) - (
a + b*x - 1)^(1/2))^6/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6 + (((a - 1)^(
1/2) - (a + b*x - 1)^(1/2))^4*(6*a^2 - 2))/((a^2 - 1)*((a + 1)^(1/2) - (a +
 b*x + 1)^(1/2))^4) - (4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*(a - 1)^
(1/2)*(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3) -
(4*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*(a - 1)^(1/2)*(a + 1)^(1/2))/(
(a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5)) - (((a - 1)^(1/2) - (a
+ b*x - 1)^(1/2))*((a*b^2)/(2*(a - 1)*(a + 1)) - (3*a*b^2*(a^2 - 1)^2)/(8*(
a - 1)^3*(a + 1)^3)))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) - (b^2*log(((a
- 1)^(1/2) - (a + b*x - 1)^(1/2))^2/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2
 - a^2 - (a^2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a + 1)^(1/2) - (a
+ b*x + 1)^(1/2))^2 + (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(a - 1)^(1
/2)*(a + 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) + 1)*(a - 1)^(1/2)
*(a + 1)^(1/2))/(2*a^4 - 4*a^2 + 2) - (a/2 + b*x)/x^2 + (b^2*(a^2 - 1)*((a
- 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/(32*((a + 1)^(1/2) - (a + b*x + 1)^(1/
2))^2*(a - 1)^(3/2)*(a + 1)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

3.282 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^4}\,dx$

Optimal. Leaf size=189

$$-\frac{ab^3\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{5/2}}+\frac{ab^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{2\left(1-a^2\right)^2x}+\frac{(a+bx-1)^{3/2}(a+bx+1)^{3/2}}{3\left(1-a^2\right)x^3}-\frac{ab\sqrt{a+bx-1}\,(a+}{2(1-a)(a+1)}$$

```
[Out] -1/3*a/x^3-1/2*b/x^2+1/3*(b*x+a-1)^(3/2)*(b*x+a+1)^(3/2)/(-a^2+1)/x^3-a*b^3
*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(
5/2)-1/2*a*b*(b*x+a+1)^(3/2)*(b*x+a-1)^(1/2)/(1-a)/(1+a)^2/x^2+1/2*a*b^2*(b
*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)^2/x
```

Rubi [A] time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5909, 14, 96, 94, 93, 205}

$$\frac{ab^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{2\left(1-a^2\right)^2x}-\frac{ab^3\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{\left(1-a^2\right)^{5/2}}+\frac{(a+bx-1)^{3/2}(a+bx+1)^{3/2}}{3\left(1-a^2\right)x^3}-\frac{ab\sqrt{a+bx-1}\,(a+bx}{2(1-a)(a+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^4,x]
```

```
[Out] -a/(3*x^3) - b/(2*x^2) + (a*b^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(2*(1
 - a^2)^2*x) - (a*b*Sqrt[-1 + a + b*x]*(1 + a + b*x)^(3/2))/(2*(1 - a)*(1 +
 a)^2*x^2) + ((-1 + a + b*x)^(3/2)*(1 + a + b*x)^(3/2))/(3*(1 - a^2)*x^3) -
 (a*b^3*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a + b
*x])])/(1 - a^2)^(5/2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
```

```
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\cosh^{-1}(a+bx)}}{x^4} dx &= \int \frac{a+bx+\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^4} dx \\
&= \int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^4}\right) dx \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^4} dx \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{(-1+a+bx)^{3/2}(1+a+bx)^{3/2}}{3\left(1-a^2\right)x^3}+\frac{(ab)\int \frac{\sqrt{-1+a+bx}\sqrt{1+a+bx}}{x^3} dx}{1-a^2} \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+bx)^{3/2}(1+a+bx)^{3/2}}{3\left(1-a^2\right)x^3}+\frac{\left(ab^2\right)\int \frac{}{x^2}}{2(1-} \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2x}-\frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+}{3} \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2x}-\frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+}{3} \\
&= -\frac{a}{3x^3}-\frac{b}{2x^2}+\frac{ab^2\sqrt{-1+a+bx}\sqrt{1+a+bx}}{2\left(1-a^2\right)^2x}-\frac{ab\sqrt{-1+a+bx}(1+a+bx)^{3/2}}{2(1-a)(1+a)^2x^2}+\frac{(-1+a+}{3}
\end{aligned}$$

Mathematica [C] time = 0.17, size = 179, normalized size = 0.95

$$\frac{1}{6}\left(-\frac{3iab^3\log\left(\frac{4\left(1-a^2\right)^{3/2}\left(\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx+1}+ia^2+iabx-i\right)}{ab^3x}\right)}{\left(1-a^2\right)^{5/2}}+\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(-2a^4-a^3bx+a^2\left(b^2x^2+\right.\right.}{\left(a^2-1\right)^2x^3}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^4,x]
```

```
[Out] ((-2*a)/x^3 - (3*b)/x^2 + (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(-2 - 2*a^4
 + a*b*x - a^3*b*x + 2*b^2*x^2 + a^2*(4 + b^2*x^2)))/((-1 + a^2)^2*x^3) - (
```

```
(3*I)*a*b^3*Log[(4*(1 - a^2)^(3/2)*(-I + I*a^2 + I*a*b*x + Sqrt[1 - a^2]*Sq
rt[-1 + a + b*x]*Sqrt[1 + a + b*x]))/(a*b^3*x)])/(1 - a^2)^(5/2))/6
```

fricas [A] time = 2.07, size = 431, normalized size = 2.28

$$\left[\frac{3\sqrt{a^2-1}\,ab^3x^3\log\left(\frac{a^2bx+a^3+\left(a^2-\sqrt{a^2-1}\,a-1\right)\sqrt{bx+a+1}\,\sqrt{bx+a-1}-\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-2\,a^7+\left(a^4+a^2-2\right)b^3x^3+6\,a^5}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/6*(3*sqrt(a^2 - 1)*a*b^3*x^3*log((a^2*b*x + a^3 + (a^2 - sqrt(a^2 - 1)*a
 - 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) - (a*b*x + a^2 - 1)*sqrt(a^2 - 1)
 - a)/x) - 2*a^7 + (a^4 + a^2 - 2)*b^3*x^3 + 6*a^5 - 6*a^3 - 3*(a^6 - 3*a^4
 + 3*a^2 - 1)*b*x - (2*a^6 - (a^4 + a^2 - 2)*b^2*x^2 - 6*a^4 + (a^5 - 2*a^3
 + a)*b*x + 6*a^2 - 2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 2*a)/((a^6 - 3
*a^4 + 3*a^2 - 1)*x^3), 1/6*(6*sqrt(-a^2 + 1)*a*b^3*x^3*arctan(-(sqrt(-a^2
+ 1)*b*x - sqrt(-a^2 + 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) -
 2*a^7 + (a^4 + a^2 - 2)*b^3*x^3 + 6*a^5 - 6*a^3 - 3*(a^6 - 3*a^4 + 3*a^2 -
 1)*b*x - (2*a^6 - (a^4 + a^2 - 2)*b^2*x^2 - 6*a^4 + (a^5 - 2*a^3 + a)*b*x
+ 6*a^2 - 2)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 2*a)/((a^6 - 3*a^4 + 3*a
^2 - 1)*x^3)]
```

giac [B] time = 0.70, size = 487, normalized size = 2.58

$$-\frac{\frac{6\,ab^4\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2\,a}{2\,\sqrt{-a^2+1}}\right)}{\left(a^4-2\,a^2+1\right)\sqrt{-a^2+1}}-\frac{4\left(12\,a^4b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^8-16\,a^5b^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^6-3\,ab^4\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^{1}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*a*b^4*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 2*a)/
sqrt(-a^2 + 1))/((a^4 - 2*a^2 + 1)*sqrt(-a^2 + 1)) - 4*(12*a^4*b^4*(sqrt(b*
x + a + 1) - sqrt(b*x + a - 1))^8 - 16*a^5*b^4*(sqrt(b*x + a + 1) - sqrt(b*
x + a - 1))^6 - 3*a*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^10 + 6*a^2*
b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8 - 56*a^3*b^4*(sqrt(b*x + a +
1) - sqrt(b*x + a - 1))^6 + 48*a^4*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a -
1))^4 + 12*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^8 - 48*a*b^4*(sqrt(b
*x + a + 1) - sqrt(b*x + a - 1))^6 + 192*a^2*b^4*(sqrt(b*x + a + 1) - sqrt(
b*x + a - 1))^4 - 96*a^3*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 14
4*a*b^4*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 + 32*a^2*b^4 + 64*b^4)/((
a^4 - 2*a^2 + 1)*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x
 + a + 1) - sqrt(b*x + a - 1))^2 + 4)^3) + (3*(b*x + a + 1)*b^4 - a*b^4 - 3
*b^4)/(b^3*x^3))/b
```

maple [B] time = 0.01, size = 374, normalized size = 1.98

$$-\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(3\sqrt{a^2-1}\,\ln\left(\frac{2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}+2a^2-2}{x}\right)x^3a\,b^3-x^2a^4b^2\sqrt{b^2x^2+2abx+a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x)
```

```
[Out] -1/6*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(3*(a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(
1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*x^3*a*b^3-x^2*a^4*b^2*(b^2*x^2
+2*a*b*x+a^2-1)^(1/2)+x*a^5*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-(b^2*x^2+2*a*b*
x+a^2-1)^(1/2)*x^2*a^2*b^2+2*a^6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-2*x*a^3*b*(b
^2*x^2+2*a*b*x+a^2-1)^(1/2)+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^2*b^2-6*a^4*(
b^2*x^2+2*a*b*x+a^2-1)^(1/2)+(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x*a*b+6*(b^2*x^2
+2*a*b*x+a^2-1)^(1/2)*a^2-2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/(b^2*x^2+2*a*b*x
+a^2-1)^(1/2)/(a^2-1)^3/x^3-1/3*a/x^3-1/2*b/x^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 zero or nonzero?
```

mupad [**B**] time = 18.89, size = 1537, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x^4,x)
```

```
[Out] ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*((3*b^3)/32 - (a^2*b^3)/32))/(((a
 + 1)^(1/2) - (a + b*x + 1)^(1/2))^2*(a^4 - 2*a^2 + 1)) - b^3/(192*(a^2 - 1
)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*((9*a^2*b^3)/8 - b^3/2 + (5*a
^4*b^3)/8))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4*(3*a^2 - 3*a^4 + a^6 -
 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*((a^2*b^3)/32 - (21*b^3)/64
 + (3*a^4*b^3)/64))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8*(3*a^2 - 3*a^4
 + a^6 - 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*((103*b^3)/96 - (12
1*a^2*b^3)/32 + (11*a^4*b^3)/32 + (67*a^6*b^3)/96))/(((a + 1)^(1/2) - (a +
b*x + 1)^(1/2))^6*(6*a^4 - 4*a^2 - 4*a^6 + a^8 + 1)) - (((a - 1)^(1/2) - (a
 + b*x - 1)^(1/2))^3*(a - 1)^(1/2)*(a + 1)^(1/2)*((17*a*b^3)/32 + (17*a^3*b
^3)/96))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3*(3*a^2 - 3*a^4 + a^6 - 1)
) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^7*(a - 1)^(1/2)*(a + 1)^(1/2)*((
3*a^3*b^3)/16 - (63*a*b^3)/32 + (9*a^5*b^3)/32))/(((a + 1)^(1/2) - (a + b*x
 + 1)^(1/2))^7*(6*a^4 - 4*a^2 - 4*a^6 + a^8 + 1)) - (((a - 1)^(1/2) - (a +
b*x - 1)^(1/2))^5*(a - 1)^(1/2)*(a + 1)^(1/2)*((17*a^3*b^3)/16 - (79*a*b^3)
/32 + (29*a^5*b^3)/32))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5*(6*a^4 - 4
*a^2 - 4*a^6 + a^8 + 1)) + (a*b^3*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(a
- 1)^(1/2)*(a + 1)^(1/2))/(32*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(a^4 -
2*a^2 + 1)))/(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3/((a + 1)^(1/2) - (a +
 b*x + 1)^(1/2))^3 + ((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^9/((a + 1)^(1/2)
 - (a + b*x + 1)^(1/2))^9 + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^5*(15*a^
2 - 3))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5) + (((a - 1)^(1/
2) - (a + b*x - 1)^(1/2))^7*(15*a^2 - 3))/((a^2 - 1)*((a + 1)^(1/2) - (a +
b*x + 1)^(1/2))^7) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*(12*a - 20*a^
3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^6*(a
^4 - 2*a^2 + 1)) - (6*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(a - 1)^(1/
2)*(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4) - (6*
a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*(a - 1)^(1/2)*(a + 1)^(1/2))/((a^
2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8)) - (a/3 + (b*x)/2)/x^3 + ((
```

```
(a - 1)^(1/2) - (a + b*x - 1)^(1/2))*((b^3*(1792*a^4 - 2048*a^2 + 256))/(40
96*(a^2 - 1)^3) - (b^3*(25*a^2 - 9))/(64*(a^2 - 1)^2) + (8*a*(a - 1)^(3/2)*
(a + 1)^(3/2)*((a*b^3*(a - 1)^(1/2)*(a + 1)^(1/2))/(8*(a^2 - 1)^2) - (a*b^3
*(a - 1)^(3/2)*(a + 1)^(3/2))/(8*(a^2 - 1)^3)))/(a^2 - 1)^2))/((a + 1)^(1/2
) - (a + b*x + 1)^(1/2)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*((a*b^3
*(a - 1)^(1/2)*(a + 1)^(1/2))/(16*(a^2 - 1)^2) - (a*b^3*(a - 1)^(3/2)*(a +
1)^(3/2))/(16*(a^2 - 1)^3)))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (b^3
*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3)/(192*(a^2 - 1)*((a + 1)^(1/2) - (
a + b*x + 1)^(1/2))^3) + (a*b^3*log(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2
/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 - a^2 - (a^2*((a - 1)^(1/2) - (a +
 b*x - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2 + (2*a*((a - 1)
^(1/2) - (a + b*x - 1)^(1/2))*(a - 1)^(1/2)*(a + 1)^(1/2))/((a + 1)^(1/2) -
 (a + b*x + 1)^(1/2)) + 1)*(a - 1)^(1/2)*(a + 1)^(1/2))/(6*a^2 - 6*a^4 + 2*
a^6 - 2) - (a*b^3*log(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2)
- (a + b*x + 1)^(1/2)))*(a - 1)^(1/2)*(a + 1)^(1/2))/(6*a^2 - 6*a^4 + 2*a^6
 - 2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**4,x)
```

```
[Out] Timed out
```

3.283 $\int \frac{e^{\cosh^{-1}(a+bx)}}{x^5} dx$

Optimal. Leaf size=238

$$-\frac{\left(4a^2+1\right)b^4\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{4\left(1-a^2\right)^{7/2}}+\frac{a\left(2a^2+13\right)b^3\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{24\left(1-a^2\right)^3x}+\frac{\left(2a^2+3\right)b^2\sqrt{a+bx-1}\,\sqrt{a+bx}}{24\left(1-a^2\right)^2x^2}$$

```
[Out] -1/4*a/x^4-1/3*b/x^3-1/4*(4*a^2+1)*b^4*arctan((1-a)^(1/2)*(b*x+a+1)^(1/2)/(
1+a)^(1/2)/(b*x+a-1)^(1/2))/(-a^2+1)^(7/2)-1/4*(b*x+a-1)^(1/2)*(b*x+a+1)^(1
/2)/x^4+1/12*a*b*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)/x^3+1/24*(2*a^2+3
)*b^2*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)^2/x^2+1/24*a*(2*a^2+13)*b^3*
(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)/(-a^2+1)^3/x
```

Rubi [A] time = 0.20, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5909, 14, 97, 151, 12, 93, 205}

$$\frac{\left(2a^2+3\right)b^2\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{24\left(1-a^2\right)^2x^2}+\frac{a\left(2a^2+13\right)b^3\sqrt{a+bx-1}\,\sqrt{a+bx+1}}{24\left(1-a^2\right)^3x}-\frac{\left(4a^2+1\right)b^4\tan^{-1}\left(\frac{\sqrt{1-a}\,\sqrt{a+bx+1}}{\sqrt{a+1}\,\sqrt{a+bx-1}}\right)}{4\left(1-a^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(3*x^3) - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(4*x^4) + (
a*b*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(12*(1 - a^2)*x^3) + ((3 + 2*a^2)
*b^2*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(24*(1 - a^2)^2*x^2) + (a*(13 +
2*a^2)*b^3*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x])/(24*(1 - a^2)^3*x) - ((1 +
 4*a^2)*b^4*ArcTan[(Sqrt[1 - a]*Sqrt[1 + a + b*x])/(Sqrt[1 + a]*Sqrt[-1 + a
 + b*x])])/(4*(1 - a^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5909

```
Int[E^(ArcCosh[u_]*(n_.))*(x_)^(m_.), x_Symbol] :> Int[x^m*(u + Sqrt[-1 + u
]*Sqrt[1 + u])^n, x] /; RationalQ[m] && IntegerQ[n] && PolynomialQ[u, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)}}{x^5}\,dx = \int \frac{a+bx+\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^5}\,dx$$

$$= \int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^5}\right)dx$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}+\int \frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{x^5}\,dx$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{1}{4}\int \frac{ab+b^2x}{x^4\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}\,dx$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\int \frac{\left(3+2a^2\right)b}{x^3\sqrt{-1+a+b}}}{12\left(1\right.}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2}{}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2}{}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2}{}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2}{}$$

$$= -\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{4x^4}+\frac{ab\sqrt{-1+a+bx}\,\sqrt{1+a+bx}}{12\left(1-a^2\right)x^3}+\frac{\left(3+2a^2\right)b^2}{}$$

Mathematica [C] time = 0.28, size = 198, normalized size = 0.83

$$\frac{1}{24}\left(-\frac{3i\left(4a^2+1\right)b^4\log\left(\frac{16i\left(1-a^2\right)^{5/2}\left(-i\sqrt{1-a^2}\sqrt{a+bx-1}\sqrt{a+bx+1}+a^2+abx-1\right)}{b^4\left(4a^2x+x\right)}\right)}{\left(1-a^2\right)^{7/2}}-\frac{\sqrt{a+bx-1}\sqrt{a+bx+1}\left(\frac{a\left(2a^2+13\right)b^3x^3}{\left(a^2-1\right)^3}-\right.}{x^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcCosh[a + b*x]/x^5,x]
```

```
[Out] ((-6*a)/x^4 - (8*b)/x^3 - (Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]*(6 + (2*a*b
*x)/(-1 + a^2) - ((3 + 2*a^2)*b^2*x^2)/(-1 + a^2)^2 + (a*(13 + 2*a^2)*b^3*x
^3)/(-1 + a^2)^3))/x^4 - ((3*I)*(1 + 4*a^2)*b^4*Log[((16*I)*(1 - a^2)^(5/2)
*(-1 + a^2 + a*b*x - I*Sqrt[1 - a^2]*Sqrt[-1 + a + b*x]*Sqrt[1 + a + b*x]))
/(b^4*(x + 4*a^2*x))])/(1 - a^2)^(7/2))/24
```

fricas [A] time = 0.95, size = 569, normalized size = 2.39

$$\left[\frac{3\left(4a^2+1\right)\sqrt{a^2-1}\,b^4x^4\log\left(\frac{a^2bx+a^3+\left(a^2+\sqrt{a^2-1}\,a-1\right)\sqrt{bx+a+1}\sqrt{bx+a-1}+\left(abx+a^2-1\right)\sqrt{a^2-1}-a}{x}\right)-6a^9-\left(2a^5+11a^3-13\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="fricas"
)
```

```
[Out] [1/24*(3*(4*a^2 + 1)*sqrt(a^2 - 1)*b^4*x^4*log((a^2*b*x + a^3 + (a^2 + sqrt
(a^2 - 1)*a - 1)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + (a*b*x + a^2 - 1)*sq
rt(a^2 - 1) - a)/x) - 6*a^9 - (2*a^5 + 11*a^3 - 13*a)*b^4*x^4 + 24*a^7 - 36
*a^5 + 24*a^3 - 8*(a^8 - 4*a^6 + 6*a^4 - 4*a^2 + 1)*b*x - (6*a^8 + (2*a^5 +
 11*a^3 - 13*a)*b^3*x^3 - 24*a^6 - (2*a^6 - a^4 - 4*a^2 + 3)*b^2*x^2 + 36*a
^4 + 2*(a^7 - 3*a^5 + 3*a^3 - a)*b*x - 24*a^2 + 6)*sqrt(b*x + a + 1)*sqrt(b
*x + a - 1) - 6*a)/((a^8 - 4*a^6 + 6*a^4 - 4*a^2 + 1)*x^4), -1/24*(6*(4*a^2
 + 1)*sqrt(-a^2 + 1)*b^4*x^4*arctan(-(sqrt(-a^2 + 1)*b*x - sqrt(-a^2 + 1)*s
qrt(b*x + a + 1)*sqrt(b*x + a - 1))/(a^2 - 1)) + 6*a^9 + (2*a^5 + 11*a^3 -
13*a)*b^4*x^4 - 24*a^7 + 36*a^5 - 24*a^3 + 8*(a^8 - 4*a^6 + 6*a^4 - 4*a^2 +
 1)*b*x + (6*a^8 + (2*a^5 + 11*a^3 - 13*a)*b^3*x^3 - 24*a^6 - (2*a^6 - a^4
- 4*a^2 + 3)*b^2*x^2 + 36*a^4 + 2*(a^7 - 3*a^5 + 3*a^3 - a)*b*x - 24*a^2 +
6)*sqrt(b*x + a + 1)*sqrt(b*x + a - 1) + 6*a)/((a^8 - 4*a^6 + 6*a^4 - 4*a^2
 + 1)*x^4)]
```

giac [B] time = 1.00, size = 817, normalized size = 3.43

$$\frac{\frac{3\left(4a^2b^5+b^5\right)\arctan\left(\frac{\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^2-2a}{2\sqrt{-a^2+1}}\right)}{\left(a^6-3a^4+3a^2-1\right)\sqrt{-a^2+1}}+\frac{2\left(128a^6b^5\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^{10}+12a^2b^5\left(\sqrt{bx+a+1}-\sqrt{bx+a-1}\right)^{14}-128a^7b^5\left(\sqrt{bx+a+1}-\sqrt{\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*(4*a^2*b^5 + b^5)*arctan(1/2*((sqrt(b*x + a + 1) - sqrt(b*x + a - 1
))^2 - 2*a)/sqrt(-a^2 + 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*sqrt(-a^2 + 1)) + 2*
(128*a^6*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^10 + 12*a^2*b^5*(sqrt(
```

```
b*x + a + 1) - sqrt(b*x + a - 1))^14 - 128*a^7*b^5*(sqrt(b*x + a + 1) - sqr
t(b*x + a - 1))^8 - 168*a^3*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^12
+ 448*a^4*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^10 + 3*b^5*(sqrt(b*x
+ a + 1) - sqrt(b*x + a - 1))^14 - 1216*a^5*b^5*(sqrt(b*x + a + 1) - sqrt(b
*x + a - 1))^8 - 42*a*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^12 + 512*
a^6*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 + 768*a^2*b^5*(sqrt(b*x +
 a + 1) - sqrt(b*x + a - 1))^10 - 2544*a^3*b^5*(sqrt(b*x + a + 1) - sqrt(b*
x + a - 1))^8 + 5632*a^4*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 84
*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^10 - 1536*a^5*b^5*(sqrt(b*x +
a + 1) - sqrt(b*x + a - 1))^4 - 312*a*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a
 - 1))^8 + 1920*a^2*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^6 - 7552*a^
3*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 + 1024*a^4*b^5*(sqrt(b*x +
a + 1) - sqrt(b*x + a - 1))^2 + 336*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a -
 1))^6 - 992*a*b^5*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 + 5888*a^2*b^5
*(sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^2 - 256*a^3*b^5 - 192*b^5*(sqrt(b*
x + a + 1) - sqrt(b*x + a - 1))^2 - 1664*a*b^5)/((a^6 - 3*a^4 + 3*a^2 - 1)*
((sqrt(b*x + a + 1) - sqrt(b*x + a - 1))^4 - 4*a*(sqrt(b*x + a + 1) - sqrt(
b*x + a - 1))^2 + 4)^4) - (4*(b*x + a + 1)*b^5 - a*b^5 - 4*b^5)/(b^4*x^4))/
b
```

maple [B] time = 0.02, size = 603, normalized size = 2.53

$$\frac{\sqrt{bx+a-1}\,\sqrt{bx+a+1}\,\left(12\sqrt{a^2-1}\,\ln\left(\frac{2abx+2\sqrt{a^2-1}\,\sqrt{b^2x^2+2abx+a^2-1}+2a^2-2}{x}\right)x^4a^2b^4-2x^3a^5b^3\sqrt{b^2x^2+2abx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x)
```

```
[Out] 1/24*(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2)*(12*(a^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^
(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/x)*x^4*a^2*b^4-2*x^3*a^5*b^3*(b^
2*x^2+2*a*b*x+a^2-1)^(1/2)+2*x^2*a^6*b^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+3*(a
^2-1)^(1/2)*ln(2*(a*b*x+(a^2-1)^(1/2)*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)+a^2-1)/
x)*x^4*b^4-11*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^3*a^3*b^3-2*x*a^7*b*(b^2*x^2+
2*a*b*x+a^2-1)^(1/2)-x^2*a^4*b^2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-6*a^8*(b^2*x
^2+2*a*b*x+a^2-1)^(1/2)+13*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^3*a*b^3+6*x*a^5*
b*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-4*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^2*a^2*b^2
+24*a^6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)-6*x*a^3*b*(b^2*x^2+2*a*b*x+a^2-1)^(1/
2)+3*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x^2*b^2-36*a^4*(b^2*x^2+2*a*b*x+a^2-1)^(
1/2)+2*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)*x*a*b+24*(b^2*x^2+2*a*b*x+a^2-1)^(1/2)
*a^2-6*(b^2*x^2+2*a*b*x+a^2-1)^(1/2))/(b^2*x^2+2*a*b*x+a^2-1)^(1/2)/(a^2-1)
^4/x^4-1/3*b/x^3-1/4*a/x^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))/x^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is a-1 zero or nonzero?
```

mupad [B] time = 28.03, size = 2347, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (a + b*x - 1)^(1/2)*(a + b*x + 1)^(1/2) + b*x)/x^5,x)
```

```
[Out] (log(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(
1/2)))*(b^4*(a - 1)^(1/2)*(a + 1)^(1/2) + 4*a^2*b^4*(a - 1)^(1/2)*(a + 1)^(
1/2)))/(48*a^4 - 32*a^2 - 32*a^6 + 8*a^8 + 8) - (a/4 + (b*x)/3)/x^4 - (log(
((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2/((a + 1)^(1/2) - (a + b*x + 1)^(1/2
))^2 - a^2 - (a^2*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2)/((a + 1)^(1/2) -
 (a + b*x + 1)^(1/2))^2 + (2*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))*(a - 1
)^(1/2)*(a + 1)^(1/2))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) + 1)*(b^4*(a -
 1)^(1/2)*(a + 1)^(1/2) + 4*a^2*b^4*(a - 1)^(1/2)*(a + 1)^(1/2)))/(48*a^4 -
 32*a^2 - 32*a^6 + 8*a^8 + 8) - ((((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^3*(
(17*a*b^4)/192 - (5*a^3*b^4)/192))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3
*(3*a^2 - 3*a^4 + a^6 - 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^11*((7
*a^3*b^4)/64 - (81*a*b^4)/128 + (3*a^5*b^4)/128))/(((a + 1)^(1/2) - (a + b*
x + 1)^(1/2))^11*(6*a^4 - 4*a^2 - 4*a^6 + a^8 + 1)) + (((a - 1)^(1/2) - (a
+ b*x - 1)^(1/2))^5*((229*a^3*b^4)/64 - (119*a*b^4)/128 + (119*a^5*b^4)/384
))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^5*(6*a^4 - 4*a^2 - 4*a^6 + a^8 +
1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^9*((1025*a*b^4)/384 - (1745*a^
3*b^4)/128 + (385*a^5*b^4)/128 + (239*a^7*b^4)/384))/(((a + 1)^(1/2) - (a +
 b*x + 1)^(1/2))^9*(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1)) + (((a - 1
)^(1/2) - (a + b*x - 1)^(1/2))^7*((1103*a*b^4)/384 - (2199*a^3*b^4)/128 + (
1039*a^5*b^4)/128 + (521*a^7*b^4)/384))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/
2))^7*(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1)) - (b^4*(a - 1)^(1/2)*(a
 + 1)^(1/2))/(1024*(a^4 - 2*a^2 + 1)) + (a*b^4*((a - 1)^(1/2) - (a + b*x -
1)^(1/2)))/(192*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))*(a^4 - 2*a^2 + 1)) +
(((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*(b^4/128 - (a^2*b^4)/384)*(a - 1)^
(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^2*(3*a^2 - 3*a^
4 + a^6 - 1)) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4*(a - 1)^(1/2)*(a +
 1)^(1/2)*((11*b^4)/512 + (249*a^2*b^4)/256 + (a^4*b^4)/1536))/(((a + 1)^(1
/2) - (a + b*x + 1)^(1/2))^4*(6*a^4 - 4*a^2 - 4*a^6 + a^8 + 1)) + (((a - 1)
^(1/2) - (a + b*x - 1)^(1/2))^10*(a - 1)^(1/2)*(a + 1)^(1/2)*(b^4/256 + (11
67*a^2*b^4)/256 - (225*a^4*b^4)/256 - (47*a^6*b^4)/256))/(((a + 1)^(1/2) -
(a + b*x + 1)^(1/2))^10*(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 - 1)) + (((
a - 1)^(1/2) - (a + b*x - 1)^(1/2))^6*(a - 1)^(1/2)*(a + 1)^(1/2)*((7*b^4)/
256 + (1665*a^2*b^4)/256 - (5365*a^4*b^4)/768 - (707*a^6*b^4)/768))/(((a +
1)^(1/2) - (a + b*x + 1)^(1/2))^6*(5*a^2 - 10*a^4 + 10*a^6 - 5*a^8 + a^10 -
 1)) - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^8*(a - 1)^(1/2)*(a + 1)^(1/2)
*((8765*a^2*b^4)/768 - (239*b^4)/1024 - (11789*a^4*b^4)/512 + (1471*a^6*b^4
)/256 + (3635*a^8*b^4)/3072))/(((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^8*(15*
a^4 - 6*a^2 - 20*a^6 + 15*a^8 - 6*a^10 + a^12 + 1)))/(((a - 1)^(1/2) - (a +
 b*x - 1)^(1/2))^4/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^4 + ((a - 1)^(1/2)
 - (a + b*x - 1)^(1/2))^12/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^12 + (((a
- 1)^(1/2) - (a + b*x - 1)^(1/2))^6*(28*a^2 - 4))/((a^2 - 1)*((a + 1)^(1/2)
 - (a + b*x + 1)^(1/2))^6) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^10*(28*
a^2 - 4))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^10) + (((a - 1)^
(1/2) - (a + b*x - 1)^(1/2))^8*(70*a^4 - 60*a^2 + 6))/(((a + 1)^(1/2) - (a
+ b*x + 1)^(1/2))^8*(a^4 - 2*a^2 + 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1
/2))^7*(24*a - 56*a^3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a +
b*x + 1)^(1/2))^7*(a^4 - 2*a^2 + 1)) + (((a - 1)^(1/2) - (a + b*x - 1)^(1/2
))^9*(24*a - 56*a^3)*(a - 1)^(1/2)*(a + 1)^(1/2))/(((a + 1)^(1/2) - (a + b*
x + 1)^(1/2))^9*(a^4 - 2*a^2 + 1)) - (8*a*((a - 1)^(1/2) - (a + b*x - 1)^(1
/2))^5*(a - 1)^(1/2)*(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x +
1)^(1/2))^5) - (8*a*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^11*(a - 1)^(1/2)*
(a + 1)^(1/2))/((a^2 - 1)*((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^11)) - (((a
 - 1)^(1/2) - (a + b*x - 1)^(1/2))^2*((5*a*(a^2 - 1)^2*((a*b^4)/(32*(a - 1)
^2*(a + 1)^2) - (5*a*b^4*(a^2 - 1)^3)/(128*(a - 1)^5*(a + 1)^5)))/((a - 1)^
(5/2)*(a + 1)^(5/2)) - (b^4*(23*a^2 - 7))/(512*(a - 1)^(5/2)*(a + 1)^(5/2))
 + (5*b^4*(a^2 - 1)*(9*a^2 - 1))/(512*(a - 1)^(7/2)*(a + 1)^(7/2))))/((a +
```

```
1)^(1/2) - (a + b*x + 1)^(1/2))^2 - (((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^
3*((a*b^4)/(96*(a - 1)^2*(a + 1)^2) - (5*a*b^4*(a^2 - 1)^3)/(384*(a - 1)^5*
(a + 1)^5)))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2))^3 + (((a - 1)^(1/2) - (a
 + b*x - 1)^(1/2))*((5*((a*b^4)/(32*(a - 1)^2*(a + 1)^2) - (5*a*b^4*(a^2 -
1)^3)/(128*(a - 1)^5*(a + 1)^5))*(9*a^2 - 1))/((a - 1)*(a + 1)) - (a*b^4)/(
16*(a - 1)^2*(a + 1)^2) - (10*a*(a^2 - 1)^2*((10*a*(a^2 - 1)^2*((a*b^4)/(32
*(a - 1)^2*(a + 1)^2) - (5*a*b^4*(a^2 - 1)^3)/(128*(a - 1)^5*(a + 1)^5)))/(
(a - 1)^(5/2)*(a + 1)^(5/2)) - (b^4*(23*a^2 - 7))/(256*(a - 1)^(5/2)*(a + 1
)^(5/2)) + (5*b^4*(a^2 - 1)*(9*a^2 - 1))/(256*(a - 1)^(7/2)*(a + 1)^(7/2)))
)/((a - 1)^(5/2)*(a + 1)^(5/2)) + (5*a*b^4*(a^2 - 1)*(3*a^4 - 4*a^2 + 1))/(
32*(a - 1)^5*(a + 1)^5)))/((a + 1)^(1/2) - (a + b*x + 1)^(1/2)) + (b^4*(a^2
 - 1)*((a - 1)^(1/2) - (a + b*x - 1)^(1/2))^4)/(1024*((a + 1)^(1/2) - (a +
b*x + 1)^(1/2))^4*(a - 1)^(5/2)*(a + 1)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a+(b*x+a-1)**(1/2)*(b*x+a+1)**(1/2))/x**5,x)
```

```
[Out] Timed out
```

3.284 $\int e^{\cosh^{-1}(a+bx)^2} x^3 \, dx$

Optimal. Leaf size=359

$$\frac{\sqrt{\pi}\, a^3 \operatorname{erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\, b^4}-\frac{\sqrt{\pi}\, a^3 \operatorname{erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\, b^4}+\frac{3\sqrt{\pi}\, a^2 \operatorname{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^4}+$$

```
[Out] -1/32*erfi(-2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(4)-1/16*erfi(-1+arccosh(b*x+
a))*Pi^(1/2)/b^4/E-3/8*a^2*erfi(-1+arccosh(b*x+a))*Pi^(1/2)/b^4/E+1/16*erfi
(1+arccosh(b*x+a))*Pi^(1/2)/b^4/E+3/8*a^2*erfi(1+arccosh(b*x+a))*Pi^(1/2)/b
^4/E+1/32*erfi(2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(4)+3/16*a*erfi(-3/2+arcco
sh(b*x+a))*Pi^(1/2)/b^4/exp(9/4)+3/16*a*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/
b^4/exp(1/4)+1/4*a^3*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-3/16*a
*erfi(1/2+arccosh(b*x+a))*Pi^(1/2)/b^4/exp(1/4)-1/4*a^3*erfi(1/2+arccosh(b*
x+a))*Pi^(1/2)/b^4/exp(1/4)-3/16*a*erfi(3/2+arccosh(b*x+a))*Pi^(1/2)/b^4/ex
p(9/4)
```

Rubi [A] time = 0.74, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5899, 6741, 12, 6742, 5512, 2234, 2204, 5514}

$$\frac{\sqrt{\pi}\, a^3 \operatorname{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\, b^4}-\frac{\sqrt{\pi}\, a^3 \operatorname{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\, b^4}+\frac{3\sqrt{\pi}\, a^2 \operatorname{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^4}+$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi]*Erfi[1 -
 ArcCosh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 - ArcCosh[a + b*x]])/(32*b
^4*E^4) + (Sqrt[Pi]*Erfi[1 + ArcCosh[a + b*x]])/(16*b^4*E) + (3*a^2*Sqrt[Pi
]*Erfi[1 + ArcCosh[a + b*x]])/(8*b^4*E) + (Sqrt[Pi]*Erfi[2 + ArcCosh[a + b*
x]])/(32*b^4*E^4) + (3*a*Sqrt[Pi]*Erfi[(-3 + 2*ArcCosh[a + b*x])/2])/(16*b^
4*E^(9/4)) + (3*a*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(1/
4)) + (a^3*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3
*a*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(1/4)) - (a^3*Sqrt[
Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^4*E^(1/4)) - (3*a*Sqrt[Pi]*Erfi[
(3 + 2*ArcCosh[a + b*x])/2])/(16*b^4*E^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int e^{\cosh^{-1}(a+bx)^2} x^3\,dx &= \frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^3 \sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{e^{x^2}(-a+\cosh(x))^3\sinh(x)}{b^3}\,dx, x, \cosh^{-1}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int e^{x^2}(-a+\cosh(x))^3\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b^4} \\
&= \frac{\text{Subst}\left(\int \left(-a^3e^{x^2}\sinh(x)+3a^2e^{x^2}\cosh(x)\sinh(x)-3ae^{x^2}\cosh^2(x)\sinh(x)+e^{x^2}\cosh\right.\right.}{b^4} \\
&= \frac{\text{Subst}\left(\int e^{x^2}\cosh^3(x)\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b^4} - \frac{(3a)\,\text{Subst}\left(\int e^{x^2}\cosh^2(x)\sinh\right.}{b^4} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{16}e^{-4x+x^2}-\frac{1}{8}e^{-2x+x^2}+\frac{1}{8}e^{2x+x^2}+\frac{1}{16}e^{4x+x^2}\right)dx, x, \cosh^{-1}(a+bx)\right)}{b^4} - \frac{(3a)\,\text{S}}{} \\
&= -\frac{\text{Subst}\left(\int e^{-4x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{16b^4} + \frac{\text{Subst}\left(\int e^{4x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{16b^4} - \frac{\text{S}}{} \\
&= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-4+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{16b^4e^4} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(4+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right.}{16b^4e^4} \\
&= \frac{\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{16b^4e} + \frac{3a^2\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8b^4e} + \frac{\sqrt{\pi}\,\text{erfi}\left(2-\cosh\right.}{32b^4e^4}
\end{aligned}
$$

Mathematica [A] time = 0.40, size = 198, normalized size = 0.55

$$\frac{\sqrt{\pi}\left(-8e^{15/4}a^3\text{erfi}\left(\cosh^{-1}(a+bx)+\frac{1}{2}\right)+12e^3a^2\text{erfi}\left(\cosh^{-1}(a+bx)+1\right)-2e^{15/4}\left(4a^2+3\right)a\text{erfi}\left(\frac{1}{2}-\cosh^{-1}(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x^3,x]
```

```
[Out] (Sqrt[Pi]*(-2*a*(3 + 4*a^2)*E^(15/4)*Erfi[1/2 - ArcCosh[a + b*x]] + 2*(1 +
6*a^2)*E^3*Erfi[1 - ArcCosh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 - ArcCosh[a +
b*x]] + Erfi[2 - ArcCosh[a + b*x]] - 6*a*E^(15/4)*Erfi[1/2 + ArcCosh[a + b*
x]] - 8*a^3*E^(15/4)*Erfi[1/2 + ArcCosh[a + b*x]] + 2*E^3*Erfi[1 + ArcCosh[
a + b*x]] + 12*a^2*E^3*Erfi[1 + ArcCosh[a + b*x]] - 6*a*E^(7/4)*Erfi[3/2 +
ArcCosh[a + b*x]] + Erfi[2 + ArcCosh[a + b*x]]))/(32*b^4*E^4)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(x^3e^{\left(\operatorname{arcosh}(bx+a)^2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] integral(x^3*e^(arccosh(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*e^(arccosh(b*x + a)^2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arccosh}(bx+a)^2}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x^3,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3*e^(arccosh(b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,e^{\operatorname{acosh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(acosh(a + b*x)^2),x)
```

```
[Out] int(x^3*exp(acosh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 e^{\operatorname{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x**3,x)
```

```
[Out] Integral(x**3*exp(acosh(a + b*x)**2), x)
```

3.285 $\int e^{\cosh^{-1}(a+bx)^2}x^2\,dx$

Optimal. Leaf size=251

$$-\frac{\sqrt{\pi}\,a^2\text{erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^3}+\frac{\sqrt{\pi}\,a^2\text{erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^3}-\frac{\sqrt{\pi}\,a\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4eb^3}-$$

[Out] 1/4*a*erfi(-1+arccosh(b*x+a))*Pi^(1/2)/b^3/E-1/4*a*erfi(1+arccosh(b*x+a))*Pi^(1/2)/b^3/E-1/16*erfi(-3/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(9/4)-1/16*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)-1/4*a^2*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/16*erfi(1/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/4*a^2*erfi(1/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(1/4)+1/16*erfi(3/2+arccosh(b*x+a))*Pi^(1/2)/b^3/exp(9/4)

Rubi [A] time = 0.49, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5899, 6741, 12, 6742, 5512, 2234, 2204, 5514}

$$-\frac{\sqrt{\pi}\,a^2\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^3}+\frac{\sqrt{\pi}\,a^2\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b^3}-\frac{\sqrt{\pi}\,a\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4eb^3}-$$

Antiderivative was successfully verified.

[In] Int[E^ArcCosh[a + b*x]^2*x^2,x]

[Out] -(a*Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(4*b^3*E) - (a*Sqrt[Pi]*Erfi[1 + ArcCosh[a + b*x]])/(4*b^3*E) - (Sqrt[Pi]*Erfi[(-3 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(9/4)) - (Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(1/4)) - (a^2*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(1/4)) + (a^2*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^3*E^(1/4)) + (Sqrt[Pi]*Erfi[(3 + 2*ArcCosh[a + b*x])/2])/(16*b^3*E^(9/4))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
```

```
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)^2}x^2\,dx = \frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}(a-\cosh(x))^2\sinh(x)}{b^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}(a-\cosh(x))^2\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(a^2e^{x^2}\sinh(x)-2ae^{x^2}\cosh(x)\sinh(x)+e^{x^2}\cosh^2(x)\sinh(x)\right)\,dx,x,\cosh^{-1}(\right.}{b^3}$$
$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh^2(x)\sinh(x)\,dx,x,\cosh^{-1}(a+bx)\right)}{b^3}-\frac{(2a)\,\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(\right.}{b^3}$$
$$= \frac{\text{Subst}\left(\int\left(-\frac{1}{8}e^{-3x+x^2}-\frac{1}{8}e^{-x+x^2}+\frac{e^{x+x^2}}{8}+\frac{1}{8}e^{3x+x^2}\right)dx,x,\cosh^{-1}(a+bx)\right)}{b^3}-\frac{(2a)\,\text{Subst}}{}$$
$$= -\frac{\text{Subst}\left(\int e^{-3x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3}-\frac{\text{Subst}\left(\int e^{-x+x^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3}+\frac{\text{S}}{}$$
$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-3+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3e^{9/4}}+\frac{\text{Subst}\left(\int e^{\frac{1}{4}(3+2x)^2}\,dx,x,\cosh^{-1}(a+bx)\right)}{8b^3e^{9/4}}$$
$$= -\frac{a\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{4b^3e}-\frac{a\sqrt{\pi}\,\text{erfi}\left(1+\cosh^{-1}(a+bx)\right)}{4b^3e}-\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-3+2\right.\right.}{16b}$$

Mathematica [A] time = 0.23, size = 136, normalized size = 0.54

$$\sqrt{\pi}\left(4e^2a^2\text{erfi}\left(\cosh^{-1}(a+bx)+\frac{1}{2}\right)+e^2\left(4a^2+1\right)\text{erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)-4e^{5/4}a\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x^2,x]
```

```
[Out] (Sqrt[Pi]*((1 + 4*a^2)*E^2*Erfi[1/2 - ArcCosh[a + b*x]] - 4*a*E^(5/4)*Erfi[
1 - ArcCosh[a + b*x]] + Erfi[3/2 - ArcCosh[a + b*x]] + E^2*Erfi[1/2 + ArcCo
sh[a + b*x]] + 4*a^2*E^2*Erfi[1/2 + ArcCosh[a + b*x]] - 4*a*E^(5/4)*Erfi[1
+ ArcCosh[a + b*x]] + Erfi[3/2 + ArcCosh[a + b*x]]))/(16*b^3*E^(9/4))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2 e^{\left(\operatorname{arcosh}(bx+a)^2\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*e^(arccosh(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(arccosh(b*x + a)^2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arccosh}(bx+a)^2} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x^2,x)
```

```
[Out] int(exp(arccosh(b*x+a)^2)*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2*e^(arccosh(b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,e^{\operatorname{acosh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(acosh(a + b*x)^2),x)
```

```
[Out] int(x^2*exp(acosh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 e^{\operatorname{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x**2,x)
```

```
[Out] Integral(x**2*exp(acosh(a + b*x)**2), x)
```

3.286 $\int e^{\cosh^{-1}(a+bx)^2} x\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\,\text{erfi}\left(\cosh^{-1}(a+bx)+1\right)}{8eb^2}+\frac{\sqrt{\pi}\,a\text{erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^2}-\frac{\sqrt{\pi}\,a\text{erfi}\left(\right.}{}$$

```
[Out] -1/8*erfi(-1+arccosh(b*x+a))*Pi^(1/2)/b^2/E+1/8*erfi(1+arccosh(b*x+a))*Pi^(
1/2)/b^2/E+1/4*a*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)-1/4*a*erfi
(1/2+arccosh(b*x+a))*Pi^(1/2)/b^2/exp(1/4)
```

Rubi [A] time = 0.28, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5899, 6741, 12, 6742, 5512, 2234, 2204, 5514}

$$\frac{\sqrt{\pi}\,\text{Erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8eb^2}+\frac{\sqrt{\pi}\,\text{Erfi}\left(\cosh^{-1}(a+bx)+1\right)}{8eb^2}+\frac{\sqrt{\pi}\,a\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b^2}-\frac{\sqrt{\pi}\,a\text{Erf}}{}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*Erfi[1 - ArcCosh[a + b*x]])/(8*b^2*E) + (Sqrt[Pi]*Erfi[1 + ArcCos
h[a + b*x]])/(8*b^2*E) + (a*Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*
b^2*E^(1/4)) - (a*Sqrt[Pi]*Erfi[(1 + 2*ArcCosh[a + b*x])/2])/(4*b^2*E^(1/4)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5514

```
Int[Cosh[v_]^(n_.)*(F_)^(u_)*Sinh[v_]^(m_.), x_Symbol] :> Int[ExpandTrigToE
xp[F^u, Sinh[v]^m*Cosh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || Pol
yQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[m, 0] && IGtQ[n,
0]
```

Rule 5899

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.))*(x_)^(m_.), x_Symbol] :>
 Dist[1/b, Subst[Int[(-(a/b) + Cosh[x]/b)^m*f^(c*x^n)*Sinh[x], x], x, ArcCo
sh[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int e^{\cosh^{-1}(a+bx)^2} x\,dx = \frac{\text{Subst}\left(\int e^{x^2}\left(-\frac{a}{b}+\frac{\cosh(x)}{b}\right)\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{x^2}(-a+\cosh(x))\sinh(x)}{b}\,dx, x, \cosh^{-1}(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}(-a+\cosh(x))\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{\text{Subst}\left(\int \left(-ae^{x^2}\sinh(x)+e^{x^2}\cosh(x)\sinh(x)\right)dx, x, \cosh^{-1}(a+bx)\right)}{b^2}$$

$$= \frac{\text{Subst}\left(\int e^{x^2}\cosh(x)\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int e^{x^2}\sinh(x)\,dx, x, \cosh^{-1}(\right.}{b^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}e^{-2x+x^2}+\frac{1}{4}e^{2x+x^2}\right)dx, x, \cosh^{-1}(a+bx)\right)}{b^2} - \frac{a\,\text{Subst}\left(\int \left(-\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)dx\right.}{b^2}$$

$$= -\frac{\text{Subst}\left(\int e^{-2x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{4b^2} + \frac{\text{Subst}\left(\int e^{2x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{4b^2} + \frac{a\,\text{Sub}}{}$$

$$= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-2+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{4b^2e} + \frac{\text{Subst}\left(\int e^{\frac{1}{4}(2+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{4b^2e} + \cdots$$

$$= \frac{\sqrt{\pi}\,\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)}{8b^2e} + \frac{\sqrt{\pi}\,\text{erfi}\left(1+\cosh^{-1}(a+bx)\right)}{8b^2e} + \frac{a\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\cosh^{-1}\right.\right.}{4b^2\sqrt[4]{e}}$$

Mathematica [A] time = 0.14, size = 76, normalized size = 0.65

$$\frac{\sqrt{\pi}\left(-2e^{3/4}a\,\text{erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)+\text{erfi}\left(1-\cosh^{-1}(a+bx)\right)-2e^{3/4}a\,\text{erfi}\left(\cosh^{-1}(a+bx)+\frac{1}{2}\right)+\text{erfi}\left(\cosh\right.\right.}{8eb^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2*x,x]
```

```
[Out] (Sqrt[Pi]*(-2*a*E^(3/4)*Erfi[1/2 - ArcCosh[a + b*x]] + Erfi[1 - ArcCosh[a +
 b*x]] - 2*a*E^(3/4)*Erfi[1/2 + ArcCosh[a + b*x]] + Erfi[1 + ArcCosh[a + b*
x]]))/(8*b^2*E)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(xe^{\left(\operatorname{arcosh}(bx+a)^2\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="fricas")
[Out] integral(x*e^(arccosh(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="giac")
[Out] integrate(x*e^(arccosh(b*x + a)^2), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int e^{\operatorname{arccosh}(bx+a)^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)*x,x)
[Out] int(exp(arccosh(b*x+a)^2)*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)*x,x, algorithm="maxima")
[Out] integrate(x*e^(arccosh(b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,e^{\operatorname{acosh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(acosh(a + b*x)^2),x)
[Out] int(x*exp(acosh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int xe^{\operatorname{acosh}^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)*x,x)
[Out] Integral(x*exp(acosh(a + b*x)**2), x)
```

3.287 $\int e^{\cosh^{-1}(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b}-\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b}$$

```
[Out] -1/4*erfi(-1/2+arccosh(b*x+a))*Pi^(1/2)/b/exp(1/4)+1/4*erfi(1/2+arccosh(b*x
+a))*Pi^(1/2)/b/exp(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5897, 5512, 2234, 2204}

$$\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)+1\right)\right)}{4\sqrt[4]{e}\,b}-\frac{\sqrt{\pi}\,\text{Erfi}\left(\frac{1}{2}\left(2\cosh^{-1}(a+bx)-1\right)\right)}{4\sqrt[4]{e}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcCosh[a + b*x]^2,x]
```

```
[Out] -(Sqrt[Pi]*Erfi[(-1 + 2*ArcCosh[a + b*x])/2])/(4*b*E^(1/4)) + (Sqrt[Pi]*Erf
i[(1 + 2*ArcCosh[a + b*x])/2])/(4*b*E^(1/4))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2234

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5512

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[{F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rule 5897

```
Int[(f_)^(ArcCosh[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[1/b,
Subst[Int[f^(c*x^n)*Sinh[x], x], x, ArcCosh[a + b*x]], x] /; FreeQ[{a, b, c
, f}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int e^{\cosh^{-1}(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int e^{x^2}\sinh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int\left(-\frac{1}{2}e^{-x+x^2}+\frac{e^{x+x^2}}{2}\right)dx, x, \cosh^{-1}(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int e^{-x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int e^{x+x^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int e^{\frac{1}{4}(-1+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}}+\frac{\text{Subst}\left(\int e^{\frac{1}{4}(1+2x)^2}\,dx, x, \cosh^{-1}(a+bx)\right)}{2b\sqrt[4]{e}} \\
&= -\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(-1+2\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}\left(1+2\cosh^{-1}(a+bx)\right)\right)}{4b\sqrt[4]{e}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.65

$$
\frac{\sqrt{\pi}\left(\text{erfi}\left(\frac{1}{2}-\cosh^{-1}(a+bx)\right)+\text{erfi}\left(\cosh^{-1}(a+bx)+\frac{1}{2}\right)\right)}{4\sqrt[4]{e}\,b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcCosh[a + b*x]^2,x]
[Out] (Sqrt[Pi]*(Erfi[1/2 - ArcCosh[a + b*x]] + Erfi[1/2 + ArcCosh[a + b*x]]))/(4
*b*E^(1/4))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$
\text{integral}\left(e^{\left(\operatorname{arcosh}(bx+a)^2\right)}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="fricas")
[Out] integral(e^(arccosh(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="giac")
[Out] integrate(e^(arccosh(b*x + a)^2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \mathrm{e}^{\operatorname{arccosh}(bx+a)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2),x)
[Out] int(exp(arccosh(b*x+a)^2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\left(\operatorname{arcosh}(bx+a)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2),x, algorithm="maxima")
[Out] integrate(e^(arccosh(b*x + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{\operatorname{acosh}(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acosh(a + b*x)^2),x)
[Out] int(exp(acosh(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\operatorname{acosh}^{2}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2),x)
[Out] Integral(exp(acosh(a + b*x)**2), x)
```

3.288 $\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{e^{\cosh^{-1}(a+bx)^2}}{x},x\right)$$

```
[Out] CannotIntegrate(exp(arccosh(b*x+a)^2)/x,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[E^ArcCosh[a + b*x]^2/x,x]
[Out] Defer[Int][E^ArcCosh[a + b*x]^2/x, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx = \int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx$$

Mathematica [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcCosh[a + b*x]^2/x,x]
[Out] Integrate[E^ArcCosh[a + b*x]^2/x, x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{\left(\text{arcosh}(bx+a)^2\right)}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="fricas")
[Out] integral(e^(arccosh(b*x + a)^2)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\text{arcosh}(bx+a)^2\right)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="giac")
[Out] integrate(e^(arccosh(b*x + a)^2)/x, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\operatorname{arccosh}(bx+a)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)/x,x)
[Out] int(exp(arccosh(b*x+a)^2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\operatorname{arcosh}(bx+a)^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x,x, algorithm="maxima")
[Out] integrate(e^(arccosh(b*x + a)^2)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{e}^{\operatorname{acosh}(a+b\,x)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acosh(a + b*x)^2)/x,x)
[Out] int(exp(acosh(a + b*x)^2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{acosh}^{2}(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)/x,x)
[Out] Integral(exp(acosh(a + b*x)**2)/x, x)
```

3.289 $\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{e^{\cosh^{-1}(a+bx)^2}}{x^2},x\right)$$

```
[Out] CannotIntegrate(exp(arccosh(b*x+a)^2)/x^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[E^ArcCosh[a + b*x]^2/x^2,x]
[Out] Defer[Int][E^ArcCosh[a + b*x]^2/x^2, x]
```

Rubi steps

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} dx = \int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} dx$$

Mathematica [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{e^{\cosh^{-1}(a+bx)^2}}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^ArcCosh[a + b*x]^2/x^2,x]
[Out] Integrate[E^ArcCosh[a + b*x]^2/x^2, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^{\left(\text{arcosh}(bx+a)^2\right)}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] integral(e^(arccosh(b*x + a)^2)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\text{arcosh}(bx+a)^2\right)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="giac")
[Out] integrate(e^(arccosh(b*x + a)^2)/x^2, x)
```

maple [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{e}^{\operatorname{arccosh}(bx+a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arccosh(b*x+a)^2)/x^2,x)
[Out] int(exp(arccosh(b*x+a)^2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\left(\operatorname{arcosh}(bx+a)^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arccosh(b*x+a)^2)/x^2,x, algorithm="maxima")
[Out] integrate(e^(arccosh(b*x + a)^2)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\mathrm{e}^{\operatorname{acosh}(a+b\,x)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(acosh(a + b*x)^2)/x^2,x)
[Out] int(exp(acosh(a + b*x)^2)/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\operatorname{acosh}^{2}(a+bx)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(acosh(b*x+a)**2)/x**2,x)
[Out] Integral(exp(acosh(a + b*x)**2)/x**2, x)
```

3.290 $\int \frac{\cosh^{-1}(a+bx)}{\frac{ad}{b}+dx} dx$

Optimal. Leaf size=60

$$\frac{\text{Li}_2\left(-e^{2\cosh^{-1}(a+bx)}\right)}{2d} - \frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(e^{2\cosh^{-1}(a+bx)}+1\right)}{d}$$

```
[Out] -1/2*arccosh(b*x+a)^2/d+arccosh(b*x+a)*ln(1+(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1
)^(1/2))^2)/d+1/2*polylog(2,-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2)/d
```

Rubi [A] time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5866, 12, 5660, 3718, 2190, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{2\cosh^{-1}(a+bx)}\right)}{2d} - \frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(e^{2\cosh^{-1}(a+bx)}+1\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] -ArcCosh[a + b*x]^2/(2*d) + (ArcCosh[a + b*x]*Log[1 + E^(2*ArcCosh[a + b*x]
)])/d + PolyLog[2, -E^(2*ArcCosh[a + b*x])]/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5866

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(
m_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*A
rcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cosh^{-1}(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{\text{Subst}\left(\int \frac{b\cosh^{-1}(x)}{dx}\,dx, x, a+bx\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh^{-1}(x)}{x}\,dx, x, a+bx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}(a+bx)\right)}{d}$$
$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}x}{1+e^{2x}}\,dx, x, \cosh^{-1}(a+bx)\right)}{d}$$
$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \log\left(1+e^{2x}\right)\,dx,$$
$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2}\right.}{2d}$$
$$= -\frac{\cosh^{-1}(a+bx)^2}{2d} + \frac{\cosh^{-1}(a+bx)\log\left(1+e^{2\cosh^{-1}(a+bx)}\right)}{d} + \frac{\text{Li}_2\left(-e^{2\cosh^{-1}(a+bx)}\right)}{2d}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.88

$$\frac{\cosh^{-1}(a+bx)\left(\cosh^{-1}(a+bx)+2\log\left(e^{-2\cosh^{-1}(a+bx)}+1\right)\right)-\text{Li}_2\left(-e^{-2\cosh^{-1}(a+bx)}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a + b*x]/((a*d)/b + d*x),x]
```

```
[Out] (ArcCosh[a + b*x]*(ArcCosh[a + b*x] + 2*Log[1 + E^(-2*ArcCosh[a + b*x])]) -
 PolyLog[2, -E^(-2*ArcCosh[a + b*x])])/(2*d)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(bx+a)}{bdx+ad}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

```
[Out] integral(b*arccosh(b*x + a)/(b*d*x + a*d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

```
[Out] integrate(arccosh(b*x + a)/(d*x + a*d/b), x)
```

maple [A] time = 0.06, size = 85, normalized size = 1.42

$$-\frac{\operatorname{arccosh}(bx+a)^2}{2d}+\frac{\operatorname{arccosh}(bx+a)\ln\left(1+\left(bx+a+\sqrt{bx+a-1}\,\sqrt{bx+a+1}\right)^2\right)}{d}+\frac{\operatorname{polylog}\left(2,-\left(bx+a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] -1/2*arccosh(b*x+a)^2/d+arccosh(b*x+a)*ln(1+(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1
)^(1/2))^2)/d+1/2*polylog(2,-(b*x+a+(b*x+a-1)^(1/2)*(b*x+a+1)^(1/2))^2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(bx+a)}{dx+\frac{ad}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(b*x + a)/(d*x + a*d/b), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a + b*x)/(d*x + (a*d)/b),x)
```

```
[Out] int(acosh(a + b*x)/(d*x + (a*d)/b), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b\int \frac{\operatorname{acosh}(a+bx)}{a+bx}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(b*x+a)/(a*d/b+d*x),x)
```

```
[Out] b*Integral(acosh(a + b*x)/(a + b*x), x)/d
```

3.291 $\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,\cosh^{-1}(x)}\,dx$

Optimal. Leaf size=3

$$\mathrm{Chi}\left(\cosh^{-1}(x)\right)$$

```
[Out] Chi(arccosh(x))
```

Rubi [A] time = 0.12, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5781, 3301}

$$\mathrm{Chi}\left(\cosh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x]*Sqrt[1 + x]*ArcCosh[x]),x]
```

```
[Out] CoshIntegral[ArcCosh[x]]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}\,\cosh^{-1}(x)}\,dx &= \mathrm{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(x)\right)\\ &= \mathrm{Chi}\left(\cosh^{-1}(x)\right)\end{aligned}$$

Mathematica [B] time = 0.06, size = 19, normalized size = 6.33

$$\frac{1}{2}(x-1)\mathrm{Chi}\left(\cosh^{-1}(x)\right)\mathrm{csch}^2\left(\frac{1}{2}\cosh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x]*Sqrt[1 + x]*ArcCosh[x]),x]
```

```
[Out] ((-1 + x)*CoshIntegral[ArcCosh[x]]*Csch[ArcCosh[x]/2]^2)/2
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{x+1}\,\sqrt{x-1}\,x}{\left(x^2-1\right)\mathrm{arcosh}(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x + 1)*sqrt(x - 1)*x/((x^2 - 1)*arccosh(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x+1}\sqrt{x-1}\ \mathrm{arcosh}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x + 1)*sqrt(x - 1)*arccosh(x)), x)
```

maple [B] time = 0.42, size = 78, normalized size = 26.00

$$-\frac{\sqrt{-2+2x}\ \sqrt{2+2x}\ \sqrt{-1+x}\ \sqrt{1+x}\ \mathrm{Ei}\left(1,\mathrm{arccosh}(x)\right)}{4\left(x^{2}-1\right)}-\frac{\sqrt{-2+2x}\ \sqrt{2+2x}\ \sqrt{-1+x}\ \sqrt{1+x}\ \mathrm{Ei}\left(1,-\mathrm{arcco}\right.}{4\left(x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x)
```

```
[Out] -1/4*(-2+2*x)^(1/2)*(2+2*x)^(1/2)*(-1+x)^(1/2)*(1+x)^(1/2)*Ei(1,arccosh(x))
/(x^2-1)-1/4*(-2+2*x)^(1/2)*(2+2*x)^(1/2)*(-1+x)^(1/2)*(1+x)^(1/2)*Ei(1,-ar
ccosh(x))/(x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x+1}\sqrt{x-1}\ \mathrm{arcosh}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(x)/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x + 1)*sqrt(x - 1)*arccosh(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.33

$$\int \frac{x}{\mathrm{acosh}(x)\sqrt{x-1}\ \sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(acosh(x)*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int(x/(acosh(x)*(x - 1)^(1/2)*(x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x-1}\ \sqrt{x+1}\ \mathrm{acosh}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(x)/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(x - 1)*sqrt(x + 1)*acosh(x)), x)
```

3.292 $\int x^3 \cosh^{-1}\left(a+bx^4\right)\,dx$

Optimal. Leaf size=54

$$\frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{a+bx^4-1}\,\sqrt{a+bx^4+1}}{4b}$$

```
[Out] 1/4*(b*x^4+a)*arccosh(b*x^4+a)/b-1/4*(b*x^4+a-1)^(1/2)*(b*x^4+a+1)^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6715, 5864, 5654, 74}

$$\frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)}{4b}-\frac{\sqrt{a+bx^4-1}\,\sqrt{a+bx^4+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCosh[a + b*x^4],x]
```

```
[Out] -(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4])/(4*b) + ((a + b*x^4)*ArcCosh[a
 + b*x^4])/(4*b)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x^3\cosh^{-1}\left(a+bx^4\right)\,dx &= \frac{1}{4}\text{Subst}\left(\int \cosh^{-1}(a+bx)\,dx,x,x^4\right)\\
&= \frac{\text{Subst}\left(\int \cosh^{-1}(x)\,dx,x,a+bx^4\right)}{4b}\\
&= \frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)}{4b}-\frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,a+bx^4\right)}{4b}\\
&= -\frac{\sqrt{-1+a+bx^4}\,\sqrt{1+a+bx^4}}{4b}+\frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.93

$$\frac{\left(a+bx^4\right)\cosh^{-1}\left(a+bx^4\right)-\sqrt{a+bx^4-1}\sqrt{a+bx^4+1}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCosh[a + b*x^4],x]
```

```
[Out] (-(Sqrt[-1 + a + b*x^4]*Sqrt[1 + a + b*x^4]) + (a + b*x^4)*ArcCosh[a + b*x^
4])/(4*b)
```

fricas [A] time = 0.51, size = 66, normalized size = 1.22

$$\frac{\left(bx^4+a\right)\log\left(bx^4+a+\sqrt{b^2x^8+2\,abx^4+a^2-1}\right)-\sqrt{b^2x^8+2\,abx^4+a^2-1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^4 + a)*log(b*x^4 + a + sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)) - sqr
t(b^2*x^8 + 2*a*b*x^4 + a^2 - 1))/b
```

giac [B] time = 0.19, size = 106, normalized size = 1.96

$$\frac{1}{4}\,x^4\log\left(bx^4+a+\sqrt{\left(bx^4+a\right)^2-1}\right)-\frac{1}{4}\,b\left(\frac{a\log\left(\left|-ab-\left(x^4|b|-\sqrt{b^2x^8+2\,abx^4+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^8+2\,a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(b*x^4 + a + sqrt((b*x^4 + a)^2 - 1)) - 1/4*b*(a*log(abs(-a*b -
(x^4*abs(b) - sqrt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1))*abs(b)))/(b*abs(b)) + sq
rt(b^2*x^8 + 2*a*b*x^4 + a^2 - 1)/b^2)
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{\left(b\,x^4+a\right)\operatorname{arccosh}\left(b\,x^4+a\right)-\sqrt{b\,x^4+a-1}\,\sqrt{b\,x^4+a+1}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(b*x^4+a),x)
```

```
[Out] 1/4/b*((b*x^4+a)*arccosh(b*x^4+a)-(b*x^4+a-1)^(1/2)*(b*x^4+a+1)^(1/2))
```

maxima [A] time = 0.30, size = 37, normalized size = 0.69

$$\frac{\left(bx^4+a\right)\operatorname{arcosh}\left(bx^4+a\right)-\sqrt{\left(bx^4+a\right)^2-1}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*((b*x^4 + a)*arccosh(b*x^4 + a) - sqrt((b*x^4 + a)^2 - 1))/b
```

mupad [B] time = 4.68, size = 295, normalized size = 5.46

$$\frac{x^4\,\mathrm{acosh}\left(b\,x^4+a\right)}{4}-\frac{\frac{4\,a\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)}{b\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)}+\frac{4\,a\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^3}{b\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^3}-\frac{8\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^2\sqrt{a-1}\,\sqrt{a+1}}{b\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^2}}{4\left(\frac{\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^4}{\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^4}-\frac{2\left(\sqrt{a-1}-\sqrt{b\,x^4+a-1}\right)^2}{\left(\sqrt{a+1}-\sqrt{b\,x^4+a+1}\right)^2}+1\right)}+\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{a-1}-\sqrt{}}{\sqrt{a+1}-\sqrt{}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*acosh(a + b*x^4),x)
```

```
[Out] (x^4*acosh(a + b*x^4))/4 - ((4*a*((a - 1)^(1/2) - (a + b*x^4 - 1)^(1/2)))/(
b*((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))) + (4*a*((a - 1)^(1/2) - (a + b*x
^4 - 1)^(1/2))^3)/(b*((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^3) - (8*((a -
1)^(1/2) - (a + b*x^4 - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a + 1
)^(1/2) - (a + b*x^4 + 1)^(1/2))^2))/(4*(((a - 1)^(1/2) - (a + b*x^4 - 1)^(
1/2))^4/((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^4 - (2*((a - 1)^(1/2) - (a
+ b*x^4 - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x^4 + 1)^(1/2))^2 + 1)) + (a
*atanh(((a - 1)^(1/2) - (a + b*x^4 - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x^4
+ 1)^(1/2))))/b
```

sympy [A] time = 0.79, size = 61, normalized size = 1.13

$$\begin{cases} \frac{a\,\mathrm{acosh}\left(a+bx^4\right)}{4b}+\frac{x^4\,\mathrm{acosh}\left(a+bx^4\right)}{4}-\frac{\sqrt{a^2+2abx^4+b^2x^8-1}}{4b} & \text{for } b\neq 0 \\ \frac{x^4\,\mathrm{acosh}\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(b*x**4+a),x)
```

```
[Out] Piecewise((a*acosh(a + b*x**4)/(4*b) + x**4*acosh(a + b*x**4)/4 - sqrt(a**2
 + 2*a*b*x**4 + b**2*x**8 - 1)/(4*b), Ne(b, 0)), (x**4*acosh(a)/4, True))
```

3.293 $\int x^{-1+n} \cosh^{-1}(a + bx^n) \, dx$

Optimal. Leaf size=55

$$\frac{(a + bx^n)\cosh^{-1}(a + bx^n)}{bn} - \frac{\sqrt{a + bx^n - 1}\sqrt{a + bx^n + 1}}{bn}$$

[Out] (a+b*x^n)*arccosh(a+b*x^n)/b/n-(-1+a+b*x^n)^(1/2)*(1+a+b*x^n)^(1/2)/b/n

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6715, 5864, 5654, 74}

$$\frac{(a + bx^n)\cosh^{-1}(a + bx^n)}{bn} - \frac{\sqrt{a + bx^n - 1}\sqrt{a + bx^n + 1}}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*ArcCosh[a + b*x^n],x]

[Out] -((Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n])/(b*n)) + ((a + b*x^n)*ArcCosh[a + b*x^n])/(b*n)

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5864

```
Int[((a_.) + ArcCosh[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Dist[1/d
, Subst[Int[(a + b*ArcCosh[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
 n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x^{-1+n}\cosh^{-1}(a + bx^n)\,dx &= \frac{\text{Subst}\left(\int \cosh^{-1}(a + bx)\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \cosh^{-1}(x)\,dx, x, a + bx^n\right)}{bn} \\
&= \frac{(a + bx^n)\cosh^{-1}(a + bx^n)}{bn} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, a + bx^n\right)}{bn} \\
&= -\frac{\sqrt{-1 + a + bx^n}\sqrt{1 + a + bx^n}}{bn} + \frac{(a + bx^n)\cosh^{-1}(a + bx^n)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.91

$$\frac{(a+bx^n)\cosh^{-1}(a+bx^n)-\sqrt{a+bx^n-1}\sqrt{a+bx^n+1}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*ArcCosh[a + b*x^n],x]
```

```
[Out] (-(Sqrt[-1 + a + b*x^n]*Sqrt[1 + a + b*x^n]) + (a + b*x^n)*ArcCosh[a + b*x^
n])/(b*n)
```

fricas [B] time = 0.71, size = 152, normalized size = 2.76

$$\frac{(b\cosh(n\log(x))+b\sinh(n\log(x))+a)\log\left(b\cosh(n\log(x))+b\sinh(n\log(x))+a+\sqrt{\frac{2ab+(a^2+b^2-1)\cosh(n}{\cosh(n\log}}\right.}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] ((b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*log(b*cosh(n*log(x)) + b*sinh(n*
log(x)) + a + sqrt((2*a*b + (a^2 + b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1
)*sinh(n*log(x)))/(cosh(n*log(x)) - sinh(n*log(x))))) - sqrt((2*a*b + (a^2
+ b^2 - 1)*cosh(n*log(x)) - (a^2 - b^2 - 1)*sinh(n*log(x)))/(cosh(n*log(x))
 - sinh(n*log(x)))))/(b*n)
```

giac [B] time = 0.73, size = 124, normalized size = 2.25

$$-\frac{b\left(\frac{a\log\left(\left|-ab-\left(x^n|b|-\sqrt{b^2x^{2n}+2abx^n+a^2-1}\right)|b|\right|\right)}{b|b|}+\frac{\sqrt{b^2x^{2n}+2abx^n+a^2-1}}{b^2}\right)-x^n\log\left(bx^n+a+\sqrt{b^2x^{2n}+2abx^n+a^2-1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="giac")
```

```
[Out] -(b*(a*log(abs(-a*b - (x^n*abs(b) - sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)
)*abs(b)))/(b*abs(b)) + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)/b^2) - x^n*
log(b*x^n + a + sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2 - 1)))/n
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-1+n}\operatorname{arccosh}(a+b\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*arccosh(a+b*x^n),x)
```

```
[Out] int(x^(-1+n)*arccosh(a+b*x^n),x)
```

maxima [A] time = 0.30, size = 39, normalized size = 0.71

$$\frac{(bx^n+a)\operatorname{arcosh}(bx^n+a)-\sqrt{(bx^n+a)^2-1}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*arccosh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] ((b*x^n + a)*arccosh(b*x^n + a) - sqrt((b*x^n + a)^2 - 1))/(b*n)
```

mupad [B] time = 1.07, size = 303, normalized size = 5.51

$$\frac{x^n \operatorname{acosh}\left(a+b\,x^n\right)}{n}-\frac{\frac{4\,a\,{\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)}^3}{b\,{\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)}^3}+\frac{4\,a\,\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)}{b\,\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)}-\frac{8\,{\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)}^2\,\sqrt{a-1}\,\sqrt{a+1}}{b\,{\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)}^2}}{n\,\left(\frac{{\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)}^4}{{\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)}^4}-\frac{2\,{\left(\sqrt{a-1}-\sqrt{a+b\,x^n-1}\right)}^2}{{\left(\sqrt{a+1}-\sqrt{a+b\,x^n+1}\right)}^2}+1\right)}+\frac{4\,a\,\operatorname{atanh}\left(\frac{\sqrt{a}\cdots}{\sqrt{a}\cdots}\right.}{b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*acosh(a + b*x^n),x)
```

```
[Out] (x^n*acosh(a + b*x^n))/n - ((4*a*((a - 1)^(1/2) - (a + b*x^n - 1)^(1/2))^3)
/(b*((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^3) + (4*a*((a - 1)^(1/2) - (a +
 b*x^n - 1)^(1/2)))/(b*((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))) - (8*((a -
1)^(1/2) - (a + b*x^n - 1)^(1/2))^2*(a - 1)^(1/2)*(a + 1)^(1/2))/(b*((a + 1
)^(1/2) - (a + b*x^n + 1)^(1/2))^2))/(n*(((a - 1)^(1/2) - (a + b*x^n - 1)^(
1/2))^4/((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^4 - (2*((a - 1)^(1/2) - (a
+ b*x^n - 1)^(1/2))^2)/((a + 1)^(1/2) - (a + b*x^n + 1)^(1/2))^2 + 1)) + (4
*a*atanh(((a - 1)^(1/2) - (a + b*x^n - 1)^(1/2))/((a + 1)^(1/2) - (a + b*x^
n + 1)^(1/2))))/(b*n)
```

sympy [A] time = 58.15, size = 76, normalized size = 1.38

$$\begin{cases} \log\left(x\right)\operatorname{acosh}\left(a\right) & \text{for}\: b = 0 \wedge n = 0 \\ \frac{x^{n}\operatorname{acosh}\left(a\right)}{n} & \text{for}\: b = 0 \\ \log\left(x\right)\operatorname{acosh}\left(a + b\right) & \text{for}\: n = 0 \\ \frac{a\operatorname{acosh}\left(a + bx^{n}\right)}{bn} + \frac{x^{n}\operatorname{acosh}\left(a + bx^{n}\right)}{n} - \frac{\sqrt{a^{2} + 2abx^{n} + b^{2}x^{2n} - 1}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*acosh(a+b*x**n),x)
```

```
[Out] Piecewise((log(x)*acosh(a), Eq(b, 0) & Eq(n, 0)), (x**n*acosh(a)/n, Eq(b, 0
)), (log(x)*acosh(a + b), Eq(n, 0)), (a*acosh(a + b*x**n)/(b*n) + x**n*acos
h(a + b*x**n)/n - sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n) - 1)/(b*n), True))
```

3.294 $\int \cosh^{-1}\left(\frac{c}{a+bx}\right) dx$

Optimal. Leaf size=58

$$\frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\tan^{-1}\left(\sqrt{\frac{c\left(1-\frac{a}{c}\right)-bx}{a+bx+c}}\right)}{b}$$

```
[Out] (b*x+a)*arcsech(a/c+b*x/c)/b-2*c*arctan((((1-a/c)*c-b*x)/(b*x+a+c))^(1/2))/
b
```

Rubi [A] time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5893, 6313, 1961, 12, 203}

$$\frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{2c\tan^{-1}\left(\sqrt{\frac{c\left(1-\frac{a}{c}\right)-bx}{a+bx+c}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[c/(a + b*x)],x]
```

```
[Out] ((a + b*x)*ArcSech[a/c + (b*x)/c])/b - (2*c*ArcTan[Sqrt[((1 - a/c)*c - b*x)
/(a + c + b*x)]])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 5893

```
Int[ArcCosh[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int
[u*ArcSech[a/c + (b*x^n)/c]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

Rule 6313

```
Int[ArcSech[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[((c + d*x)*ArcSech[c + d*
x])/d, x] + Int[Sqrt[(1 - c - d*x)/(1 + c + d*x)]/(1 - c - d*x), x] /; Free
Q[{c, d}, x]
```

Rubi steps

$$\int \cosh^{-1}\left(\frac{c}{a+bx}\right)dx = \int \operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)dx$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} + \int \frac{\sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}}{1-\frac{a}{c}-\frac{bx}{c}}\,dx$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{(4b)\operatorname{Subst}\left(\int \frac{c^2}{2b^2(1+x^2)}\,dx, x, \sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}\right)}{c}$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{1-\frac{a}{c}-\frac{bx}{c}}{1+\frac{a}{c}+\frac{bx}{c}}}\right)}{b}$$
$$= \frac{(a+bx)\operatorname{sech}^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{2c\tan^{-1}\left(\sqrt{\frac{\left(1-\frac{a}{c}\right)c-bx}{a+c+bx}}\right)}{b}$$

Mathematica [B] time = 0.56, size = 180, normalized size = 3.10

$$\frac{2c\sqrt{-\frac{a+bx-c}{a+bx+c}}\left(a\sqrt{b}\sqrt{a+bx+c}\tanh^{-1}\left(\frac{\sqrt{-bc}\sqrt{a+bx-c}}{\sqrt{bc}\sqrt{a+bx+c}}\right)-c\sqrt{-bc}\sqrt{\frac{a+bx+c}{c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx-c}}{\sqrt{2}\sqrt{bc}}\right)\right)}{\sqrt{b}\sqrt{-b^2c^2}\sqrt{a+bx-c}}+x\cosh^{-1}\left(\frac{c}{a+\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[c/(a + b*x)],x]
```

```
[Out] x*ArcCosh[c/(a + b*x)] + (2*c*Sqrt[-((a - c + b*x)/(a + c + b*x))]*(-(c*Sqr
t[-(b*c)]*Sqrt[(a + c + b*x)/c]*ArcSinh[(Sqrt[b]*Sqrt[a - c + b*x])/(Sqrt[2
]*Sqrt[b*c])]) + a*Sqrt[b]*Sqrt[a + c + b*x]*ArcTanh[(Sqrt[-(b*c)]*Sqrt[a -
 c + b*x])/(Sqrt[b*c]*Sqrt[a + c + b*x])]))/(Sqrt[b]*Sqrt[-(b^2*c^2)]*Sqrt[
a - c + b*x])
```

fricas [B] time = 0.62, size = 276, normalized size = 4.76

$$\frac{2\,bx\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+c}{bx+a}\right)-2\,c\arctan\left(\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}}{b^2x^2+2\,abx+a^2-c^2}\right)+a\log\left(\frac{(bx+a)\sqrt{-\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+\cdots}{x}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 +
2*a*b*x + a^2)) + c)/(b*x + a)) - 2*c*arctan((b^2*x^2 + 2*a*b*x + a^2)*sqrt
(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*x^2 + 2*a
*b*x + a^2 - c^2)) + a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a*b*x + a^2 - c^2)
/(b^2*x^2 + 2*a*b*x + a^2)) + c)/x) - a*log(((b*x + a)*sqrt(-(b^2*x^2 + 2*a
*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - c)/x))/b
```

giac [B] time = 13.04, size = 119, normalized size = 2.05

$$\frac{c\arcsin\left(-\frac{bx+a}{c}\right)\operatorname{sgn}(b)\operatorname{sgn}(c)}{|b|}+x\log\left(\sqrt{\frac{c}{bx+a}+1}\sqrt{\frac{c}{bx+a}-1}+\frac{c}{bx+a}\right)-\frac{a\log\left(\frac{\left|-2\,bc-2\,\sqrt{-b^2x^2-2\,abx-a^2+c^2}\,|b\cdots\right.}{\left|-2\,b^2x-2\,ab\right|}\right.}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="giac")
```

```
[Out] c*arcsin(-(b*x + a)/c)*sgn(b)*sgn(c)/abs(b) + x*log(sqrt(c/(b*x + a) + 1)*s
qrt(c/(b*x + a) - 1) + c/(b*x + a)) - a*log(abs(-2*b*c - 2*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + c^2)*abs(b))/abs(-2*b^2*x - 2*a*b))/abs(b)
```

maple [A] time = 0.05, size = 91, normalized size = 1.57

$$\operatorname{arccosh}\left(\frac{c}{bx+a}\right)x+\frac{\operatorname{arccosh}\left(\frac{c}{bx+a}\right)a}{b}+\frac{c\sqrt{\frac{c}{bx+a}-1}\,\sqrt{\frac{c}{bx+a}+1}\,\arctan\left(\frac{1}{\sqrt{\frac{c^2}{(bx+a)^2}-1}}\right)}{b\sqrt{\frac{c^2}{(bx+a)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(c/(b*x+a)),x)
```

```
[Out] arccosh(c/(b*x+a))*x+1/b*arccosh(c/(b*x+a))*a+1/b*c*(c/(b*x+a)-1)^(1/2)*(c/
(b*x+a)+1)^(1/2)/(c^2/(b*x+a)^2-1)^(1/2)*arctan(1/(c^2/(b*x+a)^2-1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,bx\log\left(\sqrt{bx+a+c}\,\sqrt{-bx-a+c}\,bx+\sqrt{bx+a+c}\,\sqrt{-bx-a+c}\,a+(bx+a)c\right)-2\,bx\log\left(bx+a\right)+(a+c)\log}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(c/(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*log(sqrt(b*x + a + c)*sqrt(-b*x - a + c)*b*x + sqrt(b*x + a + c)
*sqrt(-b*x - a + c)*a + (b*x + a)*c) - 2*b*x*log(b*x + a) + (a + c)*log(b*x
 + a + c) - 2*(b*x + a)*log(b*x + a) + (a - c)*log(-b*x - a + c))/b + integ
rate((b^2*c*x^2 + a*b*c*x)/(b^2*c*x^2 + 2*a*b*c*x + a^2*c - c^3 + (b^2*x^2
+ 2*a*b*x + a^2 - c^2)*e^(1/2*log(b*x + a + c) + 1/2*log(-b*x - a + c))), x
)
```

mupad [B] time = 1.51, size = 53, normalized size = 0.91

$$\frac{\operatorname{acosh}\left(\frac{c}{a+b\,x}\right)\,\left(a+b\,x\right)}{b}+\frac{c\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{c}{a+b\,x}-1}\,\sqrt{\frac{c}{a+b\,x}+1}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(c/(a + b*x)),x)
```

```
[Out] (acosh(c/(a + b*x))*(a + b*x))/b + (c*atan(1/((c/(a + b*x) - 1)^(1/2)*(c/(a
 + b*x) + 1)^(1/2))))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{acosh}\left(\frac{c}{a+bx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(c/(b*x+a)),x)
```

```
[Out] Integral(acosh(c/(a + b*x)), x)
```

3.295 $\int \frac{\cosh^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\cosh^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

```
[Out] arccosh((b*x^2+1)^(1/2))^(1+n)*(-1+(b*x^2+1)^(1/2))^(1/2)*(1+(b*x^2+1)^(1/2
))^(1/2)/b/(1+n)/x
```

Rubi [A] time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5895, 5676}

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\cosh^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x^
2]]^(1 + n))/(b*(1 + n)*x)
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5895

```
Int[ArcCosh[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
 :> Dist[(Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]])/(b*x), Subs
t[Int[ArcCosh[x]^n/(Sqrt[-1 + x]*Sqrt[1 + x]), x], x, Sqrt[1 + b*x^2]], x]
/; FreeQ[{b, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx &= \frac{\left(\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\right)\text{Subst}\left(\int \frac{\cosh^{-1}(x)^n}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{1+bx^2}\right)}{bx}\\ &= \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\cosh^{-1}\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.20, size = 62, normalized size = 1.00

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\cosh^{-1}\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*ArcCosh[Sqrt[1 + b*x^
2]]^(1 + n))/(b*(1 + n)*x)
```

fricas [B] time = 0.65, size = 108, normalized size = 1.74

$$\frac{\sqrt{bx^2}\cosh\left(n\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)\right)\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)+\sqrt{bx^2}\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\sinh\left(n\,\mathrm{l}\right.}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2)*cosh(n*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2))))*log(sqrt(b*x^2
 + 1) + sqrt(b*x^2)) + sqrt(b*x^2)*log(sqrt(b*x^2 + 1) + sqrt(b*x^2))*sinh(
n*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2)))))/((b*n + b)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}\left(\sqrt{b\,x^2+1}\right)^n}{\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(\sqrt{bx^2+1}\right)^n}{\sqrt{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(sqrt(b*x^2 + 1))^n/sqrt(b*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}\left(\sqrt{b\,x^2+1}\right)^n}{\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2),x)
```

```
[Out] int(acosh((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \tilde{\infty} x & \text{for}\: b = 0 \wedge n = -1 \\ 0^{n} x & \text{for}\: b = 0 \\ \int \frac{1}{\sqrt{b x^{2} + 1} \operatorname{acosh}{\left(\sqrt{b x^{2} + 1} \right)}}\, dx & \text{for}\: n = -1 \\ \frac{\sqrt{b} \sqrt{x^{2}} \operatorname{acosh}{\left(\sqrt{b x^{2} + 1} \right)} \operatorname{acosh}^{n}{\left(\sqrt{b x^{2} + 1} \right)}}{b n x + b x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh((b*x**2+1)**(1/2))**n/(b*x**2+1)**(1/2),x)
```

```
[Out] Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (Integral(1/(s
qrt(b*x**2 + 1)*acosh(sqrt(b*x**2 + 1))), x), Eq(n, -1)), (sqrt(b)*sqrt(x**
2)*acosh(sqrt(b*x**2 + 1))*acosh(sqrt(b*x**2 + 1))**n/(b*n*x + b*x), True))
```

3.296 $\int \frac{1}{\sqrt{1+bx^2}\cosh^{-1}\left(\sqrt{1+bx^2}\right)}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\log\left(\cosh^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

```
[Out] ln(arccosh((b*x^2+1)^(1/2)))*(-1+(b*x^2+1)^(1/2))^(1/2)*(1+(b*x^2+1)^(1/2))
^(1/2)/b/x
```

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{5895, 5674\}$

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\log\left(\cosh^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x^2]*ArcCosh[Sqrt[1 + b*x^2]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 +
b*x^2]]])/(b*x)
```

Rule 5674

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[Log[a + b*ArcCosh[c*x]]/(b*c*Sqrt[-
(d1*d2)]), x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && Eq
Q[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5895

```
Int[ArcCosh[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
 :> Dist[(Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]])/(b*x), Subs
t[Int[ArcCosh[x]^n/(Sqrt[-1 + x]*Sqrt[1 + x]), x], x, Sqrt[1 + b*x^2]], x]
/; FreeQ[{b, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+bx^2}\cosh^{-1}\left(\sqrt{1+bx^2}\right)}\,dx = \frac{\left(\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)}\,dx, x, \sqrt{1}\right.}{bx}$$

$$= \frac{\sqrt{-1+\sqrt{1+bx^2}}\sqrt{1+\sqrt{1+bx^2}}\log\left(\cosh^{-1}\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

Mathematica [A] time = 0.09, size = 54, normalized size = 1.00

$$\frac{\sqrt{\sqrt{bx^2+1}-1}\sqrt{\sqrt{bx^2+1}+1}\log\left(\cosh^{-1}\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x^2]*ArcCosh[Sqrt[1 + b*x^2]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[1 + b*x^2]]*Sqrt[1 + Sqrt[1 + b*x^2]]*Log[ArcCosh[Sqrt[1 +
b*x^2]]])/(b*x)
```

fricas [A] time = 1.59, size = 33, normalized size = 0.61

$$\frac{\sqrt{bx^2}\log\left(\log\left(\sqrt{bx^2+1}+\sqrt{bx^2}\right)\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*x^2)*log(log(sqrt(b*x^2 + 1) + sqrt(b*x^2)))/(b*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{arccosh}\left(\sqrt{b\,x^2+1}\right)\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

```
[Out] int(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+1}\,\operatorname{arcosh}\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 1)*arccosh(sqrt(b*x^2 + 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{acosh}\left(\sqrt{b\,x^2+1}\right)\,\sqrt{b\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh((b*x^2 + 1)^(1/2))*(b*x^2 + 1)^(1/2)),x)
```

```
[Out] int(1/(acosh((b*x^2 + 1)^(1/2))*(b*x^2 + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^2+1}\,\operatorname{acosh}\left(\sqrt{bx^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh((b*x**2+1)**(1/2))/(b*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x**2 + 1)*acosh(sqrt(b*x**2 + 1))), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```